The Empirical Laws of Galactic Rotation

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Main Collaborators:

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Talk Outline

I. Observational Evidence

1 – Rotation Curves & Galaxy Dynamics

2 – Empirical Laws of Galactic Rotation

II. Theoretical Implications

1 – Galaxy Formation in ΛCDM

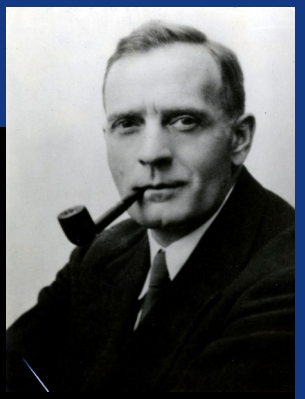
2 – Modified Gravity / Modified Inertia

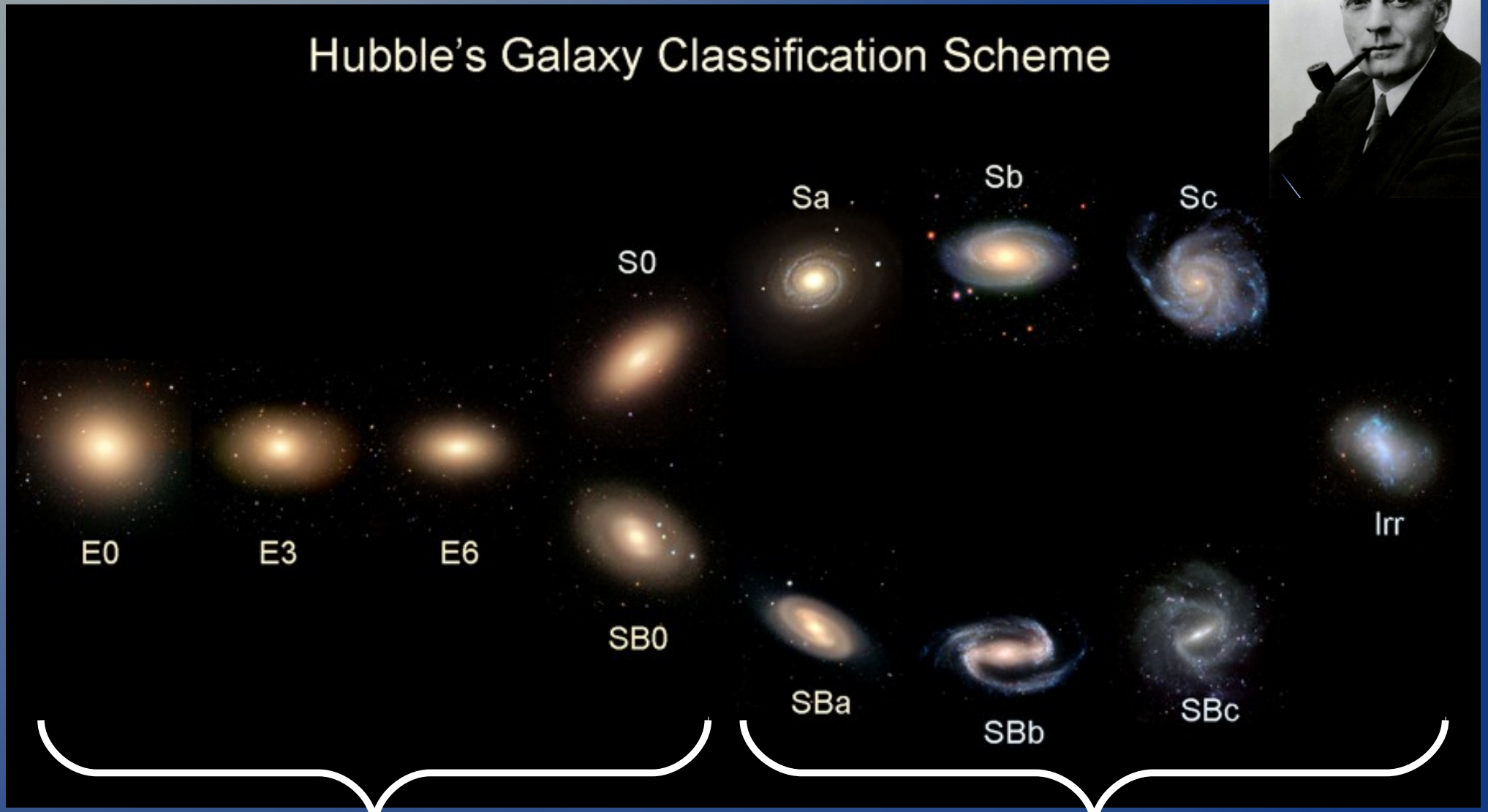
3 – Alternative Dark Matter Models

I. Observational Evidence

1. Rotation Curves & Galaxy Dynamics

Galaxies: "Island Universes"





Early-Type Galaxies (ETGs): E and S0

Little Cold Gas → No Star Formation

→ Red (only old stars)

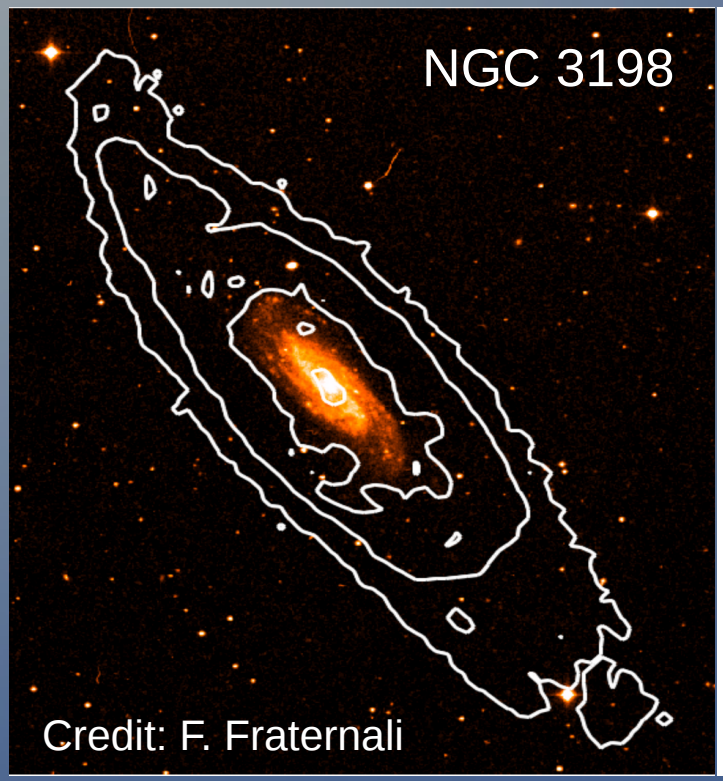
Late-Type Galaxies (LTGs): S and Irr

Disk of Cold Gas → Star Formation

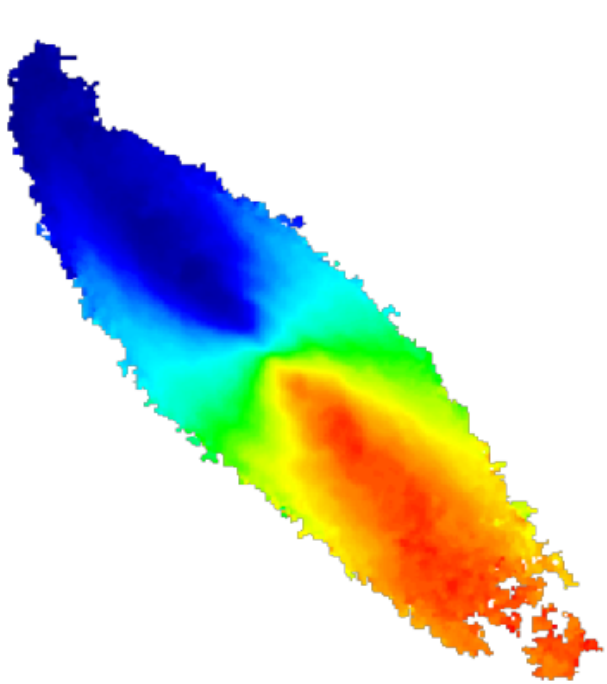
→ Blue (both young & old stars)

Cold Gas: An excellent kinematical tracer

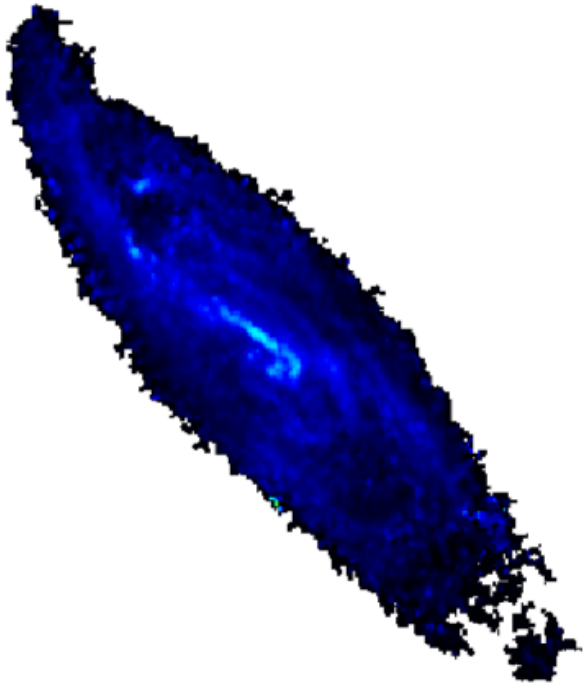
Stars + Atomic Gas



Gas Line-of-Sight Velocity

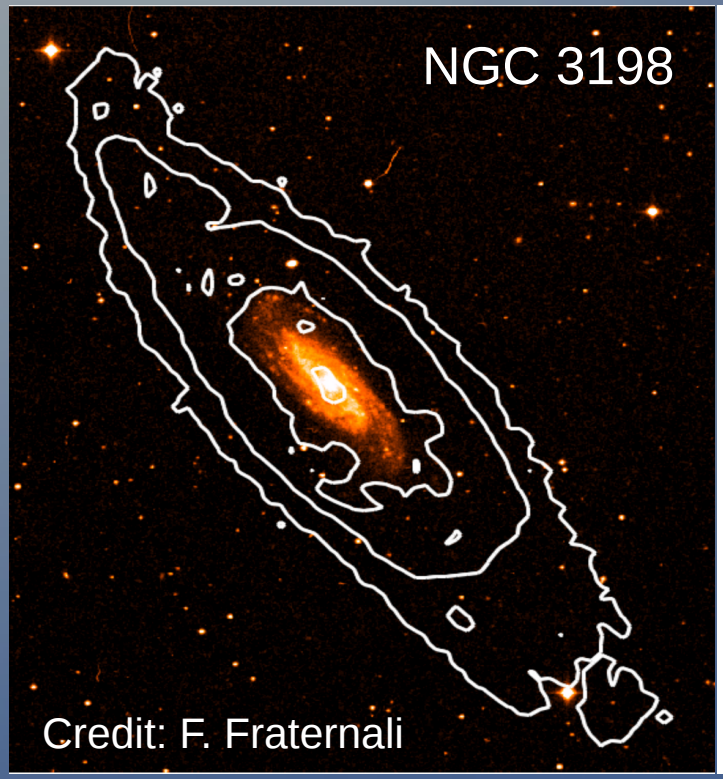


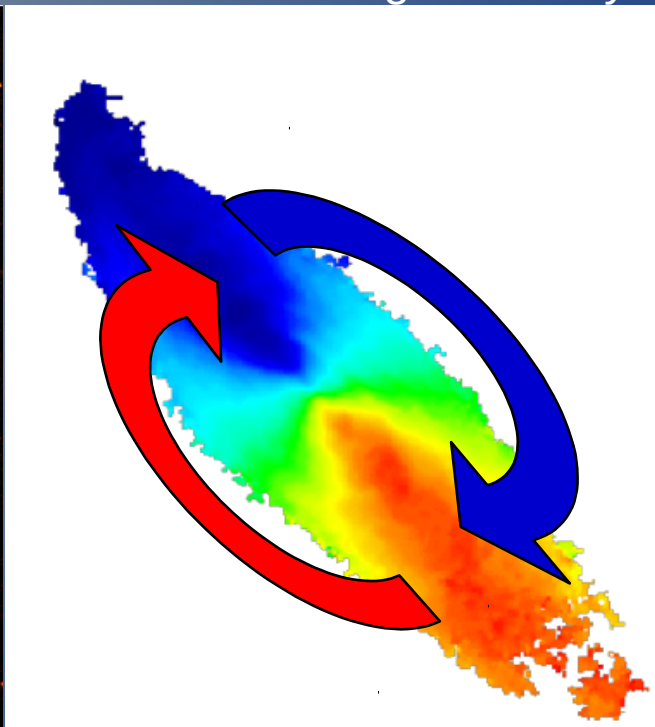
Gas Velocity Dispersion

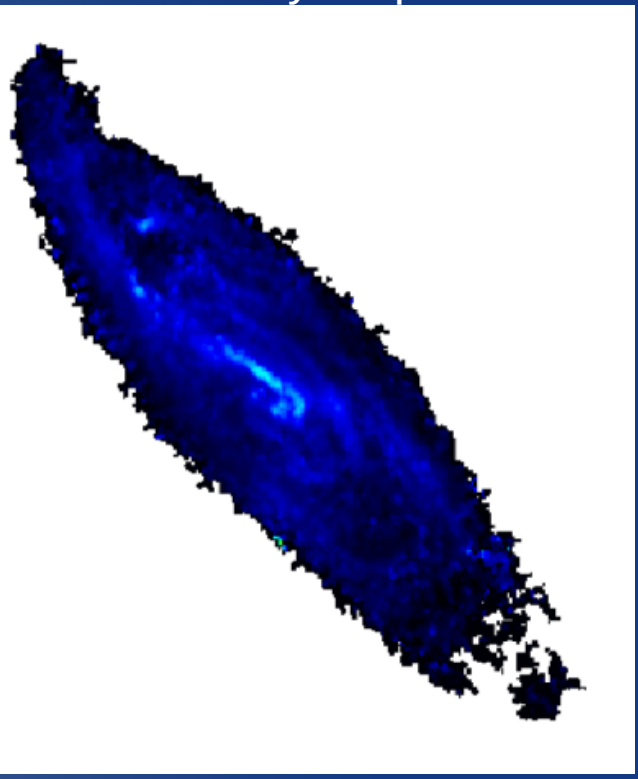


Cold Gas: An excellent kinematical tracer

Stars + Atomic Gas | Gas Line-of-Sight Velocity | Gas Velocity Dispersion





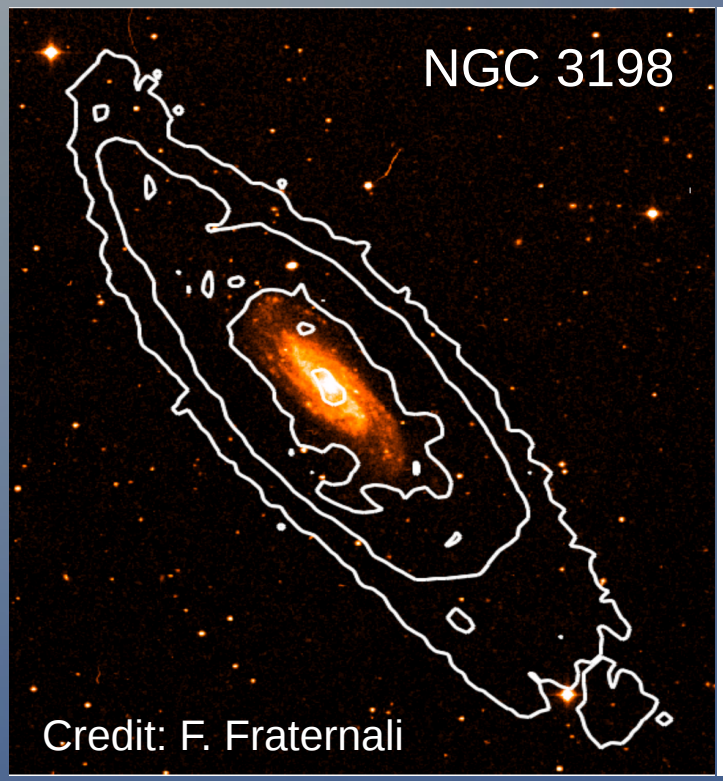


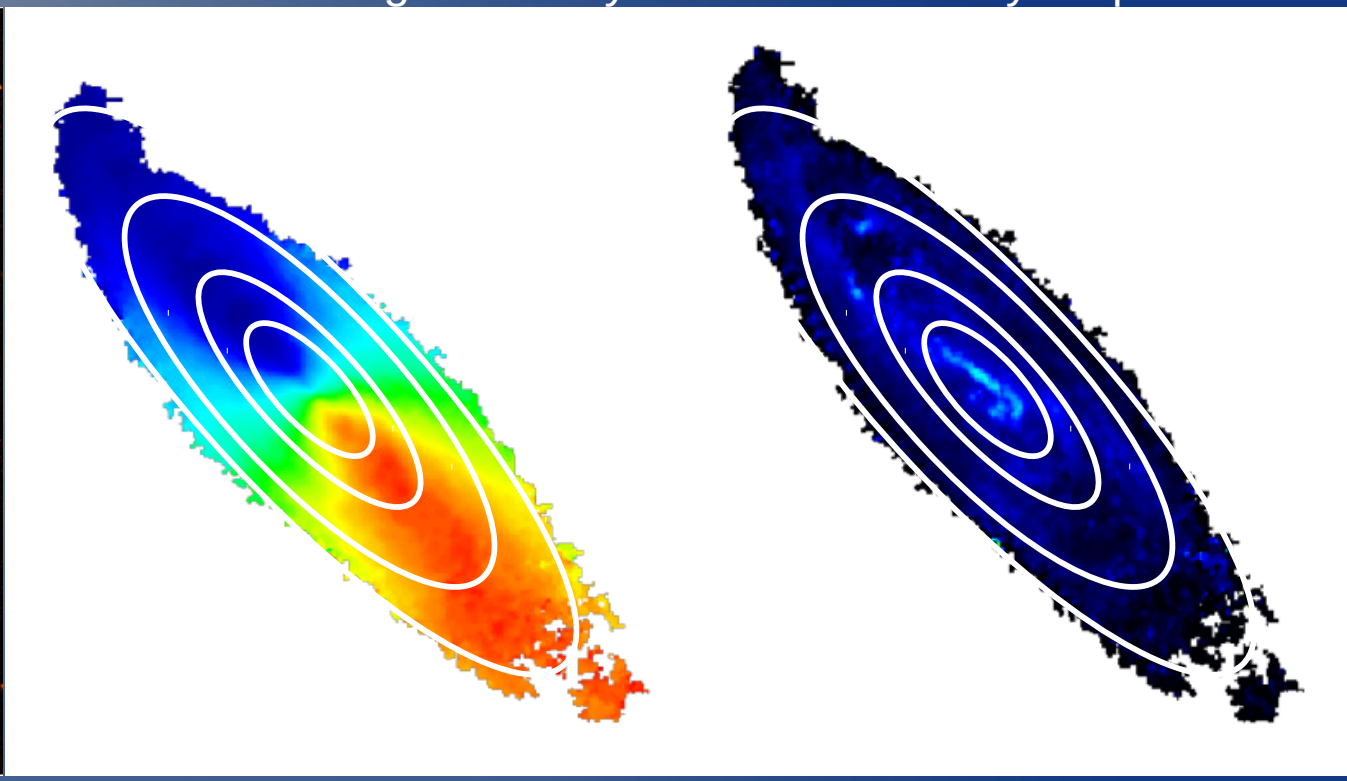
Gas velocity dispersion is small (< 10 km/s),
thus negligible (unlike stellar dynamics):

