CMBR Power Spectrum and flow fluctuations in Relativistic Heavy-Ion Collisions

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Outline:

1. Elliptic Flow in relativistic heavy-ion collision experiments (RHICE)
   Big surprize: sQGP, very small viscosity.

2. Focus on flow fluctuations: Invaluable tool for probing initial state fluctuations arising during initial pre-equilibrium stages.

3. Use tools following experience from CMBR Power spectrum analysis. Remember: CMBR Power spectrum directly probed initial quantum fluctuations of the inflaton field.

4. Important: Physics of flow fluctuations for RHICE almost the same as Inflationary fluctuations (despite absence of Gravity).
   Acoustic peaks and superhorizon suppression for RHICE, just as for CMBR: Confirmed by Hydrodynamical simulations.
   Causal structure of initial fluctuations plays crucial role here.

5. Early stage Magnetic field in RHICE: Magnetohydrodynamics Simulations: larger elliptic flow: (larger η/s than AdS/CFT limit?)

6. Magnetic field effects on QGP: Important for Instanton effects at early stages (Chiral Magnetic effect); Chiral Vortical effect

Search for Deconfined Phase of QCD

Quantum chromodynamics predicts that in extreme conditions of high density and /or temperature there should be a deconfinement of quarks and gluons, and hadrons should undergo a phase transition to a quark-gluon plasma (QGP)

QGP phase expected from Asymptotic freedom of QCD: The coupling constant becomes small at high energies/small length scales.

If nuclear matter is in a state in which the nucleon density and/or the energy density become high,

$$\rho = (10-15)\rho_0 \approx (2-3) GeV / fm^3$$

then deconfinement should occur.

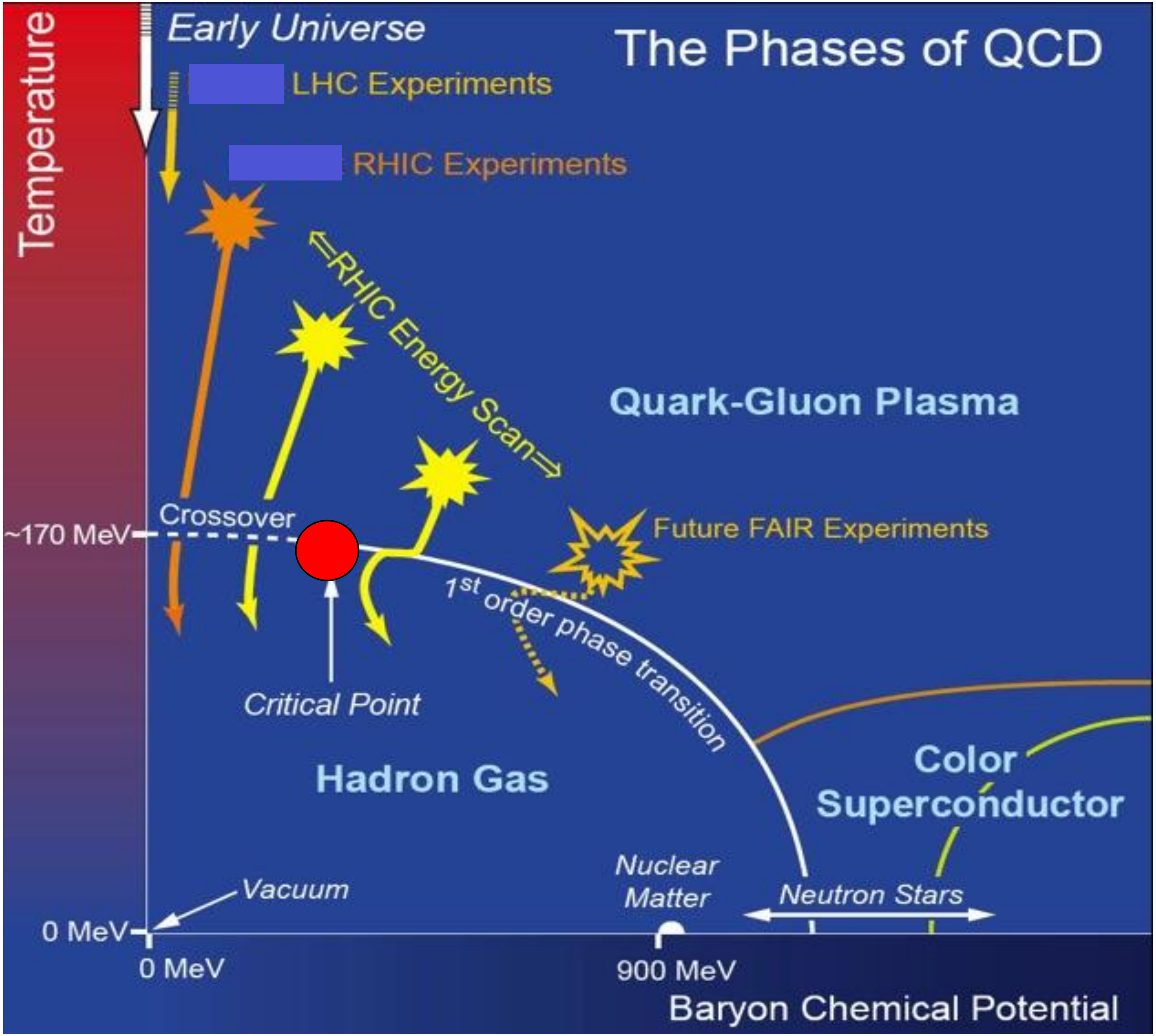
In Bag model, or string model of confinement this happens when quark separation becomes smaller than the typical size (~ 1 fm)

Expectation: Weakly interacting plasma of quarks and gluons

QGP expected to occur in the early Universe, inside neutron stars,

and in ultra-relativistic heavy-ion collision experiments

Big surprize from experiments: Elliptic Flow Measurements: QGP not like ideal gas, almost perfect fluid: sQGP


The Phases of QCD
Temperature
Early Universe
LHC Experiments
RHIC Experiments
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
Crossover
~170 MeV
Future FAIR Experiments
1st order phase transition
Critical Point
Hadron Gas
Color Superconductor
Vacuum
Nuclear Matter
Neutron Stars
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential

Relativistic heavy-ion collision experiments (RHICE):

Relativistic Heavy-Ion Collider (RHIC) at Brookhaven National Laboratory, USA:

Collision of Pb-Pb , Au-Au at 200 GeV center of mass energy.

Large Hadron Collider (LHC) at CERN:

CMS energies up to 5.5 TEV

RHIC estimates:

Energy density > 10 GeV/fm^3 , Temperature >~ 250 MeV

Lattice Calculations:

Critical temperature ~ 170 MeV

Critical energy density ~ 1 GeV/fm^3

Thus: Most likely, QGP is created in these experiments

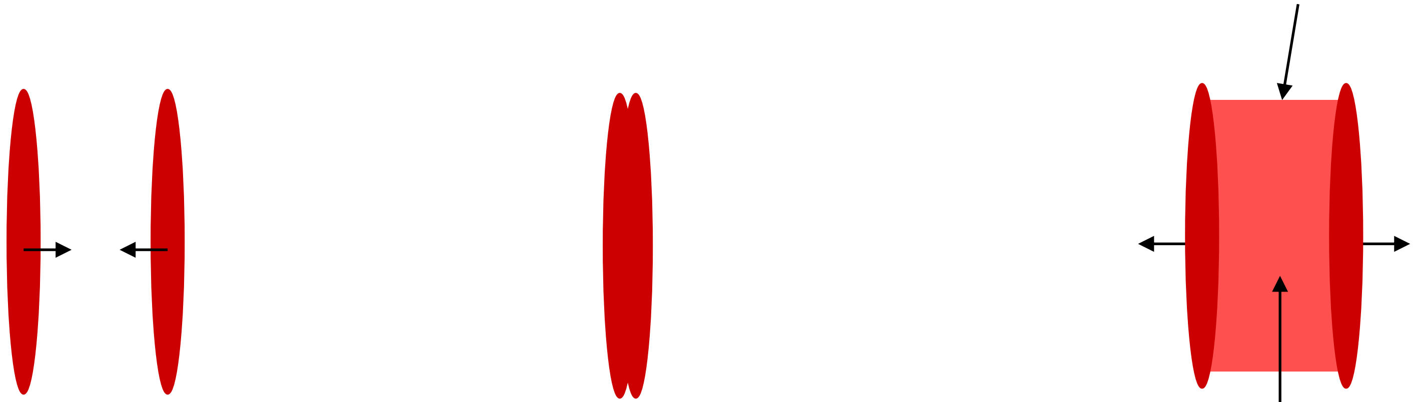
Basic Physical Picture:

Ultra relativistic heavy-ion collisions - possibility of Creating quark-gluon plasma in a transient stage

Essentially – recreating temperatures and energy densities Present in the early universe at the age ~ micro seconds

Possibility of studying quark-hadron phase transition (as occurred in the early universe) in controlled laboratory Experiments.

Initial **QGP**: Rapid Longitudinal expansion, No transverse expansion



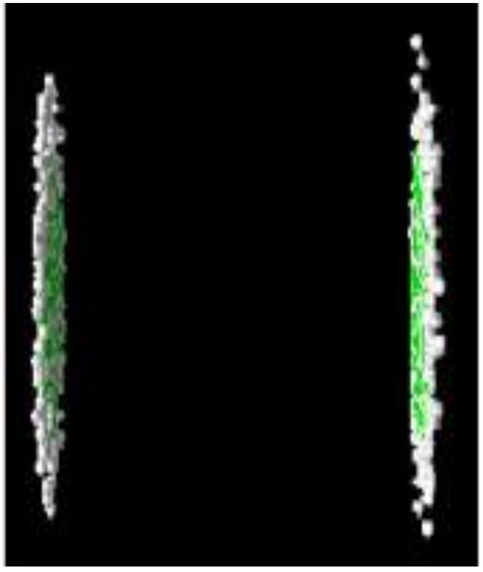
Finally: Hadrons detected

Ultra-relativistic Nuclei approaching

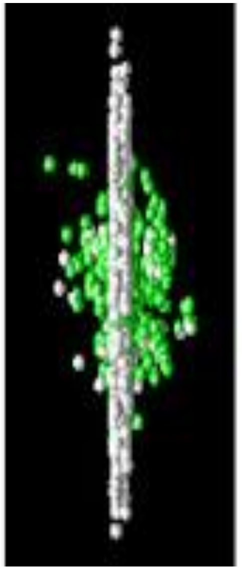
Overlap

Secondary partons thermalize And form QGP between Receding nuclei

A Pictorial View of Micro-Bangs at RHIC

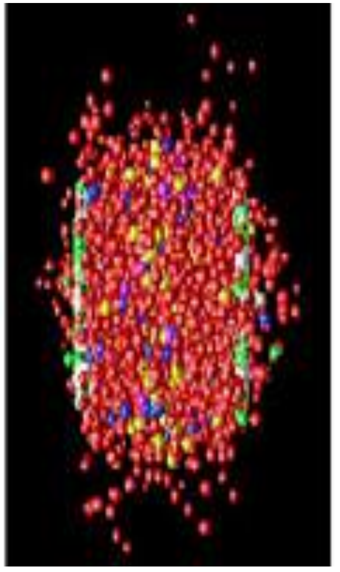


Thin Pancakes
Lorentz γ=100

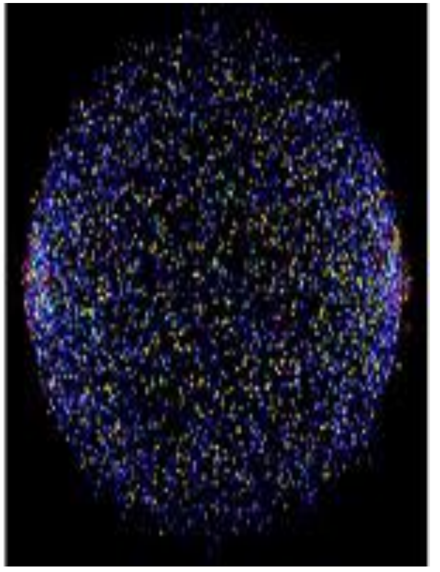


Nuclei pass throuth each other < 1 fm/c

(Asymptotic freedom)



Huge Stretch
Transverse Expansion
High Temperature (?!)



The Last Epoch:
Final Freezeout--
Large Volume

QGP phase a transient stage in lab, lasts for ~ 10^{-22} sec.

Finally only hadrons detected carrying information of the system at freezeout stages (chemical/ thermal freezeout).

Popular Talks: Quite like CMBR which carries the information at the surface of last scattering in the universe.

Just like for CMBR, one has to deduce information about The earlier stages from this information contained in hadrons Coming from the freezeout surface.

We have demonstrated that this apparent correspondence with CMBR is in fact very rigorous

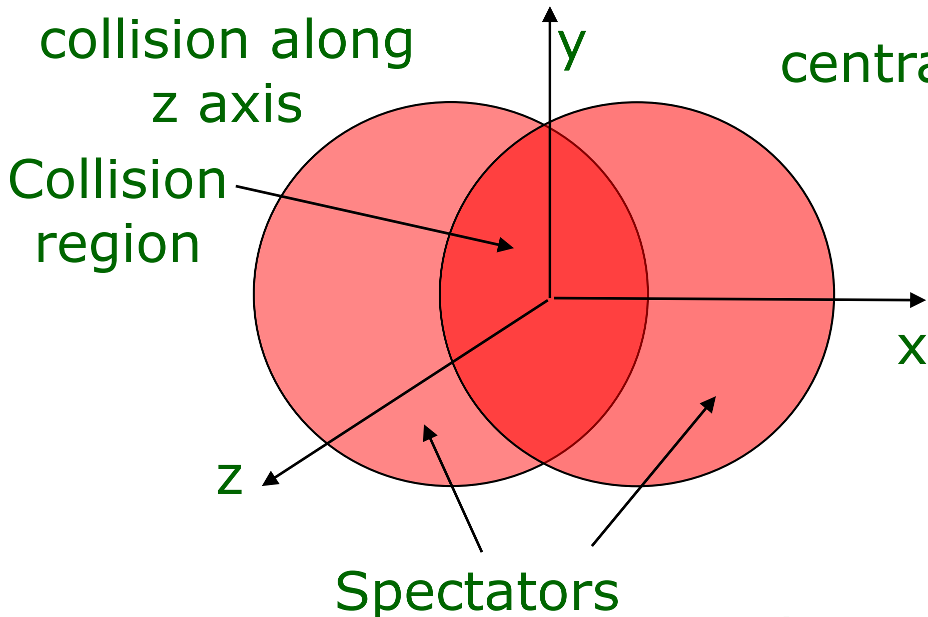
Collaborators: Ananta P. Mishra, Ranjita K. Mohapatra, and P.S. Saumia

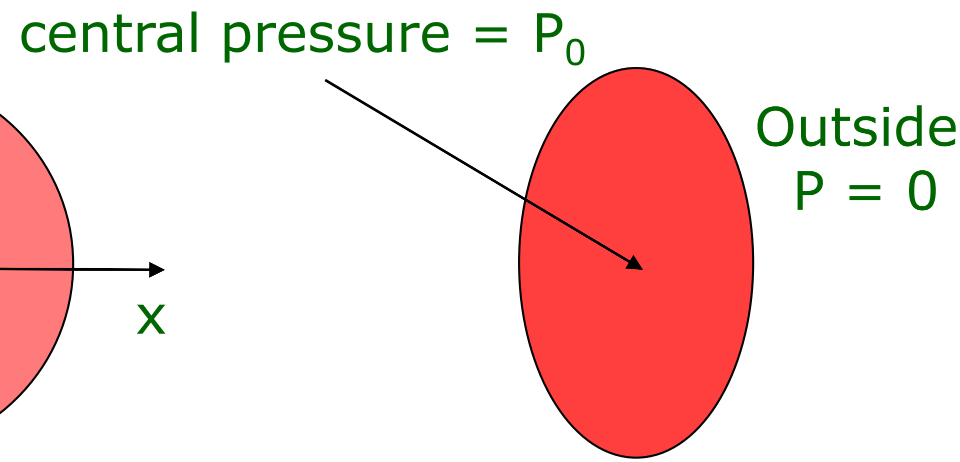
There are strong similarities in the nature of density fluctuations in the two cases (despite the obvious difference due to the absence of gravity effects for relativistic heavy-ion collision experiments).

Relativistic heavy-ion collision experiments (RHICE):

Simple picture of Elliptic Flow in RHICE:

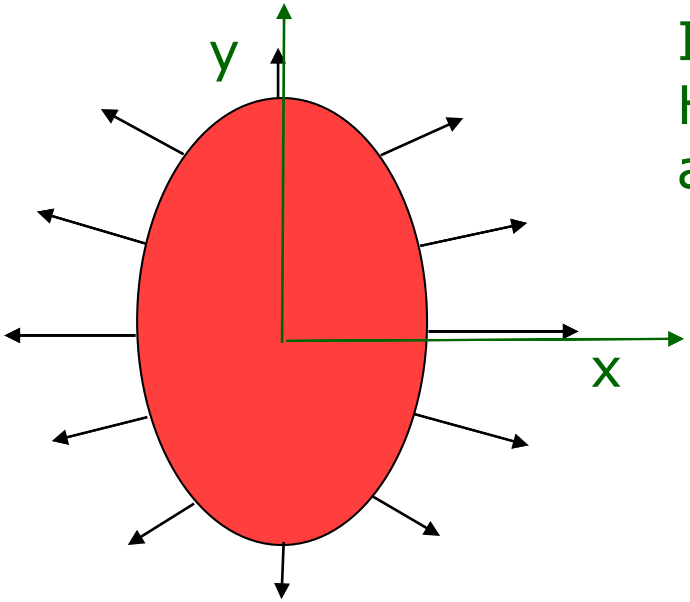
In non-central collisions: the QGP region is anisotropic





Anisotropic shape implies: $$\frac{\partial P}{\partial x} > \frac{\partial P}{\partial y}$$

Important: Initially no transverse expansion
Anisotropic pressure gradient implies:
Buildup of plasma flow larger in x direction
than in y direction



Initial particle momentum distribution
Has azimuthal isotropy : it develops
anisotropy due to larger flow in x direction

This momentum anisotropy is characterized by the 2nd Fourier coefficient V_2 (Elliptic Flow) of momentum variation in azimuthal direction

V_2 calculated by identifying orientation of ellipse (event plane)
For each event, then averaging over many events.

