Gluon Wigner distributions in the dressed quark model for different polarizations

Jai More

Indian Institute of Technology Bombay, India

in collaboration with Asmita Mukherjee, Sreeraj Nair

September 20, 2017

Outline of the talk

1 Wigner Distributions

2 Gluon Wigner Distributions

3 Numerical results and 3D plots for gluon Wigner Distributions

4 Summary

Wigner Distributions

- $\rho(\bar{k}, \bar{r})$:= phase space distribution »»»»»» classical picture

- Wigner distribution are quantum mechanical analog of classical phase space distribution.

- Quasi-probability distributions (are not positive definite).

Wigner Distributions

- $\rho(\bar{k}, \bar{r})$:= phase space distribution »»»»»» classical picture

- Wigner distribution are quantum mechanical analog of classical phase space distribution.

- Quasi-probability distributions (are not positive definite).

- Wigner distribution for 1 D quantum system is given by[Wigner 1932]

$$W(x, p) = \int dy \ e^{ip.y} \psi^*(x - y/2) \ \psi(x + y/2) \qquad \text{Nucleon state} \rightarrow 6\,D$$

- WD are related to generalized parton correlation functions (GPCFs) and generalized transverse momentum dependent distributions (GTMDs).
[Meissner, Metz and Schlegel (2009); Lorce and Pasquini (2013)]

- GPCFs: Fully unintegrated, off-diagonal, quark-quark correlaters.

GPCFs $\mathcal{F}(x, \boldsymbol{k}_\perp, k^-, \boldsymbol{\Delta}_\perp)^1$

$\downarrow \int dk^-$

GTMDs $\overset{\text{Fourier Transform}}{\underset{\boldsymbol{\Delta}_\perp \longleftrightarrow \boldsymbol{b}_\perp}{\longleftrightarrow}}$ **Wigner Distributions**

$$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$

$\int d^2\boldsymbol{k}_\perp$ $\int d^2\boldsymbol{b}_\perp$

GPDs **TMDs**

$W(x, \boldsymbol{b}_\perp)$ $W(x, \boldsymbol{k}_\perp)$

---

^{1}LF : $x^\mu = (x^+, x^-, x^\perp);$ $x^\pm = x^0 \pm x^3,$ $\boldsymbol{x}^\perp = (x^1, x^2)$
$p^\mu = (p^+, p^-, \boldsymbol{p}^\perp)$

Wigner Distribution »»»» "Mother Distribution"

- GPDs and TMDs can be related to spin and orbital angular momentum correlations of quarks and gluons.

  WD are very useful to study parton structure of hadrons as information of TMDs and GPDs are embedded.

- Measurement of quark GTMDs [Bhattacharya, Metz and Zhou 1702.04387]

- Gluon GTMDs have been discussed for small x physics in diffractive vector meson production and for Higgs production at Tevatron and LHC
  [Martin, Ryskin, Teubner, PRD 62, 014022 (2000),
  Khoze, Martin, Ryskin, EPJC 14, 525 (2000)]

- Gluon GTMDs and Wigner distributions measurement in DVCS at small x region [Hatta, Xiao and Yuan, Phys. Rev. D 95, 114026 (2017)]

See Mukherjee's Talk

Wigner distribution for a Dressed Quark Model

- Wigner distributions can be expressed as the overlap of light-front wavefunction (LFWF).
- WDs for quark has been studied in different models: constituent quark model, chiral quark soliton model, [Lorce and Pasquini (2011)], spectator model [Liu and Ma (2015)], Holographic model [Dipankar *et.al.* (2016)], etc (No gluonic dof)

Wigner distribution for a Dressed Quark Model

- Wigner distributions can be expressed as the overlap of light-front wavefunction (LFWF).

- WDs for quark has been studied in different models:
  constituent quark model, chiral quark soliton model,
  [Lorce and Pasquini (2011)], spectator model [Liu and Ma (2015)], Holographic model [Dipankar *et.al.* (2016)], etc (No gluonic dof)

- Instead of proton state, we use a simple composite spin-1/2 state i.e. a quark dressed with a gluon [Mukherjee, Nair and Ojha (2014, 2015)]
  It is a pertubative model and has gluonic degrees of freedom $\Rightarrow$ Gluon Wigner distribution

Wigner distribution for a Dressed Quark Model

- Wigner distributions can be expressed as the overlap of light-front wavefunction (LFWF).

