Soft - Collinear Effects in Threshold and Joint Resummation

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Based on

- R. Basu, E. Laenen, AM and P. Motylinslki, Phys. Rev. **76**, 014010 (2007)
- R. Basu, E. Laenen, AM and P. Motylinslki, hep-ph/1204.2503
- Work in progress with W. Benakker, E.Laenen and M. van Beekweld

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3. Threshold Resummation
4. Recoil Resummation
5. Joint Resummation
6. Soft Collinear Effects
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Resummation

What is resummation?

- Organization of large logarithms in perturbative expansion
- Construction from a subset of terms in a finite order perturbative series of an all order expression whose expansion gives at least those terms back

Motivation

- Particle Physics in high energy colliders depends on our ability to calculate cross sections with ever increasing theoretical accuracy
- Achieved by incorporating higher order corrections
- Factorization theorems of perturbative QCD play a key role in calculating these corrections

Factorization

- Cross section for a process involving two hadrons A and B can be factorized into convolutions

$$\sigma = \Sigma_{a,b} \int dx_a \phi_{a/A}(x_a, \mu) \int dx_b \phi_{b/B}(x_b, \mu) \hat{\sigma}_{ab \to FX}$$

  $\phi_{a/A}$:distribution function for parton a with momentum fraction x_a in A
  $\hat{\sigma}$: partonic cross section

- Collinear singularities are factorized order by order and absorbed into pdf's
- $\hat{\sigma}$ is IR safe to all orders(Collins, Soper, Sterman 1988)

- To calculate hadronic cross sections, we need best possible calculation of $\hat{\sigma}$
- $\hat{\sigma}$ can be calculated order by order in perturbation theory

$$\hat{\sigma} = \Sigma c_n \alpha_s^n + R_n$$

c_n's are calculated using Feynman diagrams

- LO: Lowest order usually needs tree graphs
- NLO: Next-to-leading order needs one loop graphs
- Ideally, asymptotic series converges rapidly and N $= 1, 2$ is sufficient
- Sometimes the series contains powers of some numerically large logarithm L

- The perturbation series can then take a form containing
- Single logs

$$\hat{\sigma} = \sigma_0 + \alpha_s(L + \sigma_1) + \alpha_s^2(L^2 + L + \sigma_2) + \cdot\cdot$$

- Double logs

$$\hat{\sigma} = \sigma_0 + \alpha_s(L^2 + L + \sigma_1) + \alpha_s^2(L^4 + L^3 + L^2 + L + \sigma_2) + \cdot\cdot$$

- Effective expansion parameter is $\alpha_s L$ or $\alpha_s^2 L$

Resummation of large logs

Resummation is

- technology that organizes these logs in perturbative expansions
-

$$\hat{\sigma} = \hat{\sigma}_0 exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdot\cdot \right] C(\alpha_s)$$

$g_1, g_2, \cdot\cdot$ are computable functions and lead to leading log (LL), next-to-leading-log(NLL) .....accuracy

What are these large logs?

- Depends on the observable in question
- For thrust T distribution $A = 1 - T$
- For $\frac{d\sigma(p\bar{p}\to Z+X)}{dp_T^Z}$, $A = \frac{M_Z}{p_T^Z}$
- $L^2 = ln^2(\frac{p_T^2}{M_Z^2})$ in Z boson production are called Recoil Logs
- Argument of recoil log is visible
- There can be invisible logs also in perturbation theory

- For example, for electroweak annihilation cross section, $\hat{\sigma}$ includes distributions of the kind $\alpha_s^n \left[\frac{ln^{(2n-1)}(1-z)}{(1-z)} \right]_+$ where $z = \frac{Q^2}{s}$
- These are singular at partonic threshold $\hat{s} = Q^2$, when a and b have just enough invariant mass to produce the observed final state
- For DY ($p\bar{p} \to \gamma^*(Q) + X$), $A = 1 - \frac{Q^2}{s}$, where $s = x_1 x_2 S$ is partonic cm energy Threshold double logs of the kind $L^2 = ln^2(1 - \frac{Q^2}{s})$ appear
- For heavy quark production $A = 1 - \frac{4m^2}{x_1 x_2 S}$
- Threshold resummation reorganizes the distributions containing such logs for a large class of cross sections

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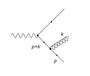
- Why Resum ?
- When L is numerically large, so that even for small α_s, the convergent behaviour of the series is endangered, resummation of the problematic terms into an anlaytic form
  a) restores the predictive power
  b) increases theoretical accuracy

Origin of large logs

From where do the large logs come?

- IR divergences cancel between real and virtual corrections
- For virtual corrections, integration is over all energies
- For real corrections, kinematics determines the phase space
- Cancellation is complete only for completely inclusive cross sections
- Near partonic threshold phase space available for gluon radiation vanishes, leading to large logarithmic corrections to partonic cross section which need to be resummed

Origin of double logs

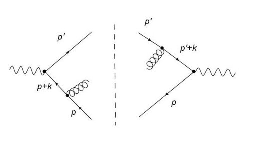


- propagator with $p^2 = k^2 = 0$. Singularities: $E_{\rm g} = 0 \longrightarrow$ *soft*; $\theta_{\rm qg} = 0 \longrightarrow$ *collinear*.

