

# Leading twist TMDs in a light-front quark-diquark model for proton

---



Tanmay Maji

Indian Institute of Technology Kanpur, India

September 20, 2017

Ref.: TM and Dipankar Chakrabarti, Phys. Rev. D 95, 074009(2017)



Frontiers in Light Front Hadron Physics: Theory and Experiment  
18<sup>th</sup> – 22<sup>nd</sup> September 2017, University of Mumbai, INDIA

# Contents

---

- Introduction
- Light-front quark-diquark model (LFQDM)
- Transverse momentum dependent distributions (TMDs)
- Quark density
- Conclusions

# Introduction

- TMDs: Transverse Momentum dependent parton Distributions.
- In 1990, D. W. Sivers proposed the concept of TMDs for the first time.

— D. W. Sivers PRD 41,(1990)83

$$p_{\perp i} \neq 0$$

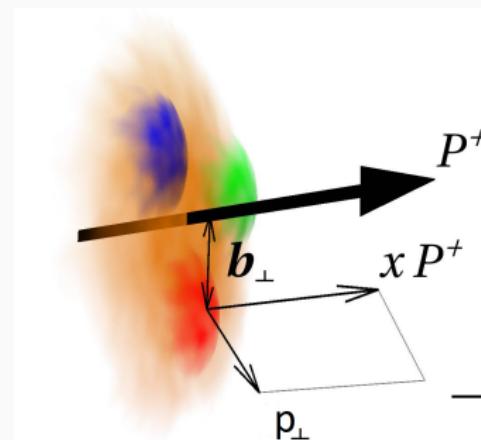
whereas,

$$\sum_i p_{\perp i} = 0 = P_{\perp}$$

$$\text{PDFs : } f^{\nu}(x)$$

↓

$$\text{TMDs : } f^{\nu}(x, \mathbf{p}_{\perp})$$



— Bernhard Musch, arXiv:0907:2381

- TMDs give probability of finding a parton with the longitudinal momentum fraction  $x$  and the transverse momentum  $\mathbf{p}_{\perp}$  inside a nucleon.
- Three dimensional structure in the momentum space provides a better understanding of the correlation between quark polarization and the nucleon polarization.

# Transverse structures in experiments

- In past few decades several experiment collaborations, e.g., HERMES, COMPASS, JLAB etc., observed azimuthal Single Spin Asymmetries(SSAs).
- SIDIS process : $\ell(\ell) + N(P) \rightarrow \ell(\ell') + h(P_h) + X$

$$A_{S_\ell S_N} = \frac{d\sigma^{\ell P^\uparrow \rightarrow \ell' h X} - d\sigma^{\ell P^\downarrow \rightarrow \ell' h X}}{d\sigma^{\ell P^\uparrow \rightarrow \ell' h X} + d\sigma^{\ell P^\downarrow \rightarrow \ell' h X}} \neq 0$$

- Collinear picture is no longer sufficient and transverse structure is need to be considered.

$$\begin{aligned} A_{UU} &\sim f_1(x, \mathbf{p}_\perp^2) \otimes D_1(z, \mathbf{k}_\perp^2) \\ A_{UT}^{\sin(\phi_h - \phi_S)} &\sim f_{1T}^\perp(x, \mathbf{p}_\perp^2) \otimes D_1(z, \mathbf{k}_\perp^2) \\ A_{UT}^{\sin(\phi_h + \phi_S)} &\sim h_1(x, \mathbf{p}_\perp^2) \otimes H_1^\perp(z, \mathbf{k}_\perp^2) \\ A_{UT}^{\sin(3\phi_h - \phi_S)} &\sim h_{1T}^\perp(x, \mathbf{p}_\perp^2) \otimes H_1^\perp(z, \mathbf{k}_\perp^2) \end{aligned}$$

—”D. Chakrabarti’s talk,” ”M. Anselmino’s talk”

- Hence TMDs have attracted a lot of attention in the last few decades.

## So far...

- Being nonperturbative in nature, the TMDs are very difficult to be calculated in full QCD. So, they have been studied in different QCD inspired models.
- **Model calculation:**  
light-cone constituent quark model( Pasquini *et. al.* PRD.78,034025),  
Spectator model (A. Bacchetta *et. al.* PRD 78,074010)  
Bag model(H. Avakian *et. al.* PRD78,114024),
- **Phenomenological extractions:** TMDs are extracted from the experimental data of SSAs considering the Gaussian ansatz—

$$\text{TMD}^\nu(x, \mathbf{p}_\perp^2) = \text{PDF}^\nu(x) \frac{e^{-\mathbf{p}_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

$h_1(x, \mathbf{p}_\perp)$  by Anselmino *et. al.* [PRD75, PRD87]

$h_{1T}^\perp(x, \mathbf{p}_\perp)$  by Prokudin *et. al.* [PRD91] etc.

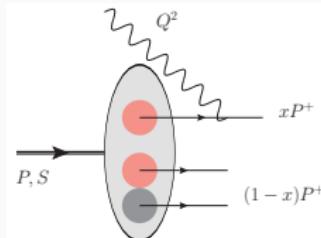
...

- We calculate the leading twist TMDs of proton, from the SIDIS process, in a light-front quark-diquark model(LFQDM) where the wave functions are taken from soft-wall AdS/QCD prediction.

