

Dynamical spin effects in the pion

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Light Cone 2017 (LC2017)
Frontiers in Light Front Hadron Physics : Theory and Experiment

18th - 22nd September 2017, University of Mumbai



Overview

Light-front holography

Quark masses and spins

Predicting pion observables

Conclusions

M. Ahmady, F. Chishtie, R. Sandapen, PRD 95 074008 (2017)

The holographic light-front Schrödinger Equation

Brodsky & de Téramond (PRL, 09)

Brodsky, de Téramond, Dosch & Erlich (Phys. Rep. 15)

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U_{\text{eff}}(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)$$

- ▶ $\zeta^2 = x(1-x)b_\perp^2$: key light-front variable for bound states
- ▶ M^2 : hadron mass squared (same as in $H_{\text{QCD}}^{\text{LF}}|\Psi\rangle = M^2|\Psi\rangle$)
- ▶ U_{eff} : effective $q\bar{q}$ potential (includes coupling of valence sector with higher Fock sectors)

Valence meson light-front wavefunction :

$$\Psi(x, \zeta, \varphi) = \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} X(x) e^{iL\varphi}$$

Assumptions behind the holographic LFSE

- ▶ no quantum loops (no Λ_{QCD}) } semiclassical approximation
- ▶ massless quarks ($m \rightarrow 0$) }
- ★ no dynamical spin effects

- ▶ QCD action is conformally invariant in semiclassical approximation
- ★ Last assumption is implicit in the suppression of helicity indices

$$\Psi(x, \zeta, \varphi, \lambda, \lambda') \rightarrow \Psi(x, \zeta, \varphi)$$

A unique confinement potential

$$U_{\text{eff}}(\zeta) = \overbrace{\kappa^4 \zeta^2}^{\text{HO}} + \overbrace{2\kappa^2(J-1)}^{\text{spin term}}$$

- ▶ Harmonic Oscillator (HO) potential comes out uniquely in the way conformal symmetry is broken (to allow emergence of a mass scale κ) in semiclassical QCD
- ▶ AdS/QCD mapping : $\zeta \leftrightarrow z_5$ (z_5 is 5th dimension of AdS) gives
 1. Spin term in potential
 2. Longitudinal mode : $X(x) = \sqrt{x(1-x)}$ by matching EM or gravitational form factor in AdS and light-front QCD

Brodsky, de Téramond & Dosch (Phys. Lett. 13)

A massless pion

Solving the holographic LF Schrödinger Equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U_{\text{eff}}(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)$$

with

$$U_{\text{eff}}(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$$

gives

$$M^2 = (4n + 2L + 2)\kappa^2 + 2\kappa^2(J - 1)$$

- ▶ Regge trajectories
- ▶ Lightest bound state ($n = L = J = 0$) is massless ($M = 0$)
- ▶ Identify with pion (as expected in the χ -limit)

Fixing the fundamental AdS/QCD scale

Regge slopes global fits

Reference	Fit	κ [MeV]
Brodsky et al. [Phys. Rep. 15]	Mesons	540–590
Brodsky et al. [IJMD, 16]	Mesons & Baryons	523

Universal $\kappa \sim 500$ MeV successfully predicts

- ▶ Diffractive ρ production Forshaw & Sandapen (PRL, 12)
- ▶ Diffractive ϕ production Ahmady, Sandapen, Sharma (PRD, 16)
- ▶ Λ_{QCD} to 5-loops in $\overline{\text{MS}}$ Brodsky, Deur, de Téramond (J.Phys, 17)

Restoring dependence on quark masses

- In momentum space, 'complete' invariant mass of $q\bar{q}$ pair

$$M_{q\bar{q}}^2 = \frac{\mathbf{k}_\perp^2}{x(1-x)} \rightarrow \frac{\mathbf{k}_\perp^2 + m_f^2}{x(1-x)}$$

Brodsky, de Téramond, Subnuclear Series Proc. (09)

- Pion holographic wavefunction becomes

$$\Psi^\pi(x, \zeta^2) = \mathcal{N} \sqrt{x(1-x)} \exp\left[-\frac{\kappa^2 \zeta^2}{2}\right] \exp\left[-\frac{m_f^2}{2\kappa^2 x(1-x)}\right]$$

- m_f : effective quark masses (because of coupling of valence sector to higher Fock sectors)

Restoring dependence on quark spins

- In semiclassical approximation :

$$\Psi^\pi(x, \zeta^2, \lambda, \lambda') = \Psi^\pi(x, \zeta^2) \times S_{\lambda, \lambda'}^{\text{non dynamical}}$$

with

$$S_{\lambda, \lambda'}^{\text{non dynamical}} = \lambda \delta_{\lambda, -\lambda'}$$

