



# Light Cone 2017 (LC2017)

Frontiers in Light Front Hadron Physics : Theory and Experiment

18<sup>th</sup> - 22<sup>nd</sup> September 2017, University of Mumbai



**Transverse degrees of freedom in QCD, momenta, spins and more**

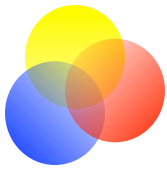
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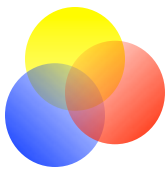
## Abstract

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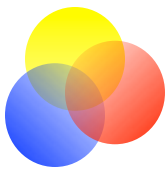
Transverse degrees of freedom in QCD: momenta, spins and more.

Light-front quantized quark and gluon states (partons) play a dominant role in high energy scattering processes. The initial state in these processes is a mixed ensemble of partons, while any produced pure partonic state appears as a mixed ensemble in the 3D world of the detector. The transition from collinear hard physics to the 3D structure including partonic transverse momenta is related to confinement and might hint at a more fundamental link between color and spatial degrees of freedom. Wilson loops, including Wilson lines along light-like directions such as used in the studies of transverse momentum dependent distribution functions (TMDs) might play a role here, establishing a direct link between transverse spatial degrees of freedom and gluonic degrees of freedom. They lead to many peculiarities among them single spin asymmetries in the physics of TMDs but they also unify and simplify our picture for gluons in the low- $x$  domain.



## Introduction

- Color & QCD
  - Distinct part of Standard Model, decoupling strong interactions
  - Color invisible: local gauge invariance! No free quarks or gluons!
  - Color visible: valence quarks,  $N$  vs  $1/N$ ,  $f \times D$  (distribution  $\times$  fragmentation), color flow (future and past pointing gauge links), ...
- Pragmatic approach: Front form quantization with good fields, dominating in OPE
$$\frac{1}{2}\gamma^-\gamma^+\psi \quad \text{and} \quad g_T^{\alpha\mu} A_\mu^a$$
- (1) A different view (entanglement & less dimensions)  
PJM – 1601.00300
- (2) Impact for strongly interacting matter
  - Example: Wilson loops and gluon TMDs



## A different view, why?

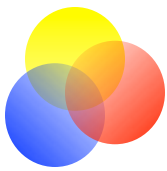
### ■ Entanglement

- Entangled pure multipartite states  $\rightarrow$  ensembles in reduced Hilbert space  
[Kharzeev & Levin \(1702.03489\)](#): how to get from a proton to parton ensemble
- And the other way: pure partonic state  $\rightarrow$  ensemble of hadrons (fragmentation)
- Maximal entanglement (MaxEnt)  
[Cervera-Liarta, Latorre, Rojo & Rottoli \(1703.02989\)](#): maximally entangled chiral left/right two-particle states are consistent with QED ( $g_A=0$ ) & electroweak ( $g_V=0$ ), at least if  $\sin \Theta_W = 1/2$
- Classical/quantum physics (['t Hooft – 1405.1548](#))

### ■ Less dimensions ( $1+3 \rightarrow 1+1$ ) advantageous

- Convergence in field theory:  $d[\phi] = (d-2)/2 \rightarrow 0$ ,  $d[\psi] = (d-1)/2 \rightarrow 1/2$ .  
[Stojkovic – 1406.2696](#): naturalness, ...
- Chirality (R/L) corresponding to right- and left-movers,  $P^+$ ,  $P^-$  eigenstates



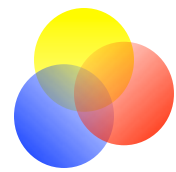


## Multipartite states (QIT)

- Multipartites live in a Hilbert space:  $\mathcal{H}^A \otimes \mathcal{H}^B \otimes \dots$  (possibly identical spaces!)
- For entangled bipartite states there is just **one class** of entangled (Bell) states (using qubits: R/L)  
 $|\text{Bell}\rangle = \frac{1}{\sqrt{2}}(|RL\rangle + |LR\rangle) \quad \text{or} \quad \frac{1}{\sqrt{2}}(|RR\rangle + |LL\rangle)$
- For entangled tripartite states there are **two classes** (Dur, Vidal, Cirac 2000)
  - aligned:  $|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$  (fragile)
  - mingled:  $|\text{W}\rangle = \frac{1}{\sqrt{3}}(|LRR\rangle + |RLR\rangle + |RRL\rangle)$  (robust)
- Multipartites and R/L basis states relevant for our purposes:
  - 1D field theory:  $\mathcal{H} = \mathcal{H}^{\otimes x}$  with right- and left-movers (chirality states)
  - Tripartite states:  $\mathcal{H}^A \otimes \mathcal{H}^B \otimes \mathcal{H}^C \left\{ \begin{array}{l} \text{space (3D): leptons, electroweak (GHZ-class)} \\ \text{color: quarks in 1D, strong (W-class)} \end{array} \right.$

Class: Equivalence under local unitary transformations:

$$U \otimes U \otimes \dots$$



# Multipartites, space-time and internal degrees of freedom

## ■ Harmonic oscillator levels ( $SO(3) \leftrightarrow$ internal symmetry & more symmetry)

level	degeneracy	$(n_x, n_y, n_z)$	$SO(3) (\ell)$	$SU(3) (\underline{n})$
0	1	(0,0,0)	0	<u>1</u>
1	3	(1,0,0), ...	1	<u>3</u>
2	6	(2,0,0), (1,1,0), ...	$0 \oplus 2$	<u>6</u>
3	10	(3,0,0), (2,1,0), (1,1,1), ...	$1 \oplus 3$	<u>10</u>
4	15	...	$0 \oplus 2 \oplus 4$	<u>15</u> <sub>s</sub>

## ■ Quark model: $SU(6) \times O(3)$

N	configuration	$SU(6) \times O(3)$ multiplets
0	$(0s)^3$	[56, 0 <sup>+</sup> ]
1	$(0s)^2(1p)$	(56, 1 <sup>-</sup> ) [70, 1 <sup>-</sup> ]
2	$(0s)^2(2s)$	(56, 0 <sup>+</sup> ) [70, 0 <sup>+</sup> ]
	$(0s)^2(2d)$	(56, 2 <sup>+</sup> ) [70, 2 <sup>+</sup> ]
	$(0s)(1p)^2$	[56, 0 <sup>+</sup> ] [56, 2 <sup>+</sup> ] (70, 0 <sup>+</sup> ) (70, 1 <sup>+</sup> ) (70, 2 <sup>+</sup> ) [20, 1 <sup>+</sup> ]

## ■ Problematic at a fundamental level



# All Possible Symmetries of the $S$ Matrix\*

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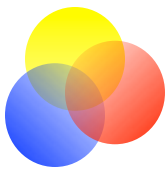
(Received 16 March 1967)

We prove a new theorem on the impossibility of combining space-time and internal symmetries in any but a trivial way. The theorem is an improvement on known results in that it is applicable to infinite-parameter groups, instead of just to Lie groups. This improvement is gained by using information about the  $S$  matrix; previous investigations used only information about the single-particle spectrum. We define a symmetry group of the  $S$  matrix as a group of unitary operators which turn one-particle states into one-particle states, transform many-particle states as if they were tensor products, and commute with the  $S$  matrix. Let  $G$  be a connected symmetry group of the  $S$  matrix, and let the following five conditions hold: (1)  $G$  contains a subgroup locally isomorphic to the Poincaré group. (2) For any  $M > 0$ , there are only a finite number of one-particle states with mass less than  $M$ . (3) Elastic scattering amplitudes are analytic functions of  $s$  and  $t$ , in some neighborhood of the physical region. (4) The  $S$  matrix is nontrivial in the sense that any two one-particle momentum eigenstates scatter (into something), except perhaps at isolated values of  $s$ . (5) The generators of  $G$ , written as integral operators in momentum space, have distributions for their kernels. Then, we show that  $G$  is necessarily locally isomorphic to the direct product of an internal symmetry group and the Poincaré group.