- WDs for quark has been studied in different models: constituent quark model, chiral quark soliton model, [Lorce and Pasquini (2011)], spectator model [Liu and Ma (2015)], Holographic model [Dipankar *et.al.* (2016)], etc (No gluonic dof)

- Instead of proton state, we use a simple composite spin-1/2 state i.e. a quark dressed with a gluon [Mukherjee, Nair and Ojha (2014, 2015)] It is a pertubative model and has gluonic degrees of freedom ⇒ Gluon Wigner distribution

- Calculated analytically in light front Hamiltonian perturbation theory.

- Quark Wigner distributions was studied for different polarization of quark in a dressed quark state [JM, Mukherjee and Nair PRD 95 (2017), 074039].

The Wigner distribution of gluons can be defined as Fourier transform of gluon-gluon correlator ($\mathcal{W}^\alpha$) [Messiner 09, Lorce 11]

$$x\, W^\alpha_{\sigma\sigma'}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{2(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp}\, \mathcal{W}^\alpha_{\sigma\sigma'}(x, \boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp)$$

with $\alpha = 1, 2, 3, 4$.

The gluon-gluon correlator is:

$$\xi = 0$$

$$\mathcal{W}^\alpha_{\sigma\sigma'}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \int \frac{dz^- d^2\boldsymbol{z}_\perp}{2(2\pi)^3 p^+} e^{ik\cdot z} \left\langle p^+, -\frac{\boldsymbol{\Delta}_\perp}{2}, \sigma' \left| \Gamma^{ij}_\alpha F^{+i}\left(-\frac{z}{2}\right) F^{+j}\left(\frac{z}{2}\right) \right| p^+, \frac{\boldsymbol{\Delta}_\perp}{2}, \sigma \right\rangle \Big|_{z^+=0}$$

$\boldsymbol{\Delta}_\perp :=$ momentum transfer of target state in transverse direction;
$\boldsymbol{b}_\perp :=$ impact parameter; $\boldsymbol{\Delta}_\perp$ and $\boldsymbol{b}_\perp$ are conjugate to each other;
$F^{+i} :=$ gluon field strength tensor; $\Gamma^{ij}_\alpha :=$ gluon operator

The Wigner distribution of gluons can be defined as Fourier transform of gluon-gluon correlator ($\mathcal{W}^\alpha$) [Messiner 09, Lorce 11]

$$x \, W^\alpha_{\sigma\sigma'}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{2(2\pi)^2} \, e^{-i\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp} \, \mathcal{W}^\alpha_{\sigma\sigma'}(x, \boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp)$$

with $\alpha = 1, 2, 3, 4$.

The gluon-gluon correlator is:

$$\mathcal{W}^\alpha_{\sigma\sigma'}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \int \frac{dz^- d^2\boldsymbol{z}_\perp}{2(2\pi)^3 p^+} e^{ik \cdot z} \left\langle p^+, -\frac{\boldsymbol{\Delta}_\perp}{2}, \sigma' \left| \Gamma^{ij}_\alpha F^{+i}\left(-\frac{z}{2}\right) F^{+j}\left(\frac{z}{2}\right) \right| p^+, \frac{\boldsymbol{\Delta}_\perp}{2}, \sigma \right\rangle \Big|_{z^+=0}$$

$\boldsymbol{\Delta}_\perp :=$ momentum transfer of target state in transverse direction;
$\boldsymbol{b}_\perp :=$ impact parameter; $\boldsymbol{\Delta}_\perp$ and $\boldsymbol{b}_\perp$ are conjugate to each other;
$F^{+i} :=$ gluon field strength tensor; $\Gamma^{ij}_\alpha :=$ gluon operator

At leading twist (expressions shown later)

