Heavy and heavy-light mesons in the Covariant Spectator Theory

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Motivation

- Intense experimental activity to explore meson structure at LHC, *BABAR*, Belle, CLEO and soon at GlueX (Jlab) and PANDA (GSI)
- Search for exotic mesons (hybrids, glueballs, … maybe $q\bar{q}$?)
- Need to understand also “conventional” $q\bar{q}$-mesons in more detail
- Study production mechanisms, transition form factors
(also important for hadronic contributions to light-by-light scattering)

Theory: a huge amount of work has already been done on meson structure (LQCD, BS/DSE, constrained dynamics two-body Dirac equation, BLFQ, relativized Schrödinger equation, …)

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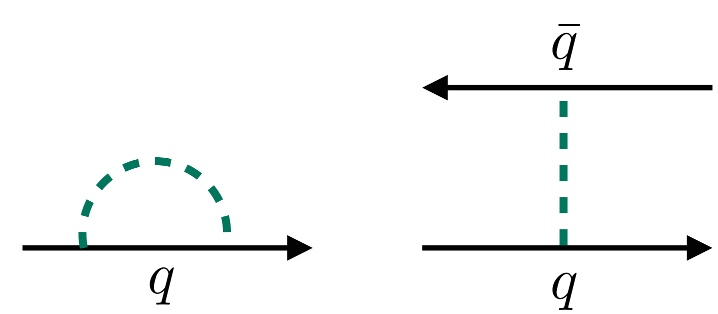
Guiding principles of our approach (CST - Covariant Spectator Theory):

- Find $q\bar{q}$ interaction that can be used in all mesons (unified model)
- Must be relativistic (relativity necessary with light quarks), and reduce to linear+Coulomb in the nonrelativistic limit
- Manifest covariance: strongly constrains spin-dependence of interactions
- Learn about the Lorentz structure of the confining interaction
- Quark masses are dynamic: self-interaction should be consistent with $q\bar{q}$ interaction

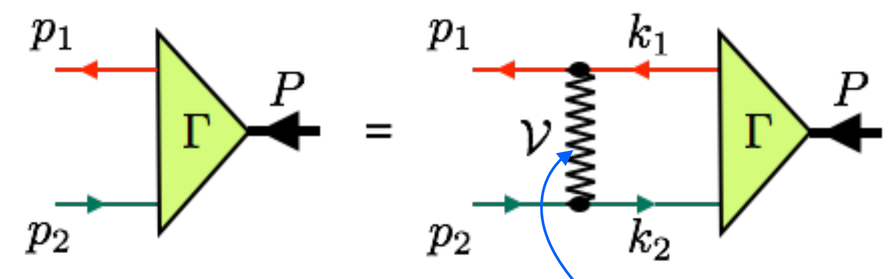
Huge mass variation:
from pions (~0.14 GeV)
to bottomonium (> 10 GeV)



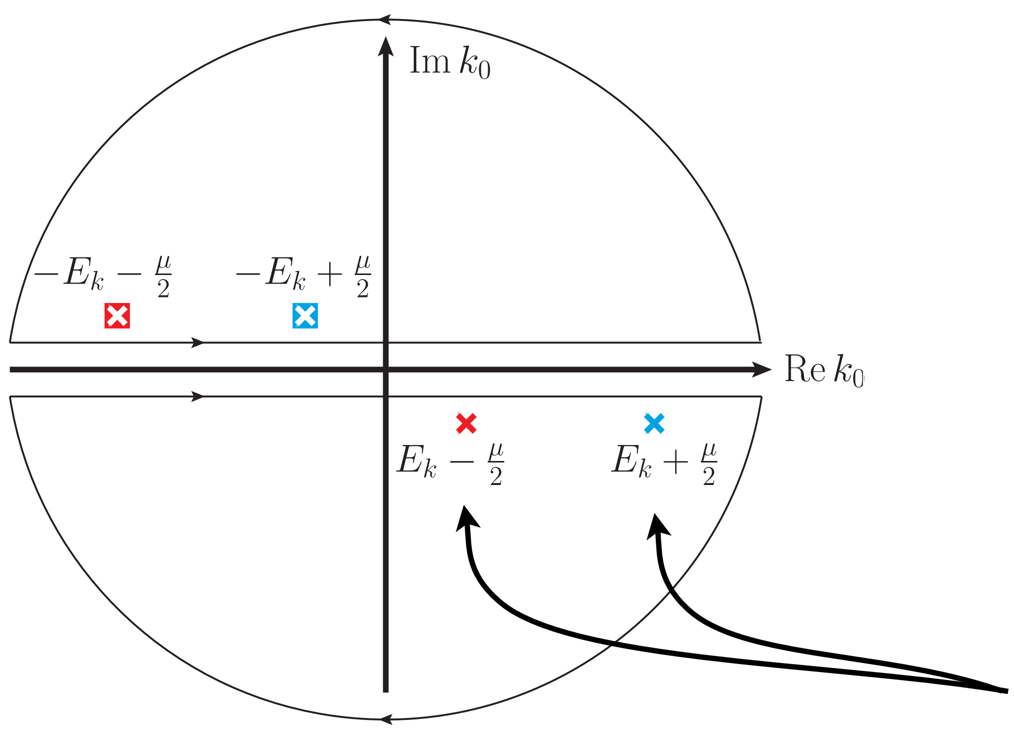
Talk by Elmar Biernat on Friday

CST equation for two-body bound states

Bethe-Salpeter equation for $q\bar{q}$ bound-state with mass μ



2PI diagrams



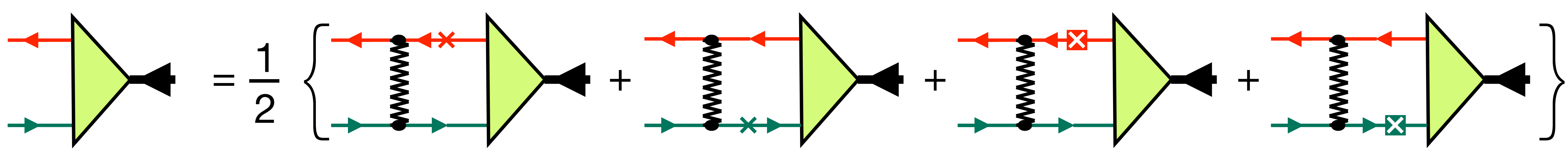
Integration over relative energy k_0:

- Keep only pole contributions from constituent particle propagators
- Poles from particle exchanges appear in higher-order kernels (usually neglected — tend to cancel)
- Reduction to 3D loop integrations, but covariant
- Correct one-body limit

If bound-state mass μ is small:
both poles are close together (both important)

Symmetrize pole contributions from both half planes: charge conjugation symmetry

BS vertex (approx.) CST vertices



Once the four CST vertices (with one quark on-shell) are all known, one can use this equation to get the vertex function for other momenta (also Euclidean).

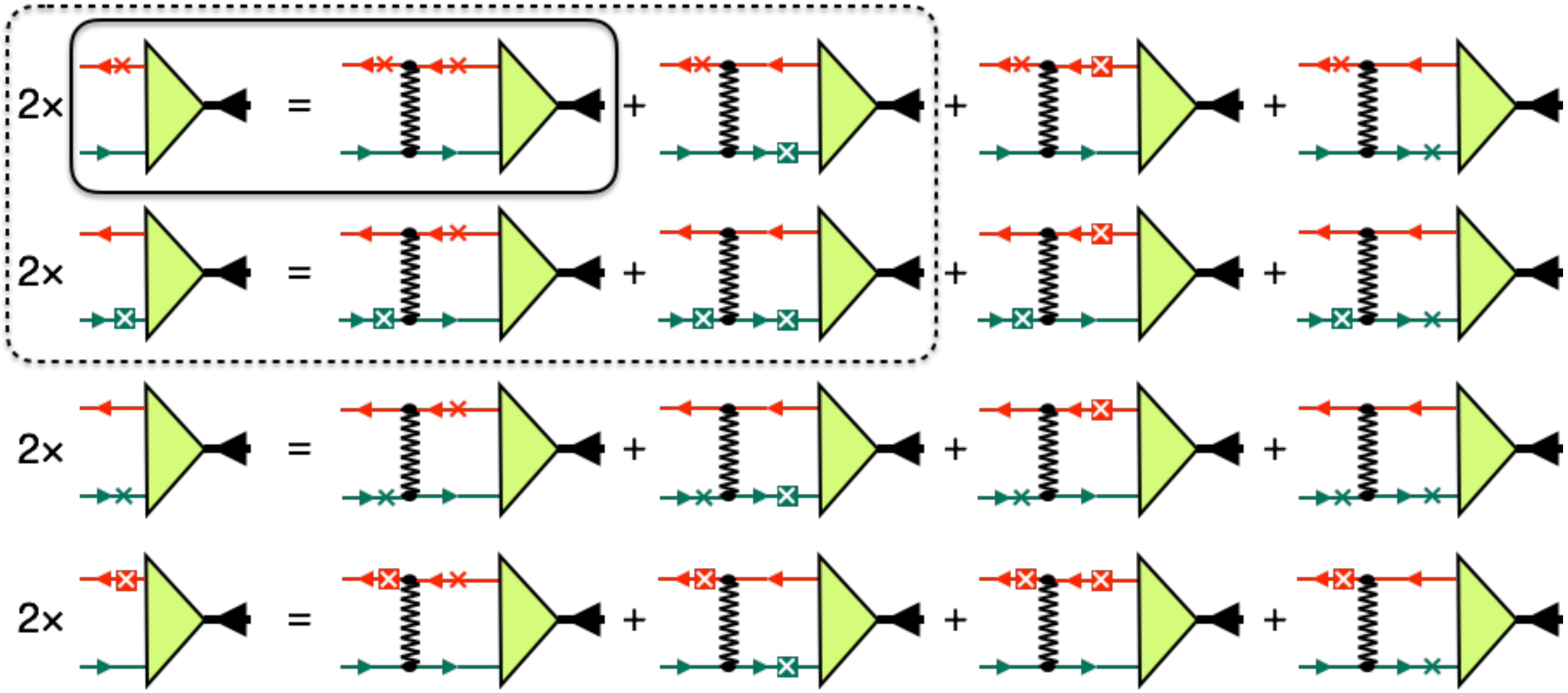
CST equations

Closed set of equations when external legs are systematically placed on-shell

1CSE

2CSE

4CSE



Solutions: bound state masses μ and corresponding vertex functions Γ

One-channel spectator equation (1CSE):

- Particularly appropriate for unequal masses
- Numerical solutions easier (fewer singularities)
- But not charge-conjugation symmetric

Two-channel spectator equation (2CSE):

- Restores charge-conjugation symmetry
- Additional singularities in the kernel

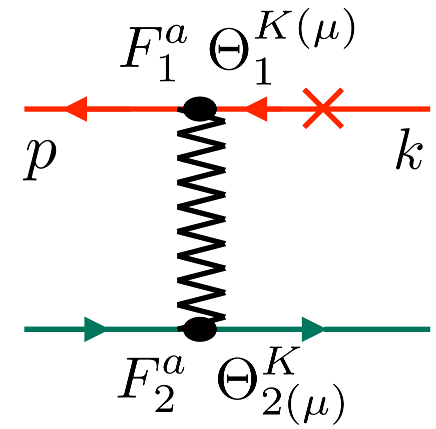
Four-channel spectator equation (4CSE):

- Necessary for light bound states (pion!)

All have smooth one-body limit (Dirac equation) and nonrelativistic limit (Schrödinger equation).