$$\frac{V_{rot}^2}{R} = -\frac{\partial \Phi_{tot}}{\partial R}$$

Cold Gas: An excellent kinematical tracer

Stars + Atomic Gas | Gas Line-of-Sight Velocity | Gas Velocity Dispersion





Deprojection from sky plane to galaxy plane:

$V_{l.o.s.} = V_{sys} + V_{rot} \sin(i) \cos(\theta)$

$\cos(\theta) = \text{fnc(center, position angle)}$

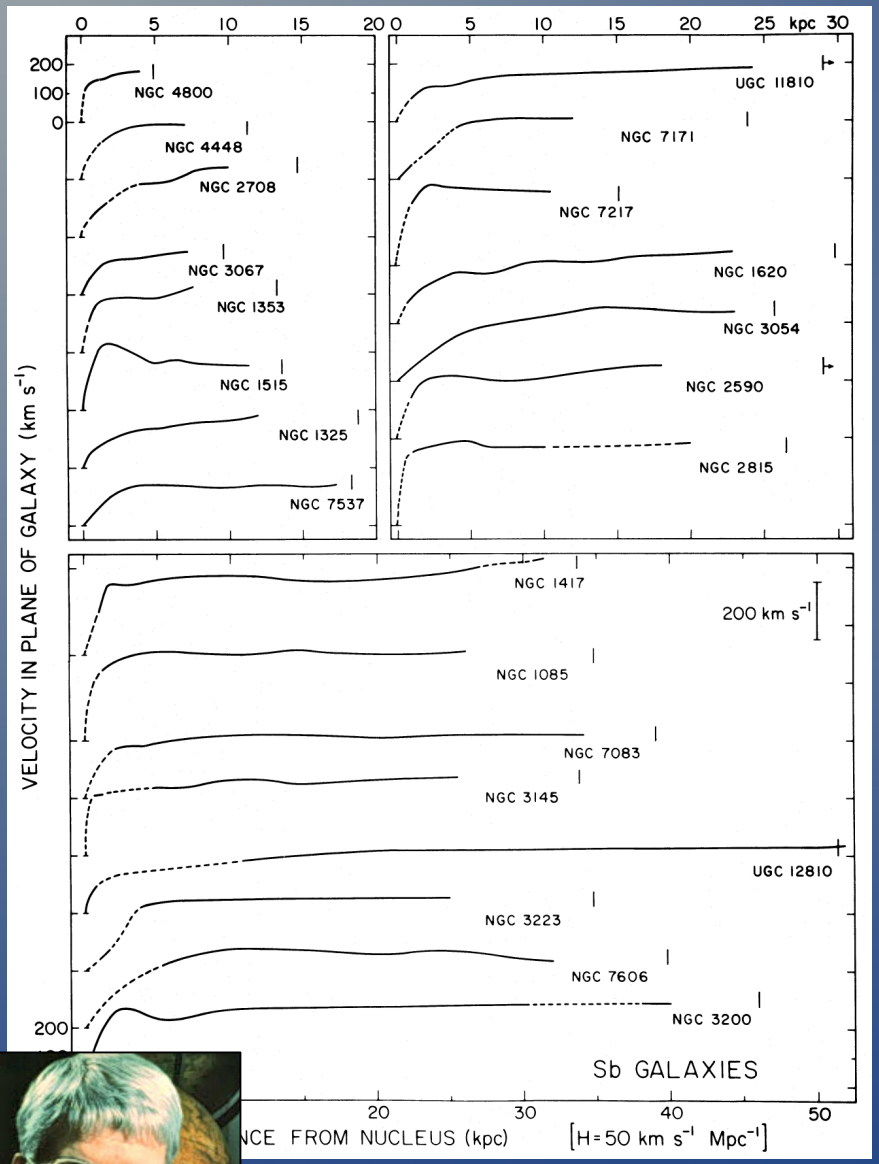
i = disk inclination angle

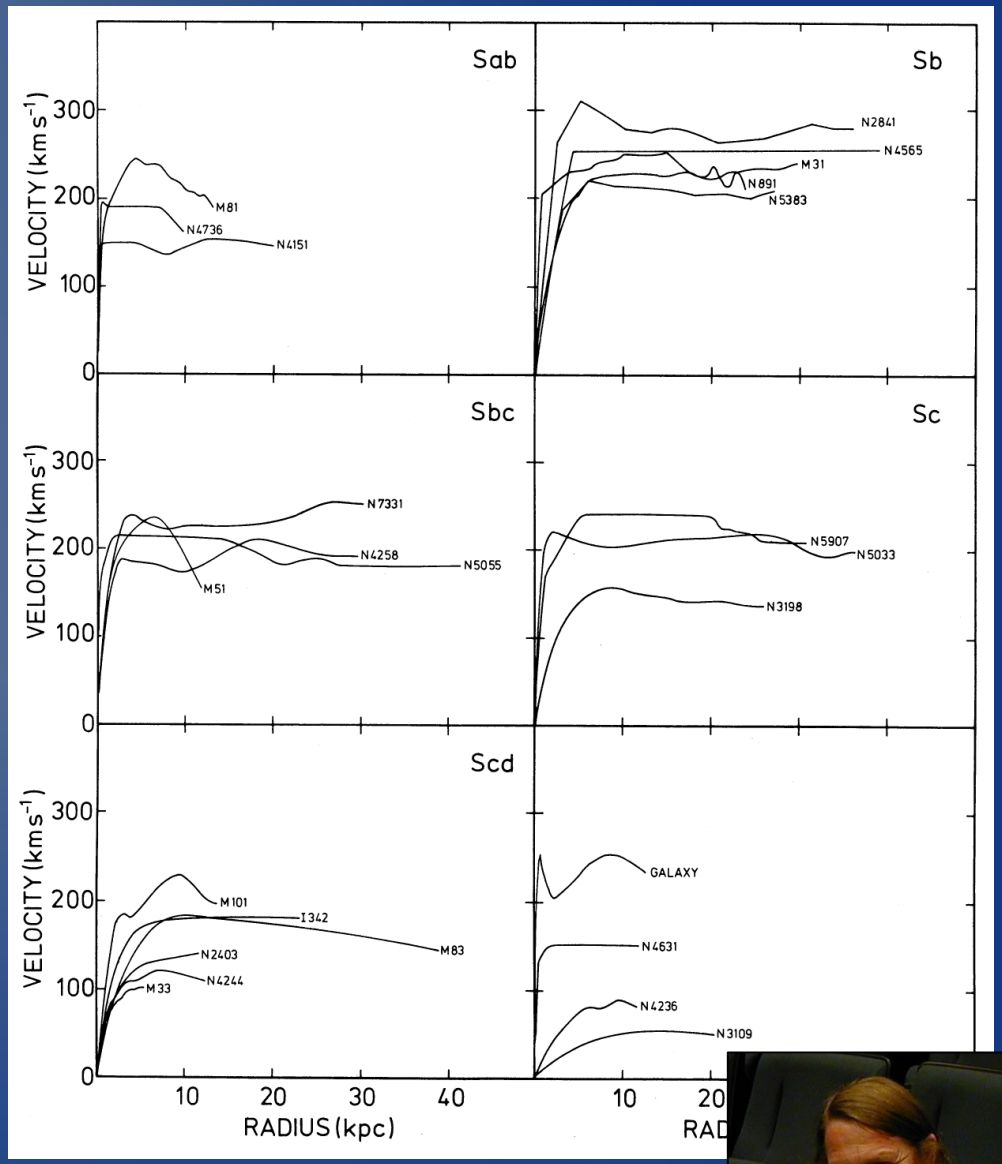
θ = azimuthal angle

V_{sys} = systemic velocity

(Hubble exp + peculiar)

Rotation Curves are Flat at Large Radii





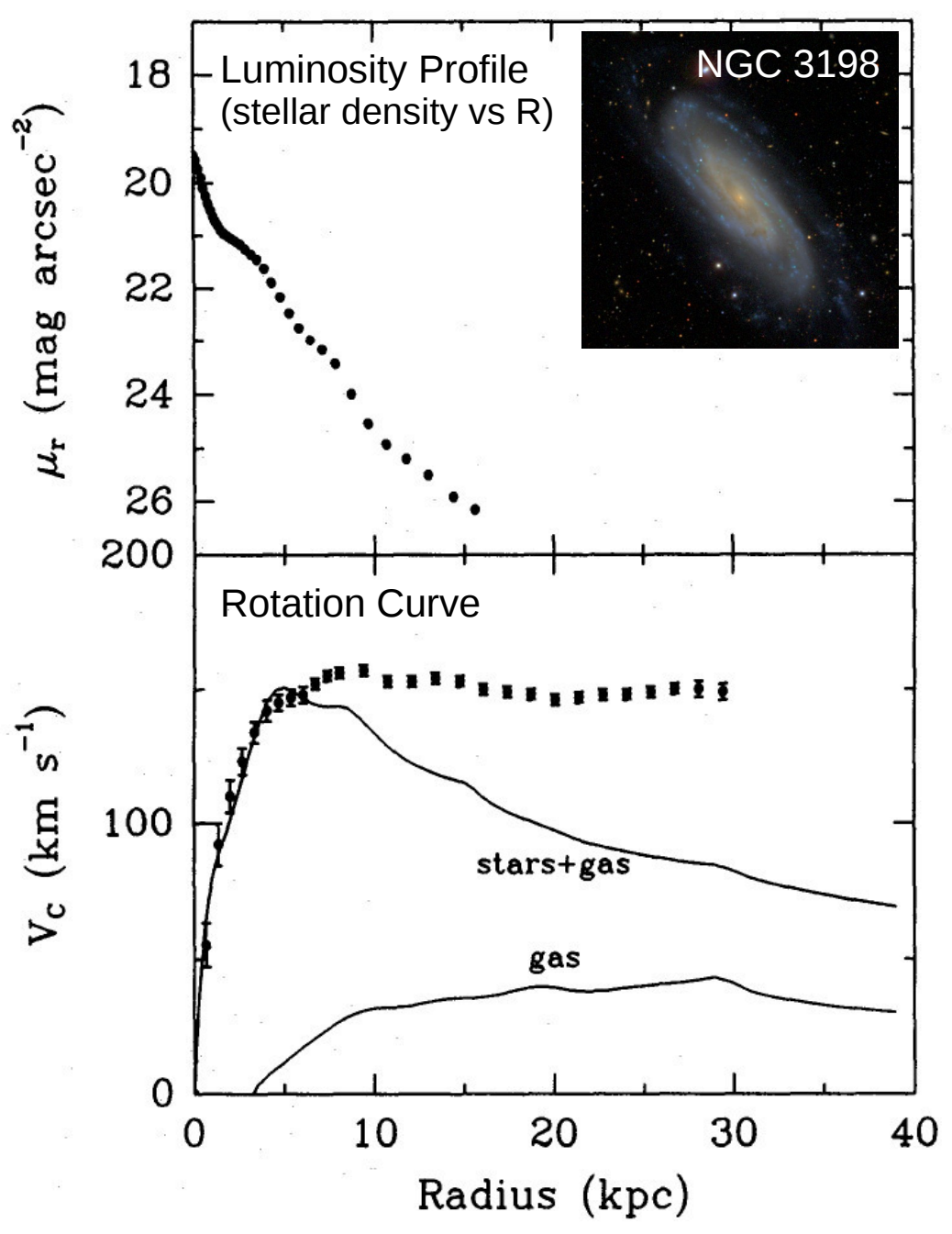
Optical - Ionized Gas (Hα)

V. Rubin et al. (1978, 1982)

Radio - Atomic Gas (HI)

A. Bosma (1978, 1982)

Mass Models for Late-Type Galaxies



Solve Poisson's Equation for each baryonic component (i = stars, gas)

$$\nabla^2 \Phi_i(R,z) = 4\pi G \rho_i(R,z)$$

Assume nominal disk thickness

$$\rho_i(R,z) = \mu_i(R)\,\nu_i(z)$$

Find expected circular velocity

$$\frac{V_i^2(R,z=0)}{R} = -\frac{\partial \Phi_i(R,z=0)}{\partial R}$$

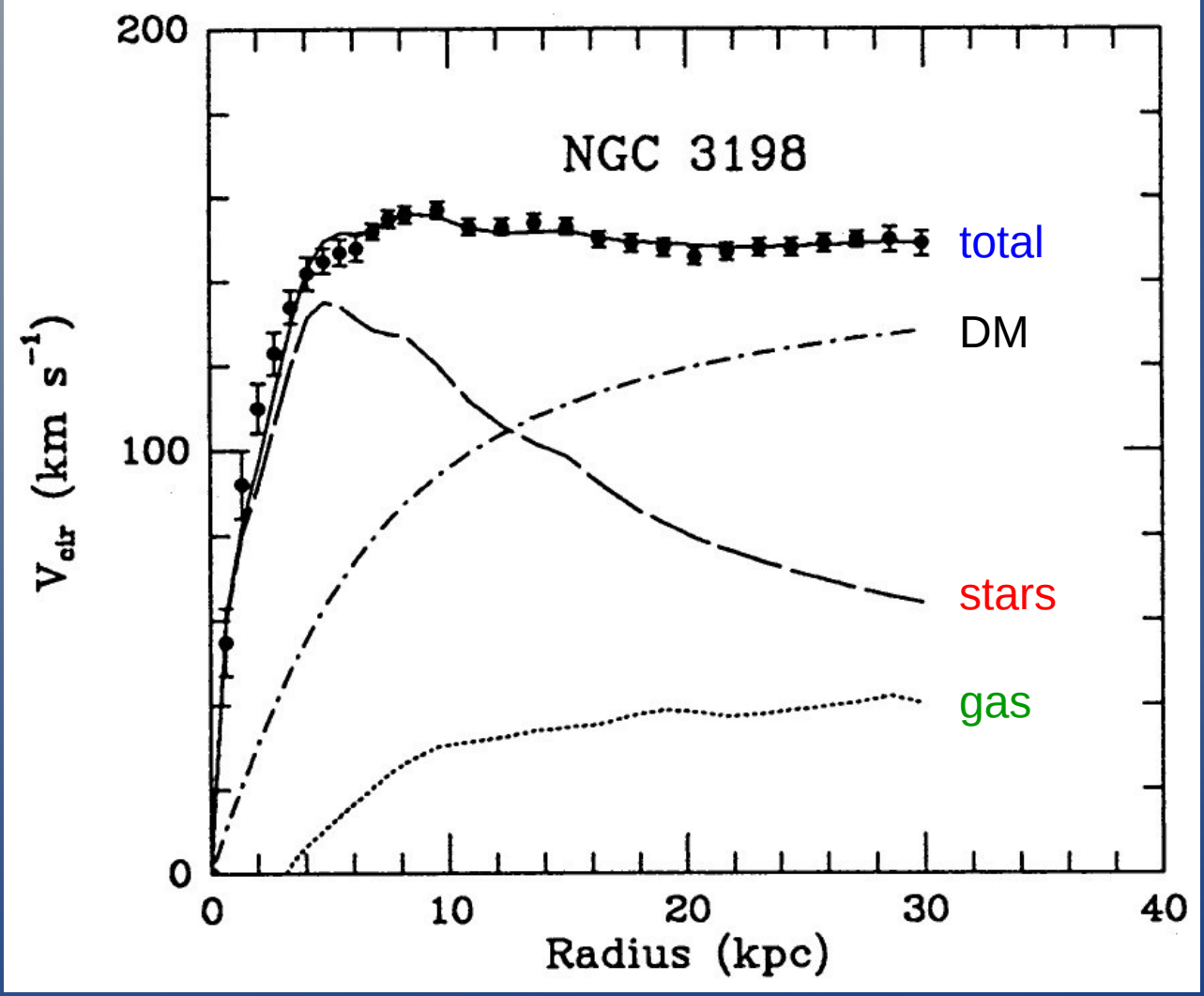
Sum over all baryonic contributions

$$V_b^2(R) = V_{stars}^2(R) + V_{gas}^2(R)$$

van Albada et al. (1985); Begeman (1987)

Mass Models with a Dark Matter Halo

$$V^2_{total} = \Upsilon_* \cdot V^2_{stars} + V^2_{gas} + V^2_{DM}(\rho_0, r_s)$$



Van Albada et al. (1985); Begeman (1987)

Spherical DM halo profile:

$$\rho_{DM} = \rho(r; \rho_0, r_s)$$

Three Free Parameters:

- ρ_0 and r_s for DM halo
- Υ_* = mass-to-light ratio for stars.

