An Exciting Odyssey in the Femto-World: QCD Critical Point

Rajiv V. Gavai
T. I. F. R., Mumbai

Importance of Being Critical

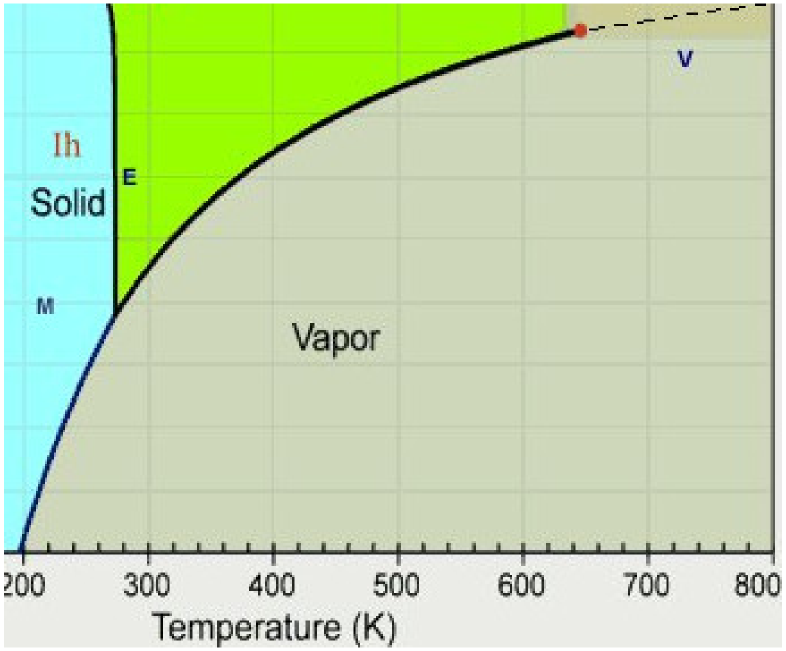
Theoretical Results

Searching Experimentally

Summary

Importance of Being Critical

Phase Diagram of Water (http://www1.lsbu.ac.uk/)



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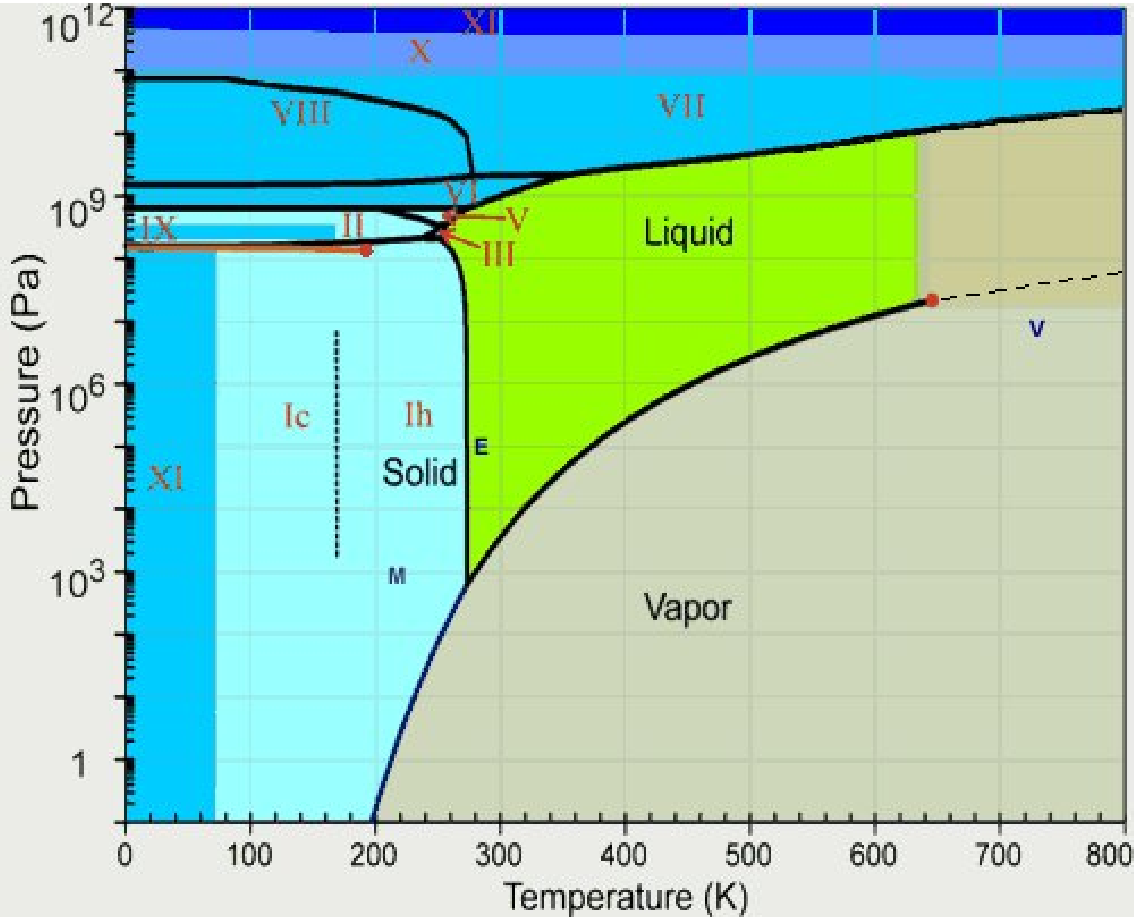

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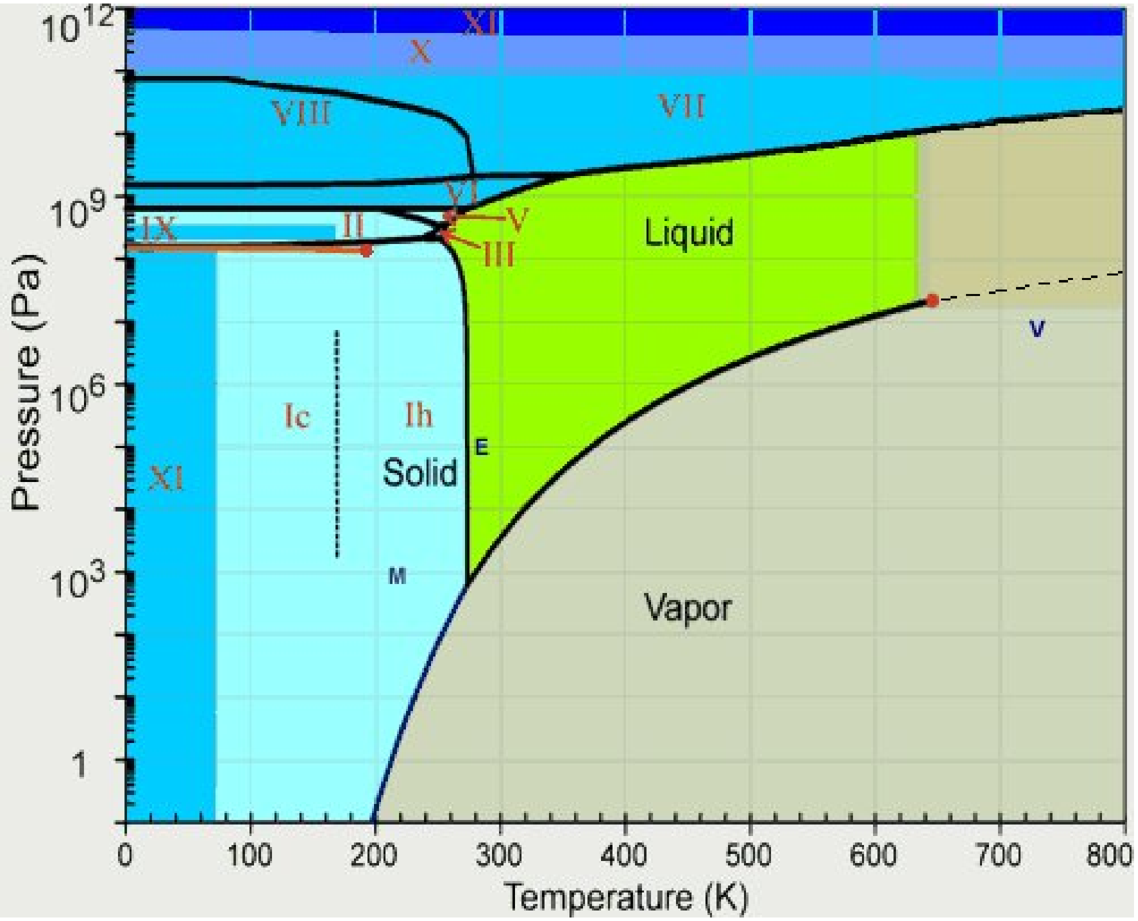

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- Extreme density fluctuations $\Longrightarrow$ Critical Opalescence (T. Andrews, Royal Society 1869).

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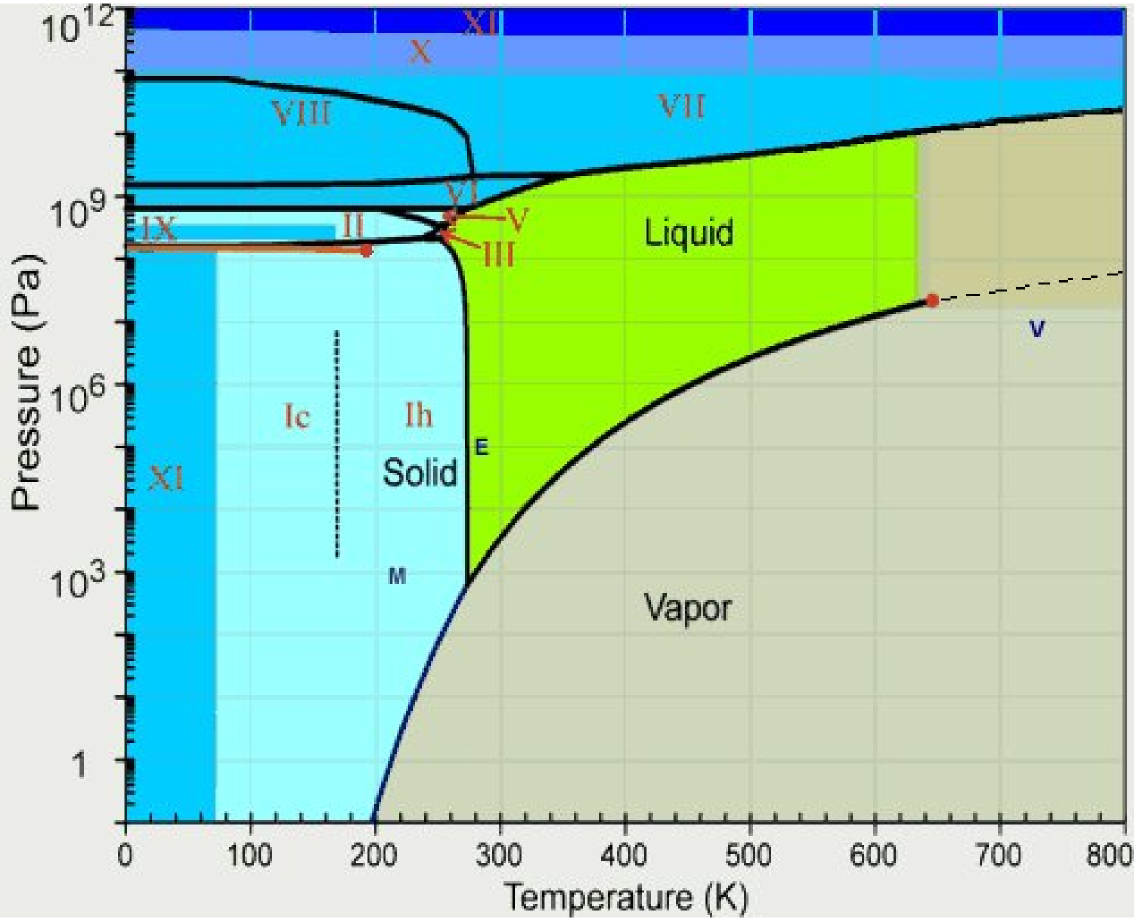
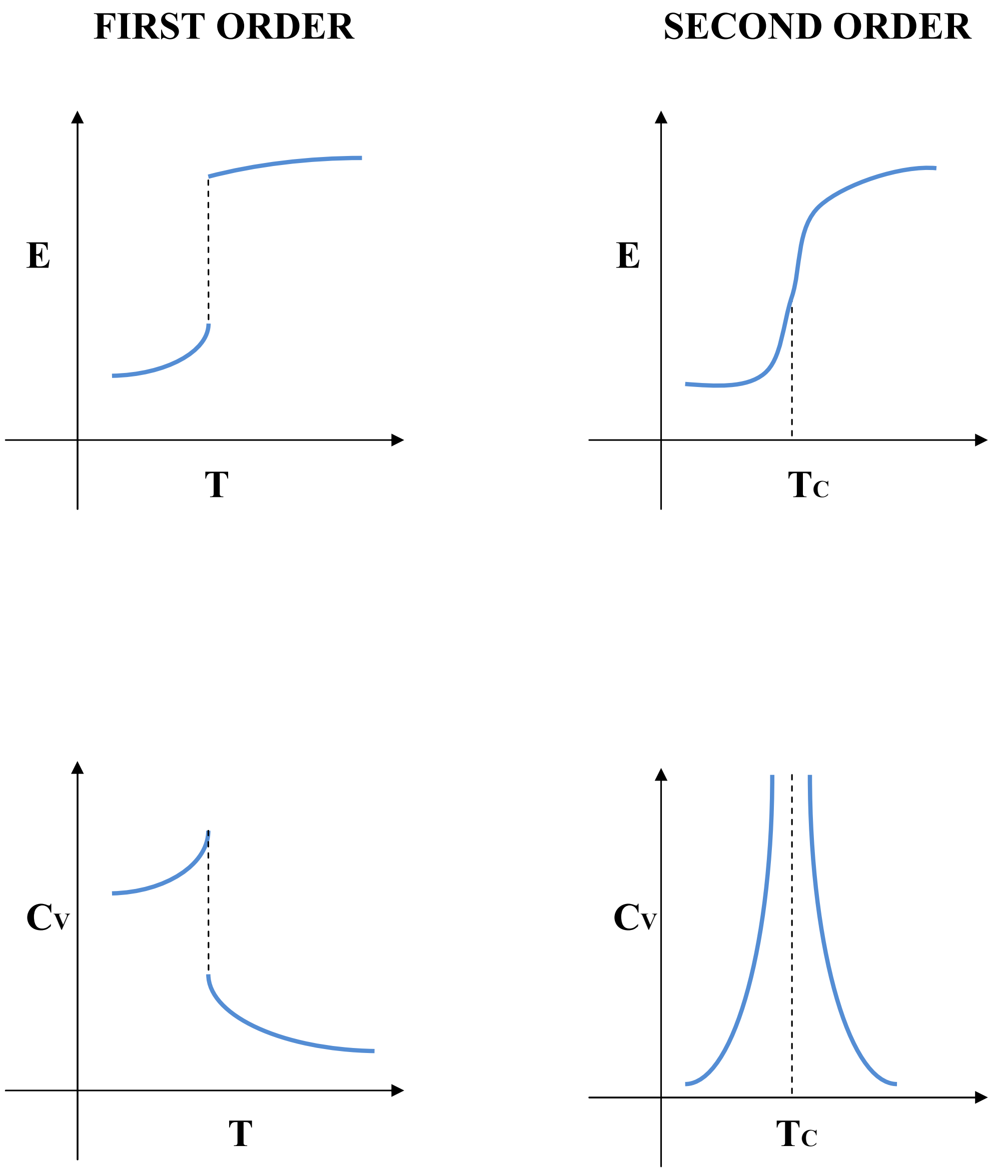
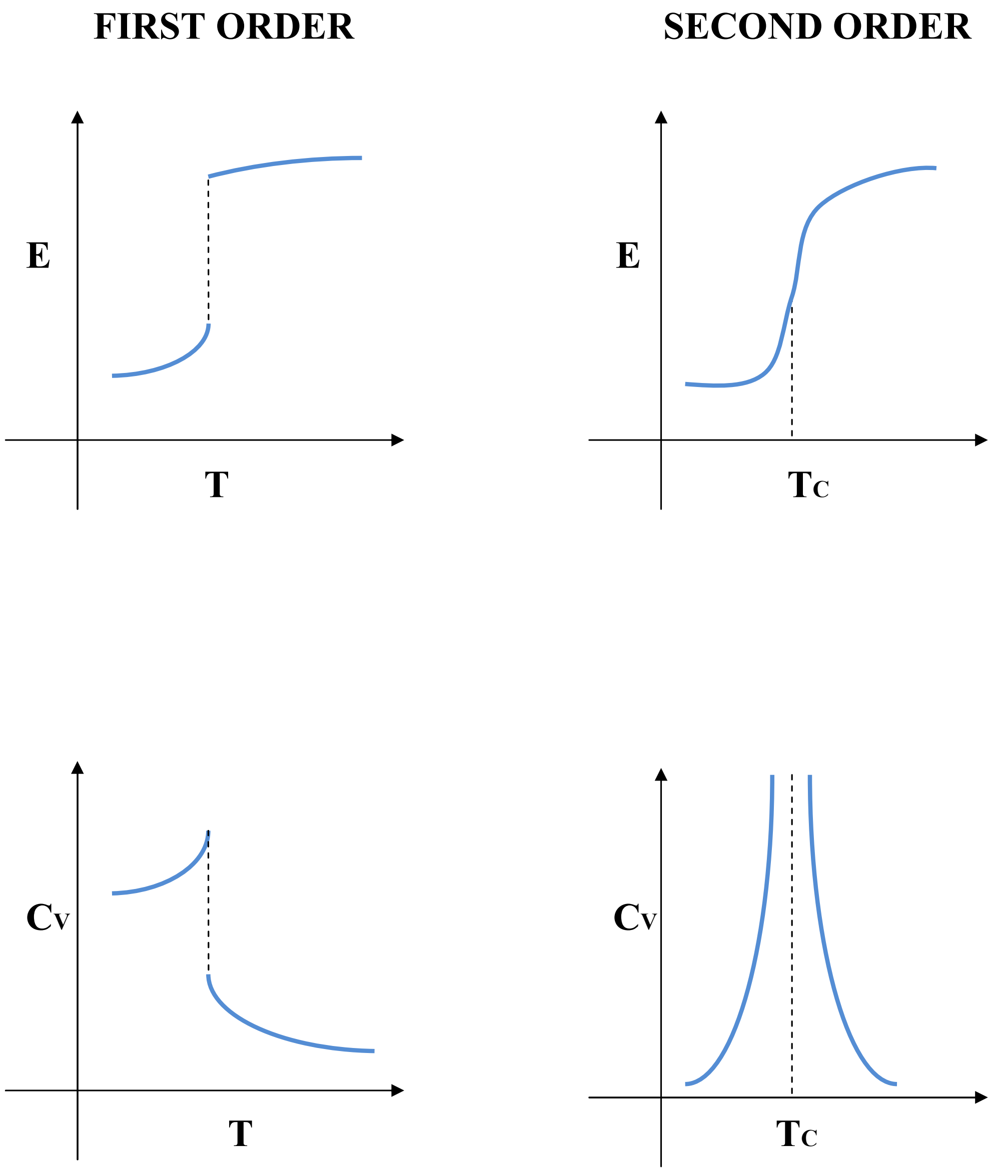


