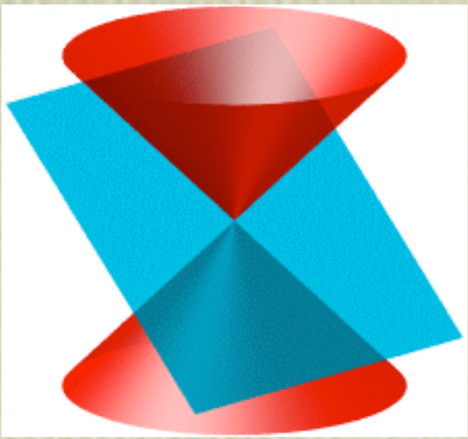
Azimuthal Spin Asymmetries in SIDIS



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Ref: Tanmay Maji, DC, O.V. Teryaev, in preparation

Introduction

Semi Inclusive DIS: $\ell\, p \rightarrow \ell'\, h X$

- spin asymmetries are observed at the angular distribution of the final hadron.

- spin asymmetries $\longrightarrow$ non-vanishing transverse momentum of the partons

- SIDIS: factorizes into TMDs and fragmentation functions

$$d\sigma^{\ell N \to \ell' h X} = \sum_{\nu} \hat{f}_{\nu/P}(x, \mathbf{p}_\perp; Q^2) \otimes d\hat{\sigma}^{\ell q \to \ell q} \otimes \hat{D}_{h/\nu}(z, \mathbf{k}_\perp; Q^2);$$

TMDs

hard scatt.

fragmentation functions

- The azimuthal asymmetry $\begin{cases} \text{SSA} \\ \text{DSA} \end{cases}$

$$A_{S_\ell S_P} = \frac{d\sigma^{\ell(S_\ell)P(S_P)\to\ell' hX} - d\sigma^{\ell(S_\ell)P(-S_P)\to\ell' hX}}{d\sigma^{\ell(S_\ell)P(S_P)\to\ell' hX} + d\sigma^{\ell(S_\ell)P(-S_P)\to\ell' hX}}.$$

- asymmetries can be written as convolutions of TMDs and fragmentation functions(FFs).

- In cross-section, each structure function comes with a definite angular coeff. contribution of a single TMD can be extracted by introducing corresponding weight-factor.

$$
\begin{aligned}
\frac{d\sigma^{\ell(S_\ell)+P(S_P)\to\ell' P_h X}}{dx_B\, dy\, dz\, d^2\mathbf{P}_{h\perp}\, d\phi_S} &= \frac{2\alpha^2}{sxy^2}\Bigg\{\frac{1+(1-y)^2}{2}F_{UU} + (2-y)\sqrt{1-y}\cos\phi_h F_{UU}^{\cos\phi_h} + (1-y)\cos 2\phi_h F_{UU}^{\cos 2\phi_h} \\
&+ S_P^L\Big[(1-y)\sin 2\phi_h F_{UL}^{\sin 2\phi_h} + (2-y)\sqrt{1-y}\sin\phi_h F_{UL}^{\sin\phi_h}\Big] \\
&+ S_P^L S_\ell^z\Big[\frac{1-(1-y)^2}{2}F_{LL} + y\sqrt{1-y}\cos\phi_h F_{LL}^{\cos\phi_h}\Big] \\
&+ S_P^T\Big[\frac{1+(1-y)^2}{2}\sin(\phi_h-\phi_S)F_{UT}^{\sin(\phi_h-\phi_S)} \\
&+ (1-y)\Big(\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)} + \sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}\Big) \\
&+ (2-y)\sqrt{(1-y)}\Big(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big)\Big] \\
&+ S_P^T S_\ell^z\Big[\frac{1-(1-y)^2}{2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)} \\
&+ y\sqrt{1-y}\Big(\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\Big)\Big]\Bigg\} \qquad (4)
\end{aligned}
$$

The weighted structure functions, $F_{S_\ell S}^{\mathcal{W}(\phi_h,\phi_S)}$, are defined as

$$
\begin{aligned}
F_{S_\ell S}^{\mathcal{W}(\phi_h,\phi_S)} &= \mathcal{C}[\mathcal{W}\hat{f}(x,\mathbf{p}_\perp)\hat{D}(z,\mathbf{k}_\perp)] \\
&= \sum_\nu e_\nu^2 \int d^2\mathbf{p}_\perp d^2\mathbf{k}_\perp \delta^{(2)}(\mathbf{P}_{h\perp} - z\mathbf{p}_\perp - \mathbf{k}_\perp)\mathcal{W}(\mathbf{p}_\perp,\mathbf{P}_{h\perp})\hat{f}^\nu(x,\mathbf{p}_\perp)\hat{D}^\nu(z,\mathbf{k}_\perp).
\end{aligned}
$$

- at leading twist 8 TMDs: 6 T-even
  2 T-odd

- 2 fragmentation functions for final unpolarized hadrons

chiral even $D_1(z, k_\perp^2)$

fragmentation of an unpolarized quark

chiral odd $H_1^\perp(z, k_\perp^2)$ Collins function

fragmentation of a transversely polarized quark

- chiral odd $h_{1L}^{\perp}(x, p_{\perp}^2)$ couples with chiral odd $H_1^{\perp}(z, P_{h\perp})$ and measured in SSA with unpolarized lepton and longitudinally polarized proton:

$$A_{UL} \sim h_{1L}^{\perp}(x, p_{\perp}^2) \otimes H_1^{\perp}(z, P_{h\perp})$$

- transversity TMD:

$$A_{UT} \sim h_1(x, p_{\perp}^2) \otimes H_1^{\perp}(z, P_{h\perp})$$

- chiral even $g_{1T}^{\perp}(x, p_{\perp}^2)$ accessed in DSA

$$A_{LT} \sim g_{1T}^{\perp}(x, p_{\perp}^2) \otimes D_1(z, P_{h\perp})$$

We consider the SIDIS for pi+ and pi- channels

SIDIS kinematics

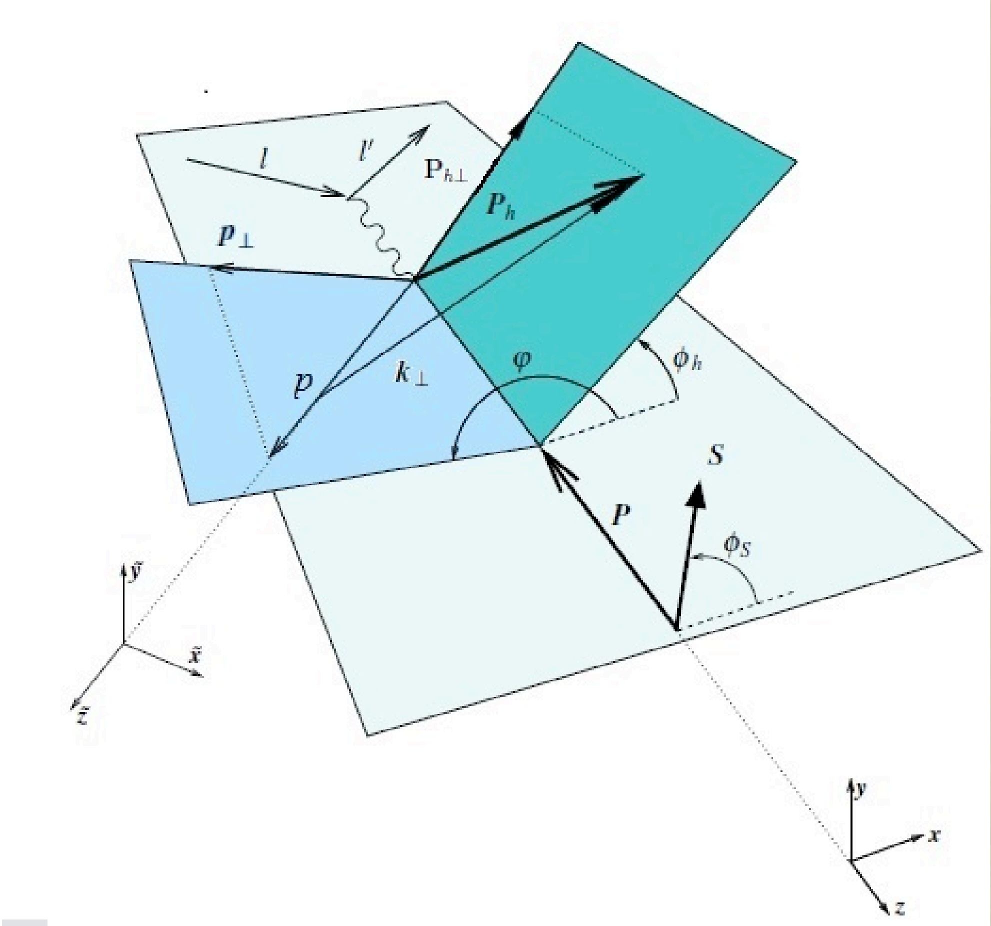
incoming proton $P \equiv (P^+, \frac{M^2}{P^+}, \mathbf{0}_\perp)$

virtual photon $q \equiv (x_B P^+, \frac{Q^2}{x_B P^+}, \mathbf{0}_\perp)$

struck quark $p \equiv (xP^+, \frac{p^2+|\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp)$

diquark $p_D \equiv ((1-x)P^+, \frac{p^2+|\mathbf{p}_\perp|^2}{(1-x)P^+}, -\mathbf{p}_\perp)$

produced hadron $\mathbf{P}_h \equiv (P^+, P^-, \mathbf{P}_{h\perp})$



$\gamma^* - P$ center of mass frame:

Bjorken variable $$x = \frac{Q^2}{2(P.q)} = x_B$$

The fractional energy transferred by the photon $$y = \frac{P.q}{P.\ell} = \frac{Q^2}{sx}$$

the energy fraction carried by the produced hadron $$z = \mathbf{P}_h^-/k^- = \frac{P.P_h}{P.q} = z_h.$$

[Fig: Anselmino et al, PRD 75,054032]

The model

- Light-front quark-diquark model considering both scalar and axial vecor diquarks: [Jakob, Mulders, Rodrigues, NPA626, 937]

$$|P;\pm\rangle = C_S|u\ S^0\rangle^\pm + C_V|u\ A^0\rangle^\pm + C_{VV}|d\ A^1\rangle^\pm.$$

S = scalar diquark A = axialvector diquark

(isospin at the superscript)

