

Electron Wigner Distributions from Light Front Wave Functions

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Light Cone 2017
18-22 September, 2017
University of Mumbai



Outline

- Parton distribution functions
- Generalized parton distributions
- Transverse momentum distributions
- Wigner distributions
- Light-front QED model
- Results

Parton Distributions Functions (PDFs)

- Parton distribution functions (pdfs) $f(x)$ were introduced by Feynman to describe the DIS and can be interpreted as probability densities to find a parton carrying a momentum fraction x .
- But important piece of information is missing in pdfs, i.e.. how partons are distributed in the plane transverse to the direction in which hadron is moving?

Generalized Parton Distributions (GPDs)

- In recent years it has become clear that appropriate exclusive scattering processes $\gamma^* p \rightarrow \gamma p$ (DVCS) can provide missing information in pdfs encoded in GPDs.

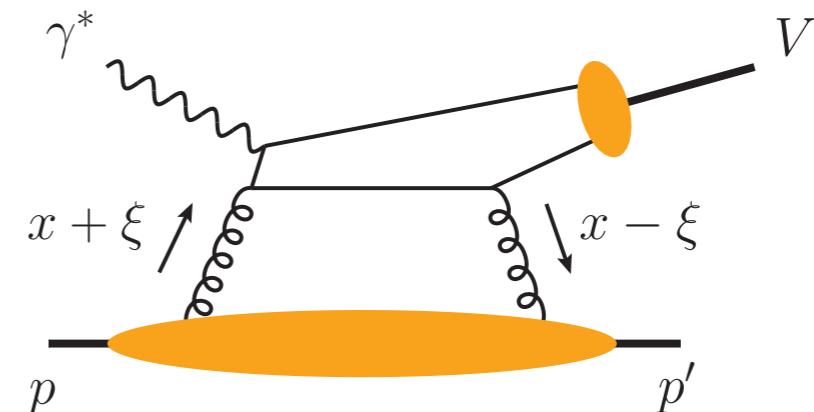
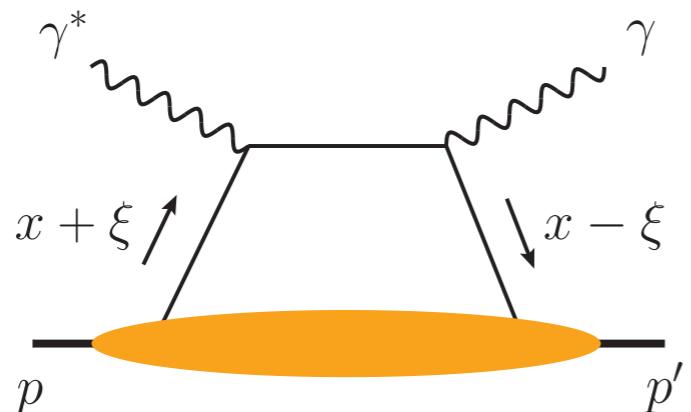


Image courtesy arXiv:1212.1701

- Unlike PDFs, GPDs have dependence on $x, \xi, t = -\vec{\Delta}_\perp^2$

GPDs continued..

- The Fourier transform of GPDs w.r.t impact parameter gives the impact parameter dependent parton distribution function (ipdpf).

$$\mathcal{H}, \mathcal{E}(x, \vec{b}_\perp) = \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} e^{-i \vec{\Delta}_\perp \cdot \vec{b}_\perp} H, E(x, t)$$

-M. Burkardt Phys. Rev. D 62 071503 (2000)

Int. Jou. Mod. Phys. A 18 173 (2002)

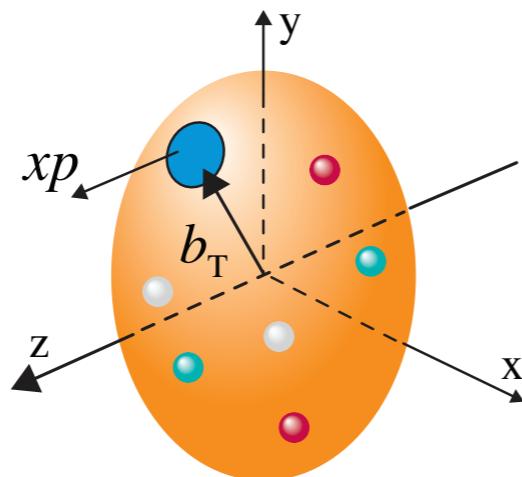


Image courtesy arXiv:1212.1701

- Thus, distribution of parton is obtained in position space i.e.. x and b . GPDs in impact parameter space as probability densities in 2 transverse coordinates and 1 longitudinal momentum.

Transverse momentum distributions (TMDs)

- But still we need information on the distribution of partons in momentum plane.
- TMDs can be accessed through semi-inclusive deep inelastic scattering $lN \rightarrow lhX$ (SIDIS) or Drell-Yan $pN \rightarrow l^+l^-X$ process.
- TMDs can be interpreted as probability densities in 3 momentum space.

8 independent TMDs

$f_1(x, \vec{p}_\perp)$

Unpolarised quarks in an unpolarised nucleon
Unintegrated unpolarised distribution

$g_{1L}(x, \vec{p}_\perp)$ Correlate longitudinal spin of quark with longitudinal spin of nucleon
Unintegrated helicity distribution

$h_{1T}(x, \vec{p}_\perp)$ Correlate transverse spin of quark with transverse spin of nucleon
Unintegrated transverse distribution

$f_{1T}^\perp(x, \vec{p}_\perp)$ Sivers function-correlate unpolarised quark with transversely polarised nucleon

$h_1^\perp(x, \vec{p}_\perp)$ Boer-Mulders function- correlate transversely polarised quark with unpolarised nucleon

$g_{1T}^\perp(x, \vec{p}_\perp)$ $h_{1L}^\perp(x, \vec{p}_\perp)$ $h_{1T}^\perp(x, \vec{p}_\perp)$

Different double spin correlations

Wigner distributions

- Wigner distributions were first introduced by E. Wigner to study quantum corrections to classical statistical mechanics.

-**E. Wigner Phys. Rev. 70 749 (1932)**
- Strict interpretation of definite position and momentum fails for quantum particle due to uncertainty principle.
- Wigner distribution can normally have negative values: smoothing the Wigner distribution (Huisimi distribution) results in a positive-semidefinite function.

Wigner distributions

- In QCD, Wigner distributions were first introduced by Xiangdong Ji

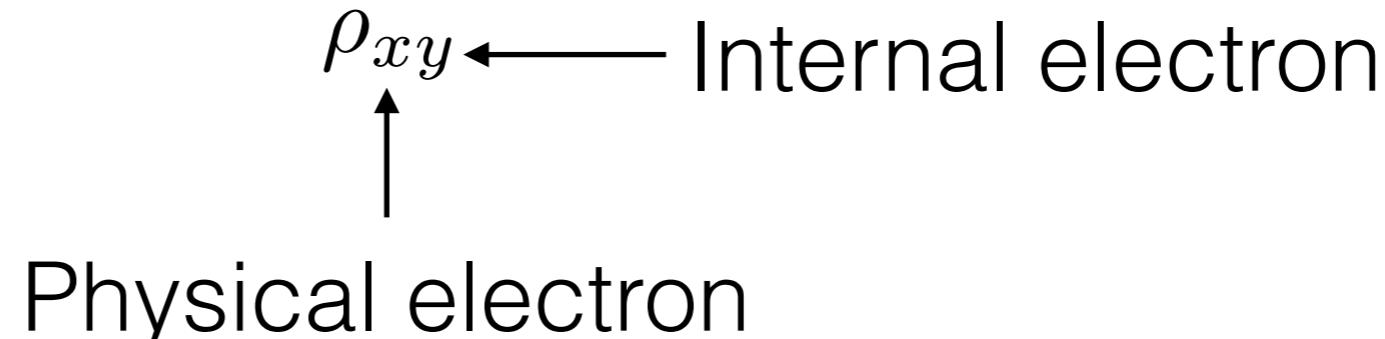
-Phys. Rev. Lett. 91 062001 (2003)

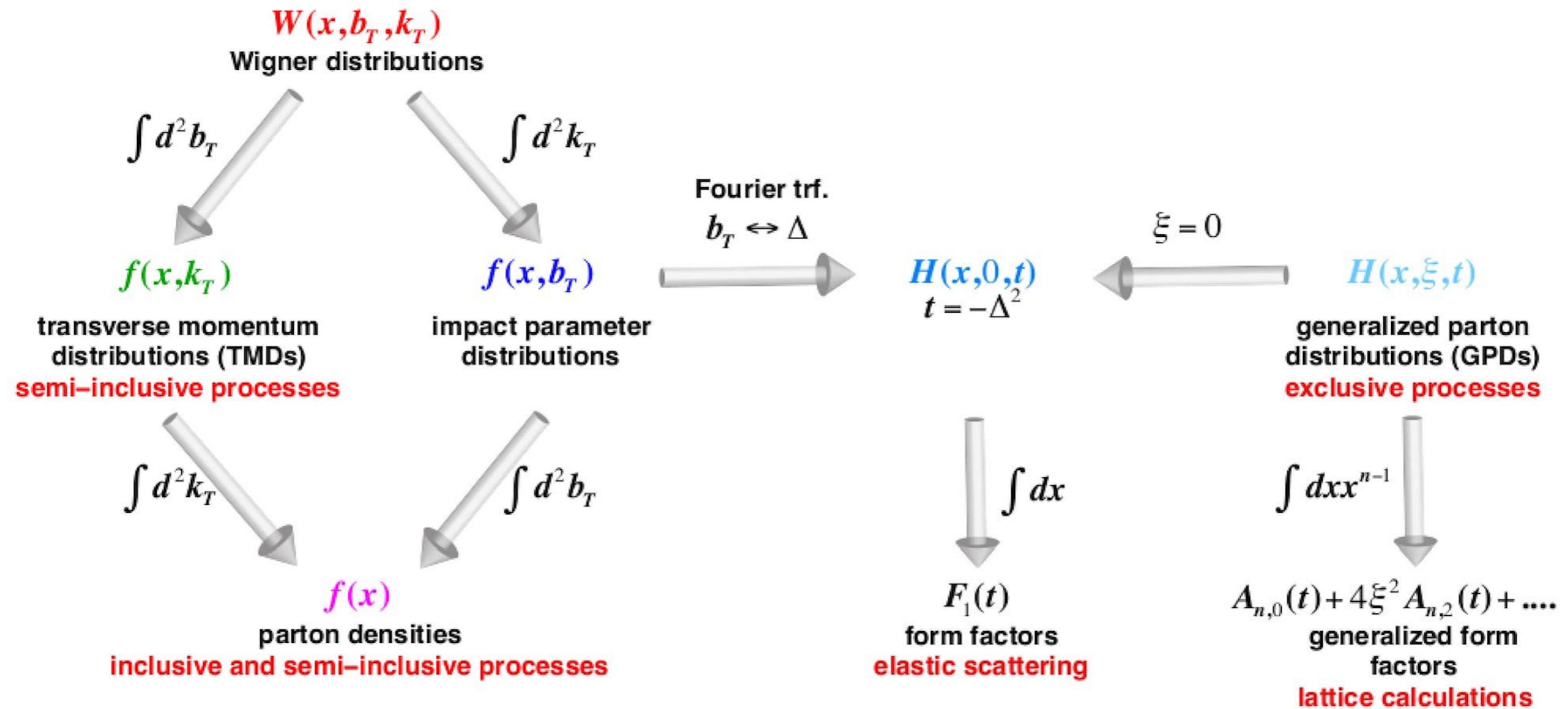
$$W^\Gamma(\vec{\Delta}_\perp, \vec{p}_\perp, x) = \int \frac{dz^- d^2 z_\perp}{(2\pi)^2} e^{ip \cdot z} \langle P' | \bar{\psi}(-z/2) \Gamma \mathcal{W}_{[-\frac{z}{2}, \frac{z}{2}]} \psi(z/2) | P \rangle$$

