# Beam Energy Scan Theory: summary of recent progress

Swagato Mukherjee

BROOKHAVEN NATIONAL LABORATORY

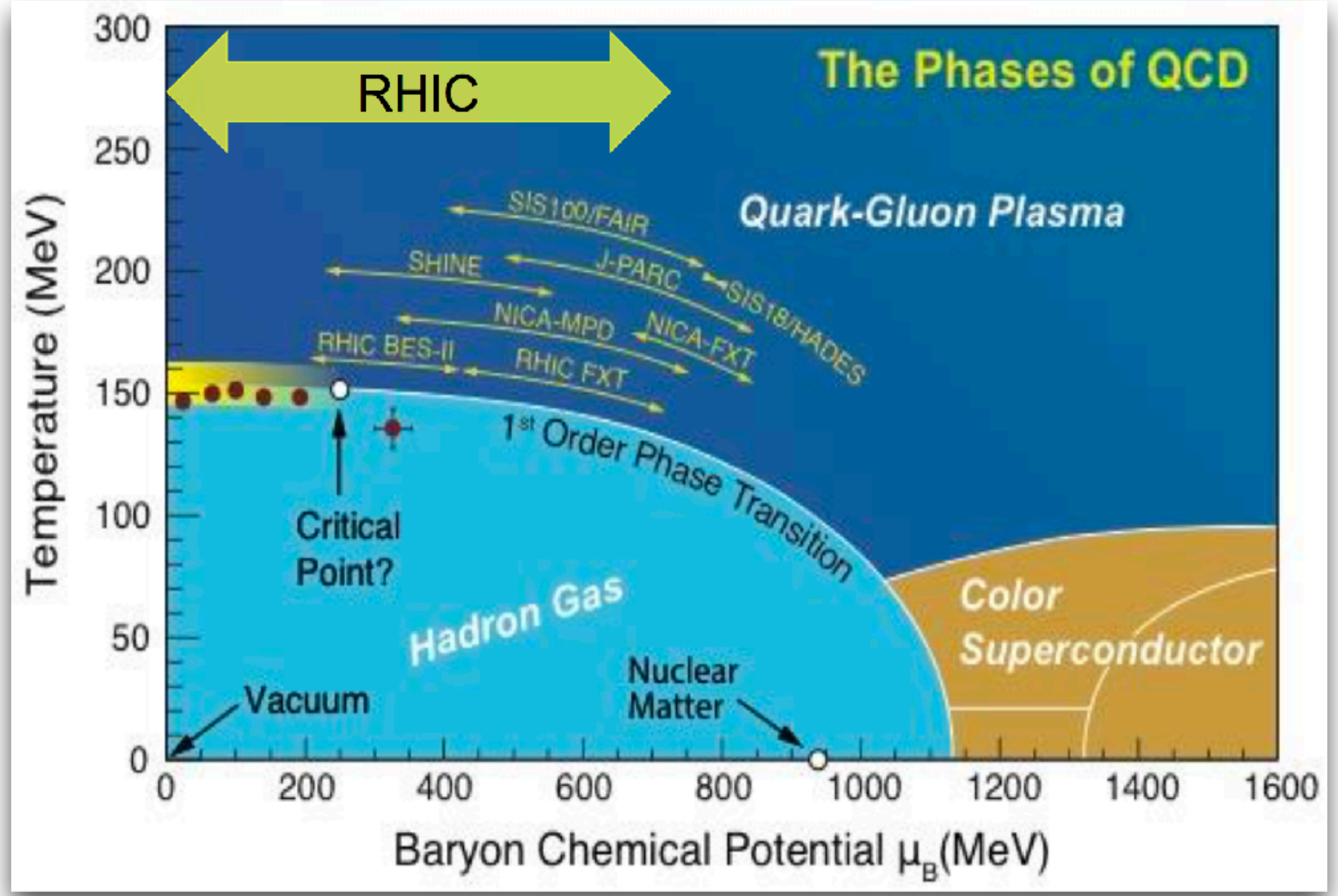
BEST COLLABORATION

August 2017, Fudan University, Shanghai

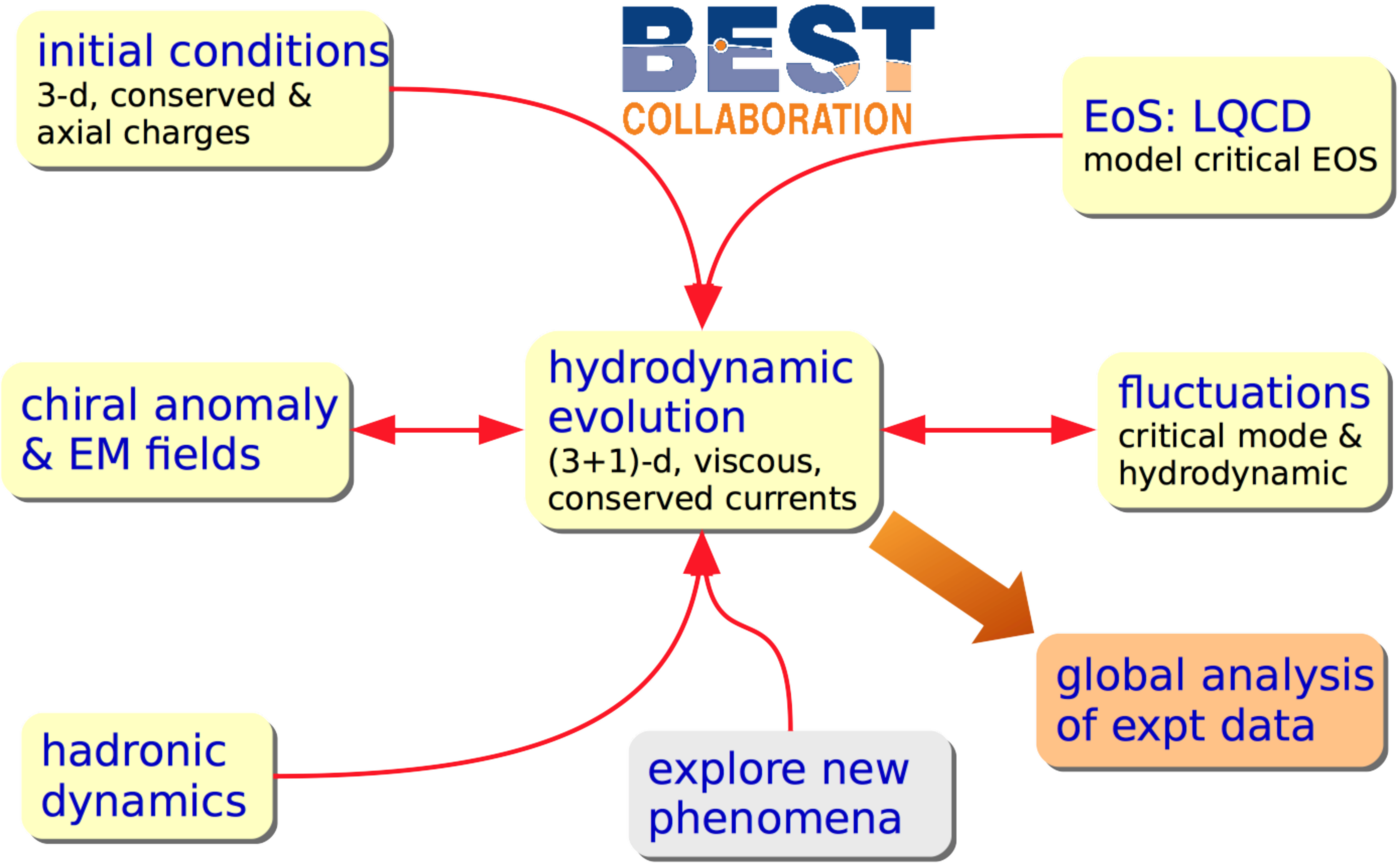
*Beam Energy Scan (BES) II @ RHIC*

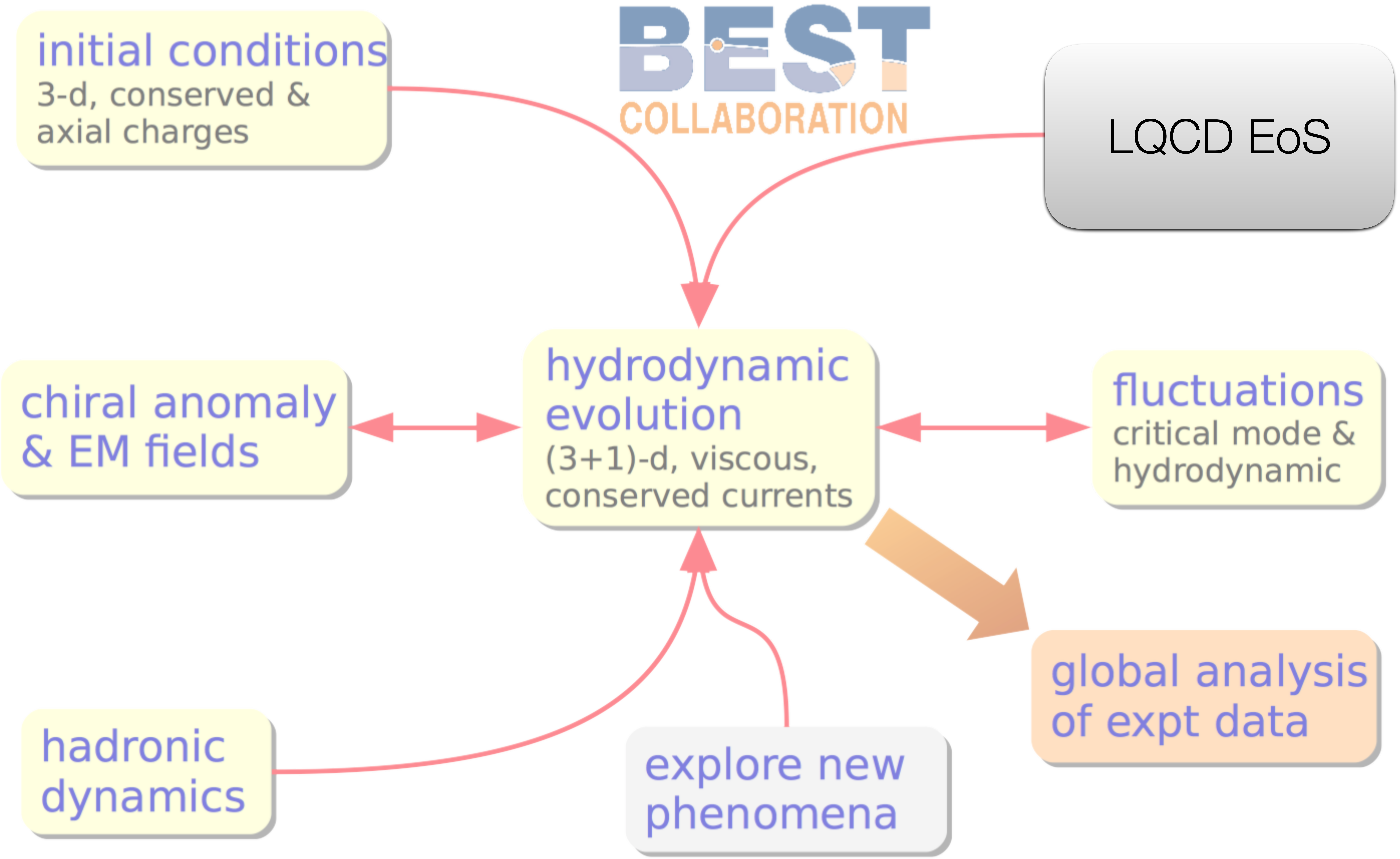
starting 2019

- QCD critical point & phase diagram
- properties of baryon-rich QGP
- onset of chiral symmetry restoration
- unexpected new phenomena




needs a comprehensive theory framework


BEST
COLLABORATION
initial conditions
3-d, conserved &
axial charges
EoS: LQCD
model critical EOS
chiral anomaly
& EM fields
hydrodynamic
evolution
(3+1)-d, viscous,
conserved currents
fluctuations
critical mode &
hydrodynamic
global analysis
of expt data
hadronic
dynamics
explore new
phenomena


BEST
COLLABORATION
initial conditions
3-d, conserved &
axial charges
LQCD EoS
chiral anomaly
& EM fields
hydrodynamic
evolution
(3+1)-d, viscous,
conserved currents
fluctuations
critical mode &
hydrodynamic
hadronic
dynamics
explore new
phenomena
global analysis
of expt data

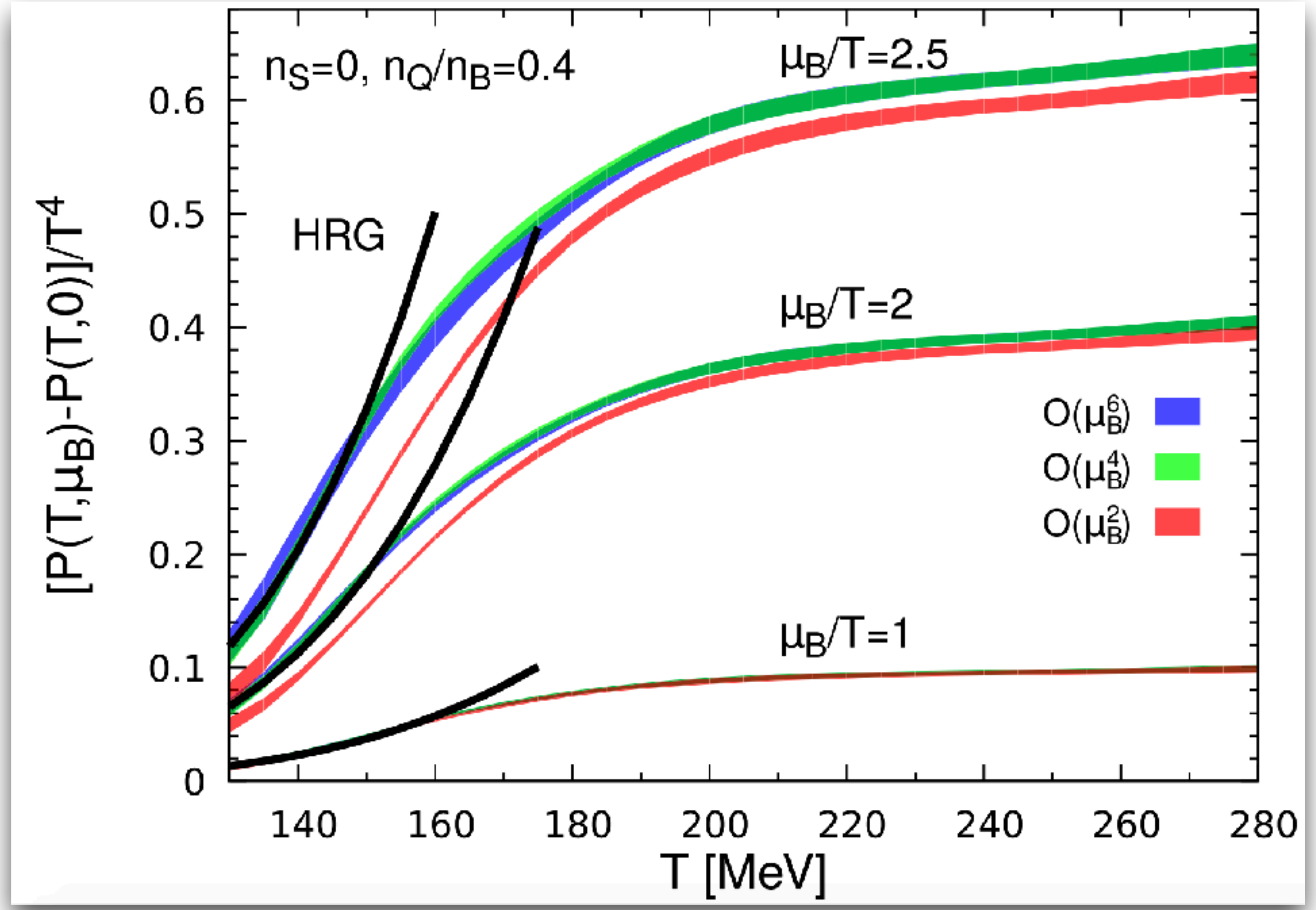
HotQCD: Phys. Rev. D95, no.5, 054504 (2017)

Taylor expansion up to $\mathcal{O}\left(\mu_B^6\right)$

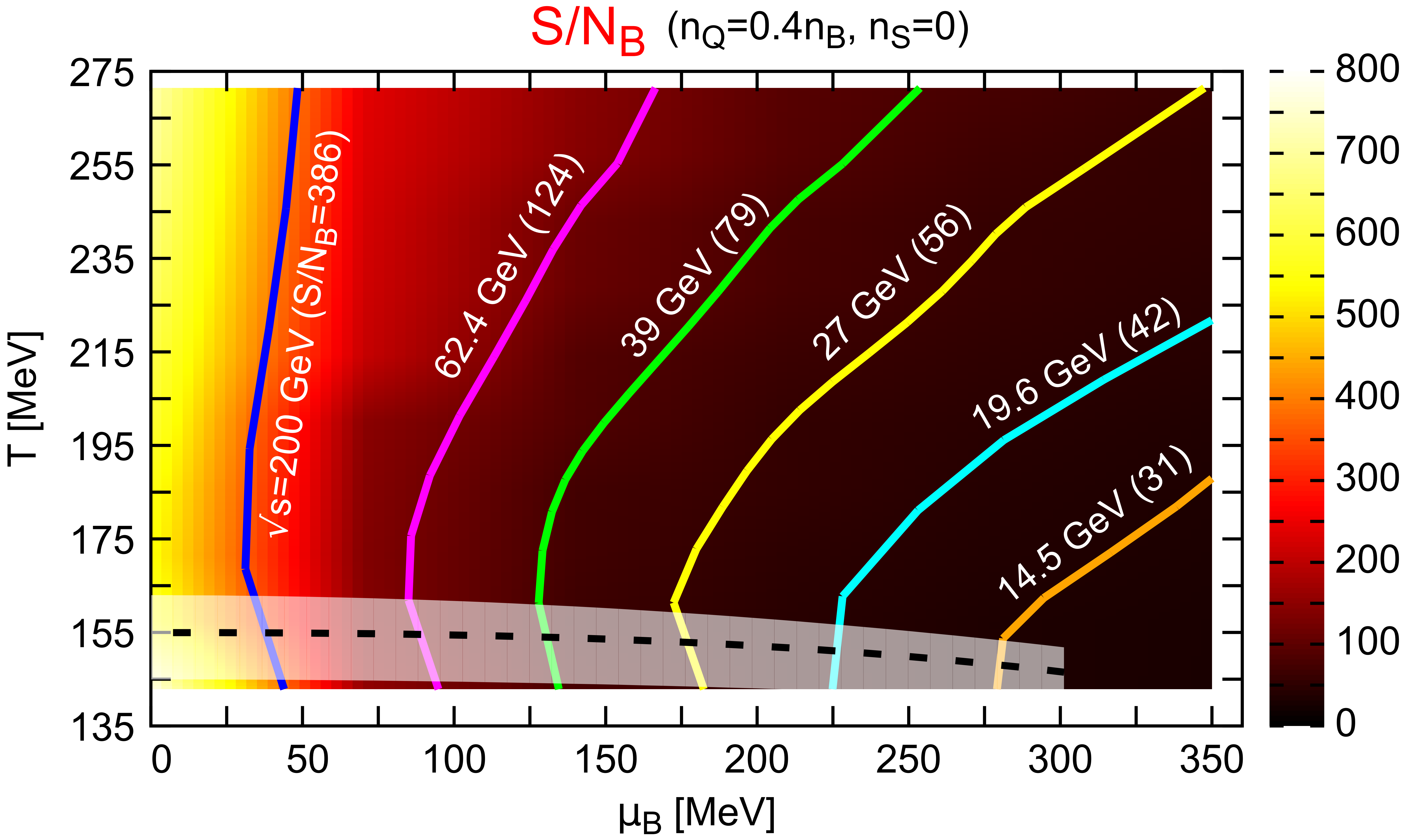
$$\frac{P(T,\mu_B)}{T^4} = \sum_{n=0,2,4,6} \frac{1}{n!}\chi_n^B(T)\left(\frac{\mu_B}{T}\right)^n$$

LQCD EoS, present reach:

$$\mu_B/T \lesssim 2$$



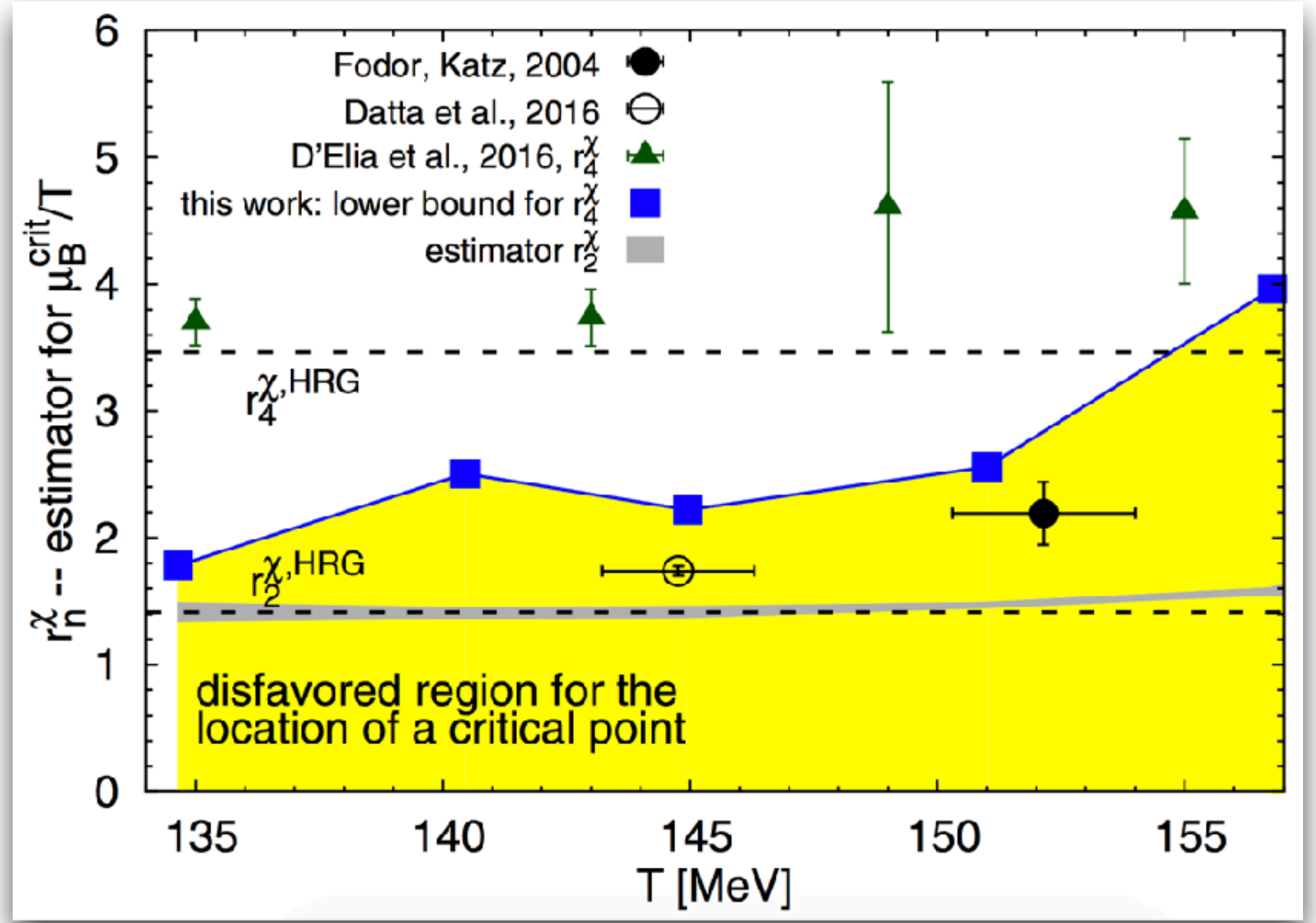
# LQCD EoS, present reach: $\sqrt{s} \gtrsim 14.5\ GeV$

