Automatic Basis Reduction for Effective Lagrangians in FeynRules

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15th MCnet Meeting

Contents

Overview

- Parametrize decoupled New Physics with SMEFT, leading effects are dimension-six operators with unknown $C_i/\Lambda_{\text{NP}}^2$
- S-matrix elements of operators $\propto$ EoM / total derivatives vanish, Fierz identities. . . $\implies$ choice of $\mathcal{L}_{\text{SMEFT}}$ not unique
- Necessary to choose a prescription to eliminate redundant operators $\implies$ define a minimal basis
- Radiative corrections, UV-matching calculations produce operators outside basis, limits set w/ physically motivated choices
- Limited tools for conversion between existing bases, but no algorithm for generic operator basis reduction

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Goal:

Explicitly implement the 'Warsaw' procedure in FeynRules to allow decomposition of general $\mathcal{L}_{\text{SMEFT}}^{(6)}$ onto a common basis

Contents

Constructing $\mathcal{L}^{(6)}_{\text{SMEFT}}$

$$\mathcal{L}_{\text{BSM}}(\{\Phi_{\text{SM}}\}, \{\mathcal{X}_{\text{NP}}\}) \to \quad \mathcal{L}_{SM}(\{\Phi_{\text{SM}}\}) \quad + \quad \frac{C_i}{\Lambda^2}\mathcal{O}_i(\{\Phi_{\text{SM}}\}) + \dots$$

UV Completion $\to$ Standard Model $+$ Dimension Six Operators

- $q^2 < m^2_{\mathcal{X}}$, heavy states decouple, generate local operators $\mathcal{O}_i$.
- The short-distance structure of the theory determines the values of the effective coupling constants $\frac{C_i}{\Lambda^2} = \frac{f(g_{\mathcal{X}})}{m^2_{\mathcal{X}}}$
- Construct most general $\mathcal{L}_{\text{SMEFT}}$ by enumerating gauge-invariant structures at each order in Λ_{NP}
- Choose to eliminate redundancies with either model-independent algorithm e.g. **Warsaw Basis** **(Grzadkowski et al. 1008.4884)**, for ease-of-interpretation in particular UV models e.g. **SILH basis** **(Giudice et al. 0703164)**, **or** correspondence with observables e.g. **Mass Basis** **(Gupta et al. 1405.0181)** $\Longrightarrow n^{(6)}_{\mathcal{O}} = 59 \ (n_f = 1)$

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Constructing $\mathcal{L}^{(6)}_{\mathrm{SMEFT}}$

$$\mathcal{L}_{\mathrm{BSM}}(\{\Phi_{\mathrm{SM}}\}, \{\boldsymbol{\chi}_{\mathrm{NP}}\}) \to \mathcal{L}_{SM}(\{\Phi_{\mathrm{SM}}\}) + \frac{C_i}{\Lambda^2}\mathcal{O}_i(\{\Phi_{\mathrm{SM}}\}) + \ldots$$

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Simple Example - Fierz Identities

Can re-express products of fields using the completeness relations for the fundamental representations of $SU(2)_L$ and $SU(3)_C$ and the Dirac algebra with $\{\Gamma_A\} \equiv \{\mathbb{I}, \gamma_5, \gamma_\mu, -\gamma_\mu\gamma_5, \frac{1}{2}\sigma_{\mu\nu}\}$ to redirect indices:

$$\tau^I_{ij}\,\tau^I_{kl} = \frac{1}{2}\,\delta_{il}\,\delta_{kj} - \frac{1}{4}\,\delta_{ij}\,\delta_{kl}$$

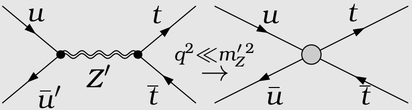
$$T^A_{ij}\,T^A_{kl} = \frac{1}{2}\,\delta_{il}\,\delta_{kj} - \frac{1}{6}\,\delta_{ij}\,\delta_{kl}$$

$$\Gamma^A_{ij}\Gamma_{A\,kl} = -\frac{1}{4^2}\sum_B C_B\,\Gamma^B_{il}\,\Gamma_{B\,kj}$$

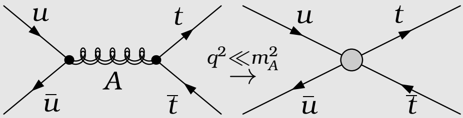
- ψ^4 operators written as products of colour octets/$SU(2)_L$ triplets $\leftrightarrow$ linear combinations of singlets with rearranged flavour indices
- $\mathcal{O}_i \in \psi^4$ **linearly dependent** - $\exists\,\{k_i\} \neq 0 :\ \sum_i k_i\,\mathcal{O}_i = 0$

$$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{q}_s\gamma_\mu T^A q_t)$$
$$= (\bar{q}_p\gamma^\mu\tau^I q_t)(\bar{q}_s\gamma_\mu\tau^I q_r) + \tfrac{1}{4}(\bar{q}_p\gamma^\mu q_t)(\bar{q}_s\gamma_\mu q_r) - \tfrac{1}{6}(\bar{q}_p\gamma^\mu q_r)(\bar{q}_s\gamma_\mu q_t)$$

Redundant Operators in UV Matching



$$(\bar{q}_R\gamma^\mu q_R)Z_\mu \rightarrow (\bar{u}\gamma^\mu u)(\bar{t}\gamma_\mu t)$$



$$(\bar{q}_R\gamma^\mu T^A q_R)A^A_\mu \rightarrow (\bar{u}\gamma^\mu T^A u)(\bar{t}\gamma_\mu T^A t)$$

- Physically transparent operators follow from UV matching to model
- Fierz constrains a linear combination of colour octet and singlet operators
- Cannot interpret all three operators as independent w/ individual coefficients
- $\Longrightarrow$ choose basis to contain two of three

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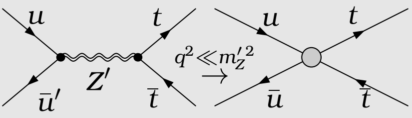

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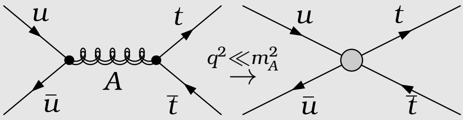

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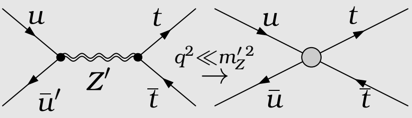

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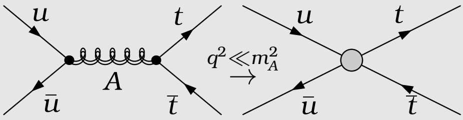

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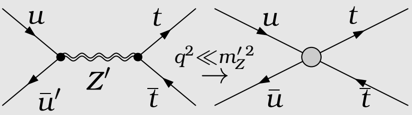

