Effective Theory of Dark Energy

Filippo Vernizzi - IPhT, CEA Saclay

Probing the Dark Sector and General Relativity
CERN, Geneva - August 14, 2017
with
Creminelli, Cusin, D’Amico, Gleyzes, Gubitosi, Langlois, Lewandowski, Mancarella, Noreña, Noui, Piazza, …
Motivations

✧ Gravity only been tested over specials ranges of scales and masses

✧ Cosmology is a window for testing gravity on very large distances
Motivations

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- Cosmology is a window for testing gravity on very large distances
- Standard model (GR): LCDM

![Planck '15 “Cosmological Parameters”](image)

![Motivations Diagram](image)
**Motivations**

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- Cosmology is a window for testing gravity on very large distances
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![Graph showing gravitational field parameters](image)

- $\xi = GM/r^3c^2$
- $\epsilon = GM/rc^2$

**Figure 1:** A parameter space for quantifying the strength of a gravitational field. The $x$-axis measures the potential $\epsilon \equiv GM/rc^2$ and the $y$-axis measures the spacetime curvature $\xi \equiv GM/r^3c^2$ of the gravitational field at a radius $r$ away from a central object of mass $M$. These two parameters provide two different quantitative measures of the strength of the gravitational fields. The various curves, points, and legends are described in the text.
Tessa Baker

Modified Gravity

Add new field content

Higher dimensions

Non-local

Higher-order

Strings & Branes

Randall-Sundrum I & II

Kaluza-Klein

Generalisations
of $S_{EH}$

Gauss-Bonnet

Lorentz violation

Horava-Lifschitz

Conformal gravity

DGP

2T gravity

Some degravitation scenarios

$R_{\mu\nu} \Box^{-1} R_{\mu\nu}$

$f \left( \frac{R}{\Box} \right)$

General $R_{\mu\nu} R^{\mu\nu}$, $\Box R$, etc.

Vector

Einstein-Aether

Lorentz violation

Bigravity

EBI

Bimetric MOND

Tensor

Massive gravity

Cuscuton

Chaplygin gases

Chern-Simons

TeVeS

$f(T)$

Einstein-Cartan-Sciama-Kibble

Torsion theories

Scalar-tensor & Brans-Dicke

Ghost condensates

Galileons

the Fab Four

KGB

Coupled Quintessence

Horndeski theories

Emergent Approaches

CDT

Padmanabhan thermo.

2T gravity

CDT

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Motivations

Many models of modified gravity, each with its own theoretical motivation and phenomenology.
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ETofDE
\[ \alpha_K(t), \alpha_B(t), \alpha_M(t), \alpha_T(t), \alpha_T(t), \ldots \]

Bridge models and observations in a minimal and systematic way
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Bridge models and observations in a minimal and systematic way

**Observationally motivated:** efficiency, now implemented in Einstein-Boltzmann codes

**Theoretically motivated:** locality, causality, diff invariance, unitarity, stability, etc...

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**Motivations**

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**Observed**

- Galaxy clustering
- Weak lensing
- CMB (ISW)
- Grav. waves

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**Models**

- Modified Gravity
- Higher dimensions
- Non-local
- Higher-order

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**Observationally motivated:**

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Observations

Galaxy clustering

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Grav. waves
Scalar-tensor theories

- Simplest models of modified gravity are based on single scalar field

- Old school theories: Quintessence, Brans-Dicke, K-essence, …

\[ \mathcal{L}(\phi, \partial_\mu \phi) \]
Scalar-tensor theories

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Scalar-tensor theories

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- Old school theories: Quintessence, Brans-Dicke, K-essence, \( \mathcal{L}(\phi, \partial_\mu \phi) \)

- Generalized theories: **Galileons** \( \mathcal{L}(\phi, \partial_\mu \phi, \nabla_\mu \nabla_\nu \phi) \)

\[
\phi \rightarrow \phi + b_\mu x^\mu + c \quad \mathcal{L} = \phi(\partial^2 \phi)^n
\]