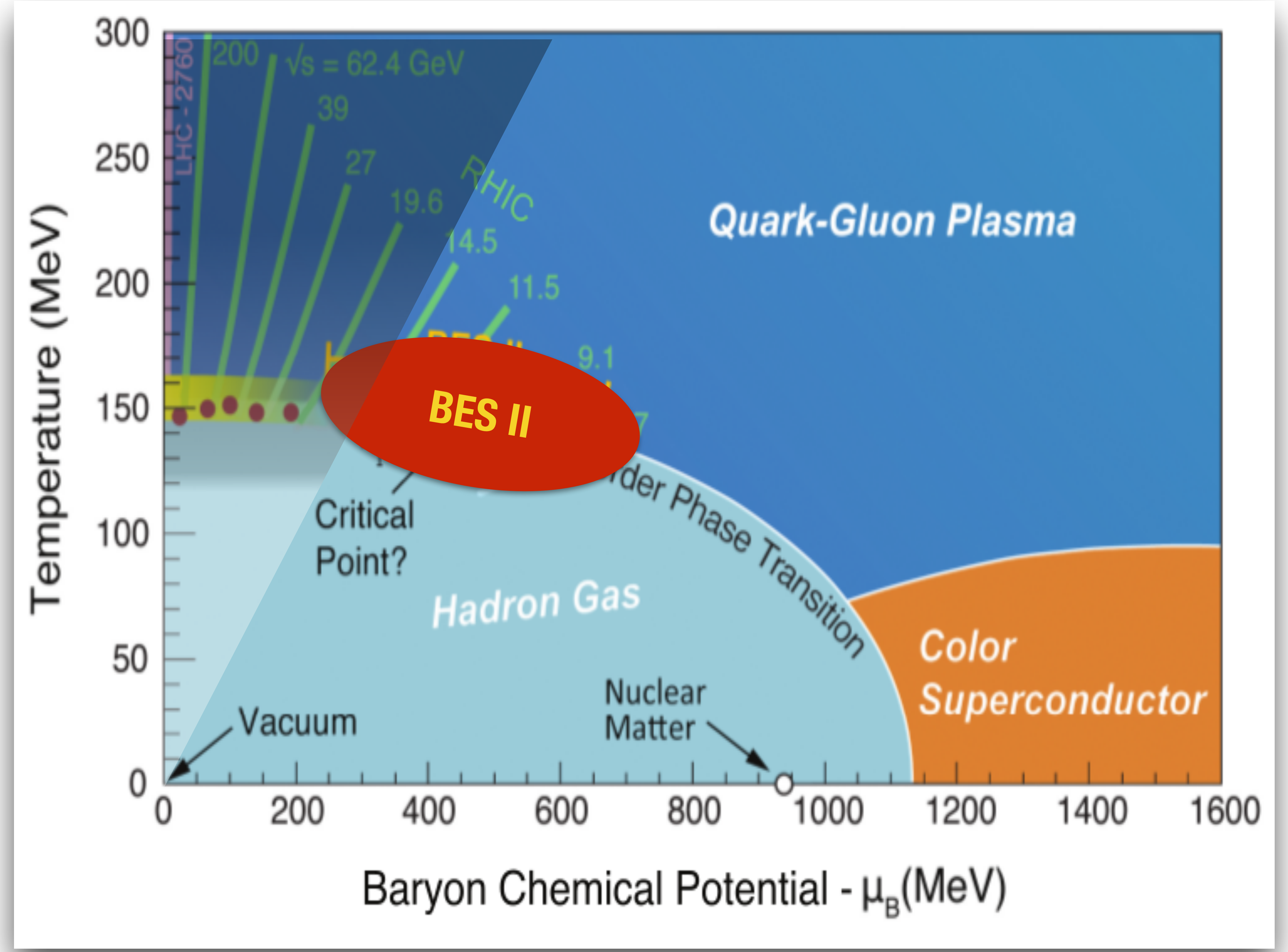



lines of const. entropy to bet-baryon number

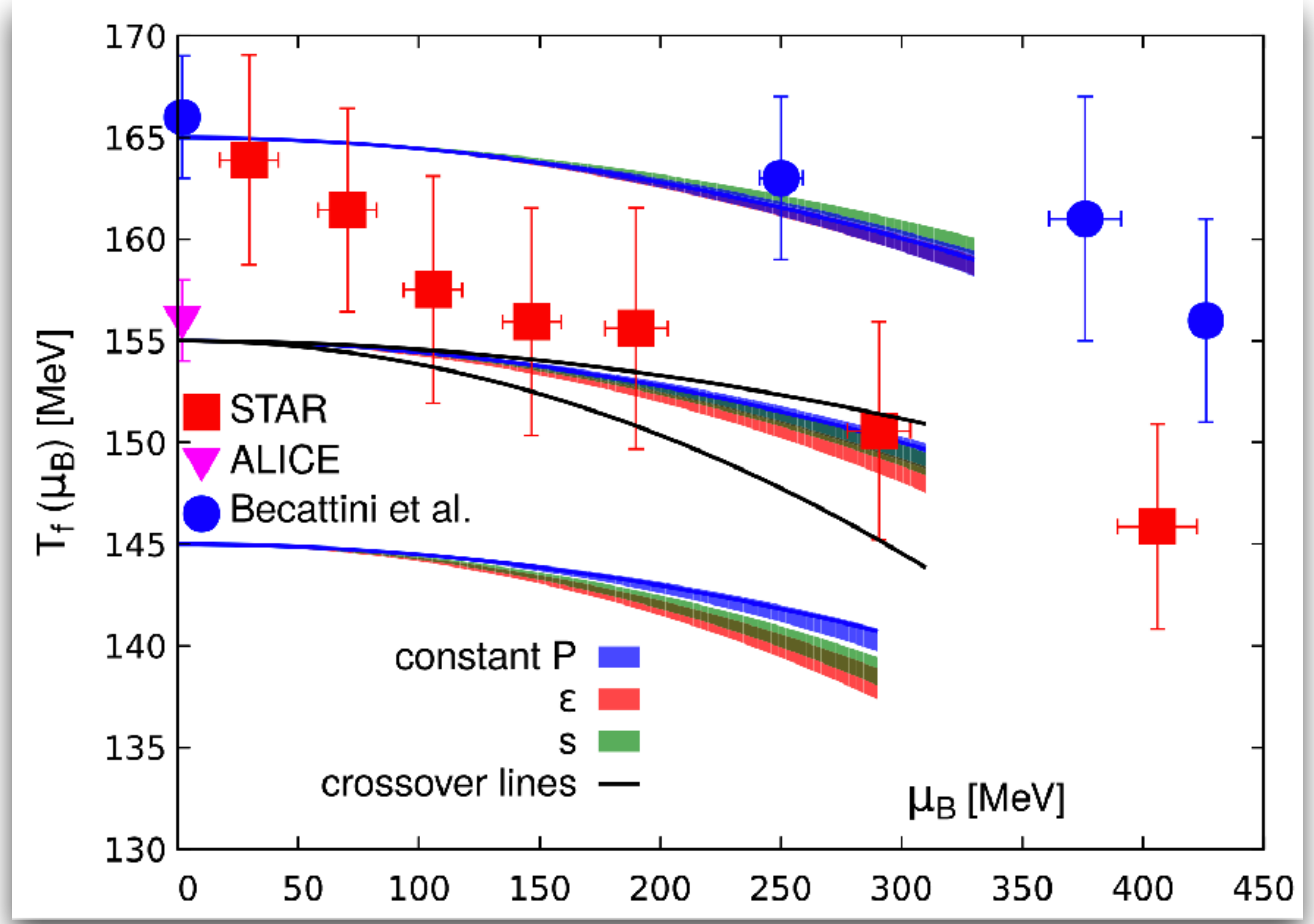
— approx. evolution trajectories of inviscid QGP

# location of critical point: $\mu_B/T \lesssim 2$ presently disfavored (consistency check)

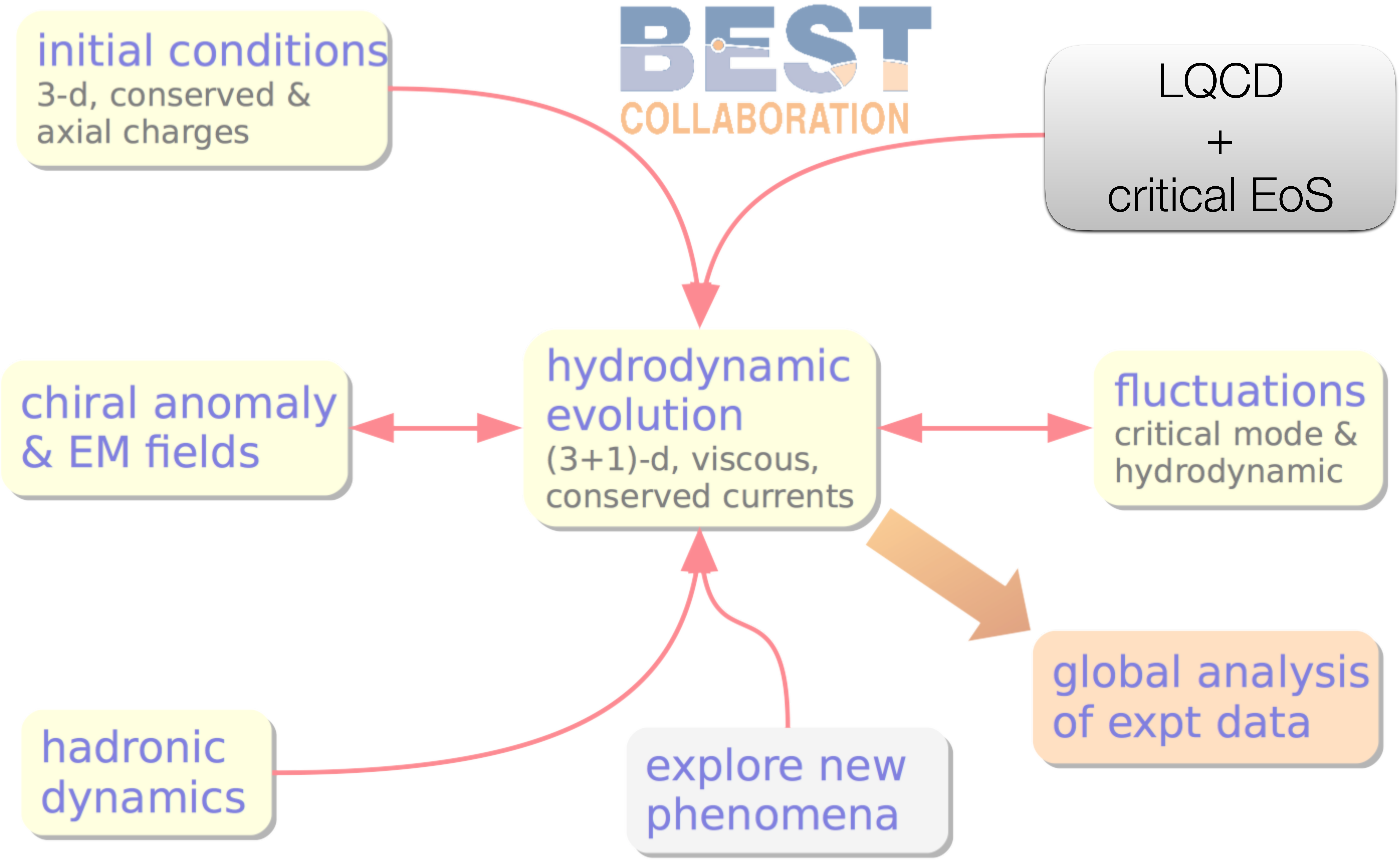



analyzing radius of convergence of Taylor expansion: $r_{2n}^\chi = \left| \frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B} \right|, \; r_c = \lim_{n\to\infty} r_{2n}^\chi$




pressure remains constant along the chiral crossover line

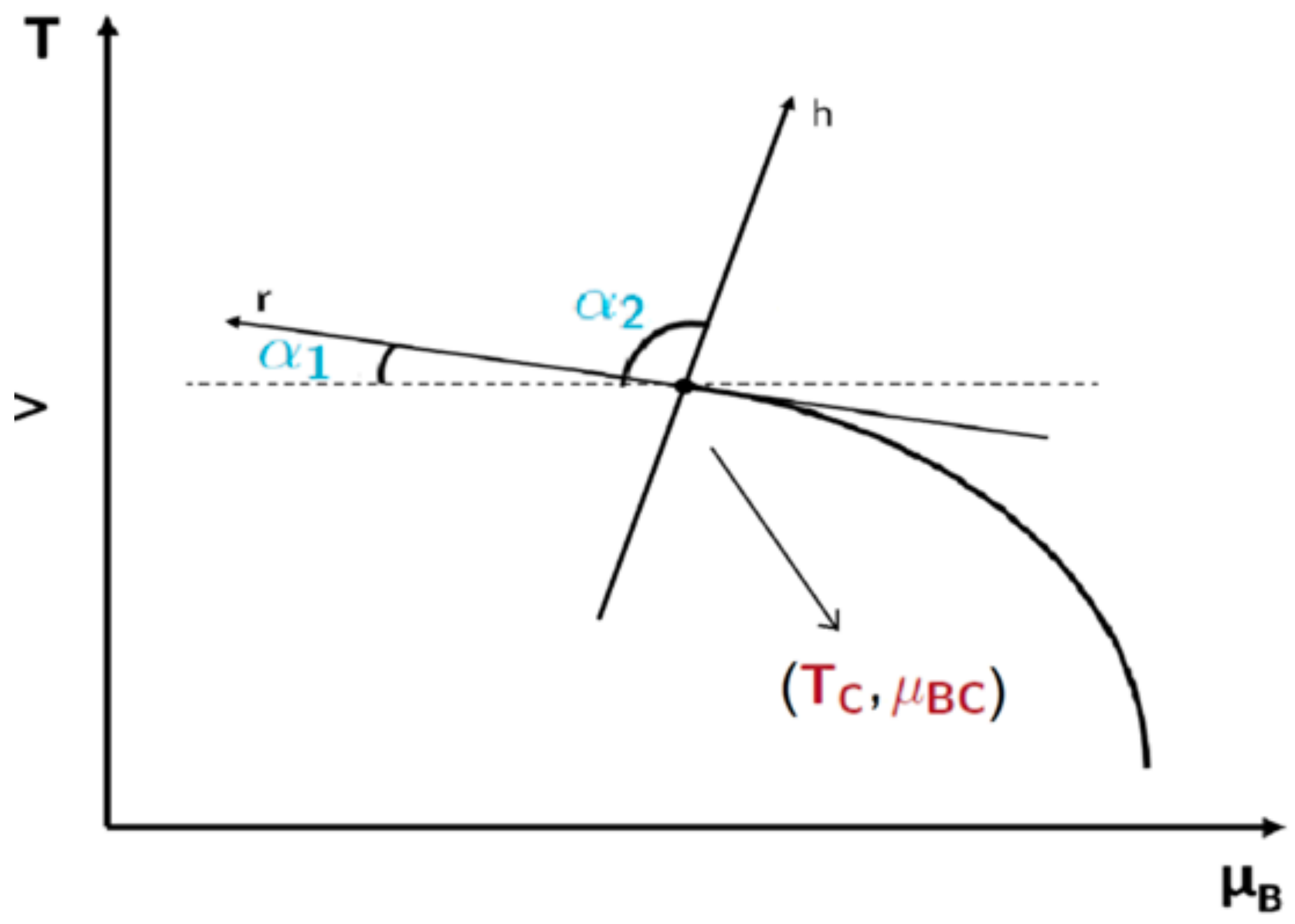


initial conditions
3-d, conserved &
axial charges
BEST
COLLABORATION
LQCD
+
critical EoS
chiral anomaly
& EM fields
hydrodynamic
evolution
(3+1)-d, viscous,
conserved currents
fluctuations
critical mode &
hydrodynamic
global analysis
of expt data
hadronic
dynamics
explore new
phenomena

parametrize critical (3-d Ising) EoS: 6 free parameters

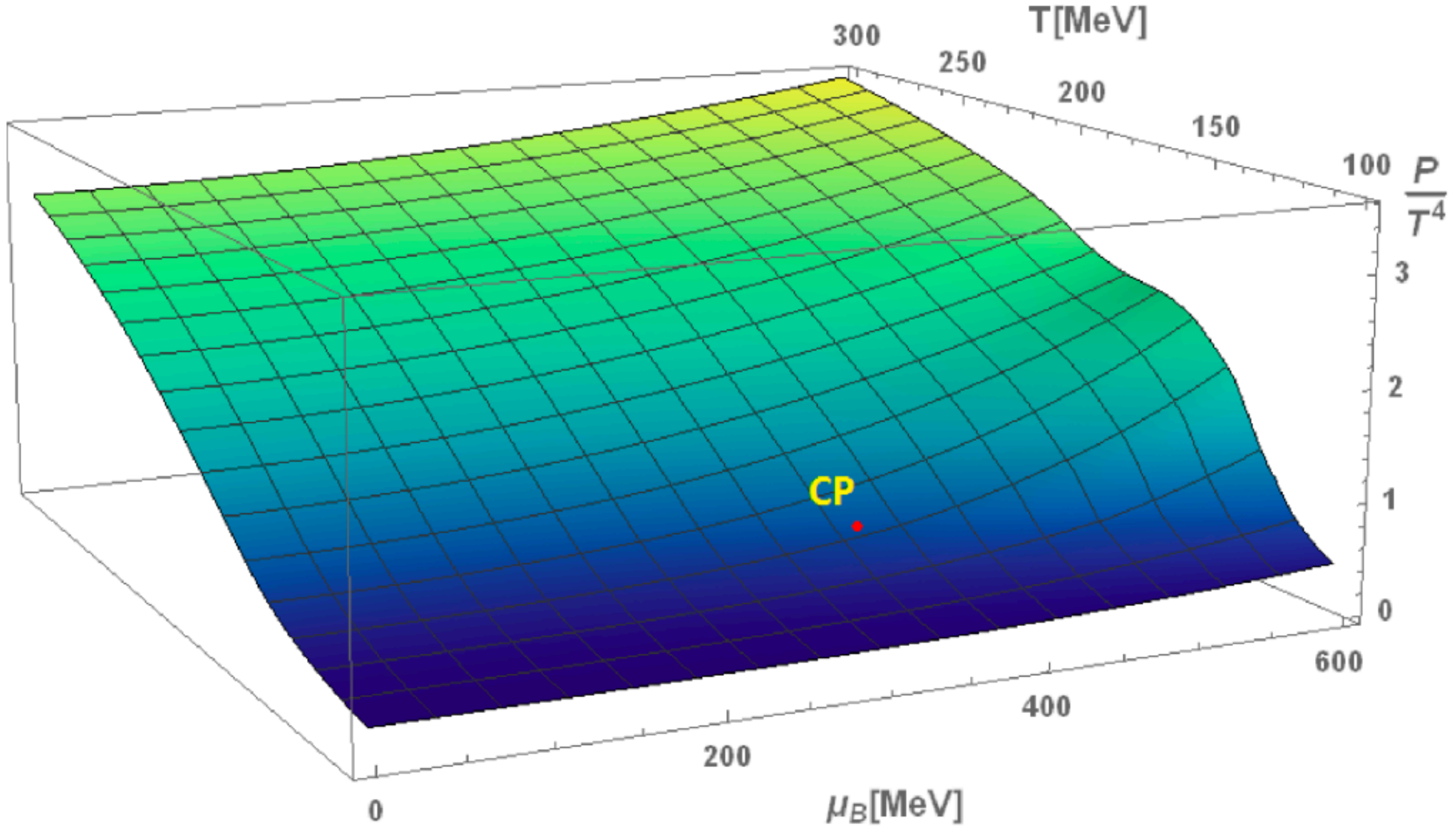
$$\frac{T - T_C}{T_C} = \mathbf{w}\,(r\rho\, \sin\alpha_1 + h\, \sin\alpha_2)$$

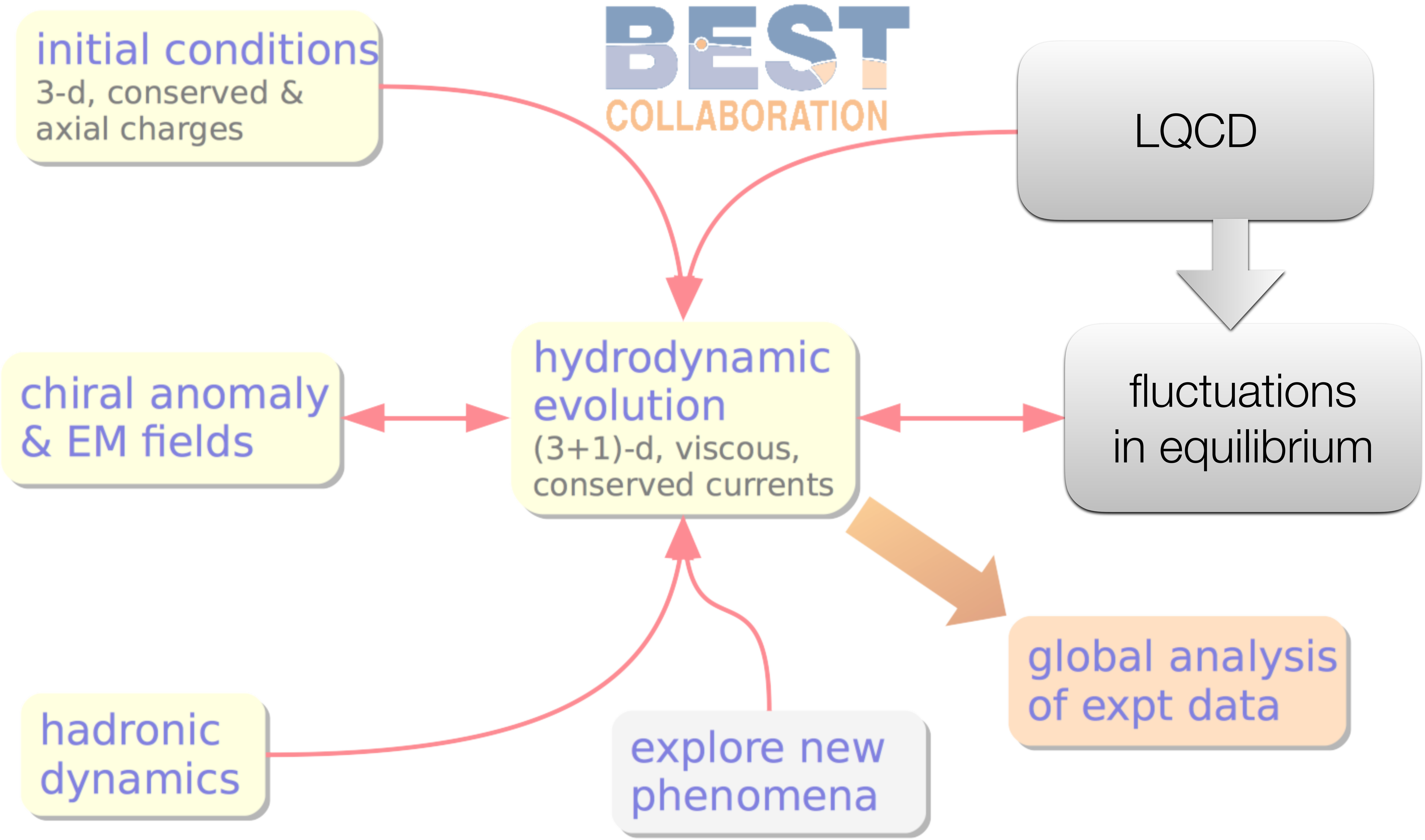
$$\frac{\mu_B - \mu_{BC}}{T_C} = \mathbf{w}\,(-r\rho\, \cos\alpha_1 - h\, \cos\alpha_2)$$




match to LQCD EOS

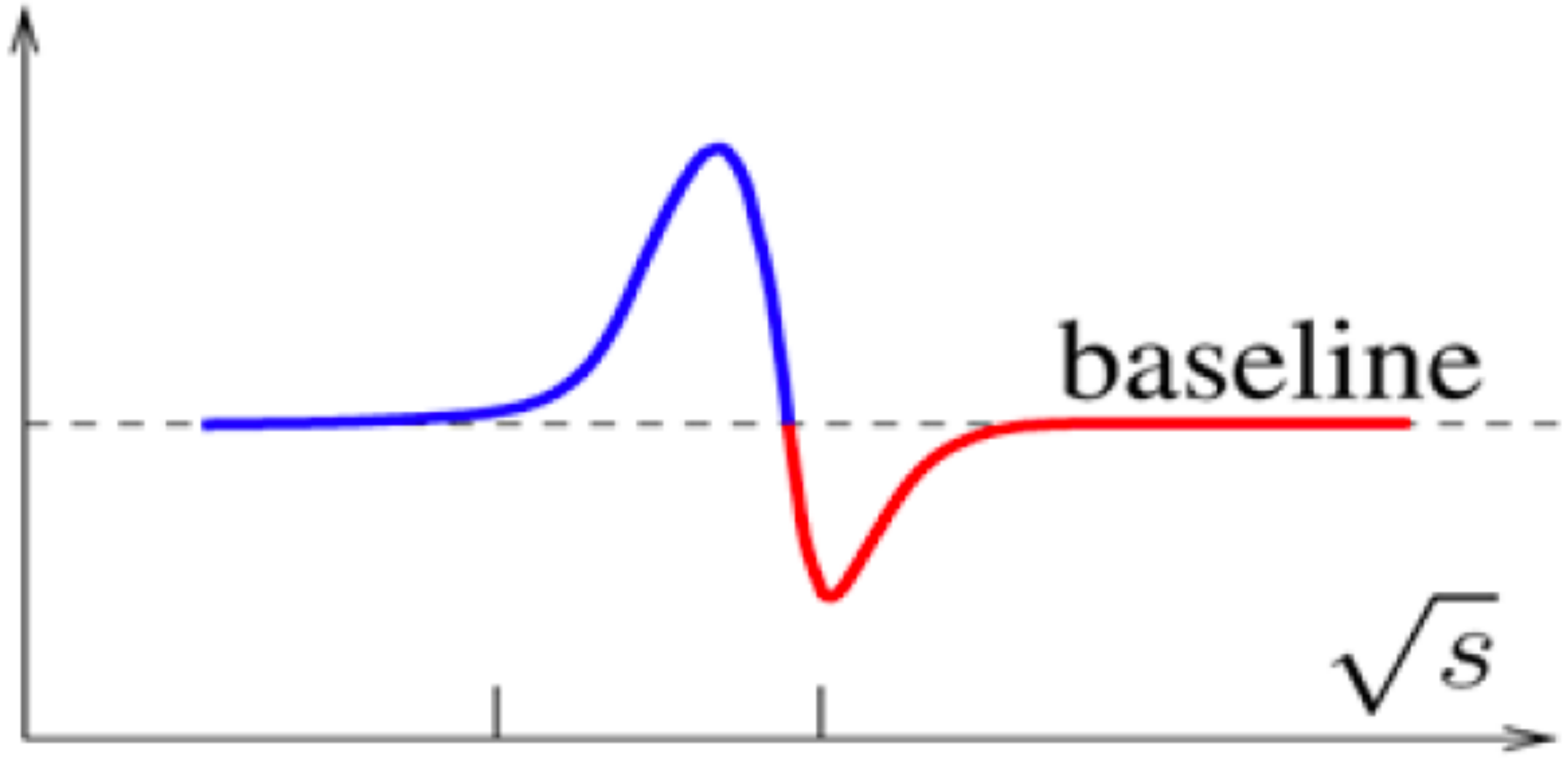
$$P(T,\mu_B) = T^4 \sum_n c^n_{\text{reg}}(T) \left(\frac{\mu_B}{T}\right)^n + T_C^4\, P_{\text{crit}}(T,\mu_B)$$




initial conditions
3-d, conserved & axial charges
BEST COLLABORATION
LQCD
chiral anomaly & EM fields
hydrodynamic evolution
(3+1)-d, viscous, conserved currents
fluctuations in equilibrium
global analysis of expt data
hadronic dynamics
explore new phenomena

static universality: $n^{th}$ cumulant ~ $\xi_{eq}^{\frac{1}{2}+\frac{5}{2}(n-1)}$

