Effective Theory of Dark Energy

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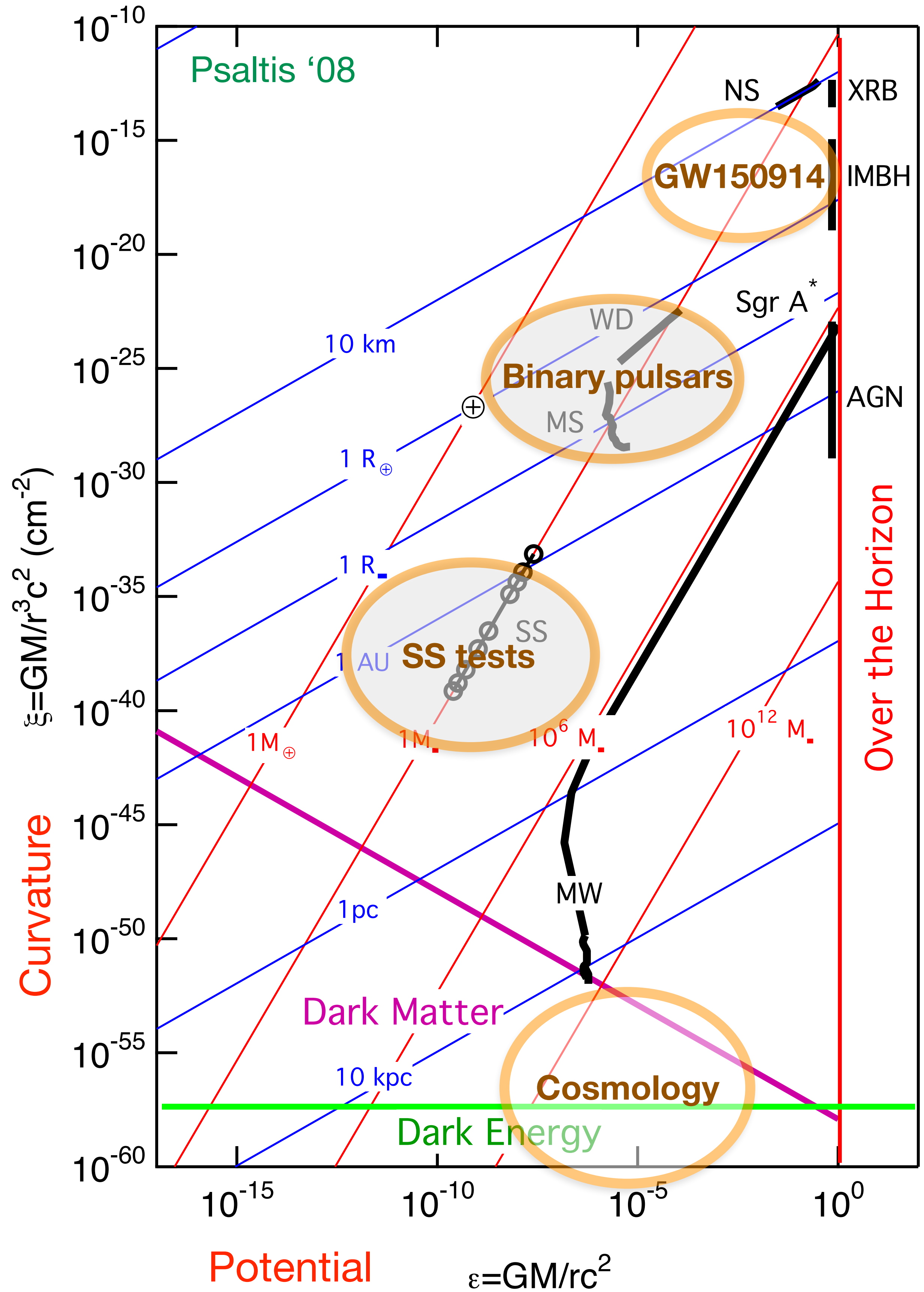
Probing the Dark Sector and General Relativity
CERN, Geneva - August 14, 2017

with
Creminelli, Cusin, D’Amico, Gleyzes,
Gubitosi, Langlois, Lewandowski,
Mancarella, Noreña, Noui, Piazza, …

Motivations

- ✦ Gravity only been tested over specials ranges of scales and masses

- ✦ Cosmology is a window for testing gravity on very large distances



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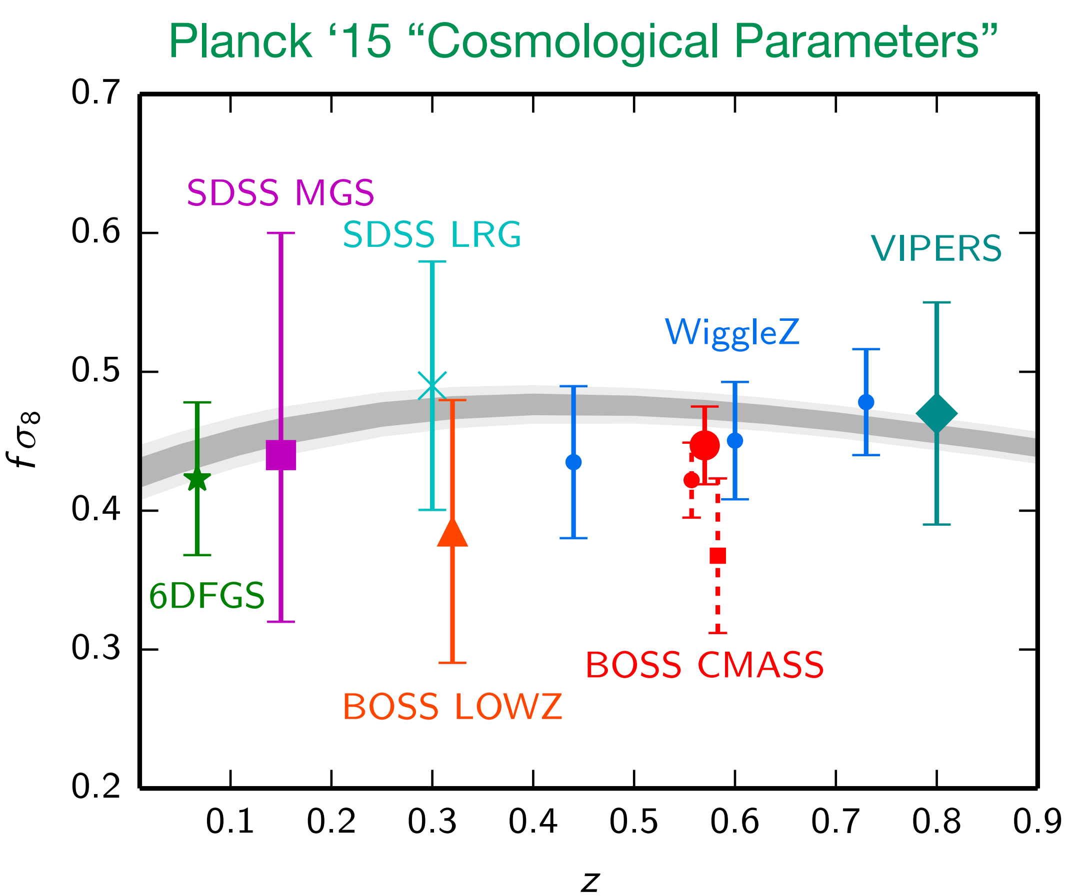

