CERN TH institute "Probing the dark sector and general relativity at all scales"
CERN 14 - 25 August 2017

Testing Fundamental Gravitation in Space:

Brief History, Recent Progress and Possible Future Directions

Slava G. Turyshev

Jet Propulsion Laboratory, California Institute of Technology
4800 Oak Grove Drive, Pasadena, CA 91009 USA

Outline: Solar System Tests of Gravity

The talk will cover:

- Theoretical Landscape in the 20th Century:
  - (brief…) History of the tests of general relativity
  - Frameworks used: the PPN formalism and Robertson-Mansouri-Sexl
  - Recent progress in the tests of general relativity
- Beginning of the 21st Century…:
  - Motivations for high-precision tests of gravity
  - What to expect in the near future? and some proposed experiments

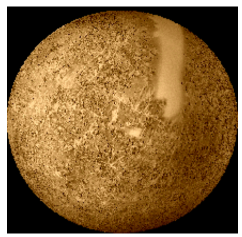
- Main objective:
  - Remind where we came from and what lessons we learned

- Themes for discussion:
  - Are the solar system tests still useful?
  - Is there a discovery potential? Or what is the importance of new improved limits?
  - What tests are most valuable?

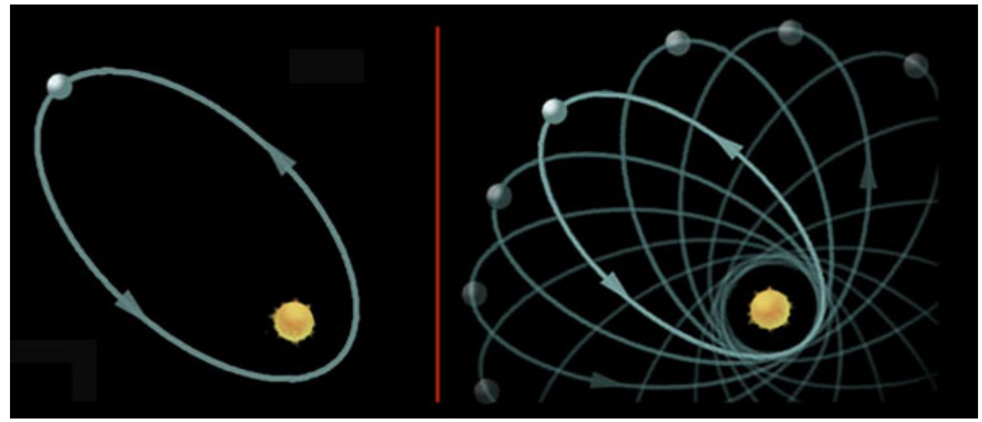
Triumph of Mathematical Astronomy in 19th Century

Discovery of Neptune: 1845

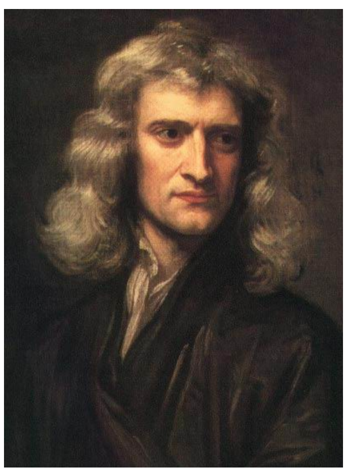
Urbain LeVerrier
(1811-1877)



- 1845: the search for Planet-X:
  - Anomaly in the Uranus' orbit → Neptune
  - Anomalous motion of Mercury → Vulcan



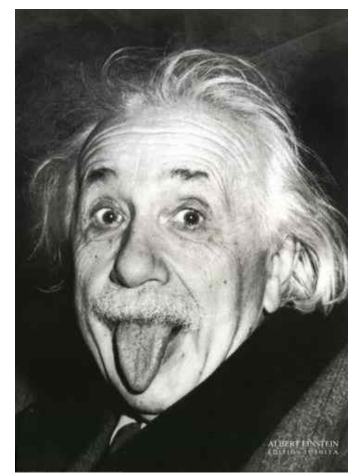
Newtonian Gravity General Relativity



Sir Isaac Newton
(1643-1727)

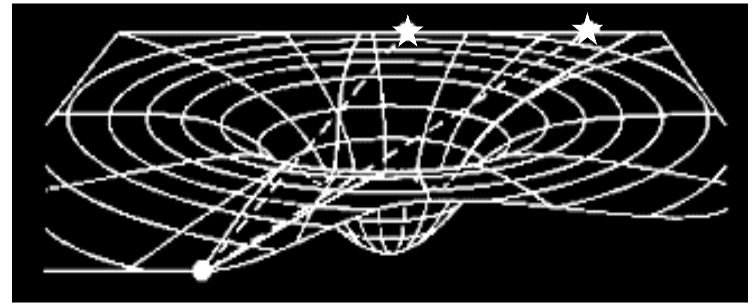
- Anomalous precession of Mercury's perihelion :
  - 43 arcsec/cy can not be explained by Newton's gravity
- Before publishing GR, in 1915, Einstein computed the expected perihelion precession of Mercury
  - When he got out 43 arcsec/cy – a new era just began!!

Almost in one year LeVerrier both confirmed the Newton's theory (Neptune) & cast doubt on it (Mercury's' anomaly).



Albert Einstein
(1879-1955)

The First Test of General Theory of Relativity



Gravitational Deflection of Light:

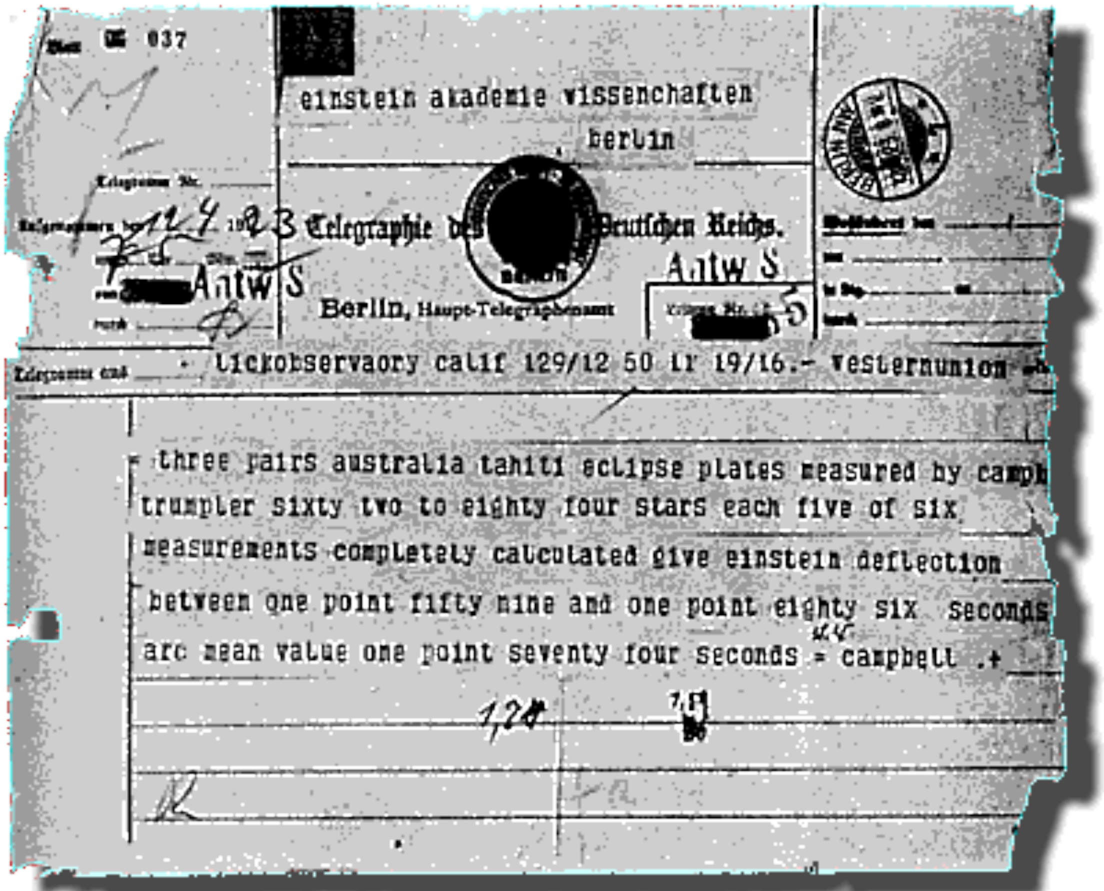
$$\theta_{gr}(b) = \frac{2(1+\gamma)GM_{\odot}}{b\,c^2} \simeq 8 \times 10^{-6}\left(\frac{1+\gamma}{2}\right)\left(\frac{R_{\odot}}{b}\right)$$

Solar Eclipse 1919: possible outcomes

Deflection = 0;

Newton = 0.87 arcsec;

Einstein = 2 x Newton = 1.75 arcsec



einstein akademie vissenchaften
berlin

Telegraphie des Deutschen Reichs.

[redacted] Antw S

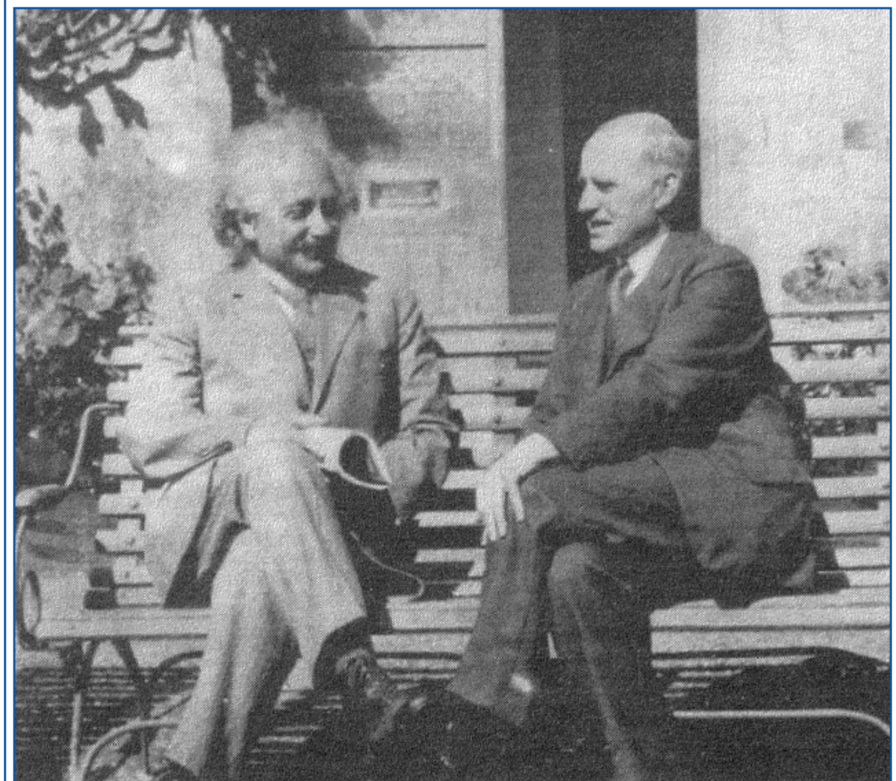
Berlin, Haupt-Telegraphenamt

[redacted]

lickobservaory calif 129/12 50 11 19/16.- vesternunion

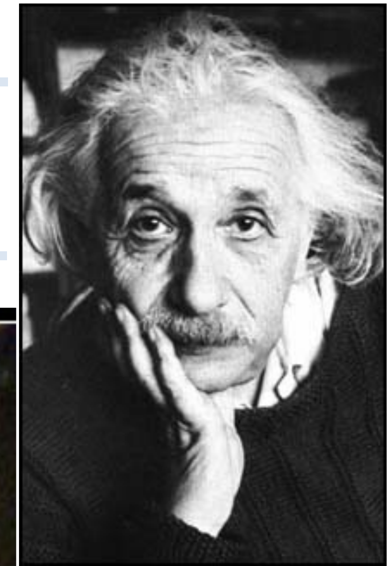
three pairs australia tahiti eclipse plates measured by campb
trumpler sixty two to eighty four stars each five of six
measurements completely calculated give einstein deflection
between one point fifty nine and one point eighty six seconds
arc mean value one point seventy four seconds = campbell .+

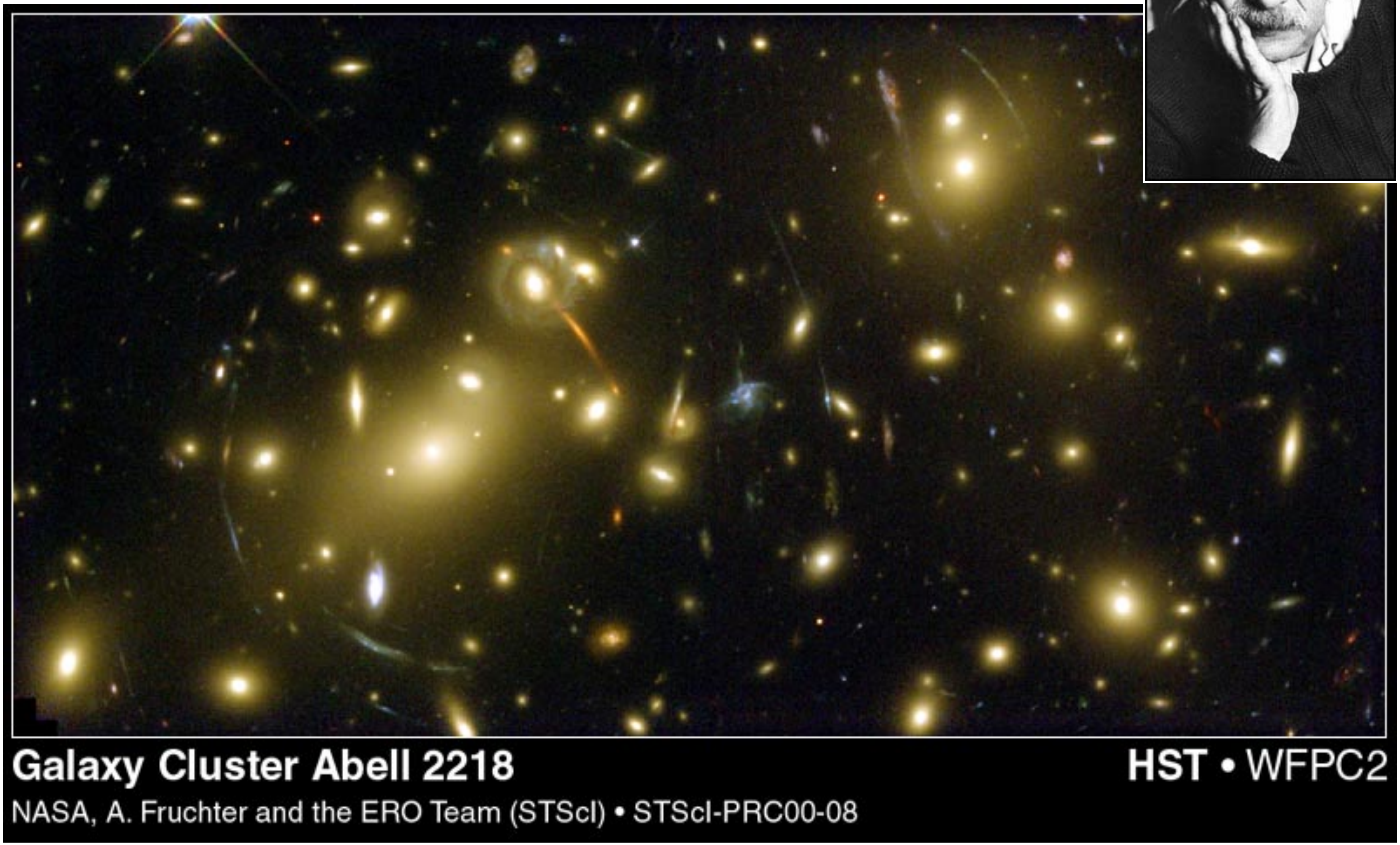
Campbell's telegram to Einstein, 1923



Einstein and Eddington, Cambridge, 1930

Gravitational Deflection of Light is a Well-Known Effect Today





Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

not a complete list…

Newton 1686 | Poincaré 1890

Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | Einstein 1915

Whitehead 1922 | Cartan 1923 | Kaluza & Klein 1932 | Fierz & Pauli 1939 | Birkhoff 1943

Milne 1948 | Thiry 1948 | Papapetrou 1954 | Jordan 1955 | Littlewood & Bergmann 1956

Brans & Dicke 1961 | Yilmaz 1962 | Whitrow & Morduch 1965 | Kustaanheimo & Nuotio 1967

Page & Tupper 1968 | Bergmann 1968 | Deser & Laurent 1968 | Nordtvedt 1970 | Wagoner 1970

Bollini et al. 1970 | Rosen 1971 | Will & Nordtvedt 1972 | Ni 1972 | Hellings & Nordtvedt 1972

Ni 1973 | Yilmaz 1973 | Lightman & Lee 1973 | Lee, Lightman & Ni 1974 | Rosen 1975

Belinfante & Swihart 1975 | Lee et al. 1976 | Bekenstein 1977 | Barker 1978 | Rastall 1979

Coleman 1983 | Hehl 1997 | Overlooked (20th century)

- Some authors proposed more than one theory, e.g. Einstein, Ni, Lee, Nordtvedt, Papapetrou, Yilmaz, etc.
- Some theories were variations of others
- Some were proposed in the 1910s/20s; many theories were in the 1960s/70s
- Overlooked: this is not a complete list!

