End Point Model for Exclusive Hadronic Processes

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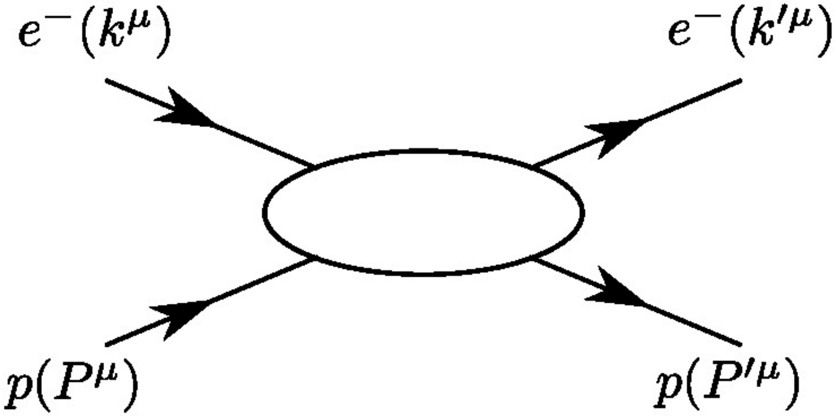
Exclusive Processes

We are interested in processes such as

- hadronic electromagnetic form factors
- p p $\rightarrow$ p p
- γ p $\rightarrow$ γ p

at high energies



Scaling Law

At large momentum transfer ($|t| = Q^2$) these show an interesting scaling law

$$\frac{d\sigma}{dt} \propto \frac{1}{s^{n-2}} f\left(\frac{t}{s}\right)$$

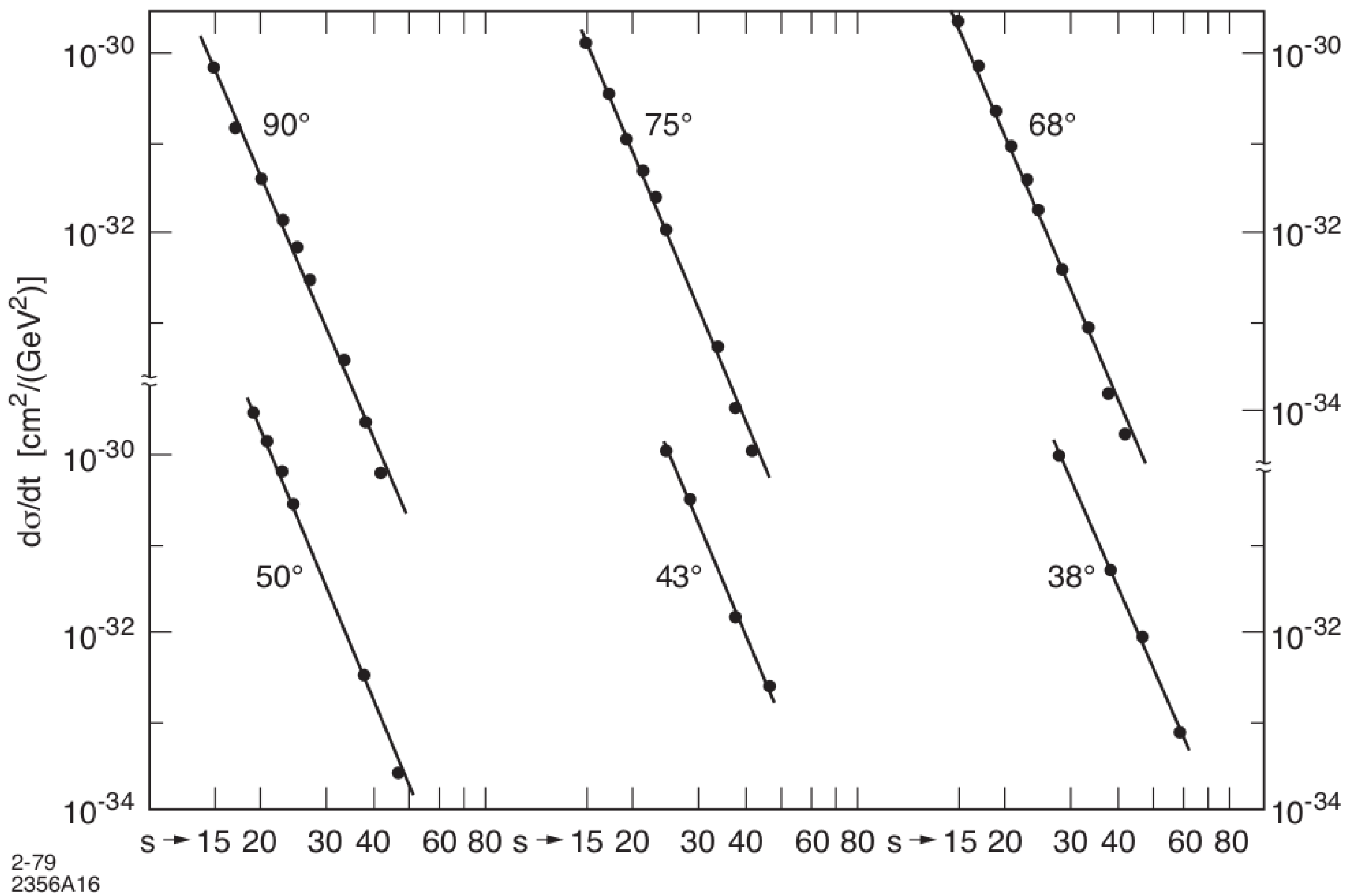
Brodsky, Farrar 1973
Matveev, Muradian, Tavkhelidze 1973