- One, possibly two, critical points.
- Extreme density fluctuations $\Longrightarrow$ Critical Opalescence (T. Andrews, Royal Society 1869).
- SCF dissolves material like liquid but passes through solid like gas.
- Dielectric constant & Viscosity $\downarrow$.

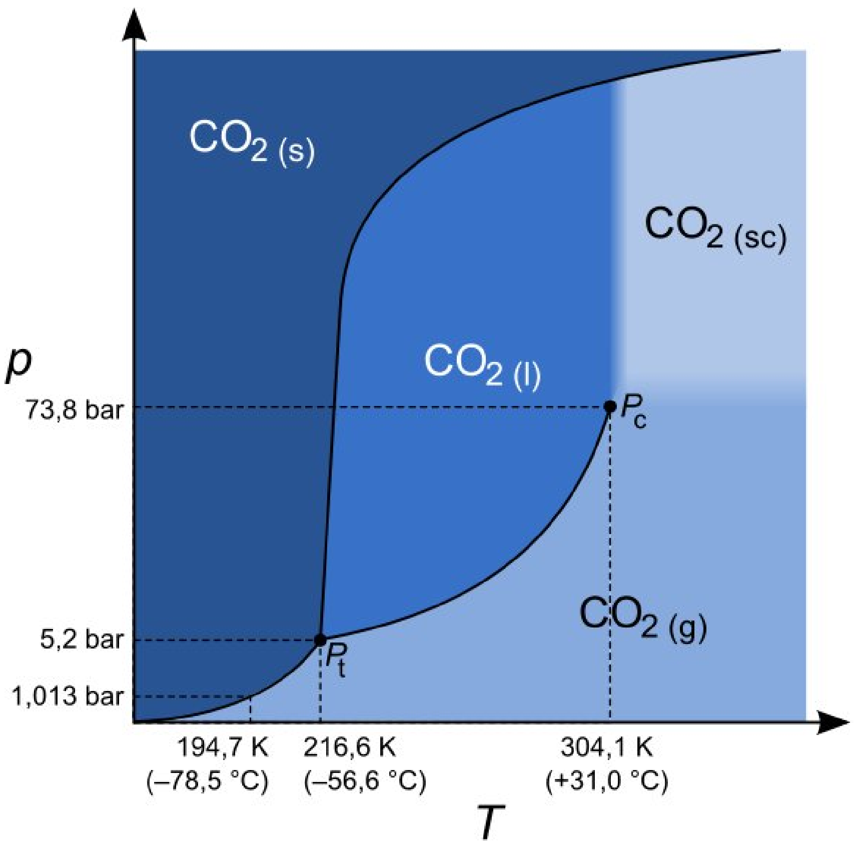


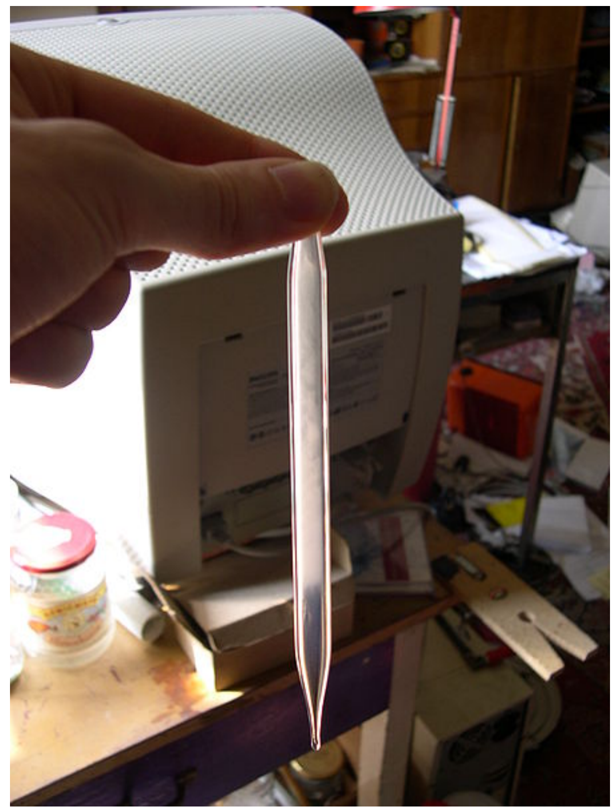
- Discontinuous ϵ – Nonzero Latent Heat– & finite C_v $\rightarrow$ First order PT.



- Discontinuous ϵ – Nonzero Latent Heat– & finite C_v $\rightarrow$ First order PT.
- Continuous ϵ, & diverging C_v $\rightarrow$ Second order PT.
- In(Finite) Correlation Length at 2nd (1st) Order transition.
- "Cross-over" – mere rapid change in ϵ, with maybe a sharp peaked C_v.

Critical Point : The meV Scale





$\uparrow$ 26 meV using

$\hbar = c = k = 1 \Longrightarrow$ 1.16 $\times 10^4\,^\circ\mathrm{K} \equiv$ 1 eV; Picts From Wikipedia

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◊ Supercritical fluid extraction is recognised as a green technology for production of essence from herbs and plants.

♡ About a third of hop extraction using supercritical CO_2!

♠ Many liquid fueled engines exploit such supercritical transitions.

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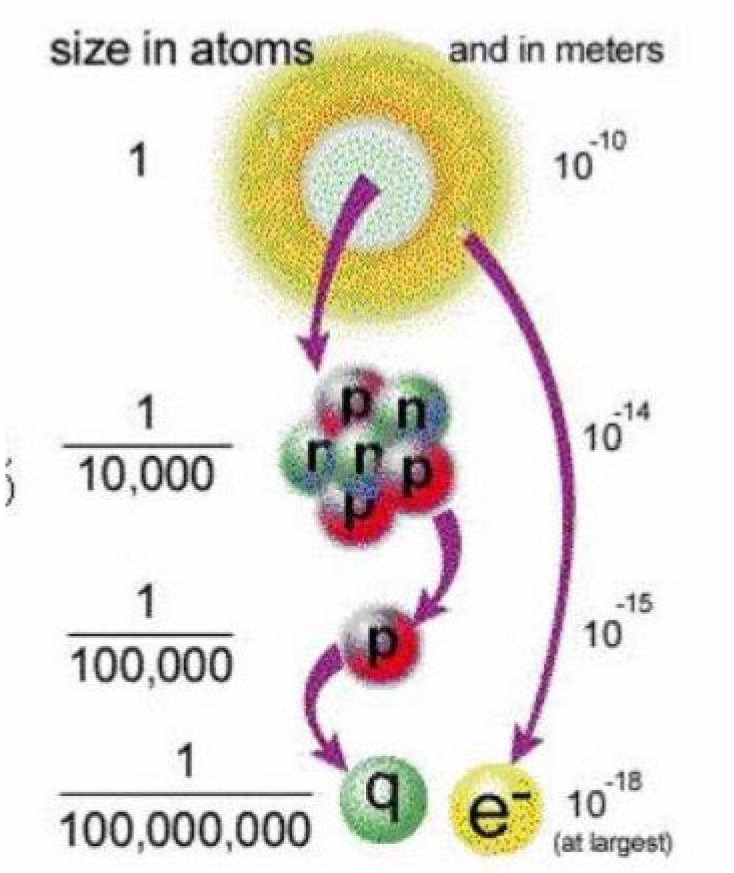
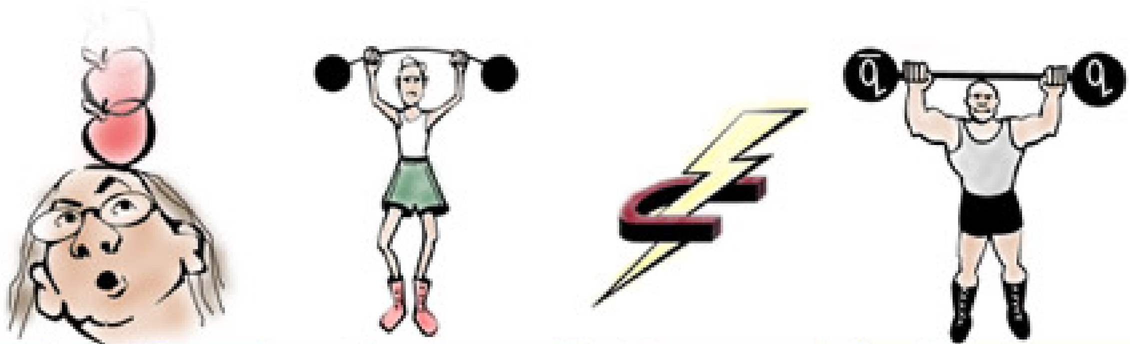
- Quarks and Leptons – Basic building blocks : Proton (uud), Neutron (udd), Pion ($u\bar{d}$)....

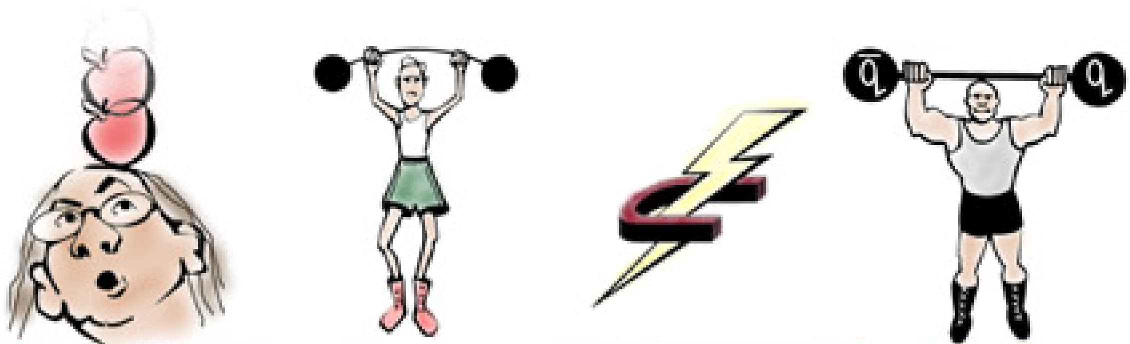
- A Variety of Vector Bosons : Carriers of forces.