- $$|u\ S\rangle^\pm = \int \frac{dx\ d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}}\Big[\psi_+^{\pm(u)}(x,\mathbf{p}_\perp,\mu)|+\frac{1}{2}\ s;xP^+,\mathbf{p}_\perp\rangle + \psi_-^{\pm(u)}(x,\mathbf{p}_\perp,\mu)|-\frac{1}{2}\ s;xP^+,\mathbf{p}_\perp\rangle\Big],$$

LF wavefunctions

$$\begin{aligned}
\psi_+^{+(\nu)}(x,\mathbf{p}_\perp,\mu) &= N_s\varphi_1^{(\nu)}(x,\mathbf{p}_\perp,\mu),\\
\psi_-^{+(\nu)}(x,\mathbf{p}_\perp,\mu) &= N_s\Big(-\frac{p^1+ip^2}{xM}\Big)\varphi_2^{(\nu)}(x,\mathbf{p}_\perp,\mu),\\
\psi_+^{-(\nu)}(x,\mathbf{p}_\perp,\mu) &= N_s\Big(\frac{p^1-ip^2}{xM}\Big)\varphi_2^{(\nu)}(x,\mathbf{p}_\perp,\mu),\\
\psi_-^{-(\nu)}(x,\mathbf{p}_\perp,\mu) &= N_s\varphi_1^{(\nu)}(x,\mathbf{p}_\perp,\mu),
\end{aligned}$$

- general form of the LF wavefunctions:

$$\psi^q_{\lambda\Lambda}(x,p_\perp) = N^q f(x,p_\perp,\lambda,\Lambda)\phi^q_i(x,p_\perp)$$

- The two-particle LF wavefunctions are adopted from AdS/QCD prediction

[Brodsky and Teramond arXiv:1203.4025]

$$\varphi_i^{(\nu)}(x,\mathbf{p}_\perp) = \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}x^{a_i^\nu}(1-x)^{b_i^\nu}\exp\left[-\delta^\nu\frac{\mathbf{p}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right].$$

$\kappa = 0.4$ GeV

a_i^ν, b_i^ν and δ^ν are fixed by fitting to EM formfactors.

[T. Maji and DC, PRD 94, 094020]

scale evolution

- QCD evolution of unpolarized TMDs and FFs

[Aybat and Rogers, PRD83, 114042]
[Aybat, Collins,Qiu and Rogers, PRD85, 034043]

TMD evolution in coord space:

TMD

$$\tilde{F}(x, \mathbf{b}_\perp; \mu) = \tilde{F}(x, \mathbf{b}_\perp; \mu_0) \tilde{R}(\mu, \mu_0, b_T) \exp\left[- g_K(b_T) \ln(\frac{\mu}{\mu_0}) \right],$$

$$g_K(b_T) = \tfrac{1}{2} g_2 b_T^2$$

$$g_2 = 0.68\ GeV^2$$

kernel

$$\tilde{R}(\mu, \mu_0, b_T) = \exp\left[\ln \frac{\mu}{\mu_0} \int_\mu^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(\mu') + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F\Big(\mu', \frac{\mu^2}{\mu'^2}\Big) \right].$$

$$\gamma_F\Big(\mu', \frac{\mu^2}{\mu'^2}\Big) = \alpha_s(\mu') \frac{C_F}{\pi} \left(\frac{3}{2} - \ln \frac{\mu^2}{\mu'^2} \right),$$

$$\gamma_K(\mu') = \alpha_s(\mu') \frac{C_F}{\pi}.$$

- parameter evolution [T. Maji and DC, PRD 94, 094020]

parameters in the model are fitted to follow DGLAP for pdfs, same scale dependence of the parameters is used for TMDs

- One can adopt the same QCD evolution for polarized TMDs to predict the asym.

- for Collins asym, we compare three schemes:

(i) f_1^ν is at $\mu^2 = 2.5\ GeV^2$ and h_1^ν is at initial scale μ_0,

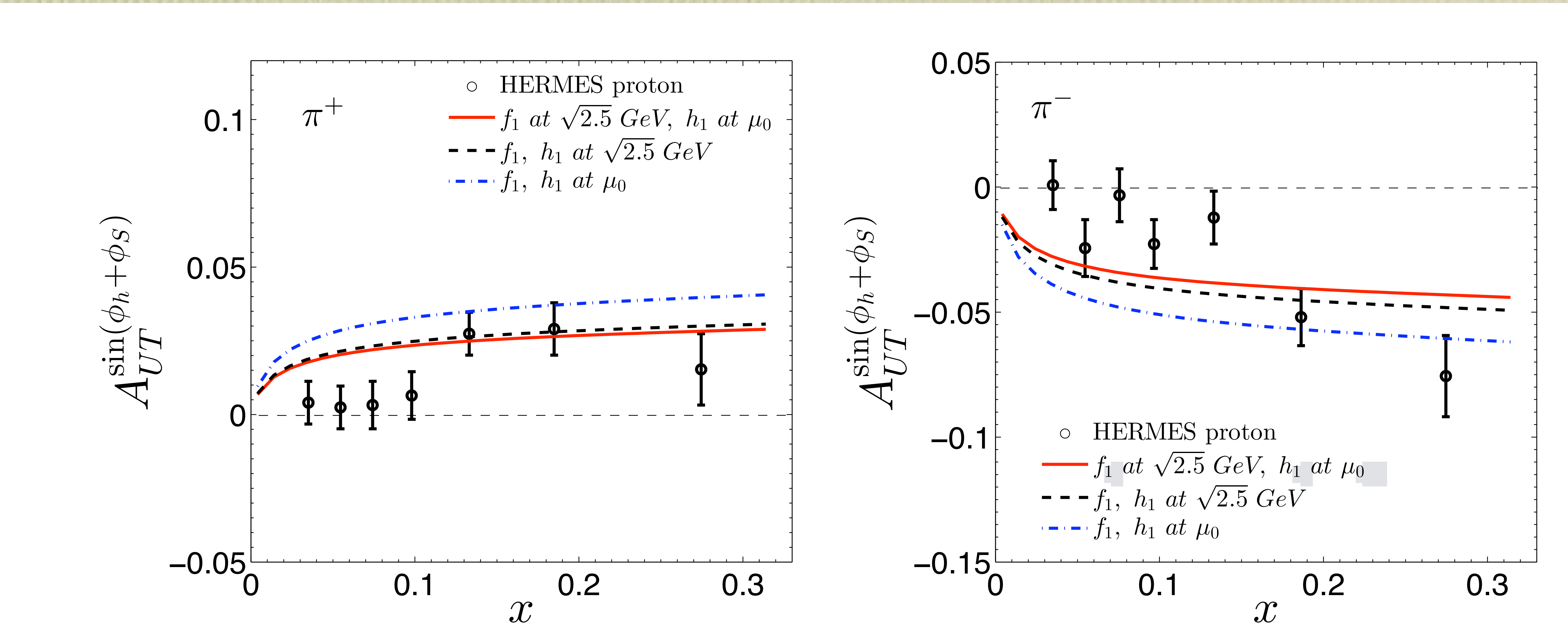
(ii) both f_1^ν and h_1^ν are at $\mu^2 = 2.5\ GeV^2$

(iii) both f_1^ν and h_1^ν are at μ_0^2.

- scheme(i) is found to be the closest to the data!

- We adopt scheme(i), the uncertainty/error is limited in the polarized TMDs only.

comparision of the different schemes



Collins asymmetry
comparison with HERMES data

- asymmetries are functions of $x, z, \mathbf{P}_{h\perp}, y$ and scale μ
  but exptl data are integrated asym for one variable at a time
- integrated asym are estimated by integrating over the variables in the corresponding kinematical limits

