

Spin-1 and Perturbative QCD

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Frontiers in Light-Front Hadron Physics

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Light-Front Motivations

- Light-Front is the Ideal Framework to Describe Hadronic Bound States
- Constituent Picture and Unambiguous Partons Content of the Hadronic System
- Light-Front Wavefunctions: Representation of Composite Systems in QFT
- Invariant Under Boosts
- Light-Front Vacuum is Trivial
- After Integrate in k^- : Bethe-Salpeter Amplitude (Wave Function)
- LF Lorentz Invariant Hamiltonian: $P^2 = P^+P^- - P_\perp^2$

Light-Front Coordinates

Four-Vector $\implies x^\mu = (x^0, x^1, x^2, x^3) = (x^+, x^-, x_\perp)$

$$x^+ = t + z \quad x^+ = x^0 + x^3 \implies \text{Time}$$

$$x^- = t - z \quad x^- = x^0 - x^3 \implies \text{Position}$$

Metric Tensor and Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

$$p^+ = p^0 + p^3, \quad p^- = p^0 - p^3, \quad p^\perp = (p^1, p^2)$$

Dirac Matrix and Electromagnetic Current

$$\begin{aligned}\gamma^+ &= \gamma^0 + \gamma^3 \implies \text{Electr. Current} & J^+ &= J^0 + J^3 \\ \gamma^- &= \gamma^0 - \gamma^3 \implies \text{Electr. Current} & J^- &= J^0 - J^3 \\ \gamma^\perp &= (\gamma^1, \gamma^2) \implies \text{Electr. Current} & J^\perp &= (J^1, J^2)\end{aligned}$$

$$p^\mu x_\mu = \frac{p^+ x^- + p^- x^+}{2} - \vec{p}_\perp \vec{x}_\perp$$

$$x^+, x^-, \vec{x}_\perp \implies p^+, p^-, \vec{p}_\perp$$

$p^- \implies$ Light-Front Energy

$$p^2 = p^+ p^- - (\vec{p}_\perp)^2 \implies p^- = \frac{(\vec{p}_\perp)^2 + m^2}{p^+}$$

On-shell

Bosons $\implies S_F(p) = \frac{1}{p^2 - m^2 + i\epsilon}$

Fermions $\implies S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + i\epsilon} + \frac{\gamma^+}{2p^+}$

Review Papers:

- Phys. Rept. 301, (1998) 299-486, Brodsky, Pauli and Pinsky
- A. Harindranath, Pramana, Journal of Indian Academy of Sciences Physics Vol.55, Nos 1 & 2, (2000) 241.

• An Introduction to Light-Front Dynamics for Pedestrians
Avaroth Harindranath

Light-Front book organizers: James Vary and Frank Wolz,(1997)

General Electromagnetic Current: Spin-1

$$J_{\alpha\beta}^{\mu} = [F_1(q^2)g_{\alpha\beta} - F_2(q^2)\frac{q_{\alpha}q_{\beta}}{2m_{\rho}^2}]p^{\mu} - F_3(q^2)(q_{\alpha}g_{\beta}^{\mu} - q_{\beta}g_{\alpha}^{\mu}),$$

- **Polarization Vectors**

$$\epsilon_x^{\mu} = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0), \quad \epsilon_y^{\mu} = (0, 0, 1, 0), \quad \epsilon_z^{\mu} = (0, 0, 0, 1),$$

$$\epsilon_x'^{\mu} = (\sqrt{\eta}, \sqrt{1+\eta}, 0, 0), \quad \epsilon_y'^{\mu} = \epsilon_y, \quad \epsilon_z'^{\mu} = \epsilon_z,$$

where $\eta = q^2/4m_{\rho}^2$

- **Breit Frame:**

$$p_i^{\mu} = (p^0, -q_x/2, 0, 0) \quad (\text{Initial}) \quad \text{where} \quad p^0 = m_{\rho}\sqrt{1+\eta}.$$

$$p_f^{\mu} = (p^0, q_x/2, 0, 0) \quad (\text{Final})$$

$$\begin{aligned}
J_{ji}^+ &= \imath \int \frac{d^4 k}{(2\pi)^4} \frac{\text{Tr}[\epsilon_j^{'\beta} \Gamma_\beta(k, k - p_f)(k - p_f + m)]}{((k - p_i)^2 - m^2 + \imath\epsilon)(k^2 - m^2 + \imath\epsilon)} \\
&\times \frac{\gamma^+(\not{k} - \not{p}_i + m)\epsilon_i^\alpha \Gamma_\alpha(k, k - p_i)(\not{k} + m)]\Lambda(k, p_f)\Lambda(k, p_i)}{((k - p_f)^2 - m^2 + \imath\epsilon)}
\end{aligned}$$

- Regulator Function

$$\Lambda(k, p_{i(f)}) = N/((p - k)^2 - m_R^2 + \imath\epsilon)^2$$

- ρ -Meson Vertex

$$\Gamma^\mu(k, p) = \gamma^\mu - \frac{m_\rho}{2} \frac{2k^\mu - p^\mu}{p.k + m_\rho m - \imath\epsilon}$$

- Mass Squared ($x = \frac{k^+}{P^+} \implies 0 < x < 1$)

$$M^2(m_a, m_b) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m_b^2}{1-x} - p_\perp^2$$

- Free Mass $M_0^2(m, m)$ and Function $M_R^2(m, m_R)$

The function M_R^2 is given by

$$M_R^2 = \frac{k_\perp^2 + m^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m_R^2}{1-x} - p_\perp^2$$

$$M_0^2 = \frac{k_\perp^2 + m^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m^2}{1-x} - p_\perp^2$$

- Wave Function

$$\Phi_i(x, \vec{k}_\perp) = \frac{N^2}{(1-x)^2(m_\rho^2 - M_0^2)(m_\rho^2 - M_R^2)^2} \vec{\epsilon}_i \cdot [\vec{\gamma} - \frac{\vec{k}}{\frac{M_0}{2} + m}]$$

Refs.