- Unique Lagrangians with 2nd order EOM:

\[
\begin{align*}
\mathcal{L}_1 &= \phi , \\
\mathcal{L}_2 &= (\partial \phi)^2 , \\
\mathcal{L}_3 &= (\partial \phi)^2 \partial^2 \phi , \\
\mathcal{L}_4 &= (\partial \phi)^2 \left[ (\partial^2 \phi)^2 - (\partial_\mu \partial_\nu \phi)^2 \right] , \\
\mathcal{L}_5 &= (\partial \phi)^2 \left[ (\partial^2 \phi)^3 - 3\partial^2 \phi (\partial_\mu \partial_\nu \phi)^2 + 2(\partial_\mu \partial_\nu \phi)^3 \right] .
\end{align*}
\]
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  \[ \phi \rightarrow \phi + b_\mu x^\mu + c \]
  \[ \mathcal{L} = \phi (\partial^2 \phi)^n \]
  (Nicolis, Rattazzi, Trincherini '08)

- Unique Lagrangians with 2nd order EOM:
  \[ \mathcal{L}_1 = \phi , \]
  \[ \mathcal{L}_2 = (\partial \phi)^2 , \]
  \[ \mathcal{L}_3 = (\partial \phi)^2 \partial^2 \phi , \]
  \[ \mathcal{L}_4 = (\partial \phi)^2 \left[ (\partial^2 \phi)^2 - (\partial_\mu \partial_\nu \phi)^2 \right] , \]
  \[ \mathcal{L}_5 = (\partial \phi)^2 \left[ (\partial^2 \phi)^3 - 3 \partial^2 \phi (\partial_\mu \partial_\nu \phi)^2 + 2 (\partial_\mu \partial_\nu \phi)^3 \right] . \]

- Can provide self-acceleration and nonlinearities (Vainshtein screening), with controlled quantum corrections and no ghost
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- Generalized theories: Galileons

- Covariantization: **Horndeski theories** (Horndeski ’73, see also Deffayet et al.’11)

\[
L_H = G_2(\phi, X) + G_3(\phi, X)\Box \phi + \\
+ G_4(\phi, X)^{(4)}R - 2G_4,_{X}(\phi, X)[(\Box \phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}] \\
+ G_5(\phi, X)^{(4)}G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_5,_{X}(\phi, X)[(\Box \phi)^3 - 3\Box \phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\nu\lambda}\phi^{;\mu}] \\
\]

\[
X \equiv \phi_{;\mu} \phi^{;\mu} \equiv \nabla_\mu \phi \nabla^\mu \phi
\]
Scalar-tensor theories

- Simplest models of modified gravity are based on single scalar field
- Old school theories: Quintessence, Brans-Dicke, K-essence, ... \( \mathcal{L}(\phi, \partial_\mu \phi) \)
- Generalized theories: Galileons \( \mathcal{L}(\phi, \partial_\mu \phi, \nabla_\mu \nabla_\nu \phi) \)
- Generally, higher-derivatives lead to extra unstable d.o.f. (Ostrogradski ghost)
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Zumalacarregui, Garcia-Bellido ’13 with Gleyzes, Langlois, Piazza ’14;
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Examples:

\[
\mathcal{L} = \frac{1}{2} \dddot{\phi}^2 + \frac{m}{2} \dddot{\phi}^2
\]

\[
\dddot{\phi} - m \dddot{\phi} = 0
\]

\[
\{ \phi(t_0), \dot{\phi}(t_0), \ddot{\phi}(t_0), \dddot{\phi}(t_0) \}
\]

\[
\frac{\partial^2 \mathcal{L}}{\partial v_a \partial v_b} = \begin{pmatrix} 1 & b \\ b & k \end{pmatrix} \quad k = b^2
\]

Degenerate!
Scalar-tensor theories

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**Diagram:**
- Horndeski
- Extra DOF

**Beyond Horndeski**
Zumalacarregui, Garcia-Bellido ’13 with Gleyzes, Langlois, Piazza ’14;

**Degenerate Higher-Order Scalar-Tensor theories**
Langlois, Noui ’15, ’16; Crisostomi, Hull et al. ’16; Crisostomi, Koyama, Tasinato ’16; Achour et al. ’16
Degenerate Higher-Order ST theories