Note: Elliptic flow strongest evidence for thermalization.
No other way to get anisotropic momentum distribution
only from spatial Anisotropy.

Led to very important results:
Strong constraints on η/s : values determined to be in range
1 -3 times AdS/CFT bound. Lower than any known liquid.
So, QGP strongly correlated system, not like ideal gas

Initial state fluctuations in relativistic heavy-ion collisions

Inhomogeneities of all scales present in heavy-ion collisions, even in central collisions: Arising from initial state fluctuations

These fluctuations were known earlier, however, discussed only for estimating errors in eccentricity for elliptic flow calculations.

We argued: Focus on these fluctuations:
Wealth of information about initial state fluctuations resulting from initial collisions (Flow sensitive to initial stages).

Recall: CMBR Fluctuations probed initial quantum fluctuations in the universe: Ruled out cosmic string models,
Observations of CMBR Power spectrum only consistent with Quantum fluctuations of the inflaton field in Inflationary models.

Lesson: Calculate Power spectrum of flow fluctuations in Relativistic heavy-ion collisions for probing initial state fluctuations

Get information about initial states of colliding nucleons

Power spectrum of flow fluctuations: New approach to flow analysis

Mishra, Mohapatra, Saumia, AMS, PRC77, 064902 (2008); 81, 034903(2009)

Early discussions of flow coefficients only for V_2, V_4, V_6 for non-central collisions
(Average values of these were calculated by finding event plane)

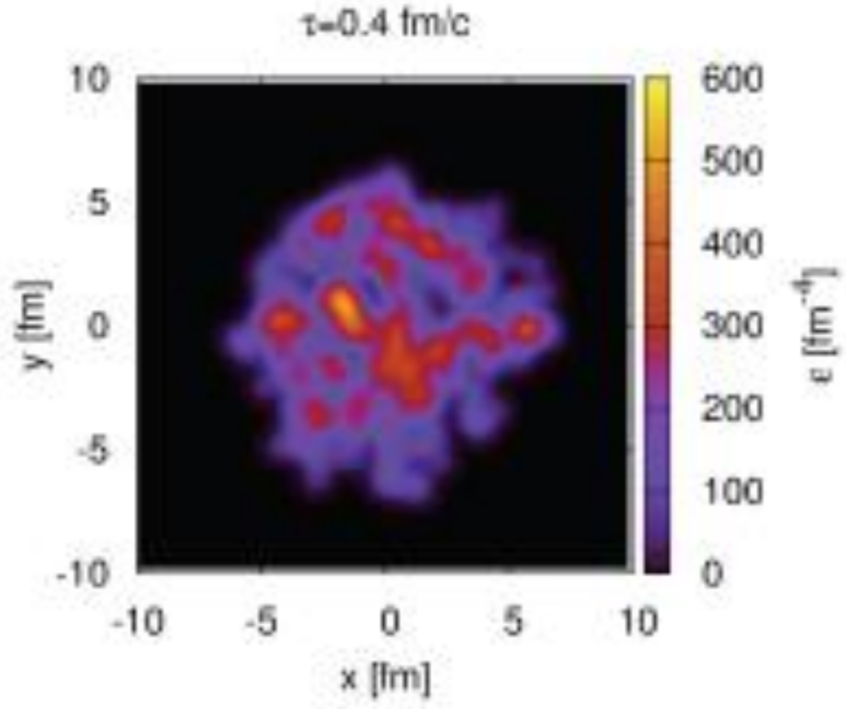
We argued that due to the initial state fluctuations all flow coefficients will be non-zero in general, even in central collisions:

Thus:
All Fourier coefficients V_n are of interest (say, n=1 to 30 -40),
including Odd harmonics, these were never discussed earlier.

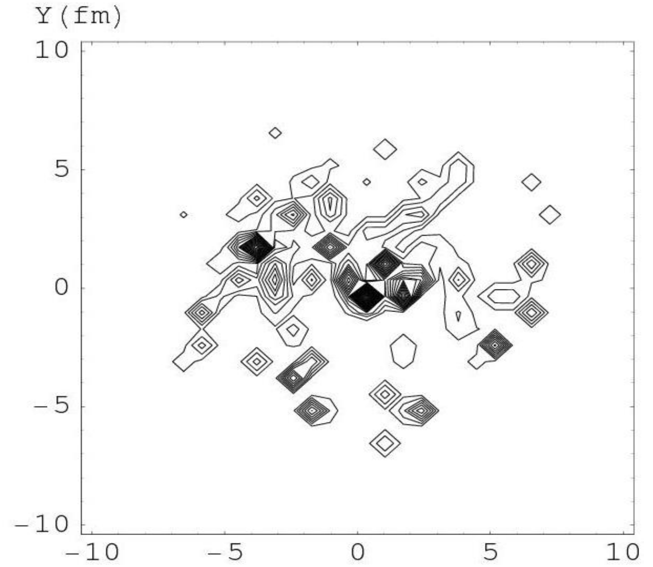
We emphasized: Learn from CMBR power spectrum analysis:

Calculate root-mean-square values of V_n , and NOT their average values.

Initial state fluctuations



(Central collision: Blaizot,
Matter at extreme conditions, 2014)



transverse energy density:
Au-Au **central collision**
at 200 GeV/A, HIJING

Thus: the equilibrated matter will also have azimuthal anisotropies (as well as radial fluctuations) of similar level.

We emphasized: Lesson from CMBR power spectrum analysis:
Plot of root-mean-square values: Enormous information about nature of fluctuations, their evolution, equation of state, etc.

Important lesson for heavy-ion collisions from CMBR analysis

CMBR temperature anisotropies analyzed using Spherical Harmonics

$$\frac{\Delta T}{T}(\theta,\phi) = a_{lm} Y_{lm}(\theta,\varphi)$$

Now: Average values of these expansions coefficients are zero due to overall isotropy of the universe

$$< a_{lm} >= 0$$

However: their standard deviations are non-zero and contain crucial information.

$$C_l =<| a_{lm} |^2>$$

This gives the celebrated Power Spectrum of CMBR anisotropies

Lesson : Apply same technique for RHICE also

For azimuthal variations of transverse momentum in central rapidity region, so calculate Fourier coefficients V_n .

For central events average values of flow coefficients will be zero

$$< V_n >= 0$$

That is why earlier calculations identify orientation of the event shape, then calculate average v_2.

(same is true even for non-central events if a coordinate frame with fixed orientation in laboratory system is used).

Following CMBR analysis,we proposed to calculate root-mean-square values of these flow coefficients using a lab fixed coordinate system, And plot it for a large range of values of n = 1, 30-40

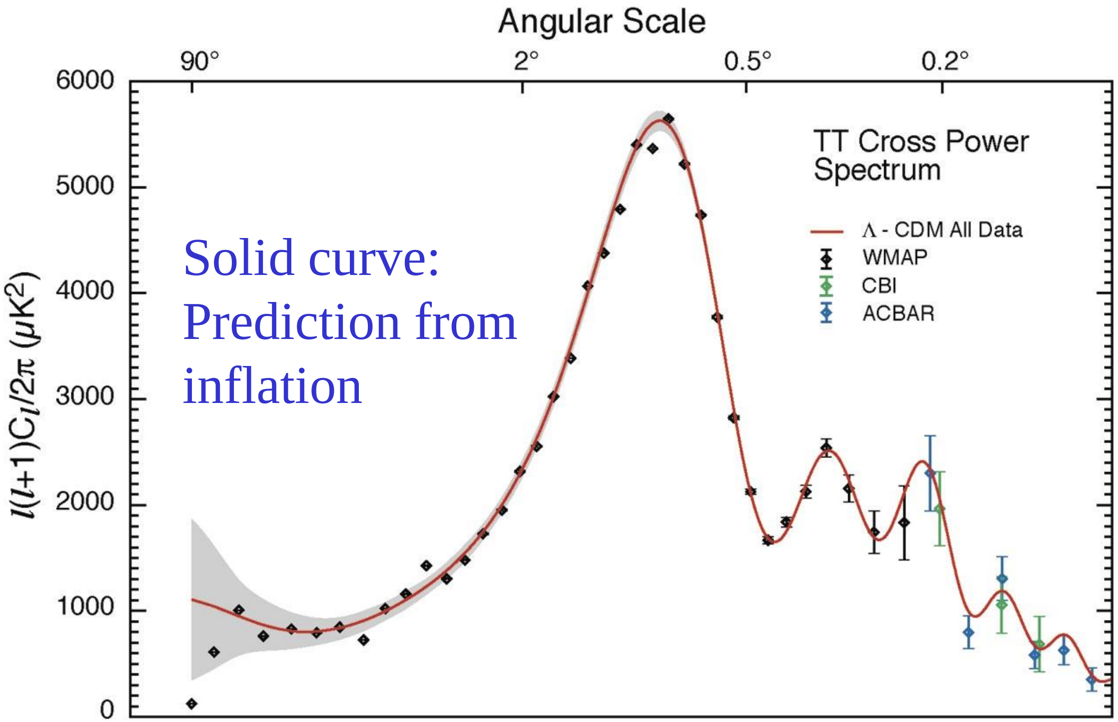
$$V_n^{rms} = \sqrt{< V_n^2 >}$$

These values will be generally non-zero for even very large n and will carry important information .

Conclusion: Plot power spectrum for V_n for entire range of values of n. The whole plot will have information about Early stages of fluctuations and plasma evolution.

A powerful tool for probing initial state fluctuations

Recall: Acoustic peaks in CMBR anisotropy power spectrum



Can such a power spectrum be expected for heavy-ion collisions ?

So far we discussed: Plot of V_n^{rms} for large values of n will give important information about initial density fluctuations.

We now discuss: Such a plot may also reveal non-trivial structure like acoustic peaks for CMBR as above.

As for CMBR, different peaks can give information about interaction between different particles (from particle specific power spectra).

Importance of causal structure of initial state fluctuations:

We have noted that initial state fluctuations of different length scales are present in Relativistic heavy-ion collisions even for central collisions

The process of equilibration will lead to some level of smoothening. However, thermalization happens quickly (for RHIC, within 1 fm)

No homogenization can be expected to occur beyond length scales larger than this.

This provides a natural concept of causal Horizon

Thus, inhomogeneities, especially anisotropies with wavelengths larger than the thermalization time scale should be necessarily present at the thermalization stage when the hydrodynamic description is expected to become applicable.

As time increases, the causal horizon (or, more appropriately, the sound horizon) increases with time.

An Important feature of flow power spectrum
Fluctuations with **superhorizon** wavelengths

Meaning of Horizon for the Universe:

Horizon size = speed of light c x age of the universe t
No physical effect possible for distances larger than this

In the universe, density fluctuations with wavelengths of superhorizon scale have their origin in the inflationary period.
Tiny fluctuations are stretched by superluminal expansion

Meaning of Horizon for Heavy-ion collisions:

System equilibrates in time τ_0 less than 1 fm/c. Horizon size = c τ_0
No physical effects possible for distances larger than c τ_0 = 1 fm.

Note: Fluctuations present of all wavelengths even at time τ_0
(arising from N-N collisions and fluctuations in nucleon positions).
All fluctuations larger than 1 fm are superhorizon at time τ_0.

At any later time τ, any fluctuation larger than cτ is superhorizon.

Inflationary Density Fluctuations:

We know: Quantum fluctuations of sub-horizon scale are stretched out to superhorizon scales during the inflationary period.

During subsequent evolution, after the end of the inflation, fluctuations of sequentially increasing wavelengths keep entering the horizon. The largest ones to enter the horizon, and grow, at the stage of decoupling of matter and radiation lead to the first peak in CMBR anisotropy power spectrum.

We have seen that superhorizon fluctuations should be present in RHICE at the initial equilibration stage itself.

Note: sound horizon, $H_s = c_s$ t here, where c_s is the sound speed, is smaller than 1 fm at t = 1 fm. At time t from the birth of the plasma, physical effects cannot propagate to distances beyond H_s

With the nucleon size being about 1.6 fm, the equilibrated matter will necessarily have density inhomogeneities with superhorizon wavelengths at the equilibration stage.

Recall: Two crucial aspects of the inflationary density fluctuations leading to the remarkable signatures of acoustic peaks in CMBR:

Coherence and Acoustic oscillations.

Note: Coherence of inflationary density fluctuations essentially results from the fact that the fluctuations initially are stretched to superhorizon sizes and are subsequently frozen out dynamically.

In the context of heavy-ion collisions, this freezing out is similar to the absence of initial transverse expansion velocity for QGP.

Fluctuations are in the spatial variation of energy density only initially, they become dynamical through hydrodynamical evolution.

For all fluctuations of certain size, it happens after a certain time when causal horizon equals the fluctuation size.

Thus coherence will be expected to hold for RHICE also.

Oscillatory behavior for the fluctuations.

Important: Small perturbations in a fluid will always propagate as acoustic waves, hence oscillations are naturally present.

Note: **The only difference from the universe is the absence of Gravity for RHICE.**

However, in the universe, the only role of attractive Gravity is to compress the initial overdensities.
Acoustic oscillations happen on top of these fluctuations.

One can say that for RHICE one will get harmonic oscillations (for a given mode) while for the Universe one gets oscillations of a forced oscillator.

(One can also argue for oscillations of the irregular shape of the boundary of the QGP region.)

Conclusion: For RHICE also, one should have acoustic oscillations, Which are coherent: just as for CMBR.

Two crucial aspects of the inflationary density fluctuations leading to the remarkable signatures of acoustic peaks in CMBR:

Coherence and Acoustic oscillations.

Note: Coherence of inflationary density fluctuations essentially results from the fact that the fluctuations initially are stretched to superhorizon sizes and are subsequently frozen out dynamically.

Thus, at the stage of re-entering the horizon, when these fluctuations start growing due to gravity, and subsequently start oscillating due to radiation pressure, the fluctuations start with zero velocity.

$$X(t) = A \cos(\omega t) + B \sin(\omega t) = C \cos(\omega t + \phi)$$

where ϕ is the phase of oscillation. Now, the velocity is:

$$dX(t)/dt = -A\omega \sin(\omega t) + B\omega \cos(\omega t) = 0 \text{ at } t = 0$$

→ $B = 0$, So only $\cos(\omega t)$ term survives in oscillations or, phase $\phi = 0$ for all oscillations, irrespective of amplitude.

So: all fluctuations of a given wavelength (ω) are **phase locked**.

This leads to clear peaks in CMBR anisotropy power spectrum

Note: for RHICE we are considering transverse fluctuations.
Main point: Transverse velocity of fluid to begin with is zero.

Transverse velocity (anisotropic part for us) arises from pressure gradients. However, for a given mode of length scale λ, pressure gradient is not effective for times $t < \lambda/c_s$. In other words, until this time, the mode is essentially frozen, just as in the universe.
(Note: This is just the condition λ > acoustic horizon size $c_s t$)

For large wavelengths, those which enter (sound) horizon at times much larger than equilibration time, build up of the radial expansion will not be negligible.
However, our interest is in oscillatory modes.
For oscillatory time dependence even for such large wavelength modes, there is no reason to expect the presence of $\sin(\omega t)$ term at the stage when the fluctuation is entering the sound horizon.

In summary: For RHICE also all fluctuations with scales larger than 1 fm should be reasonably coherent

We argued that sub-horizon fluctuations in heavy-ion collisions should display oscillatory behavior just as fluctuations for CMBR

What about super-horizon fluctuations ?

Recall: For CMBR, the importance of horizon entering is for the growth of fluctuations due to gravity.

This leads to increase in the amplitude of density fluctuations, with subsequent oscillatory evolution, leaving the imprints of these important features in terms of acoustic peaks.

Superhorizon fluctuations for universe do not oscillate (they are frozen, as we discussed earlier).

Importantly, they also do not grow,
That is: they are suppressed compared to the fluctuation which enters the horizon.

For heavy-ion collisions, there is a similar (though not the same, due to absence of gravity here) importance of horizon entering. One can argue that flow anisotropies for superhorizon fluctuations in heavy-ion collisions should be suppressed by a factor $\frac{H^{s}_{fr}}{\lambda/2}$ where H^{s}_{fr} is the sound horizon at the freezeout time t_{fr} (~ 5-10 fm for heavy-ion collisions)

This is because here spatial variations of density are not directly detected, in contrast to the Universe where one directly detects the spatial density fluctuations in terms of angular variations of CMBR.