$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_{\rm g}E_{\rm q}(1-\cos\theta_{\rm qg})}\,.$$

- phase space integration

$$\alpha_s \int \frac{d^4k}{(2\pi)^4}\,\frac{p\cdot p'}{p\cdot k\, p'\cdot k} \sim \alpha_s \int \frac{dE_{\rm g}}{E_{\rm g}} \int \frac{d\theta_{\rm qg}}{\theta_{\rm qg}} \sim \alpha_s \ln^2(\ldots)\,.$$



Factorization and Resummation

- Factorization Theorems→ a factorization of degrees of freedom

$$\sigma = J(col) \times J(col) \times S(Soft) \times G(off - shell)$$

- Factorization leads to resummation: an example(UV case)

$$G_B(p, \Lambda, g_B) = Z(\frac{\mu}{\Lambda}, g(\mu))G_R(\frac{p}{\mu}, g_R(\mu))$$

- This implies

$$\frac{d}{d\mu} lnG_R(\frac{p}{\mu}, g_R(\mu)) = -\frac{d}{d\mu} lnZ(\frac{\mu}{\Lambda}) = \gamma(g_R(\mu))$$

- which leads

$$G_R(\frac{p}{\mu}, g_R(\mu)) = G_R(1, g_R(p))exp\left[\int_p^\mu \frac{d\lambda}{\lambda}\gamma(g_R(\lambda))\right]$$

- One can do something analogous to IR + Collinear case above leading to exponentiation of large logs (Collins, Soper , Sterman)

Prompt Photon Production

The process of interest

$$H_A + H_B \to \gamma + X$$

The lowest order QCD processes producing the prompt photon at

partonic cm energy $\sqrt{s}$ are

$$q(p_a) + \bar{q}(p_b) \to \gamma(p_c) + g(p_d)\,, g(p_a) + q(p_b) \to \gamma(p_c) + q(p_d)\,.$$

Pointlike coupling of photon to quark provides a clean em probe of QCD hard scattering

Threshold Resummation For Prompt Photon Production

In perturbative QCD, rapidity integrated cross section at fixed E_T is given by the following factorised formula

$$\begin{aligned}\frac{d\sigma_\gamma(x_T, E_T)}{dE_T} &= \frac{1}{E_T^3}\sum_{a,b}\int_0^1 dx_1\ f_{a/H_1}(x_1,\mu_F^2)\int_0^1 dx_2\ f_{b/H_2}(x_2,\mu_F^2)\\ &\cdot \int_0^1 dx\left\{\delta\left(x-\frac{x_T}{\sqrt{x_1x_2}}\right)\hat\sigma_{ab\to c\gamma}(x,\alpha_s(\mu^2);E_T^2,\mu^2,\mu_F^2,\mu_f^2)\right.\\ &+ \sum_c\int_0^1 dz\ z^2\ d_{c/\gamma}(z,\mu_f^2)\\ &\cdot \left.\delta\left(x-\frac{x_T}{z\sqrt{x_1x_2}}\right)\hat\sigma_{ab\to c}(x,\alpha_s(\mu^2);E_T^2,\mu^2,\mu_F^2,\mu_f^2)\right\}\end{aligned}$$

where $x_T = 2\frac{E_T}{\sqrt{S}}$ (Direct + Fragmentation terms)

The partonic cross sections $\hat{\sigma}_{ab \to cd}$ and $\hat{\sigma}_{ab \to c\gamma}$ are computable in QCD perturbation theory

$$\hat{\sigma}_{ab \to c\gamma} = \alpha \alpha_s [\hat{\sigma}^{(0)}{}_{ab \to c\gamma(x)} + \Sigma \alpha_s^n (\mu^2) \hat{\sigma}^n_{ab \to c\gamma}]$$

$$\hat{\sigma}_{ab \to cd} = \alpha \alpha_s [\hat{\sigma}^{(0)}_{ab \to cd(x)} + \Sigma \alpha_s^n (\mu^2) \hat{\sigma}^n_{ab \to cd}]$$

$\hat{\sigma}$ has been computed to NLO in perturbation theory.
Discrepancy between theory and data needs explanation
Possible remedy: Include resummation effects

Threshold Resummation

- Near partonic threshold, $E_T^{\gamma} \to \sqrt{x_1 x_2 S}$
- $\hat{\sigma}^{(n)}(x) \sim \ln^{2n}(1-x)$ due to soft gluon radiation leading to corrections to $\frac{d\hat{\sigma}}{dp_T}$ as large as $\alpha_s^k ln^{2k}(1-\hat{x}_T^2)\hat{\sigma}^{Born}$ at order α_s^k in perturbation theory
- Threshold resummation organizes this singular(but integrable) behaviour of cross sections to all orders in perturbation theory

Threshold resummation is performed by going over to Mellin-transform space or N-space

$$\sigma_{\gamma,N}(E_T) = \int_0^1 dx_T^2 (x_T^2)^{N-1} E_T^3 \frac{d\sigma_\gamma(x_T, E_T)}{dE_T}$$