The covariant kernel

Our kernel: $\mathcal{V}(p,k;P) = \boxed{\frac{3}{4}\mathbf{F}_1 \cdot \mathbf{F}_2} \sum_K \boxed{V_K(p,k;P)} \boxed{\Theta_1^{K(\mu)} \otimes \Theta^K_{2(\mu)}}$



$F_a = \frac{1}{2}\lambda_a$

color SU(3) generators

$\boxed{1}$ for $q\bar{q}$ color singlets

momentum dependence

Dirac structure

$\Theta_i^{K(\mu)} = \mathbf{1}_i, \gamma_i^5, \gamma_i^\mu$

- Confining interaction: Lorentz (scalar + pseudoscalar) mixed with vector
  Coupling strength σ, mixing parameter y

$y = 0$	pure S+PS
$y = 1$	pure V

for correct nonrelativistic limit

$$\mathcal{V}_{\mathrm{L}}(p,k;P) = \left[(1-y)\left(\mathbf{1}_1 \otimes \mathbf{1}_2 + \gamma_1^5 \otimes \gamma_2^5\right) - y\,\gamma_1^\mu \otimes \gamma_{\mu 2}\right] V_{\mathrm{L}}(p,k;P)$$

equal weight (constraint from chiral symmetry)

→ E.P. Biernat et al., PRD **90**, 096008 (2014)

- One-gluon exchange with constant coupling strength α_s
  \+ Constant interaction (in r-space) with strength C } Lorentz vector

$$\mathcal{V}_{\mathrm{OGE}}(p,k;P) + \mathcal{V}_{\mathrm{C}}(p,k;P) = -\gamma_1^\mu \otimes \gamma_{2\mu}\left[V_{\mathrm{OGE}}(p,k;P) + V_{\mathrm{C}}(p,k;P)\right]$$

Nonrelativistic limit of the kernel

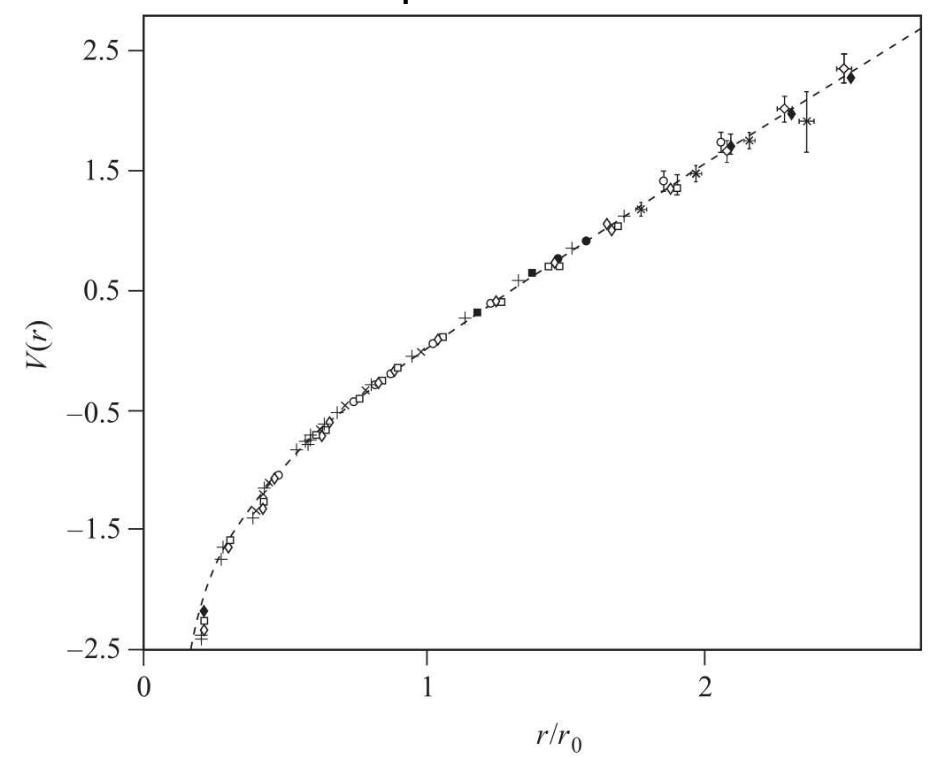
For any value of the mixing parameter y:

The nonrelativistic limit of the kernel in r-space is

$$V(r) = \sigma r - \frac{\alpha_s}{r} - C$$

(the form of the Cornell potential)

Static QCD potential from the lattice



Allton et al, UKQCD Collab., PRD **65**, 054502 (2002)

Using a confining kernel in momentum space is a bit tricky because of singularities

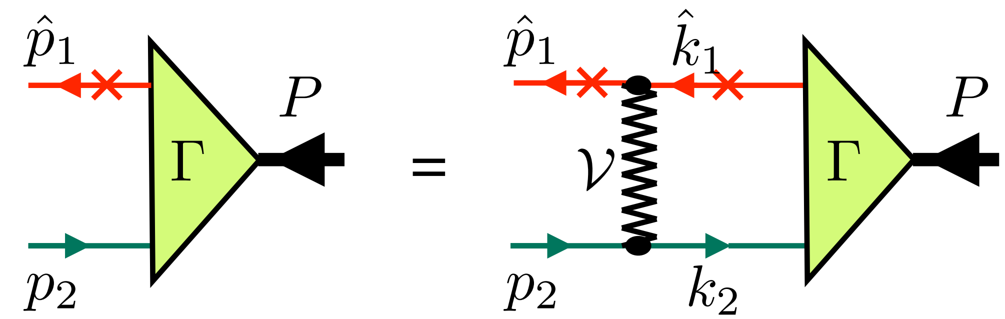
For details see:

Leitão, AS, Peña, Biernat, PRD **90**, 096003 (2014)
Gross, Milana, PRD **43**, 2401 (1991)
Savkli, Gross, PRC **63**, 035208 (2001)

The One-Channel Spectator Equation (1CSE)

We solve the 1CSE for heavy and heavy-light systems

- Should work well for bound states with at least one heavy quark
- Easier to solve numerically than 2CSE or 4CSE
- C-parity splitting small in heavy quarkonia
- For now with constant constituent quark masses (quark self-energies will be included later)



$$\Gamma(\hat{p}_1, p_2) = -\int \frac{d^3k}{(2\pi)^3} \frac{m_1}{E_{1k}} \sum_K V_K(\hat{p}_1, \hat{k}_1) \Theta_1^{K(\mu)} \frac{m_1 + \hat{\not{k}}_1}{2m_1} \Gamma(\hat{k}_1, k_2) \frac{m_2 + \not{k}_2}{m_2^2 - k_2^2 - i\epsilon} \Theta^K_{2(\mu)}$$

$$E_{ik} = \sqrt{m_i^2 + \mathbf{k}^2}$$

- Momentum-dependence of kernels is also simpler

$$V_{\rm L}(\hat{p}_1, \hat{k}_1) = -8\sigma\pi \left[\frac{1}{(\hat{p}_1 - \hat{k}_1)^4} - \frac{E_{p_1}}{m_1} (2\pi)^3 \delta^3(\mathbf{p}_1 - \mathbf{k}_1) \int \frac{d^3k_1'}{(2\pi)^3} \frac{m_1}{E_{k_1'}} \frac{1}{(\hat{p}_1 - \hat{k}_1')^4} \right]$$

$$V_{\rm OGE}(\hat{p}_1, \hat{k}_1) = -\frac{4\pi\alpha_s}{(\hat{p}_1 - \hat{k}_1)^2} \qquad V_{\rm C}(\hat{p}_1, \hat{k}_1) = (2\pi)^3 \frac{E_{k_1}}{m_1} C \delta^3(\mathbf{p}_1 - \mathbf{k}_1)$$

- Linear and OGE kernels need to be regularized
  We chose Pauli-Villars regularizations with parameter $\Lambda = 2m_1$

CST vertex functions

$$P^\mu = p_1 - p_2 \qquad \rho^\mu = \frac{p_1 + p_2}{2} \qquad \Lambda(p_i) = \frac{m_i + \not{p}_i}{2m_i}$$

Pseudoscalar mesons

$$\begin{aligned}\Gamma^P(p_1,p_2) = \Gamma_1^P(p_1,p_2)\gamma^5 + \Gamma_2^P(p_1,p_2)\Lambda(-p_1)\gamma^5 \\ + \Gamma_3^P(p_1,p_2)\gamma^5\Lambda(-p_2) + \Gamma_4^P(p_1,p_2)\Lambda(-p_1)\gamma^5\Lambda(-p_2)\end{aligned}$$

Scalar mesons

$$\Gamma^S(p_1,p_2) = \Gamma_1^S(p_1,p_2) + \Gamma_2^S(p_1,p_2)\Lambda(-p_1) + \Gamma_3^S(p_1,p_2)\Lambda(-p_2) + \Gamma_4^S(p_1,p_2)\Lambda(-p_1)\Lambda(-p_2)$$

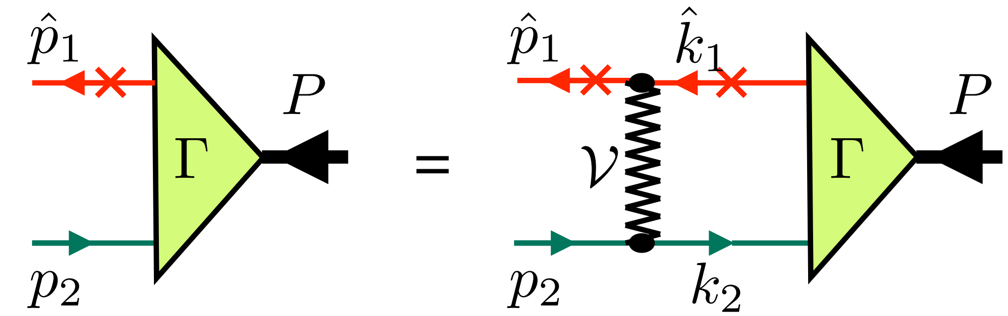
Vector mesons

$$\begin{aligned}\Gamma^{VT\mu}(p_1,p_2) =& \Gamma_1^V(p_1,p_2)\gamma^{T\mu} + \Gamma_2^V(p_1,p_2)\Lambda(-p_1)\gamma^{T\mu} + \Gamma_3^V(p_1,p_2)\gamma^{T\mu}\Lambda(-p_2) \\ &+ \Gamma_4^V(p_1,p_2)\Lambda(-p_1)\gamma^{T\mu}\Lambda(-p_2) + \Gamma_5^V(p_1,p_2)\rho^{T\mu} + \Gamma_6^V(p_1,p_2)\Lambda(-p_1)\rho^{T\mu} \\ &+ \Gamma_7^V(p_1,p_2)\rho^{T\mu}\Lambda(-p_2) + \Gamma_8^V(p_1,p_2)\Lambda(-p_1)\rho^{T\mu}\Lambda(-p_2)\end{aligned}$$

Axialvector mesons

$$\begin{aligned}\Gamma^{AT\mu}(p_1,p_2) =& \Gamma_1^A(p_1,p_2)\gamma^{T\mu}\gamma^5 + \Gamma_2^A(p_1,p_2)\Lambda(-p_1)\gamma^{T\mu}\gamma^5 + \Gamma_3^A(p_1,p_2)\gamma^{T\mu}\gamma^5\Lambda(-p_2) \\ &+ \Gamma_4^A(p_1,p_2)\Lambda(-p_1)\gamma^{T\mu}\gamma^5\Lambda(-p_2) + \Gamma_5^A(p_1,p_2)\rho^{T\mu}\gamma^5 + \Gamma_6^A(p_1,p_2)\Lambda(-p_1)\rho^{T\mu}\gamma^5 \\ &+ \Gamma_7^A(p_1,p_2)\rho^{T\mu}\gamma^5\Lambda(-p_2) + \Gamma_8^A(p_1,p_2)\Lambda(-p_1)\rho^{T\mu}\gamma^5\Lambda(-p_2)\end{aligned}$$

