Some theoretical issues of Chiral anomalous transports

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phases of QCD &BES @ Fudan Uni. Aug15-18 , 2017

Outline

I. Introduction to anomalous transport

II. CME with non-constant Axial μ_5 & B

III. Subtlety of the Wigner function used for CME

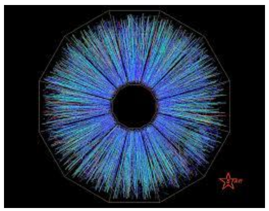
IV CME on lattice and Higher order correction

V. Conclusion and outlook

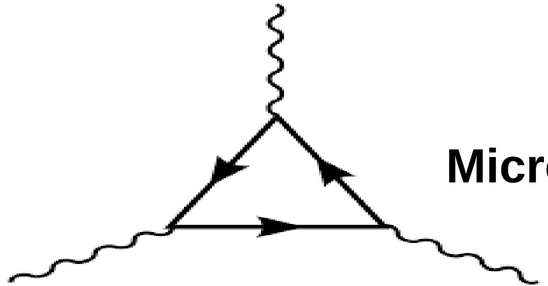
I. Introduction : Anomalous Transports

(Fukushima-Kharzeev-Warringa, Son-Zhitnitsky, Vilenkin

Micro-quantum anomaly + B/ Ω → macro-transport (CME/CVE)



Search in HIC



Micro

B **Ω**

$$\vec{J} = \sigma_5 \mu_5 \vec{B}$$

Macro

Astrophysics, cosmology

Weyl semimetal
(non-degenerated bands)

TaAs
NbAs
NbP
TaP

Dirac semimetal
(doubly degenerated bands)

$ZrTe_5$
Na_3Bi,
Cd_3As_2

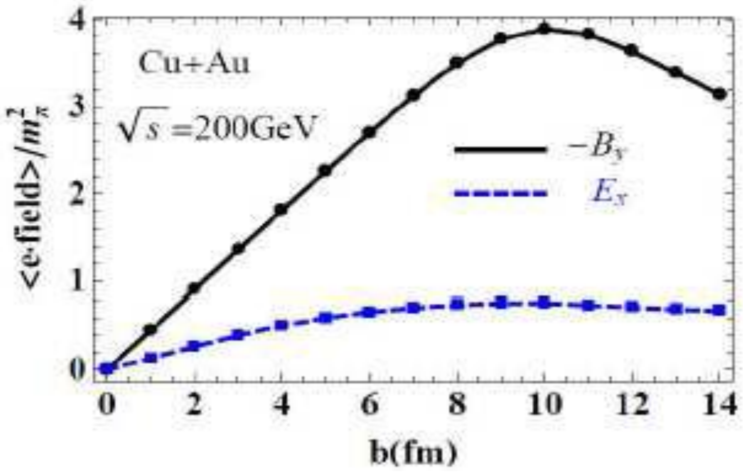
Nature Phys.12（2016）

Phys. Rev. X.5（2015）

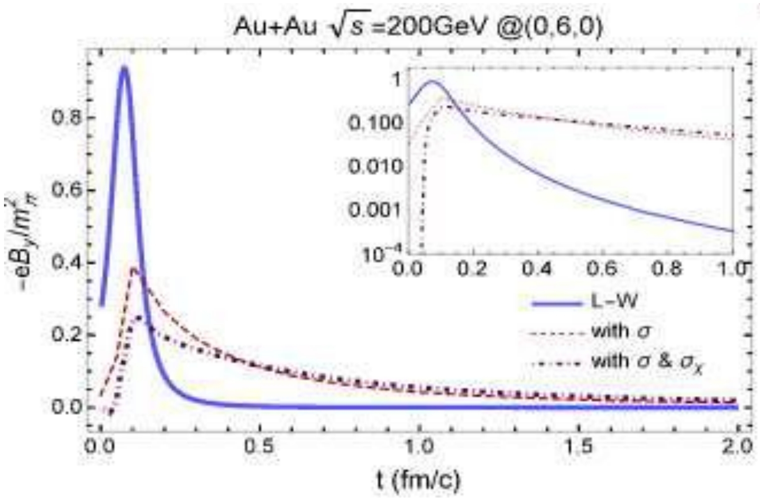
Science350（2015）413

X.Huang， Rep.prog. Phys. (2016)
Kharzeev,Liao,Voloshin,Wang: Prog.Part.Nucl.Phys.2016

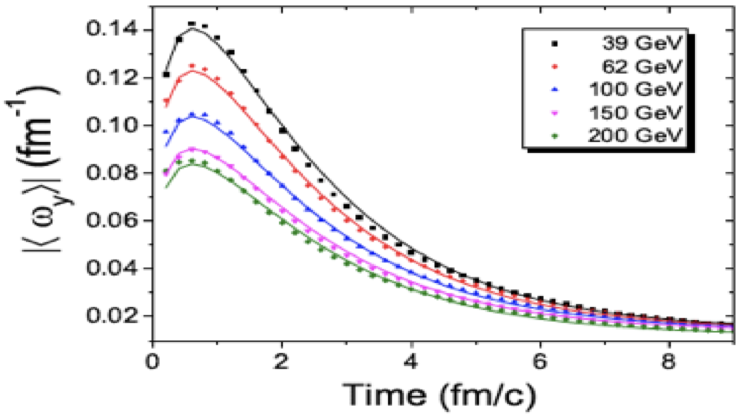
Strong EM Field/Rotation/polarization produced in HIC



Deng, Huang, 2015



Li, Sheng, Wang 2016



Jiang,Lin,Liao, 2016

Net axial charge generated

$$\partial_\mu {j_5}^\mu = -\frac{q^2 N_c}{16\pi^2} F\tilde{F} - \frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$