✦ DHOST/EST theories: most general Lorentz-invariant scalar-tensor theory with a 1 scalar and 2 tensor degrees of freedom. Many (19) functions of \((\phi, X)\)

\[ L = f_2(\phi, X)^{(4)} R + K(\phi, X) + G(\phi, X) \Box \phi + C^{\mu\nu\rho\sigma}_2(\phi, X) \partial_\mu \partial_\nu \phi \partial_\rho \partial_\sigma \phi \]

\[ + f_3(\phi, X) G_{\mu\nu} \partial_\mu \partial_\nu \phi + C^{\mu\nu\rho\sigma\alpha\beta}_3(\phi, X) \partial_\mu \partial_\nu \phi \partial_\rho \partial_\sigma \phi \partial_\alpha \partial_\beta \phi \]

✦ Kinetic matrix

\[ V \equiv t^\nu \nabla_\nu (n^\mu \nabla_\mu \phi) \quad K_{\mu\nu} \equiv \frac{1}{2} h_{\mu\nu} \equiv \frac{1}{2} \mathcal{L}_t h_{\mu\nu} \]

\[ \mathcal{L}_{\text{kin}} = (V, K_{\mu\nu}) \begin{pmatrix} \mathcal{A} & \mathcal{B}^{\rho\sigma} \\ \mathcal{B}_{\mu\nu} & \mathcal{K}_{\mu\nu, \rho\sigma} \end{pmatrix} \begin{pmatrix} V \\ K^\rho_\rho \sigma \end{pmatrix} \]

✦ Horndeski and beyond Horndeski are the simplest case: \( \mathcal{A} = 0 \). In general more complex: 3 degeneracy conditions. Degenerate Higher-Order Scalar-Tensor (DHOST) or Extended Scalar-Tensor (EST) theories.
Some degravitation scenarios – would be protected against quantum corrections and therefore remains small. This is because $m^2$ restores a symmetry (diffeomorphism invariance or general covariance). Although the simplest massive gravity theory was shown not to admit a flat FLRW universe, its bimetric generalization was indeed able to provide self-accelerating solutions $[10^4, 10^5, 10^6]$, consistent with all existing observational data at the background level $[10^7, 10^8]$. Since then, an extensive amount of work has been done to study the viability of the theories through metric perturbation theory and structure formation studies [e.g. $10^9, 10^{10}, 10^{11}, 10^{12}, 10^{13}$]. Unfortunately, although the simplest bigravity models are able to provide viable self-accelerating background expansions, all such models suffer from ghost and/or gradient instabilities $[11^7, 11^8, 11^9]$. While it is possible to push these instabilities back to unobservably early times, beyond the regime of validity of the theory, without losing self-acceleration and obtaining a technically natural acceleration parameter $[11^9]$, the theory becomes observationally indistinguishable from $\Lambda$CDM in this case. While this may render the theory less favorable from an Occam’s razor perspective, the fact that a small mass is protected by the symmetry of diffeomorphisms makes the theory more favorable than $\Lambda$CDM from the perspective of naturalness. It is then mainly a matter of subjective taste and further observational tests to decide...
Constructing the action

- Use metric quantities in uniform scalar field slicing \( \phi(t) \neq 0 \)

ADM decomposition

\[
 ds^2 = -N^2 dt^2 + h_{ij} (N^i dt + dx^i)(N^j dt + dx^j) 
\]
Constructing the action

- Use metric quantities in uniform scalar field slicing \( \dot{\phi}(t) \neq 0 \)

ADM decomposition

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ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)
\]

- Lagrangian contains all possible scalars under spatial diff's, ordered by number of perturbations and derivatives

\[
S = \int d^4x \sqrt{-g} L[t, N, K^i_j, (3)R^i_j, \ldots]
\]

- Lapse

\[
N \sim \dot{\phi} \quad (\partial \phi)^2 = -\dot{\phi}^2(t)/N^2
\]