For $s \sim |t| >> 1\ \mathrm{GeV}^2$, fixed cm scattering angle

n= number of elementary constituents participating in the reaction

Ex: proton form factor $F_1 \propto 1/Q^4$
pp elastic scattering at large cm angle $\propto 1/s^{10}$ etc

pp elastic scattering data



Scaling Law s>>|t|, fixed s

We also see a scaling behavior for fixed s, s>>|t|

For example, pp elastic scattering shows

$$\frac{d\sigma}{dt} \propto \frac{1}{|t|^8}$$

Landshoff 1974
Collins, Gault, Martin 1975
Donnachie and Landshoff 1979

Theoretical Description at large |t|

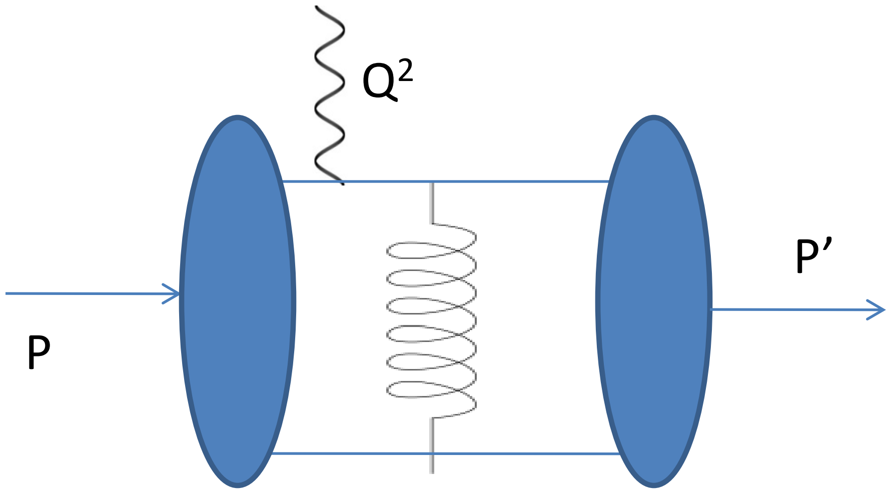
It has been argued that large momentum transfer data may be nicely explained within the framework of perturbative QCD

Theoretical Description at large |t|

The process is factorized into a perturbative part and a non-perturbative distribution amplitude φ(x) which depends on the hadron wave function

$$F_\pi(Q^2) = \int_0^1 dx dx' \, \varphi(x') T_H(x, x', Q^2) \varphi(x)$$

($|t| = Q^2$)

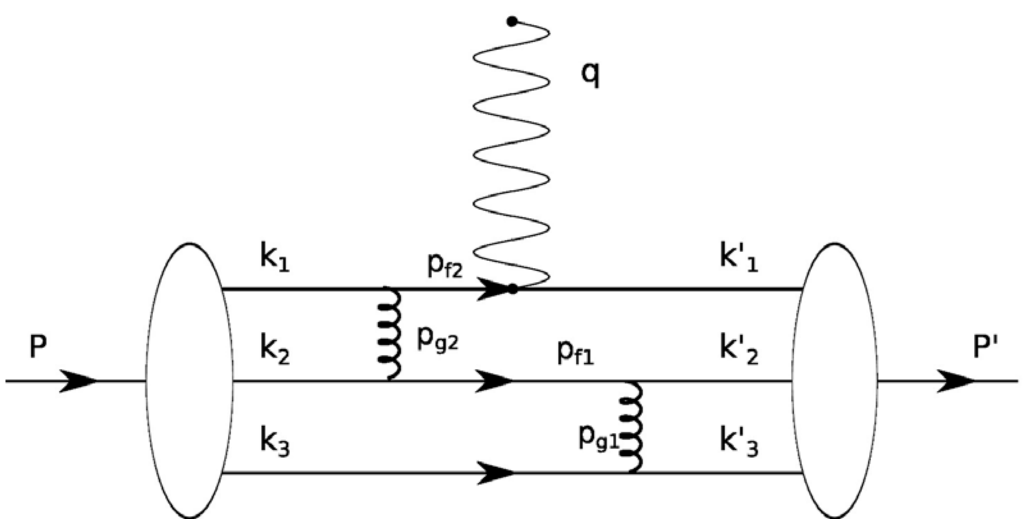


Farrar, Jackson 1979
Brodsky, Lepage 1980
Efremov, Radyushkin 1980

The scaling laws are nicely explained by perturbative QCD

These arise due to the perturbative kernel in which all quarks undergo a hard momentum exchange with another quark (or lepton).

Each gluon exchange contributes a $1/Q^2$ factor



Failure of perturbative description

However explicit calculations show that the actual magnitude of form factors predicted by pQCD does not match data at energies currently accessible in laboratory

(Isgur, Llewelyn-Smith 1984)

Hence this cannot be the right explanation for the scaling laws

Failure of perturbative description

Detailed calculations of pion and proton form factors were performed in pQCD and did show agreement with data

Li and Sterman 1992
Li 1993
Kundu, Li, Samuelsson, Jain 1999

However the dominant contribution arises from the soft region and hence the calculations are inconsistent

Failure of perturbative description

Other disagreements include:

- Violation of the helicity conservation rule
- Non-observation of a clear signal of color transparency

Soft Mechanism for Scaling Laws

We seek a soft mechanism for scaling laws.