	Gravity	Weak (Electroweak)	Electromagnetic	Strong
Carried By	Graviton (not yet observed)	W^+ W^- Z^0	Photon	Gluon
Acts on	All	Quarks and Leptons	Quarks and Charged Leptons and W^+ W^-	Quarks and Gluons

Strengths in a ratio $10^{-39} : 10^{-5} : 10^{-2} : 1$



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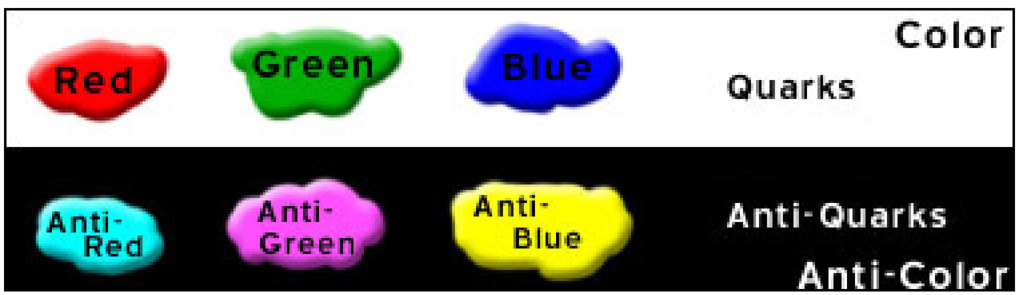
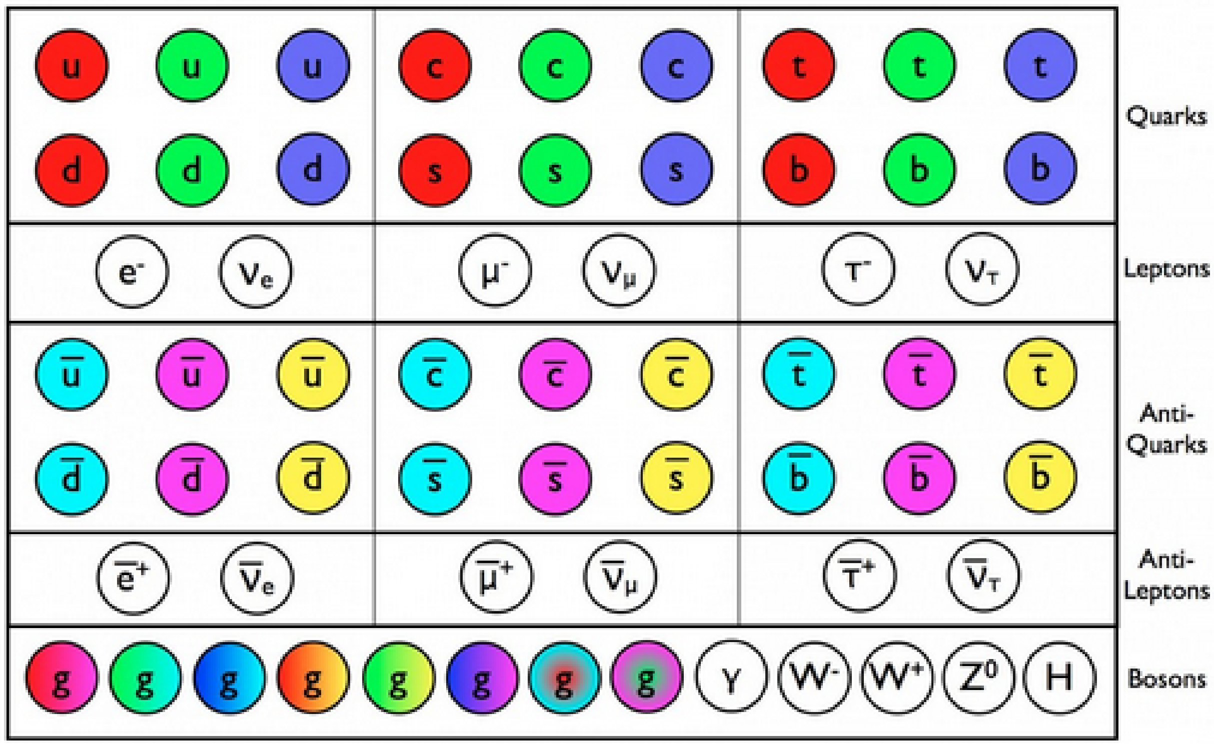


(Anti-)Quarks come in three (anti-)colours, making gluons also coloured.

Standard Model's Zoo

Family $\rightarrow$ I II III



The particles and antiparticles of the Standard Model. Image credit: E. Siegel.

Quantum Chromo Dynamics (QCD)

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- Many more "photons" (Eight) which carry colour charge & hence interact amongst themselves.
- *Unlike QED*, the coupling is usually very large : by ~ 100.
- Much richer structure : Quark Confinement, Dynamical Symmetry Breaking..
- Very high interaction (binding) energies. E.g., $M_{Proton} \gg (2m_u + m_d)$, by a factor of 100 $\rightarrow$ Understanding it is knowing where the Visible mass of Universe comes from.

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- For (N_f) massless particles, A or B do **not** change into each other: Chiral Symmetry $(SU(N_f) \times SU(N_f))$.

- Interactions can break the chiral symmetry dynamically, leading to effective masses for these particles.
- Light pions (m_π =0.14 GeV) and heavy baryons (protons/neutrons; m_N=0.94 GeV) arise this way (Y. Nambu, Physics Nobel Prize 2008).

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- Quark-Gluon Plasma is such a phase. It presumably filled our Universe a few microseconds after the Big Bang & can be produced in Relativistic Heavy Ion Collisions. QCD Critical Point arises also due to Chiral Symmetry.

- Ideally, QCD should shed light on its richer structure : Quark Confinement, Dynamical Symmetry Breaking.. But Models did that first.

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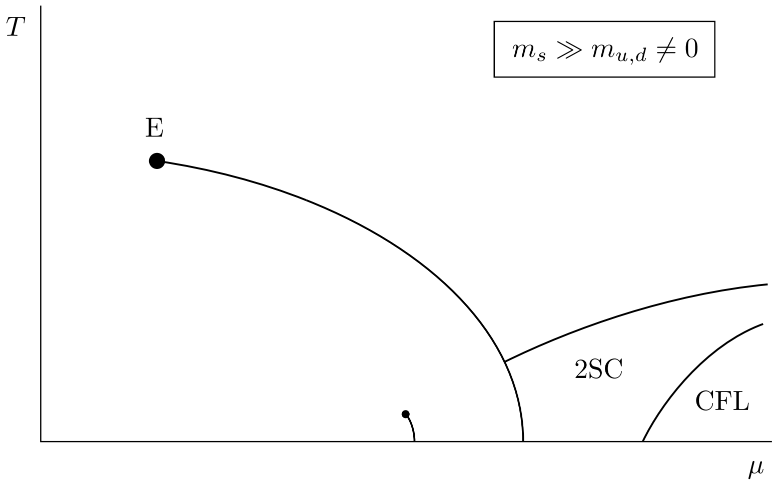

From Rajagopal-Wilczek Review, hep-ph/0011333

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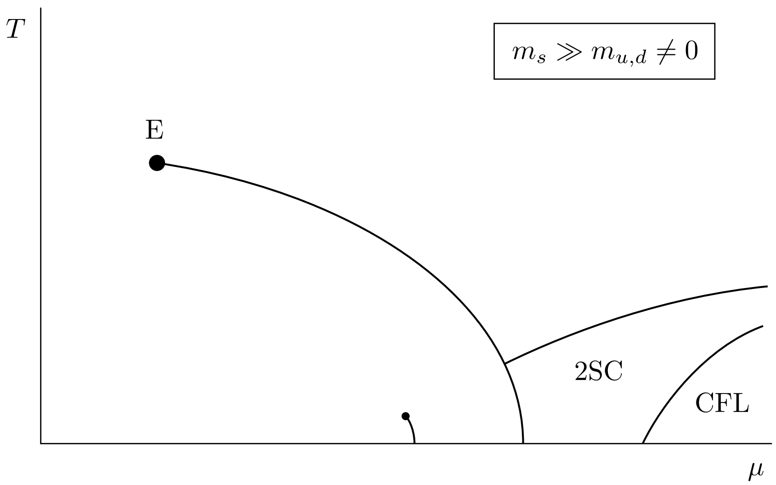

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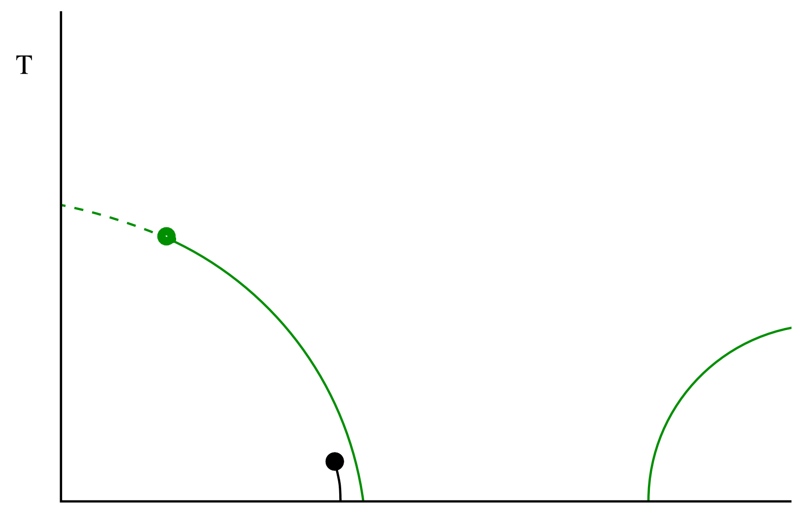

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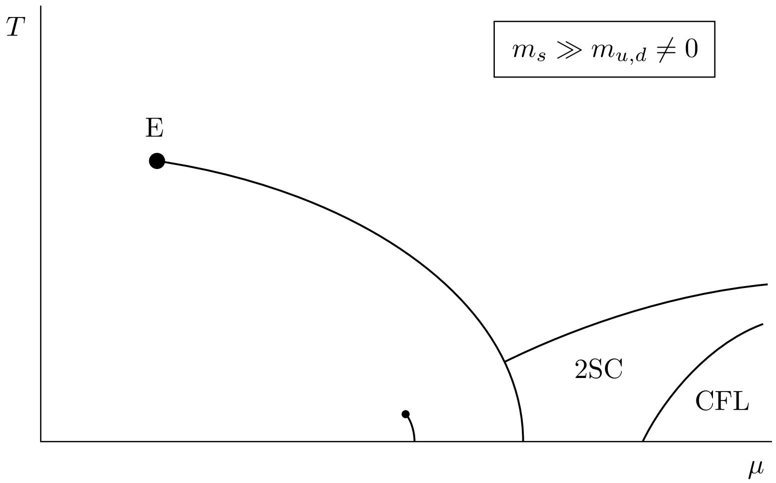

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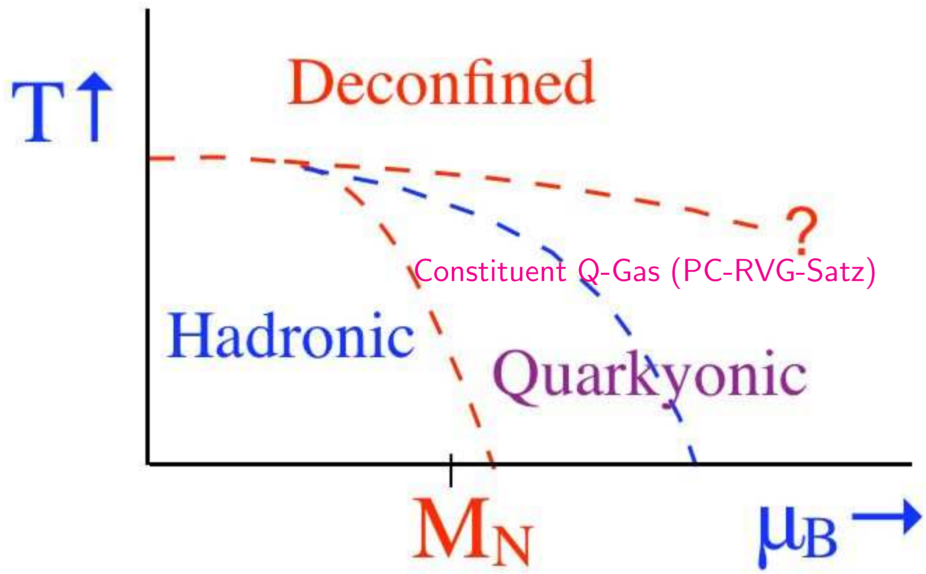

Putting QCD to Work

- QCD Partition Function : $Z_{QCD} = \mathrm{Tr}\ \exp[-(H_{QCD} - \mu_B N_B)/T]$.
- A first-principles calculation of $\epsilon(\mu, T)$ or $P(\mu, T)$ to look for phase transitions, Critical Point and many phases using the underlying theory QCD alone: NO free parameters and NO arbitrary assumptions.

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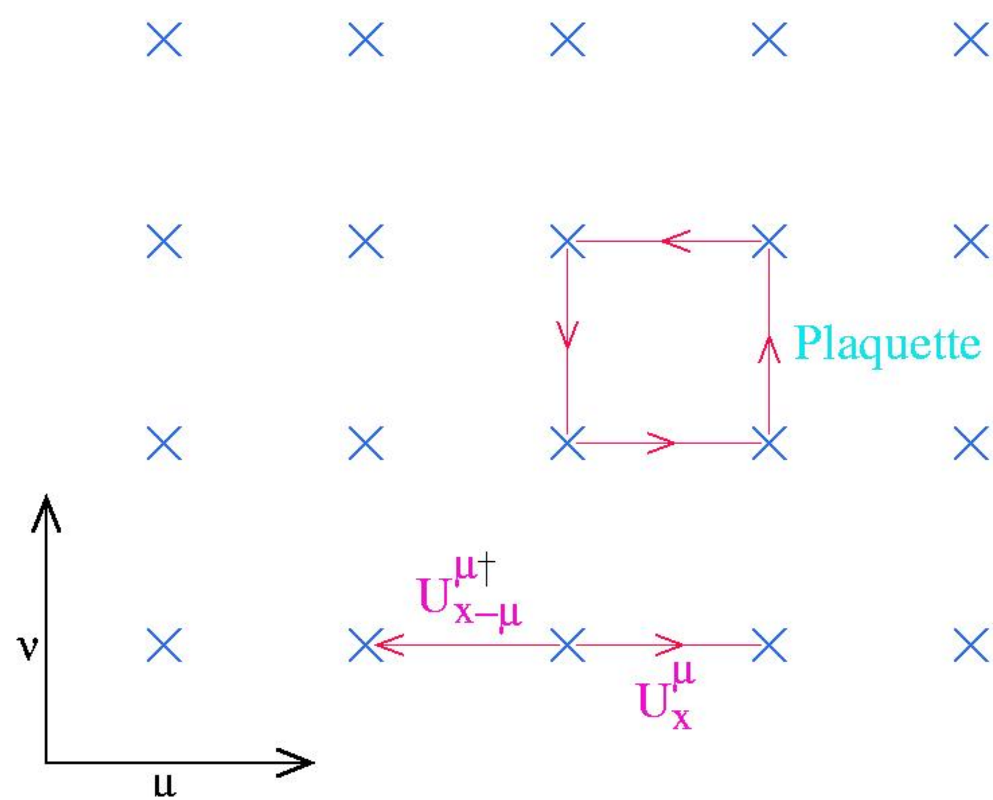
- Simpson integration trick : $\int\ dx\ F(x) = \lim_{\Delta x \to 0} \sum_i\ \Delta x\ F(x_i)$.

- Its analogue to perform functional integrations needs discretizing the space-time on which the fields are defined : Lattice Field Theory !

