Jefferson Lab
Thomas Jefferson National Accelerator Facility

Resonances in coupled channel scattering from lattice QCD

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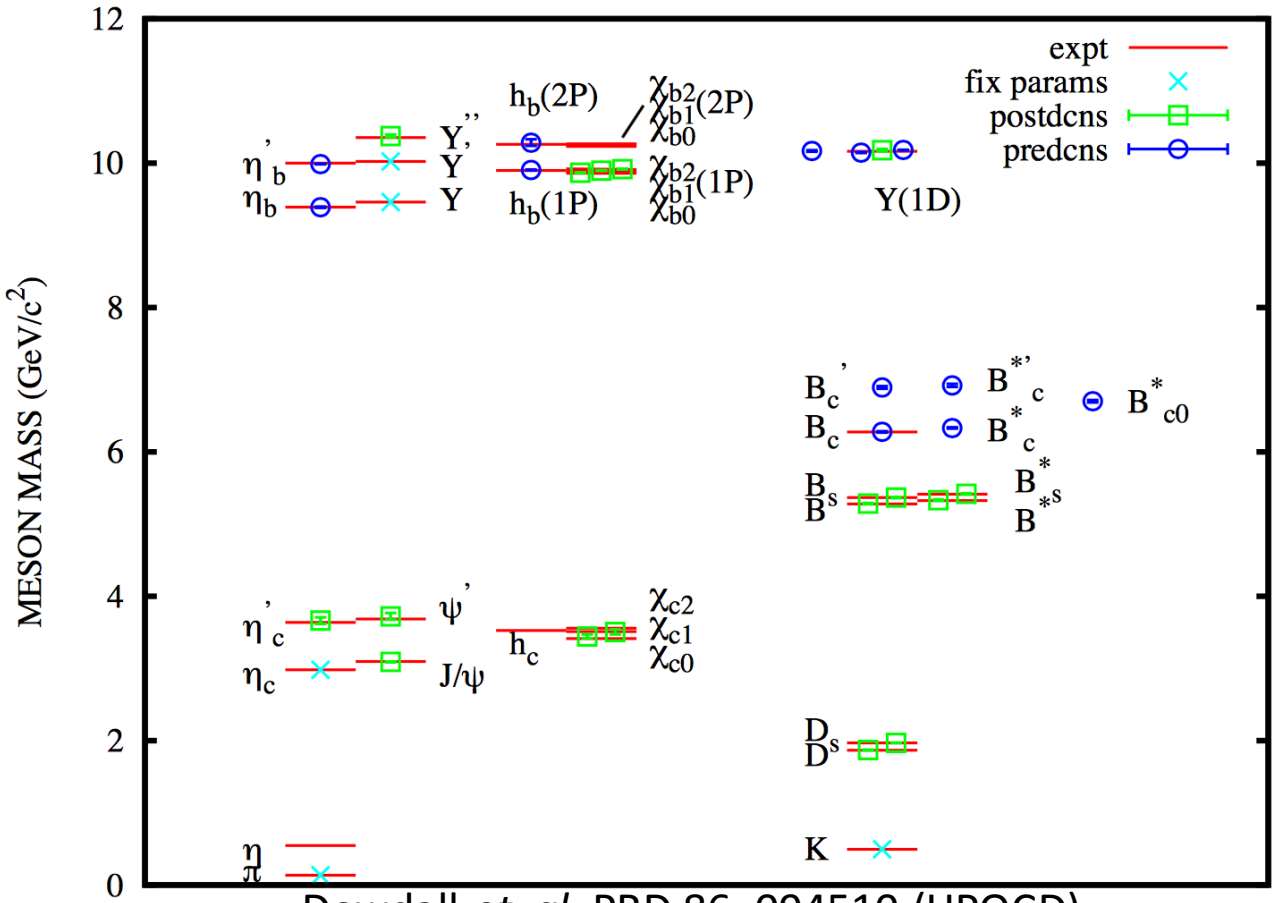
Jefferson Lab

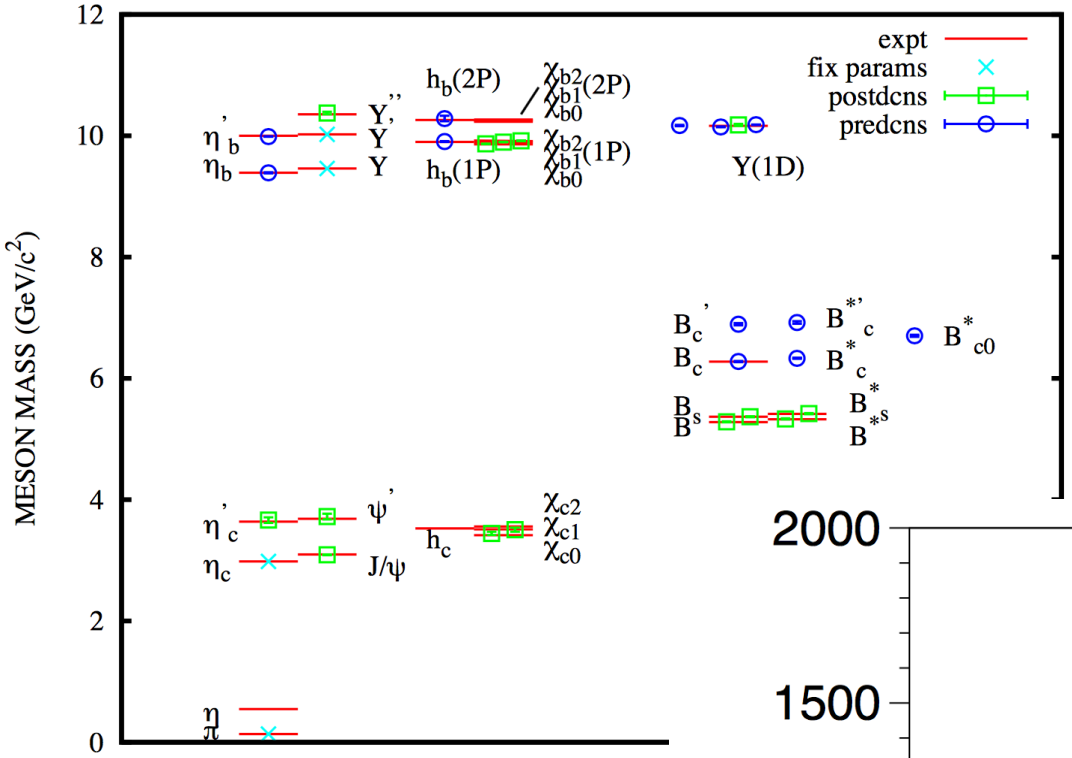
Light Cone 2017, Mumbai, India,
18th Sept, 2017.

