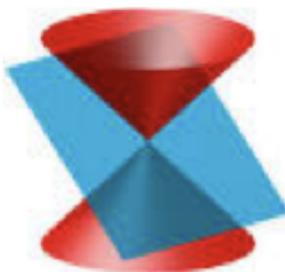


# Leading twist GPDs and spin densities in a proton

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Light Cone 2017

University of Mumbai, 22nd September, 2017

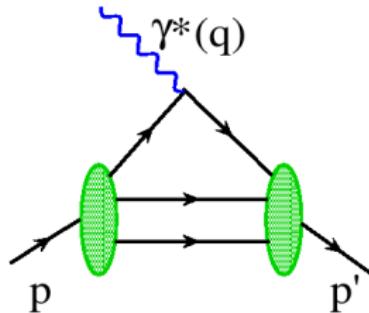
Collaborators: D. Chakrabarti, T. Maji, A. Mukherjee, X. Zhao

# Outline

- ▶ Introduction: various aspects of nucleon properties.
- ▶ Light-front quark-diquark model (LFQDM)
- ▶ *Form factors in LFQDM*
- ▶ *Generalized parton distributions (GPDs) in LFQDM*
- ▶ *Spin densities in a proton in LFQDM*
- ▶ Conclusions

# Introduction: nucleon properties

- ▶ Elastic electron scattering established the extended nature of the proton, proton radius:  $0.77 \text{ fm}$ . [R. Hofstadter, Nobel Prize 1961]
- ▶ The electromagnetic form factors can be probed through elastic scattering.

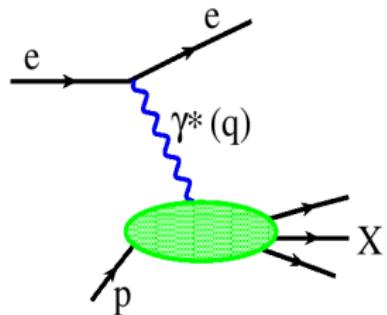


$$\langle N(p') | J^\mu(0) | N(p) \rangle = \bar{u}(p') \left[ \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}}{2m_N} q_\nu F_2(q^2) \right] u(p)$$

- ▶ The Fourier transformation of these form factors  $\Rightarrow$  spatial distributions (charge and magnetization distribution).
- ▶ Missing information: The form factors contain no dynamical information on the partons, such as their orbital angular momentum !!.
- ▶ Experimentally well established.

# Introduction: nucleon properties

- ▶ DIS discovered the existence of quasi-free point-like objects (quarks) inside the nucleon. [Friedman, Kendall, Taylor, Nobel Prize 1990]
- ▶ Parton distribution functions (PDFs) are extracted from deep inelastic scattering (DIS) processes.

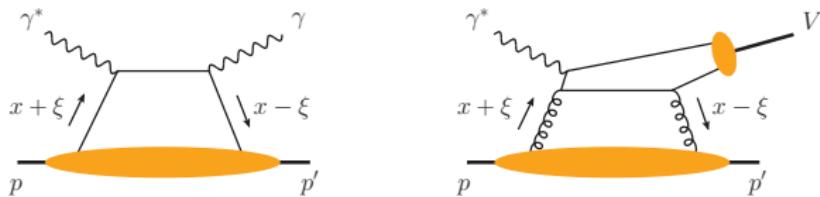


$$q(x) = \frac{p^+}{4\pi} \int dy^- e^{ixp^+y^-} \times \langle p | \underbrace{\bar{\psi}_q(0)\mathcal{O}\psi_q(y)}_{y^+=\vec{y}_\perp=0} | p \rangle$$

- ▶ PDFs encode the distribution of longitudinal momentum and polarization carried by the constituents
- ▶ Missing information: The PDFs provide no knowledge of spatial locations of parton !!

# Introduction: nucleon properties

- ▶ GPDs appear in the **exclusive processes** like deeply virtual Compton scattering (DVCS) or vector meson productions.
- ▶ The “**handbag**” diagrams for DVCS/vector meson productions process:

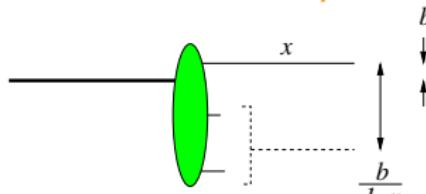


—talks by Camacho, Deshpande, Vary, Mukherjee, Kumar, Kumano

- ▶ GPDs encode the informations about the **three dimensional spatial structure of the nucleon** as well as the **spin and orbital angular momentum** of the constituents.
- ▶ Many activities are going on (**COMPASS**, **HERMES**, **ZEUS**, **JLAB** etc.) to gain insight into GPDs.

# Nucleon GPDs

- ▶ Off-forward matrix element  $\Rightarrow$  not probabilities !! [mom. space].
- ▶ GPDs [ $\zeta = 0$ ] in impact parameter space  $\Rightarrow$  distribution of parton in *transverse position space* [Burkardt, *Int J mod Phys. A18, 173 (2003)*]



- ▶ GPDs in impact parameter space:

$$\mathcal{X}(x, b) = \frac{1}{(2\pi)^2} \int d^2 \Delta e^{-i\Delta^\perp \cdot b^\perp} \mathcal{X}(x, t).$$

[Diehl et. al., *EPJC 39, 1(2005)*]

- ▶  $b = |b_\perp| \Rightarrow$  transverse distance between the struck parton and the center of momentum of the nucleon.
- ▶ The relative *distance between the struck parton and the center of momentum of the spectator system*  $\Rightarrow \frac{b}{1-x}$ .

# Nucleon GPDs

- ▶ In forward limit : GPDs  $\Rightarrow$  PDFs.
- ▶ First moment of GPDs are related to the electromagnetic FFs.
- ▶ Second moment of GPDs give gravitational FFs

$$\int_0^1 dx \ H_v^q(x, t) = F_1^q(t), \quad \int_0^1 dx \ E_v^q(x, t) = F_2^q(t)$$
$$\int_0^1 dx \ xH_v^q(x, t) = A^q(t), \quad \int_0^1 dx \ xE_v^q(x, t) = B^q(t)$$

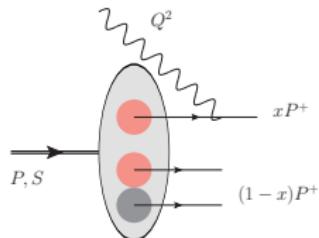
- ▶ At  $t = 0$ , 2nd moment of GPDs  $\Rightarrow$  angular momentum

$$J^q = \frac{1}{2}[A^q(0) + B^q(0)] \quad [X. Ji, PRL78, 610(1997)]$$

$$J_T^q = \frac{1}{2}[A_T^q(0) + 2\tilde{A}_T^q(0) + B_T^q(0)] \quad [M. Burkardt, PRD72, 094020(2005)]$$

# Light-front quark-diquark model

- In the quark-diquark picture, nucleon ( $p = |uud\rangle$ ,  $n = |udd\rangle$ ) is considered to be a bound state of *a single quark* and *a scalar diquark state*.
- Light front wave functions are constructed from the *AdS/QCD soft wall model wavefunctions*.