- Phy.Rev. **C55** (1997) 2043 J.P.B. C. de Melo and T. Frederico
- Phy.Lett. **B708** (2012) 87 J.P.B. C. de Melo and T. Frederico
- Few.Body.Syst. **52**(2012) 403 J.P.B. C. de Melo and T. Frederico

- Instant-Form Spin Base

$$J_{ji}^+ = \frac{1}{2} \begin{pmatrix} J_{xx}^+ + J_{yy}^+ & \sqrt{2}J_{zx}^+ & J_{yy}^+ - J_{xx}^+ \\ -\sqrt{2}J_{zx}^+ & 2J_{zz}^+ & \sqrt{2}J_{zx}^+ \\ J_{yy}^+ - J_{xx}^+ & -\sqrt{2}J_{zx}^+ & J_{xx}^+ + J_{yy}^+ \end{pmatrix}$$

- Light-Front

$$I_{m'm}^+ = \begin{pmatrix} I_{11}^+ & I_{10}^+ & I_{1-1}^+ \\ -I_{10}^+ & I_{00}^+ & I_{10}^+ \\ I_{1-1}^+ & -I_{10}^+ & I_{11}^+ \end{pmatrix}$$

$$\implies R_M^\dagger I^+ R_M^\dagger = J^+ \iff \text{Melosh}$$

- Matrix Elements

$$I_{11}^+ = \frac{J_{xx}^+ + (1 + \eta)J_{yy}^+ - \eta J_{zz}^+ - 2\sqrt{\eta}J_{zx}^+}{2(1 + \eta)}$$

$$I_{10}^+ = \frac{\sqrt{2\eta}J_{xx}^+ + \sqrt{2\eta}J_{zz}^+ - \sqrt{2}(\eta - 1)J_{zx}^+}{2(1 + \eta)}$$

$$I_{1-1}^+ = \frac{-J_{xx}^+ + (1 + \eta)J_{yy}^+ + \eta J_{zz}^+ + 2\sqrt{\eta}J_{zx}^+}{2(1 + \eta)}$$

$$I_{00}^+ = \frac{-\eta J_{xx}^+ + J_{zz}^+ - 2\sqrt{\eta}J_{zx}^+}{(1 + \eta)}$$

$$\begin{aligned}
J_{xx}^+ &= \frac{1}{1+\eta} [I_{11}^+ + 2\sqrt{2\eta} I_{10}^+ - \eta I_{00}^+ - I_{1-1}^+] \\
J_{zx}^+ &= \frac{\sqrt{2}}{1+\eta} \left[\frac{\sqrt{2\eta}}{2} I_{11}^+ + (\eta-1) I_{10}^+ + \sqrt{\frac{\eta}{2}} I_{00}^+ \right. \\
&\quad \left. - \frac{\sqrt{2\eta}}{2} I_{1-1}^+ \right] \\
J_{yy}^+ &= I_{11}^+ + I_{1-1}^+ \\
J_{zz}^+ &= \frac{1}{1+\eta} [-\eta I_{11}^+ + 2\sqrt{2\eta} I_{10}^+ + I_{00}^+ + \eta I_{1-1}^+]
\end{aligned}$$

$$\Delta(q^2) = (1 + 2\eta)I_{11}^+ + I_{1-1}^+ - \sqrt{8\eta}I_{10}^+ - I_{00}^+ = (1 + \eta)(J_{yy}^+ - J_{zz}^+) = 0$$

- Angular Condition: **Violation !!**

$$q_x \implies J_{yy}^+ = J_{zz}^+ \quad \left\{ \begin{array}{l} \text{Parity} \\ + \\ \text{Rotations} \end{array} \right.$$

$$\Delta(q^2) \neq 0$$

- Ref:
- Sov. J. Nucl. Phys. 39 (1984) 198
I.Grach and L.A. Kondratyku
- Phy. Rev. Lett. 62 (1989) 387
L.L. Frankfurt, I.Grach, L.A. Kondratyku and M. Strikman

Prescriptions

$\left\{ \begin{array}{l} FFS \text{ (Frederico, Frankfurt, Strikman)} \\ GK \text{ (Grach, Kondratyku)} \\ CCKP \text{ (Coester, Chung, Keister, Polyzou)} \\ BH \text{ (Brodsky, Hiller)} \end{array} \right.$ vs COVARIANT

- Breit Frame $\implies P^+ = P'^+, P^- = P'^-, \vec{P}'_\perp = -\vec{P}_\perp = \vec{q}/2$
- B.F: $q^+ = q^0 + q^3 = 0$
- J_ρ^+ $\left\{ \begin{array}{l} 4 \text{ Current Elements} \\ 3 \text{ Form Factors } G_0, G_1 \text{ and } G_2 \end{array} \right.$

Inna Grach Prescription: I_{00}^+

$$G_0^{GK} = \frac{1}{3}[(3 - 2\eta)I_{11}^+ + 2\sqrt{2\eta}I_{10}^+ + I_{1-1}^+] =$$

$$\frac{1}{3}[J_{xx}^+ + \eta J_{zz}^+(2 - \eta)J_{yy}^+]$$

$$G_1^{GK} = 2[I_{11}^+ - \frac{1}{\sqrt{2\eta}}I_{10}^+] = J_{yy}^+ - J_{zz}^+ - \frac{J_{zx}^+}{\sqrt{\eta}}$$

$$G_2^{GK} = \frac{2\sqrt{2}}{3}[-\eta I_{11}^+ + \sqrt{2\eta}I_{10}^+ - I_{1-1}^+] =$$

$$\frac{\sqrt{2}}{3}[J_{xx}^+ + J_{yy}^+(-1 - \eta) + \eta J_{zz}^+] .$$

$$\begin{aligned}
 G_0^{CCKP} &= \frac{1}{3(1+\eta)} \left[\left(\frac{3}{2} - \eta\right) (I_{11}^+ + I_{00}^+) + 5\sqrt{2\eta} I_{10}^+ + \left(2\eta - \frac{1}{2}\right) I_{1-1}^+ \right] \\
 &= \frac{1}{6} [2J_{xx}^+ + J_{yy}^+ + 3J_{zz}^+] \\
 G_1^{CCKP} &= \frac{1}{(1+\eta)} [I_{11}^+ + I_{00}^+ - I_{1-1}^+ - \frac{2(1-\eta)}{\sqrt{2\eta}} I_{10}^+] = -\frac{J_{zx}^+}{\sqrt{\eta}} \\
 G_2^{CCKP} &= \frac{\sqrt{2}}{3(1+\eta)} [-\eta I_{11}^+ - \eta I_{00}^+ + 2\sqrt{2\eta} I_{10}^+ - (\eta + 2) I_{1-1}^+] = \\
 &\quad \frac{\sqrt{2}}{3} [J_{xx}^+ - J_{yy}^+]
 \end{aligned}$$