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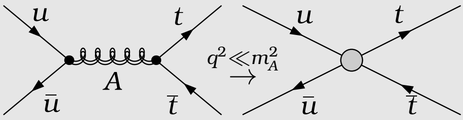

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$$(\bar{u}\gamma^\mu T^A u)(\bar{t}\gamma_\mu T^A t) = \tfrac{1}{2}(\bar{u}\gamma^\mu t)(\bar{t}\gamma_\mu u) - \tfrac{1}{6}(\bar{u}\gamma^\mu u)(\bar{t}\gamma_\mu t)$$

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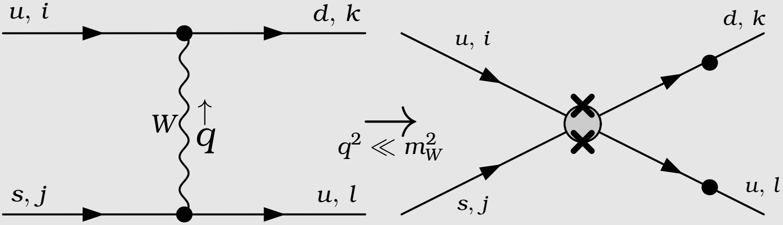
Redundant Operators via Radiative Corrections



LO: $\mathcal{L}^{(1)}_{us\to bd} = C_1\mathcal{O}_1 = C_1(\bar{u}_L\gamma^\mu s_L)(\bar{d}_L\gamma_\mu u_L)$

- Integrate out W $\implies$ colour-singlet charged current contact interaction
- Adopt natural description: colour neutral current operator(s), coefficient has simple UV interpretation $C_1 \equiv G_F \propto \frac{g_2^2}{m_W^2}$
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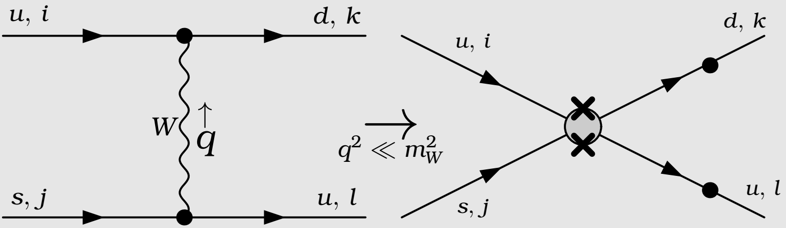

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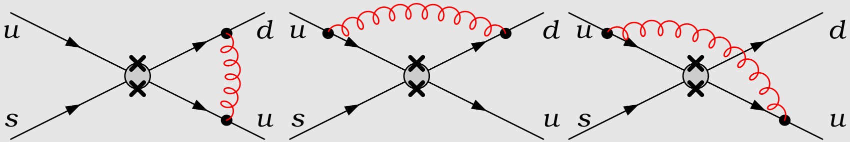

NLO: virtual gluons mix quark colour indices

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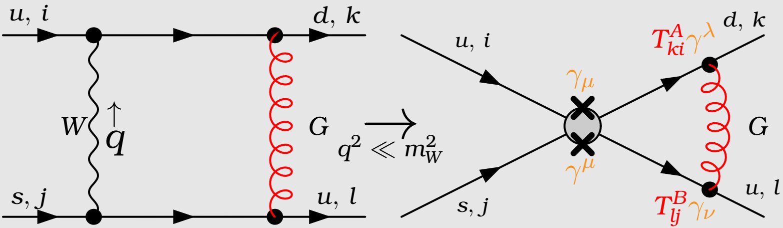

$$\mathcal{M}^{\mathcal{O}_1}_{\text{div}} \propto \frac{\alpha_s}{\bar{\epsilon}} \left\langle (\bar{d}_L \gamma^\nu \gamma^\lambda \gamma^\mu T^A u_L)(\bar{u}_L \gamma_\nu \gamma_\lambda \gamma_\mu T^A s_L) \right\rangle$$

- QCD corrections to EFT generate local terms associated with colour octet operators
- Use Fierz (Colour & Spin) to decompose back onto basis: necessary for (additive) operator renormalization, RGEs. . .

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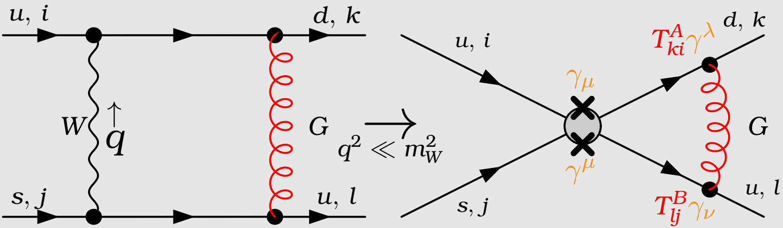

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$$\mathcal{M}^{\mathcal{O}_1}_{\text{div}} \propto \frac{\alpha_s}{\bar{\epsilon}} \left(\frac{2}{3}\mathcal{O}_1 - \mathbf{2}\boldsymbol{\mathcal{O}_2}\right)$$

$$\mathcal{O}_1 = (\bar{d}_L \gamma^\mu u_L)(\bar{u}_L \gamma_\mu s_L)$$

$$\boldsymbol{\mathcal{O}_2} = (\bar{d}_L \gamma^\mu s_L)(\bar{u}_L \gamma_\mu u_L)$$

Redundancy through Equations of Motion

Simple example: ϕ^n theory w/ Z_2-symmetry $\phi \to -\phi$:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \lambda\phi^4 + \eta g_1\phi^6 + \eta g_2\phi^3\partial^2\phi$$

Classically: $\partial^2\phi = m^2\phi + 4\lambda\phi^3 + \mathcal{O}(\eta) \implies \partial^2\phi \leftrightarrow m^2\phi + 4\lambda\phi^3$

$$\mathcal{L}_{\text{eff}} \to \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \lambda'\phi^4 + \eta g_1'\phi^6 + \mathcal{O}(\eta^2)$$

$\mathcal{O} = \phi^3\partial^2\phi$ is **redundant**: just rescales coefficients of ϕ^4 and ϕ^6 terms.