Lower-lying mesons



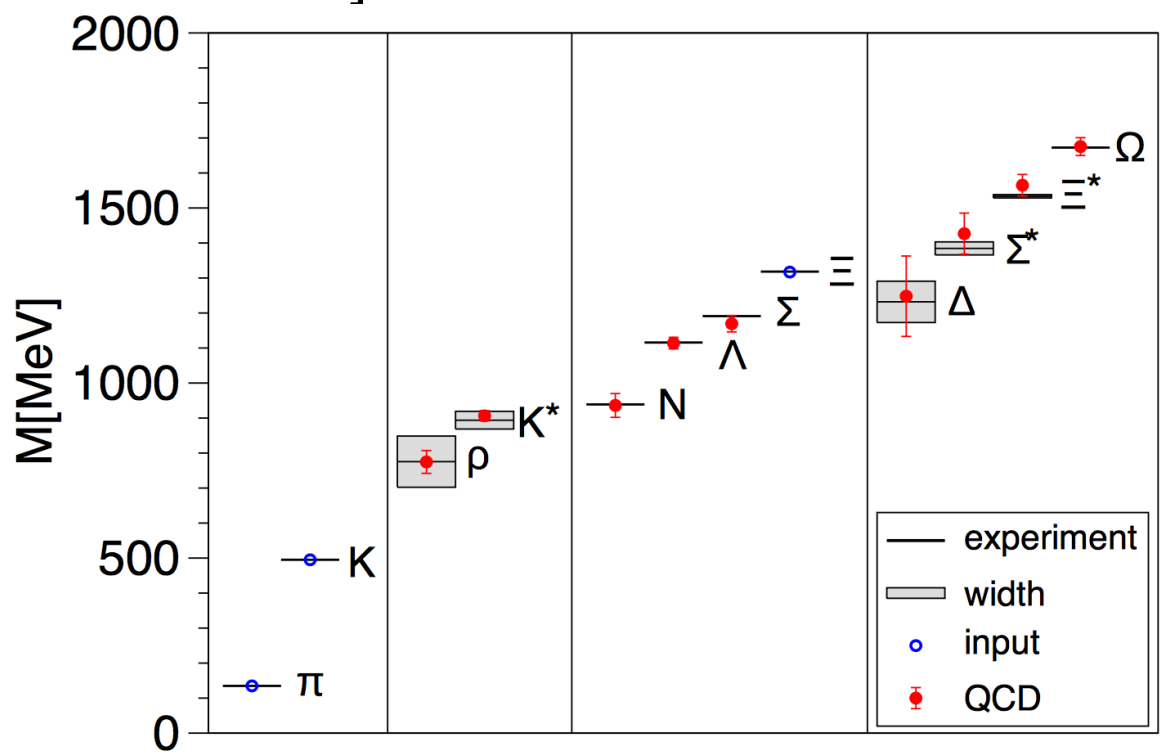
Dowdall *et. al.* PRD 86, 094510 (HPQCD)

Lower-lying mesons and baryons

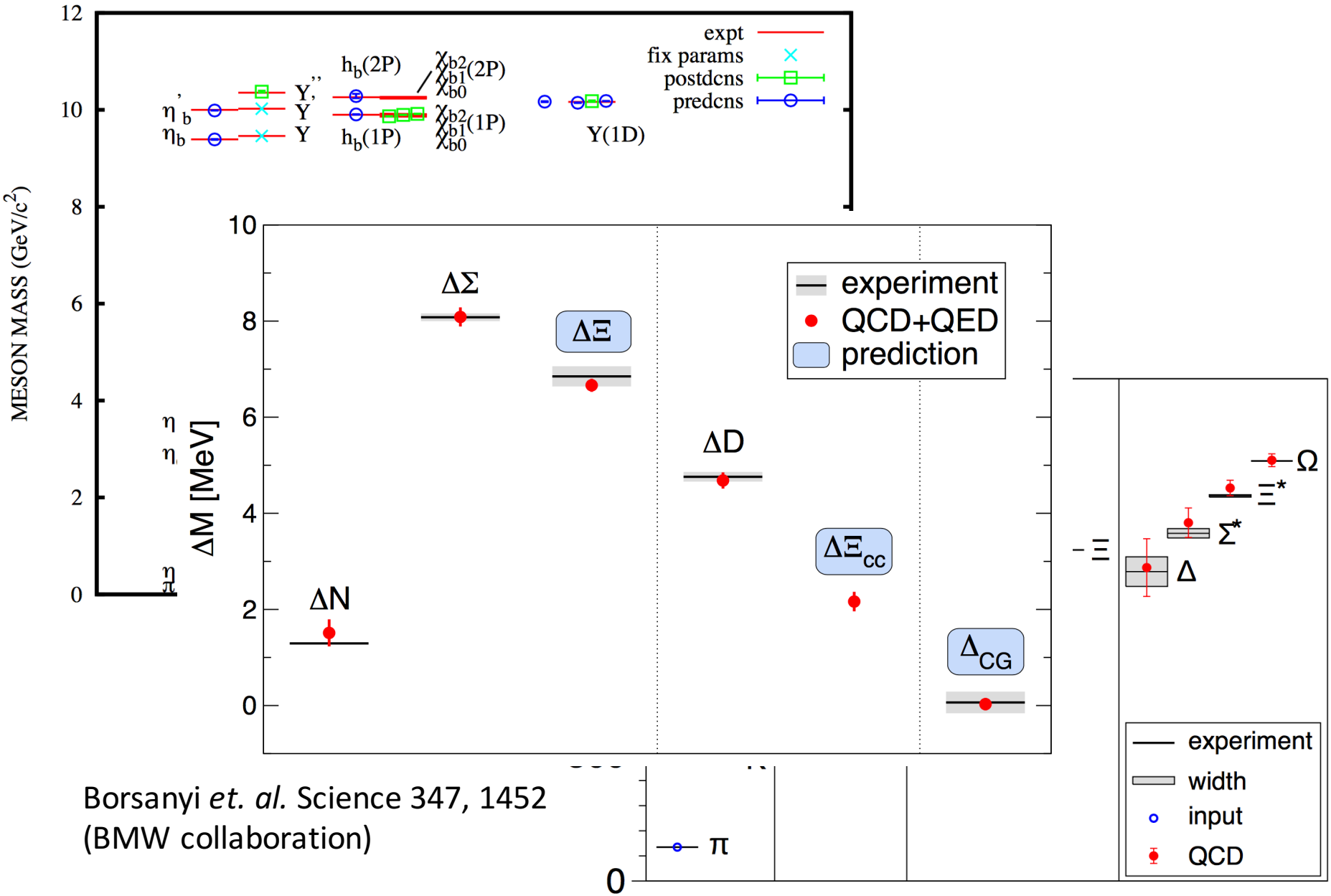


Dowdall *et. al.*
PRD 86, 094510
(HPQCD collaboration)