Numerical solution of the 1CSE



- Work in rest frame of the bound state $P = (\mu, \mathbf{0})$
- Use ρ-spin decomposition of the propagator

$$\frac{m_2 + \not{k}_2}{m_2^2 - k_2^2 - i\epsilon} = \frac{m_2}{E_{2k}} \sum_{\rho, \lambda_2} \rho \frac{u_2^\rho(\mathbf{k}, \lambda_2) \bar{u}_2^\rho(\mathbf{k}, \lambda_2)}{E_{2k} - \rho k_{20} - i\epsilon}$$

$u^+(\mathbf{k}, \lambda) \equiv u(\mathbf{k}, \lambda)$
$u^-(\mathbf{k}, \lambda) \equiv v(-\mathbf{k}, \lambda)$

ρ-spinors with helicity λ

$$E_{ik} = \sqrt{m_i^2 + \mathbf{k}^2}$$

- Project 1CSE onto ρ-spin helicity channels

$$\Gamma^{+\rho'}_{\lambda\lambda'}(p) \equiv \bar{u}_1^+(\mathbf{p}, \lambda) \Gamma(p) u_2^{\rho'}(\mathbf{p}, \lambda')$$

- Define relativistic "wave functions"

$$\Psi^{+\rho}_{\lambda\lambda'}(p) \equiv \sqrt{\frac{m_1 m_2}{E_{1p} E_{2p}}} \frac{\rho}{E_{2p} - \rho(E_{1p} - \mu)} \Gamma^{+\rho}_{\lambda\lambda'}(p)$$

The 1CSE becomes a generalized linear EV problem for the mass eigenvalues μ

- Switch to basis of eigenstates of total orbital angular momentum *L* and of total spin *S* (not necessary, but useful for spectroscopic identification of solutions)
- Expand wave functions in a basis of B-splines (modified for correct asymptotic behavior) and solve eigenvalue problem → expansion coefficients and mass eigenvalues

Data sets used in least-square fits of meson masses

	State	$J^{P(C)}$	Mass (MeV)	Data set S1	S2	S3
$b\bar{b}$	$\Upsilon(4S)$	1^{--}	10579.4 ± 1.2		•	•
	$\chi_{b1}(3P)$	1^{++}	10512.1 ± 2.3			•
	$\Upsilon(3S)$	1^{--}	10355.2 ± 0.5		•	•
	$\eta_b(3S)$	0^{-+}	10337			
	$h_b(2P)$	1^{+-}	10259.8 ± 1.2			•
	$\chi_{b1}(2P)$	1^{++}	$10255.46\pm0.22\pm0.50$			•
	$\chi_{b0}(2P)$	0^{++}	$10232.5\pm0.4\pm0.5$		•	•
	$\Upsilon(1D)$	1^{--}	10155			
	$\Upsilon(2S)$	1^{--}	10023.26 ± 0.31		•	•
	$\eta_b(2S)$	0^{-+}	9999 ± 4	•	•	•
	$h_b(1P)$	1^{+-}	9899.3 ± 0.8			•
	$\chi_{b1}(1P)$	1^{++}	$9892.78\pm0.26\pm0.31$			•
	$\chi_{b0}(1P)$	0^{++}	$9859.44\pm0.42\pm0.31$		•	•
	$\Upsilon(1S)$	1^{--}	9460.30 ± 0.26		•	•
	$\eta_b(1S)$	0^{-+}	9399.0 ± 2.3	•	•	•
$b\bar{c}$	$B_c(2S)^{\pm}$	0^-	6842 ± 6			•
	B_c^+	0^-	6275.1 ± 1.0	•	•	•
$b\bar{s}$	$B_{s1}(5830)$	1^+	5828.63 ± 0.27			•
$b\bar{q}$	$B_1(5721)^{+,0}$	1^+	5725.85 ± 1.3			•
$b\bar{s}$	B_s^*	1^-	5415.8 ± 1.5		•	•
	B_s^0	0^-	5366.82 ± 0.22	•	•	•
$b\bar{q}$	B^*	1^-	5324.65 ± 0.25		•	•
	$B^{\pm,0}$	0^-	5279.45	•	•	•

	State	$J^{P(C)}$	Mass (MeV)	Data set S1	S2	S3
$c\bar{c}$	$X(3915)$	0^{++}	3918.4 ± 1.9		•	•
	$\psi(3770)$	1^{--}	3773.13 ± 0.35		•	•
	$\psi(2S)$	1^{--}	3686.097 ± 0.010		•	•
	$\eta_c(2S)$	0^{-+}	3639.2 ± 1.2	•	•	•
	$h_c(1P)$	1^{+-}	3525.38 ± 0.11			•
	$\chi_{c1}(1P)$	1^{++}	3510.66 ± 0.07			•
	$\chi_{c0}(1P)$	0^{++}	3414.75 ± 0.31		•	•
	$J/\Psi(1S)$	1^{--}	3096.900 ± 0.006		•	•
	$\eta_c(1S)$	0^{-+}	2983.4 ± 0.5	•	•	•
$c\bar{s}$	$D_{s1}(2536)^{\pm}$	1^+	2535.10 ± 0.06			•
	$D_{s1}(2460)^{\pm}$	1^+	2459.5 ± 0.6			•
$c\bar{q}$	$D_1(2420)^{\pm,0}$	1^+	2421.4			•
	$D_0^*(2400)^0$	0^+	2318 ± 29		•	•
$c\bar{s}$	$D_{s0}^*(2317)^{\pm}$	0^+	2317.7 ± 0.6		•	•
	$D_s^{*\pm}$	1^-	2112.1 ± 0.4		•	•
$c\bar{q}$	$D^*(2007)^0$	1^-	2008.62			•
$c\bar{s}$	$D_s^{\pm}$	0^-	1968.27 ± 0.10	•	•	•
$c\bar{q}$	$D^{\pm,0}$	0^-	1867.23	•	•	•

S1: 9 PS mesons

S2: 25 PS+V+S mesons

S3: 39 PS+V+S+AV mesons

q represents a light quark (u or d)

We use $m_u = m_d \equiv m_q$

Global fits with fixed quark masses and y=0

Leitão, Stadler, Peña, Biernat, Phys. Lett. B **764** (2017) 38

First step: we perform global fits to the heavy + heavy-light meson spectrum

Adjustable model parameters: σ α_s C

Model parameters not adjusted in the fits:

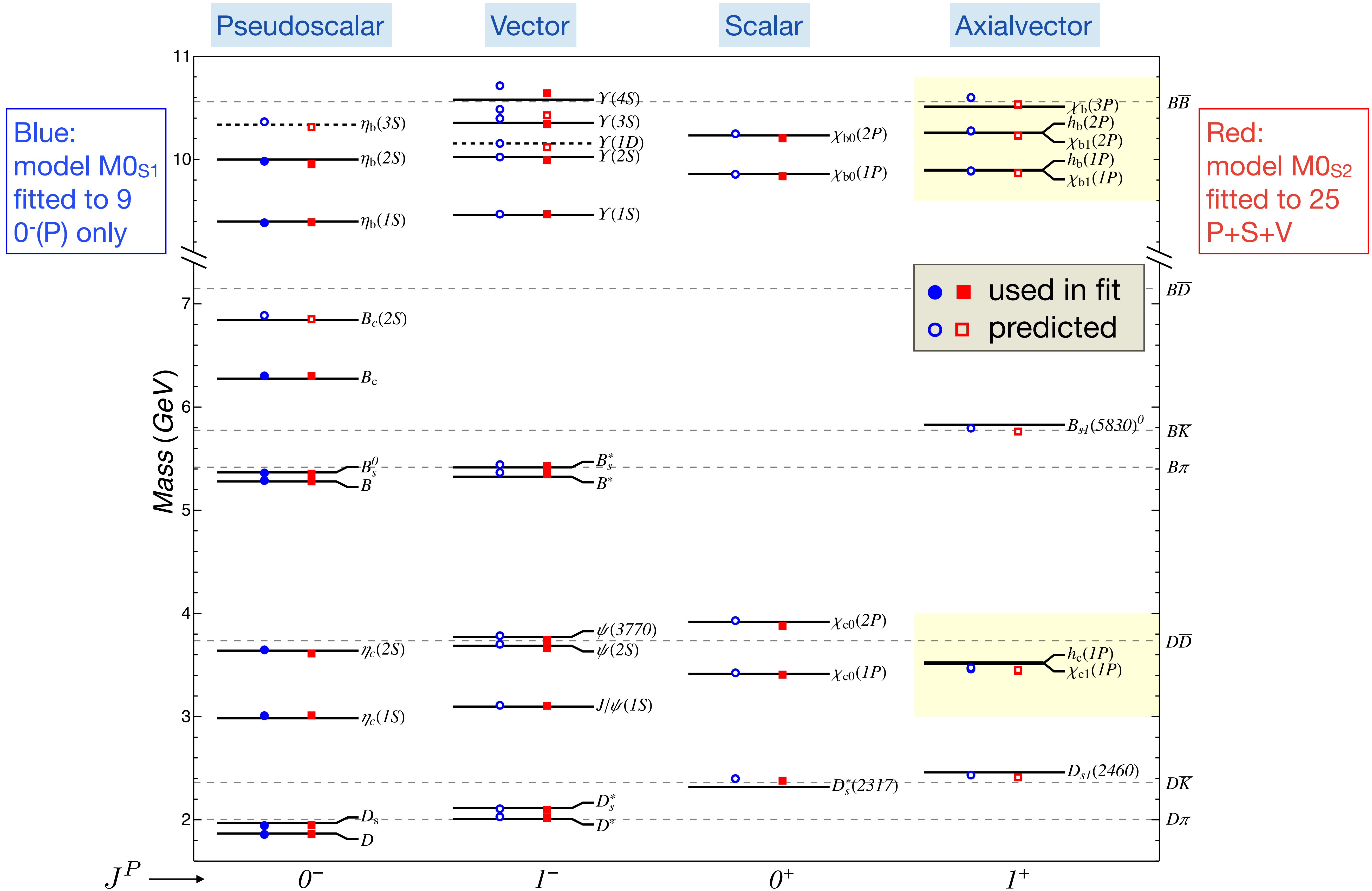
Constituent quark masses (in GeV) m_b=4.892, m_c=1.600, m_s=0.448, m_q=0.346

Scalar + pseudoscalar confinement y = 0

- Model M0$_{S1}$: fitted to 9 pseudoscalar meson masses only
- Model M0$_{S2}$: fitted to 25 pseudoscalar, vector, and scalar meson masses

(Previously called models P1 and PSV1)

Global fits with fixed quark masses and scalar confinement (y=0)



Global fits with fixed quark masses and y=0

The results of the two fits are remarkably similar!