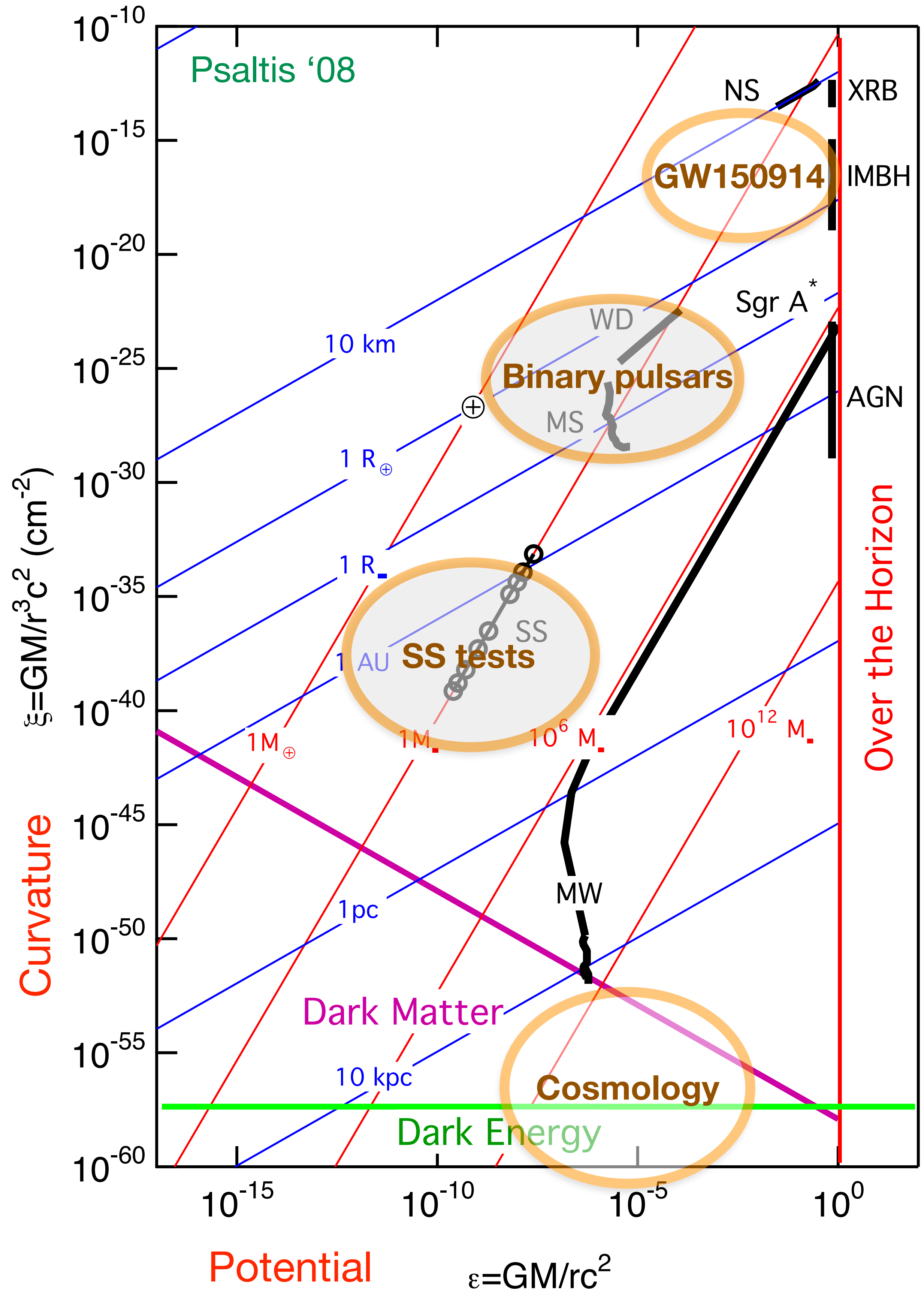

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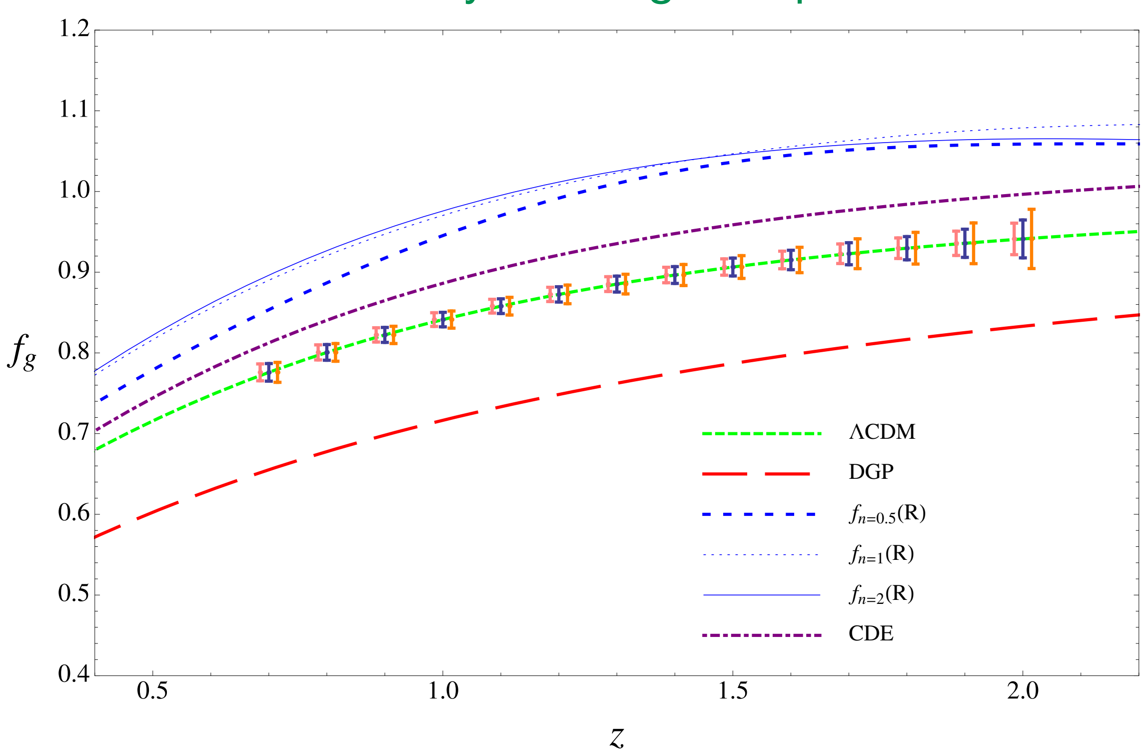
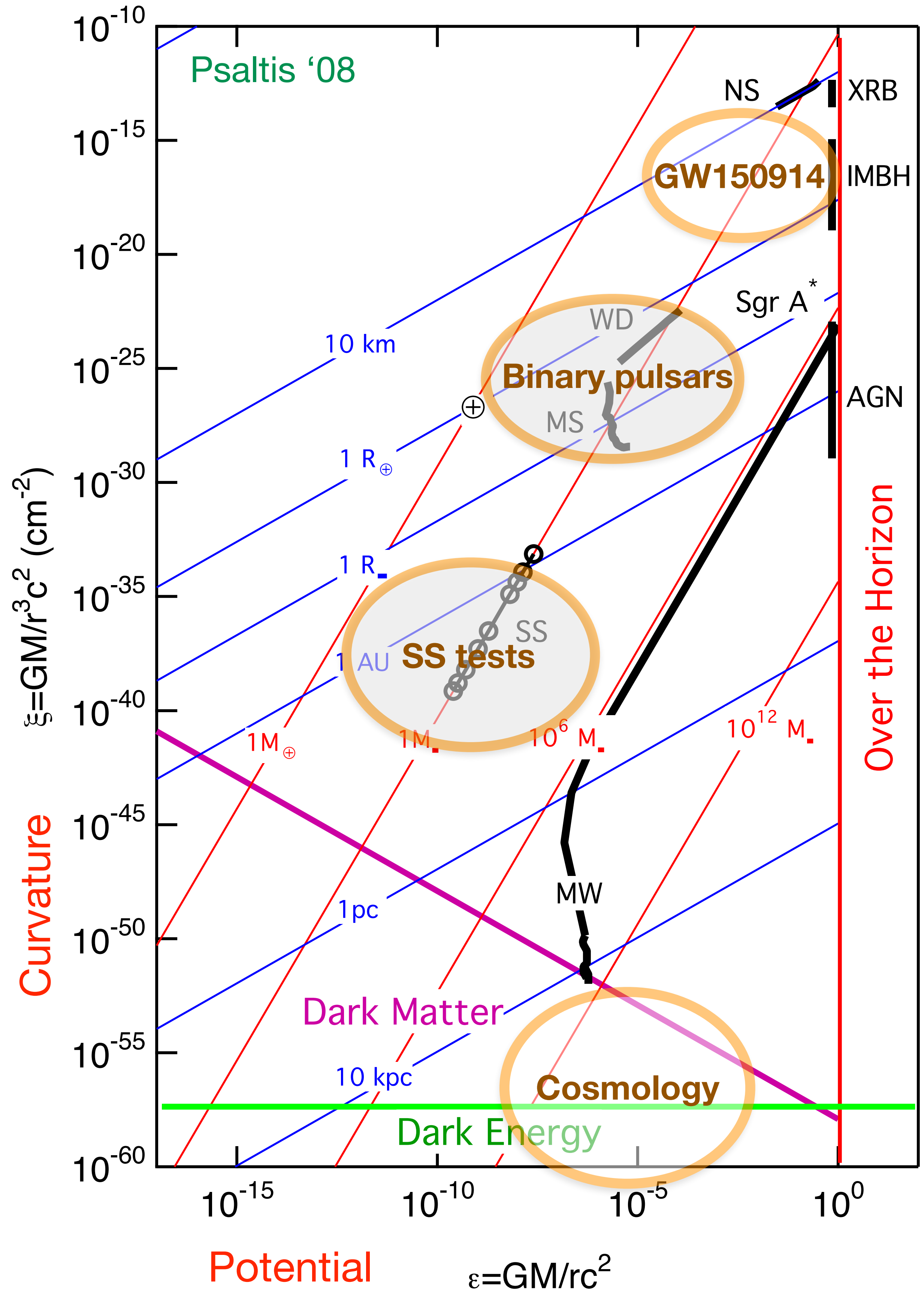
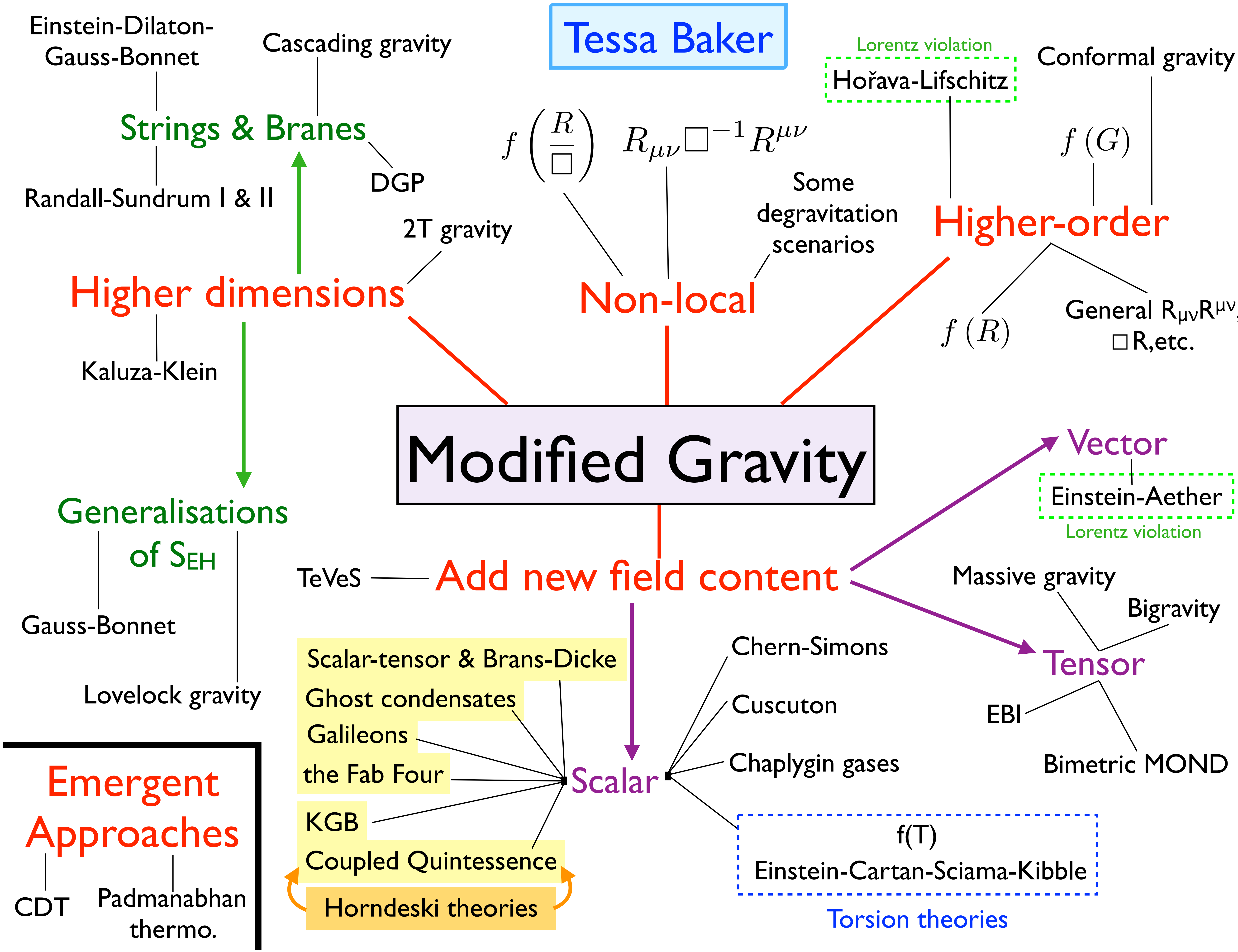
Euclid Theory Working Group '16

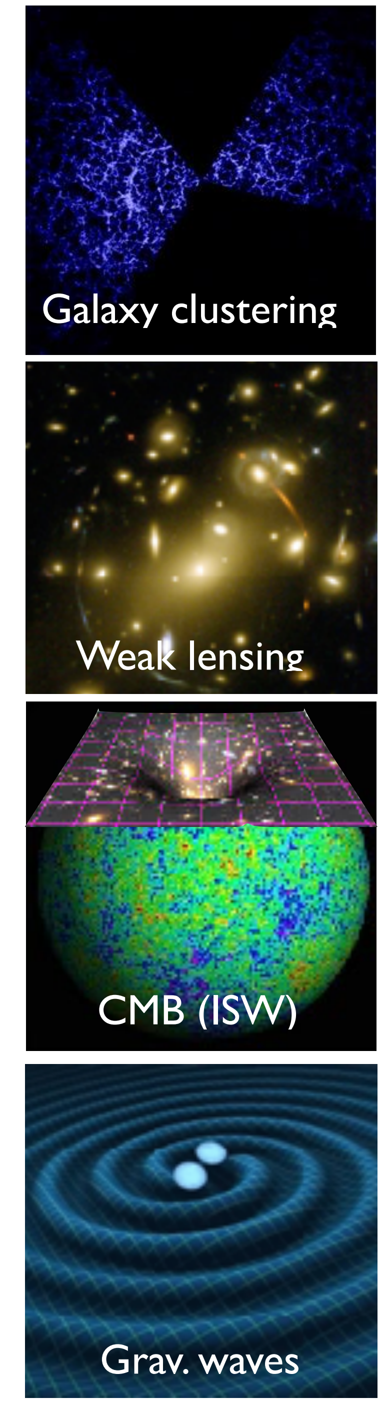




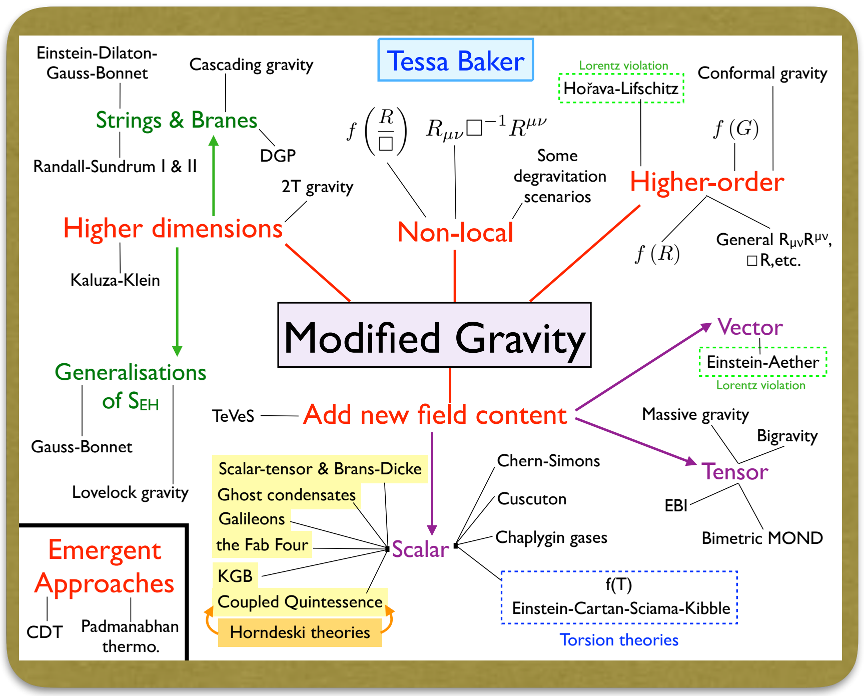

Tessa Baker
Modified Gravity
Higher dimensions
Strings & Branes
Einstein-Dilaton-Gauss-Bonnet
Cascading gravity
Randall-Sundrum I & II
DGP
2T gravity
Kaluza-Klein
Generalisations of S_{EH}
Gauss-Bonnet
Lovelock gravity
Non-local
$f\left(\frac{R}{\Box}\right)$
$R_{\mu\nu}\Box^{-1}R^{\mu\nu}$
Some degravitation scenarios
Higher-order
Lorentz violation
Hořava-Lifschitz
Conformal gravity
$f(G)$
$f(R)$
General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$,etc.
Add new field content
TeVeS
Vector
Einstein-Aether
Lorentz violation
Massive gravity
Bigravity
Tensor
EBI
Bimetric MOND
Scalar
Scalar-tensor & Brans-Dicke
Ghost condensates
Galileons
the Fab Four
KGB
Coupled Quintessence
Horndeski theories
Chern-Simons
Cuscuton
Chaplygin gases
f(T)
Einstein-Cartan-Sciama-Kibble
Torsion theories
Emergent Approaches
CDT
Padmanabhan thermo.

