Charged Compact Boson Stars in a theory of massless scalar field

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Abstract

In this work we present some new results obtained in a study of the phase diagram of charged compact boson stars in a theory involving a complex scalar field with a conical potential coupled to a U(1) gauge field and gravity. We here obtain new bifurcation points in this model. We present a detailed discussion of the various regions of the phase diagram with respect to the bifurcation points. The theory is seen to contain rich physics in a particular domain of the phase diagram.

Introduction

- In this work we study the phase diagram of charged compact boson stars in a theory involving a complex scalar field with a conical potential coupled to a U(1) gauge field and gravity [1, 2].
- A study of the phase diagram of the theory yields new bifurcation points (in addition to the one obtained earlier, cf. Refs. [1, 2]), which implies rich physics in the phase diagram of the theory.
- Our present studies extend the work of Refs. [1, 2]), performed in a theory of complex scalar field with only a conical potential, i.e., the scalar field is considered to be massless.
- We construct the boson star solutions of this theory numerically.

Action and Equations of Motion

We consider the theory defined by the following action (with $V(|\Phi|) := \lambda|\Phi|$, where λ is a constant parameter):

$$S = \int \left[\frac{R}{16\pi G} + \mathcal{L}_M\right]\sqrt{-g}\, d^4x \,,$$
$$\mathcal{L}_M = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - (D_\mu\Phi)^*(D^\mu\Phi) - V(|\Phi|)\,,$$
$$D_\mu\Phi = (\partial_\mu\Phi + ieA_\mu\Phi) \;,\; F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu). \tag{1}$$

Here
- R is the Ricci curvature scalar,
- G is Newton's gravitational constant.
- $g = det(g_{\mu\nu})$, where $g_{\mu\nu}$ is the metric tensor,
- asterisk denotes complex conjugation.

Using the variational principle, the equations of motion are obtained as:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}\,,$$
$$\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right) = -ie\sqrt{-g}[\Phi^*(D^\nu\Phi) - \Phi(D^\nu\Phi)^*]\,,$$
$$D_\mu\left(\sqrt{-g}D^\mu\Phi\right) = \frac{\lambda}{2}\sqrt{-g}\frac{\Phi}{|\Phi|}\,,$$
$$\left[D_\mu\left(\sqrt{-g}D^\mu\Phi\right)\right]^* = \frac{\lambda}{2}\sqrt{-g}\frac{\Phi^*}{|\Phi|}. \tag{2}$$

The energy-momentum tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = \Big[(F_{\mu\alpha}F_{\nu\beta}\,g^{\alpha\beta} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}) + (D_\mu\Phi)^*(D_\nu\Phi) + (D_\mu\Phi)(D_\nu\Phi)^* - g_{\mu\nu}\left((D_\alpha\Phi)^*(D_\beta\Phi)\right)g^{\alpha\beta} - g_{\mu\nu}\lambda(|\Phi|)\Big]. \tag{3}$$

To construct spherically symmetric solutions we adopt the metric

$$ds^2 = \left[-A^2N dt^2 + N^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{4}$$

This leads to the components of Einstein tensor ($G_{\mu\nu}$)

$$G^t_t = \left[\frac{-[r(1-N)]'}{r^2}\right], \qquad G^r_r = \left[\frac{2rA'N - A[r(1-N)]'}{A\,r^2}\right],$$
$$G^\theta_\theta = \left[\frac{2r[rA'N]' + [A\,r^2N']'}{2A\,r^2}\right] = G^\varphi_\varphi. \tag{5}$$

Here the arguments of the functions $A(r)$ and $N(r)$ have been suppressed. For solutions with a vanishing magnetic field, the Ansätze for the matter fields have the form:

$$\Phi(x^\mu) = \phi(r)e^{i\omega t} \;,\; A_\mu(x^\mu)dx^\mu = A_t(r)dt. \tag{6}$$

We introduce new constant parameters:

$$\beta = \frac{\lambda e}{\sqrt{2}} \;,\; \alpha^2(:= a) = \frac{4\pi G\,\beta^{2/3}}{e^2}. \tag{7}$$

Here $a := \alpha^2$ is dimensionless. We then redefine $\phi(r)$ and $A_t(r)$:

$$h(r) = \frac{(\sqrt{2}\,e\,\phi(r))}{\beta^{1/3}} \;,\; b(r) = \frac{(\omega + eA_t(r))}{\beta^{1/3}}. \tag{8}$$