Model	σ [GeV^2]	α_s	C [GeV]
M0$_{S1}$	0.2493	0.3643	0.3491
M0$_{S2}$	0.2247	0.3614	0.3377

rms differences to experimental masses (set S3):

Model	Δ_{rms} [GeV]
M0$_{S1}$	0.037
M0$_{S2}$	0.036

- Kernel parameters are already well determined through pseudoscalar states ($J^P = 0^-$)

Almost 100% L=0, S=0
(S-wave, spin singlet)

$$\langle 0^- | \mathbf{L}\cdot\mathbf{S} | 0^- \rangle = 0$$ Spin-orbit force vanishes

$$\langle 0^- | S_{12} | 0^- \rangle = 0$$ Tensor force vanishes

$$\langle 0^- | \mathbf{S}_1\cdot\mathbf{S}_2 | 0^- \rangle = -3/4$$ Spin-spin force acts in singlet only

- Good test for a covariant kernel:

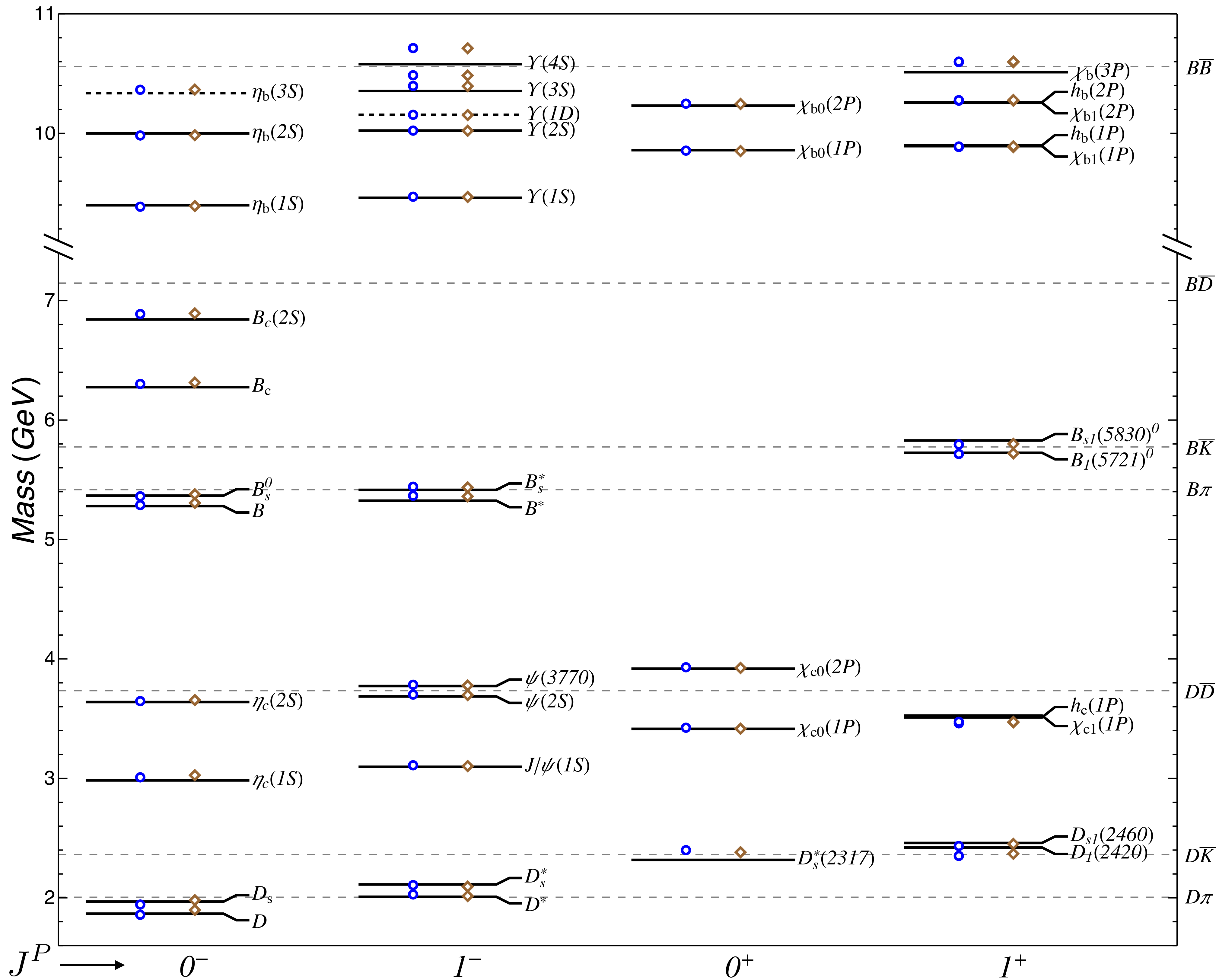
Pseudoscalar states do not constrain spin-orbit and tensor forces, and cannot separate spin-spin from central force.

But they should be determined through covariance.

Model M0$_{S1}$ indeed predicts spin-dependent forces correctly!

Leitão, AS, Peña, Biernat, Phys. Lett. B **764** (2017) 38

Importance of PS coupling in the confining kernel



Confining interaction (with y=0)

$$\left(\mathbf{1}_1 \otimes \mathbf{1}_2 + \gamma_1^5 \otimes \gamma_2^5\right) V_L$$

S PS

Model M0$_{S1}$

- S+PS
- S only (no refitting)

PS effect very small:

- a few MeV in bottomonium
- max: ~40 MeV in D mesons

Fits with variable quark masses and confinement (S+PS)-V mixing *y*

In a new series of fits we treat quark masses and mixing parameter *y* as adjustable parameters.

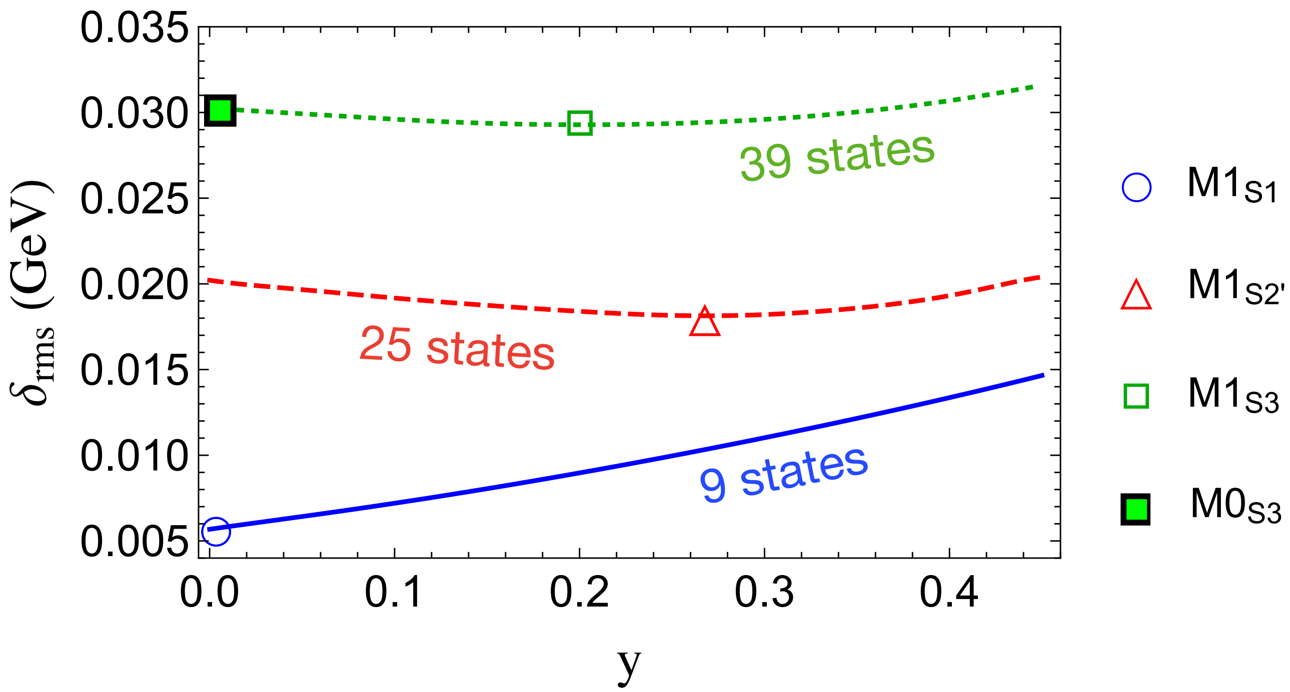
Model	Symbol	σ [GeV^2]	α_s	C [GeV]	y	m_b [GeV]	m_c [GeV]	m_s [GeV]	m_q [GeV]	N	$\delta_{\rm rms}$ [GeV]	$\Delta_{\rm rms}$ [GeV]
M0$_{\rm S1}$		0.2493	0.3643	0.3491	**0.0000**	**4.892**	**1.600**	**0.4478**	**0.3455**	9	0.017	0.037
M1$_{\rm S1}$	○	0.2235	0.3941	0.0591	0.0000	4.768	1.398	0.2547	0.1230	9	0.006	0.041
M0$_{\rm S2}$		0.2247	0.3614	0.3377	**0.0000**	**4.892**	**1.600**	**0.4478**	**0.3455**	25	0.028	0.036
M1$_{\rm S2}$		0.1893	0.4126	0.1085	0.2537	4.825	1.470	0.2349	0.1000	25	0.022	0.033
M1$_{\rm S2'}$	△	0.2017	0.4013	0.1311	0.2677	4.822	1.464	0.2365	0.1000	24	0.018	0.033
M1$_{\rm S3}$	□	0.2022	0.4129	0.2145	0.2002	4.875	1.553	0.3679	0.2493	39	0.030	0.030
M0$_{\rm S3}$	■	0.2058	0.4172	0.2821	**0.0000**	4.917	1.624	0.4616	0.3514	39	0.031	0.031

include AV states in fit

Parameters in **bold** are not varied during fit

y held fixed, other parameters refitted

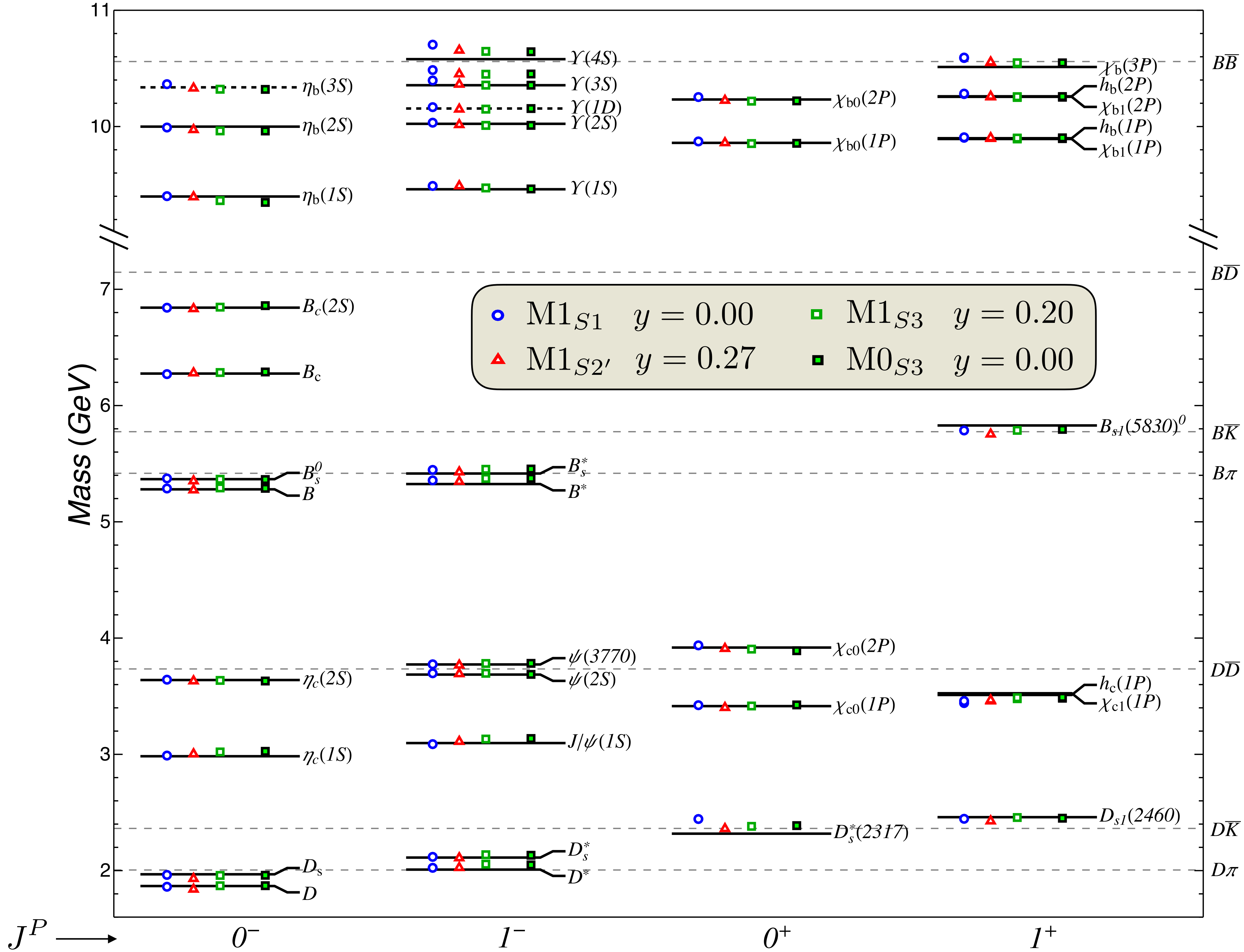
rms difference to fitted masses



- Quality of fits not much improved
- Best model M1$_{\rm S3}$ has y=0.20, but minimum is very shallow

