Relativistic studies of few-body systems using the Bethe-Salpeter approach

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Outline

General goals

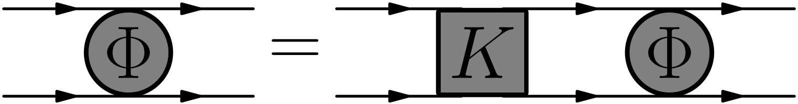
- Bethe-Salpeter equation to study non-perturbative systems;
- Fully covariant relativistic description in Minkowski space;
- Understand step-by-step the degrees of freedom within the used tools;
- Make feasible the numerics - probably the biggest challenge!
- Is the valence enough and how higher Fock contributions appears?;
- How bad is to ignore the crosses in the BSE kernel?
- Introducing color factors and the large N_c limit;
- Phenomenological approaches based on BSE and LFD;

Bethe-Salpeter equation

- The BSE for the bound state with total four momentum $p^2 = M^2$, composed of two scalar particles of mass m reads

$$\Phi(k,p) = S(p/2+k)S(p/2-k)\int \frac{d^4k'}{(2\pi)^4} iK(k,k',p)\Phi(k',p),$$

$$S(k) = \frac{i}{k^2 - m^2 + i\epsilon} \quad : \text{Feynman propagator}$$



- The kernel K is given as a sum of irreducible Feynman diagrams (ladder, cross-ladder, etc).

E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1232 (1951)

N. Nakanishi, Graph Theory and Feynman Integrals (Gordon and Breach, New York, 1971)

Nakanishi integral representation

- General representation for N-leg transition amplitudes;
- 2-point correlation function: Kallen-Lehmann spectral representation
- For the vertex function (Bound state) - 3-leg amplitude:

$$\Phi(k,p) = \int_{-1}^{1} dz' \int_{0}^{\infty} \frac{g(\gamma', z')}{(\gamma' + \kappa^2 - k^2 - (p \cdot k) z' - i\epsilon)^3}, \quad \kappa^2 = m^2 - M^2/4$$

- All dependence upon external momenta in the denominator;
- Allows to recognize the singular structure and deal with it analitically;
- Weight function $g(\gamma', z')$ is the unknown quantity to be determined numerically

T. Frederico, G. Salme and M. Viviani, Phys. Rev. D 85, 036009 (2012)

Light-front projection

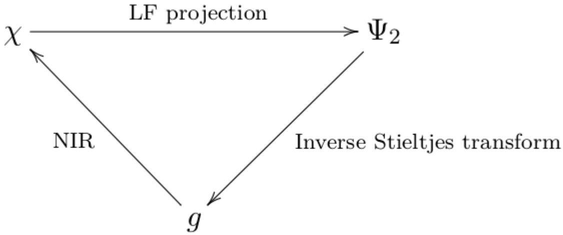
- Much easier to treat Minkowski space poles properly;
- Simpler dynamics of the propagators/amplitudes within LF (See talk by Prof. Ji)
- Easy connection with LFWF:
  - Introduce the LF variables $k_{\pm} = k_0 \pm k_z$
  - Valence LFWV from the BS amplitude:

$$\psi_{n=2/p}(\xi, \mathbf{k}_\perp) = \frac{p^+}{\sqrt{2}}\, \xi\, (1-\xi) \int_{-\infty}^{\infty} \frac{dk^-}{2\pi}\, \Phi(k,p),$$

  - Corresponding to eliminate the relative LF time $t+z=0$;
- Essential in this approach to solve BSE directly in Minkowski space;

Relations: LF, NIR and BS amplitude

- The Nakanishi integral representation (NIR) gives the Bethe-Salpeter amplitude χ (BSA) through the weight function g;
- The Light-Front projection of the BSA gives the valence light-front wave function (LFWF) Ψ_2;
- The inverse Stieltjes transform gives g from the valence LFWF (talk by Prof. Karmanov);



Valerio Gherardi, Master Thesis (2017)

Three-body within LF framework

- The Faddeev component of the three-body vertex function reads:

$$\Gamma(k_\perp, x) = F(M_{12}) \frac{1}{(2\pi)^3} \int_0^{1-x} \frac{dx'}{x'(1-x-x')} \int_0^\infty \frac{d^2 k'_\perp}{M_0^2 - M_3^2} \, \Gamma\left(k'_\perp, x'\right), \tag{1}$$

  - $F(M_{12})$ is the two-body zero-range scattering amplitude;
  - $M_{12}^2 = (1-x)M_3^2 - \frac{k_\perp^2 + (1-x)m^2}{x}$ is two-body effective mass;
  - M_0^2 is the invariant mass squared of the intermediate three-body state:

$$M_0^2 \quad = \quad \frac{\vec{k'}_\perp^2 + m^2}{x'} + \frac{\vec{k}_\perp^2 + m^2}{x} + \frac{(\vec{k'}_\perp + \vec{k}_\perp)^2 + m^2}{1-x-x'}. \tag{2}$$

- Is that enough or we need higher Fock contributions?