$F\tilde{F}$	Parallel electric and magnetic fields	condensed matter
$trG\tilde{G}$	Topological field configurations(instanton, sphaleron) Parallel chromo electric and magnetic fields	HIC
$2im\bar{\psi}\gamma^5\psi$	Explicit breaking by quark mass	HIC

All three can lead to net axial charge $N_5 = \int d^3x\, {j_5}^0$

(See S. Lin's talk on Friday)

- **Experimental situation in HIC** (See H.Z.Huang's talk)

① **Off central collisions generate inhomogeneous & transient $\vec{B}$**

② **axial charge produced via topological fluct. plus mass effect**
(See S. Lin's talk on Friday)

③ **Beyond thermal equilibrium**

- **Theoretical investigations:**

① Field theory
K. Fukushima et. al., Phy. Rev. D. 78, 074033, 2008
Hou， Liu， Ren， JHEP 1105: 046, 2011
D. Kharzeev et. al., arXiv: 1312.3348

② Holography
Yee. Rebhan et. al., JHEP 0911: 085, 2009
Shu Lin et. al., PRL114, 2015; PRD88. 2013

③ Hydrodynamics & Kinetic theory

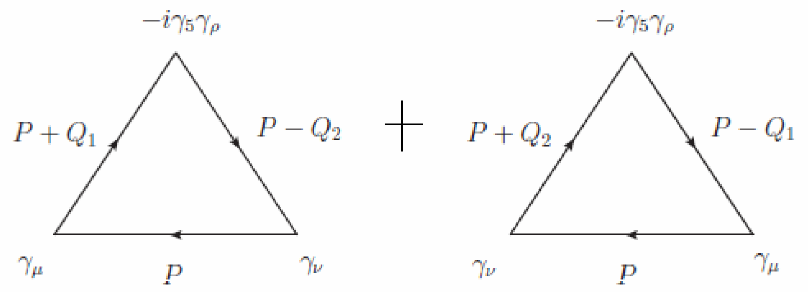
D. T. Son et. al., PRL103, 2009, PRL106, 2011; Stephanov, Yin, PRL (2012)
Gao, Liang, Pu, Q Wang, XN Wang PRL109 2012,

Anomalous Viscous Fluid Dynamics (AVFD): Jiang, Shi, Yin, JL, PLB (2015); arXiv:1611.04586.

General properties and Subtleties

Axial anomaly:

$$\Delta_{\rho\mu\nu}(Q_1, Q_2) \propto$$



$$\propto \langle J_{5\rho} J_\mu J_\nu \rangle$$

Vector Ward identity: $$Q_{1\mu}\,\Delta_{\rho\mu\nu}(Q_1, Q_2) = Q_{2\nu}\,\Delta_{\rho\mu\nu}(Q_1, Q_2) = 0 \qquad (1)$$

Naive Axial Vector Ward identity $$(Q_1 + Q_2)_\rho\,\Delta_{\rho\mu\nu}(Q_1, Q_2) = 0 \qquad (2)$$

UV diverge$\longrightarrow$ impossible to maintain （1） & （2）
Gauge invariance$\longrightarrow$ vector Ward identity

Anomaly, $$(Q_1 + Q_2)_\rho\,\Delta_{\rho\mu\nu}(Q_1, Q_2) = -i\frac{e^2}{2\pi^2}\epsilon_{\mu\nu\alpha\beta}Q_{1\alpha}Q_{2\beta}$$

Universal to all orders of coupling, all temperature & chemical potential .Necessary to explain $\pi^0 \to 2\gamma$

Anomalous, Ward identity in an electromagnetic field

$$\frac{\partial}{\partial x_\mu}\left\langle J_{5\mu}\right\rangle = i\eta\frac{e^2}{2\pi^2}\varepsilon_{\mu\nu\rho\lambda}F^l_{\mu\nu}F^l_{\rho\lambda} = \frac{\partial}{\partial x_\mu}\Omega_\mu$$

$$\Omega_\mu = Chern - Simons = i\eta\frac{e^2}{4\pi^2}\varepsilon_{\mu\nu\rho\lambda}A_\nu\frac{\partial A_\lambda}{\partial x_\rho}$$

Naïve axial charge Q_5 is not conserved

Conversed axial charge

$$Q'_5 = Q_5 + i\int d^3\vec{r}\,\Omega_4$$

$$\frac{dQ'_5}{dt} = 0$$

Q' Should be used in the equilibrium thermodynamics (Rubakov)

II. Non-constant μ_5 & B / Subtlety of Constant Limits

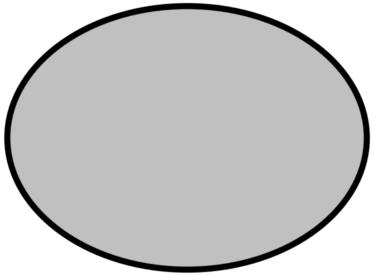
Hou, Ren, Liu JHEP 05(2011)046

CME in general

$$\mu_5(\mathbf{k}, k_0)$$

$$\Downarrow$$

$$\mathbf{J}\left(\mathbf{q}+\frac{1}{2}\mathbf{k},\ \omega+\frac{1}{2}k_0\right) \Leftarrow$$



$$\Leftarrow \mathbf{B}\left(\mathbf{q}-\frac{1}{2}\mathbf{k}, \omega-\frac{k_0}{2}\right)$$

Constant limit:

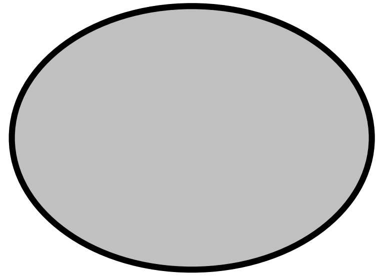
$$(\mathbf{k}, k_0) \to 0 \quad \text{and} \quad (\mathbf{q}, \omega) \to 0$$

Always gives $\mathbf{J} = \eta \frac{e^2}{2\pi^2}\mu_5 \mathbf{B}$? $\eta = N_C \sum_f q_f^2 =$ Color-flavor factor

$$\mu_5(\mathbf{k}, k_0)$$

$$\Downarrow$$

$$\mathbf{J}\left(\mathbf{q}+\frac{1}{2}\mathbf{k},\ \omega+\frac{1}{2}k_0\right) \Leftarrow$$



$$\Leftarrow \mathbf{B}\left(\mathbf{q}-\frac{1}{2}\mathbf{k}, \omega-\frac{k_0}{2}\right)$$

Constant μ_5 *, non-constant* **B:** $\mathbf{k} = k_o = 0$

$$\text{limit}_{\mathbf{q}\to 0}\text{limit}_{\omega\to 0} \Rightarrow \mathbf{J} = \eta \frac{e^2}{2\pi^2}\mu_5 \mathbf{B}$$

$$\text{limit}_{\omega\to 0}\text{limit}_{q\to 0} \Rightarrow \mathbf{J} = \frac{1}{3}\times\eta \frac{e^2}{2\pi^2}\mu_5 \mathbf{B}$$

Kharzeev & Warringar

Artifact of one-loop approximation. The ambiguity disappears with higher order corrections. (*Satow & Yee*)

Constant ***B****, non-constant* $\mu_5(\mathbf{k}, k_0)$

$$\Downarrow$$

$$\text{limit}_{\mathbf{k}\to 0}\text{limit}_{k_0\to 0} \Rightarrow \qquad \mathbf{J} = 0$$

$$\text{limit}_{k_0\to 0}\text{limit}_{\mathbf{k}\to 0} \Rightarrow \qquad \mathbf{J} = \eta\frac{e^2}{2\pi^2}\mu_5\mathbf{B}$$

Follows from the EM gauge invariance and the non-renormalization of the axial anomaly. Valid to all orders!

with T=0 and $\mu = 0$: relativistic invariance requires the two limit orders are equivalent:

$$\mathbf{J} = 0$$

$$\mathbf{J}_{CME}\left(\mathbf{q}+\frac{1}{2}\mathbf{k},\omega+\frac{1}{2}k_0\right) \quad \text{vs.} \quad \mu_5\left(\mathbf{k},k_0\right)\ \&\ \mathbf{B}\left(\mathbf{q}-\frac{1}{2}\mathbf{k},\omega-\frac{1}{2}k_0\right)$$

<table>
<tr><th></th><th colspan="2">IR limit</th><th>$\mathbf{J}_{CME}$</th><th>Higher order</th></tr>
<tr><td rowspan="2">$T=0$
$\mu=0$</td><td colspan="2">$\lim_{k_0\to 0}\lim_{\vec{k}\to 0}$</td><td rowspan="2">0</td><td rowspan="2">none</td></tr>
<tr><td colspan="2">$\lim_{\vec{k}\to 0}\lim_{k_0\to 0}$</td></tr>
<tr><td rowspan="3">$T\neq 0$
and/or
$\mu\neq 0$</td><td rowspan="2">$\lim_{\vec{k}\to 0}\lim_{k_0\to 0}$</td><td>$\lim_{\vec{q}\to 0}\lim_{\omega\to 0}$</td><td>$\eta\frac{e^2}{2\pi^2}\mu_5\mathbf{B}$</td><td>none if IR safe</td></tr>
<tr><td>$\lim_{\omega\to 0}\lim_{\vec{q}\to 0}$</td><td>$\frac{1}{3}\times\eta\frac{e^2}{2\pi^2}\mu_5\mathbf{B}$</td><td>yes</td></tr>
<tr><td colspan="2">$\lim_{k_0\to 0}\lim_{\vec{k}\to 0}$</td><td>0</td><td>none</td></tr>
</table>

III. Subtlety of the Wigner function used for CME

- **Wigner function formulation**

Vasak, Gyulassy , Elze (1987), Heinz et al (non-abelian plasma) (96)

The **Wigner function** that links various hydrodynamic quantities of the system to the Green's function

Gao et al obtain CME& CVE and axial anomalies **with a constant** μ_5

Gao, Liang, Pu, qWang, xnWang (2012)

(see S. Pu's talk)

,

inhomogeneous and transient μ_5 **?**

Wu,Hou, Ren, 2016

- **The electric current extracted from the wigner function**

$$J_\mu(x) = ie\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}W(x,p)\gamma_\mu$$

$$= ie\int d^4y\delta^4(y)U(x_+,x_-) < \overline{\psi}(x_+)\gamma_\mu\psi(x_-) >$$

$$= \lim_{y\to 0} J_\mu(x,y)$$

$$J_\mu(x,y) = ieU(x_+,x_-) < \overline{\psi}(x_+)\gamma_\mu\psi(x_-) >$$

For a constant μ_5

$$J_\mu = \eta\frac{e^2}{2\pi^2}\mu_5 B_\mu \qquad \text{with } B_\mu = \frac{1}{2}\varepsilon_{\mu\nu\rho\lambda}u_\nu F_{\rho\lambda}$$

$$u_\mu = \text{fluid velocity}$$

For a non-constant μ_5, it is problematic because of UV divergence with the limit $y\to 0$