Motivations

Observations



Models



Many models of modified gravity, each with its own theoretical motivation and phenomenology

Motivations

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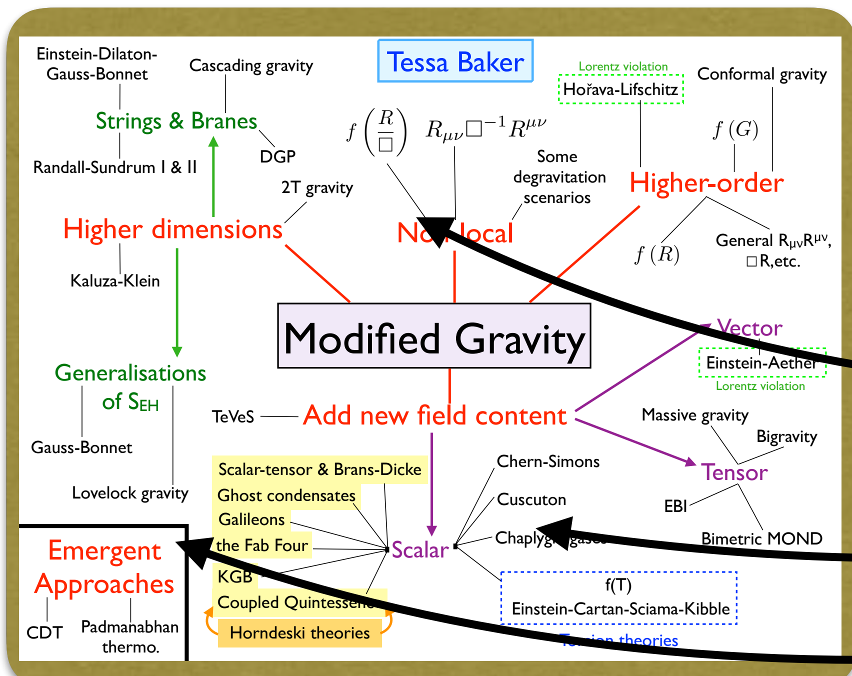

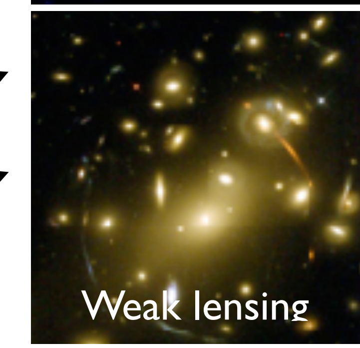

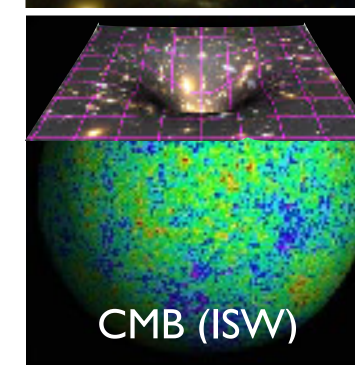

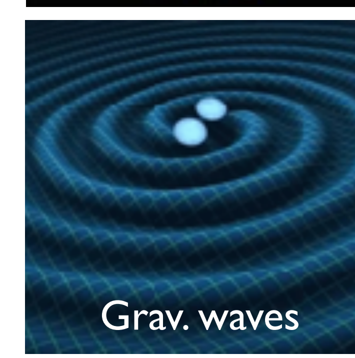

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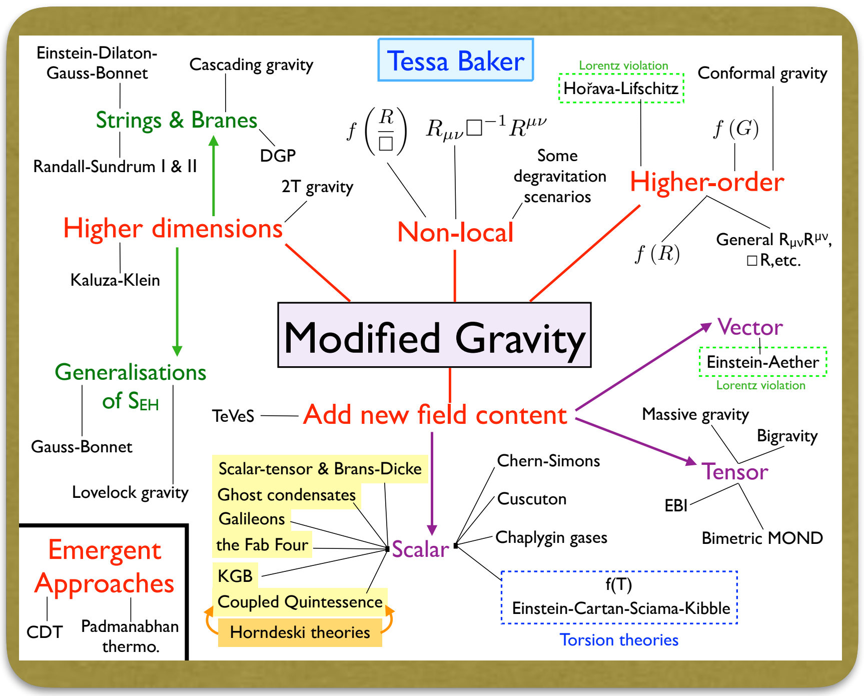
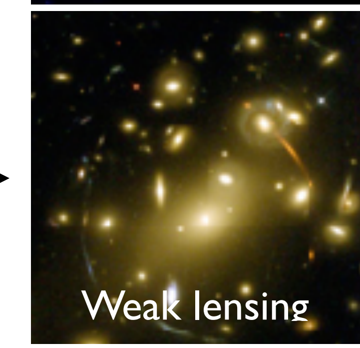
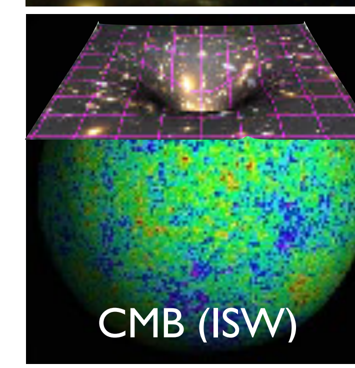


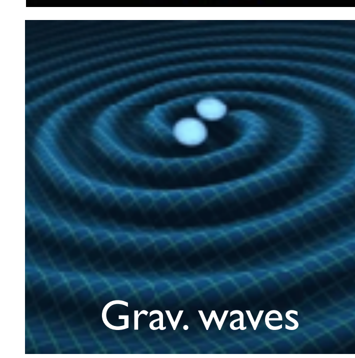
EToFDE

$\alpha_K(t),\ \alpha_B(t),\ \alpha_M(t),$
$\alpha_T(t),\ \alpha_T(t),\ \ldots$

Observations







Bridge models and observations in a minimal and systematic way

Motivations

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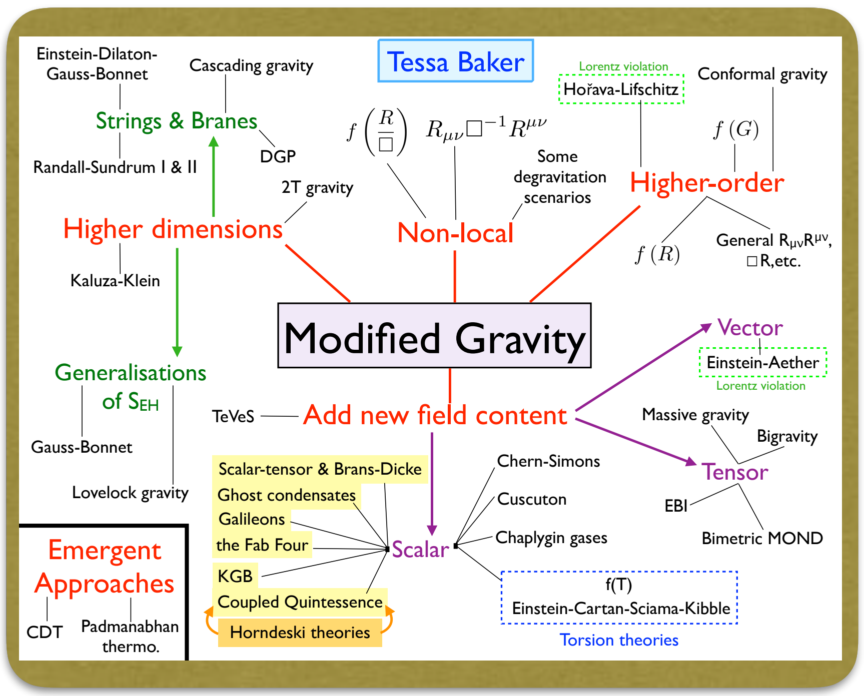

ETofDE

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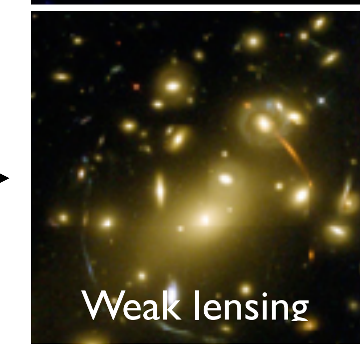
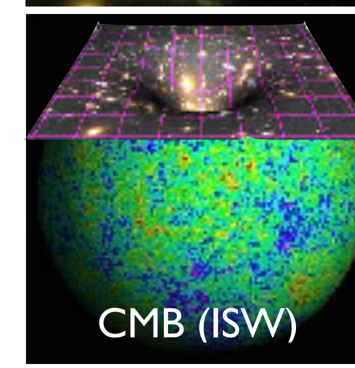
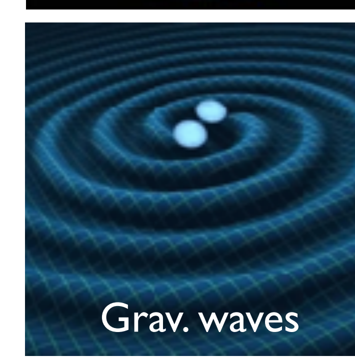