T. Frederico, Phys. Lett. B **282** (1992) 409; J. Carbonell, V.A. Karmanov, Phys. Rev. C **67** (2003) 037001.

Three-body within covariant BS equation

- The vertex function for the three-body BS equation reads

$$v(q,p) = 2iF(M_{12}) \int \frac{d^4k}{(2\pi)^4} \frac{i}{[k^2 - m^2 + i\epsilon]} \frac{i}{[(p-q-k)^2 - m^2 + i\epsilon]} v(k,p)$$

- $M_{12}^2 = (p-q)^2$.

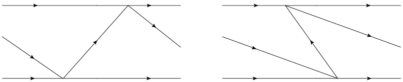


Figure: The three-body LF graphs obtained by time-ordering of the Feynman graph.

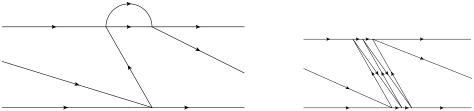


Figure: Examples of many-body intermediate state contributions to the LF three-body forces.

Results: Binding energies

- Solving the BSE by means of the Wick-rotation and comparing the solution with the LFD:

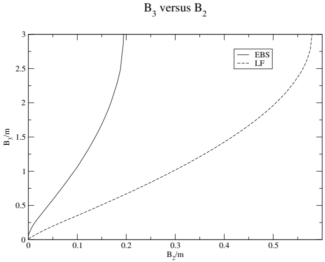
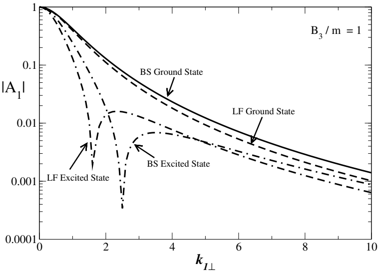


Figure: The binding energy B_3 for the first excited state vs. the two-body binding energy B_2. The solid curve is computed solving the Euclidean BS equation. The dashed curve is computed solving the LF equation.

- Additional contributions $\rightarrow$ like effective three-body force of relativistic origin.
- Comparison between BS and LFD through transverse amplitudes (talk by Prof. Frederico):

Transverse amplitudes



- Significant impact from the extra contributions also shown on the structure;
- Minkowski calculation
  - By direct integration of the poles: Very hard numerically;
  - NIR + LF projection seems to be essential!
  - Other methods in literature were too complicated to go further;

K. Kusaka, K. Simpson, and A. G. Williams, Phys. Rev. D 56, 5071 (1997).

E. Ydrefors, JHAN, V. Gigante, V. Karmanov and T. Frederico Phys. Lett. B 770 (2017) 131-137

Phenomenological application of LF framework

- $B^+ \to K^-\pi^+\pi^+$ decay amplitude + three-body final state interactions
- The full decay amplitude considering interactions between all the final states mesons reduces to

$$\mathcal{A}_0(k_i,k_j) = B_0(k_i,k_j) + \sum_\alpha \tau(s_\alpha)\xi^\alpha(k_\alpha)\ , \tag{3}$$

where the subindex in $\mathcal{A}_0$ denotes the s-wave two-meson scattering and the bachelor amplitude $\xi(k_i)$ carries the three-body rescattering effect and is represented by the connected Faddeev-like equations

$$\begin{aligned}\xi^i(k_i) &= \xi^i_0(k_i) + \int \frac{d^4q_j}{(2\pi)^4} S_j(q_j)S_k(K-k_i-q_k)\tau_j(s_j)\xi^j(q_j) \\ &+ \int \frac{d^4q_k}{(2\pi)^4} S_j(K-k_i-q_k)S_k(q_k)\tau_k(s_k)\xi^k(q_k).\end{aligned} \tag{4}$$

with $q_k = K - k_i - q_j$.

