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APPLICATIONS

EuPRAXIA

Beam Loading ASsisted maTching (BLAST) scheme for beam driven PWFA

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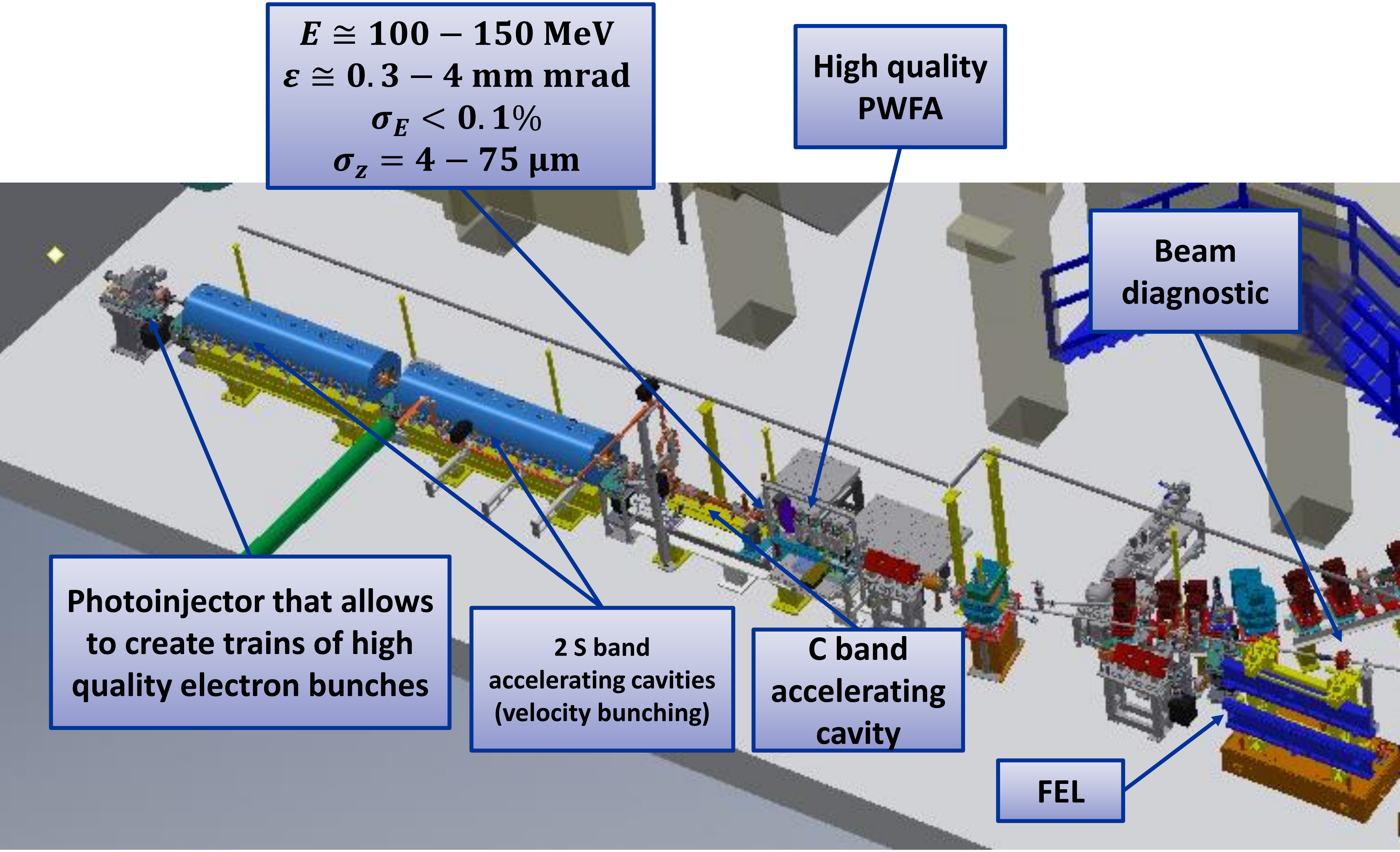
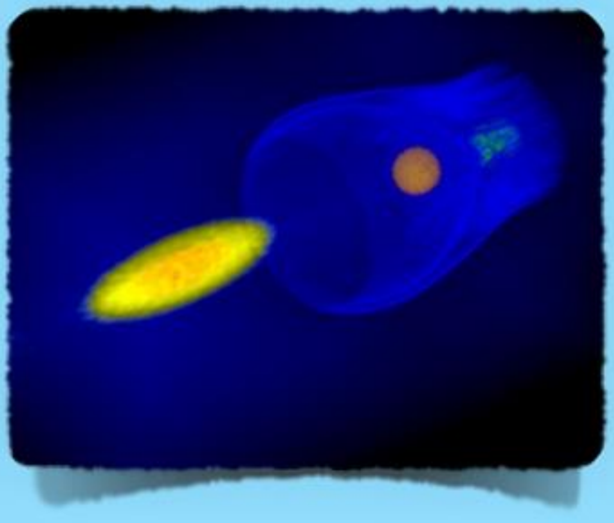
1st EuPRAXIA collaboration week, WP9, Hamburg 19-23/06/2017

This project has received funding from the European Union's Horizon 2020
research and innovation programme under grant agreement No 653782.

SPARC_LAB layout for beam driven PWFA experiments

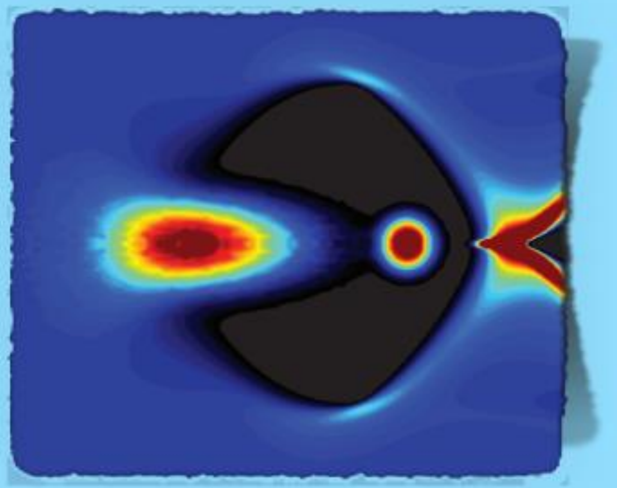


Simulation codes



ALaDyn

PIC code



Architect

hybrid code
for PWFA

Hybrid Tool: Architect

Horizon 2020

- **Bunch(es)** are treated **kinetically**
- **background plasma** as a **fluid**
- cylindrical symmetry assumed
- run time figure of merit: **1cm/h**
- **no-Quasi Static Approximation**

$$d_t\mathbf{p}_{\text{particle}} = q(\mathbf{E}+c\boldsymbol{\beta}_{\text{particle}}\times\mathbf{B})$$