Theory must be:

- **Complete**: not a law, but a theory. Derive experimental results from first principles
- **Self-consistent**: get same results no matter which mathematics or models are used
- **Relativistic**: Non-gravitational laws are those of Special Relativity
- **Newtonian**: Reduces to Newton's equation in the limit of low gravity and low velocities

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that fail already

Newton 1686	Poincaré 1890			
Einstein 1912	Nordstrøm 1912	Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915
Whitehead 1922	Cartan 1923	Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943
Milne 1948	Thiry 1948	Papapetrou 1954	Jordan 1955	Littlewood & Bergmann 1956
Brans & Dicke 1961	Yilmaz 1962	Whitrow & Morduch 1965	Kustaanheimo & Nuotio 1967	
Page & Tupper 1968	Bergmann 1968	Deser & Laurent 1968	Nordtvedt 1970	Wagoner 1970
Bollini et al. 1970	Rosen 1971	Will & Nordtvedt 1972	Ni 1972	Hellings & Nordtvedt 1972
Ni 1973	Yilmaz 1973	Lightman & Lee 1973	Lee, Lightman & Ni 1974	Rosen 1975
Belinfante & Swihart 1975	Lee et al. 1976	Bekenstein 1977	Barker 1978	Rastall 1979
Coleman 1983	Hehl 1997	Overlooked (20th century)		

- Newton (1686) – non-relativistic: implicit action at a distance - incompatible with special relativity
- Poincare (1890) and conformally flat theory of Whithrow-Morduch (1965) - incomplete: do not mesh well with non-gravitational physics (i.e., with electromagnetism of Maxwell)
- Fierz & Pauli (1939) ["spin-2 field theory"] – was inconsistent: field equations ⇒ all gravitating bodies move along straight lines, equation of motion ⇒ gravity deflects bodies
- Birkhoff (1943) – not Newtonian: demands *speed of sound = speed of light*.
- Milne (1948) – incomplete - no gravitational red-shift prediction
- Kustaanheimo-Nuotio (1967) – inconsistent: grav. redshift for photons, but not for light waves.

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that violate the Einstein's Equivalence Principle

Newton 1686 · Poincaré 1890

Einstein 1912 · Nordstrøm 1912 · Nordstrøm 1913 · Einstein & Fokker 1914 · Einstein 1915

Whitehead 1922 · Cartan 1923 · Kaluza & Klein 1932 · Fierz & Pauli 1939 · Birkhoff 1943

Milne 1948 · Thiry 1948 · Papapetrou 1954 · Jordan 1955 · Littlewood & Bergmann 1956

Brans & Dicke 1961 · Yilmaz 1962 · Whitrow & Morduch 1965 · Kustaanheimo & Nuotio 1967

Page & Tupper 1968 · Bergmann 1968 · Deser & Laurent 1968 · Nordtvedt 1970 · Wagoner 1970

Bollini et al. 1970 · Rosen 1971 · Will & Nordtvedt 1972 · Ni 1972 · Hellings & Nordtvedt 1972

Ni 1973 · Yilmaz 1973 · Lightman & Lee 1973 · Lee, Lightman & Ni 1974 · Rosen 1975

Belinfante & Swihart 1975 · Lee et al. 1976 · Bekenstein 1977 · Barker 1978 · Rastall 1979

Coleman 1983 · Hehl 1997 · Overlooked (20th century)

Einstein's Equivalence Principle (EEP):

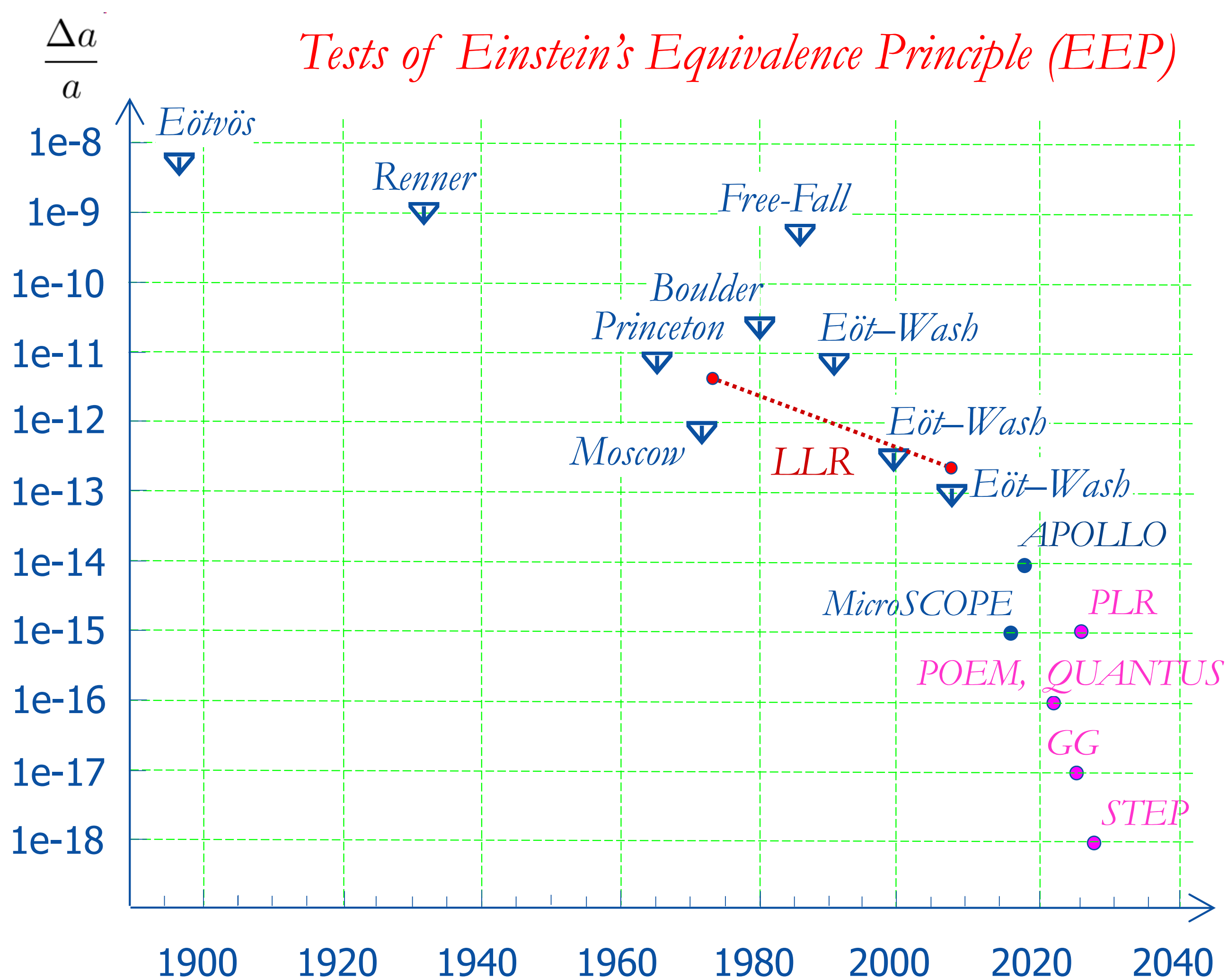
- Uniqueness of the Free Fall
- Local Lorentz Invariance
- Local Position Invariance

Only metric theories are viable:

- Belinfante & Swihart (1975): not a metric theory
- Kaluza-Klein (1932): violates EEP
- Still too many theories around…

Empirical Foundations of General Relativity:

Confrontation Between the Theory and Experiment



- ● funded projects
- ● proposed projects
- *LLR*, *APOLLO*, and *PLR* are testing the Strong Equivalence Principle (SEP)

$$\left[\frac{m_G}{m_I}\right]_{\rm SEP} = 1+\eta\left(\frac{\Omega}{mc^2}\right)$$

Uniqueness of the Free Fall

(⇒ Weak Equivalence Principle):

$$\vec{F} = m_I\vec{a} = m_G\vec{g}$$
$$\Rightarrow m_I = m_G$$

All bodies fall with the same acceleration

Define the test parameter that signifies a violation of the **WEP**

$$\frac{\Delta a}{a} = \frac{(a_1-a_2)}{\frac{1}{2}(a_1+a_2)} = \left[\frac{m_G}{m_I}\right]_1 - \left[\frac{m_G}{m_I}\right]_2$$

Let Ω is the gravitational binding energy of a test body, then the test parameter that signifies a violation of the **SEP** is

$$\frac{\Delta a}{a} = (4\beta-\gamma-3)\left\{\left[\frac{\Omega}{mc^2}\right]_1 - \left[\frac{\Omega}{mc^2}\right]_2\right\}$$

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

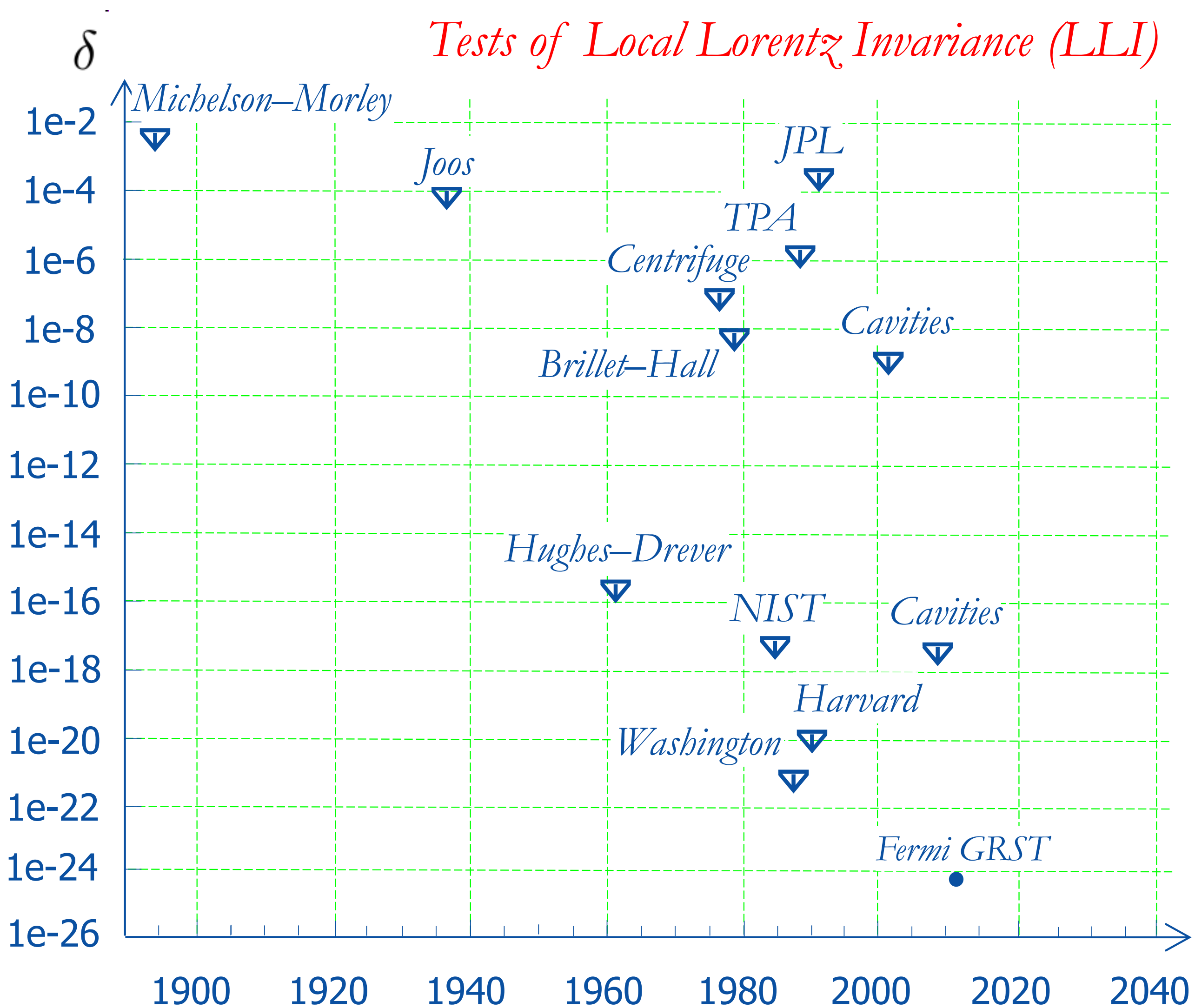
Theories that violate Local Lorentz Invariance (LLI)

Newton 1686	Poincaré 1890			
Einstein 1912	Nordstrøm 1912	Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915
Whitehead 1922	Cartan 1923	Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943
Milne 1948	Thiry 1948	Papapetrou 1954	Jordan 1955	Littlewood & Bergmann 1956
Brans & Dicke 1961	Yilmaz 1962	Whitrow & Morduch 1965	Kustaanheimo & Nuotio 1967	
Page & Tupper 1968	Bergmann 1968	Deser & Laurent 1968	Nordtvedt 1970	Wagoner 1970
Bollini et al. 1970	Rosen 1971	Will & Nordtvedt 1972	Ni 1972	Hellings & Nordtvedt 1972
Ni 1973	Yilmaz 1973	Lightman & Lee 1973	Lee, Lightman & Ni 1974	Rosen 1975
Belinfante & Swihart 1975	Lee et al. 1976	Bekenstein 1977	Barker 1978	Rastall 1979
Coleman 1983	Hehl 1997	Overlooked (20th century)		

Quasi-linear theories:

- Deser & Laurent (1968), Bollini, Giambiagi & Tiomno (1970) both predict existence of a preferred reference frame (i.e., ξ=1)
- Whitehead (1922) predicts time-dependence for ocean tides in violation of everyday experience

Empirical Foundations of General Relativity:
Confrontation Between the Theory and Experiment



– Michelson-Morley, Joos, Brillet-Hall: round-trip propagation
– Centrifuge, TPA, JPL: one-way signal propagation
– The rest are the Hughes-Drever experiments

$$\delta \equiv \frac{c}{c_0} - 1$$

Local Lorentz Invariance:

- The outcome of a (small-scale) experiment does not depend on the orientation and the velocity of the (inertial) laboratory.
- Frameworks by Kostelecky et al., Jacobson et al.