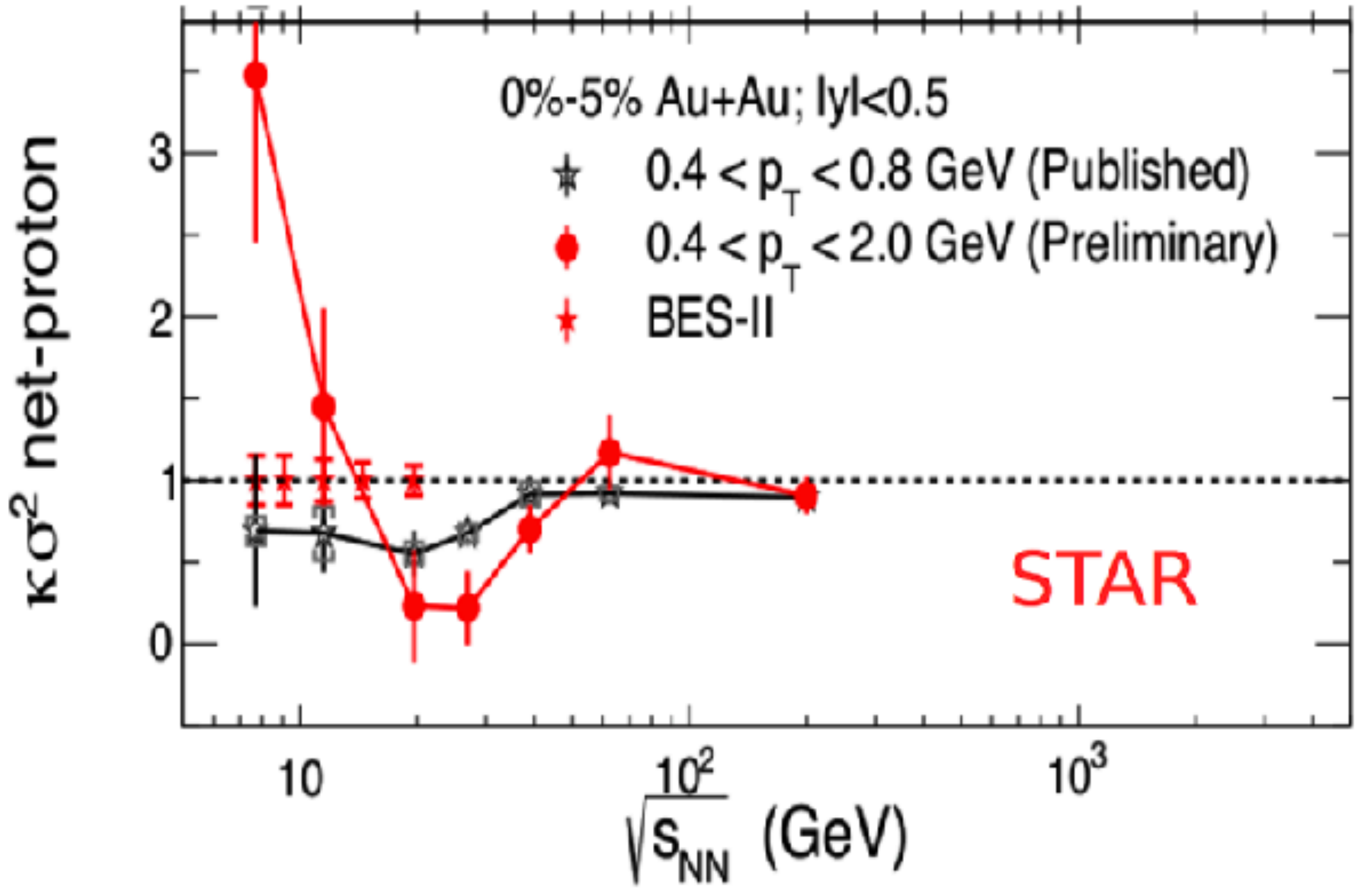
specific signs




- QCD critical point ⟷ 3-d Ising
- coupling of critical mode to baryon & proton

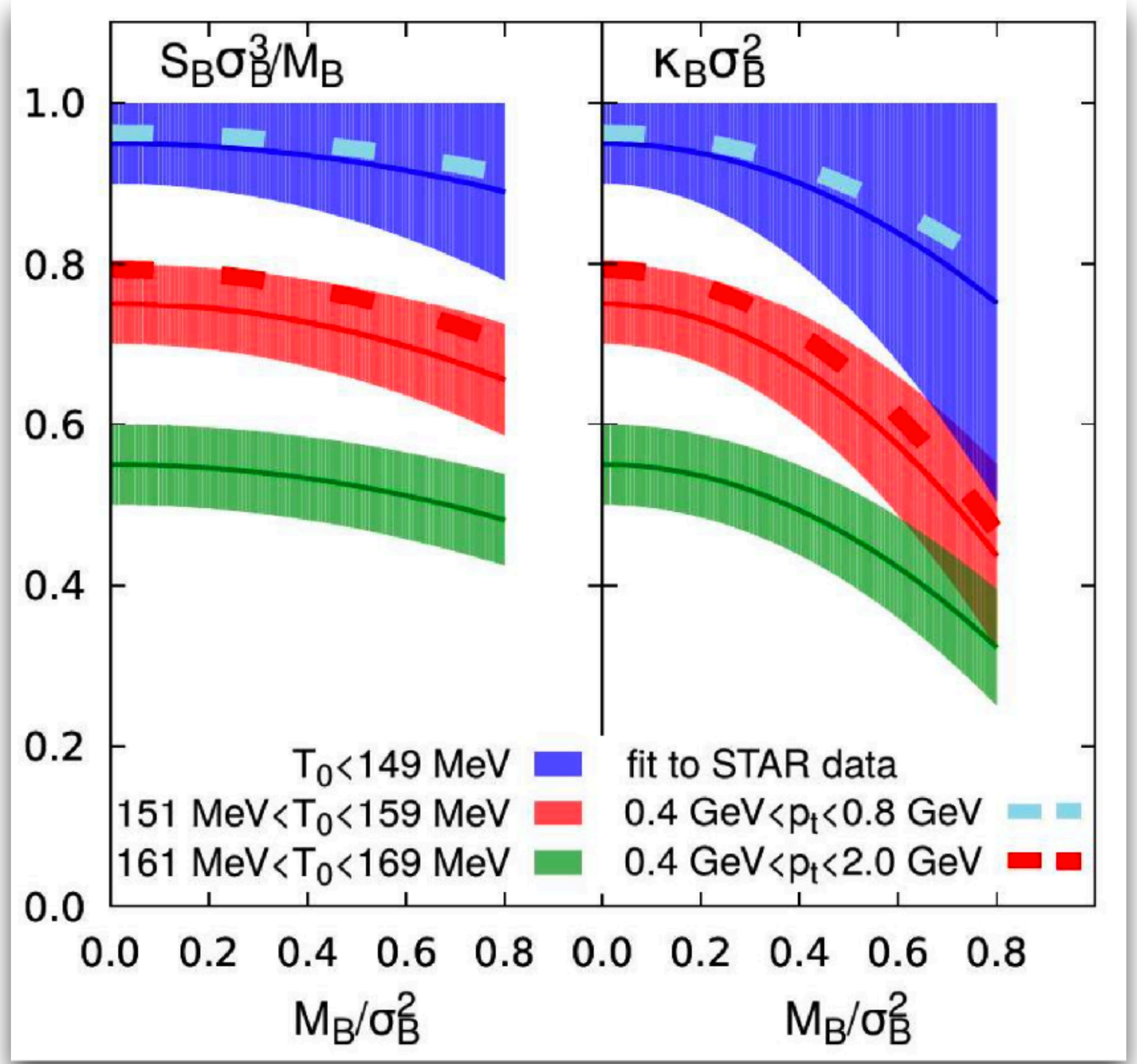
Stephanov: Phys. Rev. Lett. 107, 052301 (2011)
Stephanov: Phys. Rev. Lett. 102, 032301 (2009)
Son, Stephanov: Phys.Rev. D70 (2004) 056001 (2004)
Hatta, Stephanov: Phys. Rev. Lett. 91, 102003 (2003)

equilibrium QCD baseline ?



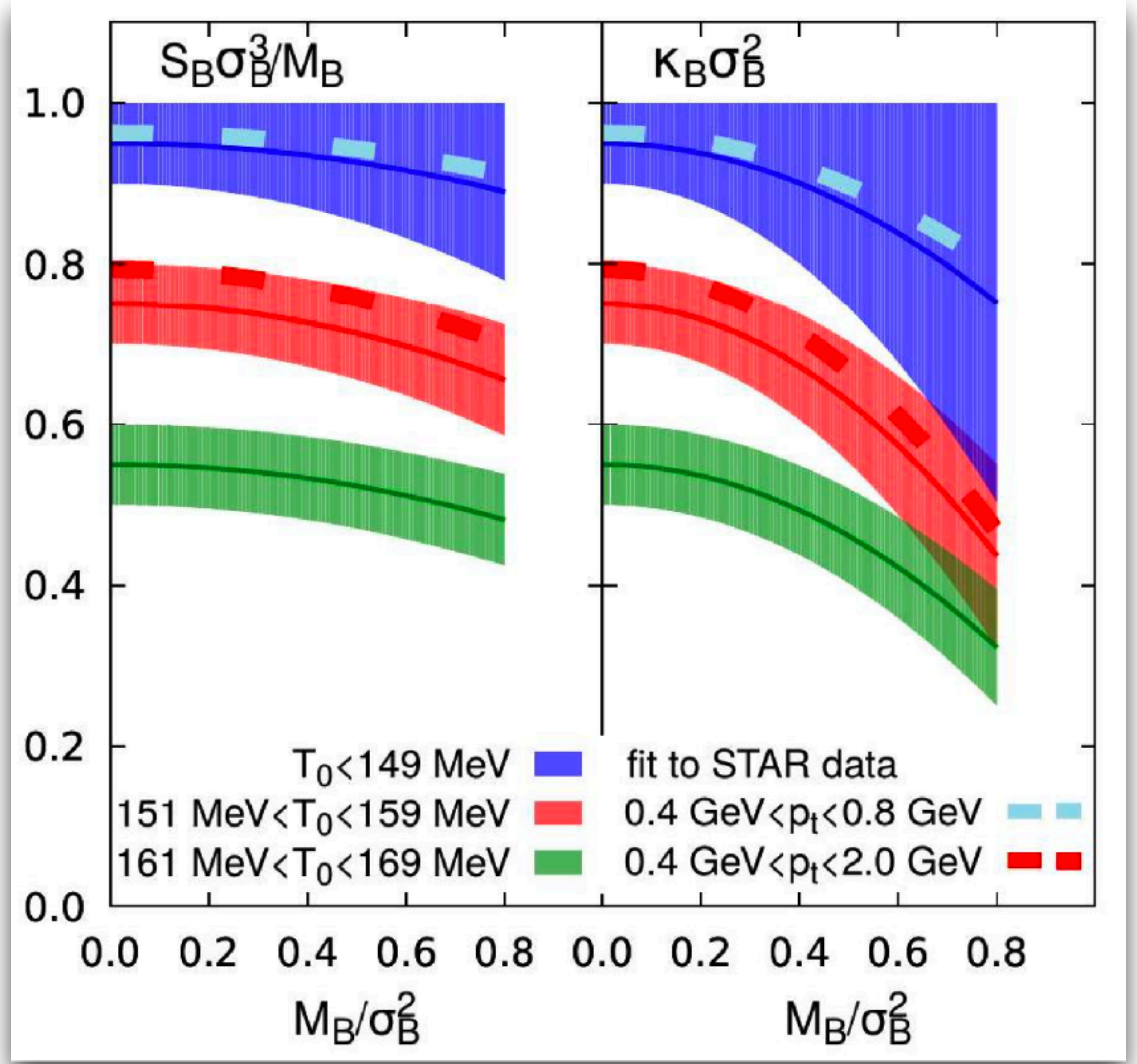
# LQCD: cumulants of net baryon fluctuations

HotQCD: arXiv:1708.04897

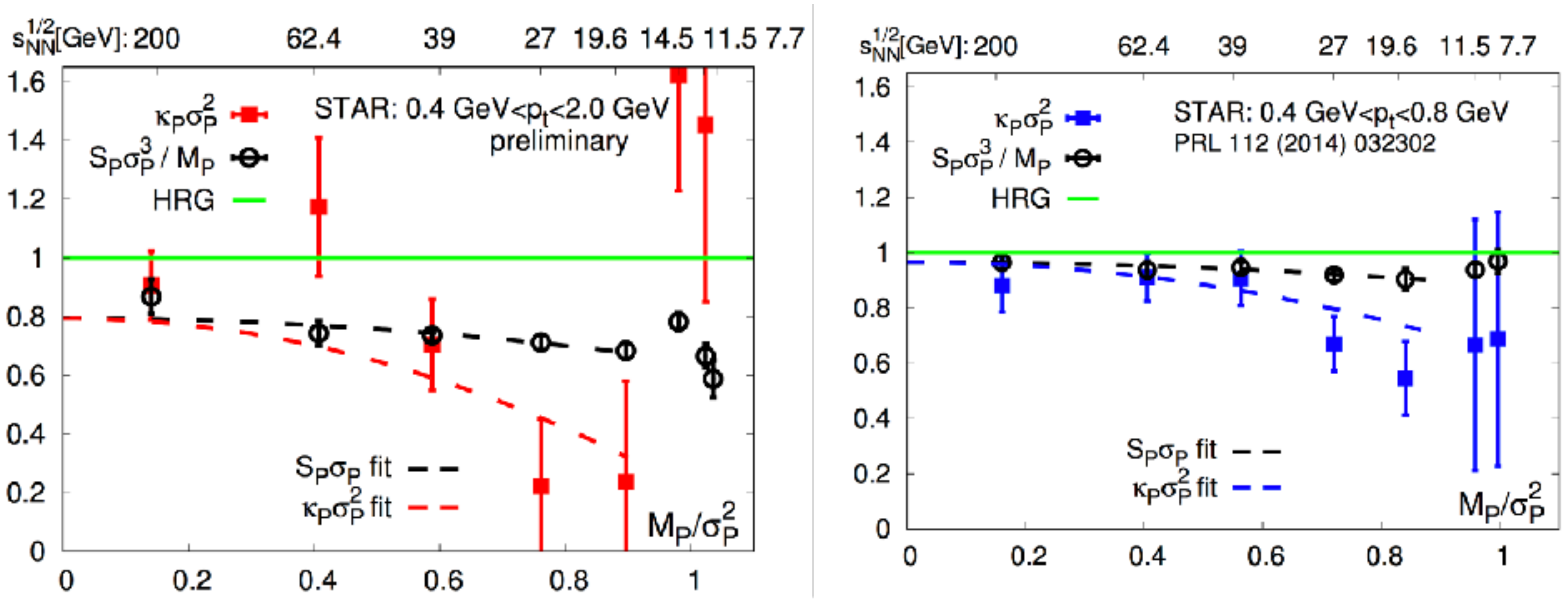


# LQCD: cumulants of net-baryon fluctuations

HotQCD: arXiv:1708.04897



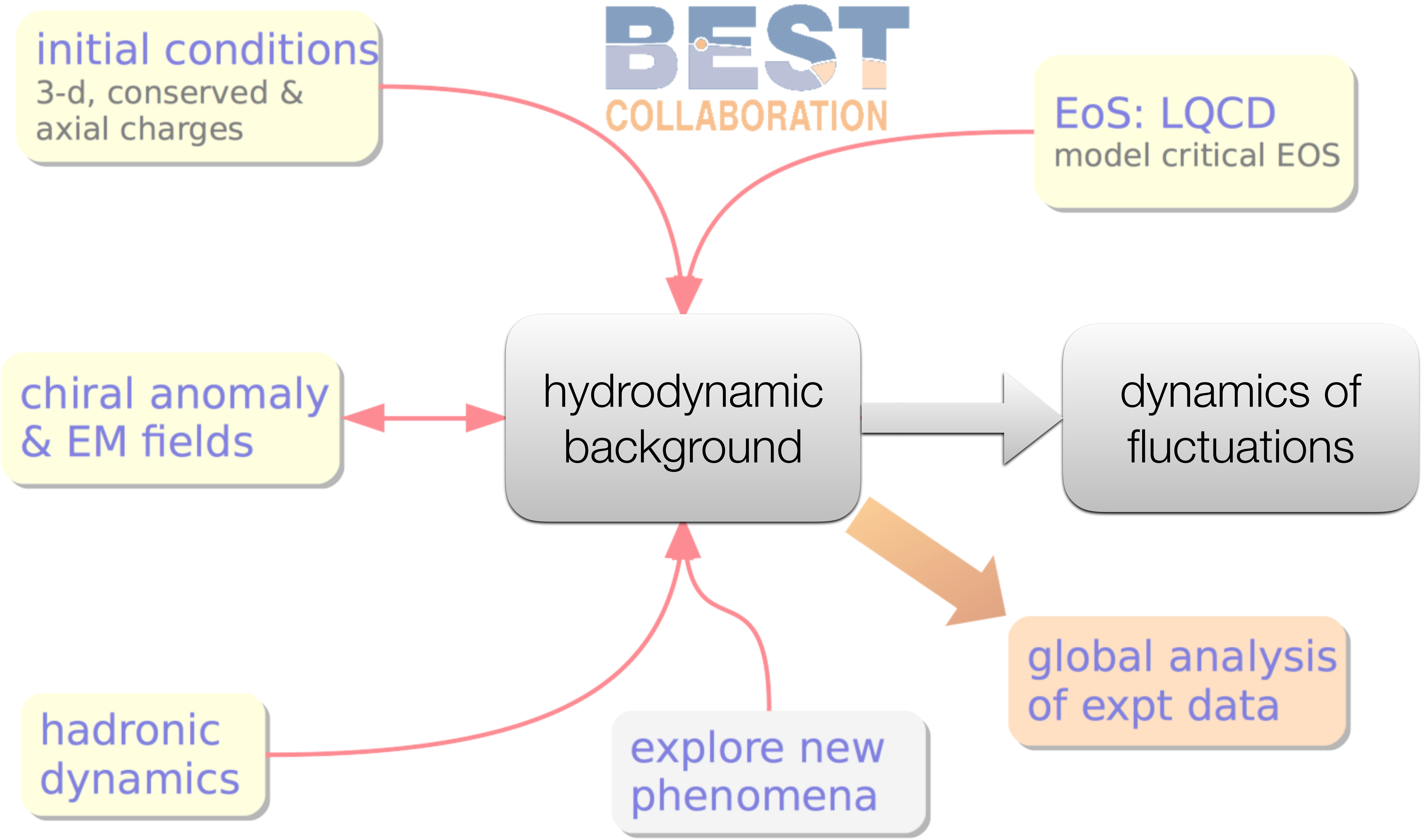

STAR: cumulants of net-p fluctuations




$\sqrt{s} \gtrsim 19.6\ GeV$ :

cumulants of net-p fluctuations are consistent with equilibrium QCD

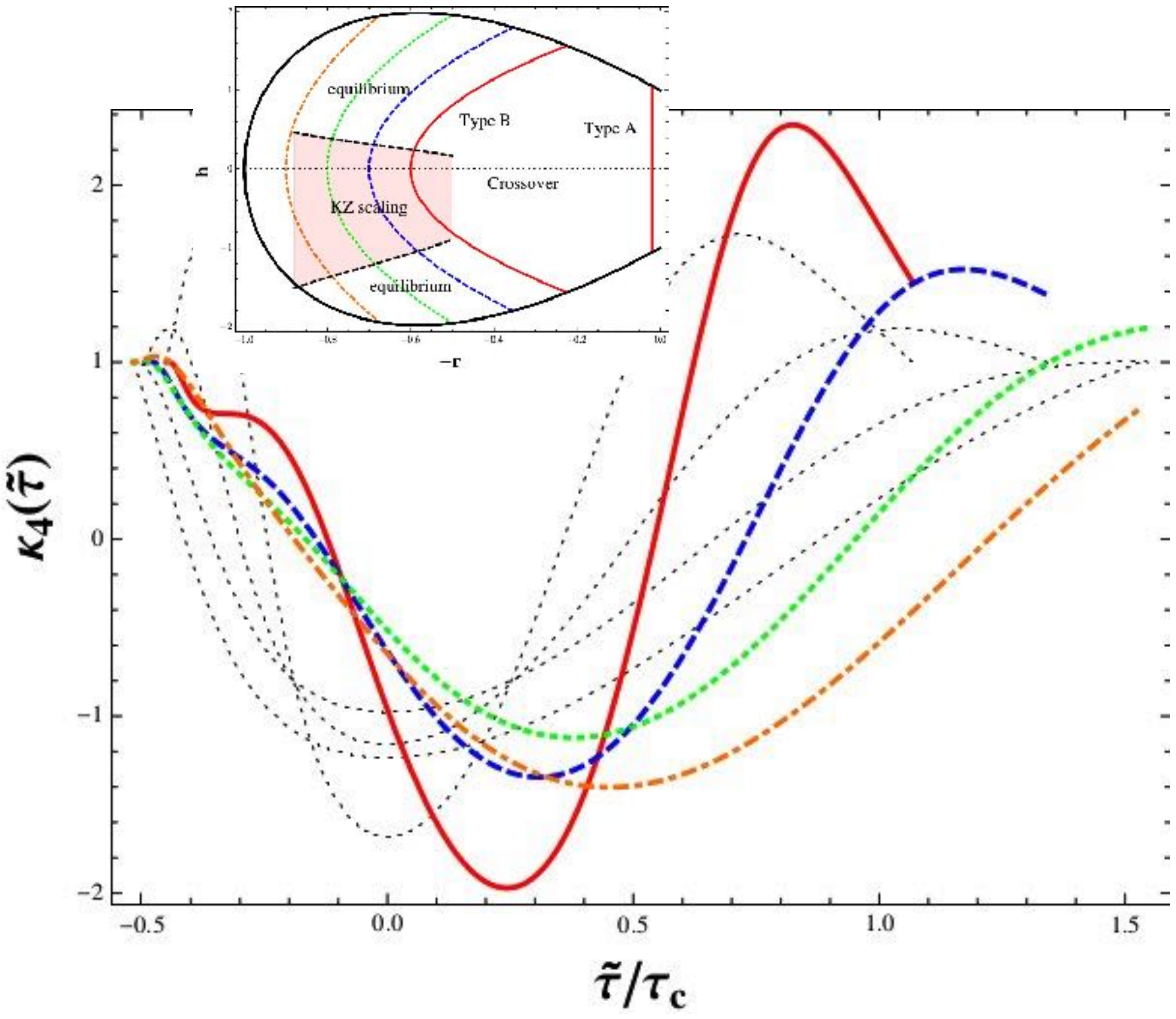
… but not an apples-to-apples comparison