Maximum disk:

Max Υ_* to reproduce inner regions with V_{stars}.

Mass Models with a Dark Matter Halo

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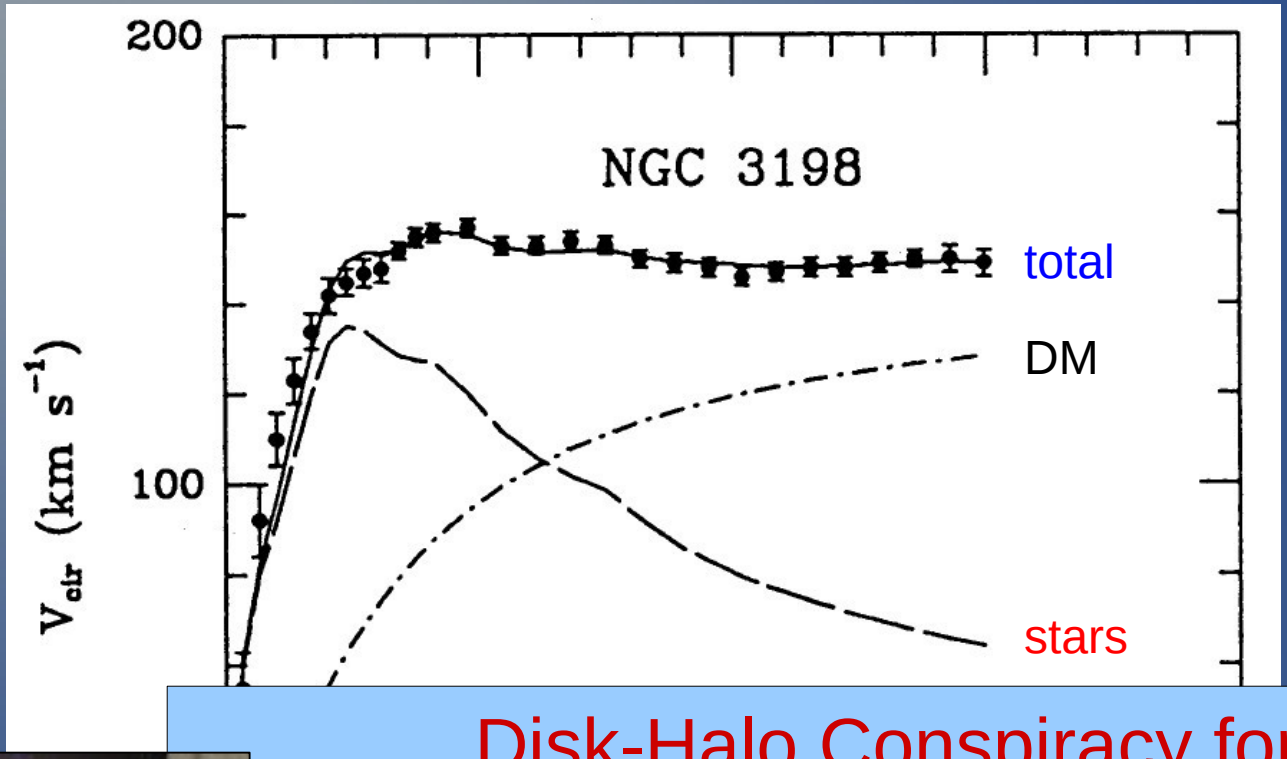

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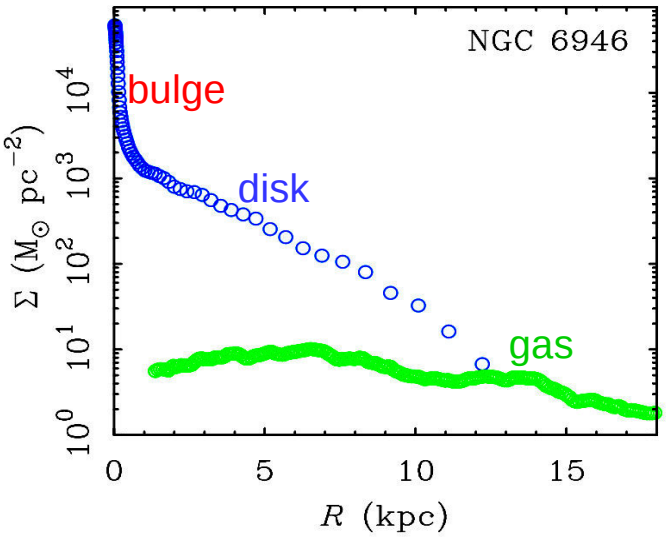
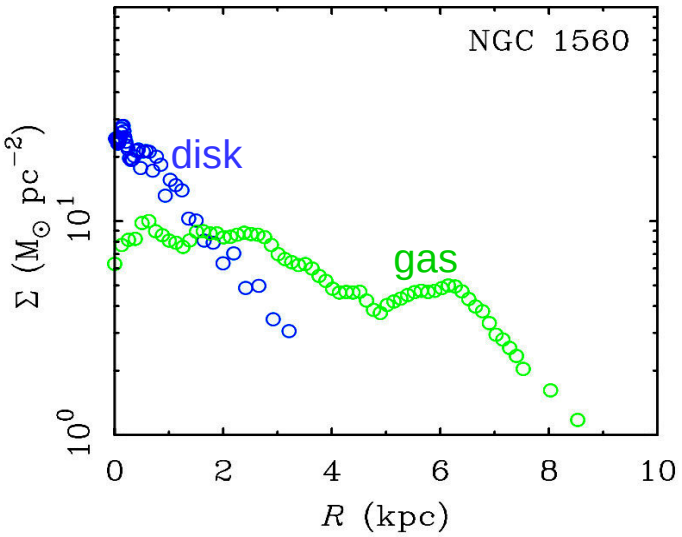
Disk-Halo Conspiracy for LTGs:

"Luminous matter and dark matter seem to conspire to produce the flat observed rotation curves in the outer region."

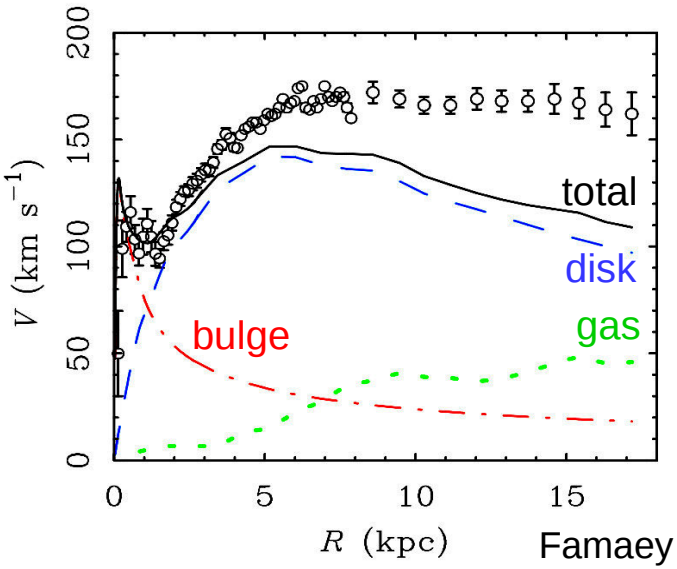
(van Albada & Sancisi 1986)

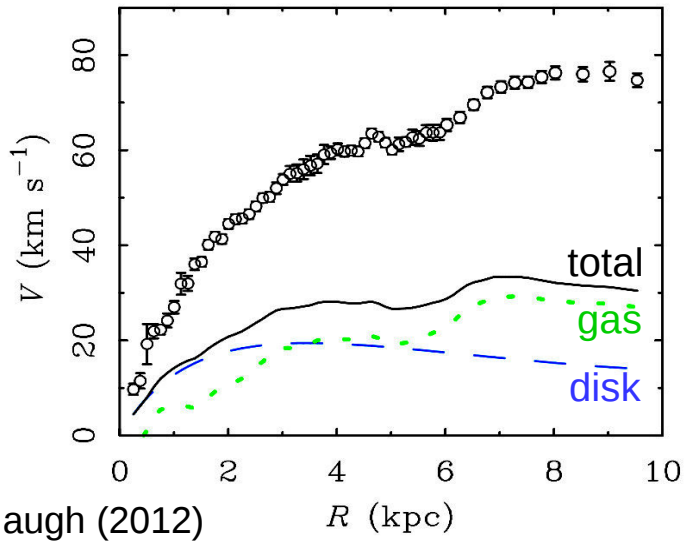
Sancisi's Rule: Baryons-DM Conspiracy?





"For any feature in the luminosity profile of a galaxy there is a corresponding feature in the rotation curve and vice versa" (Sancisi 2004)





Famaey & McGaugh (2012)

Established during 30 years: Kent 1987; Corradi & Capaccioli 1991; Casertano & van Gorkom 1991; Broeils 1992; McGaugh & de Blok 1998; McGaugh 2004, 2005; Noordermeer+2007; Swaters+2009, 2012; Lelli+2010, 2013

SPARC

Spitzer Photometry & Accurate Rotation Curves

Objective: quantify the baryon-DM coupling

Approach: look for purely empirical relations (no assumption on DM halos)

Six Papers: Lelli+(2016a,b,c); McGaugh+(2016); Lelli+(2017a,b)

Public Data: www.astroweb.cwru.edu/SPARC

SPARC

Spitzer Photometry & Accurate Rotation Curves

- HI Rotation Curves for 175 galaxies
  - 30 years of HI obs with radio interferometers
  - PhD theses from the University of Groningen

    Begeman 1987; Broeils 1992; Verheijen 1997; de Blok 1997; Swaters 1999; Noordermeer 2005; Lelli 2013 + other studies
  - Hybrid Hα/HI rotation curves for ~30% sample

    McGaugh+2001; Kuzio de Naray+2006, 2008 + other studies.

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  - Hybrid Hα/HI rotation curves for ~30% sample

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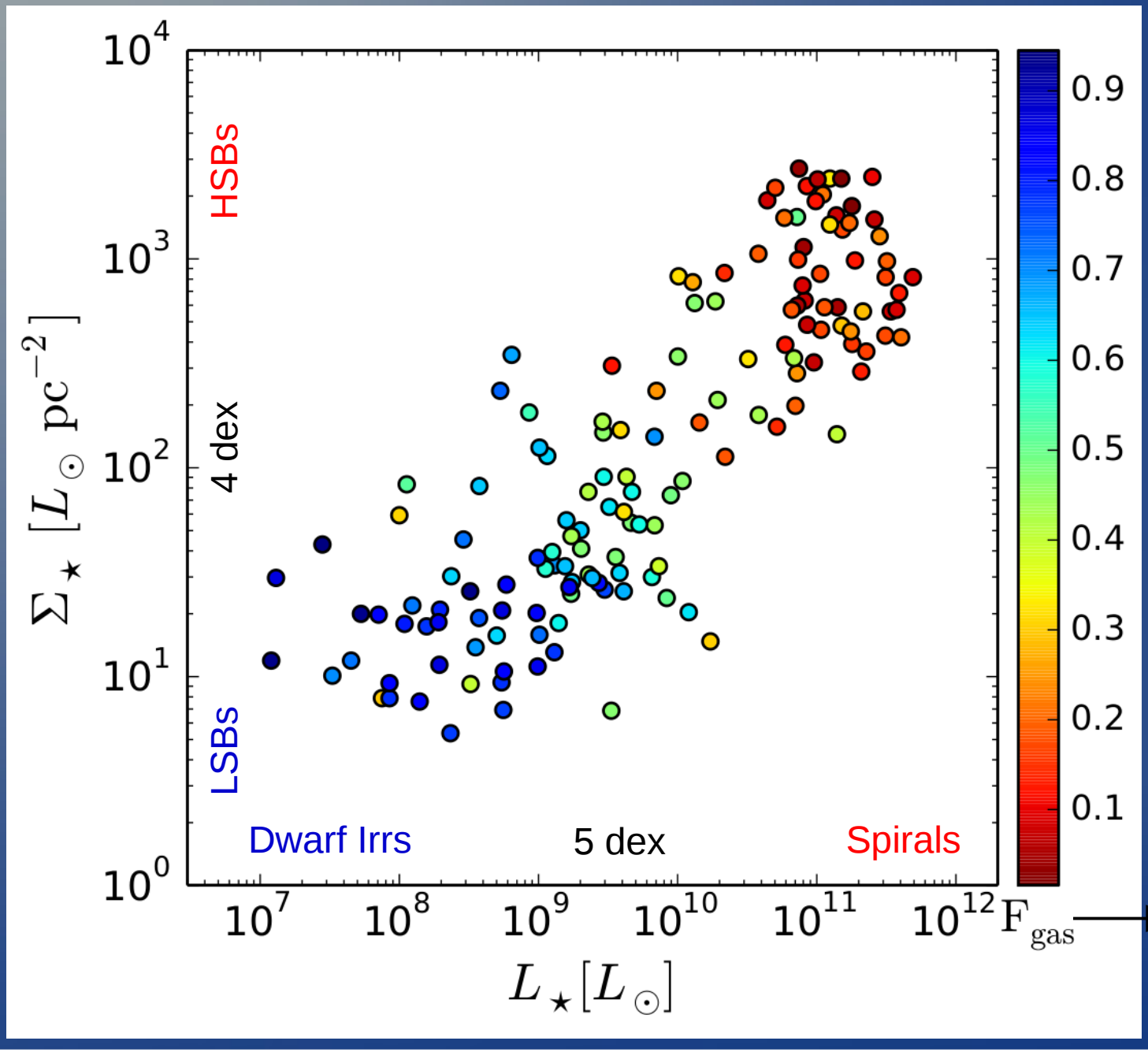
- Homogeneous imaging in the near IR
  - Optimal tracer of the stellar mass: $M_* = \Upsilon_* L$
  - Υ_* depends on stellar populations (age, metals)

    Optical: Υ_* varies strongly (factor > 10).

    NIR: Υ_* shows mild variations (factor~2).