Considering a massless Dirac field in an external A_μ and $A_{5\mu}$

* **The action:**

$$S = \int dt \int d^3x L$$

* **The Lagrangian density:**

$$L = -\bar{\psi}\gamma_\mu(\partial_\mu - ieA_\mu - i\gamma_5 A_{5\mu})\psi$$

with $A_{5\mu} = (\vec{A}_5, -i\mu_5)$

- **Closed time path Green function formation**

* **A fermion propagator**

$$S_{CTP}(x,y)=\begin{pmatrix} S_{11}(x,y) & S_{12}(x,y) \\ S_{21}(x,y) & S_{22}(x,y) \end{pmatrix}$$

$$S_{11}(x,y)_{\alpha\beta}=\left\langle T[\psi_\alpha(x)\overline{\psi}_\beta(y)]\right\rangle \quad S_{12}(x,y)_{\alpha\beta}=-\left\langle \overline{\psi}_\beta(y)\psi_\alpha(x)\right\rangle$$

$$S_{21}(x,y)_{\alpha\beta}=\left\langle \psi_\alpha(x)\overline{\psi}_\beta(y)\right\rangle \quad S_{22}(x,y)_{\alpha\beta}=\left\langle \tilde{T}[\psi_\alpha(x)\overline{\psi}_\beta(y)]\right\rangle$$

T: time ordering $\qquad \tilde{T}$: anti-time ordering

* **The electric current**

$$J_\mu(x,y)=\begin{cases} -ie\,\mathrm{tr}\,S_{11}(x_-,x_+)\gamma_\mu = J^1_\mu(x,y) & y_0\ge 0 \\ -ie\,\mathrm{tr}\,S_{22}(x_-,x_+)\gamma_\mu = J^2_\mu(x,y) & y_0<0 \end{cases}$$

* Expansion to the linear order in A_μ and $A_{5\mu}$

full propagator:

free propagator

$$S_{ab}(x_-,x_+)$$
$$=S_{ab}(x_-,x_+)-\sum_c\int d^4z S_{ac}(x_- - z)\gamma^c_{\rho 5}S_{cb}(z-x_+)A_{5\rho}(z)$$
$$-e\sum_c\int d^4z S_{ac}(x_- - z)\gamma^c_{\rho}S_{cb}(z-x_+)A_{\rho}(z)$$
$$+e\sum_{cd}\int d^4z_1\int d^4z_2 S_{ad}(x_- - z_2)\gamma^d_{\lambda 5}S_{dc}(z_2-z_1)\gamma^c_{\rho}S_{ca}(z_1-x_+)A_{\rho}(z_1)A_{5\lambda}(z_2)$$
$$+e\sum_{cd}\int d^4z_1\int d^4z_2 S_{ac}(x_- - z_2)\gamma^c_{\rho}S_{cd}(z_2-z_1)\gamma^d_{\lambda 5}S_{da}(z_1-x_+)A_{\rho}(z_2)A_{5\lambda}(z_1)$$

with $\gamma^1_\mu=\gamma_\mu,\qquad \gamma^2_\mu=-\gamma_\mu,\qquad \gamma^1_{\mu 5}=\gamma_\mu\gamma_5,\qquad \gamma^2_{\mu 5}=-\gamma_\mu\gamma_5$

gauge link:

$$U(x_-,x_+)=1+ie\int_{x_-}^{x_+}d\xi_\upsilon A_\upsilon(\xi)+O(A^2)$$

Problems of the Wigner function formalism

* **The nonconserversion of the electric current**

$$\frac{\partial}{\partial x_\mu} J_\mu(x,y) = \frac{i}{8\pi^2}\left[\varepsilon_{\mu\rho\beta\lambda} F_{\mu\rho}(x) F_{5\beta\lambda}(x) + 2\varepsilon_{\mu\rho\alpha\beta} \boxed{\frac{y_\lambda y_\alpha}{y^2}} F_{\mu\rho}(x) \frac{\partial}{\partial x_\beta} A_{5\lambda}(x)\right]$$

with $F_{\mu\upsilon}(x) = \dfrac{\partial A_\upsilon}{\partial x_\mu} - \dfrac{\partial A_\mu}{\partial x_\upsilon}$,

$$F_{5\mu\upsilon}(x) = \frac{\partial A_{5\upsilon}}{\partial x_\mu} - \frac{\partial A_{5\mu}}{\partial x_\upsilon}$$

* * **by averaging the direction of y**

$$\frac{\partial}{\partial x_\mu} J_\mu(x) = \frac{\partial}{\partial x_\mu} J_\mu(x,0) = \frac{3i}{32\pi^2} \varepsilon_{\mu\rho\beta\lambda} F_{\mu\rho}(x) F_{5\beta\lambda}(x)$$

$$\boxed{\partial_\mu J_\mu \neq 0}$$

The electric current is not conserved!

unless the axial potential is a pure gradient

$$A_{5\mu} = \frac{\partial\theta}{\partial x_\mu},$$

* An incosistency

In principle, the electric current should satisfy the consistency

$$\frac{\delta J_\mu(x)}{\delta A_\upsilon(x')} = \frac{\delta J_\upsilon(x')}{\delta A_\mu(x)}$$

However, the electric current from Wigner function:

$$\frac{\delta J_\mu(x,y)}{\delta A_\upsilon(x')} - \frac{\delta J_\upsilon(x',y')}{\delta A_\mu(x)} = \frac{i}{2\pi^2}\left(\varepsilon_{\mu\rho\beta\lambda} + \varepsilon_{\mu\rho\alpha\beta}\boxed{\frac{y_\lambda y_\alpha}{y^2}}\right)\frac{\partial A_{5\lambda}(x)}{\partial x_\beta}\delta^4(x-x')$$

Averaging the direction of y

$$\frac{\delta J_\mu(x,y)}{\delta A_\upsilon(x')} - \frac{\delta J_\upsilon(x',y')}{\delta A_\mu(x)} = \frac{3i}{16\pi^2}\varepsilon_{\mu\rho\beta\lambda}F_{5\beta\lambda}\delta^4(x-x')$$

The consistency condition broken!