BEST
COLLABORATION
initial conditions
3-d, conserved &
axial charges
EoS: LQCD
model critical EOS
chiral anomaly
& EM fields
hydrodynamic
background
dynamics of
fluctuations
hadronic
dynamics
explore new
phenomena
global analysis
of expt data

time evolution of non-Gaussian fluctuations

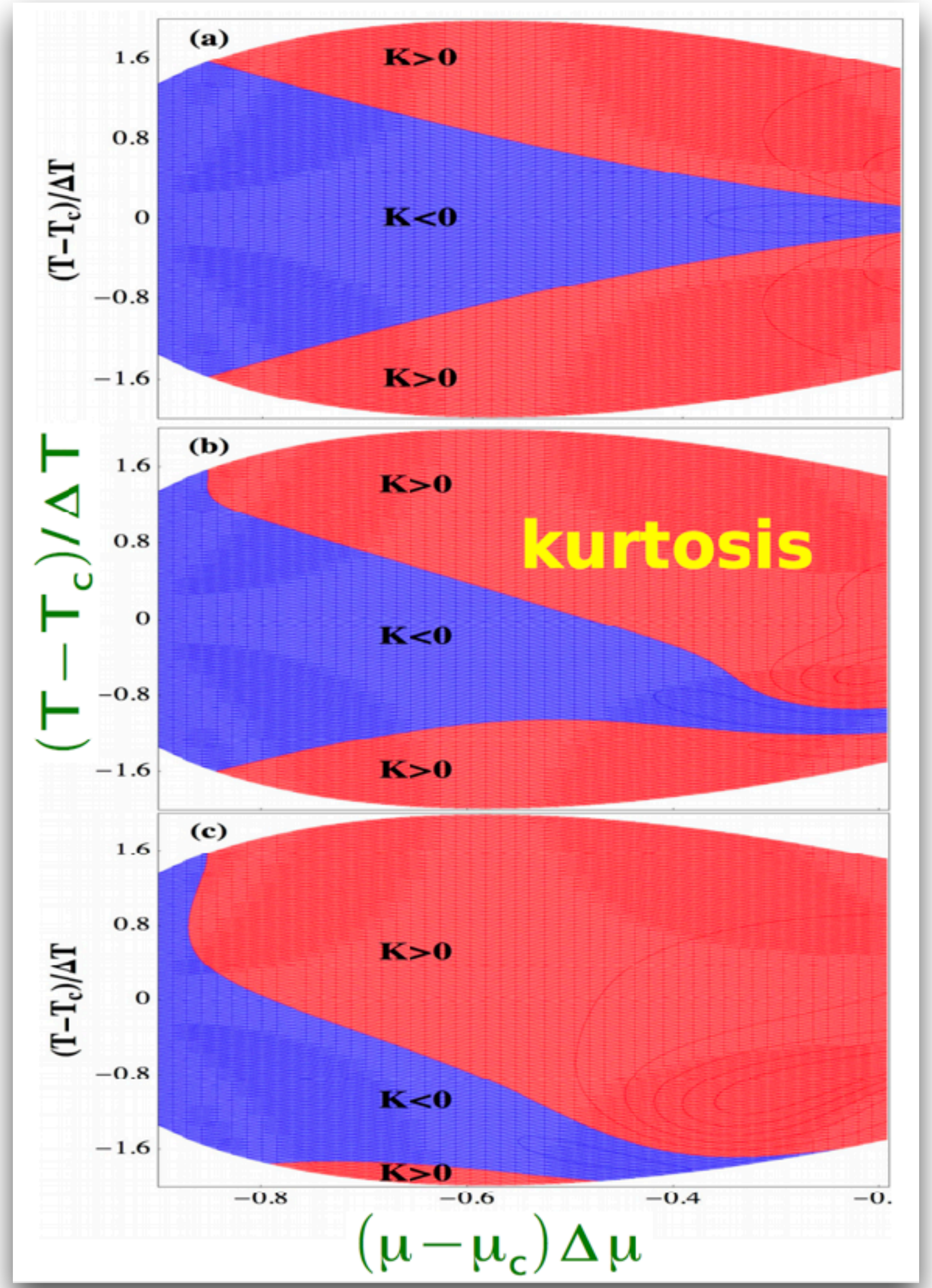
— based on generalized Fokker-Planck evolution

Mukherjee, Venugopalan, Yin: Phys. Rev. C92, no.3, 034912 (2015)

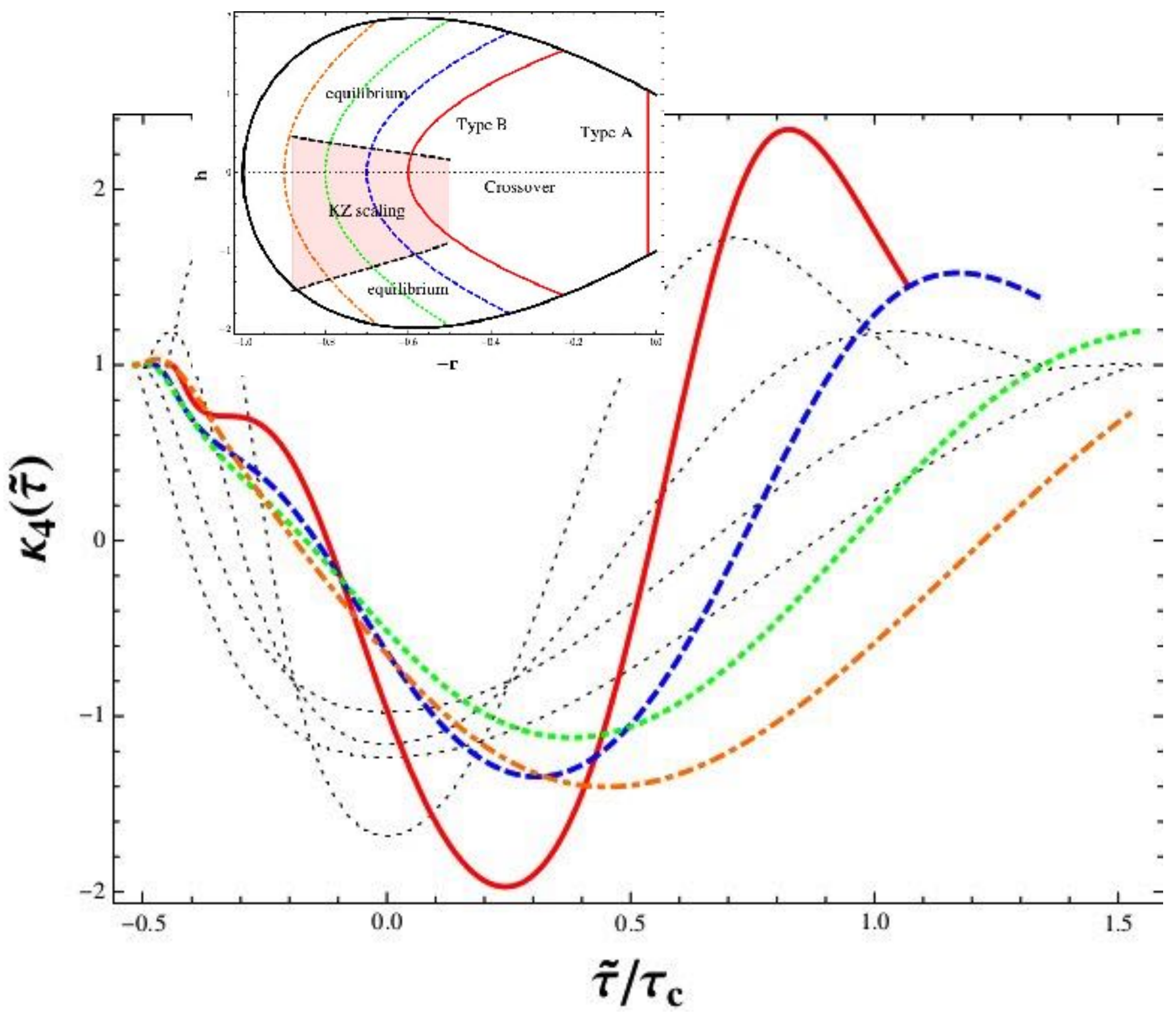



signs can be different from equilibrium

— universality lost



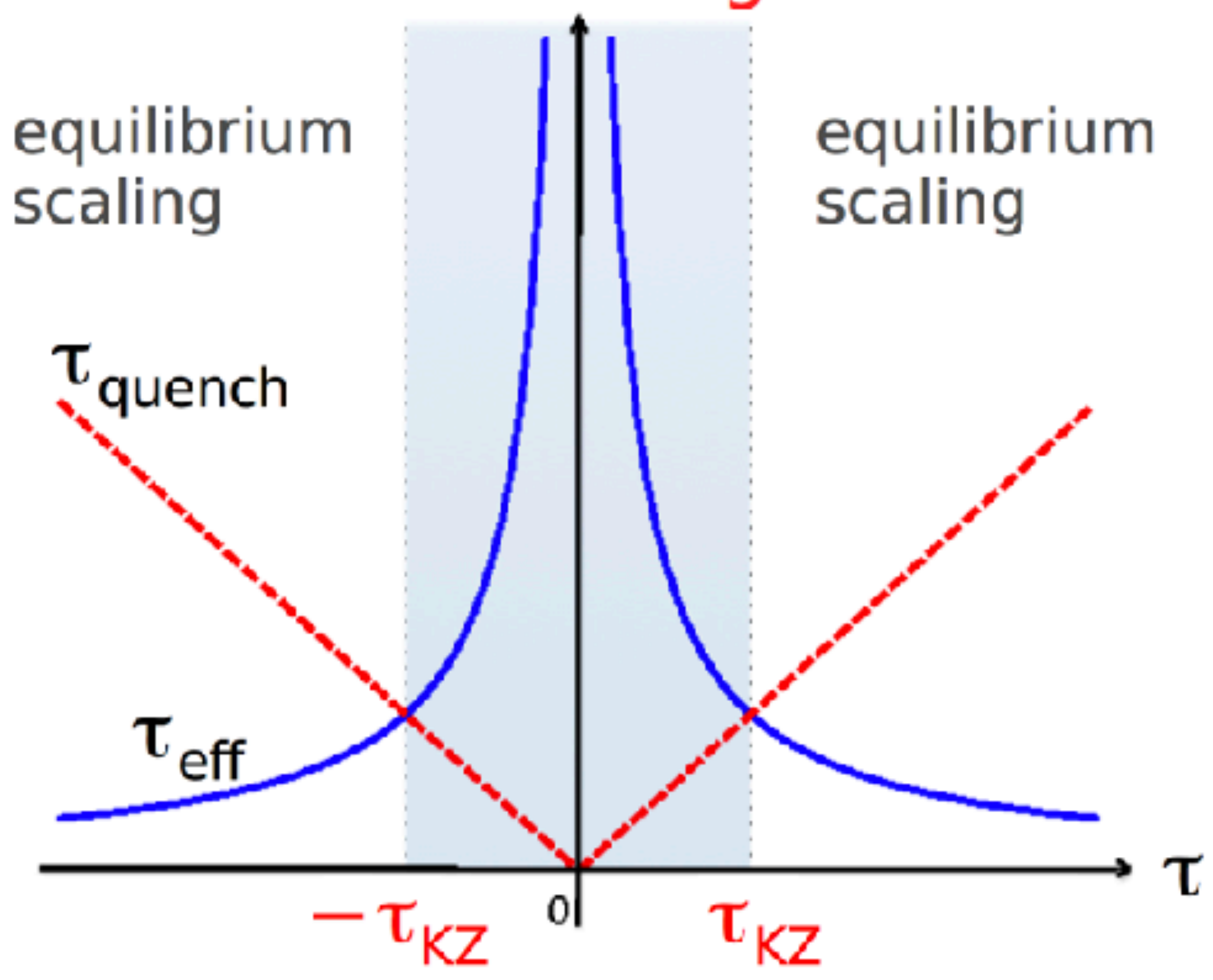
Mukherjee, Venugopalan, Yin: Phys. Rev. Lett. 117, no.22, 222301 (2016)
(editor's suggestion)

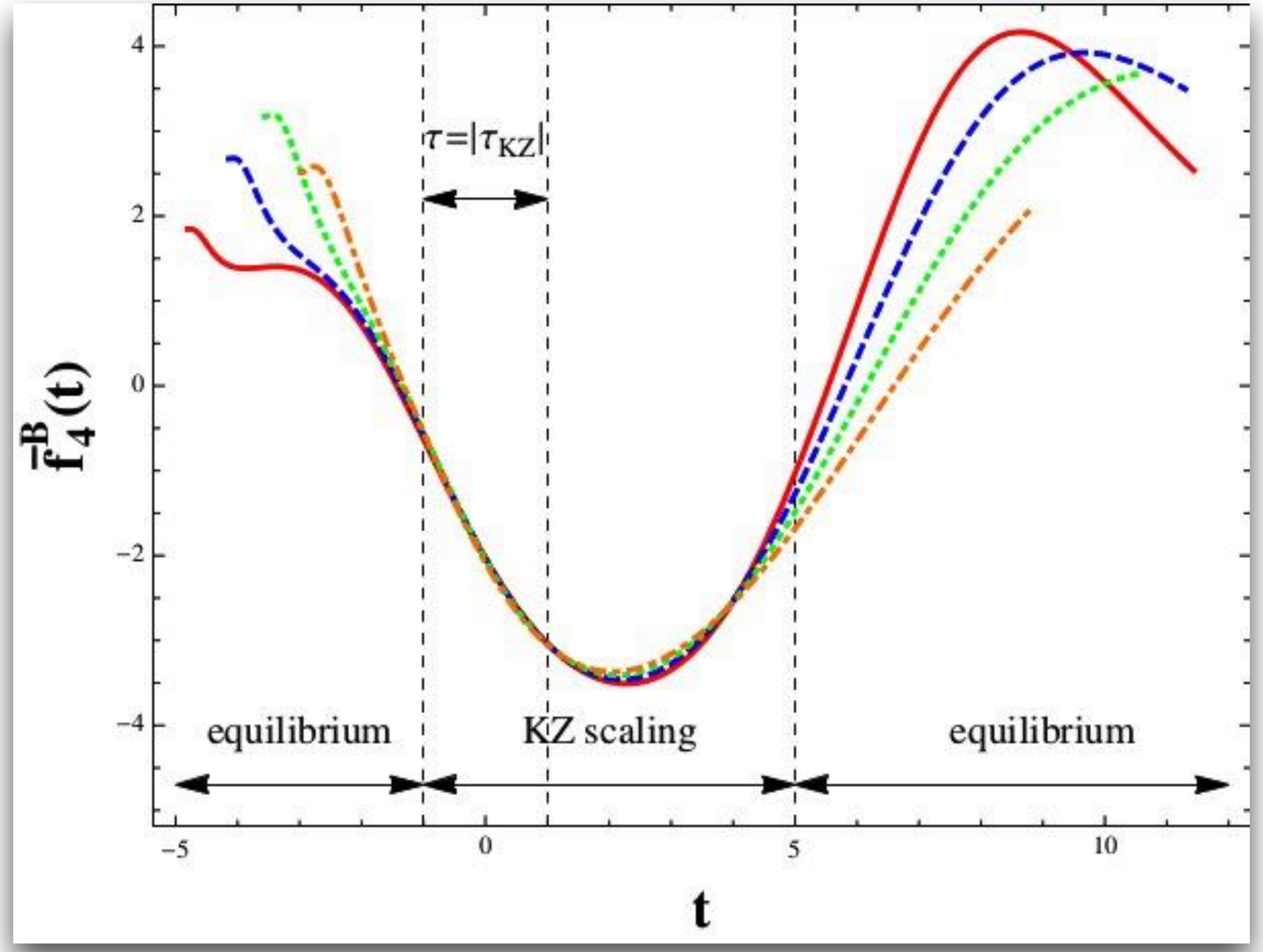



## off-equilibrium Kibble-Zurek scaling

— universality regained

emergent time scale: $\tau_{KZ} = \tau_{eff}(\tau^*) = \tau_{quench}(\tau^*)$



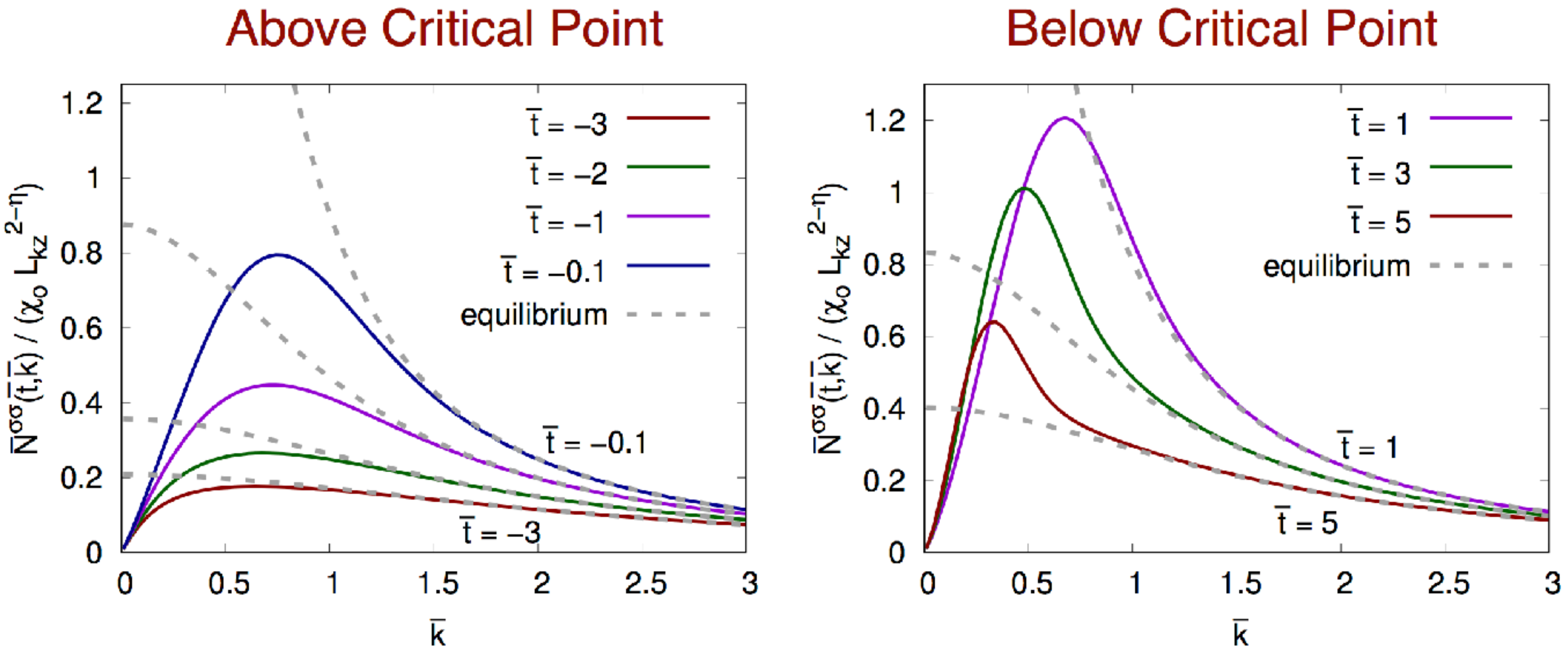

insensitive to the initial conditions

$\tilde{\tau} = \tau - \tau_c,\ t = \tilde{\tau}/\tau_{KZ}$

# KZ dynamics of the correlation function of the critical critical mode



momentum dependence




$$\bar{t} = \frac{t}{t_{\rm kz}} \quad \text{and} \quad \bar{k} = k\ell_{\rm kz} \quad \text{and} \quad \bar{\xi} = \frac{\xi}{\ell_{\rm kz}}$$

# Dynamical critical fluctuations of the sigma field

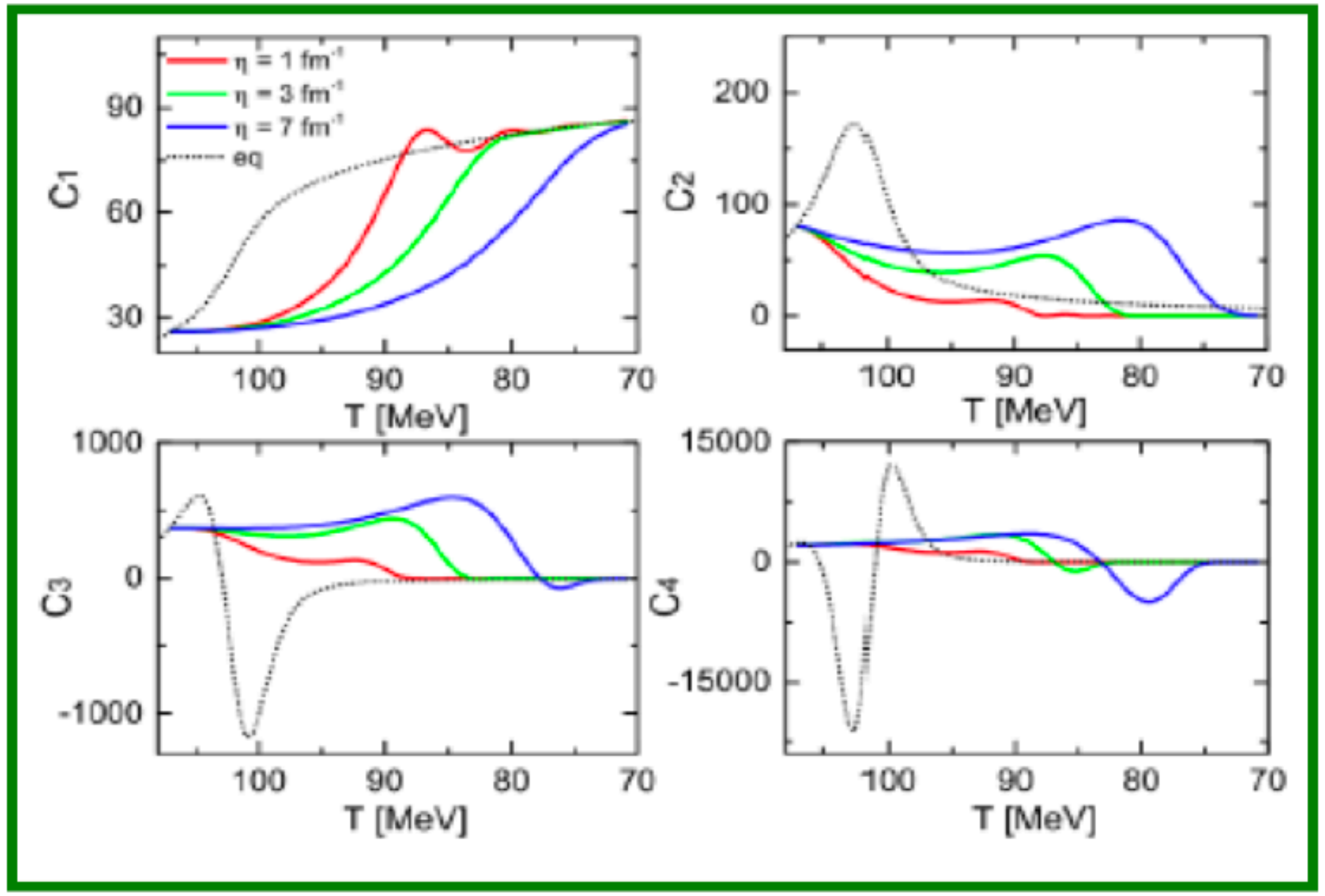
Langevin dynamics: $\partial^\mu \partial_\mu \sigma(t,x) + \eta \partial_t \sigma(t,x) + V'_{eff}(\sigma) = \xi(t,x)$ **-Model A**