- Extrinsic curvature

\[
K_{ij} \sim \partial_t g_{ij} \quad K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)
\]

- Intrinsic curvature

\[
(3)R_{ij} \sim \partial^2 g_{ij}
\]

Cheung et al. `07
Constructing the action

- Use metric quantities in uniform scalar field slicing $\dot{\phi}(t) \neq 0$

\[
d s^2 = -N^2 dt^2 + h_{ij} (N^i dt + dx^i) (N^j dt + dx^j)
\]

- ADM decomposition

- Lagrangian contains all possible scalars under spatial diffs, ordered by number of perturbations and derivatives Cheung et al. '07

\[
S = \int d^4 x \sqrt{-g} L[t; N, K^i_j, (3)R^i_j, \ldots]
\]

- Expand the action

\[
\delta N \equiv N - 1, \quad \delta K_{ij} \equiv K_{ij} - H h_{ij}, \quad (3)R_{ij}
\]

\[
L(N, K^i_j, R^i_j, \ldots) = \bar{L} + L_N \delta N + \frac{\partial L}{\partial K^i_j} \delta K^i_j + \frac{\partial L}{\partial R^i_j} \delta R^i_j + L^{(2)} + \ldots
\]
Building blocks of linear perts

\[ S^{(2)} = \int d^4 x \sqrt{h} \frac{M^2}{2} \left[ \delta K^i_j \delta K^j_i - \delta K^2 + (3) R + \delta N (3) R + \sum_i \alpha_i(t) O_i^{(2)} (\delta N, \delta K, \ldots) \right] \]

with Gleyzes, Langlois, Piazza ’13 (see also Bloomfield ’13)

✦ New operators describe deviations from GR (\( \Lambda \)CDM)

✦ Time dependent couplings (functions \( \alpha_i \)): expansion around FRW background

✦ Functions \( \alpha_i(t) \) independent of background evolution \( H(t) = \dot{a}/a \)

▶ we fit to data \( H(t) \) and \( \alpha_i(t) \) (agnostic of their time dependence and parametrization)
Building blocks of linear perturbations

\[ S^{(2)} = \int d^4x \sqrt{\gamma} \frac{M^2}{2} \left[ \delta K^i_j \delta K^j_i - \delta K^2 + (3)R + \delta N^{(3)}R + \sum_i \alpha_i(t)O_i^{(2)}(\delta N, \delta K, \ldots) \right] \]

with Gleyzes, Langlois, Piazza '13 (see also Bloomfield '13)

Notation of Bellini, Sawicki '14 for the alphas

<table>
<thead>
<tr>
<th>( \alpha_i )</th>
<th>( \alpha_K )</th>
<th>( \alpha_B )</th>
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<tr>
<td>( O_i^{(2)} )</td>
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<td>quintessence, k-essence</td>
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5 functions of time instead of 5 functions of \( \phi, (\partial \phi)^2 \); minimal number of parameters
Building blocks of linear perturbations

We impose absence of ghost and gradient stability:

\[ \mathcal{L} = + \dot{\varphi}^2 - c_s^2 (\nabla \varphi)^2 \]

- Positive kinetic energy = absence of ghosts
- Positive sound speed squared = absence of gradient instabilities

\[ h_{ij} = a^2(t) e^{2\zeta} (\delta_{ij} + \gamma_{ij}) , \quad \gamma_{ii} = 0 = \nabla_i \gamma_{ij} \]

<table>
<thead>
<tr>
<th></th>
<th>Scalar</th>
<th>Tensor</th>
</tr>
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<tbody>
<tr>
<td>No ghosts</td>
<td>( \alpha_K + 6 \alpha_B^2 ) ( M^2 &gt; 0 )</td>
<td></td>
</tr>
<tr>
<td>No gradient instability</td>
<td>( c_s^2(\alpha_i) \geq 0 ) ( \alpha_T \geq -1 )</td>
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</table>
Fisher matrix analysis

with Gleyzes, Langlois, Mancarella '15

Euclid specifications (LCDM fiducial)
Quasi-static approximation

- Background parametrization:
  \[ H^2 = H_0^2 \left[ \Omega_{m0} a^{-3} + (1 - \Omega_{m0}) a^{-3(1+w)} \right] \]

