Interpolating Quantum Electrodynamics

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LIGHT CONE 2017

Frontiers in Light Front Hadron Physics :
Theory and Experiment

In collaboration with Z. Li, B. Ma and A. Suzuki

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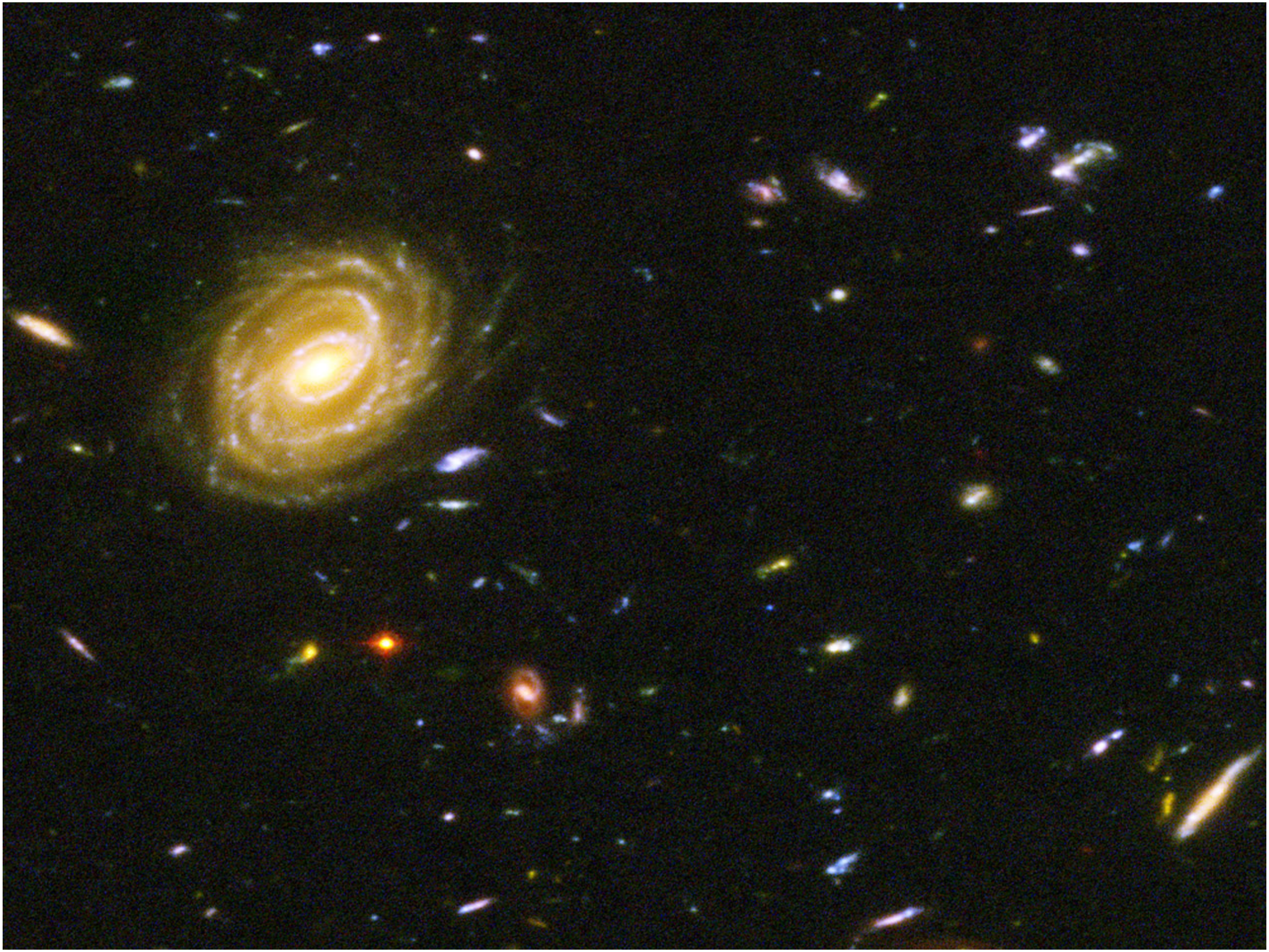
Outline

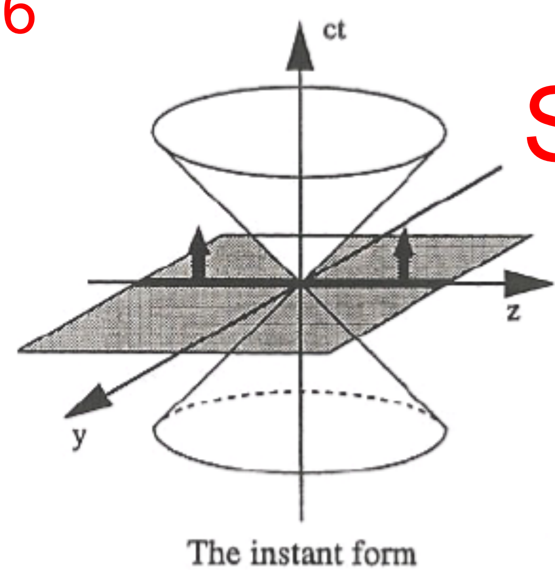
- Motivation for Interpolation between IFD and LFD
  - Time Surface
  - Physical Meaning of Stability Group
  - Interpolation of Dirac's Proposition
- Landscape between IFD and LFD
  - Clarification on IMF and LFD
  - LF Zero-mode (LFZM)
  - Link between Coulomb gauge and LF gauge
  - Jacob&Wick helicity vs. LF helicity
  - Fermion propagator
- Conclusion and Outlook



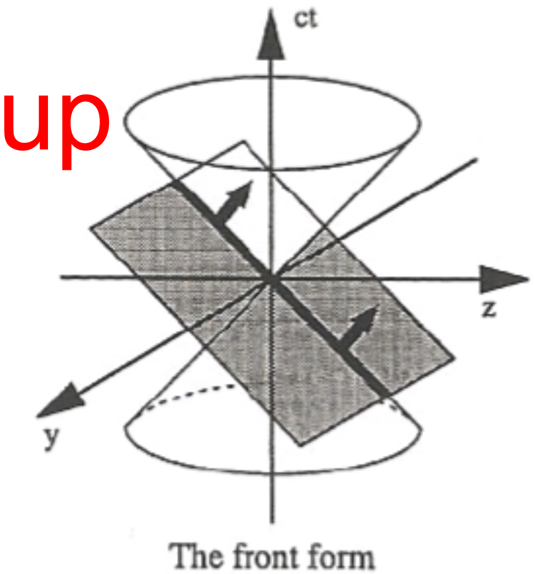
How many generators leave the time surface invariant?

6

7

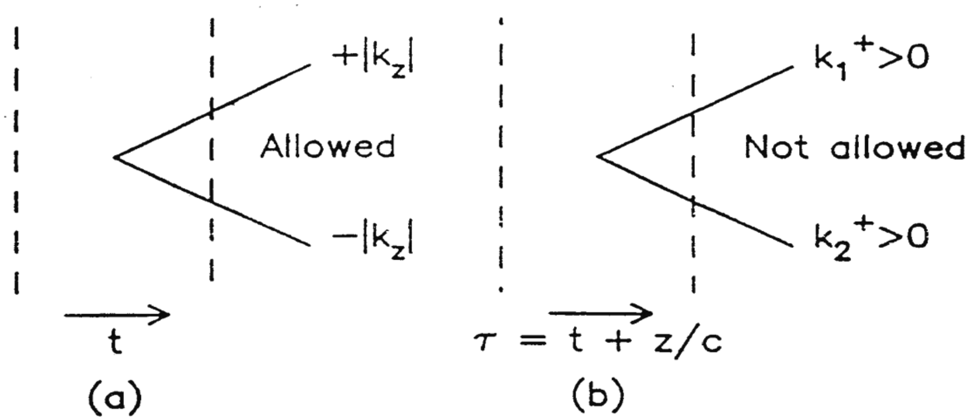


Stability Group



Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2}$$

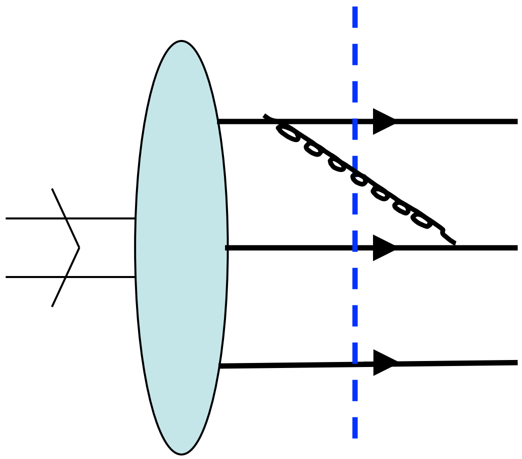


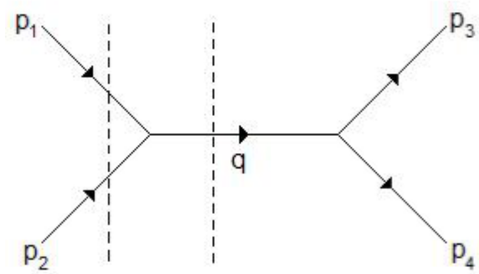
$$p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$

Zero-modes

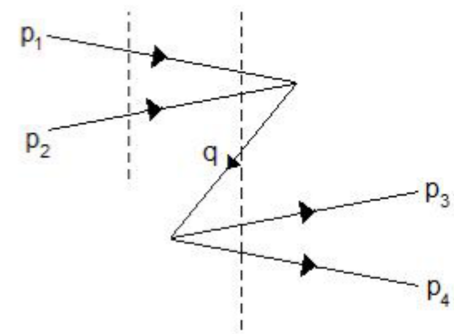
$k_1^+ = 0, k_2^+ = 0$

Physical Meaning of Stability Group





(a)



(b)

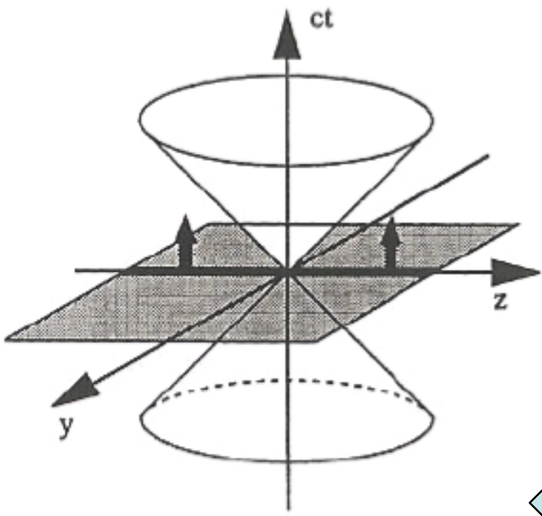
Equal-time Wavefunction

Time-ordered Scattering Amplitudes

Invariant under Stability Group Elements

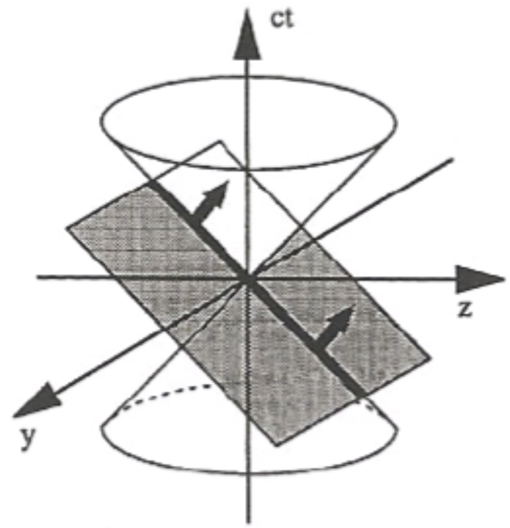
Kinematic Transformations

Dirac's Proposition



The instant form

1949



The front form

Can they be linked?

Traditional approach
evolved from NR dynamics

Innovative approach
for relativistic dynamics

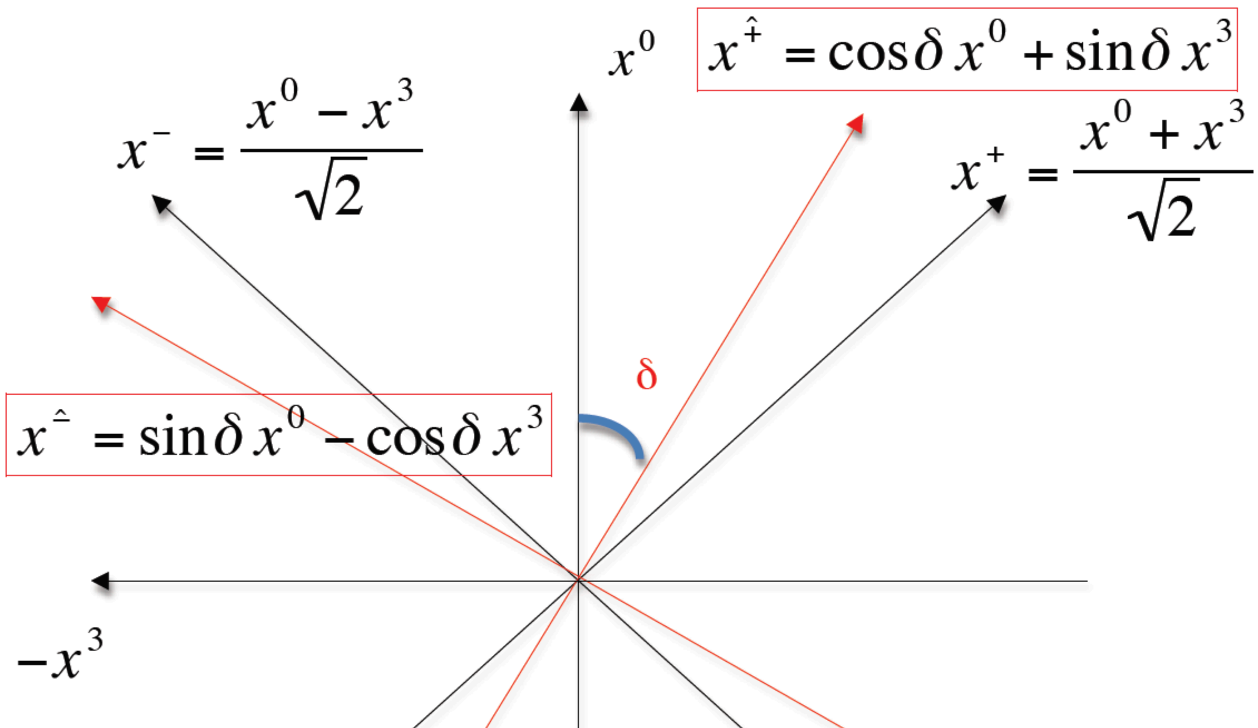
Close contact with
Euclidean space

Strictly in Minkowski space

T-dept QFT, LQCD, IMF, etc.