Quantum generalization through the Equivalence Theorem: a field redefinition (here: $\phi \to \phi + \eta g_2\phi^3$) that shifts $\mathcal{L}$ by a term proportional to classical EoM does not affect S-matrix elements (at $\mathcal{O}(\eta)$):

$$\langle \mathcal{S}\rangle_{(\mathcal{L}_0)} - \langle \mathcal{S}\rangle_{(\mathcal{L}')} = \mathcal{O}(\eta^2)\,, \qquad \mathcal{L}' = \mathcal{L}_0 + \eta\, F(\{\varphi\})\frac{\delta S_0}{\delta\varphi_i}$$

Redundancy through Integration-by-Parts

How about e.g. $\partial^\mu \phi\, \partial_\mu \partial^2 \phi$? Perturbatively, for a gauge-invariant $\mathcal{O}$:

$$0 = \int d^4x\, \partial_\mu (\mathcal{A}^\mu \mathcal{B}) \implies (\partial_\mu \mathcal{A}^\mu)\mathcal{B} = -\mathcal{A}^\mu (\partial_\mu \mathcal{B}) \quad \textbf{(IBP)}$$

$$\implies \partial^\mu \phi\, \partial_\mu \partial^2 \phi = -(\partial^2 \phi)^2 + (\text{Surface Term})$$

- Vanishing of total derivatives introduces further relationships
- EoM + IBP relations $\implies$ only one independent operator in dimension-six ϕ^n
- Choosing $\mathcal{O}_1 = \phi^6 \implies$ using EoM to eliminate derivative operators
- This defines UV-agnostic method to arrive at basis

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More Complicated - SMEFT

$$i\not{D}l - \Gamma_e e\varphi = 0$$

$$i\not{D}e - \Gamma_e^\dagger \varphi^\dagger l = 0$$

$$i\not{D}u - \Gamma_u^\dagger \widetilde{\varphi}^\dagger q = 0 \quad \Longrightarrow$$

Many relationships between operators with different field content

$$i\not{D}d - \Gamma_d^\dagger \varphi^\dagger q = 0$$

$$i\not{D}q - \Gamma_u u\widetilde{\varphi} + \Gamma_d d\varphi = 0$$

$$(D^\mu D_\mu \varphi)^j - \mu^2 \varphi^j + \lambda \left(\varphi^\dagger \varphi\right) \varphi^j + \bar{e}\,\Gamma_e^\dagger l^j - \varepsilon_{jk} \bar{q}^k\, \Gamma_u u + \bar{d}\,\Gamma_d^\dagger q^j = 0$$

$$(D^\rho G_{\rho\mu})^A - g_s \left(\bar{q}\gamma_\mu T^A q + \bar{u}\gamma_\mu T^A u + \bar{d}\gamma_\mu T^A d\right) = 0$$

$$(D^\rho W_{\rho\mu})^I - \frac{g}{2}\left(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi + \bar{l}\gamma_\mu \tau^I l + \bar{q}\gamma_\mu \tau^I q\right) = 0$$

$$\partial^\rho B_{\rho\mu} - g' Y_\varphi\, \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi - g' \sum_{\psi \in \{l,e,q,u,d\}} Y_\psi\, \bar{\psi}\gamma_\mu \psi = 0,$$

A SMEFT Example

Suppose one parametrizes NP in QCD interactions of top with:

$$\{\mathcal{O}_1, \mathcal{O}_2\} = \{\ (\bar{t}\,\gamma^\mu\, T^A\, D^\nu\, t)\, G^A_{\mu\nu} \quad , \quad (\bar{t}\,\gamma^\mu\, T^A\, t)\, D^\nu\, G^A_{\mu\nu}\ \}$$

- Through **IBP**: $\mathcal{O}_1 + \mathcal{O}_1^\dagger \overset{\text{IBP}}{=} -\mathcal{O}_2 \ldots$
- Through $\mathbf{G^A_\mu}$ **EoM**: $\mathcal{O}_2 \propto (D^\rho G_{\rho\mu})^A \overset{\text{EoM}}{=} g_s \sum_{\psi=q,u,d} (\bar{\psi}\gamma_\mu T^A \psi)$
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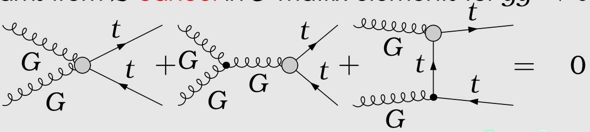
- Through **IBP**: $\mathcal{O}_1 + \mathcal{O}_1^\dagger \overset{\text{IBP}}{=} -\mathcal{O}_2 \ldots$
- Through $\mathbf{G}^{\mathbf{A}}_\mu$ **EoM**: $\mathcal{O}_2 \propto (D^\rho G_{\rho\mu})^A \overset{\text{EoM}}{=} g_s \sum_{\psi=q,u,d} (\bar{\psi}\gamma_\mu T^A\psi)$
- $\implies \mathcal{O}_1 \to$ linear combination of octet four-quark operators:

$$\mathcal{O}_1 = -g_s \sum_{\psi=q,u,d} (\bar{\psi}\gamma_\mu T^A\psi)\,(\bar{t}\,\gamma^\mu\,T^A\,t)$$

Sum of diagrams from $\mathcal{L}$ cancel in $\mathcal{S}$-matrix elements for $gg \to t\bar{t}$:



Contents

A Complete Basis for the D=6 SMEFT

$n_{\mathcal{O}}$	Paper
80	(Buchmuller & Wyler NPB 268 621, 1986)
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Complete basis identified in Warsaw, adopted as 'standard' choice, vindicated by heroic renormalization at one-loop **(Jenkins, Manohar & Trott 1308.2627,1310.4838,1312.2014)**, RGE mixing known in this basis

The Warsaw Procedure

Procedure deriving Warsaw basis settled the issue by approaching the problem systematically:

- Divide operators into classifications according to generic building blocks: $\{X, \psi, \varphi, D\}$, e.g. $(\varphi^\dagger \varphi) G^A_{\mu\nu} G^{A\,\mu\nu} \in \boxed{X^2 \varphi^2}$
- Impose a hierarchy: fewer $D_\mu \implies$ 'lower' classification
- For each classification with $n_D \geq 1$, exhaustively use the available identities to attempt to re-express operators in EoM-reducible form
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Result: proof all operators expressible as a linear combination of 59, spread over 12 classifications. EoM mappings tabulated for performing this decomposition, e.g. for previous example : $\boxed{\psi^2 XD} \to \boxed{\psi^4}$

The Warsaw Basis (I)

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

Non four-fermion dimension-six operators of the Warsaw basis

The Warsaw Basis (II)

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
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$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

Four-fermion operators of the Warsaw basis

Contents

Automating the Warsaw Procedure

Knowledge of how each classification is reduced $\neq$ explicit algorithm to reduce arbitrary operators, obtain coefficients in decomposition:

$$\mathcal{L}_{\text{eff}} \supset C_j \mathcal{O}_j^{\text{red}}, \quad \mathcal{O}_j^{\text{red}} = \sum_{i=1}^{59} k_{ij} \mathcal{O}_i^{\text{Warsaw}}$$

Why bother?:

- Automatically translate UV-matching to $\mathcal{L}_{\text{eff}}$ to common language
- Utilize w/ NLOCT (Degrande 1406.3030) to verify RGEs
- Facilitate comparison of limits set with different choices
- Can more easily expand to higher mass dimension
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Basis Reduction in FeynRules

Aim:

(Re-)write operators automatically in EoM form whenever possible, then use EoM to drop to lower classifications by trading D_μ for fields