$$\rho^\Gamma(\vec{b}_\perp, \vec{p}_\perp, x) = \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} e^{i\vec{\Delta} \cdot \vec{b}_\perp} W^\Gamma(\vec{\Delta}_\perp, \vec{p}_\perp, x)$$

$$\Gamma = \gamma^+, \gamma^+ \gamma_5, i\sigma^{+j} \gamma_5$$

- Wigner distributions are designated as ρ_{xy} , where **x** and **y** gives the polarization state of physical electron and internal electron respectively.





- Image courtesy-arXiv:1212.1701

Light front QED model

- Electron is an elementary field in QED.
- When we talk about structure of an electron. We are actually probing the fluctuations in quantum theory.
- One can consider the fluctuation $e \rightarrow e\gamma \rightarrow e$ with same quantum number.
- Bare electron is no more a single particle but surrounded by virtual clouds of e^- , e^+ and γ .
- When virtual cloud interacts with probe, parton content will resolve.

Light front wave functions in QED model

- We evaluate the results for the Wigner distribution of the physical electron by considering it as a two particle state (electron and photon). The two particle Fock state for an electron with $J^z = \pm \frac{1}{2}$ has four possible combinations.

-S.J. Brodsky et. al. Nucl. Phys. B 593 311 (2001)

$$\begin{aligned}
 \psi_{+\frac{1}{2}+1}^\uparrow(x, \vec{p}_\perp) &= -\sqrt{2} \frac{-p^1 + ip^2}{x(1-x)} \varphi & \psi_{+\frac{1}{2}+1}^\downarrow(x, \vec{p}_\perp) &= 0 \\
 \psi_{+\frac{1}{2}-1}^\uparrow(x, \vec{p}_\perp) &= -\sqrt{2} \frac{p^1 + ip^2}{(1-x)} \varphi & \psi_{+\frac{1}{2}-1}^\downarrow(x, \vec{p}_\perp) &= -\sqrt{2} \left(M - \frac{m}{x}\right) \varphi \\
 \psi_{-\frac{1}{2}+1}^\uparrow(x, \vec{p}_\perp) &= -\sqrt{2} \left(M - \frac{m}{x}\right) \varphi & \psi_{-\frac{1}{2}+1}^\downarrow(x, \vec{p}_\perp) &= -\sqrt{2} \frac{-p^1 + ip^2}{(1-x)} \varphi \\
 \psi_{-\frac{1}{2}-1}^\uparrow(x, \vec{p}_\perp) &= 0 & \psi_{-\frac{1}{2}-1}^\downarrow(x, \vec{p}_\perp) &= -\sqrt{2} \frac{p^1 + ip^2}{x(1-x)} \varphi \\
 \psi_{+\frac{1}{2}+1}^\downarrow(x, \vec{p}_\perp) &= 0
 \end{aligned}$$

$$\varphi = \varphi(x, \vec{p}_\perp) = \frac{e}{\sqrt{1-x}} \frac{1}{M^2 - \frac{\vec{p}_\perp^2 + m^2}{x} - \frac{\vec{p}_\perp^2 + \lambda^2}{1-x}} .$$

Unpolarized Wigner distribution

- Physical electron and internal electron is unpolarized

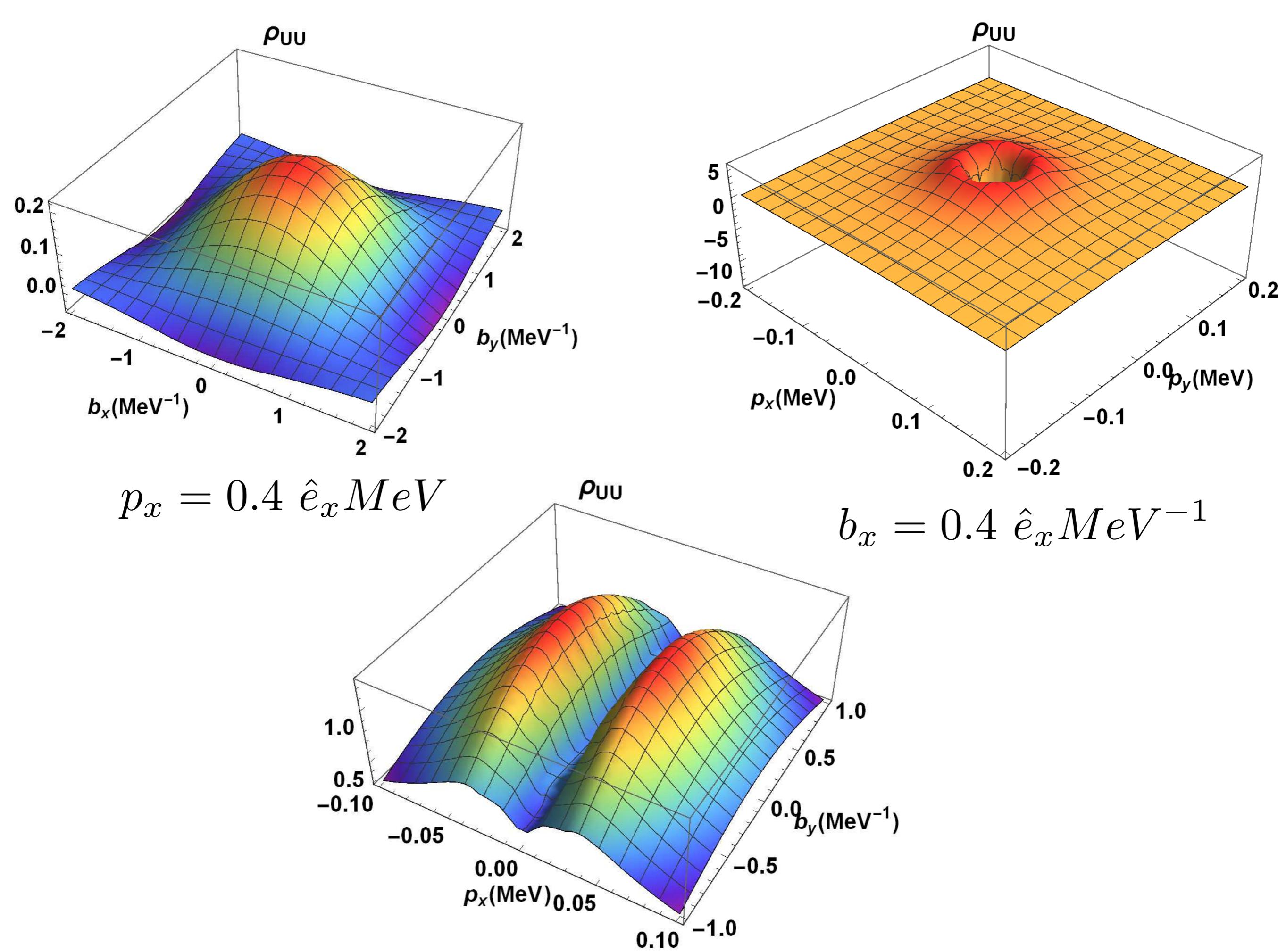
$$\rho_{UU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \frac{1}{2} \left[\rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; +\hat{e}_z) + \rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; -\hat{e}_z) \right],$$

$$\begin{aligned} \rho_{UU}(\mathbf{b}_\perp, \mathbf{p}_\perp) = & \frac{4e^2}{2(2\pi)^2 16\pi^3} \int d\Delta_x d\Delta_y \int dx \cos(\Delta_x b_x + \Delta_y b_y) \left[\frac{1+x^2}{x^2(1-x)^2} \left(\mathbf{p}_\perp^2 - \frac{(1-x)^2}{4} \boldsymbol{\Delta}_\perp^2 \right) \right. \\ & \left. + \left(M - \frac{m}{x} \right)^2 \right] \varphi^\dagger(\mathbf{p}_\perp'') \varphi(\mathbf{p}_\perp'), \end{aligned}$$

- We assume $M=0.51 \text{ MeV}$, $m=0.5 \text{ MeV}$ and $\lambda = 0.02 \text{ MeV}$

$$F_{1,1}(x, \boldsymbol{\Delta}_\perp, \mathbf{p}_\perp) = \frac{4e^2}{2(16\pi^3)} \left[\frac{1+x^2}{x^2(1-x)^2} \left(\mathbf{p}_\perp^2 - \frac{(1-\mathbf{x})^2}{4} \boldsymbol{\Delta}_\perp^2 \right) + \left(\mathbf{M} - \frac{\mathbf{m}}{\mathbf{x}} \right)^2 \right] \varphi^\dagger(\mathbf{p}_\perp'') \varphi(\mathbf{p}_\perp'),$$

- Mother distribution of TMD $f_1(x, \mathbf{p}_\perp)$ (unpolarized distribution) and GPD $\mathbf{H}(\mathbf{x}, \mathbf{0}, \mathbf{t})$.