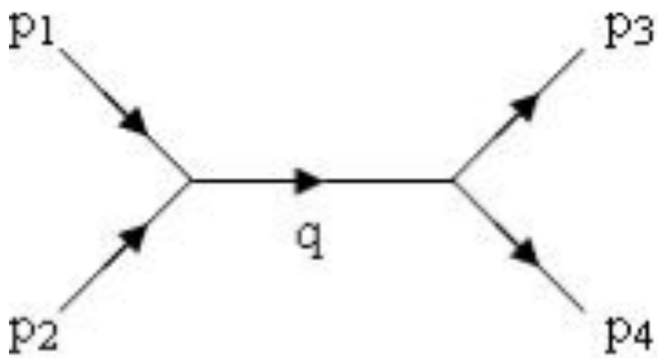
DIS, PDFs, DVCS, GPDs, etc.

Interpolation between Instant and Front Forms



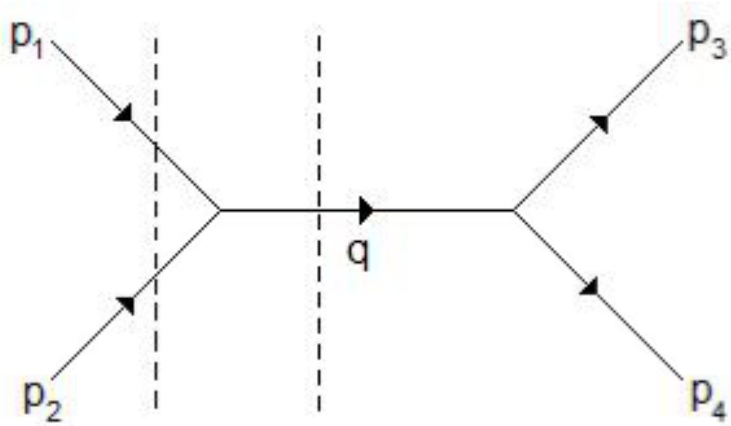
K. Hornbostel, PRD45, 3781 (1992) – RQFT
C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly
C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra
C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps
C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges
Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors
C.Ji, Z.Li, B.Ma and A.Suzuki, in prepartion – Fermion Prop.

$$\text{“}e^+e^- \to \mu^+\mu^-\text{”}$$

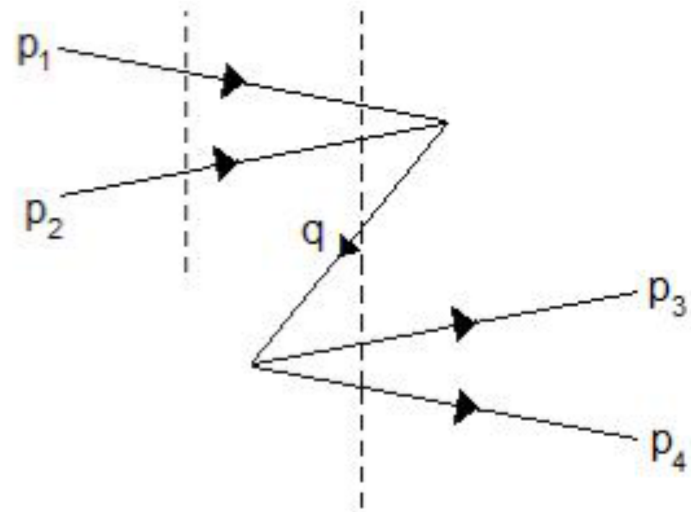


$$= \frac{1}{q^2 - m^2} = \frac{1}{s - m^2}$$

Feynman Diagram: Invariant under all Poincaré generators



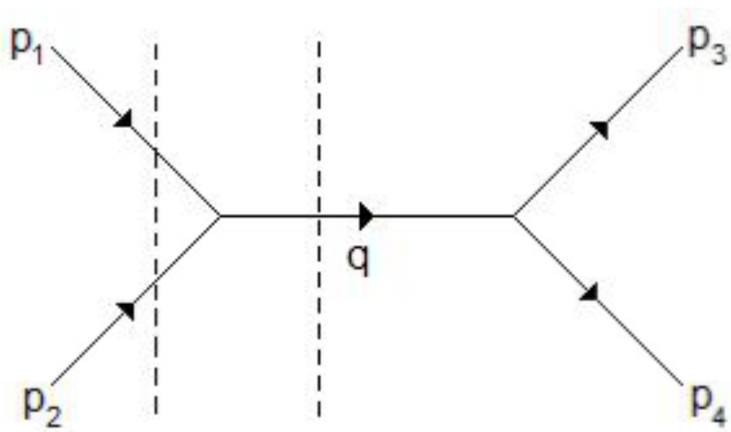
(a)



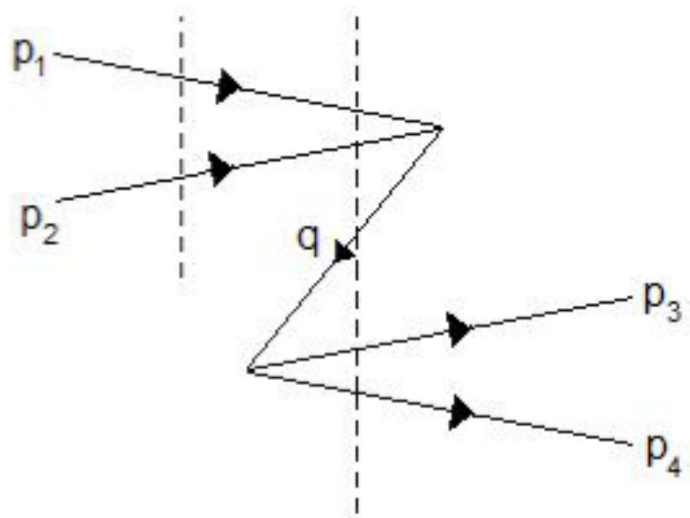
(b)

Individual Time-Ordered Diagrams: Invariant under stability group

Kinematic vs. Dynamic Generators



(a)



(b)

$$\frac{1}{E_1 + E_2 - Eq}$$

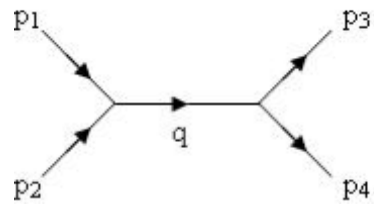
$$-\frac{1}{Eq + E_3 + E_4}$$

$$= -\frac{1}{Eq + E_1 + E_2}$$

$$\rightarrow \quad 0$$

S.Weinberg, PR158,1638(1967)
"Dynamics at Infinite Momentum"

Note however this is still in the instant form.



$$\delta = 0$$

$$p_0 = p^0$$

$$-p_3 = p^3$$

$$0 < \delta < \pi/4$$

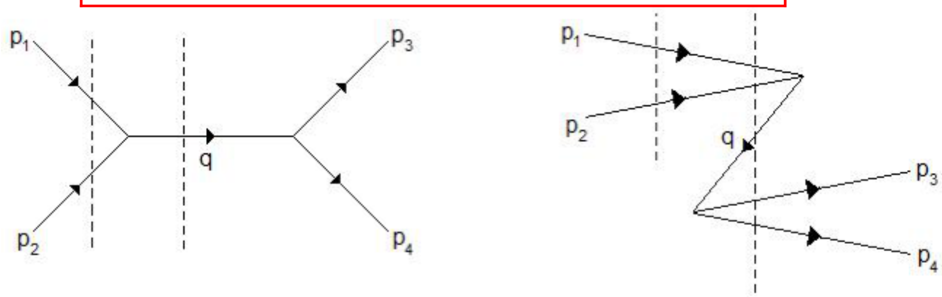
$$p_{\hat{+}} = p^0 \cos\delta - p^3 \sin\delta$$

$$p_{\hat{-}} = p^0 \sin\delta + p^3 \cos\delta$$

$$\delta = \pi/4$$

$$p_+ = p^-$$

$$p_- = p^+$$



(a) (b)

$$\frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$\frac{1}{2\omega_q}\left(\frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} - \omega_q}{\mathbb{C}}} - \frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}}}\right)$$

$$\omega_q = \sqrt{q_{\hat{-}}^2 + \mathbb{C}\left(\vec{\mathbf{q}}_{\hat{\perp}}^2 + m^2\right)}$$

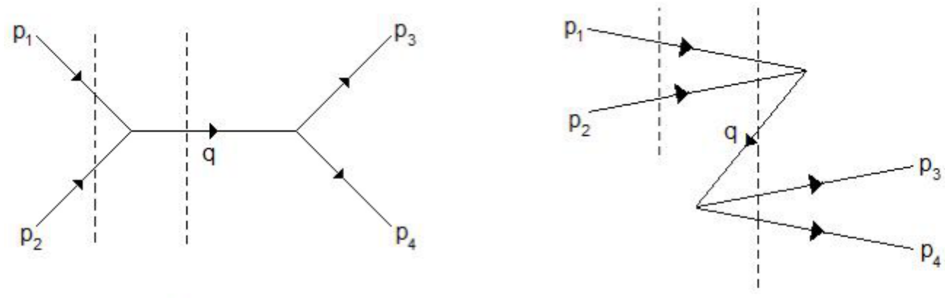
$$\mathbb{C} = \cos 2\delta$$

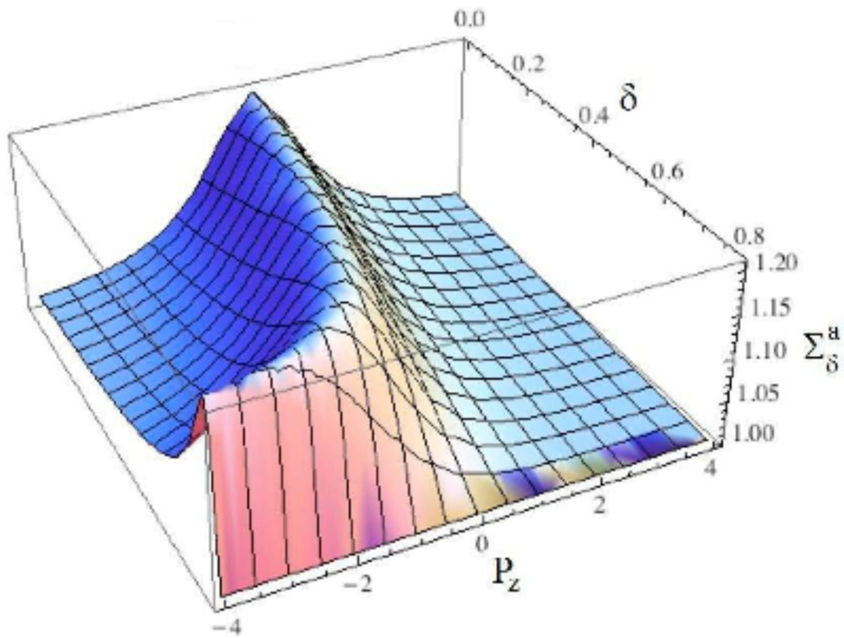
$$\mathbb{S} = \sin 2\delta$$

$$\frac{1}{P^+}\frac{1}{\left\{P^- - \frac{(\vec{\mathbf{P}}_{\perp}^2 + m^2)}{2P^+}\right\}}$$