—talks by Mukherjee, Chakrabarti, Maji

- The proton is written (for  $J^z = \pm 1/2$ ) as :

$$\underbrace{|P; \pm\rangle}_{\Lambda_S=0} = \int \frac{dx d^2\mathbf{k}_\perp}{2(2\pi)^3 \sqrt{x(1-x)}} \sum_\lambda \psi_{\lambda_q}^\pm(x, \mathbf{k}_\perp) |\lambda_q \Lambda_S; xP^+, \mathbf{k}_\perp\rangle$$

- The light-front wavefunctions

[*Gutsche et. al. PRD 89 (2014)*]

$$\begin{aligned}\psi_{+q}^+(x, \mathbf{k}_\perp) &= \varphi_q^{(1)}(x, \mathbf{k}_\perp), \\ \psi_{-q}^+(x, \mathbf{k}_\perp) &= -\frac{k^1 + ik^2}{xM} \varphi_q^{(2)}(x, \mathbf{k}_\perp), \\ \psi_{+q}^-(x, \mathbf{k}_\perp) &= \frac{k^1 - ik^2}{xM} \varphi_q^{(2)}(x, \mathbf{k}_\perp), \\ \psi_{-q}^-(x, \mathbf{k}_\perp) &= \varphi_q^{(1)}(x, \mathbf{k}_\perp),\end{aligned}$$

- $\psi_{\lambda q}^\Lambda$ : LFWF with nucleon helicity  $\Lambda$  and quark q with helicity  $\lambda$

$$\begin{aligned}\varphi_q^{(i)}(x, \mathbf{k}_\perp) &= N_q^{(i)} \frac{4\pi}{\kappa} \sqrt{\frac{\log(1/x)}{1-x}} \\ &\quad x^{a_q^{(i)}} (1-x)^{b_q^{(i)}} \exp \left[ -\frac{\mathbf{k}_\perp^2}{2\kappa^2} \frac{\log(1/x)}{(1-x)^2} \right],\end{aligned}$$

constructed from soft-wall AdS/QCD [*Brodsky-Teramond PRD 79 (2009)*]

## Electromagnetic form factor

- In the light-front formalism, for a spin- $\frac{1}{2}$  composite particle system

$$\langle P + q; + | \frac{J^+(0)}{2P^+} | P; + \rangle = F_1(q^2)$$

$$\langle P + q; + | \frac{J^+(0)}{2P^+} | P; - \rangle = -(q^1 - iq^2) \frac{F_2(q^2)}{2M}$$

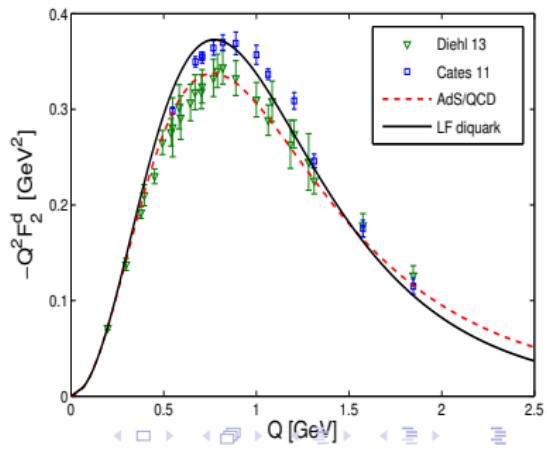
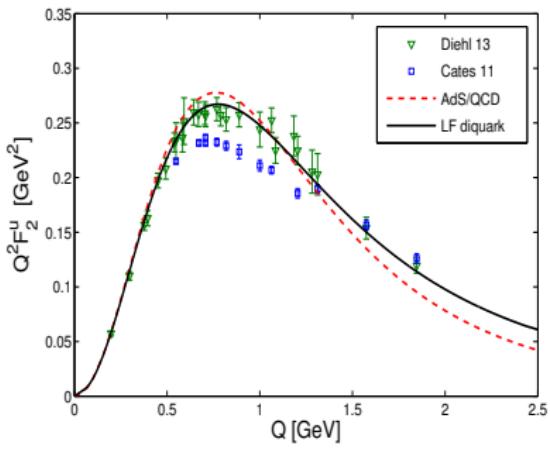
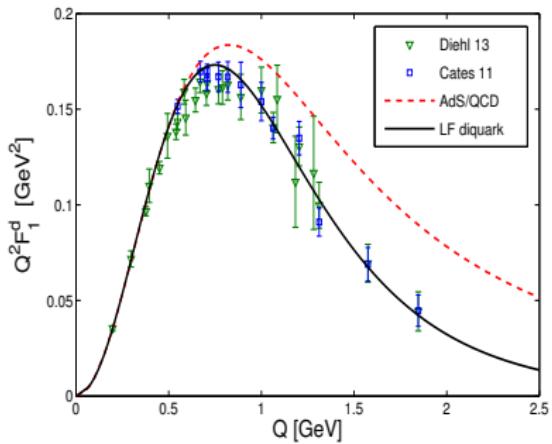
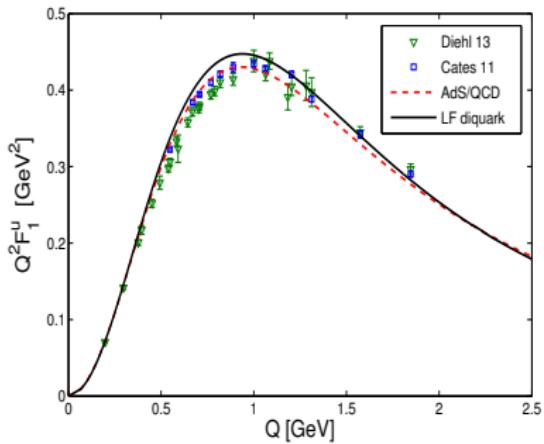
—S.J. Brodsky, S.D. Drell, PRD 22, 2236 (1980)

- The normalization of form factors for proton and neutron are  
 $F_1^p(0) = 1$ ,  $F_2^p(0) = \kappa^p = 1.793$  and  
 $F_1^n(0) = 0$ ,  $F_2^n(0) = \kappa^n = -1.913$
- Considering the charge and isospin symmetry the nucleon form factors are decomposed into flavour form factors as

$$F_i^{p(n)} = e_u F_i^{u(d)} + e_d F_i^{d(u)}.$$

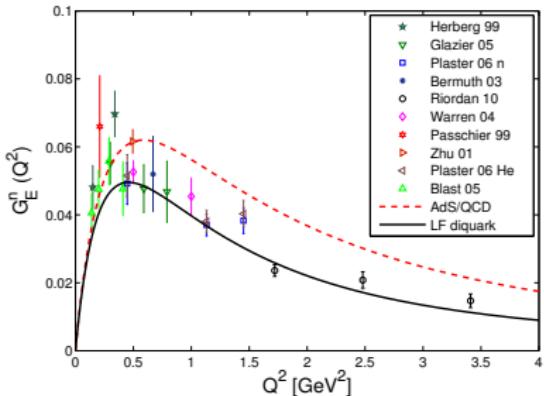
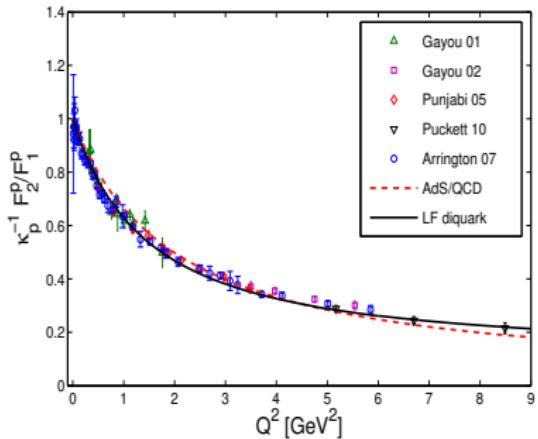
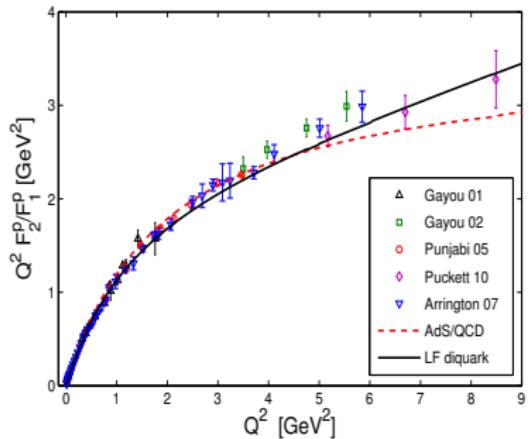
# Flavor FFs

[CM, Chakrabarti, EPJC 75 (2015)]