# **Light-front quark-diquark Model(LFQDM)**

---

# Light-front quark-diquark Model(LFQDM)



- In this model proton is considered as a bound state of a quark and a diquark with an effective mass. The diquark can have **spin-0 singlet**(scalar diquark) or **spin-1 triplet**(axial-vector diquark). The proton state is written in the spin-flavor SU(4) structure as<sup>1</sup>

$$|P; \pm\rangle = C_S |u\ S^0\rangle^\pm + C_V |u\ A^0\rangle^\pm + C_{VV} |d\ A^1\rangle^\pm$$

— R. Jakob, P. J. Mulders, j. Rodrigues NPA626(1997)937

- The two particle Fock-state expansion for  $J^z = \pm 1/2$

$$|u\ S\rangle^\pm = \int \frac{dx\ d^2\mathbf{p}_\perp}{2(2\pi)^3 \sqrt{x(1-x)}} \sum_\lambda \psi_\lambda^{\pm(u)}(x, \mathbf{p}_\perp) |\lambda\ \Lambda_S; xP^+, \mathbf{p}_\perp\rangle \Big|_{\Lambda_S=0}$$

$$|\nu\ A\rangle^\pm = \int \frac{dx\ d^2\mathbf{p}_\perp}{2(2\pi)^3 \sqrt{x(1-x)}} \sum_\lambda \sum_{\Lambda_A} \psi_{\lambda\Lambda_A}^{\pm(\nu)}(x, \mathbf{p}_\perp) |\lambda\ \Lambda_A; xP^+, \mathbf{p}_\perp\rangle \Big|_{\Lambda_A=1,0,-1}$$

---

<sup>1</sup> TM, D.Chakrabarti, PRD94,094020(2016)

# Wave function

- The light-front wave functions:

$$\psi_{\lambda\Lambda}^{\pm(\nu)}(x, \mathbf{p}_\perp) = N^\nu f(x, \mathbf{p}_\perp, \lambda, \Lambda) \varphi_i^{(\nu)}(x, \mathbf{p}_\perp) \Big|_{i=1,2}$$

- Normalized by quark counting rules.
- Modified soft-wall AdS/QCD wave function<sup>2</sup> for two particle bound state:

$$\varphi_i^{(\nu)}(x, \mathbf{p}_\perp) = \frac{4\pi}{\kappa} \sqrt{\frac{\log(1/x)}{1-x}} x^{a_i^\nu} (1-x)^{b_i^\nu} \exp \left[ -\delta^\nu \frac{\mathbf{p}_\perp^2}{2\kappa^2} \frac{\log(1/x)}{(1-x)^2} \right].$$

with the AdS/QCD scale parameter  $\kappa = 0.4 \text{ GeV}$

- We determine the parameters  $a_i^\nu$ ,  $b_i^\nu$ ,  $\delta^\nu$  by fitting the experimental data of the Dirac  $F_1(Q^2)$  and Pauli  $F_2(Q^2)$  form factors.

—T.Maji, D.Chakrabarti, PRD94,094020(2016)

---

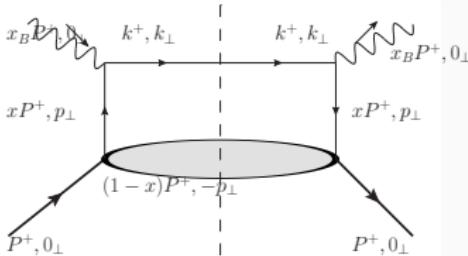
<sup>2</sup>G. F. de Teramond and S. J. Brodsky, arXiv:1203.4025 [hep-ph].

D. Chakrabarti and C. Mondal, Eur. Phys. J. C **73**, 2671 (2013).

# **Transverse momentum dependent distributions (TMDs) in the LFQDM**

---

# Definition of TMDs



- TMD correlator:

$$\Phi^\nu[\Gamma](x, \mathbf{p}_\perp; S) = \frac{1}{2} \int \frac{dz^- d^2 z_T}{2(2\pi)^3} e^{ip \cdot z} \langle P; S | \bar{\psi}^\nu(0) \Gamma \psi^\nu(z) | P; S \rangle \Big|_{z^+ = 0}$$

- Leading twist TMDs are projected out using different Dirac structure as:

$$\Phi^\nu[\gamma^+](x, \mathbf{p}_\perp; S) = f_1^\nu(x, \mathbf{p}_\perp^2) - \frac{\epsilon_T^{ij} p_\perp^i S_T^j}{M} f_{1T}^{\perp\nu}(x, \mathbf{p}_\perp^2),$$

$$\Phi^\nu[\gamma^+ \gamma^5](x, \mathbf{p}_\perp; S) = \lambda g_{1L}^\nu(x, \mathbf{p}_\perp^2) + \frac{\mathbf{p}_\perp \cdot \mathbf{S}_T}{M} g_{1T}^\nu(x, \mathbf{p}_\perp^2),$$

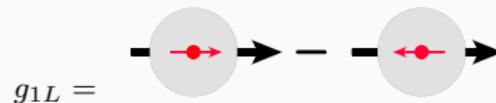
$$\begin{aligned} \Phi^\nu[i\sigma^j + \gamma^5](x, \mathbf{p}_\perp; S) &= S_T^j h_1^\nu(x, \mathbf{p}_\perp^2) + \lambda \frac{p_\perp^j}{M} h_{1L}^{\perp\nu}(x, \mathbf{p}_\perp^2) \\ &\quad + \frac{2p_\perp^j \mathbf{p}_\perp \cdot \mathbf{S}_T - S_T^j \mathbf{p}_\perp^2}{2M^2} h_{1T}^{\perp\nu}(x, \mathbf{p}_\perp^2) - \frac{\epsilon_T^{ij} p_\perp^i}{M} h_1^{\perp\nu}(x, \mathbf{p}_\perp^2). \end{aligned}$$

- Altogether 8 TMDs at the leading twist: **6 T-even** and **2 T-odd**.