Future experiments:

- Clock comparisons
- Clocks vs microwave cavities
- Time of flight of high energy photons
- Birefringence in vacuum
- Neutrino oscillations
- Threshold effects in particle physics

Test of one-way speed of light:

- Important to fundamental physics, cosmology, astronomy and astrophysics

Theoretical Landscape of the 21st Century:

Special Relativity

Laboratory tests of Lorentz Invariance: search for preferred-frame effects

frame1 : $S(T, X)$ — e.g. CMB — $v_{sol} \approx 377\ \mathrm{km/s}$

frame2 : $s(t, x)$ — laboratory — $RA, dec = (11.2, -6.4^{\circ})$

Mansouri & Sexl, 1977

$$dT = \frac{1}{a}\left(dt + \frac{v}{c^2}dx\right) \qquad a = 1 + \alpha\frac{v^2}{c^2} + \mathcal{O}(c^{-4}) \quad \text{time dilation}$$

$$dX = \frac{1}{b}dx + \frac{v}{a}\left(dt + \frac{v}{c^2}dx\right) \qquad b = 1 + \beta\frac{v^2}{c^2} + \mathcal{O}(c^{-4}) \quad \text{length} \parallel v$$

$$dY = \frac{1}{d}dy,\ dZ = \frac{1}{d}dz \qquad d = 1 + \delta\frac{v^2}{c^2} + \mathcal{O}(c^{-4}) \quad \text{length} \perp v$$

Deviations from the 2-way (round-trip) speed of light:

$$\frac{c}{c'} \sim 1 + \left(\beta - \delta - \frac{1}{2}\right)\frac{v^2}{c^2}\sin^2\theta + (\alpha - \beta + 1)\frac{v^2}{c^2}$$

SR: $\alpha = -1/2,\ \beta = 1/2,\ \delta = 0$

Clock comparison experiments:

$$P_{MM} = (\tfrac{1}{2} - \beta + \delta)$$ Michelson-Morley: orientation dependence

$$P_{KT} = (\beta - \alpha - 1)$$ Kennedy-Thorndike: velocity dependence

$$P_{IS} = |\alpha + \tfrac{1}{2}|$$ Ives-Stillwell: contraction, dilation

Precision tests of Lorentz Invariance:

$$P_{MM} = (-4 \pm 8) \times 10^{-12}$$ Herrmann et al, PRD 80 (2009) 105011

$$P_{KT} = (4.8 \pm 3.7) \times 10^{-8}$$ Toobar et al, PRD 81 (2010) 022003

$$P_{IS} \leq 8.4 \times 10^{-8}$$ Reinhardt et al, Nature Physics 3 (2007) 861

Tests of isotropy of the speed of light:

$$\Delta c_\theta / c \lesssim 1 \times 10^{-17}$$ Herrmann et al, PRD 80 (2009) 105011

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that violate Local Position Invariance (LPI)

Newton 1686 | Poincaré 1890

Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | Einstein 1915

Whitehead 1922 | Cartan 1923 | Kaluza & Klein 1932 | Fierz & Pauli 1939 | Birkhoff 1943

Milne 1948 | Thiry 1948 | Papapetrou 1954 | Jordan 1955 | Littlewood & Bergmann 1956

Brans & Dicke 1961 | Yilmaz 1962 | Whitrow & Morduch 1965 | Kustaanheimo & Nuotio 1967

Page & Tupper 1968 | Bergmann 1968 | Deser & Laurent 1968 | Nordtvedt 1970 | Wagoner 1970

Bollini et al. 1970 | Rosen 1971 | Will & Nordtvedt 1972 | Ni 1972 | Hellings & Nordtvedt 1972

Ni 1973 | Yilmaz 1973 | Lightman & Lee 1973 | Lee, Lightman & Ni 1974 | Rosen 1975

Belinfante & Swihart 1975 | Lee et al. 1976 | Bekenstein 1977 | Barker 1978 | Rastall 1979

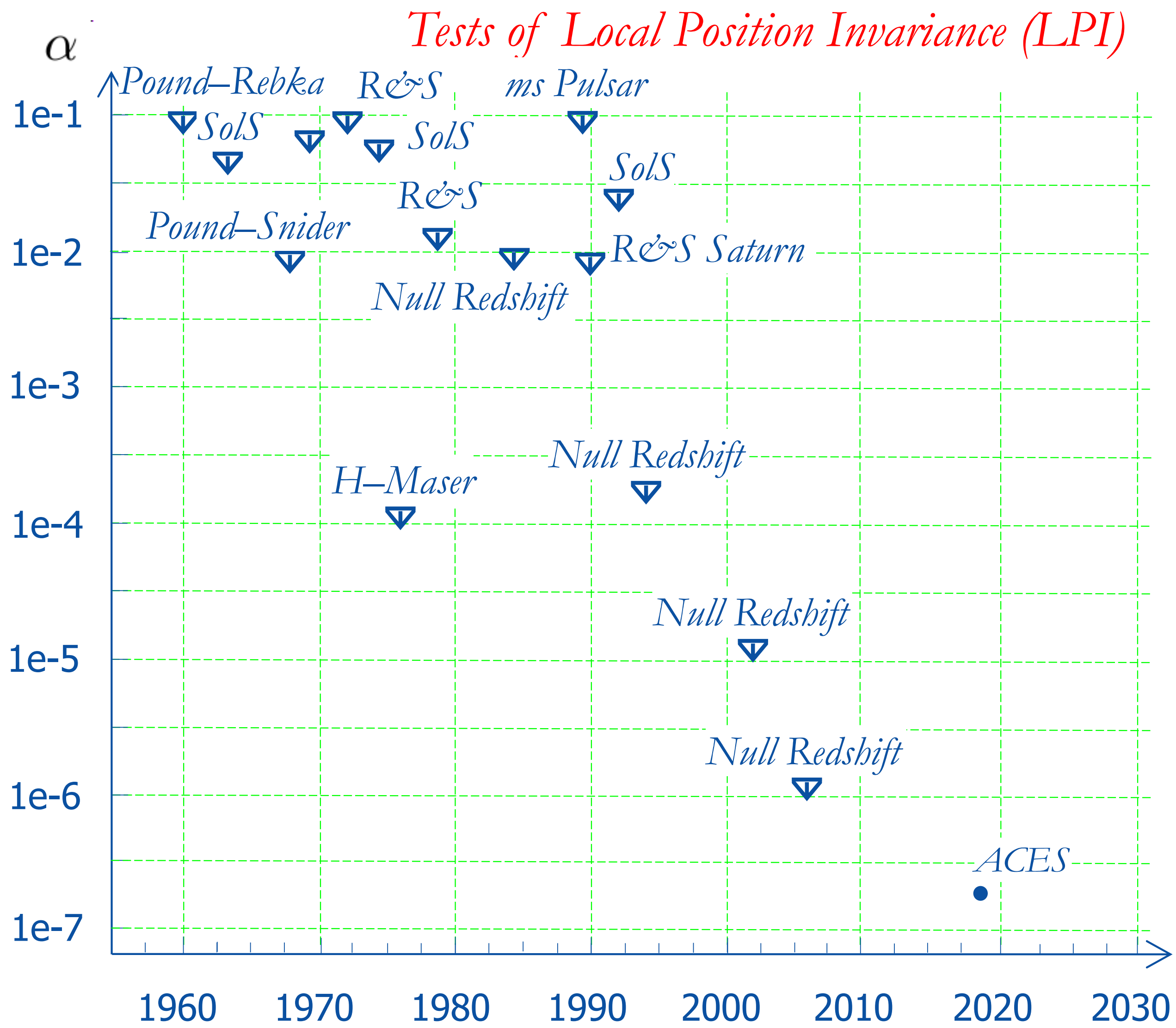
Coleman 1983 | Hehl 1997 | Overlooked (20th century)

Stratified theories with time-orthogonal time slices all predict $\xi \neq 0$:

- Einstein (1912), Papapetrou (1954) (actually two theories)
- Yilmaz (1962), Whithrow & Morduch (1965)
- Page & Tupper (1968), Rosen (1971)
- Ni (1972), Coleman (1983)

Empirical Foundations of General Relativity:

Confrontation Between the Theory and Experiment



– SolS: Solar Spectra; – R&S: Rockets and Spacecraft

– Null Redshift: comparison of different atomic clocks

Gravitational redshift:

$$\frac{\Delta\nu}{\nu} = (1+\alpha)\frac{\Delta U}{c^2}$$

Local Position Invariance:

- The outcome of any local non-gravitational experiment is independent of where & when in the universe it is performed

Splits into:

- spatial invariance
- temporal invariance
- Current best result is by Ashby et al., Phys. Rev. Lett. 98, 070802 (2007)

$$|\alpha| < 1.4 \times 10^{-6}$$

- A BEC test was attempted by Müller, Peters, and Chu, Nature 463, 926 (2010).

General Theory of Relativity and its Alternatives…

$$\mathcal{S}_{\rm G}[g_{mn}] = \frac{c^4}{16\pi G_{\rm N}}\int d^4x\sqrt{-g}R$$

Action of general relativity

$$R = g^{mn}R_{mn}$$

Ricci scalar, Ricci tensor & Christoffel symbols

$$R_{mn} = \partial_k\Gamma^k_{mn} - \partial_m\Gamma^k_{nk} + \Gamma^k_{mn}\Gamma^l_{kl} - \Gamma^k_{ml}\Gamma^l_{nk}$$

$$\Gamma^k_{mn} = \frac{1}{2}g^{kp}(\partial_m g_{pn} + \partial_n g_{pm} - \partial_p g_{mn})$$

$$\mathcal{S}_{\rm SM}[\psi, A_m, H; g_{mn}] = \int d^4x\Big[-\frac{1}{4}\sum\sqrt{-g}g^{mk}g^{nl}F^{\rm a}_{mn}F^{\rm a}_{kl} - \sum\sqrt{-g}\bar\psi\gamma^m D_m\psi$$
$$-\frac{1}{2}\sqrt{-g}g^{mn}\overline{D_m H}D_n H - \sqrt{-g}V(H) - \sum\lambda\sqrt{-g}\bar\psi H\psi - \sqrt{-g}\rho_{\rm vac}\Big]$$

Action of Standard Model

Variational principle: $\frac{\delta}{\delta g_{mn}} \otimes \Big[\mathcal{S}_{\rm tot}[\psi, A_m, H; g_{mn}] = \mathcal{S}_{\rm G}[g_{mn}] + \mathcal{S}_{\rm SM}[\psi, A_m, H; g_{mn}]\Big] \Rightarrow$

$$R_{mn} - \frac{1}{2}g_{mn}R + \Lambda g_{mn} = \frac{8\pi G_{\rm N}}{c^4}T_{mn} \qquad \Lambda = 8\pi G_{\rm N}\rho_{\rm vac}/c^4 \qquad \rho_{\rm vac} \approx (2.3\times 10^{-3}eV)^4$$

$$S = \frac{c^3}{4\pi G}\int d^4x\sqrt{-g}\left[\frac{1}{4}f(\varphi)R - \frac{1}{2}g(\varphi)\partial_\mu\varphi\partial^\mu\varphi + V(\varphi)\right] + \sum_i q_i(\varphi)\mathcal{L}_i$$

Scalar-Tensor theories of gravity

$$f(\varphi) = \varphi, \qquad g(\varphi) = \frac{\omega}{\varphi}, \qquad V(\varphi) = 0.$$

Brans and Dicke (1961)

Parameterized Post-Newtonian (PPN) formalism

PPN Formalism: Eddington, Fock, Chandrasekhar, Dicke, Nordtvedt, Thorne, Will,…

$$
\begin{aligned}
g_{00} &= 1 - \frac{2}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}} + \frac{2\beta}{c^4}\Big[\sum_{j\neq i}\frac{\mu_j}{r_{ij}}\Big]^2 - \frac{1+2\gamma}{c^4}\sum_{j\neq i}\frac{\mu_j \dot{r}_j^2}{r_{ij}} + \\
&\quad + \frac{2(2\beta-1)}{c^4}\sum_{j\neq i}\frac{\mu_j}{r_{ij}}\sum_{k\neq j}\frac{\mu_k}{r_{jk}} - \frac{1}{c^4}\sum_{j\neq i}\mu_j\frac{\partial^2 r_{ij}}{\partial t^2} + \mathcal{O}(c^{-5}) \\
g_{0\alpha} &= \frac{2(1+\gamma)}{c^3}\sum_{j\neq i}\frac{\mu_j \dot{\mathbf{r}}_j^\alpha}{r_{ij}} + \mathcal{O}(c^{-5}) \\
g_{\alpha\beta} &= -\delta_{\alpha\beta}\Big(1 + \frac{2\gamma}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}} + \frac{3\delta}{2c^4}\Big[\sum_{j\neq i}\frac{\mu_j}{r_{ij}}\Big]^2\Big) + \mathcal{O}(c^{-5})
\end{aligned}
$$

- Assumption: Local Lorentz Invariance (LLI) and local position invariance (LPI) hold, thus, preferred frame parameters α_1, α_2, α_3 are not included…
- General case, there are 10 PPN parameters: $\gamma, \beta, \zeta, \alpha_1, \alpha_2, \alpha_3, \xi_1, \xi_2, \xi_3, \xi_4$
- γ are β the Eddington's parameterized post-Newtonian (PPN) parameters:

  General relativity: $\gamma = \beta = 1$ Brans-Dicke theory: $\gamma = \frac{1+\omega}{2+\omega}$, $\beta = 1$

- δ is the post-PPN parameter – important for next generation of light propagation tests.