A soft process which is known to contribute to exclusive processes is the end point process

In this case $x \to 1$, i.e. one of the quarks carries most of the hadron momentum

This was the first mechanism proposed for this process

Feynman 1969
Drell, Yan 1970
West, 1970

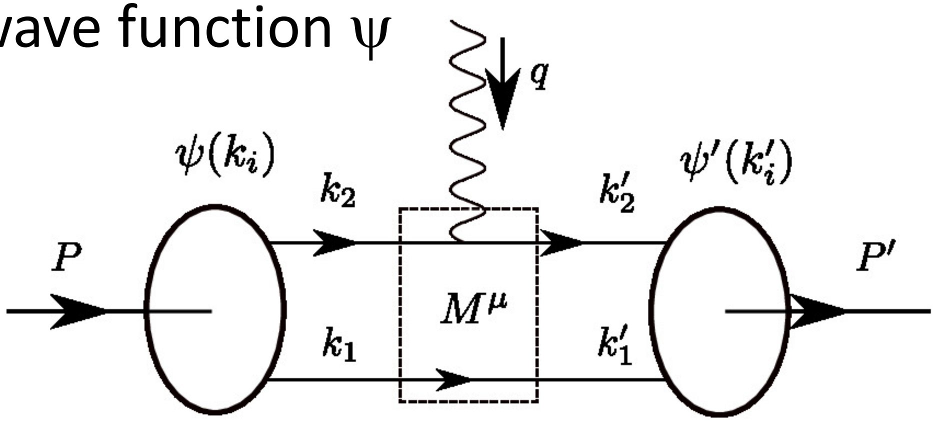
Soft End Point Mechanism

quark with $x_i \rightarrow 1$ undergoes hard scattering with the external particle $k_i = x_i P + k_{ix}\hat{x} + k_{iy}\hat{y}$

The momentum transfer between this and the rest of the quarks (spectators) is very small

Hence the spectator quarks have to be treated non-perturbatively using a model wave function ψ

The scaling laws are related to the form of ψ as $x \rightarrow 1$



Soft End Point Mechanism

- The scaling laws are related to the form of ψ as $x \to 1$. We choose the form which gives observed scaling
- As we shall see the dominant contribution comes from valence quarks
- The spectator quarks are non-perturbative. Hence we also need a model for their propagators and interaction with the active quark.
- This modeling will not affect the scaling but contributes to the actual magnitude

Pion Form Factor

$$\langle P' | J^{\mu}_{EM} | P \rangle = \int \prod_{i,j} \frac{d^4 k_j}{(2\pi)^4} \frac{d^4 k'_i}{(2\pi)^4} \Psi'^{*}(k'_i) M^{\mu} \Psi(k_j)$$

Light cone wave function

$$\psi(x_i, k_{\perp i}) = \int \prod_i d\kappa_i^- \, \delta^4\left(\sum_i \kappa_i^- - m_\pi^2 \right) \Psi(k_i)$$

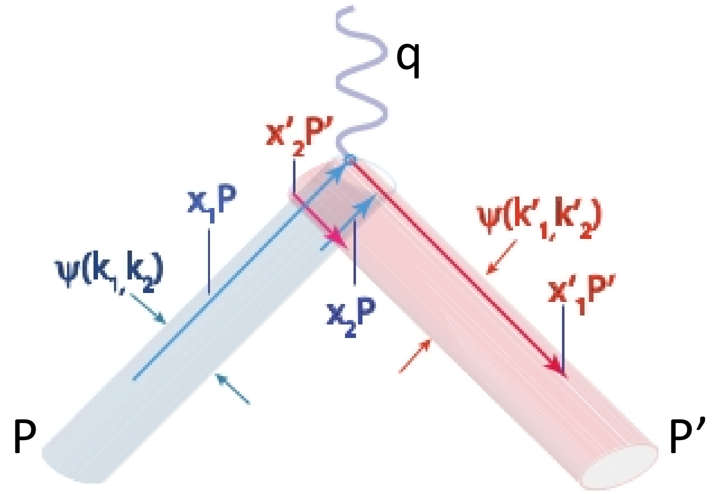
$\kappa^- = k^- p^+$

A simple model

We assume an overlap model

The spectator quark do not interact.
Their momenta remains unchanged

This is a simple model. In general one may investigate more complex models which may involve soft gluon exchanges



Frame : q = (0,Q,0,0)

$$P = \left(\sqrt{\frac{Q^2}{2}+m_\pi^2}, -\frac{Q}{2}, 0, \frac{Q}{2}\right) \qquad P' = \left(\sqrt{\frac{Q^2}{2}+m_\pi^2}, \frac{Q}{2}, 0, \frac{Q}{2}\right)$$