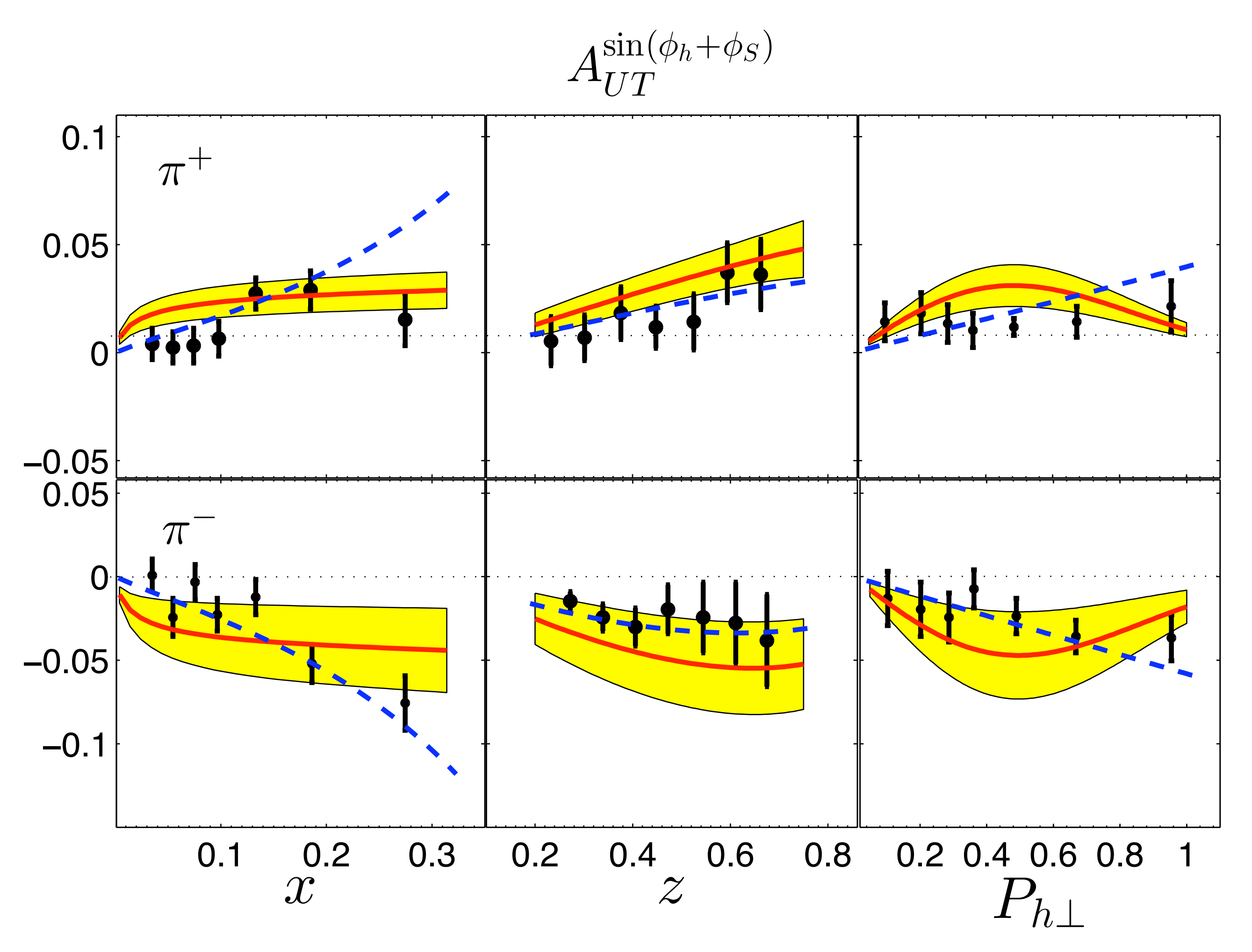
kinematical limits for HERMES

$0.023 \le x \le 0.4,$

$0.1 \le y \le 0.95$

$0.2 \le z \le 0.7$

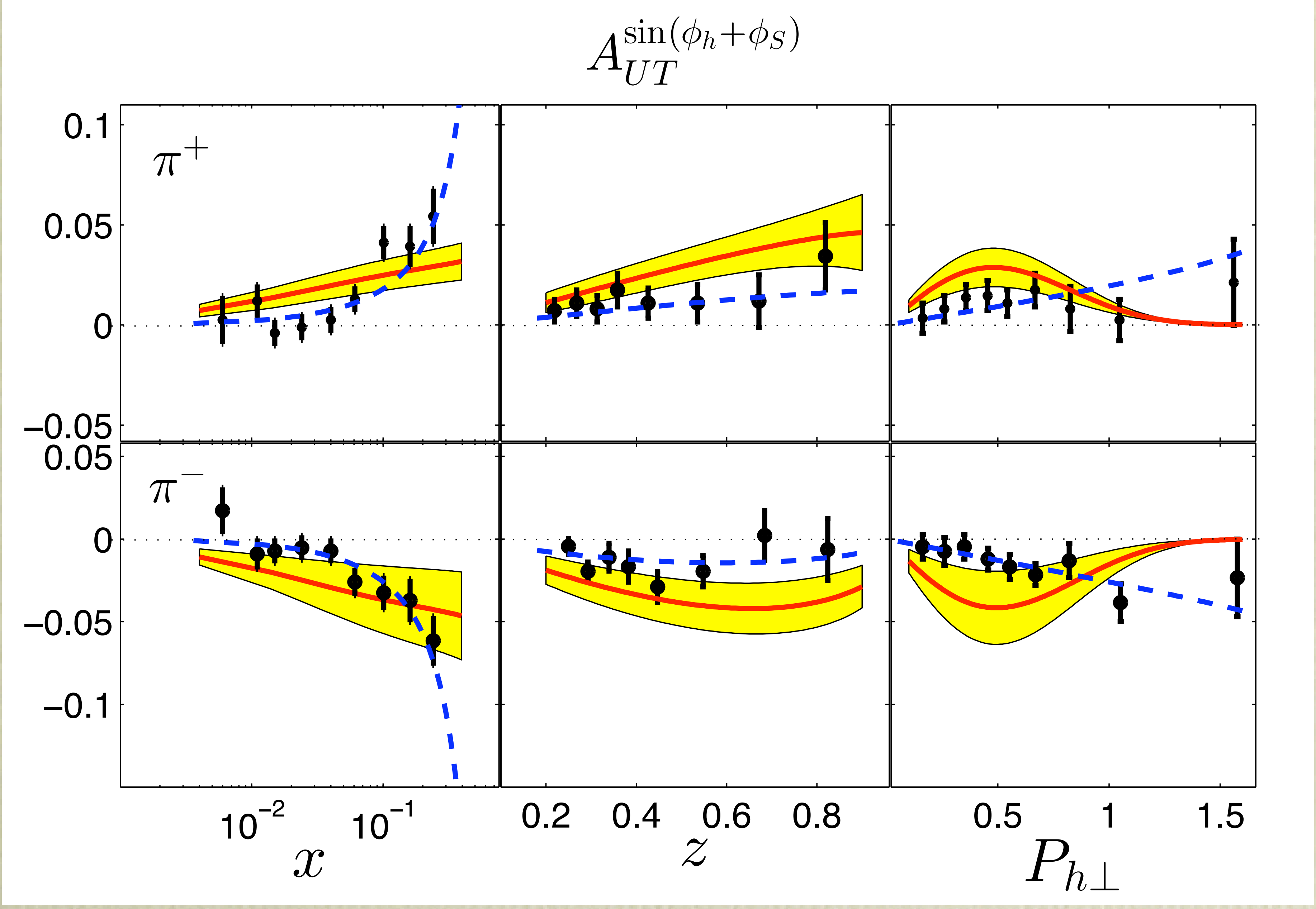
red: QCD evolution
blue: parameter evol



h_1 at initial scale

f_1^ν evolved to $\mu^2 = 2.5\ GeV^2$

comparison with COMPASS data



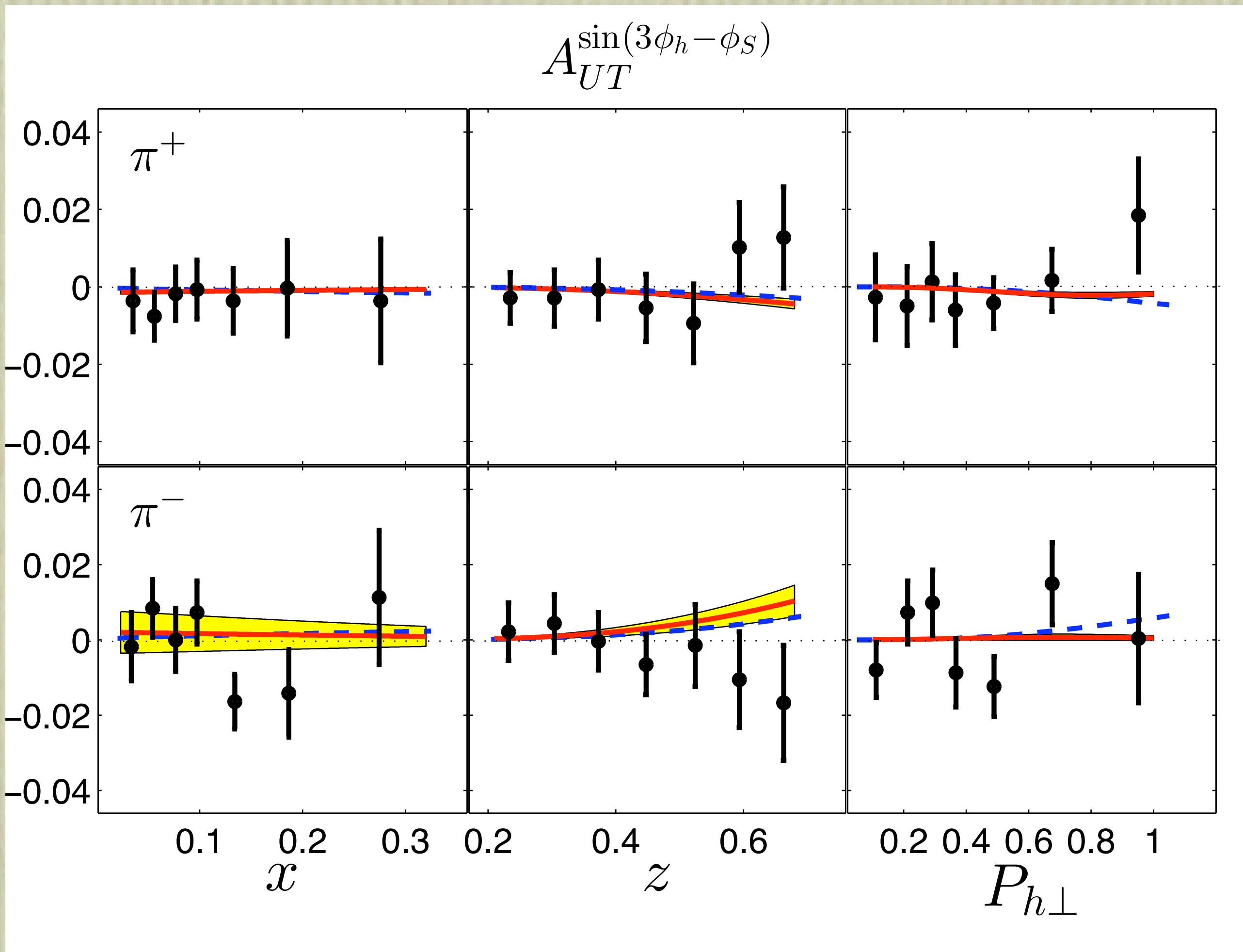
$0.003 \leq x \leq 0.7$

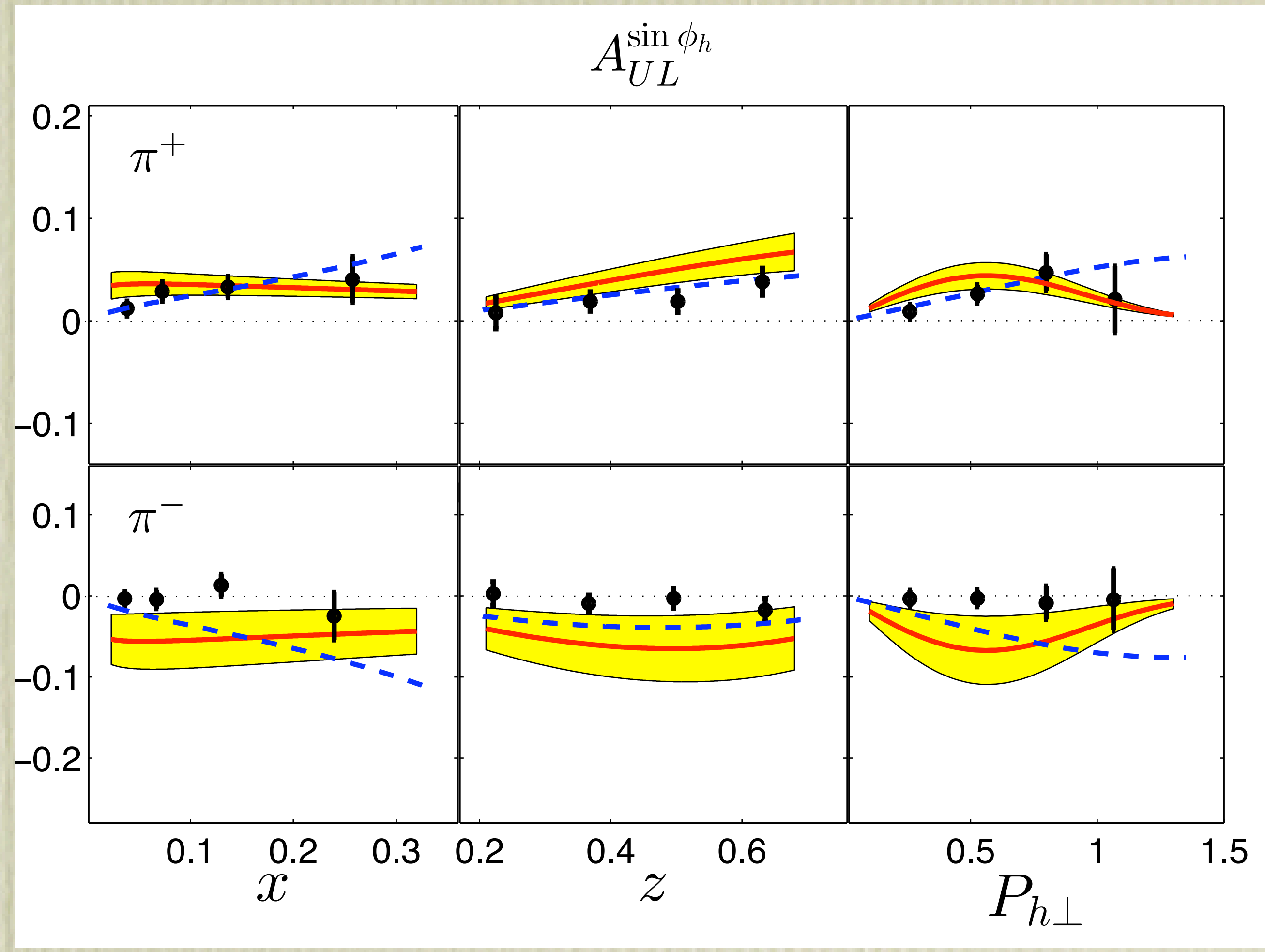
$0.1 \leq y \leq 0.9$

$0.2 \leq z \leq 1.0$

red: QCD evolution
blue: parameter evol

model predictions for other SSAs [HERMES data]

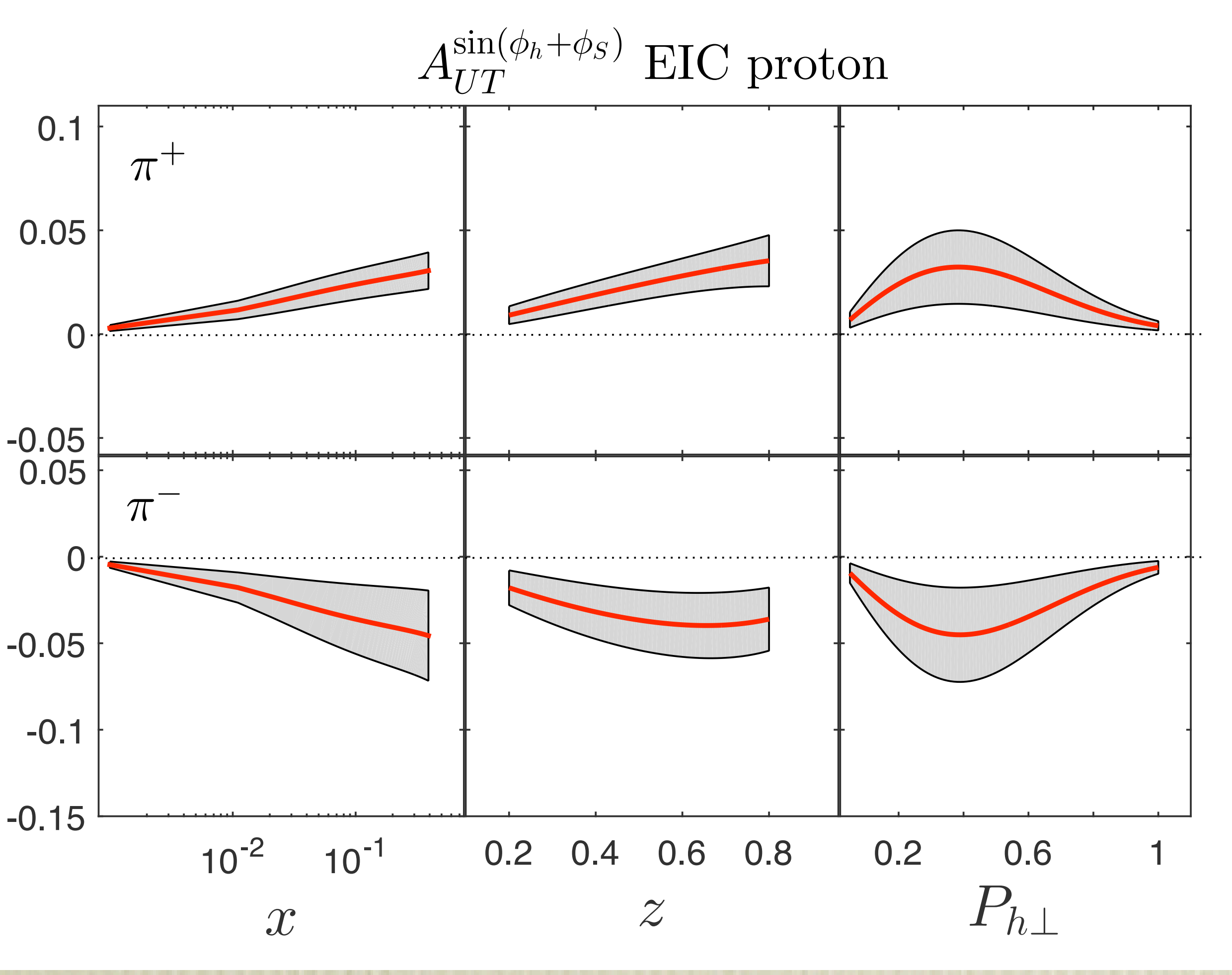




prediction for Electron-Ion collider