$\Gamma^{ij}_\alpha = \{\delta^{ij}_\perp, -i \, \epsilon^{ij}_\perp, \Gamma^{RR}, \Gamma^{LL}\}$ [Lorce and Pasquini JHEP 1309 (2013) 138]

Fock state expansion of quark state dressed with a gluon

Fock space expansion of a state with momentum 'p' and helicity 's' :

$$\left| p^+, \boldsymbol{p}_\perp, s \right\rangle = \Phi^s(p)\, b_s^\dagger(p)|0\rangle + \sum_{s_1 s_2} \int \frac{dp_1^+ d^2 \boldsymbol{p}_1^\perp}{\sqrt{16\pi^3 p_1^+}} \int \frac{dp_2^+ d^2 \boldsymbol{p}_2^\perp}{\sqrt{16\pi^3 p_2^+}} \sqrt{16\pi^3 p^+}$$

$$\times\ \delta^3(p - p_1 - p_2)\, \Phi_{s_1 s_2}^s(p; p_1, p_2)\, b_{s_1}^\dagger(p_1)\, a_{s_2}^\dagger(p_2)|0\rangle$$

$\Phi^s(p)$:= gives normalization of wavefunction

$\Phi_{s_1 s_2}^s(p; p_1, p_2)$:= two particle LFWF, related to the boost invariant wavefunction given by

$$\sqrt{P^+}\, \Phi_{s_1 s_2}^s(p; p_1, p_2) = \Psi_{s_1 s_2}^s(x_i, \boldsymbol{q}_i^\perp)$$

[Harindranath and Kundu PRD 59 116013 (1999)]

The Jacobi momenta:

$$p_i^+ = x_i \, P^+ \quad \text{and} \quad \boldsymbol{q}_i^\perp = \boldsymbol{p}_i^\perp + x_i \, \boldsymbol{P}^\perp$$

such that

$$\sum_i x_i = 1, \quad \sum_i \boldsymbol{q}_i^\perp = 0$$

The Jacobi momenta:

$$p_i^+ = x_i P^+ \text{ and } \boldsymbol{q}_i^\perp = \boldsymbol{p}_i^\perp + x_i \boldsymbol{P}^\perp$$

such that

$$\sum_i x_i = 1, \quad \sum_i \boldsymbol{q}_i^\perp = 0$$

The two particle LFWF can be calculated perturbatively as

$$
\Psi_{s_1 s_2}^{as}(x, \boldsymbol{q}^\perp) = \frac{1}{\left[m^2 - \frac{m^2 + (q^\perp)^2}{x} - \frac{(q^\perp)^2}{1-x}\right]} \frac{g}{\sqrt{2(2\pi)^3}} T^a \chi_{s_1}^\dagger \frac{1}{\sqrt{1-x}}
$$

$$
\times \left[-2\frac{\boldsymbol{q}^\perp}{1-x} - \frac{(\sigma^\perp . \boldsymbol{q}^\perp)\sigma^\perp}{x} + \frac{im\sigma^\perp(1-x)}{x}\right]\chi_s(\epsilon_{s_2}^\perp)^*
$$

χ: two component spinor; m: dressed quark mass= bare quark mass
[Harindranath and Kundu PRD 59 116013 (1999);
Zhang and Harindranath, PRD 48, 4881 (1993)]

Gluon-Gluon correlator in terms of LFWF

i) For $\Gamma^{ij} = \delta^{ij}_\perp$

$$\mathcal{W}^1_{\sigma\sigma'}(x, k_\perp, \Delta_\perp) = -\sum_{\sigma_1, \lambda_1, \lambda_2} \left[\Psi^{*\sigma'}_{\sigma_1\lambda_1}(\hat{x}, \hat{q}'_\perp) \Psi^\sigma_{\sigma_1\lambda_2}(\hat{x}, \hat{q}_\perp) \left(\epsilon^1_{\lambda_2}\epsilon^{*1}_{\lambda_1} + \epsilon^2_{\lambda_2}\epsilon^{*2}_{\lambda_1} \right) \right]$$