Overlap Model Pion form factor

$$F_\pi = \int_0^1 dx \; \Phi_\pi(x, xQ/\sqrt{2})$$

$$\Phi_\pi(x, k_x) = \int_0^1 dk_y \; \psi^*(x, k_x, k_y)\,\psi(x, k_x, k_y)$$

$$k_x = \frac{xQ}{\sqrt{2}}$$

Φ should decay rapidly for large $k_\perp$

$$\Phi_\pi(x, k_x) = e^{-|k_x|/\Lambda}\,\phi(x)$$

For large Q the dominant contribution comes from $x<\Lambda/Q$

Assume $\phi(x) \propto x$

$$\Rightarrow F_\pi \rightarrow 1/Q^2$$

Comments

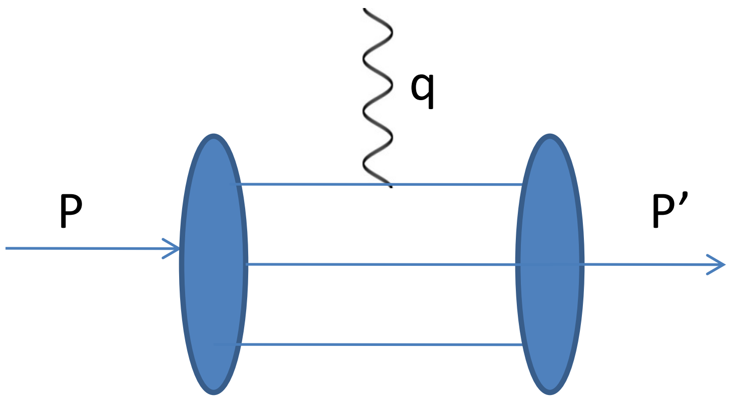
- The power law is tied to our assumed x dependence of the wave function as it approaches the end point ($x \to 0$)
- For pion it goes as $x^{1/2}$
- For every additional parton i, we have an additional integration over dx_i
- This leads to a contribution suppressed by Λ/Q
- Hence valence quarks dominate

Dagaonkar, Jain, Ralston 2014

Proton Form Factor

$$\langle P'|J^{\mu}|P\rangle = F_1(Q^2)\,(\bar{N}'\gamma^{\mu}N) + \frac{F_2(Q^2)}{2M_p}(\bar{N}'i\sigma^{\mu\nu}q_{\nu}N)$$

$$\langle P'|J^{\mu}|P\rangle = \int \prod_i dx_i\, d^2k_{\perp i} dx_i' d^2k'_{\perp i}\, \bar{Y}\, M^{\mu} Y \,\ldots$$



We assume the leading twist light cone wave function.

This serves as a model, will not affect the scaling but will affect the magnitude

$$\bar{Y}_{\alpha\beta\gamma} \propto (\gamma^{\mu}P_{\mu}C)_{\alpha\beta}(\gamma_5 N)_{\gamma}V + (\gamma^{\mu}P_{\mu}\gamma_5 C)_{\alpha\beta}N_{\gamma}A + i(\sigma_{\mu\nu}P^{\nu}C)_{\alpha\beta}(\gamma^{\mu}\gamma_5 N)_{\gamma}T$$

$$F_1 \sim \int_0^1 dx_1\, dx_2\, e^{-k_{1x}^2/\Lambda^2}\, \phi(x_1, x_2)$$

$$k_{1x} = (x_1 + x_2)\, Q/2$$

x_1, $x_2 < \Lambda/Q$

$$\phi = \psi(x_1, x_2)\, \psi(x_1, x_2)$$

$$\psi(x_1, x_2) \sim (1 - x_3)$$

$$F_1 \sim \frac{1}{Q^4}$$

Dagaonkar, Jain, Ralston 2014

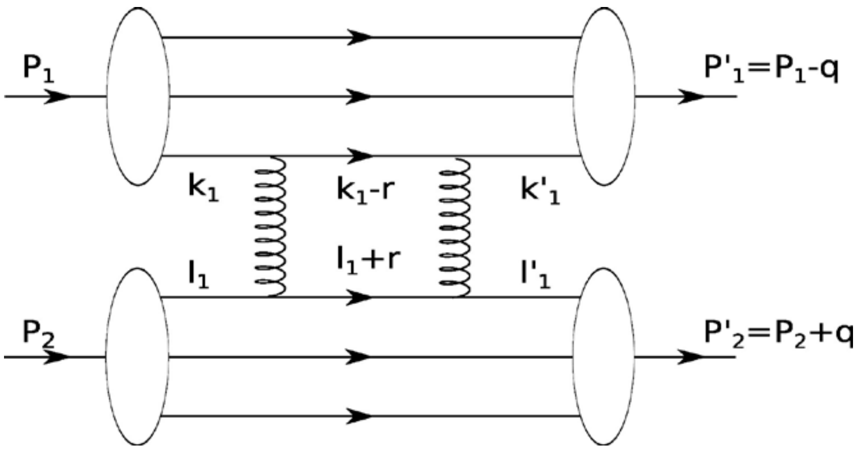
Hadron-Hadron scattering

A+B $\rightarrow$A+B $\quad\quad$ s ~ |t| >> Λ_{QCD} , fixed θ_{CM}

$$\frac{d\sigma}{dt} \propto \frac{MM^*}{s^2}$$

M ~ F_A F_B

$\therefore$ For pp $\quad \frac{d\sigma}{dt} \propto \frac{1}{s^{10}}$



We obtain the standard scaling laws expected from quark counting

Dagaonkar, Jain, Ralston 2014
Diehl, Feldmann, Jakob, Kroll, 2003

Hadron-Hadron scattering

$A+B \rightarrow A+B$

Fixed s, s >> |t| >> Λ_{QCD}

$$\frac{d\sigma}{dt} \propto \frac{MM^*}{s^2}$$

$M \sim F_A \; F_B$

$\therefore$ For pp $$\frac{d\sigma}{dt} \propto \frac{1}{|t|^8}$$

in agreement with data

Dagaonkar, Jain, Ralston 2014

Proton Pauli Form Factor F_2

- We model the spectator quark propagator as the free propagator with an effective mass of order 100 MeV
- F_2 gets contribution proportional to spectator quark mass and scales as $1/Q^5$, as observed in data