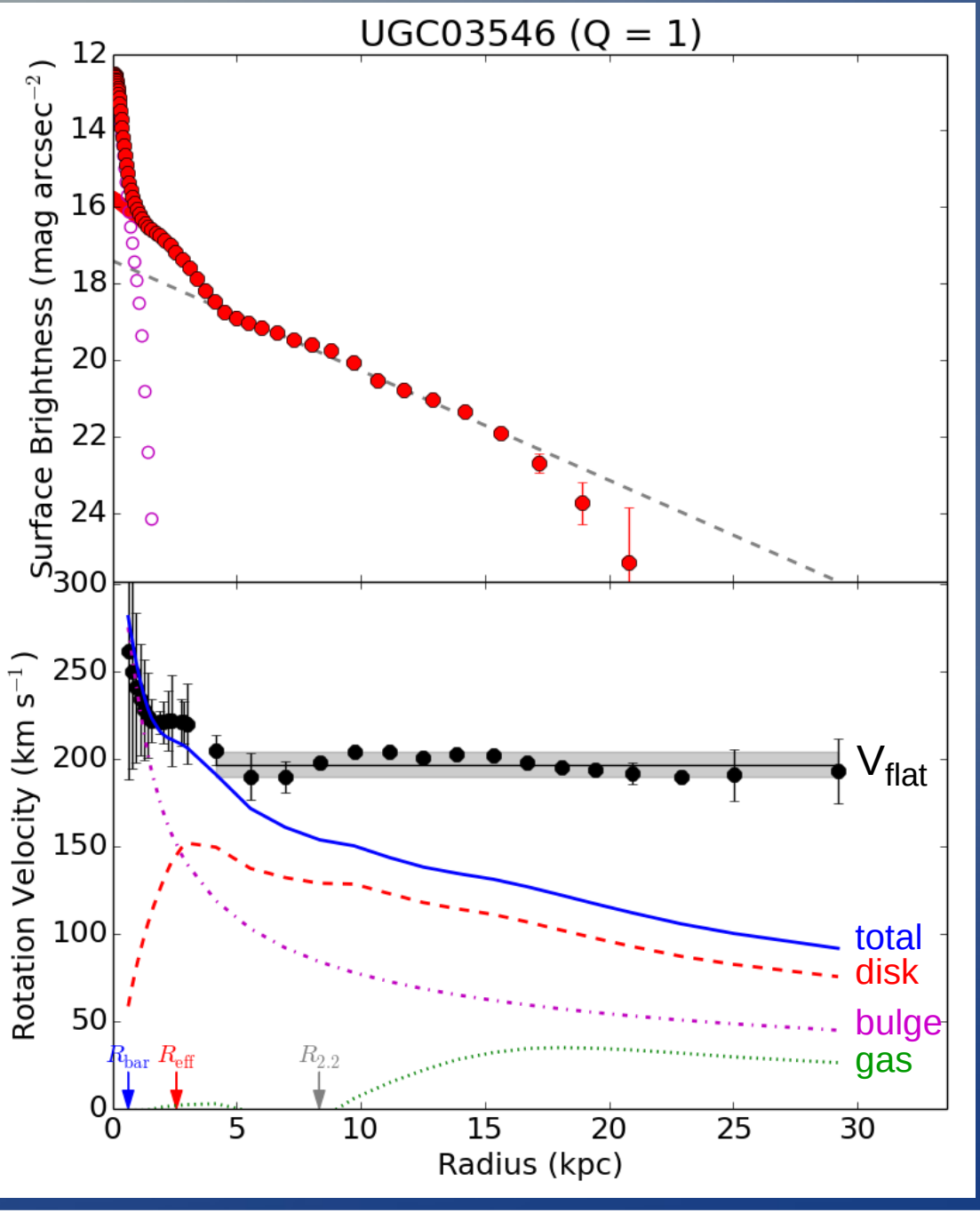
Widest possible range of disk properties

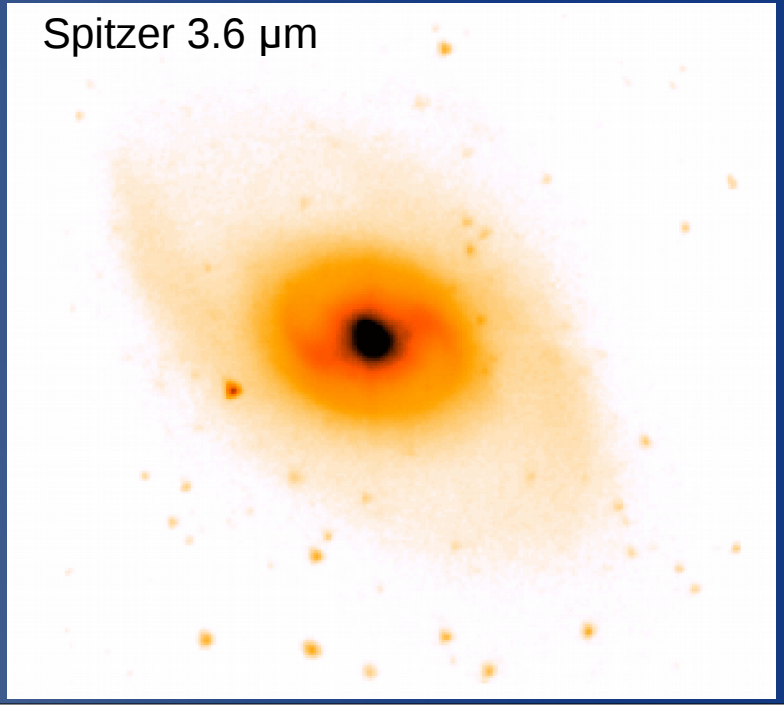


M_{gas}/M_{bar}

Basically any known galaxy type with a rotating HI disk.

Example: High-Mass, High-Density Spiral

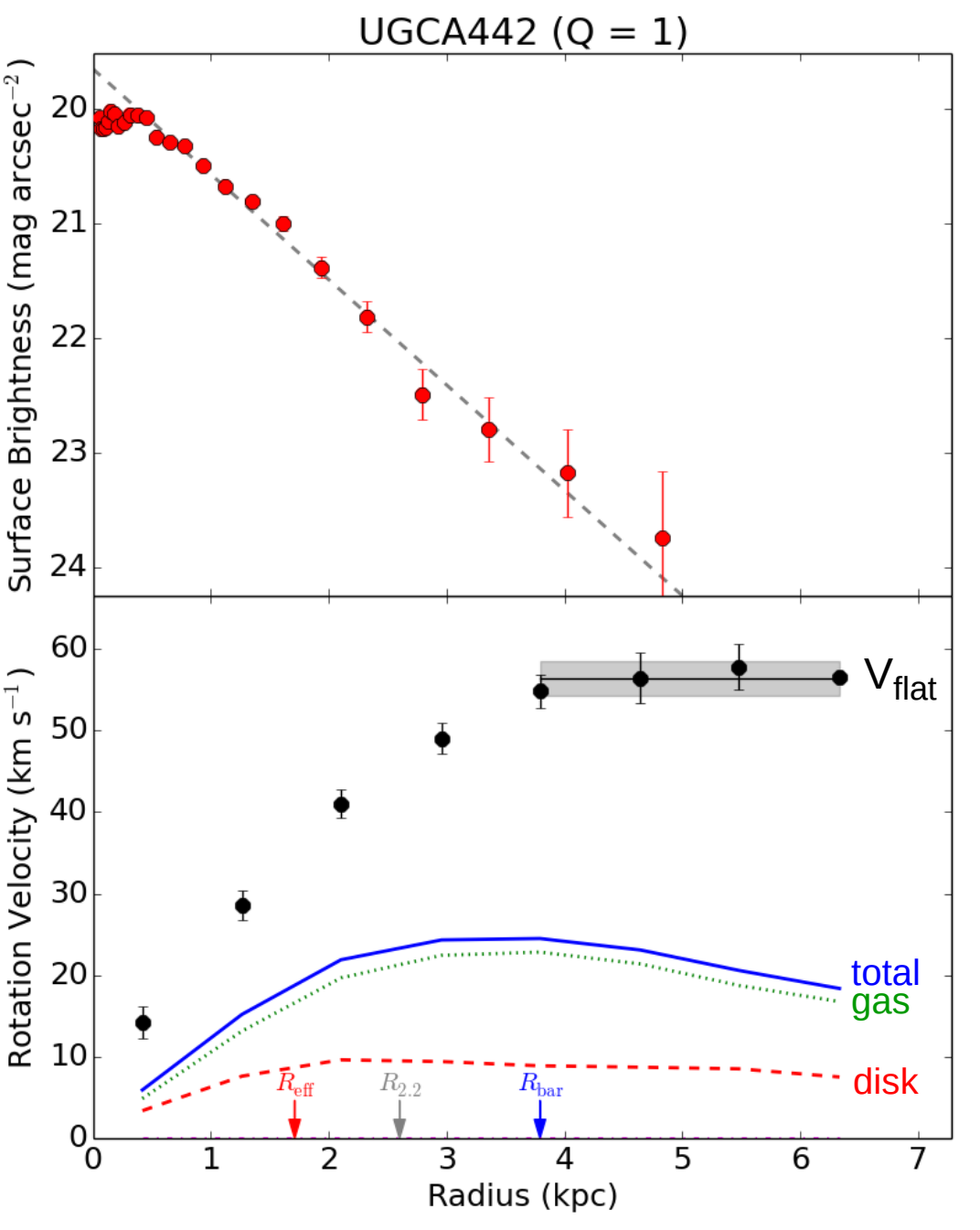


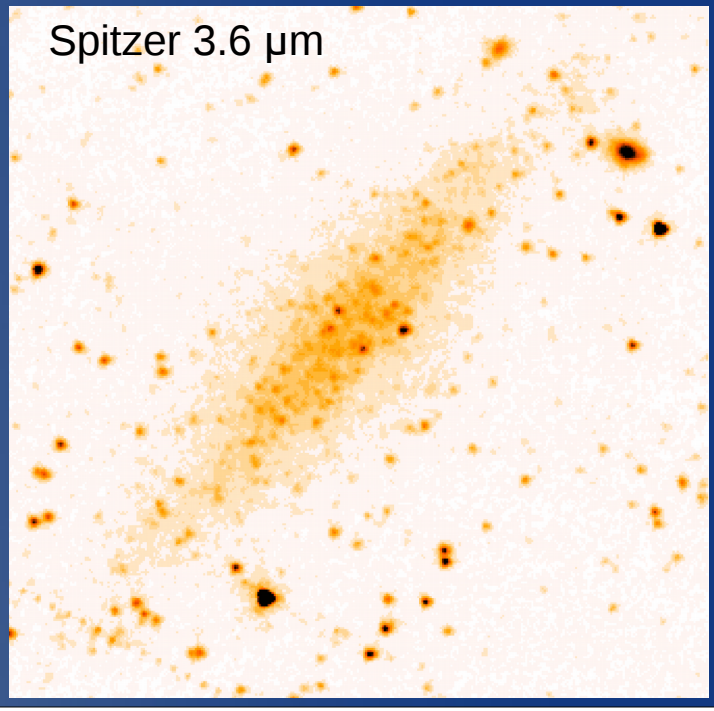


$$\nabla^2\Phi_{bar}(R,z) = 4\pi G\,\rho_{bar}(R,z)$$

- Vertical Structure:
  - Disks: $\exp(-z/h_z)$ with $h_z \propto h_R$
  - Bulges: spherical symmetry
- Stellar mass-to-light ratio:
  - $\Upsilon_* = 0.5\ M_\odot/L_\odot$ for disks
  - $\Upsilon_* = 0.7\ M_\odot/L_\odot$ for bulges

Example: Low-Mass, Low-Density Dwarf





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I. Observational Evidence

2. Empirical Laws of Galactic Rotation

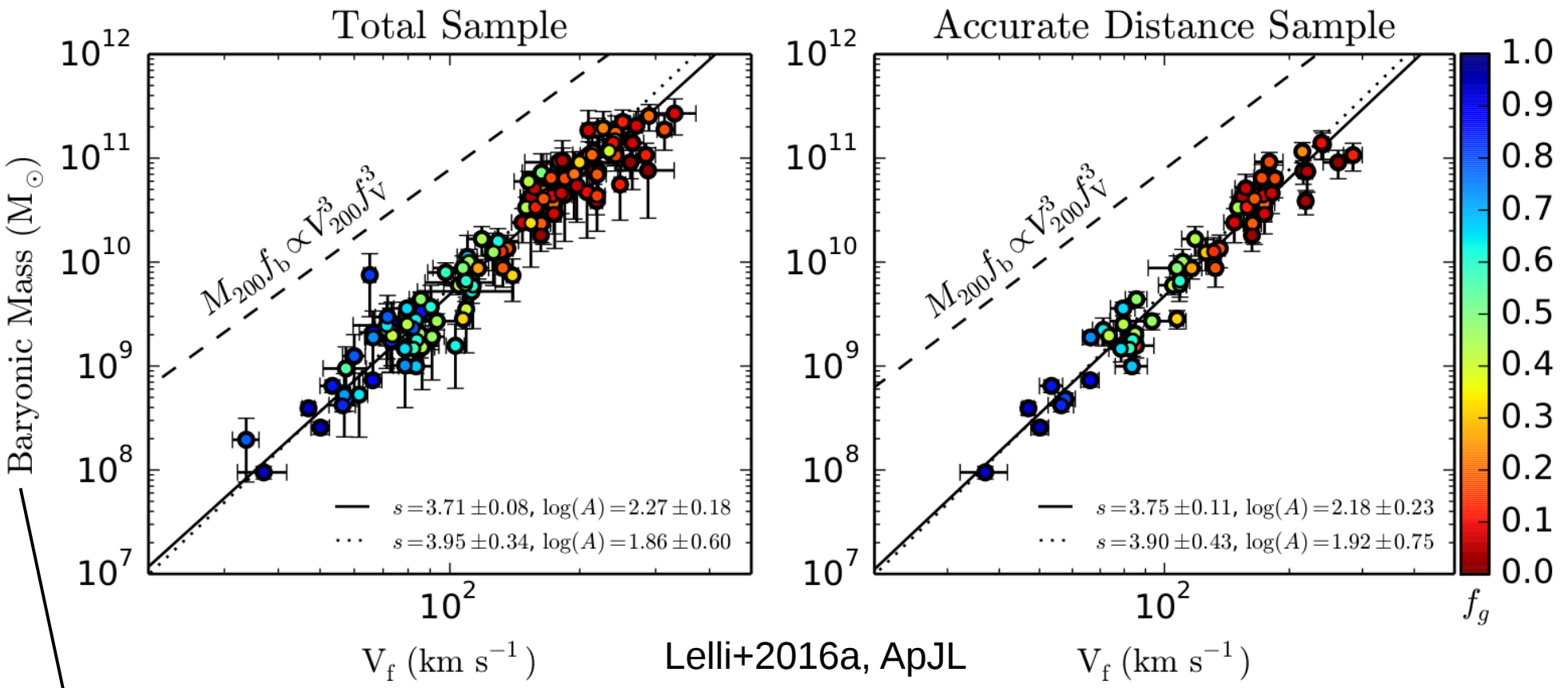
Three Laws of Galactic Rotation:

1st. Global Law: $V_{flat} \propto M_{bar}$

2nd. Central Law: $\Sigma_{dyn}(0) \propto \Sigma_{bar}(0)$

3rd. Local Law: $g_{obs}(R) \propto g_{bar}(R)$

1st Law – Baryonic Tully-Fisher Relation



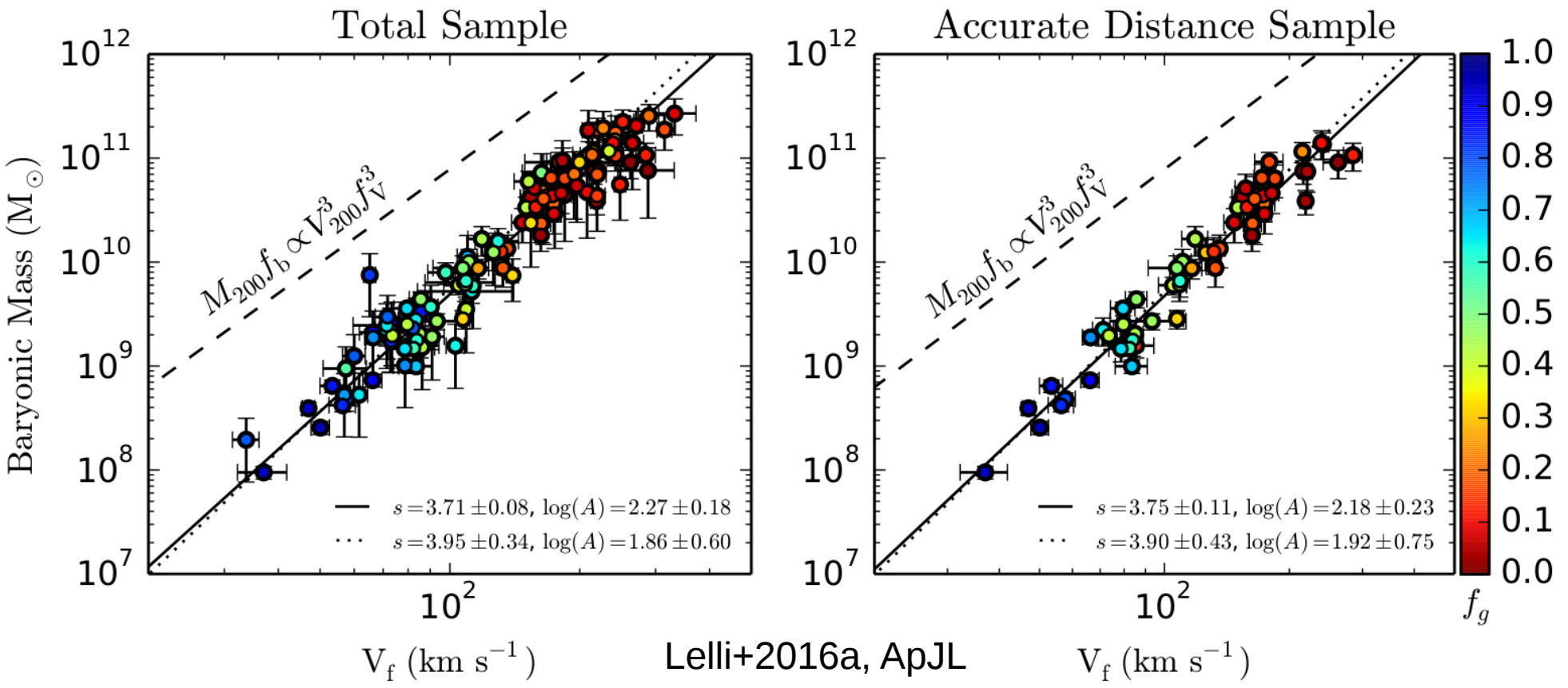
Lelli+2016a, ApJL

$M_* + M_{gas} \propto D^2$

As the data quality increases, the scatter decreases!