- EIC: future collider [Ref. A. Deshpande's talk]
- We predict the Collins asymmetry for EIC kinematics at $\sqrt{s} = 45$ GeV and $\mu^2 = 100\ GeV^2$

$$0.001 < x < 0.4, \quad 0.2 < z < 0.8,$$
$$0.05 < P_{h\perp} < 1, \quad 0.01 < y < 0.95,$$



some remarks:

- Model prediction: $A_{UT}^{\sin(3\phi_h-\phi_S)}$ is suppressed by a factor of $P_{h\perp}^2/M^2$ compared to $A_{UT}^{\sin(\phi_h+\phi_S)}$ and expected to be small,
expt result: very close to zero.

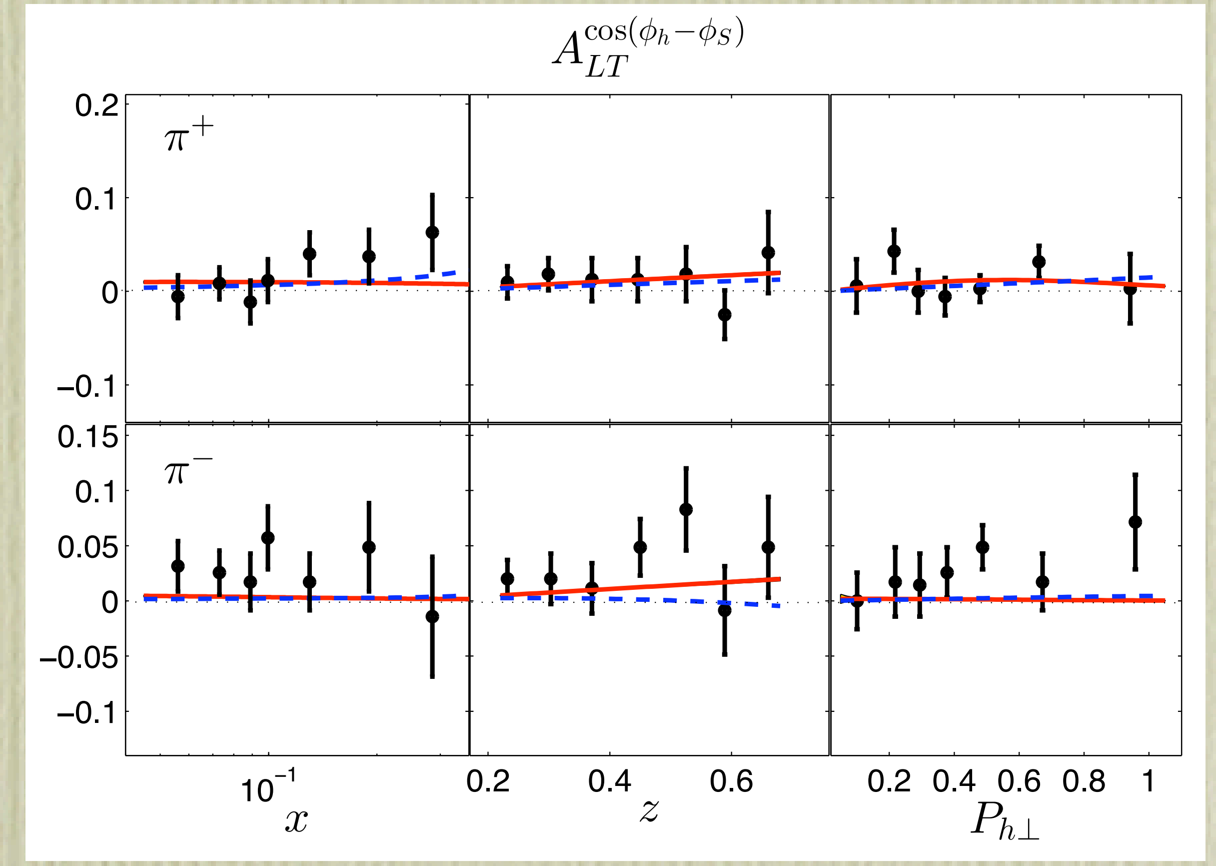
- parameter evolution: follows DGLAP evolution. But TMDs don't follow DGLAP. SSAs involve ratios of TMDs and FFs. Interestingly parameter evolution predicts SSAs very well!