Durr *et. al.*
Science 322,1224
(BMW collaboration)



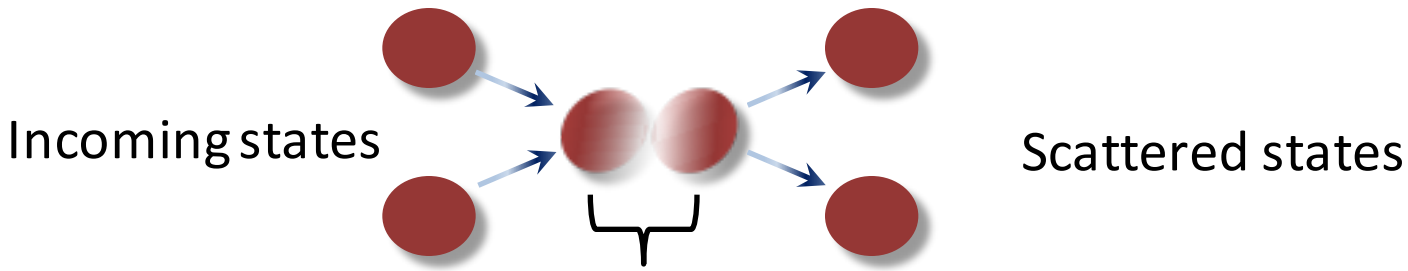
Lower-lying mesons and baryons



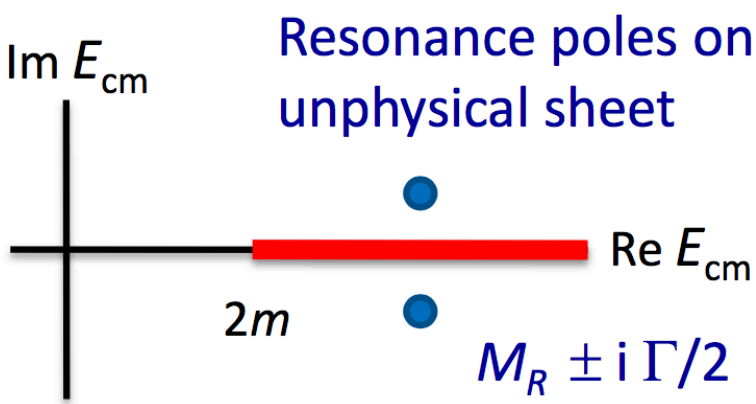
Borsanyi *et. al.* Science 347, 1452 (BMW collaboration)