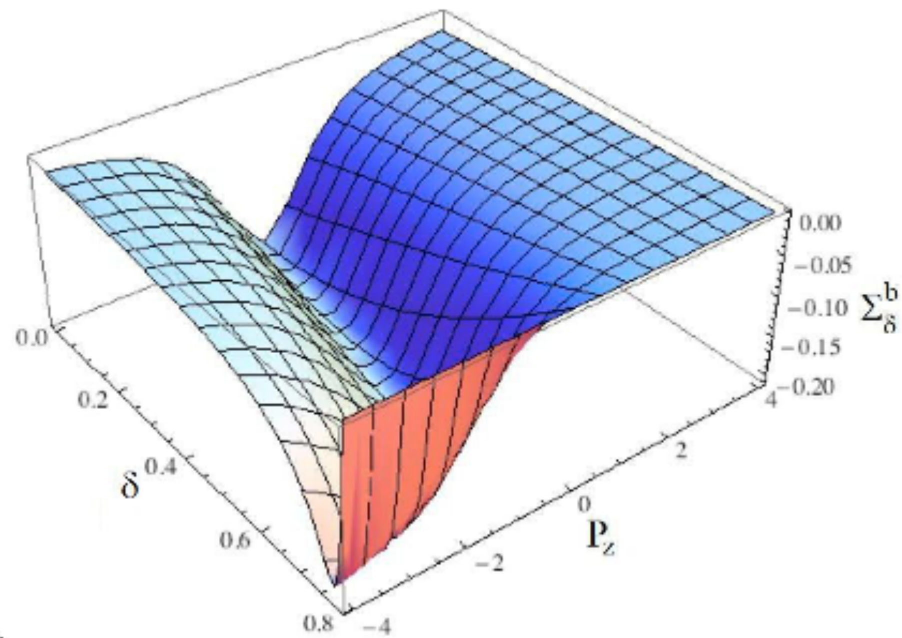
$$\frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}} \to \frac{2}{\mathbb{C}} - \frac{\vec{\mathbf{q}}_{\hat{\perp}}^2 + m^2}{2q_{\hat{-}}} + \mathcal{O}(\mathbb{C})$$

$$\to \infty \quad \text{as } \mathbb{C} \to 0$$





(a)



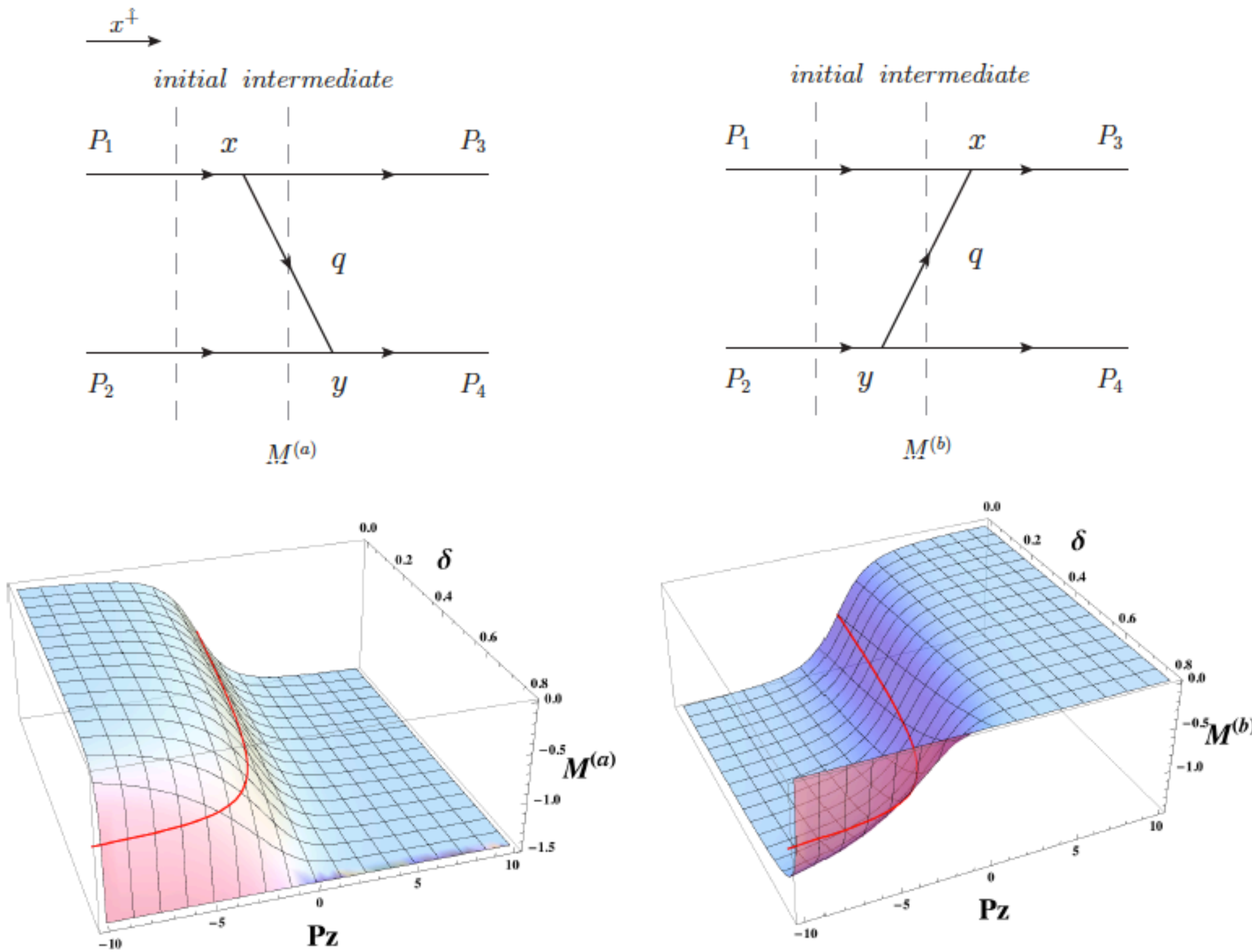
(b)

Σ(a)+Σ(b)=1/(s-m²) ; s=2 GeV², m=1GeV

J-shape peak & valley : $P_z = -\sqrt{\frac{s(1-C)}{2C}}$; $C = \cos(2\delta)$

As C → 0, $P^+ = P^0 + P_z \to 0$ leads to LF Zero-modes.

"$e\mu \to e\mu$"



$m_1 = 1$, $m_2 = 2$, $p = 3$, and $\theta = \pi/3$.

$$A^{\hat{+}} = 0,\ \partial_{\hat{-}} A_{\hat{-}} + \boldsymbol{\partial}_\perp \mathbf{A}_\perp \mathbb{C} = 0 \quad (\mathbb{C} = \cos 2\delta)$$

$\delta \to 0$ $(\mathbb{C} \to 1)$

$\delta \to \pi/4$ $(\mathbb{C} \to 0)$

C.Ji, Z. Li, and A. T. Suzuki, PRD91, 065020(2015)

$$A^0 = 0,\ \nabla \cdot \mathbf{A} = 0$$

Coulomb Gauge

$$A^+ = 0$$

Light-front Gauge

$$\sum_{\lambda=\pm} \epsilon^*_{\hat{\mu}}(\lambda)\epsilon_{\hat{\nu}}(\lambda) = -g_{\hat{\mu}\hat{\nu}} + \frac{(q\cdot n)(q_{\hat{\mu}} n_{\hat{\nu}} + q_{\hat{\nu}} n_{\hat{\mu}})}{\mathbf{q}_\perp^2 \mathbb{C} + q_{\hat{-}}^2} - \frac{\mathbb{C} q_{\hat{\mu}} q_{\hat{\nu}}}{\mathbf{q}_\perp^2 \mathbb{C} + q_{\hat{-}}^2} - \frac{q^2 n_{\hat{\mu}} n_{\hat{\nu}}}{\mathbf{q}_\perp^2 \mathbb{C} + q_{\hat{-}}^2}$$

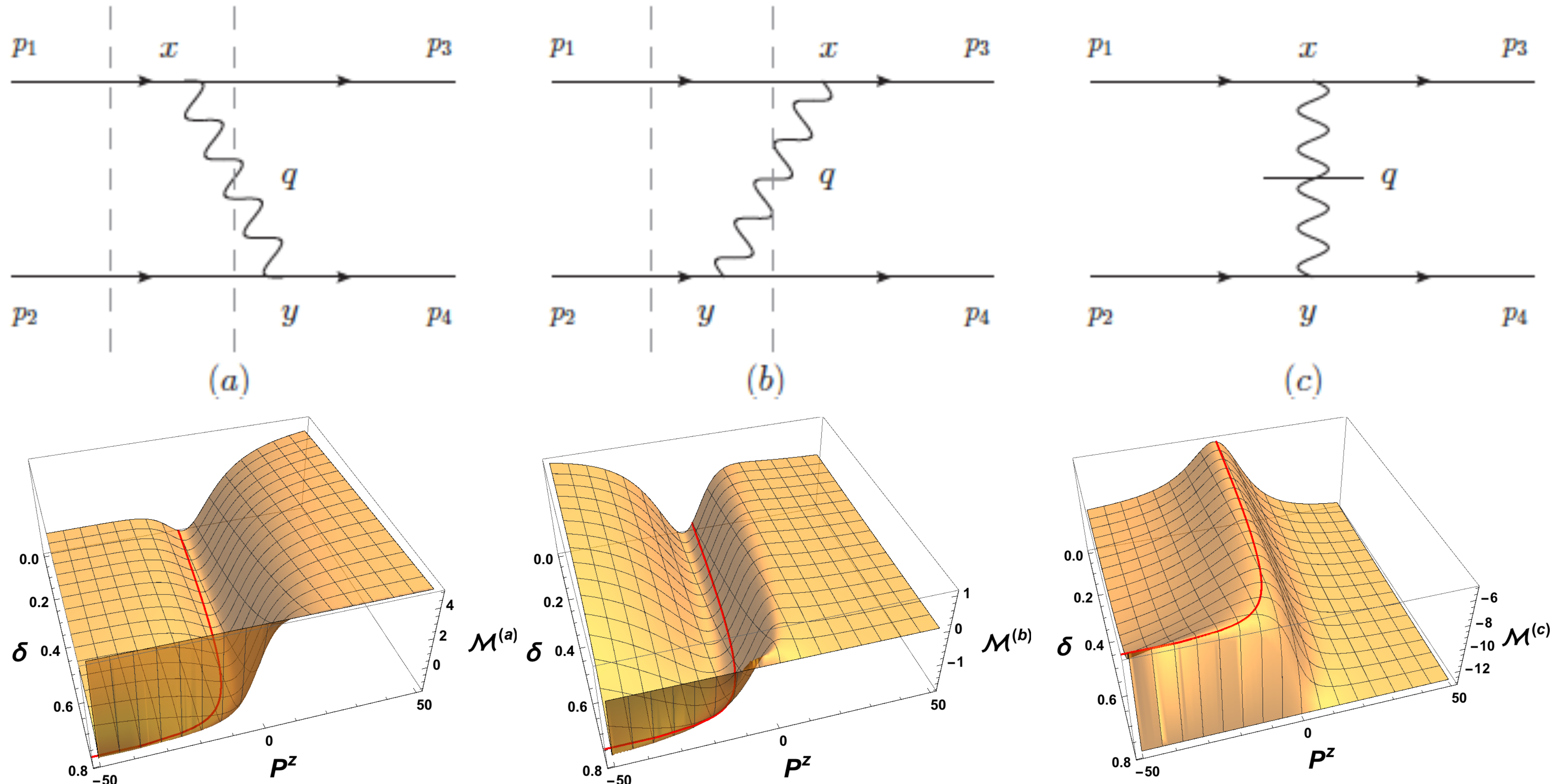
LFD

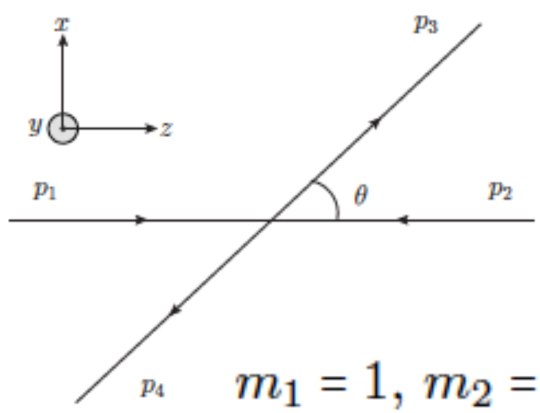
$$-g_{\mu\nu} + \frac{(q\cdot n)(q_\mu n_\nu + q_\nu n_\mu)}{(q\cdot n)^2} - \frac{q^2 n_\mu n_\nu}{(q\cdot n)^2}$$

P.Srivastava and S. Brodsky, PRD64,045006(2001)

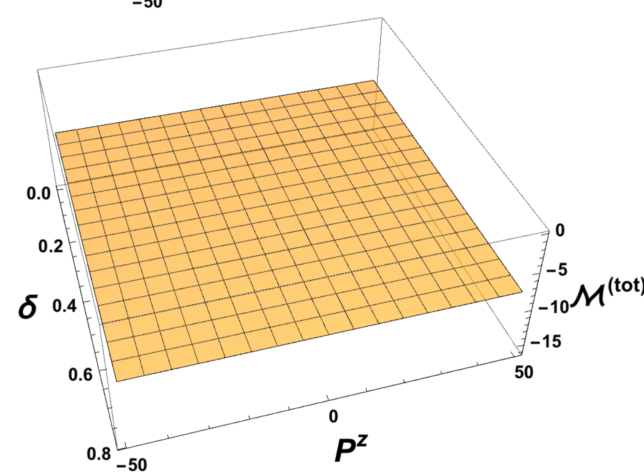
IFD

$$-\eta_{\mu\nu} + \frac{(q\cdot n)(q_\mu n_\nu + q_\nu n_\mu)}{(q\cdot n)^2 - q^2} - \frac{q_\mu q_\nu}{(q\cdot n)^2 - q^2} - \frac{q^2 n_\mu n_\nu}{(q\cdot n)^2 - q^2}$$





Total amplitude is independent of P^Z and δ as it must be.