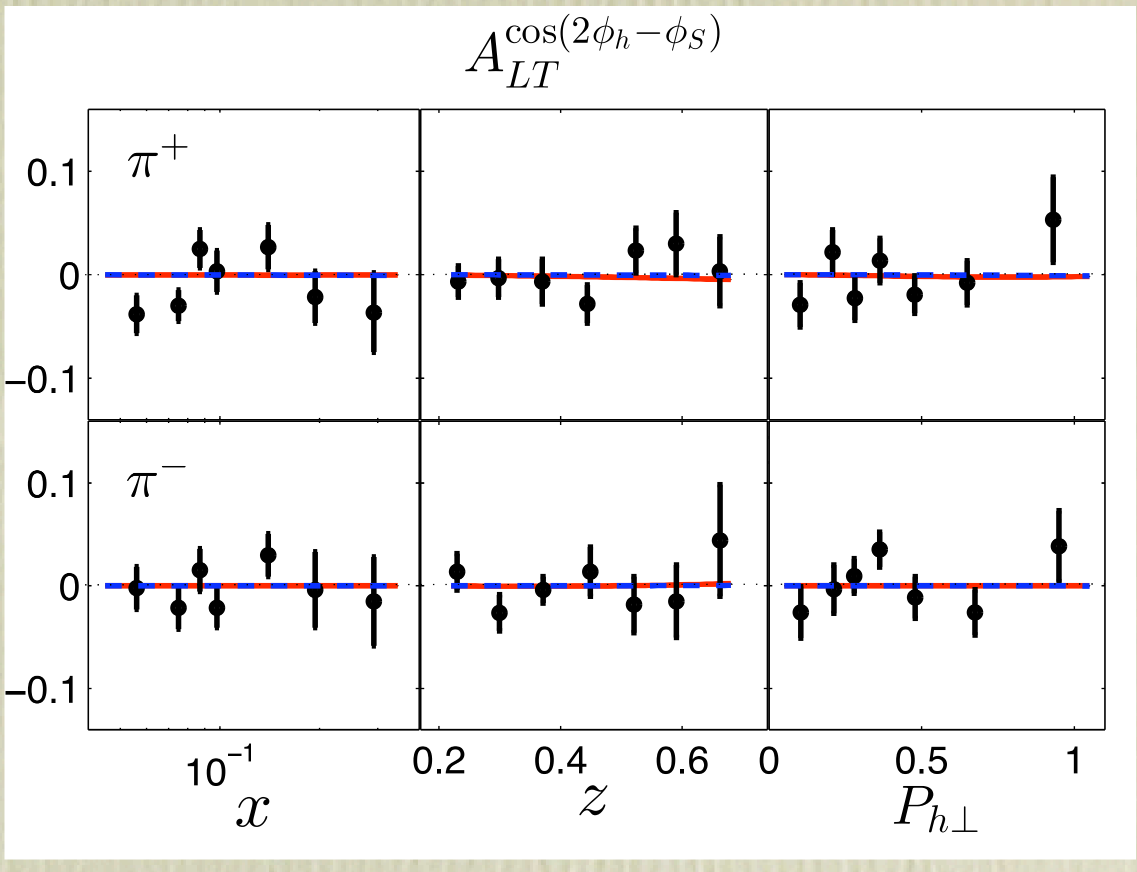
- proper QCD evolution for all polarized TMDs are required for better predictions!

Double Spin Asymmetries(DSA)

- when both the incoming lepton and the proton are polarized =>DSA
- DSAs measured in many experiments for longitudinally polarized lepton and long/ transversely polarized proton.
- SSAs discussed here are proportional to Collins function $H_1^{\perp}(z, \mathbf{k}_{\perp})$
- DSAs are proportional to chiral even FF $D_1^{h/\nu}(z, \mathbf{k}_{\perp})$

DSA: comparison with HERMES data:



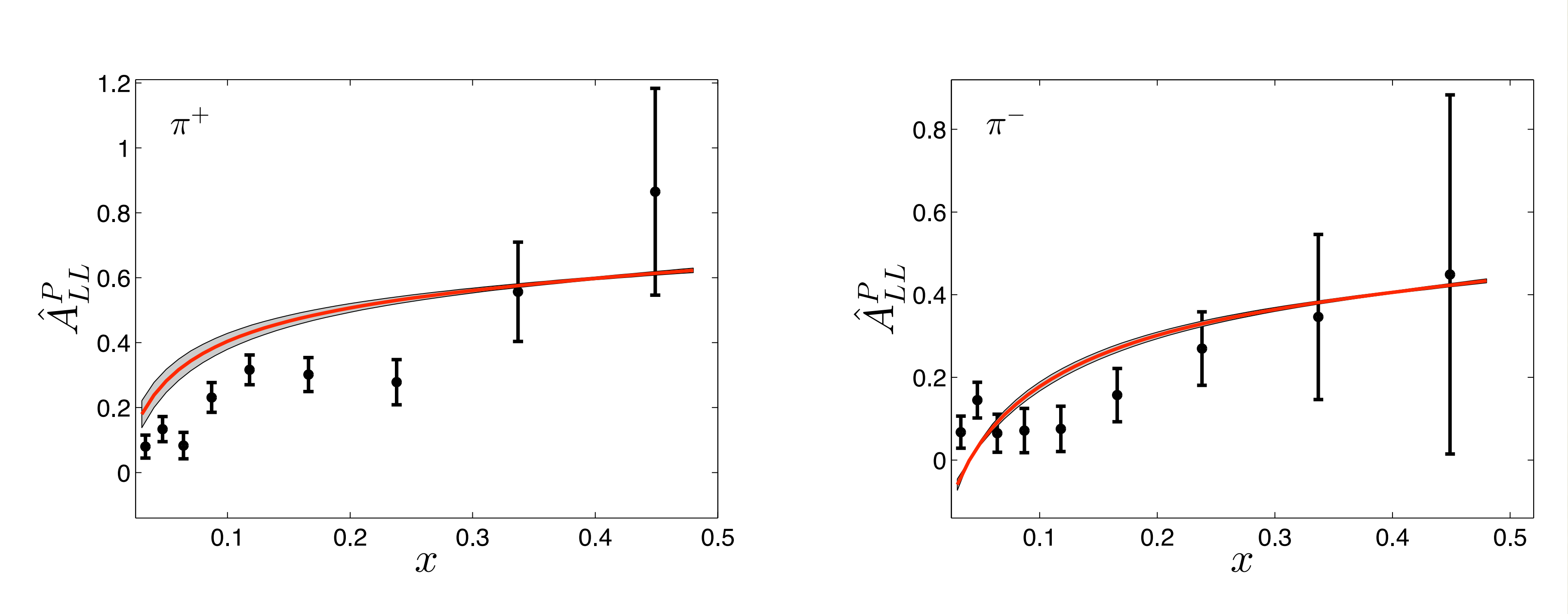


Integrated DSAs

- DSAs integrated over transverse momentum, defined in terms of helicity PDFs

$$\hat{A}^P_{LL}(x,z,\mu) = \frac{\sum_\nu e_\nu^2 g_1(x,\mu) D_1^{h/\nu}(z,\mu)}{\sum_\nu e_\nu^2 f_1(x,\mu) D_1^{h/\nu}(z,\mu)}$$

comparison with HERMES data



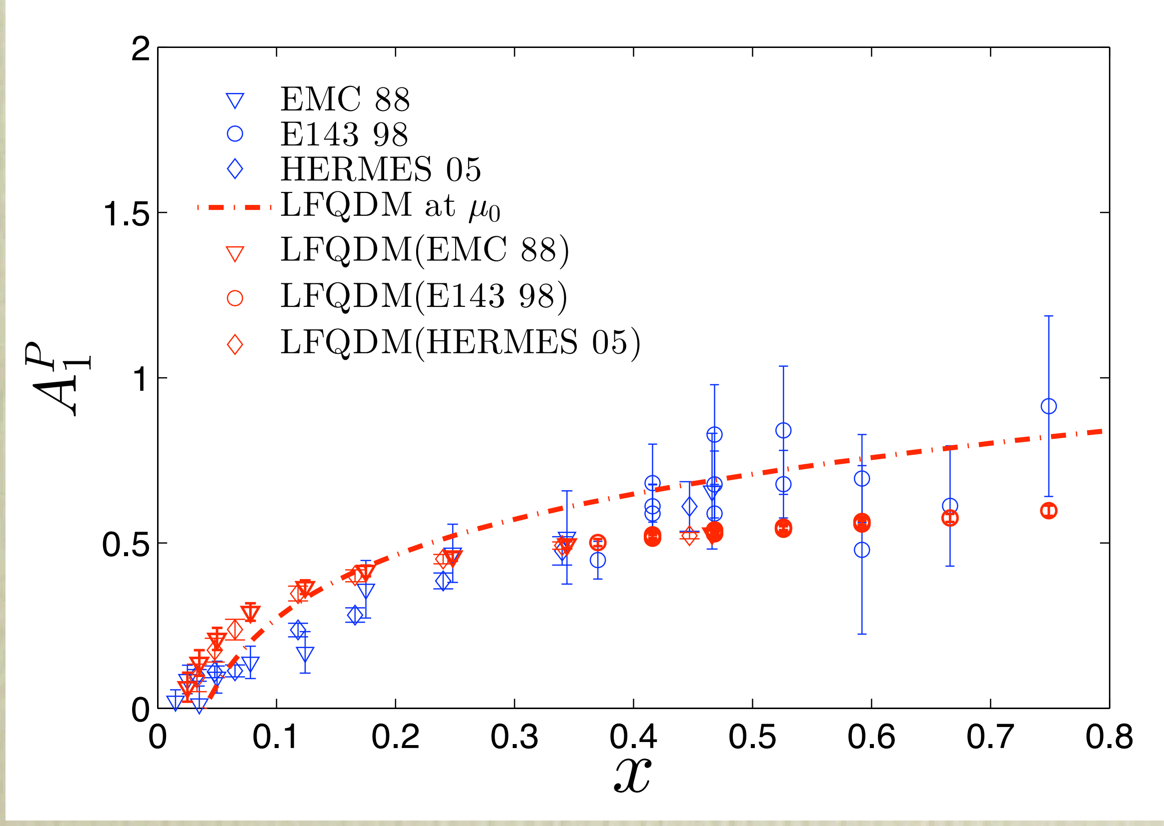
* all the distributions are taken at $\mu^2 = 2.5\ GeV^2$.