- Free functions parametrization:
  \[ \alpha_I(t) = \alpha_{I,0} \frac{1 - \Omega_m(t)}{1 - \Omega_{m,0}} \]
Higher-Order theories

\[ S^{(2)} = \int d^4 x \sqrt{\hat{h}} \frac{M^2}{2} \left[ \delta K_i^j \delta K_j^i - \delta K^2 + (3)R + \delta N(3)R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)}(\delta N, \delta K, \ldots) \right] \]

- All operators up to two derivatives

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with Langlois, Mancarella, Noui '17
Higher-Order theories

\[ S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[ \delta K^j_i \delta K^i_j - \delta K^2 + \langle 3 \rangle R + \delta N \langle 3 \rangle R + \sum_i \alpha_i(t) \mathcal{O}^{(2)}_i (\delta N, \delta K, \ldots) \right] \]

- All operators up to two derivatives

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- Generic scalar dispersion relation:
  \[ \mathcal{E}_1 \omega^4 + \mathcal{E}_2 \omega^2 k^2 + \mathcal{E}_3 \omega^2 + \mathcal{E}_4 k^4 + \mathcal{E}_5 k^2 = 0 \]

- Two types of degeneracy conditions lead to
  \[ \omega^2 - c_s^2 k^2 = 0 \]

- \( C_I : \) \( \alpha_L = 0 \), \( \beta_2 = f_2(\beta_1) \), \( \beta_3 = f_3(\beta_1) \)

- \( C_{II} : \) \( \beta_1 = f_1(\alpha_T, \alpha_H, \alpha_L) \), \( \beta_2 = f_2(\alpha_T, \alpha_H, \alpha_L) \), \( \beta_3 = f_3(\alpha_T, \alpha_H, \alpha_L) \)

  \[ c_s^2 \propto -c_T^2 \]

  ruled out!
Frame dependence

✦ Gravitational action:

\[ S_{\text{gravity}} = \int d^4 x \mathcal{L}_g (g_{\mu \nu}; \alpha_I, \beta_J), \quad I = 1, \ldots, 6, \quad J = 1, 2, 3 \]

6+3=9 parameters and 3 degeneracy conditions: 6 parameters

✦ Action transforms under metric redefinition: (most general) disformal transformation

\[ g_{\mu \nu} \rightarrow \tilde{g}_{\mu \nu} = C(\phi, X)g_{\mu \nu} + D(\phi, X)\partial_\mu \phi \partial_\nu \phi \]

\[ S^{(2)}[g_{\mu \nu}, \alpha_I] = \tilde{S}^{(2)}[\tilde{g}_{\mu \nu}, \tilde{\alpha}_I] \quad \tilde{\alpha}_I = M_I^{I'} \alpha_{I'} \]
Frame dependence

✦ Gravitational action:

\[
S_{\text{gravity}} = \int d^4 x \mathcal{L}_g(g_{\mu \nu}; \alpha_I, \beta_J), \quad I = 1, \ldots, 6, \quad J = 1, 2, 3
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\]

Bekenstein '92

\[
S^{(2)}[g_{\mu \nu}, \alpha_I] = \tilde{S}^{(2)}[\tilde{g}_{\mu \nu}, \tilde{\alpha}_I]
\]

\[
\tilde{\alpha}_I = M_I' \alpha_I'
\]

✦ Two sets of degeneracy conditions invariant under disformal transformations

✦ Class $C_1$ can be brought to Horndeski frame: $\alpha_H = 0, \beta_J = 0$

DHOST I $\xrightarrow{C(X)}$ Beyond Horndeski $\xrightarrow{D(X)}$ Horndeski

✦ Changing frame changes matter couplings (Horndeski vs Jordan): Matter matters!
Our Limited Eyes

Supernovae: $d$

13 August 2014

Modern Cosmology 2014, Benasque

Galaxy clustering

Weak lensing

CMB (ISW)

Grav. waves

ET of DE

Galaxy Counts

Galaxy Shapes/

Brightness

Some degravitation scenarios

Scalar-tensor & Brans-Dicke

Ghost condensates

the Fab Four

Coupled Quintessence

$\alpha_K(t)$, $\alpha_B(t)$, $\alpha_M(t)$,

$\alpha_T(t)$, $\alpha_T(t)$, ...