Basic Lattice QCD

- Discrete space-time : Lattice spacing a UV Cut-off.
- Quark fields $\psi(x)$, $\bar{\psi}(x)$ on lattice sites.
- Gluon Fields on links : $U_\mu(x)$



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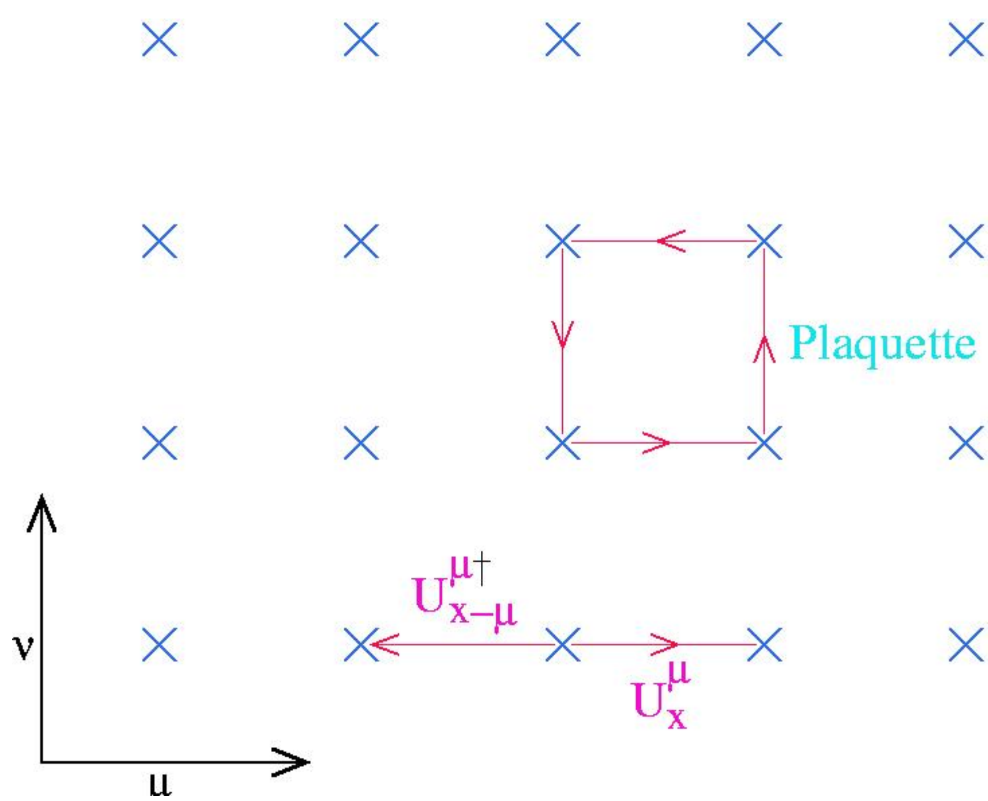
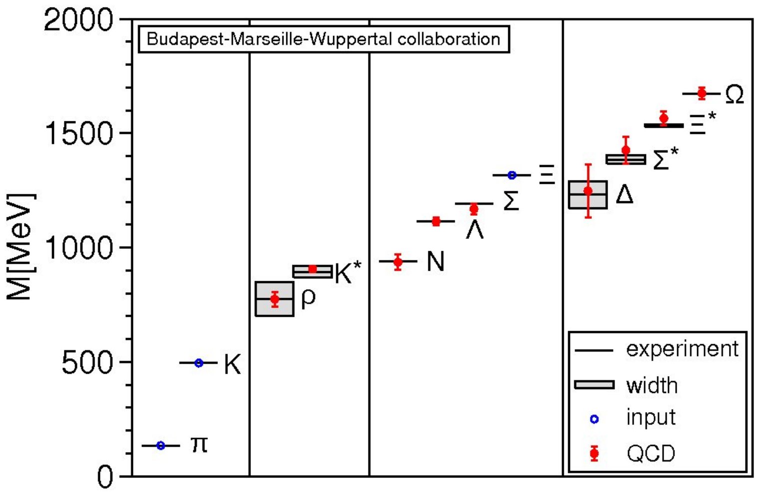
- Gauge invariance : Actions from Closed Wilson loops, e.g., plaquette.
- Fermion Actions : Staggered, Wilson, Overlap, Domain Wall..



Lattice QCD Results

- QCD defined on a space time lattice – Best and Most Reliable way to extract non-perturbative physics: Notable successes are hadron masses(S. Dürr et all, Science (2008)) & decay constants.



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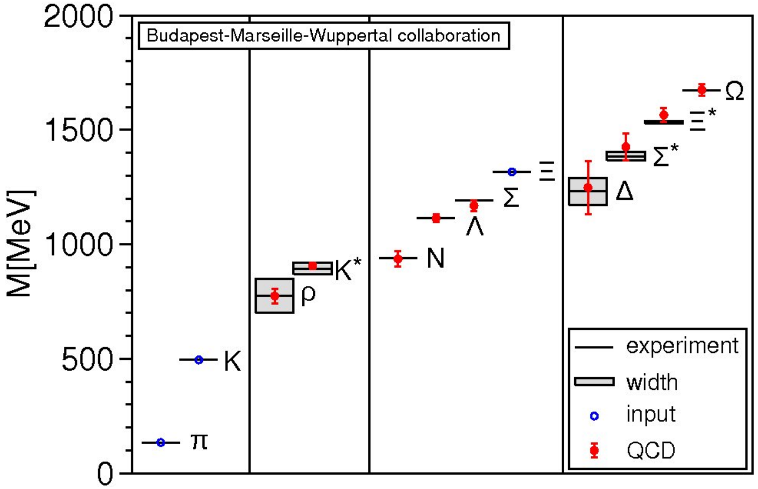

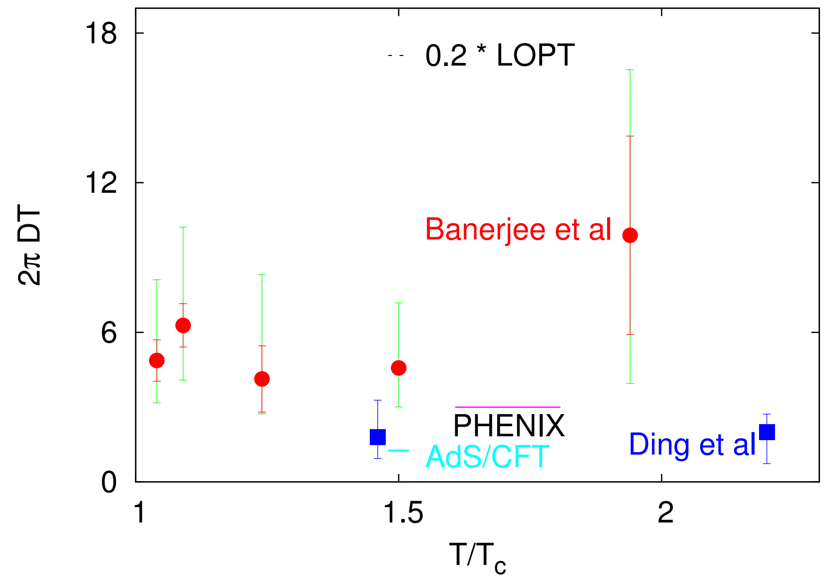

- The Transition Temperature T_c, the Equation of State, Heavy flavour diffusion coefficient D (Banerjee et al. PRD (2012)), Flavour Correlations C_{BS} and the Wróblewski Parameter λ_s are some examples for Heavy Ion Physics.

The $\mu \neq 0$ problem

Physical(thermal expectation) value of an observable $\mathcal{O}$ is

$$\langle \mathcal{O} \rangle = \int DU \left[\frac{\exp(-S_G) \ \mathrm{Det}^{N_f} \ M(m,\mu)}{\mathcal{Z}} \right] \mathcal{O},$$

where the QCD partition function $\mathcal{Z}$ is

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Typically 8-9 million dimensional integral and M is million $\times$ million. Probabilistic methods are therefore used to evaluate $\langle \mathcal{O} \rangle$.

$\Longrightarrow$ Simulations can be done IF Det^{N_f} $M > 0$ for any set of $\{U\}$. However, Det M is a complex number for all $\mu \neq 0$: The Phase/sign problem

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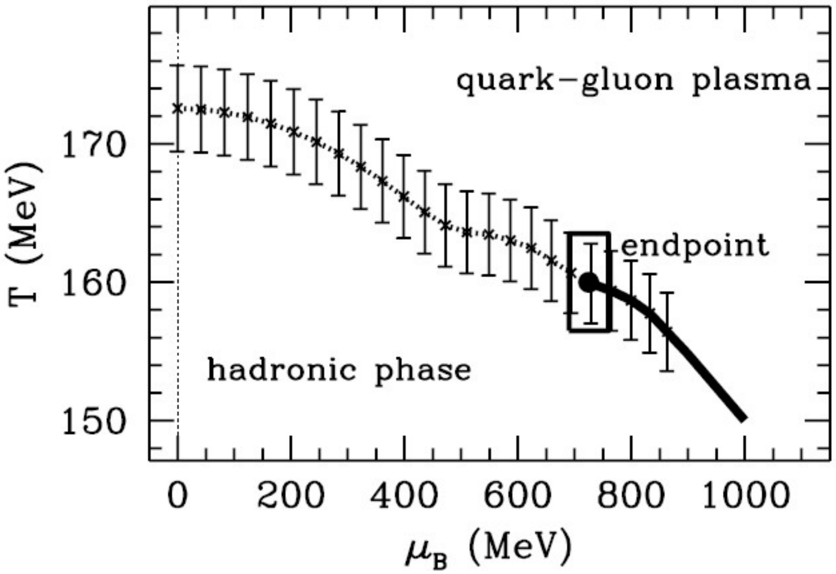
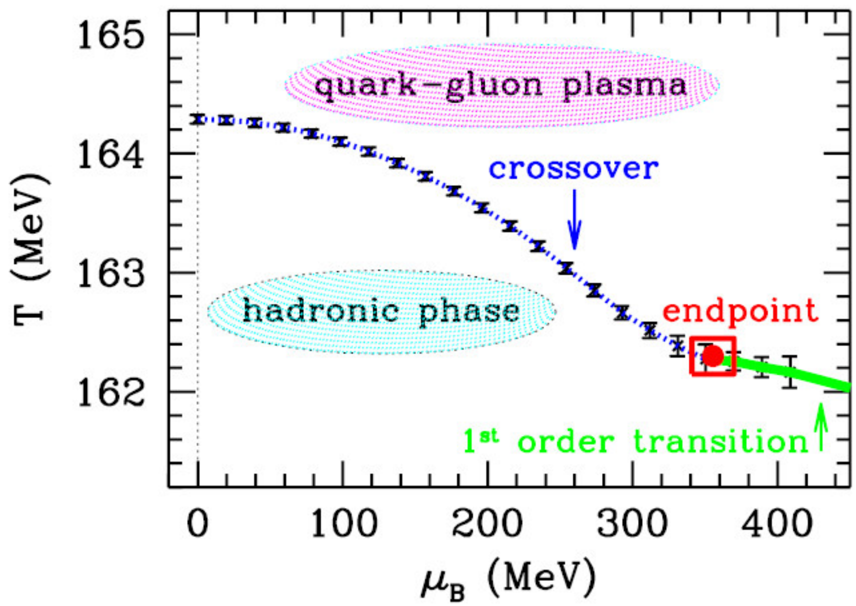
- Why Taylor series expansion? — i) Ease of taking continuum and thermodynamic limit & ii) Better control of systematic errors.

First Glimpse of QCD Critical Point





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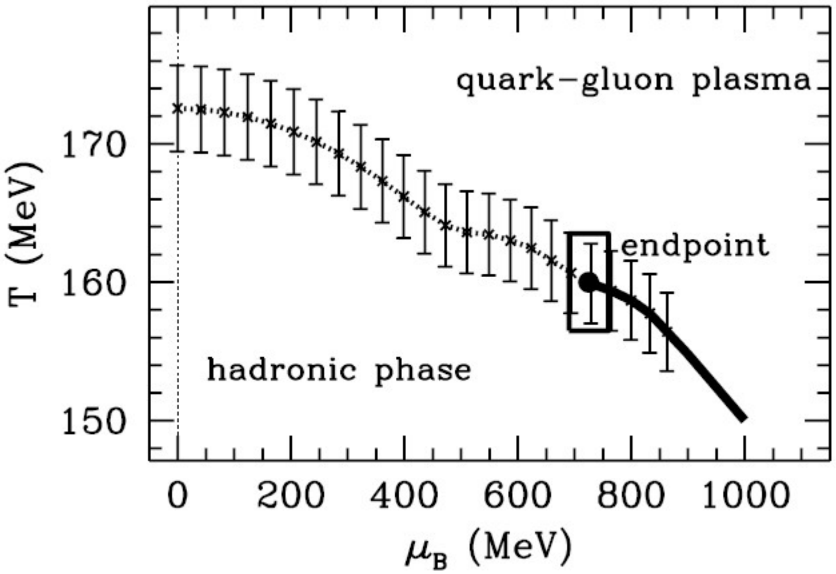

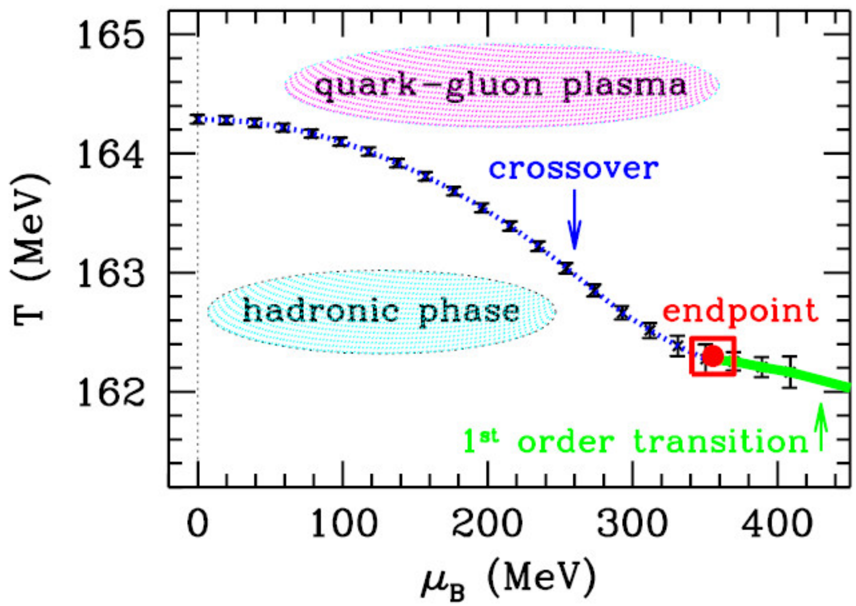

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Larger N_t or Continuum limit ?

QCD Critical Point : Taylor Expansion

- Note that 1) Specific Heat/Susceptibility diverges as one approaches critical point and 2) a series $1 + x + x^2 + x^3 .... = 1/(1 - x)$, only if $x < 1$, it diverges otherwise.

- Employ Taylor expansion of baryonic susceptibility $\chi_B(\mu, T)$ in $z = \mu/T$, and look for its radius of convergence to obtain the nearest critical point.