Brodsky-Hiller - (BH) - I_{11}^+

$$\begin{aligned} G_0^{BH} &= \frac{1}{3(2p^+)(1+2\eta)} [(3-2\eta)I_{00}^+ + 8\sqrt{2\eta}I_{10}^+ + 2(2\eta-1)I_{1-1}^+] \\ &= \frac{1}{3(2p^+)(1+2\eta)} [J_{xx}^+(1+2\eta) + J_{yy}^+(2\eta-1) + J_{zz}^+(3+2\eta)] \\ G_1^{BH} &= \frac{2}{2p^+(1+2\eta)} [I_{00}^+ - I_{1-1}^+ + \frac{(2\eta-1)}{\sqrt{2\eta}} I_{10}^+] \\ &= \frac{2}{2p^+(1+2\eta)} [\frac{J_{zx}^+}{\sqrt{\eta}} (1+2\eta) - J_{yy}^+ + J_{zz}^+] \\ G_2^{BH} &= \frac{1}{2p^+(1+2\eta)} [\sqrt{2\eta}I_{10}^+ - I_{00}^+ - (\eta+1)I_{1-1}^+] \\ &= \frac{1}{2p^+(1+2\eta)} [J_{xx}^+(1+2\eta) - J_{yy}^+(1+\eta) - \eta J_{zz}^+] \end{aligned}$$

$$\begin{aligned}G_0^{FFS} &= \frac{1}{3(1+\eta)} \left[\left(\frac{3}{2} - \eta\right) (I_{11}^+ + I_{00}^+) + 5\sqrt{2\eta} I_{10}^+ + \left(2\eta - \frac{1}{2}\right) I_{1-1}^+ \right] \\&= \frac{1}{6} [2J_{xx}^+ + J_{yy}^+ + 3J_{zz}^+] \\G_1^{FFS} &= G_1^{CCKP} \\G_2^{FFS} &= G_2^{CCKP}\end{aligned}$$

- No Zero Modes or Pair Terms Contribution for

$$I_{11}^{+Z} = 0, \quad I_{10}^{+Z} = 0, \quad I_{1-1}^{+Z} = 0$$

⇒ But $I_{00}^{+Z} = (1 + \eta) J_{zz}^{+Z}$
with $\lim_{\delta^+ \rightarrow 0_+} J_{zz}^{+Z} \neq 0$ which gives a zero-mode contribution only in this case

- Similar Results are found by Ji, Bakker and Choi

For $\Gamma(\gamma^\mu, \gamma^\nu)$ Vertex, See:

- Phy.Rev.D65 (2002) 116001
- Phy.Rev.D70 (2004) 053015

REF.:

- J.P.B.C. de Melo and T. Frederico, Phys. Lett. B708, (2012) 87
- J.P.B.C. de Melo and T. Frederico, Few Body Syst. 52 (2012) 403

pQCD // BaBar Experiment*

$$\gamma^* \rightarrow e^+ e^- \rightarrow \rho^+ \rho^-$$

- At $\sqrt{s} = 10.58$ GeV
- **BaBar Experimental Three Independent Amplitudes:**
 $|F_{00}^B|^2 : |F_{10}^B|^2 : |F_{11}^B|^2 = 0.51 \pm 0.14 \pm 0.07 : 0.10 \pm 0.04 \pm 0.01 : 0.04 \pm 0.03 \pm 0.01$

Or:

$$|F_{00}^B|^2 = 0.51 \pm 0.14 \pm 0.07$$

$$|F_{10}^B|^2 = 0.14 \pm 0.07 \pm 0.01$$

$$|F_{11}^B|^2 = 0.04 \pm 0.03 \pm 0.01$$

- **Constrained // Normalization**

$$|F_{00}^B|^2 + 4|F_{10}^B|^2 + 2|F_{11}^B|^2 = 1.$$

* **B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 78 (2008) 071103**

Brodsky-Hiller - (BH) - I_{11}^+

- Universal Ratios Spin S=1 for Large Momentum

$$G_0^{BH} = \frac{1}{3(2p^+)(1+2\eta)} [(3-2\eta)I_{00}^+ + 8\sqrt{2\eta}I_{10}^+ + 2(2\eta-1)I_{1-1}^+]$$

$$G_1^{BH} = \frac{2}{2p^+(1+2\eta)} [I_{00}^+ - I_{1-1}^+ + \frac{(2\eta-1)}{\sqrt{2\eta}} I_{10}^+]$$

$$G_2^{BH} = \frac{1}{2p^+(1+2\eta)} [\sqrt{2\eta}I_{10}^+ - I_{00}^+ - (\eta+1)I_{1-1}^+]$$

$$G_C : G_M : G_Q = \left(1 - \frac{2}{3}\eta\right) : 2 : -1$$

- Different Prescriptions, not only I_{00}^+ , but also I_{10}^+ , I_{11}^+ and I_{1-1}^+
- More General Universal Ratios for S=1

$$G_C : G_M : G_Q = \left(\alpha - \frac{2}{3}\eta\right) : \beta : -1 ,$$

- S. J. Brodsky and J. R. Hiller, Phys. Rev. D 46, (1992) 2141
- De Melo, C.-R. Ji, T. Frederico, Phy. Letts. B763 (2016) 87.

- Re-analyze the BaBar data to check the “universal ratios”, using the LF helicity amplitudes

⇒ Frame independent (or invariant under LF kinematic transformations)

⇒ Angular Condition

- Analysis based completely model independent constraints:

(i) LF angular condition implemented in the SL region, which must be satisfied for any $Q^2 > 0$;

(ii) pQCD power counting rules for the LF helicity amplitudes must work for $Q^2 \gg \Lambda_{QCD}^2$;

(iii) Analyticity that relates SL and TL regions

⇒ In order to implement (i), we considered the minimum possible sub-leading contributions ⇔ Satisfy the LF angular condition

- Time-Like region, Electromagnetic Current

$$J_\mu = (p_1 - p_2)_\mu \left[-G_1(q^2) U_1^* \cdot U_2^* + \frac{G_3(q^2)}{m_\rho^2} (U_1^* \cdot q U_2^* \cdot q - \frac{q^2}{2} U_1^* \cdot U_2^*) \right] + G_2(q^2) (U_{1\mu}^* U_2^* \cdot q - U_{2\mu}^* U_1^* \cdot q)$$

here $G_i(q)$ ($i=1,2,3$) are General complex functions in TL region
and U_1 and U_2 Polarization Vectors for Final State

- A. Dibeyssi, E. Tomasi-Gustafsson, G. I. Gakh, C. Adamuščín,
Phys. Rev. C 85 (2012) 048201.

