Effective Field Theories for Post-Newtonian Gravity with a Spin (or two)

Michèle Lévi

Institut de Physique Théorique
CEA, Université Paris-Saclay

CERN, TH workshop
August 18, 2017

<<preQFT>>
no. 639729

Outline

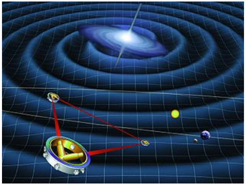
1. Formulating the tower of EFTs
   - Setup and goal
   - Degrees of freedom
   - Symmetries
2. Formulating an EFT for a spinning particle
   - Worldline spin as a further DOF
   - Restoring gauge freedom of rotational variables
   - Nonminimal couplings with spin
3. Integrating out the orbital scale
   - Disentangling particle and fields DOFs
   - NRG fields and Schwinger's time gauge
   - EOMs of spin from effective action
4. Implementing the EFTs
   - LO cubic & quartic in spin sectors
   - NNLO spin-squared interaction
   - EFTofPNG package

Gravitational Radiation

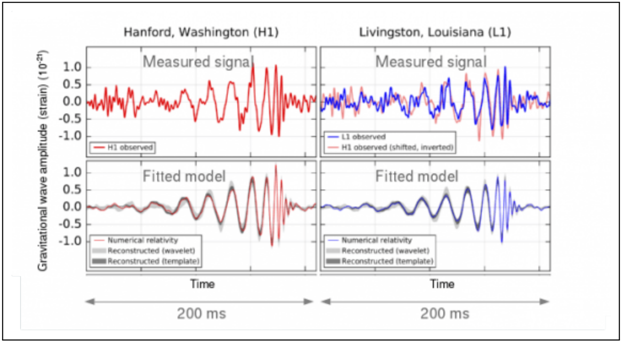
Prediction	1916 Einstein
Indirect Evidence	1974 Hulse & Taylor
Direct Observation	2015 LIGO!

Gravitational wave detectors:

- Ground-based
  - LIGO
  - GEO 600
  - Virgo
- Second generation ground-based
  - US Advanced LIGO 2015
  - EU Advanced Virgo 2017
  - Japan KAGRA 2019?
  - LIGO India 2024?
- Future space-based
  - ESA eLISA 2034?

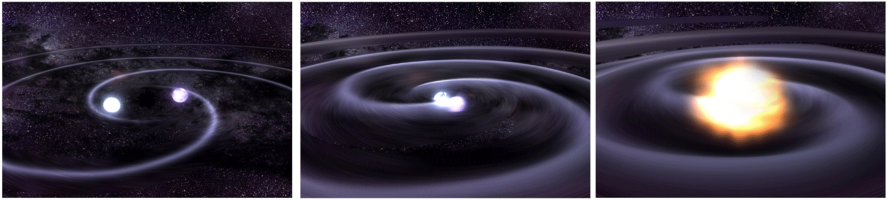


GW events – as of 2015!



1. GW150914 (shown in figure)
2. GW151226
3. GW170104

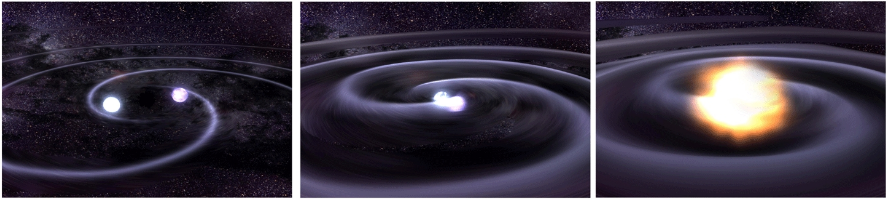
Sources of Gravitational Waves



Three phases in the life of a compact binary:

1. Inspiral
2. Merger
3. Ringdown

Sources of Gravitational Waves



Three phases in the life of a compact binary:

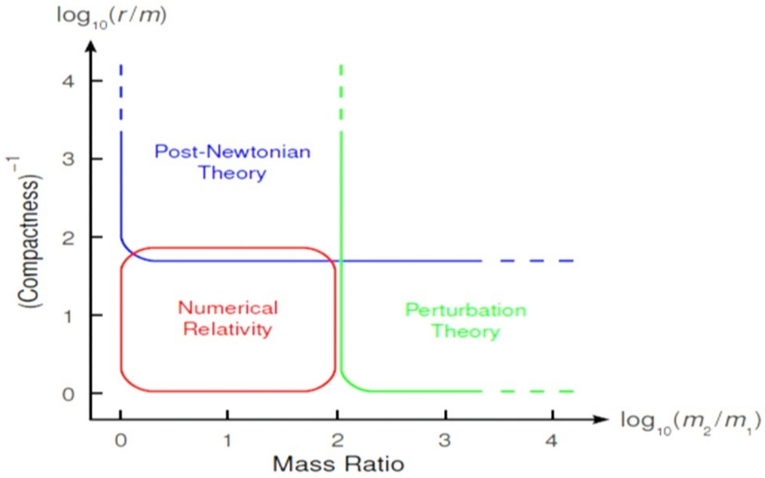
1. Inspiral
2. Merger
3. Ringdown

Detection by matched filtering

⇒ Theoretical waveform templates

⇒ Phenomenological modelling of waveforms
requires PN parameters of up to 6th order!

Methods to compute gravitational-wave templates



$\Rightarrow$ Effective-One-Body approach, Buonanno and Damour 1999

EFTs and their Setup are Universal

There is a Hierarchy of Scales

[Goldberger et al. 2007]

1. r_s, scale of internal structure, $r_s \sim m$
2. r, orbital separation scale, $r \sim \dfrac{r_s}{v^2}$
3. λ, radiation wavelength scale, $\lambda \sim \dfrac{r}{v}$

We can use EFT!

$v \ll 1$, $nPN \equiv v^{2n}$ correction in GR to Newtonian Gravity

For an EFT of PNG proceed in stages corresponding to each scale

Setup of EFT is Universal

Bottom-Up or Top-Down?

Stage 1 Remove the scale r_S of isolated compact object, bottom-up approach

$$S\left[g_{\mu\nu}\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{g}R$$

Integrate out strong field modes $g_{\mu\nu} \equiv g^s_{\mu\nu} + \bar{g}_{\mu\nu}$

$$\Rightarrow S_{\text{eff}}\left[\bar{g}_{\mu\nu}, y^\mu, e^\mu_A\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}R\left[\bar{g}_{\mu\nu}\right] + \underbrace{\sum_i C_i \int d\sigma\, O_i(\sigma)}_{\equiv S_{pp}(\sigma)\text{ with Wilson coefficients}}$$

Setup of EFT is Universal

Stage 2 Remove the orbital scale r of binary, top-down

$$\bar{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \underbrace{H_{\mu\nu}}_{\text{orbital}} + \underbrace{\widetilde{h}_{\mu\nu}}_{\text{radiation}}$$

Kenneth Wilson

$$\partial_t H_{\mu\nu} \sim \frac{v}{r} H_{\mu\nu}, \quad \partial_i H_{\mu\nu} \sim \frac{1}{r} H_{\mu\nu}, \quad \partial_\rho \widetilde{h}_{\mu\nu} \sim \frac{v}{r} \widetilde{h}_{\mu\nu}$$

$$S_{\rm eff}\left[\bar{g}_{\mu\nu}, y_1^\mu, y_2^\mu, e_{(1)}{}_A^\mu, e_{(2)}{}_A^\mu\right] = -\frac{1}{16\pi G}\int d^4x \sqrt{\bar{g}} R\left[\bar{g}_{\mu\nu}\right] + S_{(1)\rm pp} + S_{(2)\rm pp}$$

Integrate out orbital field modes

$$\Rightarrow e^{iS_{\rm eff(composite)}\left[\widetilde{h}_{\mu\nu}, y^\mu, e_A^\mu\right]} \equiv \int \mathcal{D}H_{\mu\nu}\; e^{iS_{\rm eff}\left[\bar{g}_{\mu\nu}, y_1^\mu, y_2^\mu, e_{(1)}{}_A^\mu, e_{(2)}{}_A^\mu\right]}$$

Stop here for effective action in conservative sector, that is **WITHOUT** any remaining orbital scale field DOFs

[**ML** 2010, **ML** & Steinhoff 2014]