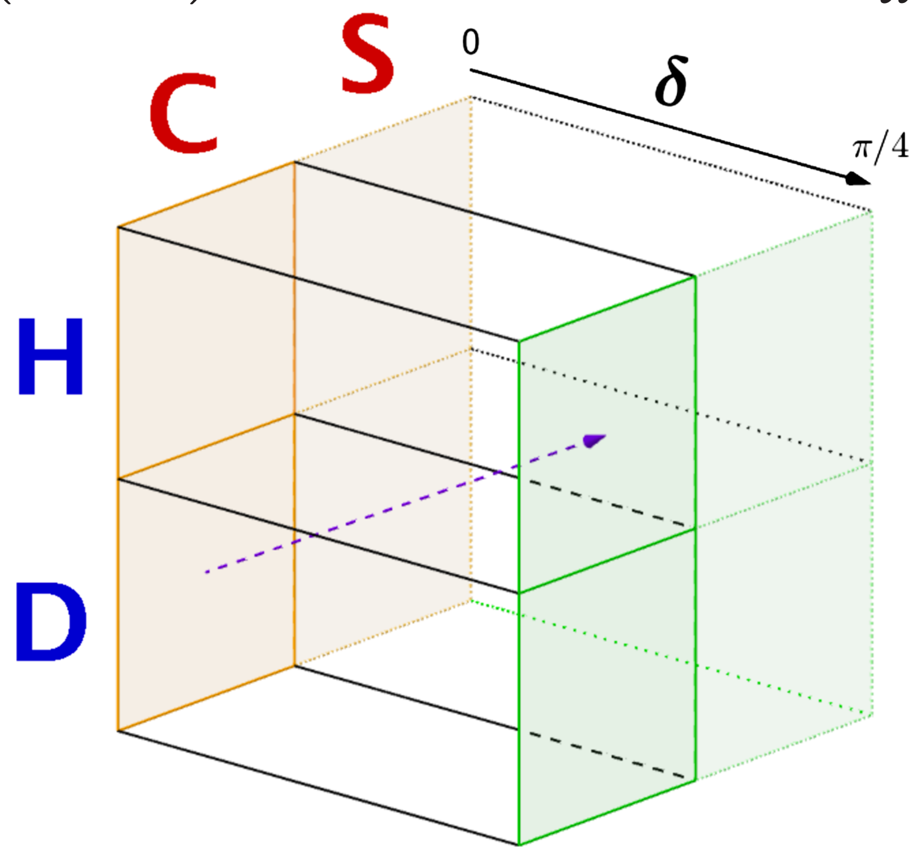


$m_1 = 1$, $m_2 = 2$, $p = 3$, and $\theta = \pi/3$.

$$S = S^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix} I & I \\ I & -I \end{pmatrix}$$

$$u_S(p) = S u_C(p),$$

$$u_C(p) = S^\dagger u_S(p),$$



$$\mathbf{A} = \frac{1}{2}(\mathbf{J} + i\mathbf{K}),$$

$$\mathbf{B} = \frac{1}{2}(\mathbf{J} - i\mathbf{K}),$$

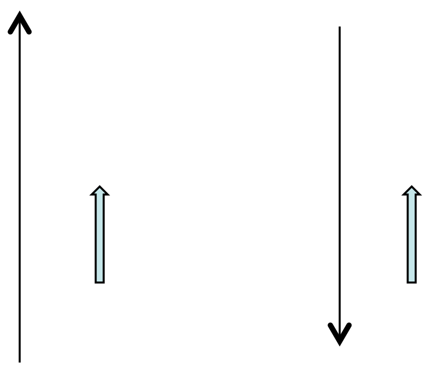
$$[A_i, A_j] = i\epsilon_{ijk}A_k,$$

$$[B_i, B_j] = i\epsilon_{ijk}B_k,$$

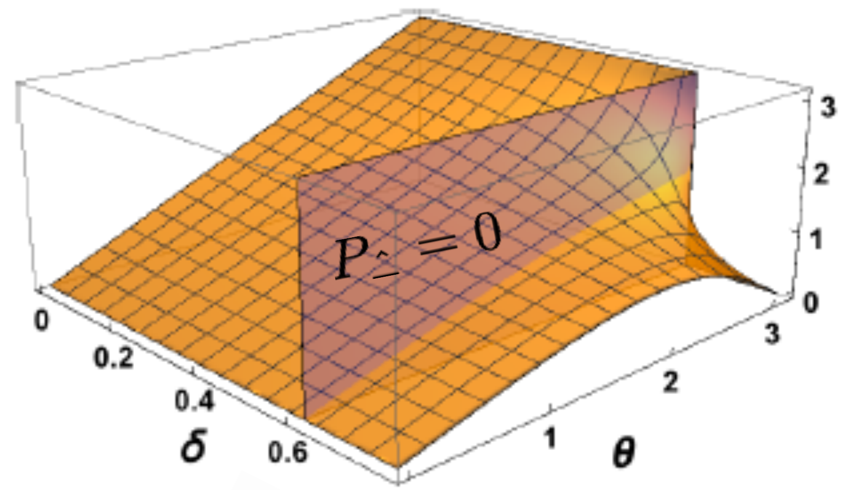
$$[A_i, B_j] = 0, \quad (i, j, k = 1, 2, 3)$$

Helicity

IFD

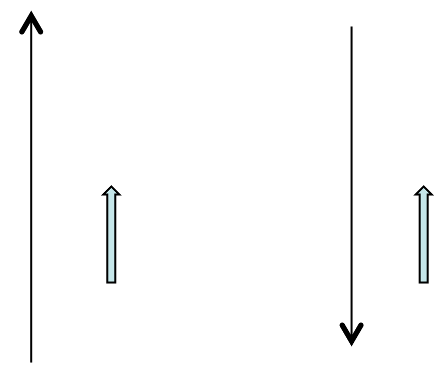


+1 -1



$\delta_c = \arctan\left(\frac{|\mathbf{P}|}{E}\right)$

LFD

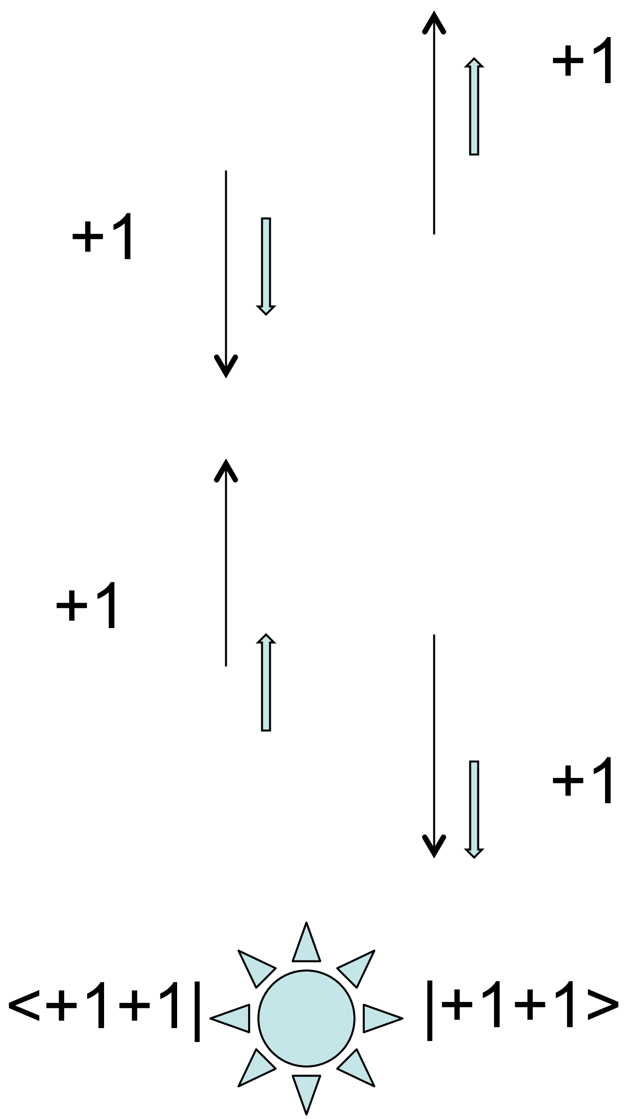


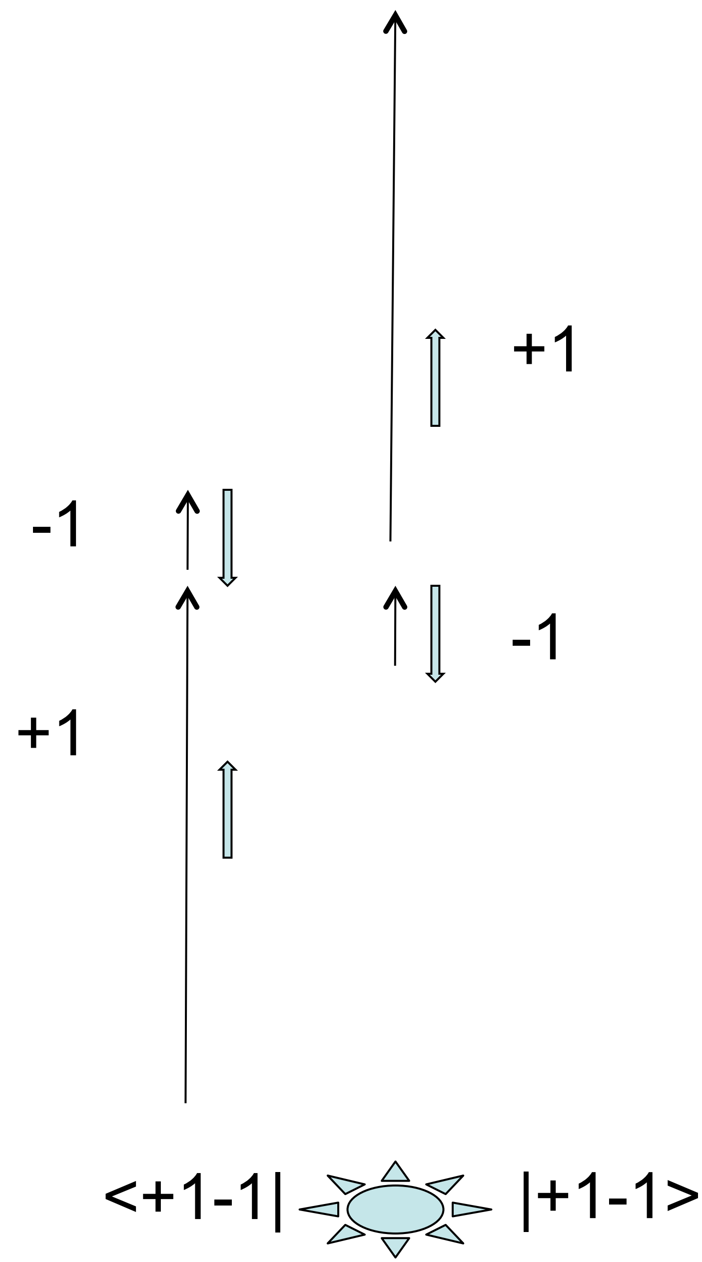
+1 +1

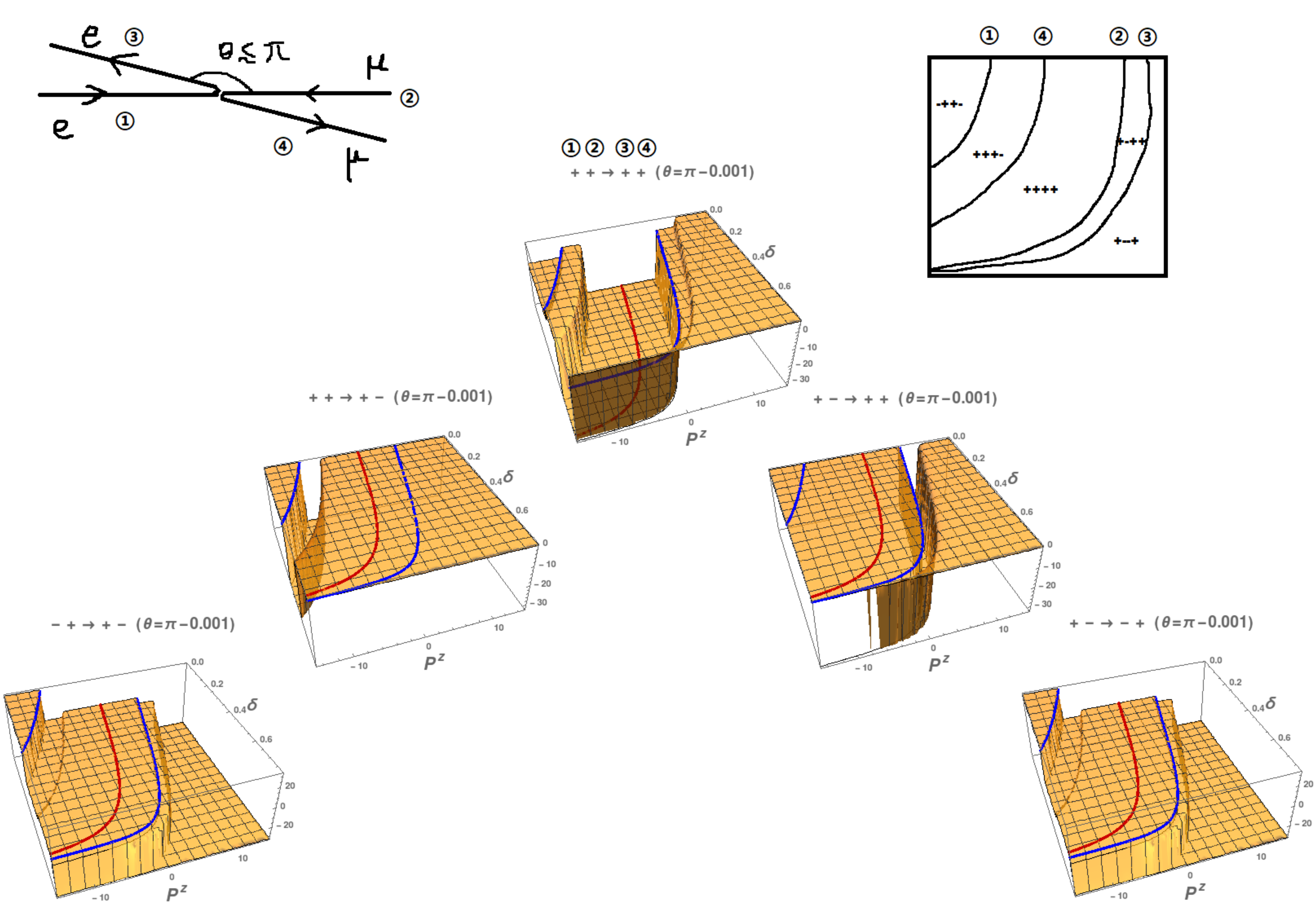
M. Jacob and G. Wick,
Ann. Phys., 7, 404 (1959)

