

# QUARK MASS FUNCTIONS AND PION STRUCTURE IN THE COVARIANT SPECTATOR THEORY

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## COLLABORATORS

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# OUTLINE

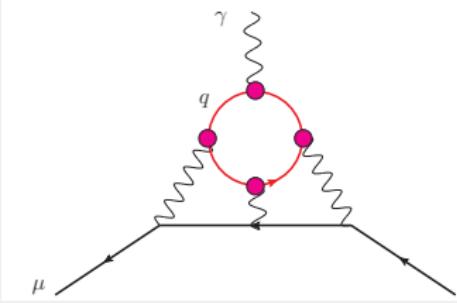
- 1 Motivation and Objectives
- 2 Dynamical quark model in the Covariant Spectator Theory
  - Confinement
  - $S\chi$ SB and  $\pi$ - $\pi$  scattering
- 3 Simplest model
  - Quark mass function
  - $\pi$  electromagnetic form factor
- 4 Conclusions

# $q\bar{q}$ -MESON PHENOMENOLOGY — MOTIVATION

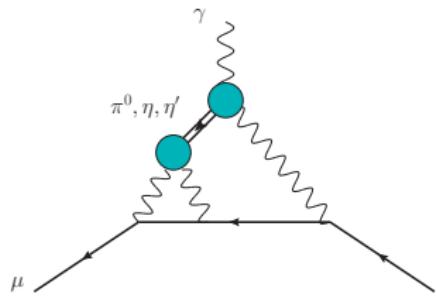
- upcoming experiments, e.g. at JLab (Halls A&D) and FAIR-GSI (Panda)
- need better theoretical understanding of  $q\bar{q}$  mesons
- **currents, form factors** needed in various processes  
e.g. hadronic **light-by-light scattering** in prediction of muon g-2:  
search for new physics



dressed quark **current & propagator**



meson **transition form factors**

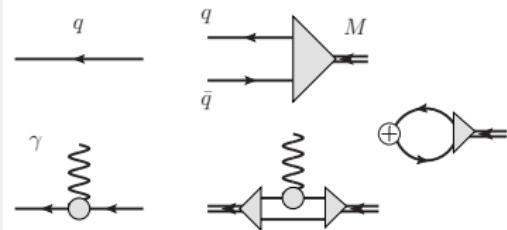


# OBJECTIVES

- find a  $q\bar{q}$  interaction for **all**  $q\bar{q}$  mesons (**unified** description)  
very good description of heavy and heavy-light mesons ✓ (ALFRED STADLER's talk)  
LEITÃO, STADLER, PEÑA, EB, PLB (2017), PRD (2017)  
to do: **light** mesons ✗
- spectrum and decay properties: information about the Lorentz structure of the **confining** interaction
- describe mass-generation mechanism of **dynamical chiral-symmetry breaking**

## Calculation of

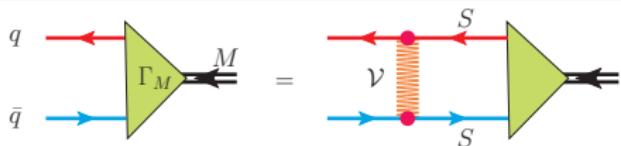
- dynamical quark mass function
- meson spectrum and vertex functions
- quark-photon vertex
- meson form factors
- meson decay properties



# COVARIANT SPECTATOR THEORY FOR $q\bar{q}$ -MESONS

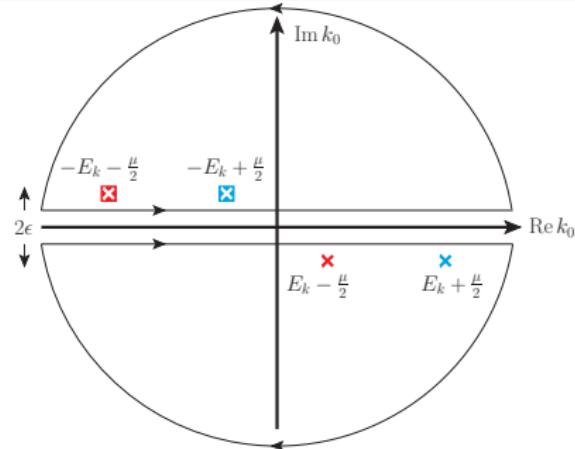
Bethe-Salpeter equation (BSE)