To construct EFT – identify Degrees of Freedom

1. *The gravitational field*
   - The metric $g_{\mu\nu}(x)$
   - The tetrad field $\eta^{ab}\tilde{e}_a{}^{\mu}(x)\tilde{e}_b{}^{\nu}(x) = g^{\mu\nu}(x)$
2. *The particle worldline coordinate*

   $y^{\mu}(\sigma)$ a function of an arbitrary affine parameter σ
   Particle worldline position does not in general coincide with object's 'center'
3. *The particle worldline rotating DOFs*

   Worldline tetrad, $\eta^{AB}e_A{}^{\mu}(\sigma)e_B{}^{\nu}(\sigma) = g^{\mu\nu}$

   $\Rightarrow$ Angular velocity $\Omega^{\mu\nu}(\sigma)$ + conjugate Spin $S_{\mu\nu}(\sigma)$

   $\Rightarrow$ Lorentz matrices, $\eta^{AB}\Lambda_A{}^{a}(\sigma)\Lambda_B{}^{b}(\sigma) = \eta^{ab}$ + conjugate local spin, $S_{ab}(\sigma)$

To construct EFT – identify Symmetries

1. *General coordinate invariance*, and *parity invariance*
2. *Worldline reparametrization invariance*
3. *Internal Lorentz invariance of local frame field*
4. *SO(3) invariance of body-fixed spatial triad*
5. *Spin gauge invariance*, that is invariance under choice of completion of body-fixed spatial triad through timelike vector
6. Assume isolated object has no intrinsic permanent multipole moments beyond the mass monopole and the spin dipole

Worldline spin as a further DOF

Effective action of a spinning particle

[Hanson & Regge 1974, Bailey & Israel 1975]

- $u^\mu \equiv dy^\mu/d\sigma$, $\Omega^{\mu\nu} \equiv e^\mu_A \frac{De^{A\nu}}{D\sigma} \Rightarrow L_{\text{pp}}\left[u_\mu, \Omega^{\mu\nu}, \bar{g}_{\mu\nu}\right]$
- $S_{\mu\nu} \equiv -2\frac{\partial L}{\partial \Omega^{\mu\nu}}$ spin as further worldline DOF – classical source

$$\Rightarrow S_{\text{pp}}(\sigma) = \int d\sigma \left[-m\sqrt{u^2} - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{\text{SI}}\left[u^\mu, S_{\mu\nu}, \bar{g}_{\mu\nu}\left(y^\mu\right)\right]\right]$$

For an EFT with spin the gauge of both rotational variables should be fixed at the level of the point particle action

[**ML** 2×PRD 2010, **ML** & Steinhoff JCAP 2014]

Start in the covariant gauge: $e^\mu_{[0]} = \frac{p^\mu}{\sqrt{p^2}}$, $\qquad S_{\mu\nu}p^\nu = 0$

- Linear momentum $p_\mu \equiv -\frac{\partial L}{\partial u^\mu} = m\frac{u^\mu}{\sqrt{u^2}} + \mathcal{O}(S^2)$

Gauge freedom of rotational variables can be restored

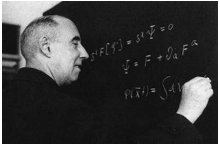
[**ML** & Steinhoff, JHEP 2015]

Introduce gauge freedom in rotational variables

Transform gauge of $e^{A\mu}$ from $e_{[0]\mu} = q_\mu \rightarrow \hat{e}_{[0]\mu} = w_\mu$
with a boost-like transformation in covariant form:

$$\hat{e}^{A\mu} = L^\mu{}_\nu(w,q)e^{A\nu}, \quad q_a,\ w_a \text{ timelike unit 4-vectors}$$

$\Rightarrow$ Generic gauge: $\hat{e}_{[0]\mu} = w_\mu,\ \hat{S}^{\mu\nu}\left(p_\nu + \sqrt{p^2}\hat{e}_{[0]\nu}\right) = 0$



Ernst Stueckelberg

$\Rightarrow$ Extra term in action appears!