ii) For $\Gamma^{ij} = -i\,\epsilon^{ij}_\perp$

$$\mathcal{W}^2_{\sigma\sigma'}(x, k_\perp, \Delta_\perp) = -i\sum_{\sigma_1, \lambda_1, \lambda_2} \left[\Psi^{*\sigma'}_{\sigma_1\lambda_1}(\hat{x}, \hat{q}'_\perp) \Psi^\sigma_{\sigma_1\lambda_2}(\hat{x}, \hat{q}_\perp) \left(\epsilon^1_{\lambda_2}\epsilon^{*2}_{\lambda_1} - \epsilon^2_{\lambda_2}\epsilon^{*1}_{\lambda_1} \right) \right]$$

iii) For $\Gamma^{RR(LL)}$

$$\mathcal{W}^{3(4)}_{\sigma\sigma'}(x, k_\perp, \Delta_\perp) = -\sum_{\sigma_1, \lambda_1, \lambda_2} \left[\Psi^{*\sigma'}_{\sigma_1\lambda_1}(\hat{x}, \hat{q}'_\perp) \Psi^\sigma_{\sigma_1\lambda_2}(\hat{x}, \hat{q}_\perp) \epsilon^{R(L)}_{\lambda_2}\epsilon^{*R(L)}_{\lambda_1} \right]$$

where $\hat{x} = (1-x)$, $\quad \hat{q}_\perp = -q_\perp \quad a^{(R)L} = a^1 \pm i\,a^2$

In symmetric frame: $q'_\perp = k_\perp + \frac{\Delta_\perp}{2}(1-x)$, $\quad q_\perp = k_\perp - \frac{\Delta_\perp}{2}(1-x)$

Unpolarized target and different gluon polarizations

We define gluon Wigner distributions as $W_{\lambda \lambda'}$ [JM, Mukherjee, Nair, arxiv 1709.00943]
where $\lambda := \{U, L, T\}$ $\qquad \lambda' := \{U, L, \mathcal{T}\}$

$$W_{UU}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{2}\Big[W^1(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, \hat{e}_z) + W^1(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, -\hat{e}_z)\Big]$$

$$W_{UL}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{2}\Big[W^2(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, \hat{e}_z) + W^2(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, -\hat{e}_z)\Big]$$

$$W_{U\mathcal{T}}^{(R)}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{2}\Big[W^3(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, \hat{e}_z) + W^3(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, -\hat{e}_z)\Big]$$

$$W_{U\mathcal{T}}^{(L)}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{2}\Big[W^4(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, \hat{e}_z) + W^4(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, -\hat{e}_z)\Big]$$

$\hat{e}_z :=$ polarization of target
The superscipt '$L(R)$' represents left (right) polarization of gluon

[JM, Mukherjee, Nair, arxiv 1709.00943]

Longitudinally polarized target and different gluon polarizations

$$W_{LU}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{2}\Big[W^1(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, \hat{\boldsymbol{e}}_z) - W^1(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, -\hat{\boldsymbol{e}}_z)\Big]$$

$$W_{LL}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{2}\Big[W^2(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, \hat{\boldsymbol{e}}_z) - W^2(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, -\hat{\boldsymbol{e}}_z)\Big]$$

$$W_{L\mathcal{T}}^{(R)}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{2}\Big[W^3(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, \hat{\boldsymbol{e}}_z) - W^3(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, -\hat{\boldsymbol{e}}_z)\Big]$$

$$W_{L\mathcal{T}}^{(L)}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{2}\Big[W^4(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, \hat{\boldsymbol{e}}_z) - W^4(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, -\hat{\boldsymbol{e}}_z)\Big]$$

[JM, Mukherjee, Nair, arxiv 1709.00943]