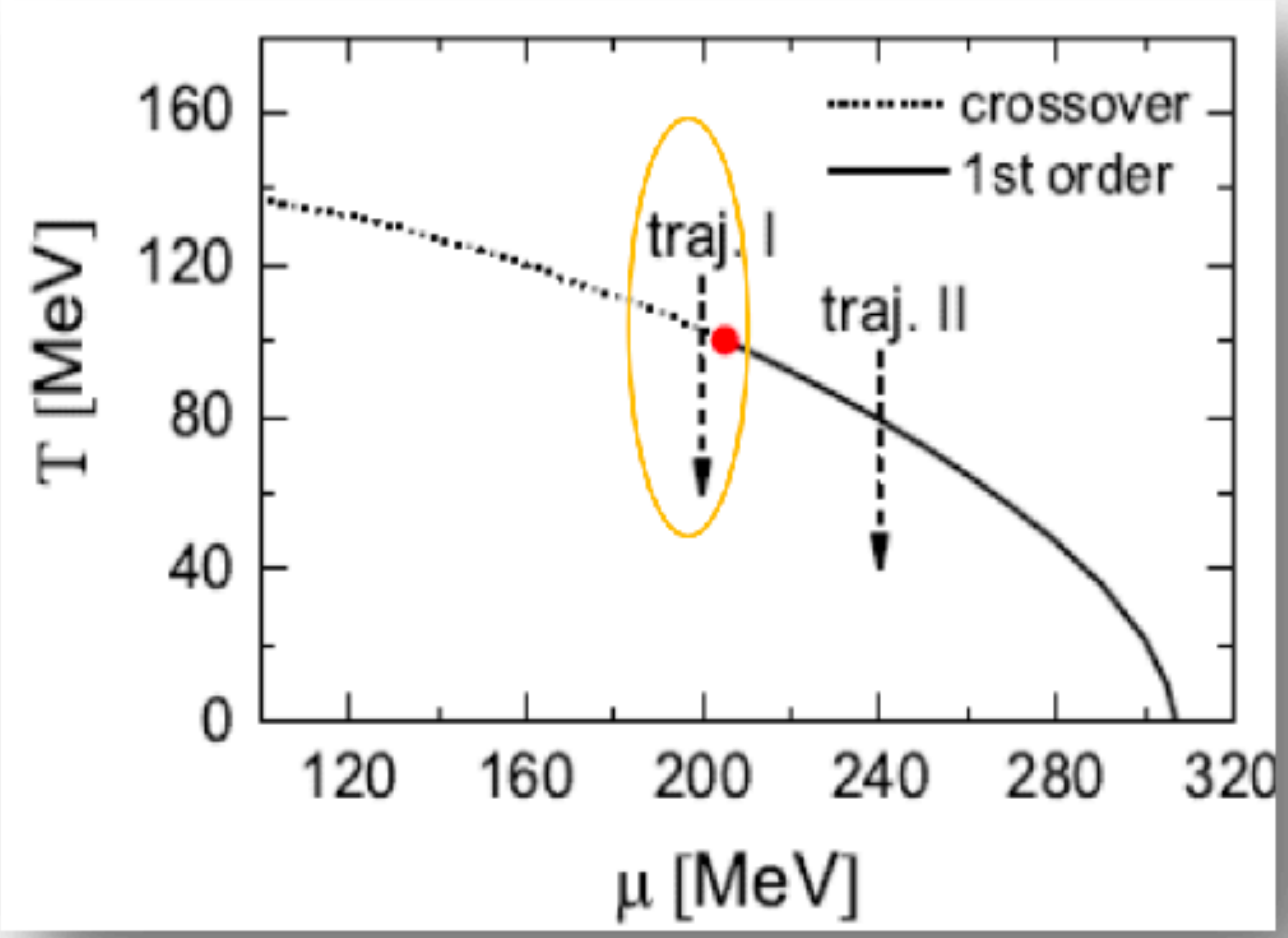
with effective potential from linear sigma model with constituent quarks

$$V_{eff}(\sigma) = U(\sigma) + \Omega_{\bar{q}q}(T,\sigma) = \frac{\lambda^2}{4}\left(\sigma^2 - \nu^2\right)^2 - h_q\sigma - U_0 - 2d_q T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + \exp\left(-\frac{E}{T}\right)\right)$$

Jiang, Wu, Song, NPA 2017, paper in preparation

**On the crossover side**

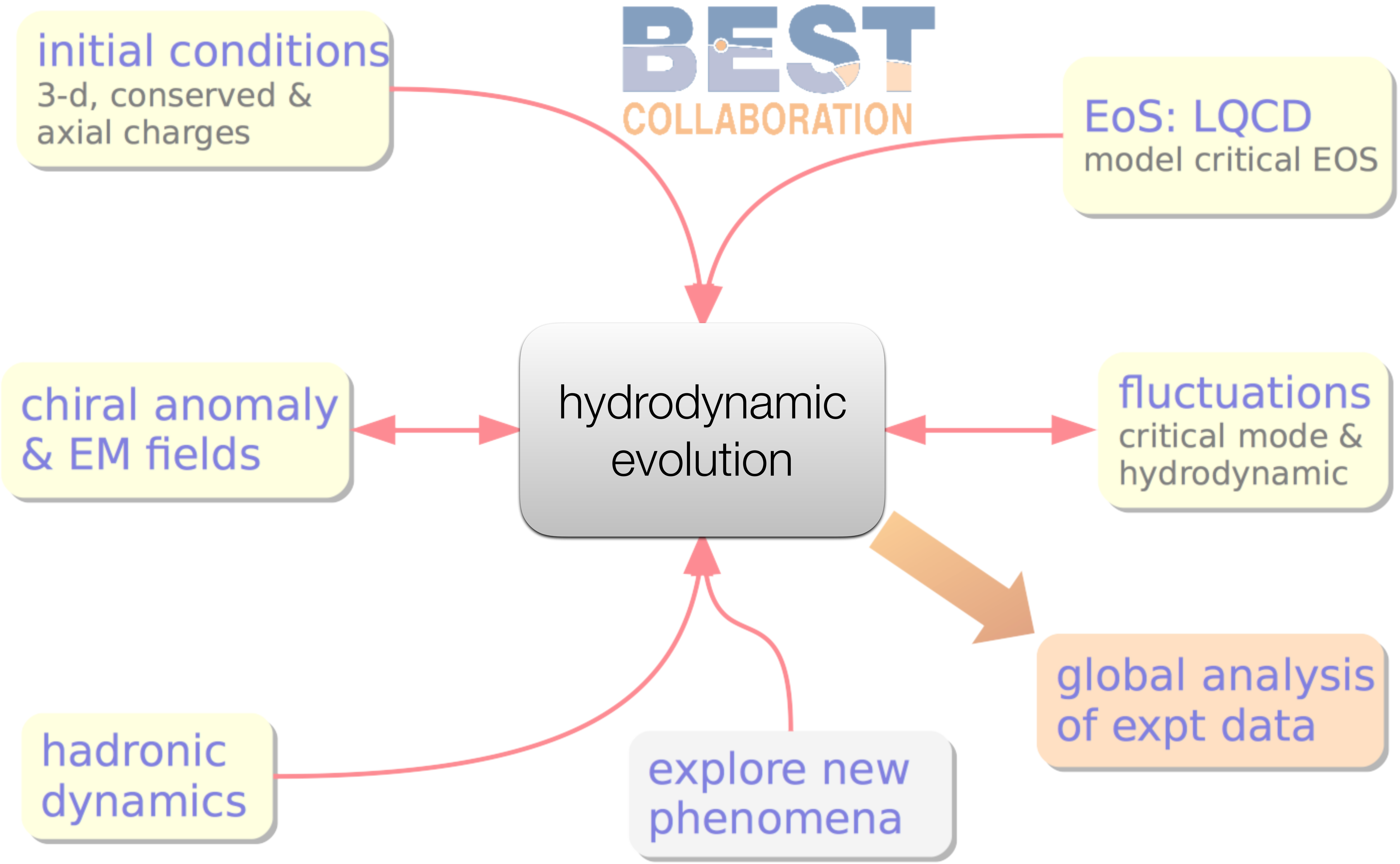



Talk by Huichao Song

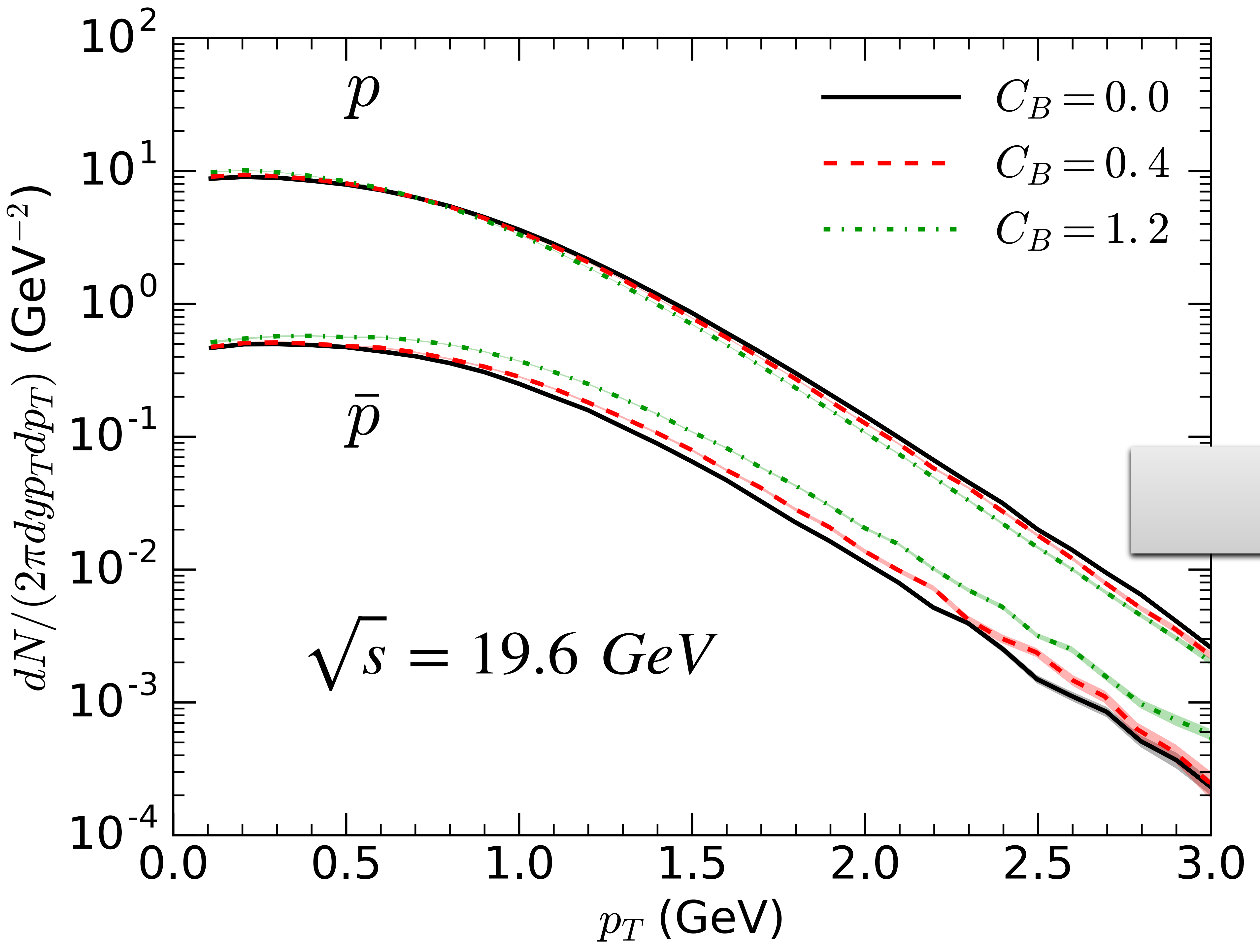
-The signs of $C_3$ & $C_4$ are different from the equil. ones due to memory effects

-in the near future: maping with 3D Ising model; extend to model B; dynamical universal behavior


BEST
COLLABORATION
initial conditions
3-d, conserved &
axial charges
EoS: LQCD
model critical EOS
chiral anomaly
& EM fields
hydrodynamic
evolution
fluctuations
critical mode &
hydrodynamic
hadronic
dynamics
explore new
phenomena
global analysis
of expt data

# MUSIC: (3+1)-d, shear & bulk viscosities, baryon current & diffusion

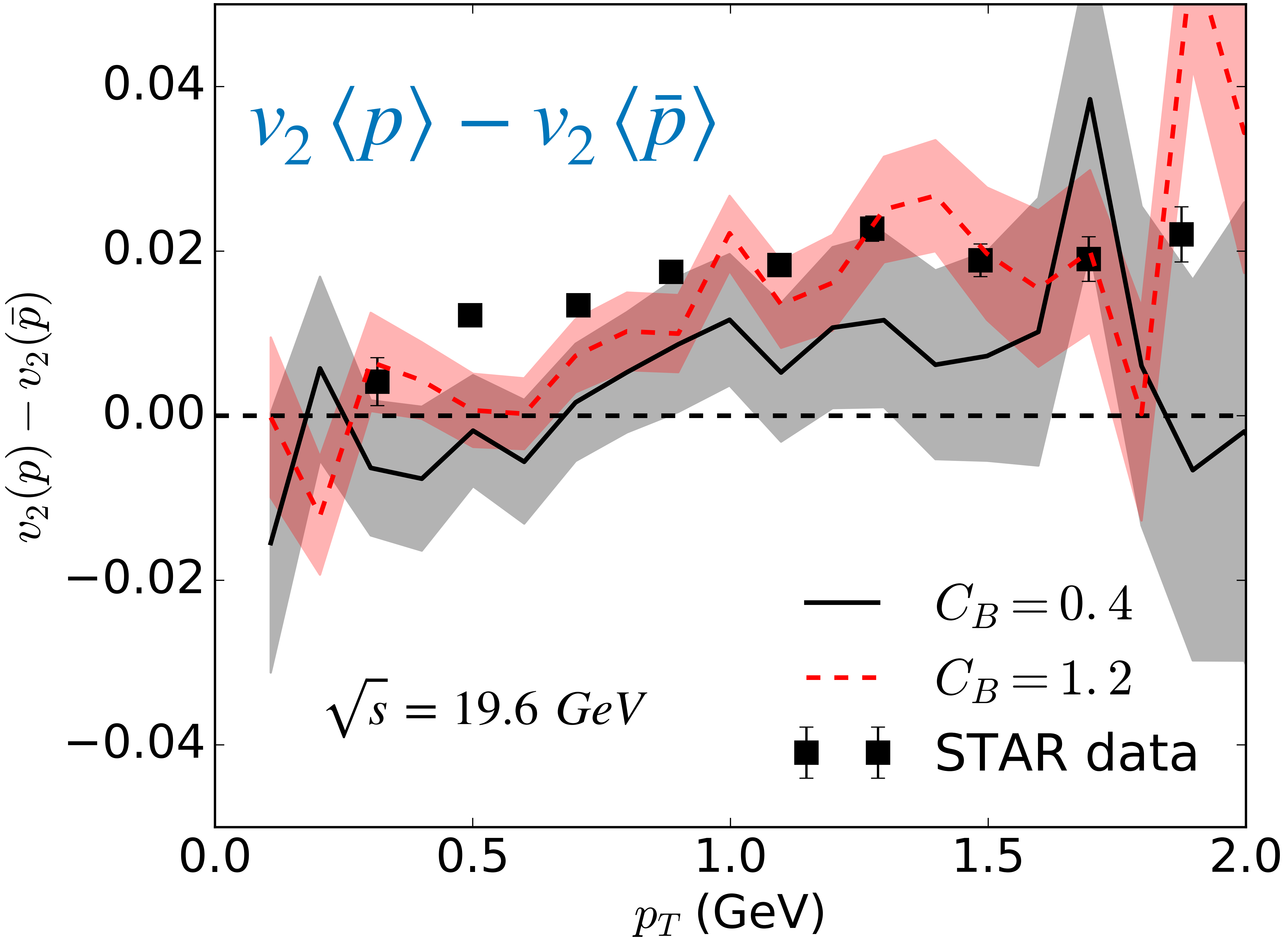
## sensitivity to baryon diffusion contant $(C_B)$




$$\langle p_\perp \rangle^{\bar{p}} - \langle p_\perp \rangle^{p}$$

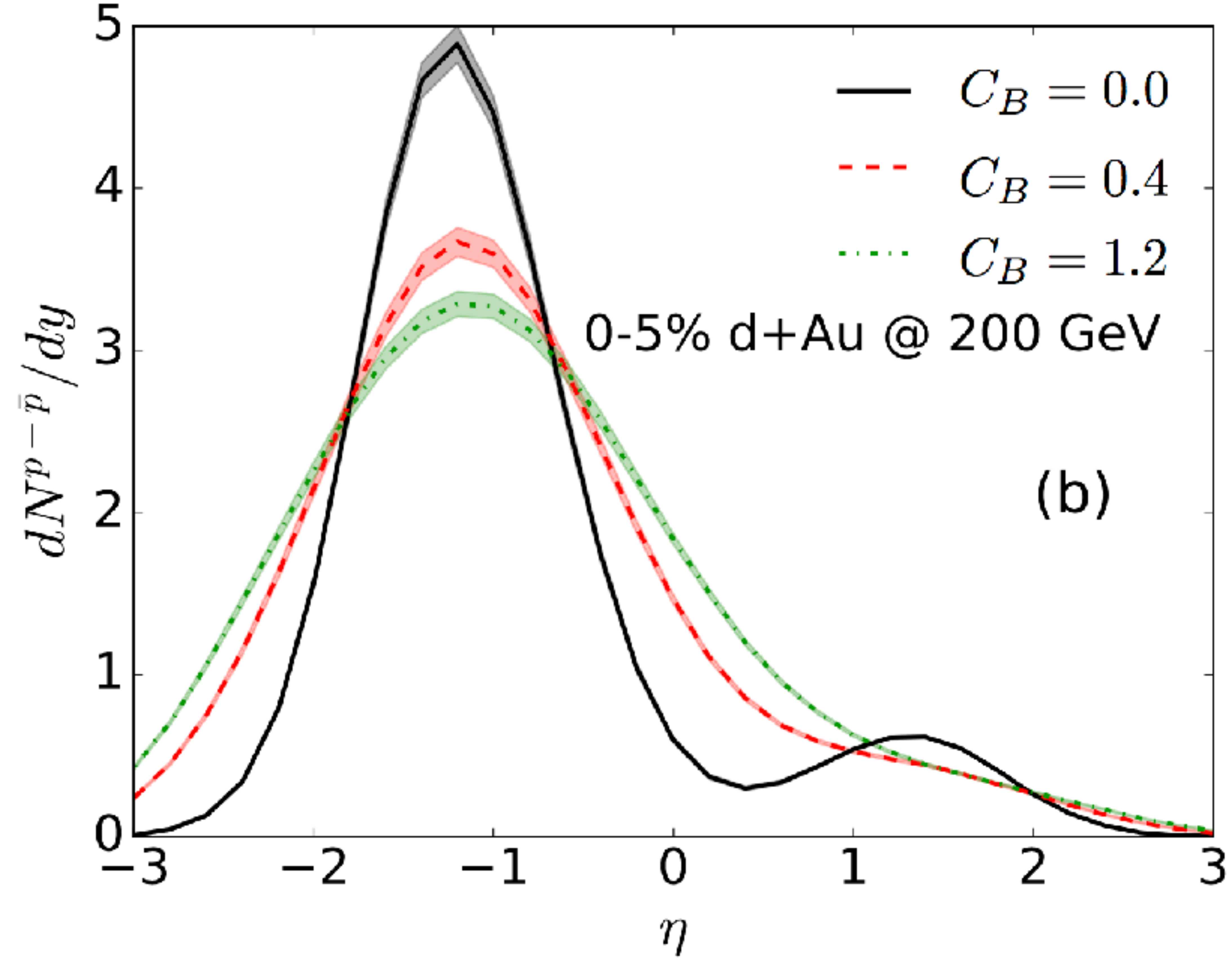
| | $C_B = 0.0$ | $C_B = 0.4$ | $C_B = 1.2$ |
|---|---|---|---|
| $\langle p_\perp \rangle^{\bar{p}} - \langle p_\perp \rangle^{p}$ (GeV) | 0.046 | 0.091 | 0.158 |

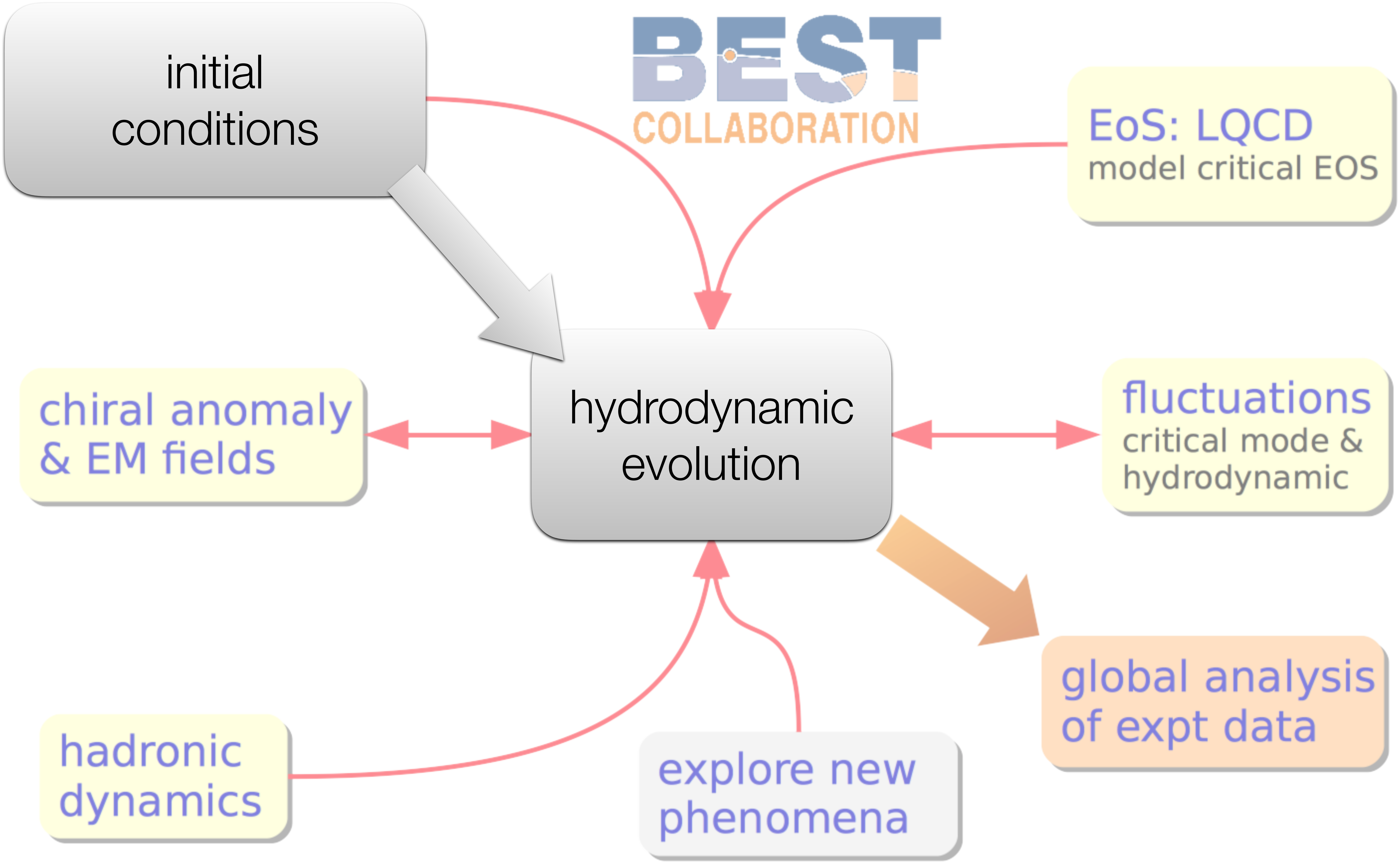
## sensitivity to baryon diffusion contant $(C_B)$