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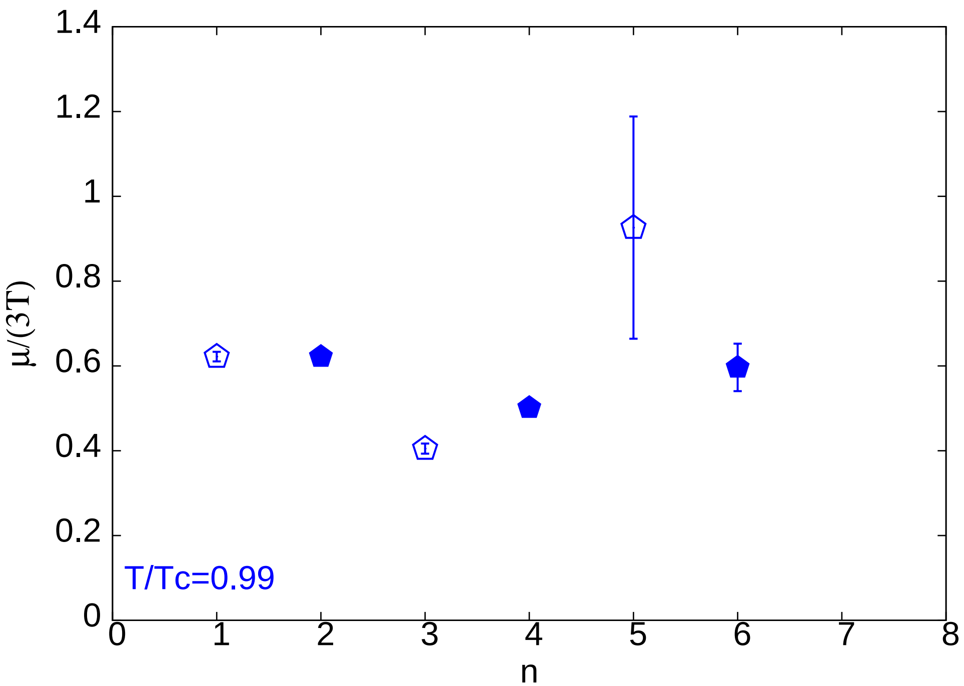
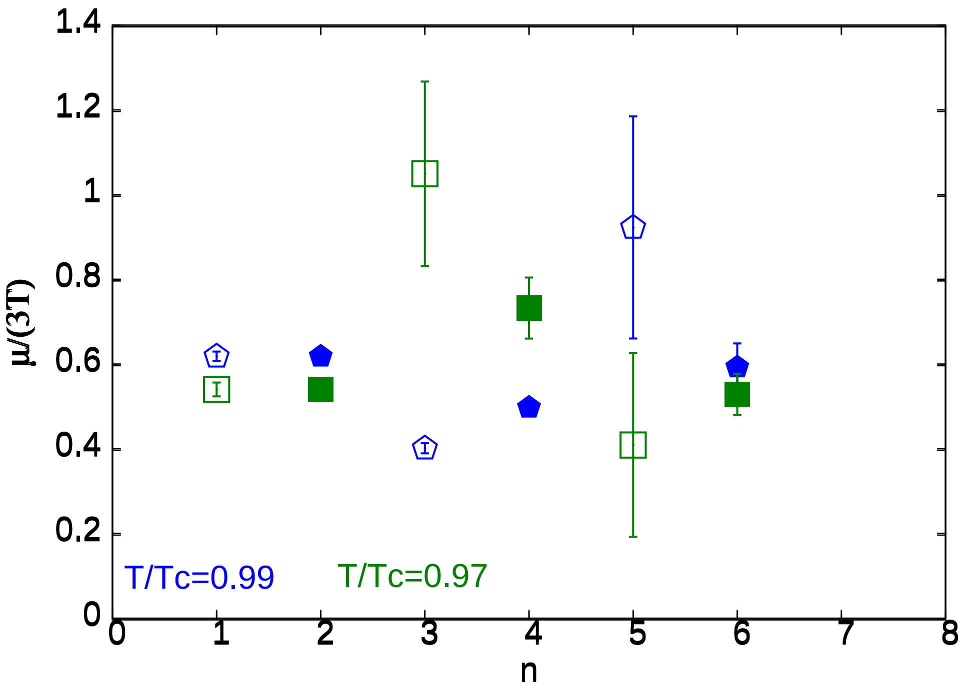
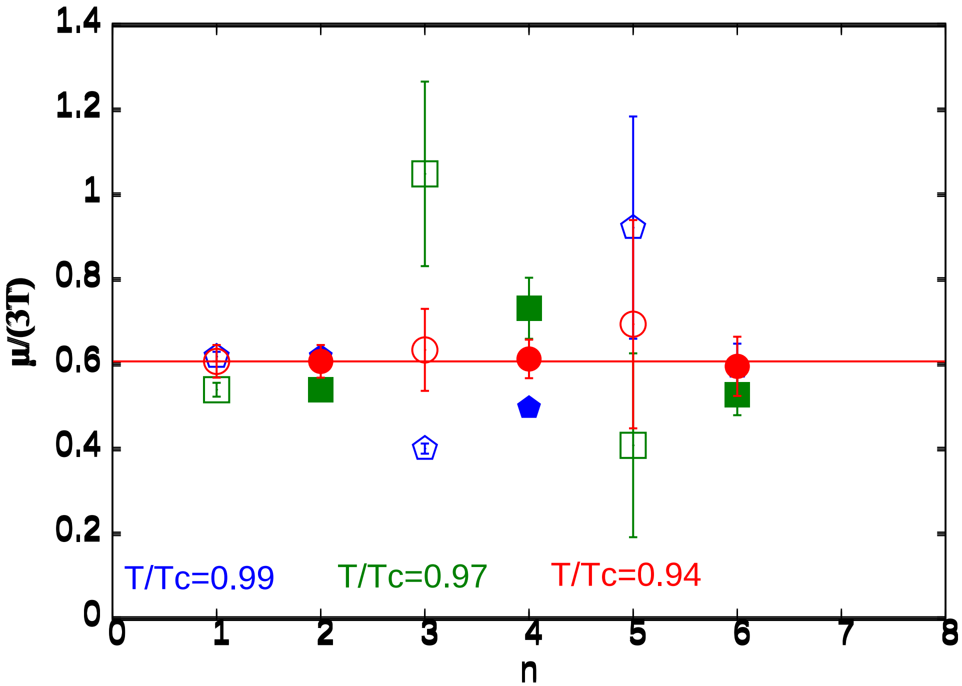
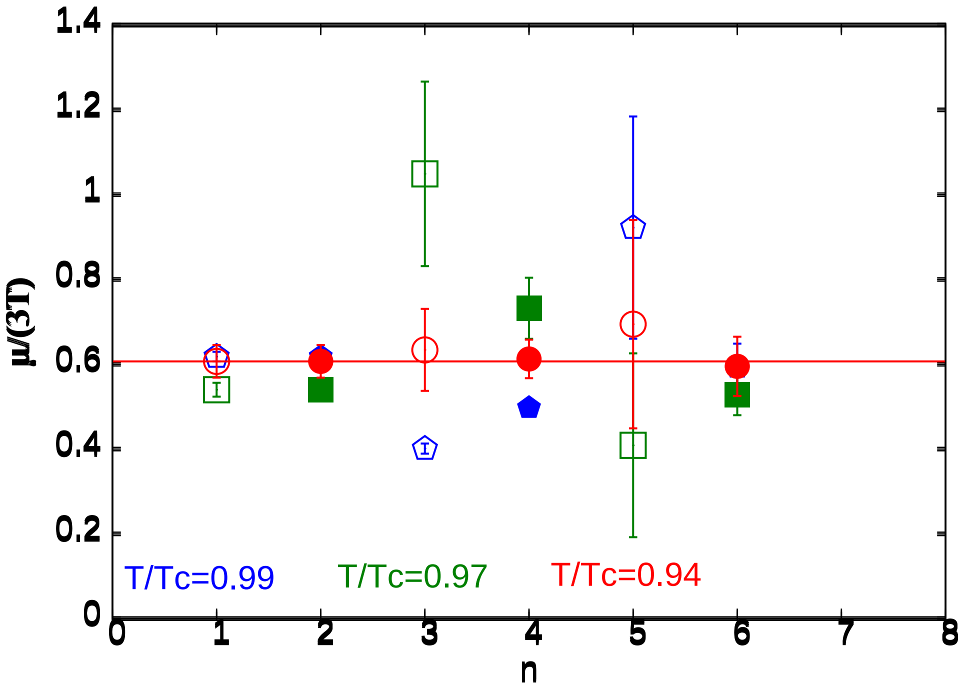
- All coefficients of the series must be POSITIVE for the critical point to be at real μ, and thus physical.

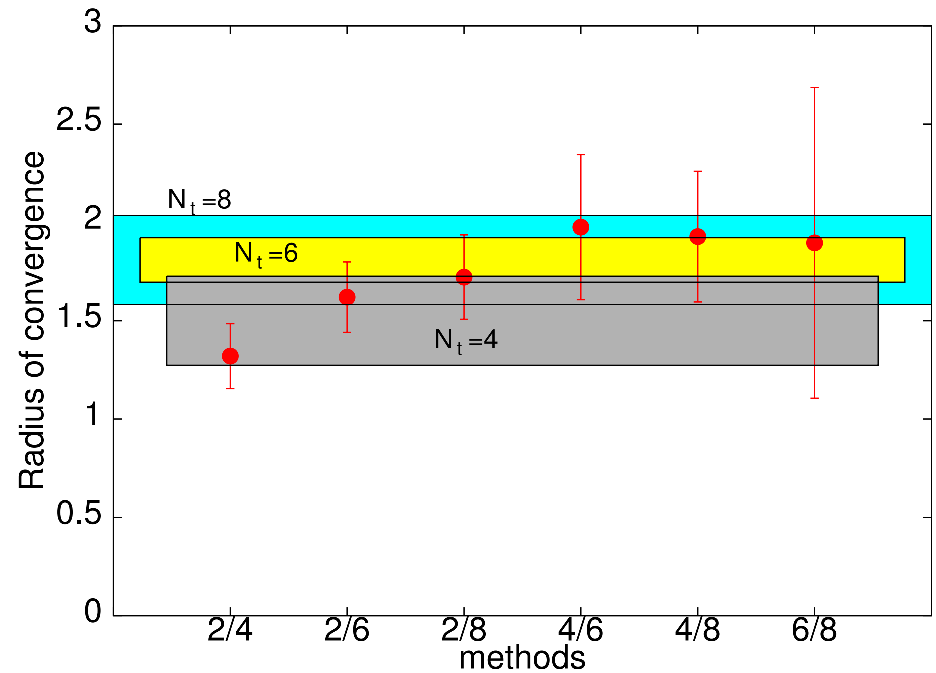

1.4
1.2
1
0.8
0.6
0.4
0.2
0
μ/(3T)
T/Tc=0.99
0
1
2
3
4
5
6
7
8
n


1.4
1.2
1
0.8
0.6
0.4
0.2
0
μ/(3T)
0
1
2
3
4
5
6
7
8
n
T/Tc=0.99
T/Tc=0.97



μ/(3T)
n
T/Tc=0.99
T/Tc=0.97
T/Tc=0.94

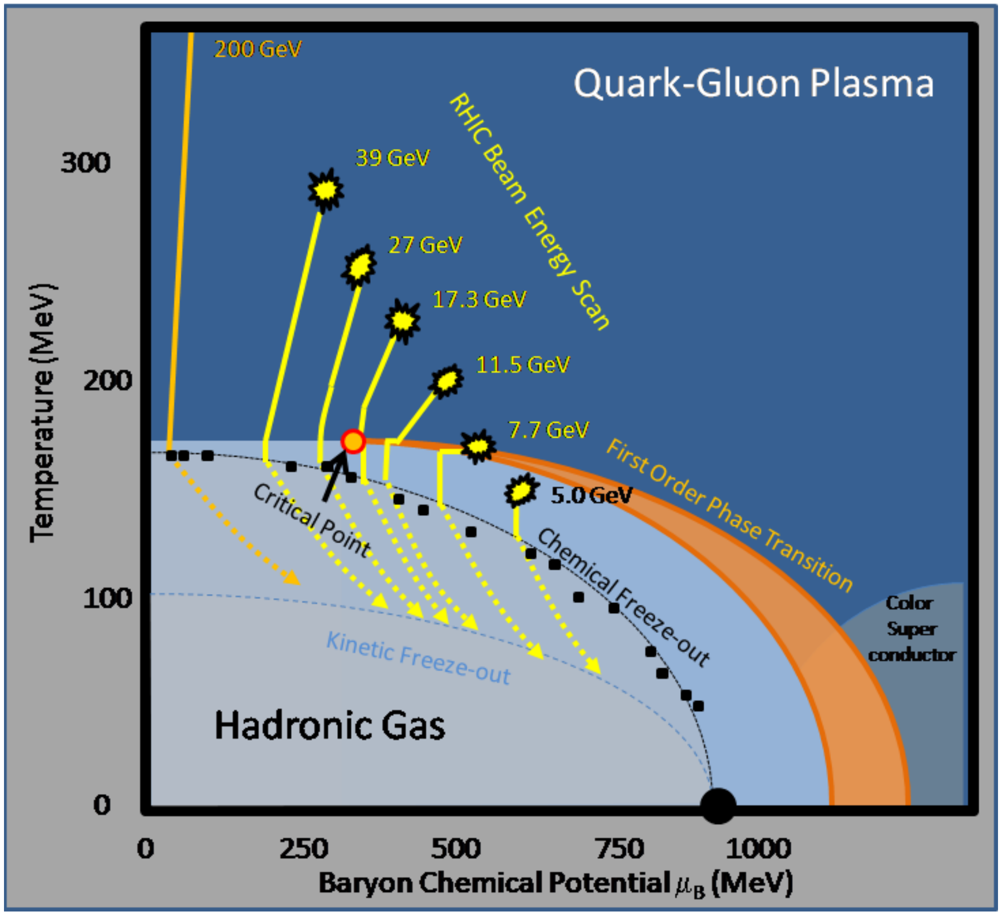




- $\frac{T^E}{T_c} = 0.94 \pm 0.01$, and $\frac{\mu_B^E}{T^E} = 1.8 \pm 0.2 (1.8 \pm 0.1)$ for the $N_t = 8(6)$ lattice (Datta-RVG-Gupta, '08, '13, '17). Recent high statistics coarser ($N_t = 4$) lattice result was $\mu_B^E/T^E = 1.5 \pm 0.2$ (Gupta-Karthik-Majumdar PRD '14).

- Critical point at $\mu_B/T \sim 1 - 2$, based on results from TIFR('05, '08, '13, '17) & Budapest-Wuppertal ('04) groups.

Searching Experimentally: Heavy Ion Collisions



STAR Collaboration, Aggarwal et al.
arXiv : 1007.2637

- Exploit the facts i) susceptibilities diverge near the critical point and ii) decreasing $\sqrt{s}$ increases μ_B (Rajagopal, Shuryak & Stephanov PRD 1999).

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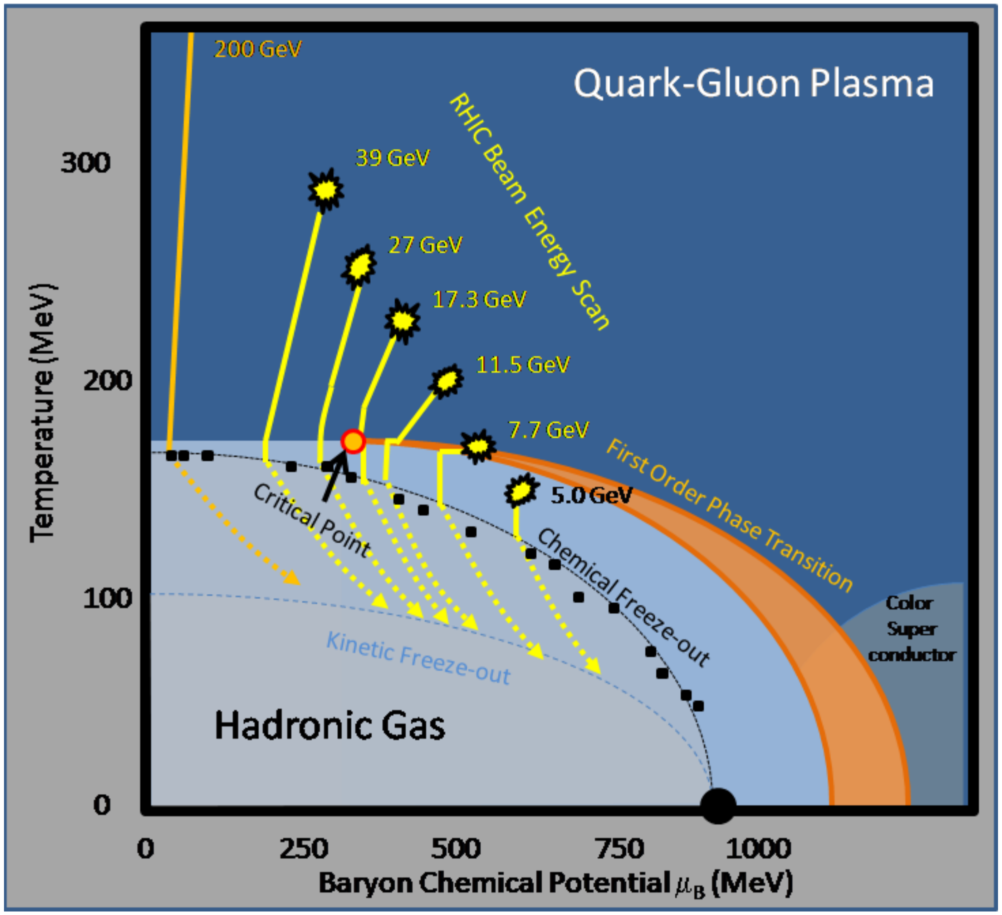


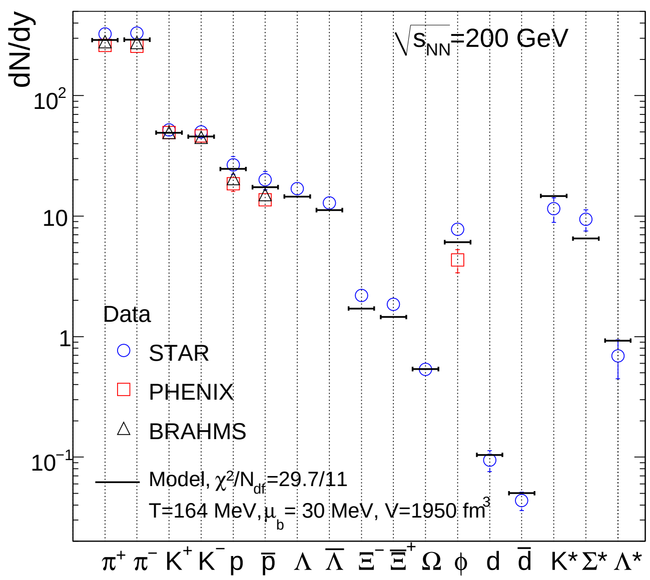
STAR Collaboration, Aggarwal et al. arXiv : 1007.2637

- Exploit the facts i) susceptibilities diverge near the critical point and ii) decreasing $\sqrt{s}$ increases μ_B (Rajagopal, Shuryak & Stephanov PRD 1999).