Some BTFR studies: Freeman 1999; Walker 1999; McGaugh+2000, 2005, 2012; Verhejen 2001; Bell & de Jong 2001; Geha+2006; Noordermeer & Verheihen+2007; Begum+2008; Avila-Reese+2008; Stark+2009; Trachternach+2010; Gurovich+2010; Catinella+2012; Zaritsky+2014; Papastergis+2016

1st Law – Baryonic Tully-Fisher Relation



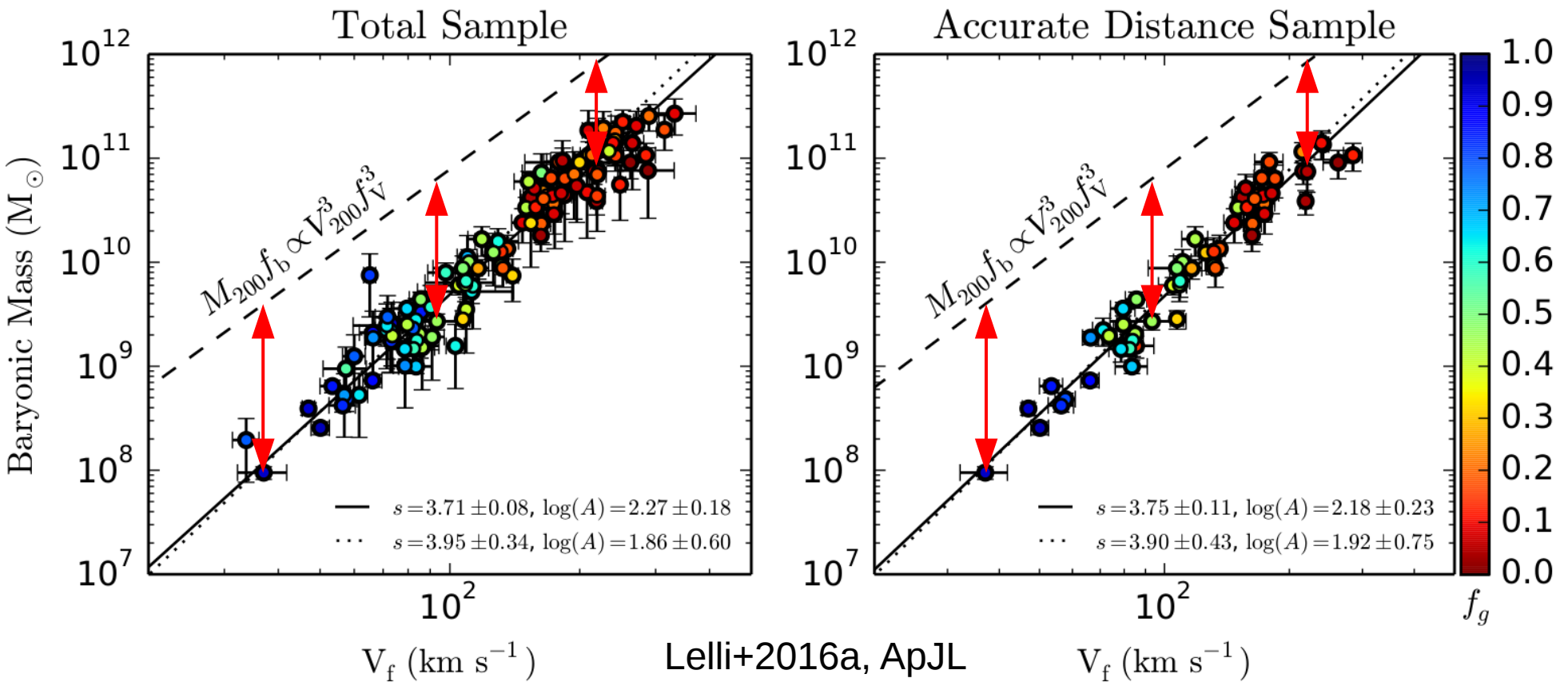
Lelli+2016a, ApJL

In ΛCDM cosmology: $M_{vir} \propto V_{vir}^3$

$f_b = M_b/M_{vir}$ $f_V = V_f/V_{vir}$ → $M_b \propto (f_b/f_V^3)\, V_f^3$

Assuming $f_V=1$ and $f_b=f_{CMB}$ → normalization and slope wrong!

1st Law – Baryonic Tully-Fisher Relation



Lelli+2016a, ApJL

To fix normalization: $f_b < f_{CMB}$ → missing baryons (hot gas?)

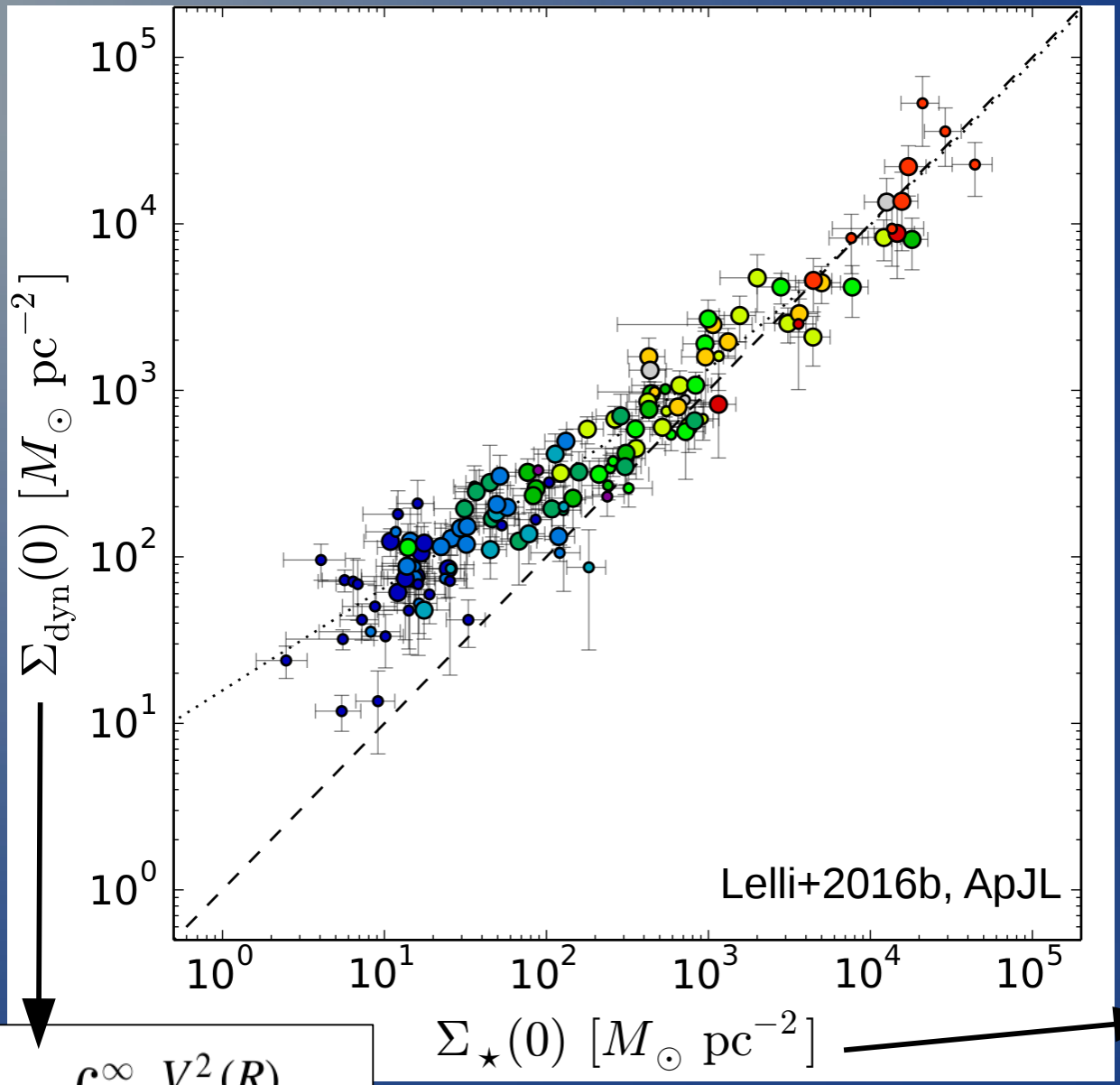
To fix slope: f_b must <u>systematically</u> vary with V_f (or M_{vir})

Small scatter (<25%): galaxy formation must be fine-tuned!

BTFR is only the tip of the iceberg!

There is much more information in the shape of the rotation curve!

2nd Law – Central Density Relation



For all galaxies:

$\Upsilon_{\rm disk} = 0.5\ M_\odot/L_\odot$

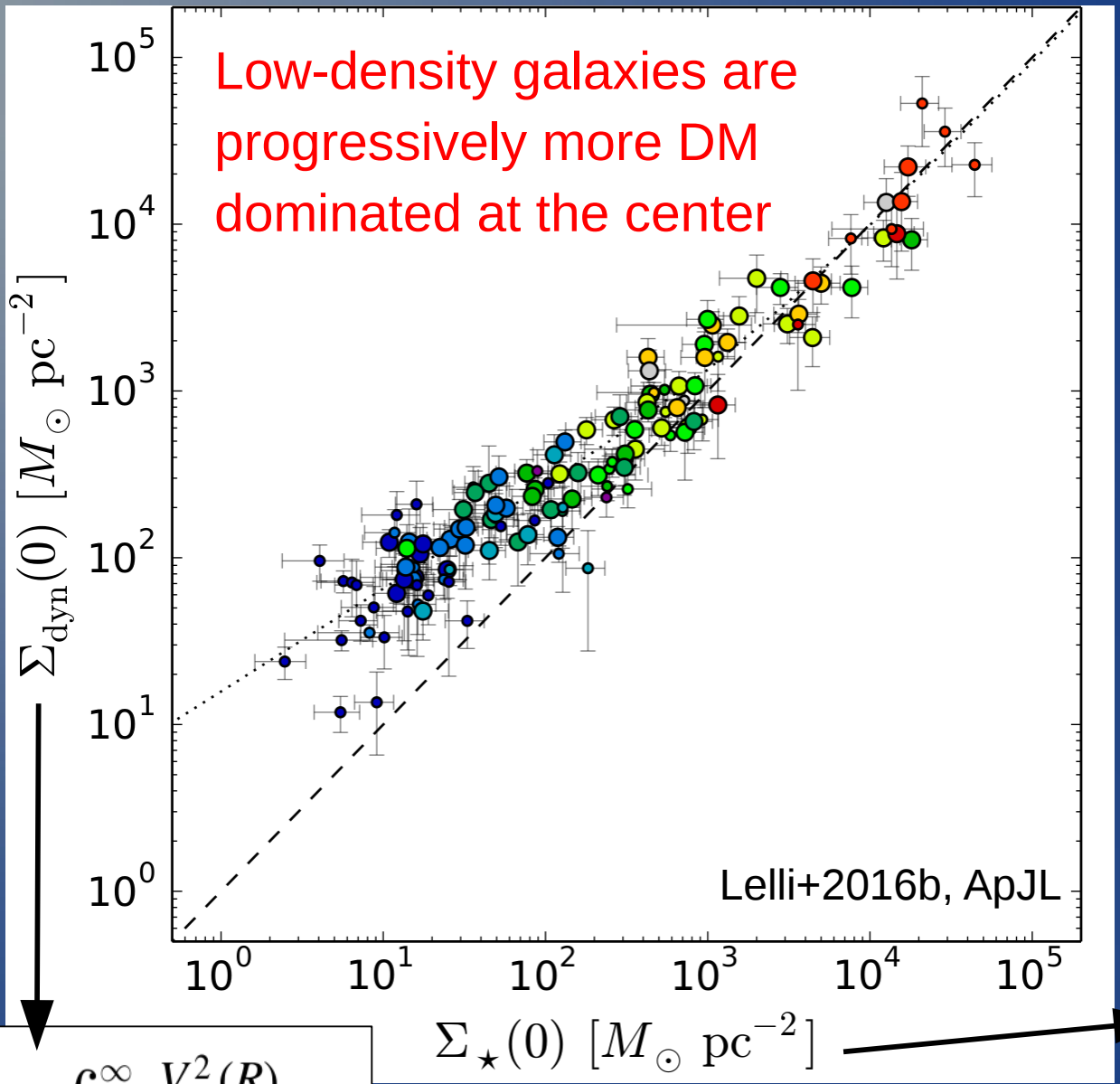
$\Upsilon_{\rm bulge} = 0.7\ M_\odot/L_\odot$

Central Stellar Surface Density

$$\Sigma_{\rm dyn}(0) = \frac{1}{2\pi G}\int_0^\infty \frac{V^2(R)}{R^2}dR,$$

Toomre (1963): Central Dynamical Density for Self-Gravitating Disks.

2nd Law – Central Density Relation



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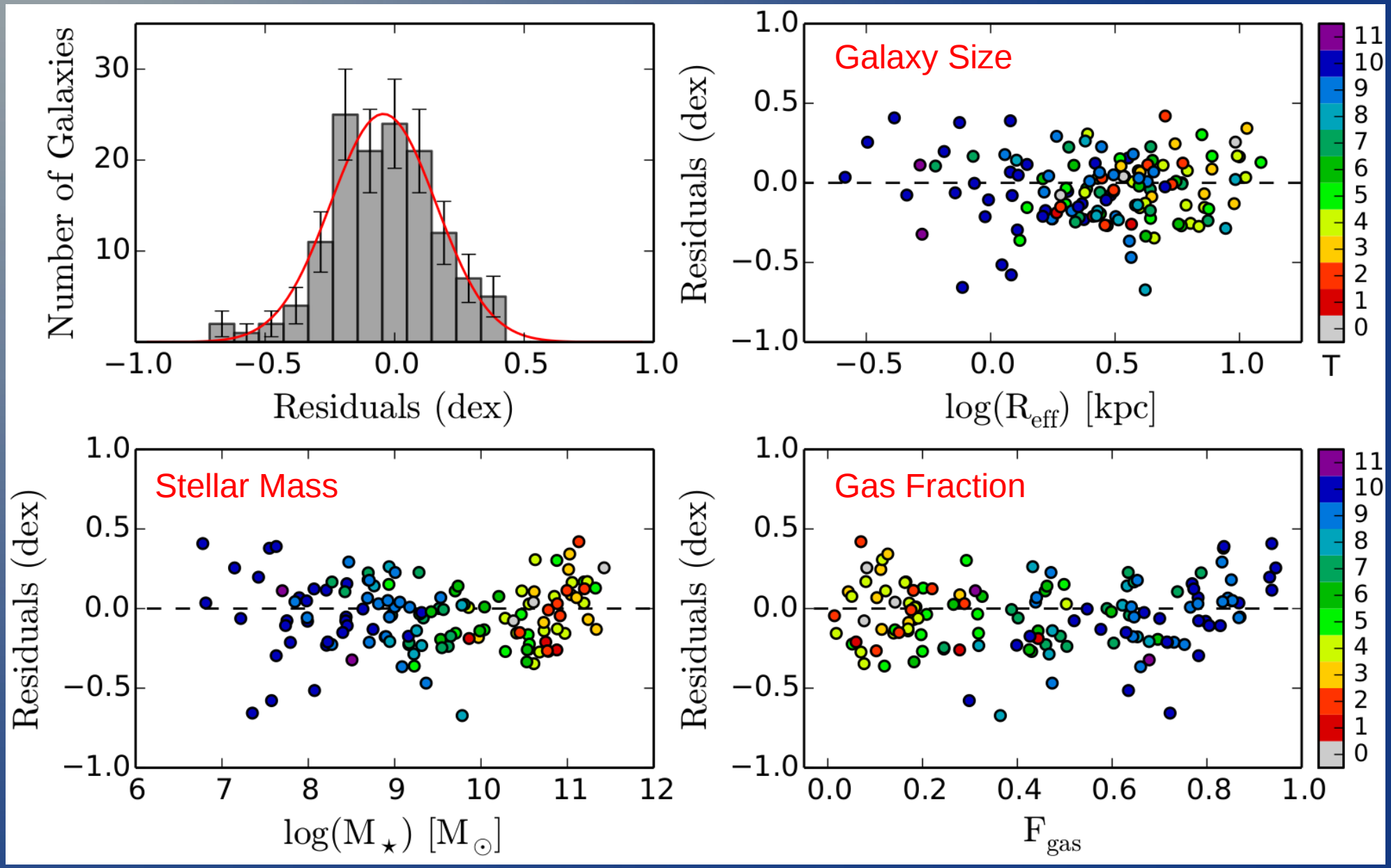
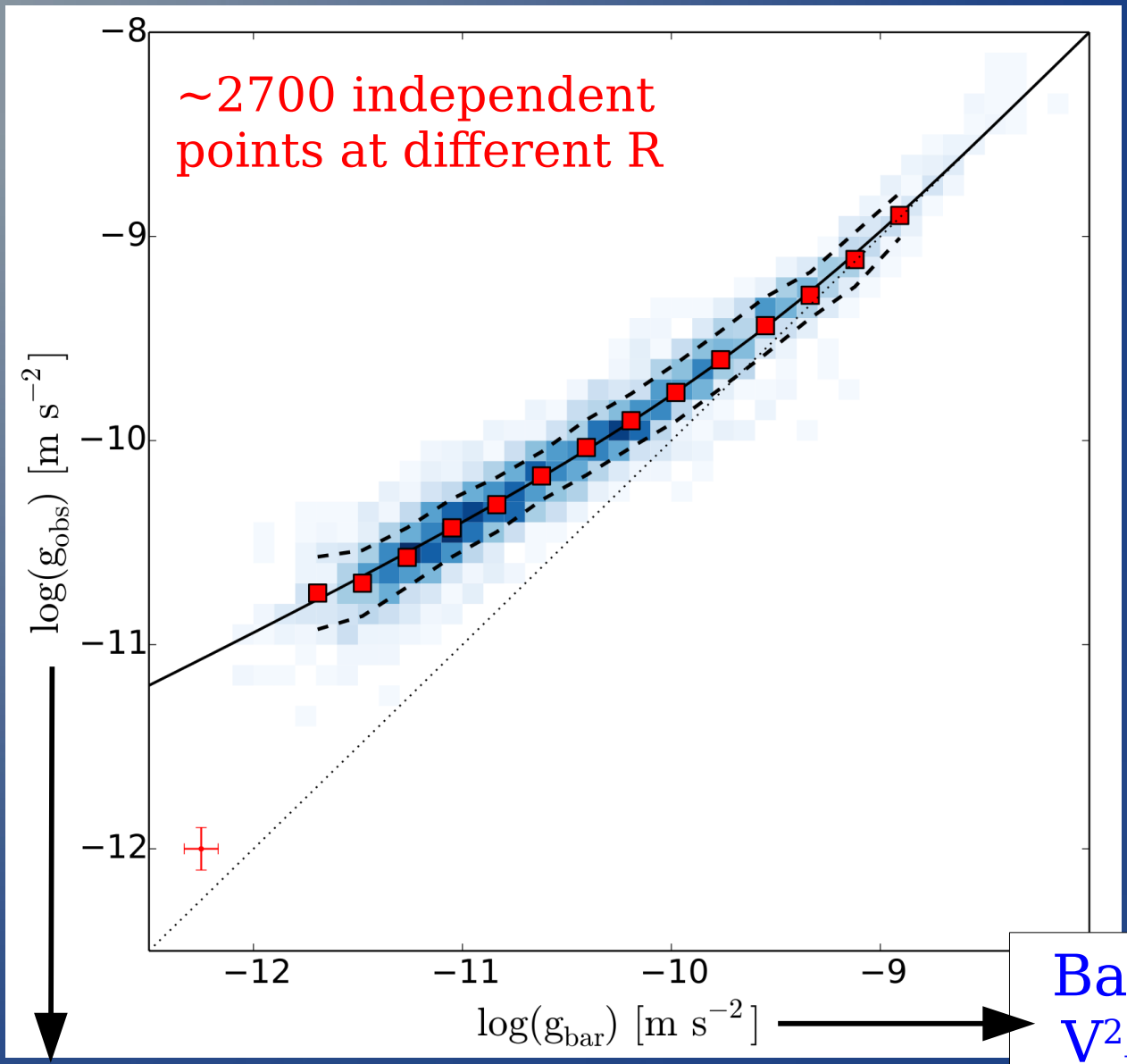
Toomre (1963): Central Dynamical Density for Self-Gravitating Disks.