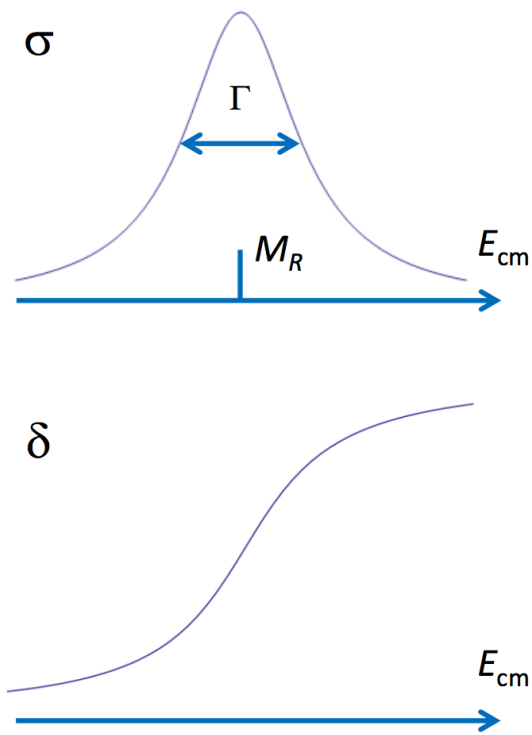
Hadron resonances

Most hadrons appear as resonances in scattering of light hadrons

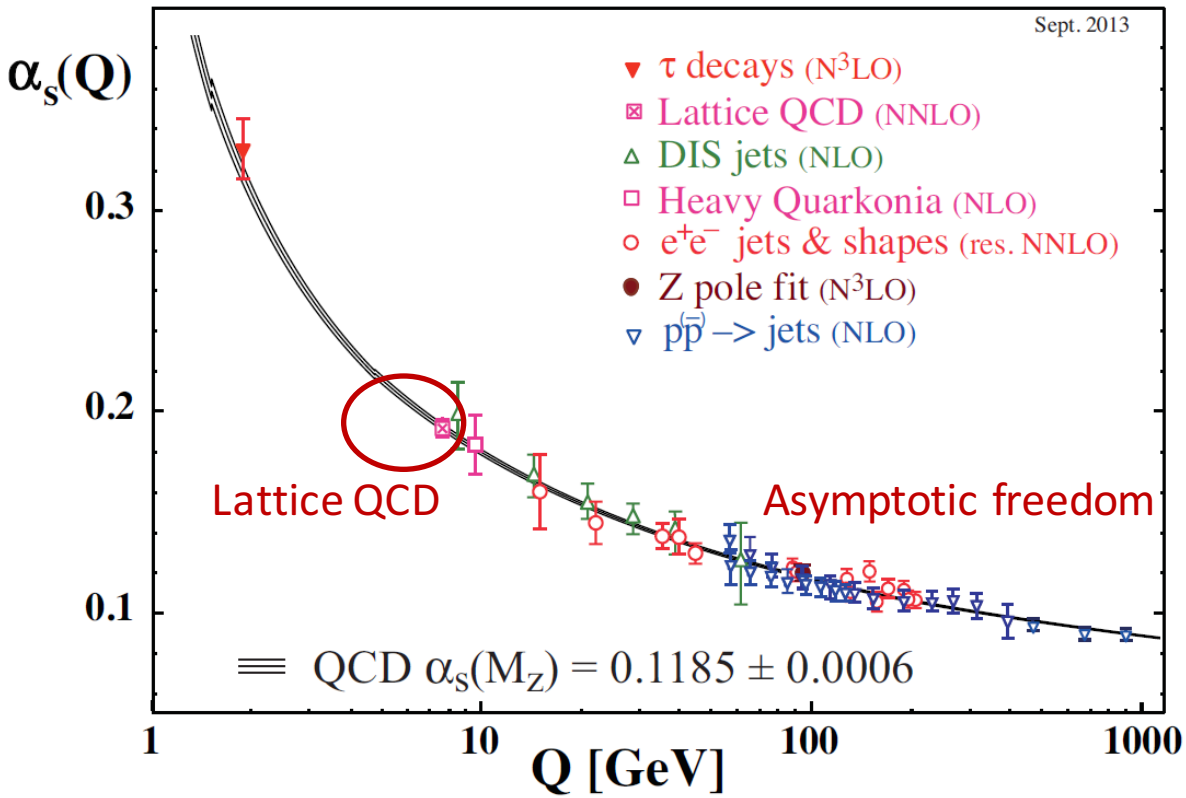


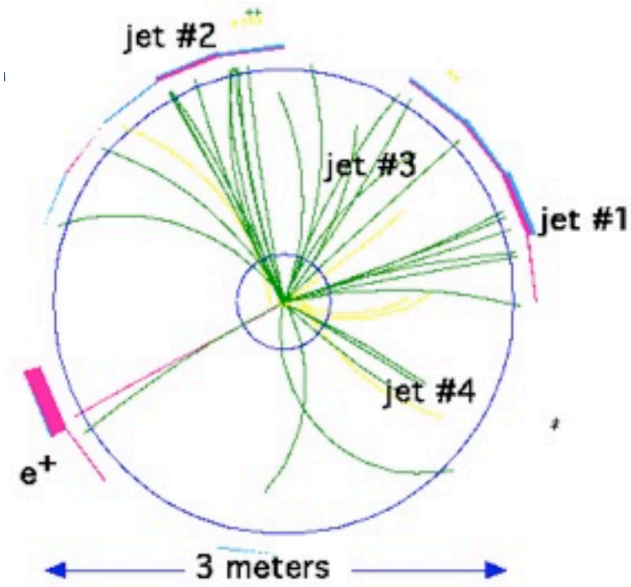
Resonant bound state - short lived





Why lattice QCD?





QCD:
both perturbative
and
Non-perturbative

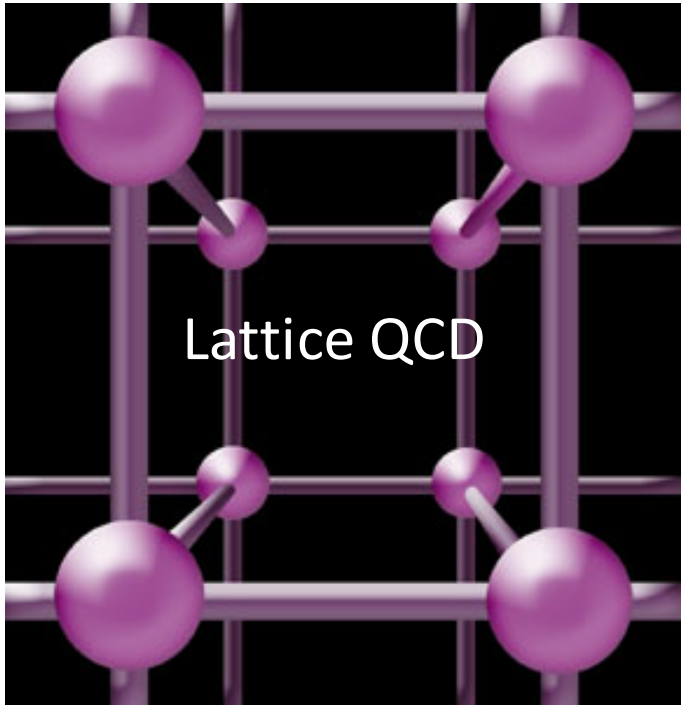
Hadrons – bound states,
non-perturbative

Non-perturbative

Model independent

First-principle

Full predictive power



Automatic regulator

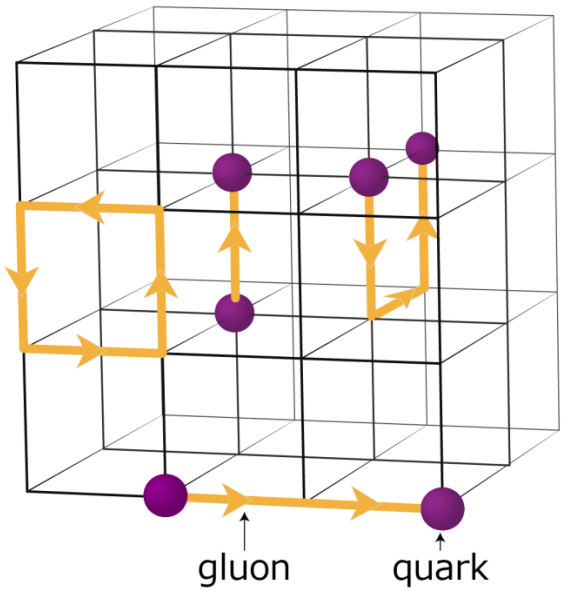
Numerical

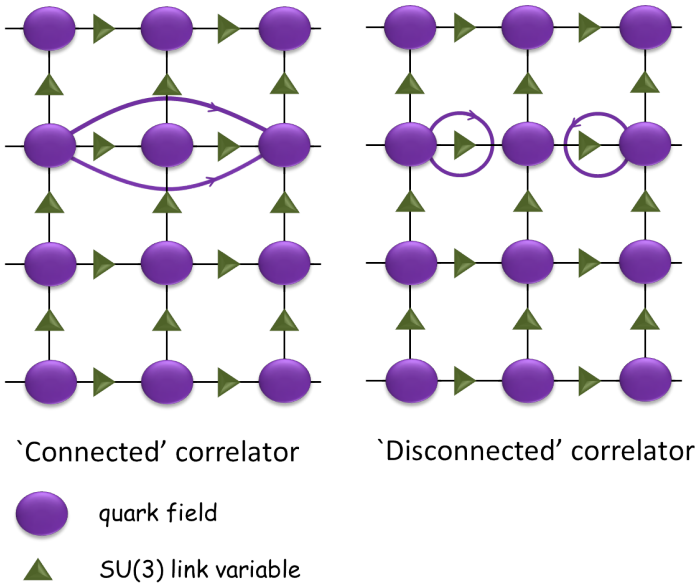
Free parameters– quark masses, Strong coupling

Controlled systematics

Hadron spectrum from lattice QCD?