Longitudinally polarized Wigner distributions

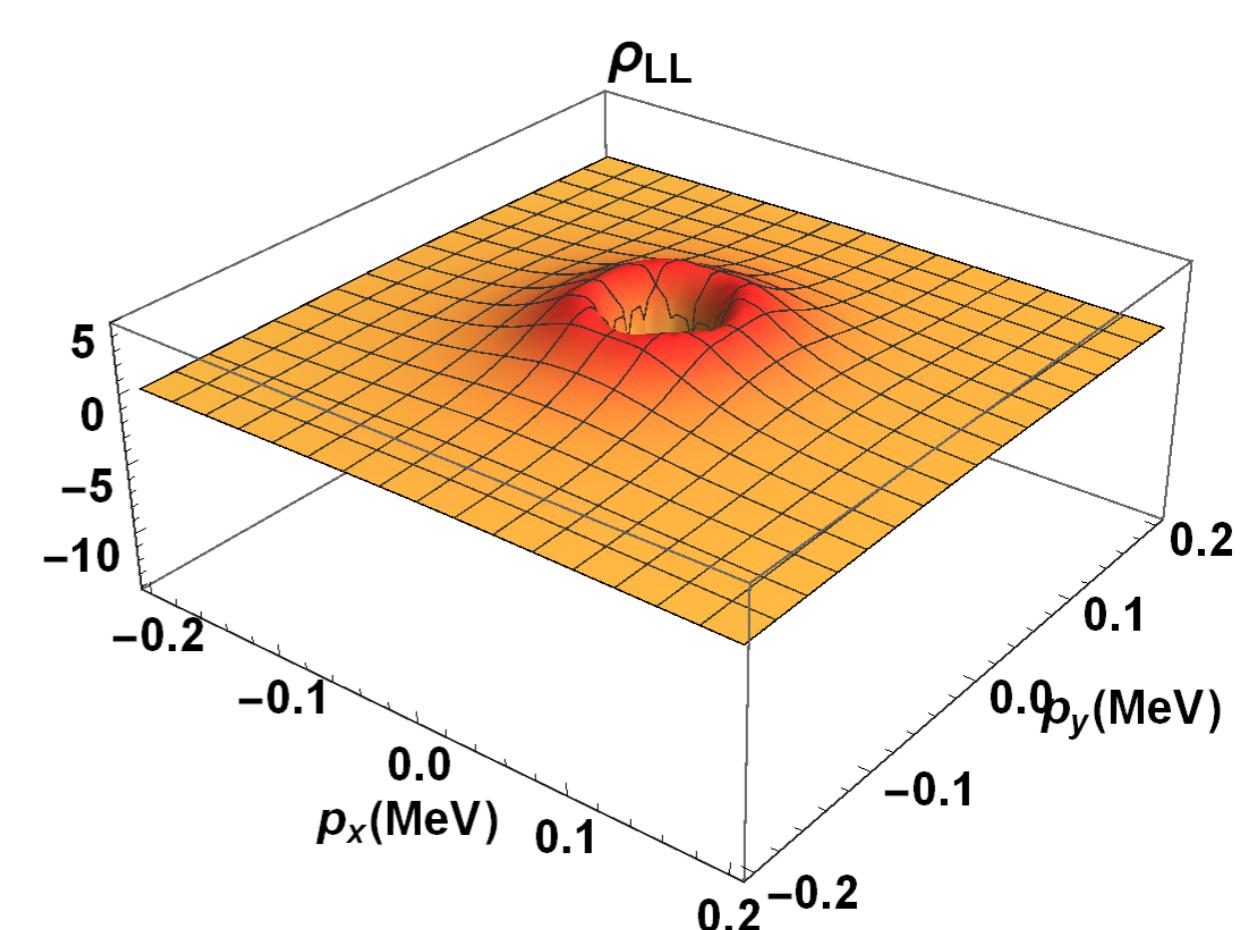
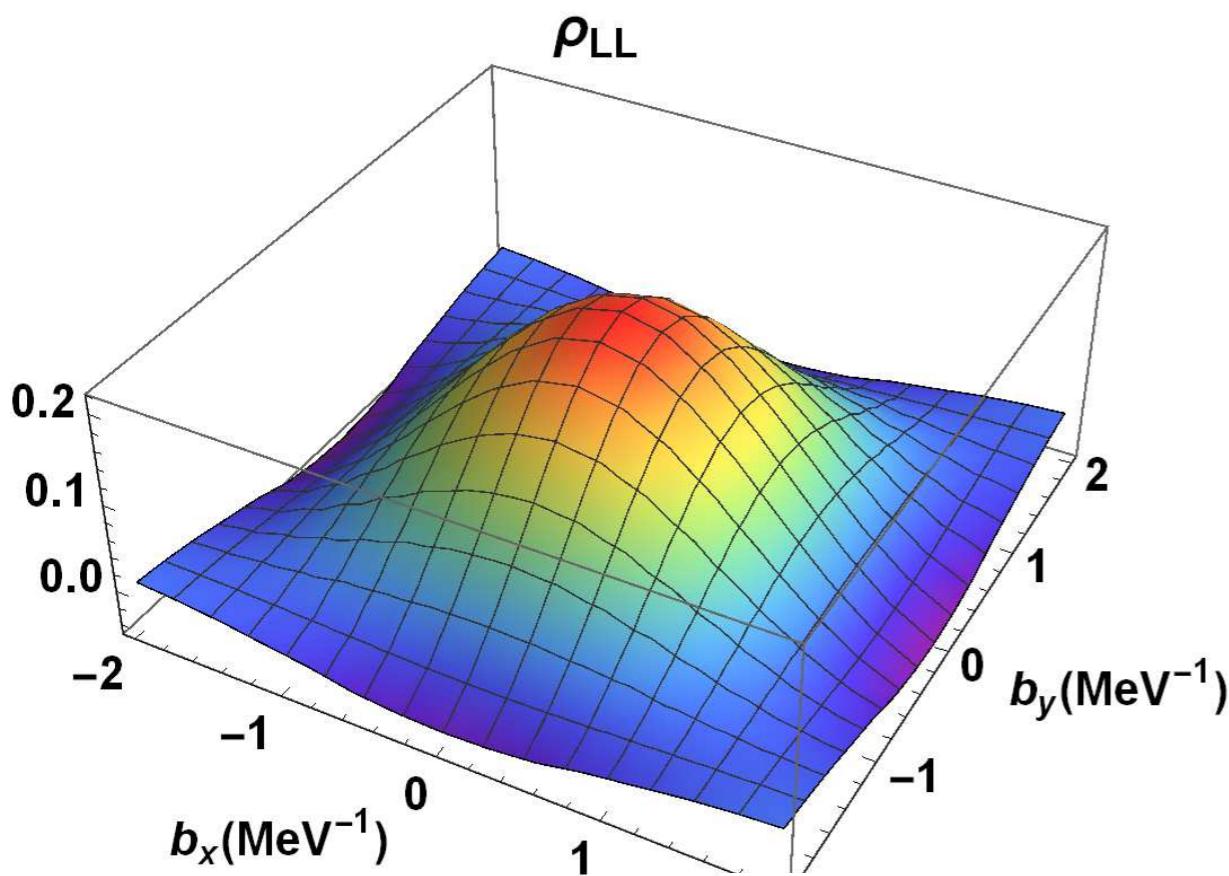
- Both physical and internal electrons are longitudinally polarized.

$$\rho_{LL}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \frac{1}{2} \left[\rho^{[\gamma^+ \gamma_5]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; +\hat{e}_z) - \rho^{[\gamma^+ \gamma_5]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; -\hat{e}_z) \right],$$

$$\begin{aligned} \rho_{LL}(\mathbf{b}_\perp, \mathbf{p}_\perp) = & \frac{4e^2}{2(2\pi)^2 16\pi^3} \int d\Delta_x d\Delta_y \int dx \cos(\Delta_x b_x + \Delta_y b_y) \left[\frac{1+x^2}{x^2(1-x)^2} \left(\mathbf{p}_\perp^2 - \frac{(1-x)^2}{4} \boldsymbol{\Delta}_\perp^2 \right) - \right. \\ & \left. \left(M - \frac{m}{x} \right)^2 \right] \varphi^\dagger(\mathbf{p}'_\perp) \varphi(\mathbf{p}'_\perp) \end{aligned}$$