Perturbative solution for the Faddeev component

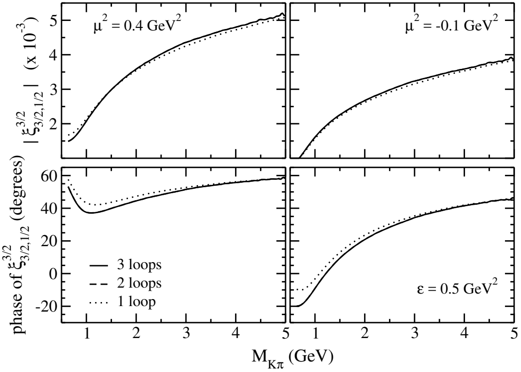


Figure: Modulus and phase of $\xi^{3/2}_{3/2,1/2}$ for $\varepsilon = 0.5\ \mathrm{GeV}^2$, $\mu^2 = 0.4\ \mathrm{GeV}^2$ (left) and $\mu^2 = -0.1\ \mathrm{GeV}^2$ (right).

- That is probably not enough without higher Fock contributions;

J.H.A.N., T. Frederico and O. Lourenço, Few Body Syst. 58 (2017) no.2, 98

BSE in Minkowski space

- Nakanishi integral representation:

$$\Phi(k,p) = \int_{-1}^{1} dz' \int_0^{\infty} \frac{g(\gamma', z')}{(\gamma' + \kappa^2 - k^2 - (p\cdot k)z' - i\epsilon)^3}, \quad \kappa^2 = m^2 - M^2/4$$

- Light-Front projection:
  - Introduce the LF variables $k_{\pm} = k_0 \pm k_z$
  - Relative LF time $t + z = 0$
  - Applying the NIR on both sides of the BSE and integration over k_- leads to the integral equation

$$\int_0^{\infty} \frac{g(\gamma', z) d\gamma'}{[\gamma + \gamma' + z^2 m^2 + (1 - z^2)\kappa^2]^2} = \int_0^{\infty} d\gamma' \int_{-1}^{1} dz' V(\gamma, z, \gamma', z') g(\gamma', z')$$

where V is expressed in terms of the BS kernel K

V. A. Karmanov and J. Carbonell, Eur. Phys. J. A 27, 1 (2006)

Coupling constants

- Solving the generalized eigenvalue problem for both L and L+CL kernels:

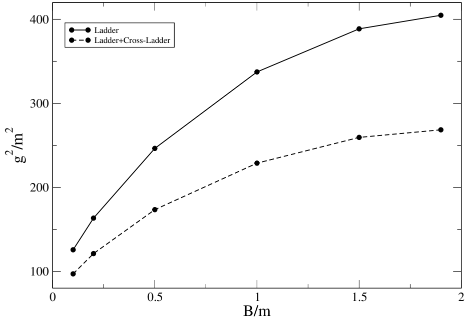


Figure: Coupling constant for various values of the binding energy B obtained by using the Bethe-Salpeter ladder (L) and ladder plus cross-ladder (CL) kernels, for an exchanged mass of $\mu = 0.5m$.

- Effect of only one cross-graph is huge!

Coupling constants and LF wave functions

B/m	μ/m	$\alpha^{(\mathrm{L+CL})}$	$\alpha^{(\mathrm{L})}$	$\alpha^{(\mathrm{L})}/\alpha^{(\mathrm{L+CL})}$	$\Psi_{\mathrm{LF}}^{(\mathrm{L})}/\Psi_{\mathrm{LF}}^{(\mathrm{L+CL})}$
1.5	0.15	4.1399	6.2812	1.5172	1.5774
	0.50	5.1568	7.7294	1.4988	1.5395
1.0	0.15	3.5515	5.3136	1.4961	1.5508
	0.50	4.5453	6.7116	1.4766	1.5094
0.5	0.15	2.5010	3.6106	1.4436	1.4805
	0.50	3.4436	4.9007	1.4231	1.4405
0.1	0.15	1.1052	1.4365	1.2997	1.2763
	0.50	1.9280	2.4980	1.2956	1.2694

- The ratios $\alpha^{(\mathrm{L})}/\alpha^{(\mathrm{L+CL})}$ and $\Psi_{\mathrm{LF}}^{(\mathrm{L})}/\Psi_{\mathrm{LF}}^{(\mathrm{L+CL})}$ are almost the same for given B and μ.
- Information about the asymptotic behavior of Ψ_{LF} for the complete kernel can be deduced from the coupling constant (which can be obtained in Euclidean space).