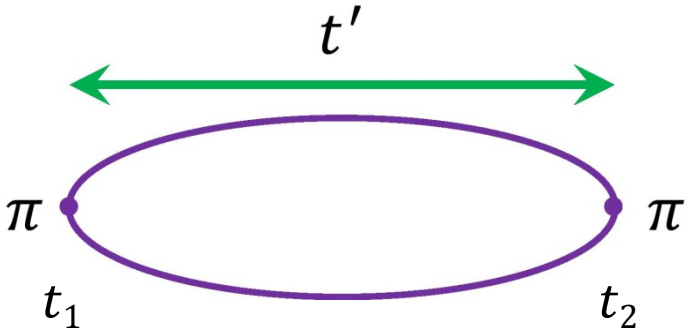
- Expectation values of observables : $\int DUD\psi D\bar{\psi} exp(-\int L_{QCD} d^4x)$
- 4-D space-time lattice – Euclidean time
- Gauge configurations : gluons + sea quarks





- Discretise : $L_q \equiv \bar{\psi}(\gamma_\mu D^\mu + m)\psi \rightarrow \bar{\psi}(\gamma.\Delta + ma)\psi$
- Inversion of Dirac matrix : propagator
- 2-point, 3-point correlation functions : extract meson properties
- Corrections for lattice artefacts, $(\mathrm{a} \rightarrow 0, \mathrm{L} \rightarrow \infty, \mathrm{T} \rightarrow \infty, \mathrm{m_q} \rightarrow \mathrm{m_q^{phys}})$

Two-point correlator construction: JLab way



$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle$$

- Basis of operators

$$\mathcal{O} \sim \bar{\psi}\Gamma \overleftrightarrow{D} \cdots \overleftrightarrow{D}\psi$$

- Optimized operator for state $|n>$

$$\Omega_{\mathfrak{n}}^\dagger = \sum_i w_i^{(\mathfrak{n})} \mathcal{O}_i^\dagger$$

in a variational sense by solving generalized eigenvalue problem-

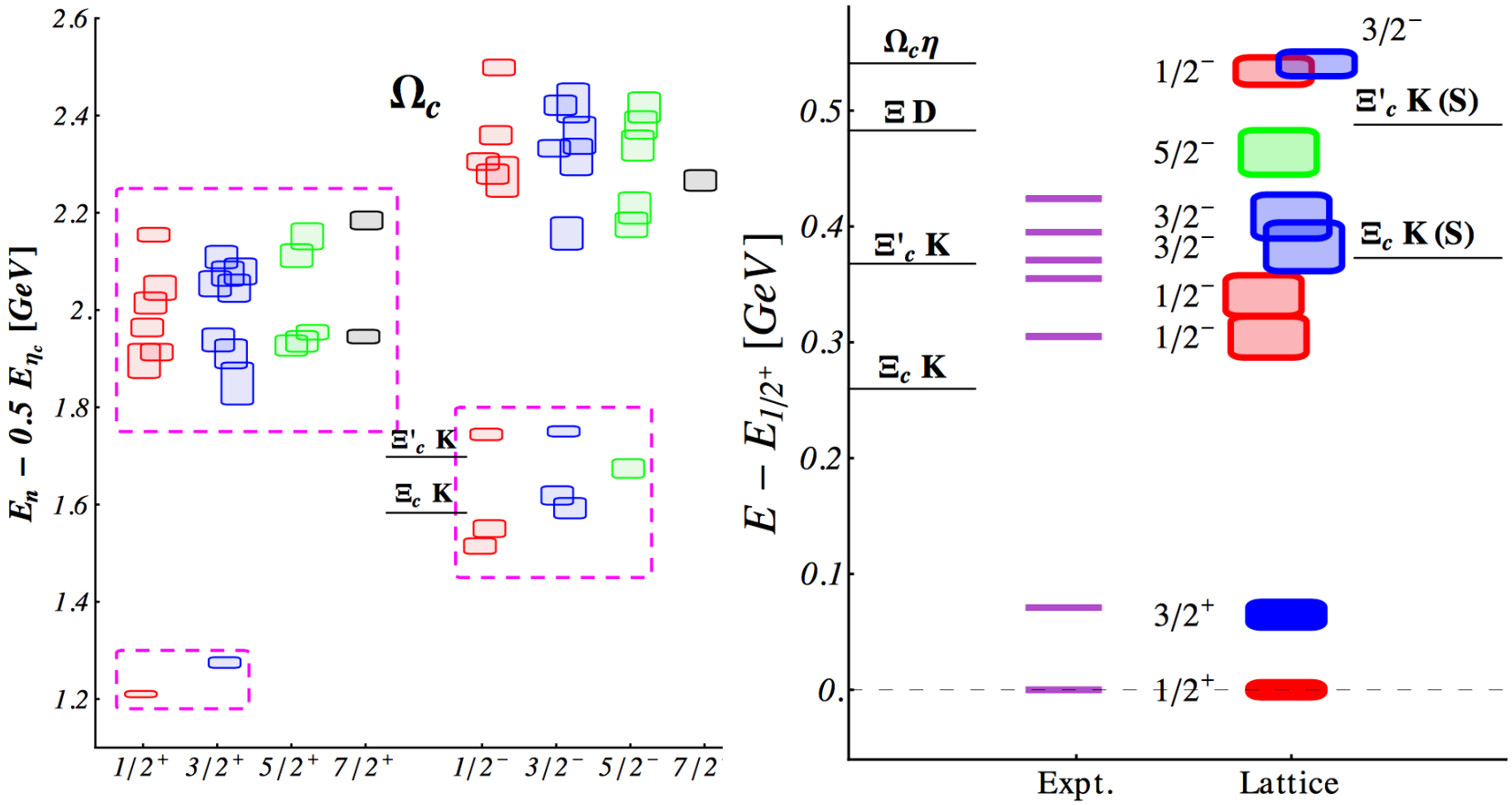
$$C(t)\, v^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t)\, C(t_0)\, v^{(\mathfrak{n})}$$

- Diagonalize the correlation matrix – eigenvalues

$$\lambda_n(t) = \exp[-E_n(t - t_0)]$$

- Correlator Construction: smearing of quark fields - 'distillation'
- Extract many low-lying energy levels

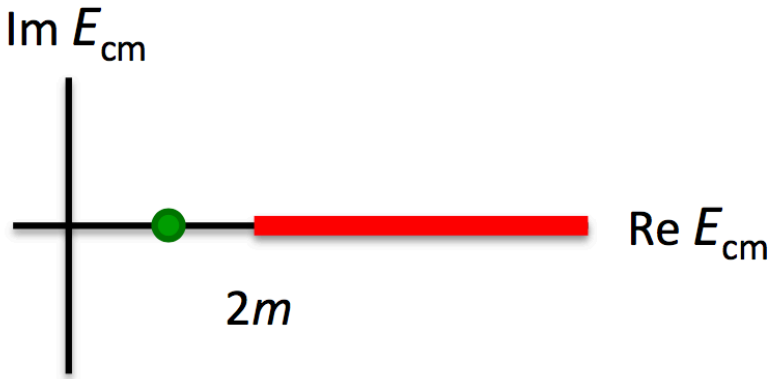
Lattice QCD predicted the spin-parity quantum numbers for the newly discovered baryons at LHCb