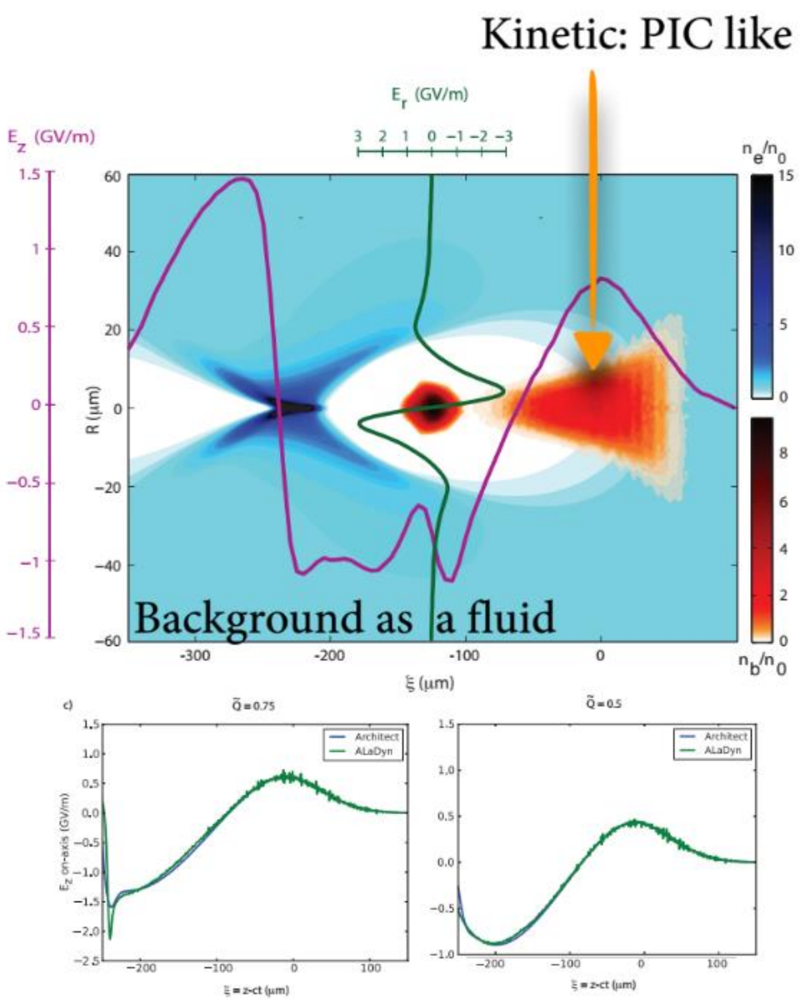
$$d_t\mathbf{x}_{\text{particle}} = \boldsymbol{\beta}_{\text{particle}}c$$

$$\partial_t n_e = -\nabla\cdot(\boldsymbol{\beta}_e c\, n_e)$$

$$\partial_t \mathbf{p}_e = -\nabla\cdot(\mathbf{p}_e \otimes \boldsymbol{\beta}_e c)+q(\mathbf{E}+c\boldsymbol{\beta}_e\times\mathbf{B})$$

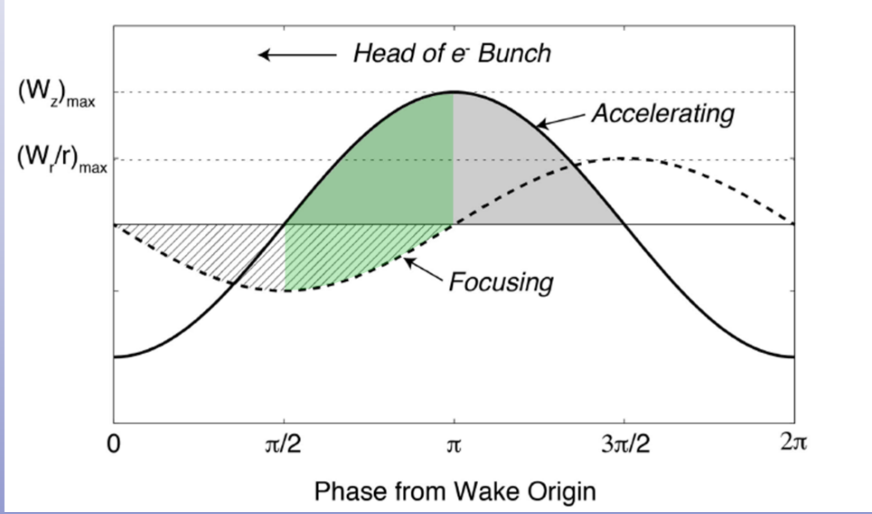
$$\partial_t\mathbf{B} = -\nabla\times\mathbf{E}$$

$$\partial_t\mathbf{E} = c^2\nabla\times\mathbf{B}-q\mu_0 c^3\left(n_e\boldsymbol{\beta}_e + n_b\boldsymbol{\beta}_b\right)$$



Code reliability respect to full PIC code showed both for linear and quasi-linear regimes **[1] [2]**

Linear/Bubble regime



$$n_b \ll n_0$$

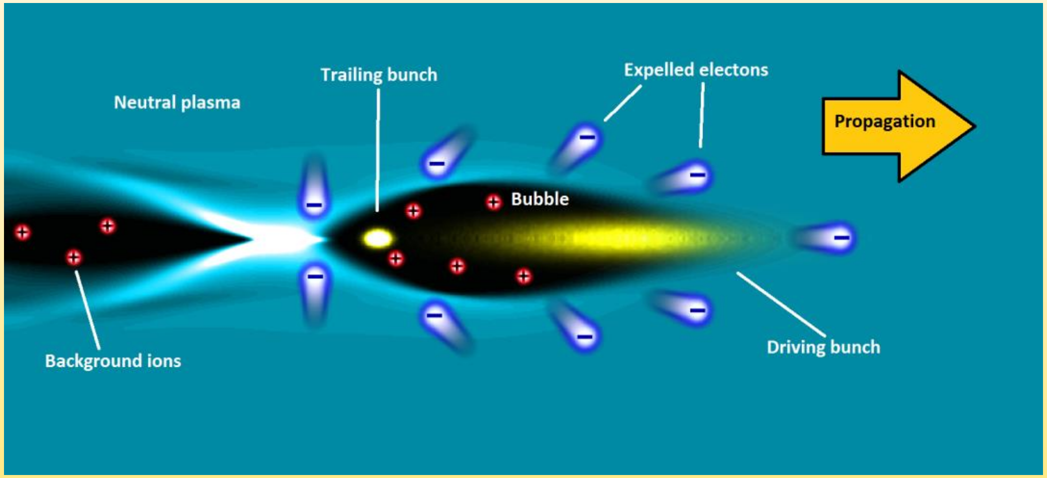
Linear field

- It is possible to inject a beam in the crest region (no focusing)
- Non linear dependency of the focusing
- Accelerating field depends on transverse position
- Lower field

$$n_b \gg n_0$$

Blow out field

- No crest region -> Spike
- Linear dependency of the focusing
- Accelerating field doesn't depend on transverse position
- Higher field