## I. INTRODUCTION

UNTIL a few years ago, most physicists believed that the exact or approximate symmetry groups of the world were (locally) isomorphic to direct products of the Poincaré group and compact Lie groups. This world-view changed drastically with the publication of the first papers on  $SU(6)$ <sup>1</sup>; these raised the dazzling possibility of a relativistic symmetry group which was not simply such a direct product. Unfortunately, all attempts to find such a group came to disastrous ends, and the situation was finally settled by the discovery of

symmetry group of the  $S$  matrix, which contains the Poincaré group and which puts a finite number of particles in a supermultiplet. Let the  $S$  matrix be nontrivial and let elastic scattering amplitudes be analytic functions of  $s$  and  $t$  in some neighborhood of the physical region. Finally, let the generators of  $G$  be representable as integral operators in momentum space, with kernels that are distributions. Then  $G$  is locally isomorphic to the direct product of the Poincaré group and an internal symmetry group. (This is a loose statement of the theorem; a more precise one follows below.)



# Basic symmetries including SUSY

## ■ Hilbert space

$$\{(a^\dagger)^n |0\rangle, b^\dagger |0\rangle\}$$

$$[a, a^\dagger] = 1, \quad \{b, b^\dagger\} = 1$$

## ■ Supercharges

$$Q_{ik}^\dagger = b_i a_k^\dagger \text{ and } Q_{ik} = b_i^\dagger a_k$$

$$\{Q_{ik}^\dagger, Q_{jl}\} = \frac{1}{2} \delta_{ij} \{a_l^\dagger, a_k\} + \frac{1}{2} \delta_{kl} [b_i^\dagger, b_j]$$

$$a_k^\dagger \xrightarrow{Q_{ik}} b_i^\dagger \quad a_k^\dagger \xleftarrow{Q_{ik}^\dagger} b_i^\dagger$$

hamiltonian/number operators (i=j, k=l)  
& **unitary rotations**

## ■ For boson and fermion fields

$$\varphi = \frac{1}{\sqrt{2\omega}} (a + a^\dagger) \quad \text{and} \quad \xi = \frac{1}{\sqrt{2}} (b + b^\dagger)$$

$$Q = \sqrt{\omega} (a^\dagger b - b^\dagger a)$$

$$\begin{aligned} [Q, \varphi] &= \xi & \{Q, \xi\} &= \{Q, [Q, \varphi]\} = F = iD\varphi \\ [Q, F] &= [Q, \{Q, \xi\}] = iD\xi \end{aligned}$$

**Free fields**

$$F = [\varphi, H] = M\varphi$$

$$iD\varphi = M\varphi = i\dot{\varphi}$$

$$iD = i\partial + gA$$

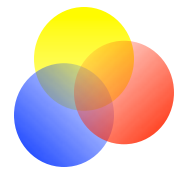


**unitary rotations**

## ■ Implement symmetries via constraints F

... and a nontrivial vacuum

$$\phi(x) = \exp\left(-i \int_0^x ds^\mu D_\mu\right) \phi$$



# Emerging symmetries of standard model

## Fields

- Real/Majorana:  $\phi$   $\xi$  and  $\langle \phi \rangle = 1$
- $\phi_{R/L}$   $\xi_{R/L}$  and  $\langle \phi_R \rangle = \langle \phi_L \rangle = 1/\sqrt{2}$   
(Wess-Zumino  $\rightarrow$  gauge theory)

- 1D:  $\phi_S$   $\phi_P \rightarrow A_3^a$   $\psi$

$$iD_\sigma \phi^i = i\partial_\sigma \phi^i + g_0 \sum_{a=1,\dots,8} A_\sigma^a (T_a)^i_j \phi^j$$

- 3D:  $\phi_S$   $A_k^a$   $\psi$

$$iD_\mu \phi^i = i\partial_\mu \phi^i + g \sum_{a=1,2,3,8} A_\mu^a (T_a)^i_j \phi^j$$

- and ....

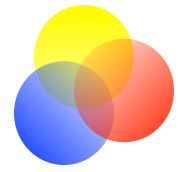
$$n_\pm^\sigma \longrightarrow n_\alpha^\mu \quad \gamma^\sigma = \begin{bmatrix} 0 & n_-^\sigma \\ n_+^\sigma & 0 \end{bmatrix} \longrightarrow \gamma^\mu = \begin{bmatrix} 0 & \bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{bmatrix}$$

in order to match space-time and field symmetries (Haag-Lupozanski-Sohnius)  
and avoid Coleman-Mandula when moving K into P(1,1) and SO(3) into P(1,3)

## Generators

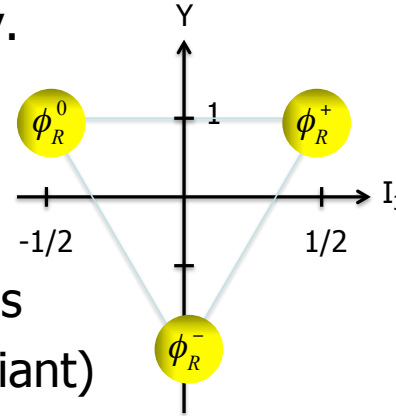
### Space-time & Internal

- H
- $P^+, P^-$  K, SU(3)
- H, P, K SU(3) = [SO(3), SU(2) x U(1)]
- H, P, K, J SU(2) x U(1)

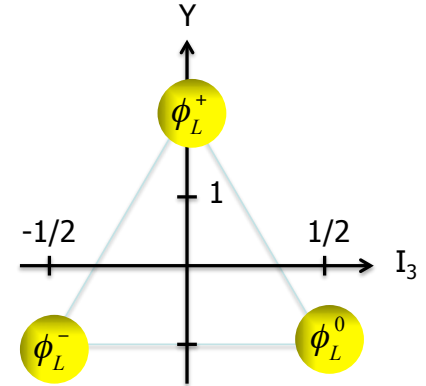


# 1D bosonic basis states and aligned tripartites

- Basis of each of the Hilbert spaces:  $P(1,1) \times SU(3)$ :  $\vec{\phi} = (\phi^1, \phi^2, \phi^3), \dots$
- Assign  $Y$ - $I_3$  using the  $SU(3)$  symmetry.