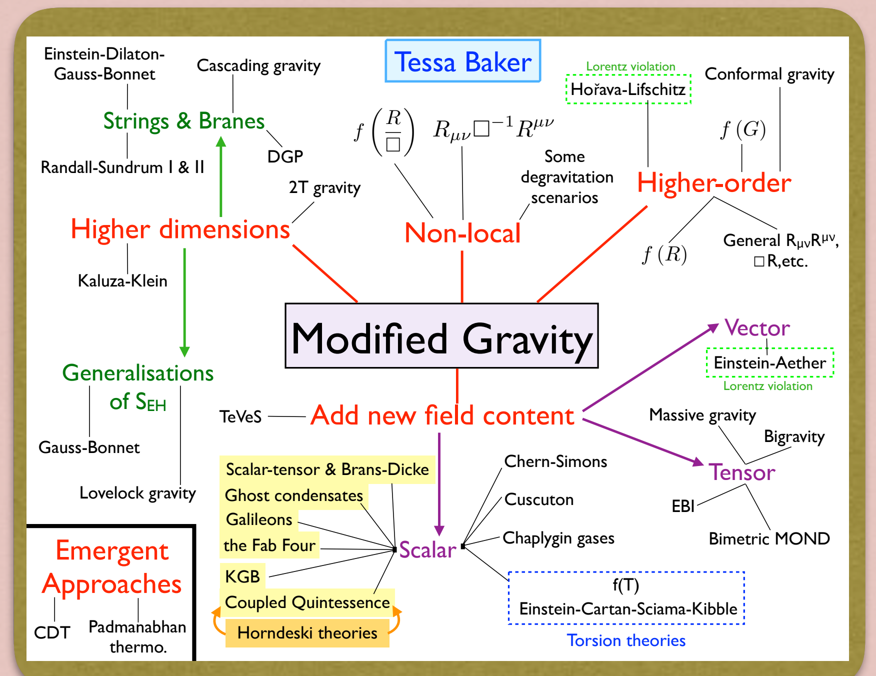


Bridge models and observations in a minimal and systematic way

Observationally motivated: efficiency, now implemented in Einstein-Boltzmann codes

Theoretically motivated: locality, causality, diff invariance, unitarity, stability, etc…

Models



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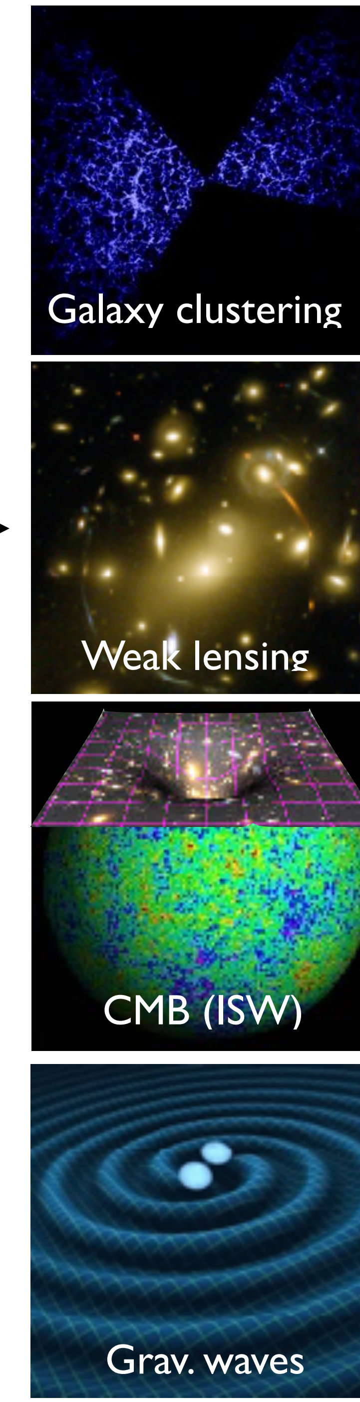

Scalar-tensor theories

- Simplest models of modified gravity are base on single scalar field

- Old school theories: Quintessence, Brans-Dicke, K-essence, … $\mathcal{L}(\phi, \partial_\mu \phi)$

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$$\phi \to \phi + b_\mu x^\mu + c \qquad \mathcal{L} = \phi(\partial^2 \phi)^n$$

(Nicolis, Rattazzi, Trincherini '08)

- Unique Lagrangians with 2nd order EOM:

$$\mathcal{L}_1 = \phi \ ,$$
$$\mathcal{L}_2 = (\partial\phi)^2 \ ,$$
$$\mathcal{L}_3 = (\partial\phi)^2 \partial^2\phi \ ,$$
$$\mathcal{L}_4 = (\partial\phi)^2 \left[(\partial^2\phi)^2 - (\partial_\mu\partial_\nu\phi)^2 \right] \ ,$$
$$\mathcal{L}_5 = (\partial\phi)^2 \left[(\partial^2\phi)^3 - 3\partial^2\phi(\partial_\mu\partial_\nu\phi)^2 + 2(\partial_\mu\partial_\nu\phi)^3 \right] \ .$$

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\end{aligned}$$

- Can provide self-acceleration and nonlinearities (Vainshtein screening), with controlled quantum corrections and no ghost

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- Generalized theories: Galileons $\mathcal{L}(\phi, \partial_\mu \phi, \nabla_\mu \nabla_\nu \phi)$
- Covariantization: **Horndeski theories** (Horndeski '73, see also Deffayet et al. '11)

$$
\begin{aligned}
L_H = & \, G_2(\phi, X) + G_3(\phi, X)\Box\phi + \\
& + G_4(\phi, X)^{(4)}R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right] \\
& + G_5(\phi, X)^{(4)}G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\nu\lambda}\phi_{;\lambda}^{\;\;;\mu}\right]
\end{aligned}
$$

$$
X \equiv \phi_{;\mu}\phi^{;\mu} \equiv \nabla_\mu \phi \nabla^\mu \phi
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- Generally, higher-derivatives lead to extra unstable d.o.f. (Ostrogradski ghost)

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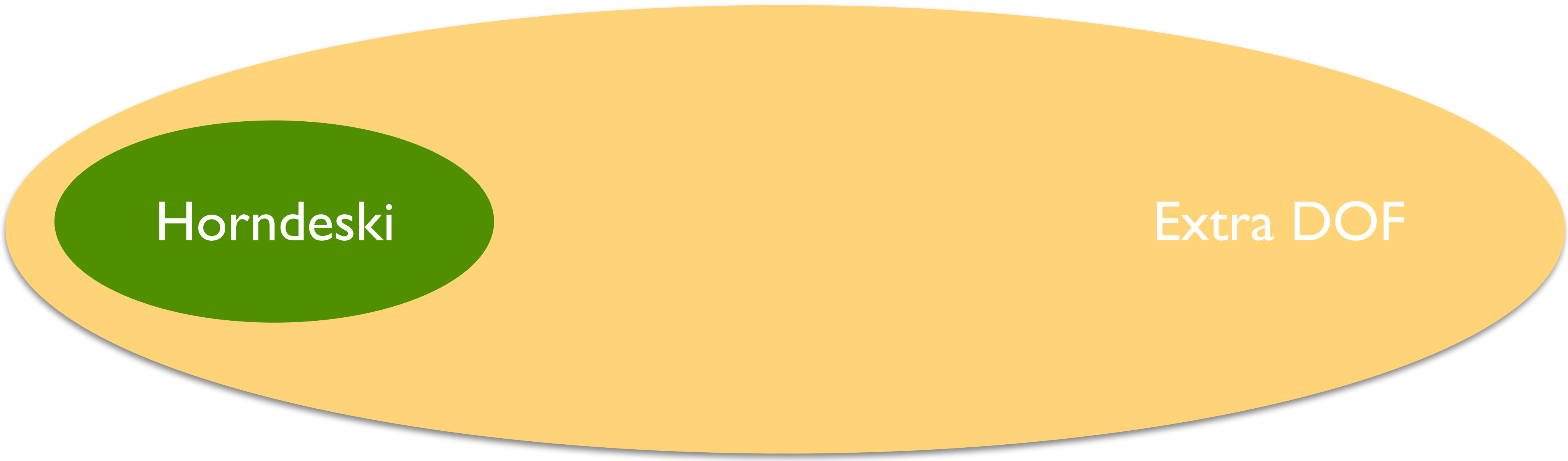

second-order
equations of motion

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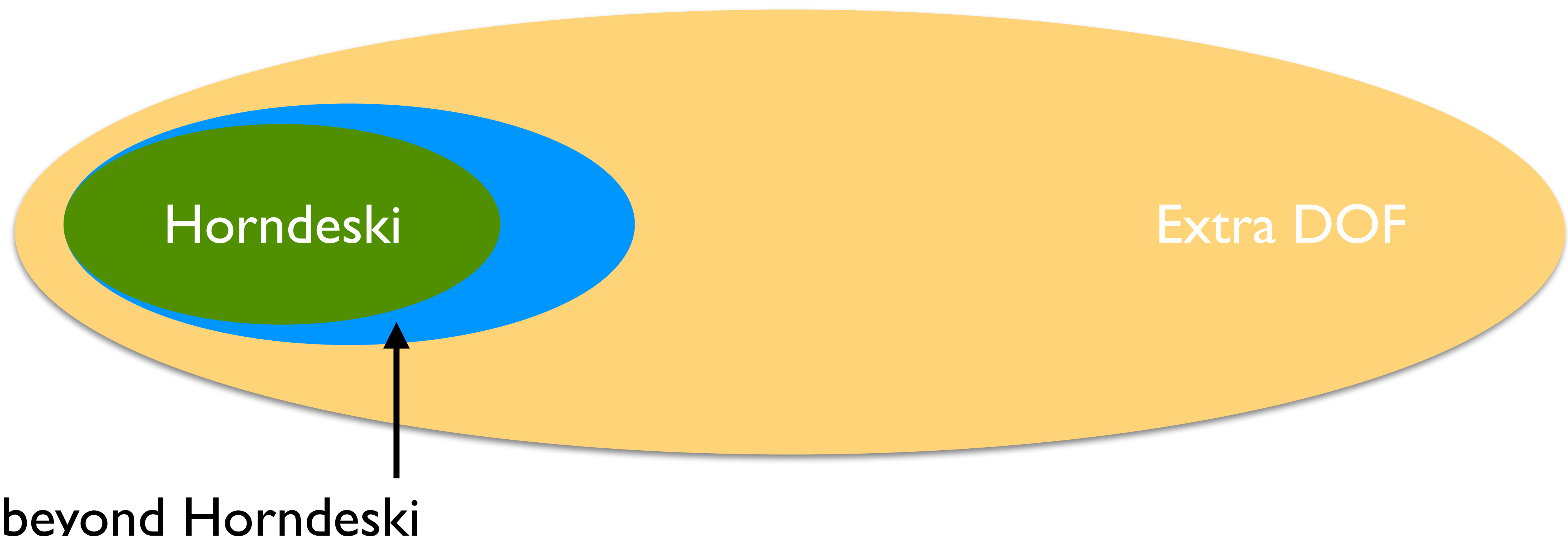

Zumalacarregui, Garcia-Bellido ’13
with Gleyzes, Langlois, Piazza ’14;

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Examples:

$$\mathcal{L} = \frac{1}{2}\ddot{\phi}^2 + \frac{m}{2}\dot{\phi}^2$$

$$\ddddot{\phi} - m\ddot{\phi} = 0$$

$$\{\phi(t_0), \dot{\phi}(t_0), \ddot{\phi}(t_0), \dddot{\phi}(t_0)\}$$

$$\mathcal{L} = \frac{1}{2}\ddot{\phi}^2 + \frac{m}{2}\dot{\phi}^2 + \frac{k}{2}\dot{\chi}^2 + b\ddot{\phi}\dot{\chi}$$

$$\ddddot{\phi} + b\dddot{\chi} - m\ddot{\phi} = 0$$

$$b\dddot{\phi} + k\ddot{\chi} = 0$$

$$\frac{\partial^2 \mathcal{L}}{\partial v_a \partial v_b} = \begin{pmatrix} 1 & b \\ b & k \end{pmatrix}$$

$k = b^2$

Degenerate!