Driver brightness and plasma regimes

$$\sigma_m = \sqrt[4]{\frac{2}{\gamma}}\sqrt{\frac{\varepsilon_n}{k_p}} \longleftrightarrow \beta_m = \frac{\sqrt{2\gamma}}{k_p}$$

After a propagation inside plasma, a driving bunch will assest to the matching condition given by β_m **[3]**

$$\varepsilon_{n,fin} = \frac{\varepsilon_{n,init}}{2}\left(\frac{1+\alpha_T^2}{\beta^*} + \beta^*\right)$$

Final emittance depends on injection condition and since $\varepsilon_{n,fin} > \varepsilon_{n,init}$ the assestment spot size $\sigma_{fin} > \sigma_m$ **[3]**

$$\alpha \gg 1 \longleftrightarrow B_n \gg \frac{2I_A}{\pi^2 \beta_m \varepsilon_n}$$

The condition over driver density becomes a condition over driver brightness

High quality acceleration in linear regime

$$C_S = -\frac{e k_p^2 Z(\xi) L}{8\sigma_{r,D}^2 \gamma m_e c^2}$$

Spherical aberration coefficient

Transverse contribution (sliced)

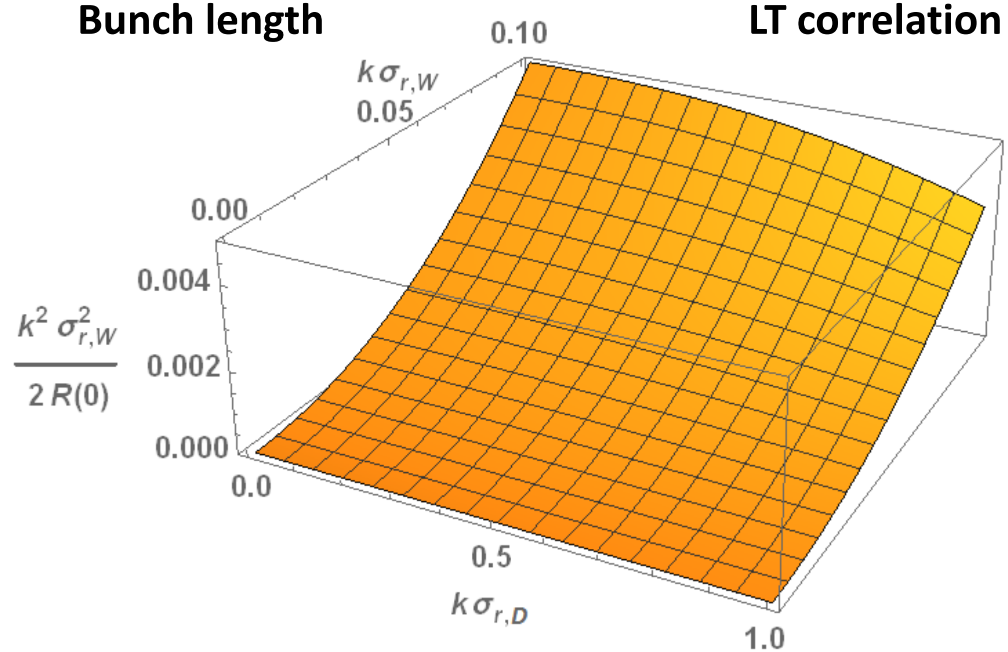
$$\sigma_{E,f} = \sqrt{\sigma_{E,i}^2 + \left(1 - \frac{\gamma_0}{\gamma}\right)^2 \left[\tan^2\phi_0\, k_p^2 \sigma_{z,w}^2 + \frac{3k_p^4\sigma_{z,w}^4}{4} + \frac{3k_p^4\sigma_{r,w}^4}{16R^2(0)} - \frac{k_p^4\sigma_{r,w}^2\sigma_{z,w}^2}{4R(0)}\right]}$$