# Helicity TMD: $g_{1L}^\nu(x, \mathbf{p}_\perp^2)$

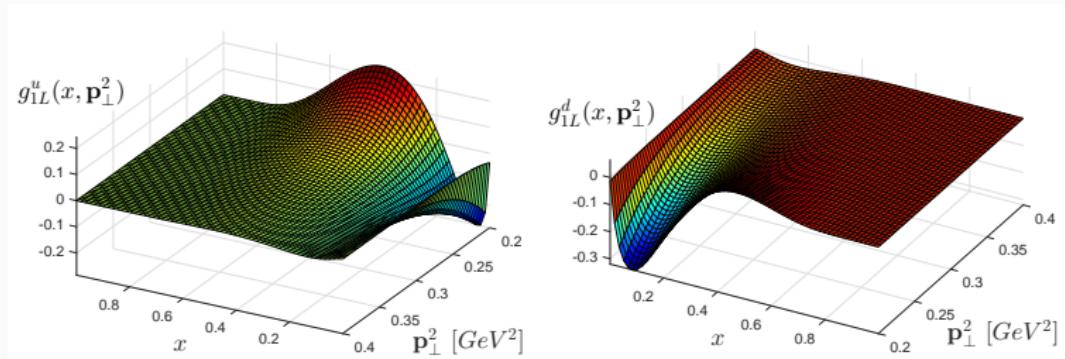
- Helicity TMD is defined as:

$$\Phi^{\nu[\gamma^+ \gamma^5]}(x, \mathbf{p}_\perp; +) = \frac{1}{2} \int \frac{dz^- d^2 z_T}{2(2\pi)^3} e^{ip.z} \langle P; + | \bar{\psi}^\nu(0) \gamma^+ \gamma^5 \psi^\nu(z) | P; + \rangle \Big|_{z^+=0}$$



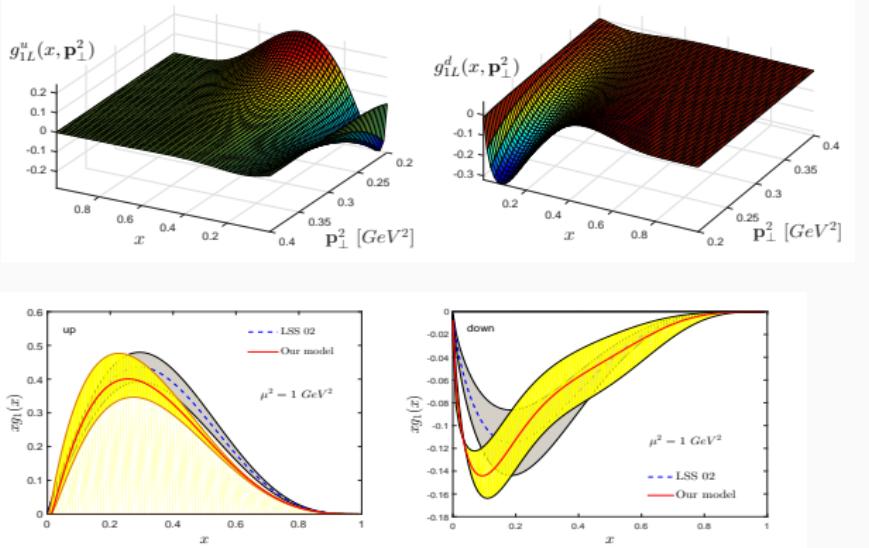
- In the LFQDM:

$$g_{1L}^\nu(x, \mathbf{p}_\perp^2) = N_{g_{1L}}^\nu \frac{\ln(1/x)}{\pi \kappa^2} \left[ x^{2a_1^\nu} (1-x)^{2b_1^\nu - 1} - \frac{\mathbf{p}_\perp^2}{M^2} x^{2a_2^\nu - 2} (1-x)^{2b_2^\nu - 1} \right] e^{-\mathbf{p}_\perp^2 \frac{\delta^\nu \ln(1/x)}{\kappa^2 (1-x)^2}}$$



# Helicity TMD: $g_{1L}^\nu(x, \mathbf{p}_\perp^2)$

$$\int d^2 \mathbf{p}_\perp \dots$$



$$\mu^2 = 1 \text{ GeV}^2$$

**Our result:**

Measured Data:

$$g_A^u$$

$$0.71 \pm 0.09$$

$$0.82 \pm 0.07$$

$$g_A^d$$

$$-0.54^{+0.19}_{-0.13}$$

$$-0.45 \pm 0.07$$

$$g_A = g_A^u - g_A^d$$

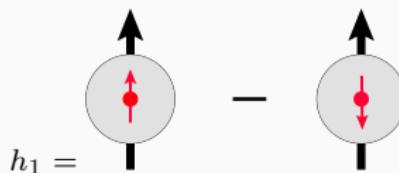
$$1.25^{+0.28}_{-0.22}$$

$$1.27 \pm 0.14$$

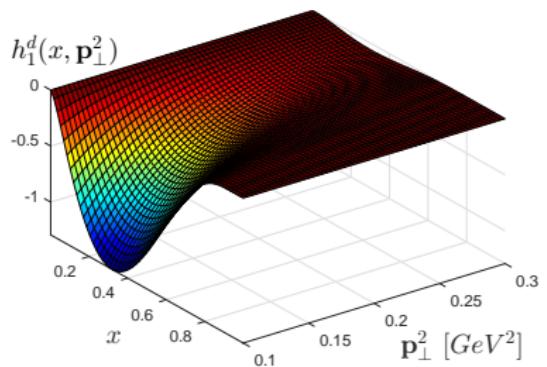
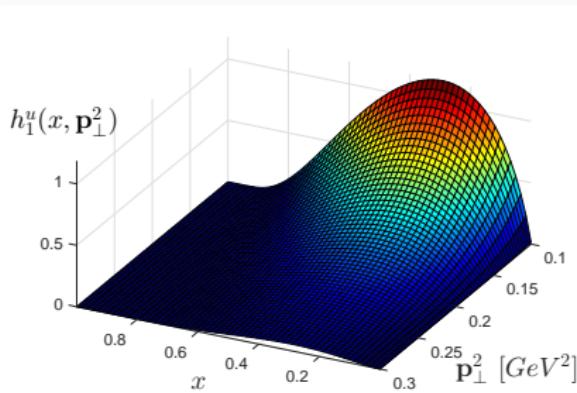
# Transversity TMD: $h_1^\nu(x, \mathbf{p}_\perp^2)$

- Transversity TMD is defined as:

$$h_1^\nu(x, \mathbf{p}_\perp^2) = \left( C_S^2 N_S^\nu - C_V^2 \frac{1}{3} N_0^\nu \right) \frac{\ln(1/x)}{\pi \kappa^2} x^{2a_1^\nu} (1-x)^{2b_1^\nu - 1} e^{-\mathbf{p}_\perp^2 \frac{\delta_1^\nu \ln(1/x)}{\kappa^2 (1-x)^2}}$$

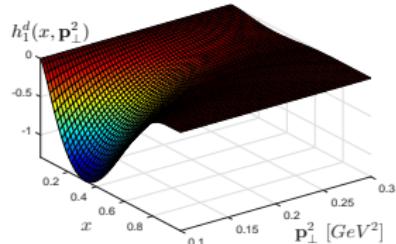
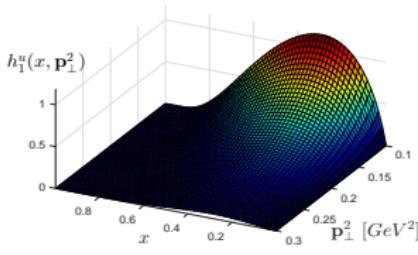


- In the LFQDM:

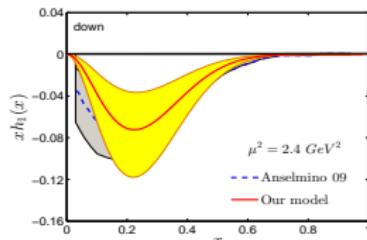
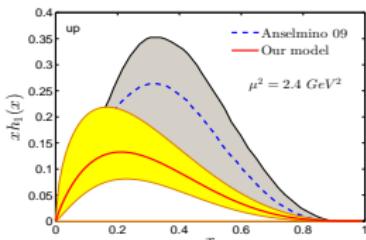


# Transversity TMD: $h_1^\nu(x, \mathbf{p}_\perp^2)$

$$\int d^2 \mathbf{p}_\perp \dots$$



$$\int dx \dots$$



$$\mu^2 = 0.8 \text{ GeV}^2$$

Our result:

Measured Data:

$$g_T^u \\ \textcolor{red}{0.37^{+0.06}_{-0.05}} \\ 0.59^{+0.14}_{-0.13}$$

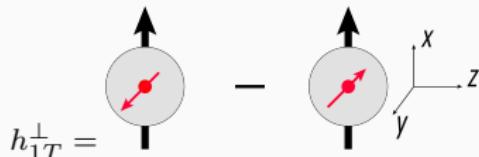
$$g_T^d \\ \textcolor{red}{-0.14^{+0.05}_{-0.06}} \\ -0.20^{+0.05}_{-0.07}$$

$$g_T = g_T^u - g_T^d \\ \textcolor{red}{0.51^{+0.12}_{-0.11}} \\ 0.79^{+0.19}_{-0.20}$$

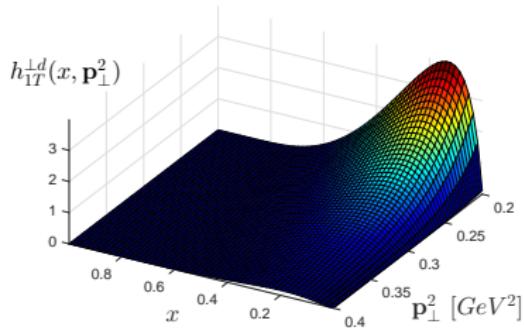
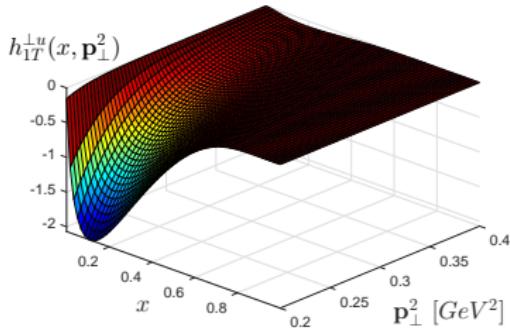
# Pretzelosity TMD: $h_{1T}^\perp(x, \mathbf{p}_\perp^2)$

- Pretzelosity TMD is defined as:

$$h_{1T}^\nu(x, \mathbf{p}_\perp^2) = - \left( C_S^2 N_S^{\nu 2} - C_V^2 \frac{1}{3} N_0^{\nu 2} \right) \frac{2 \ln(1/x)}{\pi \kappa^2} x^{2a_2^\nu - 2} (1-x)^{2b_2^\nu - 1} \\ \times \exp \left[ - \mathbf{p}_\perp^2 \delta^\nu \frac{\ln(1/x)}{\kappa^2 (1-x)^2} \right],$$