Applying to a general chiral case, the present form of the Wigner function formulation needs to be revised!

correction : the regulated Wigner function

a robust regularization scheme has to be introduced to the underlying field theory before defining the wigner function.

e.g. If the underlying field theory is regularized by PV scheme

$$L = -\bar{\psi}\gamma_\mu(\partial_\mu - ieA_\mu - i\gamma_5 A_{5\mu})\psi$$

$$J_\mu(x) = i\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}W(x,p)\gamma_\mu$$

$$= i\lim_{y\to 0} U(x_+, x_-) < \bar{\psi}(x_+)\gamma_\mu\psi(x_-) >$$

⟵ PV regulator should be included in it

$$\partial_\mu J_\mu = 0 \qquad \frac{\delta J_\mu(x)}{\delta A_\upsilon(x')} = \frac{\delta J_\upsilon(x')}{\delta A_\mu(x)}$$

But the CME current would also be cancelled.
The Bardeen like term should be added in.

$$J_\mu(x) = ie \int \frac{d^4p}{(2\pi)^4} \mathrm{tr} \left[W(x,p) + \sum_s C_s W(x,p|M_s) \right] =$$

$$J_\mu(x) = -ie\frac{1}{2} \left[\mathrm{Tr}\gamma_\mu \mathcal{S}_0(x,x) - \sum_s C_s \mathrm{Tr}\gamma_\mu \mathcal{S}_s(x,x) \right]$$

$$J_\mu(x) = e^2 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} e^{i(q_1+q_2)\cdot x} \Lambda_{\mu\rho\lambda}(q_1,q_2) A_\rho(q_1) A_{5\lambda}(q_2)$$

Wu,Hou, Ren, 2016

gives CME current :

$$\lim_{q_{20}\to 0}\lim_{\vec{q}_2\to 0}\Lambda_{ij4}(q_1,q_2) = -\frac{1}{2\pi^2}\epsilon_{ikj}q_{1k}$$

CME current canceled at thermal equilibrium.

$$\lim_{\vec{q}_2\to 0}\lim_{q_{20}\to 0}\Lambda_{ij4}(q_1,q_2) = \frac{2f(0)-1}{2\pi^2}\epsilon_{ikj}q_{1k} + O(q_1^2)$$

Phenomenological implications of the subtleties regarding the order of limits

Axial charge generated via toplogical fluctuations dictated by the stochastic Eq with a white noise

$$\left(\frac{\partial}{\partial t} - D\nabla^2 + \frac{1}{\tau}\right) n_5 = g(x)$$

In Momentum sapce

$$n_5(k) = \frac{g(k)}{-ik_0 + D\vec{k}^2 - \frac{1}{\tau}}$$

Corresponding an axial potential

$$A_{5\mu}(k) = -i\delta_{\mu 4}\frac{n_5(k)}{\chi(k)}$$

Average current vanishes, the correlation funct. $< J_i(x)J_j(y) >$ **is dominated by diffusion pole**

$$-iq_{20} + D\vec{q}_2^2 + \frac{1}{\tau} = 0$$

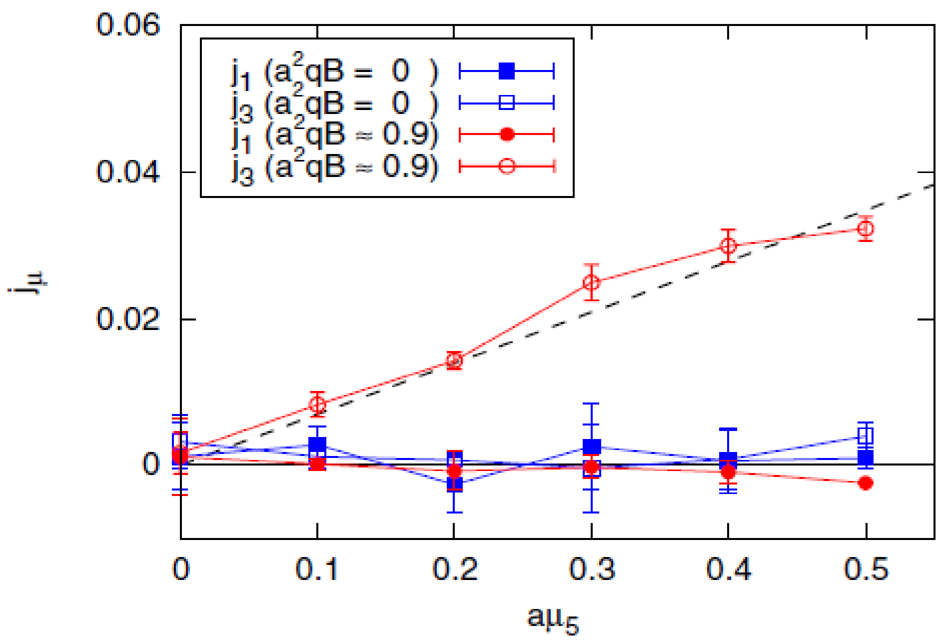
$$\frac{|q_{20}|}{|\vec{q}_2|} = D|\vec{q}_2| + \frac{1}{|\vec{q}_2|\tau} \geq \sqrt{\frac{D}{\tau}}.$$