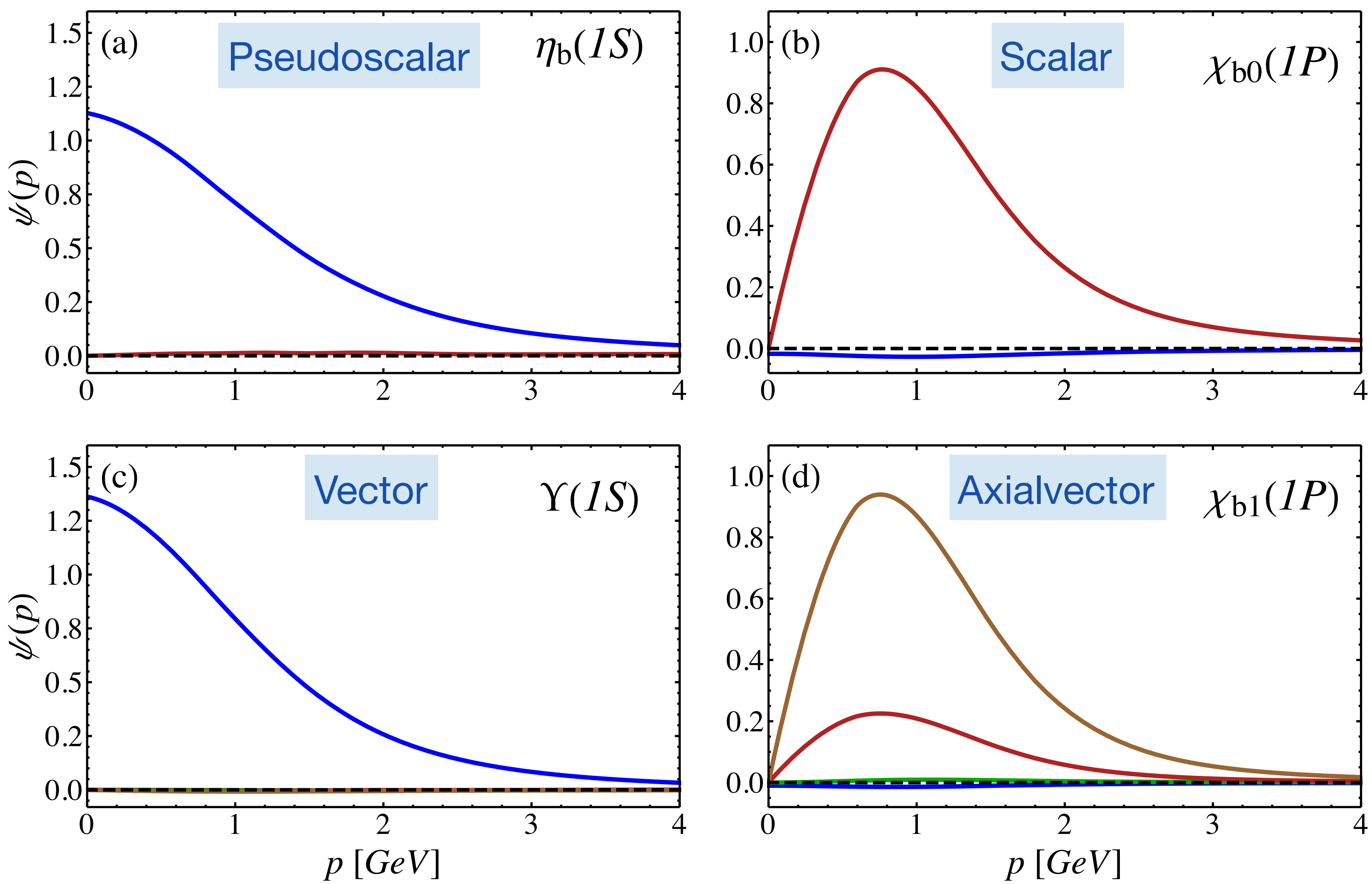
y and quark masses are not much constrained by the mass spectrum.

Mass spectra of heavy and heavy-light mesons



Bottomonium ground-state wave functions

Calculated with model $M1_{S3}$



Partial waves

- S
- P

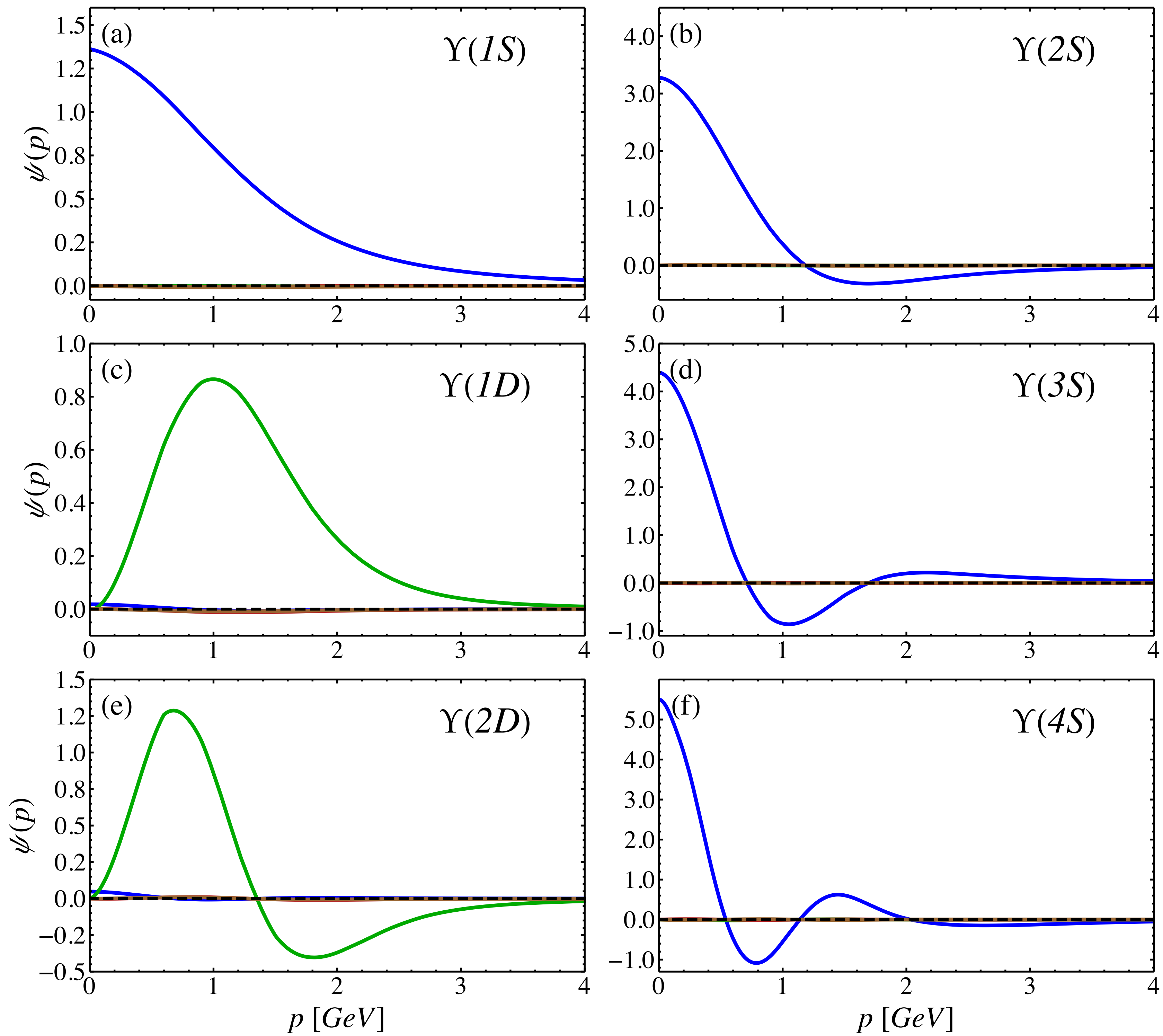
Partial waves

- S
- D
- P_t (spin triplet)
- P_s (spin singlet)

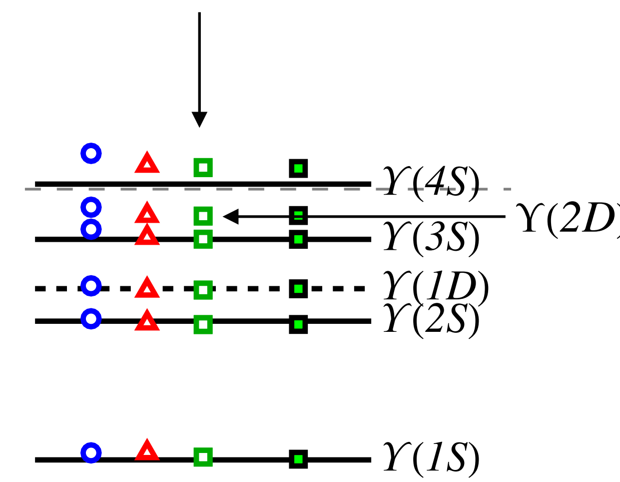
Relativistic wave function components are very small