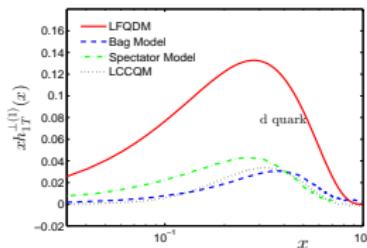
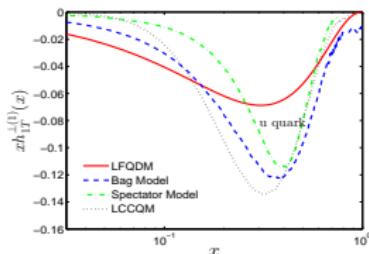
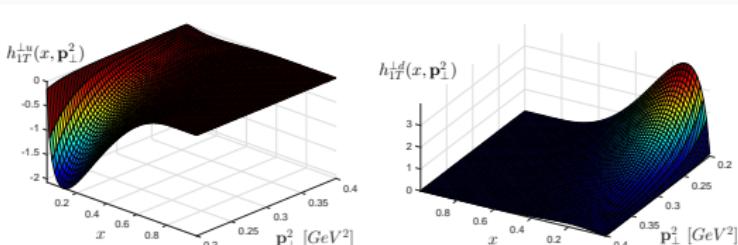


- In the LFQDM:

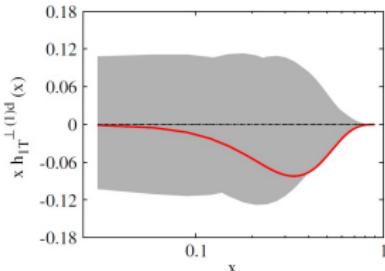
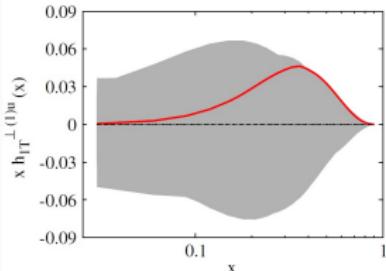


# Pretzelosity TMD: $h_{1T}^{\perp}(x, \mathbf{p}_{\perp}^2)$

$$\int d^2 \mathbf{p}_{\perp} \dots$$



Phenomenological extraction [Prokudin *et.al.* PRD91,034010(2015)]



# TMDs evolution

- A complete QCD evolutions of all the leading twist TMDs are not known.
- A series of papers by Collins(2011,2012), Aybat and Rogers(2011) have proposed QCD evolution scheme for  $f_1(x, \mathbf{p}_\perp^2)$  and  $D_1^{h/\nu}(z, \mathbf{k}_\perp^2)$ .
- The scale evolution of TMDs in the coordinate space is defined as

$$\tilde{F}(x, \mathbf{b}_\perp; \mu) = \tilde{F}(x, \mathbf{b}_\perp; \mu_0) \exp \left( \ln \frac{\mu}{\mu_0} \tilde{K}(b_\perp; \mu) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F(\mu', \frac{\mu^2}{\mu'^2}) \right),$$

Where the  $\tilde{F}(x, \mathbf{b}_\perp; \mu)$  represents the T-even TMDs at scale  $\mu$ .  $\tilde{K}(b_\perp; \mu)$  is given by

$$\tilde{K}(b_\perp; \mu) = -\frac{\alpha_s C_F}{\pi} [\ln(b_*^2 \mu_b^2) - \ln(4) + 2\gamma_E] + \left[ \int_{\mu}^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(\mu') \right] - g_K(b_T)$$

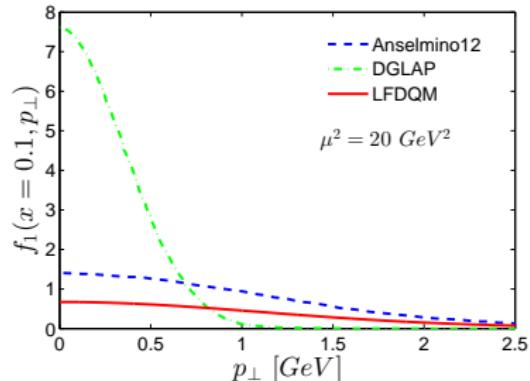
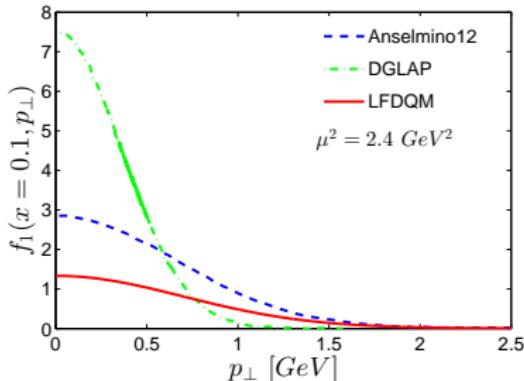
$$b_*(b_T) = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{max}^2}}} \Big|_{b_{max}=0.5 \text{ GeV}^{-1}}$$
$$\mu_b = \frac{C_1}{b_*(b_T)} \Big|_{C_1=2e^{-\gamma_E}}$$

—Collins(2011,2012), Aybat and Rogers(2011)

# TMDs evolution

- A reduced kernel approach ( $C_1/b_{max} = \mu_b (b_\perp \rightarrow \infty)$ ) for TMDs evolution is presented by Anselmino, PRD86,014028(2012) and extended to spin dependent TMDs e.g.,  $f_{1T}^\perp(x, \mathbf{p}_\perp^2)$

$$f(x, \mathbf{p}_\perp^2, \mu) = \int d^2 \mathbf{b}_\perp e^{i \mathbf{b}_\perp \cdot \mathbf{p}_\perp} \tilde{F}(x, \mathbf{b}_\perp^2, \mu)$$