- **ρ -Meson Electromagnetic Form Factors**

$$G_C = \frac{2}{3}\tau(G_2 - G_3) + \left(1 - \frac{2}{3}\tau\right)G_1$$

$$G_M = -G_2$$

$$G_Q = G_1 + G_2 + 2G_3$$

where $\tau = q^2/4m_\rho^2$

- Inverse relations

$$G_1 = G_Q + G_M - \frac{1}{\tau - 1}[G_C - G_M - (1 - \frac{2}{3}\tau)G_Q]$$

$$G_2 = -G_M$$

$$G_3 = \frac{1}{2(\tau - 1)}[G_C - G_M - (1 - \frac{2}{3}\tau)G_Q].$$

- **Jacob-Wick Helicity Amplitudes (Standard)** $F_{\lambda_1 \lambda_2}$
to $\gamma^* \rightarrow \rho^+ \rho^-$ Decay Amplitudes:

$$F_{\lambda_1 \lambda_2} = M_{\lambda_1 \lambda_2}^{\lambda} = M(\epsilon \rightarrow \epsilon^{(\lambda_{\gamma^*})}, U_1 \rightarrow U_1^{(\lambda_1)}, U_2 \rightarrow U_2^{(\lambda_2)}),$$

- **Rh-meson:** $\lambda_1 = \lambda_{\rho^+}$, $\lambda_2 = \lambda_{\rho^-}$, **Virtual Photon Helicity** $\lambda = \lambda_{\gamma^*}$
- **Conservation:** $\lambda = \lambda_1 - \lambda_2$
- **Symmetry Properties:** $F_{-1-1} = F_{11}$, $F_{10} = -F_{01} = F_{-10} = -F_{0-1}$
- \Rightarrow **Left Only Three Independent Helicity Amplitudes**
- Def. **Breit helicity amplitudes** $F_{\lambda_1 \lambda_2}^B$ To Simplify the Cross Section Analysis.
- Relations

$$F_{00}^B = F_{00} - 2\tau F_{11}, \quad F_{10}^B = F_{10}, \quad F_{11}^B = F_{11}.$$

- A. Dbeysi, E. Tomasi-Gustafsson, G. I. Gakh, C. Adamuščín,
Phys. Rev. C 85 (2012) 048201.

- Electromagnetic Form Factor ρ -meson (FFs)

$$F_{00}^B = -\frac{\sqrt{\tau-1}}{m_\rho} [q^2(G_1 + G_2 + G_3) - 2 m_\rho^2 G_1]$$

$$F_{11}^B = 2 m_\rho \sqrt{\tau-1} (G_1 + 2 \tau G_3),$$

$$F_{10}^B = -2 m_\rho \sqrt{\tau(\tau-1)} G_2.$$

- Charge, Dipole and Quadrupole Elect. Form Factors

$$F_{00}^B = 2 m_\rho \sqrt{\tau-1} \left[G_C - \frac{4}{3} \tau G_Q \right],$$

$$F_{11}^B = 2 m_\rho \sqrt{\tau-1} \left[G_C + \frac{2}{3} \tau G_Q \right],$$

$$F_{10}^B = 2 m_\rho \sqrt{\tau(\tau-1)} G_M,$$

Sub-leading contributions to the helicity matrix elements

- Inna Grach Prescription

$$G_C = \frac{1}{2p^+} \left[\frac{3-2\eta}{3} I_{11}^+ + \frac{4\eta}{3} \frac{I_{10}^+}{\sqrt{2\eta}} + \frac{1}{3} I_{1-1}^+ \right]$$

$$G_M = \frac{2}{2p^+} \left[I_{11}^+ - \frac{1}{\sqrt{2\eta}} I_{10}^+ \right]$$

$$G_Q = \frac{1}{2p^+} \left[-I_{11}^+ + 2 \frac{1}{\sqrt{2\eta}} I_{10}^+ - \frac{I_{1-1}^+}{\eta} \right]$$

- Functions $\Rightarrow G_i(I_{11}^+, I_{10}^+, I_{1-1}^+) \Rightarrow$ **not** I_{00}^+

- Angular Condition

$$(1 + 2\eta)I_{11}^+ + I_{1-1}^+ - 2\sqrt{2\eta}I_{10}^+ - I_{00}^+ = 0$$

- In terms of I_{00}^+

$$I_{11}^+ \sim \frac{1}{\eta}I_{00}^+, \quad I_{1-1}^+ \sim \frac{1}{\eta}I_{00}^+, \quad I_{10}^+ \sim \frac{1}{\sqrt{\eta}}I_{00}^+$$

Writing the following expansion, with some coefficients, a_1, a_2 and a_3 , for electromagnetic matrix element of the electromagnetic current

$$\begin{aligned} I_{11}^+ &= \left(\frac{a_1}{\eta} + \frac{a_2}{\eta\sqrt{\eta}} + \frac{a_3}{\eta^2} \right) I_{00}^+, \\ I_{1-1}^+ &= \left(\frac{b_1}{\eta} + \frac{b_2}{\eta\sqrt{\eta}} + \frac{b_3}{\eta^2} \right) I_{00}^+, \\ I_{10}^+ &= \frac{1}{\sqrt{\eta}} \left(c_1 + \frac{c_2}{\sqrt{\eta}} + \frac{c_3}{\eta} \right) I_{00}^+. \end{aligned}$$

- Angular Condition Again

$$\left[\frac{a_1}{\eta} + \frac{a_2}{\eta\sqrt{\eta}} + \frac{a_3}{\eta^2} + 2a_1 + \frac{2a_2}{\sqrt{\eta}} + \frac{2a_3}{\eta} + \frac{b_1}{\eta} + \frac{b_2}{\eta\sqrt{\eta}} + \frac{b_3}{\eta^2} - 2\sqrt{2}c_1 - 2\sqrt{2}\frac{c_2}{\sqrt{\eta}} - 2\sqrt{2}\frac{c_3}{\eta} - 1 \right] = 0$$