Radial excitations in vector bottomonium

Wave functions of excited states look reasonable



Model M1$_{S3}$

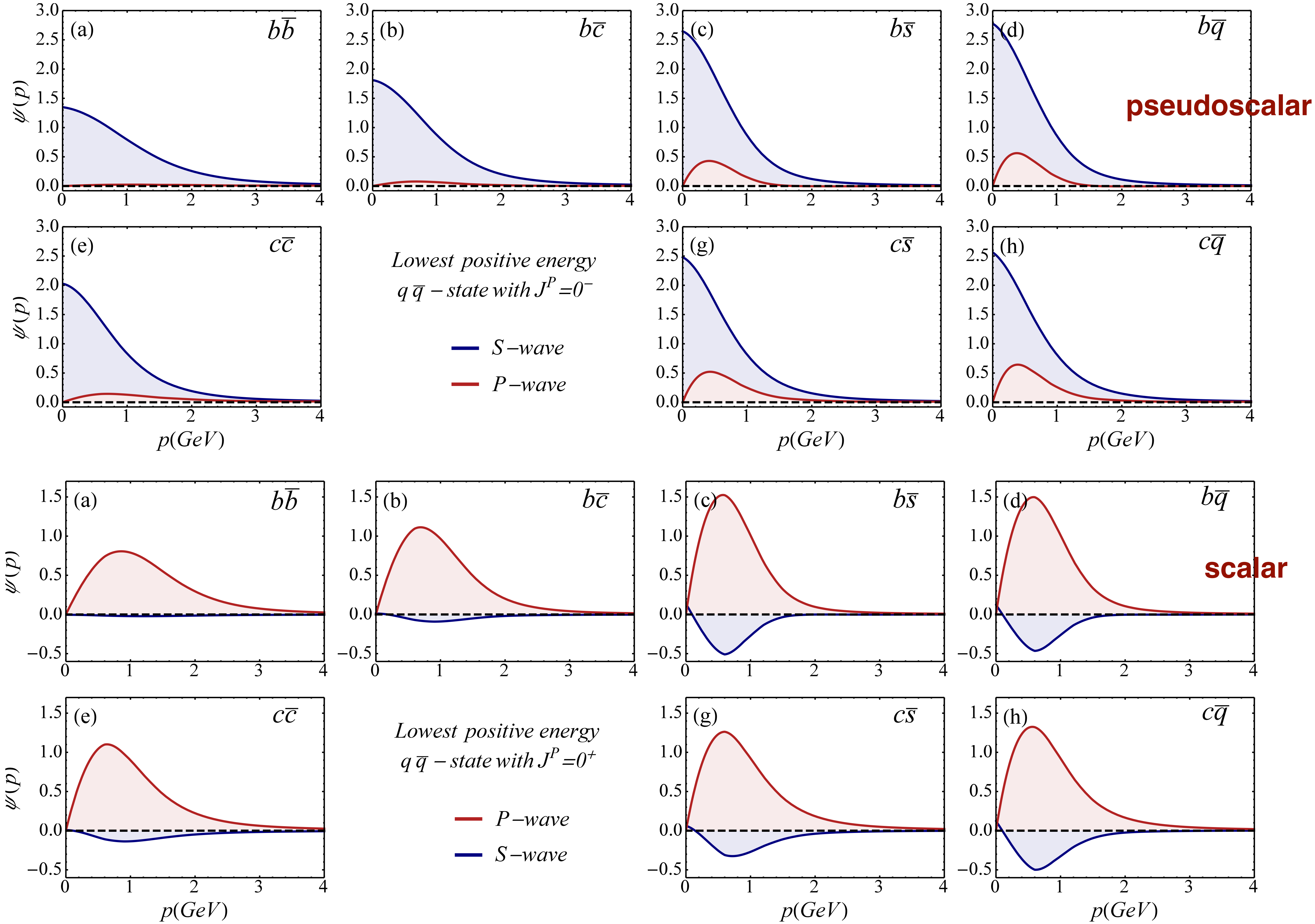


Partial waves

- S
- D
- P_t (spin triplet)
- P_s (spin singlet)

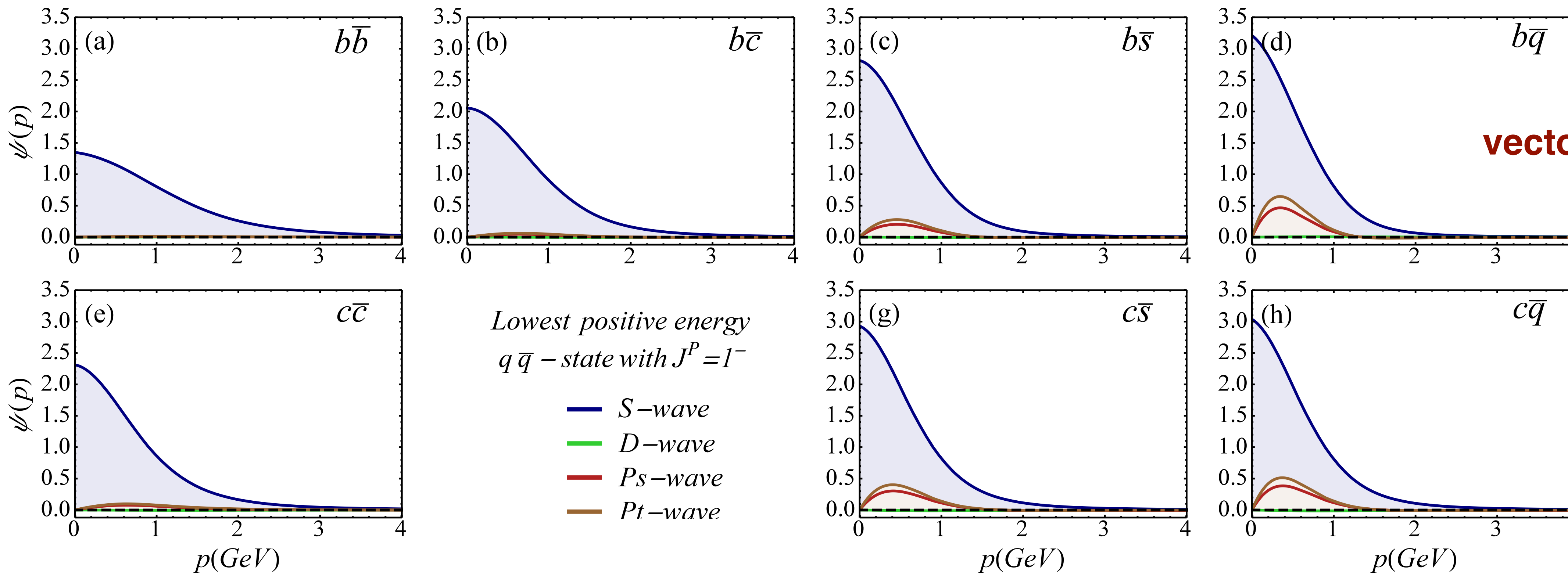
Importance of relativistic components

Ground-state wave functions of model M1$_{S3}$.

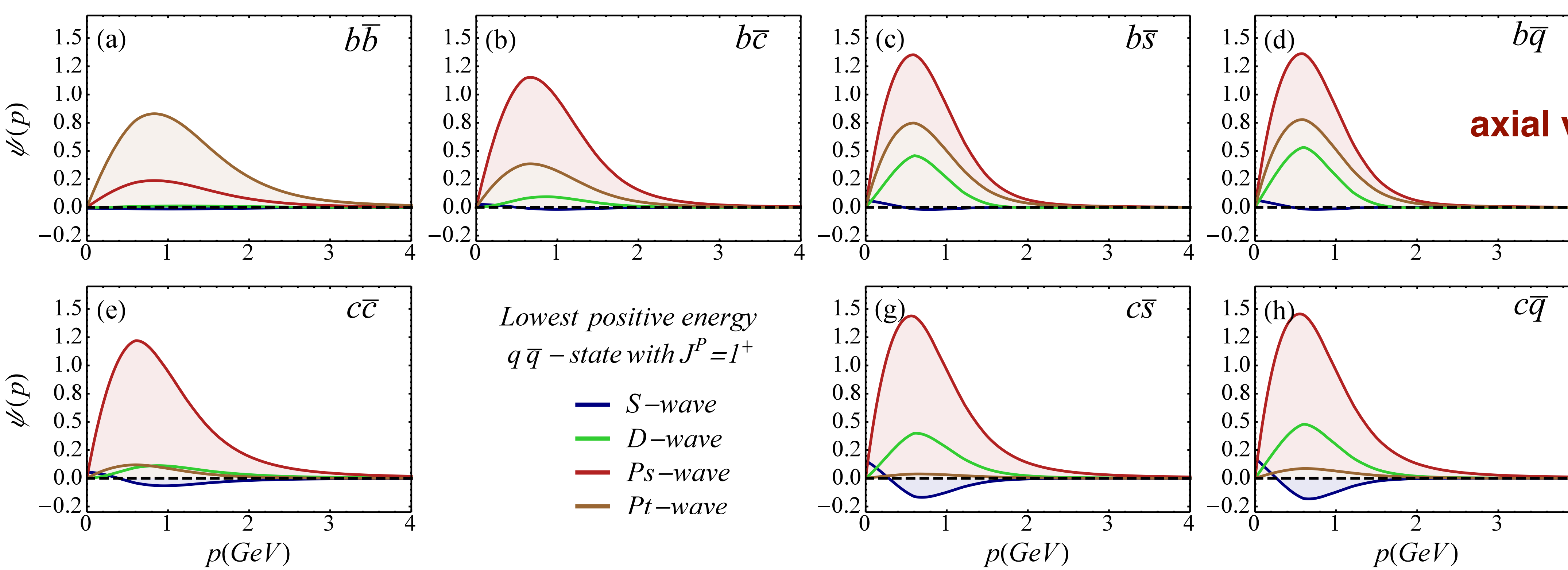


Importance of relativistic components

Ground-state wave functions of model $M1_{S3}$.