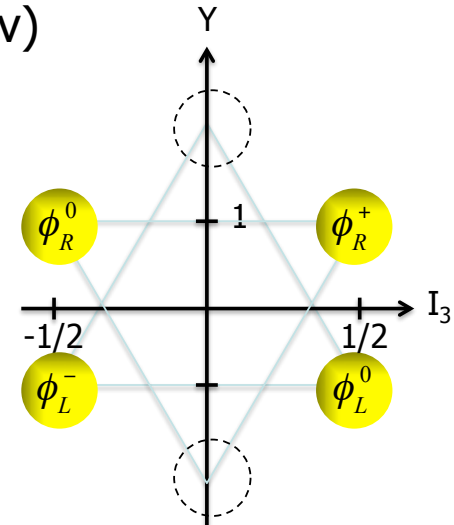
- 3D and  $P(1,3)$ : aligned tripartite states built on vacuum  $|0,0,0\rangle$  ( $SO(3)$  invariant)

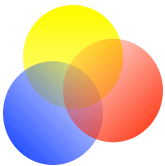


- generated by  $SU(2) \times U(1)$  from vacuum (nonzero vev)

$$\phi_R = \frac{1}{\sqrt{2}} \exp\left(+\frac{i}{2} \sum_{a=1,2,3,8} \theta^a \lambda_a\right) \begin{bmatrix} 1 + \varphi_H \\ 0 \\ 0 \end{bmatrix}$$

$$\phi_L = \frac{1}{\sqrt{2}} \exp\left(-\frac{i}{2} \sum_{a=1,2,3,8} \theta^a \lambda_a\right) \begin{bmatrix} 0 \\ 1 + \varphi_H \\ 0 \end{bmatrix}$$





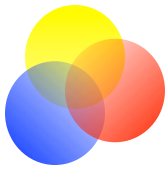
# Bosonic excitations electroweak symmetry breaking and XQCD

- 3D Electroweak symmetry breaking is  $SU(2) \times U(1) \rightarrow U(1)_{\text{QED}}$

$$\begin{aligned} iD_\mu \phi &= i\partial_\mu \phi + \frac{g}{2} \left( \sum_{i=1}^3 W_\mu^i \lambda_i + B_\mu \lambda_8 \right) \phi \\ &= i\partial_\mu \phi + \frac{g}{\sqrt{2}} (W_\mu^+ I_- + W_\mu^- I_+) \phi + (g W_\mu^0 I_3 + \frac{g}{2\sqrt{3}} B_\mu Y) \phi \end{aligned}$$

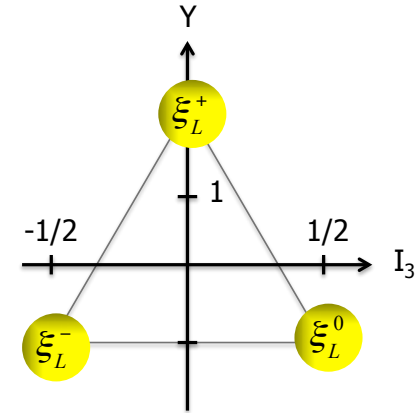
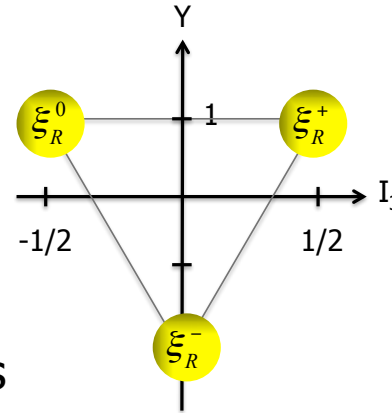
- $SU(3)$  embedding for electroweak gives embarrassingly good 'zeroth order' results:
  - Implies weak mixing angle  $\sin \theta_W = 1/2$  (Weinberg 1972)
  - $g_0 = M/2$ ,  $g^2 = 3/8$  (use  $M = M_{\text{top}}$ )
  - gives  $M_H^2 = M^2/2$ ,  $M_W^2 = 3M_Z^2/4$ ,  $M_Z = M/2$
  - $e = g/2 = (3/32)^{1/2}$  or  $1/\alpha = 134$
- 1D strong sector:  $\mathcal{L} = \frac{1}{2} \partial^\mu \varphi_S \partial_\mu \varphi_S - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}(i\not{D} - M - g_0 \varphi_S)\psi$   
 8 instantaneous gluons and a scalar field, resembling XQCD<sub>1+1</sub> (Kaplan 1306.5818)  
 and dynamics governed by  $gF_{\tau\sigma} = \delta W[C]/\delta \sigma^{\tau\sigma}$  via Wilson loop  

$$W[C] = \exp \left( -ig \oint_C ds^\mu A_\mu(s) \right)$$

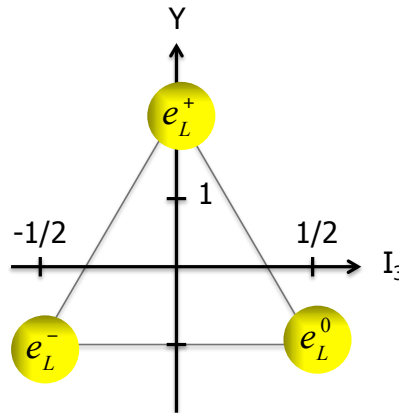
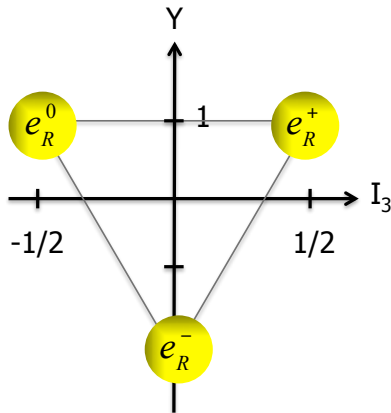


# 1D fermionic states & aligned tripartites

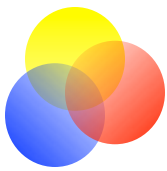
- Basis of each of the Hilbert spaces:  $P(1,1) \times SU(3)$ :  $\vec{\xi} = (\xi^1, \xi^2, \xi^3), \dots$



- 3D and  $P(1,3)$ : aligned tripartite states  
leptons: electrons & neutrinos

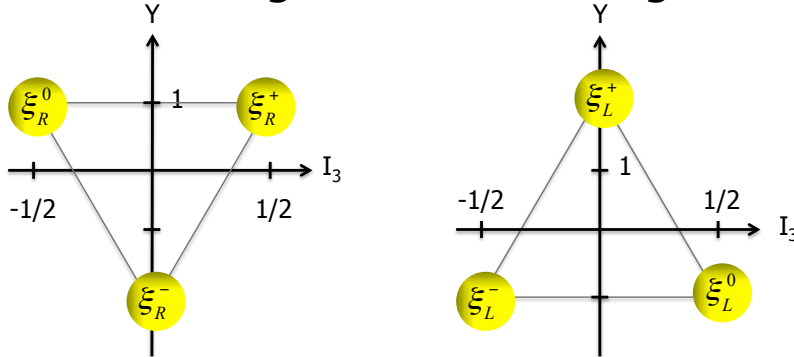






# Bosonic and fermionic excitations: lepton families

- 3D embedding fermions is straightforward:



- Families linked to three singlets of Z(3):

$$Q_{\text{ew}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad Q_{\text{mass}} = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}$$

$$Q_{\text{mass}} = U_Q^\dagger Q_{\text{ew}} U_Q \quad U_Q^\dagger = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{i}{\sqrt{2}} & 0 & \frac{-i}{\sqrt{2}} \end{bmatrix}$$

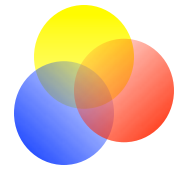
$$\langle \hat{\phi} \rangle_{\text{fam}} = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \Leftrightarrow \langle \hat{\phi} \rangle_{\text{ew}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\langle \hat{\phi} \rangle_{\text{fam}} = W \langle \hat{\phi} \rangle_{\text{ew}}$$

$$W = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ \omega^2 & 1 & \omega \\ \omega & 1 & \omega^2 \end{bmatrix}$$

$$Q_{\text{fam}} = W Q_{\text{ew}} W^\dagger = W U_Q Q_{\text{mass}} U_Q^\dagger W^\dagger \quad U_{\text{HPS}} = W U_Q = \begin{bmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{bmatrix}$$

- Lepton masses? Just note:  $M/8\pi^2 = 2 \text{ GeV}$  (factor from SO(3) group measure)



## Fermionic excitations: leptons and quarks

■ Tripartite states (R: 1 2 3 & L: 1 2 3)

■ Aligned (RRR, LLL) GHZ states

■  $SO(3) \rightarrow$  asymptotic/space

$I$ ,  $U$ , and  $V$  allowed

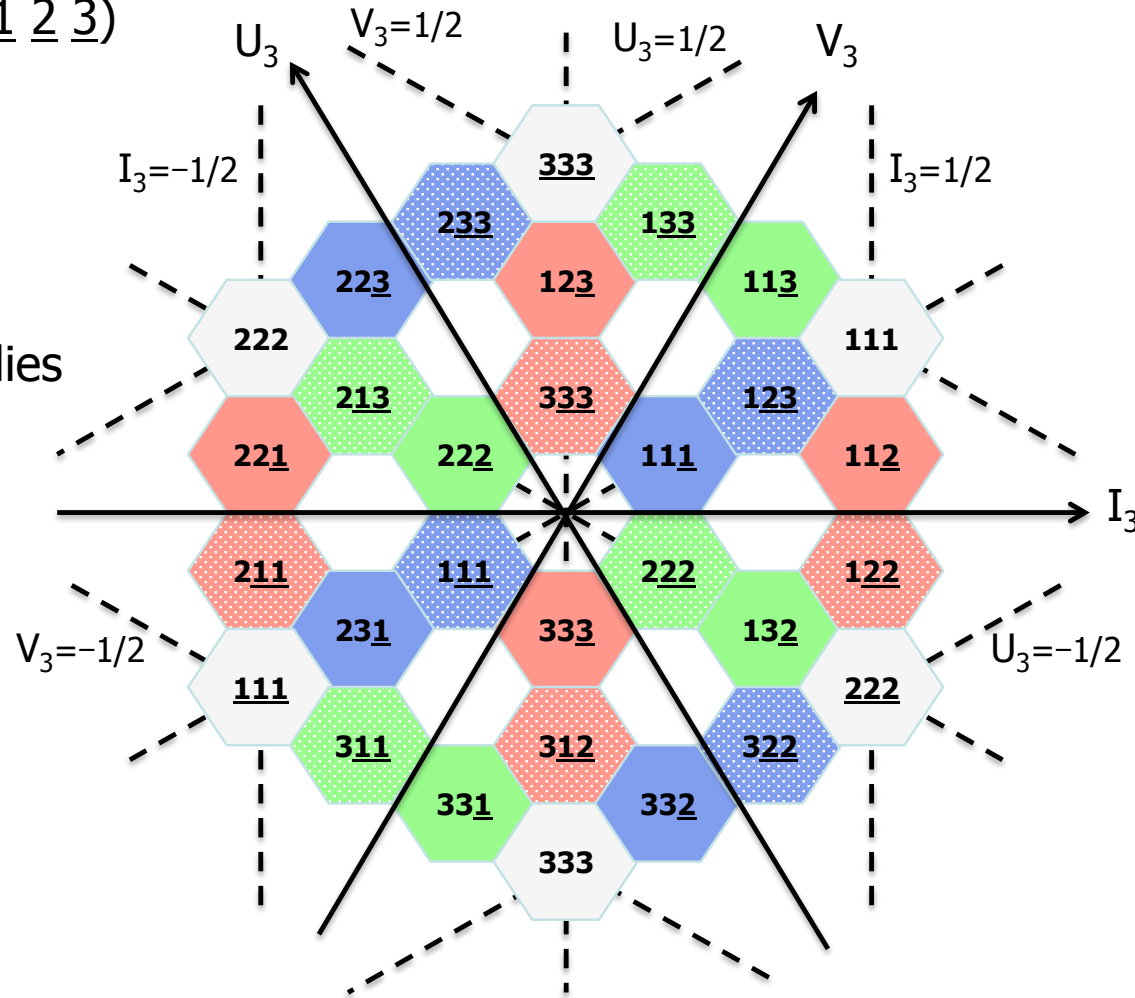
■ Three  $A(4)$  singlets  $\rightarrow$  families