Mellin moments of plus distributions give rise to powers of lnN in the Mellin N-moment expressions
In Mellin space

- Convolutions $\Rightarrow$Ordinary products
- $limit(\hat{x}_T^2 \to 1) \Rightarrow limit(N \to \infty)$
- $[\frac{ln^n(1-x)}{1-x}]_+ \Rightarrow lnN$

Convolution in factorized cross section is converted to ordinary products in Mellin space leading to simple factorised form

$$
\begin{aligned}
\sigma_{\gamma,\,N}(E_T) \quad &= \quad \sum_{a,b} f_{a/H_1,\,N+1}(\mu_F^2)\; f_{b/H_2,\,N+1}(\mu_F^2) \\
&\cdot \quad \Big\{ \hat{\sigma}_{ab\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) \\
&+ \quad \sum_c \hat{\sigma}_{ab\to c,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2)\; d_{c/\gamma,\,2N+3}(\mu_f^2) \Big\} \quad ,
\end{aligned}
$$

in terms of the moments of each of the functions.

- Leading soft gluon correction terms $\sim \alpha_s^k ln^{2k} N$
- Leading log(LL): $\alpha_s^k ln^{2k} N$
- Next to leading log(NLL): $\alpha_s^k ln^{2k-1} N$
- Threshold resummation $\Rightarrow$ Exponentiation of these logarithmic corrections

All-order resummed expressions

All order resummed expressions (log-enhanced threshold contributions)

$$\begin{aligned}\hat{\sigma}^{(\mathrm{res})}_{q\bar{q}\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) &= \alpha\, \alpha_s(\mu^2)\, \hat{\sigma}^{(0)}_{q\bar{q}\to g\gamma,\,N} \\ & \quad C_{q\bar{q}\to\gamma}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) \\ & \quad \cdot\ \Delta^{q\bar{q}\to g\gamma}_{N+1}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2)\ ,\end{aligned}$$

and similar expressions for $\hat{\sigma}^{(\mathrm{res})}_{q\bar{g}\to\gamma,\,N}$ and $\hat{\sigma}^{(\mathrm{res})}_{\bar{q}g\to\gamma,\,N}$

$C_{ab\to\gamma} \Rightarrow$ N independent and hence constant for N large

$\Delta^{q\bar{q}\to g\gamma}_{N+1} \Rightarrow$ Radiative factors $\Rightarrow$ ln N dependence fully embodied in these.

The Radiative factors

Depend on the flavour of the partons a, b, d of the LO process
They have an exponential form

$$\begin{aligned}
\Delta_N^{ab\to d\gamma}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) &= \Delta_N^a(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) \\
&\cdot \Delta_N^b(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) \\
&\cdot J_N^d(\alpha_s(\mu^2), Q^2/\mu^2) \\
&\cdot \Delta_N^{(\mathrm{int})\, ab\to d\gamma}(\alpha_s(\mu^2), Q^2/\mu^2) \ .
\end{aligned}$$

$$ln\Delta_N^a = lnNh_a^{(1)}(\lambda) + h_a^{(2)}(\lambda)$$

$$h_a^{(1)} = \frac{A_a^{(1)}}{2\pi b_0)\lambda}[2\lambda + (1-2\lambda)ln(1-2\lambda)]$$

with

$$\lambda = b_0\alpha_s(\mu^2)lnN$$

- $h_a^{(1)}(\lambda)$ and $h_a^{(2)}(\lambda)$do not depend separately on α_s and but are functions of expansion variable $b_0\alpha_s(\mu^2)lnN$.
- Thus all DL terms $c_{n,2n}\alpha_s^n ln^{2n}N$ terms in $\hat{\sigma}_N^{(n)}$ are produced by exponentiating LO contribution $c_{1,2}\alpha_s ln^2 N$.
  $h_a^{(1)}(\lambda)$ – LL exponent
  $h_a^{(2)}(\lambda)$ – NLL exponent
  Calculated by Catani etal 1999

Recoil Effects

- Another source of higher order corrections - Recoil effects
- A soft gluon may be emitted before the hard scattering
- Outgoing pair recoils against soft gluon
- $\hat{\sigma}$ in $\frac{d\sigma_{AB\to\gamma X}}{d^2Q_T dp_T}$ is singular upto $\alpha_s^n[\frac{1}{Q_T^2} ln^{(2n-1)}(\frac{Q_T^2}{Q^2})]_+$.Here, Q_T is the transverse momentum of γq pair in Compton scattering for example.
  Q_T is small compared to $Q = \frac{2p_T}{\hat{x}_T}$
  .

- Due to transverse degree of freedom of the partons, the resummation of double logarithms is performed in impact parameter b space which is conjugate to k_T space..
- In impact parameter space logs to be resummed are $ln(s^2b^2)$
$ln^2(\frac{Q_T^2}{Q^2}) \Rightarrow ln(s^2b^2)$
- Derive the Sudakov factor, Fouriour transform it back to the k_T space and convolute it with the NLO formula for direct photon production.
- Logs show up in $\frac{d\sigma_{AB\to\gamma X}}{d^2Q_T dp_T}$ and are resummed before Q_T integration .
- Approach based on parton densities unintegrated over parton transverse momentum $f(x, k_T)$ which reduce to $f(x)$ on k_T integration.