$$\Gamma(p_1, p_2) = i \int_k \mathcal{V}(p, k) S(k_1) \Gamma(k_1, k_2) S(k_2)$$



assume:  $\exists$  real quark-mass poles

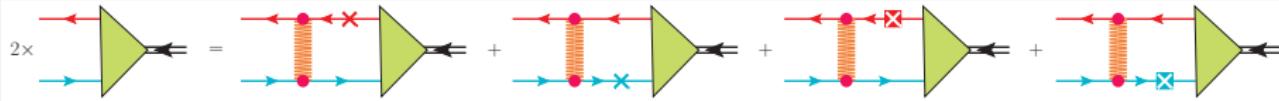
quark poles in complex  $k_0$  plane



(charge-conjugation symmetric) CST-BSE

$$\Gamma(p_1, p_2) = -\frac{1}{2} \int_{\mathbf{k}_1} \mathcal{V}(p, k) \Lambda(\hat{k}_1) \Gamma(\hat{k}_1, k_2) S(k_2) - \frac{1}{2} \int_{\mathbf{k}_2} \mathcal{V}(p, k) S(k_1) \Gamma(k_1, \hat{k}_2) \Lambda(\hat{k}_2) - \dots$$

SAVKLI, GROSS PRC (2001), EB, GROSS, PEÑA, STADLER PRD (2014)



## CST FEATURES

- CST is effective treatment of full BSE; **beyond** ladder-rainbow approximation
- manifestly **covariant** three-dimensional integral equations
- **one-body** (Dirac) and **non-relativistic** (Schrödinger) limits
- solved in **Minkowski** space
- covariant generalization of linear **confinement** plus one-gluon-exchange
- NJL-type mechanism for **dynamical chiral-symmetry breaking**

GROSS, MILANA PRD (1991)

- confining interaction with Lorentz **scalar** structures consistent with **chiral symmetry** and **AVWTI**

EB, PEÑA, RIBEIRO, STADLER, GROSS PRD (2014)

- **lattice QCD** data used to constrain parameters

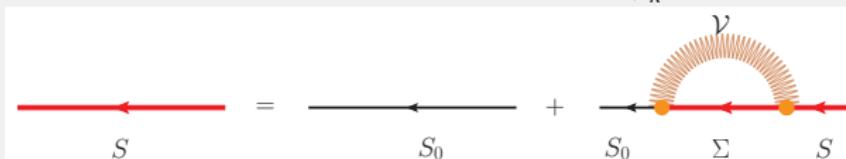
EB, GROSS, PEÑA, STADLER PRD (2014)

# CST DYSON EQUATION

EB, GROSS, PEÑA, STADLER, PRD (2014)

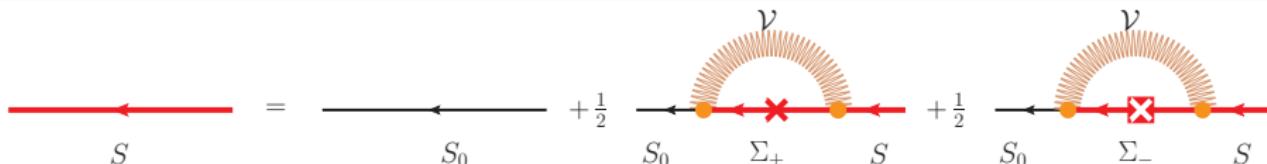
Dyson equation for **dressed quark propagator**

$$S(p) = S_0(p) + S_0(p) i \int_k \overbrace{S(k) \mathcal{V}(p, k) S(p)}^{=\Sigma(p)}$$