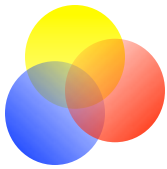
■ Mingled (RRL, RLL) W-states

■ non-asymptotic

$I$ ,  $U$ , or  $V$  allowed

■ Three  $A(4)$  triplets





# Fermionic excitations: electroweak quantum numbers

## LEPTONS

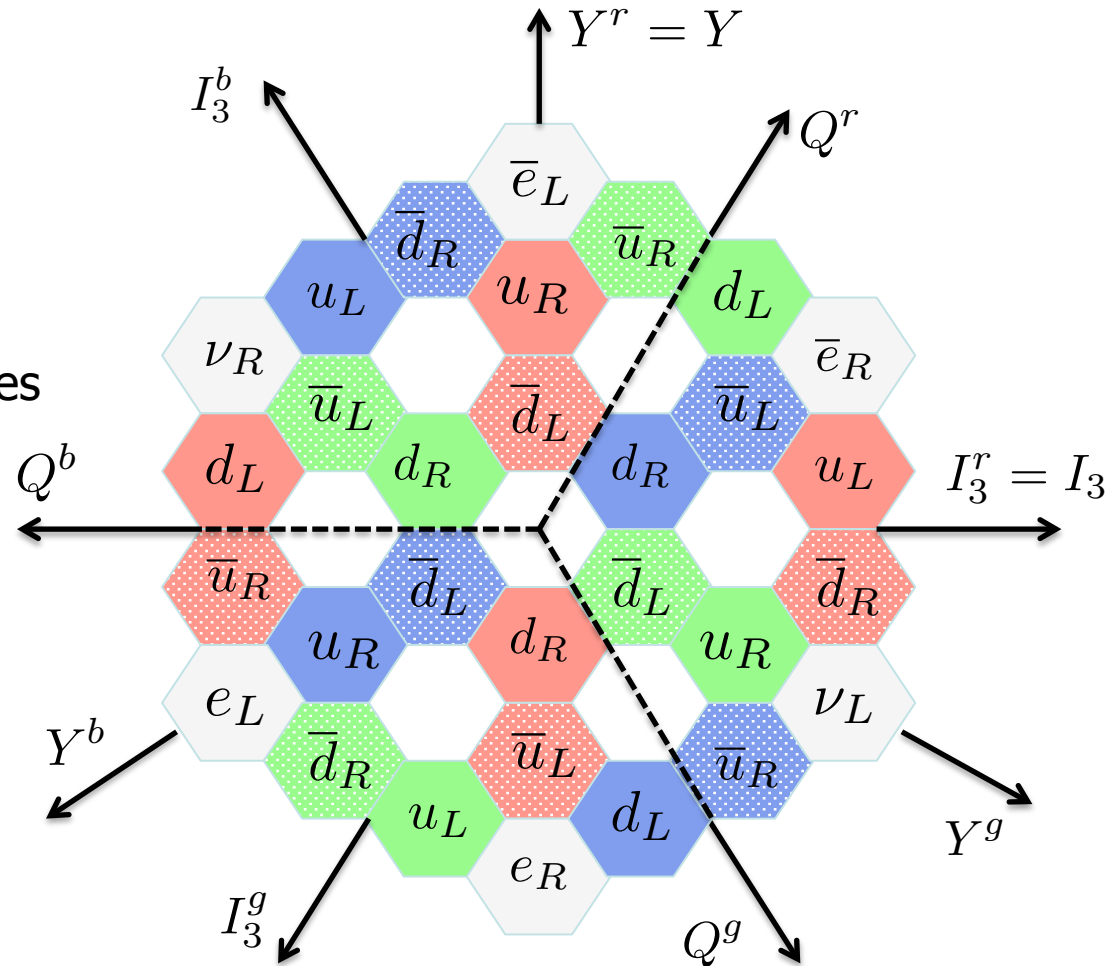
Aligned (RRR, LLL)

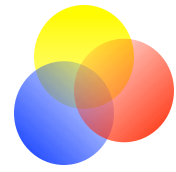
- $SO(3) \rightarrow$  asymptotic/space  
I, U, and V allowed
- Three A(4) singlets  $\rightarrow$  families
- Family mixing is tri-bimaximal

## QUARKS

Mingled (RRL, RLL)

- non-asymptotic  
I, U, or V allowed
- Three A(4) triplets
- Just one heavy quark!

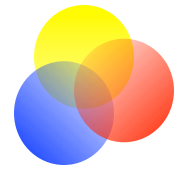




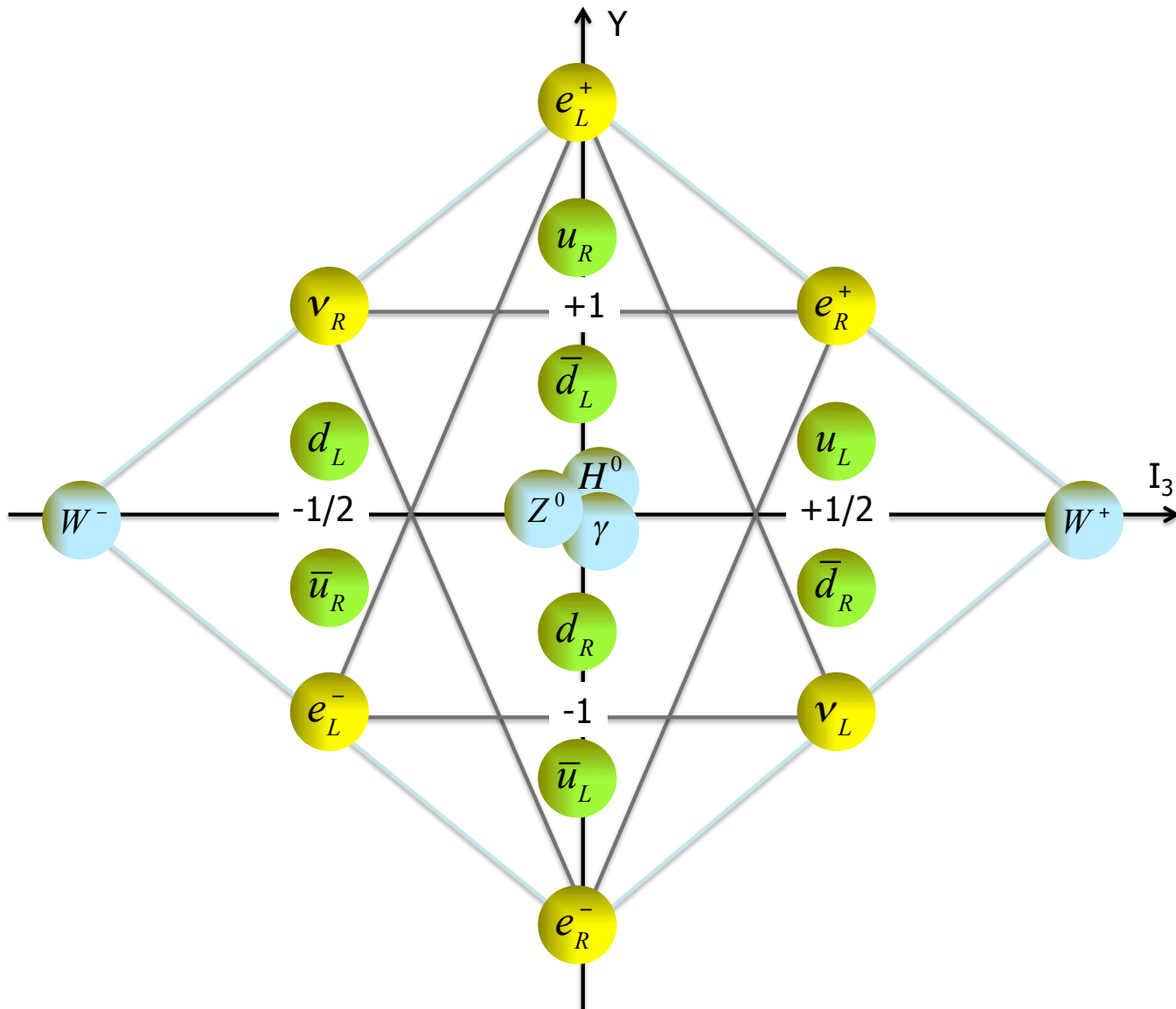
## Electroweak particle content of standard model