- Fierz identities: $M^I_{ij} M^I_{kj} \to \sum c_J M^J_{il} M^J_{kj}\,, M^J \in \{\Gamma^A, T, \tau, \delta \ldots\}$
- Integration-by-Parts: $\mathcal{A}^\mu (D_\mu \mathcal{B}) \to -(D_\mu \mathcal{A})\mathcal{B}^\mu + \boxed{T}$
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- Exploit (anti-)symmetry: $[D_\mu, D_\nu]\Phi \to g_X T_X X_{\mu\nu}\Phi$
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Procedure:

- **Inputs:** SM field content, EoMs, Identities, arbitrary $\mathcal{L}^{(6)}_{\text{eff}} = C_i \mathcal{O}_i$
- **Outputs:** $\mathcal{L}^{(6)}_{\text{eff}} = C'_i \mathcal{O}^W_i$

- Factorize $\mathcal{O}$: tensors, D_μ configuration, field content:
  $\mathcal{O}(\{\Phi\}) \equiv [SU(2)] \times [SU(3)] \times [\Gamma] \times ((\mathbf{D}) \cdot (\Phi))$
- Use applicable identities to manipulate each factor separately, find sequence of these which re-expresses this in form $O(\{\Phi\}) = F(\Phi)(D\Phi_i) + \ldots$, i.e. $\propto$ EoM for Φ_i
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- If successful, use EoM to eliminate derivatives, enforce conventions (eliminate generators with Fierz identities, require spin/gauge diagonal fermion currents. . .), repeat . . .

Basis Reduction in FeynRules Example: $\varphi^2 D^4$

Factorize $\mathcal{O} \equiv (D^\mu D_\nu D_\mu \varphi)^\dagger (D^\nu \varphi) \quad \rightarrow \quad \Big((D^\mu D_\nu D_\mu,\ D^\nu) \bullet (\varphi_i^\dagger,\ \varphi_i) \Big)$

Identify three EoM-redundant operators with the same field content:

$$(D^\mu D_\mu \varphi)^\dagger (D^\nu D_\nu \varphi)\,,\ (D^\mu \varphi)^\dagger (D_\mu D^\nu D_\nu \varphi)\,,\ (D_\nu D^\mu D_\mu \varphi)^\dagger (D^\nu \varphi)$$

Basis Reduction in FeynRules Example: $\varphi^2 D^4$

Factorize $\mathcal{O} \equiv (D^\mu D_\nu D_\mu \varphi)^\dagger (D^\nu \varphi) \quad \rightarrow \quad \Big((D^\mu D_\nu D_\mu,\ D^\nu) \bullet (\varphi_i^\dagger,\ \varphi_i) \Big)$

Identify three EoM-redundant operators with the same field content:

$$(D^\mu D_\mu \varphi)^\dagger (D^\nu D_\nu \varphi)\,,\, (D^\mu \varphi)^\dagger (D_\mu D^\nu D_\nu \varphi)\,,\, (D_\nu D^\mu D_\mu \varphi)^\dagger (D^\nu \varphi)$$

Implement reshuffling Ds w/ IBP/Commutation independently of fields:

$$(D^\mu D_\nu D_\mu,\ D^\nu) \rightarrow ([D^\mu, D_\nu] D_\mu,\ D^\nu) + (D_\nu D^\mu D_\mu,\ D^\nu)$$

If there exists a reshuffling where RHS = lower classifications, perform:

i.e. : $(D^\mu D_\nu D_\mu \varphi)^\dagger (D_\nu \varphi) = ([D^\mu, D^\nu] D_\mu \varphi)^\dagger D_\nu \varphi + (D_\nu D^\mu D_\mu \varphi)^\dagger (D^\nu \varphi)$

Basis Reduction in FeynRules Example: $\varphi^2 D^4$

Factorize $\mathcal{O} \equiv (D^\mu D_\nu D_\mu \varphi)^\dagger (D^\nu \varphi) \quad \rightarrow \quad \Big((D^\mu D_\nu D_\mu ,\ D^\nu) \bullet (\varphi_i^\dagger ,\ \varphi_i) \Big)$

Identify three EoM-redundant operators with the same field content:

$$(D^\mu D_\mu \varphi)^\dagger (D^\nu D_\nu \varphi)\,,\, (D^\mu \varphi)^\dagger (D_\mu D^\nu D_\nu \varphi)\,,\, (D_\nu D^\mu D_\mu \varphi)^\dagger (D^\nu \varphi)$$

Implement reshuffling Ds w/ IBP/Commutation independently of fields:

$$(D^\mu D_\nu D_\mu ,\ D^\nu) \rightarrow ([D^\mu , D_\nu] D_\mu ,\ D^\nu) + (D_\nu D^\mu D_\mu ,\ D^\nu)$$

If there exists a reshuffling where RHS = lower classifications, perform:

i.e. : $(D^\mu D_\nu D_\mu \varphi)^\dagger (D_\nu \varphi) = ([D^\mu , D^\nu] D_\mu \varphi)^\dagger D_\nu \varphi + (D_\nu D^\mu D_\mu \varphi)^\dagger (D^\nu \varphi)$

Reduce: $[D^\mu , D^\nu] \propto X^{\mu\nu}$, $(D^\mu D_\mu \varphi)^j \rightarrow \mu^2 \varphi^j \ - \ \lambda \left(\varphi^\dagger \varphi\right) \varphi^j \ - \ \bar{e}\,\Gamma_e^\dagger l^j \ + \ \varepsilon_{jk} \bar{q}^k \,\Gamma_u u \ - \ \bar{d}\,\Gamma_d^\dagger q^j$

$$\boxed{\varphi^2 D^4} \rightarrow \boxed{\varphi^2 X D^2} + \boxed{\varphi^4 D^2} + \boxed{\psi^2 \varphi D} + \mu^2 \boxed{\varphi^2 D^2}$$

```
DC[DC[DC[Phi†[ii1], mu], nu], mu] DC[Phi[ii2], nu] IndexDelta[ii1, ii2]
```

```
D⁴ φ²
```

Basis Reduction in FeynRules Example: $\varphi^2 D^4$ (II)