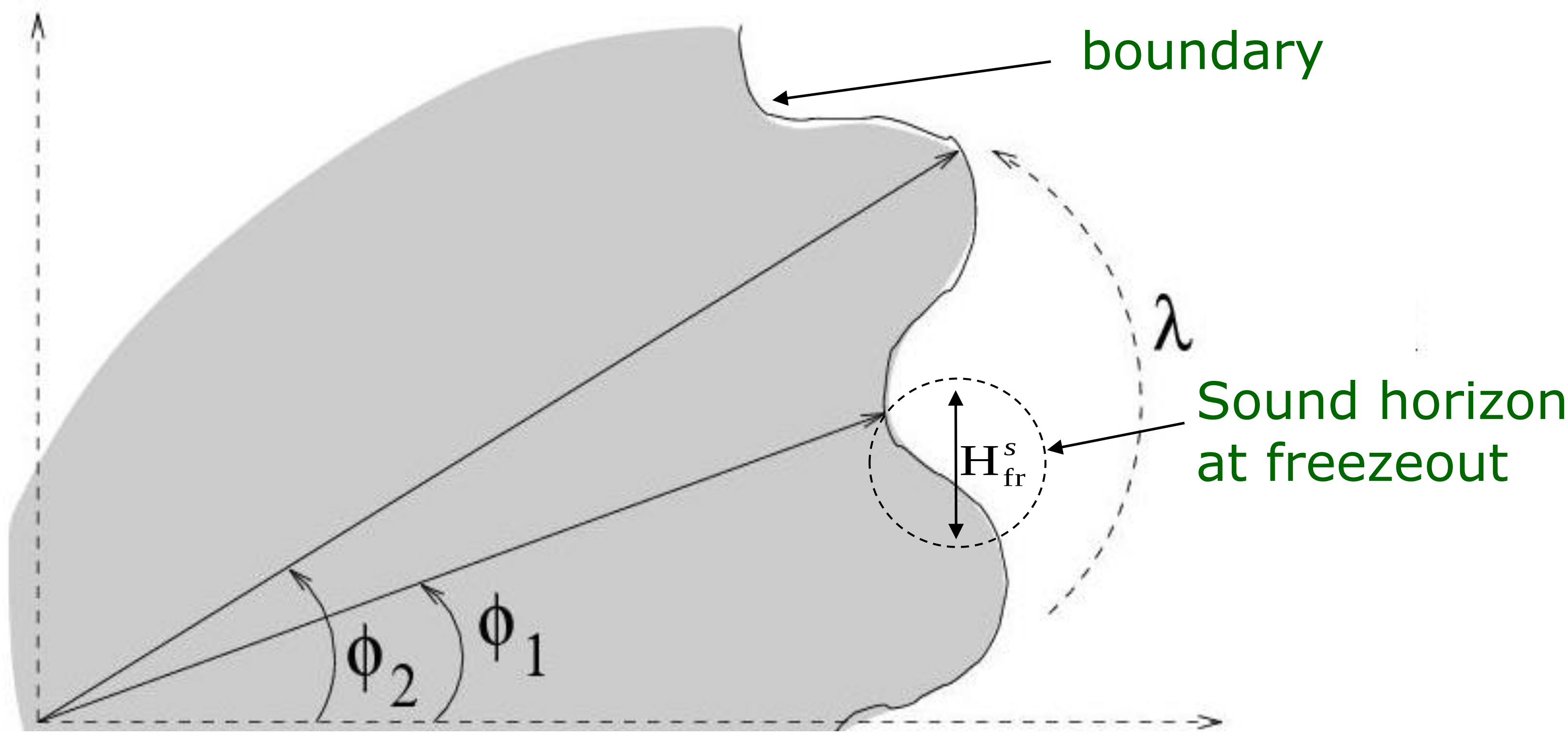
For heavy-ion collisions, spatial fluctuation of a given scale (i.e. a definite mode) has to convert to fluid momentum anisotropy of the corresponding angular scale.

This will get imprinted on the final hadrons and will be experimentally measured.

This conversion of spatial anisotropy to Momentum ansitropy (via pressure gradients) is not effective for Superhorizon modes.

Thus: Superhorizon modes will be suppressed in heavy-ion collisions

Presence of such a suppression factor can also be seen for the case when the build up of the flow anisotropies is dominated by the surface fluctuations of the boundary of the QGP region.



When $\lambda >> H^s_{fr}$, then by the freezeout time full reversal of spatial anisotropy is not possible: The relevant amplitude for oscillation is only a factor of order $H^s_{fr}/(\lambda/2)$ of the full amplitude.

Our earlier Results:

Mishra, Mohapatra, Saumia, AMS, PRC77, 064902 (2008); 81, 034903(2009)

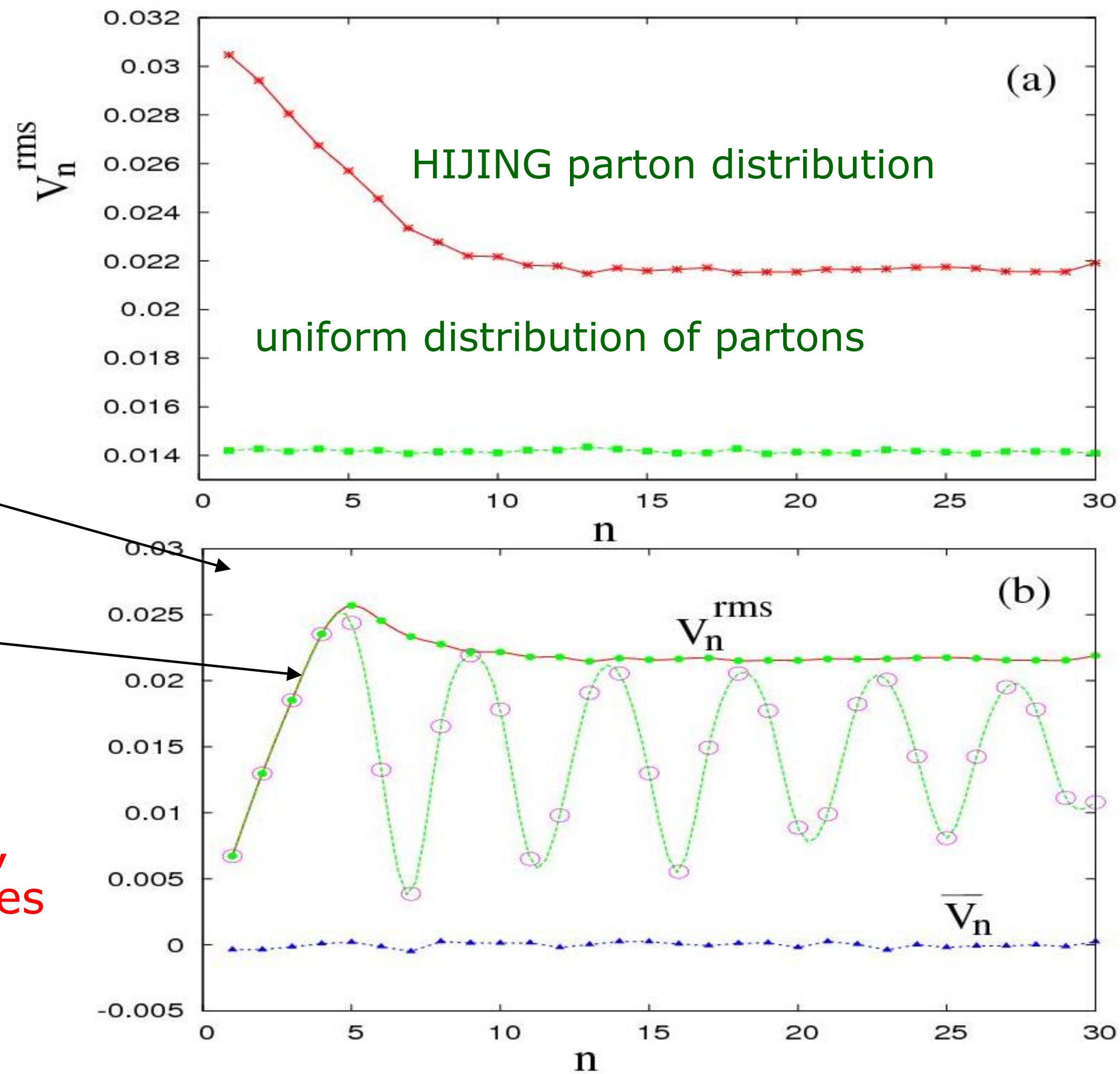
(modeling only, no hydrodynamical simulation here yet)

Errors less than ~ 2%

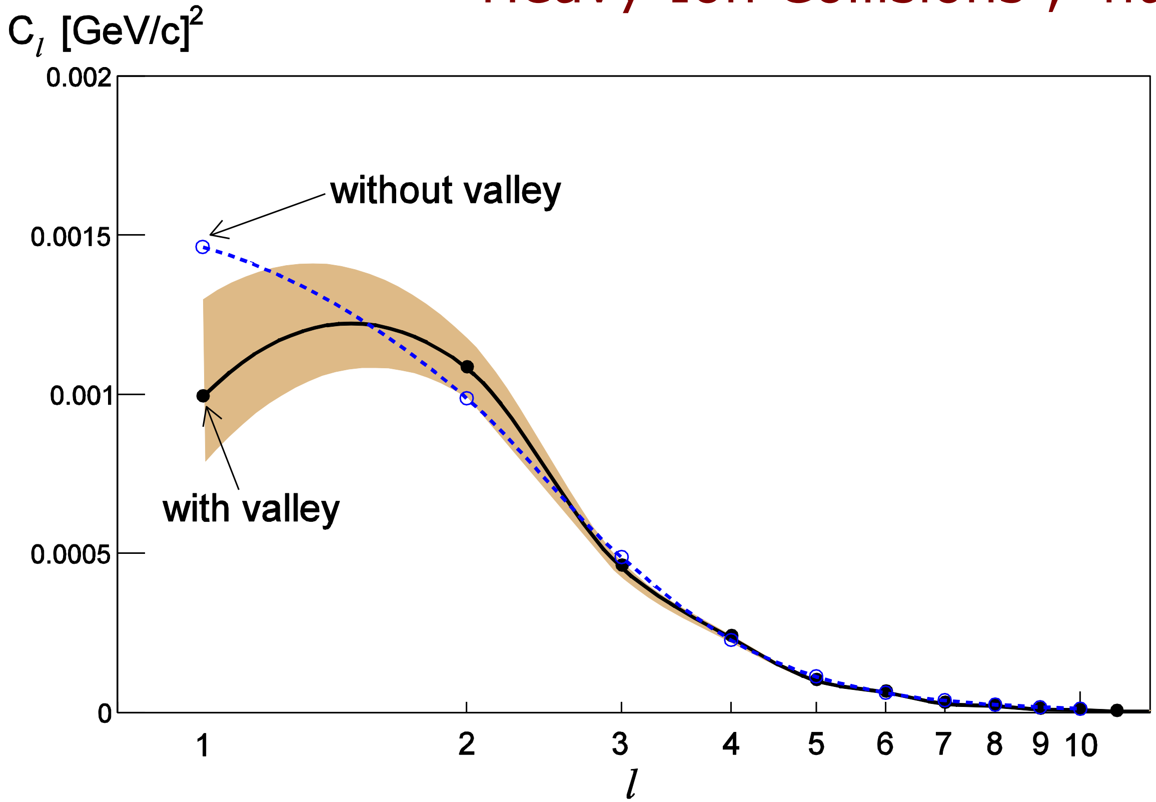
Include superhorizon suppression

Include oscillatory factor also

Note: Dissipation, e.g. from viscosity, diffusion, will damp higher n modes

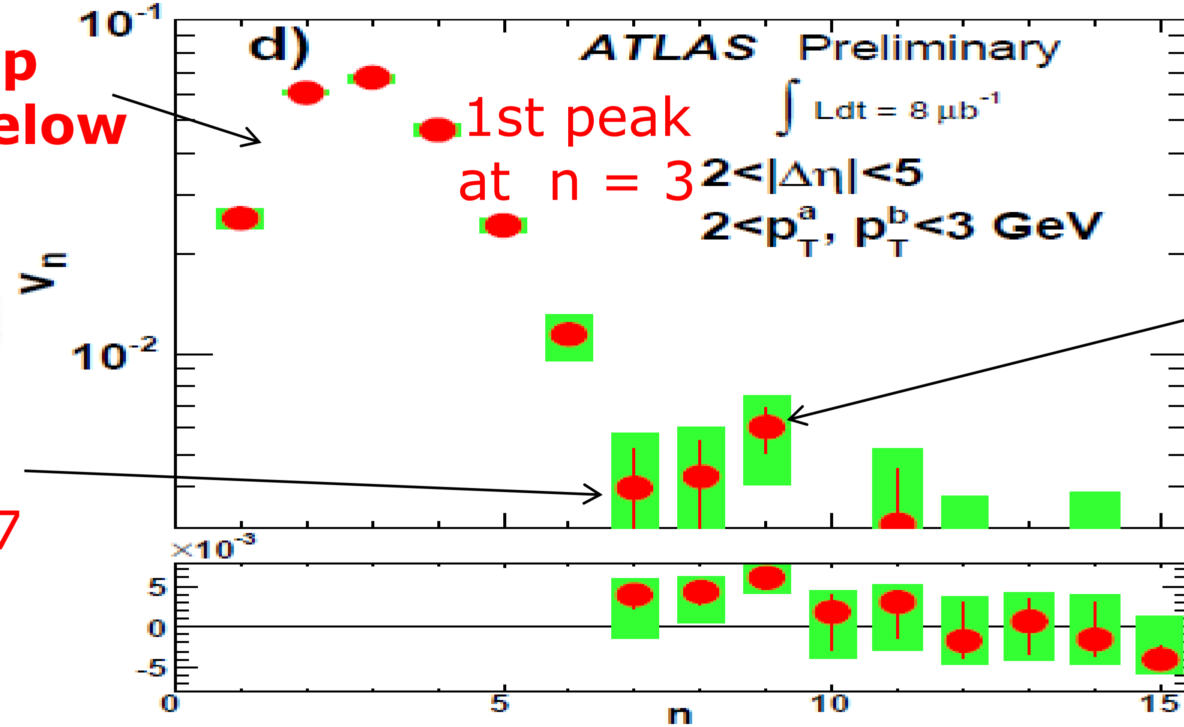


Paul Sorensen “Searching for Superhorizon Fluctuations in Heavy-Ion Collisions”, nucl-ex/0808.0503



See, also, youtube video by Sorensen from STAR:
http://www.youtube.com/watch?v=jF8QO3Cou-Q