CST Dyson equation

$$S(p) = S_0(p) - \frac{1}{2} S_0(p) \int_k \Lambda(\hat{k}) \mathcal{V}(p, \hat{k}) S(p) - \frac{1}{2} S_0(p) \int_k \Lambda(-\hat{k}) \mathcal{V}(p, -\hat{k}) S(p)$$



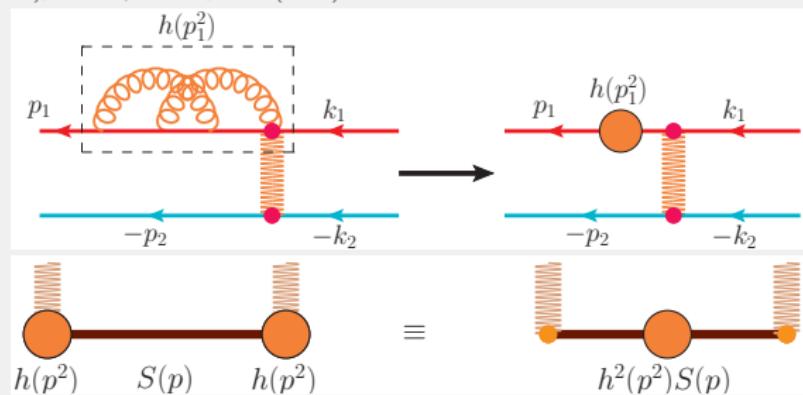
- $S_0(p) = \frac{1}{m_0 - \not{p} - i\epsilon} \rightarrow S(p) = \frac{1}{m_0 + \Sigma(p) - \not{p} - i\epsilon} \equiv \frac{\mathcal{Z}(p^2)}{M(p^2) - \not{p} - i\epsilon} = \frac{\mathcal{Z}(p^2)[M(p^2) + \not{p}]}{M^2(p^2) - p^2 - i\epsilon}$   
 quark self energy  $\Sigma(p) = A(p^2) + \not{p}B(p^2)$   
 dressed quark mass function  $M(p^2) = \frac{A(p^2) + m_0}{1 - B(p^2)}$
- constituent quark mass obtained from pole condition  $m = M(p^2 = m^2)$

# INTERACTION KERNEL: MOMENTUM DEPENDENCE

use strong **quark form factors** for each quark line at vertex:

$$\mathcal{V}(p, k) = h(p_1^2)h(p_2^2)h(k_1^2)h(k_2^2)\mathcal{V}_R(p - k)$$

GROSS, RISKA PRC (1987); SURYA, GROSS, PRC (1996)



- CST 'linear confinement':

$$\int_k \mathcal{V}_L(p, k)\psi(k) = \sigma h(p_1^2)h(p_2^2) \int_k h(m^2) \frac{h(k_2^2)\psi(k) - h(p_{R2}^2)\psi(p_R)}{(p_1 - \hat{k})^4}$$

CST-BSE  $\Rightarrow$  **both quarks cannot be on-shell simultaneously** SAVKLI, GROSS PRC (2001)  
 $\int_k \mathcal{V}_L(p, \hat{k}) = 0$

- one-gluon exchange:  $\mathcal{V}_{RG}(p, \hat{k}) = \frac{\alpha_s}{(p - \hat{k})^2}$

- constant:  $\mathcal{V}_{RC}(p, \hat{k}) = 2C \frac{E_k}{m} \delta^3(\vec{p} - \vec{k})$

# AVWTI AND LORENTZ STRUCTURE OF THE KERNEL

EB, PEÑA, RIBEIRO, STADLER, GROSS PRD (2014)

- consistency with chiral symmetry and its breaking:  
**axial-vector Ward-Takahashi identity (AVWTI)**

$$-i(p_1 - p_2)_\mu \Gamma_R^{5\mu}(p_1, p_2) + 2m_0 \Gamma_R^5(p_1, p_2) \equiv \Gamma_R^A(p_1, p_2) = \tilde{S}^{-1}(p_1)\gamma_5 + \gamma_5 \tilde{S}^{-1}(p_2)$$