- Operators with many $D \rightarrow$ linear combination of many operators of lower classifications, with coefficients dictated by terms in EoM, $[D_\mu, D_\nu]\varphi \rightarrow g_1 Y_\varphi B_{\mu\nu}\varphi + \ldots$
- Rinse and repeat, attempt to massage each into form $\propto$ EoM for any of the fields present as far as possible &: $\mathcal{O}_i \rightarrow \sum_{j=1,\ldots,59} k_{ij} \mathcal{O}_j^W$

```
DC[Phi[j], μ] DC[Phi†[l], μ] IndexDelta[j, i] IndexDelta[l, k] Phi[k] Phi†[i] +
 del[del[Phi[l] Phi†[k], mu], mu] IndexDelta[j, i] IndexDelta[l, k] Phi†[i] Phi†[j] +
 lR̄[s1, p].LL[s2, j, r] yl[p, r]† IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k]
  Phi[l] Phi†[i] Phi†[k] + dR̄[s1, p, a].QL[s2, j, r, b] yd[p, r]† IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k] Phi[l] Phi†[i] Phi†[k] -
 μ² IndexDelta[j, i] IndexDelta[l, k] Phi[j] Phi[l] Phi†[i] Phi†[k] -
 uR̄[s1, r, a].QL[s2, m, p, b] yu[p, r]† Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] -
 μ² IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[j] Phi†[l] + lam IndexDelta[j, i]
  IndexDelta[l, n] IndexDelta[m, k] Phi[j] Phi[l] Phi[m] Phi†[i] Phi†[k] Phi†[n] +
 lam IndexDelta[i, j] IndexDelta[k, m] IndexDelta[l, n] Phi[i] Phi[k] Phi[l]
  Phi†[j] Phi†[m] Phi†[n] + QL̄[s1, j, r, a].dR[s2, p, b] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] yd[p, r] +
 LL̄[s1, j, r].lR[s2, p] IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i]
  Phi[k] Phi†[l] yl[p, r] - QL̄[s1, m, p, a].uR[s2, r, b] Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[j, k] Phi[l] Phi†[i] Phi†[k] yu[p, r]
```

```
{D² φ², D² X φ², D² X φ², D² φ ψ², D² φ ψ², D² φ ψ², D² φ ψ², D² φ⁴, D² φ⁴, D² φ⁴, D² φ⁴,
 D² φ ψ², D² X φ², D² X φ², D² φ ψ², D² φ ψ², D² φ ψ², D² φ ψ², D² φ ψ², D² φ ψ², D² φ ψ²}
```

Basis Reduction in FeynRules Example: $\varphi^2 D^4$ (II)

- Operators with many $D \to$ linear combination of many operators of lower classifications, with coefficients dictated by terms in EoM, $[D_\mu, D_\nu]\varphi \to g_1 Y_\varphi B_{\mu\nu}\varphi + \dots$
- Rinse and repeat, attempt to massage each into form $\propto$ EoM for any of the fields present as far as possible &: $\mathcal{O}_i \to \sum_{j=1,\dots,59} k_{ij}\mathcal{O}_j^W$

```
DC[Phi[j], μ] DC[Phi†[l], μ] IndexDelta[j, i] IndexDelta[l, k] Phi[k] Phi†[i] +
 del[del[Phi[l] Phi†[k], mu], mu] IndexDelta[j, i] IndexDelta[l, k] Phi†[i] Phi†[j] +
 lR[s1, p].LL[s2, j, r] yl[p, r]† IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k]
  Phi[l] Phi†[i] Phi†[k] + dR[s1, p, a].QL[s2, j, r, b] yd[p, r]† IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k] Phi[l] Phi†[i] Phi†[k] -
 μ² IndexDelta[j, i] IndexDelta[l, k] Phi[j] Phi[l] Phi†[i] Phi†[k] -
 uR[s1, r, a].QL[s2, m, p, b] yu[p, r]† Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] -
 μ² IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[j] Phi†[l] + lam IndexDelta[j, i]
  IndexDelta[l, n] IndexDelta[m, k] Phi[j] Phi[l] Phi[m] Phi†[i] Phi†[k] Phi†[n] +
 lam IndexDelta[i, j] IndexDelta[k, m] IndexDelta[l, n] Phi[i] Phi[k] Phi[l]
  Phi†[j] Phi†[m] Phi†[n] + QL[s1, j, r, a].dR[s2, p, b] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] yd[p, r] +
 LL[s1, j, r].lR[s2, p] IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i]
  Phi[k] Phi†[l] yl[p, r] - QL[s1, m, p, a].uR[s2, r, b] Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[j, k] Phi[l] Phi†[i] Phi†[k] yu[p, r]
```

```
{D² φ², D² X φ², D² X φ², D² φ ψ², D² φ ψ², D² φ ψ², D² φ ψ², D² φ⁴, D² φ⁴, D² φ⁴, D² φ⁴,
 D² φ ψ², D² X φ², D² X φ², D² φ ψ², D² φ ψ², D² φ ψ², D² φ ψ², D² φ ψ², D² φ ψ², D² φ ψ²}
```

Repeat. . .

- Pick $(\varphi^\dagger\varphi)(D_\mu\varphi^\dagger D^\mu\varphi)$
- $[\delta_{ij}\delta_{kl}]_{SU(2)} \times (\mathbb{I},\ \mathbb{I},\ D_\mu,\ D^\mu) \cdot (\varphi_i^\dagger,\ \varphi_j,\ \varphi_k^\dagger,\ \varphi_l)$
- Can use Fierz & IBP. . .
- $\boxed{\varphi^4 D^2} \to \boxed{\varphi^4 D^2} + \boxed{\varphi^3\psi^2} + \boxed{\varphi^6} + \mu^2\boxed{\varphi^4}$
- No further reduction possible via EoM; all operators expressed in lowest possible classifications

```
DC[Phi[j], μ] DC[Phi†[l], μ] IndexDelta[j, i] IndexDelta[l, k] Phi[k] Phi†[i] +
 del[del[Phi[l] Phi†[k], mu], mu] IndexDelta[j, i] IndexDelta[l, k] Phi†[i] Phi†[j] +
 l̄R[s1, p].LL[s2, j, r] yl[p, r]† IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k]
  Phi[l] Phi†[i] Phi†[k] + d̄R[s1, p, a].QL[s2, j, r, b] yd[p, r]† IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k] Phi[l] Phi†[i] Phi†[k] -
 μ² IndexDelta[j, i] IndexDelta[l, k] Phi[j] Phi[l] Phi†[i] Phi†[k] -
 ūR[s1, r, a].QL[s2, m, p, b] yu[p, r]† Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] -
 μ² IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[j] Phi†[l] + lam IndexDelta[j, i]
  IndexDelta[l, n] IndexDelta[m, k] Phi[j] Phi[l] Phi[m] Phi†[i] Phi†[k] Phi†[n] +
 lam IndexDelta[i, j] IndexDelta[k, m] IndexDelta[l, n] Phi[i] Phi[k] Phi[l]
  Phi†[j] Phi†[m] Phi†[n] + Q̄L[s1, j, r, a].dR[s2, p, b] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] yd[p, r] +
 L̄L[s1, j, r].lR[s2, p] IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i]
  Phi[k] Phi†[l] yl[p, r] - Q̄L[s1, m, p, a].uR[s2, r, b] Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[j, k] Phi[l] Phi†[i] Phi†[k] yu[p, r]
```

```
{D² φ⁴, φ², φ³ ψ², φ³ ψ², φ⁴, φ³ ψ², φ⁶, φ³ ψ², φ³ ψ², φ³ ψ²}
```

Repeat. . .