- For minimal coupling $\frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} = \frac{1}{2}\hat{S}_{\mu\nu}\hat{\Omega}^{\mu\nu} + \frac{\hat{S}^{\mu\rho}p_\rho}{p^2}\frac{Dp_\mu}{D\sigma}$
- Extra term contributes to finite size effects,
  yet carries no Wilson coefficient
- Beyond minimal coupling $S_{\mu\nu} = \hat{S}_{\mu\nu} - \frac{\hat{S}_{\mu\rho}p^\rho p_\nu}{p^2} + \frac{\hat{S}_{\nu\rho}p^\rho p_\mu}{p^2}$

Nonminimal coupling action with spin must be constrained

[**ML** & Steinhoff, JHEP 2014, JHEP 2015]

Building blocks $\sim$ Riemann $\times$ Spin-induced higher multipoles

Spin-induced higher multipoles

$$S^{\mu} \equiv *S^{\mu\nu}\frac{p_{\nu}}{\sqrt{p^2}} \simeq *S^{\mu\nu}\frac{u_{\nu}}{\sqrt{u^2}}, \quad *S_{\alpha\beta} \equiv \frac{1}{2}\epsilon_{\alpha\beta\mu\nu}S^{\mu\nu}$$

$$\Rightarrow \; S_{\mu}S^{\mu} = -\frac{1}{2}S_{\mu\nu}S^{\mu\nu} \equiv -S^2$$

- $SO(3)$ invariance of body-fixed triad
  $\Rightarrow$ Spin multipoles are $SO(3)$ irrep tensors
- Initially, covariant gauge $e_{[0]}{}^{\mu} = u^{\mu}/\sqrt{u^2}$, $e_{[i]}{}^{\mu}u_{\mu} = 0$

$\Rightarrow$ Spin-induced higher multipoles are symmetric, traceless, and spatial, constant tensors in body-fixed frame

$\Rightarrow$ From parity invariance:
Even and odd combinations of spin vector S^{μ} couple to even and odd parity electric and magnetic curvature tensors, and their covariant derivatives, respectively.

Nonminimal coupling action with spin must be constrained

[**ML** & Steinhoff, JHEP 2014, JHEP 2015]

Curvature tensors

Electric component, $E_{\mu\nu} \equiv R_{\mu\alpha\nu\beta} u^\alpha u^\beta$,
Magnetic component, $B_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\alpha\beta\gamma\mu} R^{\alpha\beta}{}_{\delta\nu} u^\gamma u^\delta$,
and their spatial body-fixed covariant derivatives, $D_{[i]} = e^\mu_{[i]} D_\mu$

- Field is vacuum solution at LO, properties of Riemann, Bianchi identities
  $\Rightarrow E_{\mu\nu}$, $B_{\mu\nu}$, symmetric, traceless, and orthogonal to u^μ,
  also when projected to body-fixed frame, where they are spatial
- At most linear in Riemann
  $\Rightarrow$ Time derivative $D_{[0]} = u^\mu D_\mu \equiv D/D\sigma$ can be ignored
- Analogous to Maxwell's equations

$$\begin{aligned} \epsilon_{[ikl]} D_{[k]} E_{[lj]} &= \dot{B}_{[ij]} \quad \Rightarrow D_{[i]} E_{[ij]} = D_{[i]} B_{[ij]} = 0, \\ \epsilon_{[ikl]} D_{[k]} B_{[lj]} &= -\dot{E}_{[ij]} \qquad \Box E_{[ij]} = \Box B_{[ij]} = 0 \end{aligned}$$

$\Rightarrow$ Indices of covariant derivatives would be symmetrized
with respect to indices of electric and magnetic tensors
$\Rightarrow$ Covariant derivatives of these tensors are also traceless

LO nonminimal couplings to all orders in spin are fixed

[**ML** & Steinhoff, JHEP 2014, JHEP 2015]

LO nonminimal couplings to all orders in spin

New spin-induced Wilson coefficients:

$$L_{\rm SI} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

LO spin couplings up to 4PN order

$nPN \equiv v^{2n}$ correction in GR to Newtonian Gravity

- $L_{ES^2} = -\frac{C_{ES^2}}{2m} \frac{E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu$, Quadrupole @2PN
- $L_{BS^3} = -\frac{C_{BS^3}}{6m^2} \frac{D_\lambda B_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda$, Octupole @3.5PN
- $L_{ES^4} = \frac{C_{ES^4}}{24m^3} \frac{D_\lambda D_\kappa E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda S^\kappa$, Hexadecapole @4PN