- Li99: possibility of simultaneous resummation in both threshold and transverse momentum logs?
- In Q_T resummation, the double logs, $ln^2(p^+b)$ are organized in Sudakov factor $exp[-3\int_{\frac{1}{b}}^{xp^+}\frac{dp}{p}\int_{\frac{1}{b}}^{p}\frac{d\mu}{\mu}\gamma_K(\alpha_s(\mu)]$
- In threshold resummation, double logs in $ln^2(\frac{1}{N})$ are organised in the exponent as $exp[-2\int_0^1 dz\frac{z^{N-1}-1}{1-z}\int_{1-z}^{1}\frac{d\lambda}{\lambda}\gamma_K(\alpha_s(\lambda p^+)]$
- In Colins-Soper formalism, the two different types of logs are summed by choosing appropriate cutoffs in the evaluation of soft gluon corrections.
- If transverse degree of freedom of the partons are included, $\frac{1}{b}$ will serve as IR cutoff giving rise to $ln^2(p^+b)$
- In endpoint region, $z \to 1$, we keep longitudinal cutoff $(1-z)p^+$or $\frac{p^+}{N}$ in moment space and integrate out the transverse degrees of freedom and get $ln^2(\frac{1}{N})$.

- If $(p^+b) >> N$, neglect longitudinal cutoff and sum $ln^2(p^+b)$
- If $N >> p^+b$, neglect transverse cutoff and sum $ln^2\frac{1}{N}$
- Li1998: Retain both longitudinal and transverse cutoff simultaneously
- Joint resummation of both kinds of logarithms

Joint Resummation

Proposed by Laenen, Sterman, Vogelsang 2000
Generalize threshold resummation to include recoil effects.

- Based on generalization of threshold resummation
- Reoganizes logs within collinear factorization - standard parton distributions are used.
- Derived from refactorization of partonic cross sections
- Reproduces threshold resummation when recoil effects are neglected
- Reproduces Q_T resummation at low Q_T when threshold logs are supressed.

JR expression for resummed cross section is

$$p_T^3 \frac{d\sigma_{AB\to\gamma}}{dp_T} = \sum_{ij} \frac{p_T^4}{8\pi S^2} \int_{\mathcal{C}} \frac{dN}{2\pi i} \, \tilde{\phi}_{i/A}(N) \tilde{\phi}_{j/B}(N) \int_0^1 d\tilde{x}_T^2 \left(\tilde{x}_T^2\right)^N \frac{|M_{ij}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}}$$

$$C_\delta^{(ij\to\gamma k)}(\alpha_s, \tilde{x}_T^2) \int \frac{d^2\mathbf{Q}_T}{(2\pi)^2} \, \Theta\left(\bar{\mu} - Q_T\right) \left(\frac{S}{4\mathbf{p}_T'^2}\right)^{N+1} P_{ij}(N, \mathbf{Q}_T, Q)$$

$\tilde{x}_T^2 = 4|\mathbf{p}_T - \mathbf{Q}_T/2|^2/\hat{s}$,
$\tilde{\phi}_{i/A}(N)$ Mellin moments of the parton distributions,
$|M_{ij}|^2$ the Born amplitudes,
$\bar{\mu}$ a cut-off restricting $\mathbf{Q}_T$ to sufficiently small values for resummation to be relevant.
$C_\delta^{(ij\to\gamma k)}$:infrared safe coefficient functions, which include short-distance dynamics at the scale Q.

$P_{ij}(N, \mathbf{Q}_T, Q)$:Profile function is a profile of $\mathbf{Q}_T$ -dependence for fixed N.

$$P_{ij}(N, \mathbf{Q}_T, Q) = \int d^2\mathbf{b} e^{-i\mathbf{b}\cdot\mathbf{Q}_T} e^{E_{ij\to\gamma k}}$$

where $e^{E_{ij\to\gamma k}}$ is the perturbative exponent given by

$$\begin{aligned} E^{\mathrm{PT}}_{ij\to\gamma k}(N, b, Q, \mu, \mu_F) = E^{\mathrm{PT}}_i(N, b, Q, \mu, \mu_F) + E^{\mathrm{PT}}_j(N, b, Q, \mu, \mu_F) \\ + F_k(N, Q, \mu) + G_{ijk}(N, \mu)\,. \end{aligned}$$

Integral form of the initial state NLL exponent can be written as

$$E_i^{PT}(N, b, Q, \mu, \mu_F) = -\int_{\frac{Q^2}{\chi^2}}^{Q^2} \frac{dk_T^2}{k_T^2}[A_i(\alpha_s(k_T)) ln(\frac{Q}{\tilde{N}k_T})] \\ -2ln\tilde{N}\int_{\mu_F^2}^{Q^2} \frac{dk_T^2}{k_T^2} A_i(\alpha_s(k_T))$$

The function $\chi(N, b)$ defines the N- and b-dependent scale of soft gluons to be included in the resummation, and is chosen as

$$\chi(N, b) = \tilde{b} + \frac{\tilde{N}}{1 + \frac{\eta\tilde{b}}{\tilde{N}}},$$

$N \to \infty, b = 0$ reproduces threshold resummation.
$b \to \infty, N = 0$ reproduces recoil resummation.