- Pick $(\varphi^\dagger\varphi)(D_\mu\varphi^\dagger D^\mu\varphi)$
- $[\delta_{ij}\delta_{kl}]_{SU(2)} \times (\mathbb{I},\ \mathbb{I},\ D_\mu,\ D^\mu) \bullet (\varphi_i^\dagger,\ \varphi_j,\ \varphi_k^\dagger,\ \varphi_l)$
- Can use Fierz & IBP. . .
- $\boxed{\varphi^4 D^2} \to \boxed{\varphi^4 D^2} + \boxed{\varphi^3\psi^2} + \boxed{\varphi^6} + \mu^2\boxed{\varphi^4}$
- No further reduction possible via EoM; all operators expressed in lowest possible classifications

```
DC[Phi[j], μ] DC[Phi†[l], μ] IndexDelta[j, i] IndexDelta[l, k] Phi[k] Phi†[i] +
 del[del[Phi[l] Phi†[k], mu], mu] IndexDelta[j, i] IndexDelta[l, k] Phi†[i] Phi†[j] +
 lR[s1, p].LL[s2, j, r] yl[p, r]† IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k]
  Phi[l] Phi†[i] Phi†[k] + dR[s1, p, a].QL[s2, j, r, b] yd[p, r]† IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k] Phi[l] Phi†[i] Phi†[k] -
 μ² IndexDelta[j, i] IndexDelta[l, k] Phi[j] Phi[l] Phi†[i] Phi†[k] -
 uR[s1, r, a].QL[s2, m, p, b] yu[p, r]† Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] -
 μ² IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[j] Phi†[l] + lam IndexDelta[j, i]
  IndexDelta[l, n] IndexDelta[m, k] Phi[j] Phi[l] Phi[m] Phi†[i] Phi†[k] Phi†[n] +
 lam IndexDelta[i, j] IndexDelta[k, m] IndexDelta[l, n] Phi[i] Phi[k] Phi[l]
  Phi†[j] Phi†[m] Phi†[n] + QL[s1, j, r, a].dR[s2, p, b] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] yd[p, r] +
 LL[s1, j, r].lR[s2, p] IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i]
  Phi[k] Phi†[l] yl[p, r] - QL[s1, m, p, a].uR[s2, r, b] Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[j, k] Phi[l] Phi†[i] Phi†[k] yu[p, r]
```

```
{D² φ⁴, φ², φ³ ψ², φ³ ψ², φ⁴, φ³ ψ², φ⁶, φ³ ψ², φ³ ψ², φ³ ψ²}
```

Repeat. . .

- Pick $(\varphi^\dagger\varphi)(D_\mu\varphi^\dagger D^\mu\varphi)$
- $[\delta_{ij}\delta_{kl}]_{SU(2)} \times (\mathbb{I},\ \mathbb{I},\ D_\mu,\ D^\mu) \cdot (\varphi_i^\dagger,\ \varphi_j,\ \varphi_k^\dagger,\ \varphi_l)$
- Can use Fierz & IBP. . .
- $\boxed{\varphi^4 D^2} \to \boxed{\varphi^4 D^2} + \boxed{\varphi^3\psi^2} + \boxed{\varphi^6} + \mu^2\boxed{\varphi^4}$
- No further reduction possible via EoM; all operators expressed in lowest possible classifications

```
DC[Phi[j], μ] DC[Phi†[l], μ] IndexDelta[j, i] IndexDelta[l, k] Phi[k] Phi†[i] +
 del[del[Phi[l] Phi†[k], mu], mu] IndexDelta[j, i] IndexDelta[l, k] Phi†[i] Phi†[j] +
 l̄R[s1, p].LL[s2, j, r] yl[p, r]† IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k]
  Phi[l] Phi†[i] Phi†[k] + d̄R[s1, p, a].QL[s2, j, r, b] yd[p, r]† IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k] Phi[l] Phi†[i] Phi†[k] -
 μ² IndexDelta[j, i] IndexDelta[l, k] Phi[j] Phi[l] Phi†[i] Phi†[k] -
 ūR[s1, r, a].QL[s2, m, p, b] yu[p, r]† Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] -
 μ² IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[j] Phi†[l] + lam IndexDelta[j, i]
  IndexDelta[l, n] IndexDelta[m, k] Phi[j] Phi[l] Phi[m] Phi†[i] Phi†[k] Phi†[n] +
 lam IndexDelta[i, j] IndexDelta[k, m] IndexDelta[l, n] Phi[i] Phi[k] Phi[l]
  Phi†[j] Phi†[m] Phi†[n] + Q̄L[s1, j, r, a].dR[s2, p, b] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] yd[p, r] +
 L̄L[s1, j, r].lR[s2, p] IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i]
  Phi[k] Phi†[l] yl[p, r] - Q̄L[s1, m, p, a].uR[s2, r, b] Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[j, k] Phi[l] Phi†[i] Phi†[k] yu[p, r]
```

$\{D^2\,\varphi^4,\ \varphi^2,\ \varphi^3\,\psi^2,\ \varphi^3\,\psi^2,\ \varphi^4,\ \varphi^3\,\psi^2,\ \varphi^6,\ \varphi^3\,\psi^2,\ \varphi^3\,\psi^2,\ \varphi^3\,\psi^2\}$

Repeat. . .

- Pick $(\varphi^\dagger\varphi)(D_\mu\varphi^\dagger D^\mu\varphi)$
- $[\delta_{ij}\delta_{kl}]_{SU(2)} \times (\mathbb{I},\ \mathbb{I},\ D_\mu,\ D^\mu) \bullet (\varphi_i^\dagger,\ \varphi_j,\ \varphi_k^\dagger,\ \varphi_l)$
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- No further reduction possible via EoM; all operators expressed in lowest possible classifications

```
DC[Phi[j], μ] DC[Phi†[l], μ] IndexDelta[j, i] IndexDelta[l, k] Phi[k] Phi†[i] +
 del[del[Phi[l] Phi†[k], mu], mu] IndexDelta[j, i] IndexDelta[l, k] Phi†[i] Phi†[j] +
 lR[s1, p].LL[s2, j, r] yl[p, r]† IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k]
  Phi[l] Phi†[i] Phi†[k] + dR[s1, p, a].QL[s2, j, r, b] yd[p, r]† IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k] Phi[l] Phi†[i] Phi†[k] -
 μ2 IndexDelta[j, i] IndexDelta[l, k] Phi[j] Phi[l] Phi†[i] Phi†[k] -
 uR[s1, r, a].QL[s2, m, p, b] yu[p, r]† Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] -
 μ2 IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[j] Phi†[l] + lam IndexDelta[j, i]
  IndexDelta[l, n] IndexDelta[m, k] Phi[j] Phi[l] Phi[m] Phi†[i] Phi†[k] Phi†[n] +
 lam IndexDelta[i, j] IndexDelta[k, m] IndexDelta[l, n] Phi[i] Phi[k] Phi[l]
  Phi†[j] Phi†[m] Phi†[n] + QL[s1, j, r, a].dR[s2, p, b] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] yd[p, r] +
 LL[s1, j, r].lR[s2, p] IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i]
  Phi[k] Phi†[l] yl[p, r] - QL[s1, m, p, a].uR[s2, r, b] Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[j, k] Phi[l] Phi†[i] Phi†[k] yu[p, r]
```

```
{D2 φ4, φ2, φ3 ψ2, φ3 ψ2, φ4, φ3 ψ2, φ6, φ3 ψ2, φ3 ψ2, φ3 ψ2}
```

Repeat. . .