C. Carlson and C.Ji,
PRD, 67, 116002 (2003)

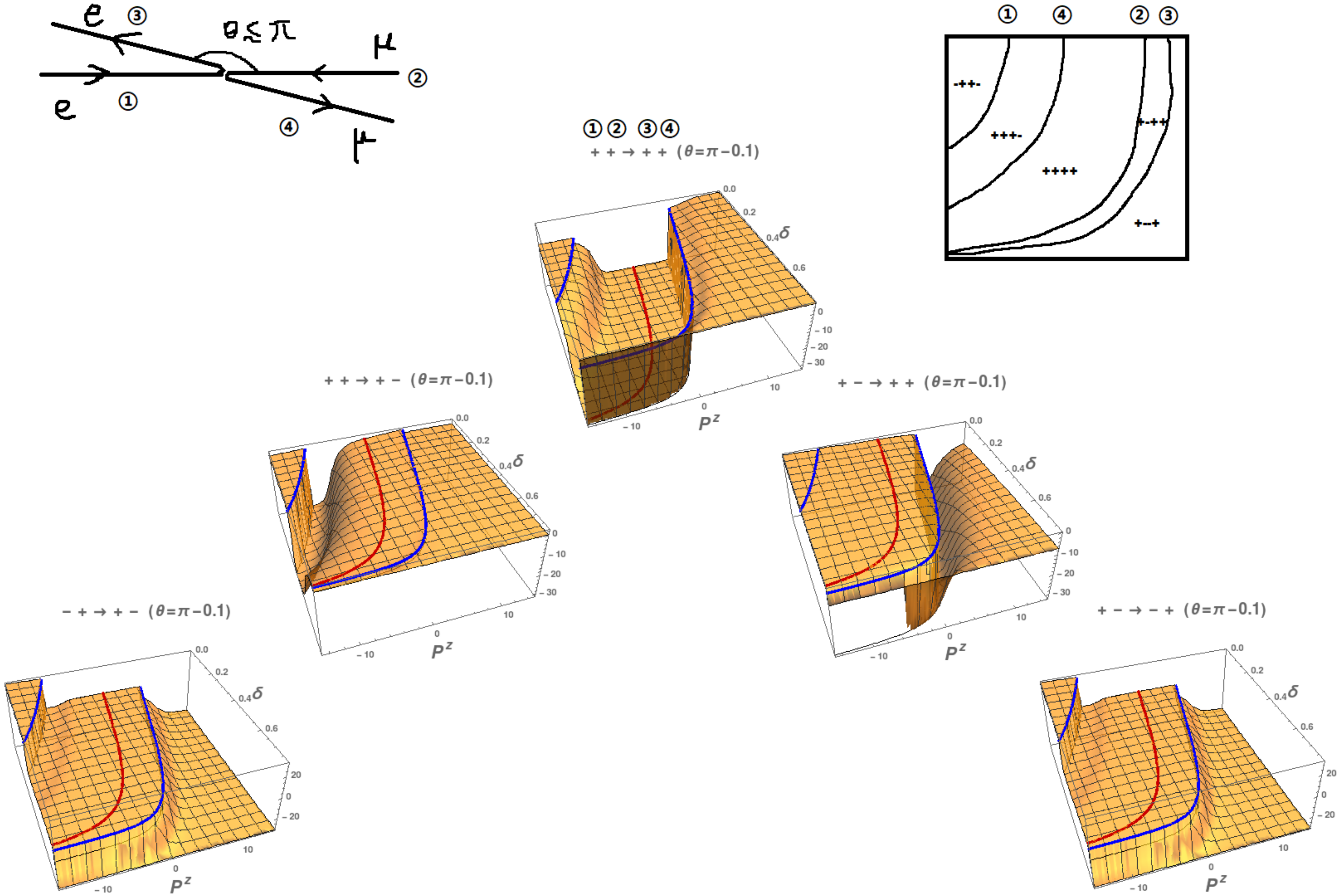
K_z Dependent vs. K_z Independent

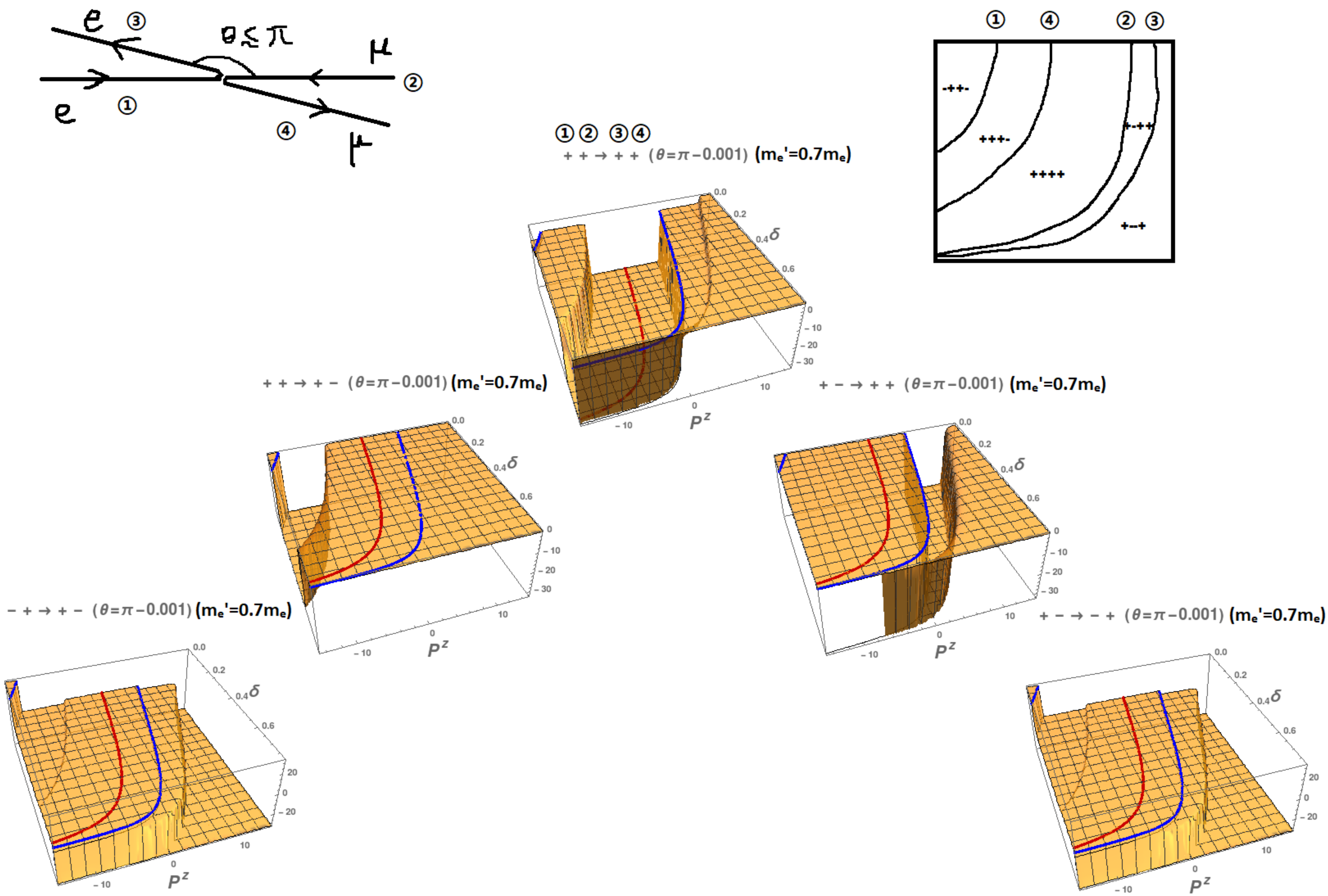

+1
+1
+1
+1
<+1+1|
|+1+1>


+1
-1
-1
+1
<+1-1|
|+1-1>

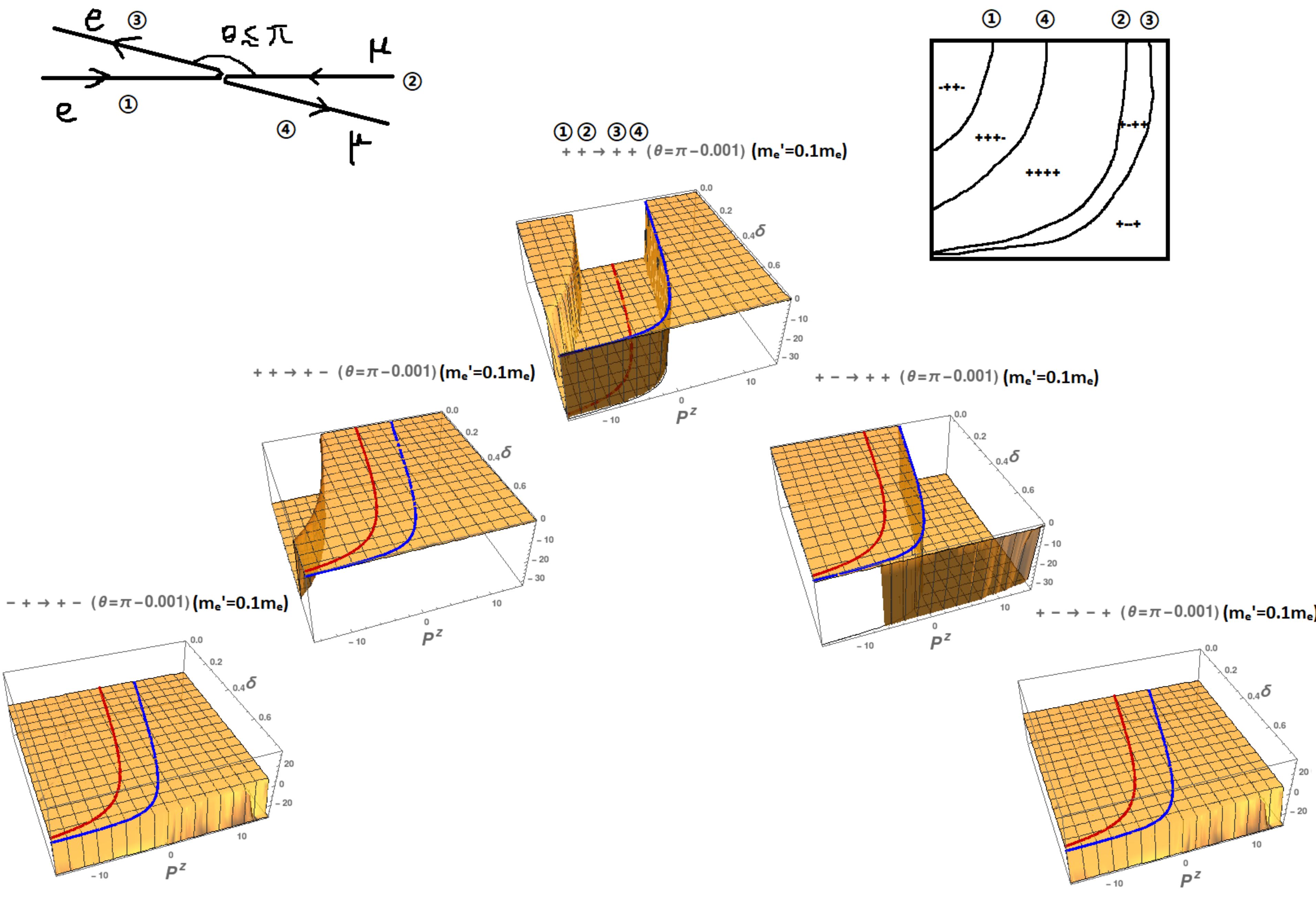


e ③
θ ≲ π
μ ②
e ①
④ μ
① ④ ② ③
-++-
+++-
++++
+-++
+--+
① ② ③ ④
+ + → + + (θ=π−0.001)
+ + → + − (θ=π−0.001)
+ − → + + (θ=π−0.001)
− + → + − (θ=π−0.001)
+ − → − + (θ=π−0.001)
δ
P^z