- constrains **scalar**, **pseudoscalar** and **tensor** structures of kernel

$$\begin{aligned}\mathcal{V}_R(p-k) = & V_{RL}(p-k) \left[ \lambda_S(\mathbf{1} \otimes \mathbf{1}) + \lambda_S(\gamma^5 \otimes \gamma^5) + \lambda_V(\gamma^\mu \otimes \gamma_\mu) \right. \\ & \left. + \lambda_A(\gamma^5 \gamma^\mu \otimes \gamma^5 \gamma_\mu) + \frac{\lambda_T}{2}(\sigma^{\mu\nu} \otimes \sigma_{\mu\nu}) \right] \\ & + V_{RCG}(p-k) \left[ \kappa_V(\gamma^\mu \otimes \gamma_\mu) + \kappa_A(\gamma^5 \gamma^\mu \otimes \gamma^5 \gamma_\mu) \right]\end{aligned}$$

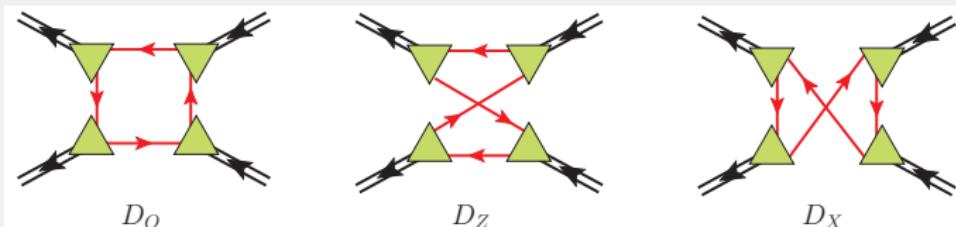
$\Rightarrow$  if  $\mathcal{V}_L$  has **scalar**, it must also have **equally-weighted pseudoscalar structure!**

- 'soft pion' limit  $P \rightarrow 0$ :  $\Gamma_R^A(p, p) \rightarrow \Gamma_R^\pi(p, p) \sim \frac{A(p^2)}{h^2(p^2)} \gamma^5 \Rightarrow \pi$  becomes **massless** ✓

# CHIRAL SYMMETRY AND $\pi$ - $\pi$ SCATTERING

- chiral symmetry  $\Rightarrow$  'Adler consistency-zero':  $\pi$ - $\pi$  scattering amplitude at threshold  $\mathcal{A}^{\pi\pi} \sim \frac{m_\pi^2}{f_\pi^2} \rightarrow 0$  in  $\chi$ -limit  
WEINBERG PRL (1966)

## (lowest order) impulse approximation



$\chi$ -limit and at threshold:  $\mathcal{A}_{\text{impulse}}^{\pi\pi} = D_O + D_Z + D_X \neq 0!$   
 $\Rightarrow$  direct contribution **violates** chiral symmetry  $\times$

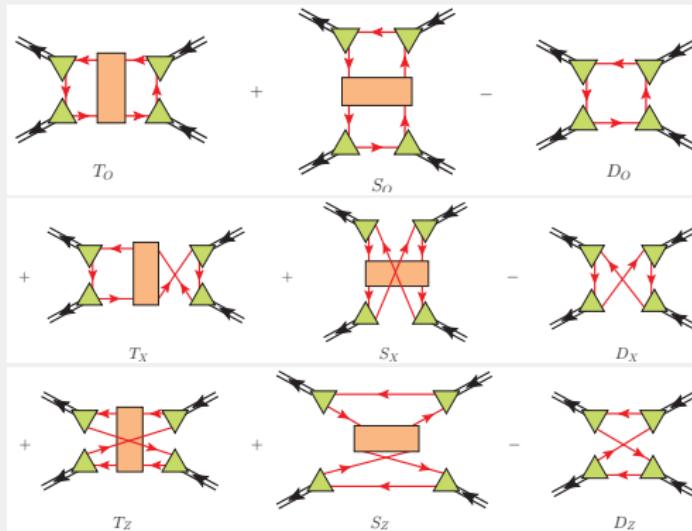
BICUDO ET AL. PRD (2002)

also the case in CST  $\times$

EB, PEÑA, RIBEIRO, STADLER, GROSS PRD (2014)