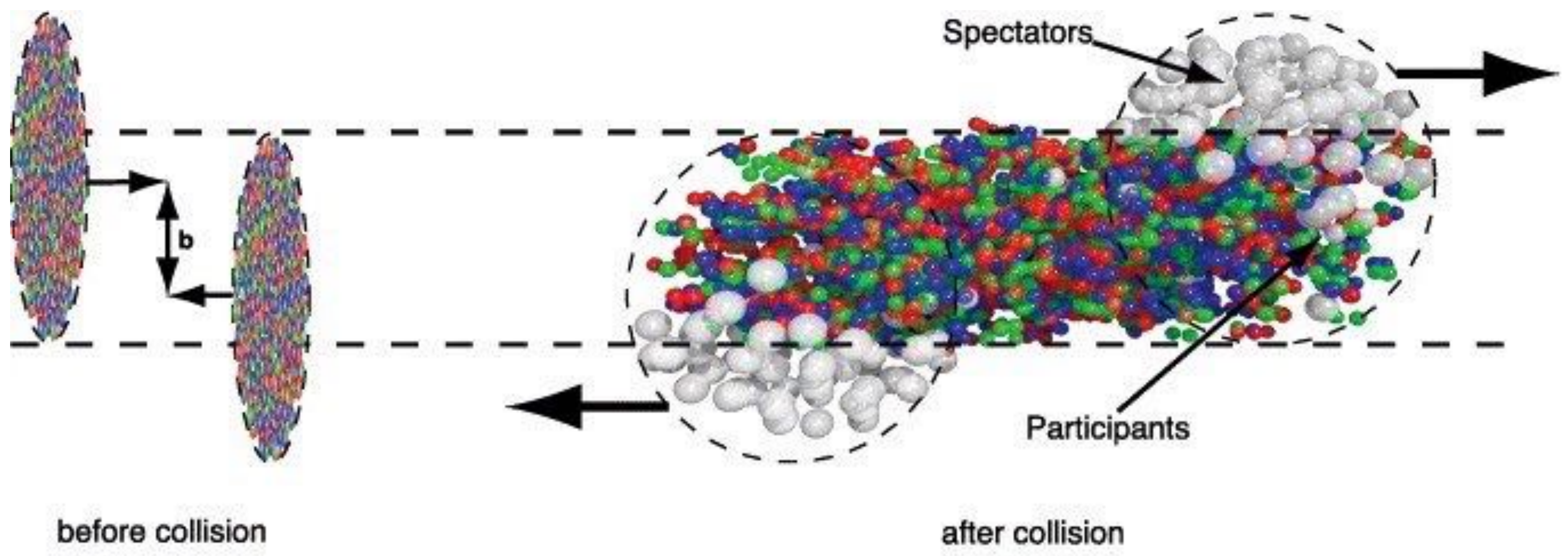


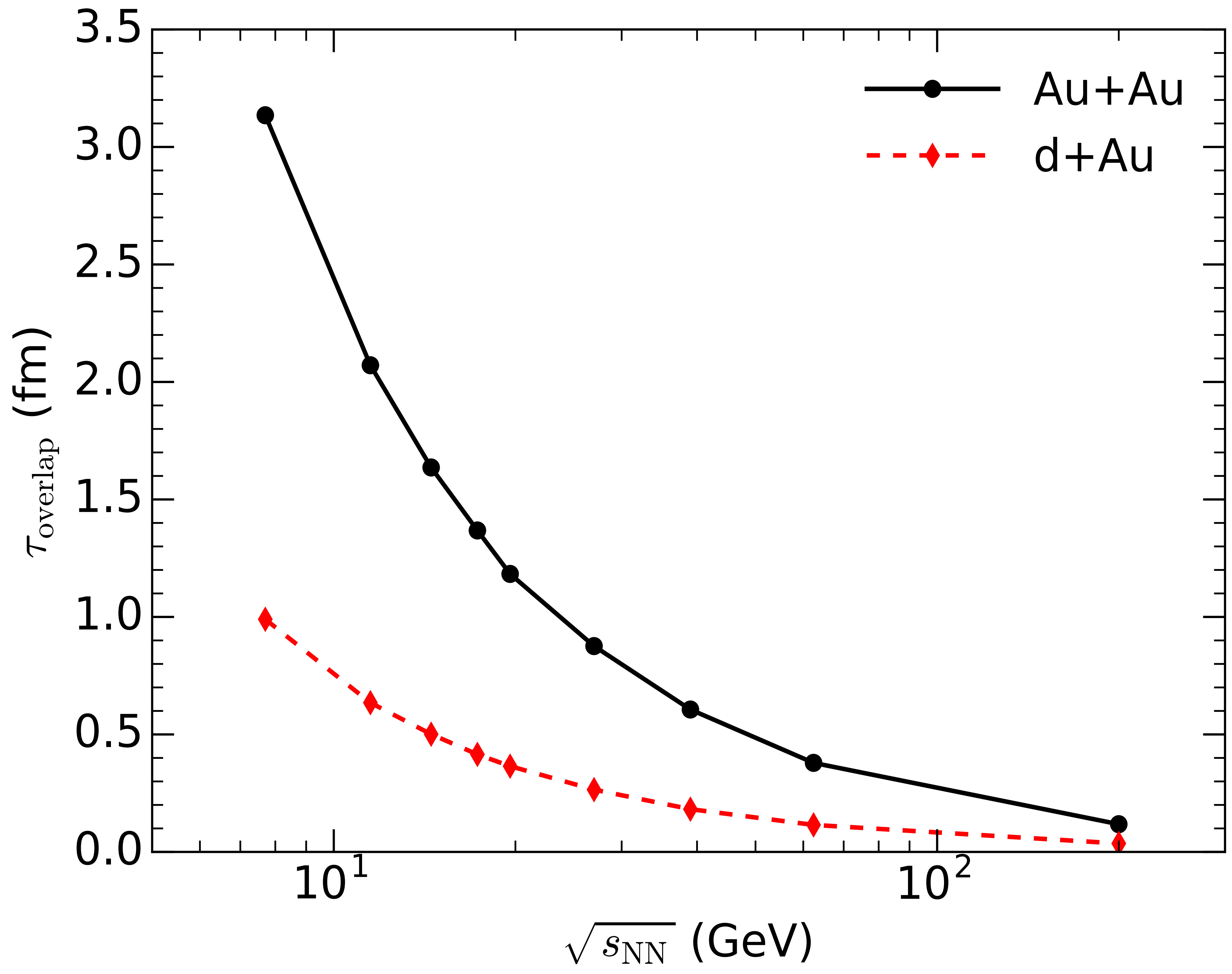

excluding all effects of chiral anomaly

# sensitivity to baryon diffusion contant $(C_B)$




initial conditions
BEST
COLLABORATION
EoS: LQCD
model critical EOS
chiral anomaly & EM fields
hydrodynamic evolution
fluctuations
critical mode & hydrodynamic
global analysis of expt data
hadronic dynamics
explore new phenomena

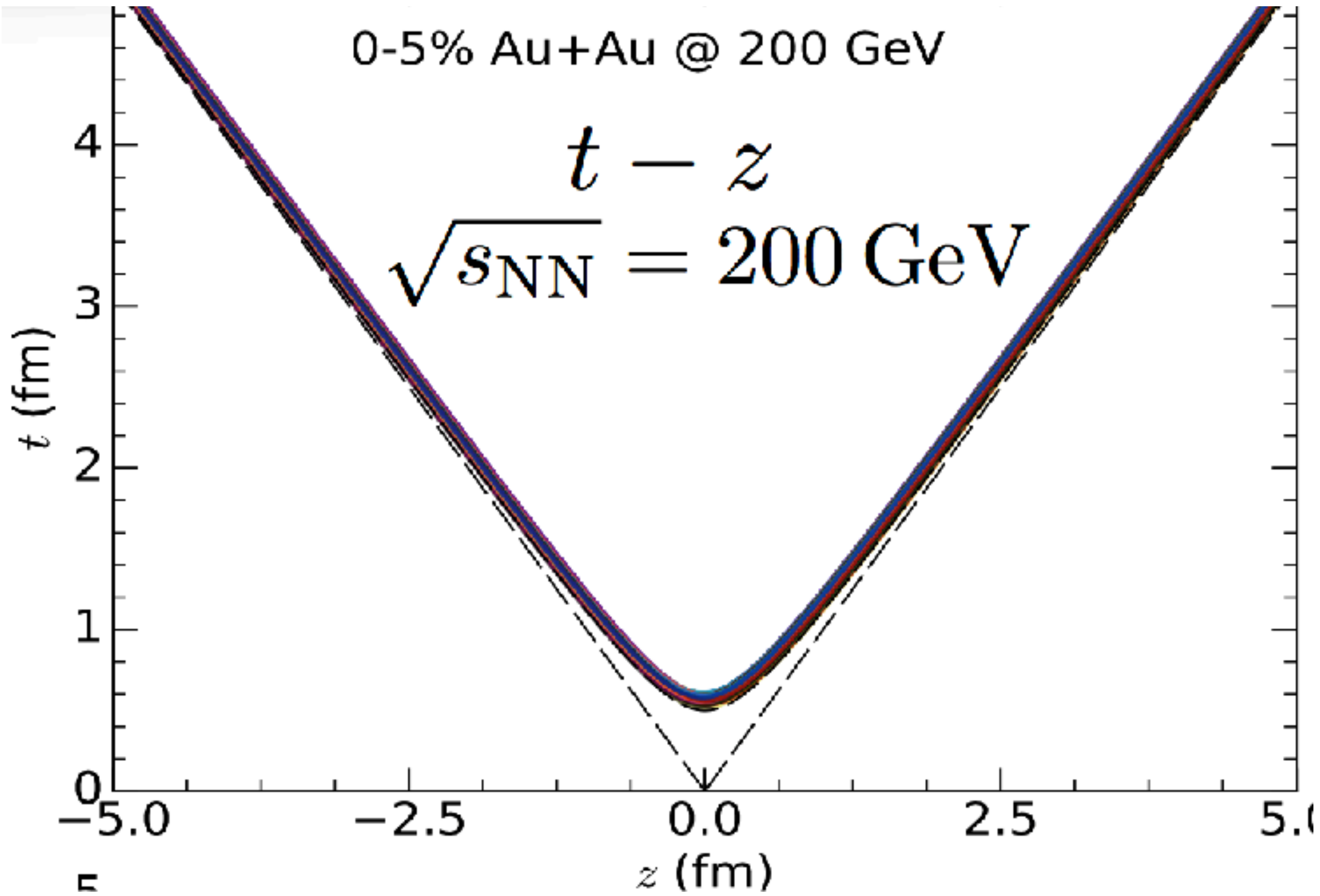


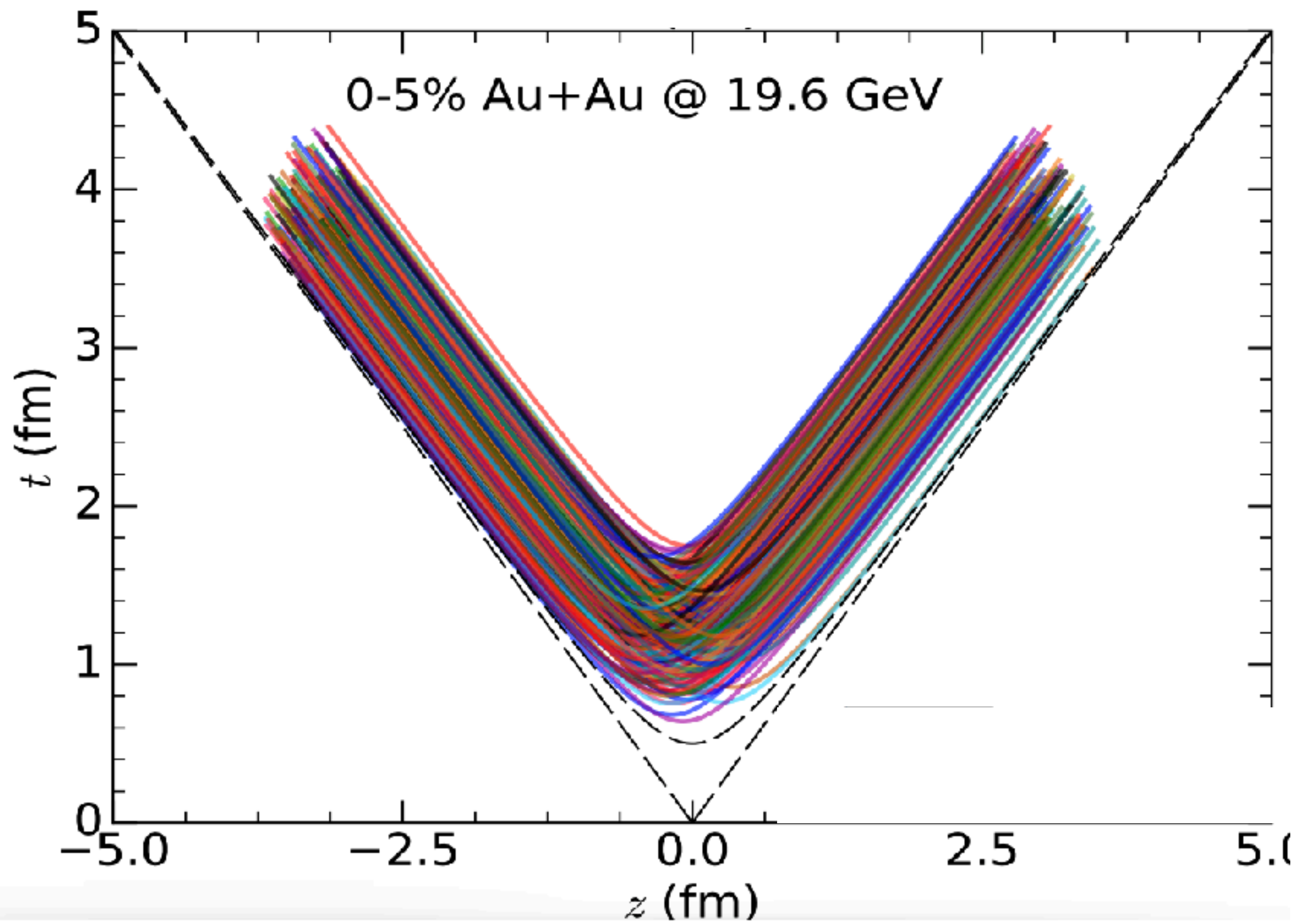




nuclei overlap time large at low $\sqrt{s}$

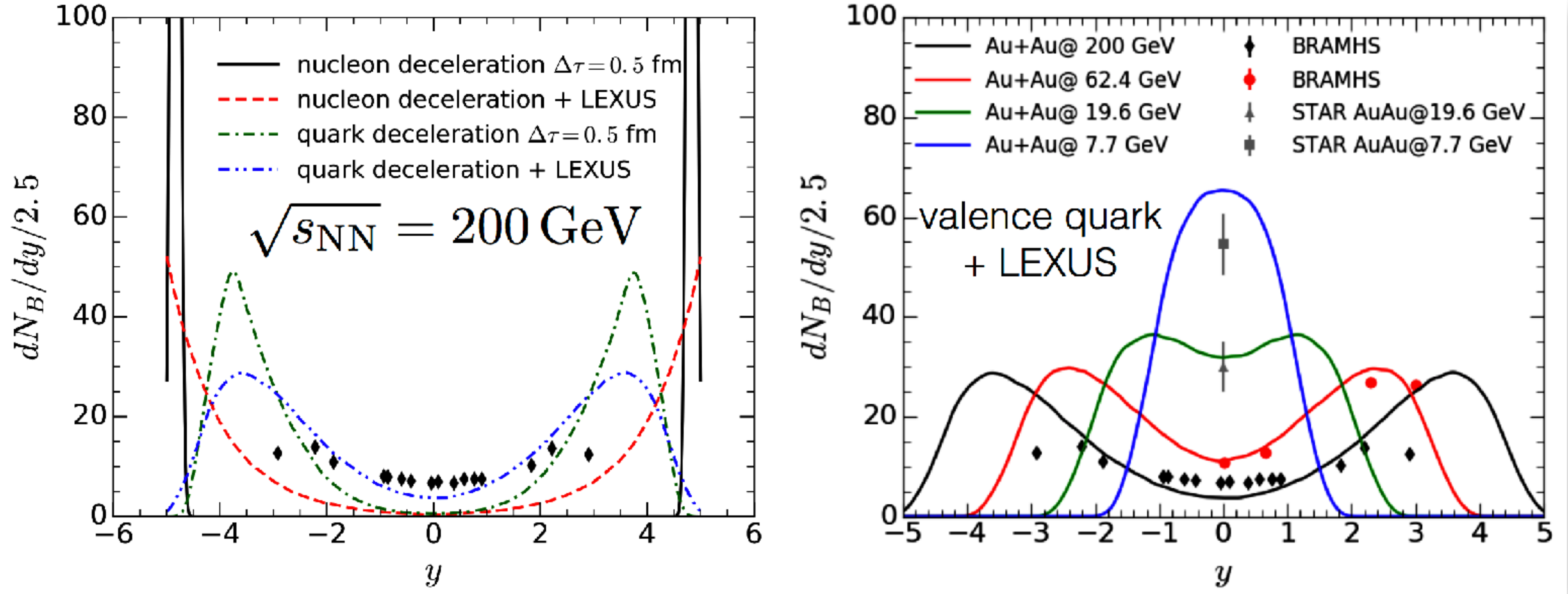
- suitable boost non-invariant initial state model
- pre-equilibrium dynamics
- dynamical initialization of hydrodynamics

# initial state model: 3-d MC quark Glauber, rapidity loss, string declarations, …




- sample valance quarks from incoming participants
- sample rapidity loss is determined using the LEXUS model Jeon, Kapusta: PRC 56, 468 (1997)
- collision time & 3-d spatial positions determined for every binary collision
- QCD strings are randomly produced from collision points Bialas, Bzdak, Koch: arXiv:1608.07041
- strings are decelerated with a constant string tension 1 GeV/fm before thermalize into medium



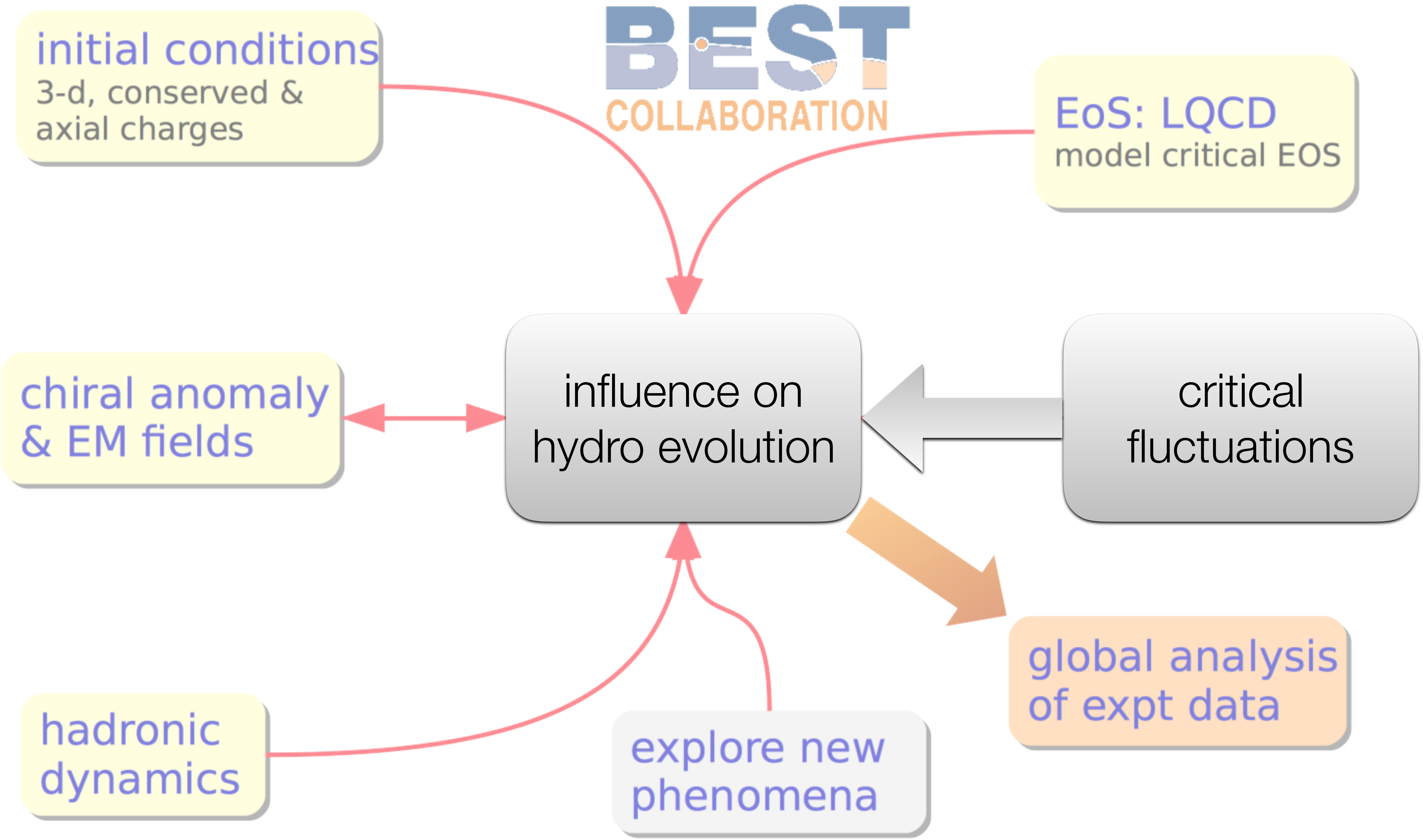
nucleon deceleration $\Delta\tau = 0.5$ fm
nucleon deceleration + LEXUS
quark deceleration $\Delta\tau = 0.5$ fm
quark deceleration + LEXUS
$\sqrt{s_{NN}} = 200$ GeV
$dN_B/dy/2.5$
$y$
Au+Au@ 200 GeV
Au+Au@ 62.4 GeV
Au+Au@ 19.6 GeV
Au+Au@ 7.7 GeV
BRAMHS
BRAMHS
STAR AuAu@19.6 GeV
STAR AuAu@7.7 GeV
valence quark
+ LEXUS