- Normalization

$$\sum_{\lambda, \lambda'} \int d^2 \mathbf{b} dx |\Psi^\pi(x, \zeta^2, \lambda, \lambda')|^2 = P_{q\bar{q}}$$

$P_{q\bar{q}} \leq 1$: inequality allows for higher Fock sectors contribution

Earlier predictions for pion

Reference	κ [MeV]	$m_{u/d}$ [MeV]	$P_{q\bar{q}}$
Vega et al [PRD, 09]	787	330	0.279
Branz et al [PRD, 10]	550	420	0.6
Swarnkar & Chakrabarti [PRD, 15]	550	330	0.61

- ▶ Constituent quark masses with non-dynamical spin
- ▶ $P_{q\bar{q}} < 1$: important contribution of higher Fock sectors
- ▶ Special status for the pion compared to other light mesons

Dynamical spin effects

$$\Psi(x, \mathbf{k}_\perp) \rightarrow \Psi(x, \mathbf{k}_\perp, \lambda, \lambda') = \overbrace{\Psi(x, \mathbf{k}_\perp)}^{\text{Non pert.}} \times \overbrace{S_{\lambda\lambda'}(x, \mathbf{k}_\perp)}^{\text{Pert.}}$$

- ▶ For pion :

$$S_{\lambda\lambda'}^{\text{dynamical}}(x, \mathbf{k}_\perp) = \frac{\bar{v}_{\lambda'}(x, \mathbf{k}_\perp)}{\sqrt{1-x}} [A[\not{p}]_+ \gamma^5 + BM_\pi \gamma^5] \frac{u_\lambda(x, \mathbf{k}_\perp)}{\sqrt{x}}$$

instead of

$$S_{\lambda\lambda'}^{\text{non dynamical}} = \frac{1}{\sqrt{2}} \lambda \delta_{\lambda, -\lambda'}$$

- ▶ Note :

1. $[A = 1, B = 0]$: $S_{\lambda\lambda'}^{\text{dynamical}}(x, \mathbf{k}_\perp) \rightarrow S_{\lambda\lambda'}^{\text{non dynamical}}$
2. $B \neq 0$: going beyond the semiclassical approximation

Normalization, AdS/QCD scale and quark masses

To generate predictions for pion observables, we use

- ▶ the normalization condition

$$\sum_{\lambda, \lambda'} \int d^2 \mathbf{b}_\perp dx |\Psi^\pi(x, \zeta^2, \lambda, \lambda')|^2 = 1$$

- ▶ $\kappa = 523$ MeV and $m_{u/d} = 330$ MeV

Pion radius

$$\sqrt{\langle r_\pi^2 \rangle} = \left[\frac{3}{2} \int dx d^2\mathbf{b}_\perp [b_\perp(1-x)]^2 |\Psi^\pi(x, \mathbf{b}_\perp)|^2 \right]^{1/2}$$

TABLE I. Our predictions for the pion radius using the holographic wavefunction with $\kappa = 523$ MeV and $m_{u/d} = 330$ MeV. The datum is from PDG 2014 [53].

	$\sqrt{\langle r_\pi^2 \rangle}$ [fm]
Original	0.544
Spin-improved ($A = 0, B = 1$)	0.683
Spin-improved ($A = 1, B = 1$)	0.673
Experiment [53]	0.672 ± 0.008

Pion decay constant

$$\langle 0 | \bar{\Psi}_d \gamma^\mu \gamma_5 \Psi_u | \pi^+ \rangle = f_\pi P^\mu$$

$$f_\pi = 2\sqrt{\frac{N_c}{\pi}} \int dx \{ A((x(1-x)M_\pi^2) + Bm_f M_\pi) \} \frac{\Psi^\pi(x, \zeta)}{x(1-x)} \Big|_{\zeta=0}$$

TABLE II. Our predictions for the pion decay constant using the holographic wavefunction with $\kappa = 523$ MeV and $m_{u/d} = 330$ MeV. The datum is from PDG 2014 [53].