- to respect chiral symmetry in  $\pi$ - $\pi$  scattering and obtain Adler zero must go **beyond** impulse approximation

Correct description of  $\pi$ - $\pi$  scattering: include **full ladder sum** BICUDO ET AL. PRD (2002)



$\chi$ -limit:

$$T_{O,x,z} + S_{O,x,z} - D_{O,x,z} \rightarrow 0 \quad \checkmark$$

EB, PEÑA, RIBEIRO, STADLER, GROSS PRD  
(2014)

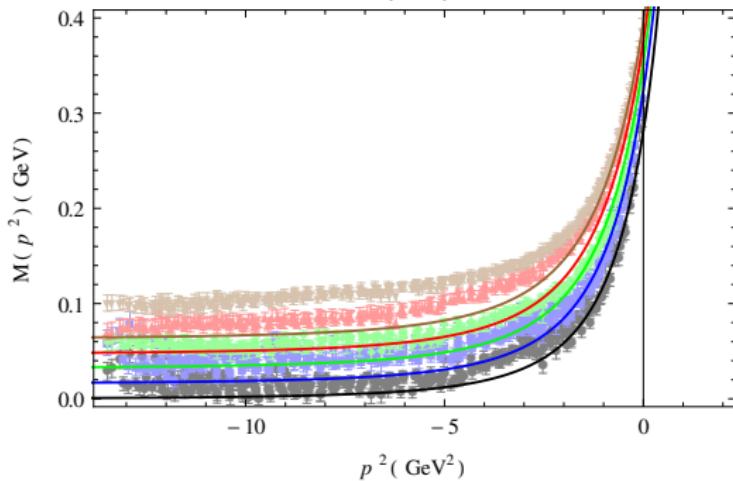


$$\text{because } \int_k V_L(p, \hat{k}) = 0$$

## SIMPLEST MODEL: THE MASS FUNCTION

- $\mathcal{V}_R = \mathcal{V}_{LR} + \mathcal{V}_{CR}$  with  $\mathcal{V}_{LR} = [\mathbf{1} \otimes \mathbf{1} + \gamma_5 \otimes \gamma_5] V_{LR} \Rightarrow \Sigma_L = 0!$   
 $\mathcal{V}_{CR} = [\gamma^\mu \otimes \gamma_\mu] C 2 \frac{E_k}{m} \delta^3(p - k)$  only contributes to  $A(p^2)$
- mass function  $M(p^2) = C h^2(m^2) h^2(p^2) + m_0$  with  $h(p^2) = \left( \frac{\Lambda_x^2 - m_x^2}{\Lambda_x^2 - p^2} \right)^2$
- $m_0 = 0$ : fix  $m_x$  and  $\Lambda_x$  by fit to extrapolated  $\chi$ -limit LQCD data
- $m_0 > 0$ : solve  $M(m^2) = m$  and global fit  $C$  to LQCD data

EB, GROSS, PEÑA, STADLER, PRD (2014) ; lattice data: BOWMAN *et al* PRD (2005)