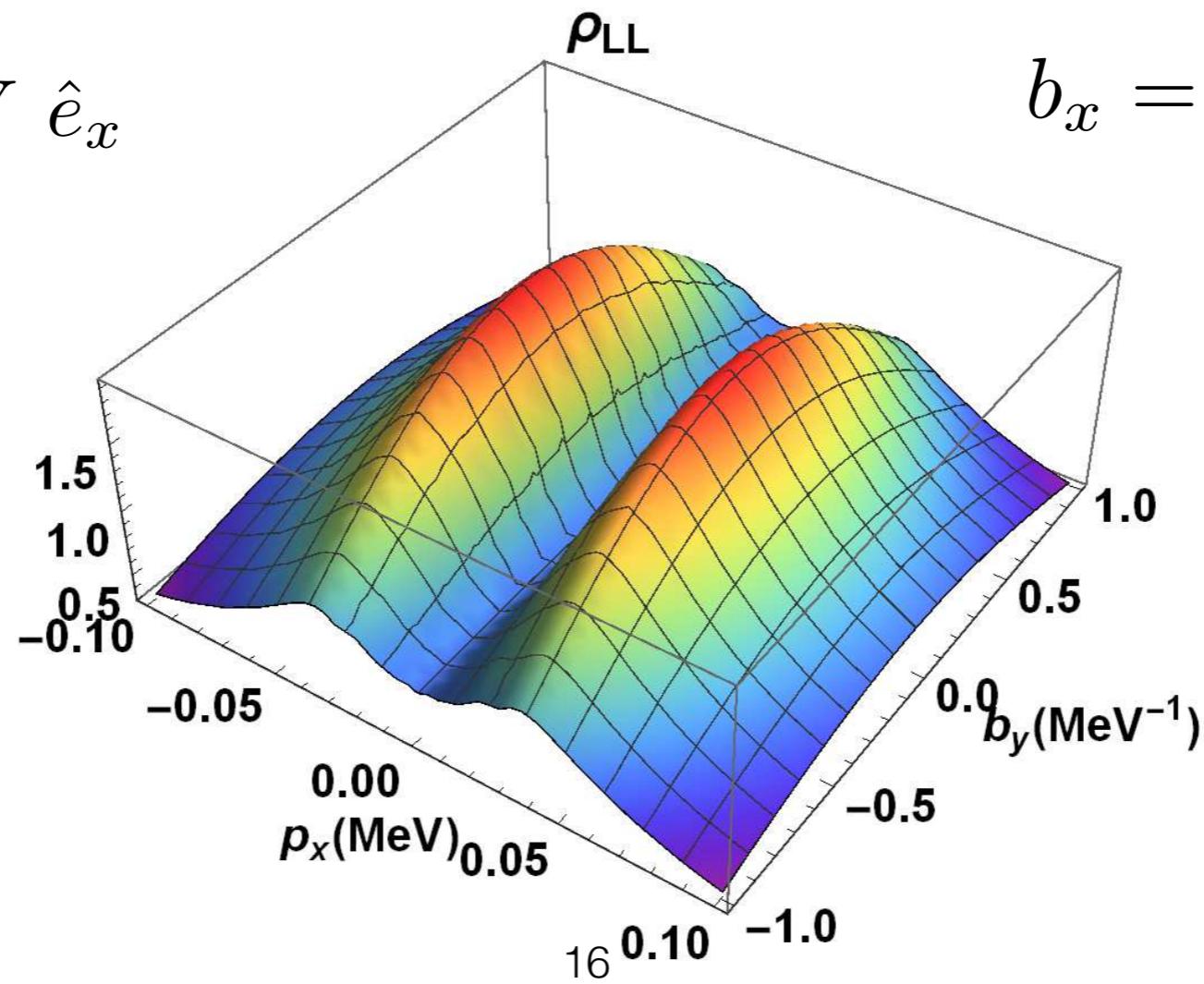
$$\begin{aligned} G_{1,4}(x, \boldsymbol{\Delta}_\perp, \mathbf{p}_\perp) = & \frac{4e^2}{2(16\pi^3)} \left[\frac{1+x^2}{x^2(1-x)^2} \left(\mathbf{p}_\perp^2 - \frac{(1-\mathbf{x})^2}{4} \boldsymbol{\Delta}_\perp^2 \right) - \left(\mathbf{M} - \frac{\mathbf{m}}{\mathbf{x}} \right)^2 \right] \\ & \varphi^\dagger(\mathbf{p}''_\perp) \varphi(\mathbf{p}'_\perp), \end{aligned}$$

- Mother distribution of TMD g_{1L} and GPD $\tilde{H}(x, 0, t)$.



$$p_x = 0.4 \text{ MeV } \hat{e}_x$$

$$b_x = 0.4 \text{ MeV}^{-1} \hat{e}_x$$



Longitudinally-unpolarized Wigner distributions

- Physical electron is longitudinally polarized and internal electron is unpolarized.

$$\rho_{LU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \frac{1}{2} \left[\rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; +\hat{e}_z) - \rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; -\hat{e}_z) \right],$$

$$\rho_{LU}(\mathbf{b}_\perp, \mathbf{p}_\perp) = \frac{4 e^2}{2(2\pi)^2 16\pi^3} \int d\Delta_x d\Delta_y \int dx \sin(\Delta_x b_x + \Delta_y b_y) \frac{(\Delta_x p_y - \Delta_y p_x)}{x^2(1-x)} (x^2 - 1) \varphi^\dagger(\mathbf{p}'_\perp) \varphi(\mathbf{p}'_\perp),$$

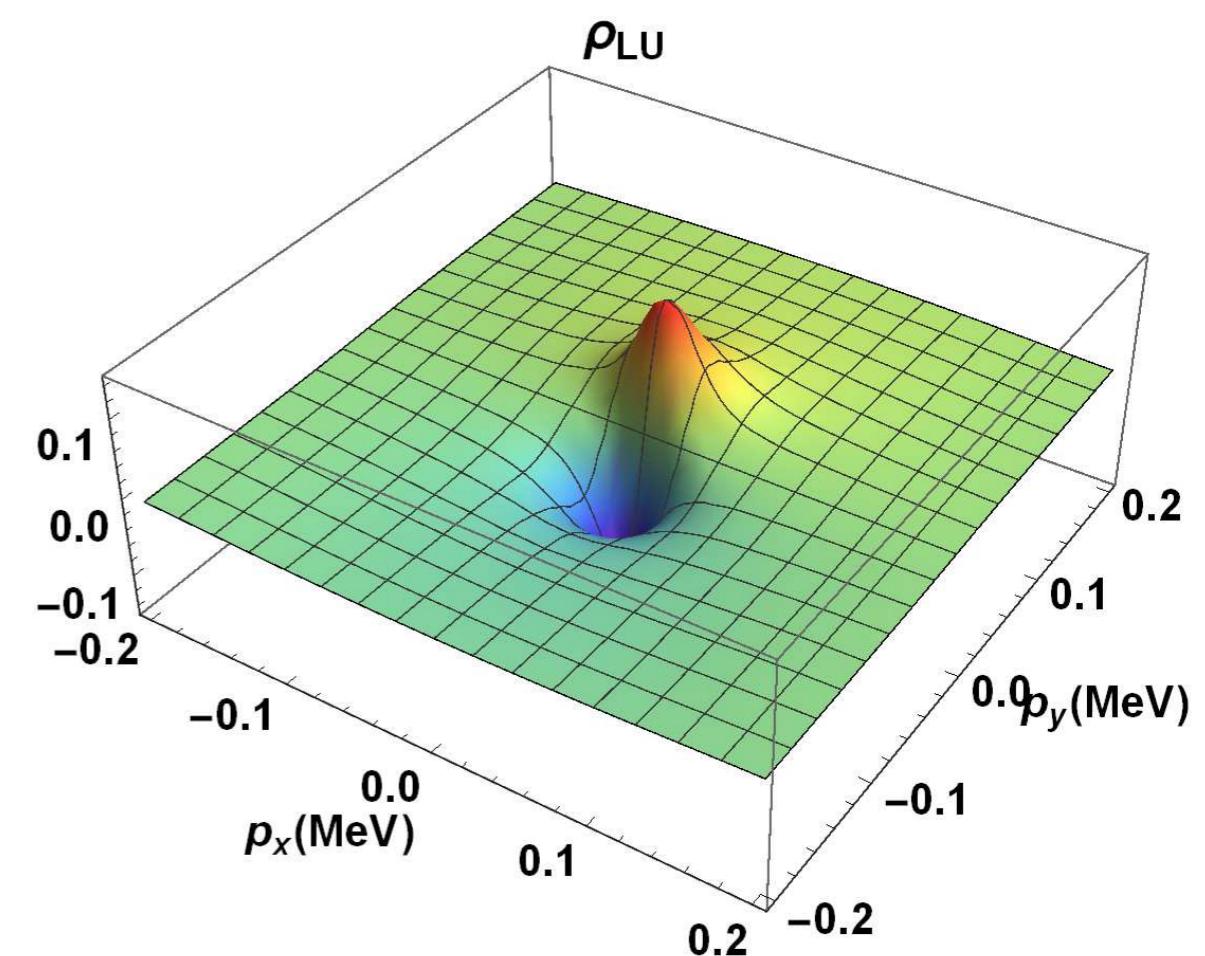
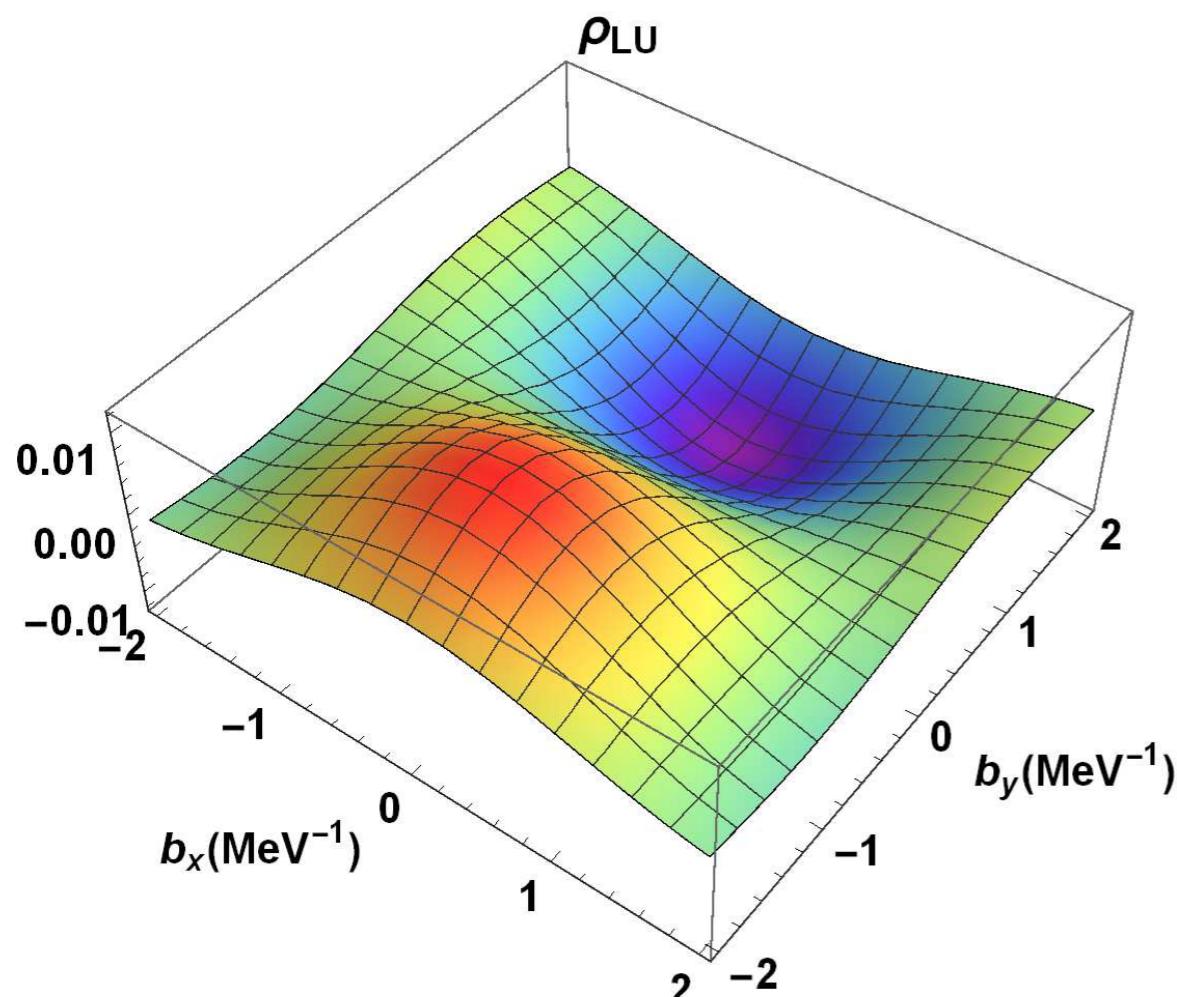
- Connection with orbital-angular momentum issues.