Particle DOFs should be disentangled from field DOFs

[**ML**, 2×PRD 2010, **ML** & Steinhoff, JCAP 2014, JHEP 2015]

Worldline tetrad contains both rotational and field DOFs

- $\hat{e}_A{}^{\mu} = \hat{\Lambda}_A{}^b \tilde{e}_b{}^{\mu}$: $\eta^{AB}\hat{\Lambda}_A{}^a\hat{\Lambda}_B{}^b = \eta^{ab}$, tetrad field $\eta_{ab}\tilde{e}^a{}_\mu\tilde{e}^b{}_\nu = g_{\mu\nu}$
- $\frac{1}{2}\hat{S}_{\mu\nu}\hat{\Omega}^{\mu\nu} = \frac{1}{2}\hat{S}_{ab}\hat{\Omega}^{ab}_{\text{flat}} + \frac{1}{2}\hat{S}_{ab}\omega_\mu{}^{ab}u^\mu$, $\omega_\mu{}^{ab} \equiv \tilde{e}^b{}_\nu D_\mu \tilde{e}^{a\nu}$ Ricci rotation coefficients

$\Rightarrow$ New rotational variables: $\hat{\Omega}^{ab}_{\text{flat}} = \hat{\Lambda}^{Aa}\frac{d\hat{\Lambda}_A{}^b}{d\sigma}$, $\hat{S}_{ab}$

Separation of field from particle worldline DOFs is not complete

- $\hat{\Lambda}_{[0]}{}^a = w^a = \tilde{e}^a{}_\mu w^\mu$ may contain further field dependence
- $\hat{S}_{0i}$ contain further field dependence

$\Rightarrow$ Field completely disentangled from worldline DOFs only once gauge for rotational variables is fixed

Fixing the gauge of rotational variables

[**ML** & Steinhoff, JHEP 2015]

$$\hat{\Lambda}_{[0]a} = w_a, \qquad \hat{S}^{ab}\left(p_b + \sqrt{p^2}\hat{\Lambda}_{[0]b}\right) = 0$$

3 sensible gauges

1. Covariant gauge

$$\hat{\Lambda}_{[0]a} = \frac{p_a}{\sqrt{p^2}} \Rightarrow {\Lambda_{[0]}}^a = \frac{p^a}{\sqrt{p^2}}$$

2. Canonical gauge

$$\hat{\Lambda}_{[0]}{}^a = \delta^a_0 \Leftrightarrow \hat{\Lambda}_A{}^{(0)} = \delta^0_A \Rightarrow \qquad \hat{S}^{ab}\left(p_b + \sqrt{p^2}\delta_{0b}\right) = 0$$

Generalization of Pryce-Newton-Wigner SSC from flat spacetime

3. No mass dipole gauge

$$\hat{\Lambda}_{[0]}{}^a = \frac{2p_0\delta^a_0 - p^a}{\sqrt{p^2}} \Rightarrow \qquad \hat{S}_{a0} = 0$$

NRG fields and Schwinger's time gauge

Reduction over time dimension à la Kaluza-Klein

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \equiv e^{2\phi}(dt - A_i dx^i)^2 - e^{-2\phi}\gamma_{ij} dx^i dx^j$$

- $\phi, A_i, \gamma_{ij} \equiv \delta_{ij} + \sigma_{ij}$, Non Relativistic Gravitational (NRG) fields
- Advantageous, preferable e.g. over Lorentz covariant, ADM decompositions...

[w spin **ML**, PRD 2010 etc.]