* considered bin averaged value of $z = 0.46$.

- If no final hadron is observed [DIS], the DSA for proton is given by

$$A_1^P = \frac{\sum_\nu e_\nu^2 g_1(x)}{\sum_\nu e_\nu^2 f_1(x)}$$

- does not involve any frag. function.

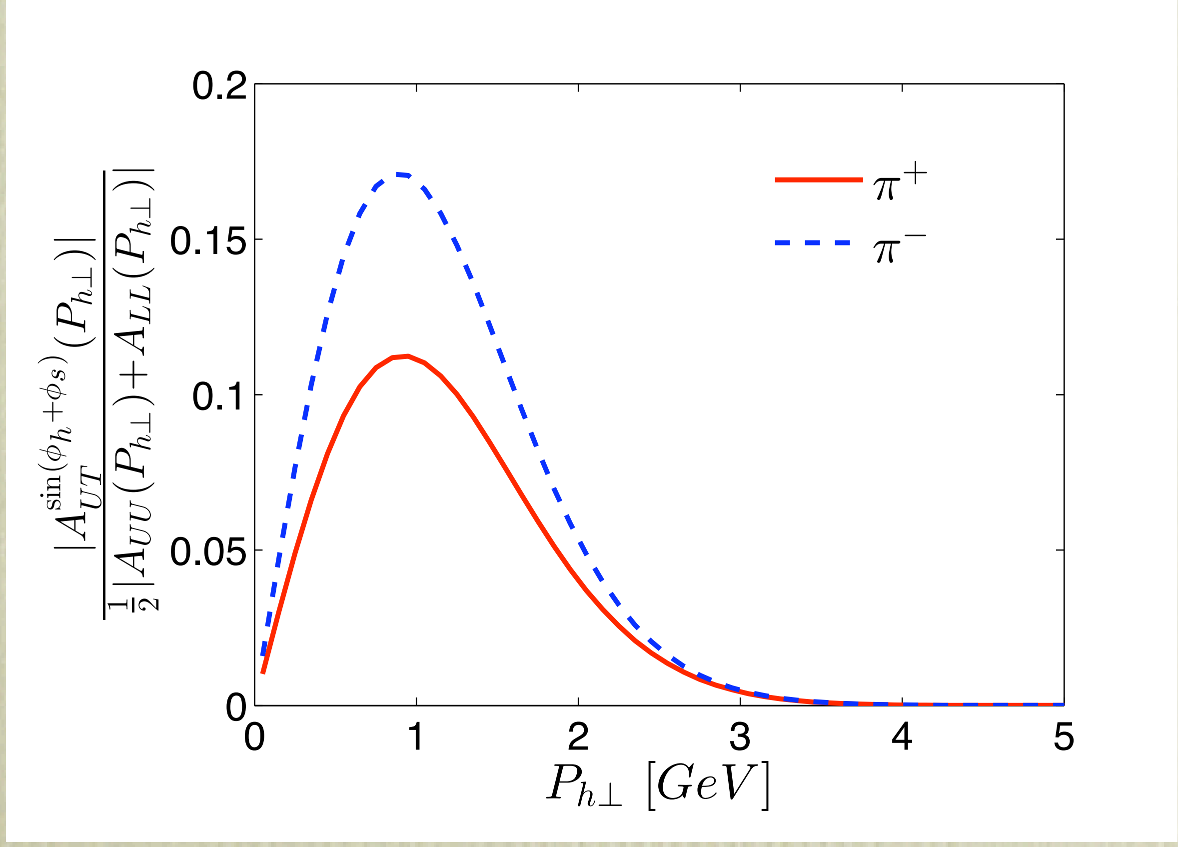


inequalities

★ SSA and DSA satisfy a Soffer bound type inequality

$$A_{UT}^{\sin(\phi_h+\phi_s)}(P_{h\perp}) \leq \frac{1}{2}|A_{UU}(P_{h\perp}) + A_{LL}(P_{h\perp})|$$

$\hat{h}_1^\nu$ f_1^ν g_1^ν



another inequality

$$|\frac{\mathbf{P}_{h\perp}^2}{2M^2} A_{UT}^{\sin(3\phi_h-\phi_s)}(P_{h\perp})| \leq \frac{1}{2}|A_{UU}(P_{h\perp}) - A_{LL}(P_{h\perp})|$$

some equalities:

★ ratio of asymmetries associated with same TMDs

$$\frac{A_{UL}^{\sin(2\phi_h)}\Big/(zP_{h\perp})}{A_{UL}^{\sin(\phi_h)}\langle P_{h\perp}^2\rangle_C\Big/\langle\hat{m}_\perp^2\rangle} = (-Q)\frac{1-y}{2(2-y)\sqrt{1-y}}$$

$$\frac{A_{LL}}{A_{LL}^{\cos\phi_h}\langle P_{h\perp}^2\rangle\Big/(zP_{h\perp}\langle p_\perp^2\rangle_x)} = (-Q)\frac{1-(1-y)^2}{4y\sqrt{1-y}}$$

$$\frac{A_{LT}^{\cos(\phi_h-\phi_S)}\Big/(zP_{h\perp})}{A_{LT}^{\cos\phi_S}\langle P_{h\perp}^2\rangle\Big/\langle\hat{n}_\perp^2\rangle} = (-Q)\frac{1-(1-y)^2}{2y\sqrt{1-y}}$$

$$\frac{A_{LT}^{\cos(\phi_h-\phi_S)}}{A_{LT}^{\cos(2\phi_h-\phi_S)}\langle P_{h\perp}^2\rangle\Big/(zP_{h\perp}\langle p_\perp^2\rangle_x)} = (-Q)\frac{1-(1-y)^2}{2y\sqrt{1-y}}$$

RHS indep of x,z,P_h

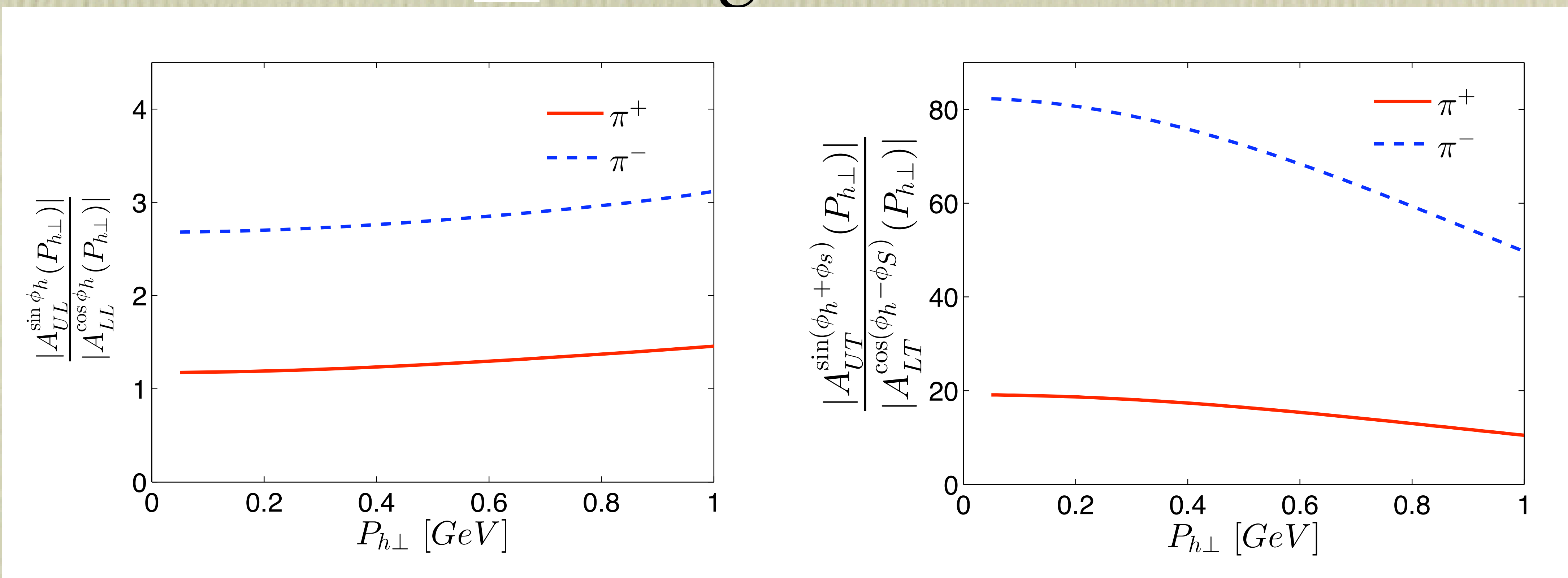
where

$$\langle\hat{m}_\perp^2\rangle = \Big[\langle k_\perp^2\rangle_C\langle P_{h\perp}^2\rangle_C + z\langle p_\perp^2\rangle_x(P_{h\perp}^2 - \langle P_{h\perp}^2\rangle_C)\Big].$$