-C. Lorcé and B. Pasquini , PRD 84 014015 (2011)

C. Lorcé et. al., PRD 85 114006 (2012)

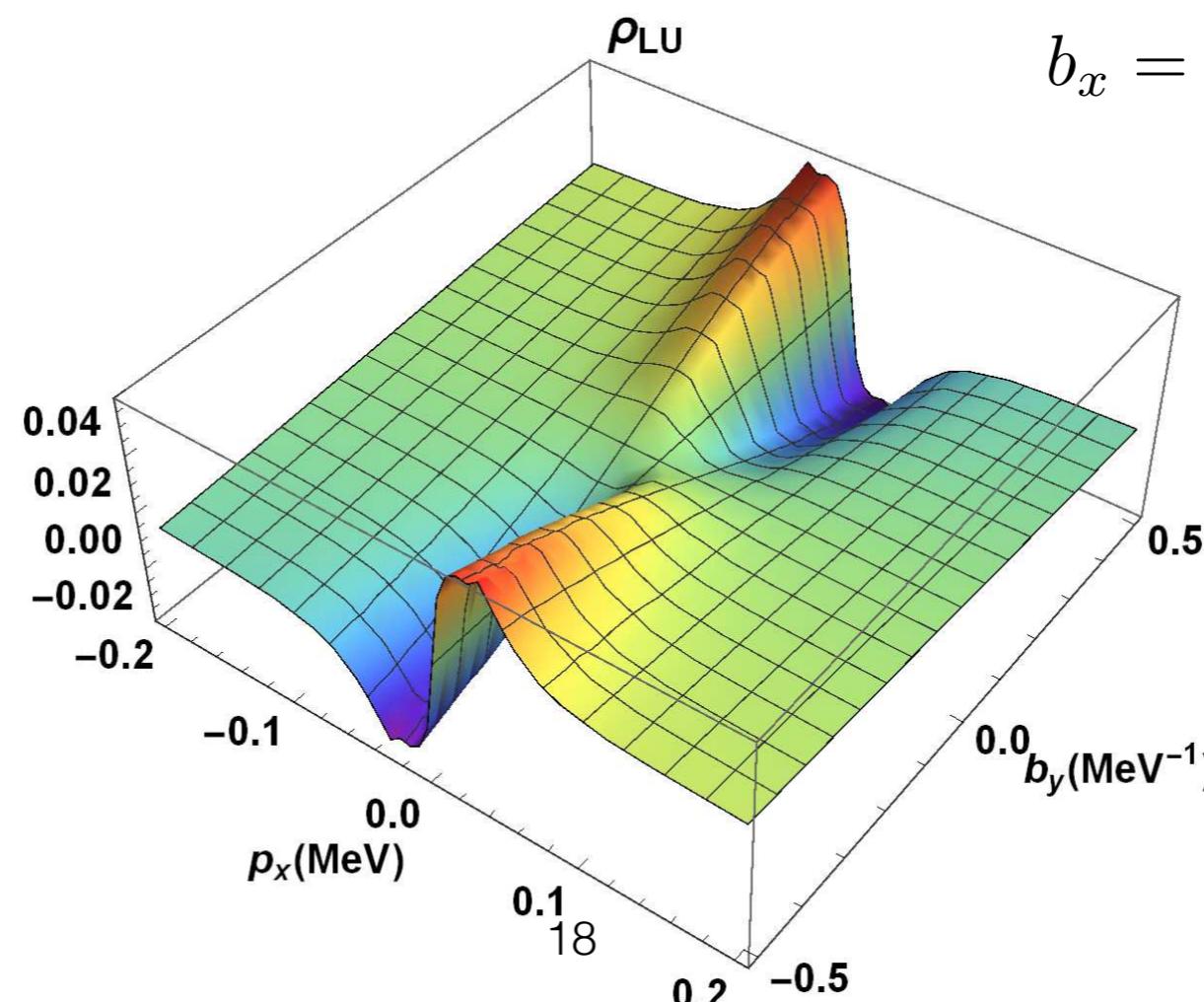
- No connection with twist-2 TMDs or IPDs.

$$F_{1,4}(x, \Delta_\perp, \mathbf{p}_\perp) = \frac{4e^2}{2(16\pi^3)} \frac{M^2(1+x)}{x^2} \varphi^\dagger(\mathbf{p}''_\perp) \varphi(\mathbf{p}'_\perp),$$



$$p_x = 0.4 \text{ MeV } \hat{e}_x$$

$$b_x = 0.4 \text{ MeV}^{-1} \hat{e}_x$$



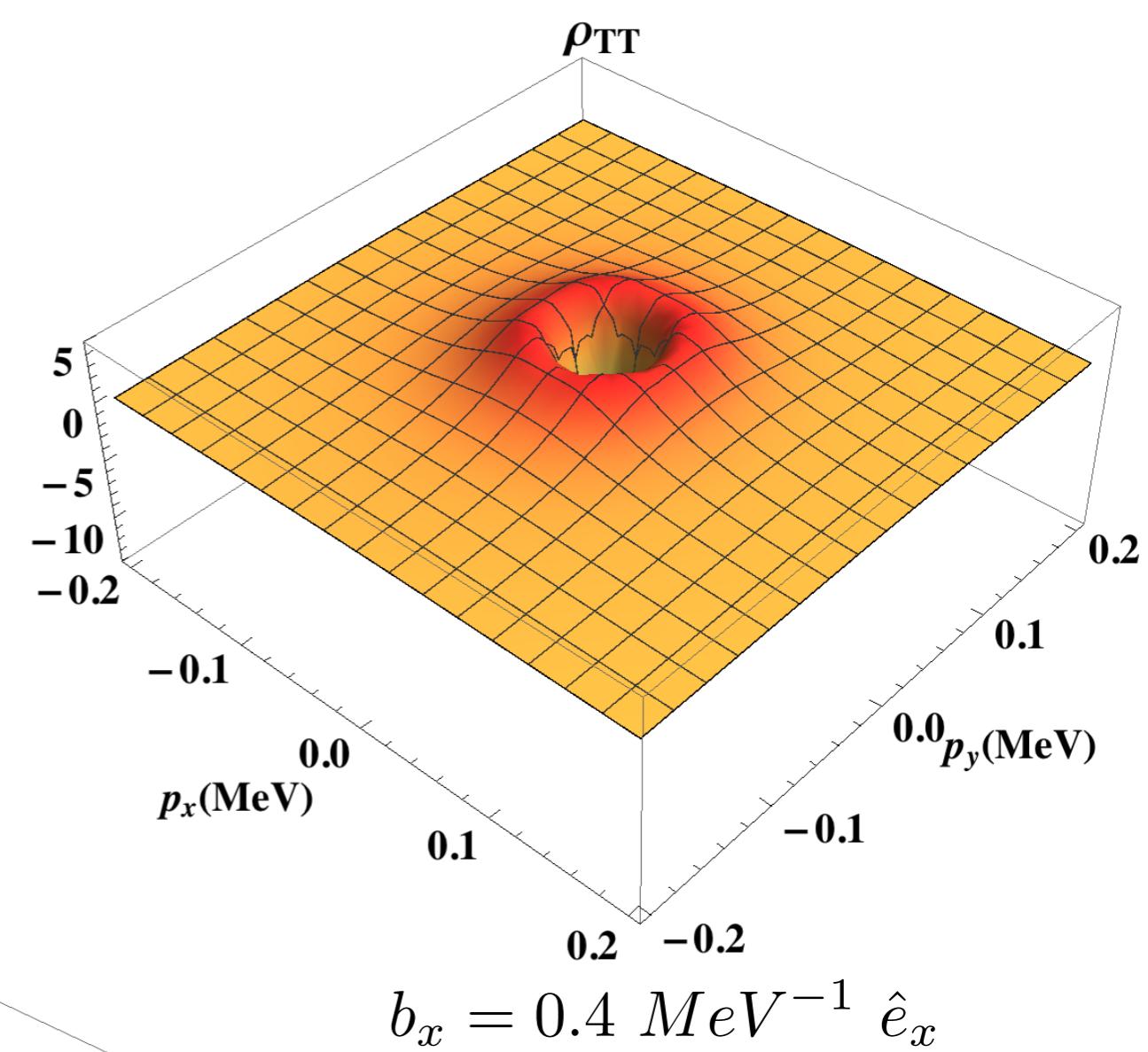
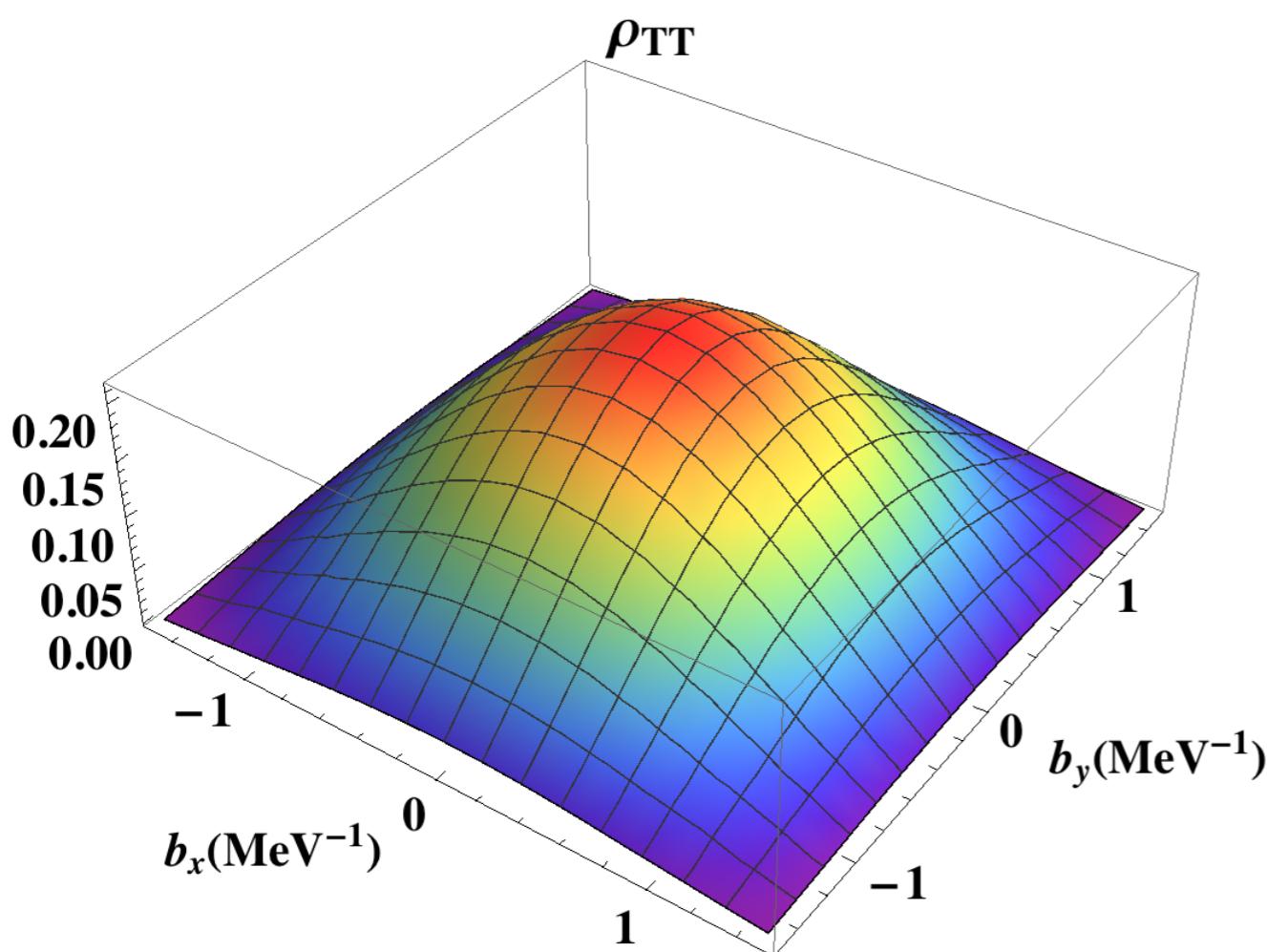
Tranversely polarized Wigner distribution