Introducing a dimensionless coordinate $\hat r$ defined by $\hat r := \beta^{1/3}r$ (implying $\frac{d}{dr} = \beta^{1/3}\frac{d}{d\hat r}$), Eq. (8) reads:

$$h(\hat r) = \frac{(\sqrt{2}\,e\,\phi(\hat r))}{\beta^{1/3}} \;,\; b(\hat r) = \frac{(\omega + eA_t(\hat r))}{\beta^{1/3}}. \tag{9}$$

The equations of motion in terms of $h(\hat r)$ and $b(\hat r)$ (where the primes denote differentiation with respect to $\hat r$, and $\mathrm{sign}(h)$ denotes the usual signature function) read:

$$\left[AN\hat r^2 h'\right]' = \frac{\hat r^2}{AN}\left(A^2N\mathrm{sign}(h) - b^2h\right) \;,\; \left[\frac{\hat r^2 b'}{A}\right]' = \frac{bh^2\hat r^2}{AN}. \tag{10}$$

We thus obtain the set of equations:

$$N' = \left[\frac{1-N}{\hat r} - \frac{\alpha^2\hat r}{A^2N}\left(A^2N^2h'^2 + Nb'^2 + 2A^2Nh + b^2h^2\right)\right], \tag{11}$$
$$A' = \left[\frac{\alpha^2\hat r}{AN^2}\left(A^2N^2h'^2 + b^2h^2\right)\right], \tag{12}$$
$$h'' = \left[\frac{\alpha^2}{A^2N}\hat r h'\left(2A^2h + b^2\right) - \frac{h'(N+1)}{\hat rN} + \frac{A^2N\mathrm{sign}(h) - b^2h}{A^2N^2}\right] \tag{13}$$
$$b'' = \left[\frac{\alpha^2}{A^2N^2}\hat r b'\left(A^2N^2h'^2 + b^2h^2\right) - \frac{2b'}{\hat r} + \frac{bh^2}{N}\right]. \tag{14}$$

Boundary Conditions

For the metric function $A(\hat r)$ we choose the boundary condition $A(\hat r_o) = 1$, where $\hat r_o$ is the outer radius of the star. For constructing globally regular ball-like boson star solutions, we choose:

$$N(0) = 1 \;,\; b'(0) = 0 \;,\; h'(0) = 0 \;,\; h(\hat r_o) = 0 \;,\; h'(\hat r_o) = 0. \tag{15}$$

In the exterior region $\hat r > \hat r_o$ we match the Reissner-Nordström solution.

Charge and Mass

The theory has a conserved Noether current:

$$j^\mu = -i\,e\left[\Phi(D^\mu\Phi)^* - \Phi^*(D^\mu\Phi)\right] \;, \qquad j^\mu_{\;;\mu} = 0\,. \tag{16}$$

The charge Q of the boson star is given by

$$Q = -\frac{1}{4\pi}\int_0^{\hat r_o} j^t\sqrt{-g}\,dr\,d\theta\,d\phi\,, \qquad j^t = -\frac{h^2(\hat r)b(\hat r)}{A^2(\hat r)N(\hat r)}. \tag{17}$$

For all boson star solutions we obtain the mass M (in the units employed):

$$M = \left(1 - N(\hat r_o) + \frac{\alpha^2Q^2}{\hat r_o^2}\right)\frac{\hat r_o}{2}. \tag{18}$$

Results and Discussion

- We study the numerical solutions of Eqs. (11)-(14) with the boundary conditions defined by $A(\hat r_o) = 1$ and Eq. (15)
- We determine their domain of existence for a sequence of specific values of the parameter a.
- The theory is seen to possess rich physics in the domain $a = 0.22$ to $a \simeq +0.16$.
- We observe very interesting phenomena (in the phase diagram) near specific values of a, where the system is seen to have bifurcation points B_1, B_2 and B_3 corresponding to the following values of a: $a_{c_1} \simeq 0.198926$, $a_{c_2} \simeq 0.169311$ and $a_{c_3} \simeq 0.168308$, respectively.
- Possibility of further bifurcation points is not ruled out.
- In this work our focus is on the bifurcations.
- The phase diagram is divided into different regions in the vicinity of bifurcation points (as explained in the caption of Fig. 1).