$m_0$ (GeV)	$m$ (GeV)
0	0.308
0.016	0.363
0.032	0.403
0.047	0.434
0.063	0.462

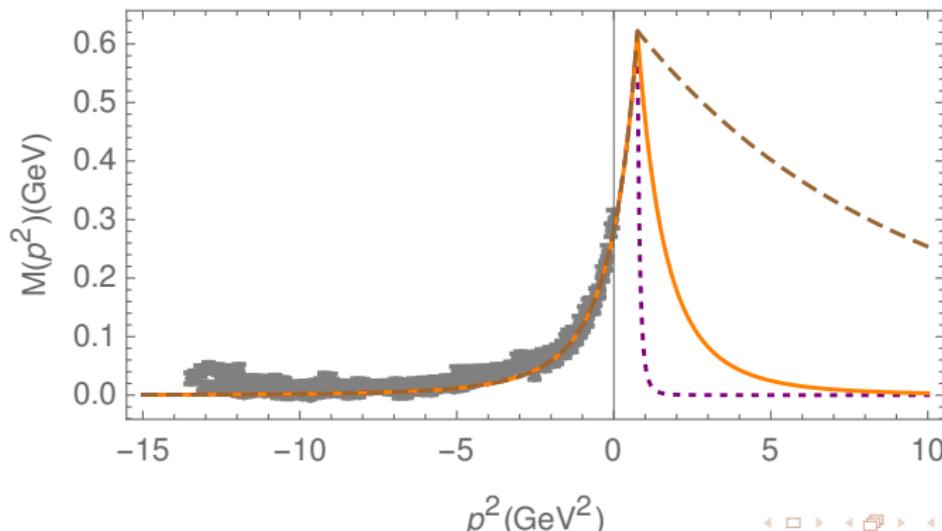
# MASS FUNCTION IN TIMELIKE REGION

EB, GROSS, PEÑA, STADLER PRD (2015)

- piecewise form

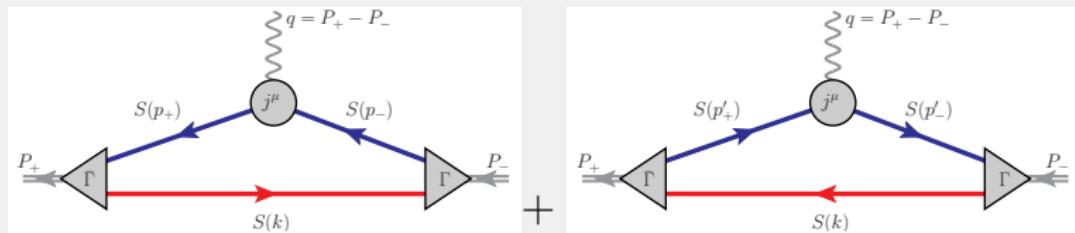
$$h(p^2) = \begin{cases} \left( \frac{\Lambda_\chi^2 - m_\chi^2}{\Lambda_\chi^2 - p^2} \right)^2 & \text{if } p^2 < s_+ \\ \mathcal{N}(\alpha) \left( \frac{\alpha^2 \Lambda_\chi^2 - m_\chi^2}{\alpha^2 \Lambda_\chi^2 + p^2 - 2s_+} \right)^2 & \text{if } p^2 > s_+ \end{cases}$$

- chiral limit,  $\alpha = 3, 1, 0.5$



# $\pi^+$ ELECTROMAGNETIC CURRENT

EB, GROSS, PEÑA, STADLER PRD (2015); PRD (2014)



2 **spectator** and 4 **active** quark pole contributions

⇒ charge-conjugation invariant complete impulse approximation (C-CIA)