- Both physical and internal electrons are transversely polarized

$$\rho_{TT}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \frac{1}{2} \delta_{ij} \left[\rho^{[i\sigma^{+j}\gamma_5]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; +\hat{e}_i) - \rho^{[i\sigma^{+j}\gamma_5]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; -\hat{e}_i) \right],$$

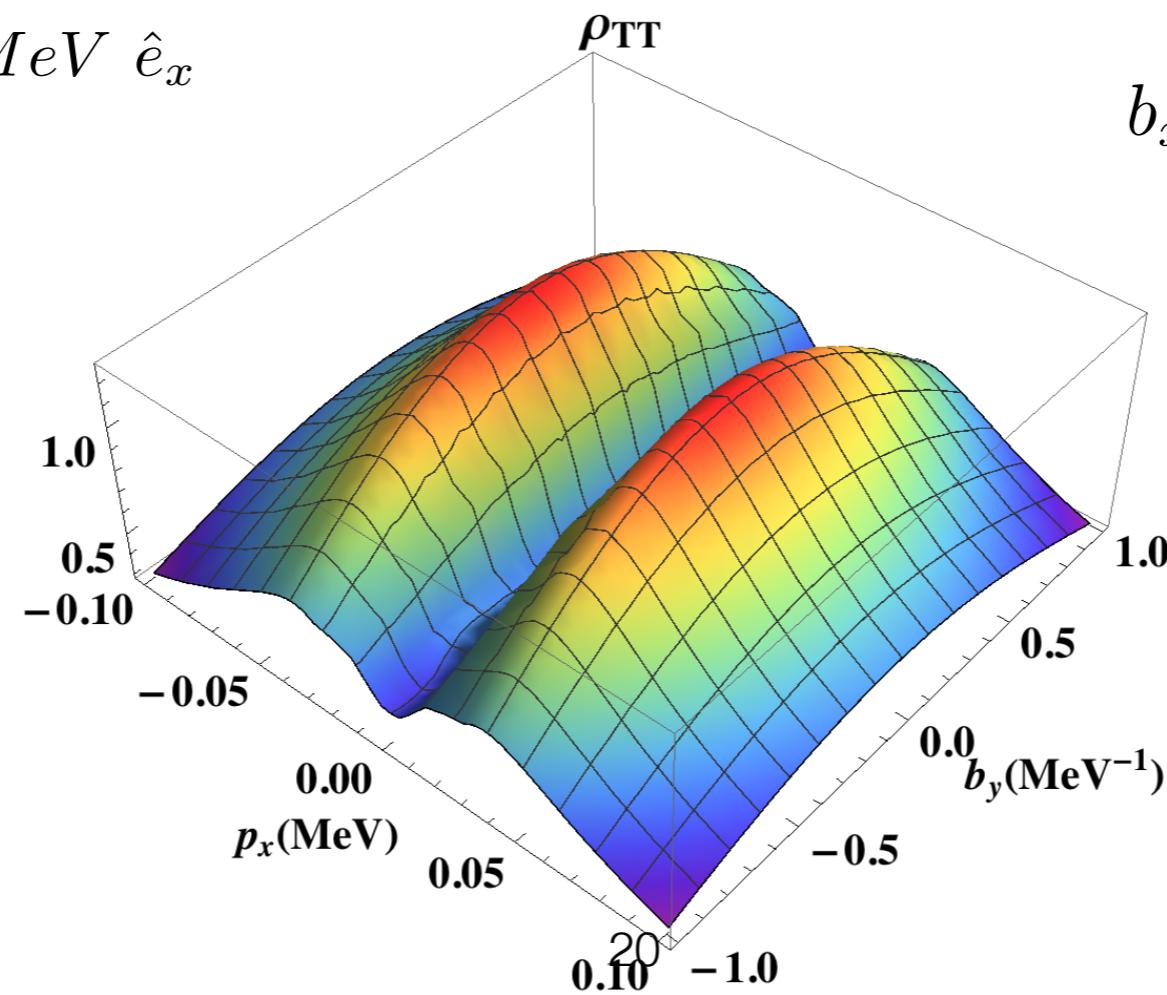
$$\rho_{TT}(\mathbf{b}_\perp, \mathbf{p}_\perp) = \frac{4e^2}{16\pi^3(2\pi)^2} \int d\Delta_x \, d\Delta_y \int dx \cos(\Delta_x b_x + \Delta_y b_y) \frac{1}{x(1-x)^2} \left(\mathbf{p}_\perp^2 - \frac{(1-x)^2}{4} \boldsymbol{\Delta}_\perp^2 \right) \varphi^\dagger(\mathbf{p}'_\perp) \varphi(\mathbf{p}'_\perp),$$

- In the TMD limit it reduces to transversity distribution $h_1(x, \mathbf{p}_\perp)$ and it is connected with linear combinations of transversity GPDs.



$$p_x = 0.4 \text{ MeV } \hat{e}_x$$

$$b_x = 0.4 \text{ MeV}^{-1} \hat{e}_x$$



Spin-spin correlations

- The Wigner distributions with the composite system helicity Λ and fermion constituent helicity λ is defined for $\Gamma = \gamma^+ \frac{1 + \lambda \gamma^5}{2}$ and $\mathbf{S} = \Lambda \hat{S}_z$

$$\rho_{\Lambda\lambda}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \frac{1}{2} [\rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; \Lambda \hat{S}_z) \\ + \lambda \rho^{[\gamma^+ \gamma^5]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; \Lambda \hat{S}_z)],$$

which can be decomposed as

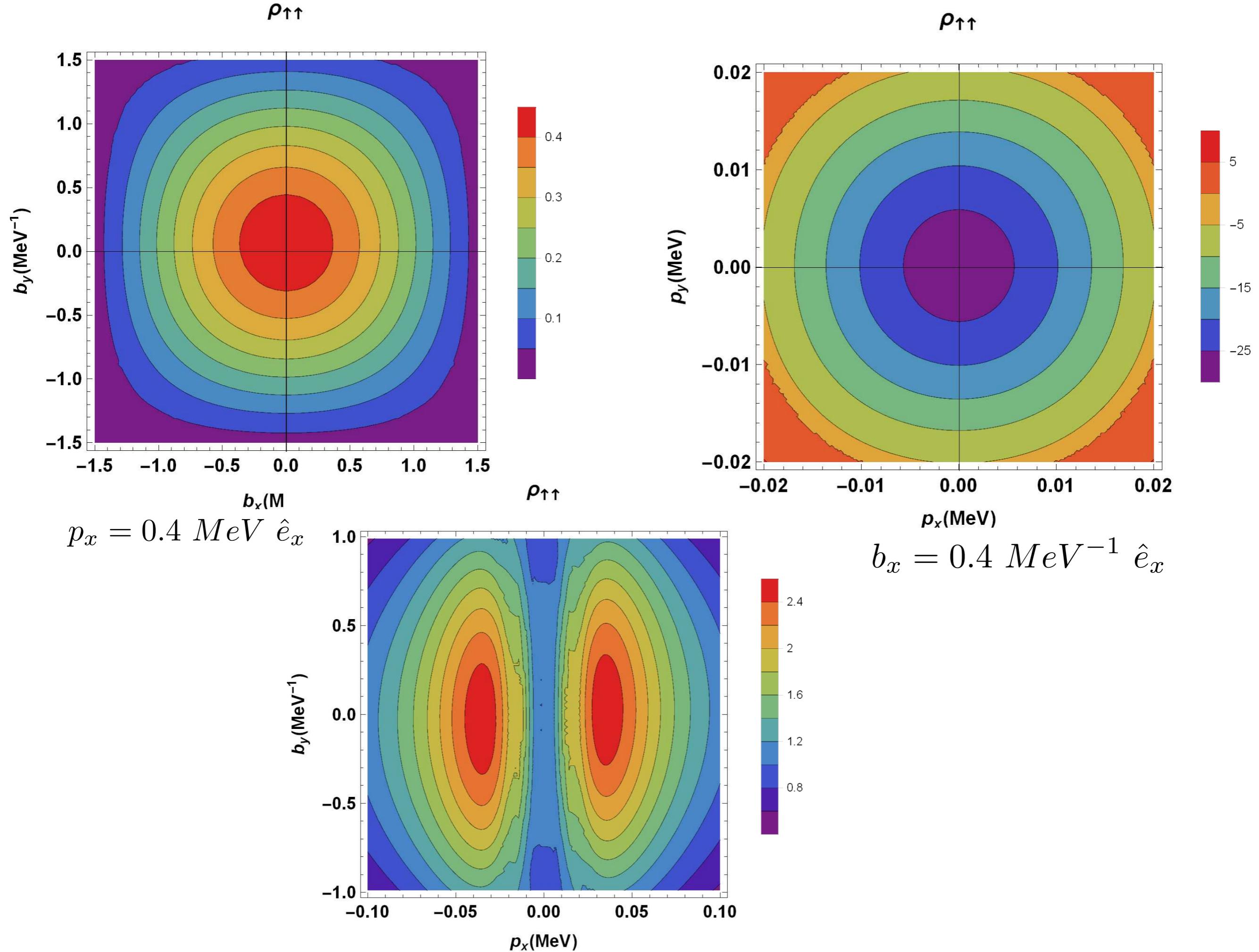
$$\rho_{\Lambda\lambda}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \frac{1}{2} [\rho_{UU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \Lambda \rho_{LU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \\ \lambda \rho_{UL}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \Lambda \lambda \rho_{LL}(\mathbf{b}_\perp, \mathbf{p}_\perp, x)],$$