2nd Law – No Residual Correlations



Lelli et al. (2016b, ApJL). See also Milgrom (2016, PRL) for a possible MOND explanation.

3rd Law – Radial Acceleration Relation



For all galaxies:

$\Upsilon_{disk} = 0.5\ M_\odot/L_\odot$

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McGaugh+2016, PRL

Lelli+2017a, ApJ

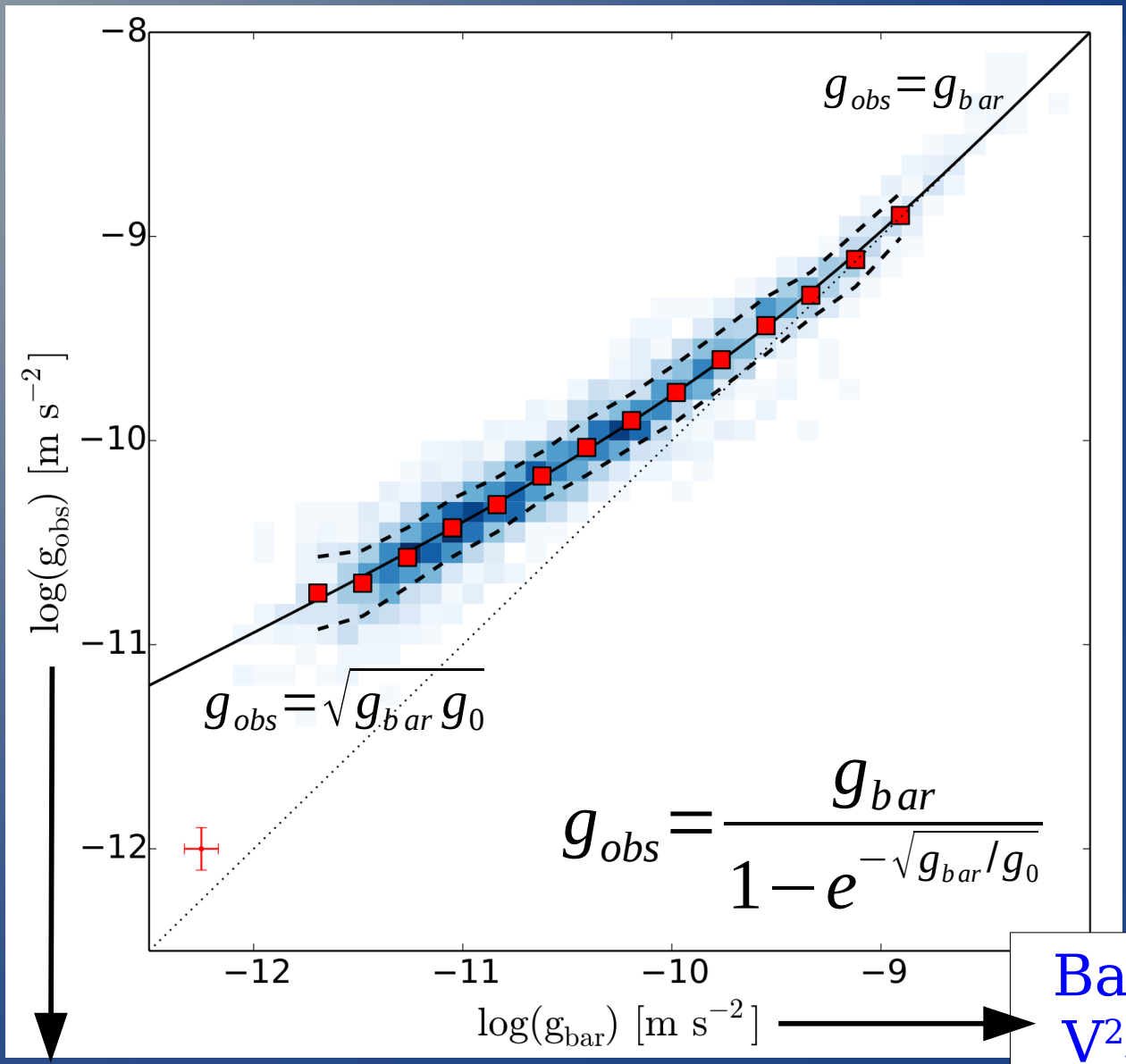
Total Acceleration: $V^2_{obs}/R = -\nabla\Phi_{tot}$

Baryonic Force:

$V^2_{bar}/R = -\nabla\Phi_{bar}$

$\nabla^2\Phi_{bar} = 4\pi G\ \rho_{bar}$

3rd Law – Radial Acceleration Relation



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McGaugh+2016, PRL

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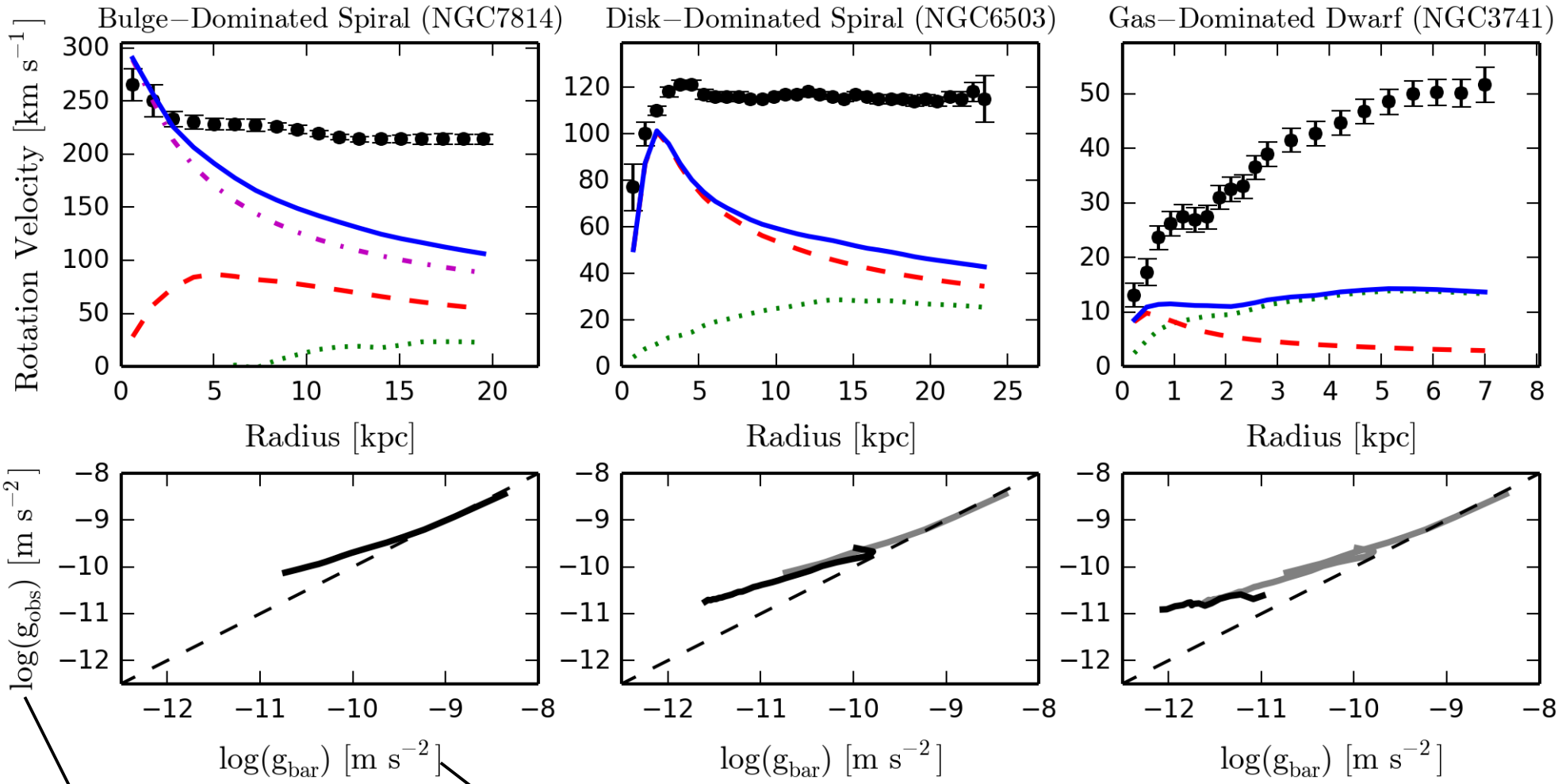
Baryonic Force:

$V^2_{bar}/R = -\nabla\Phi_{bar}$

$\nabla^2\Phi_{bar} = 4\pi G\, \rho_{bar}$

Very different galaxies but ONE relation



$V^2_{obs}/R = -\nabla\Phi_{tot}$

$V^2_{bar}/R = -\nabla\Phi_{bar}$

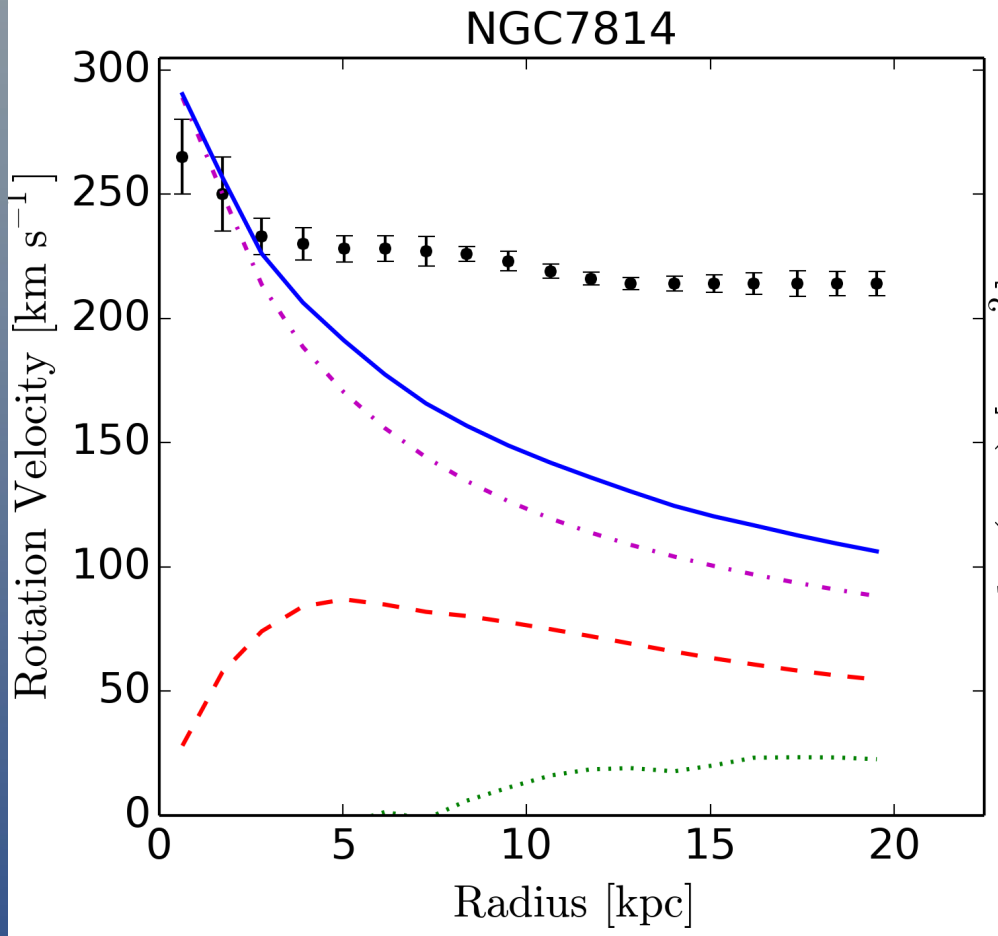
$\nabla^2\Phi_{bar} = 4\pi G\,\rho_{bar}$

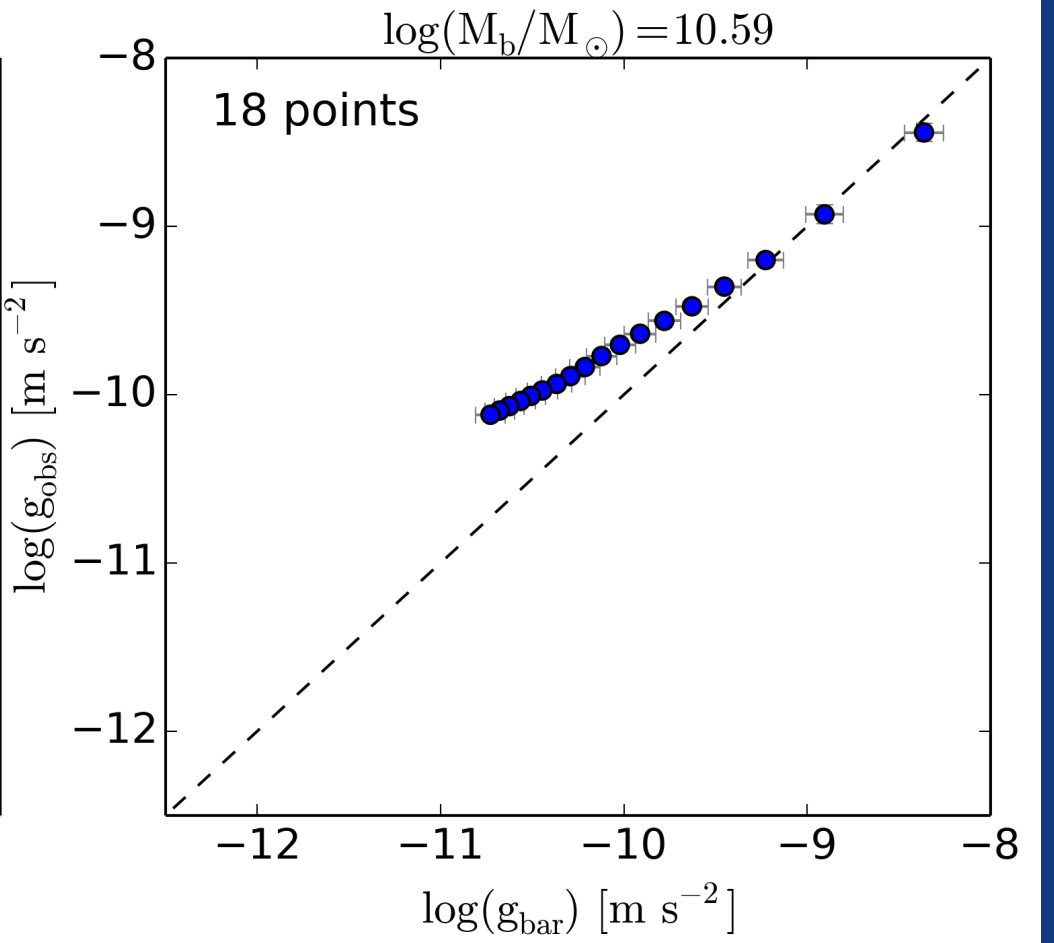
McGaugh, Lelli, Schombert (2016)