vector



axial vector

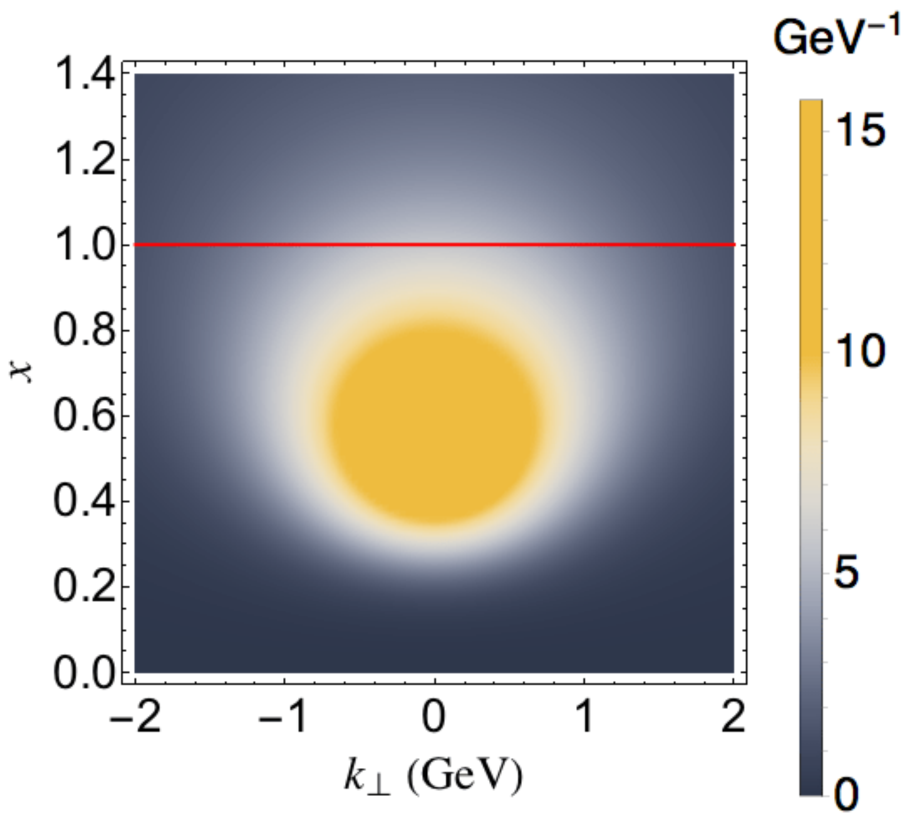
CST light-front wave functions

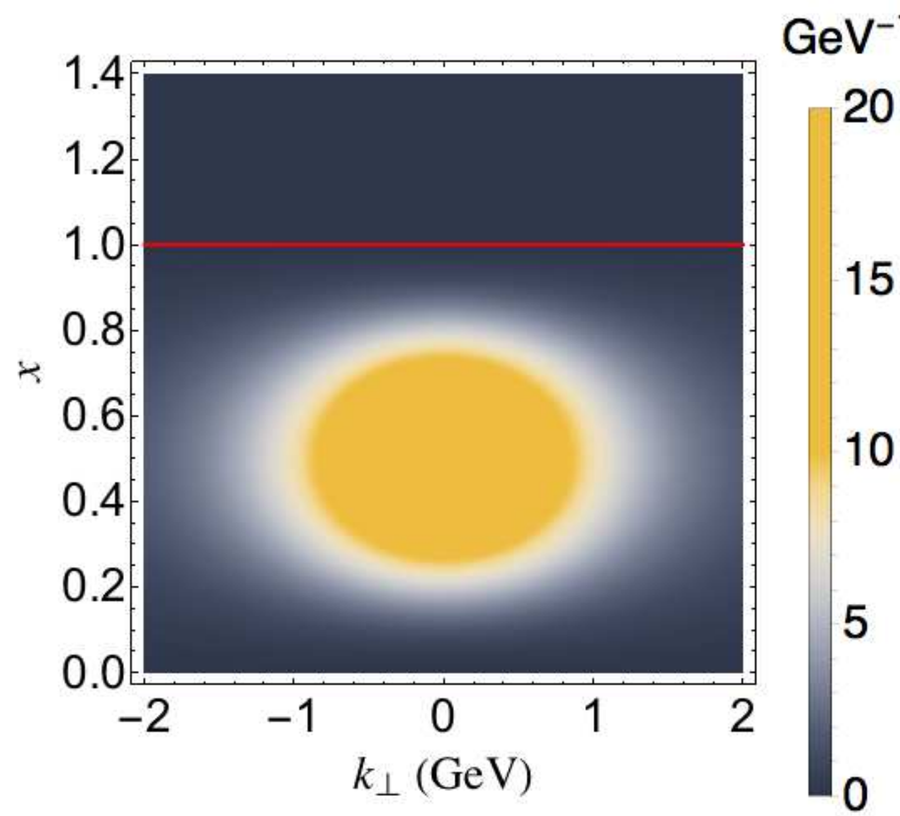
Leitão, Li, Maris, Peña, AS, Vary, Biernat, arXiv:1705.06178

Comparison of CST and BLFQ wave functions

Calculated CST-LFWF, mapped with the Brodsky-Huang-Lepage prescription (map.)

Example: wave function of J/ψ (1S) with λ=0





$$x=\frac{k_1^+}{P^+}=\frac{E_k+k^3}{M}=\frac{\sqrt{m^2+\boldsymbol{k}_\perp^2+(k^3)^2}+k^3}{M}$$

$$x=\frac{k^+}{P^+}\equiv\frac{E_k+k^3}{2E_k}=\frac{1}{2}+\frac{k^3}{2\sqrt{k_\perp^2+(k^3)^2+m^2}}$$

BHL prescription

Heavy quarkonia decay constants

Comparison between two calculations of quarkonia decay constants:

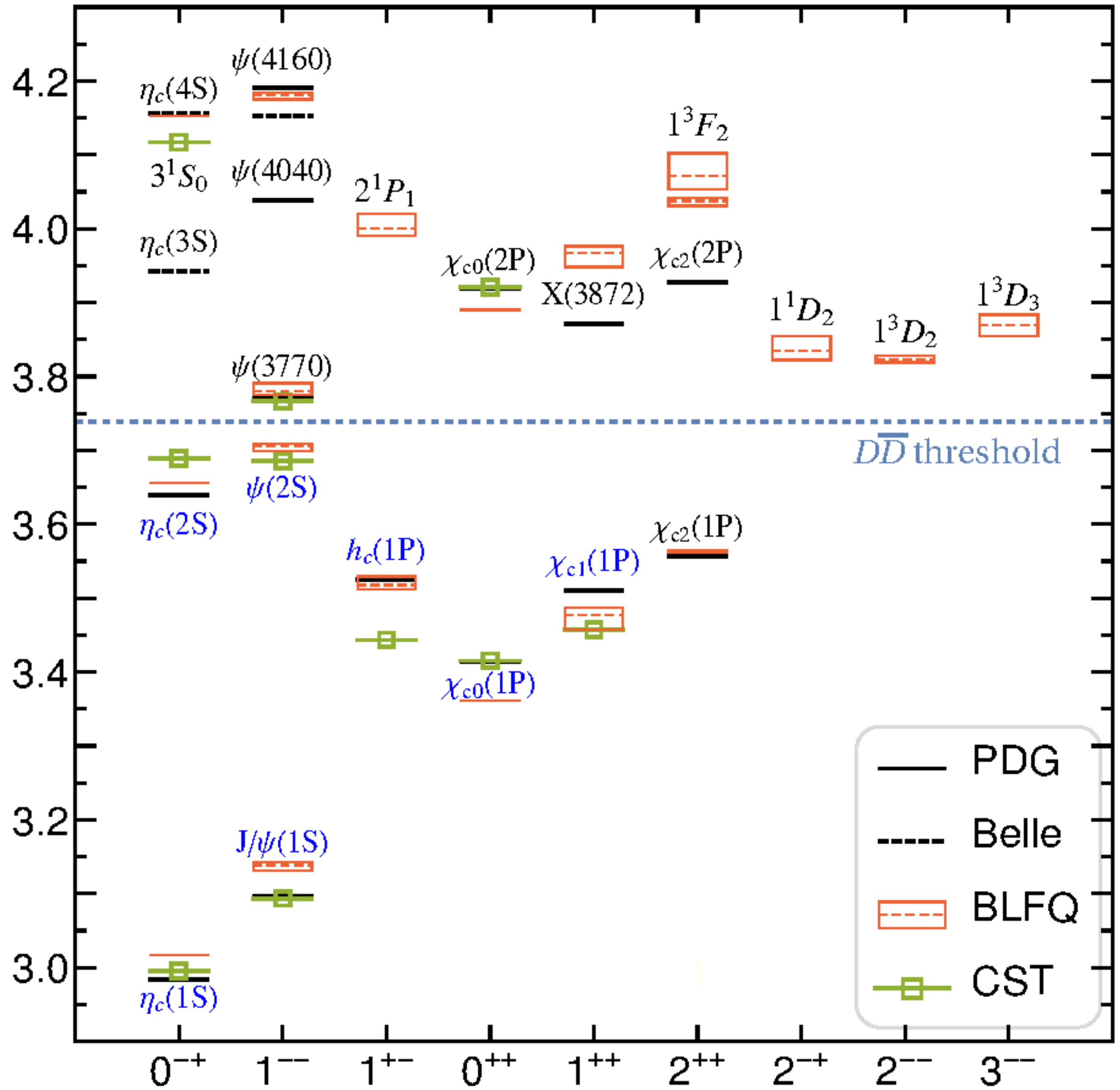
1. Calculated with CST-LFWF, mapped with the Brodsky-Huang-Lepage prescription (map.)
2. Calculated directly in the CST formalism (dir.)

Charmonium				Bottomonium			
	map.	dir.	δ		map.	dir.	δ
η_c	359(10)	343(9)	16	η_b	655(14)	664(15)	9
η_c'	277(2)	251(2)	26	η_b'	427(21)	432(23)	5
				η_b''	372(9)	373(15)	1
J/ψ	295(4)	280(3)	15	Υ	480 (10)	480(17)	0
ψ'	259(3)	229(3)	30	Υ'	351(18)	347(20)	4
				Υ''	316(2)	309(6)	7
$\psi(3770)$	38(1)	12(1)	26	1^3D_1	12(1)	4(1)	8

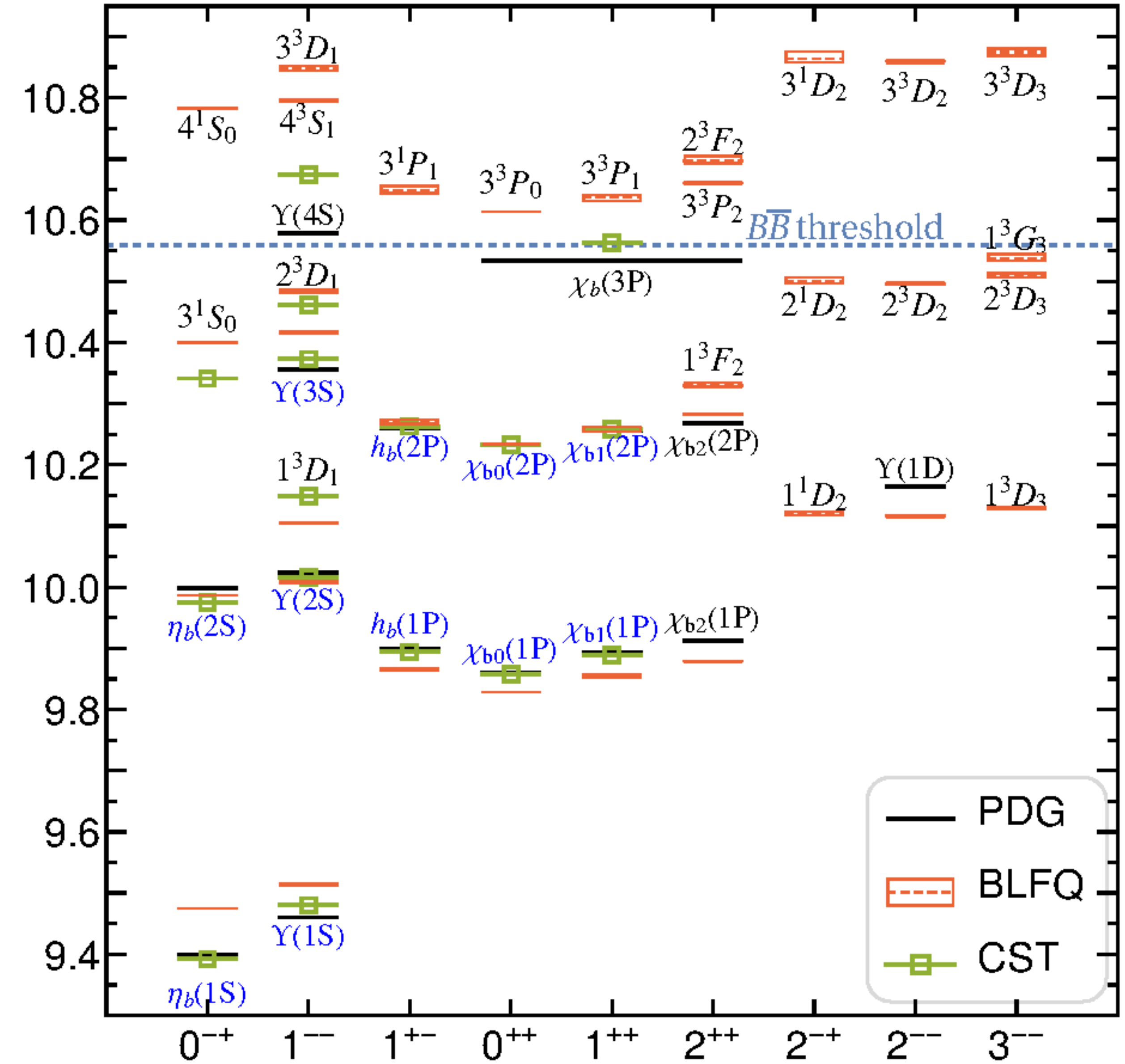
Decay constants in MeV (δ = map.-dir.)