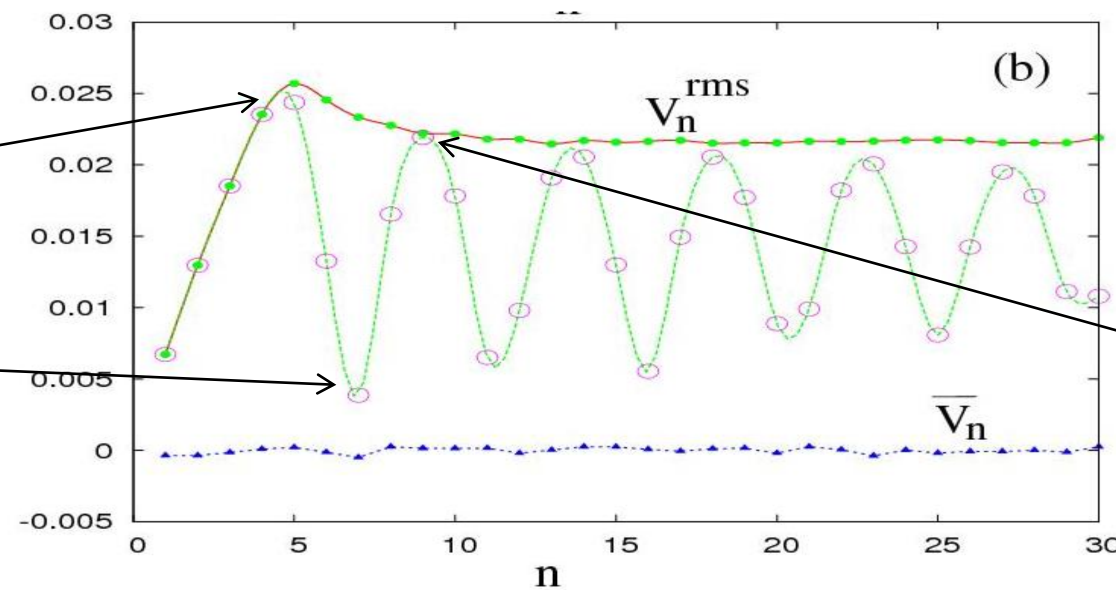
Focus on dip for low n below First peak



1st peak at n = 3

2nd peak at n = 9

1^{st} dip At n = 7

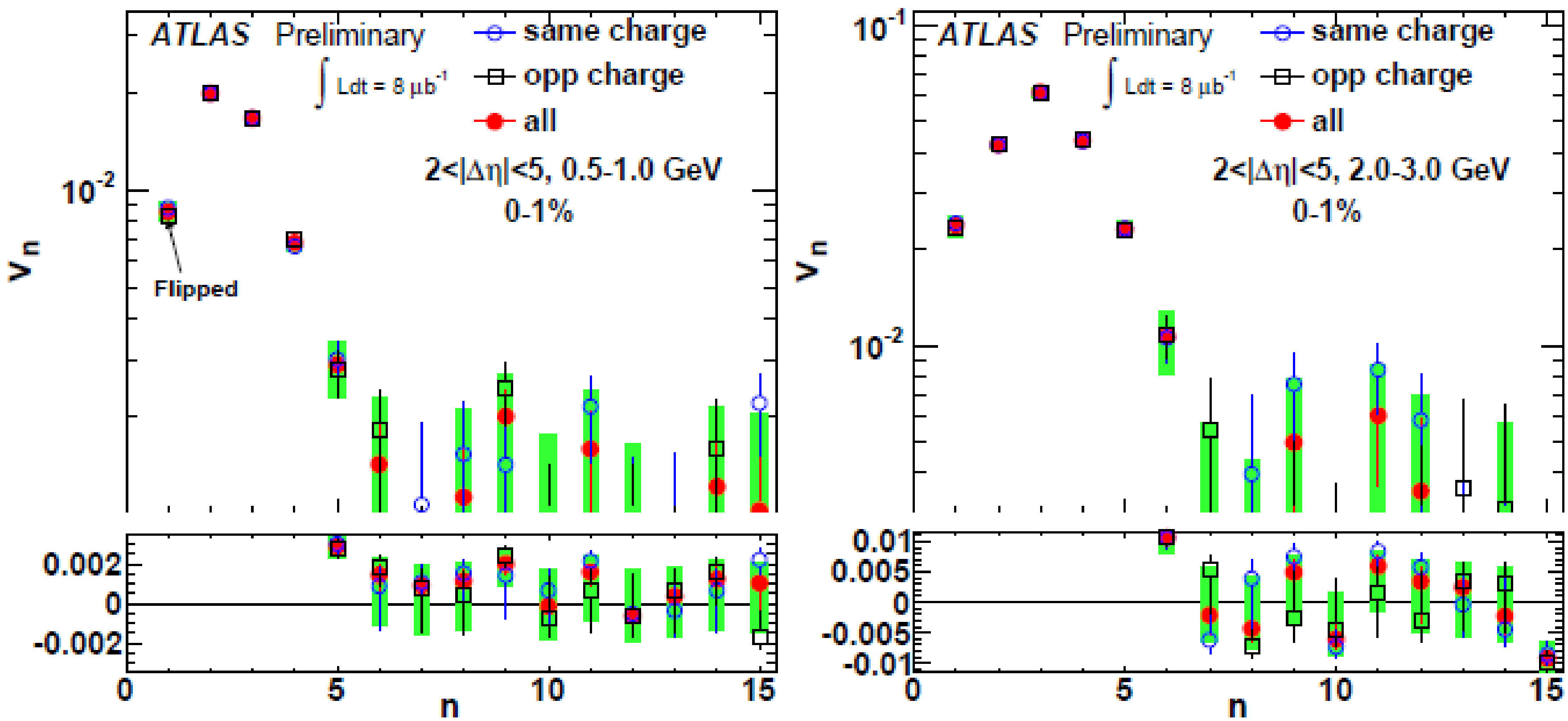


1st peak at n = 5

1^{st} dip at n = 7

2nd peak at n = 9

arXiv: 1107.1468, LHC Pb-Pb at 2.76 TeV



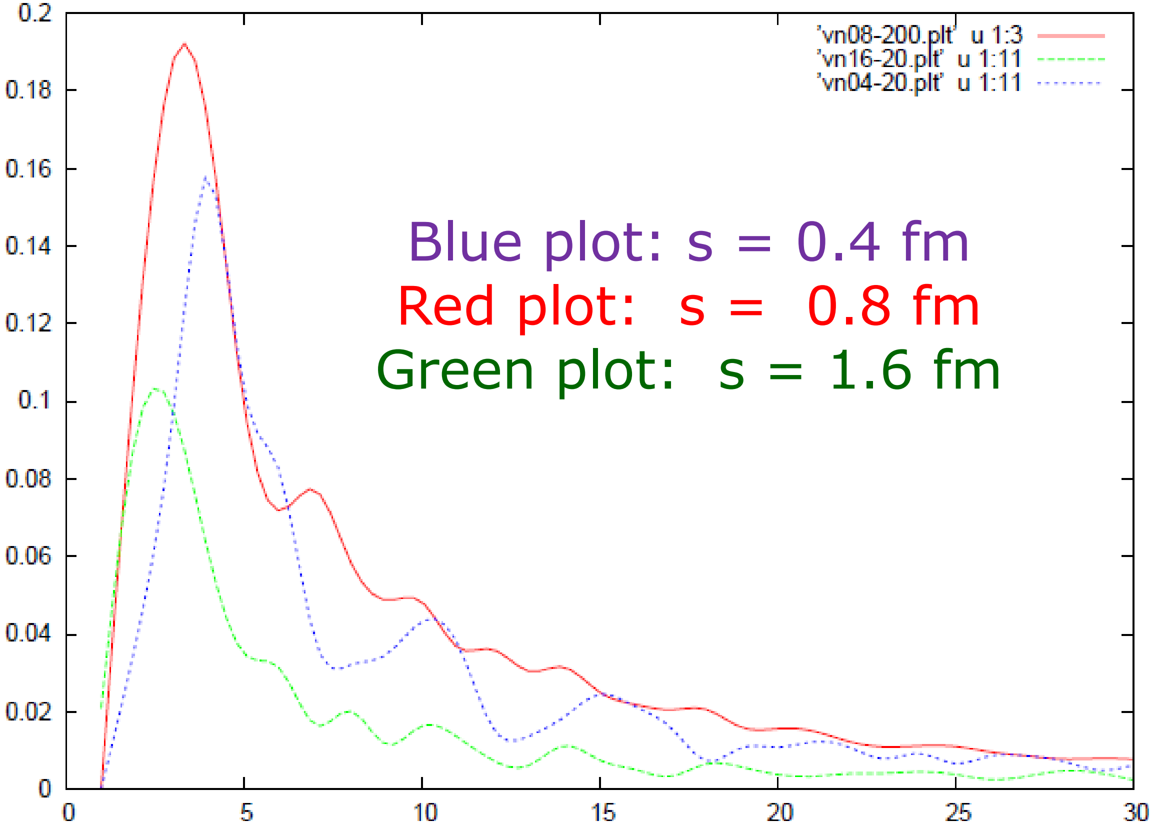
Note: Very important to understand suppression of low n harmonics
It contains the information about freezeout horizon size:

Signals presence of initial fluctuations on superhorizon scales

Relativistic Hydrodynamics Simulations:

(Saumia P.S., AMS, Mod. Phys. Lett. A31, 1650197 (2016))

Plots of V_n^{rms} vs. n for Gaussian fluctuations of width s

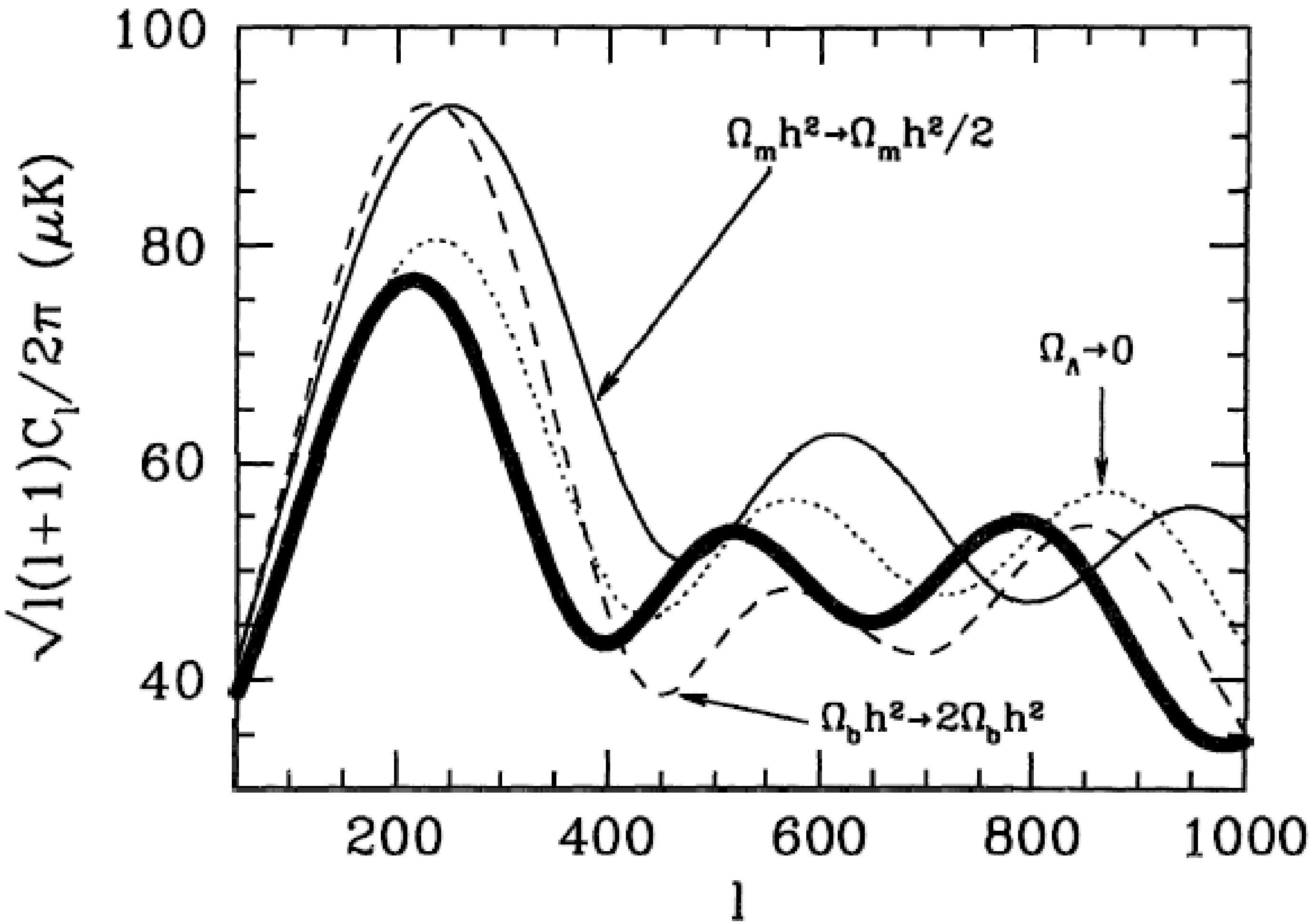


Note: Here peak position gives information about length scale of fluctuations

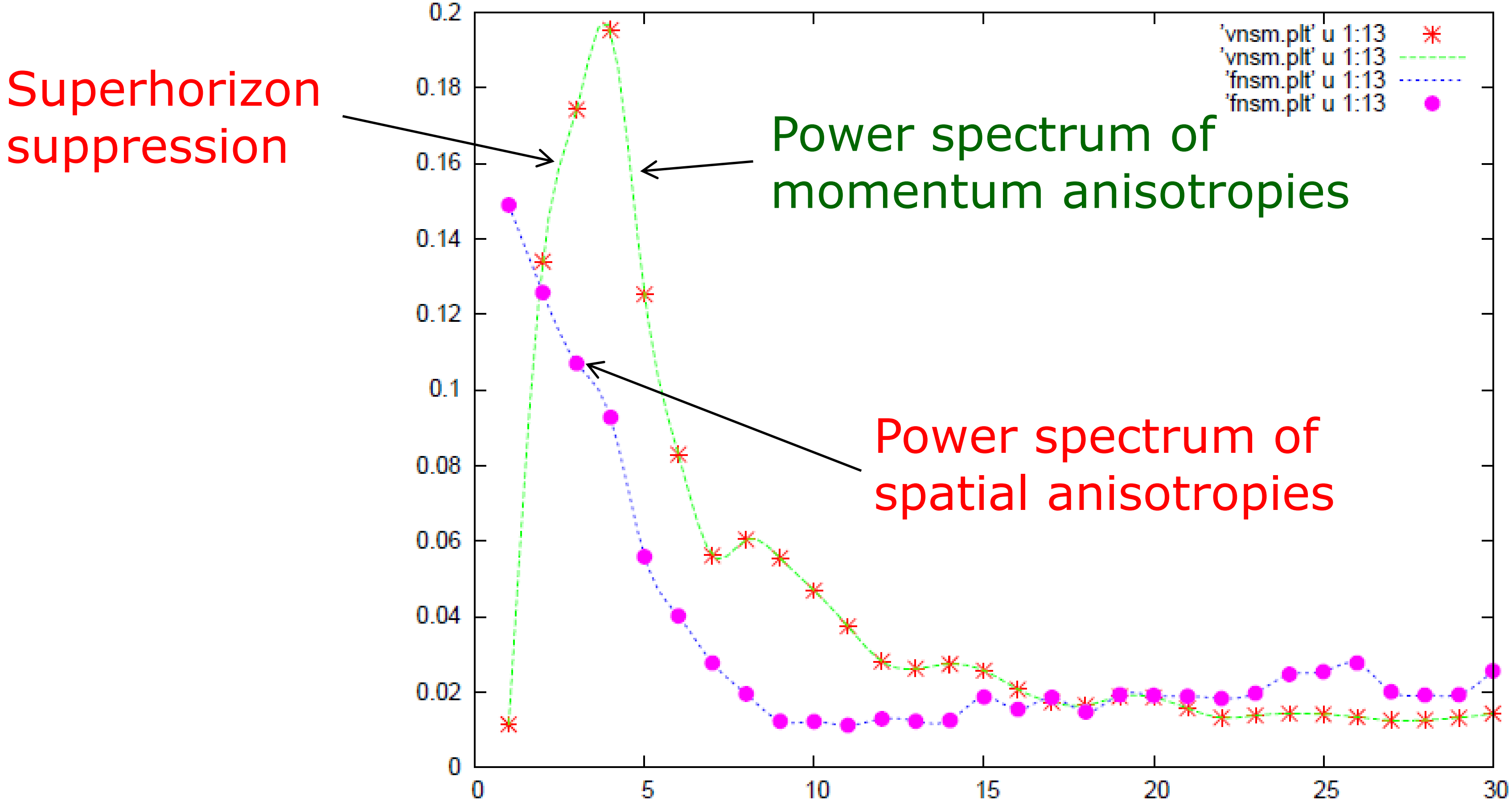
Just as for CMBR, where first peak location directly gives size of largest fluctuation at last scattering surface

Woods-Saxon density profile, 2 fm radius with 10 Gaussian fluctuations, T_0 =500 MeV

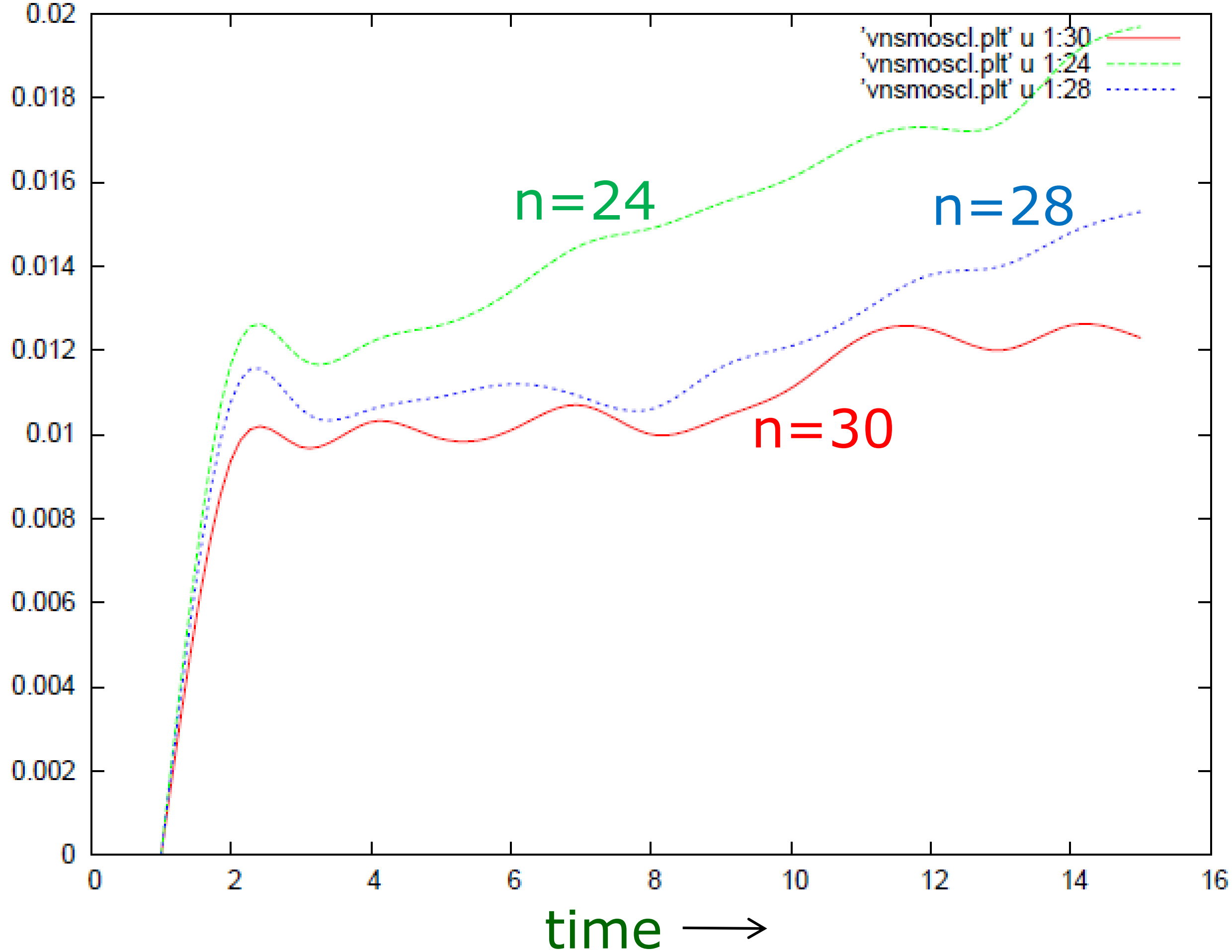
CMBR: Changes in the location of peaks with energy-matter density of the Universe, (apparent horizon size changes)