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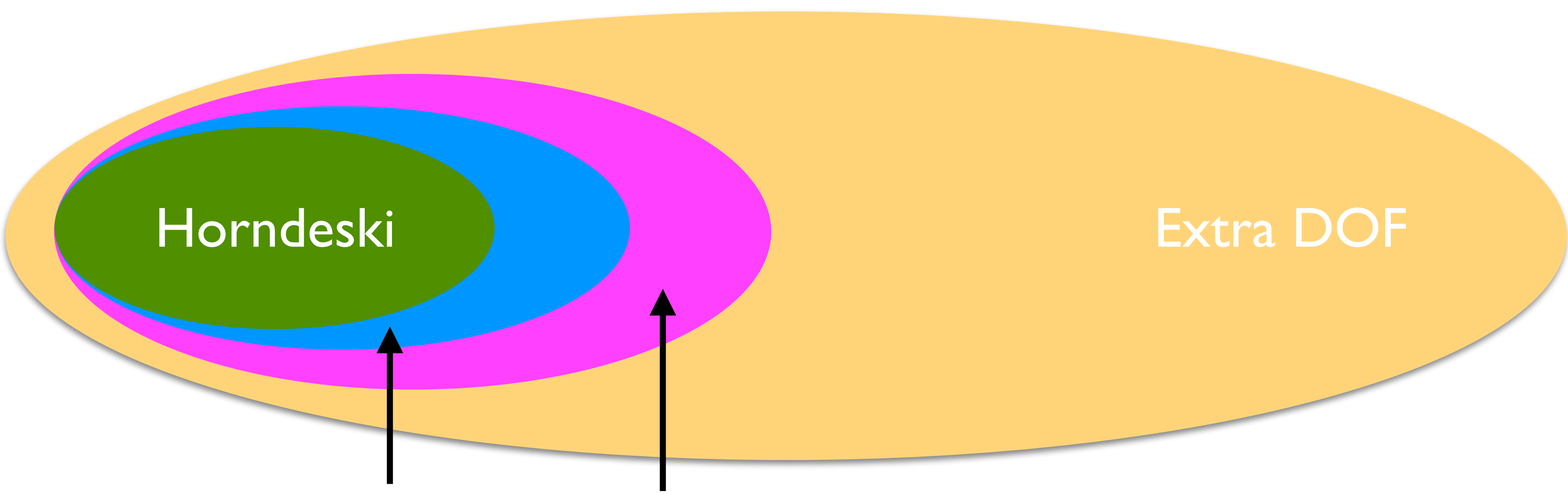


beyond Horndeski
Zumalacarregui, Garcia-Bellido '13
with Gleyzes, Langlois, Piazza '14;

Degenerate Higher-Order
Scalar-Tensor theories

Langlois, Noui '15, '16;
Crisostomi, Hull et al. '16;
Crisostomi, Koyama, Tasinato '16;
Achour et al. '16

Degenerate Higher-Order ST theories

- DHOST/EST theories: most general Lorentz-invariant scalar-tensor theory with a 1 scalar and 2 tensor degrees of freedom. Many (19) functions of (ϕ, X)

Langlois and Noui '15, '16; Crisostomi, Koyama, Tasinato '16

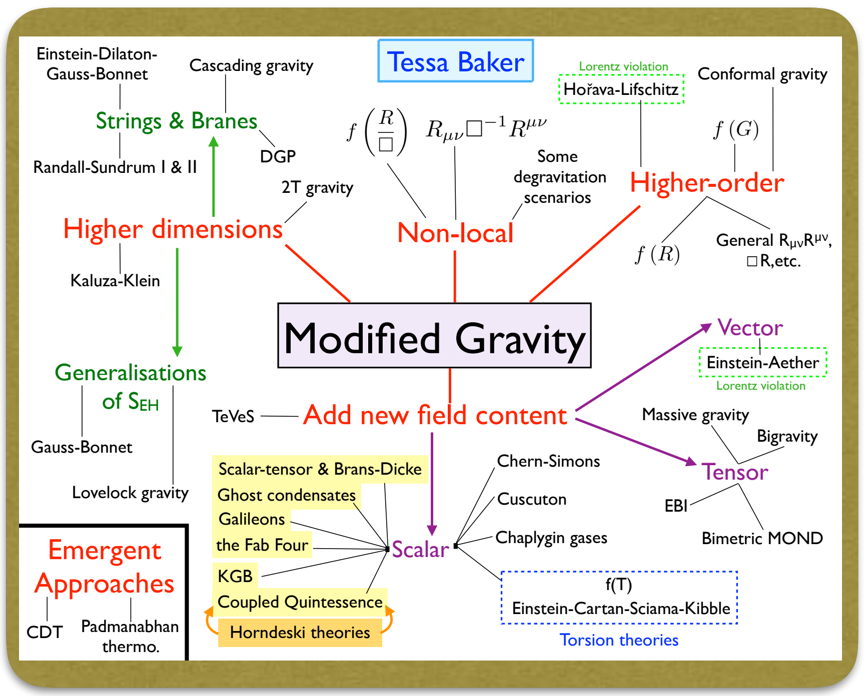
$$L = f_2(\phi, X)^{(4)}R + K(\phi, X) + G(\phi, X)\Box\phi + C_2^{\mu\nu\rho\sigma}(\phi, X)\partial_\mu\partial_\nu\phi\partial_\rho\partial_\sigma\phi$$
$$+ f_3(\phi, X)G_{\mu\nu}\partial^\mu\partial^\nu\phi + C_3^{\mu\nu\rho\sigma\alpha\beta}(\phi, X)\partial_\mu\partial_\nu\phi\partial_\rho\partial_\sigma\phi\partial_\alpha\partial_\beta\phi$$

- Kinetic matrix $\quad V \equiv t^\nu\nabla_\nu(n^\mu\nabla_\mu\phi) \qquad K_{\mu\nu} \equiv \frac{1}{2}\dot{h}_{\mu\nu} \equiv \frac{1}{2}\mathcal{L}_t h_{\mu\nu}$

$$\mathcal{L}_{\rm kin} = (V, K_{\mu\nu}) \begin{pmatrix} \mathcal{A} & \mathcal{B}^{\rho\sigma} \\ \mathcal{B}^{\mu\nu} & \mathcal{K}^{\mu\nu,\rho\sigma} \end{pmatrix} \begin{pmatrix} V \\ K_{\rho\sigma} \end{pmatrix}$$

- Horndeski and beyond Horndeski are the simplest case: $\mathcal{A} = 0$. In general more complex: 3 degeneracy conditions. Degenerate Higher-Order Scalar-Tensor (DHOST) or Extended Scalar-Tensor (EST) theories.

Models



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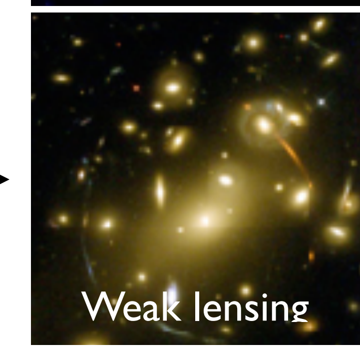
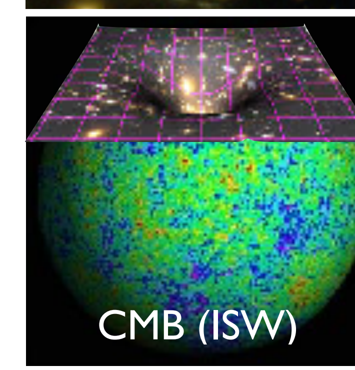
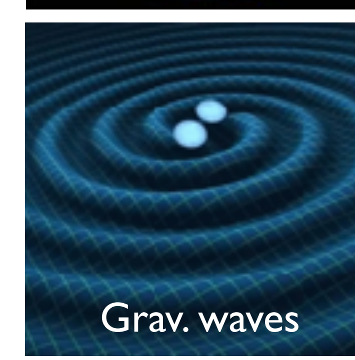
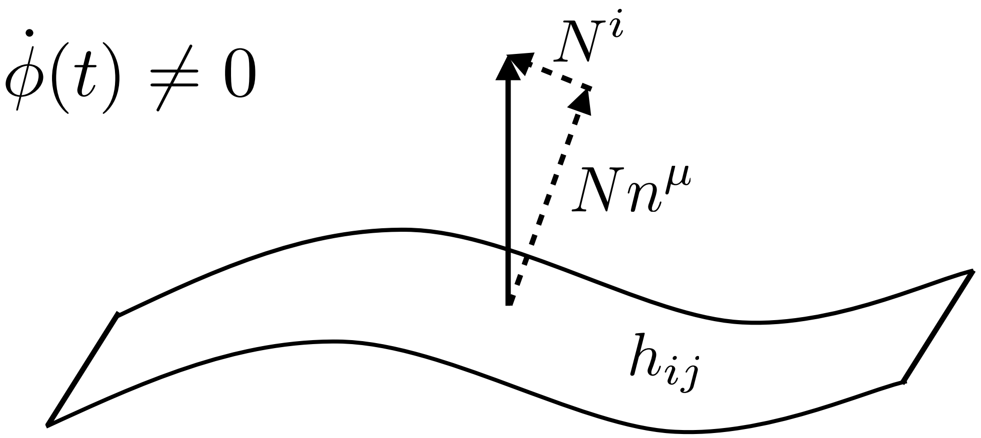






Constructing the action

✦ Use metric quantities in uniform scalar field slicing $\dot{\phi}(t) \neq 0$



▶ ADM decomposition

$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

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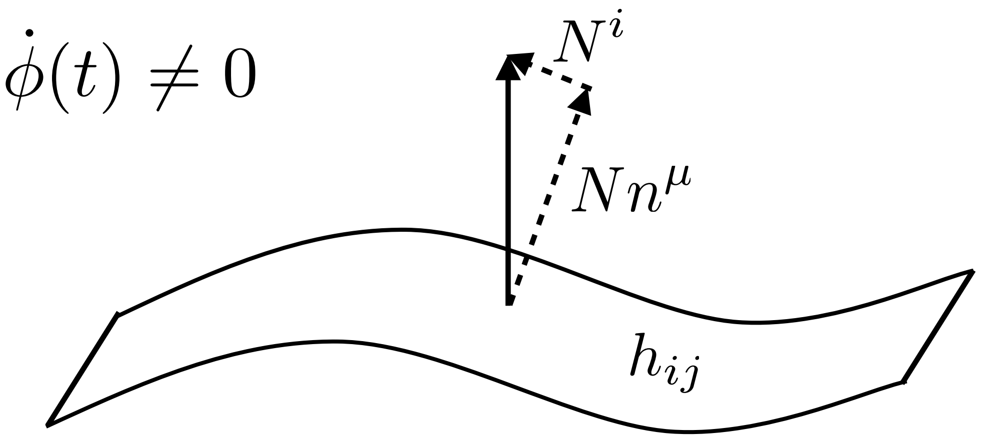

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✦ Lagrangian contains all possible scalars under spatial diffs, **ordered by number of perturbations and derivatives** Cheung et al. \`07

$$S = \int d^4x \sqrt{-g} L[t; N, K^i_j, {}^{(3)}R^i_j, \ldots]$$

Lapse	N	$\sim \dot{\phi}$	$(\partial\phi)^2 = -\dot{\phi}_0^2(t)/N^2$
Extrinsic curvature	K_{ij}	$\sim \partial_t g_{ij}$	$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$
Intrinsic curvature	${}^{(3)}R_{ij}$	$\sim \partial^2 g_{ij}$	

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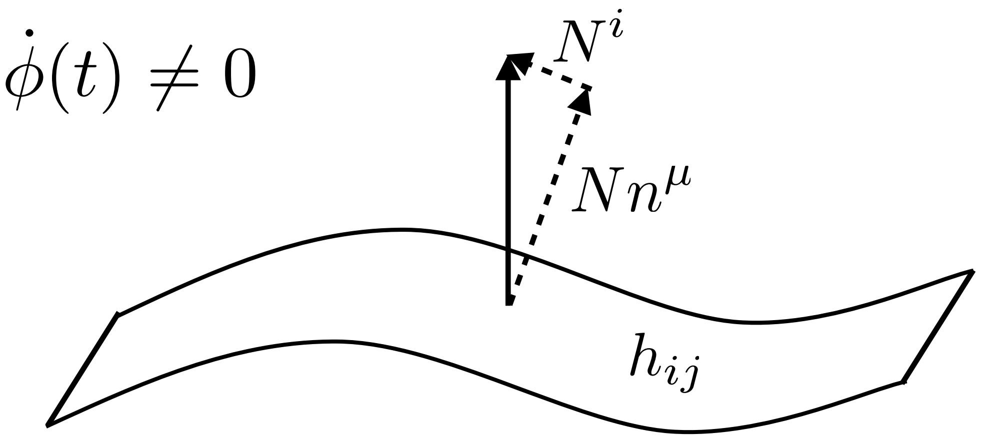