- Look for nonmonotonic dependence of the event-by-event fluctuations with colliding energy. No indications in early such results for π, K-mesons. E.g., CERN NA49 results (C. Roland NA49, J.Phys. G30 (2004) S1381-S1384).

Lattice predictions along the freezeout curve

- Hadron yields well described using Statistical Hadronization Models, leading to the freezeout curve in the T-μ_B plane. (Andronic, Braun-Munzinger & Stachel, PLB 2009 ; Oeschler, Cleymans, Redlich & Wheaton, 2009)



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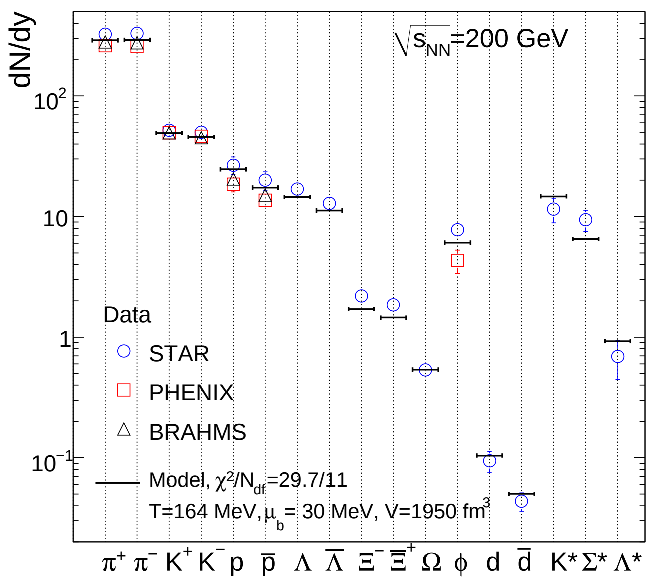
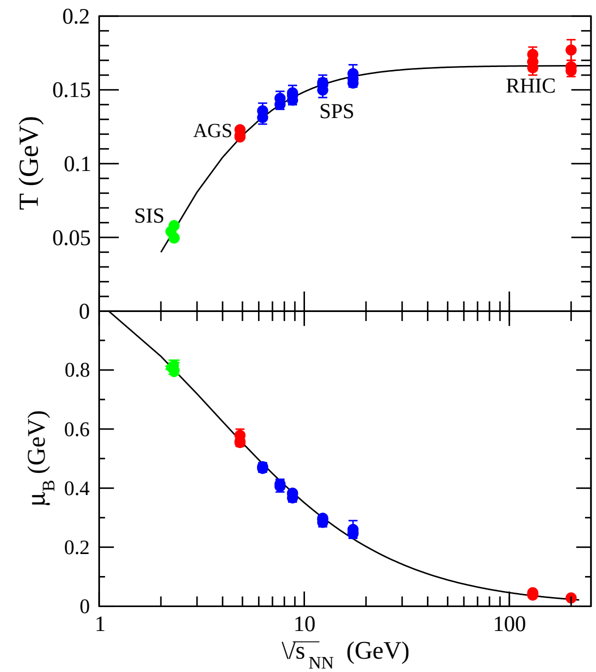
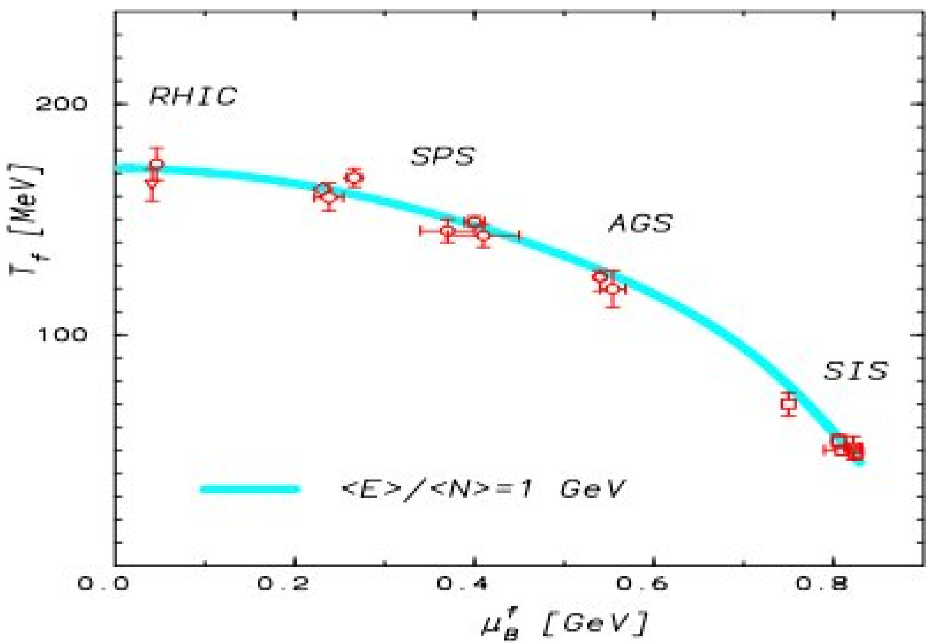
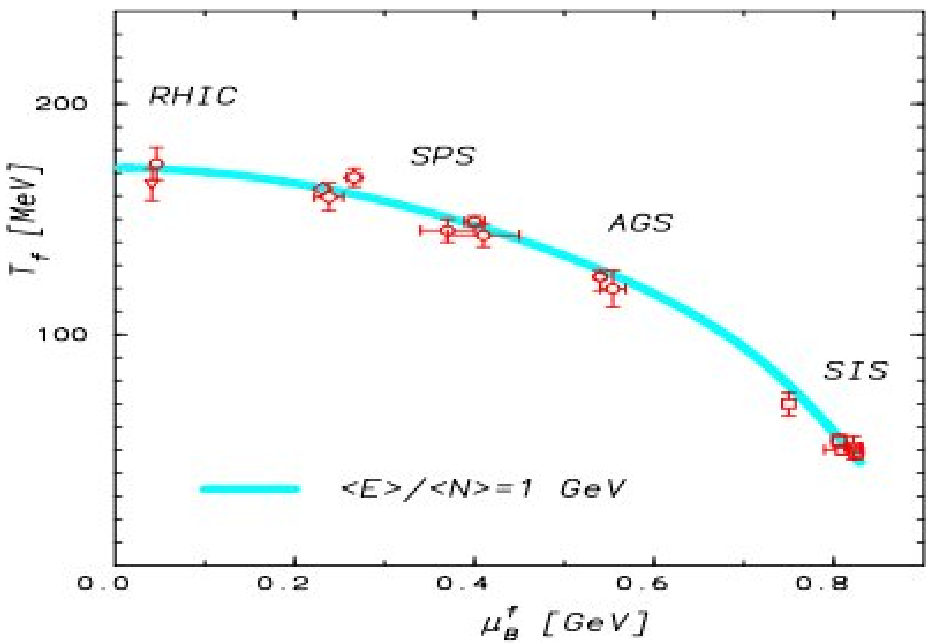




- Plotting these results in the T-μ_B plane, one has the freezeout curve, which was shown to correspond the $\langle E \rangle / \langle N \rangle \simeq 1$. (Cleymans and Redlich, PRL 1998)



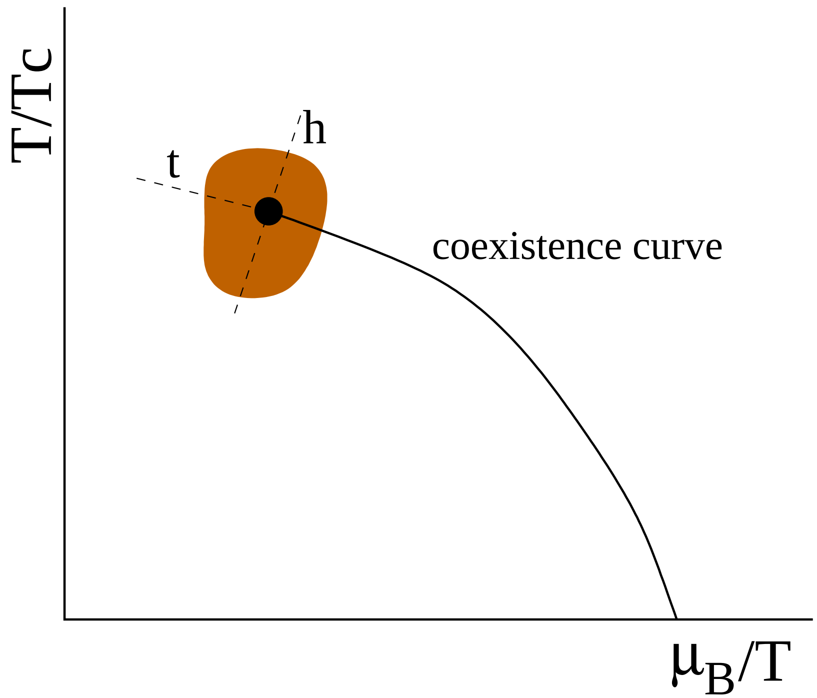
(From Braun-Munzinger, Redlich and Stachel nucl-th/0304013)

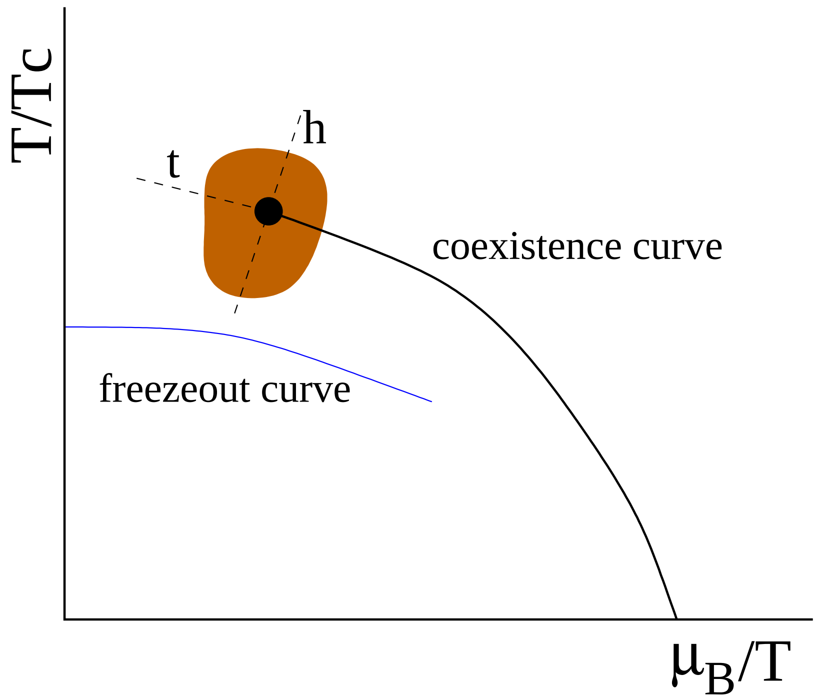


(From Braun-Munzinger, Redlich and Stachel nucl-th/0304013)

- Note : Freeze-out curve is based solely on data on hadron yields, & gives the (T, μ) accessible in heavy-ion experiments.

- Our Key Proposal : Use the freezeout curve from hadron abundances to *predict baryon* fluctuations using lattice QCD along it. (Gavai-Gupta, TIFR/TH/10-01, arXiv 1001.3796)