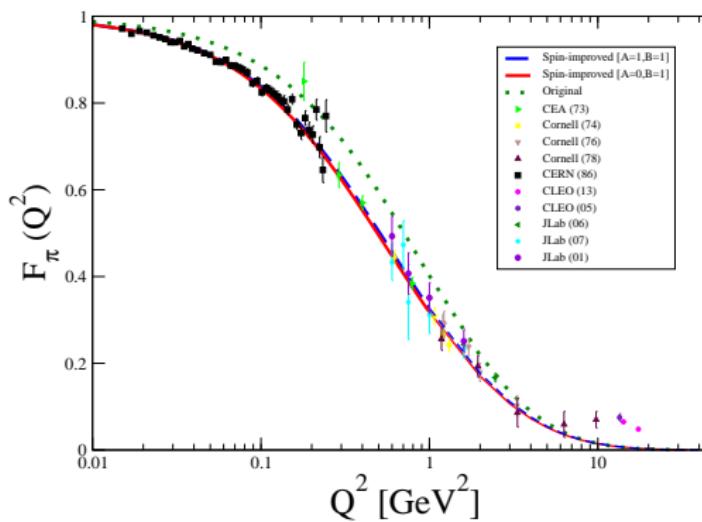
	f_π [MeV]
Original	161
Spin-improved ($A = 0, B = 1$)	135
Spin-improved ($A = 1, B = 1$)	138
Experiment [53]	$130.4 \pm 0.04 \pm 0.2$

Pion EM spacelike form factor

Drell & Yan (PRL, 70); West (PRL, 70)

$$\langle \pi^+ : P' | J_{\text{em}}^\mu(0) | \pi^+ : P \rangle = 2(P + P')^\mu F_\pi(Q^2)$$

$$F_\pi(Q^2) = 2\pi \int dx db_\perp b_\perp J_0[(1-x)b_\perp Q] |\Psi^\pi(x, \mathbf{b}_\perp)|^2$$



Pion Distribution Amplitude

$$\langle 0 | \bar{\Psi}_d(z) \gamma^+ \gamma_5 \Psi_u(0) | \pi^+ \rangle = f_\pi P^+ \int dx e^{ix(P \cdot z)} \varphi_\pi(x, \mu)$$

$$f_\pi \varphi_\pi(x, \mu) \propto \int db_\perp J_0(\mu b_\perp) b_\perp \{ A((x(1-x)M_\pi^2) + B m_f M_\pi) \} \frac{\Psi^\pi(x, \zeta)}{x(1-x)}$$

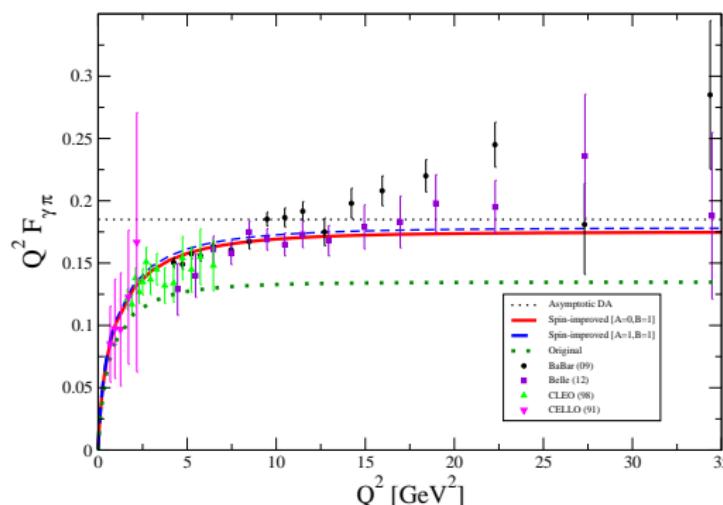
DA	μ [GeV]	$\langle \xi_2 \rangle$	$\langle \xi_4 \rangle$
Asymptotic	∞	0.2	0.085
LFH (original)	~ 1	0.151	0.050
LFH (spin-improved)	~ 1	0.199, 0.195	0.078, 0.076
Sum Rules	1	0.24	0.11
Lattice	2	0.2361(41)(39)	
Dyson-Schwinger[RL,DB]	2	0.280, 0.251	0.151, 0.128

Improved agreement with standard non-perturbative QCD methods

Photon-to-Pion transition form factor

Lepage & Brodsky, PRD (80)

$$F_{\gamma\pi}(Q^2) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 dx \overbrace{\frac{\varphi_\pi(x, xQ)}{Q^2 x}}^{\text{pion DA}}$$



Conclusions

- ▶ Systematic improvement in the description of all pion data when dynamical spin effects are taken into account
- ▶ Universal AdS/QCD scale with constituent light quark masses

Acknowledgements

- ▶ NSERC (Discovery Grant SAPIN-2017-00031) for funding



- ▶ Organizers of LC2017