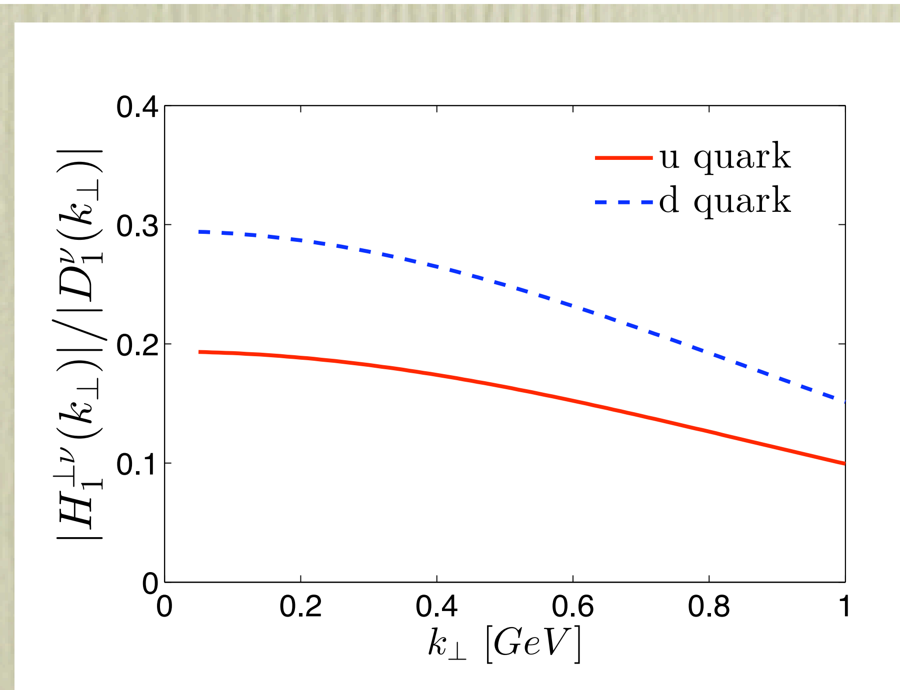
$$\langle\hat{n}_\perp^2\rangle = [\langle k_\perp^2\rangle\langle p_\perp^2\rangle + z^2P_{h\perp}^2\langle p_\perp^2\rangle]$$

ratios of SSA and DSA

- Ratio for π^- is larger than π^+



favored fragmentations:
u -> pi+
d -> pi-

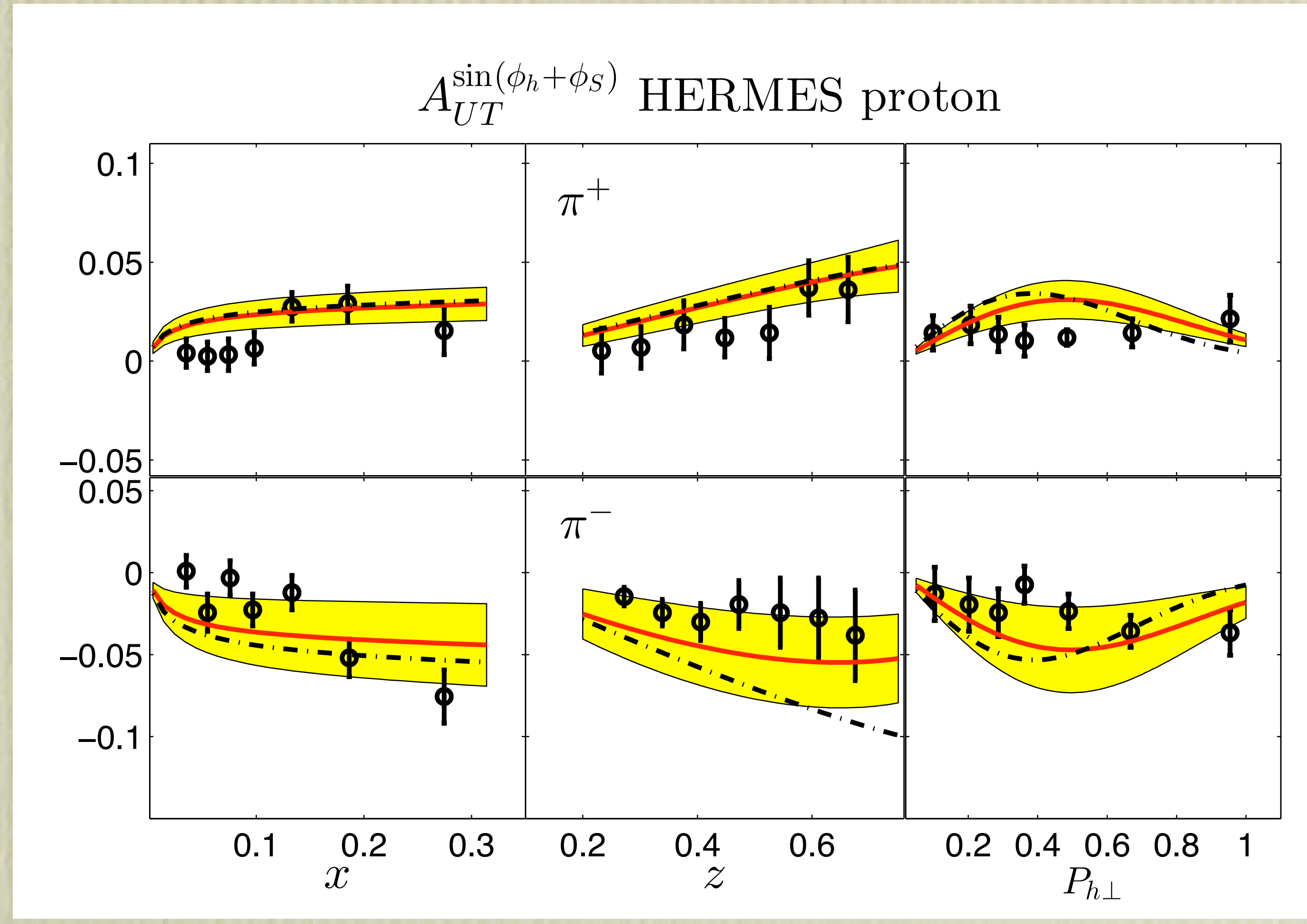


ratio of the FFs for d is larger than u

how much the axial vector diquark contribute?

- we evaluate the SSAs with $C_{VV} = 0$ i.e., without uu - axial vector diquark.

black dot-dashed line: $C_{VV} = 0$



- for pi+ channel: uu contributes in unfavored FF
  for pi- channel: uu contributes in favored FF

Sivers & Boer-Mulders Asymmetries

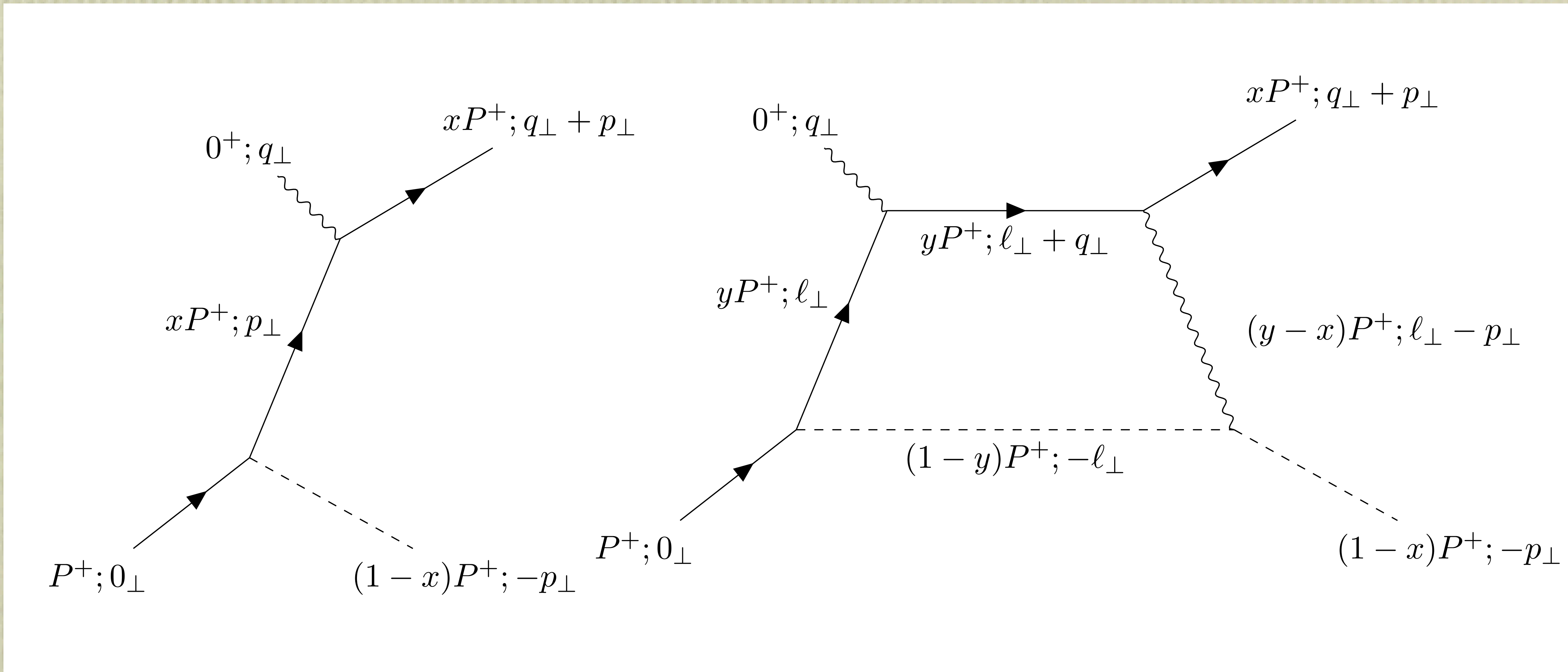
{DC,T. Maji, A. Mukherjee, in preparation}

- Sivers and Boer-Mulders functions are T-odd.
- Require a complex phase in the LFWFs.
- Sivers function: distibution of unpolarized quark inside a transversely polarized proton.
- Boer-Mulders function: transversely polarized quark inside an unpolarized proton.
- Sivers/Boer-Mulders asymmetries: experimentally observed.