Conformal gravity

Some degravitation scenarios

f(R)

f(R)

f(G)

Modified Gravity

Non-local

Higher-order

Higher dimensions

Tessa Baker

Einstein-Dilaton-Gauss-Bonnet

Cascading gravity

Stringy Brane

Randall-Sundrum I & II

Kaluza-Klein

Low-curvature gravity

Generalisations of SH

Strings & Branes

Generalisation of SH

Adding new field content

Einstein-Aether

Massive gravity

Bigravity

EBI

Bimetric MOND

Horndeski theories

Torsion theories

KGB

Calabi-Yau

Horndeski theories

Einstein-Hořava-Lifschitz

Conformal gravity

Strings & Branes

Generalisations

Emergent Approaches

CDT

Pathfinder theories

Observations

Galaxy clustering

Weak lensing

CMB (ISW)

Grav. waves
Phenomenology

• Undo unitary gauge: \[ t \rightarrow t + \pi(t, \vec{x}) \]

• Newtonian gauge (scalar fluct): \[ dt^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2 \]

Note: the 3-dim quantities on the right are defined with respect to the new time hypersurfaces.
• Undo unitary gauge: \( t \rightarrow t + \pi(t, \vec{x}) \)

• Newtonian gauge (scalar fluctuations):
  \[
  dt^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2
  \]

• Quasi-static approximations — valid on scales \( k \gg aHc_s^{-1} \).
  Sawicki, Bellini '15
  E.g., for surveys such as Euclid \( c_s \gtrsim 0.1 \).

\[
\nabla^2 (\Psi + \Phi) = 8\pi Ga^2 \rho_m \delta_m \\
\nabla^2 \Psi = 4\pi Ga^2 \rho_m \delta_m \\
\dot{\vec{v}}_m + H\vec{v}_m = -\nabla \Phi \\
\dot{\delta}_m + \nabla \vec{v}_m = 0
\]
• Undo unitary gauge: 
\[ t \to t + \pi(t, \vec{x}) \]

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• Quasi-static approximations — valid on scales \( k \gg aHc_s^{-1} \). E.g., for surveys such as Euclid \( c_s \gtrsim 0.1 \).

\[ \nabla^2(\Psi + \Phi) = 8\pi G(1 + \gamma_{\text{lens}})a^2 \rho_m \delta_m \]

\[ \gamma_{\text{lens}} = \gamma_{\text{lens}}(\alpha_i) \]

\[ \nabla^2 \Psi = 4\pi G(1 + \gamma_{\Psi})a^2 \rho_m \delta_m \]

\[ \gamma_{\Psi} = \gamma_{\Psi}(\alpha_i) \]

\[ \dot{\delta}_m + \nabla \vec{v}_m = 0 \]

\[ \dot{\vec{v}}_m + H\vec{v}_m = -\nabla \Phi \]

Sawicki, Bellini ‘15
Small scale limit

<table>
<thead>
<tr>
<th>$\alpha_K$</th>
<th>$\alpha_B$</th>
<th>$\alpha_M$</th>
<th>$\alpha_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta N^2$</td>
<td>$\delta N \delta K$</td>
<td>$\frac{dM^2}{d \ln a}$</td>
<td>$^{(3)}R$</td>
</tr>
</tbody>
</table>