Building up the Radial Acceleration Relation

Large Diversity in Rotation Curves

Regularity in Acceleration Plane





Video available at astroweb.cwru.edu/SPARC/

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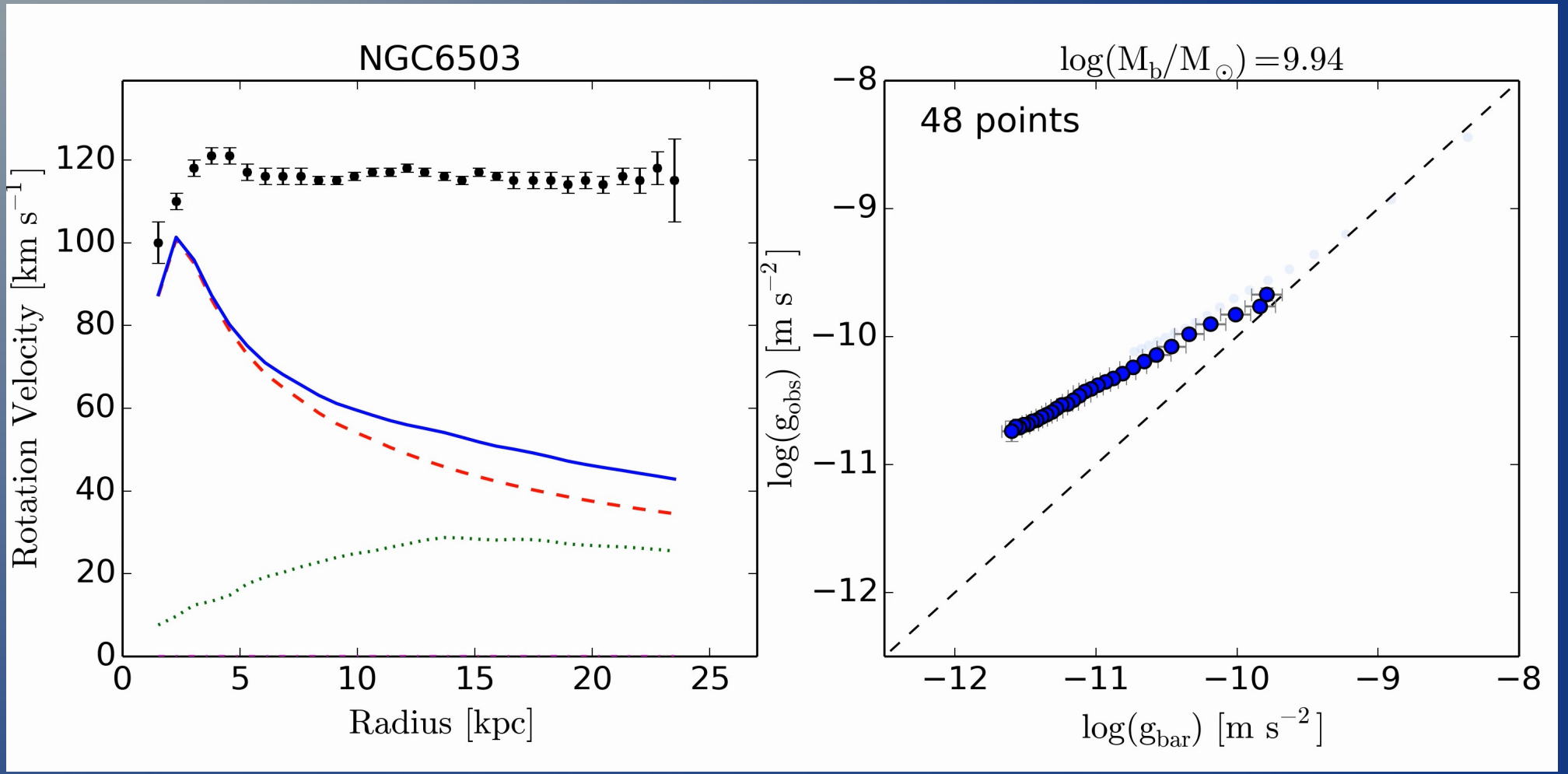
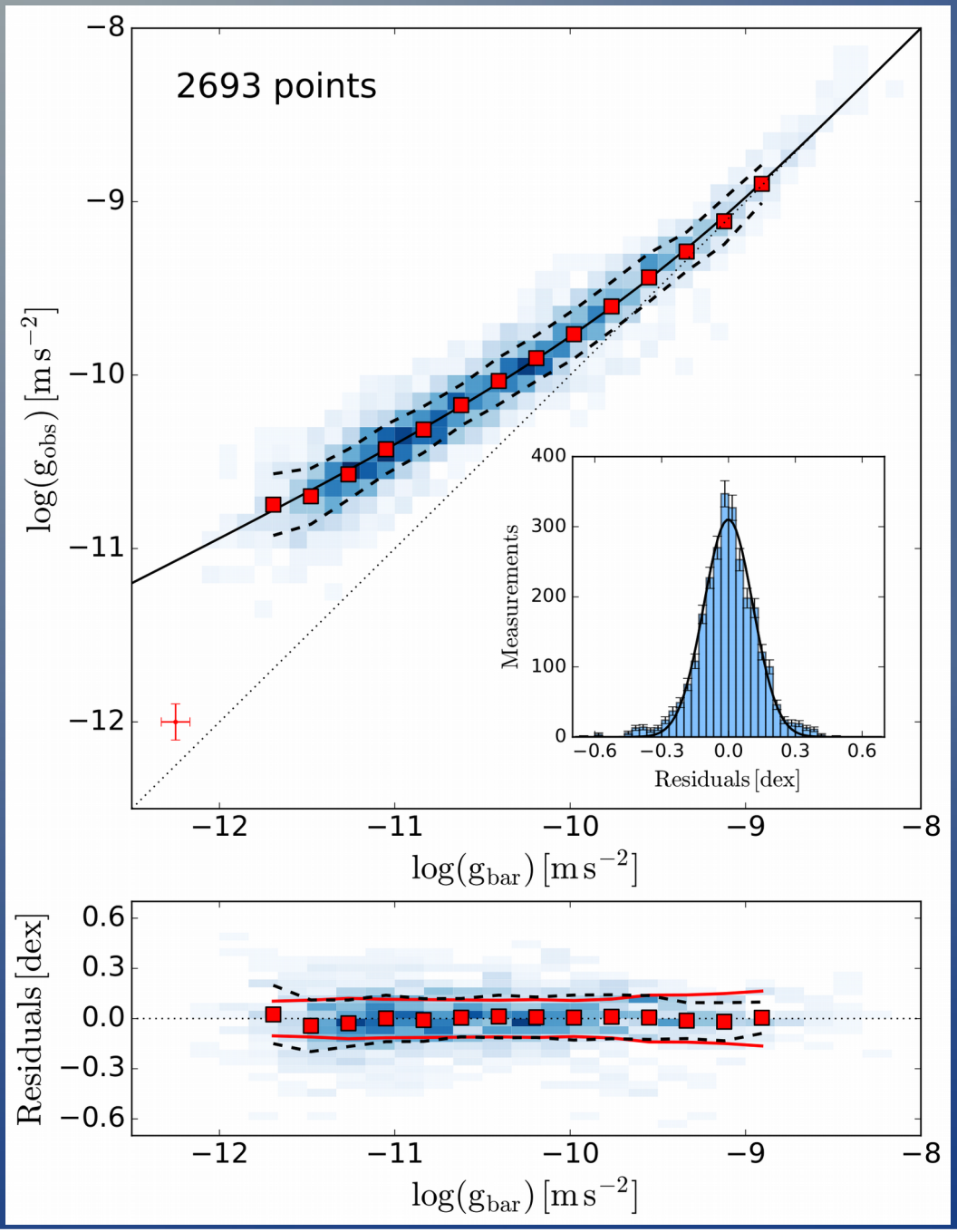
Large Diversity in Rotation Curves

Regularity in Acceleration Plane



Video available at astroweb.cwru.edu/SPARC/

Is There Any Intrinsic Scatter?



Uncertainties drive scatter!

$err(g_{bar}) \rightarrow \Upsilon_*$, 3D geometry

$err(g_{obs}) \rightarrow$ Dist, Inc, V_{rot}

$$\sigma_{obs}^2 = \sigma_{err}^2 + \sigma_{int}^2$$

$\sigma_{obs} \rightarrow$ measured rms

$\sigma_{err} \rightarrow$ error propagation

$\sigma_{int} \rightarrow$ consistent with zero!

McGaugh+2016, PRL; Lelli+2017, ApJ

Three Laws of Galactic Rotation:

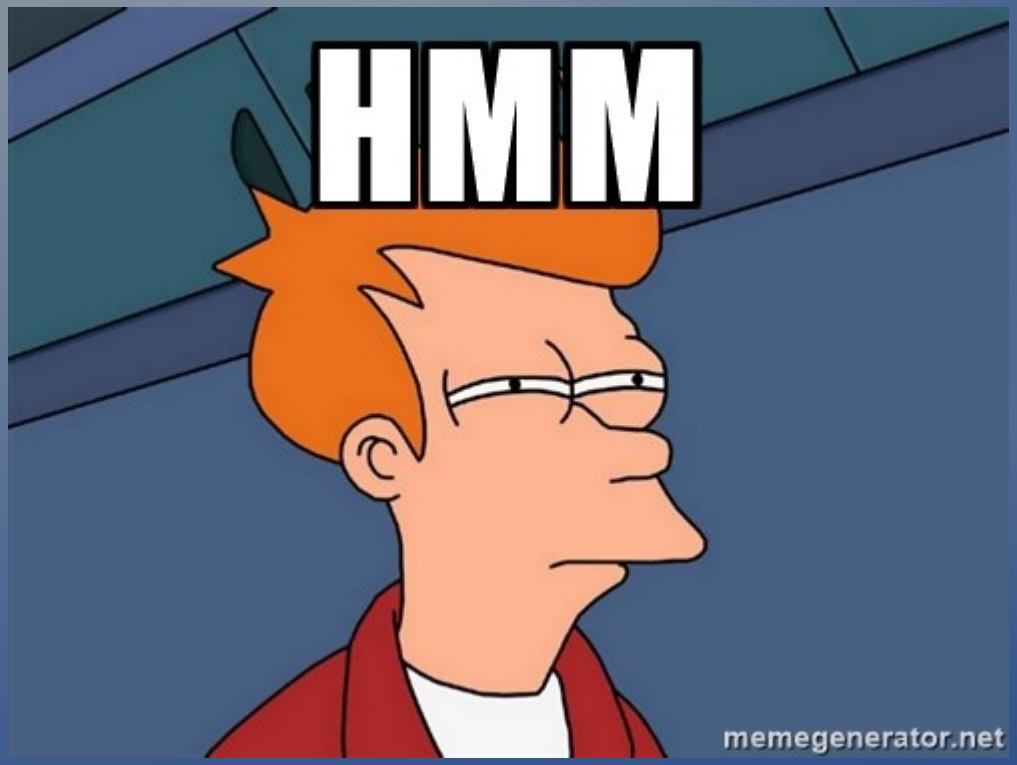
1st. Global Law: $V_{flat} \propto M_{bar}$

2nd. Central Law: $\Sigma_{dyn}(0) \propto \Sigma_{bar}(0)$

3rd. Local Law: $g_{obs}(R) \propto g_{bar}(R)$

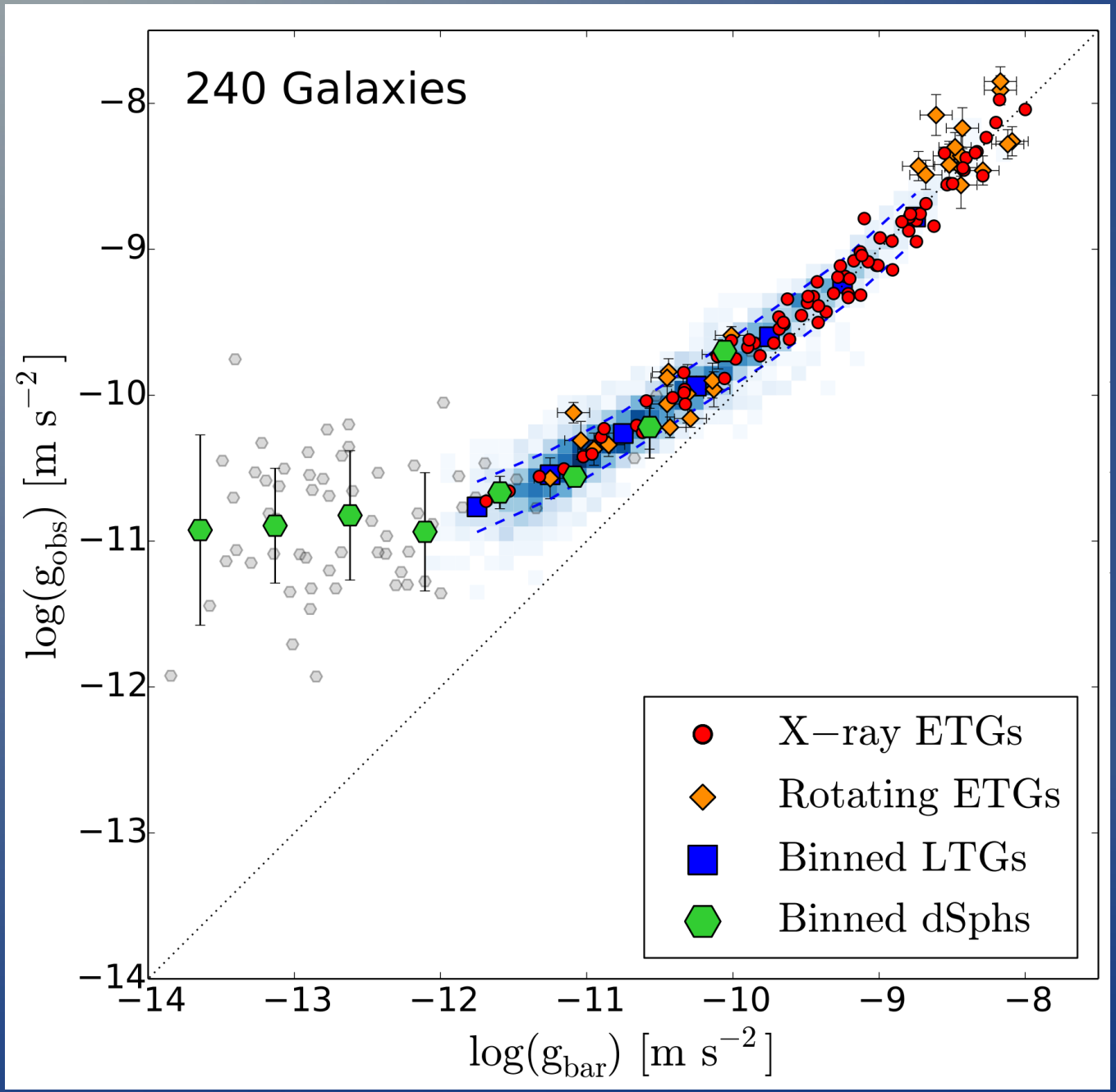
Minimum set of assumptions: M_*/L & Poisson's Equation.

σ_{int} is consistent with zero. No residual correlations.



OK. This works for LTGs but... what about ETGs?

Radial Acceleration Relation for ETGs



X-rays ETGs:

g_{obs} from hot gas haloes in hydrostatic equilibrium

Rotating ETGs:

g_{obs} from stellar kinematics + Jeans Axisymmetric Models

Dwarf Spheroidals:

g_{obs} from stellar kinematics + Jeans Spherical Models

Lelli+2017a, ApJ

We can infer the DM distribution from g_{bar}!

From the observations: $g_{DM} = g_{tot} - g_{bar} = F(g_{bar})$

For a spherical DM halo: $M_{DM}(R) = \frac{R^2}{G} F(g_{bar})$

For our fiducial fitting F: $M_{DM}(R) = \frac{R^2}{G} \frac{g_{bar}}{\exp(\sqrt{g_{bar}/g_0}) - 1}$

Purely empirical relations (accuracy ~30%).

II. Theoretical Implications

ΛCDM, Modified Gravity or New Dark Matter Models?