C. M. Will, Living Rev. Relat. **9** (2006), gr-qc/0510072

PPN Equations of Motion (a part of the model)

$$
\begin{aligned}
\ddot{\mathbf{r}}_i \;=\; & \sum_{j\neq i} \frac{Gm_j(\mathbf{r}_j-\mathbf{r}_i)}{r_{ij}^3}\Big\{ \Big[\frac{m_G}{m_I}\Big]_i - \frac{2(\beta+\gamma)}{c^2}\sum_{l\neq i}\frac{Gm_l}{r_{il}} - \frac{2\beta-1}{c^2}\sum_{k\neq j}\frac{Gm_k}{r_{jk}} + \\
& + \gamma\Big(\frac{\dot{r}_i}{c}\Big)^2 + (1+\gamma)\Big(\frac{\dot{r}_j}{c}\Big)^2 - \frac{2(1+\gamma)}{c^2}\dot{\mathbf{r}}_i\dot{\mathbf{r}}_j + \frac{\dot{G}\cdot t}{G} - \\
& - \frac{3}{2c^2}\Big[\frac{(\mathbf{r}_i-\mathbf{r}_j)\dot{\mathbf{r}}_j}{r_{ij}}\Big]^2 + \frac{1}{2c^2}(\mathbf{r}_j-\mathbf{r}_i)\ddot{\mathbf{r}}_j\Big\} + \\
& + \frac{1}{c^2}\sum_{j\neq i}\frac{Gm_j}{r_{ij}^3}\Big\{\Big[\mathbf{r}_i-\mathbf{r}_j\Big]\cdot\Big[(2+2\gamma)\dot{\mathbf{r}}_i-(1+2\gamma)\dot{\mathbf{r}}_j\Big]\Big\}(\dot{\mathbf{r}}_i-\dot{\mathbf{r}}_j) + \\
& + \frac{3+4\gamma}{2c^2}\sum_{j\neq i}\frac{Gm_j\ddot{\mathbf{r}}_j}{r_{ij}} + \sum_{m=1}^{3}\frac{Gm_m(\mathbf{r}_m-\mathbf{r}_i)}{r_{im}^3} + \sum_{c,s,m}\mathbf{F}_{\text{asteroids}}
\end{aligned}
$$

Possible EP violation

Possible temporal dependence of *G*

$$\Big[\frac{m_G}{m_I}\Big]_{\text{SEP}} = 1 + \eta\Big(\frac{\Omega}{mc^2}\Big)$$

$$\eta = 4\beta - \gamma - 3$$

$$\Omega_i = -\frac{G}{2}\int_i d^3x\rho_i U_i = -\frac{G}{2}\int_i d^3x d^3x' \frac{\rho_i(\mathbf{r})\rho_i(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

- In general theory of relativity $\beta = \gamma = 1$, thus $\eta = 0$ (this is not the case for scalar-tensor theories of gravity, for instance, where these parameters can have different values).

$$t_2 - t_1 = \frac{r_{12}}{c} + (1+\gamma)\sum_i \frac{\mu_i}{c^3}\ln\left[\frac{r_1^i + r_2^i + r_{12}^i + \frac{(1+\gamma)\mu_i}{c^2}}{r_1^i + r_2^i - r_{12}^i + \frac{(1+\gamma)\mu_i}{c^2}}\right] + \mathcal{O}(c^{-5})$$

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Cassini (2003): $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$

Theories that predict $\gamma = 0$ or $\gamma = 1$ fail

Newton 1686	Poincaré 1890			
Einstein 1912	Nordstrøm 1912	Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915
Whitehead 1922	Cartan 1923	Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943
Milne 1948	Thiry 1948	Papapetrou 1954	Jordan 1955	Littlewood & Bergmann 1956
Brans & Dicke 1961	Yilmaz 1962	Whitrow & Morduch 1965	Kustaanheimo & Nuotio 1967	
Page & Tupper 1968	Bergmann 1968	Deser & Laurent 1968	Nordtvedt 1970	Wagoner 1970
Bollini et al. 1970	Rosen 1971	Will & Nordtvedt 1972	Ni 1972	Hellings & Nordtvedt 1972
Ni 1973	Yilmaz 1973	Lightman & Lee 1973	Lee, Lightman & Ni 1974	Rosen 1975
Belinfante & Swihart 1975	Lee et al. 1976	Bekenstein 1977	Barker 1978	Rastall 1979
Coleman 1983	Hehl 1997	Overlooked (20th century)		

The Parameterized Post-Newtonian Formalism (PPN):

- Solar system is the main arena to test weak gravity:
- Expand the metrics; identify various potentials
- They have 10 PPN parameters in front

$$\gamma, \beta, \zeta, \alpha_1, \alpha_2, \alpha_3, \xi_1, \xi_2, \xi_3, \xi_4$$

- Calculate those parameters & Compare with experiments

[**2017**: A need for a Cosmological "PPN formalism"?]

Conformally-flat theories fail test of time delay and deflection of light:

- Nordstrom (1912)
- Nordstrom (1913)
- Einstein & Fokker (1914)
- Littlewood & Bergmann (1956)
- Ni (1972)

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Unlikely Scalar-Tensor Theories

Newton 1686 | Poincaré 1890

Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | Einstein 1915

Whitehead 1922 | Cartan 1923 | Kaluza & Klein 1932 | Fierz & Pauli 1939 | Birkhoff 1943

Milne 1948 | Thiry 1948 | Papapetrou 1954 | Jordan 1955 | Littlewood & Bergmann 1956

Brans & Dicke 1961 | Yilmaz 1962 | Whitrow & Morduch 1965 | Kustaanheimo & Nuotio 1967

Page & Tupper 1968 | Bergmann 1968 | Deser & Laurent 1968 | Nordtvedt 1970 | Wagoner 1970

Bollini et al. 1970 | Rosen 1971 | Will & Nordtvedt 1972 | Ni 1972 | Hellings & Nordtvedt 1972

Ni 1973 | Yilmaz 1973 | Lightman & Lee 1973 | Lee, Lightman & Ni 1974 | Rosen 1975

Belinfante & Swihart 1975 | Lee et al. 1976 | Bekenstein 1977 | Barker 1978 | Rastall 1979

Coleman 1983 | Hehl 1997 | Overlooked (20th century)

Scalar-Tensor theories are extremely constrained by Viking (1976) and Cassini (2003) results on γ:

– Thiry (1948), Jordan 1955
– Brans & Dicke (1961): ω >6500 (Viking, 1976), ω >40,000 (Cassini, 2003)
– Bergmann (1968), Nordtvedt (1970)
– Wagoner (1970), Bekenstein (1977)
– Barker (1978)

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Mercury's Perihelion: Theories that fail

Newton 1686 | Poincaré 1890

Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | **Einstein 1915**

Whitehead 1922 | **Cartan 1923** | Kaluza & Klein 1932 | Fierz & Pauli 1939 | Birkhoff 1943

Milne 1948 | Thiry 1948 | Papapetrou 1954 | Jordan 1955 | Littlewood & Bergmann 1956

Brans & Dicke 1961 | Yilmaz 1962 | Whitrow & Morduch 1965 | Kustaanheimo & Nuotio 1967

Page & Tupper 1968 | Bergmann 1968 | Deser & Laurent 1968 | Nordtvedt 1970 | Wagoner 1970

Bollini et al. 1970 | Rosen 1971 | **Will & Nordtvedt 1972** | Ni 1972 | **Hellings & Nordtvedt 1972**

Ni 1973 | **Yilmaz 1973** | **Lightman & Lee 1973** | **Lee, Lightman & Ni 1974** | **Rosen 1975**

Belinfante & Swihart 1975 | **Lee et al. 1976** | Bekenstein 1977 | Barker 1978 | **Rastall 1979**

Coleman 1983 | **Hehl 1997** | **Overlooked (20th century)**

Stratified theories predict preferred frame effects on perihelion shift:

- Ni (1973)
- Lee, Lightman & Ni (1974)

$$\dot{\pi} = (2 + 2\gamma - \beta)\frac{GM_\odot n_M}{c^2 a_M (1 - e_M^2)} + \frac{3}{4}\left(\frac{R_\odot}{a_M}\right)^2 \frac{J_{2\odot} n_M}{(1 - e_M^2)^2}(3\cos^2 i_M - 1), \quad ''/\text{cy}$$

$$\dot{\pi} = 42''.98\left[\frac{1}{3}(2 + 2\gamma - \beta) + 0.296 \cdot J_{2\odot} \times 10^4\right], \quad ''/\text{cy}$$

The value $J_{2\odot} = (2.02 \pm 01) \times 10^{-7}$ was obtained from spacecraft (Konopliv et al., 2010)

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

GW & Binary Pulsar: Theories that fail

Newton 1686 | Poincaré 1890

Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | **Einstein 1915**

Whitehead 1922 | **Cartan 1923** | Kaluza & Klein 1932 | Fierz & Pauli 1939 | Birkhoff 1943

Milne 1948 | Thiry 1948 | Papapetrou 1954 | Jordan 1955 | Littlewood & Bergmann 1956

Brans & Dicke 1961 | Yilmaz 1962 | Whitrow & Morduch 1965 | Kustaanheimo & Nuotio 1967

Page & Tupper 1968 | Bergmann 1968 | Deser & Laurent 1968 | Nordtvedt 1970 | Wagoner 1970

Bollini et al. 1970 | Rosen 1971 | **Will & Nordtvedt 1972** | Ni 1972 | **Hellings & Nordtvedt 1972**

Ni 1973 | **Yilmaz 1973** | **Lightman & Lee 1973** | Lee, Lightman & Ni 1974 | **Rosen 1975**

Belinfante & Swihart 1975 | **Lee et al. 1976** | Bekenstein 1977 | Barker 1978 | **Rastall 1979**

Coleman 1983 | **Hehl 1997** | **Overlooked (20th century)**

Bi-metric Theories predict a dipole radiation. This can't be…:

- Rosen (1975)
- Lee et al. (1976)
- Rastall (1979)
- Lightman & Lee (1973)

List of PPN Parameters for Competing Theories

Competing theories of Gravity	γ	β	ξ	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
Einstein (1915) GR	1	1	0	0	0	0	0	0	0	0
Scalar Field theories – Note: in Page-Tupper (1968): parameter *d* is defined as $\Delta = 1-\gamma$										
Einstein (1912) [not GR]	0	0	–	–4	0	–2	0	–1	0	0*
Whitrow-Morduch (1965)	0	–1	–	–4	0	0	0	–3	0	0*
Rosen (1971)	λ	$\frac{3}{4}+\frac{\lambda}{4}$	–	$-4(1-\lambda)$	0	–4	0	–1	0	0
Papapetrou (1954a, 1954b)	1	1	–	–8	-4	0	0	2	0	0
Ni (1972) (stratified)	1	1	–	–8	0	0	0	2	0	0
Yilmaz (1958, 1962)	1	1	–	–8	0	–4	0	–2	0	–1*
Page-Tupper (1968)	γ	β	–	-4Δ	0	-2Δ	0	ζ_2	0	ζ_4
Nordström (1912, 1913)	–1	½	–	0	0	0	0	0	0	0*
Einstein-Fokker (1914)	–1	½	–	0	0	0	0	0	0	0
Ni (1972) (flat)	–1	$1-q$	–	0	0	0	0	ζ_2	0	0*
Whitrow-Morduch (1960)	–1	$1-q$	–	0	0	0	0	q	0	0*
Littlewood (1953), Bergman (1956)	–1	½	–	0	0	0	0	–1	0	0*

– Note: * The theory is incomplete, and ζ_4 can take one of two values. The value closest to zero is listed.

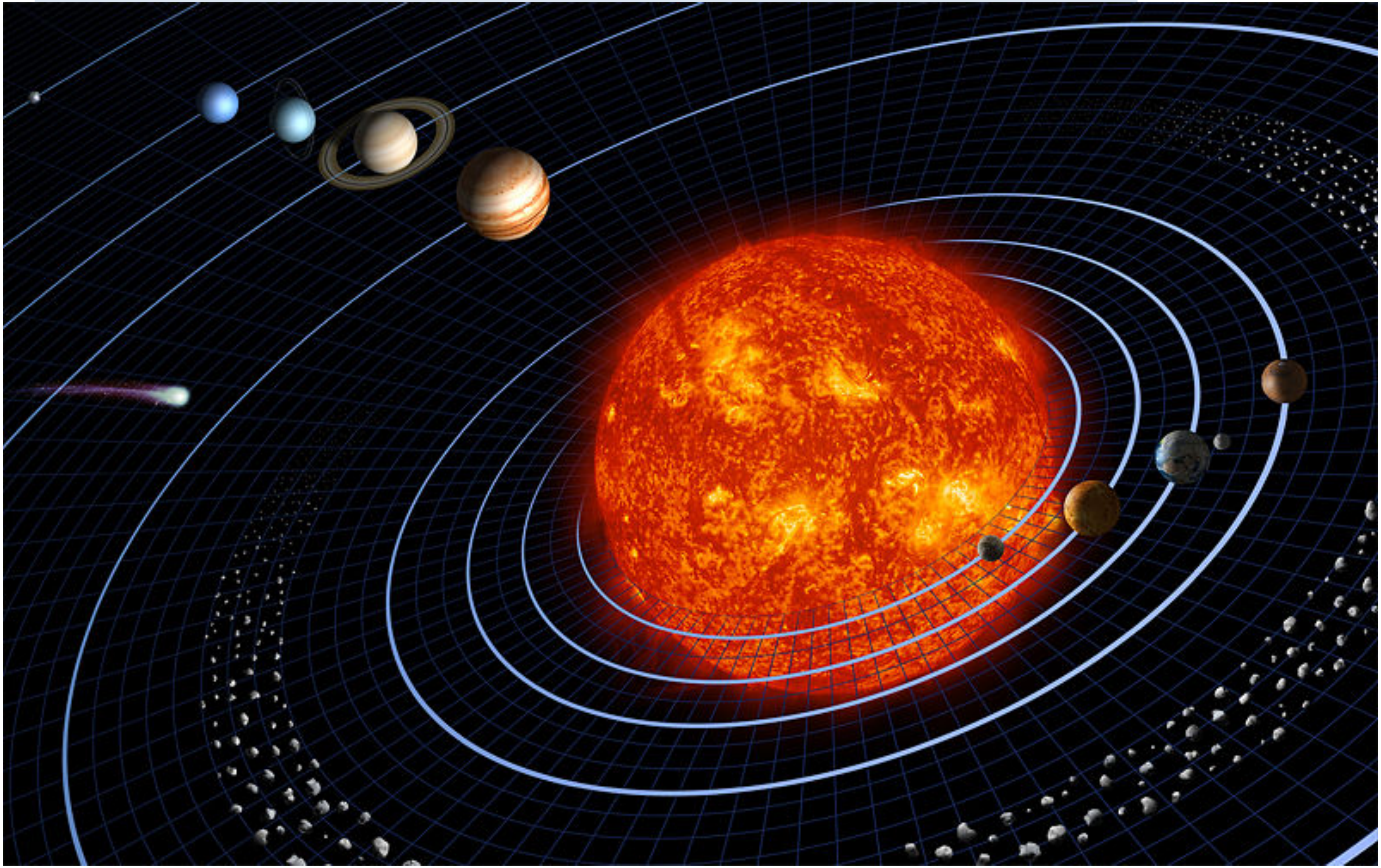
List of PPN Parameters for Competing Theories

Competing theories of Gravity	γ	β	ξ	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
Einstein (1915) GR	1	1	0	0	0	0	0	0	0	0
Scalar-Tensor theories										
Bergmann (1968), Wagoner (1970)	$\frac{1+\omega}{2+\omega}$	β	0	0	0	0	0	0	0	0
Nordtvedt (1970), Bekenstein (1977)	$\frac{1+\omega}{2+\omega}$	β	0	0	0	0	0	0	0	0
Brans-Dicke (1961)	$\frac{1+\omega}{2+\omega}$	1	0	0	0	0	0	0	0	0
Vector-Tensor theories										
Hellings-Nordtvedt (1973)	γ	β	0	α_1	α_2	0	0	0	0	0
Will-Nordtvedt (1972)	1	1	0	0	α_2	0	0	0	0	0
Bimetric theories – Note: in Rosen (1975): parameter k_2 is defined as $k_2 = (c_0/c_1) - 1$										
Rosen (1975)	1	1	0	0	k_2	0	0	0	0	0
Rastall (1979)	1	1	0	0	α_2	0	0	0	0	0
Lightman-Lee (1973)	γ	β	0	α_1	α_2	0	0	0	0	0
Stratified theories										
Lee-Lightman-Ni (1974)	ac_0/c_1	β	ξ	α_1	α_2	0	0	0	0	0
Ni (1973)	ac_0/c_1	bc_0	0	α_1	α_2	0	0	0	0	0