- In the limit $k \to \infty$:

\[ \nabla^2 \Phi = \frac{3}{2} \Omega_m H^2 a^2 \delta_m \left( 1 + \alpha_T + \frac{\xi^2}{\nu} \right) \]

\[ \nabla^2 \Psi = \frac{3}{2} \Omega_m H^2 a^2 \delta_m \left( 1 + \frac{\xi \alpha_B}{\nu} \right) \]

\[ \xi = \alpha_B (1 + \alpha_T) + \alpha_T - \alpha_M \]

\[ \nu = - \left\{ (1 + \alpha_B) \left[ \alpha_B (1 + \alpha_T) + \alpha_T - \alpha_M + \frac{\dot{H}}{H^2} \right] + \frac{\dot{\alpha}_B}{H} + \frac{3}{2} \Omega_m \right\} = \frac{c_s^2 \alpha}{2} > 0 \]
Boltzmann codes

• Full Einstein-Boltzmann solver:

\[ \frac{df_I}{d\eta} = C_I [f_I], \quad I = \gamma, \nu, b, \text{CDM} \]

\[ \frac{\delta S^{(2)}}{\delta \pi} = 0 \quad \& \quad G_{ij}^{\text{modified}} = 8\pi G \sum_I T_{ij}^{(I)} \]
Boltzmann codes

• Full Einstein-Boltzmann solver: 

\[
\frac{df_I}{d\eta} = C_I[f_I], \quad I = \gamma, \nu, b, \text{CDM}
\]

\[
\frac{\delta S^{(2)}}{\delta \pi} = 0 \quad \& \quad G_{ij}^{\text{modified}} = 8\pi G \sum_I T_{ij}^{(I)}
\]

• EFTCAMB (from CMBFAST) (Hu, Raveri, Frusciante, Silvestri et al.)
• hi_class (from CLASS) (Zumalacarregui, Bellini, Sawicki, Lesgourgues et al.)
• COOP (indep. code, Zhiqi Huang) (with D’Amico, Huang and Mancarella)
• LVDM-CLASS (from CLASS) (Blas, Ivanov, Sibiryakov)
• others …
Boltzmann codes

• Full Einstein-Boltzmann solver:
  \[ \frac{df_I}{d\eta} = C_I[f_I], \quad I = \gamma, \nu, b, \text{CDM} \]

\[ \frac{\delta S^{(2)}}{\delta \pi} = 0 \quad \& \quad G_{ij}^{\text{modified}} = 8\pi G \sum_I T_{ij}^{(I)} \]
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  \[ \frac{\delta S^{(2)}}{\delta \pi} = 0 \quad \& \quad G^\text{modified}_{ij} = 8\pi G \sum_I T^{(I)}_{ij} \]

Bellini et multi alii, in prep.

![Graph showing multipole moments and power spectra](image)
Deviations from $\Lambda$CDM

$$\alpha_X = c_X \frac{\Omega_{DE}(z)}{\Omega_{DE}(z = 0)}$$

Alonso et al. ‘16

<table>
<thead>
<tr>
<th>Case</th>
<th>$&gt; \omega_{BD}$, 95% C.L.</th>
<th>$\sigma(c_B)$</th>
<th>$\sigma(c_M)$</th>
<th>$\sigma(c_T)$</th>
<th>$\sigma(c_K)$</th>
<th>$\sigma(w)$</th>
<th>$\sigma(\Sigma m_\nu)$ [meV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>S4</td>
<td>$2.9 \times 10^3$</td>
<td>0.796</td>
<td>0.746</td>
<td>1.26</td>
<td>4.9</td>
<td>0.112</td>
<td>71</td>
</tr>
<tr>
<td>LSST</td>
<td>$1.2 \times 10^4$</td>
<td>0.193</td>
<td>0.089</td>
<td>0.205</td>
<td>8.8</td>
<td>0.016</td>
<td>45</td>
</tr>
<tr>
<td>S4+LSST</td>
<td>$1.3 \times 10^4$</td>
<td>0.169</td>
<td>0.072</td>
<td>0.179</td>
<td>3.5</td>
<td>0.011</td>
<td>22</td>
</tr>
</tbody>
</table>

$\sigma(\alpha_X) \sim \mathcal{O}(0.1)$

Cassini (Bertotti et al. 03): $\omega_{BD} > 40\,000$

This work: $\omega_{BD} > 20\,000$
Mildly nonlinear scales

- Nonlinear scales are difficult!