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$$S = \int d^4x \sqrt{-g} L[t; N, K^i_j, {}^{(3)}R^i_j, \ldots]$$

✦ Expand the action

$$\delta N \equiv N - 1\,, \qquad \delta K_{ij} \equiv K_{ij} - H h_{ij}\,, \qquad {}^{(3)}R_{ij}$$

$$L(N, K^i_j, R^i_j, \ldots) = \bar{L} + L_N \delta N + \frac{\partial L}{\partial K^i_j}\delta K^i_j + \frac{\partial L}{\partial R^i_j}\delta R^i_j + L^{(2)} + \ldots$$

Building blocks of linear perts

$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[\delta K^j_i \delta K^i_j - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}^{(2)}_i(\delta N, \delta K, \ldots) \right]$$

with Gleyzes, Langlois, Piazza '13 (see also Bloomfield '13)

- ✦ New operators describe deviations from GR (ΛCDM)
- ✦ Time dependent couplings (functions α_i): expansion around FRW background
- ✦ Functions $\alpha_i(t)$ independent of background evolution $H(t) = \dot{a}/a$
  - we fit to data $H(t)$ and $\alpha_i(t)$ (agnostic of their time dependence and parametrization)

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Notation of Bellini, Sawicki '14 for the alphas

α_i	α_K	α_B	α_M	α_T	α_H
$\mathcal{O}^{(2)}_i$	δN^2	$\delta N \delta K$	$\frac{dM^2}{d \ln a}$	${}^{(3)}R$	$\delta N \, {}^{(3)}R$
quintessence, k-essence	✓				
Cubic Galileon	✓	✓			
Brans-Dicke, f(R)	✓	✓	✓		
Horndeski	✓	✓	✓	✓	
Beyond Horndeski	✓	✓	✓	✓	✓

5 functions of time instead of 5 functions of $\phi, (\partial\phi)^2$; minimal number of parameters

Building blocks of linear perts

✦ We impose absence of ghost and gradient stability:

$$\mathcal{L} = +\dot{\varphi}^2 - c_s^2 (\nabla \varphi)^2$$

positive kinetic energy = absence of ghosts

positive sound speed squared = absence of gradient instabilities

scalar

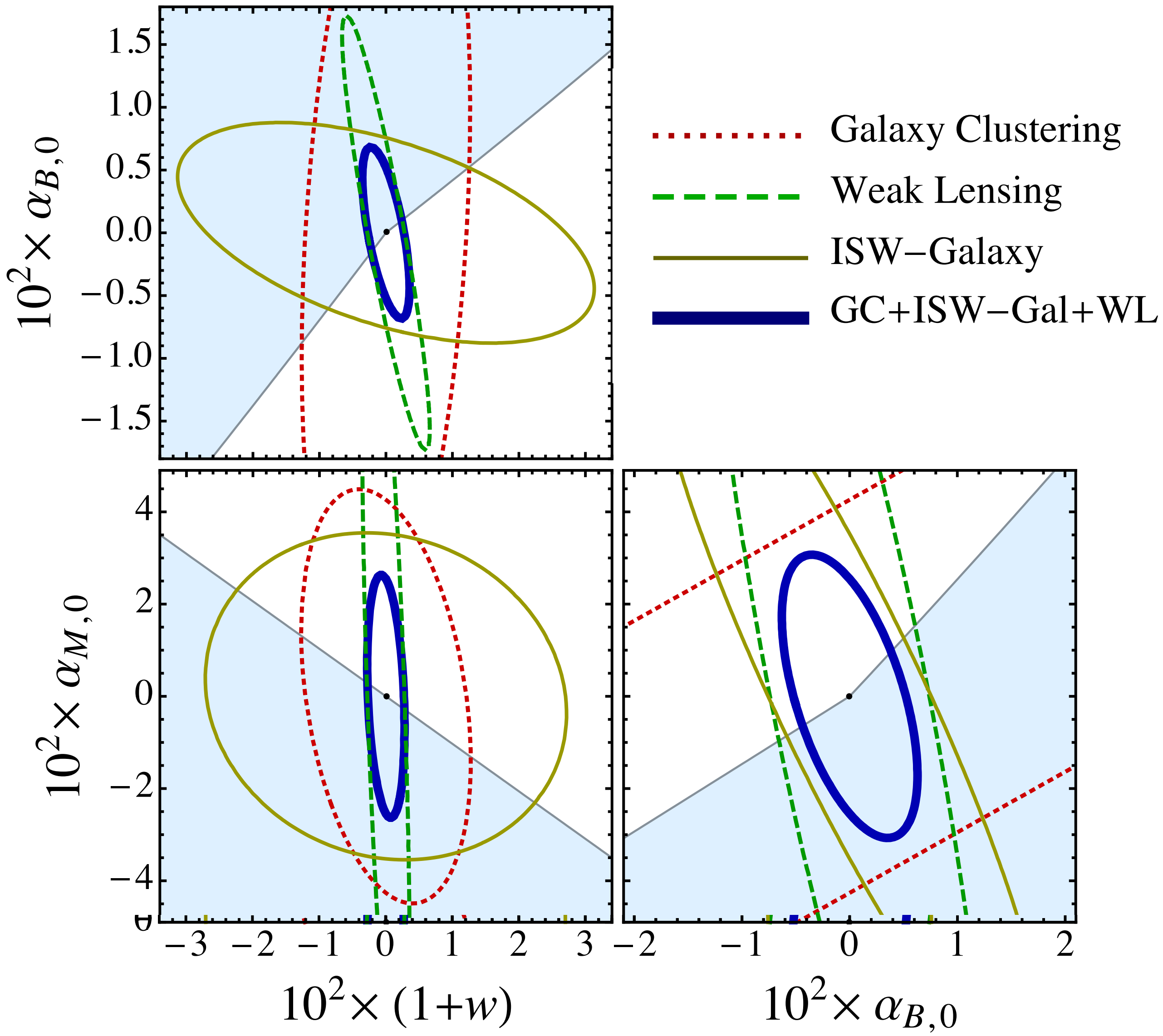
tensor

$$h_{ij} = a^2(t) e^{2\zeta} (\delta_{ij} + \gamma_{ij}) \ , \quad \gamma_{ii} = 0 = \nabla_i \gamma_{ij}$$

	Scalar	Tensor
No ghosts	$\alpha_K + 6\alpha_B^2$	$M^2 > 0$
No gradient instability	$c_s^2(\alpha_i) \geq 0$	$\alpha_T \geq -1$

Fisher matrix analysis

with Gleyzes, Langlois, Mancarella '15



Euclid specifications (LCDM fiducial)

Quasi-static approximation

- Background parametrization:

$$H^2 = H_0^2 \left[\Omega_{m0} a^{-3} + (1 - \Omega_{m0}) a^{-3(1+w)} \right]$$

- Free functions parametrization:

$$\alpha_I(t) = \alpha_{I,0} \frac{1 - \Omega_m(t)}{1 - \Omega_{m,0}}$$

Higher-Order theories

$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N \, {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)}(\delta N, \delta K, \ldots) \right]$$

- ✦ All operators up to two derivatives

with Langlois, Mancarella, Noui '17

α_i	α_K	α_B	α_M	α_T	α_H	α_L	β_1	β_2	β_3
$\mathcal{O}_i^{(2)}$	δN^2	$\delta N \delta K$	$\frac{dM^2}{d\ln a}$	${}^{(3)}R$	$\delta N \, {}^{(3)}R$	δK^2	$\delta \dot{N}^2$	$\delta \dot{N} \delta K$	$(\partial_i \delta N)^2$
quintessence, k-essence	✓								
Cubic Galileon	✓	✓							
Brans-Dicke, f(R)	✓	✓	✓						
Horndeski	✓	✓	✓	✓					
Beyond Horndeski	✓	✓	✓	✓	✓				
DHOST/EST theries	✓	✓	✓	✓	✓	✓	✓	✓	✓

Higher-Order theories

$$S^{(2)} = \int d^4x\sqrt{h}\frac{M^2}{2}\left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}\!R + \delta N\,{}^{(3)}\!R + \sum_i \alpha_i(t)\mathcal{O}_i^{(2)}(\delta N, \delta K, \ldots)\right]$$

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- ✦ Generic scalar dispersion relation: $\mathcal{E}_1\omega^4 + \mathcal{E}_2\omega^2k^2 + \mathcal{E}_3\omega^2 + \mathcal{E}_4k^4 + \mathcal{E}_5k^2 = 0$

- ✦ Two types of degeneracy conditions lead to $\omega^2 - c_s^2k^2 = 0$

$$\mathcal{C}_{\rm I}: \quad \alpha_L = 0\,, \qquad \beta_2 = f_2(\beta_1)\,, \qquad \beta_3 = f_3(\beta_1)$$

$$\mathcal{C}_{\rm II}: \quad \beta_1 = f_1(\alpha_T, \alpha_H, \alpha_L)\,, \quad \beta_2 = f_2(\alpha_T, \alpha_H, \alpha_L)\,, \quad \beta_3 = f_3(\alpha_T, \alpha_H, \alpha_L)$$

$c_s^2 \propto -c_T^2$ **ruled out!**

Frame dependence

- Gravitational action:

$$S_{\rm gravity} = \int d^4x \mathcal{L}_g(g_{\mu\nu}; \alpha_I, \beta_J) \ , \qquad I = 1, \ldots, 6 \ , \quad J = 1, 2, 3$$

6+3=9 parameters and 3 degeneracy conditions: 6 parameters

- Action transforms under metric redefinition: (most general) disformal transformation

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = C(\phi, X) g_{\mu\nu} + D(\phi, X) \partial_\mu \phi \partial_\nu \phi$$

Bekenstein '92

$$S^{(2)}[g_{\mu\nu}, \alpha_I] = \tilde{S}^{(2)}[\tilde{g}_{\mu\nu}, \tilde{\alpha}_I] \qquad \tilde{\alpha}_I = M_I^{I'} \alpha_{I'}$$

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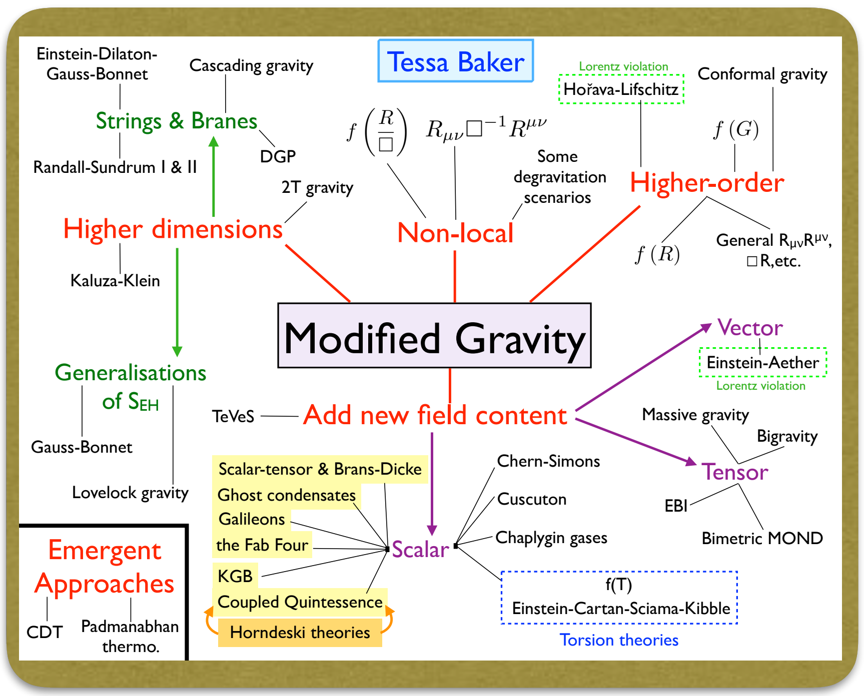
✦ Two sets of degeneracy conditions invariant under disformal transformations

✦ Class $\mathcal{C}_{\rm I}$ can be brought to Horndeski frame: $\alpha_H = 0, \ \beta_J = 0$

DHOST I $\xrightarrow{C(X)}$ Beyond Horndeski $\xrightarrow{D(X)}$ Horndeski

✦ Changing frame changes matter couplings (Horndeski vs Jordan): Matter matters!