We have the following equations

$$\begin{cases} 2a_1 - 2\sqrt{2}c_1 - 1 = 0 \\ 2a_2 - 2\sqrt{2}c_2 = 0 \\ a_1 + 2a_3 + b_1 - 2\sqrt{2}c_3 = 0 \\ a_2 + b_2 = 0 \\ a_3 + b_3 = 0 \end{cases}$$

- The equations above, have the solution:

$$\left\{ \begin{array}{l} a_1 = \sqrt{2}c_1 + \frac{1}{2} \\ a_2 = \sqrt{2}c_2 \\ b_1 = -a_1 - 2a_3 + 2\sqrt{2}c_3 \\ \quad = -\frac{1}{2} - \sqrt{2}c_1 - 2a_3 + 2\sqrt{2}c_3 \\ b_2 = -a_2 = -\sqrt{2}c_2 \\ b_3 = -a_3 \end{array} \right.$$

- Electromagnetic Matrix elements, $I_{m'm}^+$:

$$\begin{aligned} I_{11}^+ &= \frac{1}{\eta} \left[\sqrt{2}c_1 + \frac{1}{2} \frac{\sqrt{2}c_2}{\sqrt{\eta}} + \frac{a_3}{\eta} \right] I_{00}^+, \\ I_{1-1}^+ &= \frac{1}{\eta} \left[2 \left(\sqrt{2}c_3 - a_3 \right) - \left(\sqrt{2}c_1 + \frac{1}{2} \right) - \frac{\sqrt{2}c_2}{\sqrt{\eta}} - \frac{a_3}{\eta} \right] I_{00}^+, \\ I_{10}^+ &= \frac{1}{\sqrt{\eta}} \left[c_1 + \frac{c_2}{\sqrt{\eta}} + \frac{c_3}{\eta} \right] I_{00}^+. \end{aligned} \tag{1}$$

- Suppose, we take $a_3 = 0$, to remove $\frac{1}{\eta^2}$ term in G_Q , (and call $c_3 = c_2$)

$$\begin{aligned} I_{11}^+ &= \frac{1}{\eta} \left[\sqrt{2}c_1 + \frac{1}{2} \right] I_{00}^+, \\ I_{1-1}^+ &= \frac{1}{\eta} \left[2\sqrt{2}c_2 - \sqrt{2}c_1 - \frac{1}{2} \right] I_{00}^+ \\ I_{10}^+ &= \frac{1}{\sqrt{\eta}} \left[c_1 + \frac{c_2}{\eta} \right] I_{00}^+ \end{aligned}$$

- The Angular Condition is exactly using the matrix elements above!!

- Space-Like Form Factor in terms of c_1, c_2

$$G_C = \left[1 + 2\sqrt{2}c_1 + 4\sqrt{2}c_2 - \eta \right] \frac{f_{00}^+}{6},$$

$$G_M = \left[1 + \sqrt{2}c_1 - \frac{\sqrt{2}}{\eta}c_2 \right] \frac{f_{00}^+}{2}$$

$$G_Q = \left[-1 + \frac{1 + 2\sqrt{2}(c_1 - c_2)}{\eta} \right] \frac{f_{00}^+}{4},$$

- ★ Here $f_{00} = I_{00}^+/(p^+\eta)$

- **Asymptotic Ratios (General)**

$$G_C : G_M : G_Q = \left(\alpha - \frac{2}{3}\eta \right) : \beta : -1$$

where

$$\alpha = \frac{2}{3} \left(1 + 2\sqrt{2}c_1 + 4\sqrt{2}c_2 \right), \quad \beta = 2(1 + \sqrt{2}c_1),$$

- **And c_1 and c_2 in terms of α and β**

$$c_1 = \frac{\beta - 2}{2\sqrt{2}} \quad \text{and} \quad c_2 = \frac{1 + \frac{3}{2}\alpha - \beta}{4\sqrt{2}}.$$

- Electromagnetic Form Factors in terms of (α, β)

$$G_C = \left[\alpha - \frac{2}{3}\eta \right] \frac{f_{00}^+}{4}, \quad G_M = \left[\beta + \frac{1}{2\eta}(\beta - 3\alpha - 2) \right] \frac{f_{00}^+}{4},$$
$$G_Q = - \left[1 + \frac{3}{2\eta}(1 + \alpha - \beta) \right] \frac{f_{00}^+}{4}$$

Time-like BaBar data with sub-leading contributions

- **Analitic Continuation**

$$\implies \eta \rightarrow -\tau, \quad \eta = Q^2/2m_\rho, \quad \tau = q^2/4m_\rho, \quad \text{and} \quad Q^2 = -q^2$$

- **Form Factors:**

$$G_C = \left[\alpha + \frac{2}{3}\tau \right] \frac{f_{00}^+}{4}$$

$$G_M = \left[\beta + -\frac{1}{2\tau}(\beta - 3\alpha - 2) \right] \frac{f_{00}^+}{4}$$

$$G_Q = - \left[1 - \frac{3}{2\tau}(1 + \alpha - \beta) \right] \frac{f_{00}^+}{4}$$

- where $f_{00}^+ \implies q^2 > 0$

- Breit Helicity Amplitudes

$$F_{10}^B = \frac{m_\rho}{8} \sqrt{1 - \tau^{-1}} [2 + 3\alpha + \beta(4\tau - 2)] f_{00}^+$$

$$F_{00}^B = m_\rho \sqrt{\tau - 1} [\beta + \tau - 1] f_{00}^+$$

$$F_{11}^B = \frac{m_\rho}{4} \sqrt{\tau - 1} [2 + 3\alpha - 2\beta] f_{00}^+$$

- Our analysis of the BaBar data for $e^+e^- \rightarrow \rho^+\rho^-$ at $\sqrt{s} = 10.58$ GeV
- $\Rightarrow \alpha = 1$ and $\beta = 2$ ("Universal Ratios of FFs")

$$|F_{00}^B|^2 : |F_{10}^B|^2 : |F_{11}^B|^2 = (4 + 4\tau)^2 : \left(\frac{1}{2\sqrt{\tau}} + 4\sqrt{\tau}\right)^2 : 1$$

- BaBar Data (Remember)

$$|F_{00}^B|^2 : |F_{10}^B|^2 : |F_{11}^B|^2 = 0.51 \pm 0.14 \pm 0.07 : 0.10 \pm 0.04 \pm 0.01 : 0.04 \pm 0.03 \pm 0.01$$