If $\sqrt{D/\tau} >> 1$ **the homog. \mu_5 is a good approximation and classic form of CME current emerges ---Noneq. Phenom.**

Towards equilibrium, $\tau \to \infty,\qquad \frac{|q_{20}|}{|\vec{q}_2|} = D|\vec{q}_2| \sim \frac{D}{|\vec{x}-\vec{y}|} \to 0$

Inverse limit-order prevails, and CME current disappears,

CME on Lattice



See Bo Feng"s talk

Yamamoto,PRL(2011)

using lattice QCD with Wilson term

$$I = -\sum_x \sum_\mu \frac{1}{2a}\left[\bar{\psi}(x)\left(\frac{1}{i}\gamma_\mu - r\right)U_\mu(x)\psi(x+a_\mu)\right.$$
$$\left. - \bar{\psi}(x+a_\mu)\left(\frac{1}{i}\gamma_\mu + r\right)U_\mu^\dagger(x)\psi(x)\right]$$
$$-\sum_x M\bar{\psi}(x)\psi(x) + \cdots$$

Karsten and Smit (1981)

$$J_i(p) = -\Pi_{ij}(p) A_j(p)$$

One-loop self-energy on lattice of size $N_s^3 \times N_t$

$$\Pi_{ij}^{(1)}(p) = \mathcal{I} \sum_k \epsilon_{ikj} p_k + \mathcal{O}(a)$$

CME vanishes at continu. limit .

At zero temperature

$$\Pi_{ij}(q) \equiv \Lambda_{ij4}(q)$$
$$= -\lim_{q_4 \to 0} \frac{1}{q_4} \sum_\rho \frac{2}{a} \sin \frac{1}{2} a (Q_1 + Q_2)_\rho \Lambda_{ij\rho}(Q_1, Q_2)$$

$$\Pi_{ij}(q) = \frac{e^2}{2\pi^2} \sum_k \epsilon_{ijk} q_k$$

Feng,Hou, Liu, Ren, Wu, PRD95,(2017)

umerical calculations

Lattice size	$\mathcal{I}$
$N_s = 6, N_t = 4$	1.347×10^{-2}
$N_s = 12, N_t = 4$	2.439×10^{-4}
$N_s = 20, N_t = 4$	8.886×10^{-7}
$N_s = 50, N_t = 8$	4.512×10^{-9}

nalytical calculations(In the limit $N_s \to \infty$)

$$\mathcal{I} = 12 \frac{1}{N_t} \sum_{l_4} \int \frac{d^3\mathbf{l}}{(2\pi)^3} \frac{\mathcal{N}(l)}{\left[\sin^2 l + \mathcal{M}^2(l)\right]^3} = 0$$

Higher order correction to CVE

Under B & vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{v}$

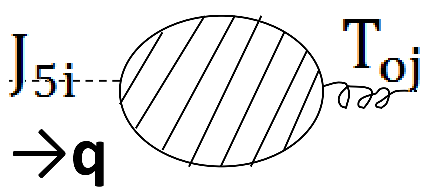
$$\vec{J}_{\rm em} = \sigma^B_{\rm em}\vec{B} + \sigma^V_{\rm em}\vec{\omega},$$
$$\vec{J}_{\rm b} = \sigma^B_{\rm b}\vec{B} + \sigma^V_{\rm b}\vec{\omega},$$
$$\vec{J}_5 = \sigma^B_5\vec{B} + \sigma^V_5\vec{\omega},$$

Son & Surowka

$\boldsymbol{\sigma}^{\rm B}_{\rm em}, \boldsymbol{\sigma}^{\rm B}_{\rm b}, \boldsymbol{\sigma}^{\rm B}_5 \rightarrow \text{CME}$

$\boldsymbol{\sigma}^{\rm v}_{\rm em}, \ \boldsymbol{\sigma}^{\rm v}_{\rm b}, \ \boldsymbol{\sigma}^{\rm v}_5 \rightarrow \text{CVE}$

Kubo formula：



$\mathbf{q} \rightarrow 0$: $\sigma^{\rm v}_5 \varepsilon_{ijk} q_k$

Triangle anomaly & hydrodynamics & thermodynamics

$$\sigma_5^V = \frac{\mu_5^2}{2\pi^2} + cT^2$$

D. T. Son & P. Surowka

Y. Neiman & Y. Oz

One-loop calculation *Landsteiner et. al.* C=1/12

Kinetic theory , *Stephenov, Gao et.al.*

Any higher order connection to c ?

- Possible relation with the gravity anomaly→ No （*Landsteiner et.al*）
- Coleman-Hill theorem for a field theory without gauge degrees of freedom at all→（*Golkan & Son*）
- A field theory with gauge degrees of freedom？

Higher order correction to CVE

Field Theoretic Formulation:

QED Lagrangian density

$$\mathcal{L} = -\frac{1}{4e^2}V^{\mu\nu}V_{\mu\nu} - i\bar{\psi}\gamma^{\mu}D_{\mu}\psi + \frac{1}{2}h^{\mu\nu}T_{\mu\nu} + A^{\mu}J_{5\mu}$$

$$V_{\mu\nu} = \partial_{\mu}V_{\nu} - \partial_{\nu}V_{\mu}$$

$$D_{\mu} = \partial_{\mu} - iV_{\mu}$$

$$T_{\mu\nu} = V_{\mu}{}^{\rho}\,V_{\nu\rho} - \frac{1}{4}g_{\mu\nu}V^{\rho\lambda}V_{\rho\lambda} + \frac{1}{4}\left(-D_{\mu}\bar{\psi}\gamma_{\nu}\psi - D_{\nu}\bar{\psi}\gamma_{\mu}\psi + \bar{\psi}\gamma_{\mu}D_{\nu}\psi + \bar{\psi}\gamma_{\nu}D_{\mu}\psi\right)$$

$$J_5^{\mu} = i\bar{\psi}\gamma_{\mu}\gamma_5\psi$$

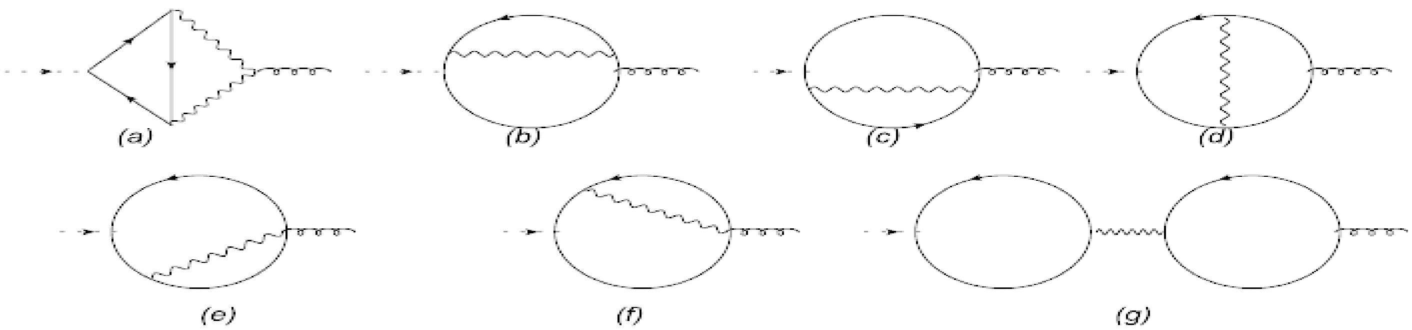
Anomalous Ward identity

$$\partial_{\mu}J_5^{\mu} = \frac{e^2}{16\pi^2\sqrt{-g}}\epsilon^{\mu\nu\rho\lambda}V_{\mu\nu}V_{\rho\lambda}$$

Kubo formula for CVE

$$\mathcal{G}_{ij}(\vec{q}) = -\int_0^{\infty}dt\int d^3\vec{r}\,e^{-i\vec{q}\cdot\vec{r}}\,\frac{\mathrm{Tr}e^{-\beta H}\left[J_{5i}(\vec{r},t),T_{0j}(0,0)\right]}{\mathrm{Tr}e^{-\beta H}} \underset{\vec{q}\to 0}{\longrightarrow} \sigma_V\epsilon_{ijk}q_k$$

Hou,Liu,Ren, PRD86 (2012) 121703®



$$\xi_5 = \frac{\mu_5^2}{2\pi^2} + cT^2$$

Are there any corrections from higher orders ?

S. Golkar and D. T. Son, arXiv:1207.5806 : No (Yes)

$$c = \frac{1}{12} + \frac{N_c^2 - 1}{2N_c}\frac{g_0^2}{48\pi^2} \xrightarrow[N_c \to \infty]{} \frac{1}{12} + \frac{\lambda}{96\pi^2} \qquad c = \frac{1}{12} + \frac{e_0^2}{48\pi^2}$$

V. Concluding Remarks

- The zero P & zero E limits of μ_5 do not commute and the difference is robust against Higer Order correction

- While the CSE is expected in RHIC, its magnitude may not reach the ideal value $\mathbf{J} = \eta \frac{e^2}{2\pi^2} \mu_5 \mathbf{B}$ because of inhomogeneity

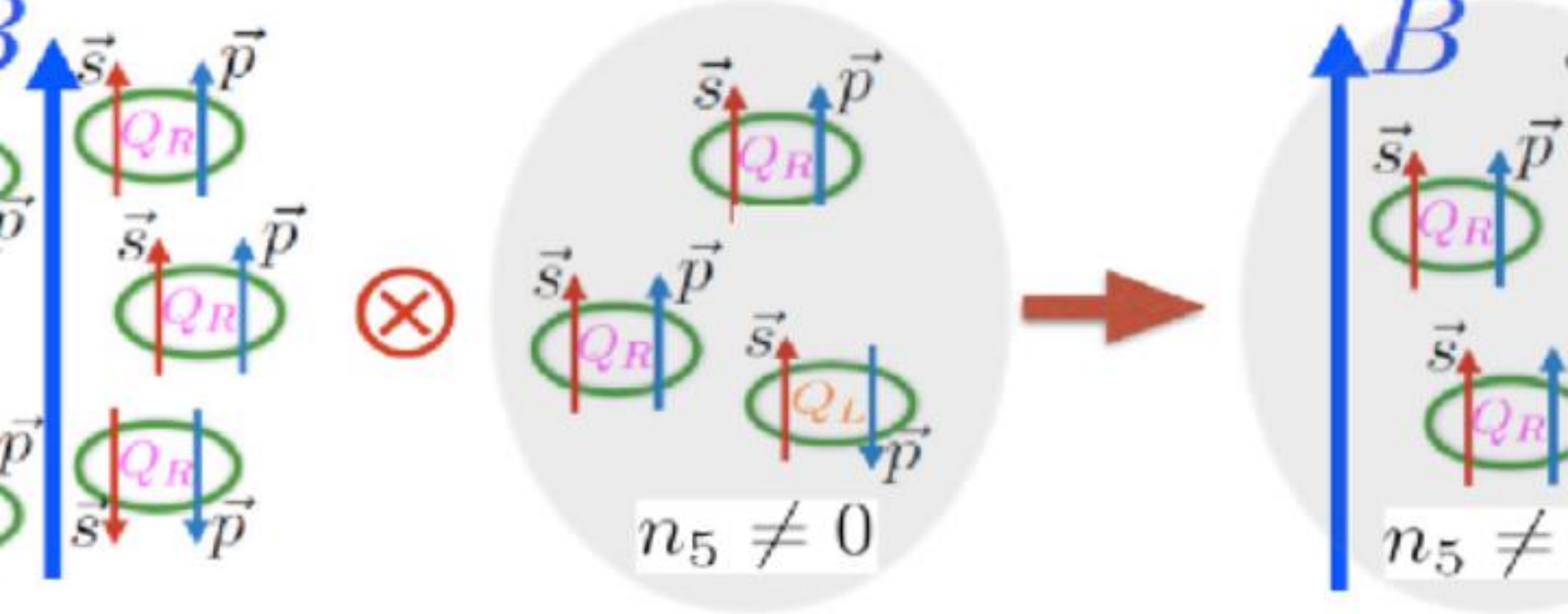
- Nonrenormalization is true for most but not for all anomal. transp. coefs . We obtained 2-loop correction to CVE coef.