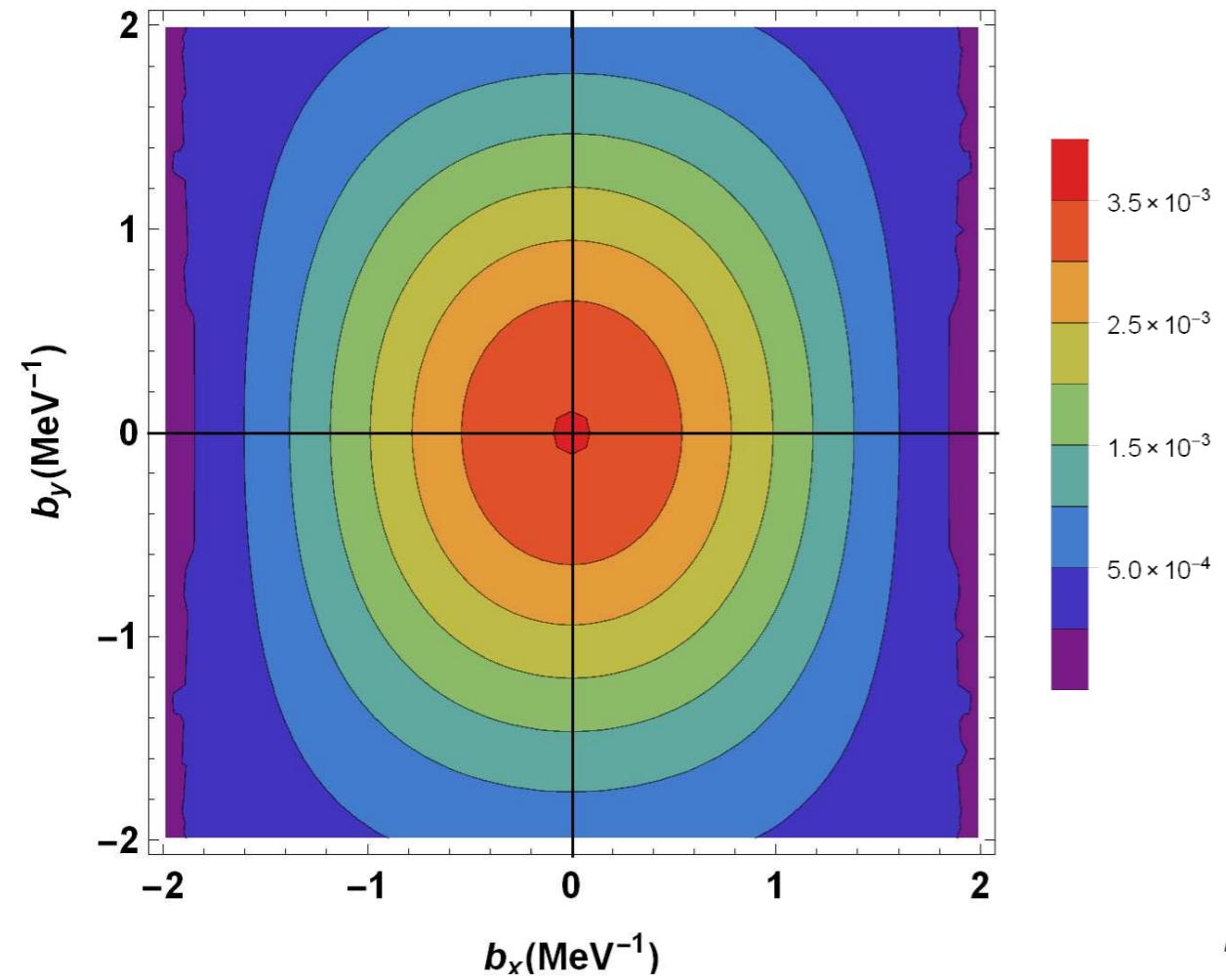
- In this model

$$\rho_{UL} = \rho_{LU}$$

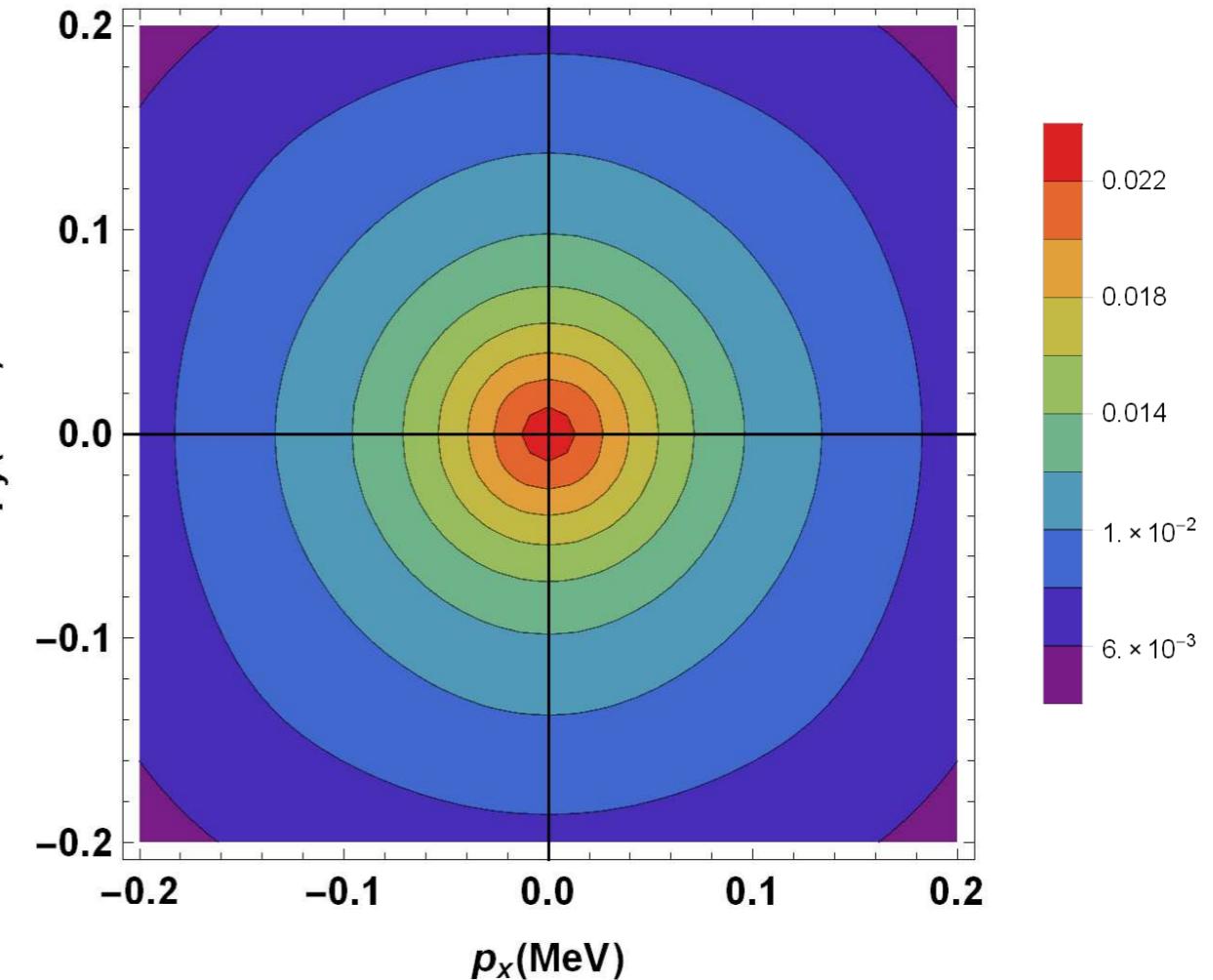
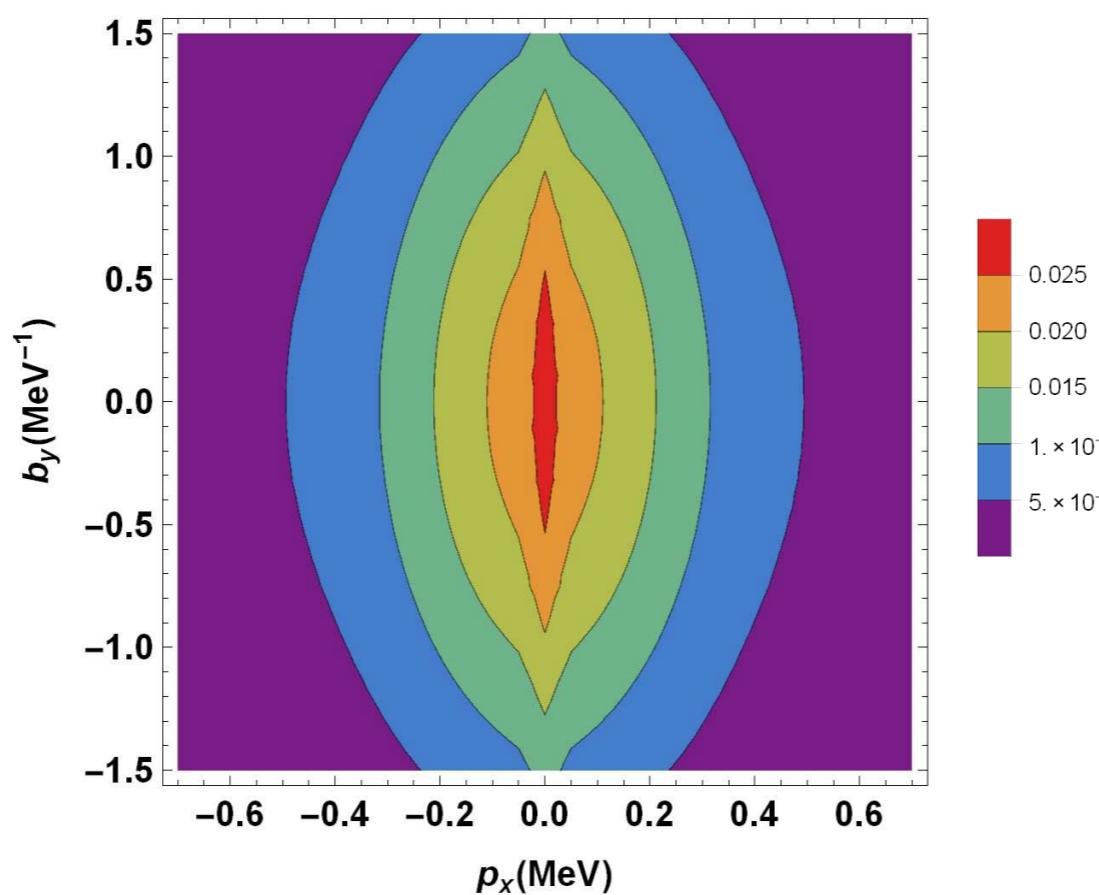
$$\rho_{\uparrow\uparrow} = \frac{1}{2} [\rho_{UU} + 2\rho_{LU} + \rho_{LL}],$$