LL and NLL exponents can be written in terms of functions $h^{(1)}$ and $h^{(2)}$ as before where now

$$h_a^{(1)}(\lambda, \beta) = \frac{A_a^{(1)}}{2\pi b_0 \beta}[2\beta + (1-2\lambda) ln(1-2\beta)]$$

etc, with
$\lambda = b_0 \alpha_s(\mu) ln(\tilde{N})$
$\beta = b_0 \alpha_s(\mu) ln(\chi)$
Preliminary analysis based on this approach showed substantial effects for prompt photon production.

Soft - Collinear Effects

Another important class of potentially large terms are of the form

$$\alpha_s^i \sum_j^{2i-1} d_{ij} \frac{\ln^j N}{N} .$$

which have soft-collinear origin.
Two kinds of sources for such terms

- The singular plus distributions $[\ln^{2j-1}(1-z)/(1-z)]_+$, which can be included by keeping the subleading terms in Mellin transform of plus distributions
- For example

$$\int_0^1 dz z^N \left[\frac{ln(1-z)}{1-z}\right]_+ = \frac{1}{2} ln^2 N - \frac{1}{2}(lnN+1)\frac{1}{N} + \cdot$$

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- The singular but integrable $\ln^{2j-1}(1-z)$.
  Have purely collinear origin
- Can be incorporated by including the regular part of Altarelli -Parisi splitting function
  Kramer, Laenen,Spira1997, Catani etal1998

Effect incorporated in threshold resummation by the replacement

$$\frac{z^{N-1}-1}{1-z}A_i^{(1)} \to \left[\frac{z^{N-1}-1}{1-z} - p_i z^{N-1}\right] A_i^{(1)} + \mathcal{O}(\frac{1}{N^2})\,,$$

in each of the radiative factors
($p_q = 1, p_g = 2$),
The replacement is equivalent to exchanging at order j one soft collinear gluon (corresponding to one factor $\alpha_s \ln^2 N$) for a hard-collinear one (corresponding to a factor $\alpha_s \ln N/N$

$$\alpha_s^k \ln^{2k} N \to \alpha_s^k \frac{\ln^{2k-1} N}{N}\,.$$

E Laenen, S. Majhi, AM and R. Basu, 2004

The extra term can be cast in a convenient form using

$$z^{N-1} = \frac{z^{N-1} - 1 - (z^N - 1)}{1 - z}$$

and the replacement

$$z^{N-1} - 1 \to \theta(1 - z - \frac{1}{N})$$

leading to leading $\frac{lnN}{N}$ terms

$$f_q' = \frac{A^{(1)_q}}{2_0} exp\left(- \frac{\lambda}{\alpha_s b_0} \right) [ln(1 - 2\lambda) - ln(1 - \lambda)]$$

$$f_g' = \frac{3A^{(1)_g}}{2_0} exp\left(- \frac{\lambda}{\alpha_s b_0} \right) [ln(1 - 2\lambda) - ln(1 - \lambda)]$$

$\frac{lnN}{N}$ terms in JR

The initial state related $\alpha_s^k \ln^{2k-1} N/N$ terms can also be generated in the context of joint resummation by extending evolution of parton densities to a soft scale.
Based on the observation that in JR resummed expression

$$E_i^{\mathrm{PT}}(N, b, Q, \mu, \mu_F) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left\{ A_i\left((k_T)\right) \ln\left(\frac{Q}{k_T}\right) + B_i\left((k_T)\right) \right\} + \int_{\mu_F^2}^{Q^2/\chi^2} \frac{dk_T^2}{k_T^2} \left\{ -\ln \bar{N} A_i\left((k_T)\right) - B_i\left((k_T)\right) \right\} .$$

the second term represents flavor-conserving evolution to NLL accuracy (the integrand consists of the $\ln N$ and constant terms for the anomalous dimension matrix $\gamma_{i/j}(N)$ for $j = i$) from the hard scale μ_F to the soft scale Q/χ.

The first term in this expression leads to

$$E_i^{\mathrm{PT}}(N,b,Q,\mu) = \frac{1}{\alpha_s(\mu)} h_i^{(0)}(\beta) + h_i^{(1)}(\beta,Q,\mu))$$

In second term replace

$$(-A_i(\alpha_s)\ln(N) - B_i(\alpha_s)) \longrightarrow \gamma_{i/i}(N)(\alpha_s)$$

This includes the leading, flavor-diagonal $\ln N/N$ effects generated by the k_T integral (the $1/N$ part of $\gamma_{i/i}$ combines with the $\ln N$ terms). Can also include the off-diagonal contributions via the replacement

$$\delta_{ig}\ \exp\left[\frac{-A_g^{(1)}\ln\bar{N} - B_g^{(1)}}{2\pi b_0}\, s(\beta)\right]\ f_{g/H}(N,\mu_F) \longrightarrow\ \mathcal{E}_{ik}\left(N,Q/\chi,\mu_F\right)\ f_{k/H}(N,\mu_F)\,.$$

where $s(\beta) = \ln(1-2\beta)$ plus NLL corrections.