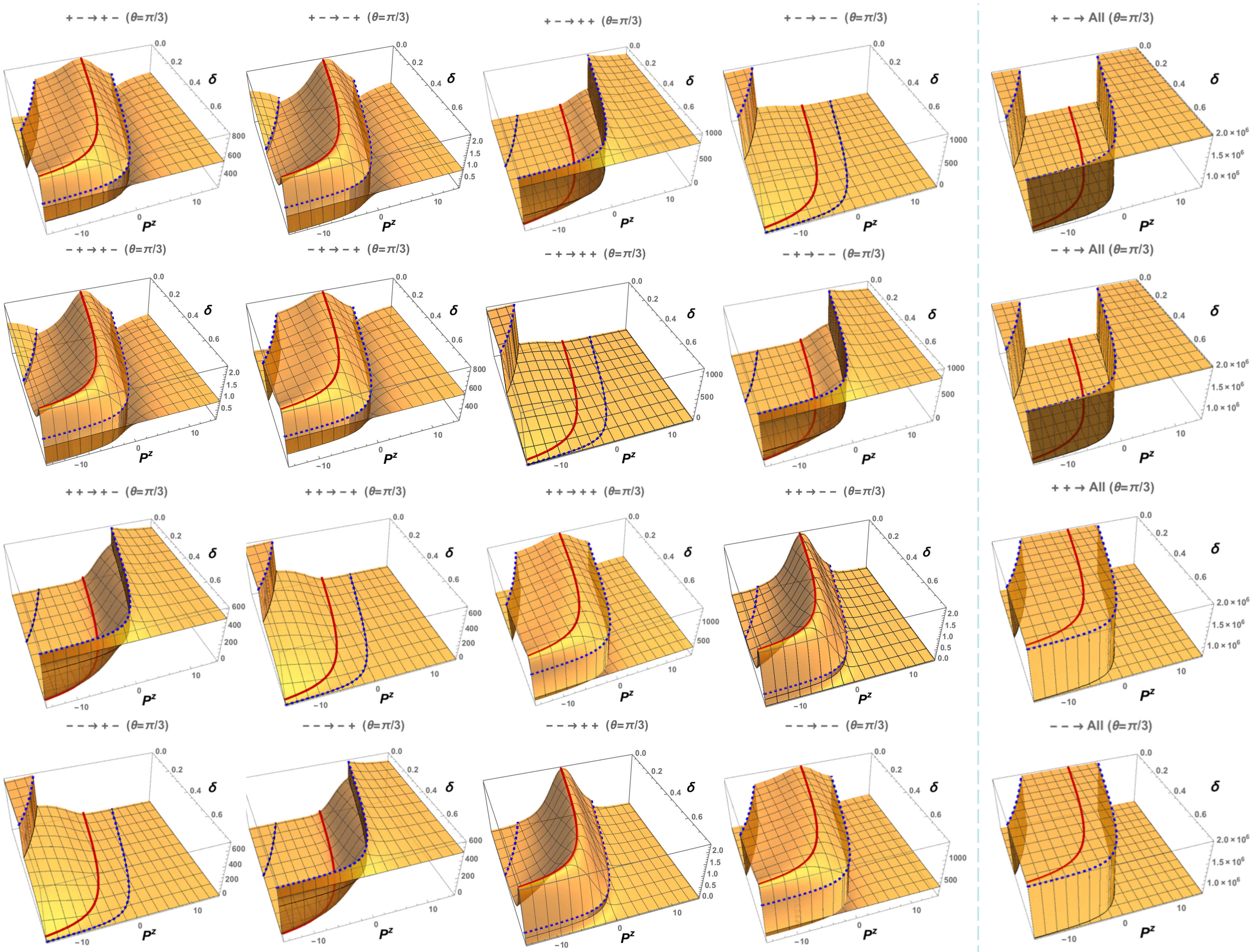

e ③
θ ≲ π
μ
②
e
①
④
μ
① ④ ② ③
-++-
+++-
++++
+-++
+--+
① ② ③ ④
+ + → + + (θ=π−0.1)
+ + → + − (θ=π−0.1)
+ − → + + (θ=π−0.1)
− + → + − (θ=π−0.1)
+ − → − + (θ=π−0.1)
δ
P^z



e ③
θ ≲ π
μ
②
e ①
④
μ
① ④ ② ③
-++-
+++-
++++
+-++
+--+
① ② ③ ④
+ + → + + (θ=π−0.001) (m_e'=0.7m_e)
+ + → + − (θ=π−0.001) (m_e'=0.7m_e)
+ − → + + (θ=π−0.001) (m_e'=0.7m_e)
− + → + − (θ=π−0.001) (m_e'=0.7m_e)
+ − → − + (θ=π−0.001) (m_e'=0.7m_e)
δ
P^z



e ③
θ ≲ π
μ
②
e ①
④
μ
① ④ ② ③
-++-
+++-
++++
+-++
+--+
① ② ③ ④
+ + → + + (θ=π−0.001) (m_e'=0.1m_e)
+ + → + − (θ=π−0.001) (m_e'=0.1m_e)
+ − → + + (θ=π−0.001) (m_e'=0.1m_e)
− + → + − (θ=π−0.001) (m_e'=0.1m_e)
+ − → − + (θ=π−0.001) (m_e'=0.1m_e)
δ
P^z


+ − → + − (θ=π/3)
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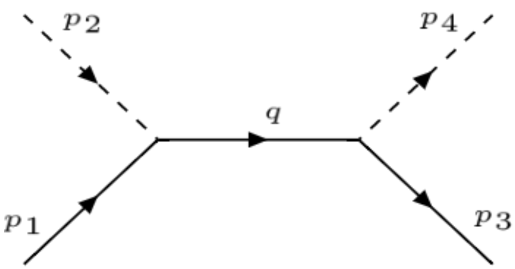
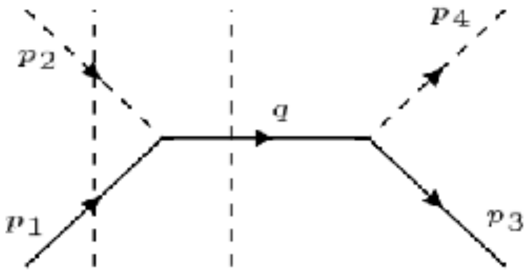
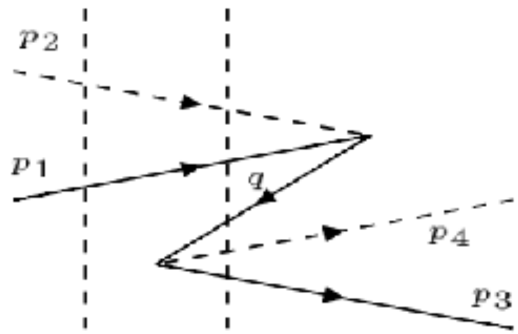
− + → + − (θ=π/3)
− + → − + (θ=π/3)
− + → + + (θ=π/3)
− + → − − (θ=π/3)
− + → All (θ=π/3)
+ + → + − (θ=π/3)
+ + → − + (θ=π/3)
+ + → + + (θ=π/3)
+ + → − − (θ=π/3)
+ + → All (θ=π/3)
− − → + − (θ=π/3)
− − → − + (θ=π/3)
− − → + + (θ=π/3)
− − → − − (θ=π/3)
− − → All (θ=π/3)
δ
P^z



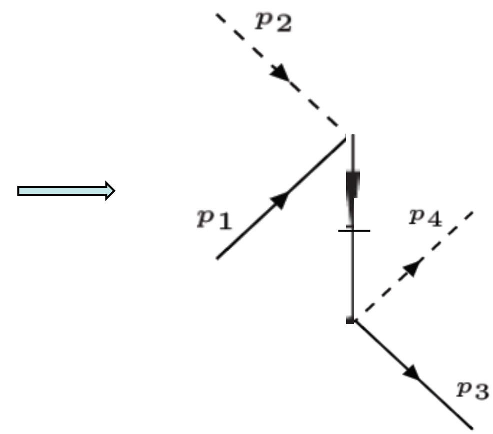
Fermion Propagator



(a)



(b)



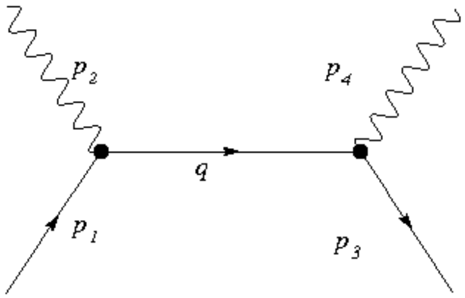
$$\Sigma_a^{\text{IFD}} + \Sigma_b^{\text{IFD}} = \frac{1}{2q^0_{on}}\left(\frac{\not{q}+m}{q^0-q^0_{on}} - \frac{\not{q}+m}{q^0+q^0_{on}}\right) = \frac{1}{2q^0_{on}}\frac{2q^0_{on}(\not{q}+m)}{(q^0)^2-(q^0_{on})^2} = \frac{\not{q}+m}{q^2-m^2}$$

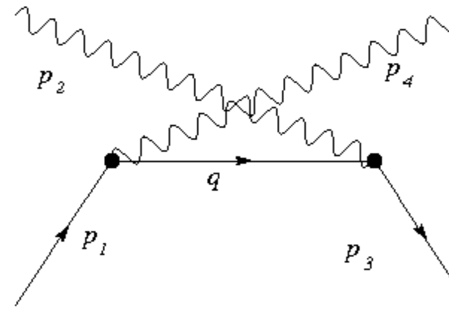
$$\Sigma_{a,\delta\to\frac{\pi}{4}} = \frac{\not{q}_{on}+m}{q^2-m^2}$$

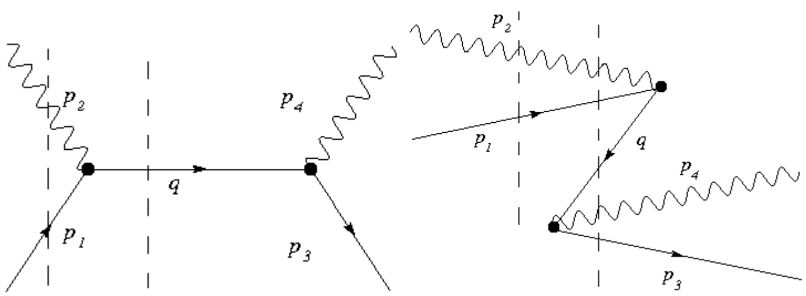
$$\Sigma_{b,\delta\to\frac{\pi}{4}} = \frac{\gamma^+}{2q^+}$$

$$\frac{1}{\not{q}-m} = \frac{\sum_s u(q,s)\overline{u}(q,s)}{q^2-m^2} + \frac{\gamma^+}{2q^+}$$

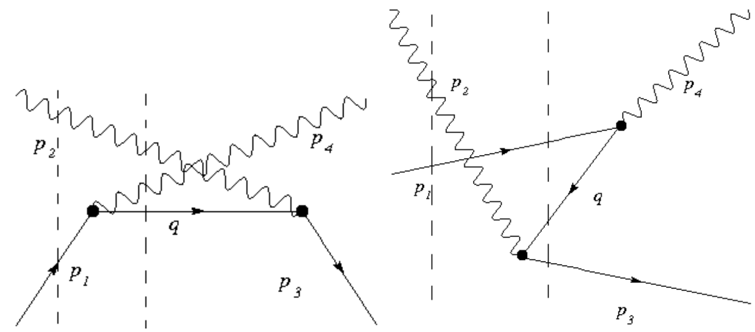
S.-J.Chang and T.-M.Yan, PRD7,1147(1973)