Transversely polarized target and different gluon polarizations

$$W_{TU}^i(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{2}\Big[W^1(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, \hat{\boldsymbol{e}}_i) - W^1(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, -\hat{\boldsymbol{e}}_i)\Big]$$

$$W_{TL}^i(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{2}\Big[W^2(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, \hat{\boldsymbol{e}}_i) - W^2(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, -\hat{\boldsymbol{e}}_i)\Big]$$

$$W_{TT}^{i(R)}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{2}\Big[W^3(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, \hat{\boldsymbol{e}}_i) - W^3(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, -\hat{\boldsymbol{e}}_i)\Big]$$

$$W_{TT}^{i(L)}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{2}\Big[W^4(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, \hat{\boldsymbol{e}}_i) - W^4(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, -\hat{\boldsymbol{e}}_i)\Big]$$

where $i :=$ transverse directions (we choose $i := x$)
$\hat{e}_i :=$ transverse polarization of the target state and can be expressed as a linear combination of helicity states.
[JM, Mukherjee, Nair, arxiv 1709.00943]

Six linearly independent gluon Wigner distributions:

$$
\begin{aligned}
W_{UU}(x, \mathbf{k}_\perp, \mathbf{b}_\perp) &= N \int \frac{d^2\mathbf{\Delta}_\perp}{2(2\pi)^2} \frac{\cos(\mathbf{\Delta}_\perp \mathbf{b}_\perp)}{D(\mathbf{q}_\perp)D(\mathbf{q}'_\perp)} \\
&\quad \times \left[-\frac{4m^2x^4 + (x^2 - 2x + 2)\left(4\mathbf{k}_\perp{}^2 - \mathbf{\Delta}_\perp{}^2(1-x)^2\right)}{(1-x)^2x^3} \right] \\
W_{UL}(x, \mathbf{k}_\perp, \mathbf{b}_\perp) &= N \int \frac{d^2\mathbf{\Delta}_\perp}{2(2\pi)^2} \frac{\sin(\mathbf{\Delta}_\perp \mathbf{b}_\perp)}{D(\mathbf{q}_\perp)D(\mathbf{q}'_\perp)} \left[\frac{4(x^2 - 2x + 2)(\Delta_y k_x - \Delta_x k_y)}{(1-x)x^3} \right] \\
W_{LU}(x, \mathbf{k}_\perp, \mathbf{b}_\perp) &= N \int \frac{d^2\mathbf{\Delta}_\perp}{2(2\pi)^2} \frac{\sin(\mathbf{\Delta}_\perp \mathbf{b}_\perp)}{D(\mathbf{q}_\perp)D(\mathbf{q}'_\perp)} \left[\frac{4(2-x)(\Delta_y k_x - \Delta_x k_y)}{(1-x)x^2} \right]
\end{aligned}
$$

[JM, Mukherjee, Nair, arxiv 1709.00943]

$$
\begin{aligned}
W_{LL}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) &= N \int \frac{d^2 \boldsymbol{\Delta}_\perp}{2(2\pi)^2} \frac{\cos(\boldsymbol{\Delta}_\perp \boldsymbol{b}_\perp)}{D(\boldsymbol{q}_\perp) D(\boldsymbol{q}'_\perp)} \\
&\quad \times \left[-\frac{4m^2 x^3 + (2-x)\left(4\boldsymbol{k}_\perp{}^2 - \boldsymbol{\Delta}_\perp{}^2(1-x)^2\right)}{(1-x)^2 x^2} \right]
\end{aligned}
$$

$$
W_{TU}^x(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = N \int \frac{d^2 \boldsymbol{\Delta}_\perp}{2(2\pi)^2} \frac{\sin(\boldsymbol{\Delta}_\perp \boldsymbol{b}_\perp)}{D(\boldsymbol{q}_\perp) D(\boldsymbol{q}'_\perp)} \left[\frac{4m\Delta_x}{x} \right]
$$

$$
W_{TL}^x(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = N \int \frac{d^2 \boldsymbol{\Delta}_\perp}{2(2\pi)^2} \frac{\cos(\boldsymbol{\Delta}_\perp \boldsymbol{b}_\perp)}{D(\boldsymbol{q}_\perp) D(\boldsymbol{q}'_\perp)} \left[\frac{8mk_y}{x(1-x)} \right]
$$

where,

$$
N = \frac{g^2 C_F}{(2\pi)^3}, \quad C_F := \text{ color factor}, \quad D(\boldsymbol{q}_\perp) = \left[m^2 - \frac{m^2 + \boldsymbol{q}_\perp^2}{x} - \frac{\boldsymbol{q}_\perp^2}{1-x} \right]
$$