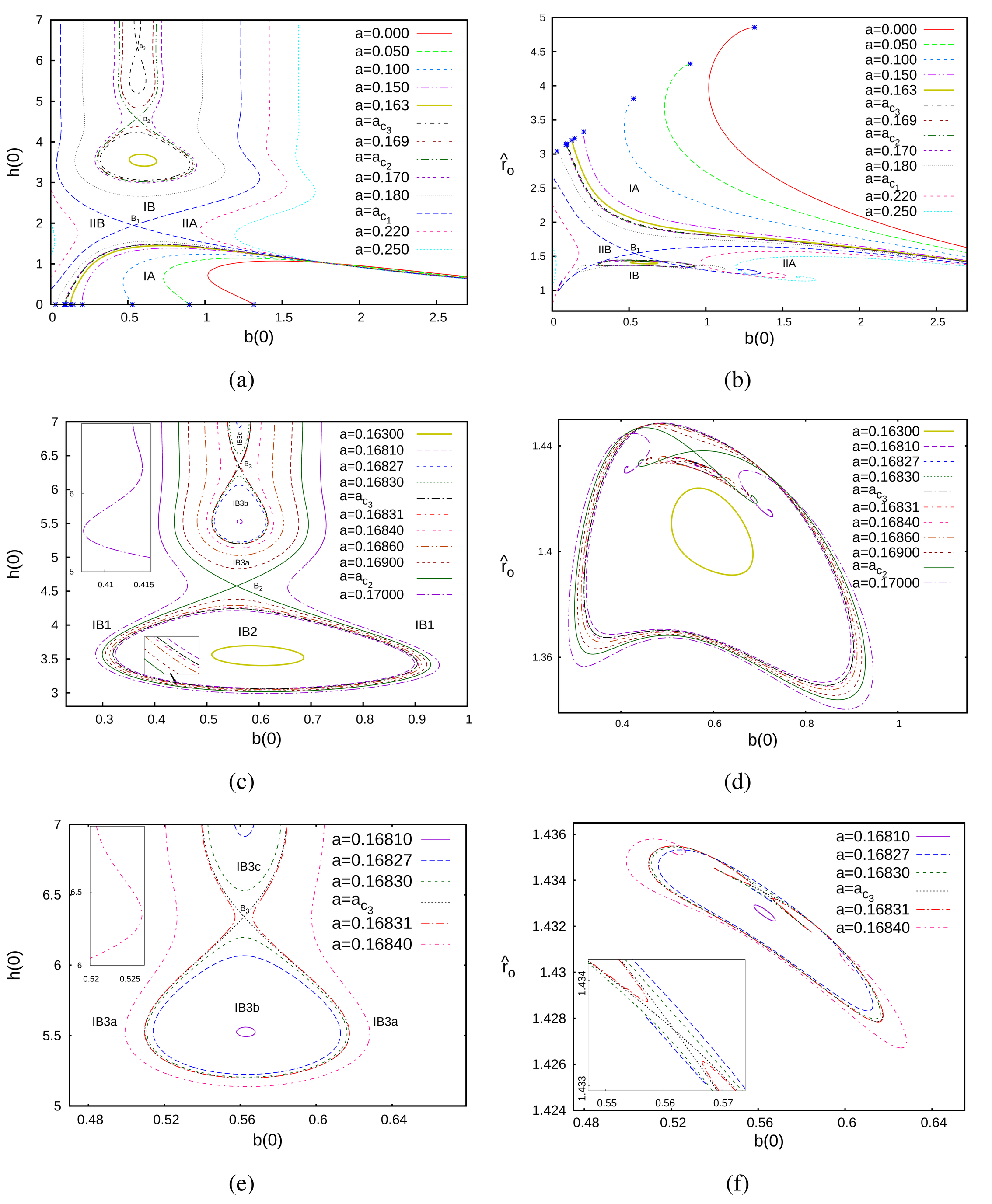

Figure 1: Fig. (a) depicts the phase diagram of the theory in terms of the vector field at the center of the star $b(0)$ and the scalar field at the center of the star $h(0)$ for different values of the parameter a in the range $a = 0$ to $a = 0.225$. The points B_1, B_2 and B_3 represent three bifurcation points. The entire region depicted in the phase diagram in Fig. (a) is divided into four regions IA, IB and IIA, IIB in the vicinity of B_1. The region IB of the phase diagram shown in Fig. (a) is separately depicted in detail in Fig. (c). The region IB of the phase diagram is subdivided into three regions IB1, IB2 and IB3 in the vicinity of B_2. The region IB3 of the phase diagram shown in Fig. (b) is separately depicted in detail in Fig. (e). It is subdivided into three regions IB3a, IB3b and IB3c in the vicinity of B_3. Figs. (b), (d) and (f) shows the radius $\hat r_o$ of the boson star versus the vector field at the center of the star $b(0)$ for different values of a. The spiral behaviour of the solutions is visible in the regions IA and IIB. The asterisks shown in Fig. (a), corresponding to $h(0) = 0$, represent the transition points from the boson stars to boson shells. The insets in Figs. (b) and (c) represent parts of these phase diagrams with higher resolution.