Evidence for Superhorizon suppression from hydro simulations



Development of acoustic oscillations:
Larger n oscillate first, lower n oscillate later



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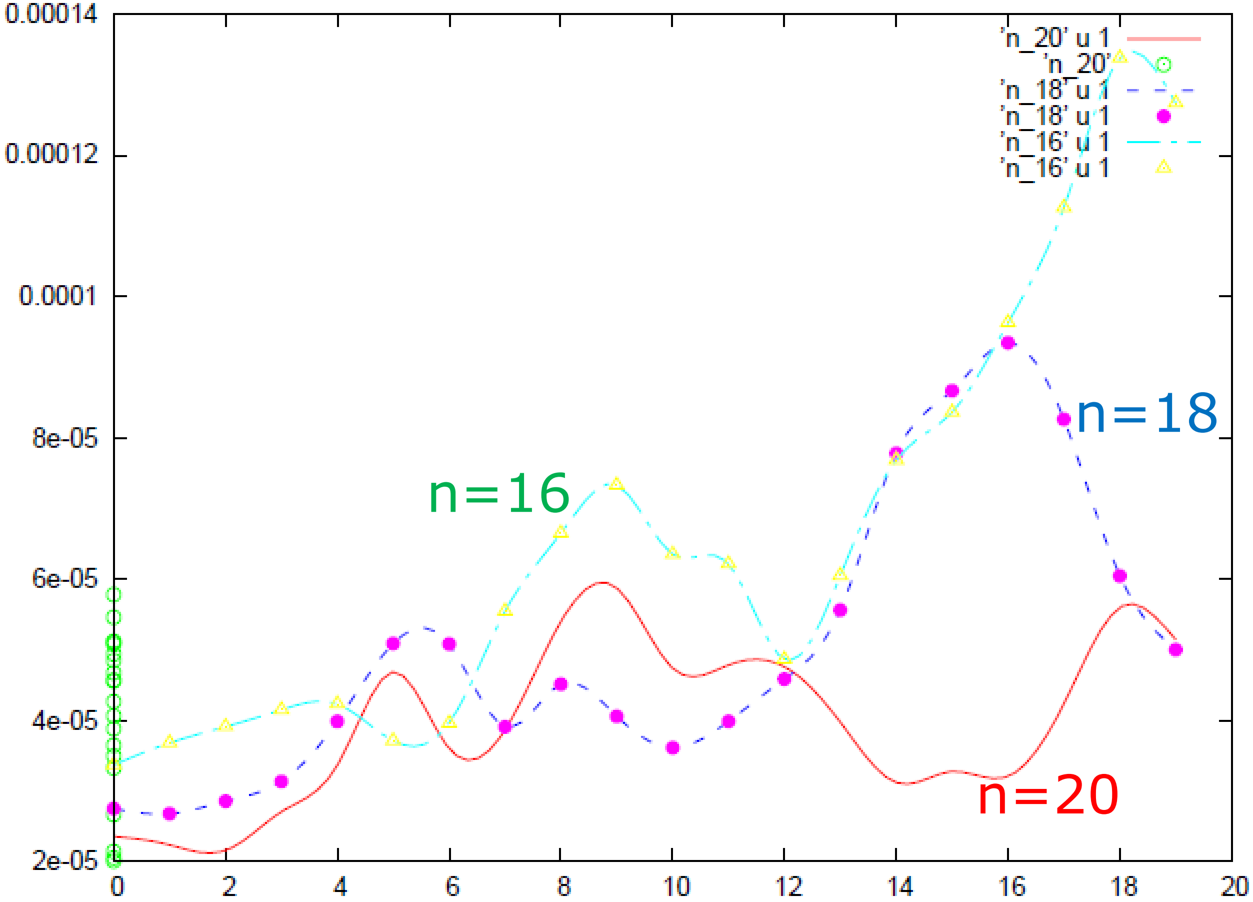


Effect of Magnetic Field

On acoustic peaks of CMBR:

Primordial magnetic fields are present in the universe. Plasma will evolve according to the equations of magnetohydrodynamics.

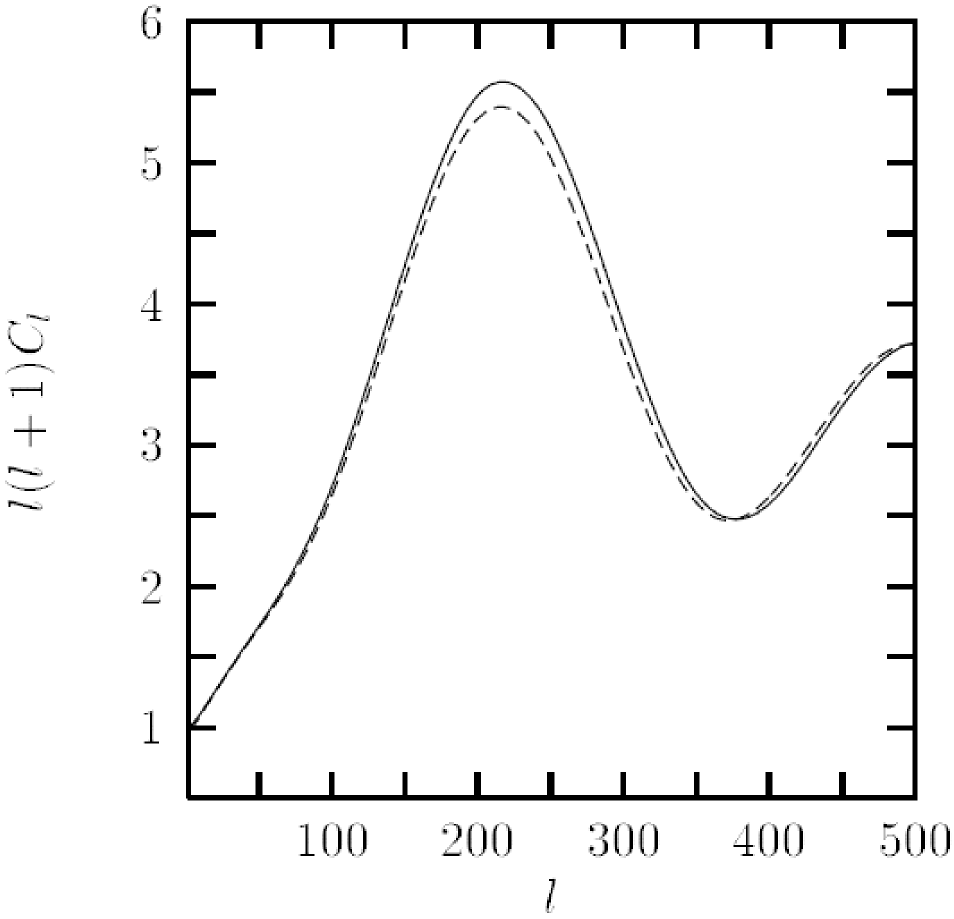
In presence of magnetic field, there are three types of waves in the plasma in place of ordinary sound waves.

Fast magnetosonic waves: Generalised sound waves with significant contributions from the magnetic pressure. Their velocity is given by $c^2_+ \sim c^2_s + v^2_A \sin^2\theta$ where θ is the angle between the magnetic field B_0 and the wave vector and the Alfven velocity $v_A = B_0/\sqrt{4\pi\rho}$

Slow magnetoacoustic waves: Sound waves with strong magnetic guidance. $c^2_- = v^2_A \cos^2\theta$

Alfven waves: Propagation of magnetic field perturbations.

The magnetic field effects distort the CMBR acoustic peaks. The distortion can be seen as an effect due to the modified sound velocity (fast magnetosonic waves) with some modulation from slow magnetosonic waves.

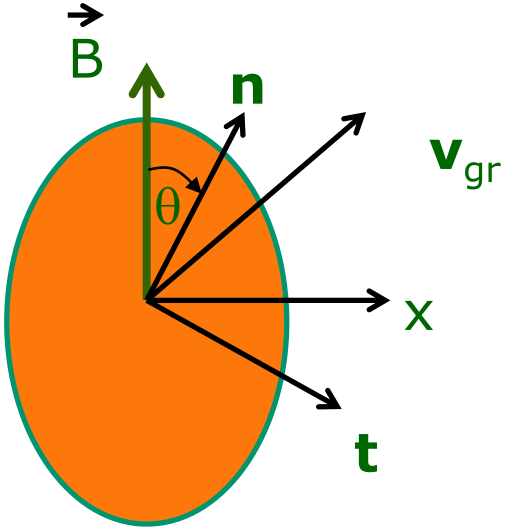


Jenni Adams et al. (1996)

We studied the effect of magnetic field on flow Anisotropies in relativistic Heavy-ion collisions

The expression for group velocity in relativistic MHD

$$\boldsymbol{v}_{gr} = vp_h \left[\boldsymbol{n} + \boldsymbol{t} \frac{\sigma \pm 2\delta\left(a \mp (1+\delta\cos^2\theta)\right)\sin\theta\cos\theta}{2(1+\delta\cos^2\theta \pm a)a} \right]$$



$$\boldsymbol{v}_{ph} = \boldsymbol{n} \frac{\left[\left(\frac{4\rho}{3\omega}\right)c_s^{\,2} + vA^2\right]^{1/2}}{2} (1+\delta\cos^2\theta \pm a)^{1/2}$$

$$\boldsymbol{n} = \boldsymbol{k}/k,\ \boldsymbol{t} = \left[\left(\frac{\boldsymbol{B}}{B}\right) \times \boldsymbol{n}\right] \times \boldsymbol{n}$$

$$a^2 = (1+\delta\cos^2\theta) - \sigma\cos^2\theta),$$

$$\delta = \frac{c_s^{\,2} v_A^{\,2}}{\left(\frac{4\rho}{3\omega}\right)c_s^{\,2} + vA^2},\ \sigma = \frac{4\ c_s^{\,2} v_A^{\,2}}{\left[\left(\frac{4\rho}{3\omega}\right)c_s^{\,2} + vA^2\right]^2},\ \omega = \left(\frac{4\rho}{3}\right) + B^2$$

Note: Direction of group velocity depends on the coefficient of t above, which depends on local pressure (for a given B).

Thus: with pressure variations, direction keeps changing,

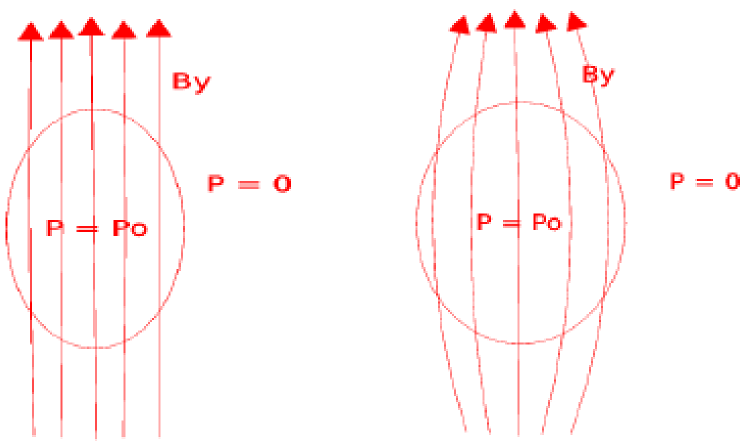
Thus: Complex flow pattern even with radial expansion

Magnetic field enhances elliptic flow

Mohapatra, Saumia, AMS, MPLA 26, 2477 (2011)

Basic physics of the effect:
In presence of magnetic field, there are different types of waves in the plasma. Fast magnetosonic waves: Generalised sound waves with significant contributions from the magnetic pressure.

Basically, distortions of magnetic field in transverse direction costs energy, equation of state stiffer in that direction



Expect larger sound speed in transverse direction.

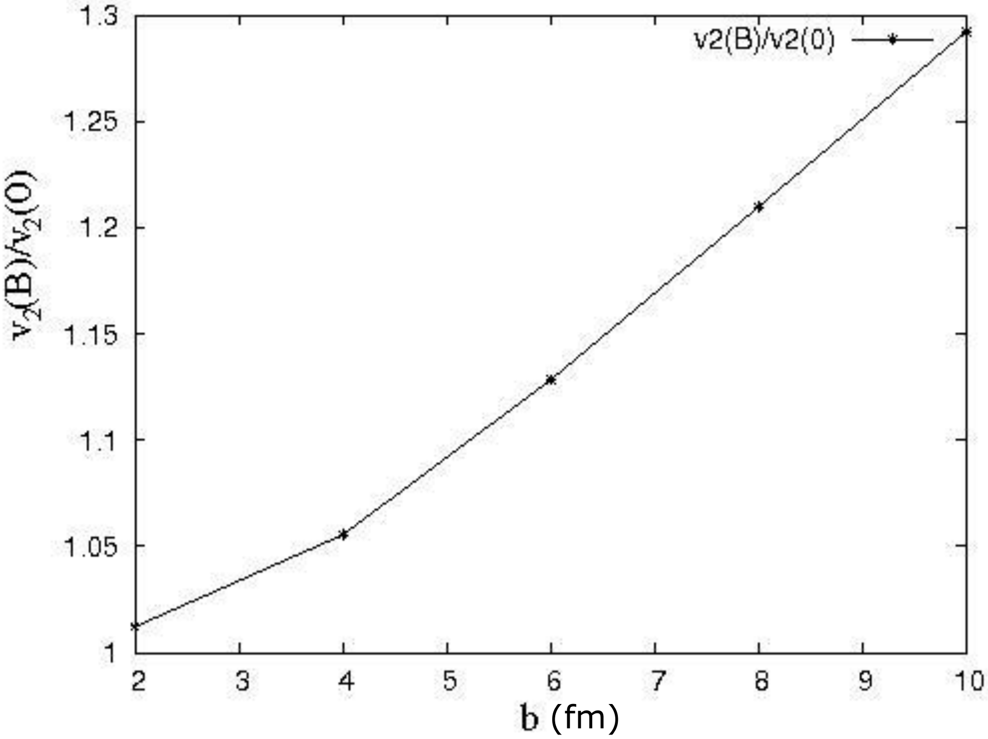
Flow velocity proportional to c_s^2, so we argued: Flow in x direction will be enhanced, while in y direction will not change:

Conclusion: B increases v_2 .

(larger η/s than AdS/CFT limit?)

Enhancement of elliptic flow v_2 with magnetic field (up to 30 %) (for RHIC energies)

R.K.Mohapatra, P.S. Saumia, AMS, MPLA26, 2477 (2011).



Subsequent analysis: Results in agreement with our prediction

K. Tuchin, J.Phys. G 39, 025010 (2012)

Important: A recent simulation did not find any such effect

(Note: for large impact parameter, this will be important).

Inghirami etc al: arXiv: 1609.03042

We have now done Magnetohydrodynamics simulations and our results explain the discrepancies between different works.

It turns out, the physics of elliptic flow in presence of magnetic field is not that simple. Other factors can be present.

For example, it is known that under certain situations, expansion of a conducting plasma into regions of magnetic field gets hindered.

One can expect it from Lenz's law: expanding conductor squeezes magnetic flux, which should oppose expansion of plasma (cause of squeezing). Such an argument will imply suppression of v_2 due to B. This will be expected when magnetic field extends well beyond plasma region.

However, this is also not correct, as this completely misses the factor of distortion of magnetic field costing energy (which was the argument we used in our paper arguing for increase of v_2 .) We can expect that to hold true when magnetic field is entirely contained inside plasma region.

In general, all such factors are present. As we will see later, in some situation one factor will dominate, while in another, the other factor. Along with these two factors, fluctuations also play important role. Final effect is a combination of all these factors.

Relativistic Magneto-Hydrodynamics Simulations:

Arpan Das, Shreyansh S. Dave, Saumia P.S., AMS, PRC 96, 034902 (2017)

Due to conductivity (Tuchin) magnetic field does not decay very rapidly in the plasma, field diffusion time at least several fm.

We take an initial value of the field, at a given time after the collision, calculated by taking uniformly charged nuclei (spherical or ellipsoidal for deformed case), and Lorentz transforming for oppositely moving nuclei with required impact parameter.

We carry out 3+1 dimensional simulation using Glauber-like initial conditions for QGP, with profile in z-direction being Woods-Saxon with appropriate size. We work in the limit of infinite conductivity: so use equations of Ideal Relativistic MHD:

Due to computer limitation: simulation limited to lower energy Collisions, cms energy of 20 GeV.