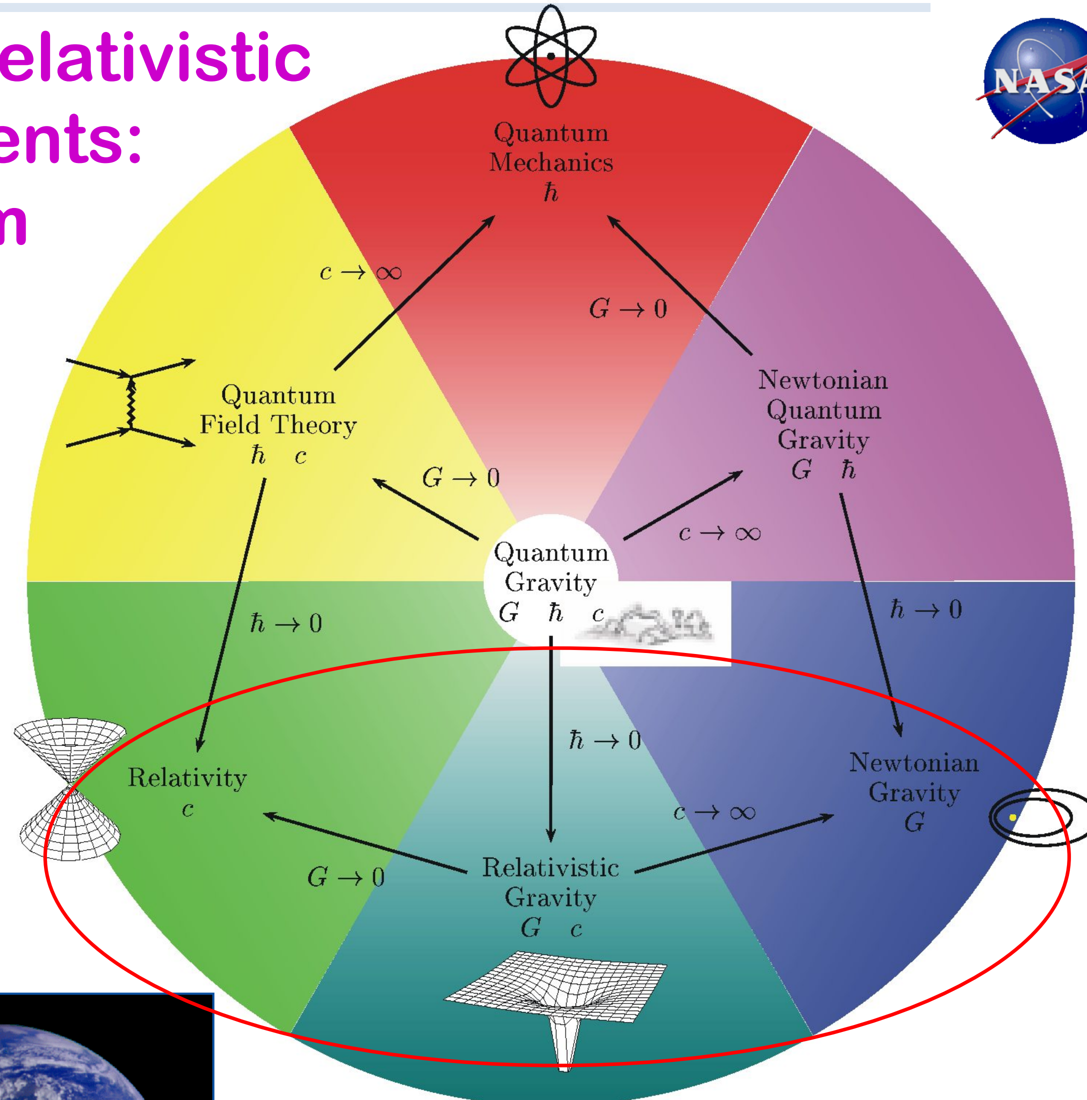
The Current Values of the PPN Parameters (2017)

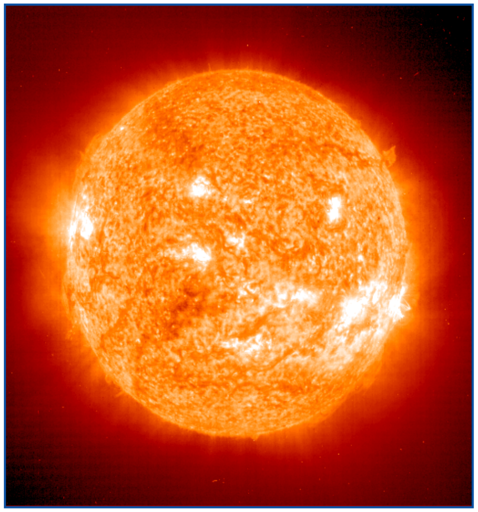
Para-meter	What is measured relative to General Relativity?	Current value	Effects	Experiments
$\gamma-1$	Measure of space curvature produced by unit mass	2.3×10^{-5}	Time delay, light deflection	Cassini tracking
$\beta-1$	Measure of non-linearity in gravitational superposition	8.0×10^{-5}	Perihelion shift	Solar system planetary and spacecraft tracking
ξ	Measure of existence of preferred location effects	4×10^{-9}	Spin precession	Millisecond pulsars
α_1	Measure the existence of preferred frame effects	4×10^{-5}	Orbit polarization	PSR J1738+0333
α_2		2×10^{-9}	Spin precession	Millisecond pulsars
α_3		4×10^{-20}	Self-acceleration	Pulsar spin-down statistics
ζ_1	Measure (plus α_3) of the failure of conservation laws of energy, momentum and angular momentum	2×10^{-2}	–	Combined PPN bounds
ζ_2		4×10^{-5}	Binary pulsar acceleration	Pulsar: PSR 1913+16
ζ_3		1×10^{-8}	Newton's 3rd law	Lunar acceleration
ζ_4		6×10^{-3}	–	Kreuzer experiment

Our solar system and tests of gravity



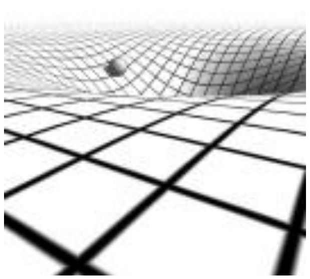
Laboratory for Relativistic Gravity Experiments: Our Solar System





Strongest gravity potential

$$\frac{GM_{Sun}}{c^2 R_{Sun}} \sim 10^{-6}$$



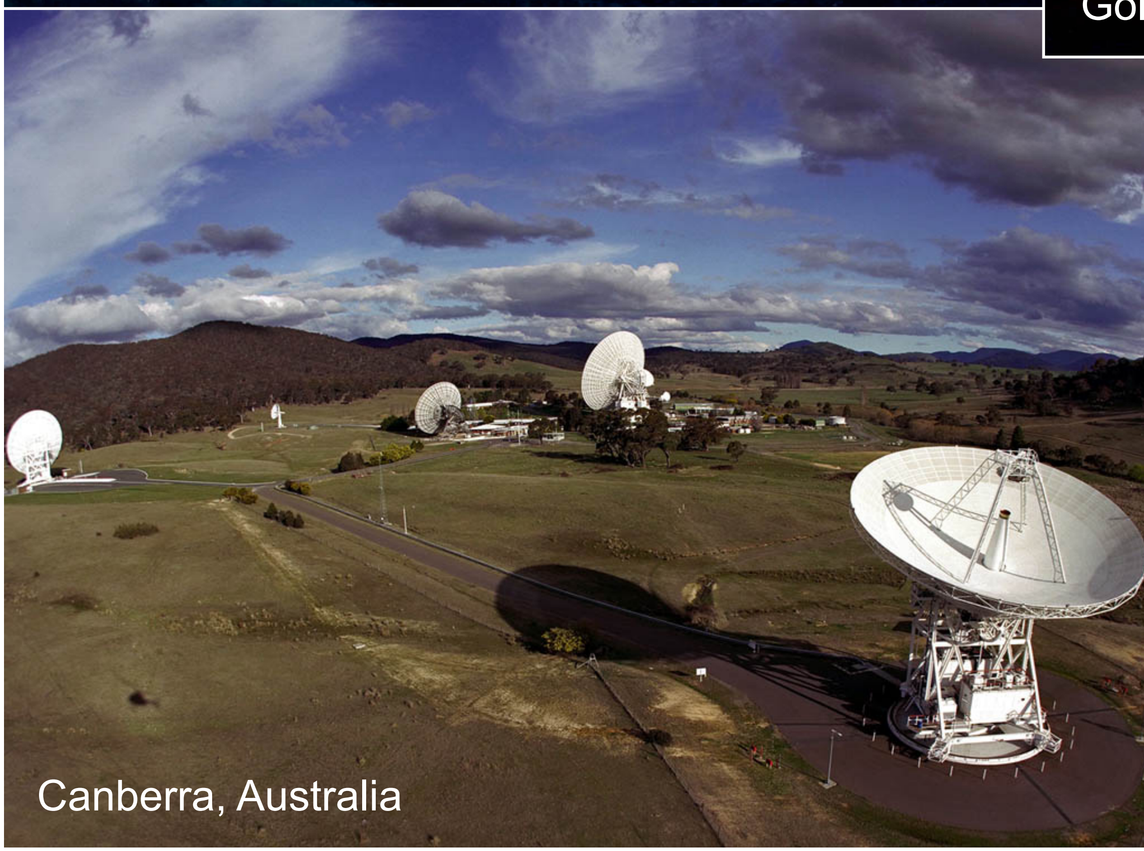
$$\frac{GM_{\oplus}}{c^2 R_{\oplus}} \sim 10^{-9}$$

Most accessible region for gravity tests in space:

- ISS, LLR, SLR, free-fliers

Technology is available to conduct tests in the immediate solar proximity

Deep Space Network



40+ Years of Solar System Gravity Tests

Techniques for Gravity Tests:

Radar Ranging:
- Planets: Mercury, Venus, Mars
- s/c: Mariners, Vikings, Pioneers, Cassini, Mars Global Surveyor, Mars Orbiter, etc.
- VLBI, GPS, etc.

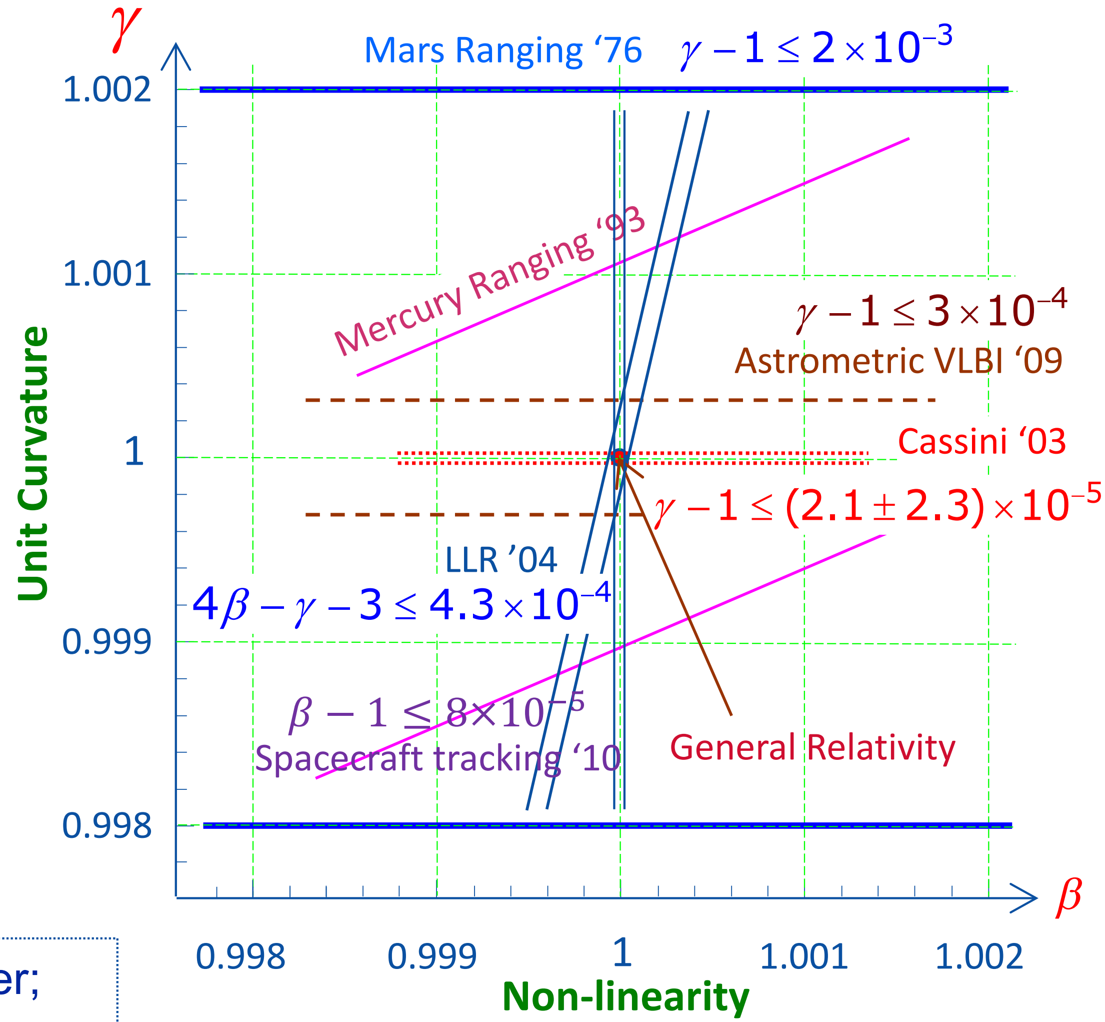
Laser:
- SLR, LLR, interplanetary, etc.

Dedicated Gravity Missions:

- **LLR (1969 - on-going!!)**
- GP-A, '76; LAGEOS, '76,'92; GP-B, '04; LARES, '12; MicroSCOPE, '16, ACES, '18; eLISA, 2030+(?)

New Engineering Discipline – Applied General Relativity:

- Daily life: GPS, geodesy, time transfer;
- Precision measurements: deep-space navigation & µas-astrometry (ESA's Gaia).



A factor of 100 in 40 years is impressive, but is not enough for the near future!

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Some Theories resist to fail

Newton 1686 | Poincaré 1890

Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | **Einstein 1915**

Whitehead 1922 | **Cartan 1923** | Kaluza & Klein 1932 | Fierz & Pauli 1939 | Birkhoff 1943

Milne 1948 | Thiry 1948 | Papapetrou 1954 | Jordan 1955 | Littlewood & Bergmann 1956

Brans & Dicke 1961 | Yilmaz 1962 | Whitrow & Morduch 1965 | Kustaanheimo & Nuotio 1967

Page & Tupper 1968 | Bergmann 1968 | Deser & Laurent 1968 | Nordtvedt 1970 | Wagoner 1970

Bollini et al. 1970 | Rosen 1971 | **Will & Nordtvedt 1972** | Ni 1972 | **Hellings & Nordtvedt 1972**

Ni 1973 | **Yilmaz 1973** | Lightman & Lee 1973 | Lee, Lightman & Ni 1974 | Rosen 1975

Belinfante & Swihart 1975 | Lee et al. 1976 | Bekenstein 1977 | Barker 1978 | Rastall 1979

Coleman 1983 | **Hehl 1997** | **Overlooked (20th century)**

- Will & Nordtvedt (1972) and Hellings & Nordtvedt (1972) are vector-tensor theories. Deviations can only be significant in high energy regime (e.g. Planck-scale energy)
- Yilmaz (1973) was mathematically inconsistent, but now is fixed. Does not predict black holes
- Cartan (1923), Hehl (1997) introduces matter spin

"Aesthetics-Based" Conclusion for 20th Century

Newton 1686 · Poincaré 1890

Einstein 1912 · Nordstrøm 1912 · Nordstrøm 1913 · Einstein & Fokker 1914 · **Einstein 1915**

Whitehead 1922 · Cartan 1923 · Kaluza & Klein 1932 · Fierz & Pauli 1939 · Birkhoff 1943

Milne 1948 · Thiry 1948 · Papapetrou 1954 · Jordan 1955 · Littlewood & Bergmann 1956

Brans & Dicke 1961 · Yilmaz 1962 · Whitrow & Morduch 1965 · Kustaanheimo & Nuotio 1967

Page & Tupper 1968 · Bergmann 1968 · Deser & Laurent 1968 · Nordtvedt 1970 · Wagoner 1970

Bollini et al. 1970 · Rosen 1971 · Will & Nordtvedt 1972 · Ni 1972 · Hellings & Nordtvedt 1972

Ni 1973 · Yilmaz 1973 · Lightman & Lee 1973 · Lee, Lightman & Ni 1974 · Rosen 1975

Belinfante & Swihart 1975 · Lee et al. 1976 · Bekenstein 1977 · Barker 1978 · Rastall 1979

Coleman 1983 · Hehl 1997 · Overlooked (20th century)

– "Among all bodies of physical law none has ever been found that is simpler and more beautiful than Einstein's geometric theory of gravity"

  – Misner, Thorne and Wheeler, 1973

– "[...] Unfortunately, any finite number of effects can be fitted by a sufficiently complicated theory. [...] Aesthetic or philosophical motives will therefore continue to play a part in the widespread faith in Einstein's theory, even if all tests verify its predictions."