Energy spread growth

Chirp

Bunch length

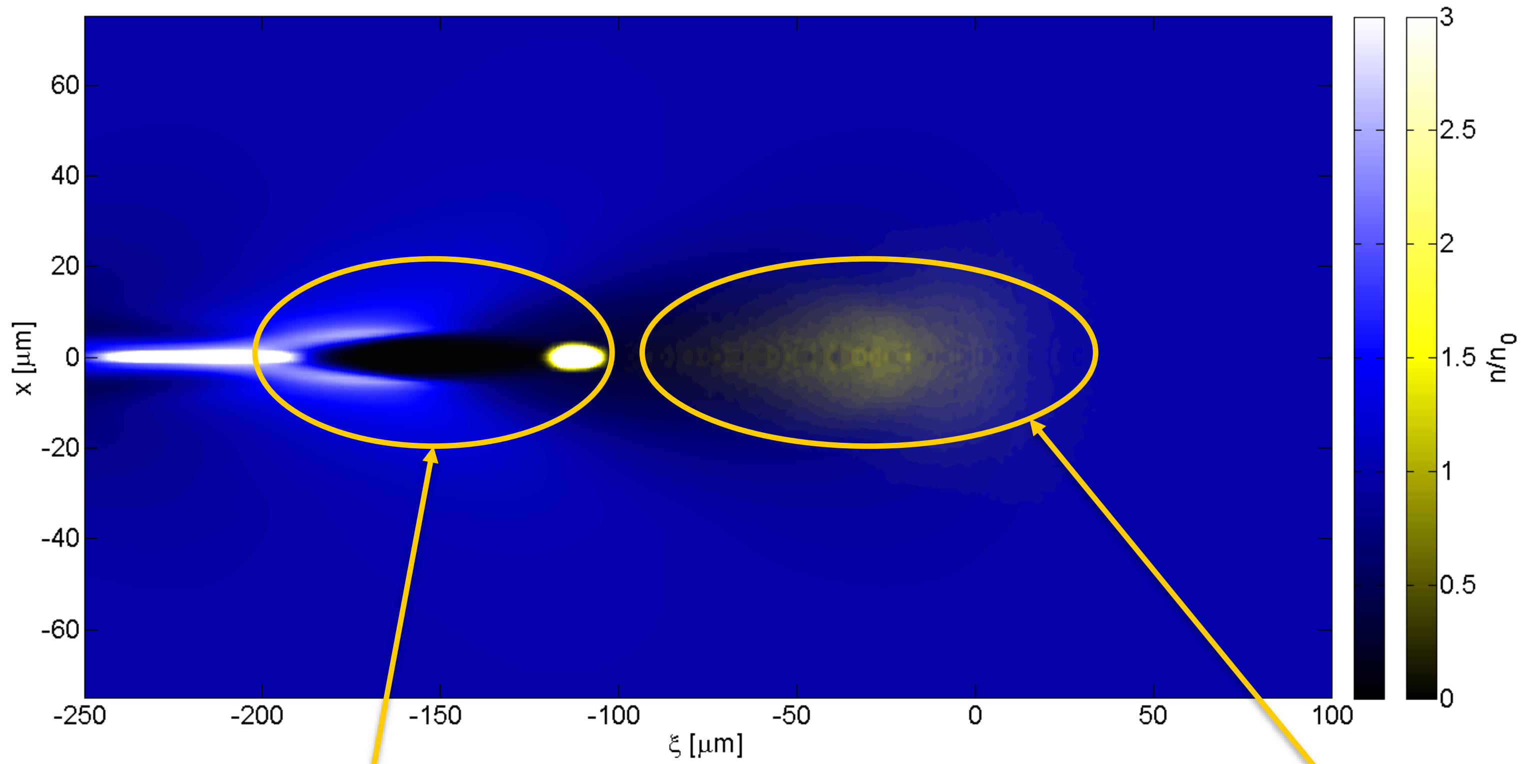
LT correlation



High quality requires:

- $\sigma_{r,w} \ll \sigma_{r,D}$
- $k_p^2\sigma_{z,W}^2 \ll 1$
- In many realistic cases, beam loading is not negligible

BLAST scheme



The field generated by the witness is non linear

- Focusing is guaranteed by a beam loading effect

$$\sigma_x = \sqrt[4]{\frac{1}{\gamma}}\sqrt{\frac{2\varepsilon_n}{k_p}} \rightarrow \beta_x = \frac{2\sqrt{\gamma}}{k_p}$$

The driver generates a linear field

- Witness can be injected on crest

$$\sigma_x = \sqrt{\frac{4}{\gamma \Lambda Z}}\varepsilon_n$$

Energy spread growth

- The longitudinal field generated by a low charge high density bunch can be described by linear equations **[4]**
- The beam loading compensation occurs when the first derivative of the accelerating field in the center of the witness is 0 **[5]**
- From the analytical model of field it is possible to evaluate the energy spread **[6]**

Beam loading compensation $$\frac{\partial E_z}{\partial \xi} = 0 \longleftrightarrow \sin\phi_0 = -k_p \frac{Z(0)R(0)}{E_0}$$

For any witness in a great range of parameters it's possible to find an injection distance that guarantees beam loading compensation

$$\sigma_{E,f} = \sqrt{\sigma_{E,i}^2 + \left(1 - \frac{\gamma_0}{\gamma}\right)^2 \frac{3k_p^4 \sigma_{z,w}^4}{4}}$$

Energy spread growth depends only on witness length

BLAST scheme transverse matching

A correlated focusing on longitudinal dimension causes emittance growth

$$\varepsilon_{n,c} = \frac{k^2 \sigma_x^2 L}{c} \sqrt{\langle f^2(\xi) \rangle}$$

$$\frac{\partial (E_r - cB_\theta)}{\partial \xi} = 0$$

Hypotesis of energy spread compensation using beam loading (NO BUNCH SHAPING)

$$\frac{\partial E_z}{\partial \xi} = 0$$

Hypotesis of quasi-linearity $J_z = 0$ **[7]**

Consequences $J_r = 0$ $\quad \frac{\partial \rho}{\partial r, \xi} = 0$ $\quad \frac{\partial (E_r - cB_\theta)}{\partial \xi} = 0$

Ion column model can be applied $n = \frac{n_0}{2}$ **[8]** $\quad \sigma_m = \sqrt[4]{\frac{1}{\gamma}} \sqrt{\frac{2\varepsilon_n}{k_p}}$

Working point parameters

	SPARC_LAB Working Point	
	Driver Injection	Witness Injection
γ	200	200
$\sigma_r\,[\mu\mathrm{m}]$	10.3	1.26
$\sigma_z\,[\mu\mathrm{m}]$	37.2	3
$\sigma_E\,[\%]$	0.1	0.1
$\varepsilon_n\,[\mu\mathrm{m}]$	17	0.3
$Q\,[\mathrm{pC}]$	200	10
$I\,[\mathrm{kA}]$	0.68	0.42
$\tilde{Q}$	1.2	0.06
$n_0\,[\mathrm{cm}^{-3}]$	$2 \cdot 10^{16}$	
$E_z\,[\mathrm{GV/m}]$	2	
$\Delta s\,[\mathrm{cm}]$	5	

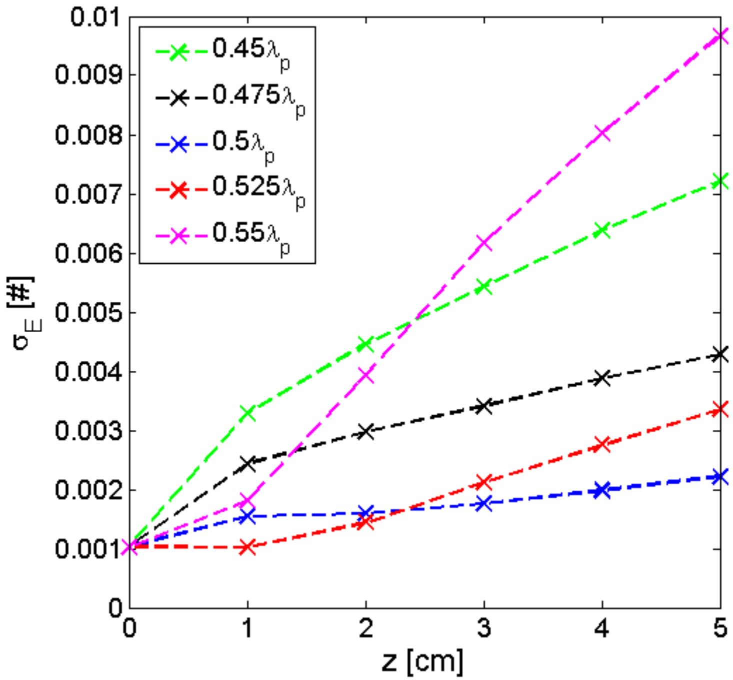
$\boldsymbol{\lambda_p \approx 235\ \mu m}$