We follow formalism from: Mignone and Bodo, Mon. Not. R. Astron. Soc. (2005)

Brief summary of the formalism:

Conservation of total energy-momentum tensor (perfect fluid QGP + magnetic field):

$$\partial_\alpha[(\rho + p_g + |b|^2)u^\alpha u^\beta - b^\alpha b^\beta + (p_g + \frac{|b|^2}{2})\eta^{\alpha\beta}] = 0$$

Maxwell's equations:

$$\partial_\alpha(u^\alpha b^\beta - b^\alpha u^\beta) = 0$$

Where:

$$b^\alpha = \gamma[\vec{v}.\vec{B}, \frac{\vec{B}}{\gamma^2} + \vec{v}(\vec{v}.\vec{B})]$$

and:

$$u^\alpha b_\alpha = 0, \text{ and } |b|^2 \equiv b^\alpha b_\alpha = \frac{|\vec{B}|^2}{\gamma^2} + (\vec{v}.\vec{B})^2$$

For simulation, these equations are cast in the following form

$$\frac{\partial U}{\partial t} + \sum_k \frac{\partial F^k}{\partial x^k} = 0$$

Where different quantities are defined as:

$$U = (m_x, m_y, m_z, B_x, B_y, B_z, E)$$

$$m_k = [\rho h \gamma^2 + |\vec{B}|^2] v_k - (\vec{v}.\vec{B}) B_k$$

$$E = \rho h \gamma^2 - p_g + \frac{|\vec{B}|^2}{2} + \frac{v^2 |\vec{B}|^2 - (\vec{v}.\vec{B})^2}{2}$$

$$F^x = \begin{pmatrix} m_x v_x - B_x \frac{b_x}{\gamma} + p \\ m_y v_x - B_x \frac{b_y}{\gamma} \\ m_z v_x - B_x \frac{b_z}{\gamma} \\ 0 \\ B_y v_x - B_x v_y \\ B_z v_x - B_x v_z \\ m_x \end{pmatrix}$$

$$p = p_g + \frac{|b|^2}{2}$$

($F^{y,z}$ are similarly defined by appropriate change of indices)

Note: From U at each stage, independent variables $(p_g, \vec{v}, \vec{B})$ have to be extracted.

$(p_g, \vec{v}, \vec{B})$ Are extracted by defining:

$$S = \vec{m}.\vec{B} \qquad W = \rho h \gamma^2$$

And writing

$$E = W - p_g + (1 - \frac{1}{2\gamma^2})|\vec{B}|^2 - \frac{S^2}{2W^2}$$

$$|m|^2 = (W + |\vec{B}|^2)^2(1 - \frac{1}{\gamma^2}) - \frac{S^2}{W^2}(2W + |\vec{B}|^2)$$

These equations are written eventually as a single equation for one unknown W (by rewriting equation for $|m|^2$:

$$\gamma = \left(1 - \frac{S^2(2W + |\vec{B}|^2) + |m|^2 W^2}{(W + |\vec{B}|^2)^2 W^2}\right)^{-1/2}$$

$$f(W) \equiv W - p_g + (1 - \frac{1}{2\gamma^2})|\vec{B}|^2 - \frac{S^2}{2W^2} - E = 0$$

This equation is solved using Newton-Raphson method to get W, from which other independent variables are obtained using above equations.

Limitations of the simulation:

Due to computer limitations, we use small lattice (200x200x200) so small nuclei used (copper), also for small times only up to maximum of 3 fm time, sometime much shorter time. We use smaller energy CMS energy of 20 GeV, for very large energies magnetic field becomes very large near receding nuclei (it is 3+1 dimensional simulation), causing problem

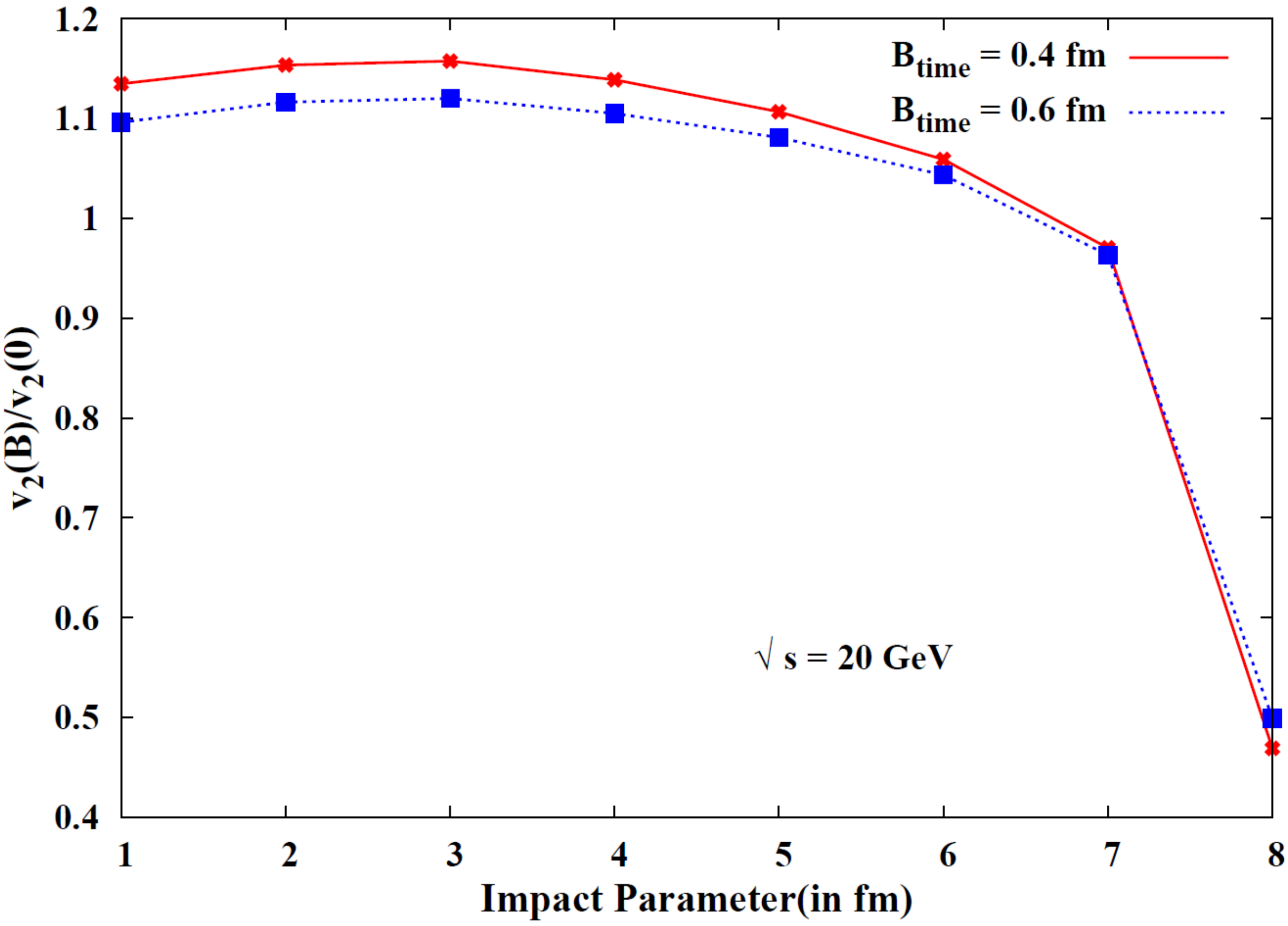
With fluctuations difficult to run for long times.

General problem when magnetic field energy density becomes much larger than the plasma density.
Same problem was found in other simulation also (Inghrami et al. where no effect on v_2 was found)

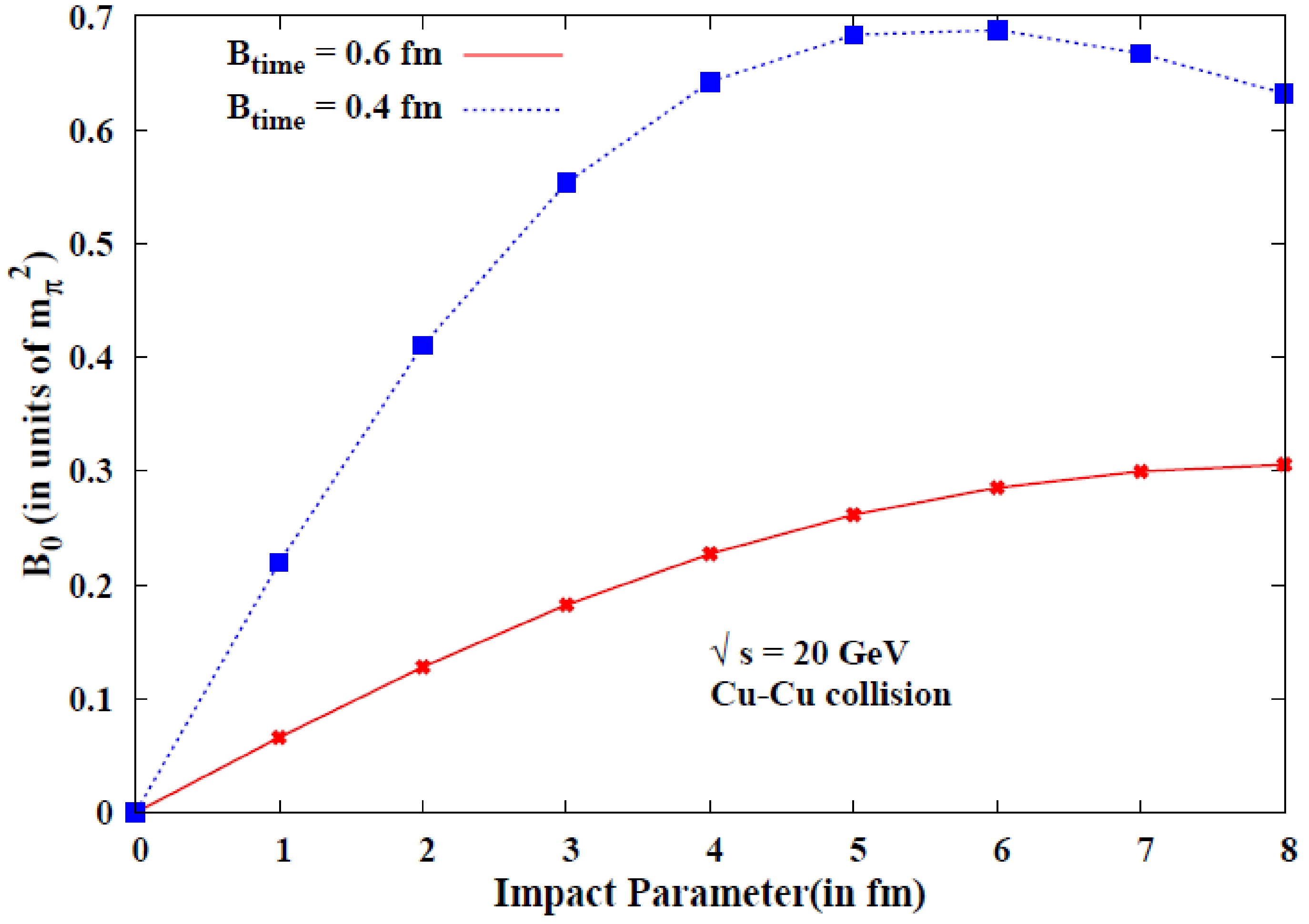
For elliptic flow, we have studied details of the dependence of elliptic flow on magnetic field, and it seems to crucially depend on the relative profiles of B and plasma density.

We first present these results

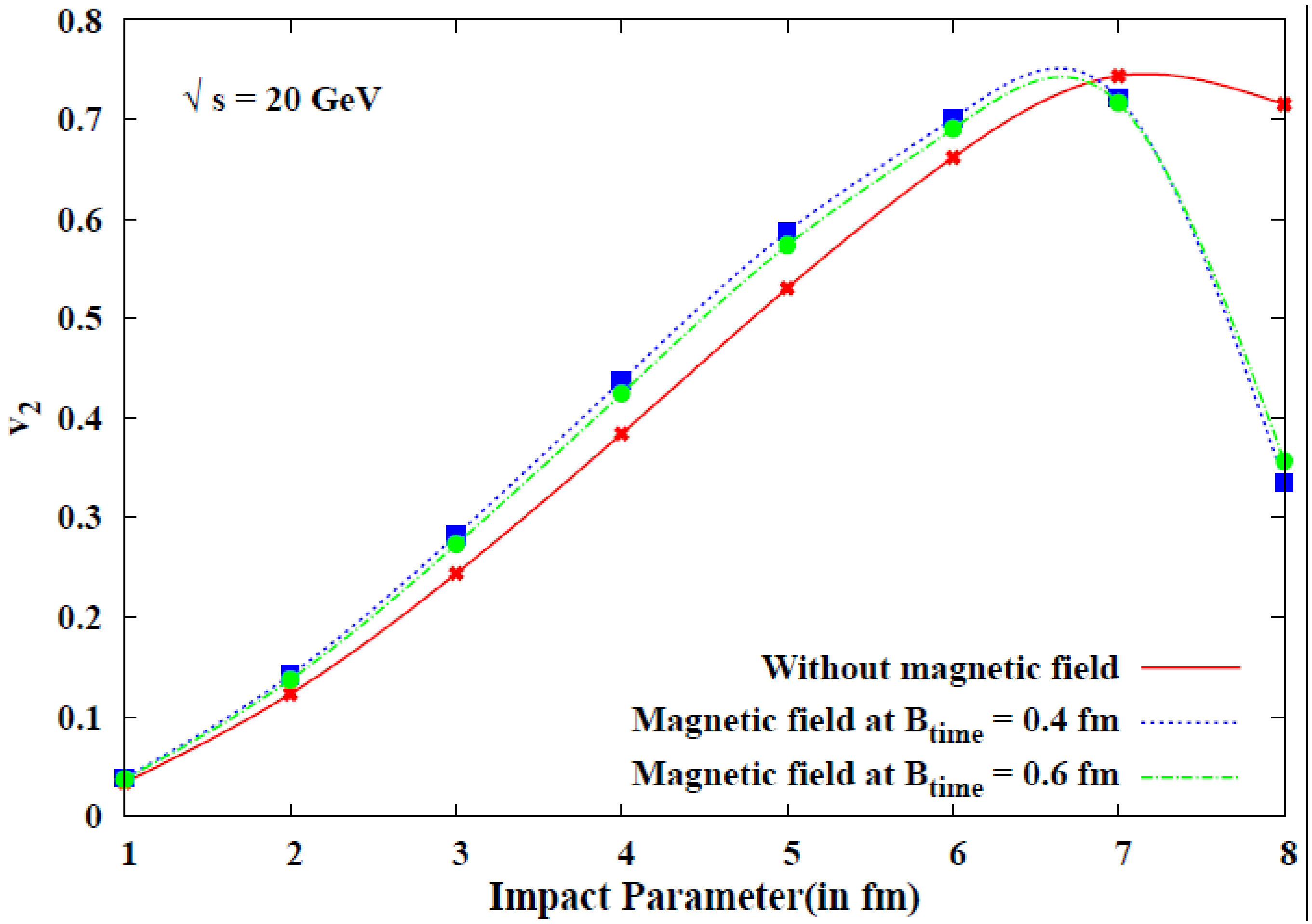
Result-1: Elliptic flow in the presence of magnetic field:



We see that magnetic field enhances v_2 , but only up to impact parameter of about 6 fm, after that B suppresses it. Also enhancement peaks for small impact parameter. Why?

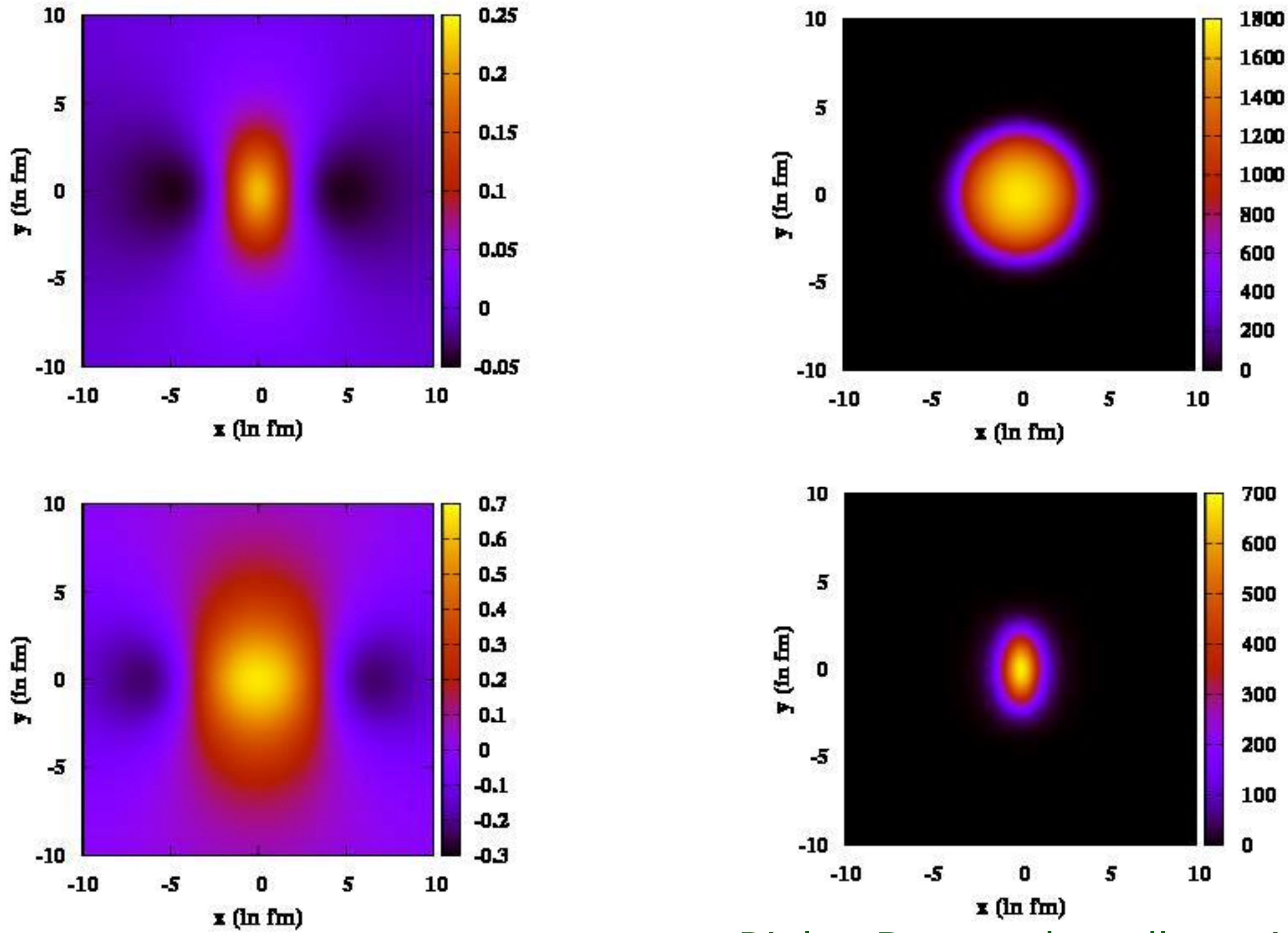


Note: Magnetic field almost monotonically increases with impact parameter.