T/Tc
h
t
coexistence curve
μ_B/T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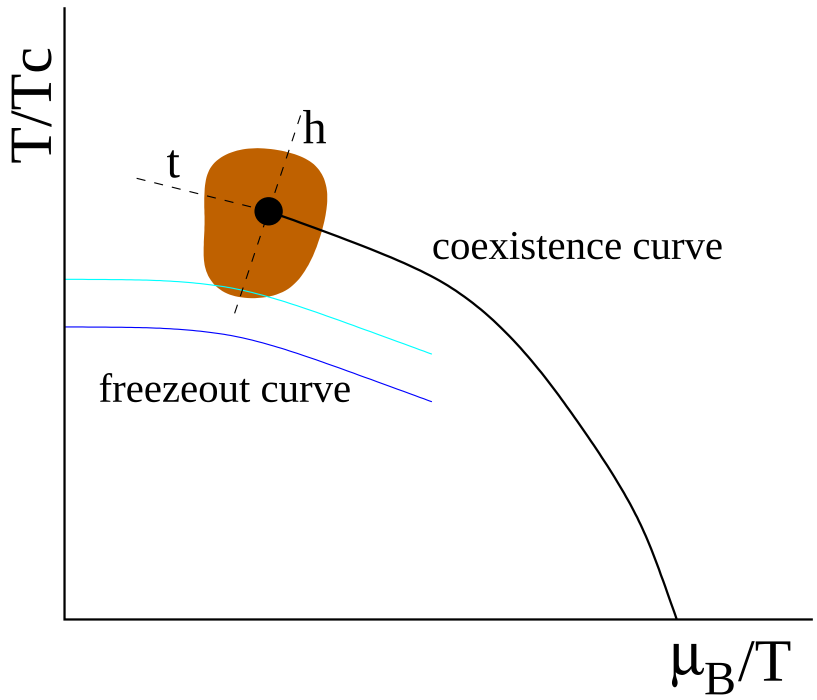
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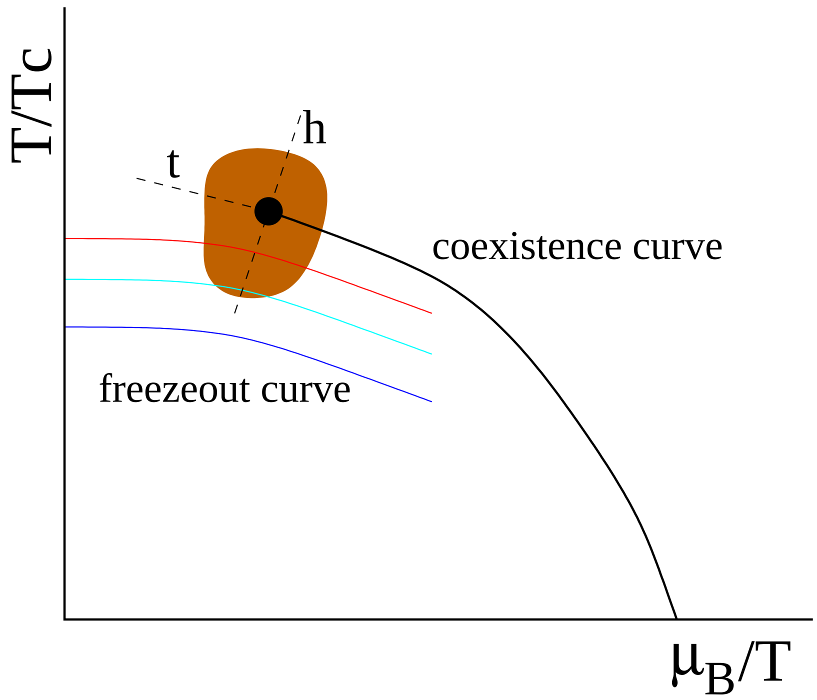
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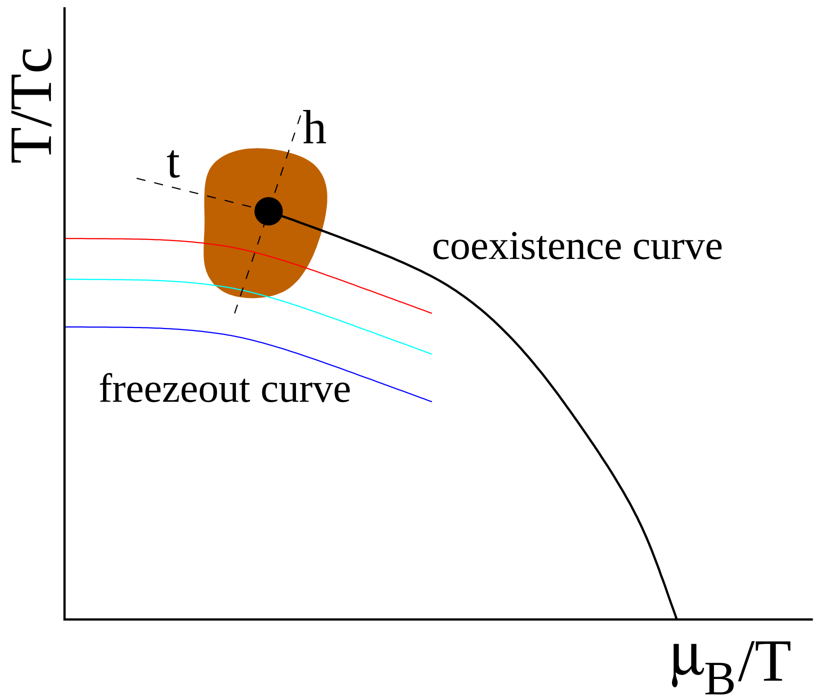
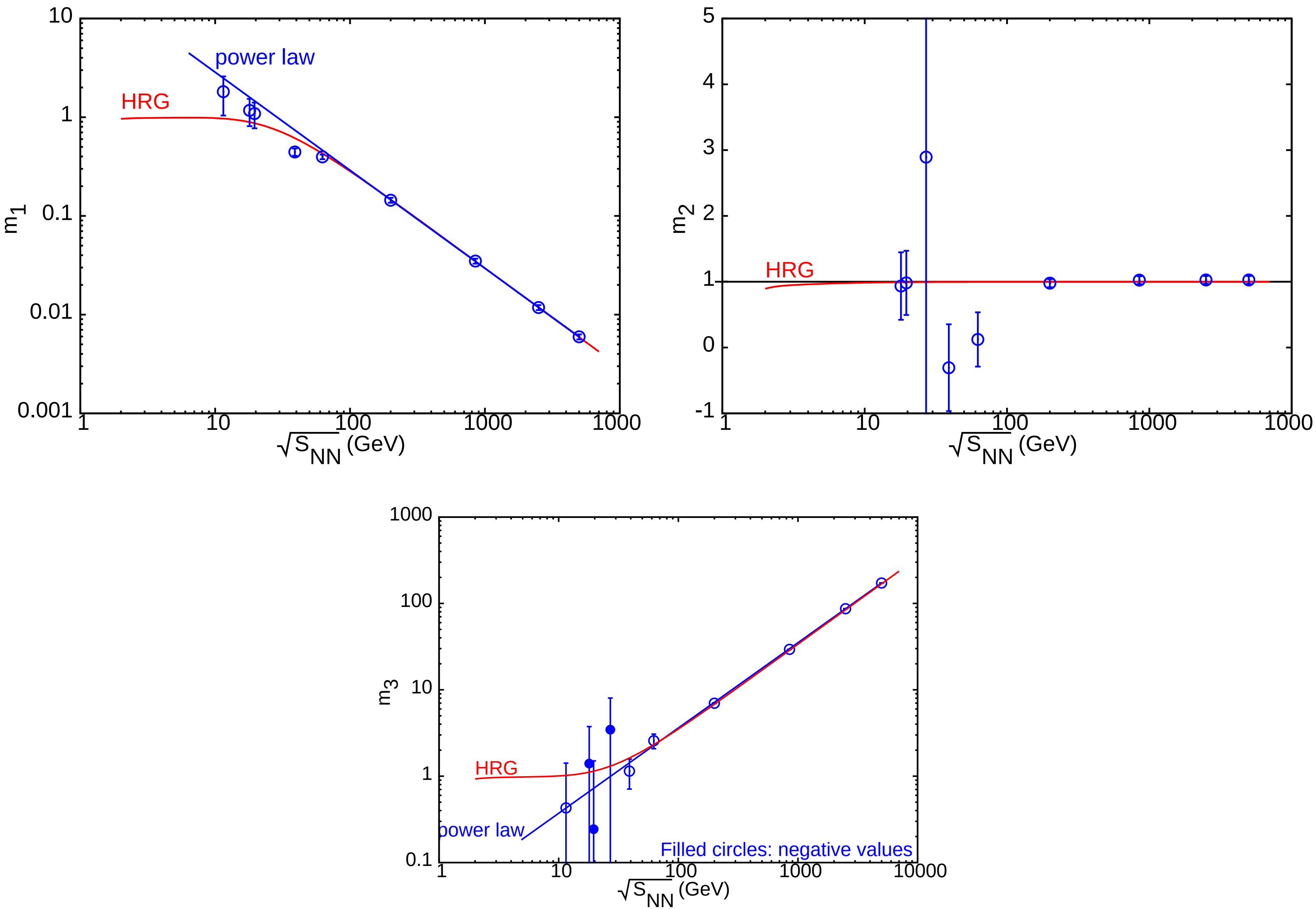
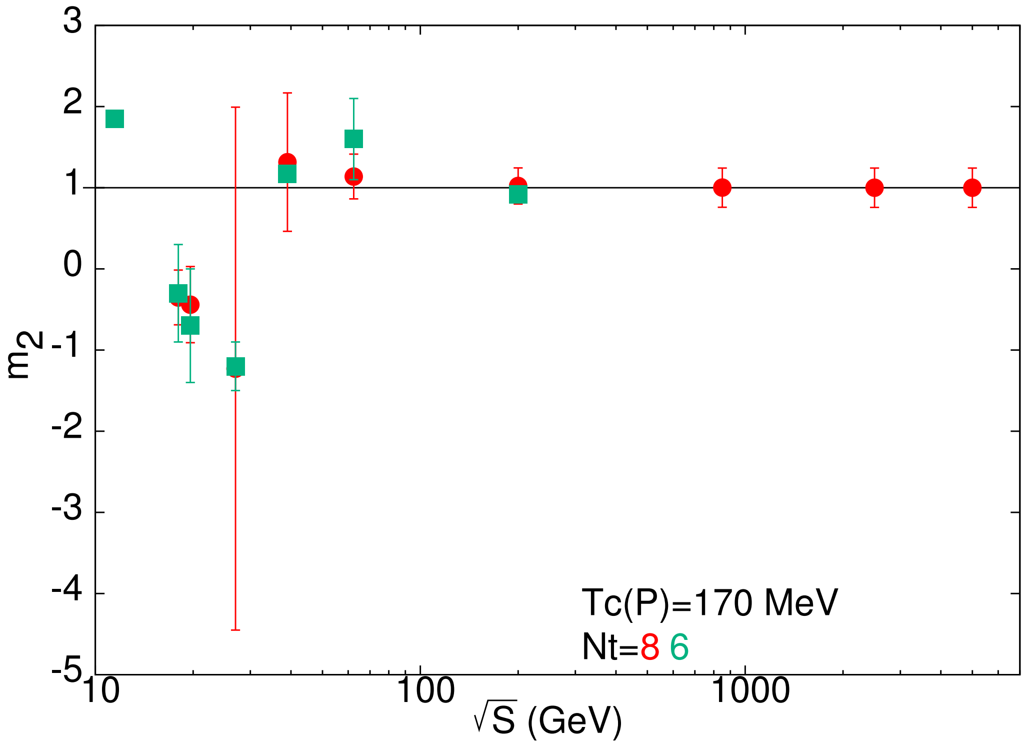


- Use the freezeout curve to relate $(T, \mu_B)$to $\sqrt{s}$ and employ lattice QCD predictions along it. (Gavai-Gupta, TIFR/TH/10-01, arXiv 1001.3796)

- Define $m_1 = \frac{T\chi^{(3)}(T,\mu_B)}{\chi^{(2)}(T,\mu_B)}$, $m_3 = \frac{T\chi^{(4)}(T,\mu_B)}{\chi^{(3)}(T,\mu_B)}$, and $m_2 = m_1 m_3$ and use the Padè method to construct them.



♠ Used $T_c(\mu = 0) = 170$ MeV (Gavai & Gupta, arXiv: 1001.3796).

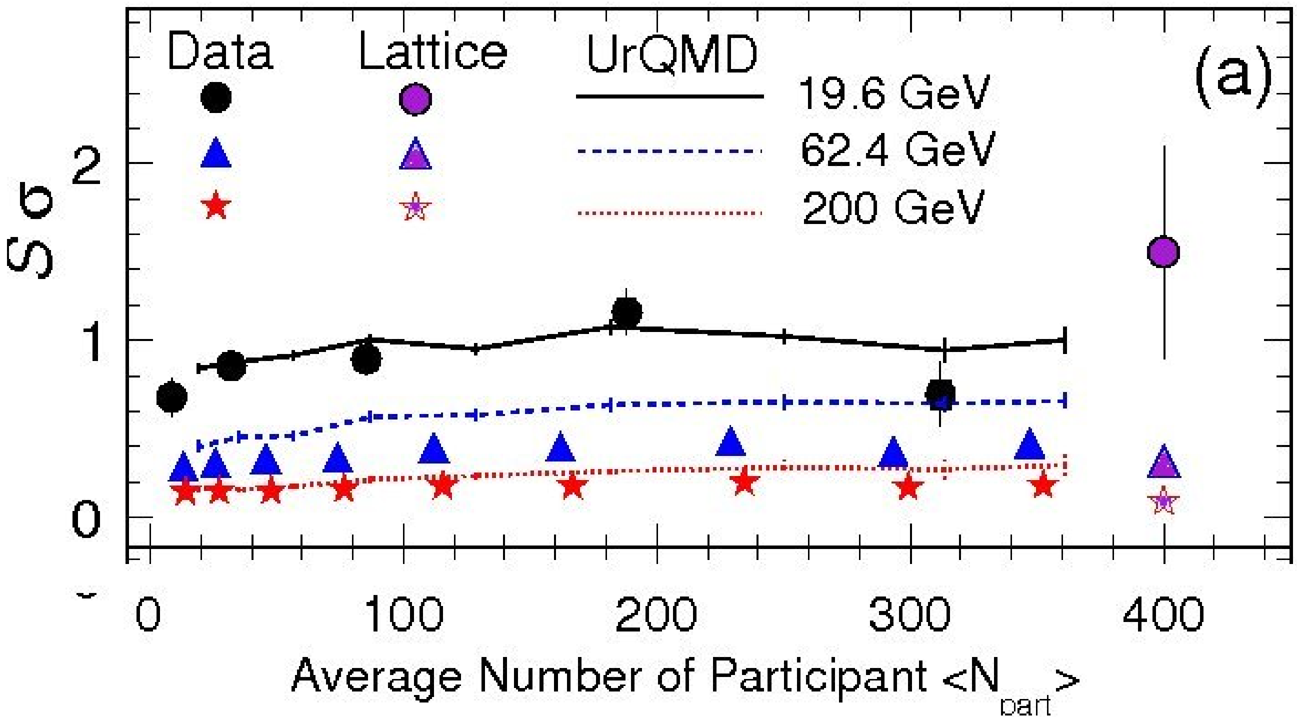


Gavai-Gupta, '10 & Datta-Gavai-Gupta, Lattice 2013

- Smooth & monotonic behaviour for large $\sqrt{s}$: $m_1 \downarrow$, $m_3 \uparrow$, and $m_2 \sim$ constant.

- Note that even in this smooth region, an experimental comparison is exciting : Direct Non-Perturbative test of QCD in hot and dense environment.

$$S\sigma \equiv m_1$$



Aggarwal et al., STAR Collaboration, arXiv : 1004.4959

- Reasonable agreement with our lattice results. Where is the critical point ?

- Our estimated critical point suggests non-monotonic behaviour in all m_i, which should be accessible to the low energy scan of RHIC BNL !

- Caution : Experiments measure *only* proton number fluctuations.

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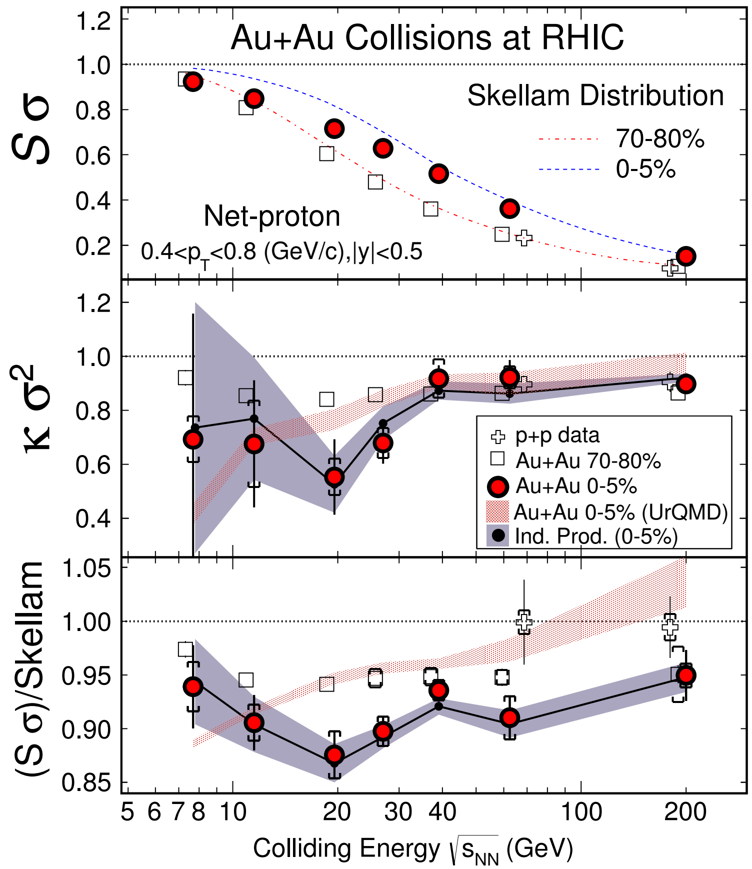
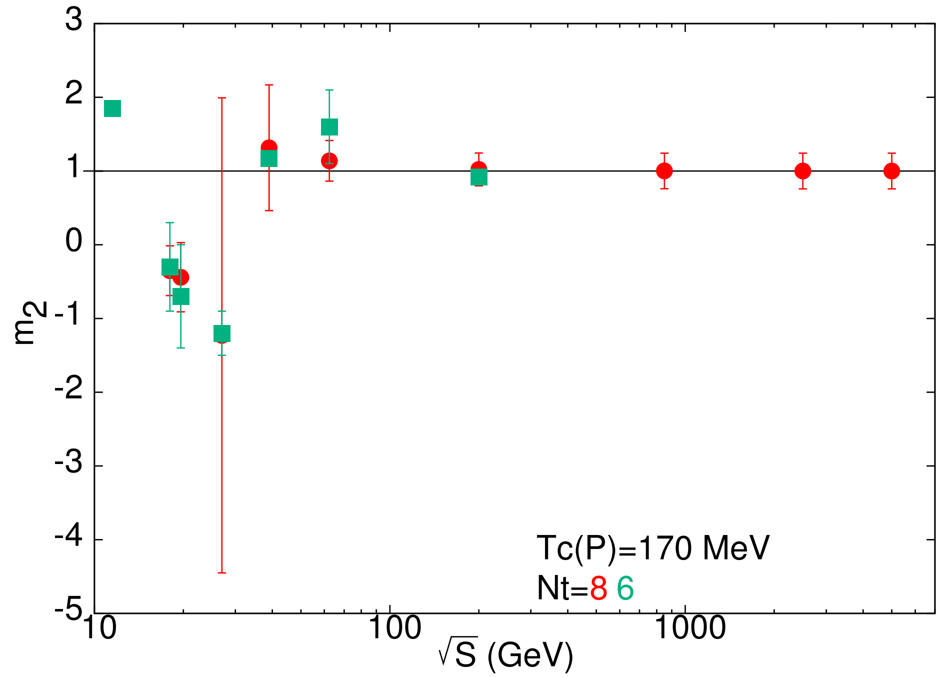
- Neat idea : Since diverging baryonic susceptibility at the critical point is linked to σ mode, which cannot mix with any isospin modes, expect χ_I to be regular.