—M. Anselmino *et.al.* PRD 86, 014028 (2012)

TMDs evolution from DGLAP :

$$\text{TMD}^\nu(x, \mathbf{p}_\perp^2; \mu) = \text{PDF}^\nu(x; \mu) \frac{e^{-\mathbf{p}_\perp^2 / \langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

# **Quark density in LFQDM**

---

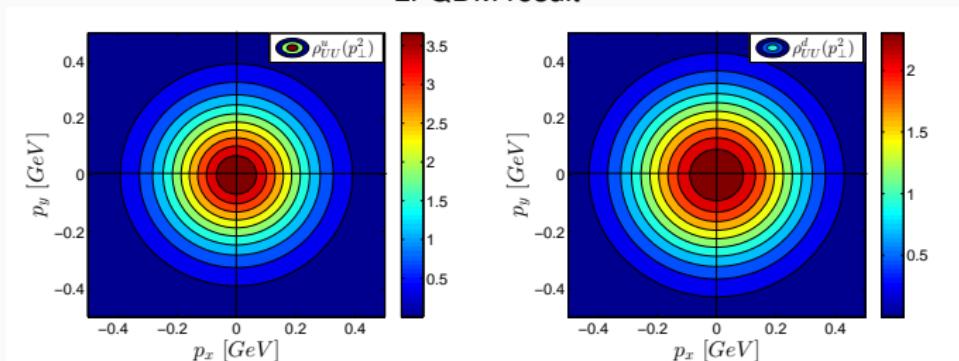
# Quark density

- Mellin moments of the TMDs can be interpreted as a quark density inside the proton as:

$$\begin{aligned}\rho_{UU}^\nu(\mathbf{p}_\perp) &= f_1^{\nu(1)}(\mathbf{p}_\perp^2), \\ \rho_{TL}^\nu(\mathbf{p}_\perp; \mathbf{S}_\perp, \lambda) &= \frac{1}{2} f_1^{\nu(1)}(\mathbf{p}_\perp^2) + \frac{\lambda}{2} \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M} g_{1T}^{\nu(1)}(\mathbf{p}_\perp^2).\end{aligned}$$

- Density of unpolarised quark in an unpolarised proton,  $\rho_{UU}^\nu(\mathbf{p}_\perp)$ , is circularly symmetric for u and d quarks.

LFQDM result

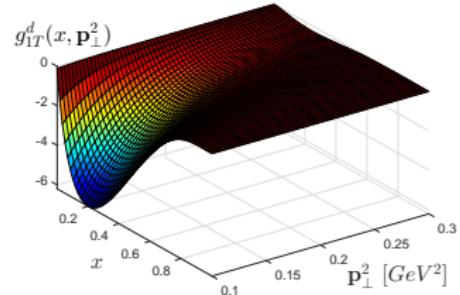
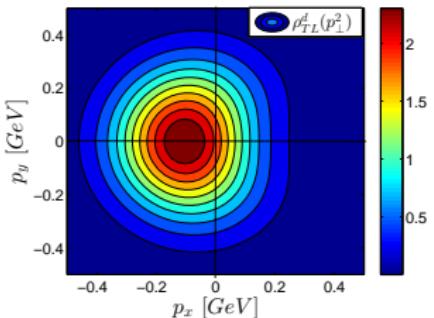
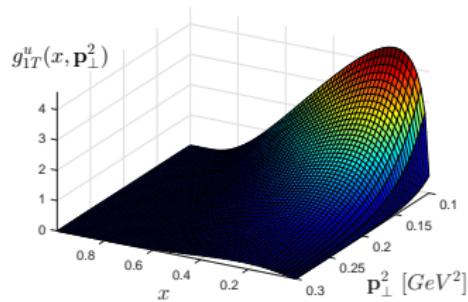
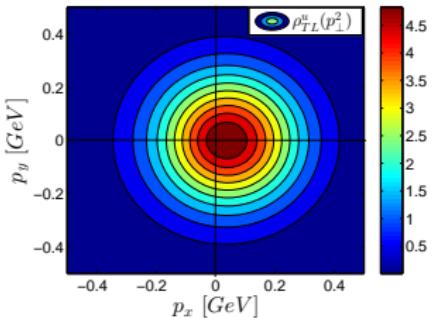


$\rho_{XY}^\nu(\mathbf{p}_\perp) : X = \text{polarization of Proton}; Y = \text{Polarization of quark}$

# Quark density

$$\rho_{TL}^\nu(\mathbf{p}_\perp; \mathbf{S}_\perp, \lambda) = \frac{1}{2} f_1^{\nu(1)}(\mathbf{p}_\perp^2) + \frac{\lambda}{2} \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M} g_{1T}^{\nu(1)}(\mathbf{p}_\perp^2).$$

- For  $\mathbf{S}_\perp \equiv (1, 0)$ , the density of longitudinally polarised quarks(L) in a transversely polarised(T) proton are