0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
v_2
√ s = 20 GeV
Without magnetic field
Magnetic field at B_{time} = 0.4 fm
Magnetic field at B_{time} = 0.6 fm
1
2
3
4
5
6
7
8
Impact Parameter(in fm)

Magnetic field plots (top) and plasma density plots (bottom) for small 1 fm (left) and large 7 fm (right) impact parameter



Left: B contained entirely within plasma region, expect v_2 enhancement from anisotropic sound speed

Right: B extends well outside plasma region, expect v_2 suppression from flux squeezing

Summary of results for effect of magnetic field on elliptic flow:

If magnetic field is contained almost entirely within the plasma region, then elliptic flow is enhanced with increasing magnetic field. This happens for small values of the impact parameter.

This is in accordance with the original argument of having a stiffer equation of state transverse to magnetic field direction.

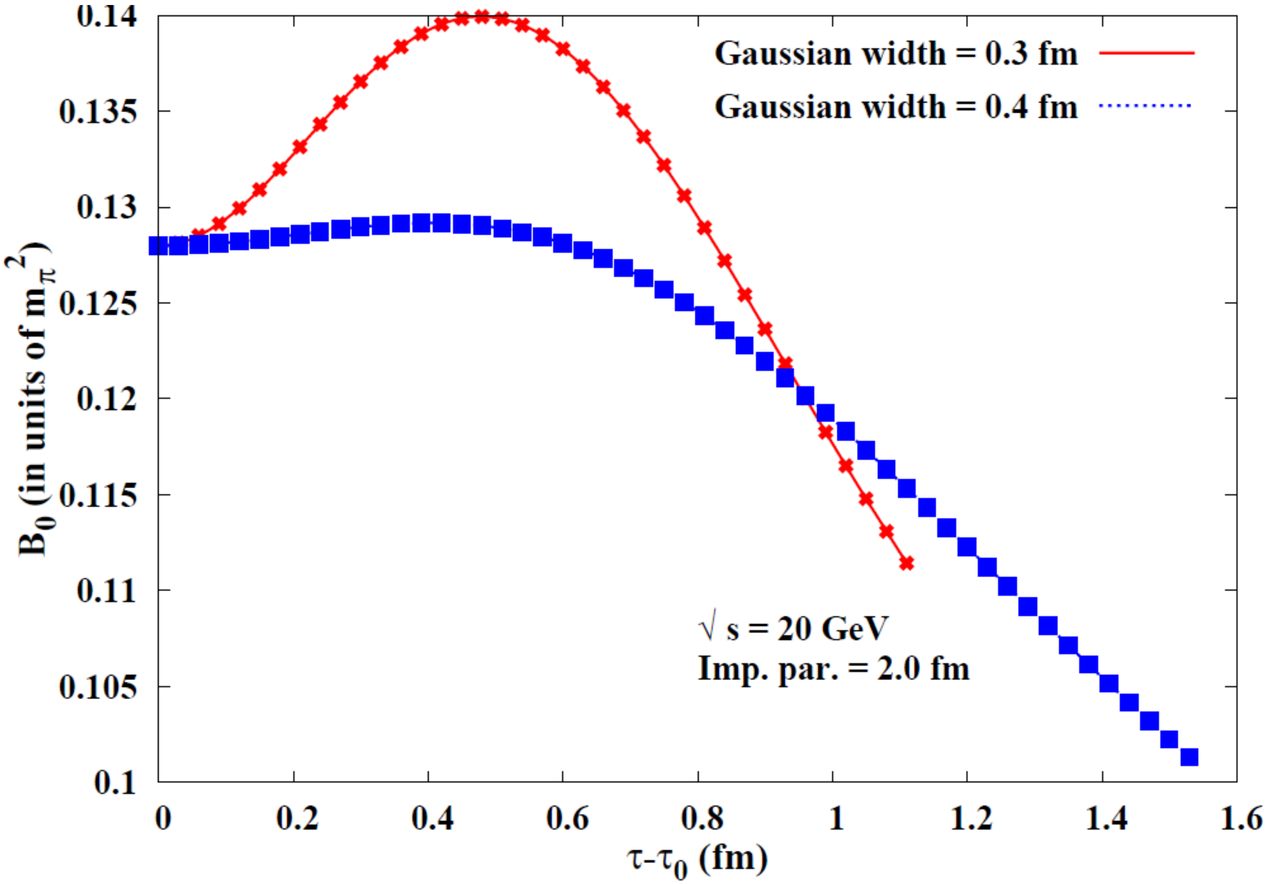
However, if the magnetic field extends well beyond the plasma region, then elliptic flow is suppressed by the magnetic field. This will be in accordance with the Lenz's law. This happens when impact parameter is large.

Note: The simulations of Inghirami et al. (arXiv:1609.03042) was for large impact parameter, hence may have been affected by this Lenz's law suppression.

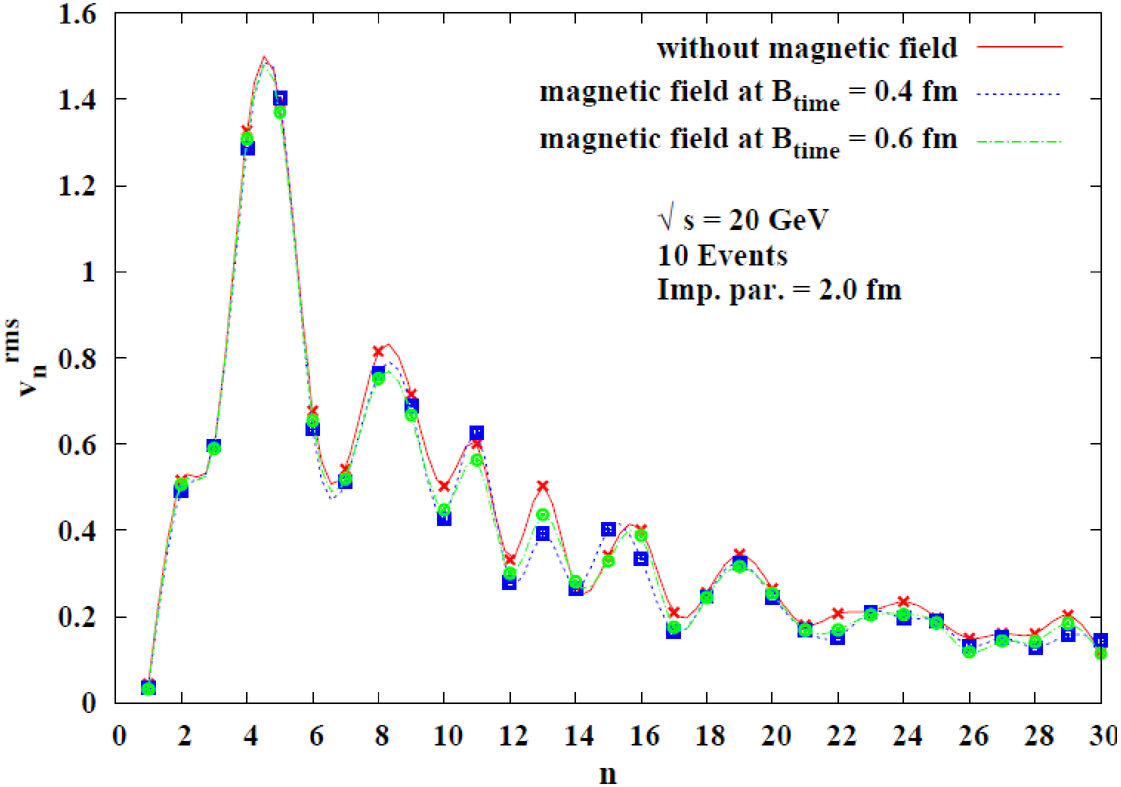
Important to check that simulation for small values of impact parameters.

Result-2: Temporary increase of magnetic field due to flux-rearrangement by evolving plasma density fluctuations

Evolving fluctuations can push around flux lines, leading to temporary, localized concentration of flux.



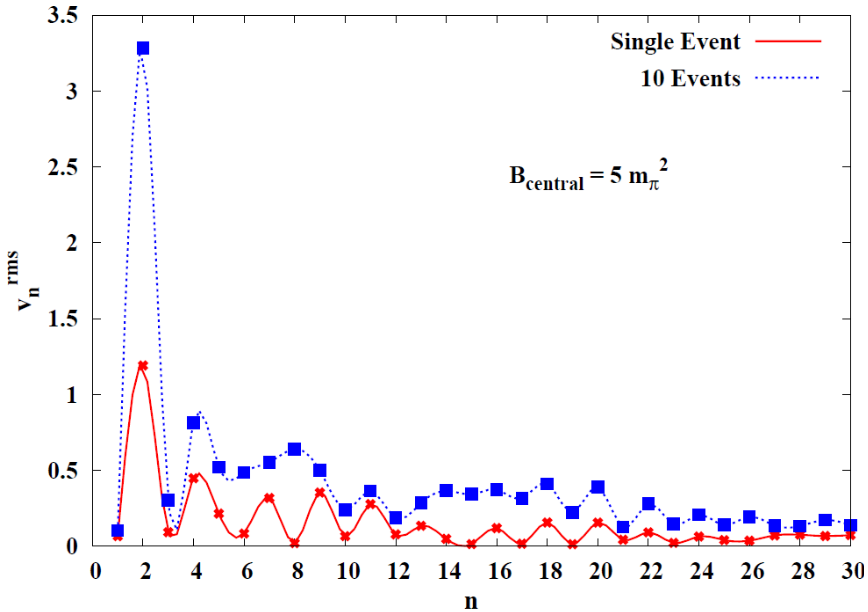
Important for chiral magnetic effect **which is sensitive to local magnetic field (instanton size regions).** We could only study small fluctuations, for large fluctuations effect can be stronger



Result-3:

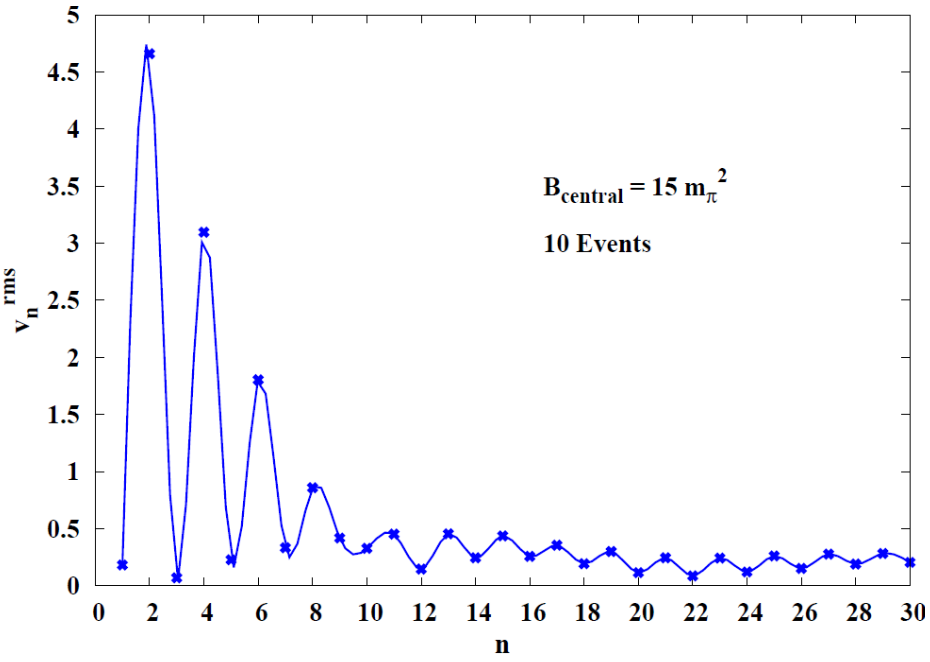
Magnetic field affects flow power spectrum qualitatively:

Left: small magnetic field: 0.1 – 0.4 m_π^2 . Very tiny effect



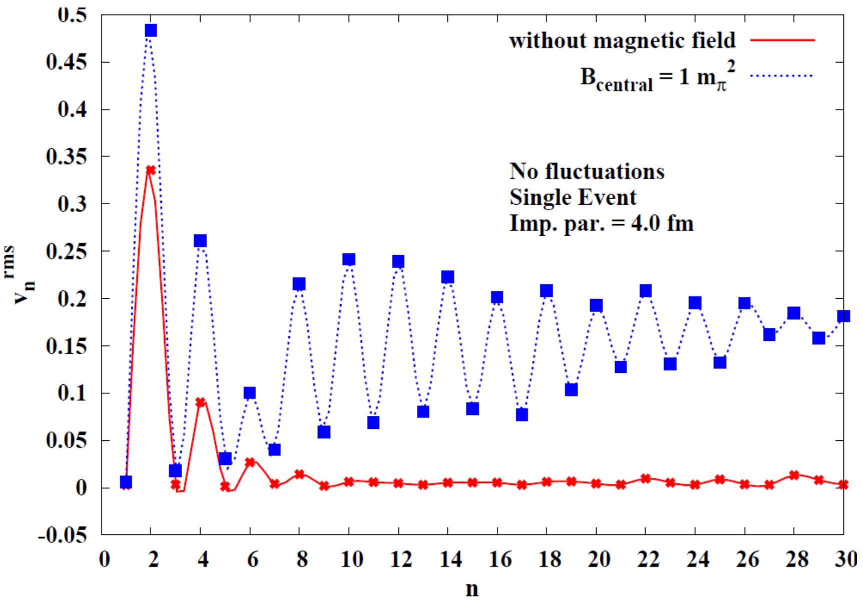
Strong magnetic field 5 m_π^2 . Significant effect, note first few flow coefficients show some even-odd power difference.

Here magnetic field put in by hand (also taken constant along Y direction for stable simulation, for Gauss' law)



Much stronger magnetic field: 15 m_π^2 . Very clear even-odd power difference. Qualitative in nature. Arises from reflection symmetry about the axis of magnetic field, so clear effect only when B dominates over random fluctuations.

(important implications for low l modes of CMBR power spectrum)



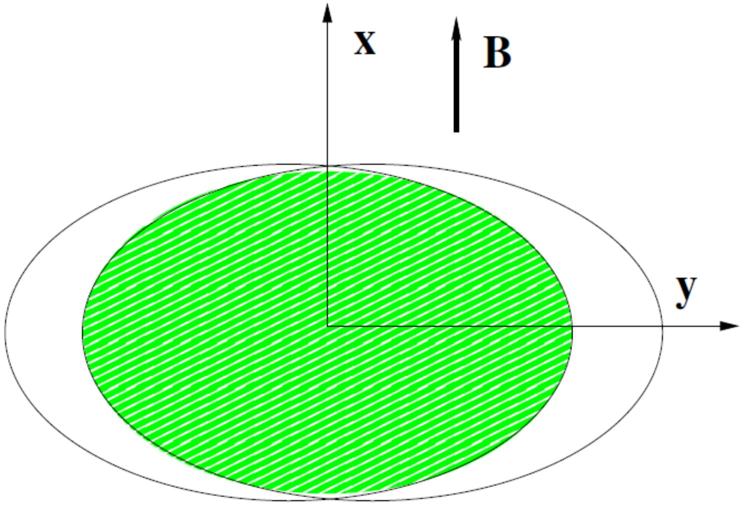
To show that fluctuations mask this signal we show power spectrum in the presence of small magnetic field (1 m_π^2), but in the absence of any initial state fluctuations. Note: even-odd effect still very strong.

Result-4: New possibilities with deformed nucleus

different collision geometries will lead to anomalous elliptic flow:

At present: we represent these situations by producing QGP region and magnetic field independently using suitable impact parameters, and then combine the two profiles as per our choice.

First consider: Elliptical plasma region, but magnetic field induces larger flow along semi-minor axis of the ellipse,
This should suppress v_2 .(for B contained within plasma region)

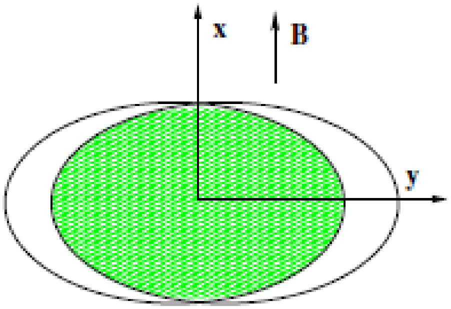


(a)

Expect: larger flow along x axis from pressure gradient. Usual Elliptic flow.