p2
p4
q
p1
p3

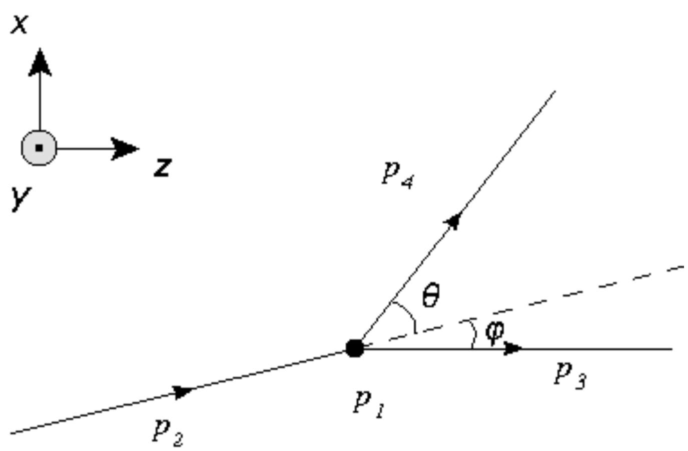

p2
p4
q
p1
p3

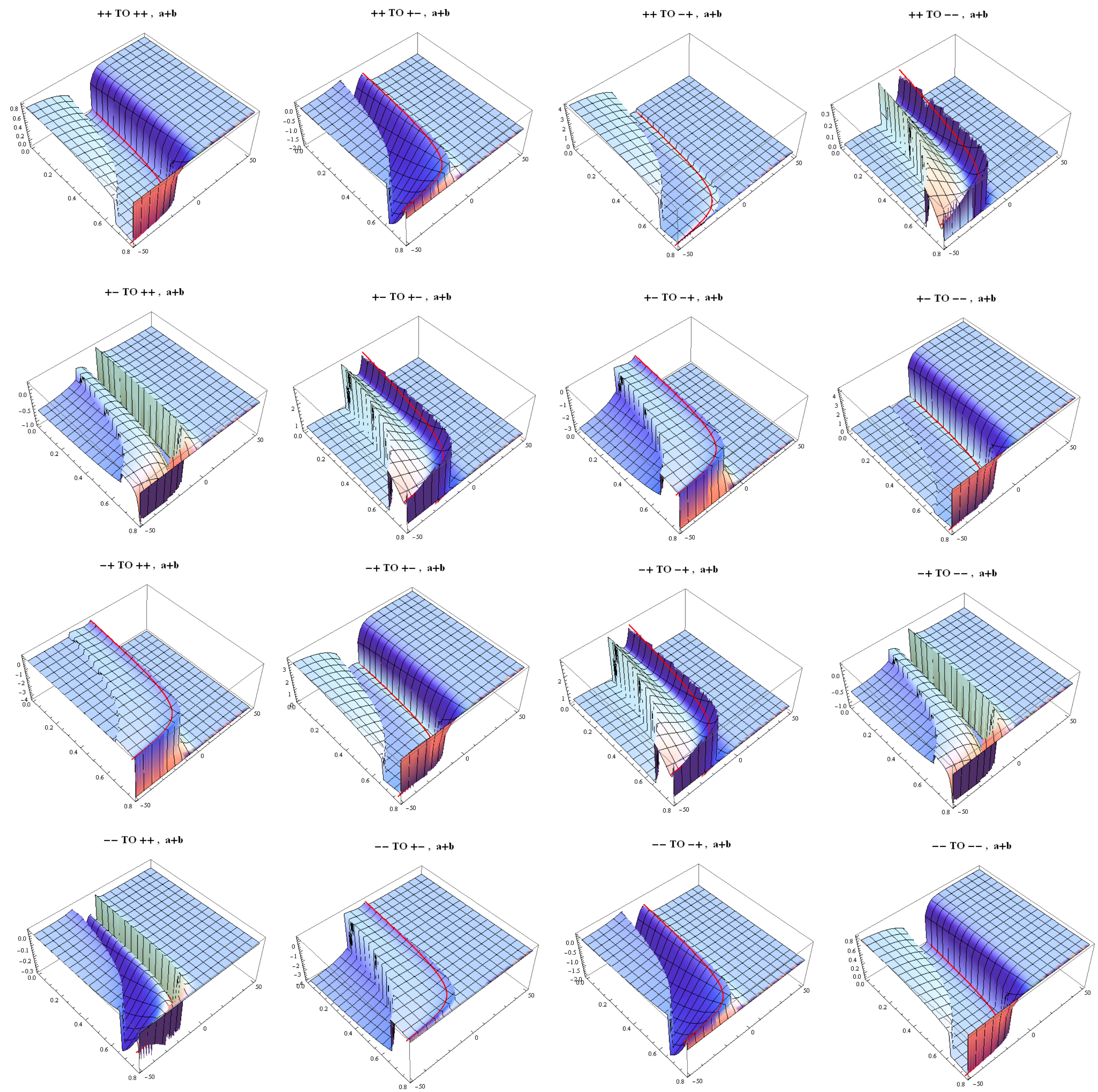

p2
p4
q
p1
p3
p2
p1
q
p4
p3

(a) (b)

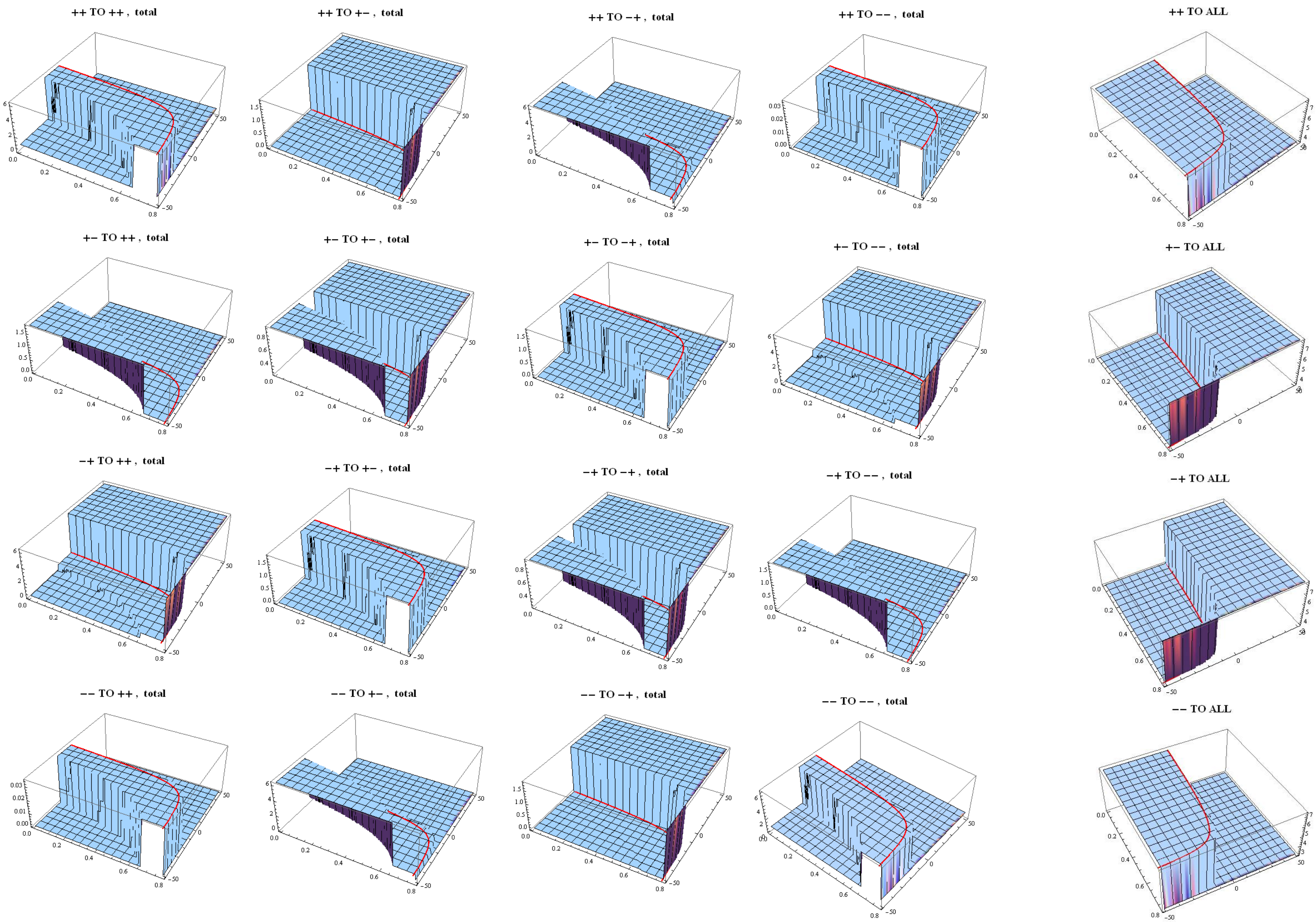

p2
p4
q
p1
p3
p2
p4
p1
q
p3

(a) (b)

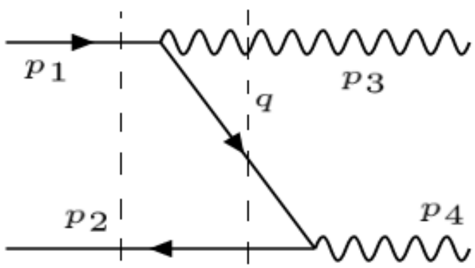

x
z
y
p4
θ
φ
p3
p1
p2



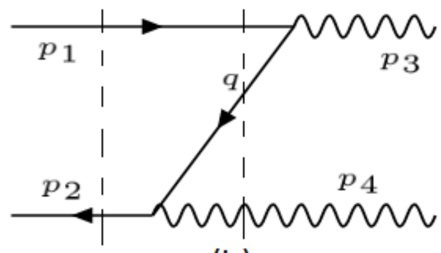
++ TO ++ , a+b
++ TO +− , a+b
++ TO −+ , a+b
++ TO −− , a+b
+− TO ++ , a+b
+− TO +− , a+b
+− TO −+ , a+b
+− TO −− , a+b
−+ TO ++ , a+b
−+ TO +− , a+b
−+ TO −+ , a+b
−+ TO −− , a+b
−− TO ++ , a+b
−− TO +− , a+b
−− TO −+ , a+b
−− TO −− , a+b


++ TO ++ , total
++ TO +− , total
++ TO −+ , total
++ TO −− , total
++ TO ALL
+− TO ++ , total
+− TO +− , total
+− TO −+ , total
+− TO −− , total
+− TO ALL
−+ TO ++ , total
−+ TO +− , total
−+ TO −+ , total
−+ TO −− , total
−+ TO ALL
−− TO ++ , total
−− TO +− , total
−− TO −+ , total
−− TO −− , total
−− TO ALL

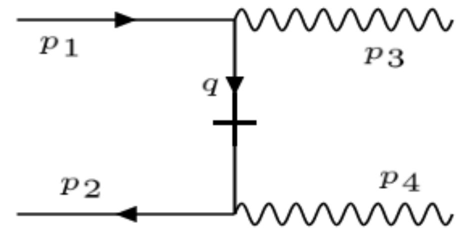
Example Application: The Annihilation of Electron- positron Pair into Two Photons



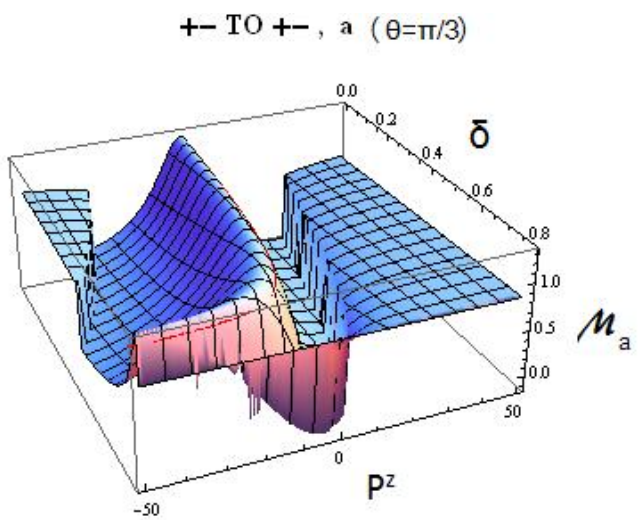
(a)

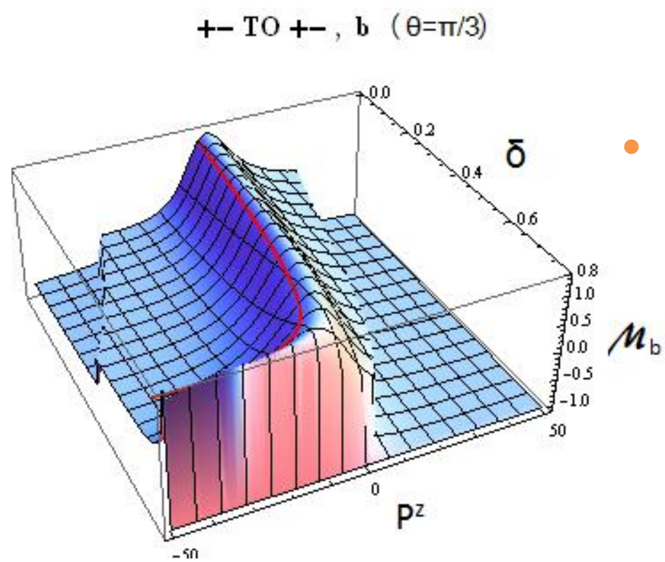


(b)

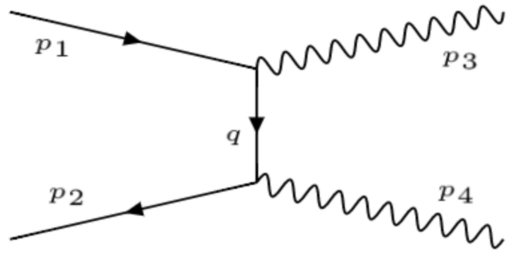


(c)

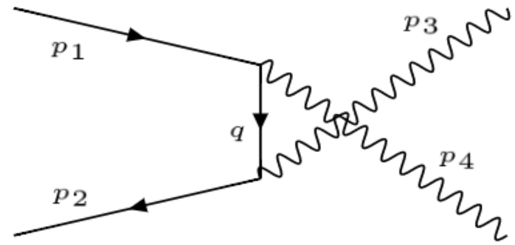




- Diagram (c) only exists in LFD
- Only one of (a) and (b) is allowed in LFD and the other one changes to instantaneous interaction in LFD