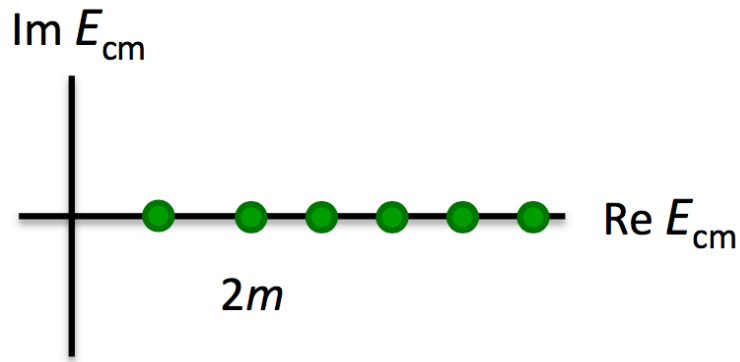


Madanagopalan & Mathur (Hadron Spectrum Collaboration)
Phys. Rev. Lett. 119 (2017) no.4, 042001

Scattering in lattice QCD

Infinite volume – continuous energy levels above threshold





Finite volume discrete spectrum

- Single particle in a box – finite volume effect suppresses exponentially ~ exp ($m_\pi L$)
- Can be neglected by taking $m_\pi L = 4$

Finite volume effects

- Two-particle system – Lüscher formalism (and its extensions) :

relate finite volume energy levels to infinite volume scattering t-matrix $\mathbf{t}(E_{\text{cm}})$

$$\det\left[\underbrace{\mathbf{t}^{-1}(E)}_{\text{scattering matrix}} + i\underbrace{\boldsymbol{\rho}(E)}_{\text{phase space}} - \underbrace{\mathbf{M}(E,L)}_{\text{known functions}}\right] = 0$$

Quantization condition

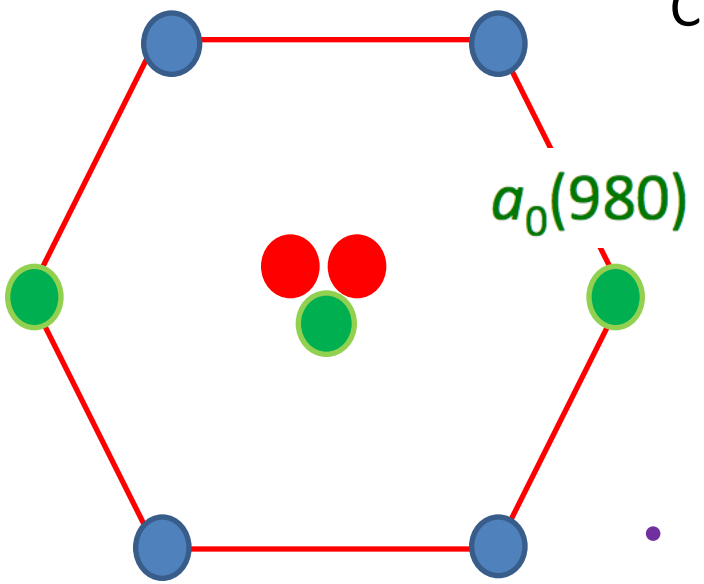
$$\mathbf{t} = \begin{bmatrix} t_{\pi\eta\to\pi\eta} & t_{\pi\eta\to K\overline{K}} \\ t_{K\overline{K}\to\pi\eta} & t_{K\overline{K}\to K\overline{K}} \end{bmatrix}$$

- Determinant equations for $t(E_{cm})$ at each E_{cm}
- Under-constrained problem
- Different parametrisation of E_{cm} dependence of t-matrix, e.g. Breit-Wigner
- Lattice – reduced symmetries:
  proper subduction needed, partial wave mixing