# Nucleon FFs

[CM, Chakrabarti, EPJC 75 (2015)]



## Electromagnetic radii

$$\langle r_E^2 \rangle^N = -6 \frac{dG_E^N(Q^2)}{dQ^2} \Big|_{Q^2=0},$$

$$\langle r_M^2 \rangle^N = -\frac{6}{G_M^N(0)} \frac{dG_M^N(Q^2)}{dQ^2} \Big|_{Q^2=0}$$

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4M_N^2} F_2^N(Q^2),$$

$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2).$$

Quantity	AdS/QCD	LF diquark	Measured data
$r_E^p$ (fm)	0.810	0.786	$0.877 \pm 0.005$
$r_M^p$ (fm)	0.782	0.772	$0.777 \pm 0.016$
$\langle r_E^2 \rangle^n$ (fm $^2$ )	-0.088	-0.085	$-0.1161 \pm 0.0022$
$r_M^n$ (fm)	0.796	0.7596	$0.862^{+0.009}_{-0.008}$

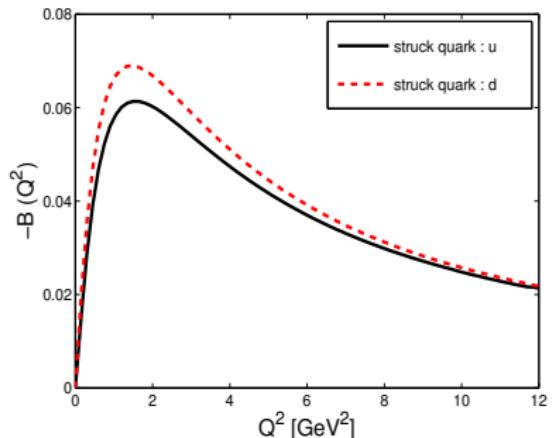
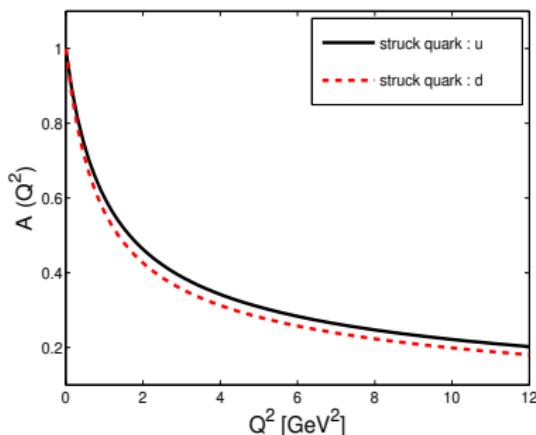
# Gravitational form factors

In the light-front:

[Chakrabarti, CM, Mukherjee, PRD 92 (2015)]

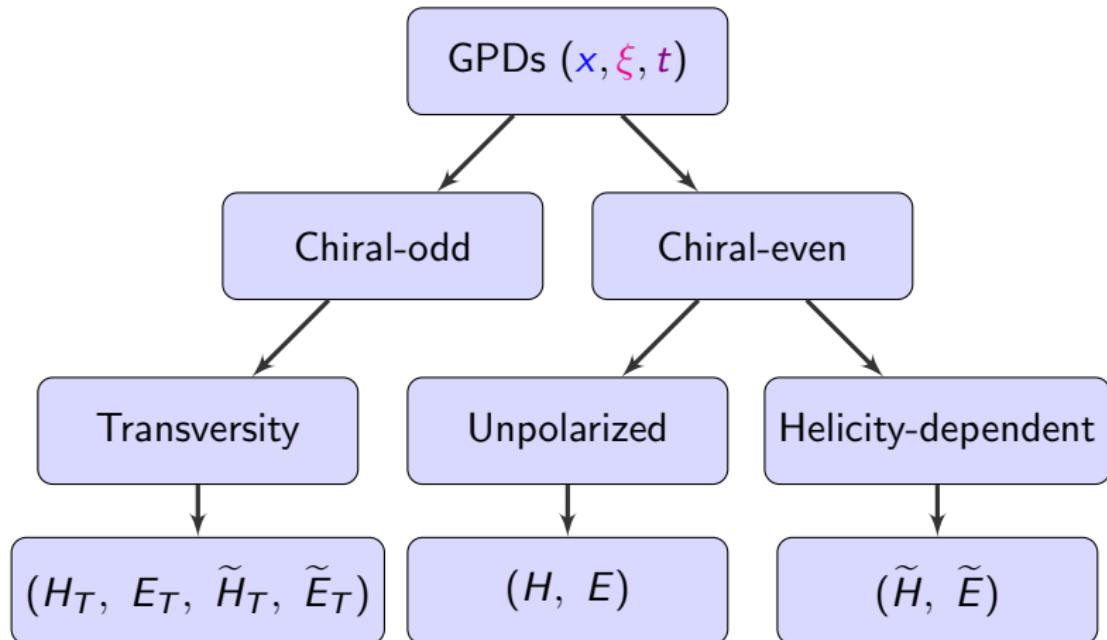
$$\langle P + q, \uparrow | \frac{T_i^{++}(0)}{2(P^+)^2} | P, \uparrow \rangle = A_i(q^2),$$

$$\langle P + q, \uparrow | \frac{T_i^{++}(0)}{2(P^+)^2} | P, \downarrow \rangle = -(q^1 - iq^2) \frac{B_i(q^2)}{2M}.$$



Nucleon GFFs = Quark GFFs + diquark GFFs

# Nucleon GPDs



**Unpolarized :** 
$$\frac{P^+}{2\pi} \int dy^- e^{ixP^+y^-} \langle p' | \bar{\psi}_q(-y/2) \underbrace{\gamma^+}_{\text{blue}} \psi_q(y/2) | p \rangle \Big|_{y^+ = \vec{y}_\perp = 0}$$

$$= \mathbf{H}^q(\mathbf{x}, \xi, \mathbf{t}) \bar{u}(p') \gamma^+ u(p) + \mathbf{E}^q(\mathbf{x}, \xi, \mathbf{t}) \bar{u}(p') i\sigma^{+\nu} \frac{\Delta_\nu}{2M_n} u(p)$$

**Helicity :** 
$$\frac{P^+}{2\pi} \int dy^- e^{ixP^+y^-} \langle p' | \bar{\psi}_q(-y/2) \underbrace{\gamma^+ \gamma_5}_{\text{blue}} \psi_q(y/2) | p \rangle \Big|_{y^+ = \vec{y}_\perp = 0}$$

$$= \tilde{\mathbf{H}}^q(\mathbf{x}, \xi, \mathbf{t}) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{\mathbf{E}}^q(\mathbf{x}, \xi, \mathbf{t}) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p),$$