- Pick $(\varphi^\dagger\varphi)(D_\mu\varphi^\dagger D^\mu\varphi)$
- $[\delta_{ij}\delta_{kl}]_{SU(2)} \times (\mathbb{I},\ \mathbb{I},\ D_\mu,\ D^\mu) \cdot (\varphi_i^\dagger,\ \varphi_j,\ \varphi_k^\dagger,\ \varphi_l)$
- Can use Fierz & IBP. . .
- $\boxed{\varphi^4 D^2} \rightarrow \boxed{\varphi^4 D^2} + \boxed{\varphi^3\psi^2} + \boxed{\varphi^6} + \mu^2\boxed{\varphi^4}$
- No further reduction possible via EoM; all operators expressed in lowest possible classifications

```
DC[Phi[j], μ] DC[Phi†[l], μ] IndexDelta[j, i] IndexDelta[l, k] Phi[k] Phi†[i] +
 del[del[Phi[l] Phi†[k], mu], mu] IndexDelta[j, i] IndexDelta[l, k] Phi†[i] Phi†[j] +
 lR[s1, p].LL[s2, j, r] yl[p, r]† IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k]
  Phi[l] Phi†[i] Phi†[k] + dR[s1, p, a].QL[s2, j, r, b] yd[p, r]† IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[l, k] Phi[l] Phi†[i] Phi†[k] -
 μ^2 IndexDelta[j, i] IndexDelta[l, k] Phi[j] Phi[l] Phi†[i] Phi†[k] -
 uR[s1, r, a].QL[s2, m, p, b] yu[p, r]† Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] -
 μ^2 IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[j] Phi†[l] + lam IndexDelta[j, i]
  IndexDelta[l, n] IndexDelta[m, k] Phi[j] Phi[l] Phi[m] Phi†[i] Phi†[k] Phi†[n] +
 lam IndexDelta[i, j] IndexDelta[k, m] IndexDelta[l, n] Phi[i] Phi[k] Phi[l]
  Phi†[j] Phi†[m] Phi†[n] + QL[s1, j, r, a].dR[s2, p, b] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i] Phi[k] Phi†[l] yd[p, r] +
 LL[s1, j, r].lR[s2, p] IndexDelta[s1, s2] IndexDelta[i, j] IndexDelta[k, l] Phi[i]
  Phi[k] Phi†[l] yl[p, r] - QL[s1, m, p, a].uR[s2, r, b] Eps[j, m] IndexDelta[a, b]
  IndexDelta[s1, s2] IndexDelta[j, i] IndexDelta[j, k] Phi[l] Phi†[i] Phi†[k] yu[p, r]
```

$\{D^2\varphi^4,\ \varphi^2,\ \varphi^3\psi^2,\ \varphi^3\psi^2,\ \varphi^4,\ \varphi^3\psi^2,\ \varphi^6,\ \varphi^3\psi^2,\ \varphi^3\psi^2,\ \varphi^3\psi^2\}$

Reduction of Four-Fermion Operators

Warsaw classifications of ψ^4 operators are reproduced by encoding three conditions:

- **1**: Require all fermion billinears are simultaneously diagonal in gauge/spin indices (i.e. no billinears like $(\bar{\psi}_a \Gamma \psi_b)(\bar{\psi}_b \Gamma \psi_a)$)
- **2**: Require operators be written as product of $Y = 0$ currents if possible
- **3**: As far as is possible while satisfying **1** & **2**, use Fierz identities to eliminate $\sigma_{\mu\nu} \otimes \sigma^{\mu\nu} > T^A \otimes T^A > \tau^I \otimes \tau^I$ in favour of scalar/singlet currents

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These uniquely fix the basis operators found within each chiral ψ^4 classification. . .

Four-Fermion Example

$$(\bar{q}_p\gamma^\mu T^A\tau^I q_r)(\bar{q}_s\gamma_\mu T^A\tau^I q_t) \rightarrow [\tau^I_{ij}\tau^I_{kl}]_{SU(2)} \times [T^A_{ab}T^A_{cd}]_{SU(3)} \times [\gamma^\mu_{s_1 s_2}\gamma_{\mu\, s_3 s_4}]_{\text{Spin}} \times \{\bar{q}^{i\,a}_{s_1},\, q^{j\,b}_{s_2},\, \bar{q}^{k\,c}_{s_3},\, q^{l\,d}_{s_4}\}$$

```
Q̄L[s1, d, p, a].QL[s2, e, r, b] Q̄L[s3, f, s, c].QL[s4, g, t, d]
 Ga[mu, s1, s2] Ga[mu, s3, s4] T[AA, a, b] T[AA, c, d] Ta[II, d, e] Ta[II, f, g]
```

```
(Lbar.L)^2
```

$$\frac{3}{16}(\bar{q}_p\gamma^\mu\tau^I q_t)(\bar{q}_s\gamma_\mu\tau^I q_r) - \frac{1}{6}(\bar{q}_p\gamma^\mu q_r)(\bar{q}_s\gamma_\mu q_t) - \frac{1}{4}(\bar{q}_p\gamma^\mu q_t)(\bar{q}_s\gamma_\mu q_r)$$

```
3/16 Q̄L[s1, i, p, a].QL[s4, l, t, d] Q̄L[s3, k, s, c].QL[s2, j, r, b] Ga[μ, s1, s4]
 Ga[μ, s3, s2] IndexDelta[a, d] IndexDelta[c, b] IndexDelta[i, l] IndexDelta[k, j] -
1/4 Q̄L[s1, i, p, a].QL[s4, l, t, d] Q̄L[s3, k, s, c].QL[s2, j, r, b] Ga[μ, s1, s4]
 Ga[μ, s3, s2] IndexDelta[a, d] IndexDelta[c, b] Ta[II, i, l] Ta[II, k, j] -
1/6 Q̄L[s1, i, p, a].QL[s2, j, r, b] Q̄L[s3, k, s, c].QL[s4, l, t, d] Ga[μ, s1, s2]
 Ga[μ, s3, s4] IndexDelta[a, b] IndexDelta[c, d] Ta[II, i, j] Ta[II, k, l]
```

```
{(Lbar.L)^2, (Lbar.L)^2, (Lbar.L)^2}
```