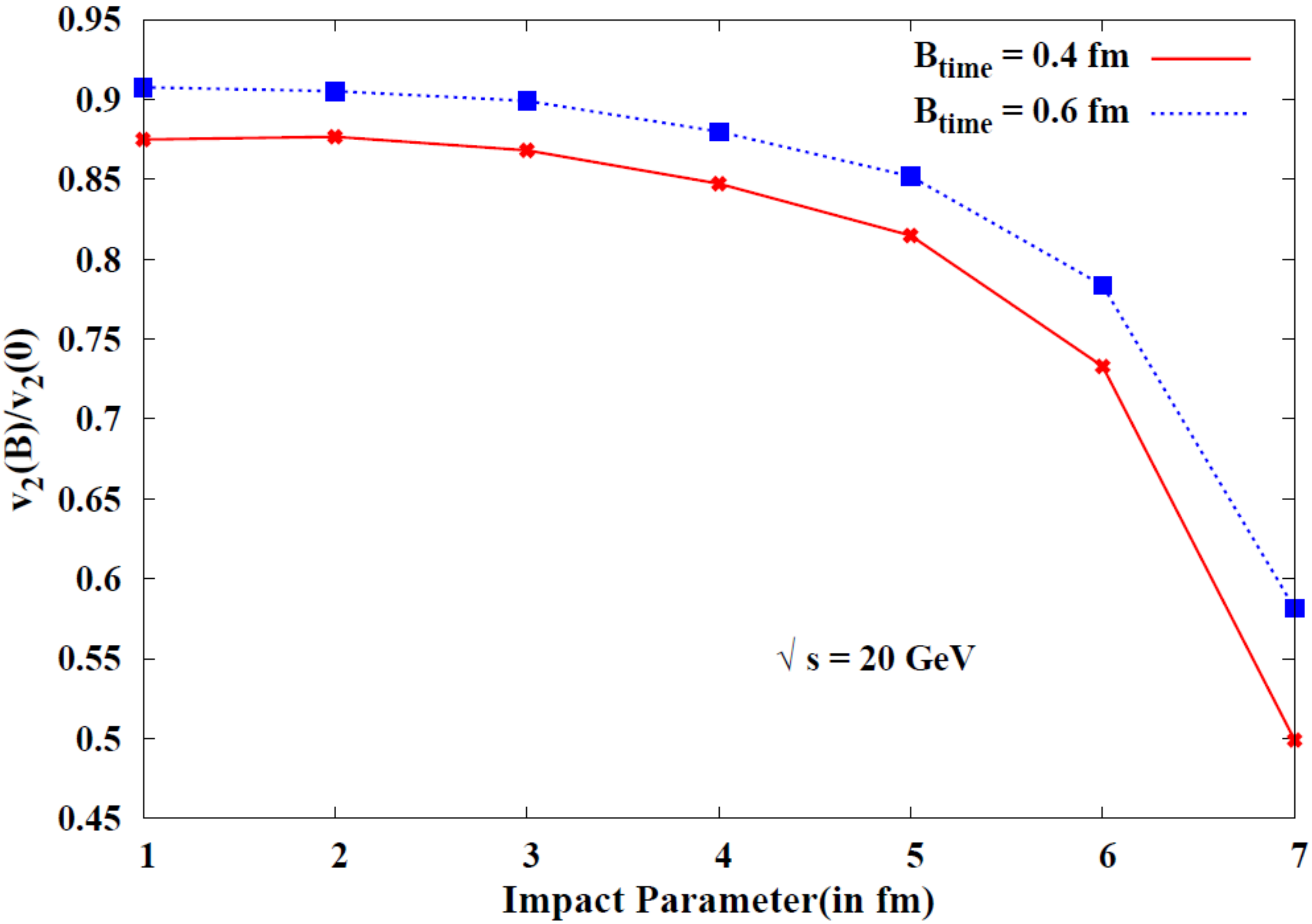
But B along x axis gives stiffer eqn. of state along y axis: This should give larger flow along y axis.

Result: suppression of elliptic flow along x axis. For strong magnetic field case, it can lead to negative elliptic flow.



Simulation result:
Suppression of v_2 due to magnetic field pointing along x axis (semi-minor axis).

Full simulation for deformed nuclei:
In progress



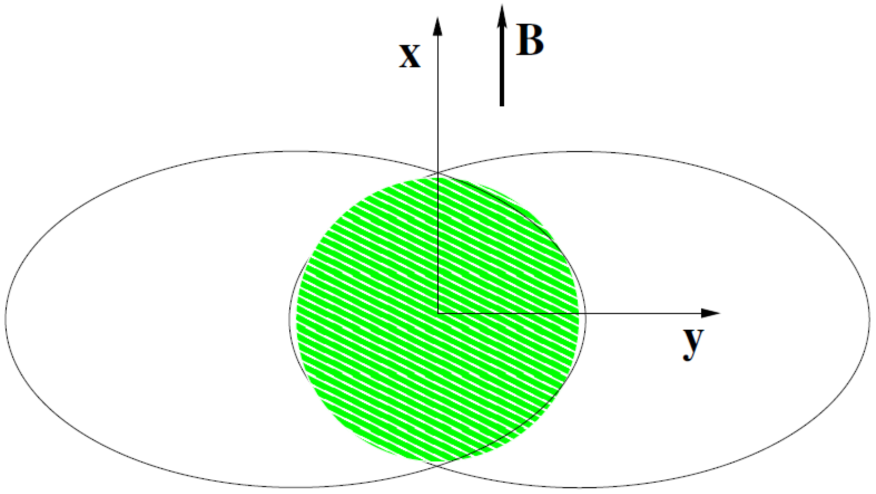
Another interesting possibility:

Isotropic QGP region but B gives rise to significant non-zero elliptic flow, even without any fluctuations.

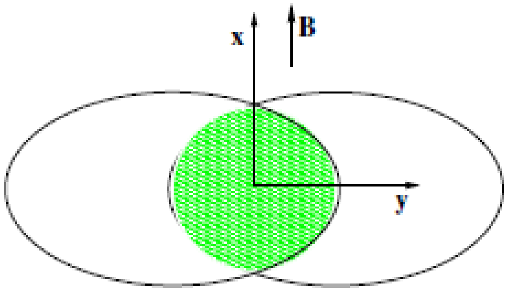
May provide a good signal for presence of initial magnetic field

Note: non-zero magnetic field arising from spectators, even when QGP region is roughly isotropic.

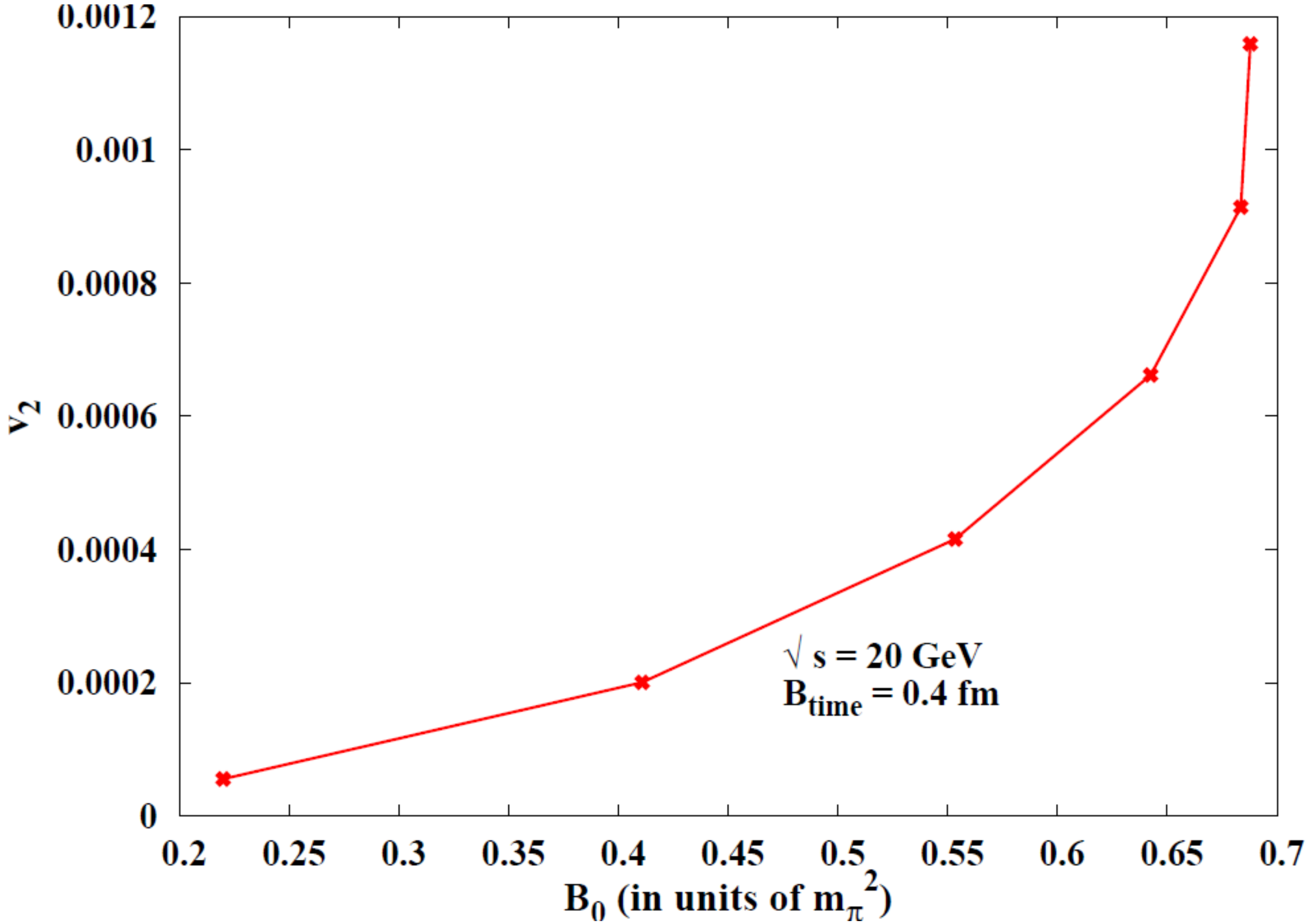
This magnetic field induces non-zero elliptic flow



(b)



Isotropic QGP region, but non-zero v_2 due B, which increases monotonically with B.

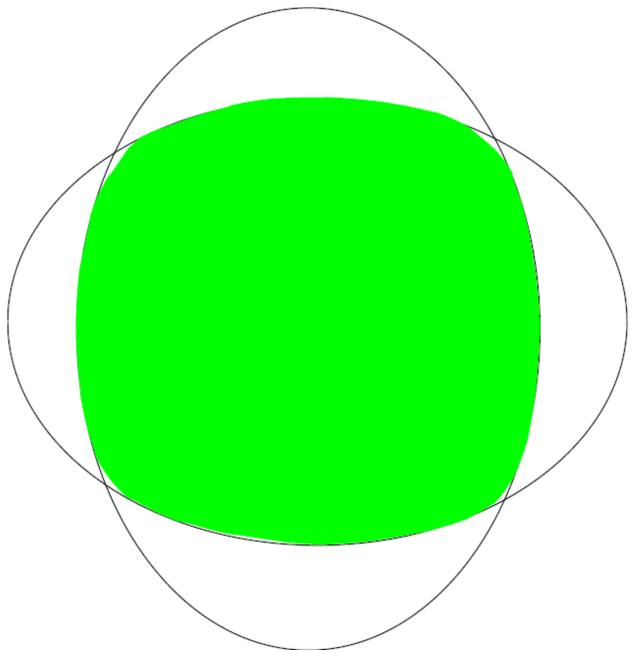


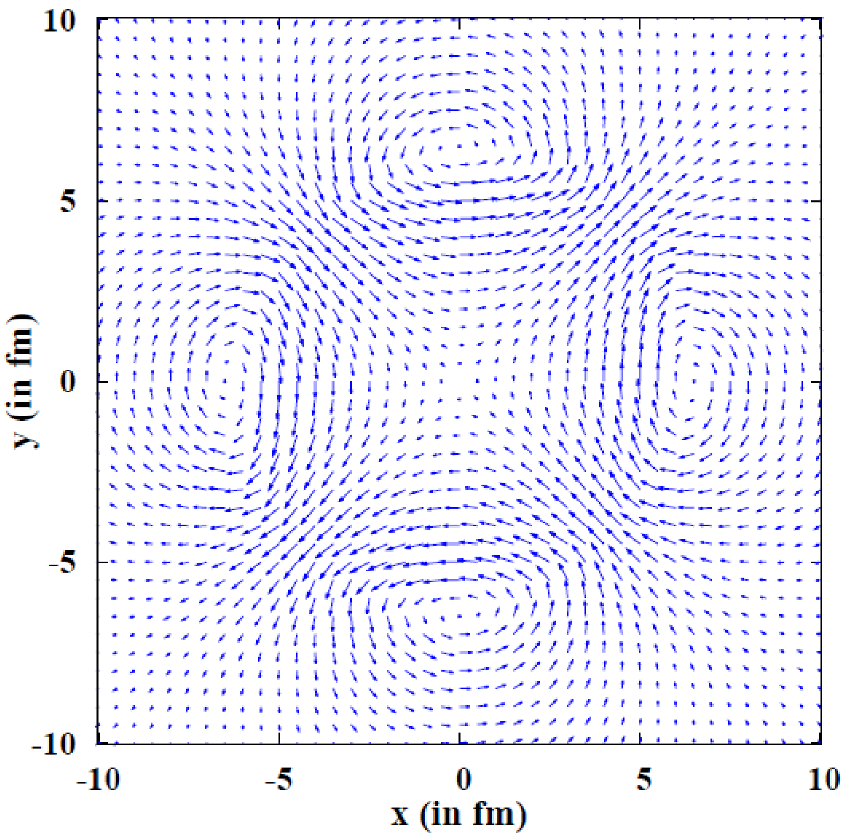
A very interesting possibility: Crossed-configuration for deformed (Uranium) nuclei. Produced magnetic field is quadrupolar.

Expect phenomena arising from beam focusing (along longitudinal direction), along with very large v_4 .

Deviations from Bjorken scaling?

Under investigation at present.





Concluding remarks:
Plots of v_n^{rms} may reveal important information:

1) It Probes initial state fluctuations directly. The first peak contains information about the freezeout stage, and the dominant scale of fluctuations (just as they do for CMBR)

2) Other peaks can give more details of equation of state, and interaction of different particles with particle specific power spectrum.

3) One important factor which can affect the shape and inter-spacings of these peaks, is the nature and presence of the quark-hadron transition (e.g. via speed of sound).

4) Magnetic field leads to complex flow patterns: Non-trivial effects on elliptic flow (AdS/CFT bound on η/s ?)

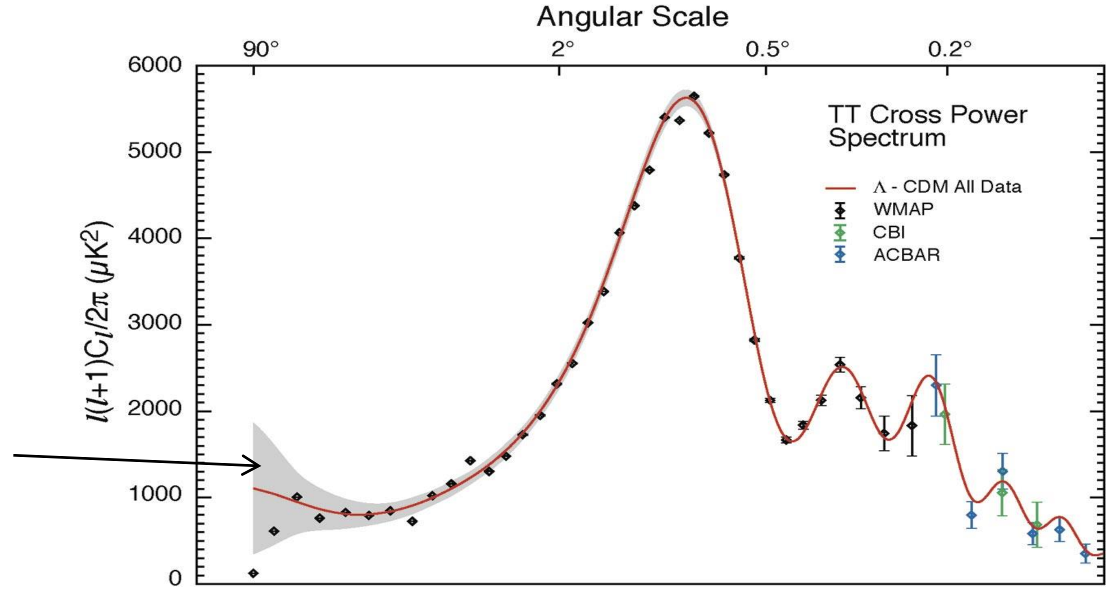
5) Magnetic field effects on plasma important to understand for probing chiral magnetic effect, chiral vertical effect etc.

One important difference in favor of RHICE:

For CMBR, for each ℓ, only $2\ell+1$ independent measurements are available, as there is only one CMBR sky to observe.

This limits accuracy by the so called cosmic variance.

Cosmic variance



In contrast, for RHICE: Each nucleus-nucleus collision (with same parameters like collision energy, centrality etc.) provides a new sample event (in some sense like another universe). Therefore with large number of events, it should be possible to resolve any signal present in these events as discussed here.

Thank You