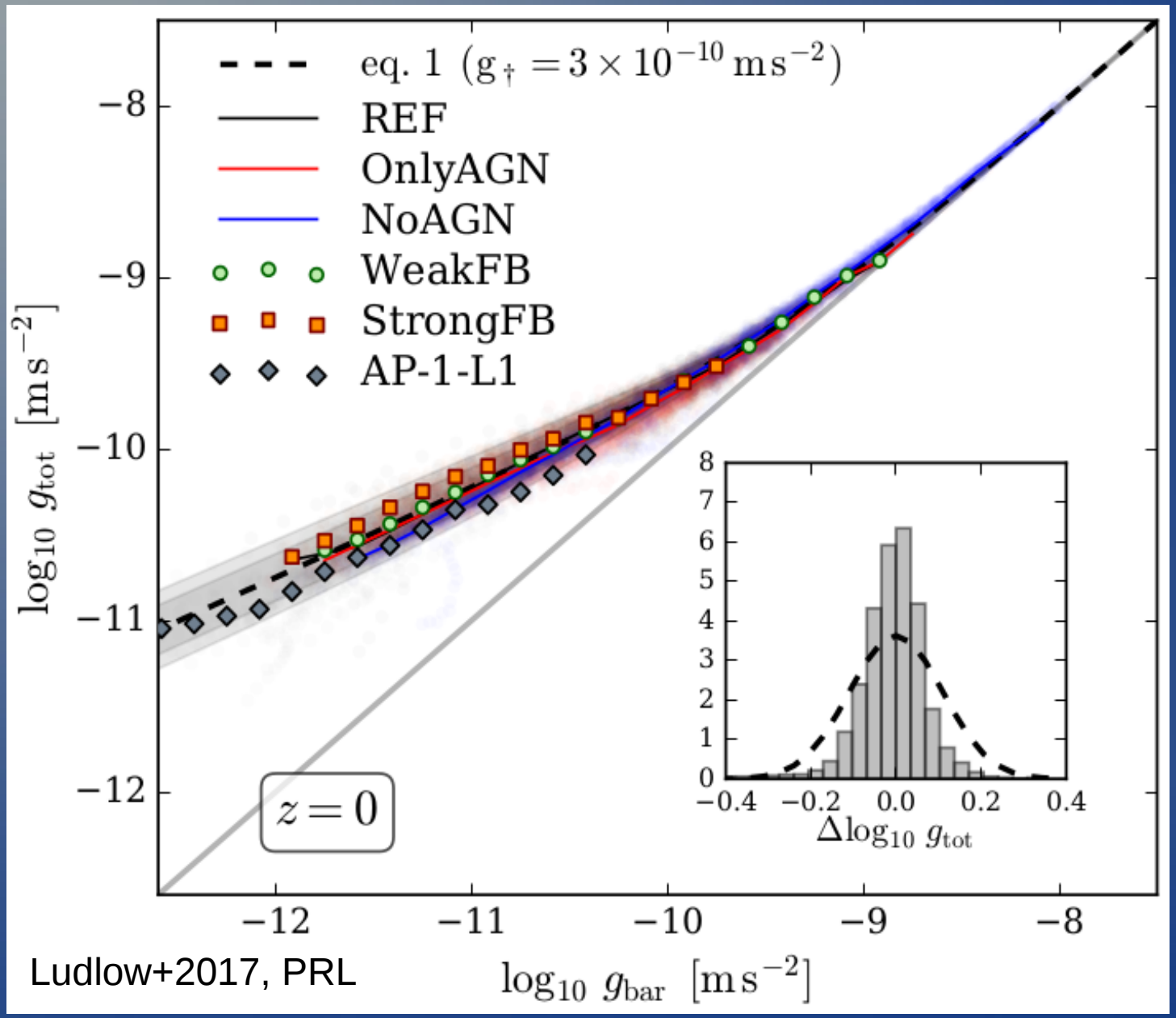
Three Possible Interpretations:

1 - End Product of Galaxy Formation in ΛCDM?

2 - New Dynamical Laws instead of Dark Matter?

3 - New Physics in the Dark Sector?

RAR from Hydrodynamical Simulations



Ludlow+2017, PRL

Analytic Models:

Di Cintio & Lelli 2016

Navarro+2016

Desmond 2017

Numerical Sims:

Keller & Wadsley 2016

Ludlow+2017

Tenneti+2017

A similar relation is found but the real problem is the tightness:

$$\sigma_{obs}^{\ 2} = \sigma_{int}^{\ 2} + \sigma_{err}^{\ 2}$$

Can't forget errors!

Key Conceptual Issues:

In ΛCDM, galaxy formation is complex and stochastic:

Merging of DM halos, Inflows/Outflows of gas,

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Can regularity emerge from these random processes?

Why there should be an acceleration scale in ΛCDM?

2- New Dynamical Laws instead of DM?

Modified Newtonian Dynamics (MOND)

Milgrom (1983) predicted the existence of these laws before the data existed, before I was born.

The MOND paradigm

Three Postulates (Milgrom 1983, 2009):

(1) New constant of Nature: a_0 (like *c* in GR, *h* in QM)

(2) $a \gg a_0 \rightarrow a = g_N$ (correspondence principle)

(3) $a \ll a_0 \rightarrow$ Dynamics is scale invariant: $(\mathbf{x}, t) \rightarrow (\lambda\mathbf{x}, \lambda t)$

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Two General Results (Milgrom 2009):

(1) $v = dx/dt$ is scale invariant $\rightarrow$ flat rotation curves

(2) $a = \sqrt{(g_N\, a_0)} \rightarrow$ outer slope of RAR is 0.5 (as observed!)

"Working" versions of MOND (Milgrom 1983)

(1) $a \cdot \mu(a/a_0) = g_N$ μ = interpolation function (arbitrary)

μ ~ Planck's law in QM, γ in SR, Schwarzschild's or Kerr's solutions in GR

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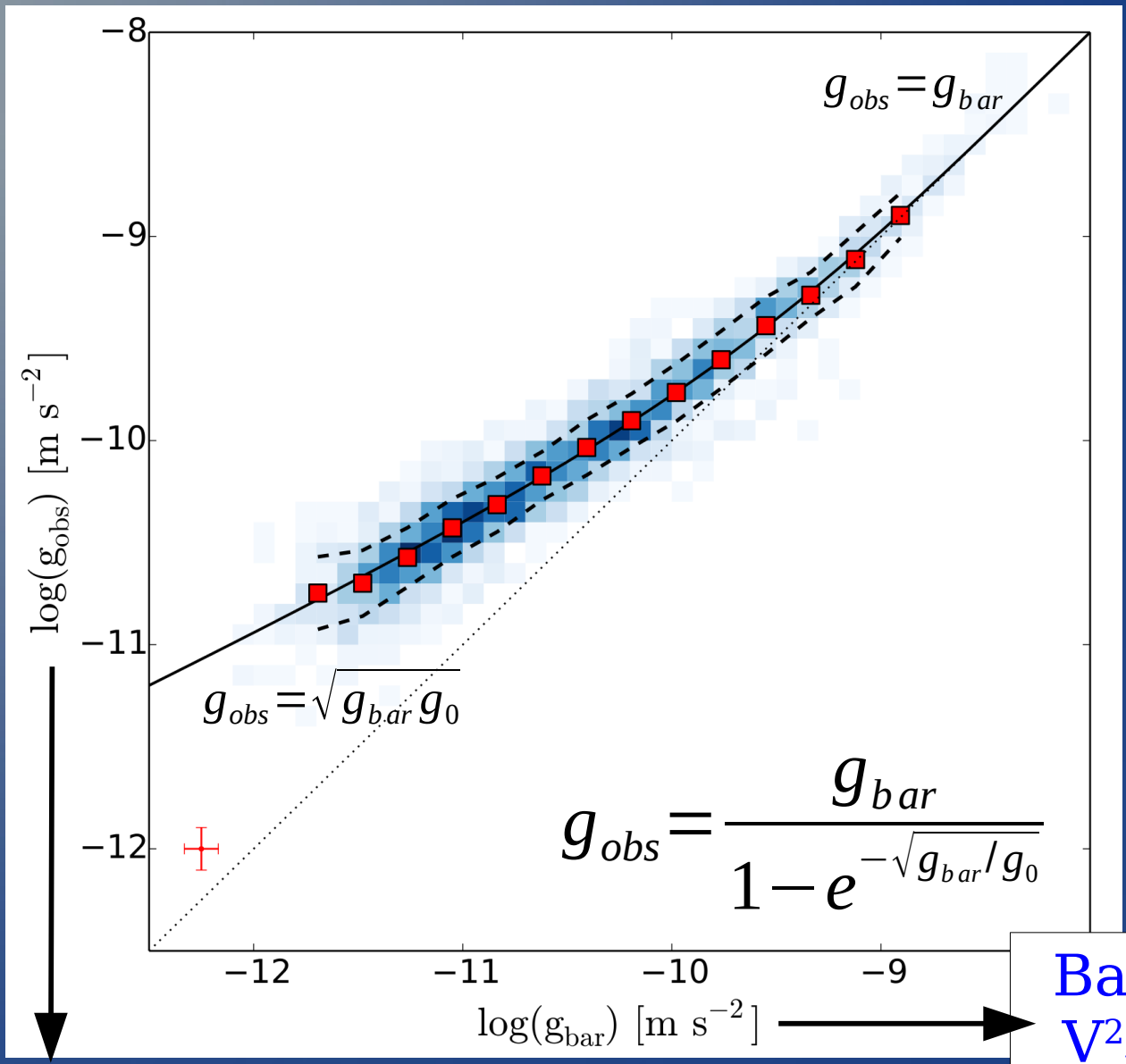
Lagrangian MOND modified-gravity theories

- Non-Relativistic: AQUAL (Bekenstein & Milgrom 1984), QMOND (Milgrom 2009)
- Relativistic: TeVeS (Bekenstein 2004), BIMOND (Milgrom 2010)

In all these theories, $\nu = \mu^{-1}$ is a free function to specify...

RAR constrains $\nu = \mu^{-1}$ for circular orbits



For all galaxies:

$\Upsilon_{disk} = 0.5\ M_\odot/L_\odot$

$\Upsilon_{bulge} = 0.7\ M_\odot/L_\odot$

McGaugh+2016, PRL

Lelli+2017a, ApJ

Total Acceleration: $V^2_{obs}/R = -\nabla\Phi_{tot}$

Baryonic Force:

$V^2_{bar}/R = -\nabla\Phi_{bar}$

$\nabla^2\Phi_{bar} = 4\pi G\ \rho_{bar}$

Emergent Gravity (Verlinde 2010, 2016)

- Numerical Coincidence: $a_0 \simeq cH_0$ (Milgrom 1994)

- Spacetime and gravity may emerge from an underlying microscopic theory (like thermodynamics)

- Entropy due to quantum vacuum in AdS ($\Lambda<0$) (Verlinde 2010):

  $S \propto A$ (like Bekenstein-Hawking S for Black Holes) $\rightarrow$ GR

- Entropy due to cosmo horizon in dS ($\Lambda>0$) (Verlinde 2016):

  $S \propto V \rightarrow$ Additional force due to DE plays the role of DM!

Emergent Gravity: Key Equations

For a static, spherical system (Verlinde 2016):

$$\int_0^r \frac{G\mathrm{M}_\mathrm{D}^2(\hat{r})}{\hat{r}^2} d\hat{r} = \mathrm{M}_\mathrm{b}\mathrm{a}_0 r.$$

Apparent DM distribution can be predicted from baryons!

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For a galaxy with an exponential profile (Lelli+2017b, MNRAS):

$$g_t = g_b \left\{ 1 + \sqrt{\frac{a_0}{g_b}} \sqrt{1 + \frac{\exp(-2\tilde{R})}{\tilde{R}[I_0(\tilde{R})K_0(\tilde{R}) - I_1(\tilde{R})K_1(\tilde{R})]}} \right\}$$

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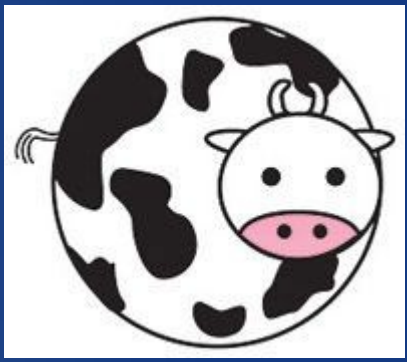
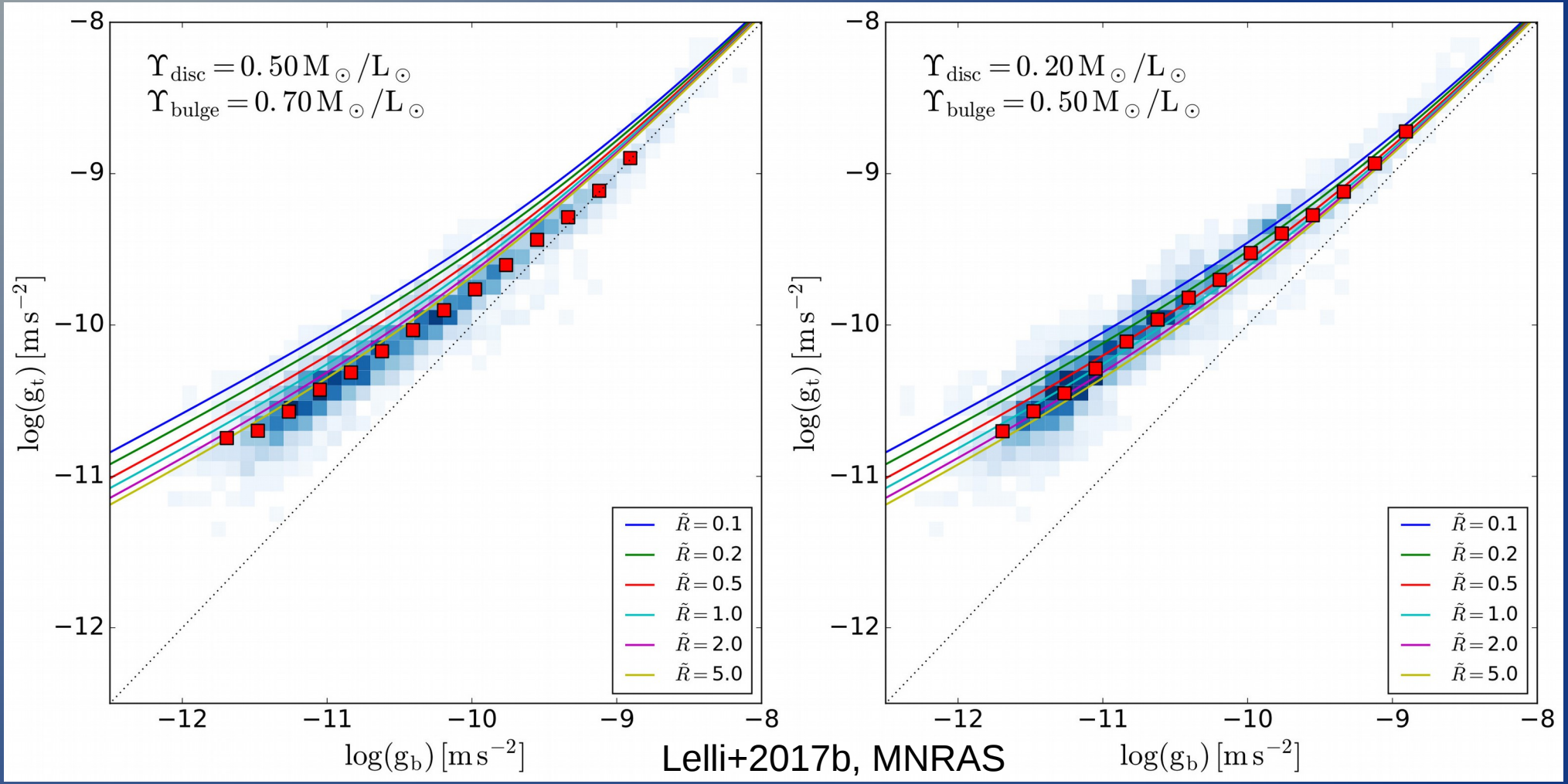


In general, we can write EG in a MOND-like fashion:

$$g_t = f(g_b/a_0; r) \cdot g_b$$

f is specified by the theory BUT there is an explicit dependence on r

RAR vs Emergent Gravity



Lelli+2017b, MNRAS

EG implies: (1) low M/L on average
(2) some intrinsic scatter
(3) residual corr. with R

$$g_t = f(g_b/a_0; r) \cdot g_b$$

3- New Physics in the Dark Sector?

Philosophy: reproduce ΛCDM on cosmological scales, while getting MOND-like behaviour on galaxy scales.

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1. Dipolar Dark Matter (Blanchet & Le Tiec 2008, 2009)

   Dielectric analogy: $\mathbf{E}\cdot\mu(\mathbf{E}) = k_e Q/r^2$ μ = permittivity of the medium

   Identify: $\mathbf{E} = \mathbf{a}$; $k_e Q/r^2 = g_N \rightarrow \mathbf{a}\cdot\mu(\mathbf{a}) = g_N$

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2. Dark Matter is a fluid (Berezhiani & Khoury 2015, Khoury 2016)

   Superfluid analogy: weird kinetic term $(\nabla\Phi)^{3/2}$ in deep-MOND Lagrangian.

   Phonons can give weird kinetic terms (Unitary Fermi Gas has $X^{5/2}$)

$V_{200} \sim T$ $T>T_c$ Normal fluid → ΛCDM (CMB, Galaxy Clusters)

$T<T_c$ Superfluid → MOND (Galaxies)

Summary on Theoretical Implications:

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Good: Hybrids ΛCDM + MOND that explain both large and small scales.

Bad: 'Good' is good by construction. New predictions?

Questions?