Another Four-Fermion Example

$$(\bar{q}_p T^A u_r)(\bar{u}_s T^A q_t) \to [\delta_{\bar{i}\bar{j}}]_{SU(2)} \times [T^A_{ab} T^A_{cd}]_{SU(3)} \times [\gamma^\mu_{s_1 s_2} \gamma_{\mu\, s_3 s_4}]_{\text{Spin}} \times \{\bar{q}^{i\,a}_{s_1},\, u^b_{s_2},\, \bar{u}^c_{s_3},\, q^{j\,d}_{s_4}\}$$

```
-2 QLbar[ss1, d1, p, aa].uR[ss4, t, dd] uRbar[ss3, s, cc].QL[ss2, d2, r, bb]
 IndexDelta[ss1, ss4] IndexDelta[ss3, ss2] IndexDelta[d1, d2] T[AA, aa, dd] T[AA, cc, bb]
```

```
Lbar.R Rbar.L
```

$$\frac{4}{9}(\bar{q}_p \gamma^\mu T^A q_t)(\bar{u}_s \gamma_\mu T^A u_r) - \frac{1}{3}(\bar{q}_p \gamma^\mu T^A q_t)(\bar{u}_s \gamma_\mu T^A u_r)$$

```
4/9 QLbar[s1, i, p, a].QL[s2, j, r, b] uRbar[s3, s, c].uR[s4, t, d]
 Ga[μ, s1, s2] Ga[μ, s3, s4] IndexDelta[a, b] IndexDelta[c, d] IndexDelta[i, j] -
 1/3 QLbar[s1, i, p, a].QL[s2, j, r, b] uRbar[s3, s, c].uR[s4, t, d] Ga[μ, s1, s2]
 Ga[μ, s3, s4] IndexDelta[i, j] T[AA, a, b] T[AA, c, d]
```

```
{Lbar.L Rbar.R, Lbar.L Rbar.R}
```

Contents

Status & Goals

Now:

- All operator-level identities implemented in symbolic framework
- Automatic reduction of $\boxed{\psi^4}$ class operators complete, tested
- Bosonic classes $\boxed{\varphi^2 D^4}$, $\boxed{\varphi^4 D^2}$, $\boxed{\varphi^2 X D^2}$, $\boxed{X^2 D^2}$ ... $\approx$ complete, testing. . .
- Single current classes $\boxed{\psi^2 D^3}$, $\boxed{\psi^2 \varphi D^2}$, $\boxed{\psi^2 X D}$, $\boxed{\psi^2 X \varphi}$ underway. . .

Physics applications:

- Interface with NLOCT $\implies$ verify SMEFT RGEs
- Check linear independence of arbitrary operators
- Generalize beyond SMEFT, apply to $D = 8$ SMEFT. . .

Summary

- EFT $\Longrightarrow$ operator redundancy, prevelant in calculations
- $\mathcal{L}^{(6)}_{\text{SMEFT}}$ complex, non-trivial operator redundancies eliminated by Warsaw algorithm
- Proof-of-principle: laborious bookkeeping procedure can be fully automated for the SMEFT using FeynRules/Mathematica
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Single Fermionic Current Operators

Single fermionic current operators can be redundant through fermion EoMs e.g. :

$$i\not{D}q = -\Gamma_u u\widetilde{\varphi} - \Gamma_d d\varphi$$

$$(\bar{q}\gamma^\mu T^A D^\nu q)G^A_{\mu\nu} \to [\delta_{\bar{i}j}]_{SU(2)} \times [T^A_{ab}]_{SU(3)} \times [\gamma^\mu_{s_1 s_2}]_{\text{Spin}} \times [\eta^{\nu\rho}]_{\text{Lor}} \times \left(\{\mathbb{I},\, D_\rho,\, \mathbb{I}\} \cdot \{\bar{q}^{i\,a}_{s_1},\, q^{j\,b}_{s_2},\, G^A_{\mu\rho}\}\right)$$

Dirac algebra identities determine whether spacetime and spin indices can be redistributed to project out $\not{D}q$...

Use: $\eta_{\mu\nu}\gamma_\rho \to \gamma_\nu\eta_{\mu\rho} + i\gamma_\mu\sigma_{\nu\rho} + i\epsilon_{\nu\rho\mu\sigma}\gamma^\sigma\gamma_5$, so...

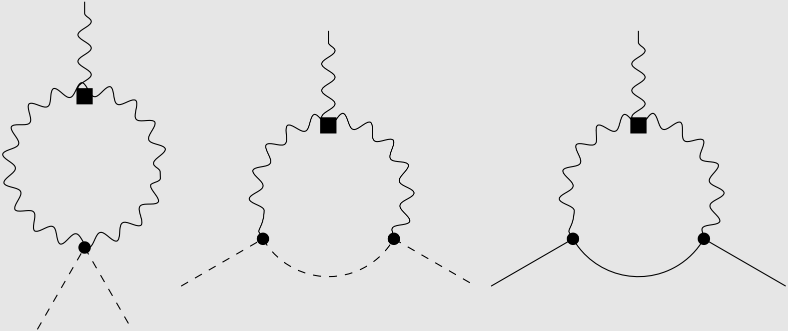
$$(\bar{q}\gamma^\mu T^A D^\nu q)G^A_{\mu\nu} + h.c. \to -i(\bar{q}\gamma^\nu T^A D^\mu q)\widetilde{G}^A_{\mu\nu} - i(\bar{q}\sigma^{\mu\nu} T^A \not{D}q)G^A_{\mu\nu}$$

```
QL̄[s1, i, p, a].DC[QL[s2, i, r, b], nu] FS[G, mu, nu, AA] Ga[mu, s1, s2] T[AA, a, b]
```

```
QL̄[s1, i, p, a].DC[QL[s2, i, r, b], mu2] FS[G, mu2, nu, AA] Ga[nu, s1, s2] T[AA, a, b] -
 ⅈ QL̄[s1, i, p, a].DC[QL[s2, i, r, b], nu2] Eps[mu2, nu, nu2, sig$852000]
  FS[G, mu2, nu, AA] Ga[sig$852000, s1, s2] T[AA, a, b] -
 ⅈ QL̄[s1, i, p, a].DC[QL[s2, i, r, b], nu2] FS[G, mu2, nu, AA]
  Ga[nu2, dumsp$851999, s2] Sig[mu2, nu, s1, dumsp$851999] T[AA, a, b]
```

A SMEFT Example

EoM redundancy extensive in one-loop SMEFT renormalization **(Alonso et al. 1312.2014)**, important in structure of mixing, done by hand. . .



Graphs with X^3 operator insertions which cancel after using the EoM

$$A_1 \propto D^\mu \varphi^\dagger \tau^I D^\nu \varphi W^I_{\mu\nu}, \quad A_{2,3} \propto D^\mu W^I_{\mu\nu} j_\psi^{I\,\nu} + D^\mu G^A_{\mu\nu} j_\psi^{A\,\nu}$$