- Possible strategy: conservative cutoff on small scales. But certain observables require (mildly) nonlinear modelling. E.g. redshift-space distortions, baryon acoustic oscillations, etc.

- Ample information on nonlinear scales: many more modes and possible new signatures (screening mechanism, nonlinear couplings, etc.)

- Many developments in numerical simulations including DE/MG
  - Only developed for some models (e.g. DGP, f(R))
  - Time consuming and non-standard models difficult to implement

- Many developments in analytical perturbative methods

  Codes: ECOSMOG, MG-GADGET, ISIS, DGPM, …

  (Winther et al 15)

  Baldauf, Bernardeau, Bertolini, Blas, Carrasco, Crocce, Garny, Ivanov, Pajer, Peloso, Pietroni, Scoccimarro, Senatore, Sibiryakov, Valageas, Zaldarriaga and many others
Nonlinear ET of DE

\[ S = \int d^4x \sqrt{h} \frac{M^2}{2} \left[ \delta K^i_j \delta K^j_i - \delta K^2 + (3)^R + \delta N^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)} + \sum_i \alpha_i(t) \mathcal{O}_i^{(3)} \right] \]

- In the short-scale limit, a finite number of operators dominate

Example: Horndeski has only 3 cubic operators and nothing more

Bellini, Jimenez, Verde '15
Nonlinear ET of DE

\[ S = \int d^4 x \sqrt{\frac{h}{2}} \left[ \delta K^j_i \delta K^i_j - \delta K^2 + R^{(3)} + R^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}^{(2)}_i + \sum_i \alpha_i(t) \mathcal{O}^{(3)}_i \right] \]

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  Bellini, Jimenez, Verde ‘15

- Standard Perturbation Theory
  \[
  \dot{\delta}_m + \nabla \left[ (1 + \delta_m) \vec{v}_m \right] = 0 \\
  \dot{\vec{v}}_m + H \vec{v}_m + \vec{v}_m \cdot \nabla \vec{v}_m = -\nabla \Phi
  \]

- GR case: Poisson equation
  \[
  \nabla^2 \Phi = \frac{3}{2} a^2 H^2 \Omega_m \delta_m
  \]

\[
\delta_m \sim 1 \quad H_0^{-1}
\]
Nonlinear ET of DE

\[ S = \int d^4 x \sqrt{h} \frac{M^2}{2} \left[ \delta K_i^j \delta K^i_j - \delta K^2 + \delta N^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)} + \sum_i \alpha_i(t) \mathcal{O}_i^{(3)} \right] \]

♦ In the short-scale limit, a finite number of operators dominate

Example: Horndeski has only 3 cubic operators and nothing more  
Bellini, Jimenez, Verde ‘15

♦ Standard Perturbation Theory

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\begin{align*}
\dot{\delta}_m + \nabla [(1 + \delta_m) \vec{v}_m] &= 0 \\
\dot{\vec{v}}_m + H \vec{v}_m + \vec{v}_m \cdot \nabla \vec{v}_m &= -\nabla \Phi
\end{align*}
\]

♦ Modifications of gravity encoded in Poisson-like equation

\[
k^2 \Phi = -\frac{3}{2} a^2 H^2 \Omega_m \mu_\Phi,1 \delta_m - \frac{9}{4} a^2 H^2 \Omega_m^2 \mu_\Phi,2 (\vec{k}_1, \vec{k}_2) \delta_m (\vec{k}_1) \ast \delta_m (\vec{k}_2) + \ldots
\]

large nonlinearities, screening, …

\[ \delta_m \sim 1 \]

mildly NL scales

\[ H_0^{-1} \]
Conclusions

- Unifying description for scalar-tensor theories, including higher-order degenerate ones (and more)

- Analysis of (degenerate higher-order) theories highly simplified

- Linear regime worked out! Issue of time dependence of α’s when comparing to data

- Straightforward connection to mildly and fully nonlinear regime