[JM, Mukherjee, Nair, arxiv 1709.00943]

Model dependent relations:

$$W_{U\mathcal{T}}^{(R)}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = W_{UU}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) - W_{UL}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$

$$W_{U\mathcal{T}}^{(L)}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = W_{UU}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) + W_{UL}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$

$$W_{L\mathcal{T}}^{(R)}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = -W_{LL}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) + W_{LU}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$

$$W_{L\mathcal{T}}^{(L)}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = W_{LL}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) + W_{LU}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$

$$W_{T\mathcal{T}}^{(R)x}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = -W_{TL}^x(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) + W_{TU}^x(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$

$$W_{T\mathcal{T}}^{(L)x}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = W_{TL}^x(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) + W_{TU}^x(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$

Numerical results and 3D plots
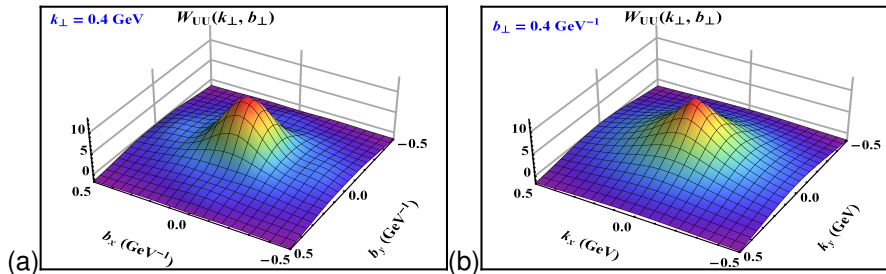
Numerical strategy

Levin method:

- ✓ Good convergence
- ✓ The result does not depend on the cutoff of the $\Delta_\perp$ integration.
- ✓ $\int dx$.........Transverse Wigner distribution
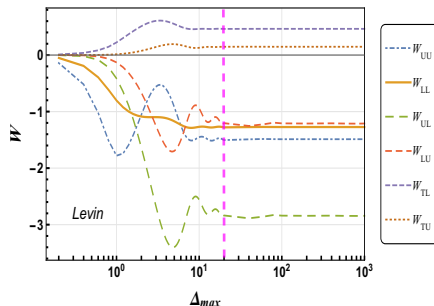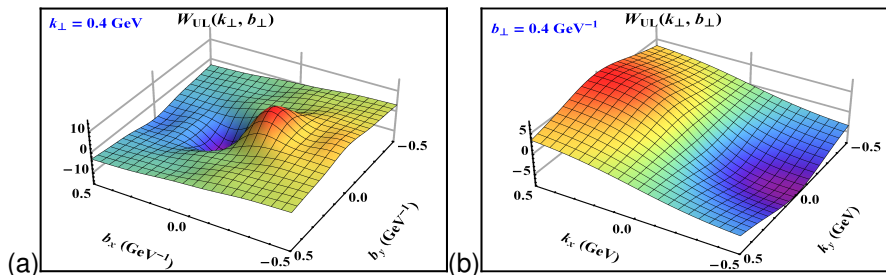

20 GeV

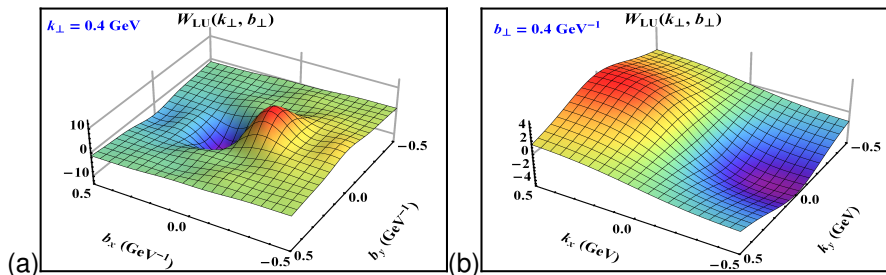# Plots of W_{UU} in transverse position and momentum space