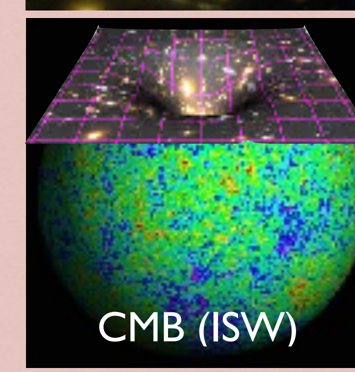
Models



ETofDE

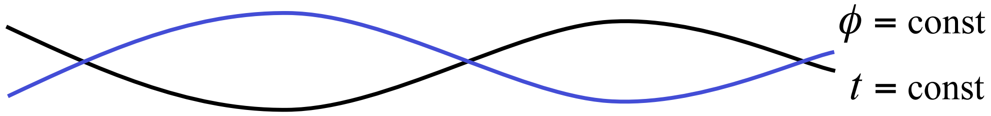
$$\alpha_K(t),\ \alpha_B(t),\ \alpha_M(t),$$
$$\alpha_T(t),\ \alpha_T(t),\ \ldots$$

Observations



Phenomenology

- Undo unitary gauge: $t \to t + \pi(t, \vec{x})$



- Newtonian gauge (scalar flucts): $dt^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2$

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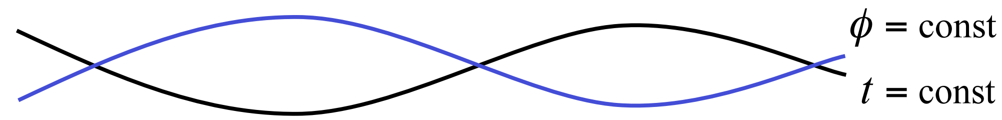

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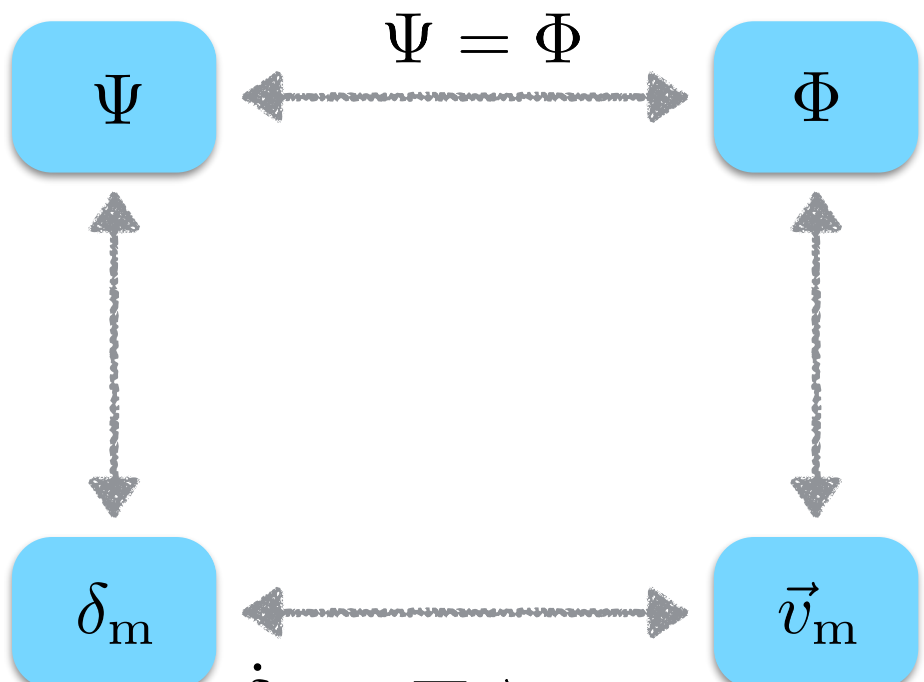
- Quasi-static approximations — valid on scales $k \gg aHc_s^{-1}$. Sawicki, Bellini '15

E.g., for surveys such as Euclid $c_s \gtrsim 0.1$.

$$\nabla^2(\Psi + \Phi) = 8\pi G a^2 \rho_m \delta_m$$

$$\Psi = \Phi$$



$$\nabla^2 \Psi = 4\pi G a^2 \rho_m \delta_m$$

$$\dot{\vec{v}}_m + H\vec{v}_m = -\nabla\Phi$$

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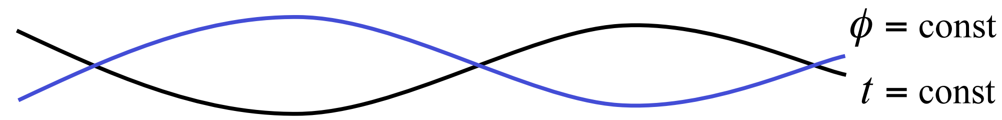

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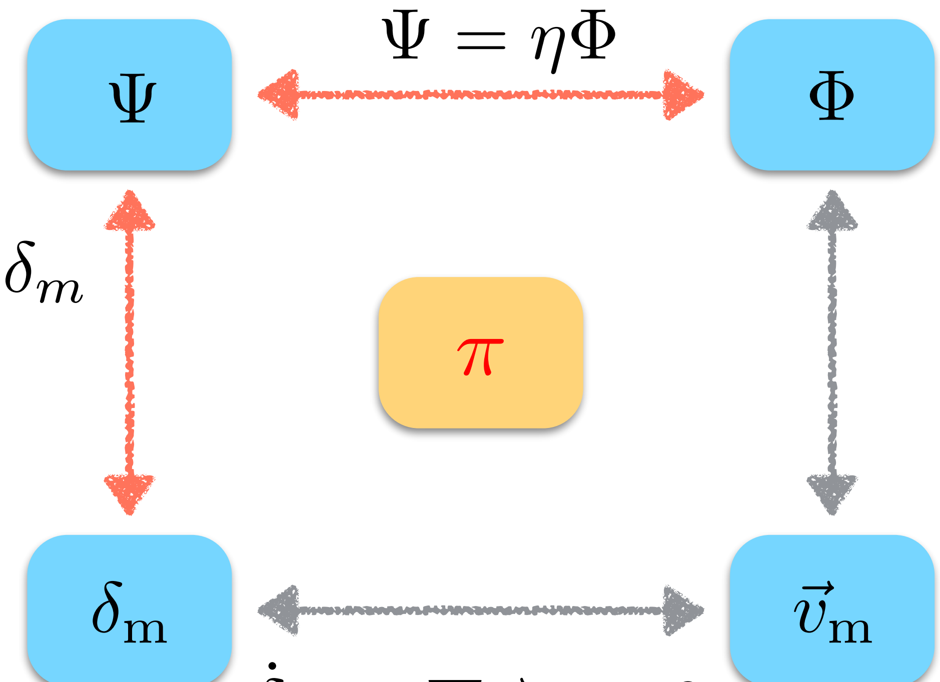
E.g., for surveys such as Euclid $c_s \gtrsim 0.1$.

$$\nabla^2(\Psi + \Phi) = 8\pi G(1 + \Upsilon_{\rm lens})a^2 \rho_m \delta_m$$

$$\Upsilon_{\rm lens} = \Upsilon_{\rm lens}(\alpha_i)$$

$$\Psi = \eta\Phi$$



$$\nabla^2\Psi = 4\pi G(1 + \Upsilon_\Psi)a^2 \rho_m \delta_m$$

$$\Upsilon_\Psi = \Upsilon_\Psi(\alpha_i)$$

$$\dot{\vec{v}}_m + H\vec{v}_m = -\nabla\Phi$$

$$\dot{\delta}_m + \nabla\vec{v}_m = 0$$

Small scale limit

α_K	α_B	α_M	α_T
δN^2	$\delta N \delta K$	$\frac{dM^2}{d\ln a}$	${}^{(3)}R$

- In the limit $k \to \infty$:

$$\nabla^2\Phi = \frac{3}{2}\Omega_m H^2 a^2 \delta_m \left(1 + \alpha_T + \frac{\xi^2}{\nu}\right)$$

$$\nabla^2\Psi = \frac{3}{2}\Omega_m H^2 a^2 \delta_m \left(1 + \frac{\xi\alpha_B}{\nu}\right)$$

$$\xi = \alpha_B(1+\alpha_T) + \alpha_T - \alpha_M$$

$$\nu = -\left\{(1+\alpha_B)\left[\alpha_B(1+\alpha_T) + \alpha_T - \alpha_M + \frac{\dot{H}}{H^2}\right] + \frac{\dot{\alpha}_B}{H} + \frac{3}{2}\Omega_m\right\} = \frac{c_s^2\alpha}{2} > 0$$

Boltzmann codes

- Full Einstein-Boltzmann solver: $$\frac{df_I}{d\eta} = C_I[f_I] \, , \quad I = \gamma, \nu, b, \mathrm{CDM}$$

$$\frac{\delta S^{(2)}}{\delta \pi} = 0 \qquad \& \qquad G_{ij}^{\mathrm{modified}} = 8\pi G \sum_I T_{ij}^{(I)}$$

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- EFTCAMB (from CMBFAST) (Hu, Raveri, Frusciante, Silvestri et al.)
- hi_class (from CLASS) (Zumalacarregui, Bellini, Sawicki, Lesgourgues et al.)
- COOP (indep. code, Zhiqi Huang) (with D'Amico, Huang and Mancarella)
- LVDM-CLASS (from CLASS) (Blas, Ivanov, Sibiryakov)
- others …

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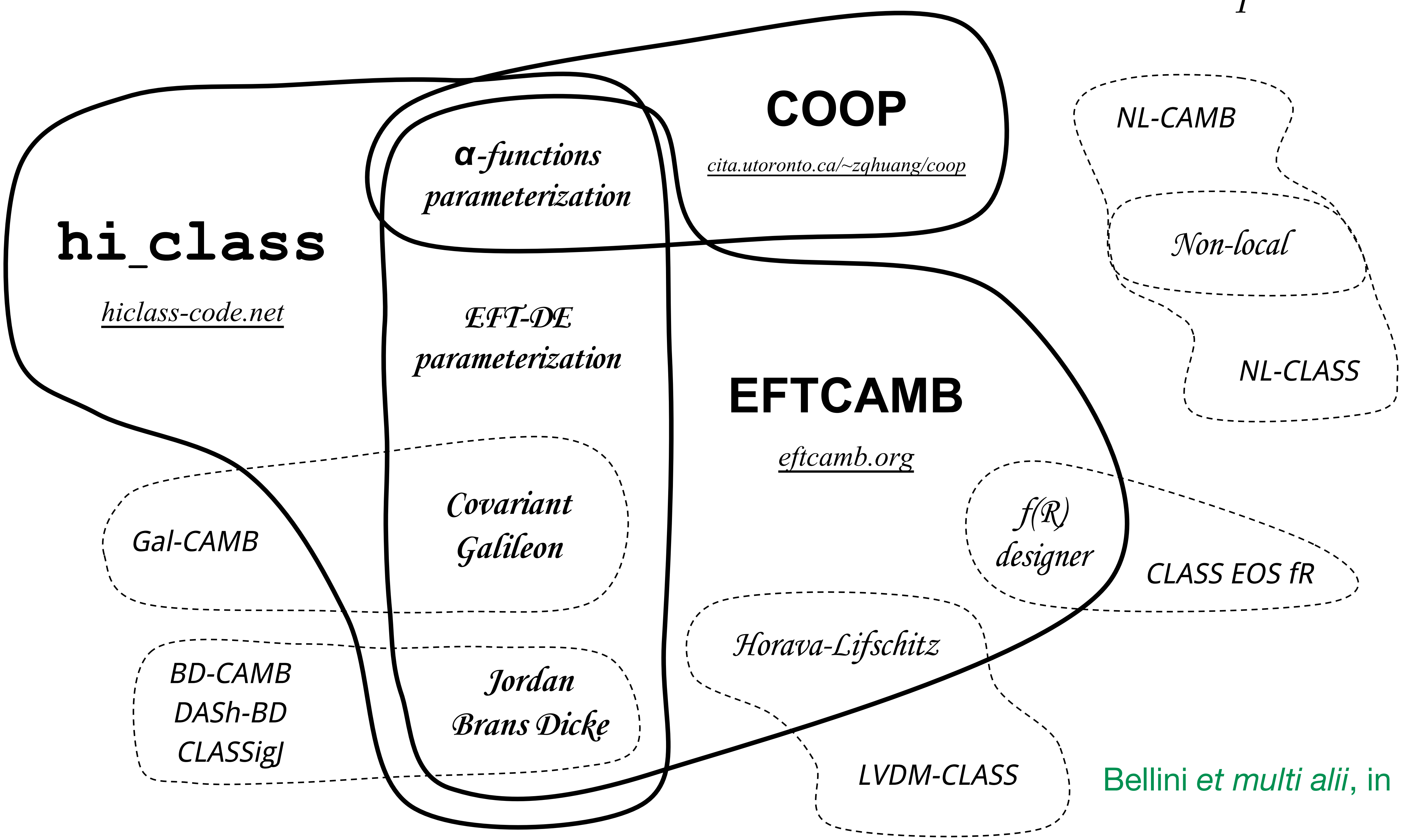

Bellini *et multi alii*, in prep.