HE, JHEP 0507 011 HANSEN, PRD86 016007
BRICENO, PRD88 094507 GUO, PRD88 014051

Light scalar mesons sector – an example of resonance

$a_0(980)$ resonance in $\pi\eta$, $K\overline{K}$



Channels, $J^P = 0^+$, I = 1

- Sharp experimental enhancement near $K\overline{K}$ decaying to $\pi\eta$

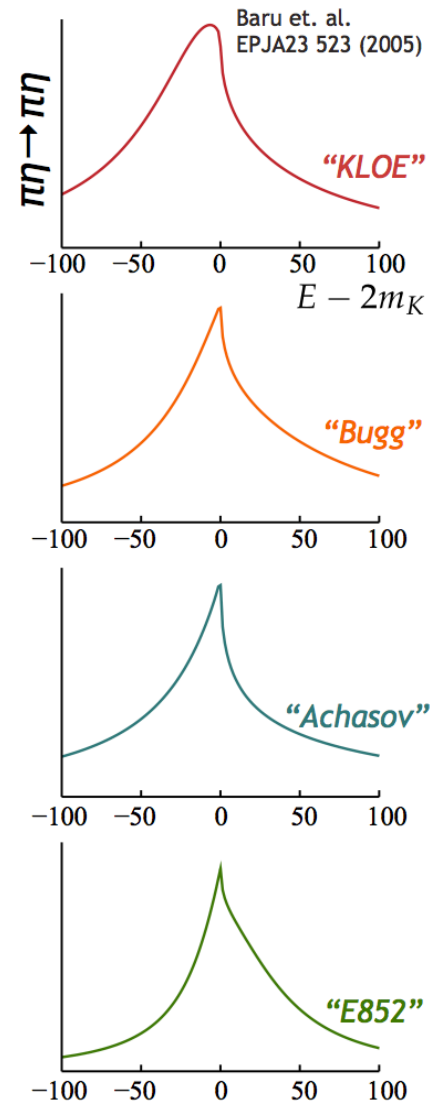
- Amplitude model Typically gives

$$\frac{g^2(K\overline{K})}{g^2(\pi\eta)} \sim 1$$

- Typically observed in ``more complex" Production processes

$p\bar{p} \to \pi\pi\eta$

$\phi \to \gamma\pi\eta$



Correlator constructions

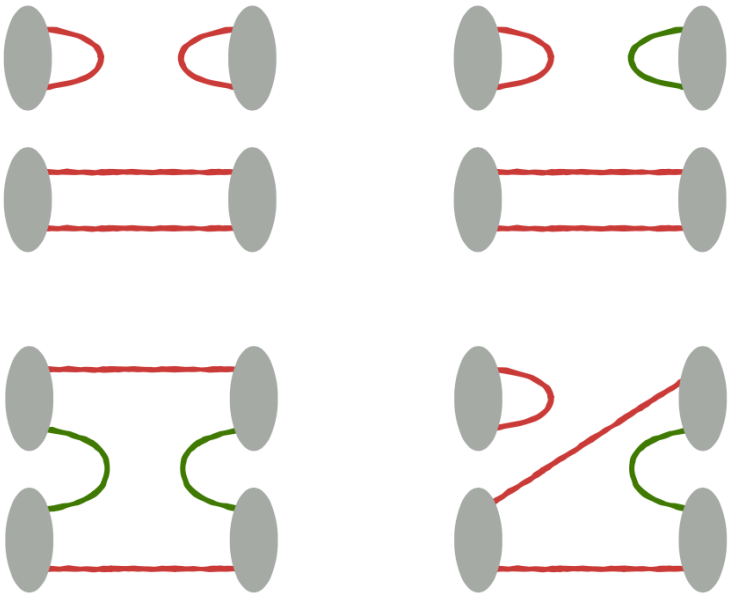
Matrices of correlation function with large operator basis

$\pi\eta$-like $\sum_{\hat{p}_1,\hat{p}_2} C(\vec{p}_1,\vec{p}_2;\vec{P})\,\pi(\vec{p}_1)\eta(\vec{p}_2)$

$K\bar{K}$-like $\sum_{\hat{p}_1,\hat{p}_2} C(\vec{p}_1,\vec{p}_2;\vec{P})\,K(\vec{p}_1)\bar{K}(\vec{p}_2)$

$\pi\eta'$-like $\sum_{\hat{p}_1,\hat{p}_2} C(\vec{p}_1,\vec{p}_2;\vec{P})\,\pi(\vec{p}_1)\eta'(\vec{p}_2)$

Many wick contractions required for creation and annihilation

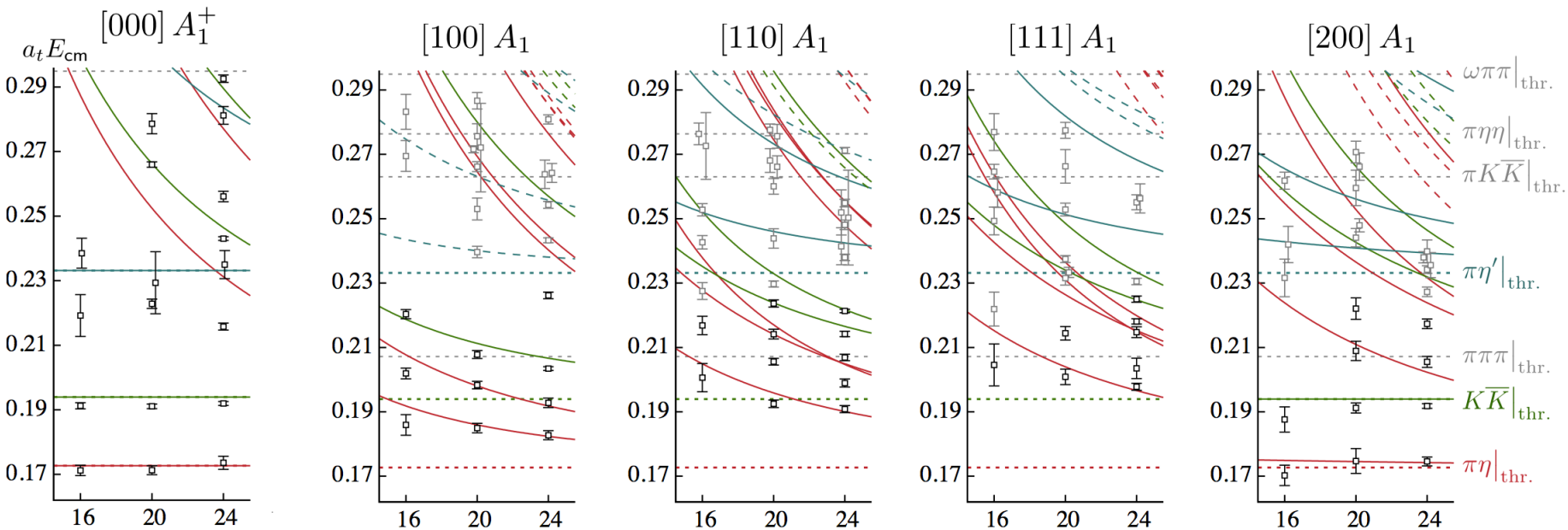


First calculation of a_0 resonance in Coupled channel scattering by Hadron Spectrum Collaboration