NRG tetrad field gauge

[**ML** & Steinhoff JHEP 2015]

Schwinger's time gauge

$$\tilde{e}_{(i)}{}^{0}(x) = 0$$

Using internal Lorentz symmetry

$$\tilde{e}^a{}_\mu = \begin{pmatrix} e^\phi & -e^\phi A_i \\ 0 & e^{-\phi}\sqrt{\gamma}_{ij} \end{pmatrix}, \quad \sqrt{\gamma}_{ij} \text{ symmetric square root of } \gamma_{ij}$$

EOMs for the precession of spin

[**ML** & Steinhoff, JCAP 2014]

From variation of the final effective action – like those of position

$$S_{\text{eff(spin)}} = \int dt \left[-\frac{1}{2} \sum_{I=1}^{2} S_{Iij} \Omega_I^{ij} - V\left(\vec{x}_I, \dot{\vec{x}}_I, \ddot{\vec{x}}_I, \ldots, S_{Iij}, \dot{S}_{Iij}, \ldots \right) \right]$$

Lorentz matrices and spin are independent variational variables!

$$\Rightarrow \dot{S}_I^{ij} = -\, 4 S_I^{k[i} \delta^{j]l} \frac{\delta \int dt\, V}{\delta S_I^{kl}} = -4 S_I^{k[i} \delta^{j]l} \left[\frac{\partial V}{\partial S_I^{kl}} - \frac{d}{dt} \frac{\partial V}{\partial \dot{S}_I^{kl}} + \ldots \right]$$

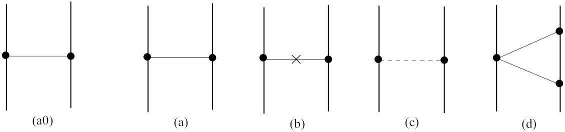
If rotational gauge is not fixed

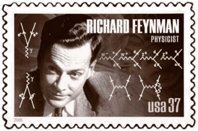
$$S_{\text{(spin)}} = \int dt \left[-\frac{1}{2} \sum_{I=1}^{2} S_{Iab} \Omega_I^{ab} - V\left(\vec{x}_I, \dot{\vec{x}}_I, \ddot{\vec{x}}_I, \ldots, S_{Iab}, \dot{S}_{Iab}, \ldots \right) \right]$$

$$\Rightarrow \dot{S}_I^{ab} = 4 S_I^{c[a} \eta^{b]d} \frac{\delta \int dt\, V}{\delta S_I^{cd}} = 4 S_I^{c[a} \eta^{b]d} \left[\frac{\partial V}{\partial S_I^{cd}} - \frac{d}{dt} \frac{\partial V}{\partial \dot{S}_I^{cd}} + \ldots \right]$$

LO sectors beyond Newtonian

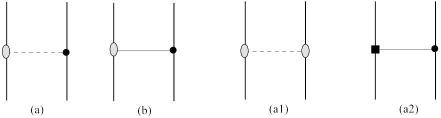
Feynman graphs of non-spinning sector to 1PN order





One-loop diagram – absent from 1PN with NRG fields

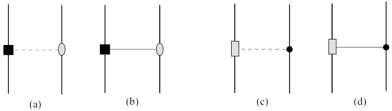
LO Feynman diagrams with spin – to quadratic in spin



LO cubic and quartic in spin sectors

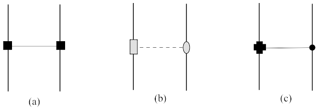
[**ML** & Steinhoff, JHEP 2014]

Feynman diagrams of LO **cubic** in spin sector



- On left pair – quadrupole-dipole, on right – octupole-monopole
- Note analogy of each pair with LO spin-orbit

Feynman diagrams of LO **quartic** in spin sector

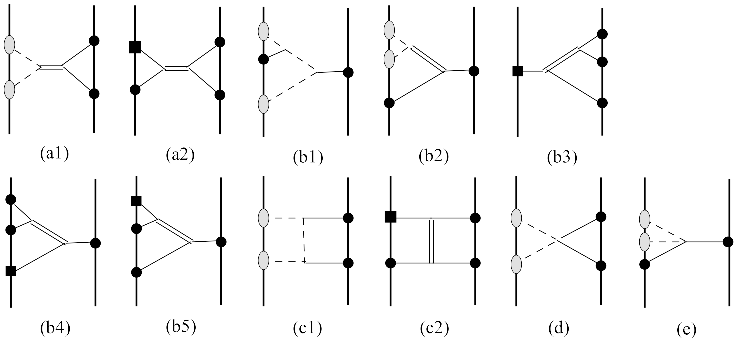


- On left and right – quadrupole-quadrupole and hexadecapole-monopole
- Each is analogous to LO spin-squared
- In middle – octupole-dipole analogous to LO spin1-spin2