Dagaonkar, Jain, Ralston 2016

- Hadron helicity conservation rule is violated due to non-perturbative effects arising from spectator quarks

Numerical Fitting of Proton Form Factor

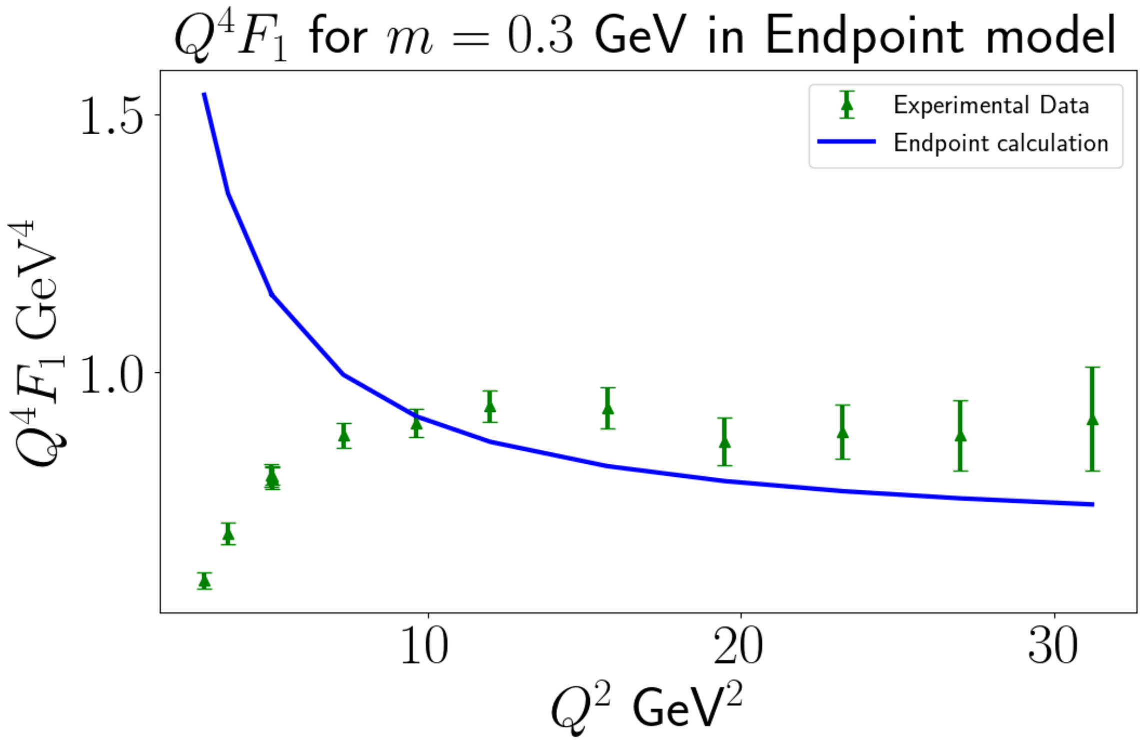
- We assume leading twist proton wave function as a model
- We assume a mass of order 100 MeV for spectator quarks
- We make a detailed fit to form factors F_1, F_2
- Parameters: spectator quark masses m,