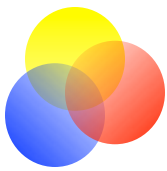
particle	space			isospin		hypercharge	charge	color
	$L$	$T_1$	$T_2$	$I$	$I_3$	$Y$	$Q$	$\underline{c}$
$\nu_L$	$\xi_L^0$	$\xi_L^0$	$\xi_L^0$	1/2	+1/2	-1	0	$\underline{1}$
$e_L^-$	$\xi_L^-$	$\xi_L^-$	$\xi_L^-$	1/2	-1/2	-1	-1	$\underline{1}$
$e_L^+$	$\xi_L^+$	$\xi_L^+$	$\xi_L^+$	0	0	+2	+1	$\underline{1}$
$\nu_R$	$\xi_R^0$	$\xi_R^0$	$\xi_R^0$	1/2	-1/2	+1	0	$\underline{1}$
$e_R^+$	$\xi_R^+$	$\xi_R^+$	$\xi_R^+$	1/2	+1/2	+1	+1	$\underline{1}$
$e_R^-$	$\xi_R^-$	$\xi_R^-$	$\xi_R^-$	0	0	-2	-1	$\underline{1}$
$u_L$	$\xi_L^0$	$(\xi_R^+ \ \xi_R^+)$	$(\xi_R^+ \ \xi_R^+)$	1/2	+1/2	+1/3	+2/3	$\underline{3}$
$d_L$	$\xi_L^-$	$(\xi_R^0 \ \xi_R^0)$	$(\xi_R^0 \ \xi_R^0)$	1/2	-1/2	+1/3	-1/3	$\underline{3}$
$\bar{u}_L$	$\xi_L^0$	$(\xi_L^- \ \xi_R^-)$	$(\xi_L^- \ \xi_R^-)$	0	0	-4/3	-2/3	$\underline{3}^*$
$\bar{d}_L$	$\xi_L^+$	$(\xi_L^0 \ \xi_R^0)$	$(\xi_L^0 \ \xi_R^0)$	0	0	+2/3	+1/3	$\underline{3}^*$
$\bar{u}_R$	$\xi_R^0$	$(\xi_L^- \ \xi_L^-)$	$(\xi_L^- \ \xi_L^-)$	1/2	-1/2	-1/3	-2/3	$\underline{3}^*$
$\bar{d}_R$	$\xi_R^+$	$(\xi_L^0 \ \xi_L^0)$	$(\xi_L^0 \ \xi_L^0)$	1/2	+1/2	-1/3	+1/3	$\underline{3}^*$
$u_R$	$\xi_R^0$	$(\xi_L^+ \ \xi_R^+)$	$(\xi_L^+ \ \xi_R^+)$	0	0	+4/3	+2/3	$\underline{3}$
$d_R$	$\xi_R^-$	$(\xi_L^0 \ \xi_R^0)$	$(\xi_L^0 \ \xi_R^0)$	0	0	-2/3	-1/3	$\underline{3}$

Resembles rishon model (Harari & Seiberg 1982), but no compositeness!



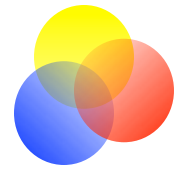
# Standard model particle content





## Using 1D to 3D transition (plan)

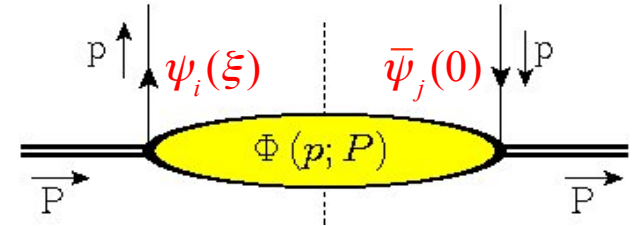
- Parton distribution functions: collinear PDFs to TMDs and role of Wilson loop  
(with Daniel Boer, Tom van Daal, Sabrina Cotogno)
- Dominance of gluons at low  $x$  (dipole picture, color glass condensate, ...)  
(with Elena Petreska)



## TMDs: matrix elements (with gauge links)

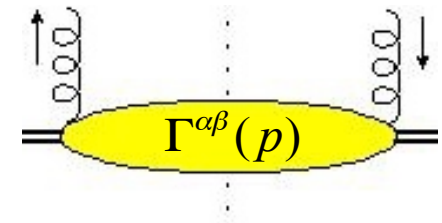
- quark-quark:  $u_i(k)\bar{u}_j(k) \implies$

$$\Phi_{ij}^{[U]}(x, p_T; n) = \int \frac{d\xi \cdot P d^2\xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle P, S | \bar{\psi}_j(0) U_{[0, \xi]} \psi_i(\xi) | P, S \rangle \Big|_{\xi \cdot n = 0}$$

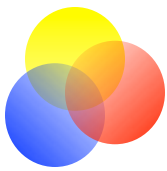


- gluon-gluon:  $\epsilon^\alpha(k)\epsilon^{\beta*}(k) \implies$

$$\Gamma^{[U, U']}_{\mu\nu}(x, p_T; n) = \int \frac{d\xi \cdot P d^2\xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle P, S | F^{n\mu}(0) U_{[0, \xi]} F^{n\nu}(\xi) U'_{[\xi, 0]} | P, S \rangle \Big|_{\xi \cdot n = 0}$$



- Relevant matrix elements at high energies project on 'good' fermion and transverse gauge fields and naturally represent densities of these.
- Noncollinearity requires nontrivial Wilson lines / gauge links.



## TMDs and color gauge invariance (gauge links)

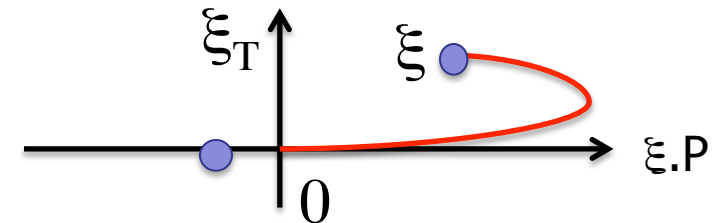
- Gauge invariance in a non-local situation requires a gauge link  $U(0,\xi)$

$$\bar{\psi}(0)\psi(\xi) = \sum_n \frac{1}{n!} \xi^{\mu_1} \dots \xi^{\mu_N} \bar{\psi}(0) \partial_{\mu_1} \dots \partial_{\mu_N} \psi(0)$$

$$U(0,\xi) = \mathcal{P} \exp \left( -ig \int_0^\xi ds^\mu A_\mu \right)$$

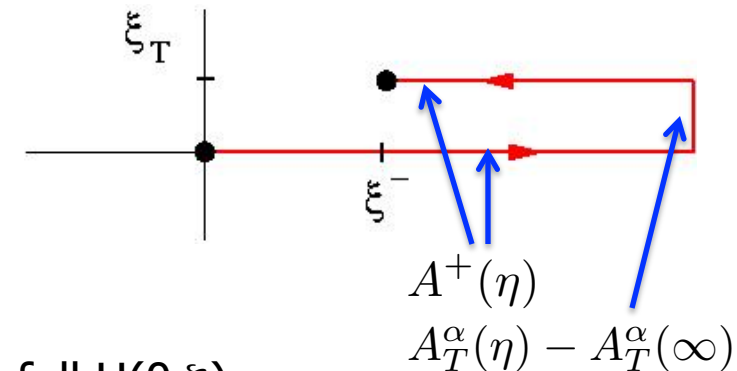
$$\bar{\psi}(0) U(0,\xi) \psi(\xi) = \sum_n \frac{1}{n!} \xi^{\mu_1} \dots \xi^{\mu_N} \bar{\psi}(0) D_{\mu_1} \dots D_{\mu_N} \psi(0)$$