- $\pi$  vertex function  $\Gamma(p_1, p_2) \sim h(p_1^2)h(p_2^2)\gamma^5$

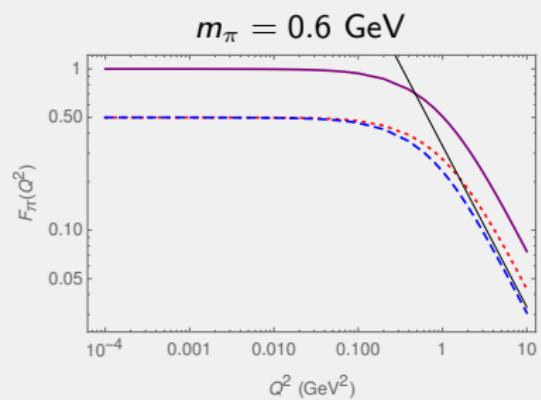
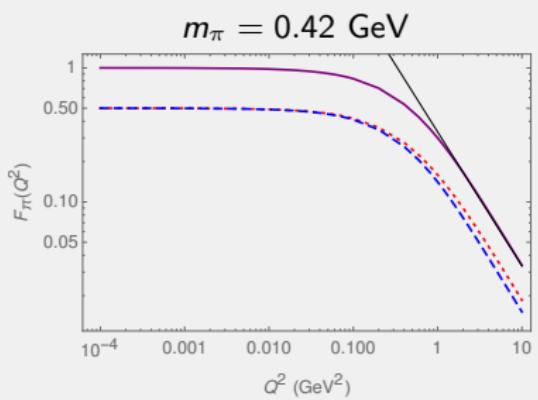
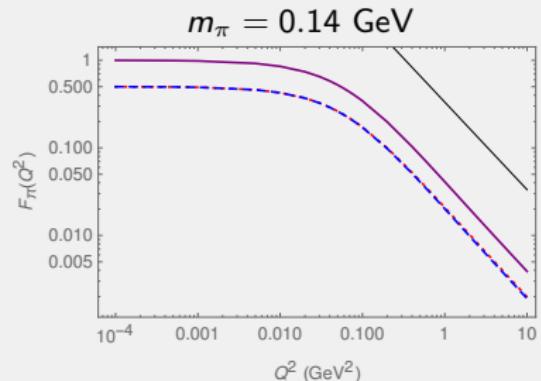
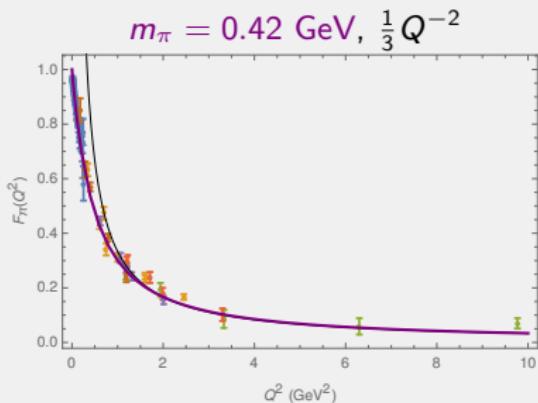
(Reduced) off-shell **quark current**



$$j_R^\mu = f(\gamma^\mu + \kappa \frac{i\sigma^{\mu\nu}q_\nu}{2m}) + \delta'\Lambda'\gamma^\mu + \delta\gamma^\mu\Lambda + g\Lambda'\gamma^\mu\Lambda \quad \text{with } \Lambda = \frac{M(p)-\phi}{2M(p)}$$

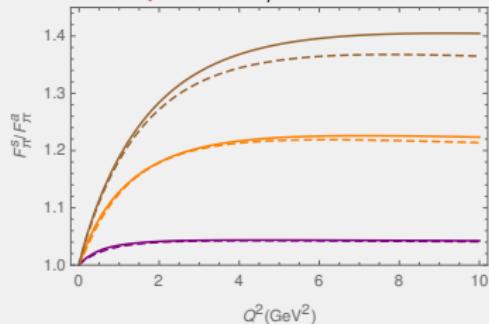
- satisfies **(vector) Ward-Takahashi identity** ⇒ pion current conserved ✓
- differs in chiral limit by transverse component from Ball-Chiu current

## RESULTS: SPECTATOR VS. ACTIVE VS. (TOTAL) C-CIA



## RESULTS: EFFECT OF DYNAMICAL QUARK MASS

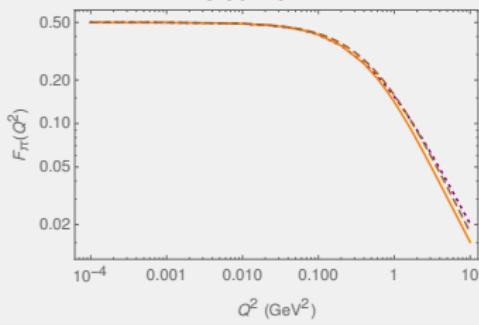
spectator/active



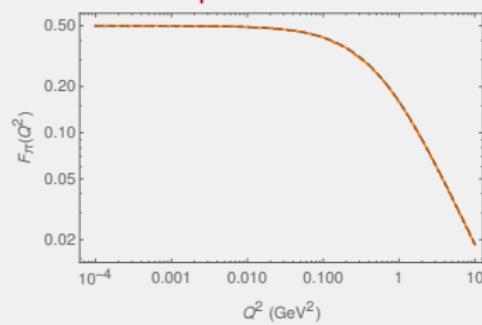
solid: with dynamical mass  
dashed: with fixed mass  
 $m_\pi = 0.6, 0.42, 0.14$  GeV