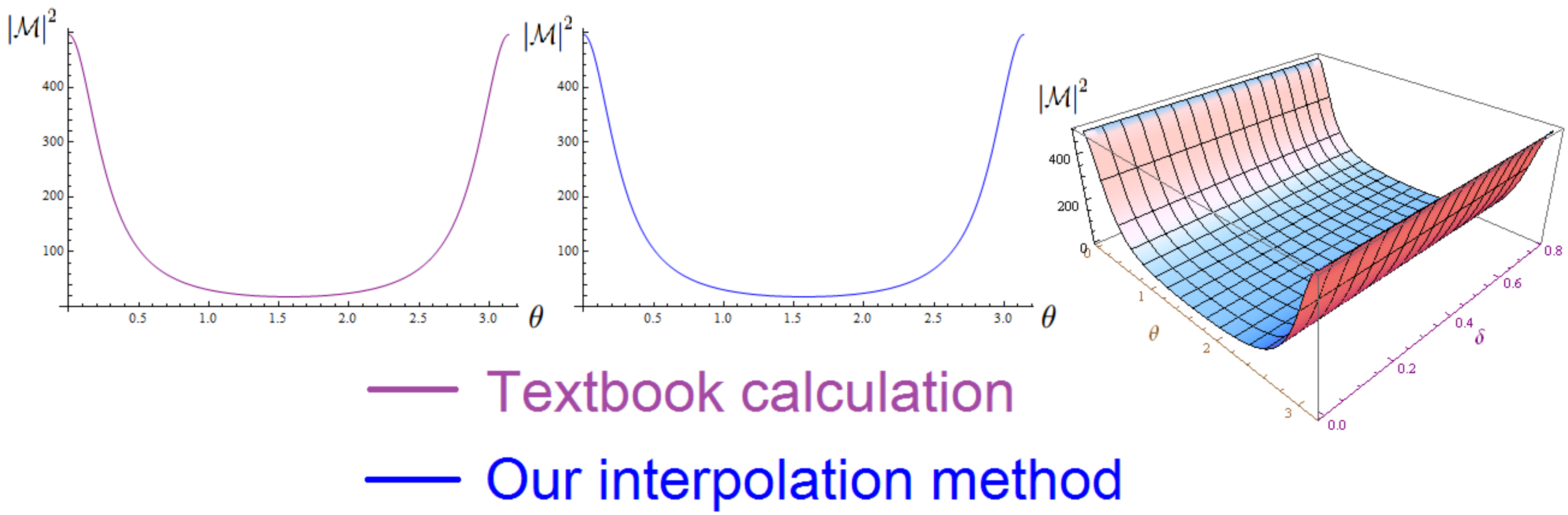


Direct Diagram



Exchanged Diagram

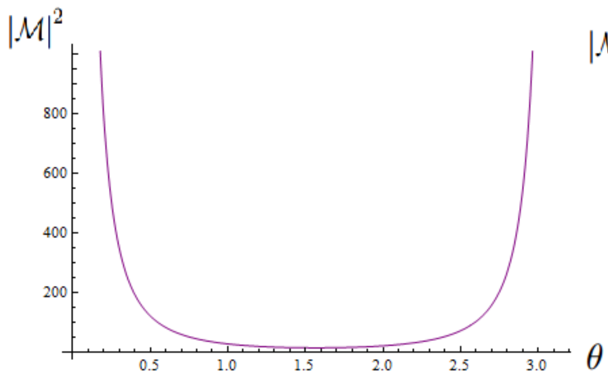
Including electron mass

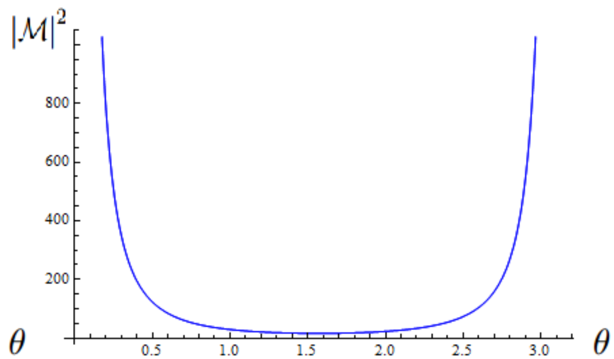


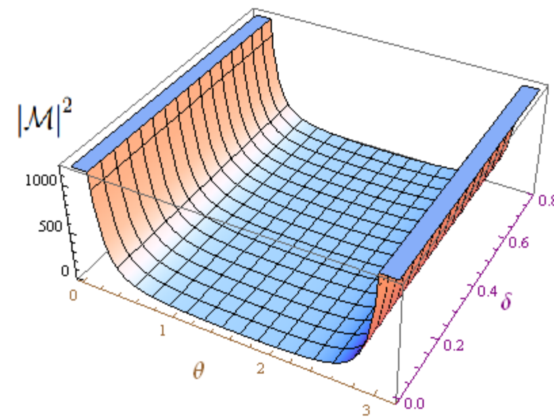
$$|\mathcal{M}|^2 = 2e^4\left[\frac{u_m}{t_m} + \frac{t_m}{u_m} + 2m^2\left(\frac{s_m}{t_m u_m} - \frac{1}{t_m} - \frac{1}{u_m}\right) - 4m^4\left(\frac{1}{{t_m}^2} + \frac{1}{{u_m}^2}\right)\right]$$

where $t_m \equiv t - m^2$, $u_m \equiv u - m^2$, and $s_m \equiv s - 4m^2$.

Taking electron mass zero





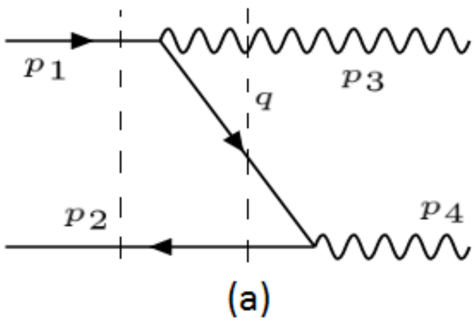


— Textbook calculation

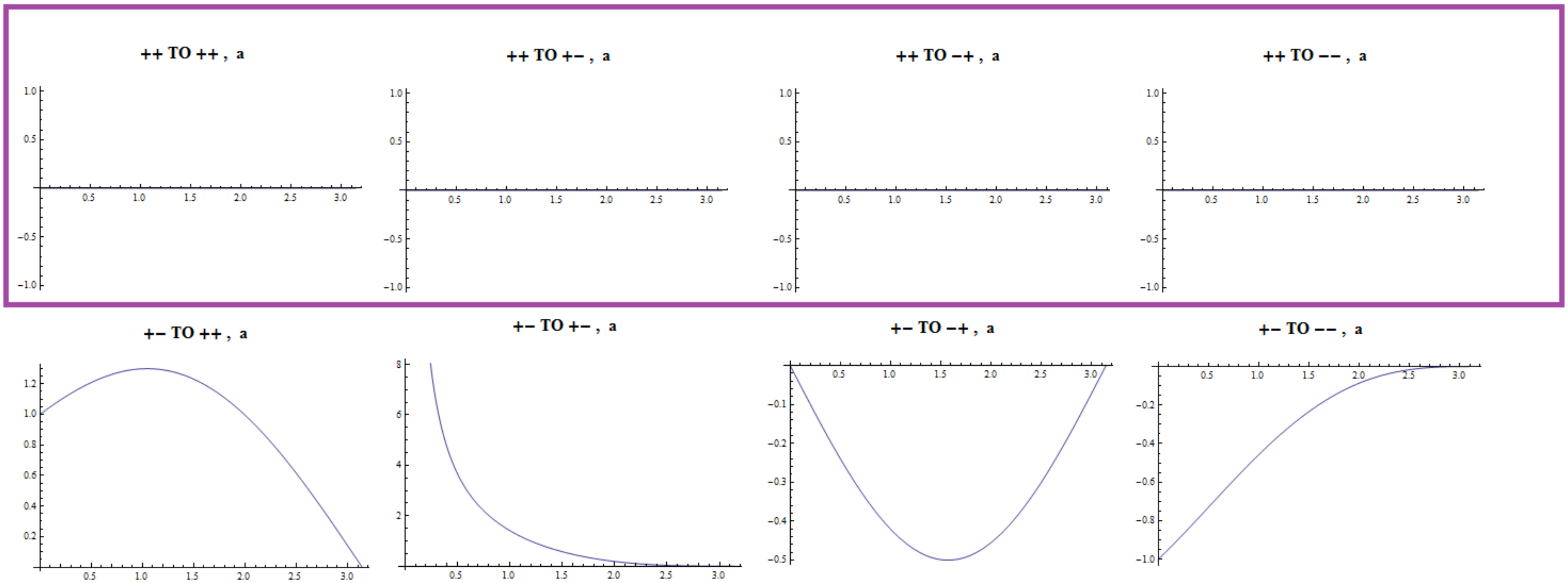
— Our interpolation method

$$|\mathcal{M}|^2 = 2e^4\left(\frac{u}{t}+\frac{t}{u}\right)$$

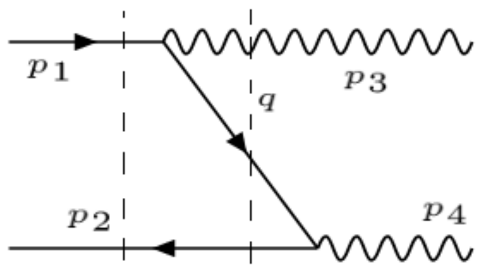
Scattering Angle Dependence of the Annihilation Amplitudes: Chirality



When m_e=0, chirality is conserved.

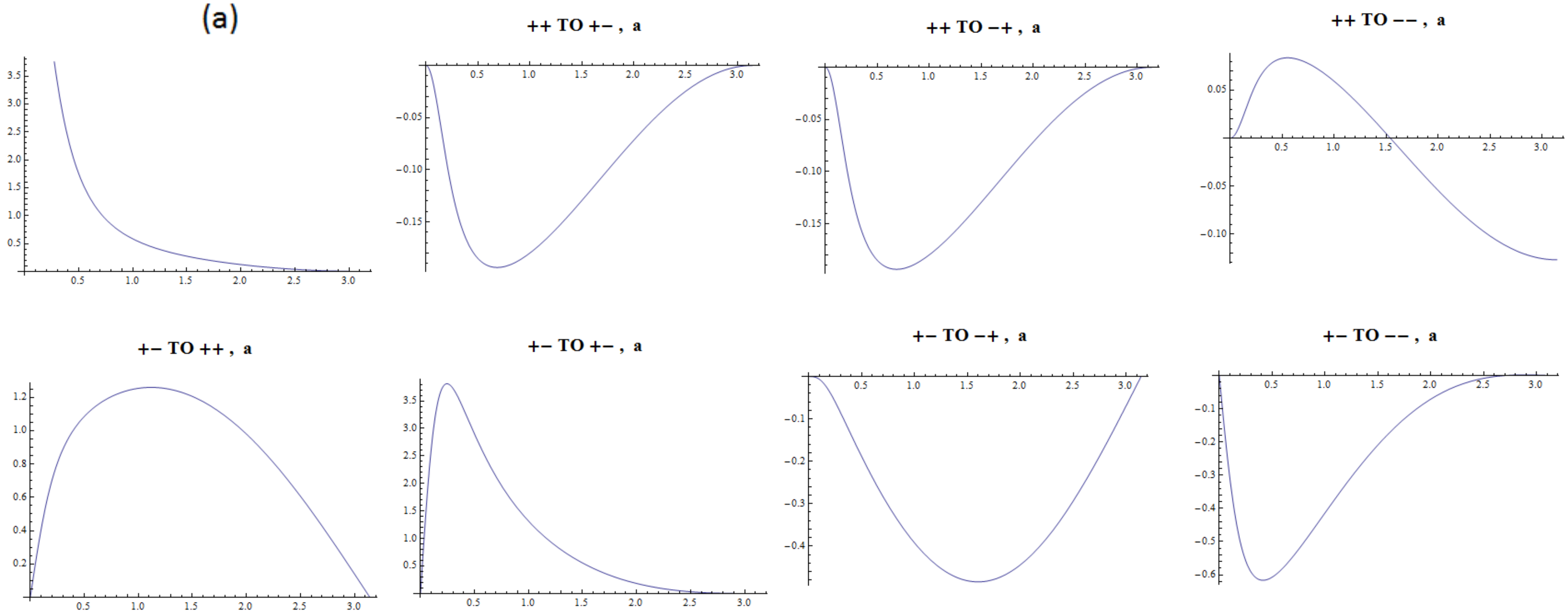


Scattering Angle Dependence of the Annihilation Amplitudes: Chirality



(a)

When $m_e \neq 0$, no such property.



Conclusion and Outlook

- Whole landscape between IFD and LFD has been revealed in QED tree-level with interpolating spinors,gauge bosons,their propagators.
- Maximal stability group of LFD saves significant dynamic efforts.
- Interpolating quantum field theory appears useful in resolution of theoretical issues, e.g. LFZM.
- Loop level applications are underway, e.g. to investigate the mass gap equation in QCD.

Kinematic Operators
(Members of Stability Group)

$$Exp\left(-i\omega\hat{\aleph}^i\right)|x^{\hat{+}}> \propto |x^{\hat{+}}>$$

$$[\hat{\aleph}^i, P^{\hat{+}}] = 0$$

$$\hat{\aleph}^i = \hat{F}^i \cos 2\delta - \hat{E}^i \sin 2\delta$$

$\delta = 0$

$-J^2$

J^1

$$\hat{\aleph}^1 = -J^2 \cos\delta - K^1 \sin\delta$$

$$\hat{\aleph}^2 = J^1 \cos\delta - K^2 \sin\delta$$

$\delta = \pi/4$

$-E^1 = -(J^2 + K^1)/\sqrt{2}$

$E^2 = (J^1 - K^2)/\sqrt{2}$

$$(J^3, P^1, P^2, P_{\hat{-}})$$

particle at rest

$$p^0 = M\,,\ p^1 = p^2 = p^3 = 0$$

$$(p_{\hat{+}} = M\cos\delta\,, p_{\hat{-}} = M\sin\delta)$$

same p^0 ↙ Under $\hat{\aleph}^i$ transformation ↘ $p^0 + p^3$ same

remain at rest

$$P^0 = M;\ p^3 = 0$$

can move

$$P^0 = M + \frac{\vec{p}_\perp^{\,2}}{2M}\,;\ p^3 = -\frac{\vec{p}_\perp^{\,2}}{2M}$$

↓

$$(p^0)^2 - (p^3)^2 = (M + \frac{\vec{p}_\perp^{\,2}}{2M})^2 - (-\frac{\vec{p}_\perp^{\,2}}{2M})^2 = M^2 + \vec{p}_\perp^{\,2} = 2p^+p^- > 0$$

Rational Energy-Momentum Dispersion Relation

Vacuum gets simpler in LFD.