  – Malcolm MacCallum, 1976

Theoretical Landscape of the 21st Century:

Competing Theories of Gravity

First decade of 21'st century… they are back!

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 Cartan 1923 **Kaluza & Klein 1932** Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 Thiry 1948 Papapetrou 1954 Jordan 1955 Littlewood & Bergmann 1956

Brans & Dicke 1961 Yilmaz 1962 Whitrow & Morduch 1965 Kustaanheimo & Nuotio 1967

Page & Tupper 1968 Bergmann 1968 Deser & Laurent 1968 Nordtvedt 1970 Wagoner 1970

Bollini et al. 1970 Rosen 1971 Will & Nordtvedt 1972 Ni 1972 Hellings & Nordtvedt 1972

Ni 1973 Yilmaz 1973 Lightman & Lee 1973 Lee, Lightman & Ni 1974 Rosen 1975

Belinfante & Swihart 1975 Lee et al. 1976 Bekenstein 1977 Barker 1978 Rastall 1979

Coleman 1983 Hehl 1997 Overlooked (20thcentury) **Scalar-Tensor Theories**

Arkani-Hamed, Dimopoulos & Dvali 2000 **Dvali, Gabadadze & Poratti 2003** **Strings theory?**

Bekenstein 2004 **Moffat 2005** **Multiple f(R) models 2003-10** **Bi-Metric Theories**

Need for new theory of gravity:

- Classical GR description breaks down in regimes with large curvature
- If gravity is to be quantized, GR will have to be modified or extended

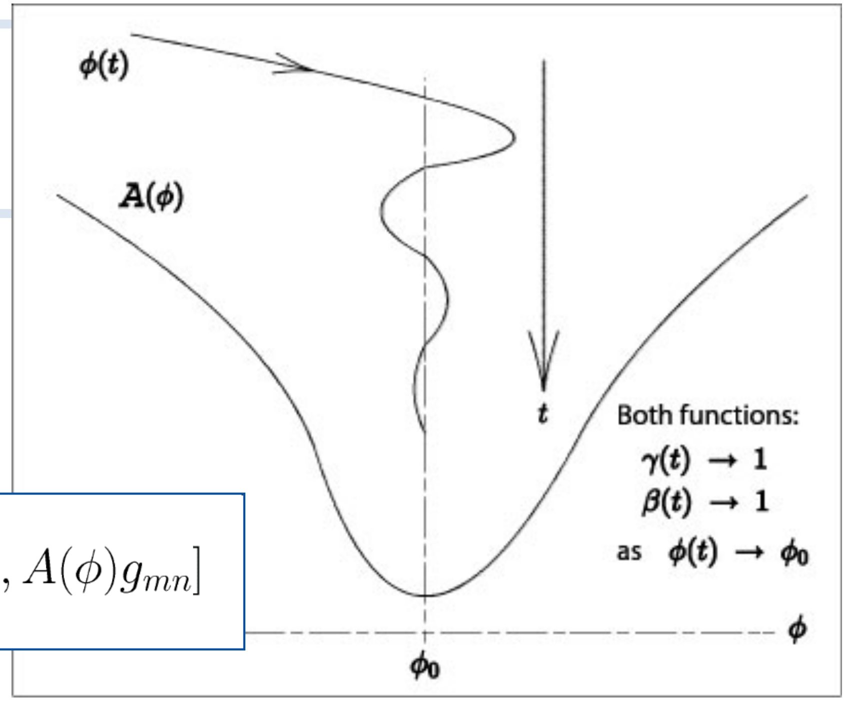
Other challenges:

- Dark Matter
- Dark Energy

Motivations for new tests of GR:

- GR is a fundamental theory
- Alternative theories & models
- New ideas & techniques require comprehensive investigations

Theoretical Motivation for New Gravity Tests



Long-range massless [or low-mass] scalar:

The low-energy limit of the String Theory in 'Einstein Frame' (Damour-Nordtvedt-Polyakov 1993) suggests:

$$S = -\frac{1}{16\pi G}\int dx^4\sqrt{-g}\Big(R - 2g^{mn}\nabla_m\phi\nabla_n\phi\Big) + S_M[\psi_M, A(\phi)g_{mn}]$$

Expansion *A(ϕ)* around background value ϕ_0 of the scalar leads:

$$\ln A(\varphi) = \ln A(\varphi_0) + \alpha_0(\varphi-\varphi_0) + \frac{1}{2}k_0(\varphi-\varphi_0)^2 + \mathcal{O}(\Delta\varphi^3)$$

Slope α_0 measures the coupling strength of interaction between matter and the scalar.

$$\gamma - 1 \ = \ \frac{-2\alpha_0^2}{1+\alpha_0^2} \simeq -2\alpha_0^2$$

$$\beta - 1 = \frac{1}{2}\frac{\alpha_0^2 k_0}{(1+\alpha_0^2)^2} \simeq \frac{1}{2}\alpha_0^2 k_0 \simeq \frac{1}{4}(1-\gamma)k_0$$

Scenario for cosmological evolution of the scalar (Damour, Piazza & Veneziano 2002):

$$\gamma - 1 \sim 7.3\times10^{-7}\left(\frac{H_0}{\Omega_0^3}\right)^{\frac{1}{2}} \quad \Longrightarrow \quad \gamma - 1 \sim 10^{-5} - 10^{-7}$$

The unit curvature PPN parameter γ is the most important quantity to test

Modifications of Einstein Gravity

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_m[g_{\mu\nu},\psi]$$

Carroll et al, PRD 70 (2004) 043528
…

Modification of PPN Gravity

$$\gamma - 1 = -\frac{f''(R)^2}{f'(R) + 2f''(R)^2},$$

$$\beta - 1 = \frac{1}{4}\frac{f'(R)\cdot f''(R)}{2f'(R) + 3f''(R)^2}\frac{d\gamma}{dR}.$$

Analogy between scalar-tensor and higher-order gravity

Constraints on … $f(R)$ from solar system experiments…

…tight restrictions on the form of the gravitational Lagrangian

Need for cosmological "PPN formalism"

Capozziello, Stabile, Trosi, gr-qc/0603071

Modified Gravity: f(R) theories

- **A broad class of alternative theories**

$$\mathcal{L}_\phi = -\frac{M^2}{2}\omega(\phi)(\partial\phi)^2 - V(\phi)$$

$$S = \int d^n x\sqrt{-g}\Big[\frac{1}{2}f(R,\phi) + \mathcal{L}_\phi(g_{\mu\nu},\phi,\partial\phi) + \mathcal{L}_m(g_{\mu\nu},\Psi)\Big]$$

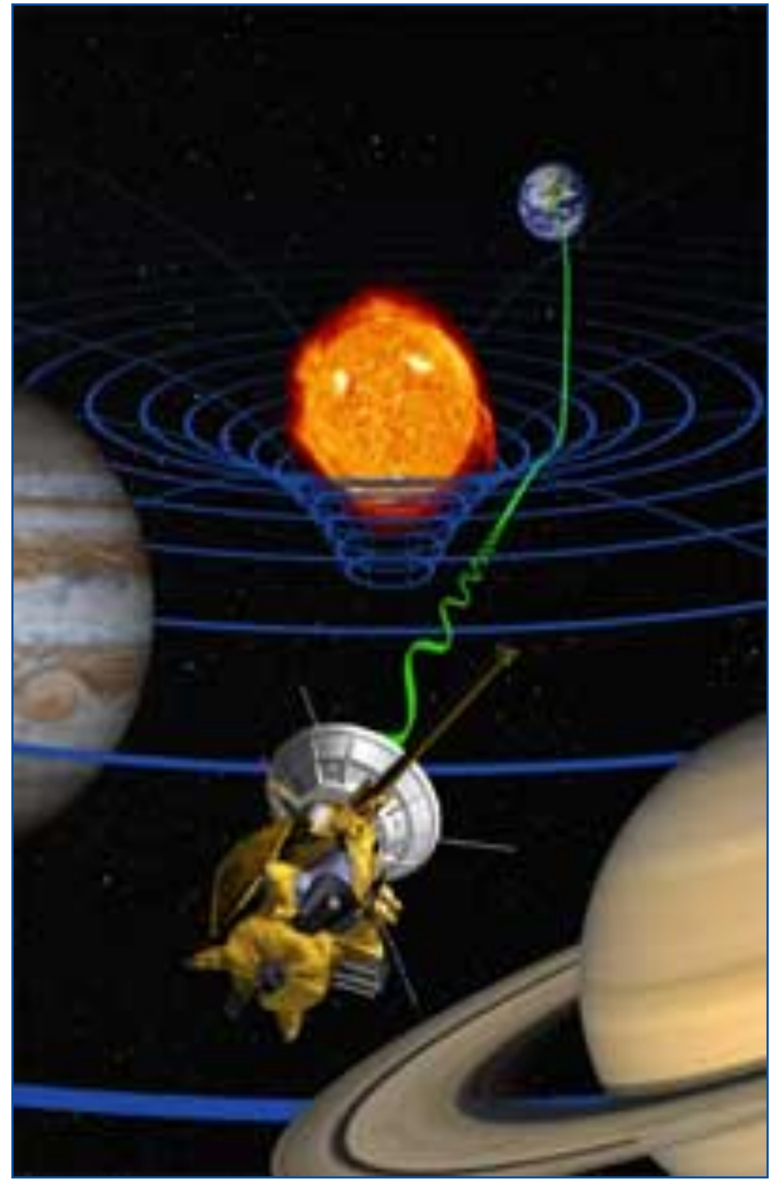
$$(\partial\phi)^2 = \nabla_\mu\phi\nabla^\mu\phi \qquad F(R,\phi) = \partial f(R,\phi)/\partial R$$

Generalized gravity	$\frac{1}{2}f(R,\phi)$	$\mathcal{L}_\phi(\phi,\partial\phi)$	$p(R,\phi)$	φ	$\hat{V}(\varphi)$
Nonlinear gravity	$\frac{1}{2}f(R)$	$\omega=0,\ V=0$	$p=F(R)$	$\sqrt{\frac{3}{2}}\ln F$	$\frac{FR-f}{2F^2}$
R^2-gravity	$\frac{1}{2}(R+\alpha R^2)$	$\omega=0,\ V=0$	$p=1+2\alpha R$	$\sqrt{\frac{3}{2}}\ln F$	$\frac{FR-f}{2F^2}$
$1/R$-gravity	$\frac{1}{2}(R-\mu^4/R)$	$\omega=0,\ V=0$	$p=1+\mu^4/R^2$	$\sqrt{\frac{3}{2}}\ln F$	$\frac{FR-f}{2F^2}$
Scalar-tensor theory	$\frac{1}{2}F(\phi)R$	$\omega(\phi),\ V(\phi)$	$p=F(\phi)$	$\int\sqrt{\frac{\omega}{F}+\frac{3}{2}\frac{F'^2}{F^2}}d\phi$	$\frac{V}{F^2}$
Brans-Dicke theory	ϕR	$\omega(\phi)=2\frac{\omega}{\phi},\ V=0$	$p=\phi$	$\int\sqrt{\frac{\omega}{F}+\frac{3}{2}\frac{F'^2}{F^2}}d\phi$	0
Dilaton	$\frac{1}{2}e^{-\phi}R$	$\omega(\phi)=e^{-\phi},\ V=0$	$p=e^{-\phi}$	$\frac{5}{2}\phi$	0
NMC scalar	$\frac{1}{2}(1+\xi\phi^2)R$	$\omega=1,\ V(\phi)$	$p=1+\xi\phi^2$	$\int\frac{\sqrt{1+\xi(6\xi-1)\phi^2}}{1-\xi\phi^2}d\phi$	$\frac{V}{1-\xi\phi^2}$
CC ($\xi=\frac{1}{6}$)	$\frac{1}{2}(1+\frac{1}{6}\phi^2)R$	$\omega=1,\ V(\phi)$	$p=1+\frac{1}{6}\phi^2$	$\sqrt{6}\tanh^{-1}\frac{\phi}{\sqrt{6}}$	$\frac{V}{1-\frac{1}{6}\phi^2}$
Induced Gravity	$\frac{1}{2}\epsilon\phi^2 R$	$\omega=1,\ V(\phi)$	$p=\epsilon\phi^2$	$\sqrt{6+\frac{1}{\epsilon}}\ln\phi$	$\frac{V}{\epsilon\phi^2}$
GR with a scalar	$\frac{1}{2}R$	$\omega=1,\ V(\phi)$	$p=1$	ϕ	V

ψ – are matter fields; ϕ – is a scalar field

too many references…

Cassini 2003: Where Do We Go From Here?



Cassini Conjunction Experiment:

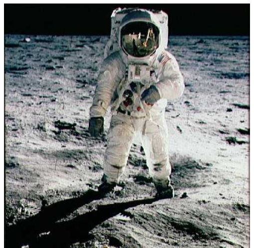
- Spacecraft—Earth separation > 1 billion km
- Doppler/Range: X~7.14GHz & Ka~34.1GHz
- Result: $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$

Possible with Existing Technologies?!

- VLBI [current $\gamma = 3 \times 10^{-4}$]: limited to $\sim 1 \times 10^{-4}$:
  - uncertainty in the radio source coordinates
- LLR [current $\eta = 4 \times 10^{-4}$]: in 5 years $\sim 3 \times 10^{-5}$:
  - mm accuracies [APOLLO] & modeling efforts
- μ-wave ranging to a lander on Mars $\sim 6 \times 10^{-6}$
- GRACE-FO in Earth's orbit (2017): $\sim 5 \times 10^{-6}$
- tracking of BepiColombo s/c at Mercury $\sim 2 \times 10^{-6}$
- Optical astrometry [current $\gamma = 3 \times 10^{-3}$]:
  - ESA's Gaia mission (2013) $\sim 1 \times 10^{-6}$ (2018?)

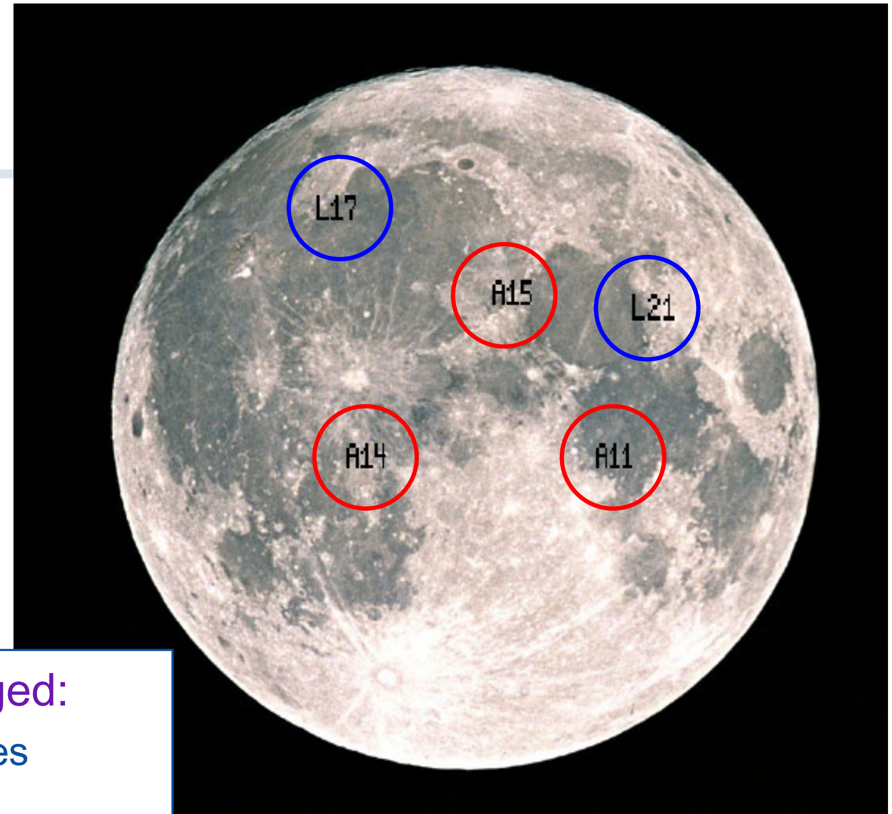
One needs a dedicated mission to explore accuracies better than 10^{-6} for both PPN parameters γ (and β). Interplanetary laser ranging is a possibility.