- To understand the stability of the boson stars, we consider the mass M versus the charge Q, as shown in Fig. 3(a) and 3(c) or the mass per unit charge M/Q versus the charge, as shown in Figs. 3(b) and 3(d).
- For the value a_{c_1} the two branches of solutions, limiting the region IA, possess lower masses than the the two branches of solutions, limiting the region IB, and should therefore be more stable.
- The two branches of solutions, limiting the region IB, might be classically stable as well, until the first extrema of mass and charge are encountered.
- Quantum mechanically, however, they would be unstable, since tunneling might occur. Beyond these extrema, unstable modes should be present, and thus the solutions should also be classically unstable.
- The curves shown in region IIB represent the lowest mass solutions for a given charge, they should be stable as well.
- In the region IIA, however, the solutions exhibiting oscillating/spiral behavior translates into the presence of a sequence of spikes, as seen in the insets of Figs. 3(a) and 3(b). Here the solutions should be stable only on their fundamental branch, reaching up to a maximal value of the mass and the charge, where a first spike is encountered. With every following spike a new unstable mode is expected to arise, as we conclude by analogy with the properties of non-compact boson stars.

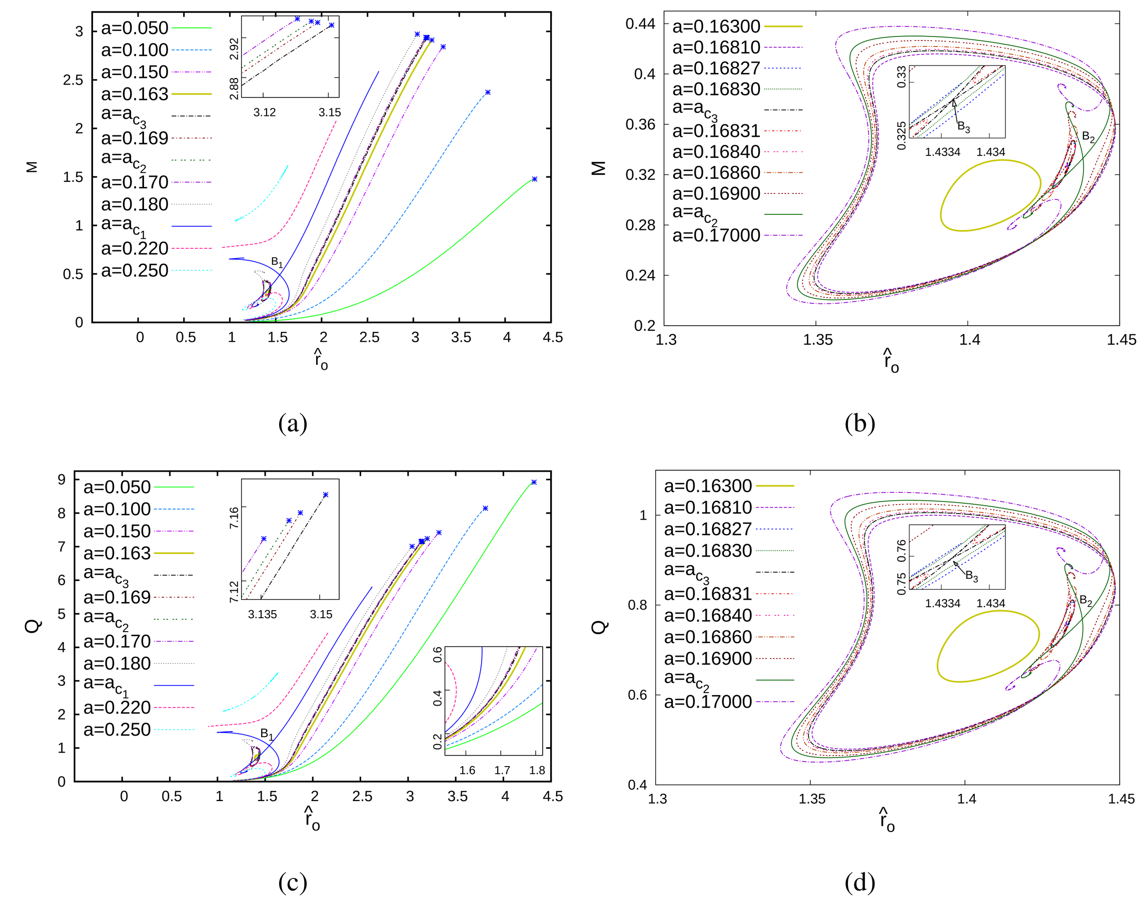

Figure 2: Fig. (a) depicts the mass M versus the radius of the star $\hat r_o$ for the same sequence of values of the parameter a. As before, the asterisks represent the transition points from the boson stars to boson shells, and the insets magnify parts of the diagram. Fig. (b) zooms into the region of the bifurcations, with the inset giving a magnified view of the bifurcation B3. Fig. (c) and (d) are the analog of Fig. (a) and (b) respectively for the charge Q.