  wave function parameters v, t in the end point region

$$V = v(1 - x_i)\, e^{-k_T^2/\Lambda^2}$$

$$T = t(1 - x_i)\, e^{-k_T^2/\Lambda^2}$$

Proton Form Factor F_1



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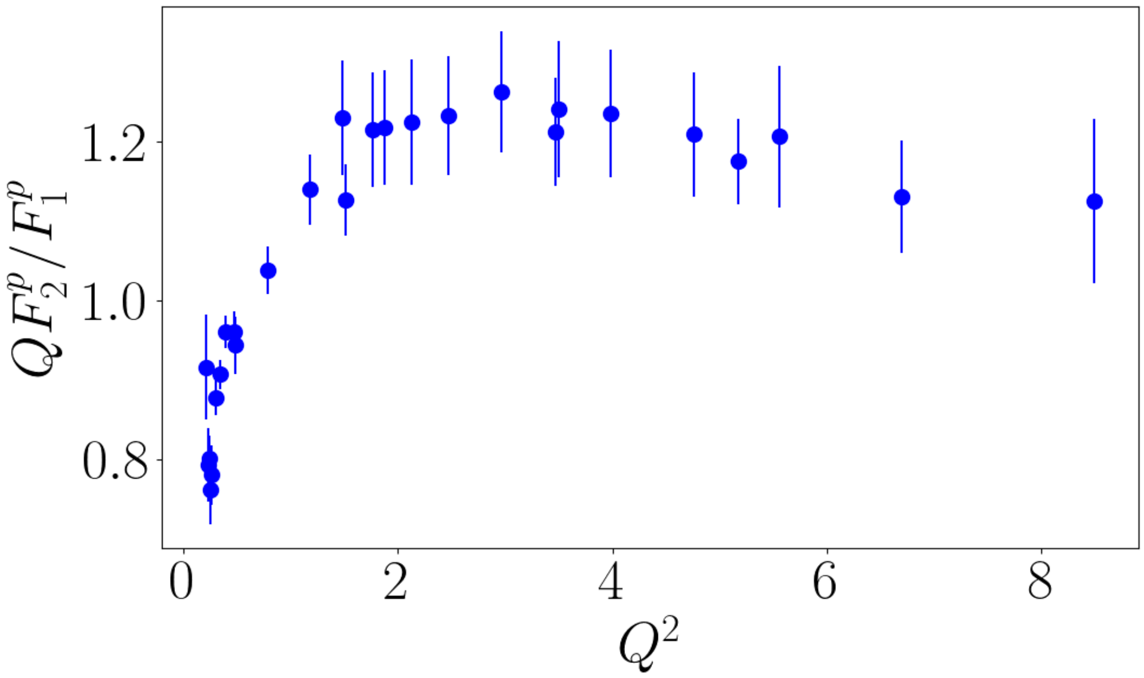
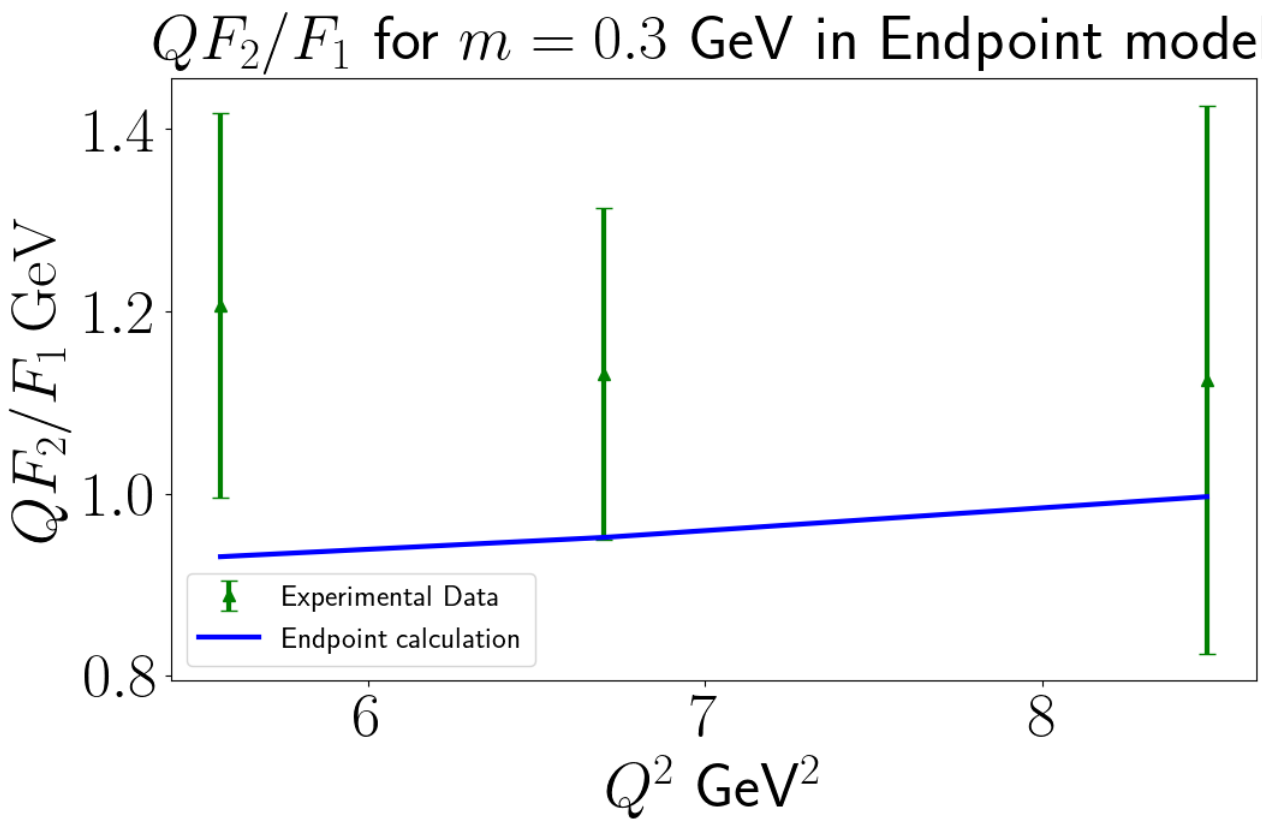
Proton Form Factor ratio F_2/F_1



Jones et al 2000; Punjabi et al 2005; Ron et al 2010; Puckett et al 2012;