Lunar Laser Ranging



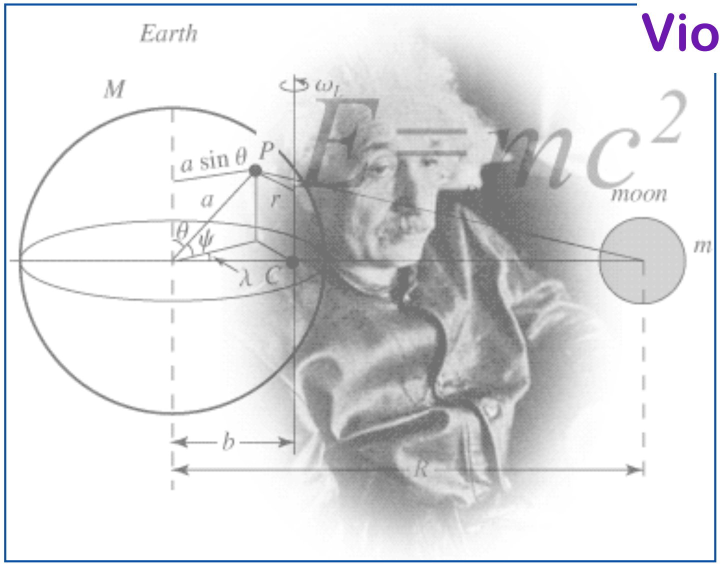
It is all begun ~48 years ago…

Laser Ranges between observatories on the Earth and retroreflectors on the Moon started by Apollo in 1969 and continue to the present



- All 5 reflectors are ranged:
  - Apollo 11, 14 & 15 sites
  - Lunakhod 2 Rover
  - Lunakhod 1 Rover (>2010)
- LLR conducted primarily from 3 observatories:
  - McDonald (Texas, USA)
  - OCA (Grasse, France)
  - Haleakala (Hawaii, USA)
- New LLR stations:
  - Apache Point, (NM, USA)
  - Matera (Matera, Italy)
  - South Africa, former OCA LLR
  - OCTL, Table Mountain, CA

Testing General Relativity with LLR



Violation of the Equivalence Principle in PPN formalism:

$$\frac{\Delta a}{a} \equiv \frac{2(a_1 - a_2)}{(a_1 + a_2)} = \left(\frac{m_G}{m_I}\right)_1 - \left(\frac{m_G}{m_I}\right)_2, \qquad \frac{m_G}{m_I} = 1 + (4\beta - \gamma - 3)\frac{\Omega}{mc^2}$$

$$\frac{\Delta a}{a} = \eta \cdot \left(\frac{\Omega_e}{m_e c^2} - \frac{\Omega_m}{m_m c^2}\right) = -\eta \cdot 4.45 \times 10^{-10}, \qquad \eta \equiv 4\beta - \gamma - 3.$$

If η =1, this would produce a 13 m displacement of lunar orbit.
By 2010, range accuracy is ~1.4 cm, the effect was not seen.

LLR results (April 2010):

17,471 normal points through May 29, 2010, including 247APOLLO points plus MLRS, OCA, and HALA

$$\Delta\left(\frac{m_G}{m_I}\right) = (-0.95 \pm 1.30) \times 10^{-13}$$ – corrected for solar radiation pressure from Vokrouhlicky (1997).

$$\frac{\Delta a}{a} = (-1.95 \pm 1.91) \times 10^{-13}$$ – test of the Strong Equivalence Principle with Adelberger (2001) results for WEP

$$\eta = 4\beta - \gamma - 3 = (4.4 \pm 4.3) \times 10^{-4}$$

Using Cassini '03 result $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5} \Rightarrow \beta - 1 = (1.2 \pm 1.1) \times 10^{-4}$

$K_{GP} = -0.0007 \pm 0.0047$ – Geodetic / de Sitter-Fokker precession

$$\dot{G}/G = (4.9 \pm 5.7) \times 10^{-13}\ \mathrm{yr}^{-1}$$

Advanced LLR: anticipated results

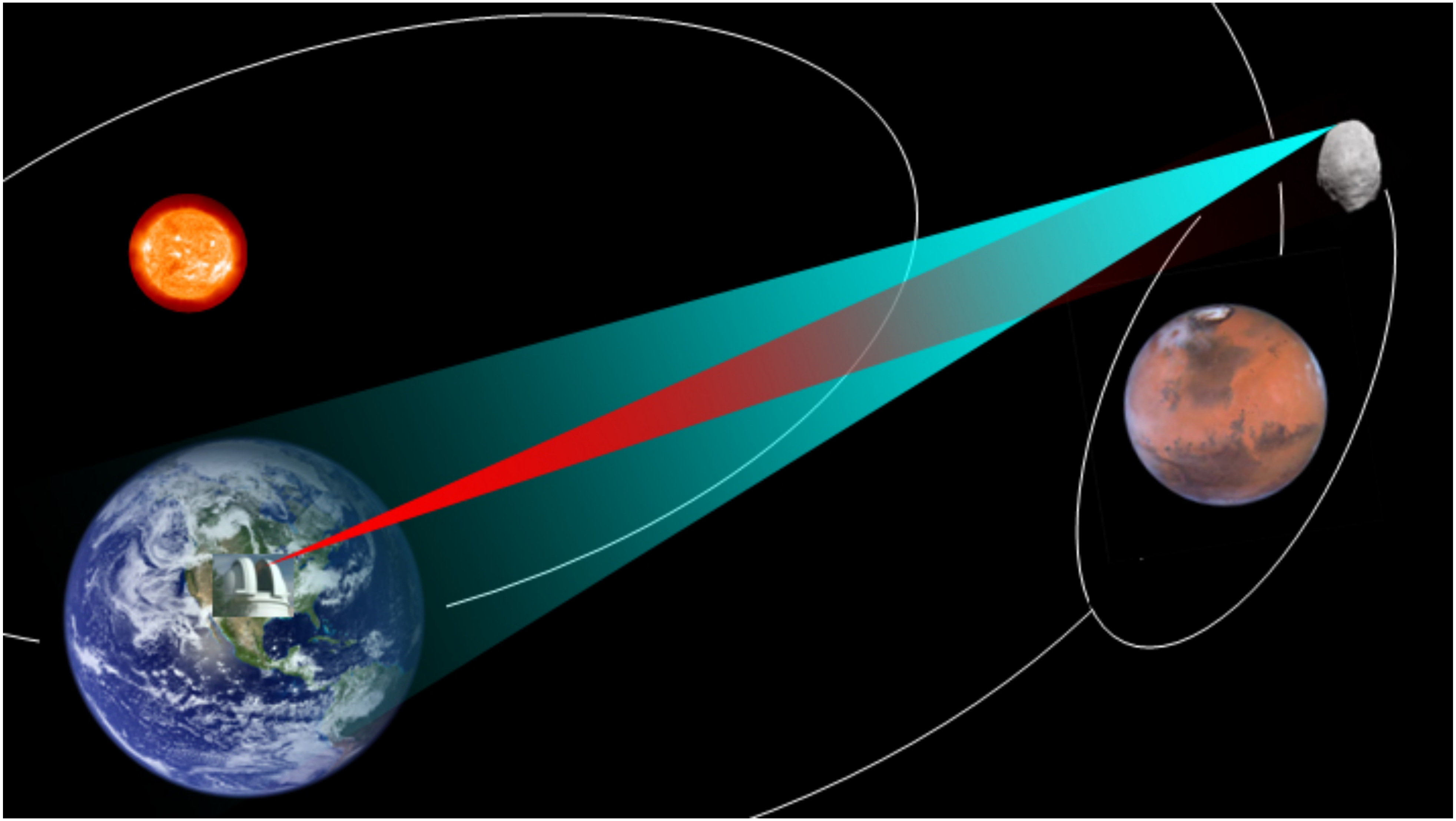
Tests of GR

Science	Timescale	Current (cm)	1 mm	0.1 mm
Weak Equivalence Principle	Few years	$\|\Delta a/a\|<1.3\times10^{-13}$	10^{-14}	10^{-15}
Strong Equivalence Principle	Few years	$\|\eta\|<4.3\times10^{-4}$	3×10^{-5}	3×10^{-6}
PPN parameter β	Few years	$\|\beta-1\|<1.1\times10^{-4}$	10^{-5}	10^{-6}
Time variation of G	~10 years	5.7×10^{-13} yr^{-1}	5×10^{-14}	5×10^{-15}
Inverse Square Law	~10 years	$\|\alpha\|<3\times10^{-11}$	10^{-12}	10^{-13}

Lunar science

Effect	Current	Future Goals
Positions on Moon	yes	More locations
Low-degree gravity field	yes	Distinguish mantle from inner core for gravity and moments
3 free libration mantle modes	yes	Seek stimulating events
Solid-body tides	yes	Improve Love number accuracies
Tidal dissipation	yes	Improve tidal Q vs frequency
Core/mantle boundary dissipation	yes	Improve uncertainty, used to limit fluid core size
Core/mantle boundary flattening	yes	Improve uncertainty
Fluid core moment of inertia	no	Detect and determine
Fluid core free precession mode	no	Detect mode, determine amplitude & period
Inner solid core	no	Detect inner core, determine gravity
3 inner core free libration modes	no	Detect modes, determine amplitudes & periods
Inner core boundary dissipation	no	Limit inner core size

Phobos Laser Ranging Architecture



Next Step – Interplanetary Laser Ranging

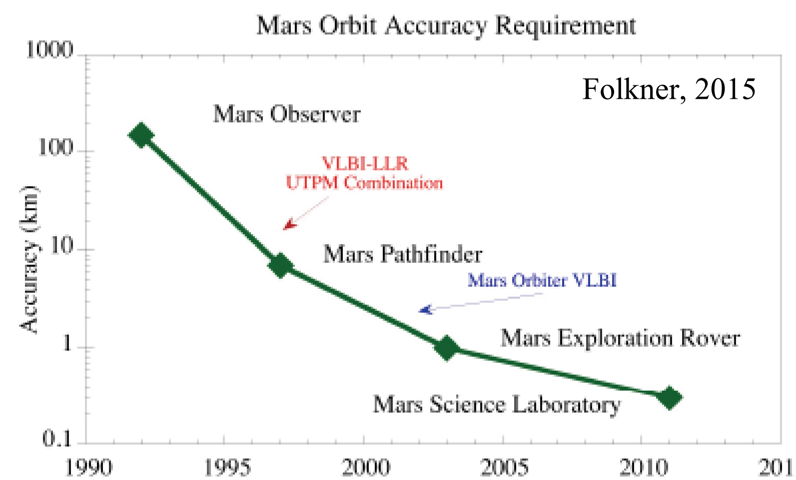
Navigation & Science Data Accuracy Goals

Tracking Error Source (1σ Accuracy)	units	Current X-band capability		Current Ka-band capability	Current DSOC[a] capability	Near-term DOT[a] capability	
		value	Int.time			value	Int.time
Optical comm w/o radio nav	–	n/a			–	yes	
Range	m	2	$10–10^3$ s	0.1	–[b]	5 mm	$10–10^2$ s
Doppler (range-rate)	μm/s	30	$10–10^3$ s	10[c]	–	10[e]	$10–10^2$ s
Astrometry from space[d]	mas	500	1-10 s		–	1-10	3-300 s

[a] Assumed: 0.2m aperture at 2AU distance, non-coherent detection;
[b] Ranging for the current DSOC architecture is yet TBD;
[c] Demonstrated on BeppiColombo; [d] Hubble camera;
[e] Range-rate (computed as Range/Int.time = non-coherent Doppler).

Towards navigational capabilities with DOTs:

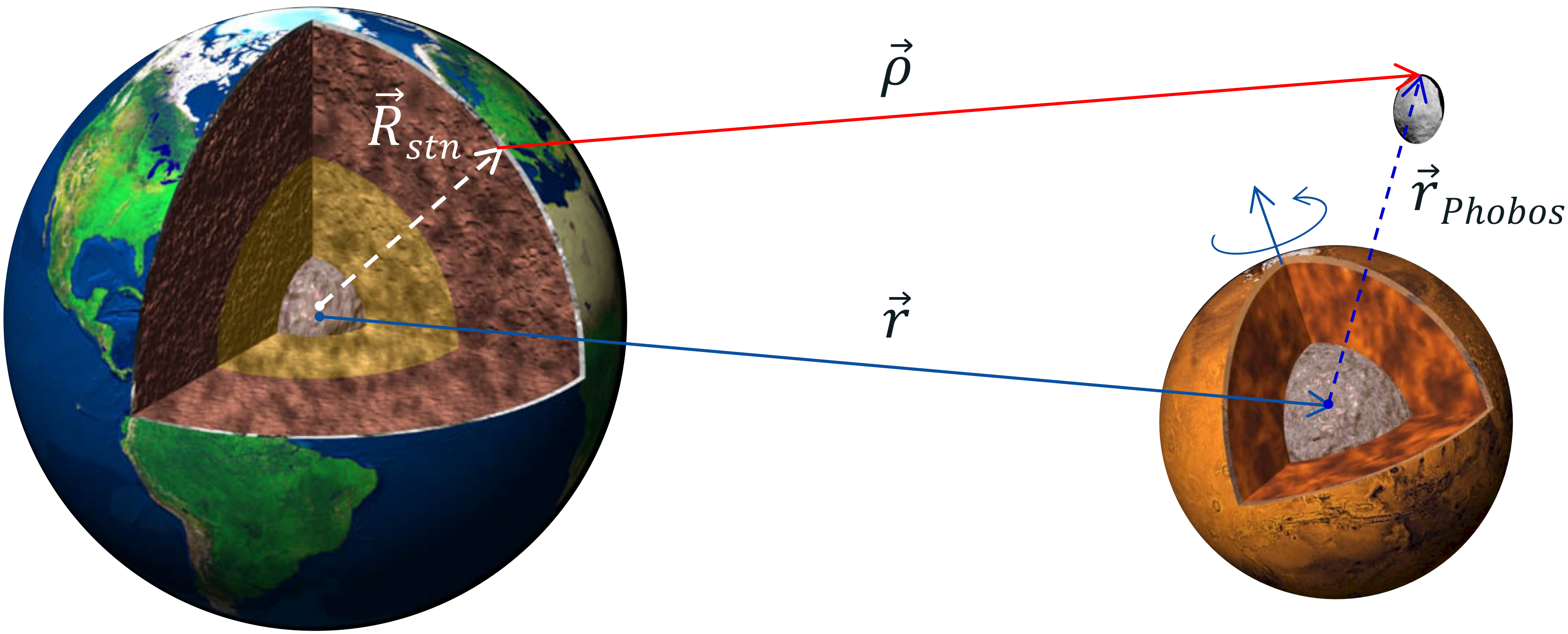
- X-band capabilities are the current state-of-the-art for deep space navigation;
- At present, there are no missions requiring nav accuracy beyond that of the X-band;
- Thus, capabilities beyond those of X-band driven by science, but available to navigation.