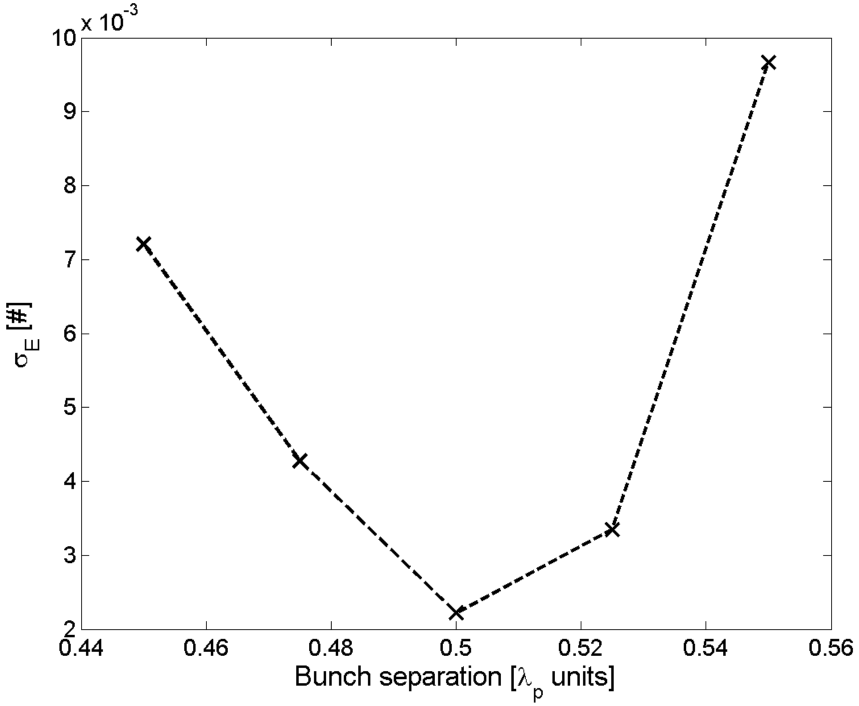
Bunch separation scan
(low resolution)

$$\sin\phi_0 = -k_p \frac{Z(0)R(0)}{E_0}$$

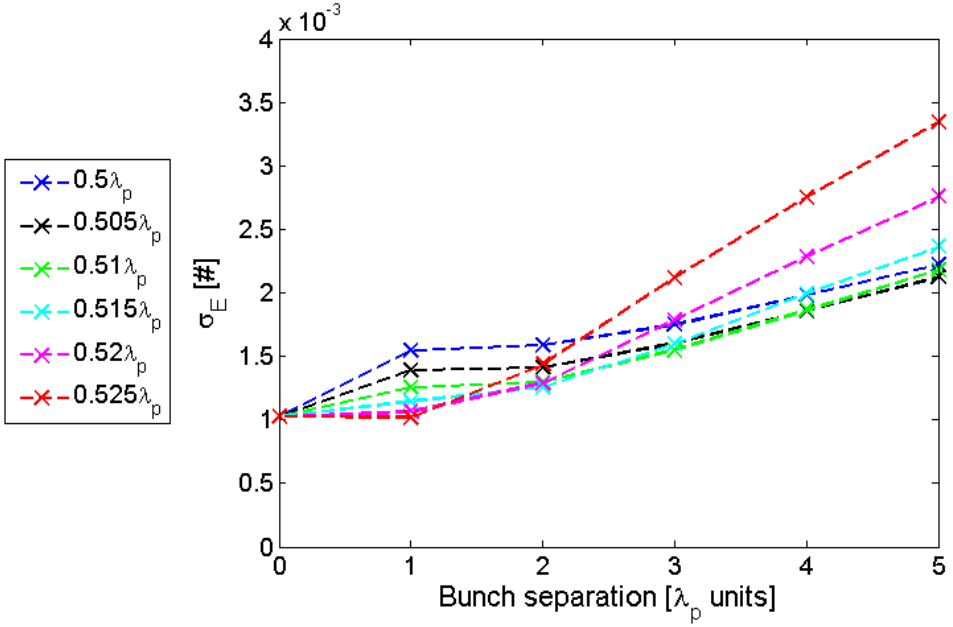
The model developed for BLAST scheme doesn't forsee the driver head erosion effects

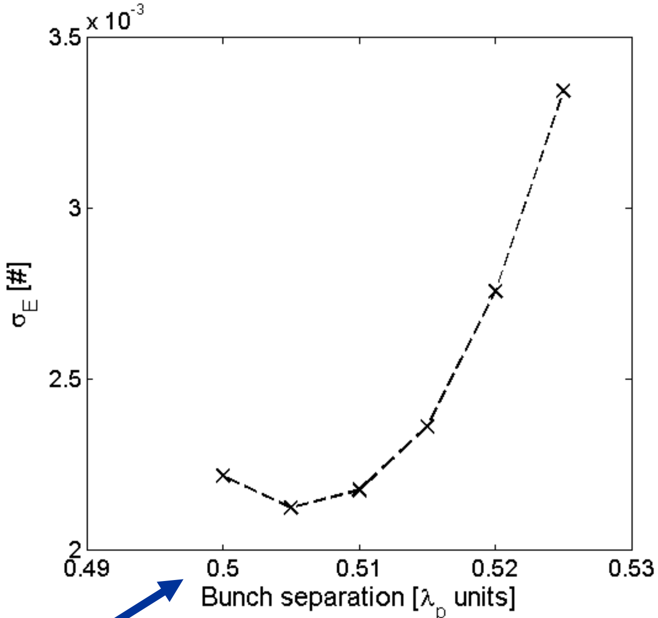
A simulation scan is required in order to evaluate E_0 and ϕ_0