- Introduces path dependence in  $\Phi^{[U]}(x, p_T)$



- 'Dominant' paths: along lightcone connected at lightcone infinity (staples)

- Reduces to 'straight line' for  $\Phi(x)$



$$\Phi^{[U]}(x, p_T) \Rightarrow \Phi(x)$$

(no gluon dynamics)

- Be aware that one needs all orders in  $g$  to obtain full  $U(0,\xi)$



# Non-universality because of process dependent gauge links

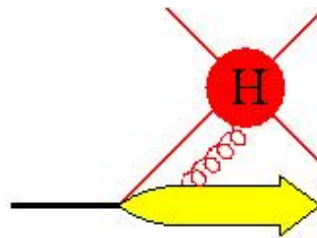
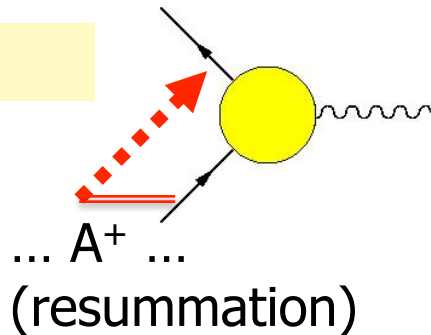
$$\Phi_{ij}^{q[C]}(x, p_T; n) = \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{i p \cdot \xi} \langle P | \bar{\psi}_j(0) U_{[0, \xi]}^{[C]} \psi_i(\xi) | P \rangle_{\xi \cdot n = 0}$$

TMD

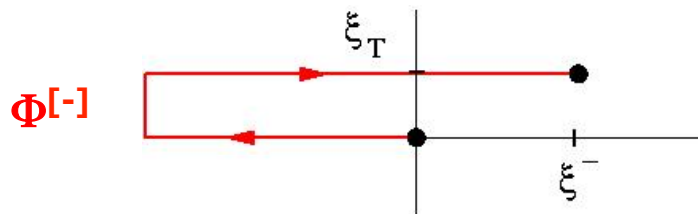
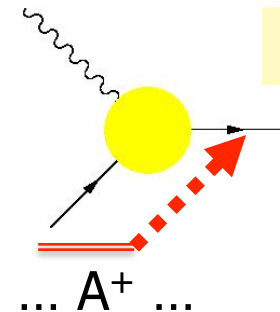
path dependent gauge link

- ◆ Gauge links associated with dimension zero (not suppressed!) collinear  $A^n = A^+$  gluons, leading for TMD correlators to **process-dependence**:

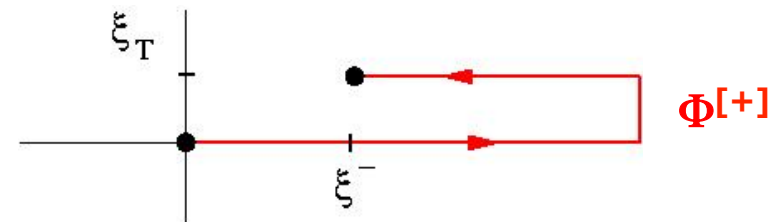
DY



SIDIS



Time reversal



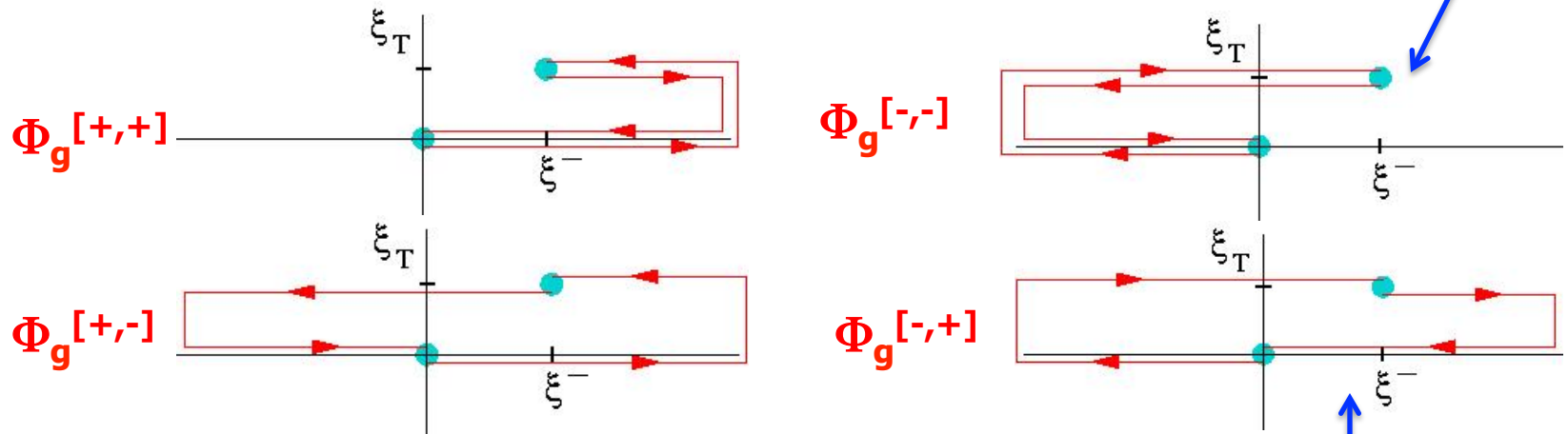
# Non-universality because of process dependent gauge links

$$\Phi_g^{\alpha\beta[C,C']}(x, p_T; n) = \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{i p \cdot \xi} \left\langle P \left| U_{[\xi, 0]}^{[C]} F^{n\alpha}(0) U_{[0, \xi]}^{[C']} F^{n\beta}(\xi) \right| P \right\rangle_{\xi \cdot n = 0}$$

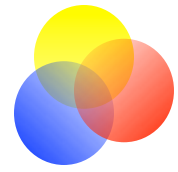
- ◆ The TMD gluon correlators contain **two** links, which can have different paths. Note that standard field displacement involves  $C = C'$

$$F^{\alpha\beta}(\xi) \rightarrow U_{[\eta, \xi]}^{[C]} F^{\alpha\beta}(\xi) U_{[\xi, \eta]}^{[C]}$$

- ◆ Basic (simplest) gauge links for gluon TMD correlators:



- ◆ Collinear gluon PDFs: straight line 'octet' link



## Gluon correlators in a polarized target (up to spin 1/2)

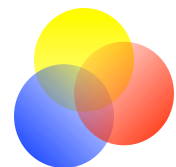
### ■ Unpolarized target

$$\Gamma^{ij[U]}(x, k_T) = \frac{x}{2} \left\{ -g_T^{ij} f_1^{[U]}(x, k_T^2) + \frac{k_T^{ij}}{M^2} h_1^{\perp [U]}(x, k_T^2) \right\}$$