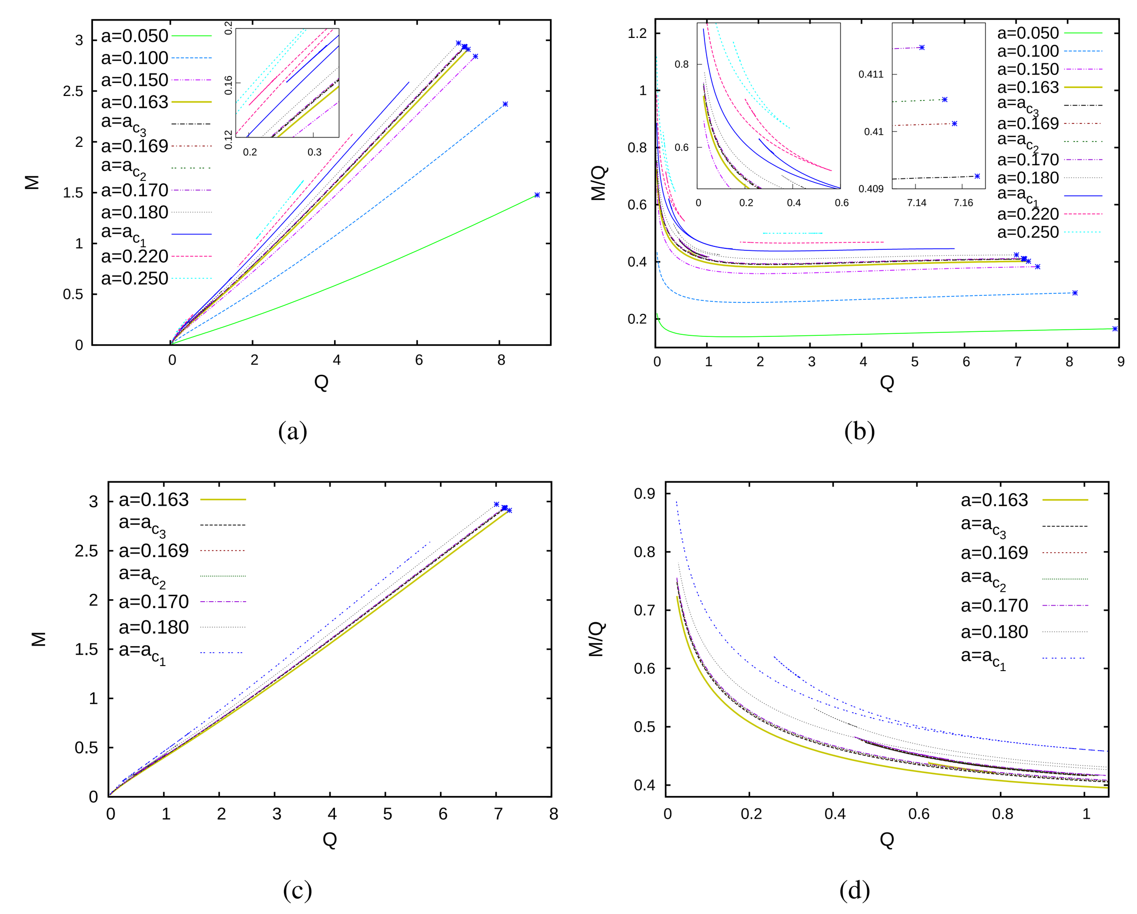

Figure 3: Fig. (a) depicts the mass M versus the charge Q for the same set of solutions. As before, the asterisks represent the transition points from the boson stars to boson shells, and the inset magnifies a part of the diagram. Fig. (b) depicts the mass per unit charge M/Q versus the charge Q. Again the insets magnify parts of the diagram. Fig. (c) and (d) zooms further into the region of the bifurcations.

Conclusion

- We have studied in this work a theory of a complex scalar field with a conical potential, coupled to a U(1) gauge field and gravity.
- We have shown that the theory has rich physics in the domain $a = 0.22$ to $a \simeq 0.16$, where we have identified three bifurcation points B_1, B_2 and $B_3$of possibly a whole sequence of further bifurcations.
- We have investigated the physical properties of the solutions, including their mass, charge and radius.
- By considering the mass versus the charge (or the mass per unit charge versus the charge) we have given arguments concerning the stability of the solutions.

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