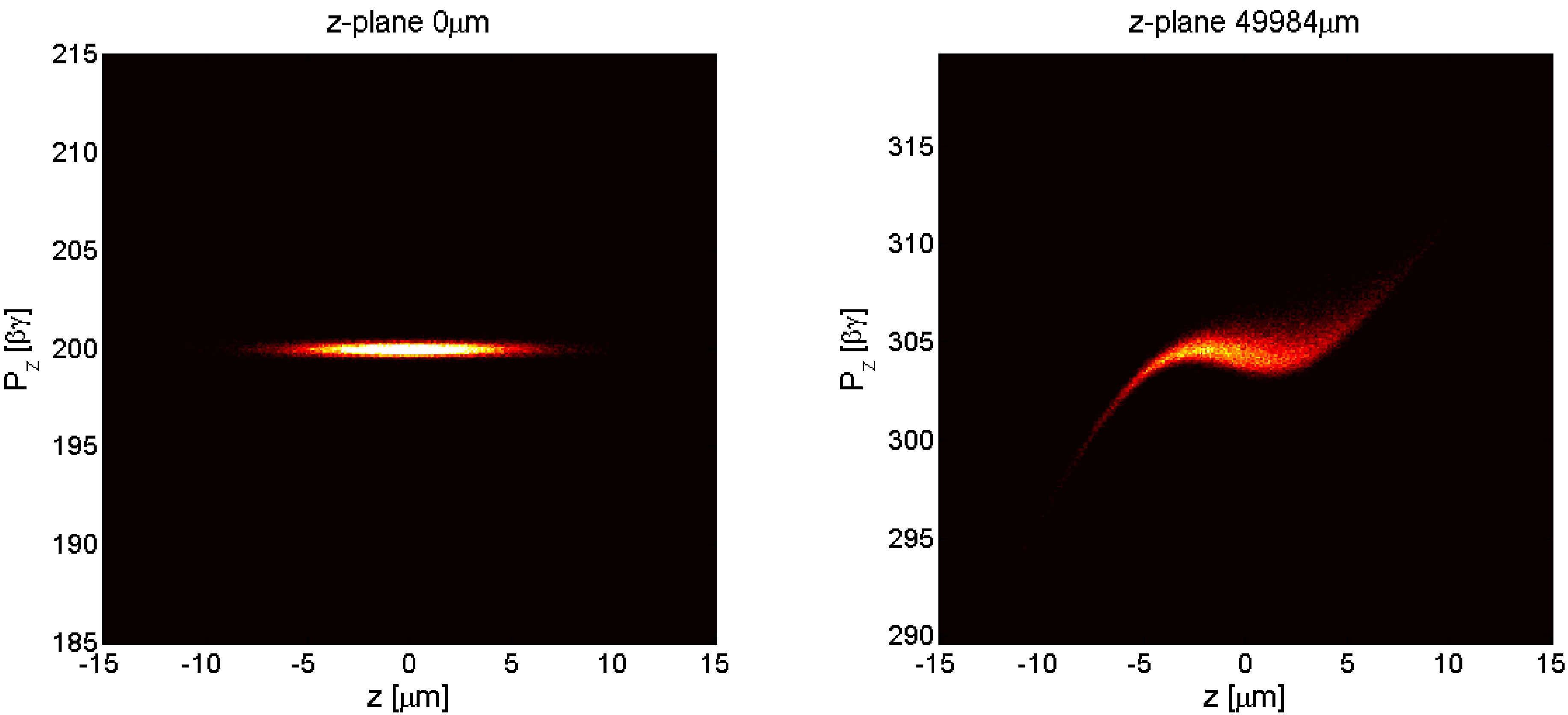
Bunch separation scan
(high resolution)





Optimal injection distance $0.505\lambda_p$

Witness LPS

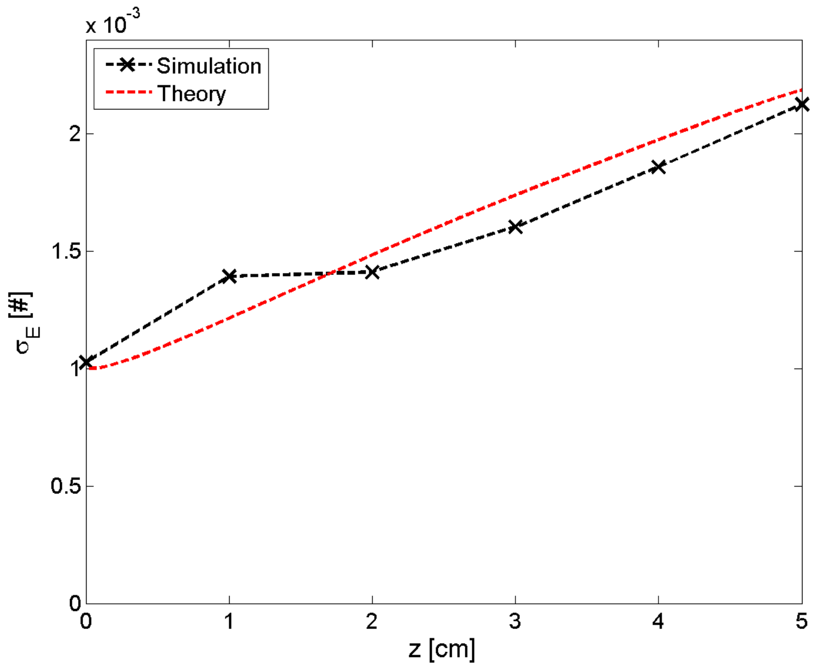


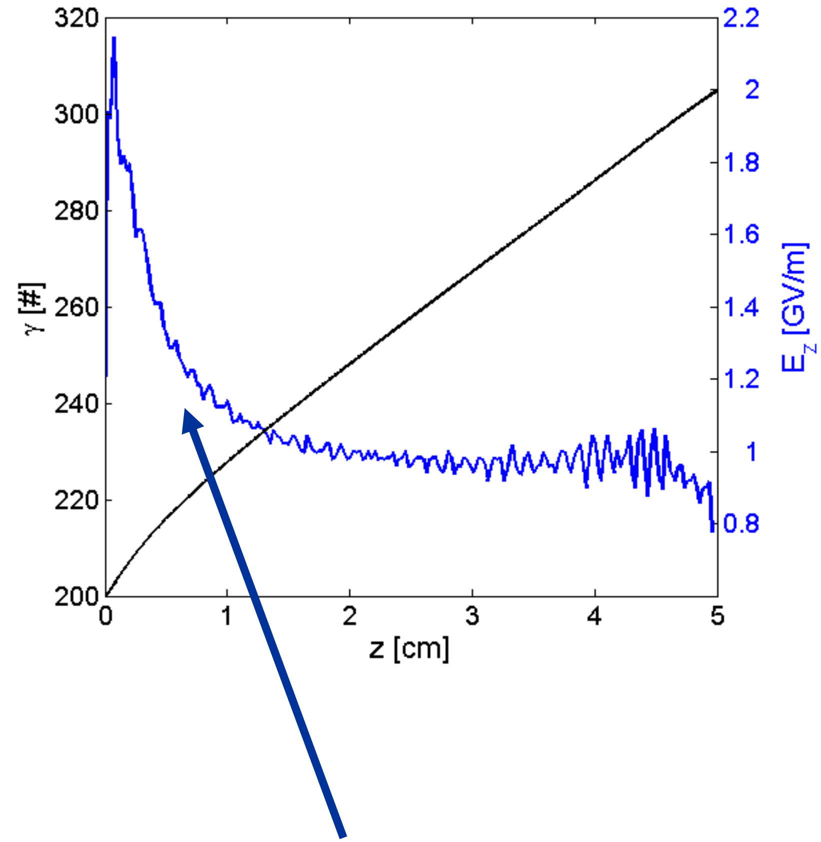
z-plane 0μm
215
210
205
200
195
190
185
-15
-10
-5
0
5
10
15
z [μm]
P_z [βγ]
z-plane 49984μm
315
310
305
300
295
290
-15
-10
-5
0
5
10
15
z [μm]
P_z [βγ]