# dynamically initialized hydrodynamic evolution



energy-momentum & net-baryon density are fed into hydro evolution as sources


time evolution of net baryon density @ $\sqrt{s} = 19.6\ GeV$

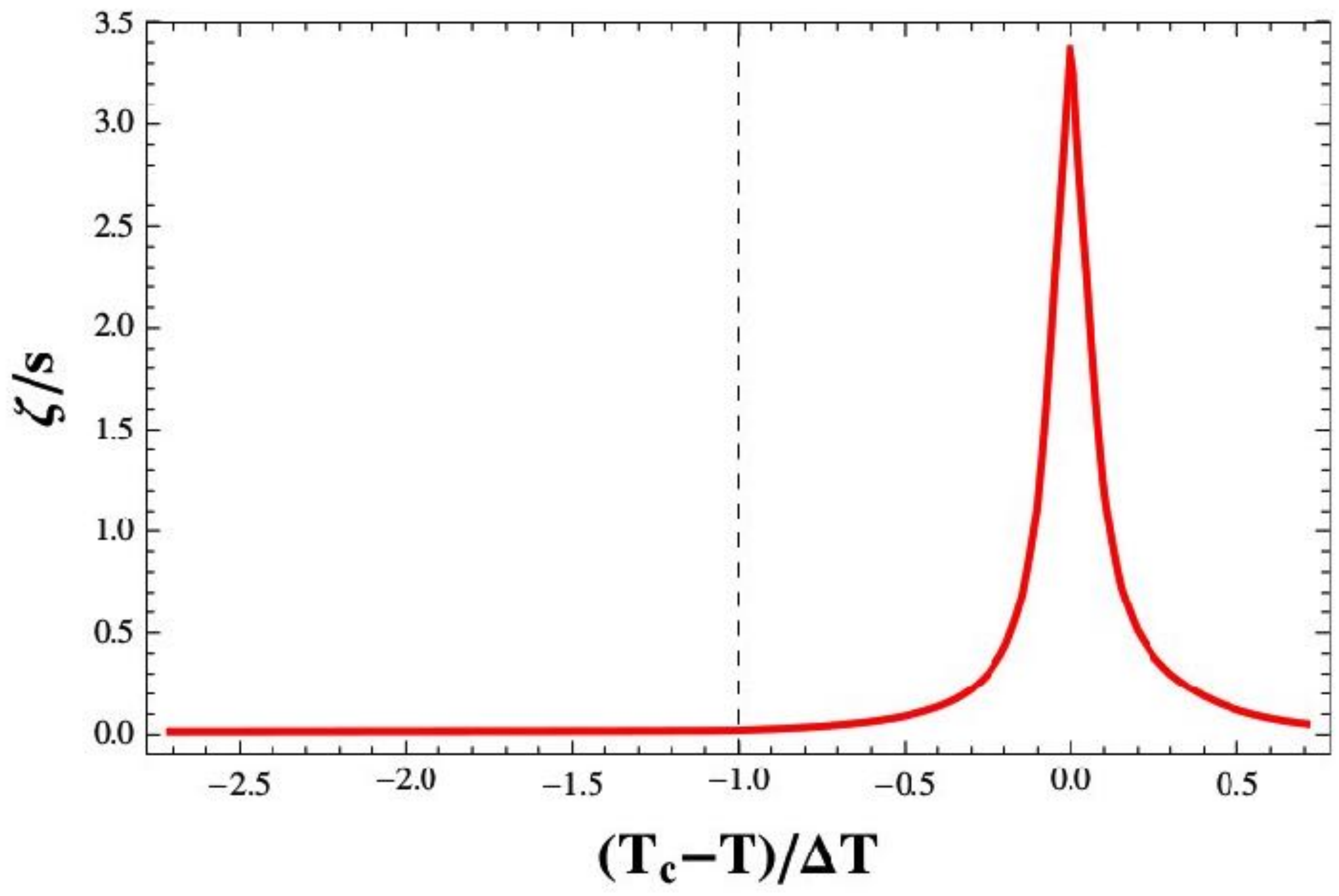

BEST
COLLABORATION
initial conditions
3-d, conserved &
axial charges
EoS: LQCD
model critical EOS
chiral anomaly
& EM fields
influence on
hydro evolution
critical
fluctuations
hadronic
dynamics
explore new
phenomena
global analysis
of expt data

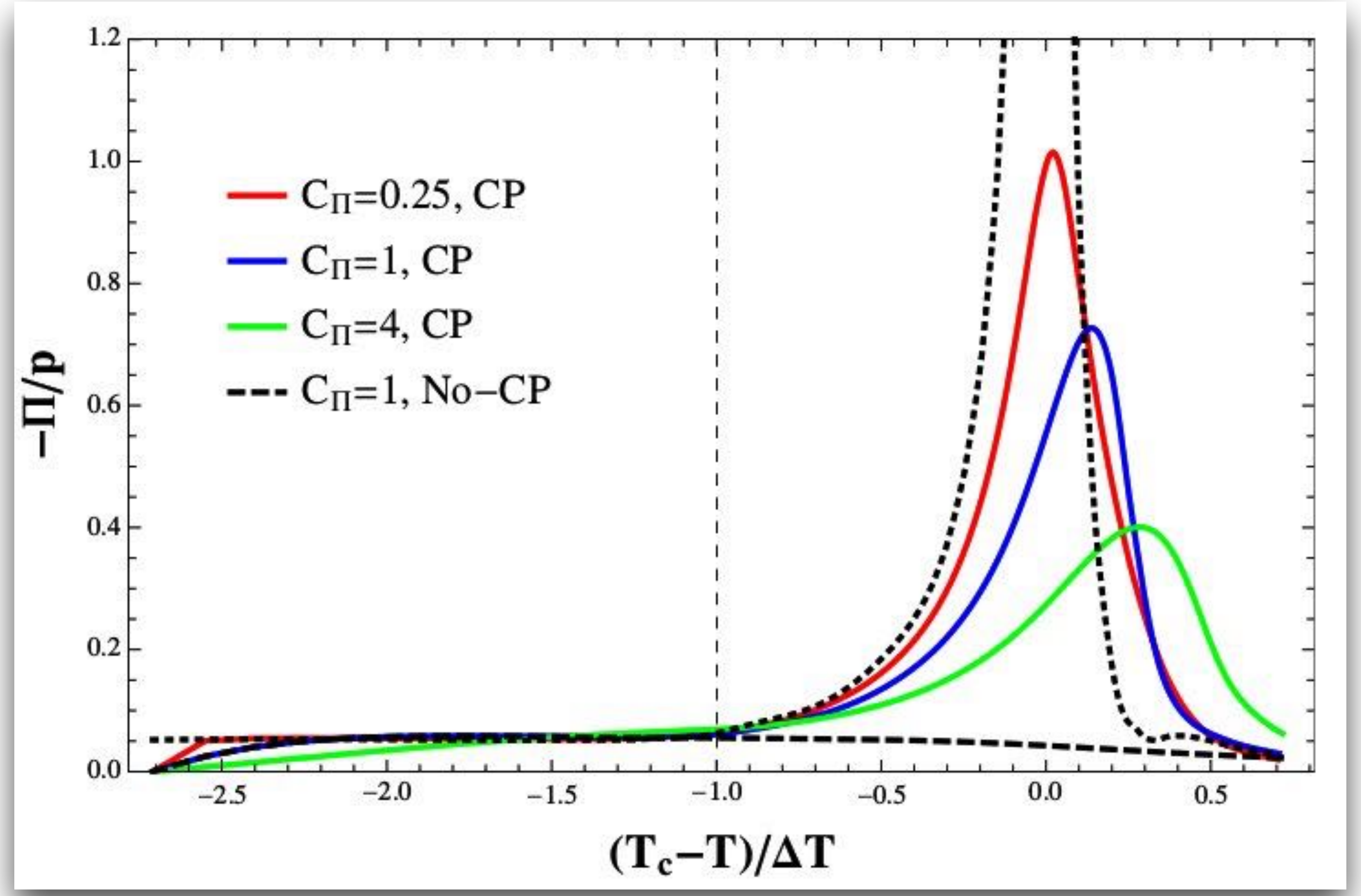
# diverging bulk viscosity at QCD critical point

Monnai, Mukherjee, Yin: Phys. Rev. C95 no.3, 034902 (2017)

$$\zeta \sim \tau_\Pi \sim \tau_\sigma \sim \xi^3$$

critical fluctuations leads to break down of ordinary hydrodynamics for: $k \sim \xi^{-3}$



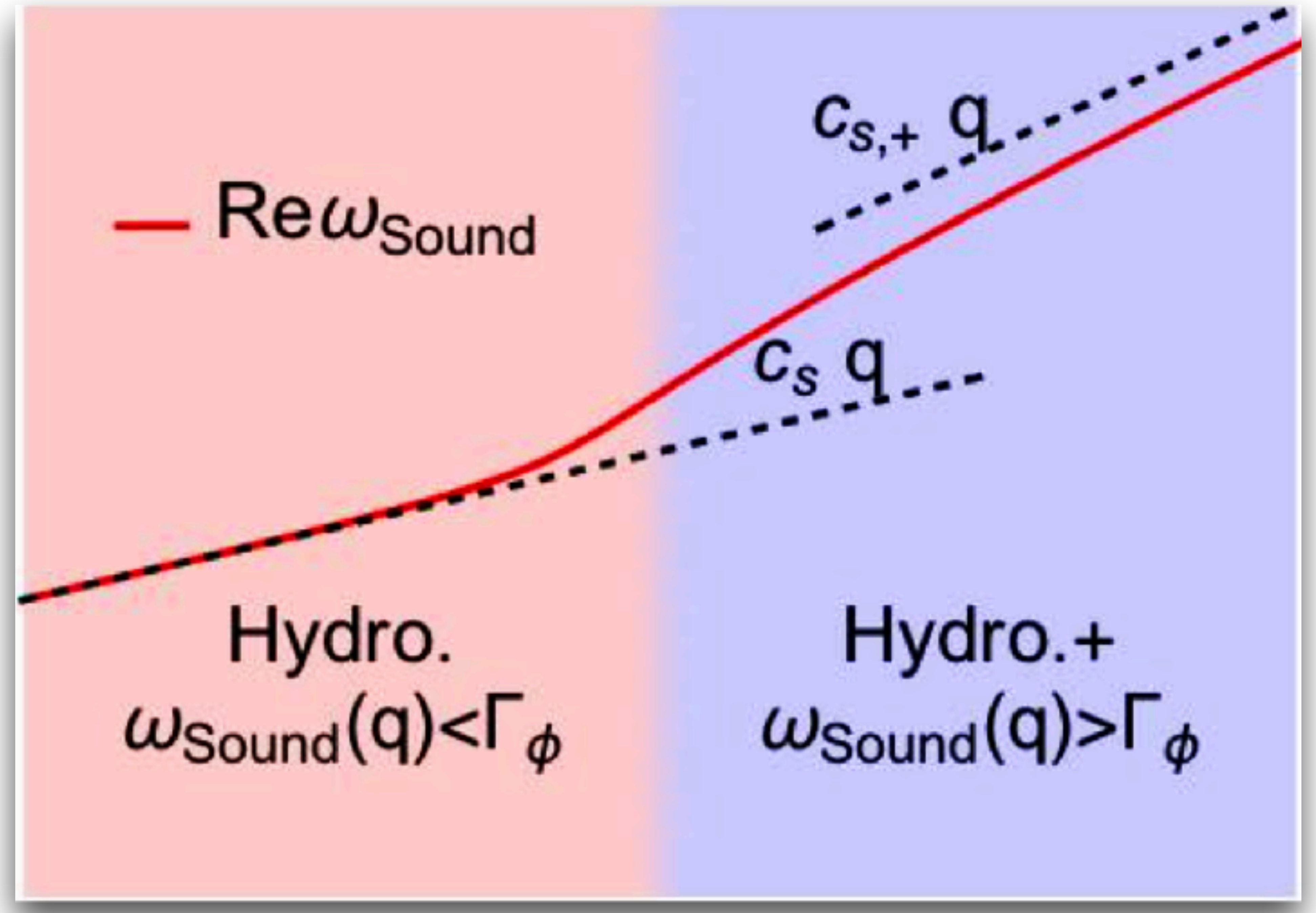

bulk viscous pressure

(1+1)D, Israel-Stewart hydro

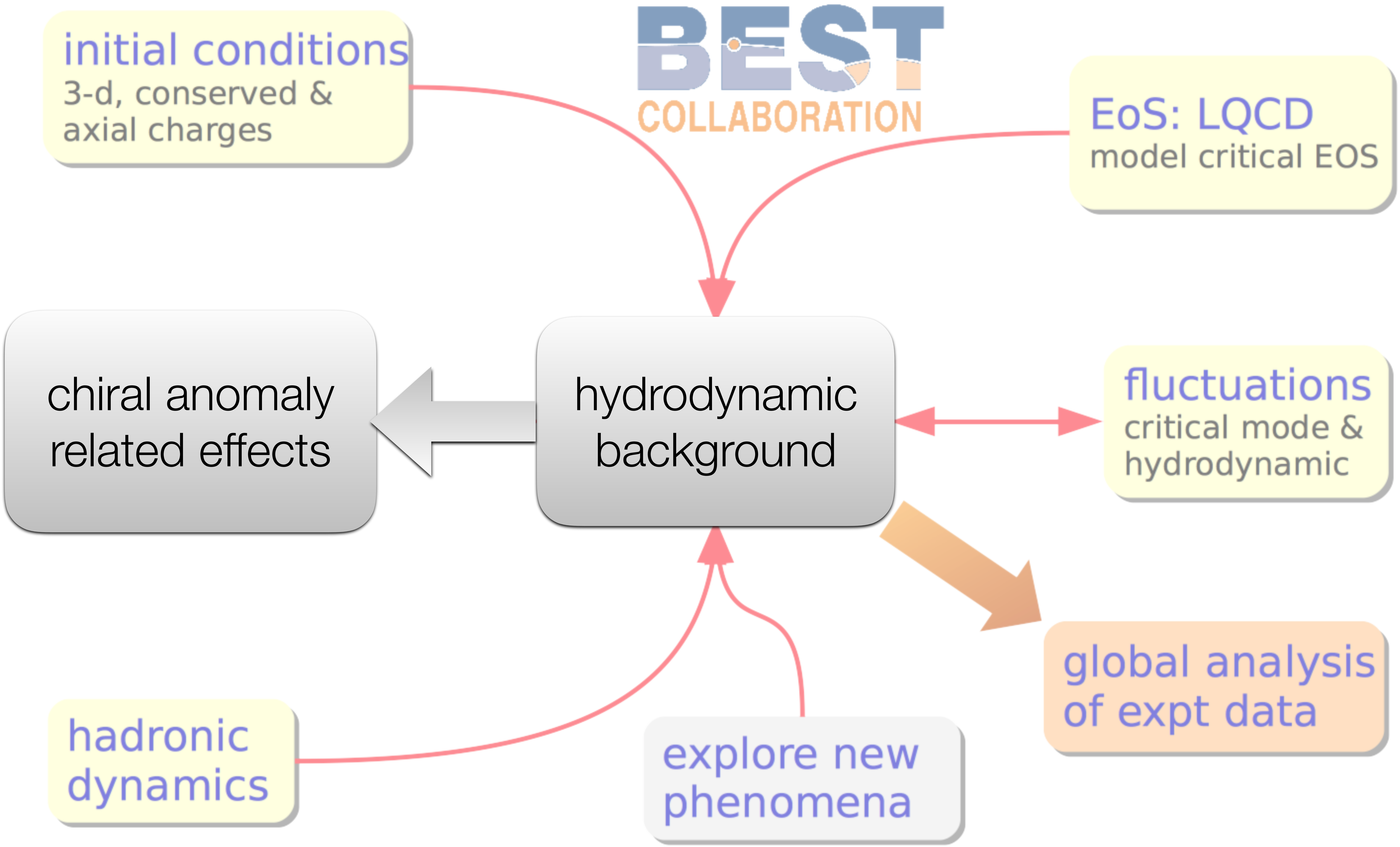
# Hydro+: hydrodynamics with critical fluctuations

extends the validity of hydro down to scales: $\omega \sim \xi^{-1}$ (from $\omega \sim \xi^{-3}$)

- by coupling fluctuations of slow critical mode to hydro
- critical slowdown modifies the speed of sound




$\omega \sim 1/\xi^3$



BEST
COLLABORATION
initial conditions
3-d, conserved &
axial charges
EoS: LQCD
model critical EOS
chiral anomaly
related effects
hydrodynamic
background
fluctuations
critical mode &
hydrodynamic
global analysis
of expt data
hadronic
dynamics
explore new
phenomena

# anomalous-viscous fluid dynamics

Talk by Shuzhe Shi: CPOD-2017

## chiral anomalous current on hydro + EM background

Jiang, Shi, Yin, Liao: arXiv:1611.04586
Shi, Jiang, Lilleskov, Liao: in final preparation

$$D_\mu J_R{}^\mu = +\frac{N_c q^2}{4\pi^2} E_\mu B^\mu \qquad D_\mu J_L{}^\mu = -\frac{N_c q^2}{4\pi^2} E_\mu B^\mu$$

$$J_R{}^\mu = n_R\, u^\mu + \nu_R{}^\mu + \frac{N_c q}{4\pi^2} \mu_R\, B^\mu$$

$$J_L{}^\mu = n_L\, u^\mu + \nu_L{}^\mu - \frac{N_c q}{4\pi^2} \mu_L\, B^\mu$$

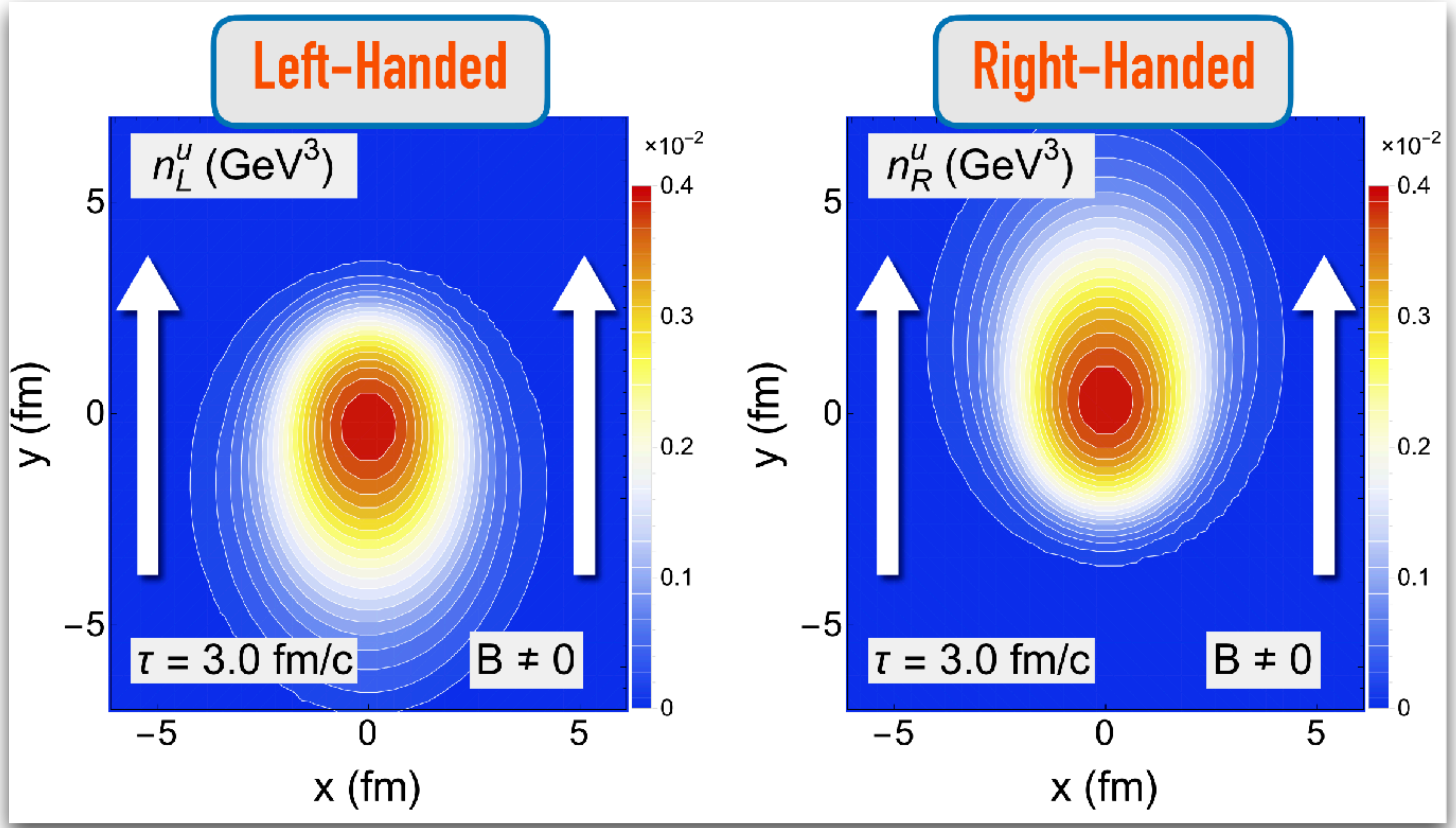
CME

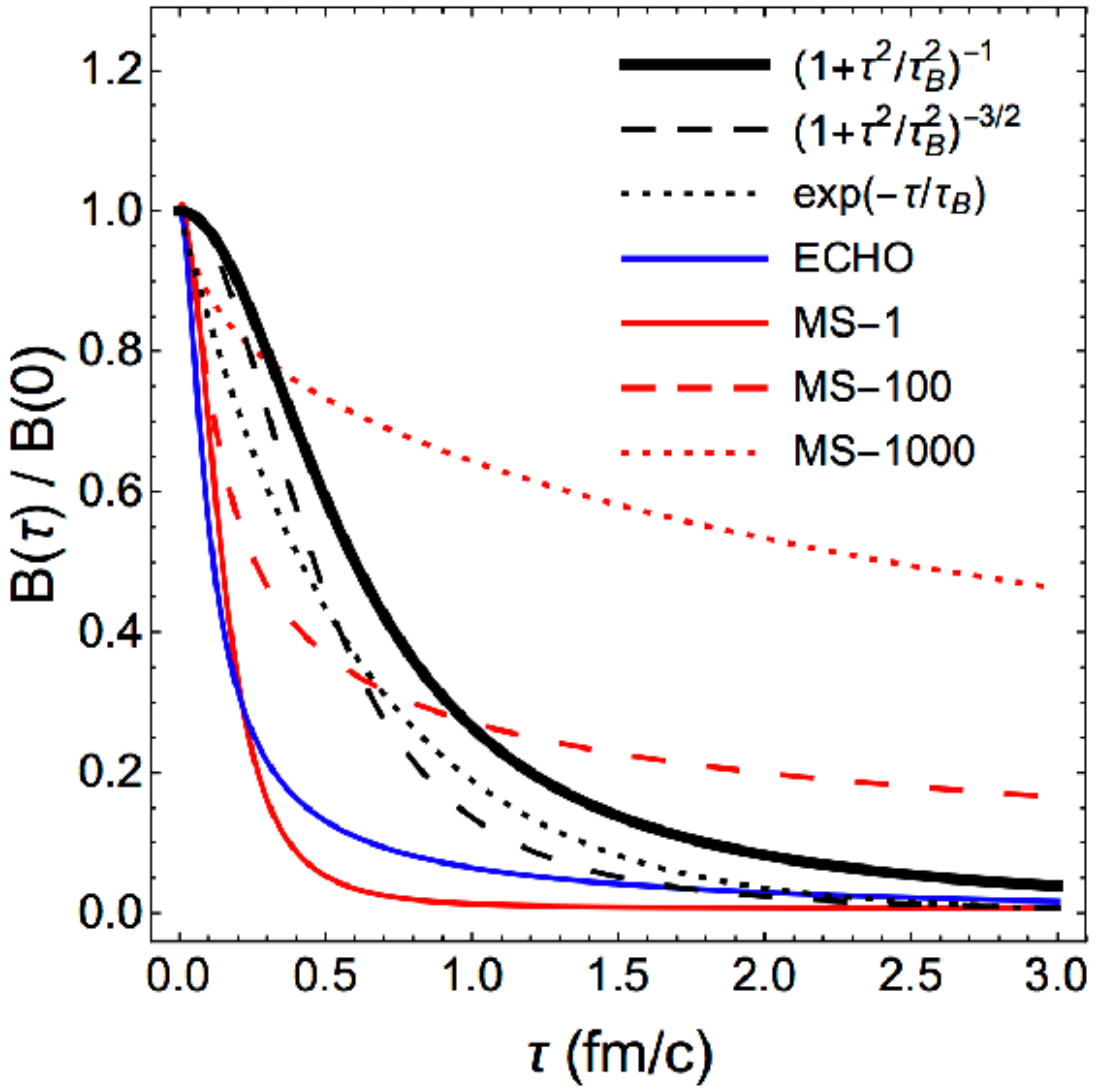
Viscous Effect

$$\Delta^\mu{}_\nu\, d\, \nu_{R,L}{}^\nu = -\frac{1}{\tau_{rlx}} (\nu_{R,L}{}^\mu - \nu_{NS}{}^\mu)$$