⊛ peak at the center
⊛ TMD limit f_1^g
⊛ GPD limit H^g

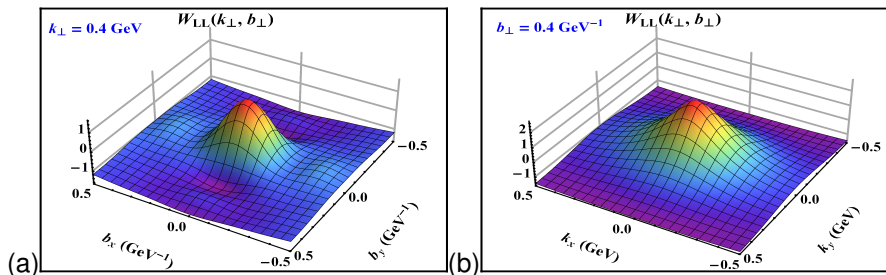# Plots of W_{UL} in transverse position and momentum space

(a)

(b)

⊛ Dipole like behavior in both $b_\perp$ and $k_\perp$ space

⊛ W_{UL} is related to gluon spin-orbit corelations.

Plots of W_{LU} in transverse position and momentum space
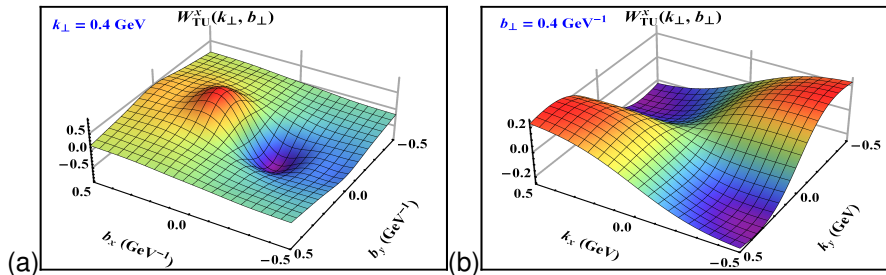

(a) (b)

⊛ Shows similar behavior as W_{UL}

⊛ W_{LU} is related to gluon orbital angular momentum.

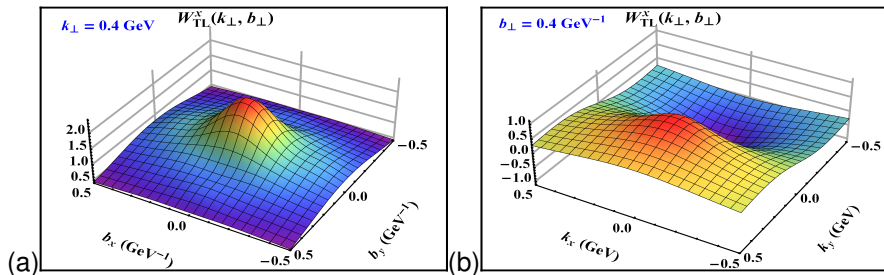# Plots of W_{LL} in transverse position and momentum space

(a) (b)

⊛ TMD limit g_{1L}^g
⊛ GPD limit $\tilde{H}^g$

Plots of W^x_{TU} in transverse position and momentum space

(a)   (b)

⊛ b space: dipole like behavior
⊛ k space: quadrapule like behavior
⊛ TMD limit $f^{\perp g}_{1T}$
⊛ GPD limit H^g_T, E^g_T

Plots of W_{TL}^x in transverse position and momentum space

(a) (b)

⊛ k space: dipole like behavior
⊛ TMD limit g_{1T}^g
⊛ GPD limit $\tilde{H}_T^g, \tilde{E}_T^g$

Summary

- We have studied gluon Wigner distribution for unpolarized, longitudinal and transversely polarized target state, in the LF dressed quark model.

- We obtain six linearly independent distributions in our model.

- Numerical strategy we used removes the cutoff dependence of the integration.

- Wigner distributions do not have probabilistic interpretation however one can obtain probability interpretation by integrating out some variables.

- Future plan is to study Wigner distribution and TMDs including transverse gauge link.

Thank you!