NNLO spin-squared sector

[**ML** & Steinhoff, JCAP 2016]

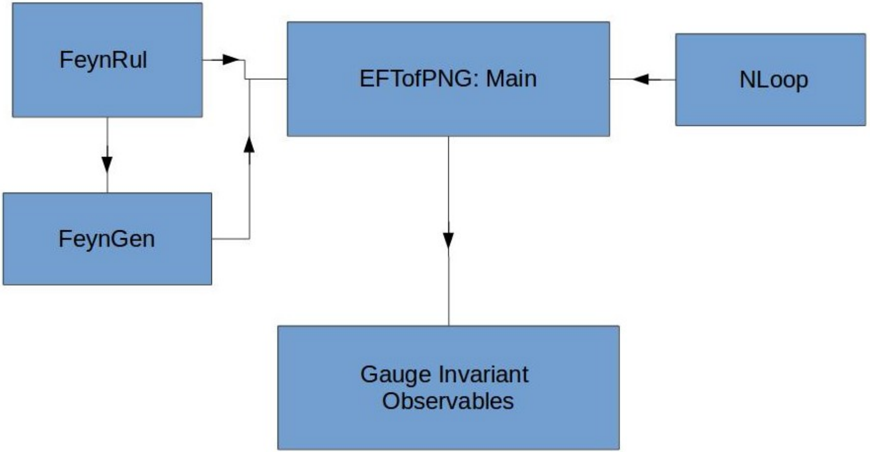
Feynman diagrams of order G^3 with two loops



- Five 2-loop topologies actually fall into 3 kinds
- H topology (graphs c1,c2) – irreducible kind – is the nasty one!

EFTofPNG package version 1.0

[**ML** & Steinhoff, arXiv:1705.06309]



Public repository URL: https://github.com/miche-levi/pncbc-eftofpng

Conclusions

EFT of PNG with Spin: Summary of formal results

[**ML** & Steinhoff, JHEP 2015; **ML**, arXiv:1705.07515]

- EFT formulation for spinning gravitating objects
- Spin-induced nonminimal coupling to all orders in spin
- EOMs and Hamiltonians straightforward to derive

EFT of PNG with Spin: Summary of applications

[**ML**, arXiv:1705.07515]

- NLO S1-S2, SO [**ML**, 2×PRD 2010], S^2 [**ML** & Steinhoff, JHEP 2015]
- NNLO S1-S2 [**ML**, PRD 2011], SO, S^2 [**ML** & Steinhoff, 2×JCAP 2016]
- LO S^3+S^4 [**ML** & Steinhoff, JHEP 2014]
- Public package with spin + observables pipeline [**ML** & Steinhoff, 2017]

⇒ Spinning & non-spinning PN accuracy – unprecedently in sync!

Prospective work

- Formal matching of spin-induced Wilson coefficients
- Radiative sector with spin: Formulation of EFT → Implementation


thank you
danke
рахмат
謝謝
ngiyabonga
teşekkür ederim
Баярлалаа
спасибо
gracias
merci
obrigado
bedankt
dziękuję
go raibh maith agat
mochchakkeram
sukriya
kop khun krap
terima kasih
grazie
arigatô
dakujem
takk
tapadh leat
hvala
mauruuru
sagolun
taiku
dank je
mersi
barka
welalin
tack
spas
kia ora
faafetai lava
vinaka
спасибі
blagodaram
misaotra
matondo
paldies
grazzi
mahalo
хвала
asante
manana
obrigada
murakoze
tenki
chokrane
kiitos
dankie
dhanyavad
nanni
nandri
bayarlalaa
gracie
koszonom
enkosi
sobodi
dėkuji
mėsi
didi madloba
najis tuke
kam sah hamnida
rahmat
chnorakaloutioun
gratias ago
gracies
sulpay
akun
dankon
aciū
djiere dieuf
tau
дякую
mamnun
trugarez
ありがとう
감사합니다
tanemirt
rahmet
xiexie
ευχαριστώ
diolch
dhanyavadagalu
shukriya
merce
мерси