- Leads to a ratio $\chi_Q{:}\chi_I{:}\chi_B =$ 1:0:4

- Assuming protons, neutrons, pions to dominate, both χ_Q and χ_B can be shown to be fully reflected in proton number fluctuations.

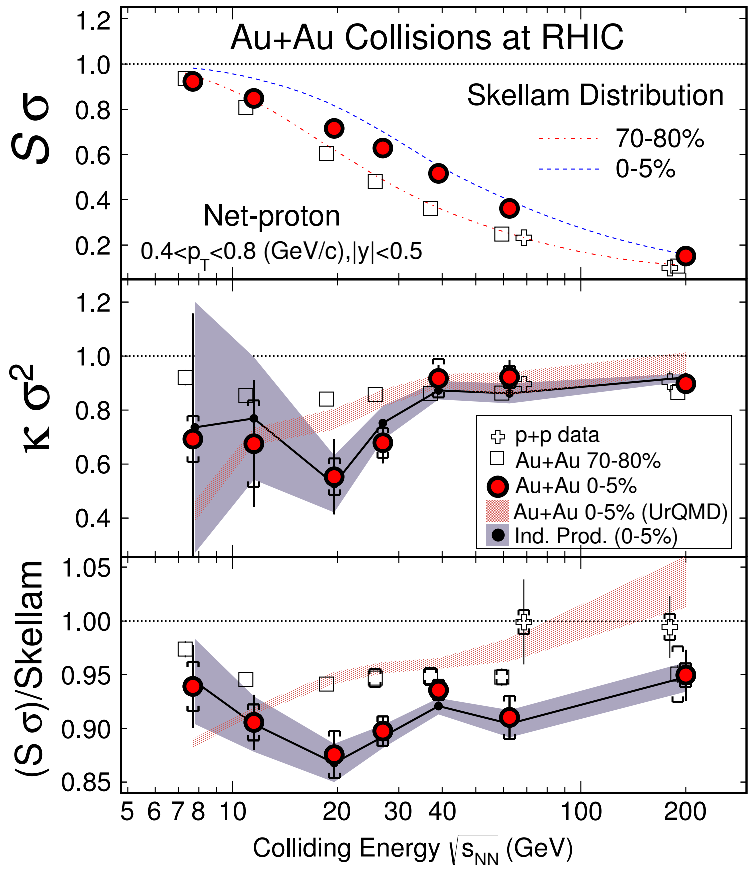


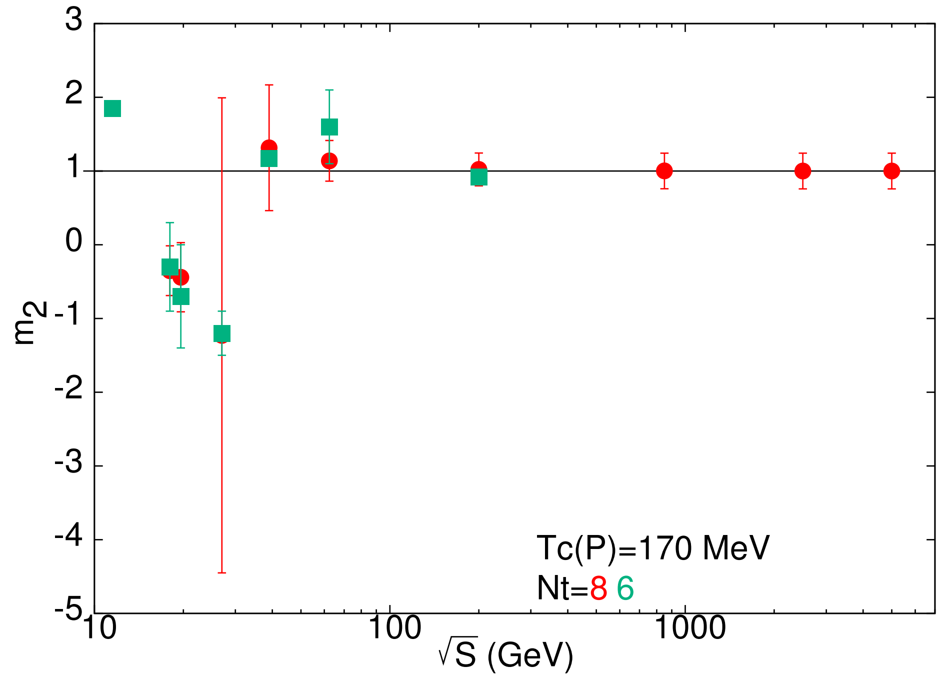


L. Adamczyk *et al.*
STAR Collaboration PRL (2014)

Gavai-Gupta, '10
Datta-Gavai-Gupta, Lattice 2013

$\mathcal{S}\sigma \equiv m_1$ and $\kappa\sigma^2 \equiv m_2$.





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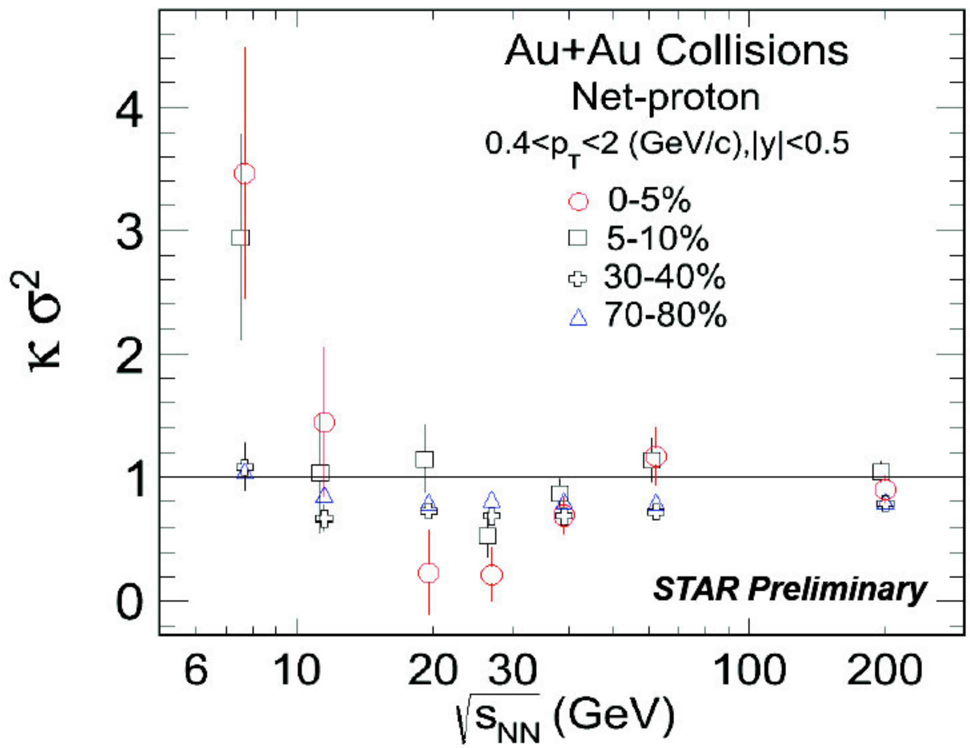
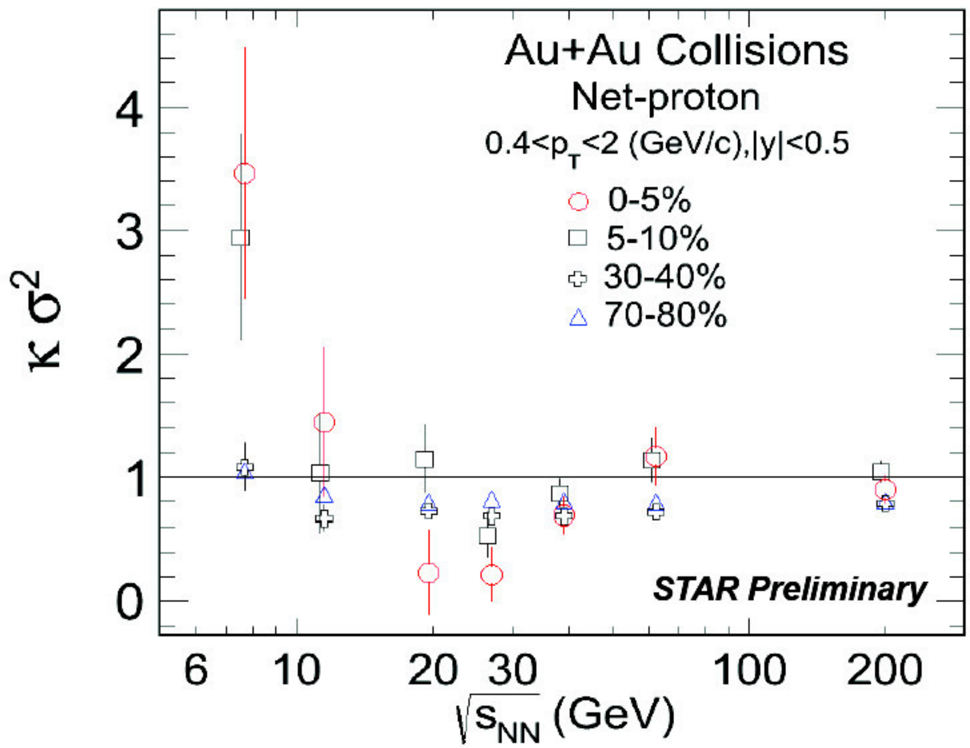
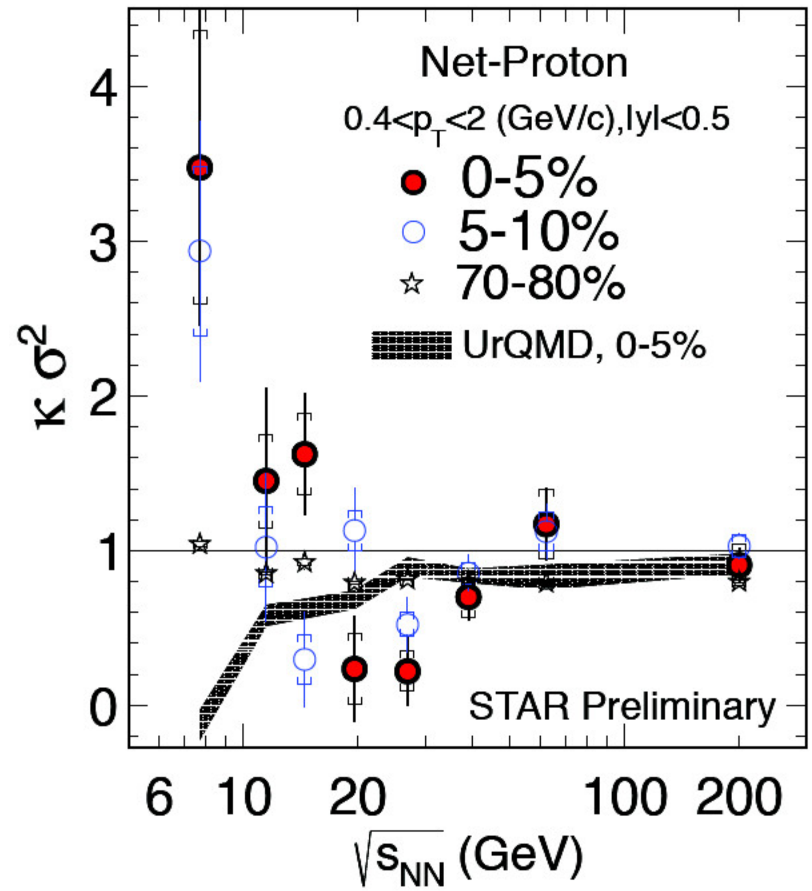
"These observables show a centrality and energy dependence, which are neither reproduced by non-CP transport model calculations, nor by a hadron resonance gas model. " — STAR Collaboration PRL (2014).



Increasing Δp_T deepens the structure !

X. Luo, CPOD 2014, Bielefeld, STAR Collab.





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Interesting Oscillations !!

X. Luo, Quark Matter 2015, Kobe, Japan

Summary

- Phase diagram in $T-\mu$ has begun to emerge: Different methods, $\rightsquigarrow$ similar qualitative picture. Critical Point at $\mu_B/T \sim 1-2$.

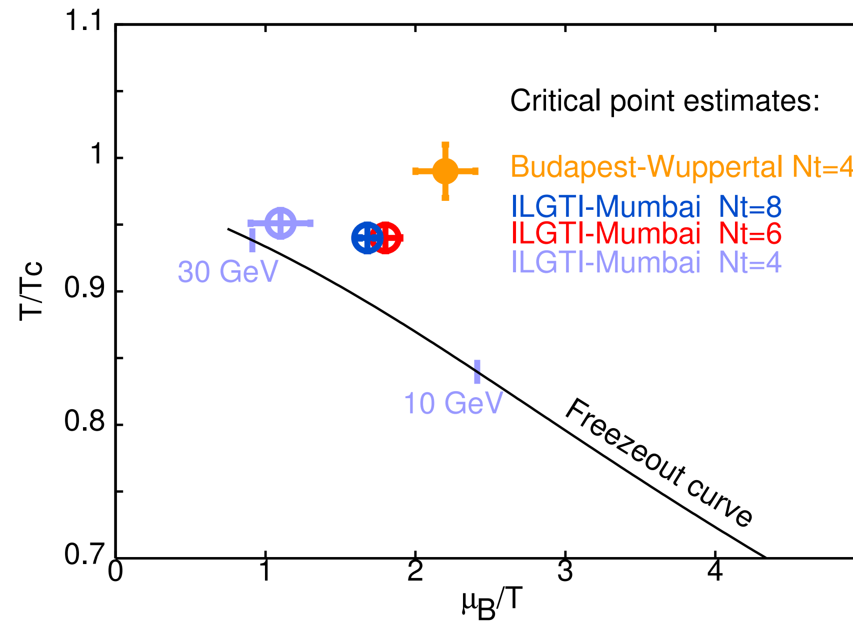
- Our results for $N_t = 8$ first to begin the inching towards continuum limit.

Summary

- Phase diagram in $T - \mu$ has begun to emerge: Different methods, $\rightsquigarrow$ similar qualitative picture. Critical Point at $\mu_B/T \sim 1 - 2$.

- Our results for $N_t = 8$ first to begin the inching towards continuum limit.

- We showed that Critical Point leads to structures in m_i on the Freeze-Out Curve. Possible Signature ?



♡ STAR, BNL results appear to agree with our Lattice QCD predictions. ☺