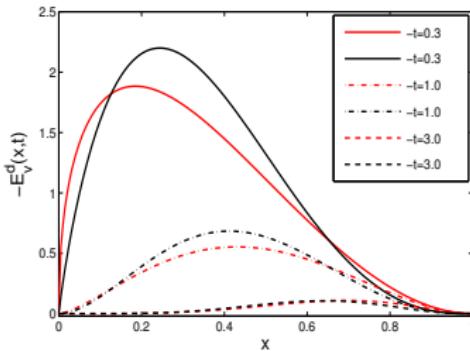
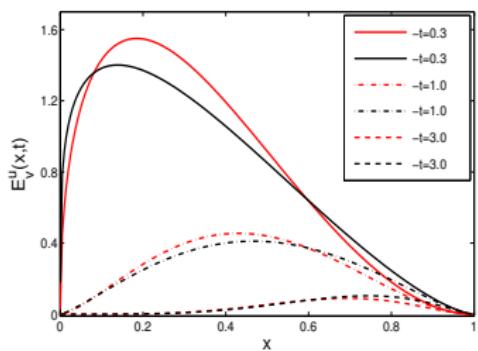
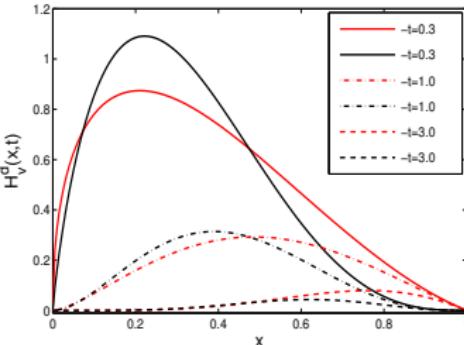
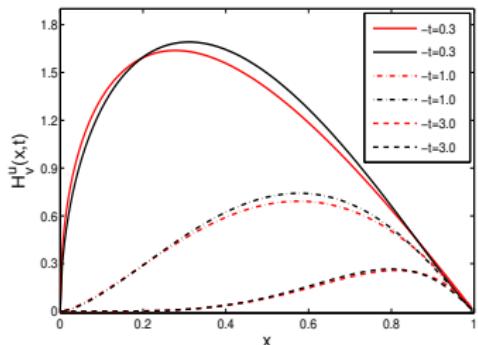
**Transversity :** 
$$\frac{P^+}{2\pi} \int dy^- e^{ixP^+y^-} \langle p' | \bar{\psi}_q(-y/2) \underbrace{\sigma^{+i}}_{\text{blue}} \gamma_5 \psi_q(y/2) | p \rangle \Big|_{y^+ = \vec{y}_\perp = 0}$$

$$= \bar{u}(p', \lambda') \left[ \mathbf{H}_T^q \sigma^{+i} \gamma_5 + \tilde{\mathbf{H}}_T^q \frac{\epsilon^{+i\alpha\beta} \Delta_\alpha P_\beta}{M^2} \right.$$

$$\left. + \mathbf{E}_T^q \frac{\epsilon^{+i\alpha\beta} \Delta_\alpha \gamma_\beta}{2M} + \tilde{\mathbf{E}}_T^q \frac{\epsilon^{+i\alpha\beta} P_\alpha \gamma_\beta}{M} \right] u(p, \lambda),$$

# Unpolarized GPDs [ $\xi = 0$ ]

[CM, Chakrabarti, EPJC 75 (2015)]



Red lines  $\Rightarrow$  AdS/QCD [Chakrabarti, CM PRD 88 (2013)]

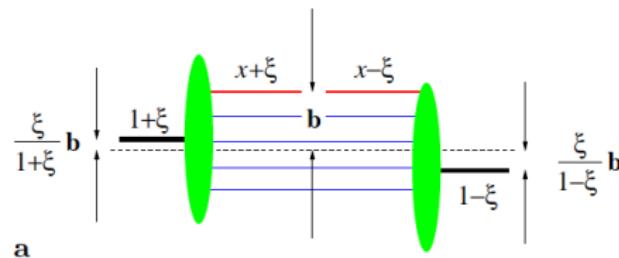
black lines  $\Rightarrow$  quark-diquark model inspired by AdS/QCD

## GPDs in transverse position space:

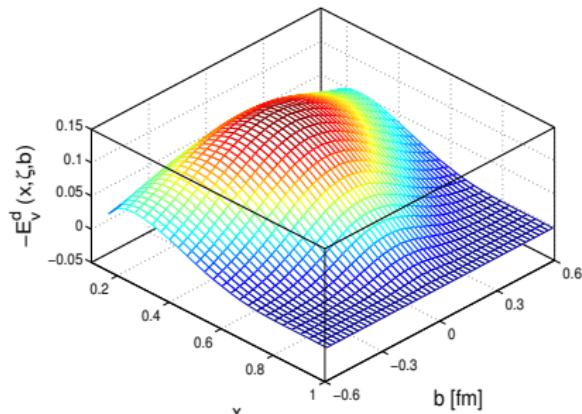
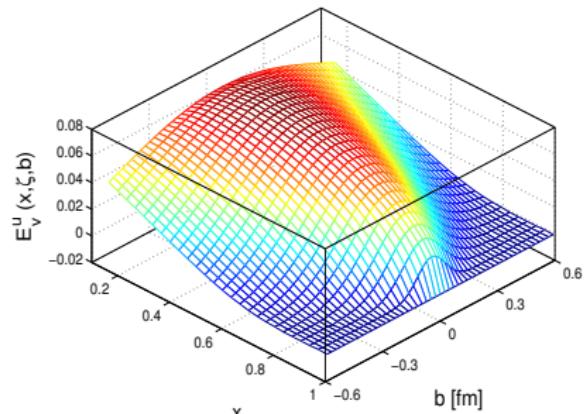
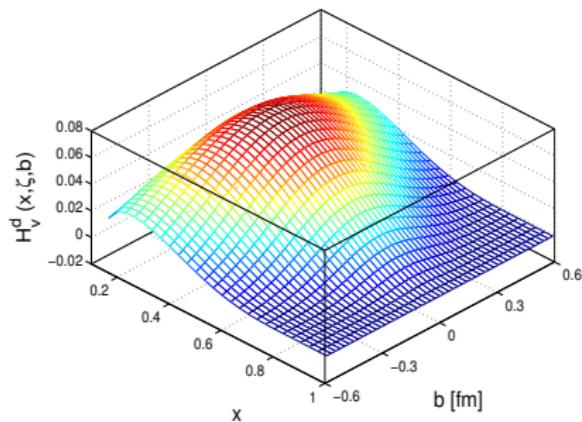
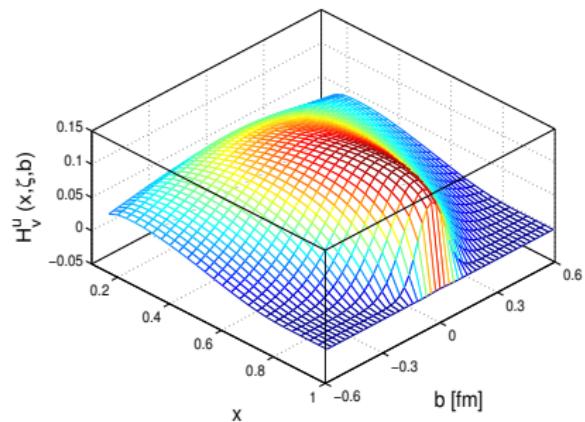
GPDs in transverse impact parameter space are defined by a two-dimensional Fourier transform in  $\Delta_{\perp}$ :

$$\mathcal{X}(x, \xi, b_{\perp}) = \frac{1}{(2\pi)^2} \int d^2 \Delta_{\perp} e^{-i \Delta_{\perp} \cdot b_{\perp}} \mathcal{X}(x, \xi, t),$$