As a result, we can replace the combination

$$f_{i/A}(\mu_F, N) f_{j/B}(\mu_F, N) \exp\left[E_i^{\mathrm{PT}}(N, b, Q, \mu, \mu_F) + E_j^{\mathrm{PT}}(N, b, Q, \mu, \mu_F)\right]$$

by

$$\mathcal{C}_{i/A}(Q, b, N)\, \mathcal{C}_{j/B}(Q, b, N)\ \exp\left[E_i^{\mathrm{PT}}(N, b, \mu, Q) + E_j^{\mathrm{PT}}(N, b, \mu, Q)\right]$$

where

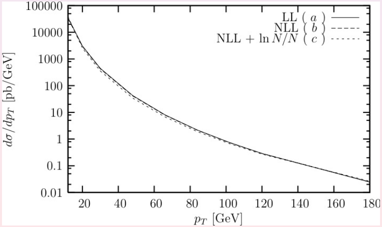
$$\mathcal{C}_{i/H}(Q, b, N) = \sum_k \mathcal{E}_{ik}\left(N, Q/\chi, \mu_F\right)\, f_{k/H}(N, \mu_F)\ .$$

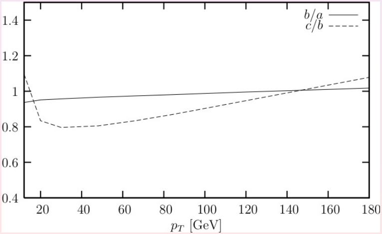
The matrix $\mathcal{E}$ implements evolution from scale μ_F to scale Q/χ, and is normalized to be the unit matrix if these two scales are equal.

Results : Effect of lnN/N- Direct process only

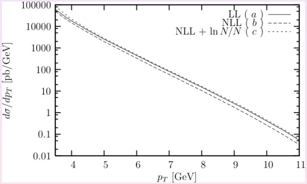
R.Basu, E.Laenen,AM,P.Motylinski 2007: Effect of including $\frac{lnN}{N}$ corrections in case of prompt photon - Direct process JR with and

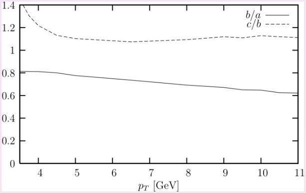
without $\frac{lnN}{N}$ *contribution*(*Tevatron*)



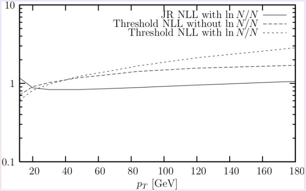


JR with and without $\frac{lnN}{N} contribution(E706)$

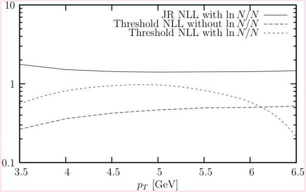
vspace0.5cm 



Threshold vs JR(Tevatron)



Threshold vs JR(E706)



Fragmentation contribution

- Apart from the direct partonic sub processes, there are contributions from $2 \to 2$ hard scattering processes also in which one of the final state partons fragments into photon

$$q(p_a) + \bar{q}(p_b) \to q_(p_c) + \bar{q}(p_d) \; ,$$

where the parton c subsequently fragments into a photon.

- Fagmentation component also contributes at $O(\alpha\alpha_s)$, expected to contribute to the cross section substantially (deFlorian et al 2005).
- Taking into account the fragmentation component, the p_T distribution of prompt photons in hadronic collisions is

$$\frac{p_T^3 d\sigma^{(\mathrm{resum})}_{AB\to\gamma+X}}{dp_T} = \frac{p_T^3 d\sigma^{(\mathrm{direct})}_{AB\to\gamma+X}}{dp_T} + \frac{p_T^3 d\sigma^{(\mathrm{frag})}_{AB\to\gamma+X}}{dp_T}$$

- The fragmentation component is given by (Laenen, Sterman, Vogensang 2000)

$$\frac{p_T^3 d\sigma^{(\mathrm{frag})}_{AB\to\gamma+X}}{dp_T} = \sum_{abc} \frac{p_T^4}{8\pi S^2} \int_{\mathcal{C}} \frac{dN}{2\pi i}\, f_{a/A}(N,\mu_F) f_{b/B}(N,\mu_F) D_{\gamma/c}(2N+3,\mu_F^2)$$
$$\times \int_0^1 d\tilde{x}_T^2 \left(\tilde{x}_T^2\right)^N \frac{|M_{ab\to cd}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}}\; C^{(ab\to cd)}((\mu),\tilde{x}_T^2)$$
$$\times \int \frac{d^2\mathbf{Q}_T}{(2\pi)^2}\, \Theta\left(\bar{\mu}-Q_T\right) \left(\frac{S}{4\mathbf{p}_T'^{\,2}}\right)^{N+1}$$
$$\times \int d^2\mathbf{b}\, \mathrm{e}^{i\mathbf{b}\cdot\mathbf{Q}_T}\, \exp\left[E_{ab\to cd}\left(N, b, \frac{4p_T^2}{\tilde{x}_T^2}, \mu_F\right)\right]$$