Quarkonium spectrum with BLFQ and CST

Charmonium



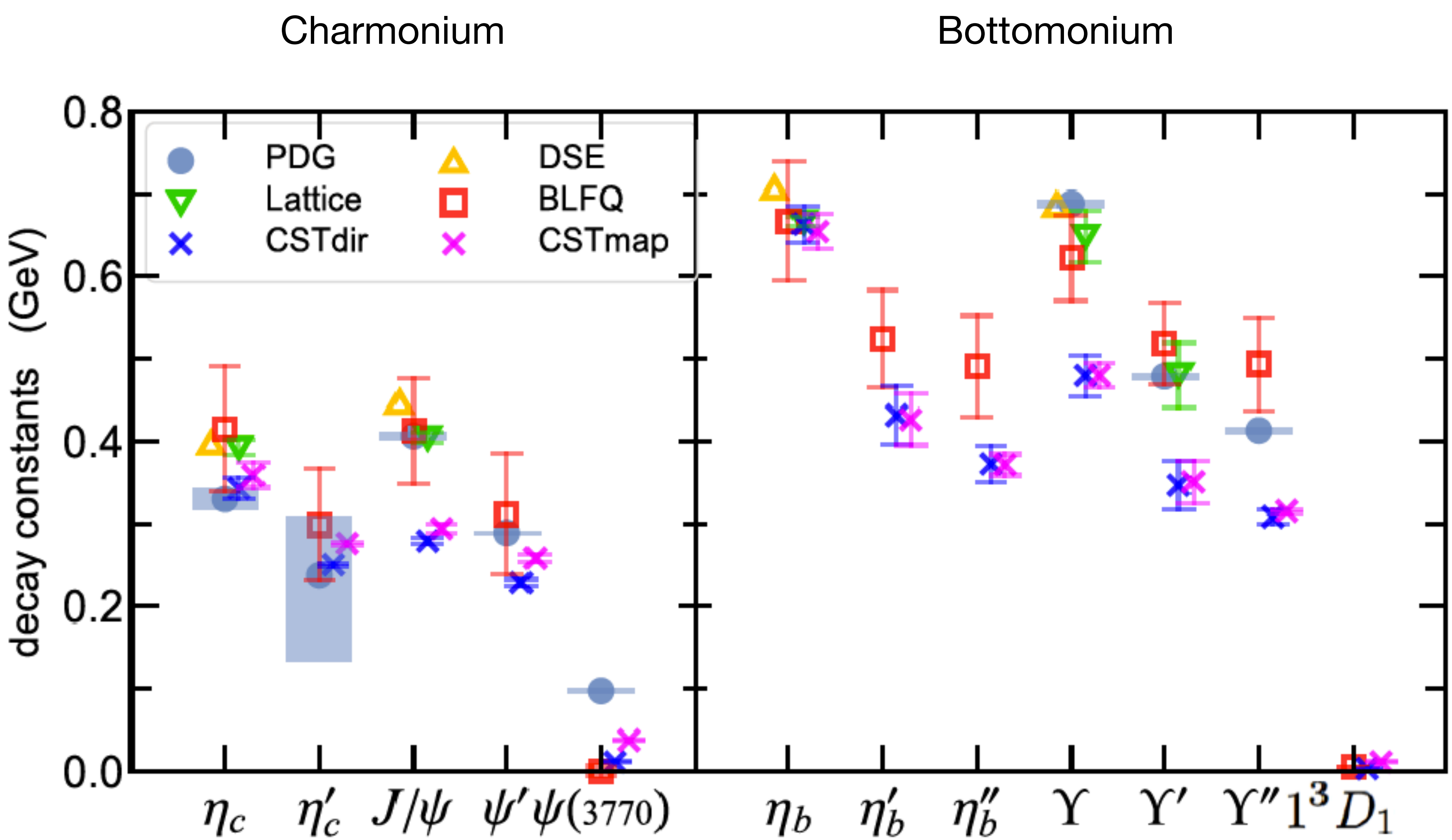
Bottomonium



Rms differences (in MeV) between calculated and experimental masses shown in blue

	Charmonium	Bottomonium
BLFQ	33	39
CST	42	11

Quarkonium decay constants with BLFQ and CST

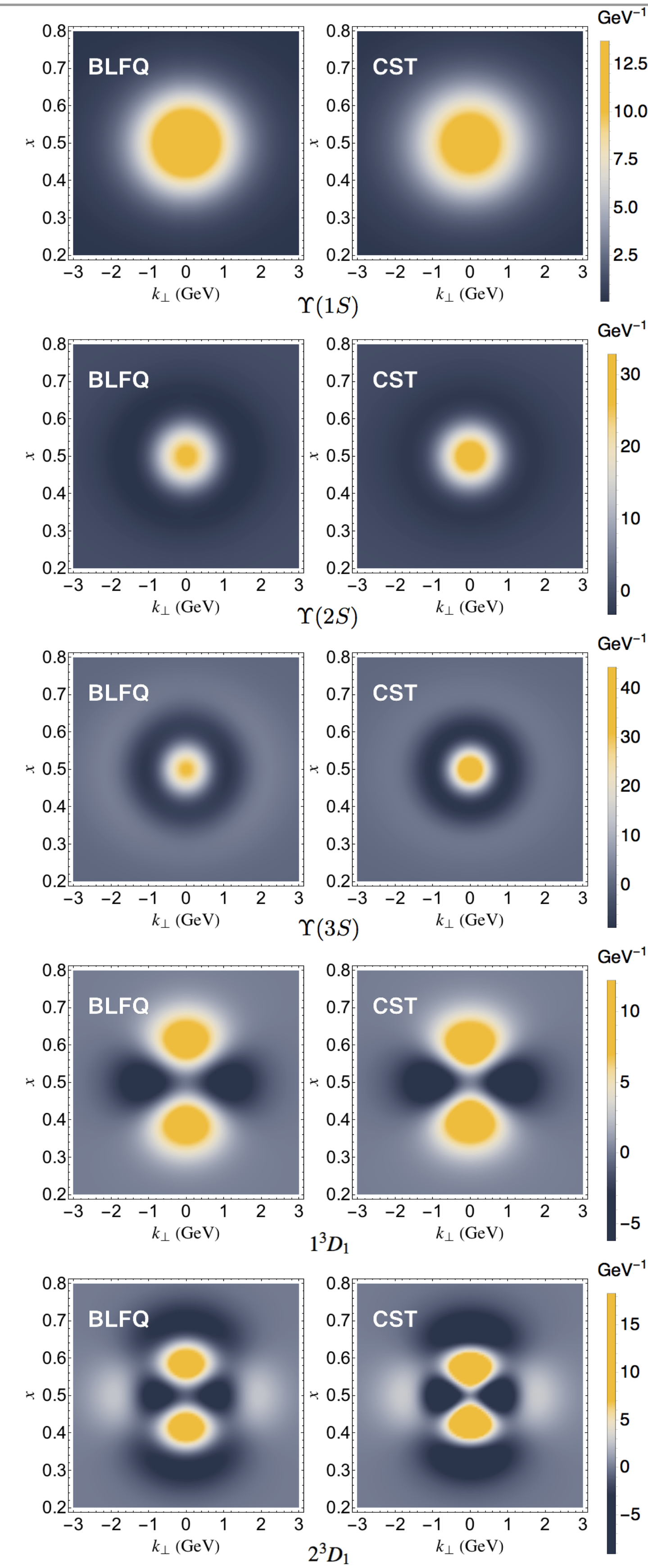


Comparison between BLFQ and CST light front wave functions

BLFQ: Basis Light Front Quantization

Y. Li, P. Maris, J. Vary, PRD **96**, 016022 (2017)

Vector bottomonium wave functions,
dominant components (S=1)



BLFQ and CST distribution amplitudes

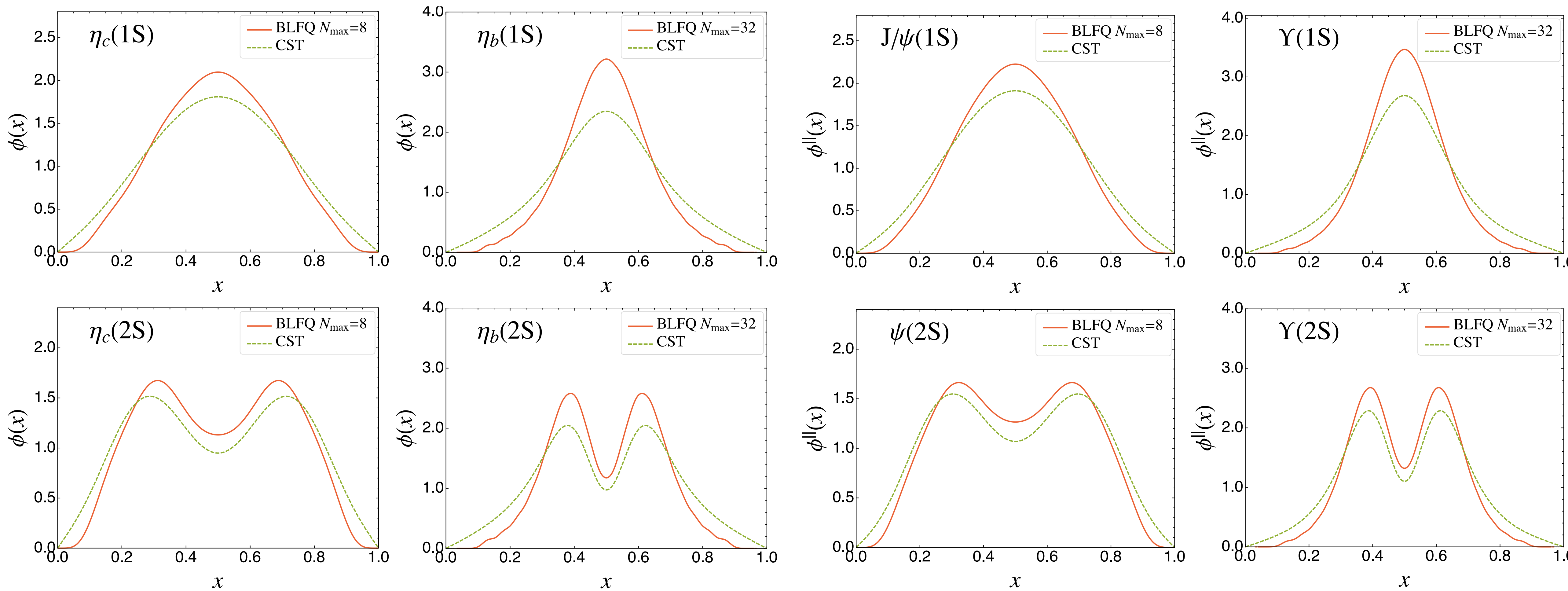
Leading twist distribution amplitudes from BLFQ and CST (map.) wave functions

$$\frac{f_{P,V}}{2\sqrt{2Nc}}\phi_{P,V^{||}}(x;\mu) = \frac{1}{\sqrt{x(1-x)}}\int_0^{k_\perp\leq\mu}\frac{d^2\mathbf{k}_\perp}{2(2\pi)^3}\psi^{\lambda=0}_{\uparrow\downarrow\mp\downarrow\uparrow}(\mathbf{k}_\perp,x)$$

- PS
+ V

Pseudoscalar quarkonia | Vector quarkonia

Charmonium | Bottomonium | Charmonium | Bottomonium



Summary

- With the simplest, one-channel CST equation and a few global parameters, we get a very nice description of the heavy and heavy-light meson spectrum
- (S+PS) confining kernel with ~ 0%—30% admixture of V coupling is compatible with the data
- In heavy quarkonia, we find remarkable similarities between CST LFWF (with BHL prescription) and BLFQ LFWF by Li, Vary, Maris, even in excited states

Next steps:

- Study other constraints on Lorentz structure of confining interaction
- Calculation of tensor mesons (spin ≥ 2)
- Inclusion of running quark-gluon coupling
- Extension of current model to the light-quark sector (requires 4-channel eq.)
- Calculation of parton distribution functions
- Calculate relativistic quark-antiquark states with exotic J^{PC}