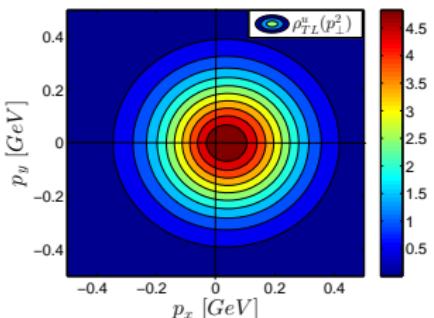


# Quark density

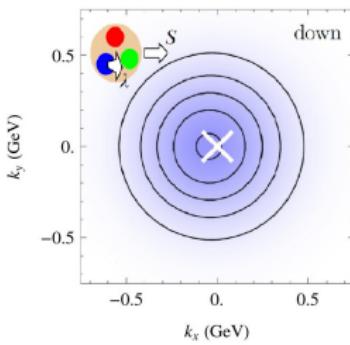
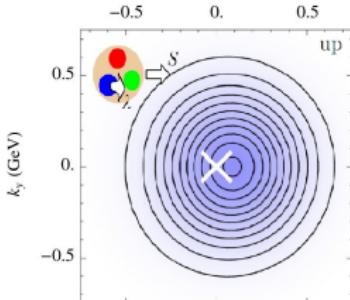
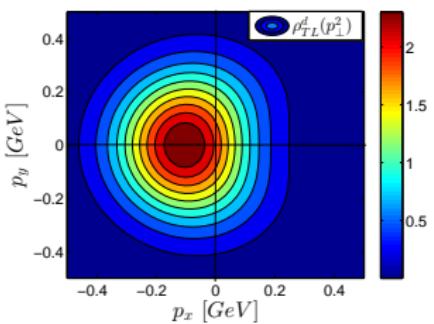
$$\rho_{TL}^\nu(\mathbf{p}_\perp; \mathbf{S}_\perp, \lambda) = \frac{1}{2} f_1^{\nu(1)}(\mathbf{p}_\perp^2) + \frac{\lambda}{2} \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M} g_{1T}^{\nu(1)}(\mathbf{p}_\perp^2).$$

- For  $\mathbf{S}_\perp \equiv (1, 0)$ , the density of longitudinally polarised quarks(L) in a transversely polarised(T) proton are

LFQDM



Lattice [B. Musch, arXiv:0907.2381]



# TMD inequalities

- Soffer bound:

$$|h_1^\nu(x)| \leq \frac{1}{2} \left| f_1^\nu(x) + g_1^\nu(x) \right|$$

- The Soffer bound for TMDs:

$$|h_1^\nu(x, \mathbf{p}_\perp^2)| < \frac{1}{2} \left| f_1^\nu(x, \mathbf{p}_\perp^2) + g_{1L}^\nu(x, \mathbf{p}_\perp^2) \right|.$$

- Relations valid in QCD and all models:

$$\begin{aligned} f_1^\nu(x, \mathbf{p}_\perp^2) &> 0, \\ |g_{1L}^\nu(x, \mathbf{p}_\perp^2)| &< |f_1^\nu(x, \mathbf{p}_\perp^2)|. \end{aligned}$$

- Relations found in other models like Bag model, LCCQM and are generic for diquark models

$$\begin{aligned} |f_1^\nu(x, \mathbf{p}_\perp^2)| &> |h_1^\nu(x, \mathbf{p}_\perp^2)|, \\ |f_1^\nu(x, \mathbf{p}_\perp^2)| &> |h_{1T}^\nu(x, \mathbf{p}_\perp^2)|. \end{aligned}$$

## **Summary and Conclusion**

---

## Summary and conclusions

- The T-even TMDs are discussed in a light-front quark-diquark model of the proton. The model includes both scalar and vector diquarks and the light front wave functions are constructed from soft-wall AdS/QCD predictions.
- PDF limits of TMDs have reasonably good agreement with phenomenology.
- For transversely polarized proton the quark densities are found to be non-spherical and a right(left) shift is observed for u quark(d quark) like Lattice result.
- The transversity TMD is found to satisfy the Soffer bound for TMDs and other inequalities.



Thank you!



## Form factor fitting and the parameters

- In the light-front formalism, for a spin- $\frac{1}{2}$  composite particle system the Dirac and Pauli form factors are defined as

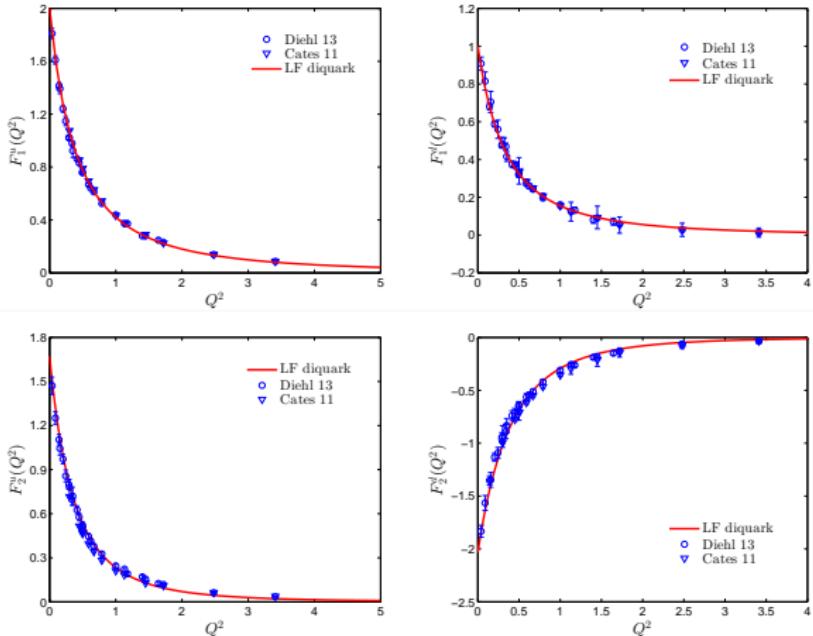
$$\begin{aligned}\langle P + q; + | \frac{J^+(0)}{2P^+} | P; + \rangle &= F_1(q^2) \\ \langle P + q; + | \frac{J^+(0)}{2P^+} | P; - \rangle &= -(q^1 - iq^2) \frac{F_2(q^2)}{2M}\end{aligned}$$

Where the  $q^2$  is square of the momentum transferred to the nucleon of mass  $M$ .