- Rewrite

$$\frac{p_T^3 d\sigma^{(\mathrm{frag})}_{AB\to\gamma+X}}{dp_T} = \sum_{abc} \frac{p_T^4}{8\pi S^2} \int_{\mathcal{C}} \frac{dN}{2\pi i}\, f_{a/A}(N,\mu_F) f_{b/B}(N,\mu_F) D_{\gamma/c}(2N+3,\mu_F^2)$$
$$\times \int \frac{d^2\mathbf{Q}_T}{(2\pi)^2}\, \Theta\left(\bar{\mu}-Q_T\right) \left(\frac{S}{4\mathbf{p}_T'^{\,2}}\right)^{N+1} \int d^2\mathbf{b}\, \mathrm{e}^{i\mathbf{b}\cdot\mathbf{Q}_T}\, \Sigma^{(\mathrm{resum})}_{ab\to cd}$$

- $$\Sigma^{(\mathrm{resum})}_{ab\to cd}(N,b) = exp\Big[E_a^{\mathrm{PT}}(N,b,Q,\mu,\mu_F) + E_b^{\mathrm{PT}}(N,b,Q,\mu,\mu_F) + E_c^{\mathrm{PT}}(N,b,Q,\mu,\mu_F) + F_d(N,Q,\mu)\Big] \times Tr\left[\tilde{H}\bar{P}exp\left(\int_{p_T}^{\frac{P_T}{N}} \frac{d\mu'}{\mu'}\Gamma_S^{\dagger}(\alpha_s(\mu'^2))\right) \times \tilde{S}(\alpha_s(\frac{p_T^2}{N^2}))Pexp\left(\int_{p_T}^{\frac{P_T}{N}} \frac{d\mu'}{\mu'}\Gamma_S(\alpha_s(\mu'^2))\right)\right.$$

- $E_a^{\mathrm{PT}}(N,b,Q,\mu,\mu_F)$, $E_b^{\mathrm{PT}}(N,b,Q,\mu,\mu_F)$ and $E_c^{\mathrm{PT}}(N,b,Q,\mu,\mu_F)$ represent the effects of soft gluon radiation collinear to initial partons a and b and the observed final state parton c respectively. $E_d(N,Q,\mu)$ represents the collinear, soft or hard, emission by the non observed parton d.

- Additional radiative factor for the final state parton c which fragments into photon
- Radiative term for fragmenting parton is similar to the initial state parton
- The underlying hard process is $2 \to 2$ scattering involving only partons and hence involves, unlike direct case any color tensor that may be constructed from color representations of the incoming partons. The different color structures may mix due to soft gluon emission hence the radiative factor for wide angle soft radiation is a matrix in color space.
- In a color basis, where the soft anomalous dimension matrix Γ_S is diagonal, the path-ordered exponentials of matrices reduce to a sum of simple exponentials

- Resummed exponent after diagonalization

$$\Sigma^{(\mathrm{resum})}_{ab\to cd}(N-1,b) = C_{ab\to cd}\, exp\Big[E_a^{\mathrm{PT}}(N,b,Q,\mu,\mu_F)$$

$$+E_b^{\mathrm{PT}}(N,b,Q,\mu,\mu_F) + E_c^{\mathrm{PT}}(N,b,Q,\mu,\mu_F) + F_d(N,Q,\mu)\Big]$$

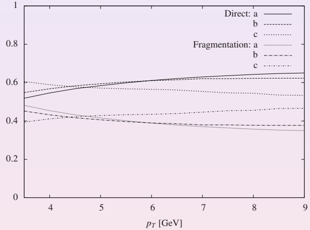
$$\times\left[\sum G^I_{ab\to cd}\, exp\left(\Gamma_{IN}^{(int)ab\to cd}\right)\right]\sigma^{(\mathrm{Born})}_{ab\to cd}(N-1,b)$$

- The sum runs over all possible color configurations I with $G^I_{ab\to cd}$ representing a weight for each color configuration such that $\sum G^I_{ab\to cd} = 1$.

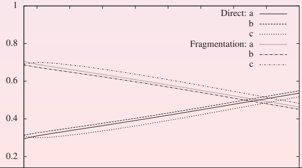
Results : Effect of lnN/N- Direct vs Fragmentation

Rahul Basu, E . Laenen, AM, P. Motylinski, hep-ph 1204.2503

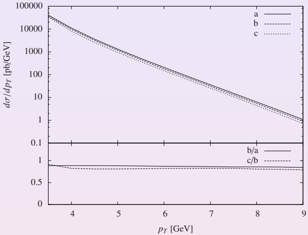
- Direct vs fragmentation : E706 kinematics



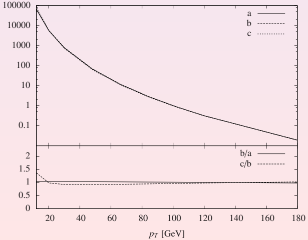
- Direct vs fragmentation : Tevatron



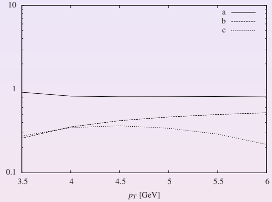
- only LL vs NLL vs lnN/N : E706 kinematics