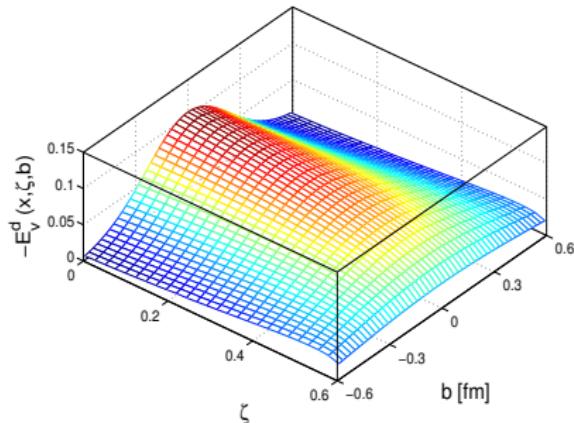
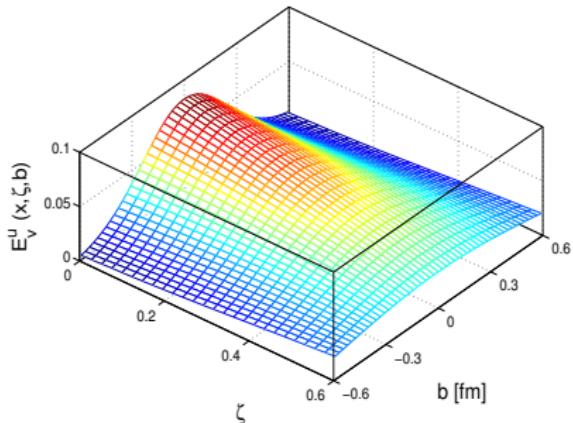
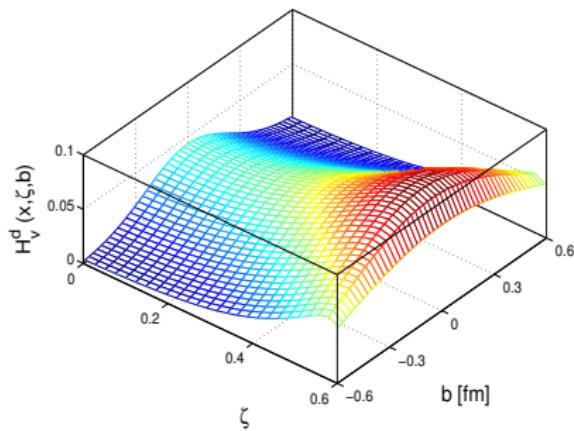
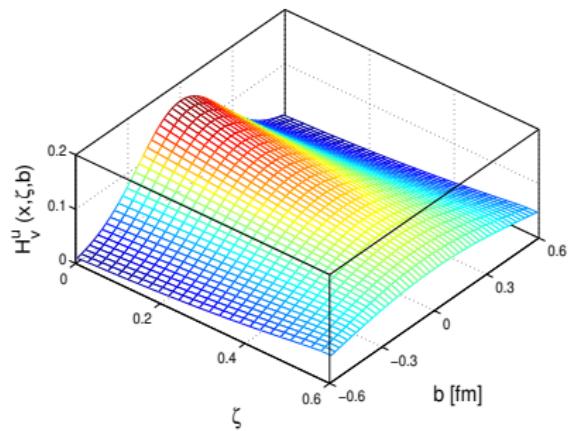
where  $\Delta_{\perp}^2 = -t(1 - \xi) - M_n^2 \xi^2$ .



# Unpolarized GPDs in transverse position space: $\xi = 0.15$

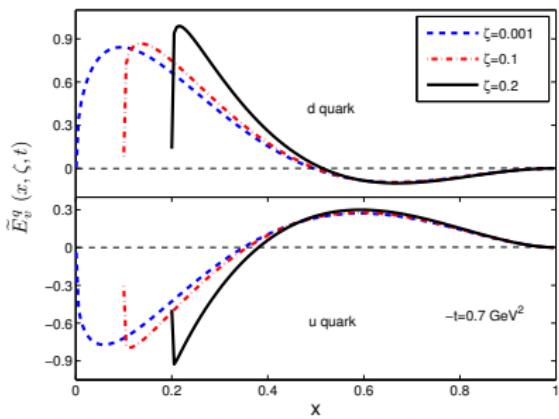
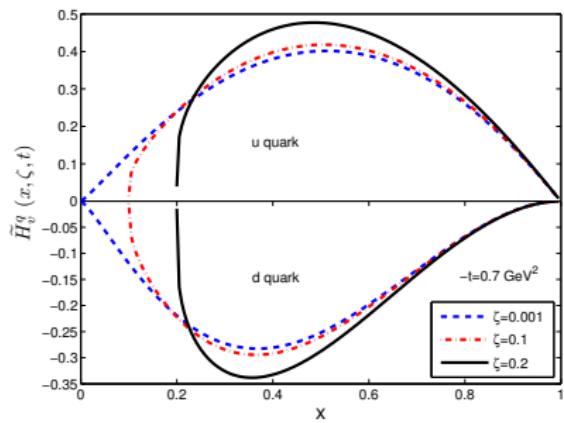


# Unpolarized GPDs in transverse position space: $x = 0.6$



# Helicity-dependent GPDs for $x > \zeta$

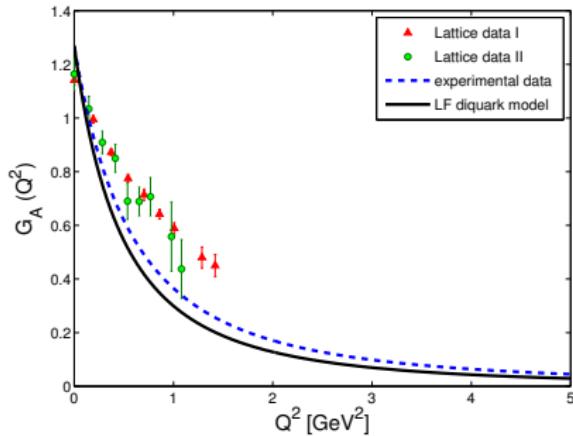
[to appear in EPJC]



# Axial form factor

[to appear in EPJC]

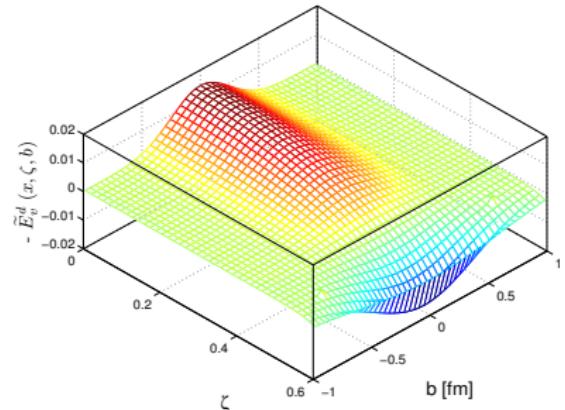
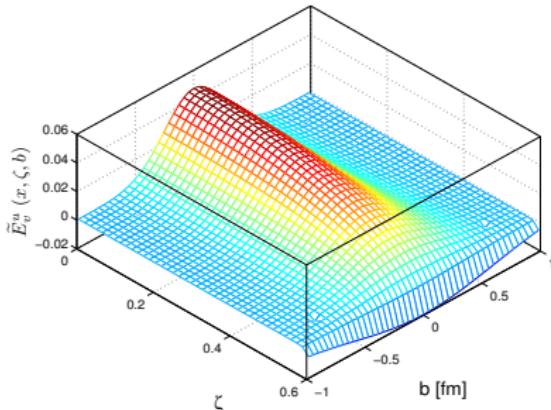
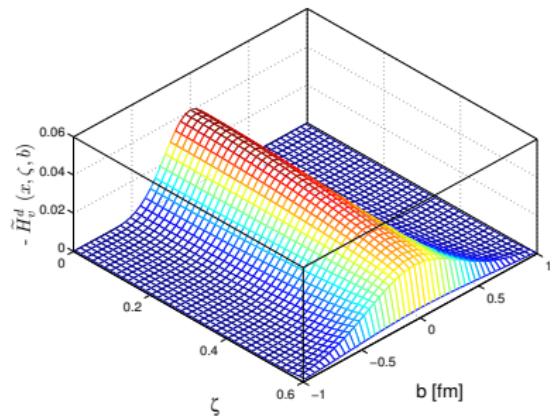
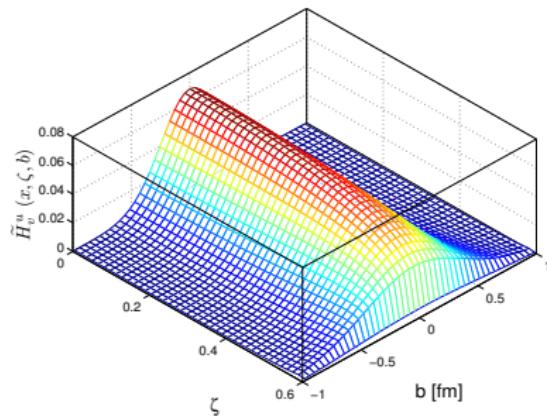
$$G_A(Q^2) = \int_0^1 dx [\tilde{H}_v^u(x, 0, t) - \tilde{H}_v^d(x, 0, t)],$$