- At $\sqrt{s} = 10.58$ GeV and Normalization:

$$|F_{00}^B|^2 + 4|F_{10}^B|^2 + 2|F_{11}^B|^2 = 1$$

- BaBar Amplitude Ratios (Exp.)

$$\left| \frac{F_{00}^B}{F_{11}^B} \right|^2 = 12.75 \pm 10.27$$

$$\left| \frac{F_{10}^B}{F_{11}^B} \right|^2 = 2.5 \pm 2.13$$

$$\left| \frac{F_{00}^B}{F_{10}^B} \right|^2 = 5.1 \pm 2.5$$

$\implies 0.51$ for $|F_{00}^B|^2$ (with Same BaBar Normalization)

- Asymptotic theoretical ratio with $\alpha = 1$ and $\beta = 2$

$$|F_{00}^B|^2 : |F_{10}^B|^2 : |F_{11}^B|^2 = 0.51 : 1.1 \times 10^{-2} : 1.4 \times 10^{-5}, \quad (2)$$

\implies Asymptotic region has not yet been reached at
 $\sqrt{s} = 10.58$ GeV in the BaBar Experiment

- With BaBar Data and Breit Helicity Amplitudes
- \Rightarrow Extracted $\Rightarrow (\alpha, \beta)$
- 4 Solutions !!

Table: Extracted values of α and β from the BaBar ratios BaBar data at $\sqrt{s} = 10.58$ GeV for $\gamma^* \rightarrow \rho^+ + \rho^-$ using the expressions for the helicity amplitudes with the subleading contributions. The last line gives the zero of G_C in the SL region. Two sets of $\{\alpha, \beta\}$ values with $\alpha < 0$, i.e. (III) and (IV), have no zero of G_C in the SL region.

Solution	(I)	(II)	(III)	(IV)
α	23.1 ± 8.3	10.7 ± 6.4	-15.6 ± 8.3	-19.2 ± 6.4
β	6.4 ± 2.0	-5.4 ± 1.2	7.2 ± 2.0	-5.0 ± 1.2
Q_0 [GeV]	9.1 ± 1.6	6.2 ± 1.9	-	-

- Charge Electromagnetic Form Factor Zero: $Q_0^2 = 6m_\rho^2\alpha$

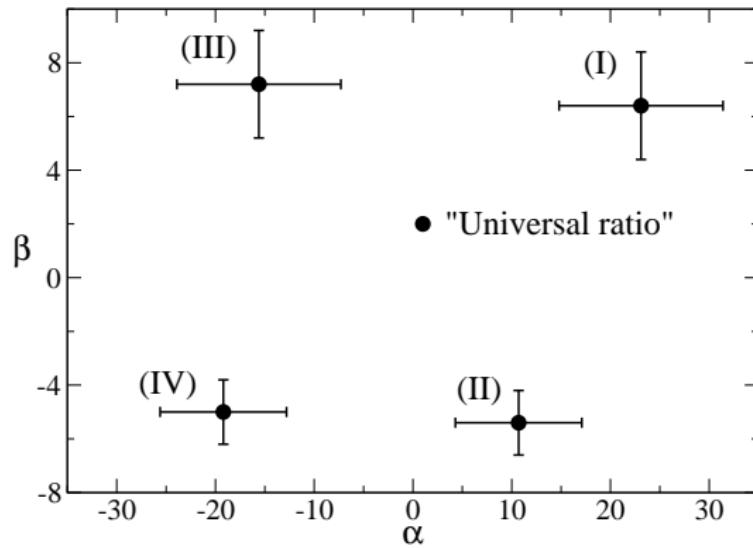


Figure: Parameter sets $\{\alpha, \beta\}$ with the corresponding errors from the fits (I) to (IV) given in Table 1. The central point is the parameter set for the “universal ratios”, i.e. $\{1, 2\}$. The sets (I) and (II) indicate a zero in G_C in the SL region.

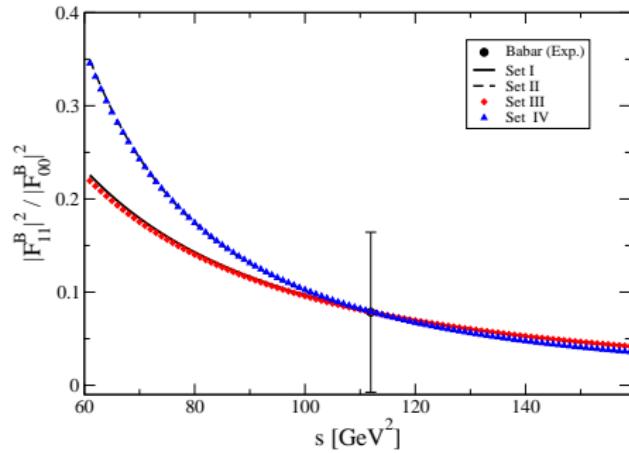
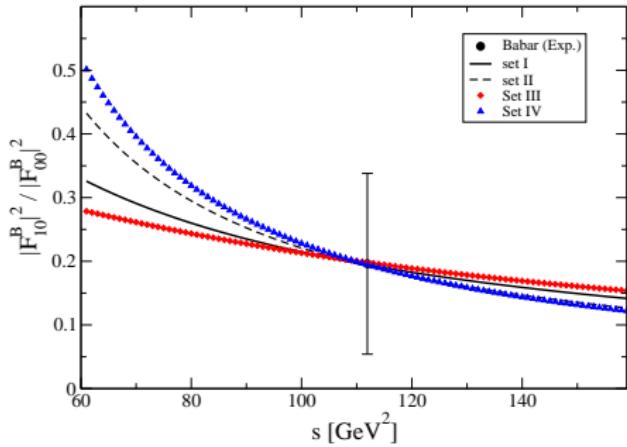