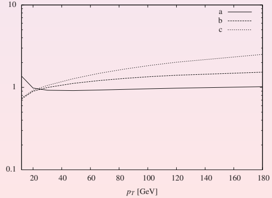
- Fragmentation only LL vs NLL vs lnN/N : Tevatron



- Comparison of JR without lnN/N with JR with lnN/N (a), Threshold without lnN/N (b) and Threshold with lnN/N(c) : E706 kinematics



- Comparison of JR without lnN/N with JR with lnN/N (a), Threshold without lnN/N (b) and Threshold with lnN/N(c) : Tevatron



- Evolution method is tricky due to the singularity at $\mathbf{p_T} = \frac{\mathbf{Q_T}}{2}$ which arises because one is extrapolating the resummation to a region where JR is no longer exact ($Q_T shouldbesmallerthan \frac{p_T}{N}$ for JR)

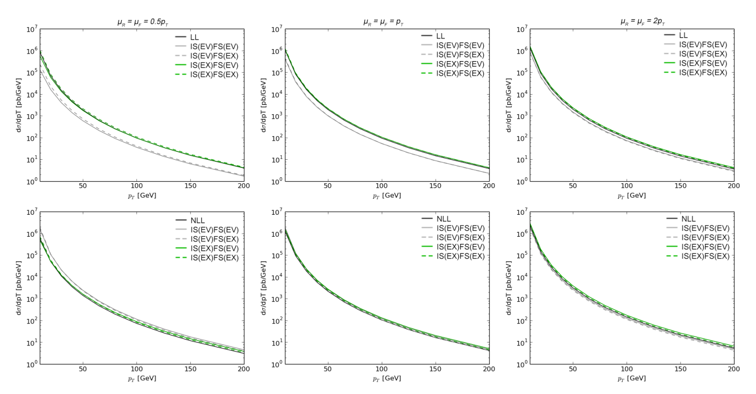
- Singularity dealt with by putting a cutoff on Q_T so the resummed expression is valid in the region of integration

- Alternative way : Approximate the kinematical factor
$$\left(\frac{S}{4(\bar{p}_T - \frac{\bar{Q}_T}{2})^2}\right)^{N+1} = \left(\frac{4p_T^2}{S}\right)^{-N-1}\left(1 - \frac{\bar{p}_T \cdot \bar{Q}_T}{p_T^2} - \cdot\right)^{-N-1}$$
which can be approximated by
$$\left(\frac{S}{4(\bar{p}_T - \frac{\bar{Q}_T}{2})^2}\right)^{N+1} = (x_T^2)^{-N-1} e^{(N+1)\frac{\mathbf{p_T}\cdot\mathbf{Q_T}}{p_T^2}}\left[1 + O\left(\frac{(NQ_T^2)}{p_T^2}\right)\right]$$

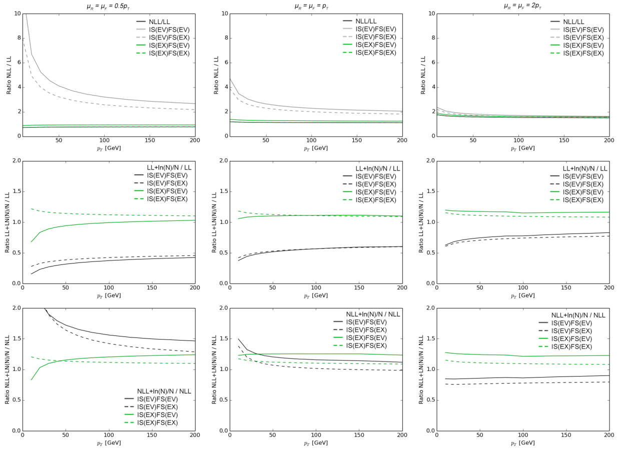
- Under this approximation, one can perform the b integral analytically leading to a delta function thus setting
$\mathbf{b} = -\frac{i(N+1)\mathbf{p_T}}{p_T^2}$ in the resummed exponent

- In preparation with W. Benakker, E.Laenen and M. van Beekweld,

- Comparison of exponentiation and evolution methods including fragmentation in JR with lnN/N at LL and NLL : LHC kinematics :(Preliminary)



- Ratio NLL/LL using exponentiation and evolution methods including fragmentation in JR with lnN/N : LHC



Summary

- Joint resummation extends QCD's predictive power beyond LO, NLO, NNLO. . . ·, Improvements possible in joint as well as threshold resummation through summing purely collinear enhancements ($\ln^i N/N$))
- Effect of including the leading ($\ln N/N$) term in resummed cross section is substantial both for direct and fragmentation processes
- Including fragmentation contribution in lnN/N accuracy, substantial contribution in threshold but small effect in JR - corrections due to recoil effects overshadow soft-collinear effects
- For final state the difference in results obtained using exponentiation and evolution methods is small, but larger initial state exponent
- It will be interesting to analytically show what is the difference between doing an evolution and an exponentiation (Work in progress)
- It may be worthwhile to include sub leading terms of the kind $\ln^i N/N$ and assess their impact

THANK YOU