$$G_A(Q^2) = \frac{g_A}{(1 + Q^2/M_A^2)^2}$$

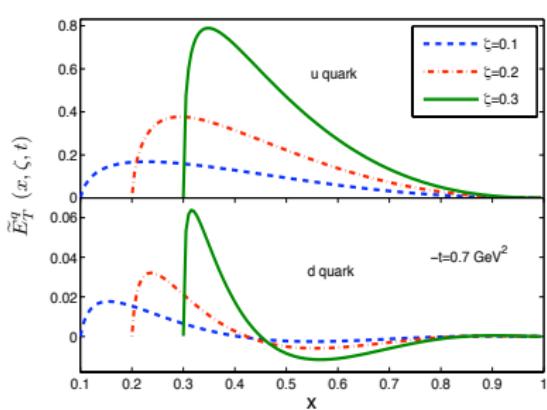
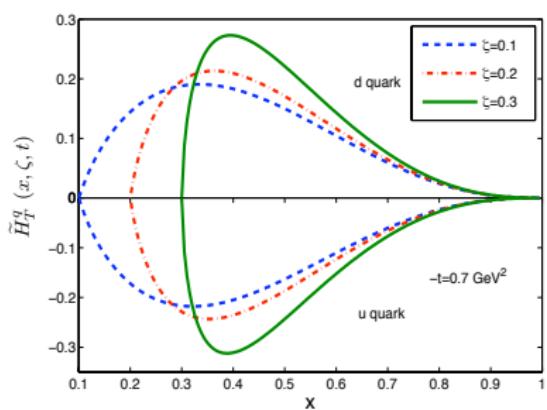
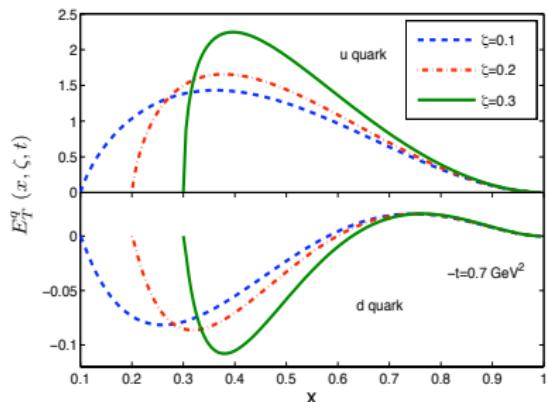
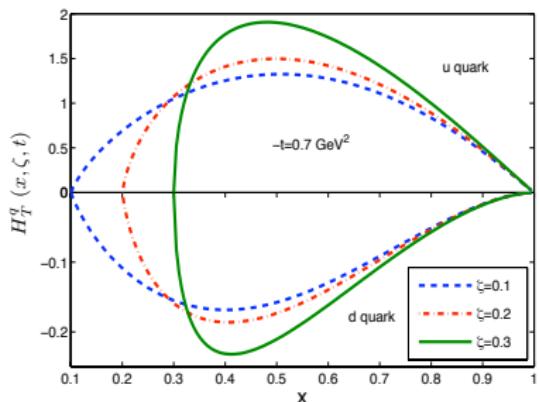
$g_A = 1.2673$  and  $M_A = 1.069$  GeV. [Alexandrou et. al. PRD 88, (2013)]

# Helicity GPDs in transverse position space: $x = 0.6$



## Chiral-odd GPDs for $x > \zeta$

[Chakrabarti, CM PRD 92 (2015)]

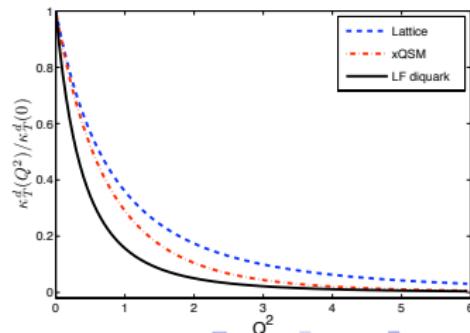
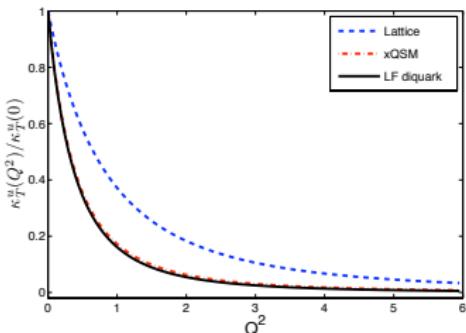
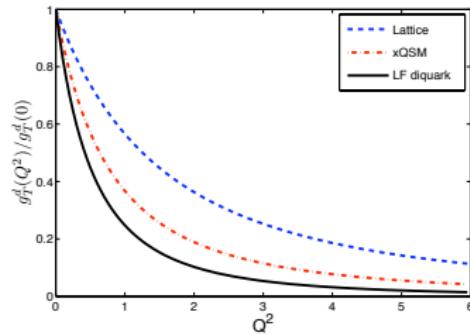
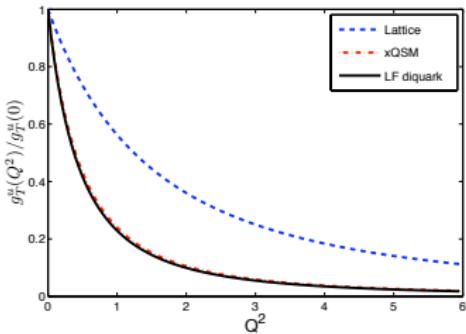


# Tensor form factors

[Chakrabarti, CM PRD 92 (2015)],

[lattice: Gckeler et. al PRL 98 (2007); xQSM: Ledwig-Kim PRD 85 (2012)]

$$g_T^q(t) = \int_0^1 dx H_T^q(x, 0, t), \quad k_T^q(t) = \int_0^1 dx (E_T^q + 2\tilde{H}_T^q)$$



# Proton spin densities

- ▶ For  $\xi = 0$ , GPDs in the impact parameter space  $\Rightarrow$  densities of quarks with longitudinal momentum fraction  $x$  and transverse location  $\mathbf{b}_\perp$ .
- ▶ The densities are given as

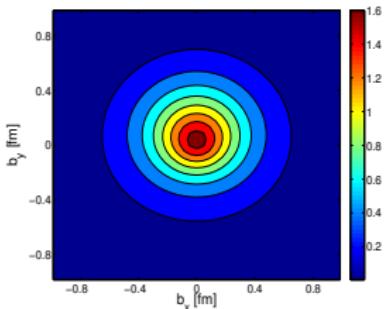
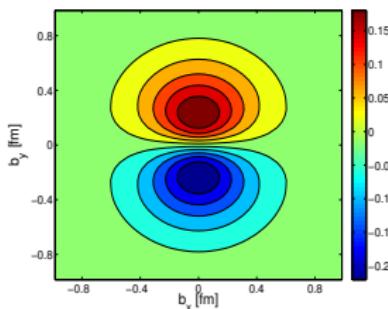
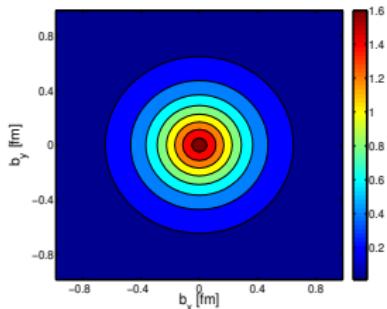
$$\rho(x, \mathbf{b}_\perp, \lambda, \Lambda) = \frac{1}{2} \left[ H(x, b^2) + b^j \varepsilon^{ji} S^i \frac{1}{M} E'(x, b^2) + \lambda \Lambda \tilde{H}(x, b^2) \right],$$

$$\begin{aligned} \rho(x, \mathbf{b}_\perp, s, S) &= \frac{1}{2} \left[ H + s^i S^i \left( H_T - \frac{1}{4M^2} \Delta_b \tilde{H}_T \right) \right. \\ &\quad + \frac{b^j \varepsilon^{ji}}{M} \left( S^i E' + s^i [E'_T + 2\tilde{H}'_T] \right) \\ &\quad \left. + s^i (2b^i b^j - b^2 \delta_{ij}) S^j \frac{1}{M^2} \tilde{H}''_T \right], \end{aligned}$$