- Naive Wigner function can not be applicable to the case with non-constant μ_5 . The problem stems from axial anomaly . The PV regulated WF leads consistent results

. We examine the issues raised here with lattice formulation we obtained the same results as that in continuous case with QFT and Wigner function method .

B.Feng, H. Liu, H-c, Ren, Y. Wu

Thank you very much for your attention!



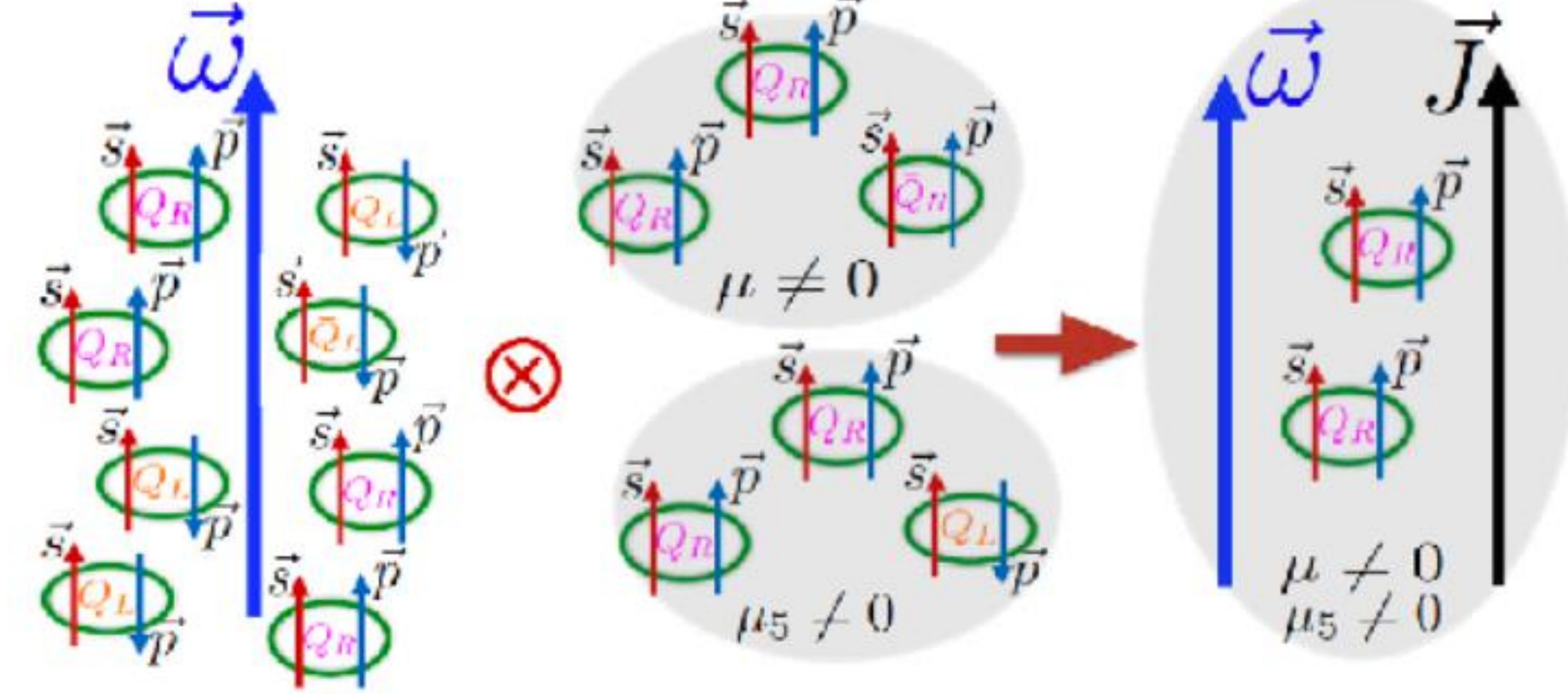
Intuitive understanding of CME:

agnetic polarization —>
orrelation between micro.
PIN & EXTERNAL FORCE

$\otimes$

Chiral imbalance —>
correlation between directions of
SPIN & MOMENTUM

Current along external B field!



Intuitive understanding of CVE:

otational polarization —>
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⊗

Chiral imbalance —>
correlation between directions of
SPIN & MOMENTUM

Current along fluid rotation axis