J. Dudek *et. al.*
Phys.Rev. D93 (2016) no.9, 094506

Spectra obtained

Using variational method



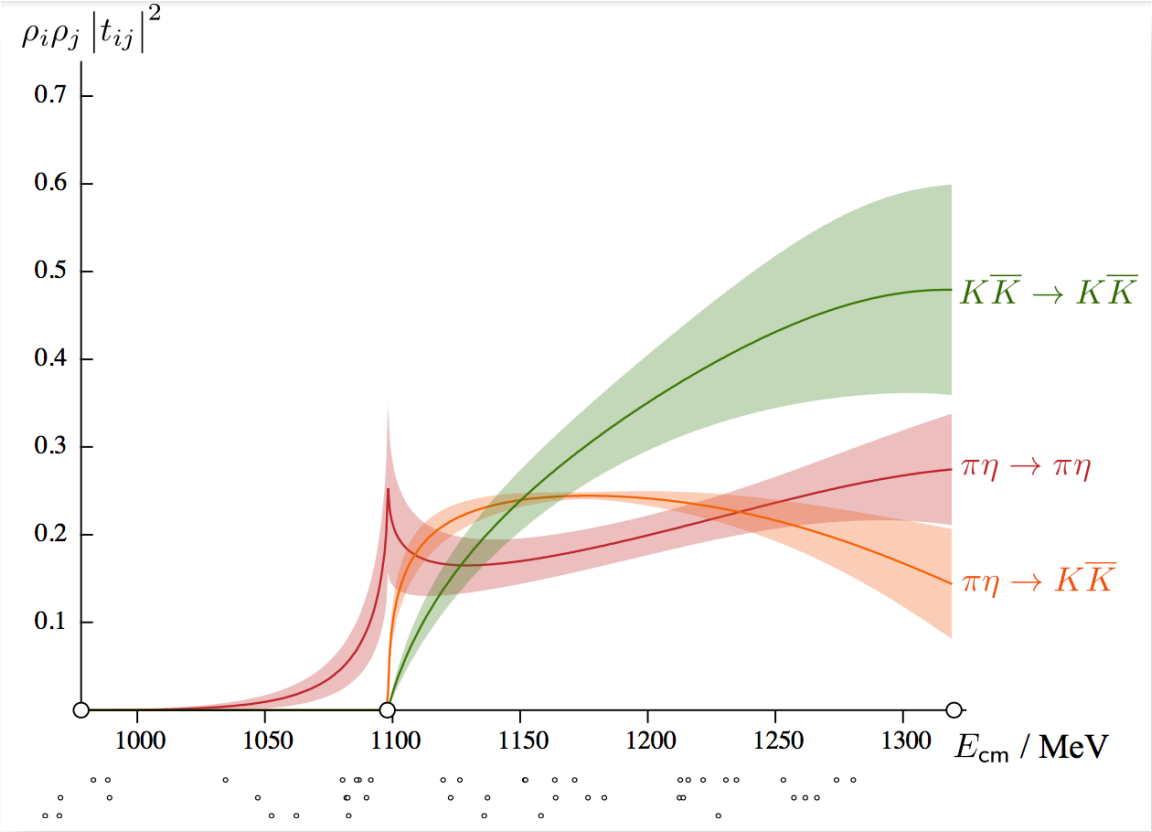
$m_\pi \sim 391\,\mathrm{MeV}$

16^3, 20^3, 24^3

a_s ~ 0.12 fm

a_t ~ $a_s/3.5$

Hadron Spectrum Collaboration
J. Dudek *et. al.*
Phys.Rev. D93 (2016) no.9, 094506

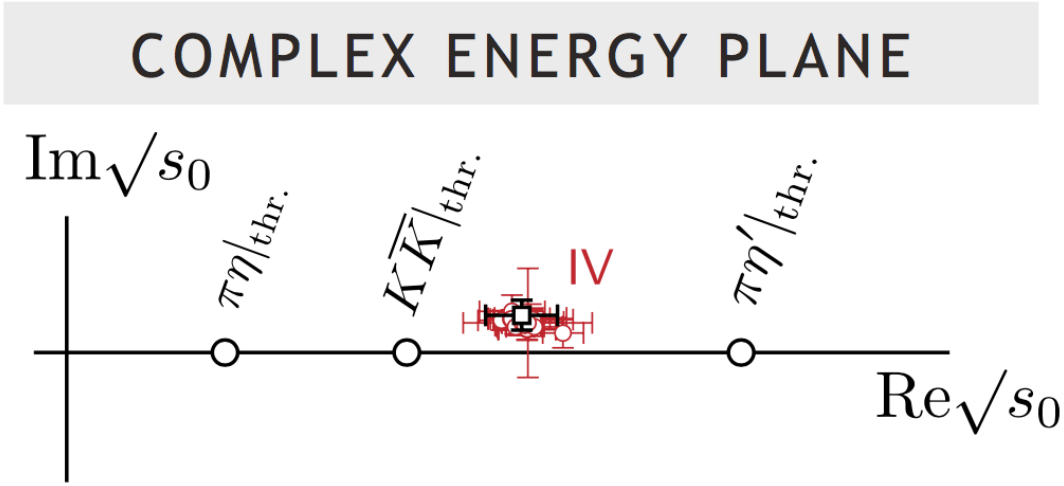


- Strong cusp in $\pi\eta$ at $K\overline{K}$ threshold
- Rapid turn on $K\overline{K}$ amplitudes
- Indicating resonance – look for pole singularity at complex energy

$$t_{ij}(s) \sim \frac{g_i g_j}{s_0 - s}$$

Hadron Spectrum Collaboration
J. Dudek *et. al.*
Phys.Rev. D93 (2016) no.9, 094506

Found a single dominant nearby pole – in ``unphysical" fourth Riemann sheet



Pole coupling ratio

$$\left|\frac{g_{K\bar{K}}}{g_{\pi\eta}}\right| = 1.3(4)$$

Hadron Spectrum Collaboration
J. Dudek *et. al.*
Phys.Rev. D93 (2016) no.9, 094506

Pole position

$$\sqrt{s_0} = \left((1177 \pm 27) + \frac{i}{2}(49 \pm 33)\right) \mathrm{MeV}$$

$$\mathrm{Re}\sqrt{s_0} - 2m_K = 79 \pm 27$$

Conclusion and outlook

- Resonance spectroscopy is a powerful tool to probe the nature of strong force.
- Lattice QCD plays crucial role in studying resonance.
- For studying coupled-channel scattering: extract many energy levels reliably
  → map out scattering amplitudes.
- More recent works by Hadron Spectrum Collaboration.
- Work in progress with closer to physical pion mass.
- Three-hadron scattering channels are being included.
- Also, elastic and transition form factors are being studied.

Extra slides

Two-point correlator construction : JLab way

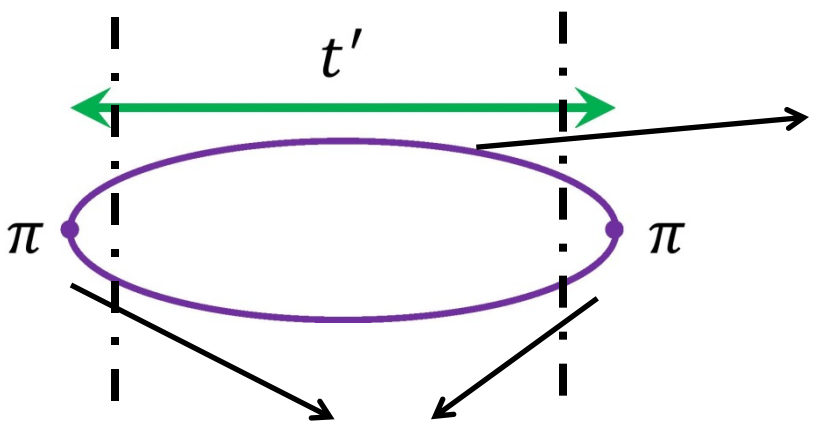
Correlator Construction: smearing of quark fields - 'distillation' with

$$\Box_{\vec{x}\vec{y}}(t) = \sum_{n=1}^{N_D} \xi^{(n)}_{\vec{x}}(t)\, \xi^{(n)\dagger}_{\vec{y}}(t)$$

Low lying hadron states

Meson creation operator :

$$\mathcal{O}^\dagger(\vec{p}) = \bar{\psi}_{\vec{x}} \Box_{\vec{x}\vec{y}}\, e^{-i\vec{p}\cdot\vec{y}}\, \mathbf{\Gamma}_{\vec{y}\vec{z}}\, \Box_{\vec{z}\vec{w}} \psi_{\vec{w}}$$



Parambulators by inverting the Dirac matrix

+

Operator construction with momentum projection