Energy spread evolution and accelerating gradient

$$\sigma_{E,f} = \sqrt{\sigma_{E,i}^2 + \left(1 - \frac{\gamma_0}{\gamma}\right)^2 \frac{3k_p^4\sigma_{z,w}^4}{4}}$$

Energy spread growth depends only on witness length





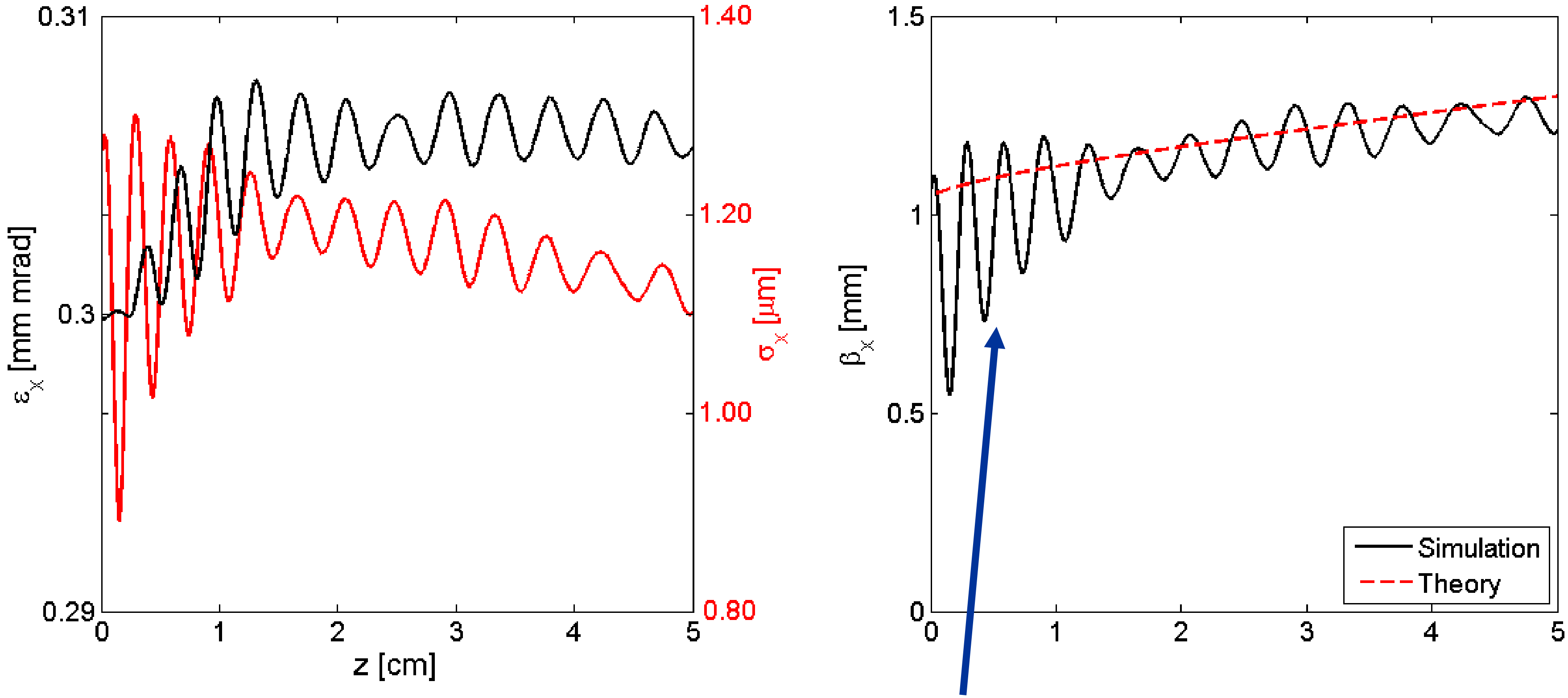
Final accelerating gradient 1.07 GV/m

E_0 is not constant for the first cm

Witness envelope

$$\epsilon_{n,fin} = \frac{\epsilon_{n,init}}{2}\left(2 + \frac{s^2}{\beta_w}\right)$$

Assuming an error in waist position of the witness equal to λ_p the emittance growth results to be of 3% **[5]**



Envelope doesn't follow the prevision for the first cm

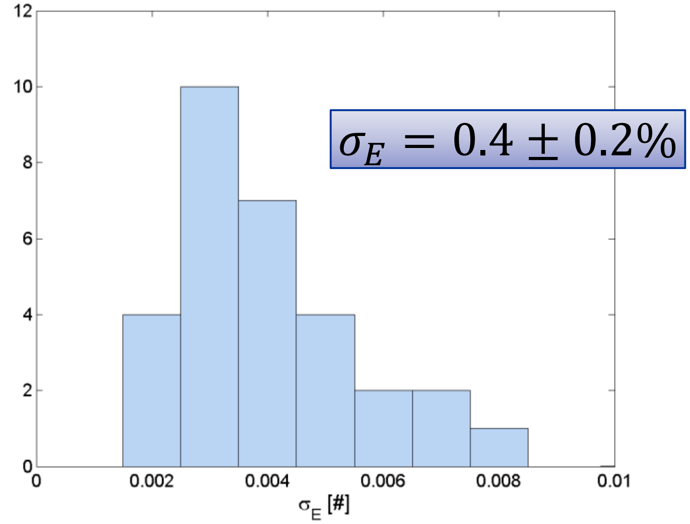
Working point results

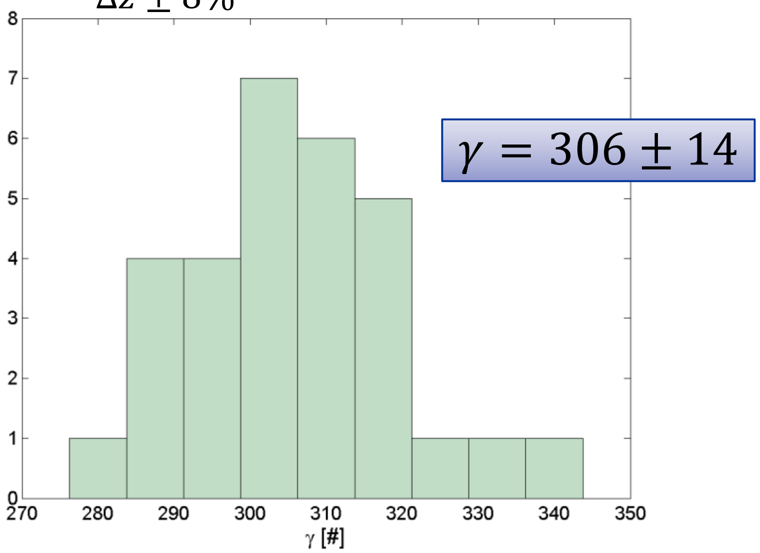
Incoming witness	Outcoming witness
$\gamma = 200$	$\gamma = 300$
$\varepsilon_n = 0.3\ mm\ mrad$	$\varepsilon_n = 0.308\ mm\ mrad$
$\sigma_E = 0.1\%$	$\sigma_E = 0.22\%$