DOT 2.0 has to match the X-band nav capabilities, but may go beyond, if science-driven

Phobos Laser Ranging: Principle

1 mm range accuracy with PLR is possible



Impact on:

- Test of general relativity
- The science of Phobos, especially its interior

@ $550M (FY 2009 $)

Gravity Tests with PLR vs Experiment Duration

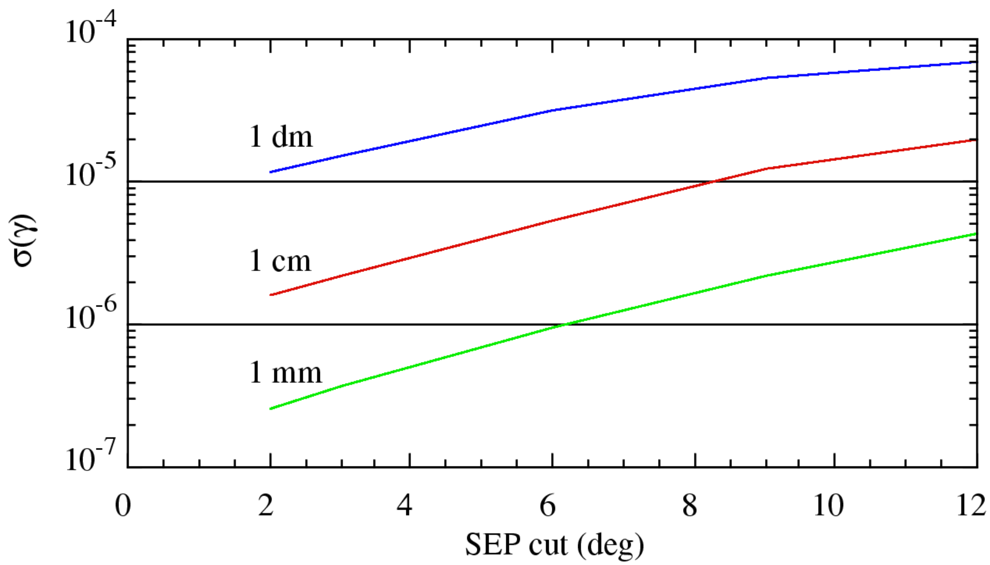
Relativistic Effect	Current best	Mission duration / N of conjunctions		
		1 yr / 1 cnj	3 yr / 2 cnj	6 yr / 3 cnj
PPN parameter γ	2.3×10^{-5}	3.1×10^{-7}	1.4×10^{-7}	7.9×10^{-8}
PPN parameter β	1.1×10^{-4}	4.3×10^{-4}	1.6×10^{-4}	9.4×10^{-5}
Test of Strong Equiv. Principle, η	4.3×10^{-4}	1.5×10^{-3}	2.8×10^{-4}	8.8×10^{-5}
Solar oblateness, J_2	2.0×10^{-7}	6.9×10^{-8}	3.2×10^{-8}	2.3×10^{-8}
Search for time variation in the grav. constant G, $dG/dt/G$, yr^{-1}	7×10^{-13}	1.7×10^{-14}	2.8×10^{-15}	1.0×10^{-15}
Gravitational inverse square law	2×10^{-9} @ 1.5 AU	4×10^{-11} @ 1.5 AU	2×10^{-11} @ 1.5 AU	1×10^{-11} @ 1.5 AU

Estimated uncertainties for parameters of interest as a function of Phobos lander mission duration, with 1 mm laser ranging once per day with 2° SEP cut-off and 67 asteroid mass parameters estimated.

Simulations by W.M. Folkner, JPL

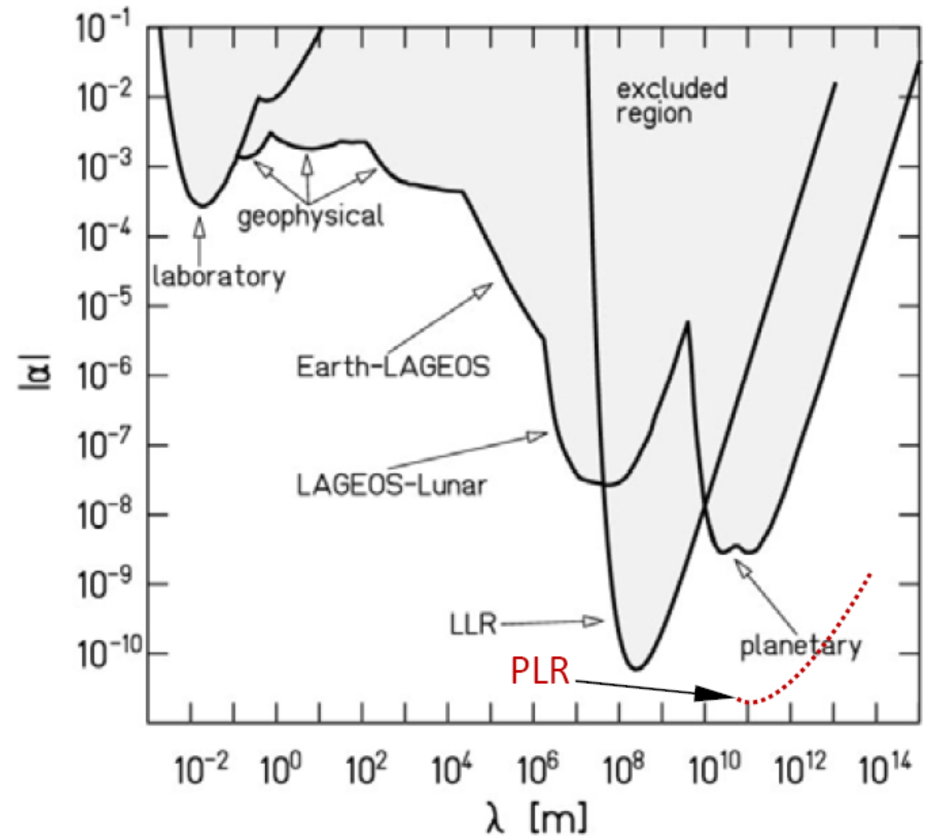
Gravity Tests with PLR: PPN γ and the ISL

Estimated uncertainty in PPN γ



Estimated uncertainty in PPN γ as a function of data accuracy and data cut-off with angular separation from the Sun as viewed from Earth.

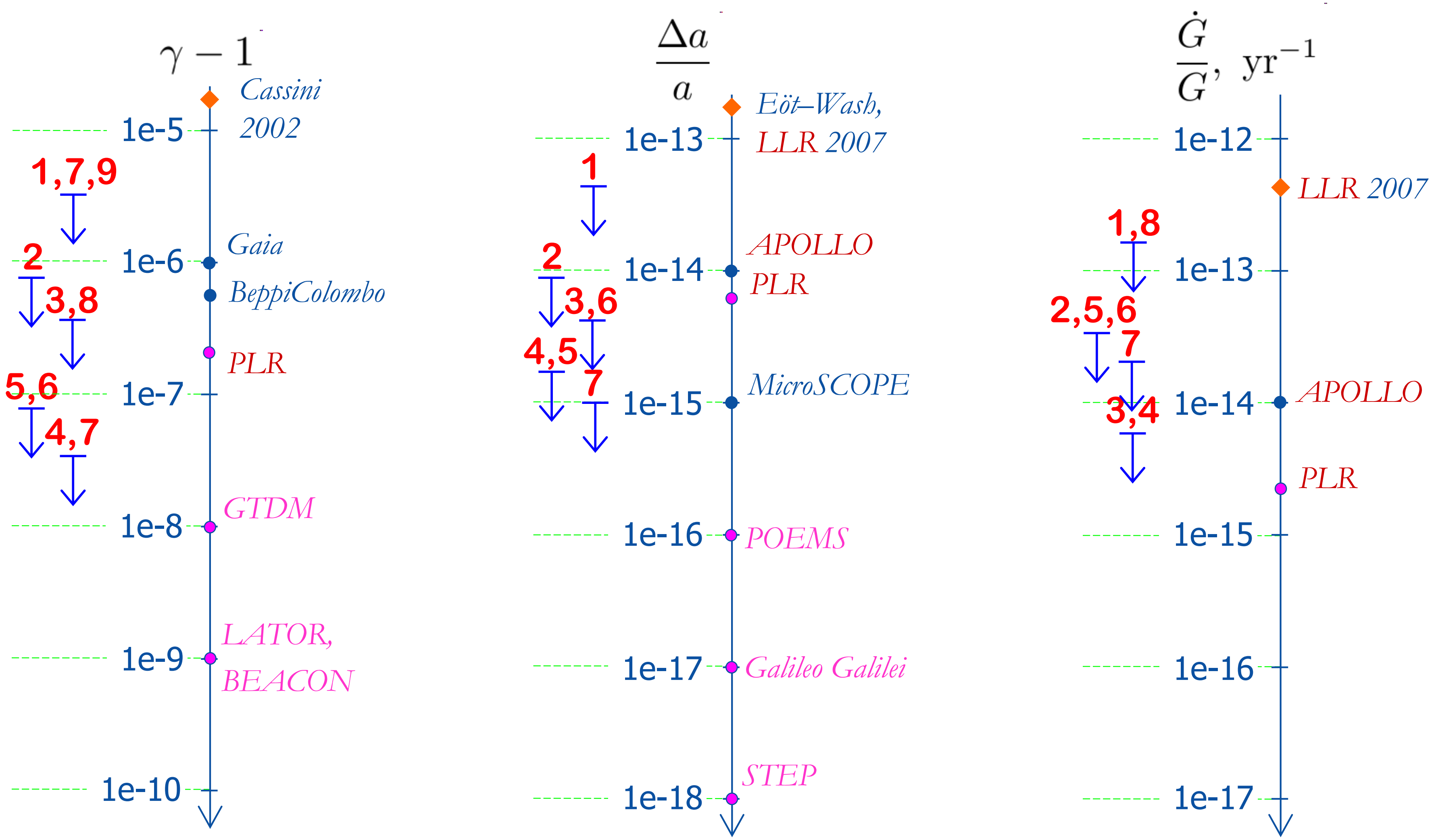
Limits on the ISL violations

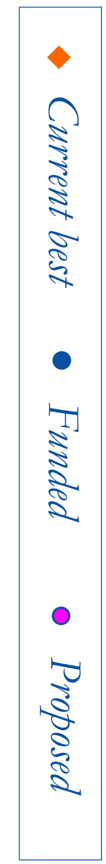


Simulations by W.M. Folkner;
background graphics from (Adelberger et al., 2003)

Theoretical Landscape of the 21th Century:

Confrontation Between Theory and Experiment



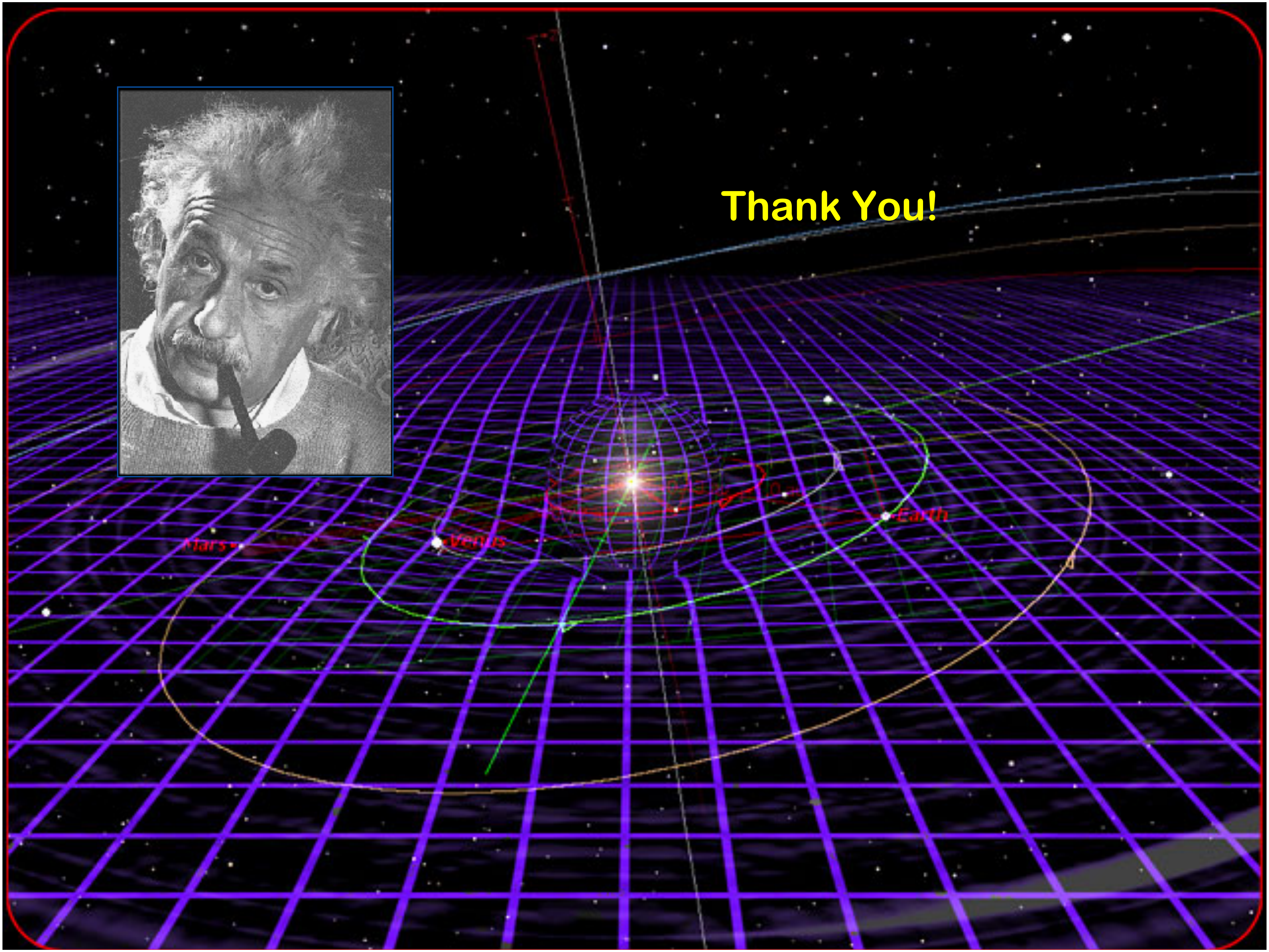


New Theories & Future Tests

1 *Damour-Polyakov-Nordtvedt 1993*
2 *Damour-Esposito-Farese 1996*
3 *Damour-Piazza-Veneziano 2002*
4 *Arkani-Dimopoulos-Dvali 2000*
5 *Dvali-Gabadadze-Poratti 2003*
6 *F(R) gravity models 2003-07*
7 *Bekenstein 2004*
8 *Moffat 2005*
9 *Jaekel-Reynaud 2006*

Conclusions

- Recent technological progress: **arXiv:0902.3004 [gr-qc]**
  - Resulted in new instruments with unique performance
  - Could lead to major improvements in the tests of relativistic gravity
  - Already led to a number of recently proposed gravitational experiments
- Challenges for solar system tests of gravity:
  - Dedicated space-based experiments are very expensive – the science must worth the cost… – *EP, G-dot and PPN γ tests are most relevant.*
  - Motivation for the tests in a weak gravity field is a challenge: there is no strong expectation to see deviations from GR in the solar system (we are looking for anomalies…) – *access to strong(er) gravity regime is needed!*
  - GR is very hard to modify, embed, extend or augment (whatever your favorite verb is…) – *thus, perhaps, those anomalies are important…*
  - PPN formalism becomes less relevant for modern gravity research…
  - Looking to Cosmos for help? There is none: Little or no correspondence between cosmological tests and physical principles in the foundation of tests of PPN gravity – *EP, LLI, LPI, energy-momentum conservation, etc…*


Thank You!
Mars
Venus
Earth