- both are process dependent.
- Both are studied in diff. models.
- LFWFs modified to incorporate the FSI.

modified LFWFs

[D.S Hwang, 1003.0867]



LFWF $\psi^q_{\lambda\Lambda}(x, p_\perp) = N^q f(x, p_\perp, \lambda, \Lambda)\phi^q_i(x, p_\perp)$ modifies to

$$\psi^q_{\lambda\Lambda}(x, p_\perp) = N^q f(x, p_\perp, \lambda, \Lambda)(1 + i\frac{e_1 e_2}{8\pi}(p_\perp^2 + B)g_i)\phi^q_i(x, p_\perp)$$

$$g_1 = \int_0^1 d\alpha \frac{-1}{\alpha(1-\alpha)\mathbf{p}_\perp^2 + \alpha m_g^2 + (1-\alpha)B}$$

$$g_2 = \int_0^1 d\alpha \frac{-\alpha}{\alpha(1-\alpha)\mathbf{p}_\perp^2 + \alpha m_g^2 + (1-\alpha)B}$$

$$B = x(1-x)(-M^2 + \frac{m_q^2}{x} + \frac{m_D}{1-x})$$

Sivers & Boer-Mulders functions

Sivers
$$f_{1T}^{\perp\nu}(x,\mathbf{p}_\perp^2) \;=\; \left(C_S^2 N_S^{\nu 2} - C_A^2 \frac{1}{3} N_0^{\nu 2}\right) f^\nu(x,\mathbf{p}_\perp^2)$$

Boer-Mulders
$$h_1^{\perp\nu}(x,\mathbf{p}_\perp^2) \;=\; \left(C_S^2 N_S^{\nu 2} + C_A^2 \Big(\frac{1}{3} N_0^{\nu 2} + \frac{2}{3} N_1^{\nu 2}\Big)\right) f^\nu(x,\mathbf{p}_\perp^2),$$

$$f^\nu(x,\mathbf{p}_\perp^2) \;=\; -C_F\alpha_s\left[\mathbf{p}_\perp^2 + x(1-x)(-M^2 + \frac{m_D^2}{1-x} + \frac{m_q^2}{x})\right]\frac{1}{\mathbf{p}_\perp^2}\ln\left[1 + \frac{\mathbf{p}_\perp^2}{x(1-x)(-M^2 + \frac{m_D^2}{1-x} + \frac{m_q^2}{x})}\right]$$
$$\times \frac{\ln(1/x)}{\pi\kappa^2} x^{a_1^\nu + a_2^\nu - 1}(1-x)^{b_1^\nu + b_2^\nu - 1}\exp(-\frac{\mathbf{p}_\perp^2 \ln(1/x)}{\kappa^2(1-x)^2}), \tag{18}$$

write $$h_1^{\perp\nu}(x,\mathbf{p}_\perp^2) \simeq \lambda^\nu f_{1T}^{\perp\nu}(x,\mathbf{p}_\perp^2).$$

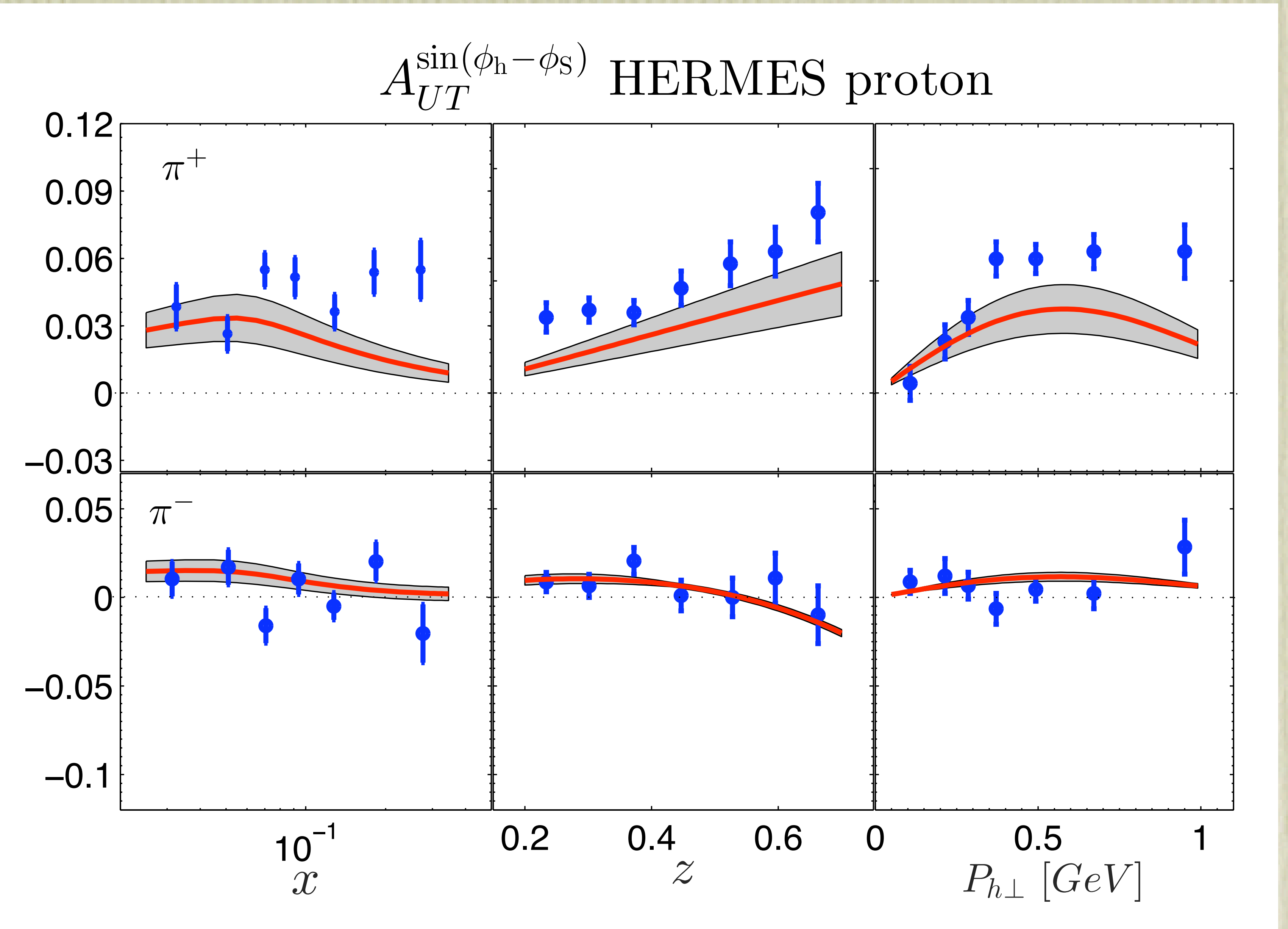
	λ^u	λ^d
LFQDM	2.29	-1.08
Phenomenological fit	2.1 ± 0.1	-1.11 ± 0.02

fit to HERMES/ COMPASS data

[Barone,Melis, Prokudin,PRD81,114026]

Sivers Asymmetry

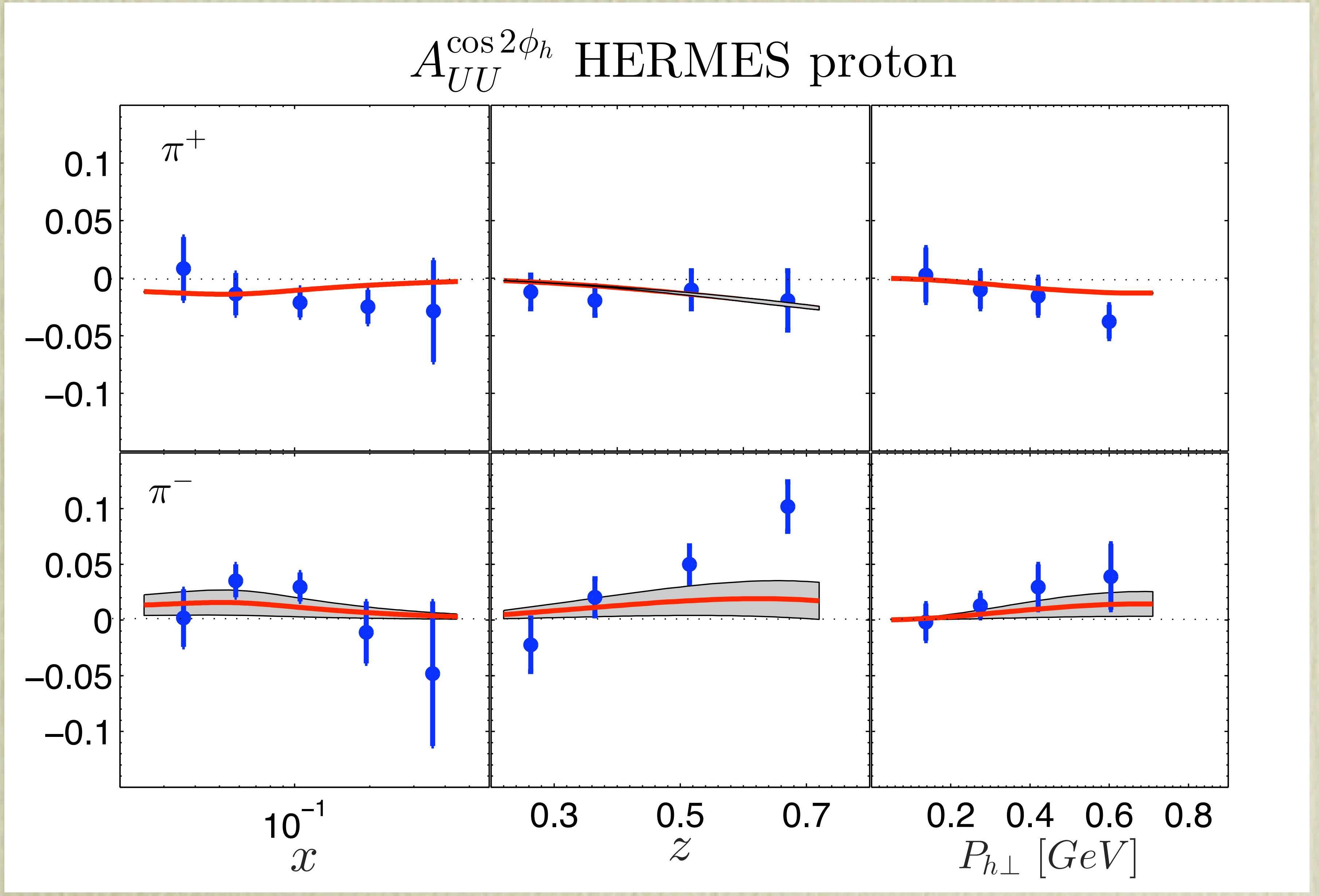
- Sivers asym is extracted by the weigh factor $\sin(\phi_h - \phi_S)$



[A. Airapetian et al.[HERMES Coll], PRL 103,152002]

Boer-Mulders Asymmetry

- extracted with the weight factor $\cos 2\phi_h$



[F. Giordano et al.[HERMES coll], AIP conf. proc.1149, 423]

summary and conclusion

- We presented results for both SSA and DSA in a light front quark-diquark model.
- scale evolution of all TMDs are not known.
- polarized TMDs are taken at initial scale. Two diffeent evol. scheme used for unpol TMD.
- SSA and DSA are compared with HERMES and COMPASS data. Good agreement!
- Different relations among SSA and DSA are found. Interesting to check in other models.

- LFWFs modified to have complex phase factor which is required for Sivers & Boer Mulders functions.
- Sivers -->Lensing function $\simeq \frac{1}{4(1-x)}$
- Sivers & Boer-Mulders asymmetries are consistent with experimental data.

THANK

YOU