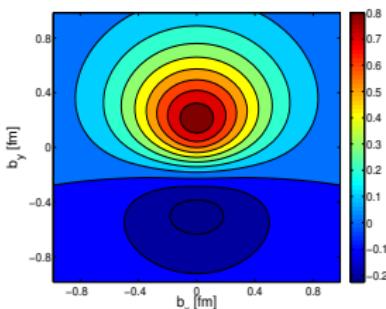
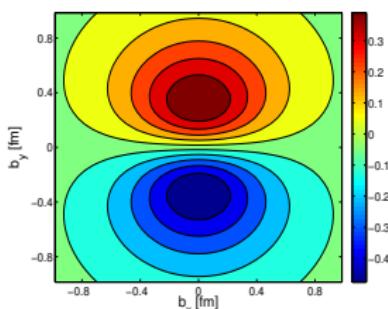
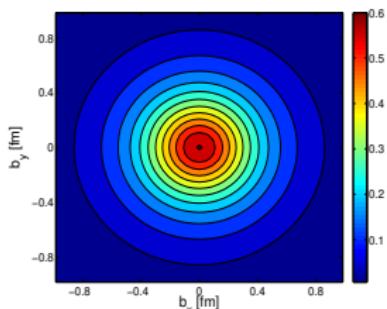
Shorthand notations  $f' = \frac{\partial}{\partial b^2} f$ ,  $f'' = \left( \frac{\partial}{\partial b^2} \right)^2 f$ ,  $\Delta_b f = \frac{\partial}{\partial b^i} \frac{\partial}{\partial b^i} f$   
[Diehl-Hagler, EPJC 44, 87 (2005); Pasquini-Boffi, PLB 653, 23 (2007)]

# Unpolarized proton & polarized quark( $\hat{x}$ )

*u quark*



*d quark*



$$\frac{1}{2}H$$

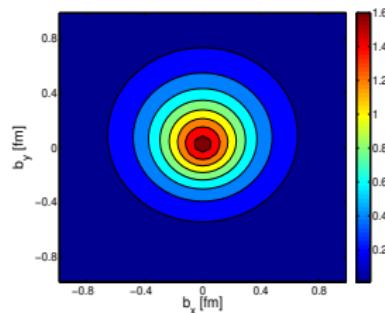
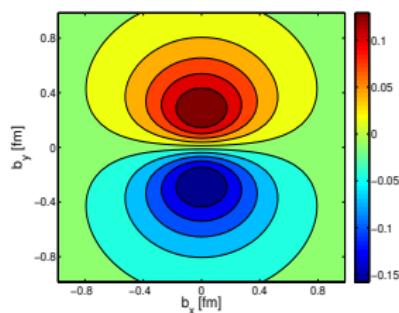
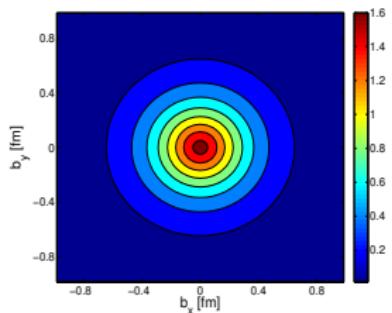
$$-\frac{1}{2}\mathbf{s}_x b_y (E'_T + 2\tilde{H}'_T)/M$$

*sum*

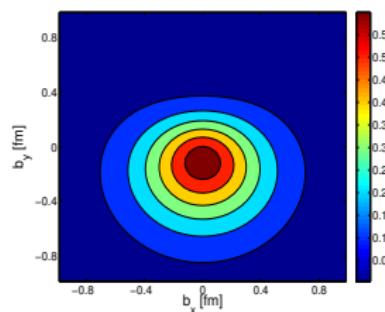
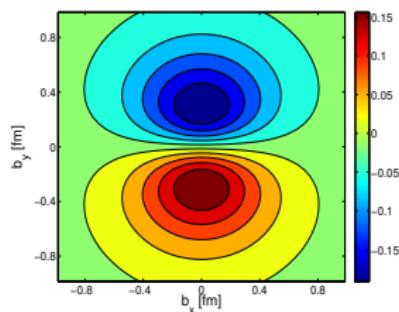
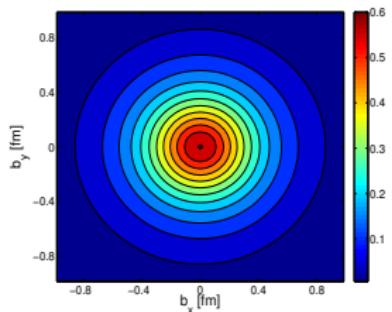
[*T. Maji, CM, D. Chakrabarti PRD 96, 013006 (2017)*]