Figure: ρ -meson helicity amplitudes ratios

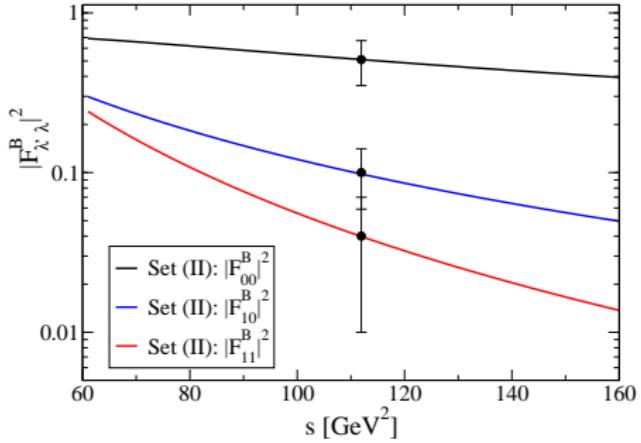
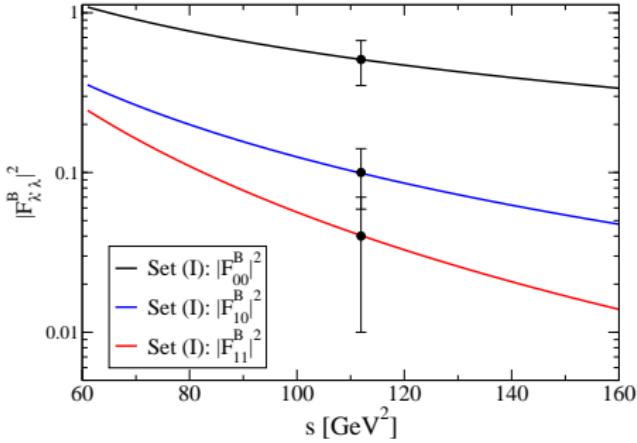


Figure: ρ -meson helicity amplitudes

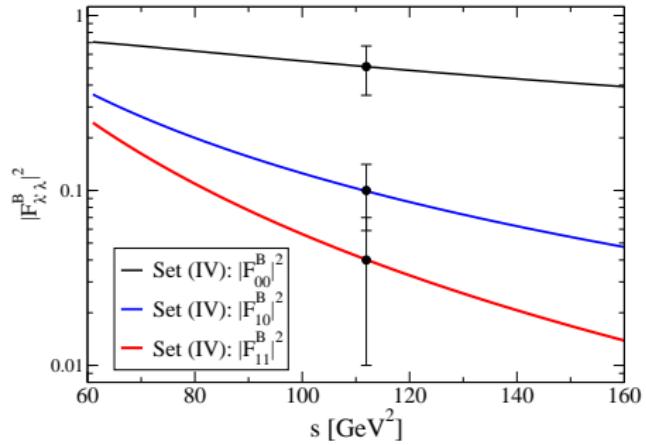
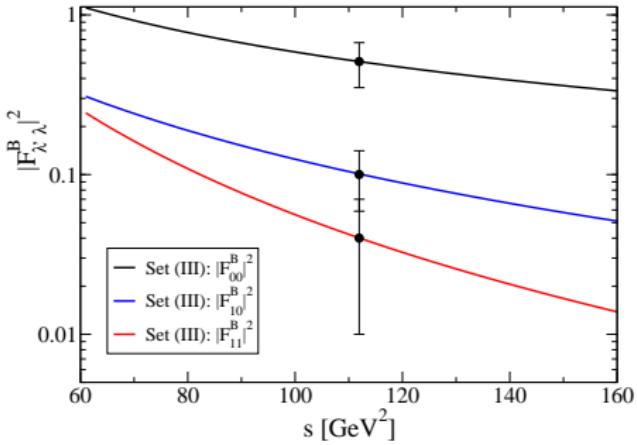


Figure: ρ -meson helicity amplitudes

Cross-section: $e^+e^- \rightarrow \rho^-\rho^+$

- The Cross-section in terms of the time-like FFs:

$$\sigma_{e^+e^- \rightarrow \rho^+\rho^-} = \frac{\pi \bar{\alpha}^2 \bar{\beta}^3}{3q^2} \left(3|G_C|^2 + 4\tau|G_M|^2 + \frac{8}{3}\tau^2|G_Q|^2 \right),$$

- Fine structure constant $\bar{\alpha} = 1/137$ and $\bar{\beta} = \sqrt{1 - \tau^{-1}}$ is the vector meson velocity in the center of mass system of the e^+e^- collision in units of the speed of light
- Babar Collaboration

$$\sigma_{e^+e^- \rightarrow \rho^+\rho^-} = 19.5 \pm 1.6 \text{ (stat)} \pm 3.21 \text{ (syst)} \text{ fb}.$$

- With the Breit helicity amplitudes

$$\sigma_{e^+ e^- \rightarrow \rho^+ \rho^-} = \sigma_0 \left(|F_{00}^B|^2 + 4|F_{10}^B|^2 + 2|F_{11}^B|^2 \right),$$

where

$$\sigma_0 = \frac{\pi \bar{\alpha}^2 \bar{\beta}^3}{3q^2} \frac{1}{4m_\rho^2(\tau - 1)}. \quad (3)$$

- Differential Cross-section

$$\frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \sigma_0 \left[|F_{10}^B|^2 (1+z^2) + \frac{1}{2}(1-z^2) \left(|F_{00}^B|^2 + 2|F_{11}^B|^2 \right) \right],$$

where $z = \cos \theta$.

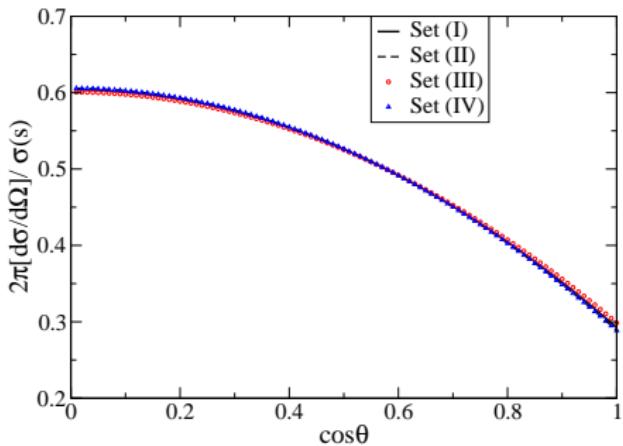
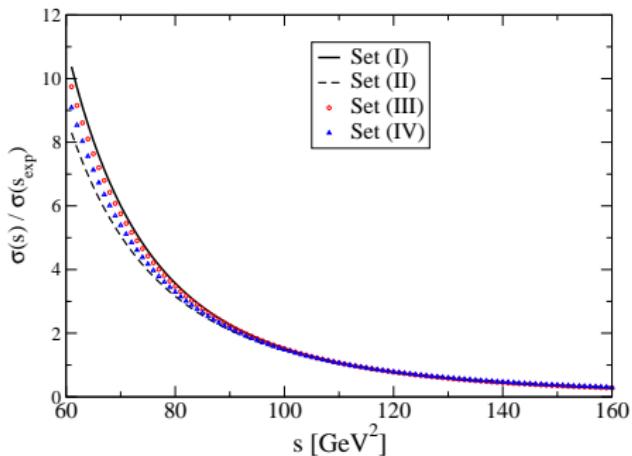