- The normalization of form factors for proton and neutron are given as  $F_1^p(0) = 1$ ,  $F_2^p(0) = \kappa^p = 1.793$  and  $F_1^n(0) = 0$ ,  $F_2^n(0) = \kappa^n = -1.913$
- Considering the charge and isospin symmetry the nucleon form factors are decomposed into flavour form factors as

$$F_i^{p(n)} = e_u F_i^{u(d)} + e_d F_i^{d(u)}.$$

# Value of the parameters



$\nu$	$a_1^\nu$	$b_1^\nu$	$a_2^\nu$	$b_2^\nu$	$\delta^\nu$
$u$	$0.280 \pm 0.001$	$0.1716 \pm 0.0051$	$0.84 \pm 0.02$	$0.2284 \pm 0.0035$	1.0
$d$	$0.5850 \pm 0.0003$	$0.7000 \pm 0.0002$	$0.9434^{+0.0017}_{-0.0013}$	$0.64^{+0.0082}_{-0.0022}$	1.0

—T.Maji, D.Chakrabarti, PRD94,094020(2016)

# Parameret evolution approach

- In this model:

$$f_1^\nu(x, \mathbf{p}_\perp^2) = N_{f_1}^\nu \frac{\ln(1/x)}{\pi \kappa^2} \left[ x^{2a_1^\nu} (1-x)^{2b_1^\nu - 1} + \frac{\mathbf{p}_\perp^2}{M^2} x^{2a_2^\nu - 2} (1-x)^{2b_2^\nu - 1} \right] e^{-\mathbf{p}_\perp^2 \frac{\delta^\nu \ln(1/x)}{\kappa^2 (1-x)^2}}$$

$$a_i^\nu(\mu) \quad b_i^\nu(\mu) \quad \delta^\nu(\mu)$$

- Scale variation of the parameters is determined by fitting the DGLAP evolution for unpolarised PDFs. .

$$a_i^\nu(\mu) = a_i^\nu(\mu_0) + A_i^\nu(\mu),$$

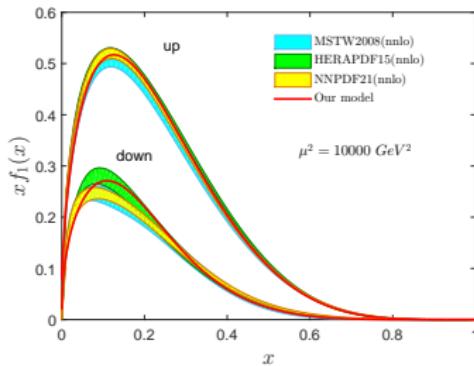
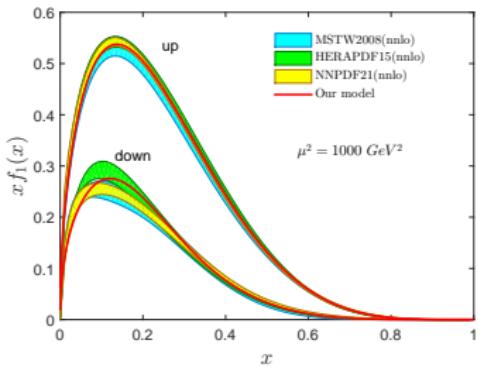
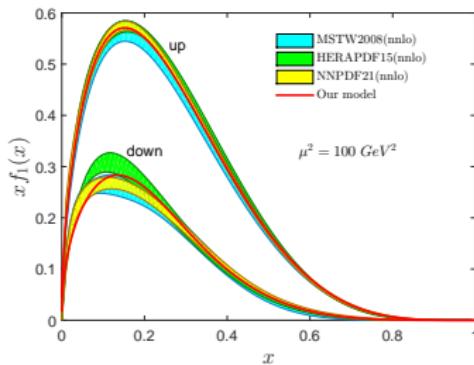
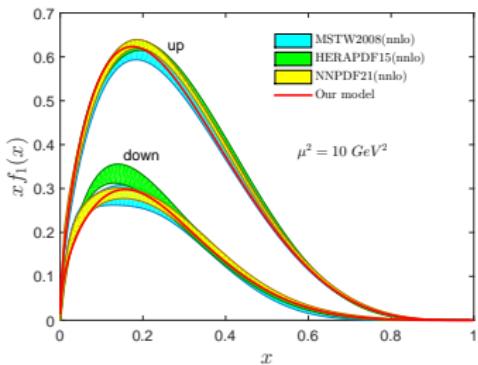
$$b_i^\nu(\mu) = b_i^\nu(\mu_0) - B_i^\nu(\mu) \frac{4C_F}{\beta_0} \ln \left( \frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)} \right),$$

$$\delta^\nu(\mu) = \exp \left[ \delta_1^\nu \left( \ln(\mu^2/\mu_0^2) \right)^{\delta_2^\nu} \right],$$

$$P_i^\nu(\mu) = \alpha_{P,i}^\nu \mu^{2\beta_{P,i}^\nu} \left[ \ln \left( \frac{\mu^2}{\mu_0^2} \right) \right]^{\gamma_{P,i}^\nu} \Big|_{P=A,B \text{ at } i=1,2},$$

TM, D.Chakrabarti, Phys. Rev. D 94,094020(2016)

# Unpol. PDFs evolution upto $\mu^2 = 10^4 \text{ GeV}^2$



- This model evolution predicts the unpolarised PDFs accurately for a wide range of scale.