# Polarized proton( $\hat{x}$ ) & unpolarized quark

*u* quark



*d* quark



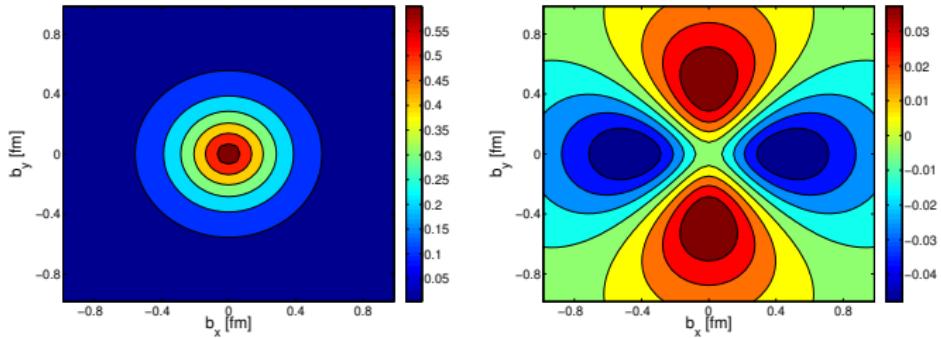
$$\frac{1}{2}H$$

$$-\frac{1}{2}S_x b_y E' / M$$

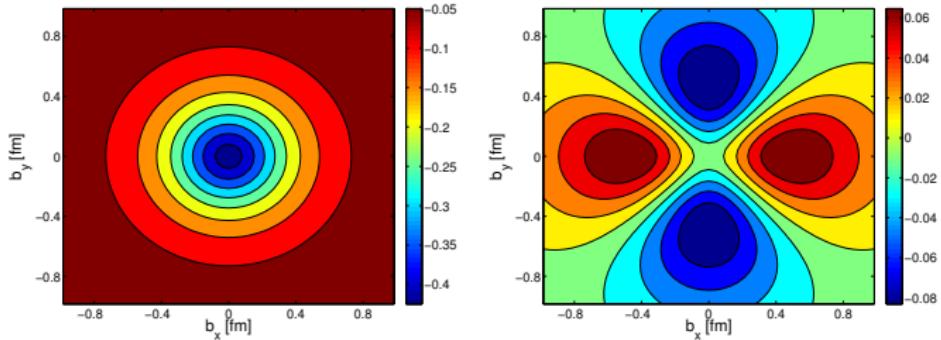
*sum*

# Polarized proton( $\hat{x}$ ) & polarized quark( $\hat{x}$ )

*u quark*



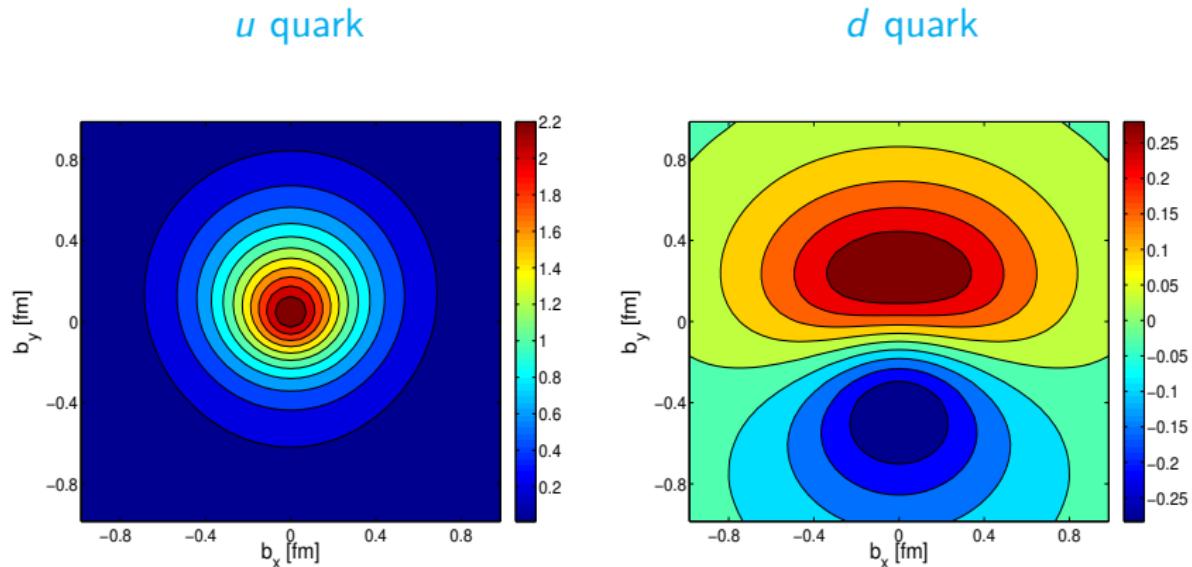
*d quark*



$$\frac{1}{2} s_x S_x (H_T - \Delta_b \tilde{H}_T / 4M^2)$$

$$\frac{1}{2} s_x S_x (b_x^2 - b_y^2) \tilde{H}_T'' / M^2$$

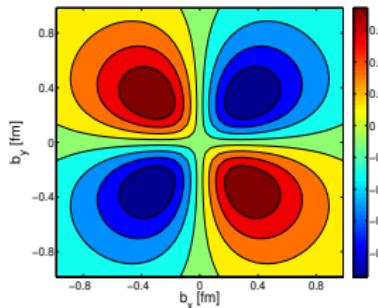
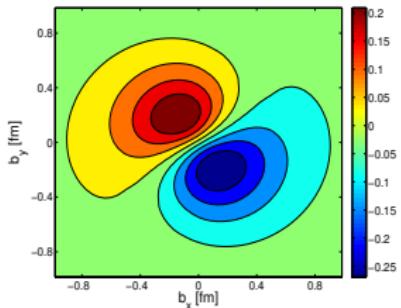
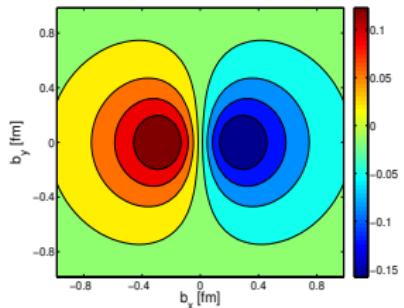
# Polarized proton( $\hat{x}$ ) & polarized quark( $\hat{x}$ )



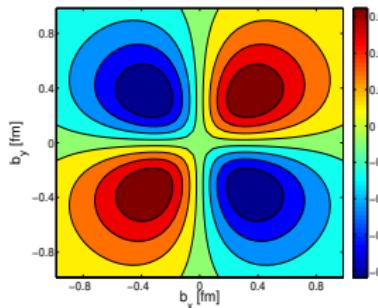
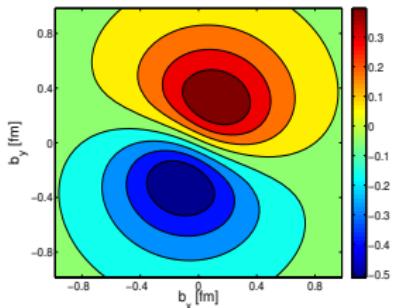
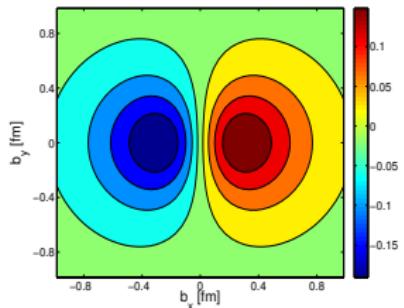
Total spin distribution  $\rho(\mathbf{b}_\perp, \mathbf{s}_x, \mathbf{S}_x)$ :  
*monopole* + *dipole* + *quadrupole*

# Polarized proton( $\hat{y}$ ) & polarized quark( $\hat{x}$ )

*u* quark



*d* quark



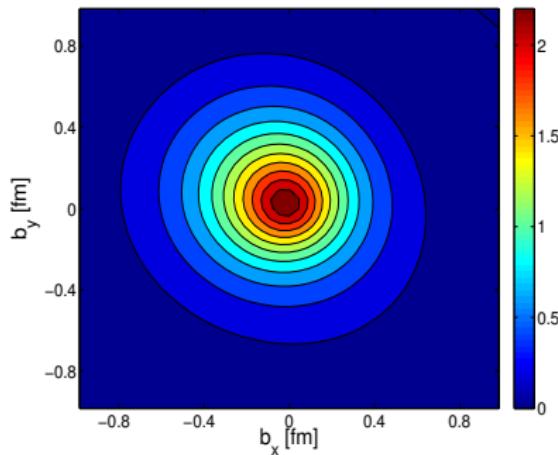
$$\frac{1}{2} S_y b_x E' / M$$

$$\frac{1}{2} [S_y b_x E' - s_x b_y (E'_T + 2\tilde{H}'_T)] / M$$

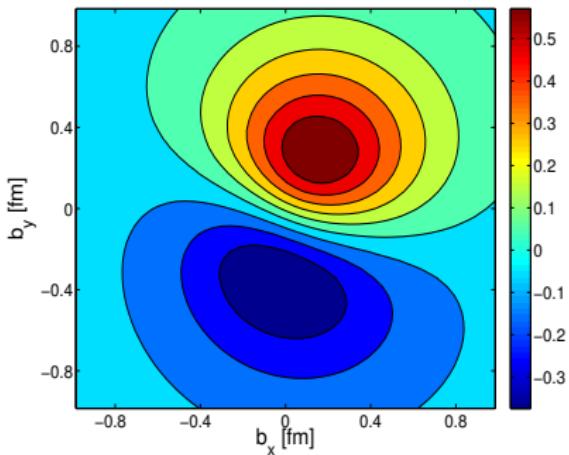
$$s_x S_y b_x b_y \tilde{H}''_T / M^2$$

# Polarized proton( $\hat{y}$ ) & polarized quark( $\hat{x}$ )

*u* quark



*d* quark



Total spin distribution  $\rho(\mathbf{b}_\perp, \mathbf{s}_x, \mathbf{S}_y)$ :  
*monopole* + *dipole* + *quadrupole*

# Conclusions

- ▶ Using a recently proposed light-front quark-diquark model for the proton we have presented FFs and both the chiral even and odd leading twist GPDs.
- ▶ The spin densities for different proton polarizations have been presented. Though for longitudinally polarized proton, only the chiral even GPDs contribute, for transversely polarized proton both chiral even and odd GPDs and their derivatives are required to study the spin densities.
- ▶ Our study reveals how different GPDs are contributing to the proton spin densities for different polarizations of the quark and proton. Monopole, dipole and quadrupole contributions to the spin densities are shown separately.
- ▶ For  $u$  quark, monopole contribution to spin density is large and dipole and quadrupole distortions are relatively small, whereas for  $d$  quark the distortions are found to be significantly large.

*Thank You*