Figure: Cross-section of $e^+e^- \rightarrow \rho^+\rho^-$ as a function of s normalized to the experimental value at $\sqrt{s} = 10.58$ GeV [BaBar Data] is shown in the left panel. The differential angular cross-section $2\pi[d\sigma/d\Omega]/\sigma(s)$ as a function of $\cos\theta$ normalized to the cross-section for $s = 112$ GeV 2 is shown in the right panel. The calculations are performed for the parametrizations (I)-(IV) from Table

- The predicted cross-section after $s > 80$ GeV 2 are quite similar
 ⇒ Fitting BaBar helicity amplitudes $F_{\lambda'\lambda}^B$

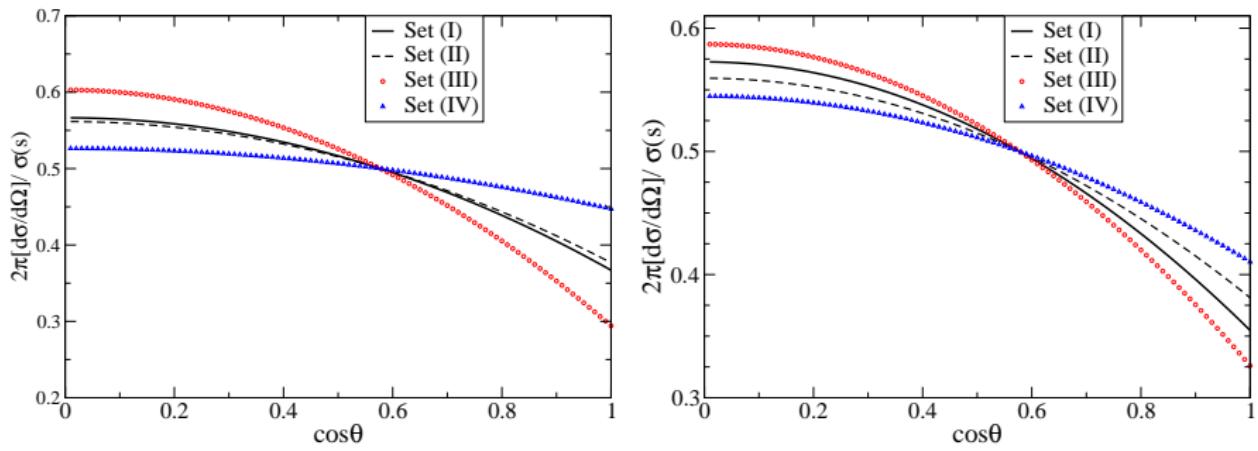


Figure: Differential angular cross-section $2\pi[d\sigma/d\Omega]/\sigma(s)$ of $e^+e^- \rightarrow \rho^+\rho^-$ as a function of $\cos\theta$ normalized to the cross-section for $s = 36 \text{ GeV}^2$ (left panel) and for $s = 60 \text{ GeV}^2$ (right panel). The calculations are performed for the parametrizations (I)-(IV) from Table

- Limits:
- Sub-leading term is not relevant for I_{10}^+

$$\Rightarrow |c_1| \gg |c_2|/\tau$$

$$\frac{I_{10}^+}{I_{00}^+} = \frac{c_1}{\sqrt{\eta}} + \frac{c_2}{\eta\sqrt{\eta}}, \quad \frac{I_{11}^+}{I_{00}^+} = \frac{\sqrt{2}c_1 + \frac{1}{2}}{\eta}, \quad \frac{I_{1-1}^+}{I_{00}^+} = \frac{2\sqrt{2}c_2 - (\sqrt{2}c_1 + \frac{1}{2})}{\eta}$$

and,

$$c_1 = \frac{\beta - 2}{2\sqrt{2}} \quad \text{and} \quad c_2 = \frac{1 + \frac{3}{2}\alpha - \beta}{4\sqrt{2}}.$$

- We have at the end:

$$\sqrt{s} \gg \sqrt{s_{pqcd}} = 2m_\rho \sqrt{\frac{|1 + \frac{3}{2}\alpha - \beta|}{|2\alpha - 4|}} = \begin{cases} 2.8 \text{ GeV / (I)} \\ 1.9 \text{ GeV / (II)} \\ 2.6 \text{ GeV / (III)} \\ 2.0 \text{ GeV / (IV)} \end{cases}$$

- \Rightarrow Indicative for $\sqrt{s_{pqcd}} [\text{GeV}] \sim 10 \Lambda_{QCD}$

- From the Universal Ratios

$$G_C : G_M : G_Q = \left(\alpha - \frac{2}{3}\eta \right) : \beta : -1 ,$$

- We have $\Rightarrow \sqrt{s} >> m_\rho \sqrt{6|\alpha|}$
 - With the parameters (α, β) we have $\Rightarrow 4.0 < s < 11 \text{ GeV}$
 - That results, support $\sqrt{s} >> 40 \Lambda_{QCD} \text{ GeV}$
- \Rightarrow Dominance leading pQCD behavior for that process

Summary

- Annihilation/production process $e^+ + e^- \rightarrow \rho^+ + \rho^-$ from the BaBar experiment at $\sqrt{s} = 10.58$ GeV analyzed \Rightarrow universal pQCD predictions.
- Sub-leading contributions to the helicity matrix elements of the electromagnetic current beyond the universal leading pQCD amplitudes are important
- Matrix elements of the ρ -meson electromagnetic current satisfy the constraint dictated by the light-front angular condition.
- Estimative cross-section $60 < q^2 < 160$ $\text{GeV}^2 \Rightarrow$ subleading order are important.
- At low s , however, the differential cross-section show sensitivity to solutions found here
- Important to check "universal ratios" for Spin S=1 from pQCD

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