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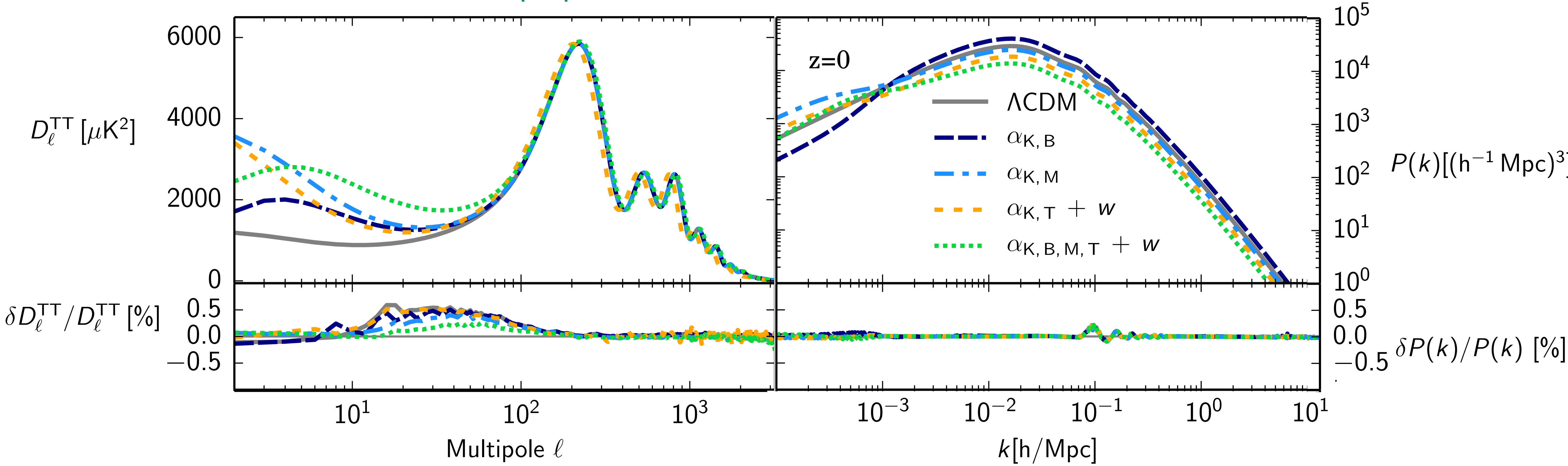
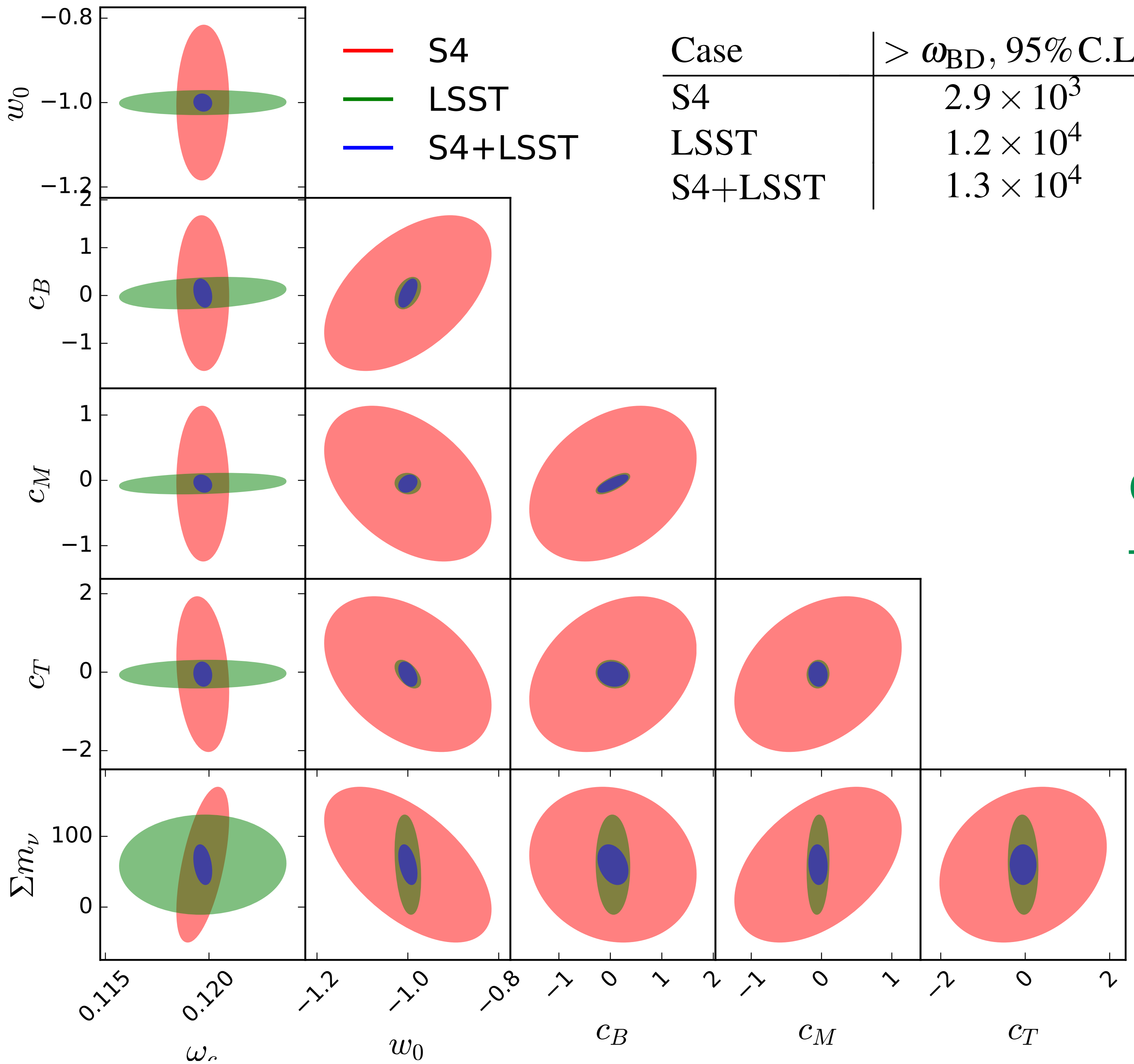


Deviations from ΛCDM

Alonso et al. '16

$$\alpha_X = c_X \frac{\Omega_{\rm DE}(z)}{\Omega_{\rm DE}(z=0)}$$



Case	$>\omega_{\rm BD}$, 95% C.L.	$\sigma(c_B)$	$\sigma(c_M)$	$\sigma(c_T)$	$\sigma(c_K)$	$\sigma(w)$	$\sigma(\sum m_\nu)$ [meV]
S4	2.9×10^3	0.796	0.746	1.26	4.9	0.112	71
LSST	1.2×10^4	0.193	0.089	0.205	8.8	0.016	45
S4+LSST	1.3×10^4	0.169	0.072	0.179	3.5	0.011	22

$$\sigma(\alpha_X) \sim \mathcal{O}(0.1)$$

Cassini (Bertotti et al. 03): $\omega_{\rm BD} > 40\,000$

This work: $\omega_{\rm BD} > 20\,000$

Mildly nonlinear scales

- ✦ Nonlinear scales are difficult!
- ✦ Possible strategy: conservative cutoff on small scales. But certain observables require (mildly) nonlinear modelling. E.g. redshift-space distortions, baryon acoustic oscillations, etc.
- ✦ Ample information on nonlinear scales: many more modes and possible new signatures (screening mechanism, nonlinear couplings, etc.)
- ✦ Many developments in numerical simulations including DE/MG
  - Only developed for some models (e.g. DGP, f(R))
  - Time consuming and non-standard models difficult to implement

Codes: ECOSMOG, MG-GADGET, ISIS, DGPM, …

(Winther et al 15)

- ✦ Many developments in analytical perturbative methods

Baldauf, Bernardeau, Bertolini, Blas, Carrasco, Crocce, Garny, Ivanov, Pajer, Peloso, Pietroni, Scoccimarro, Senatore, Sibiryakov, Valageas, Zaldarriaga and many others

Nonlinear ET of DE

$$S = \int d^4x \sqrt{h} \frac{M^2}{2} \left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)} + \sum_i \alpha_i(t) \mathcal{O}_i^{(3)} \right]$$

- In the short-scale limit, a finite number of operators dominate

Example: Horndeski has only 3 cubic operators and nothing more   Bellini, Jimenez, Verde '15

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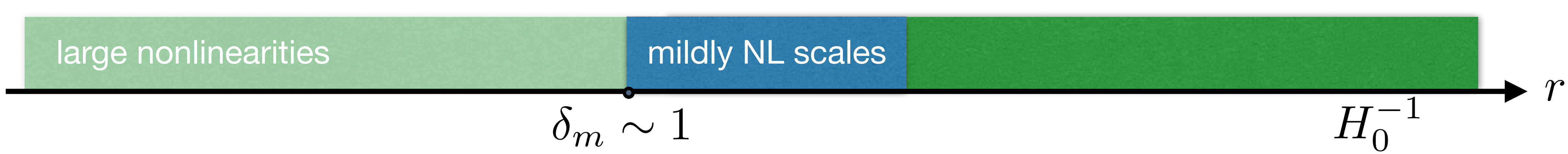
- Standard Perturbation Theory

$$\dot{\delta}_m + \nabla \left[(1 + \delta_m) \vec{v}_m \right] = 0$$

$$\dot{\vec{v}}_m + H \vec{v}_m + \vec{v}_m \cdot \nabla \vec{v}_m = -\nabla \Phi$$

- GR case: Poisson equation

$$\nabla^2 \Phi = \frac{3}{2} a^2 H^2 \Omega_{\rm m} \delta_{\rm m}$$



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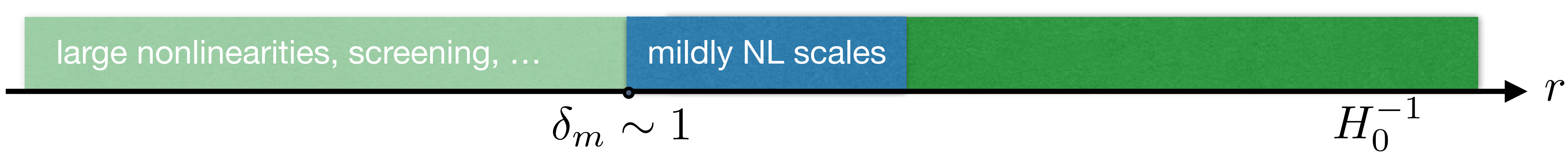
- Modifications of gravity encoded in Poisson-like equation

$$k^2 \Phi = -\frac{3}{2} a^2 H^2 \Omega_{\rm m} \mu_{\Phi,1} \delta_{\rm m} - \frac{9}{4} a^2 H^2 \Omega_{\rm m}^2 \mu_{\Phi,2}(\vec{k}_1, \vec{k}_2) \delta_{\rm m}(\vec{k}_1) \star \delta_{\rm m}(\vec{k}_2) + \ldots$$



Conclusions

- ✳ Unifying description for scalar-tensor theories, including higher-order degenerate ones (and more)
- ✳ Analysis of (degenerate higher-order) theories highly simplified
- ✳ Linear regime worked out! Issue of time dependence of α's when comparing to data
- ✳ Straightforward connection to mildly and fully nonlinear regime