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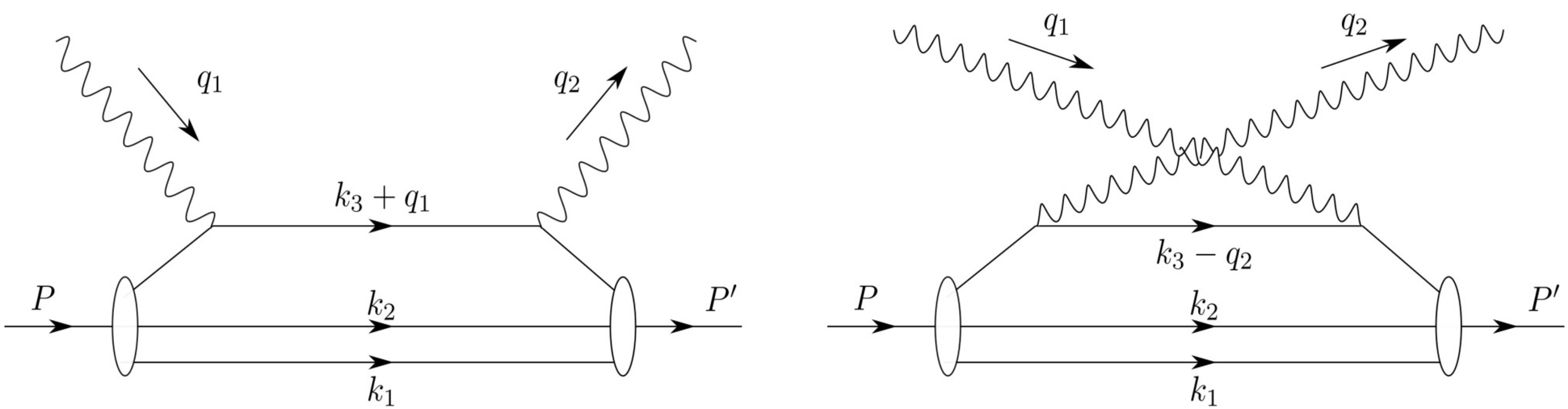
Compton Scattering

- The fitted parameters can now be used to predict real Compton scattering cross section



scaling

- $s \sim |t| >> \Lambda_{QCD}$, fixed θ_{CM}

$$\frac{d\sigma}{dt} \sim \frac{1}{s^6}$$

Not seen in data, which goes close to $1/s^8$

- Fixed s, $s >> |t| >> \Lambda_{QCD}$

$$\frac{d\sigma}{dt} \sim \frac{1}{t^4}$$

in approximate agreement to observations

Compton scattering: model calculations

- Perform a model calculation using parameters extracted from fitting the proton form factors

Fixed s, s>>Q^2

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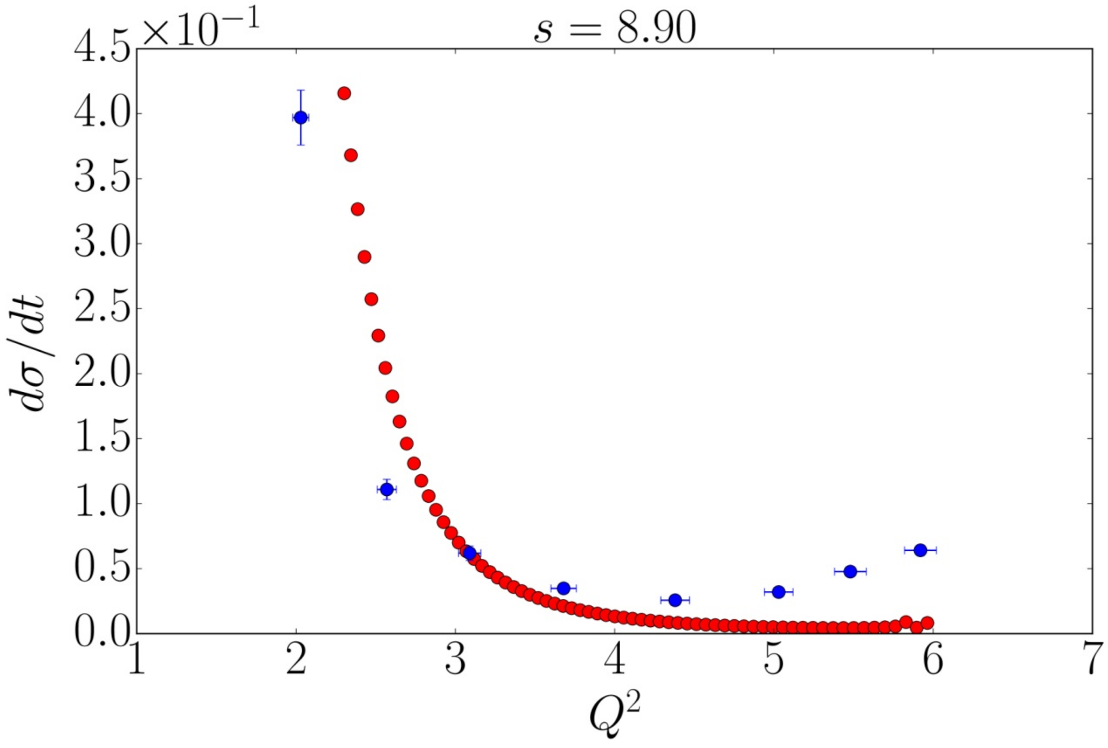
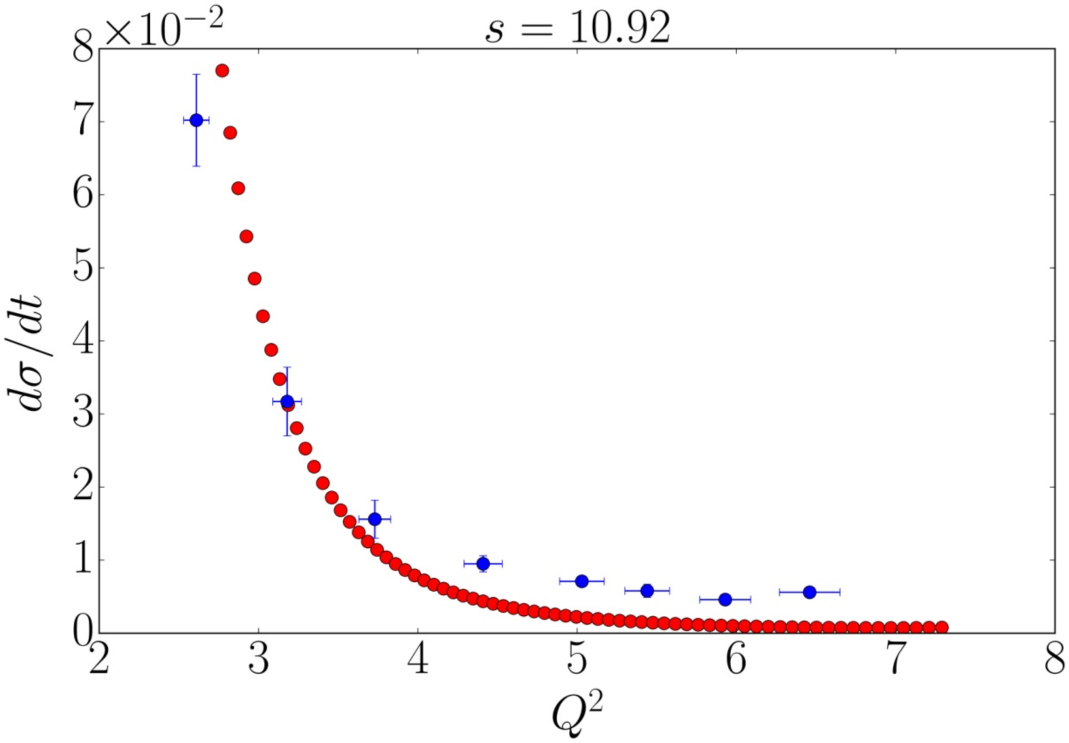