V. Gigante, JHAN, E. Ydrefors, C. Gutierrez, V. Karmanov, T. Frederico, Phys. Rev. D 95, (2017) 056012

Full set of crossed graphs

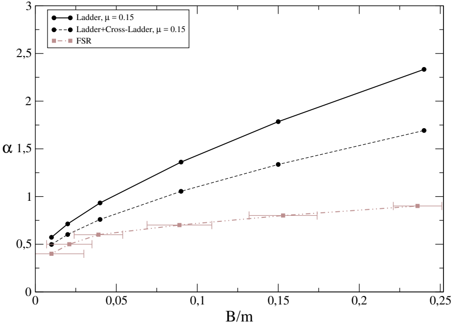
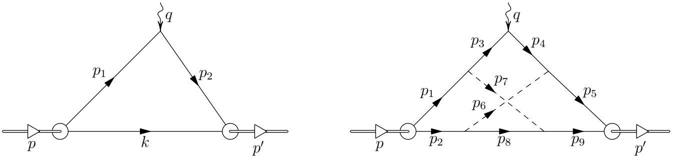


Figure: Coupling constant versus B fot the BSE with ladder (L) and ladder plus cross-ladder (CL) kernels, for an exchanged mass of $\mu = 0.15m$. Calculation within Feynman-Schwinger representation considering all crossed-ladder graphs is also presented.

- We can use information from the coupling to predict the tail of the LFWV.

T. Nieuwenhuis and J. A. Tjon, Phys. Rev. Lett. 77 (1996) 814

Elastic electromagnetic form factor



- Electromagnetic current for spinless system

$$J_\mu = (p_\mu + p'_\mu)F_1(Q^2) + (p_\mu - p'_\mu)F_2(Q^2), \quad F_2 = 0 \quad \text{(elastic case)}$$

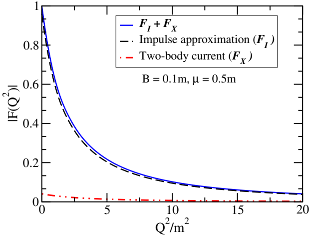
- Impulse-approximation contribution (one-body type)

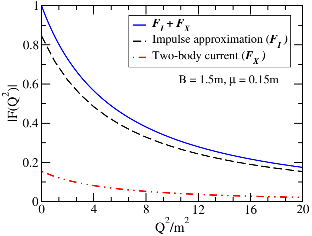
$$F_I(Q^2) = \frac{1}{2^7\pi^3} \int_0^\infty d\gamma \int_{-1}^1 dz g(\gamma,z) \int_0^\infty d\gamma' \int_{-1}^1 dz' g(\gamma',z') \times \int_0^1 dy y^2(1-y)^2 \frac{f_{\text{num}}}{f_{\text{den}}^4}$$

- Two-body current contribution

$$F_X(Q^2) = -\frac{3\alpha^2 m^4}{(2\pi)^5}\int_0^\infty d\gamma \int_{-1}^1 dz \int_0^\infty d\gamma' \int_{-1}^1 dz' \prod_{i=1}^6 \int_0^1 dy_i$$

$$\Theta(1-\sum_{j=i+1;i<4} y_j)(1-y_5)^2 y_5^2 (1-y_6)^2 y_6^3 \frac{f^X_{\text{num}}}{[f^X_{\text{den}}]^5} g(\gamma,z) g(\gamma',z')$$





- The two-body current diagram has a significant contribution for large B.
- The impulse-approximation contribution dominates for large values of Q^2. Asymptotics: $F_I(Q^2) \sim Q^{-4}$, $F_X(Q^2) \sim Q^{-6}$

Large-N_c limit

- 't Hooft - A planar diagram theory for strong interactions;
  - In the $N_c \to \infty$ limit non-planar graphs are suppressed in QCD_{1+1};
- Light-cone-quantized QCD_{1+1}
  - QCD light-cone Hamiltonian diagonalized;
  - Numerical test of approximation for large-N_c expansion;
  - Able to produce hadron spectrum, wave functions, ...
- We can introduce the color degree of freedom in our BS kernel;
- Numerical test in a simple 3+1 dynamical model of the large-N_c approximation;
  - How big N_c is needed to suppress the huge effect from the crossed-kernel?