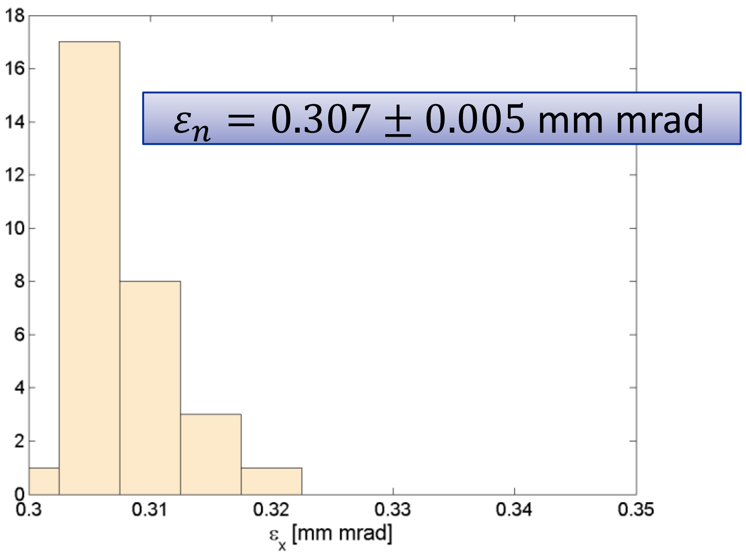
Acceleration parameters
$\Delta s = 5cm$
$E_z = 1.07GV/m$

Stability analysis

- A scan of 30 simulation was performed using the latin hypercube sample in order to analyze the working point robustness
- The chosen parameter range of the scan is consistent with measurements performed at SPARC_LAB injector (worst results)
  - $\sigma_r \pm 15\%$
  - $\sigma_z \pm 8\%$
  - $Q \pm 10\%$
  - $\Delta z \pm 8\%$







Rescaling to EuPRAXIA parameters

	SPARC_LAB Working Point			EuPRAXIA		
	Driver Injection	Witness Injection	Witness Extraction	Driver Injection	Witness Injection	Exp. Witness Extraction
γ	200	200	300	980	980	**1960**
$\sigma_r\,[\mu\text{m}]$	10.3	1.26	1.3	5.8	0.67	0.67
$\sigma_z\,[\mu\text{m}]$	37.2	3	3	16.6	1	1
$\sigma_E\,[\%]$	0.1	0.1	0.22	0.1	0.1	**0.2**
$\varepsilon_n\,[\mu\text{m}]$	17	0.3	0.31	34	0.3	**0.3**
$Q\,[\text{pC}]$	200	10	10	200	30	**30**
$I\,[\text{kA}]$	0.68	0.42	0.42	1.5	**3.8**	**3.8**
$\tilde{Q}$	1.2	0.06	0.06	2.72	0.4	0.4
$n_0\,[\text{cm}^{-3}]$	$2 \cdot 10^{16}$			$\mathbf{10^{17}}$		
$E_z\,[\text{GV/m}]$	1 (2 exp.)			**5.2**		
$\Delta s\,[\text{cm}]$	5			**15**		

Conclusions

Results

- Low energy working point fullfilled
- Theoretical basis for working point forecasts estabilished

Future perspectives

- Model for head erosion
- Cross-check of the results with full PIC/3D codes
- Simulation scan in order to propose a possible EuPRAXIA working point is on chart

Acknowledgements:

SPARC_LAB GROUP, EuPRAXIA collaboration, WP9

List of references

[1] Massimo, F., S. Atzeni, and A. Marocchino. "Comparisons of time explicit hybrid kinetic-fluid code Architect for Plasma Wakefield Acceleration with a full PIC code. " *Journal of Computational Physics* 327 (2016): 841-850.

[2] Marocchino, A., et al. "Efficient modeling of plasma wakefield acceleration in quasi-non-linear-regimes with the hybrid code Architect.“ *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 829 (2016): 386-391.

[3] Mehrling, T., et al. "Transverse emittance growth in staged laser-wakefield acceleration." *Physical Review Special Topics-Accelerators and Beams* 15.11 (2012): 111303.

[4] Barov, Nick, et al. "Energy loss of a high-charge bunched electron beam in plasma: Analysis." *Physical Review Special Topics-Accelerators and Beams* 7.6 (2004): 061301.

[5] Chiou, T. C., and T. Katsouleas. "High beam quality and efficiency in plasma-based accelerators." *Physical review letters* 81.16 (1998): 3411.

[6] LCLS, CDR, ch07. *SLAC*

[7] Lu, W., et al. "A nonlinear theory for multidimensional relativistic plasma wave wakefields a." *Physics of Plasmas* 13.5 (2006): 056709.

[8] Stupakov, G., et al. "Wake excited in plasma by an ultrarelativistic pointlike bunch." Physical Review Accelerators and Beams 19.10 (2016): 101302.

Brightness demonstration

$$\beta_m = \frac{\sqrt{2\gamma}}{k_p}$$

$$\sigma_m = \sqrt{\frac{\beta_m \varepsilon_n}{\gamma}}$$

$$\alpha \leq \frac{\Lambda}{k_p^2 \sigma_m^2}$$

$$\Lambda = \frac{2I}{I_A}$$

$$B_n = \frac{2I}{\pi^2 \varepsilon_n^2}$$

→

$$\frac{\Lambda}{k_p^2 \sigma_m^2} \geq \alpha \gg 1$$

$$\frac{\pi^2}{2} \beta_m \varepsilon_n \frac{B_n}{I_A} \gg 1$$

→

$$B_n \gg \frac{2I_A}{\pi^2 \beta_m \varepsilon_n}$$

Half density ion column

$$\frac{\partial E_z}{\partial \xi} = 0$$

$$\frac{\partial E_z}{\partial r} = \mu_0 c J_r$$

$$\frac{\partial (E_r - cB_\theta)}{\partial \xi} = \mu_0 c J_r = 0$$

$$\frac{\partial J_r}{\partial \xi} = 0$$

$$\frac{\partial \rho}{\partial \xi} = 0$$

$$J_r = 0$$

$$J_z = 0$$

$$\rho = \frac{n_0}{2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}[r(E_r - cB_\theta)] = \frac{\rho}{\epsilon_0} - \mu_0 c J_z$$