Data from Danagoulian et al 2007

Fixed s, s>>Q^2



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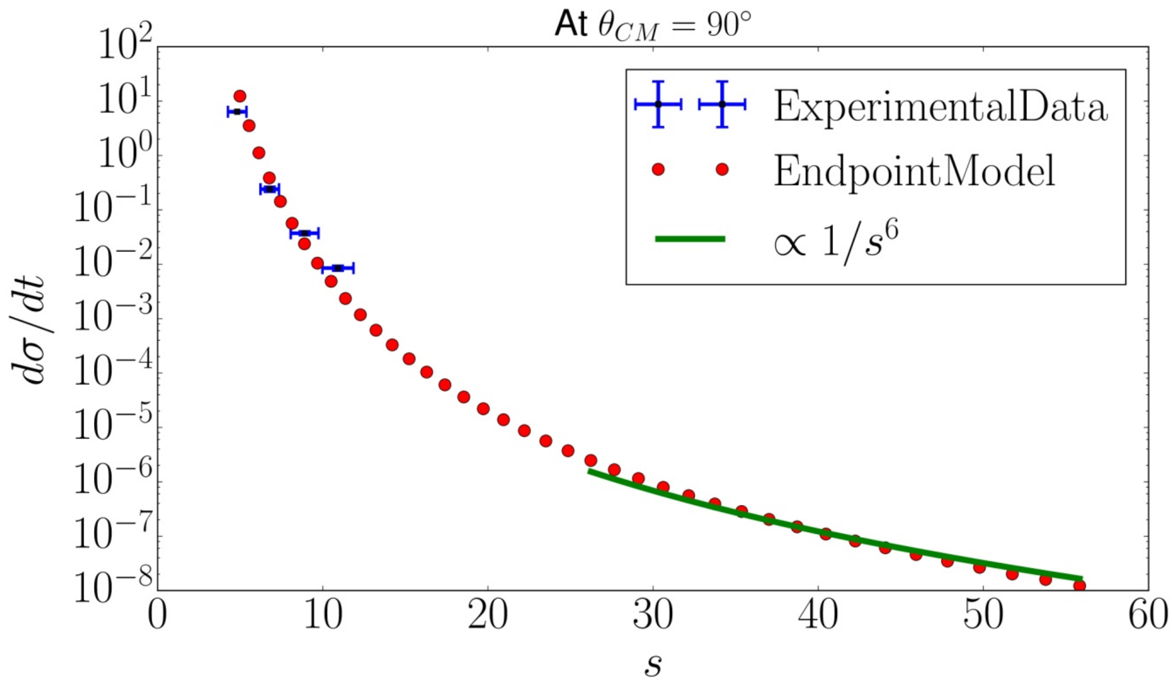
Fixed θ_{CM}



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Data from Danagoulian et al 2007

Compton scattering fixed θ_{CM}

- The model calculation correctly reproduces the observed scaling at low s
- It predicts that asymptotic scaling will set in at $s>25\ GeV^2$

Conclusions

- End point model nicely explains the scaling observed in hard exclusive processes both at

  fixed θ_{CM} ; s, $|t| \gg \Lambda_{QCD}$

  fixed s; $s \gg |t| \gg \Lambda_{QCD}$

- It can also be used to perform explicit model calculations of these processes
- Results for proton form factors and real Compton scattering show reasonable agreement with data