G.'t Hooft, Nucl. Phys. B 75 (1974) 461

K. Hornbostel, S. J. Brodsky and H.-C. Pauli, Phys. Rev. D 41 (1990) 3814.

Coupling constants with color factor

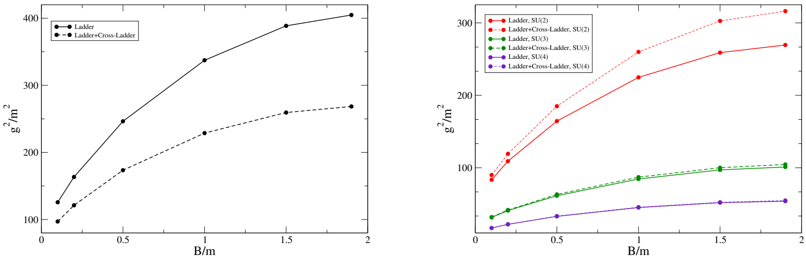


Figure: Coupling constant for various values of the binding energy B obtained by using the Bethe-Salpeter ladder (L) and ladder plus cross-ladder (CL) kernels, for an exchanged mass of $\mu = 0.5m$. In the upper panels are shown the results computed with no color factors. Similarly, in the lower panels are compared the results for $N = 2, 3$ and 4 colors.

- Suppression is already pretty good for $N_c = 3$ - that may support ladder truncations...at least within this system.

JHAN, C.-R. Ji, E. Ydrefors and T. Frederico, in preparation

Suppression in the LFWF

- Checking also the suppression in the structure:

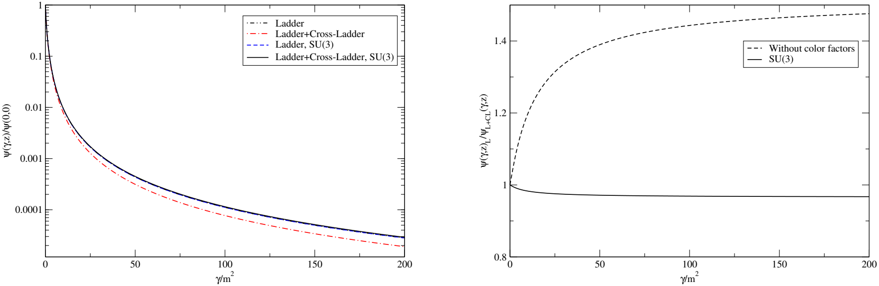


Figure: (left) Valence LFWF as a function of $\gamma = k_\perp^2$ computed with the BSA for L and L+CL. Results for $N = 3$ colors are compared with the ones where no color factors are included. Used values of the input parameters: $B = 1.0m$, $\mu = 0.5m$ and $z = 0.0$. (right) Ratio between the valence wave functions calculated with the L and L+CL.

Outlook

- BSE in Minkowski space :Quark-diquark model for the nucleon;
- Suppression of non-planar diagrams in other systems;
  - Unequal mass case and BSE one-body limit;
- Solve Schwinger-Dyson equation in 3+1 directly in Minkowski space by means of spectral representations (project with Prof. Peter Maris);
  - Avoid Euclidean models (spacelike region) for the dressed-gluon propagator and quark-gluon vertex;
- Phenomenological applications using presented ideas to take into account higher-Fock contributions;
- Fermion-fermion system within BSE in Minkowski space was solved (talk by Prof. Frederico)
  - Spectrum, momentum distributions, form factors of hadrons and so on in this approach

Conclusions

- Three-body system with zero-range interaction:
  - Solved in both Euclidean BSE and LFD approaches;
  - Higher-Fock components leads to a much stronger binding compared to the LF truncated equation;
  - Higher-Fock components - effective three-body forces;
  - Minkowski space calculation through direct integration of the poles + numerical treatment is very tough;
- NIR and the LF projection $\rightarrow$ Very helpful to study the BSE in Minkowski;
  - Many systems already treated and many issues already solved;
  - Easy connection with LF dynamics;
  - No truncation on the Fock expansion;
  - Crossed graphs have huge effects;
- Color degree of freedom shows to suppress non-planar diagrams very well already for $N_c = 3$;
  - This may be a way to support ladder truncations;
  - It is almost impossible to consider more than one cross-ladder in pratical calculations;

Thank you!