### ■ Vector polarized target

$$\Gamma_L^{ij[U]}(x, k_T) = \frac{x}{2} \left\{ i\epsilon_T^{ij} S_L g_1^{[U]}(x, k_T^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha}}{M^2} S_L h_{1L}^{\perp [U]}(x, k_T^2) \right\}$$

$$\Gamma_T^{ij[U]}(x, k_T) = \frac{x}{2} \left\{ \frac{g_T^{ij} \epsilon_T^{kS_T}}{M} f_{1T}^{\perp [U]}(x, k_T^2) - \frac{i\epsilon_T^{ij} k_T \cdot S_T}{M} g_{1T}^{[U]}(x, k_T^2) \right. \\ \left. - \frac{\epsilon_T^{k\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} k_T^{j\}}}{4M} h_1(x, k_T^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^{\perp}(x, k_T^2) \right\}$$



# Structure of gluon TMDs in targets (up to spin 1)

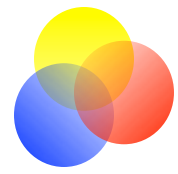
		PARTON SPIN		
TARGET SPIN	GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \dots$
	U	$f_1^g$		$h_1^{\perp g}$
	L		$g_1^g$	$h_{1L}^{\perp g}$
	T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_1^g \quad h_{1T}^{\perp g}$
	LL	$f_{1LL}^g$		$h_{1LL}^{\perp g}$
	LT	$f_{1LT}^g$	$g_{1LT}^g$	$h_{1LT}^g \quad h_{1LT}^{\perp g}$
	TT	$f_{1TT}^g$	$g_{1TT}^g$	$h_{1TT}^g \quad h_{1TT}^{\perp g} \quad h_{1TT}^{\perp\perp g}$

Jaffe & Manohar, Nuclear gluonometry, PL B223 (1989) 218

PJM & Rodrigues, PR D63 (2001) 094021

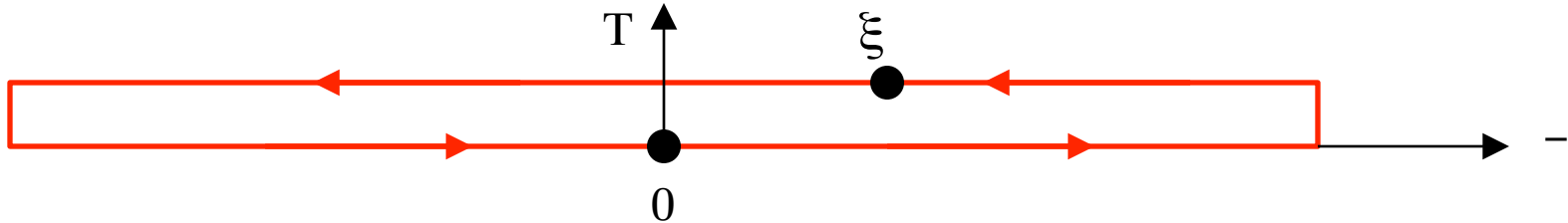
Meissner, Metz and Goeke, PR D76 (2007) 034002

D Boer, S Cotogno, T van Daal, PJM, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654



## Gluons at low x – link color-space

- Interestingly, there is a Wilson loop linking transverse spatial structure and transverse gluons

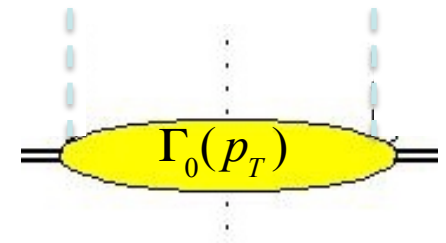


$$F^{\alpha\beta} = \frac{\delta W[C]}{\delta \sigma_{\alpha\beta}}$$

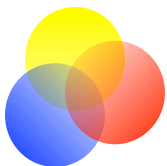
$$A_T^\alpha \longleftrightarrow F^{+\alpha} = \frac{\delta W[C]}{\delta \sigma_{\alpha+}}$$

- Matrix element of single Wilson loop correlator just represents a 'TMD'

- Relevant for diffraction and as  $x \rightarrow 0$  limit of TMDs  
Even without probing color, interactions can be induced  
Link with dipole picture



- Differentiation gives  $\Gamma^{[+-]}$  gluon TMD for zero momentum ( $x = 0$ )



## Small x physics in terms of TMDs

- Note limit  $x \rightarrow 0$  for gluon TMDs linked to Wilson loop correlator  $\Gamma_0$

$$\Gamma_0(k_T) = \frac{1}{2M^2} \left\{ e(k_T^2) - \frac{\epsilon^{kS_T}}{M} e_T(k_T^2) \right\}$$

- Dipole correlators: at **small x** only two structures for unpolarized and transversely polarized nucleons: pomeron & odderon structure

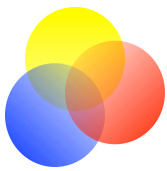
$$\begin{aligned} x f_1^{[+,-]}(x, k_T^2) &\longrightarrow \frac{k_T^2}{2M^2} e^{[+,-]}(k_T^2) \\ x h_1^{\perp[+,-]}(x, k_T^2) &\longrightarrow e^{[+,-]}(k_T^2) \end{aligned}$$

$$\begin{aligned} x f_{1T}^{\perp[+,-]}(x, k_T^2) &\longrightarrow \frac{k_T^2}{2M^2} e_T^{[+,-]}(k_T^2) \\ x h_1^{[+,-]}(x, k_T^2) &\longrightarrow \frac{k_T^2}{2M^2} e_T^{[+,-]}(k_T^2) \\ x h_{1T}^{\perp[+,-]}(x, k_T^2) &\longrightarrow e_T^{[+,-]}(k_T^2) \end{aligned}$$

Dominguez, Xiao, Yuan 2011

D Boer, MG Echevarria, PJM, J Zhou, PRL 116 (2016) 122001, ArXiv 1511.03485

D Boer, S Cotogno, T van Daal, PJM, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654



## Summary and implications for standard model & QCD

- 'Different view' does not invalidate the standard model field theoretical results
  - it may affect way that (QCD+EW) loop corrections are implemented
  - It does away with the confinement issue: quarks are not asymptotic states.
  - Only for color singlet composites, rotational invariance can be employed in analogy to the lepton sector, implying that for valence quarks and antiquarks in hadrons a swap has to be made from  $SU(3)_{\text{local}}$  in 1D to  $SU(3)_{\text{global}}$  in 3D
- Provides a new view for many phenomena in QCD (confinement, Bloom-Gilman duality, separation of hard/soft modes in SCET, jet physics, color-kinematic duality, multitude of effective models for QCD, CFT approaches a la Brodsky, de Téramond, Dosch, Lorcé getting to effective SUSY for baryons/mesons)
- It could shed light on the transition from collinear  $\rightarrow$  3D picture
  - At level of partons/good fields: transition from PDFs to TMDs with staple gauge links
  - Role of Wilson loops in unifying dipole and TMD pictures at small  $x$
- Many open ends remain!