$$\rho_{\uparrow\downarrow} = \frac{1}{2} [\rho_{UU} - \rho_{LL}].$$



$\rho_{\uparrow\downarrow}$ 

$$p_x = 0.4 \text{ MeV } \hat{e}_x$$

 $\rho_{\uparrow\downarrow}$  $\rho_{\uparrow\downarrow}$ 

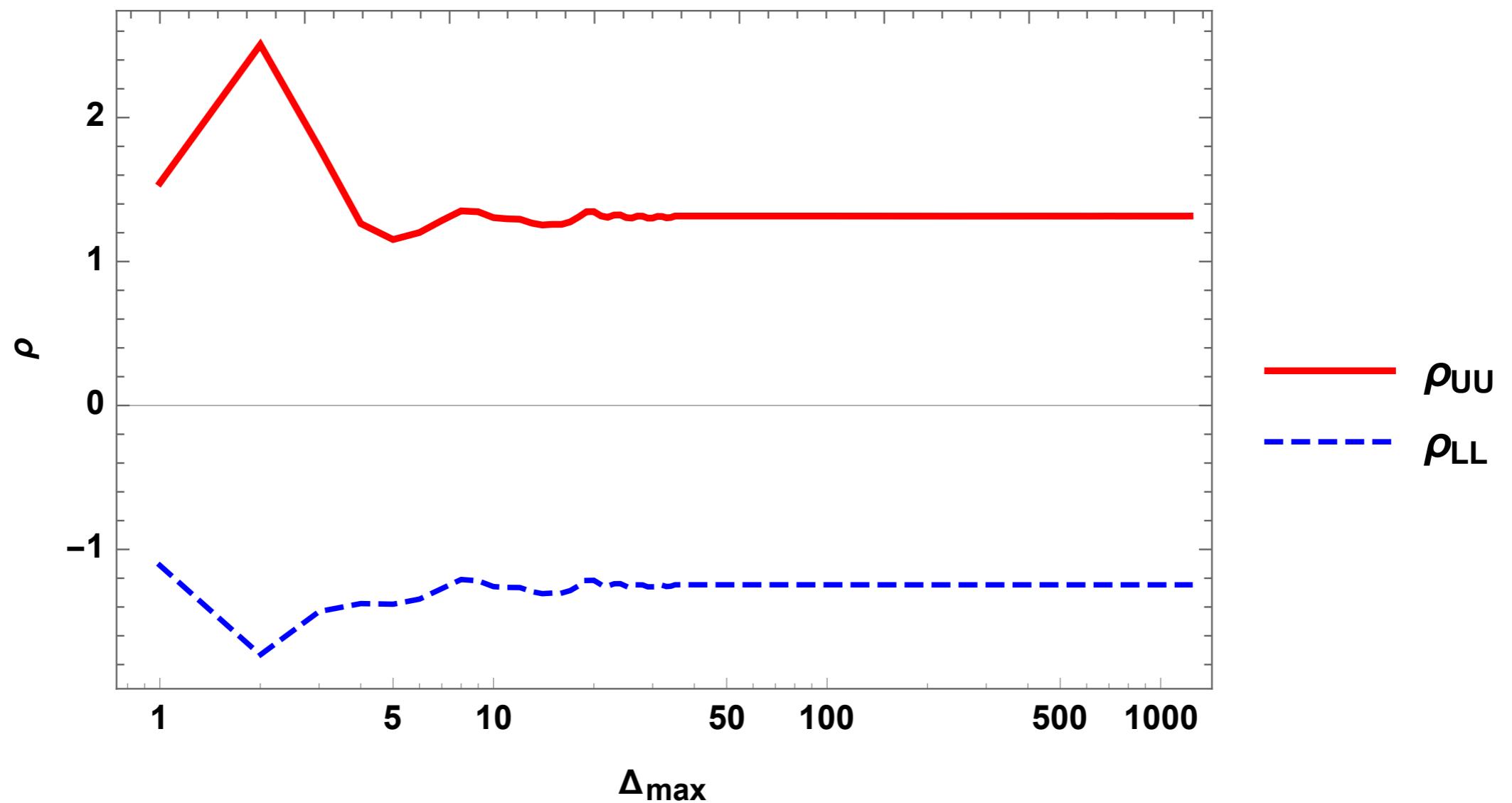
$$b_x = 0.4 \text{ MeV}^{-1} \hat{e}_x$$

Conclusions

- Wigner distributions of an electron are calculated which provides the multi-dimensional images of electron.
- We consider different polarization configurations.
- The transverse spin-spin correlations and GTMDs (not presented here) are also studied.
- Results provide rich and interesting information on the distribution of QED partons.
- See arXiv:hep-ph-1705.03183

Thank You

Back up slides



GTMDs

$$W_{\lambda\lambda'}^{[\gamma^+]} = \frac{1}{2M} \bar{u}(p', \lambda') \left[F_{1,1} + \frac{i\sigma^{i+} k_\perp^i}{P^+} F_{1,2} \right.$$

$$\left. + \frac{i\sigma^{i+} \Delta_\perp^i}{P^+} F_{1,3} + \frac{i\sigma^{ij} k_\perp^i \Delta_\perp^j}{M^2} F_{1,4} \right] u(p, \lambda)$$

$$W_{\lambda\lambda'}^{[\gamma^+ \gamma_5]} = \frac{1}{2M} \bar{u}(p', \lambda') \left[- \frac{i\varepsilon_\perp^{ij} k_\perp^i \Delta_\perp^j}{M^2} G_{1,1} + \frac{i\sigma^{i+} \gamma_5 k_\perp^i}{P^+} \right.$$

$$\left. G_{1,2} + \frac{i\sigma^{i+} \gamma_5 \Delta_\perp^i}{P^+} G_{1,3} + i\sigma^{+-} \gamma_5 G_{1,4} \right] u(p, \lambda)$$

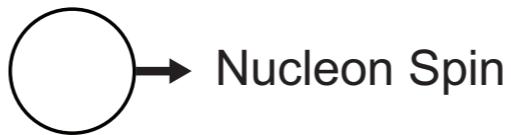
$$W_{\lambda\lambda'}^{[i\sigma^{j+} \gamma_5]} = \frac{1}{2M} \bar{u}(p', \lambda') \left[- \frac{i\varepsilon_\perp^{ij} k_\perp^i}{M} H_{1,1} - \frac{i\varepsilon_\perp^{ij} \Delta_\perp^i}{M} H_{1,2} \right.$$

$$\left. + \frac{M i\sigma^{j+} \gamma_5}{P^+} H_{1,3} + \frac{k_\perp^j i\sigma^{k+} \gamma_5 k_\perp^k}{M P^+} H_{1,4} \right.$$

$$\left. + \frac{\Delta_\perp^j i\sigma^{k+} \gamma_5 k_\perp^k}{M P^+} H_{1,5} + \frac{\Delta_\perp^j i\sigma^{k+} \gamma_5 \Delta_\perp^k}{M P^+} H_{1,6} \right.$$

$$\left. + \frac{k_\perp^j i\sigma^{+-} \gamma_5}{M} H_{1,7} + \frac{\Delta_\perp^j i\sigma^{+-} \gamma_5}{M} H_{1,8} \right] u(p, \lambda),$$

Leading Twist TMDs



		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \bullet$		$h_1^\perp = \bullet \downarrow - \bullet \uparrow$ Boer-Mulders
	L		$g_{1L} = \bullet \rightarrow - \bullet \leftarrow$ Helicity	$h_{1L}^\perp = \bullet \nearrow - \bullet \searrow$
	T	$f_{1T}^\perp = \bullet \uparrow - \bullet \downarrow$ Sivers	$g_{1T}^\perp = \bullet \uparrow - \bullet \leftarrow$	$h_{1T}^\perp = \bullet \nearrow - \bullet \nearrow$ Transversity