With different mass functions  $\alpha = 3, 1, 0.5$

active



spectator

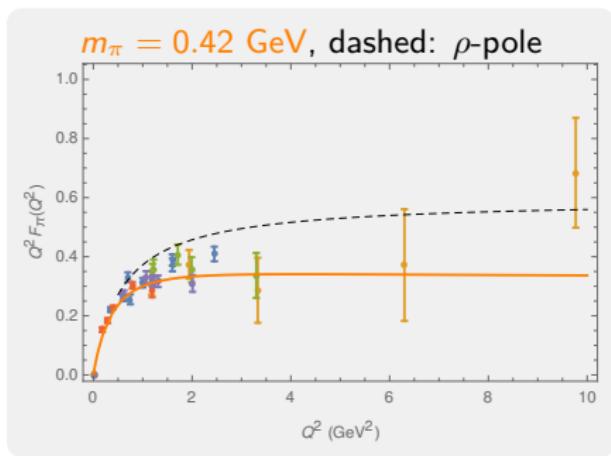
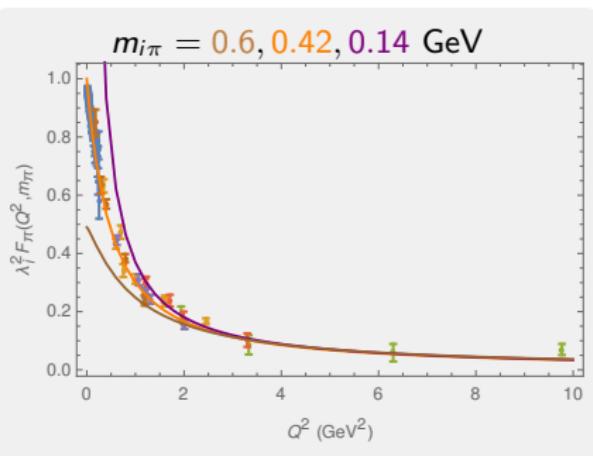


## RESULTS: SCALING RELATIONS AND COMPARISON WITH $\rho$ -POLE

$$F_\pi(Q^2, \lambda_i m_\pi) \stackrel{Q^2 \gg m_\pi^2}{\simeq} \lambda_i^2 F_\pi(Q^2, m_\pi)$$

$$\lambda_i = m_{i\pi}/m_\pi, \quad m_\pi = 0.42 \text{ GeV}$$

Comparison with  $\rho$ -pole:



## CONCLUSIONS AND OUTLOOK

- dynamical model for  $q\bar{q}$  mesons in CST:
  - AVWTI: *Lorentz scalar part in confining kernel requires equally-weighted pseudoscalar counterpart*
  - $\pi\pi$  scattering in  $\chi$ -limit satisfies Adler zero constraint ✓
- dressed quark **mass function** in Minkowski space with Euclidean LQCD data used to fix parameters
- qualitative CST study of **pion electromagnetic form factor** in C-CIA with simple pion vertex function and dressed quark current:  
reasonable results for large  $Q^2$  ✓  
issue at small  $Q^2$  and small  $m_\pi$  ✗

### Outlook and work in progress:

- ① use dynamically calculated dressed quark-photon vertex and pion vertex function from solving the CST-BSE: expect to fix form factor at small  $Q^2$
- ② mass function from kernel with vector structures for  $V_L$ ; add one-gluon exchange
- ③  $\pi\pi$  scattering away from  $\chi$  limit: expect deviation from Weinberg result
- ④ solve CST bound-state equations for all mesons and fit meson spectrum

## ACKNOWLEDGEMENTS/SUPPORT



**THANK YOU!**