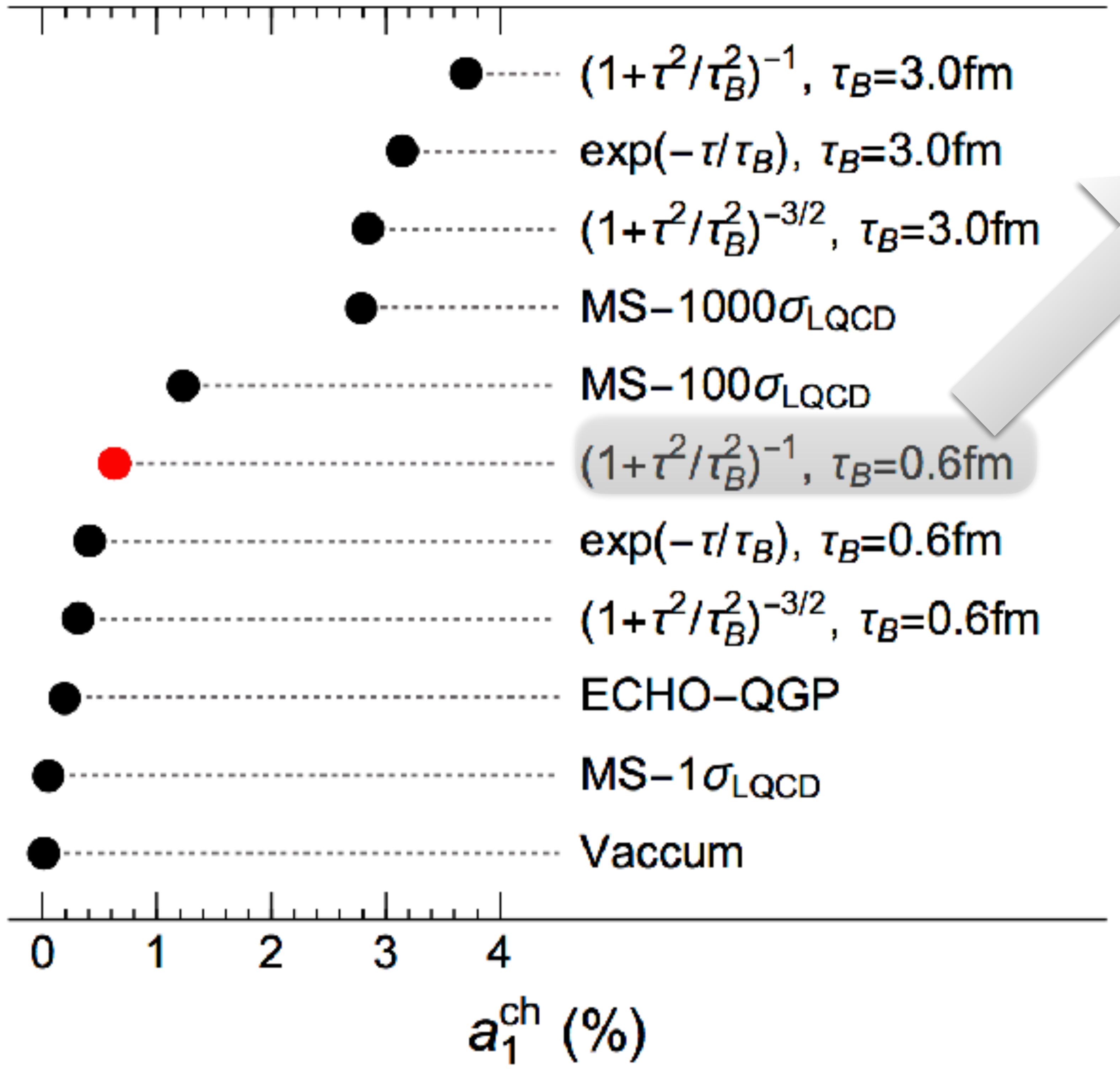
$$\nu_{NS}{}^\mu = \frac{\sigma}{2} T \Delta^{\mu\nu} \partial_\nu \frac{\mu}{T} + \frac{\sigma}{2} q\, E^\mu$$

hydro background:
(2+1)-d viscous VISHNU
(Heinz et. al.)

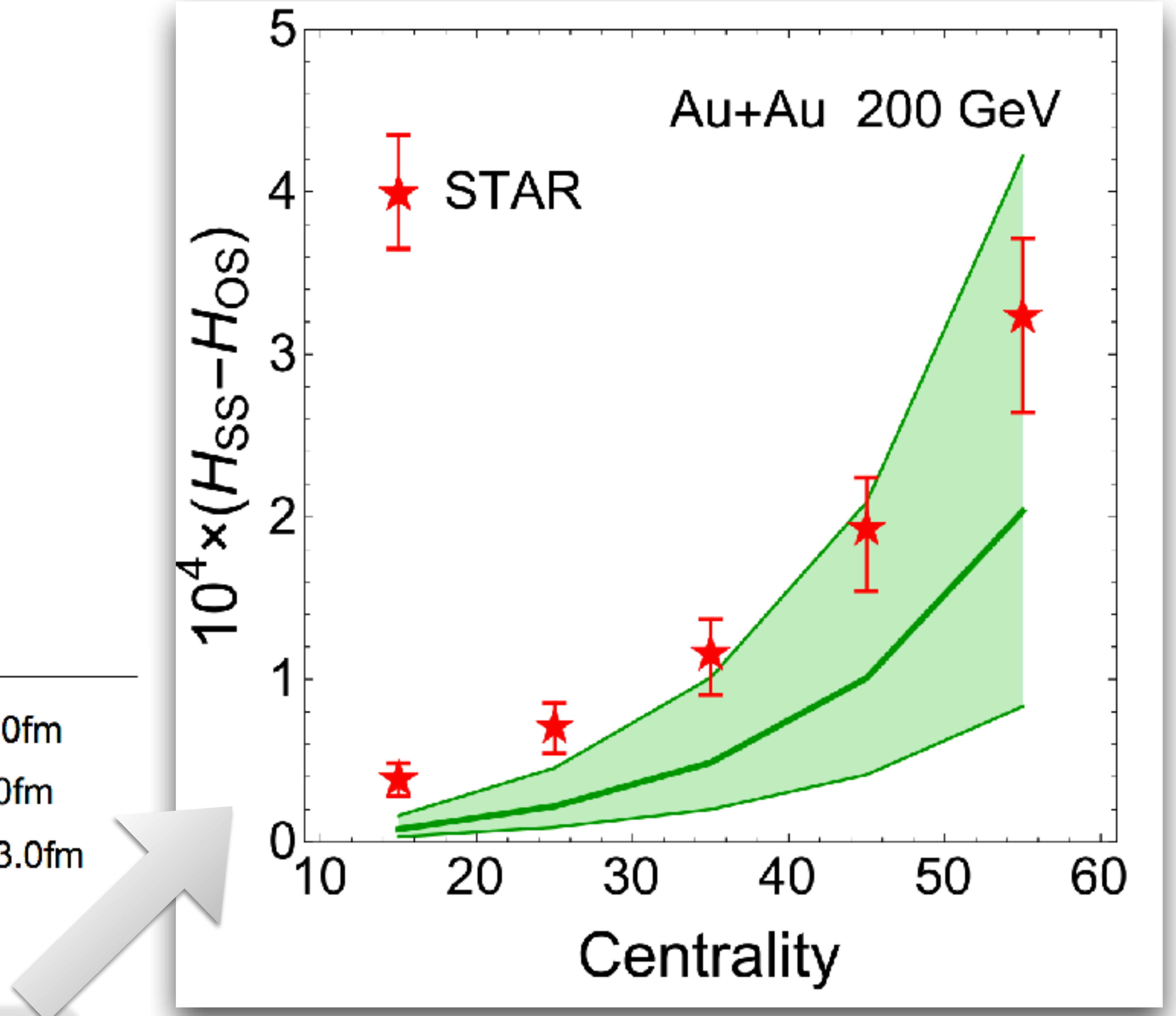

Left-Handed
Right-Handed
$n_L^u$ (GeV$^3$)
$n_R^u$ (GeV$^3$)
×10$^{-2}$
0.4
0.3
0.2
0.1
0
5
0
−5
y (fm)
x (fm)
τ = 3.0 fm/c
B ≠ 0



$(1+\tau^2/\tau_B^2)^{-1}$
$(1+\tau^2/\tau_B^2)^{-3/2}$
$\exp(-\tau/\tau_B)$
ECHO
MS−1
MS−100
MS−1000
$B(\tau) / B(0)$
$\tau$ (fm/c)

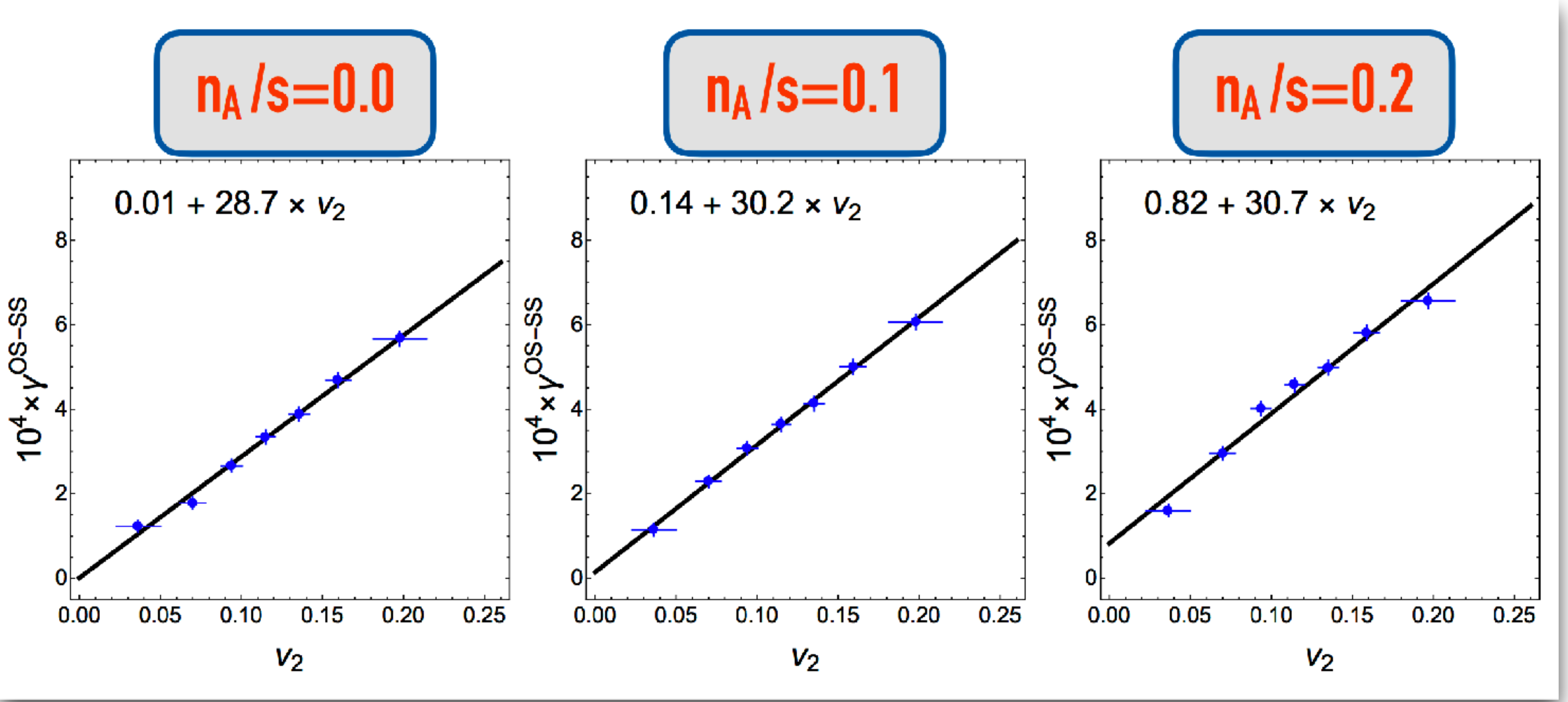



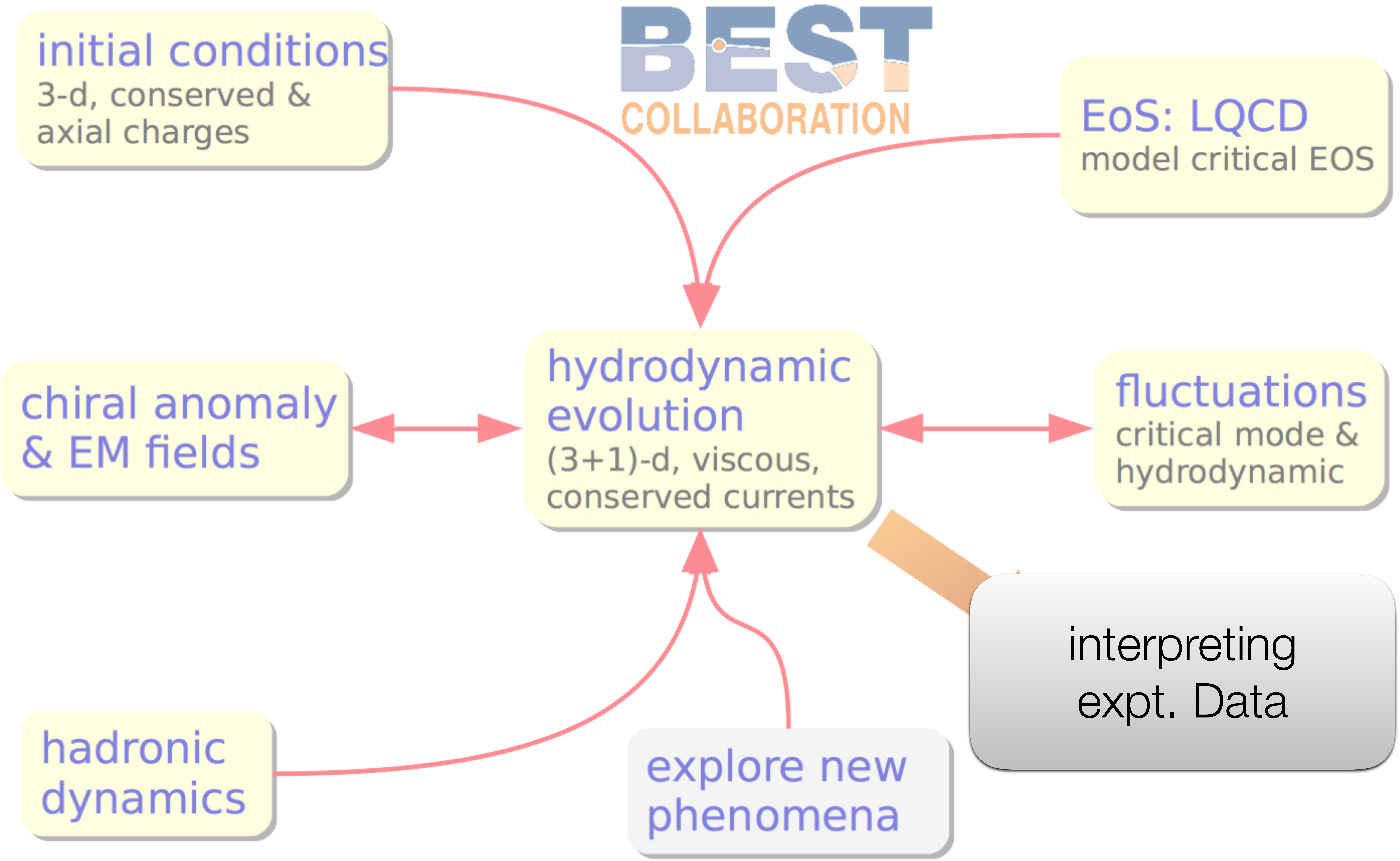
$(1+\tau^2/\tau_B^2)^{-1}$, $\tau_B$=3.0fm
$\exp(-\tau/\tau_B)$, $\tau_B$=3.0fm
$(1+\tau^2/\tau_B^2)^{-3/2}$, $\tau_B$=3.0fm
MS−1000$\sigma_{LQCD}$
MS−100$\sigma_{LQCD}$
$(1+\tau^2/\tau_B^2)^{-1}$, $\tau_B$=0.6fm
$\exp(-\tau/\tau_B)$, $\tau_B$=0.6fm
$(1+\tau^2/\tau_B^2)^{-3/2}$, $\tau_B$=0.6fm
ECHO−QGP
MS−1$\sigma_{LQCD}$
Vaccum
$a_1^{ch}$ (%)

Au+Au 200 GeV
STAR
$10^4 \times (H_{SS} - H_{OS})$
Centrality

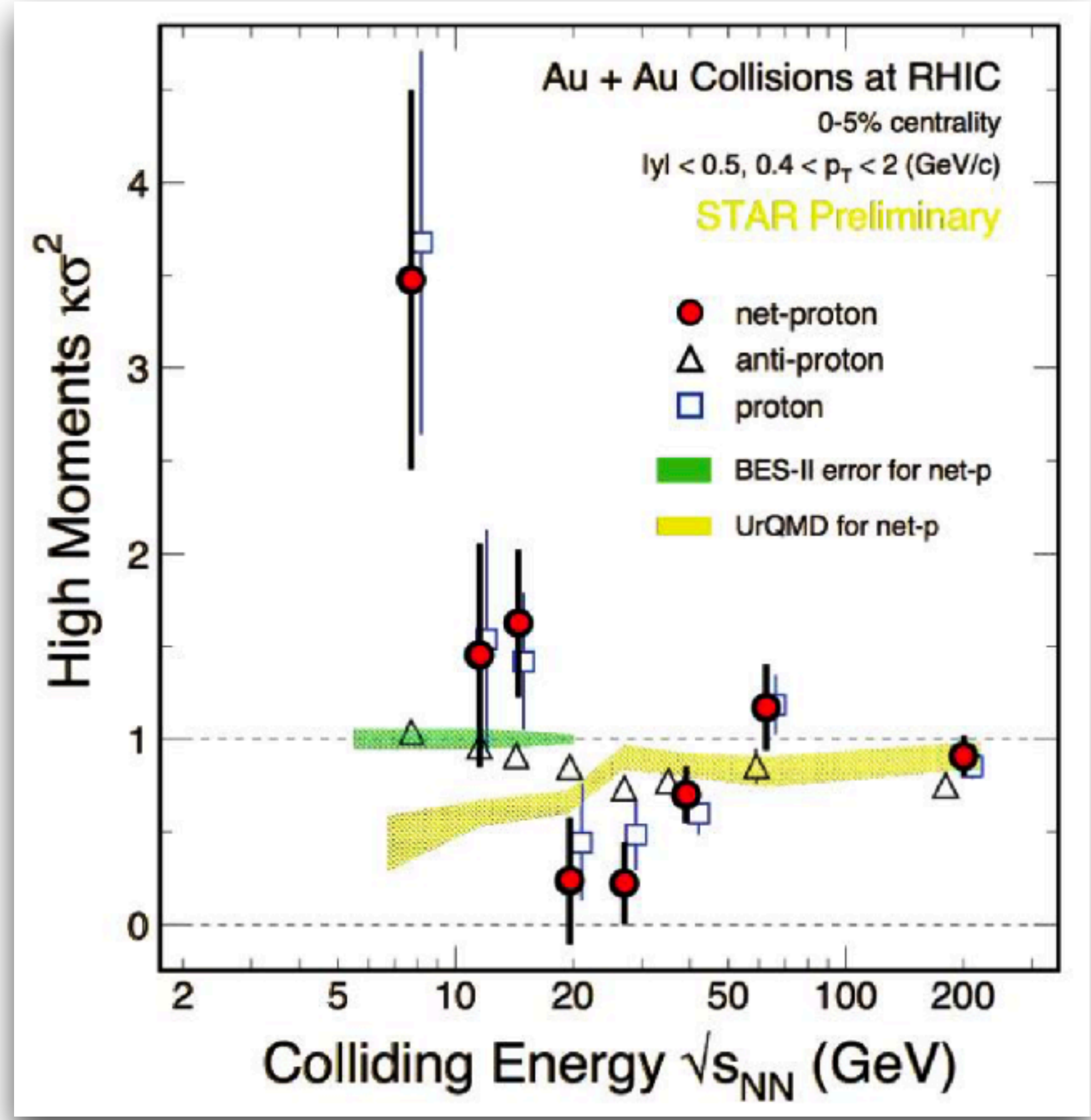
extended to event-by-event





BEST
COLLABORATION
initial conditions
3-d, conserved &
axial charges
EoS: LQCD
model critical EOS
hydrodynamic
evolution
(3+1)-d, viscous,
conserved currents
chiral anomaly
& EM fields
fluctuations
critical mode &
hydrodynamic
interpreting
expt. Data
hadronic
dynamics
explore new
phenomena

# what can we learn from these data ?




Talk by Adam Bzdak

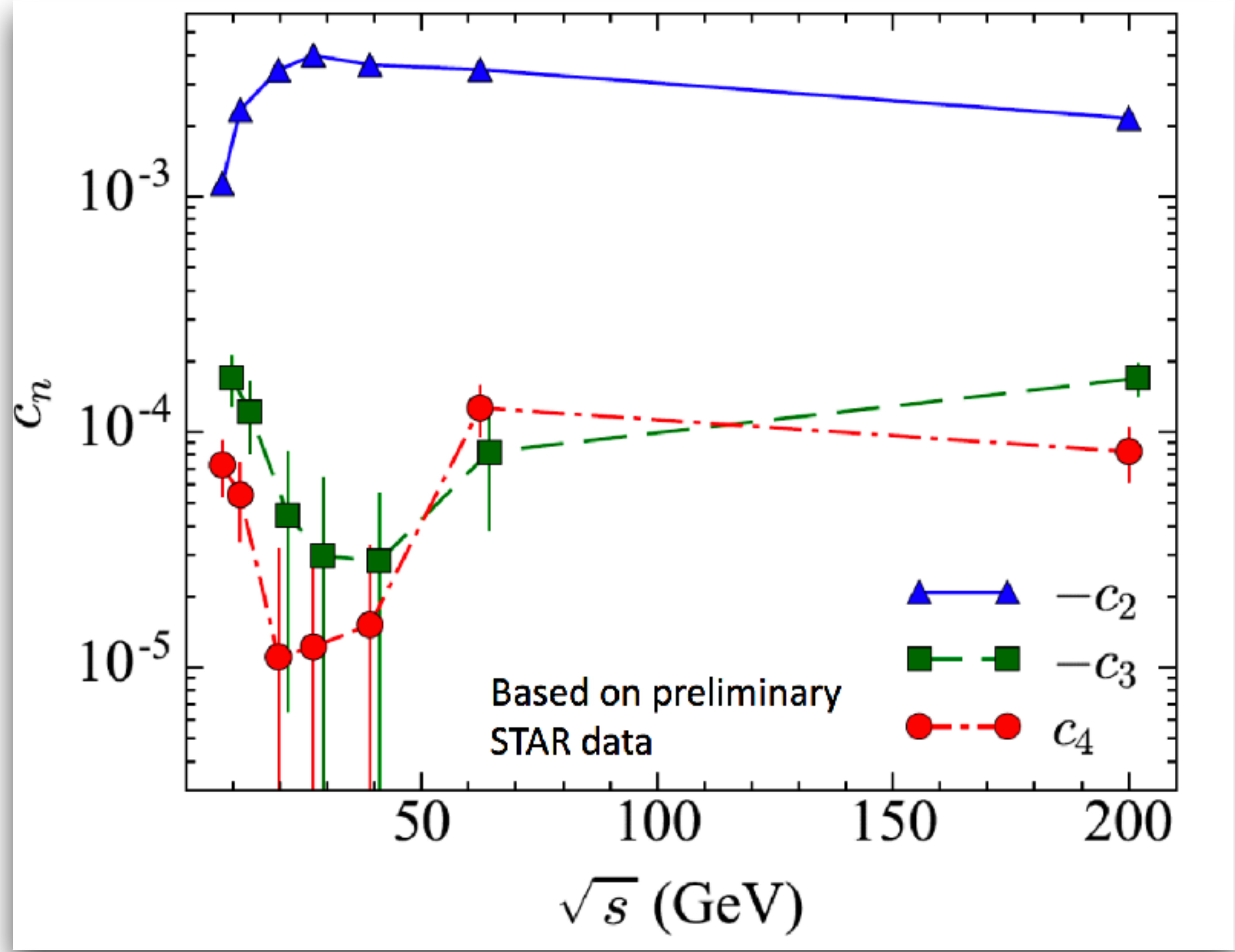
Bzdak, Koch, Strodthoff: arXiv:1607.07375
Bzdak, Koch, Skokov: arXiv:1612.05128
Bzdak, Koch: arXiv: 1707.02640
Bzdak, Koch: in progress

# from cumulants to reduced correlation functions (factorial cumulants)

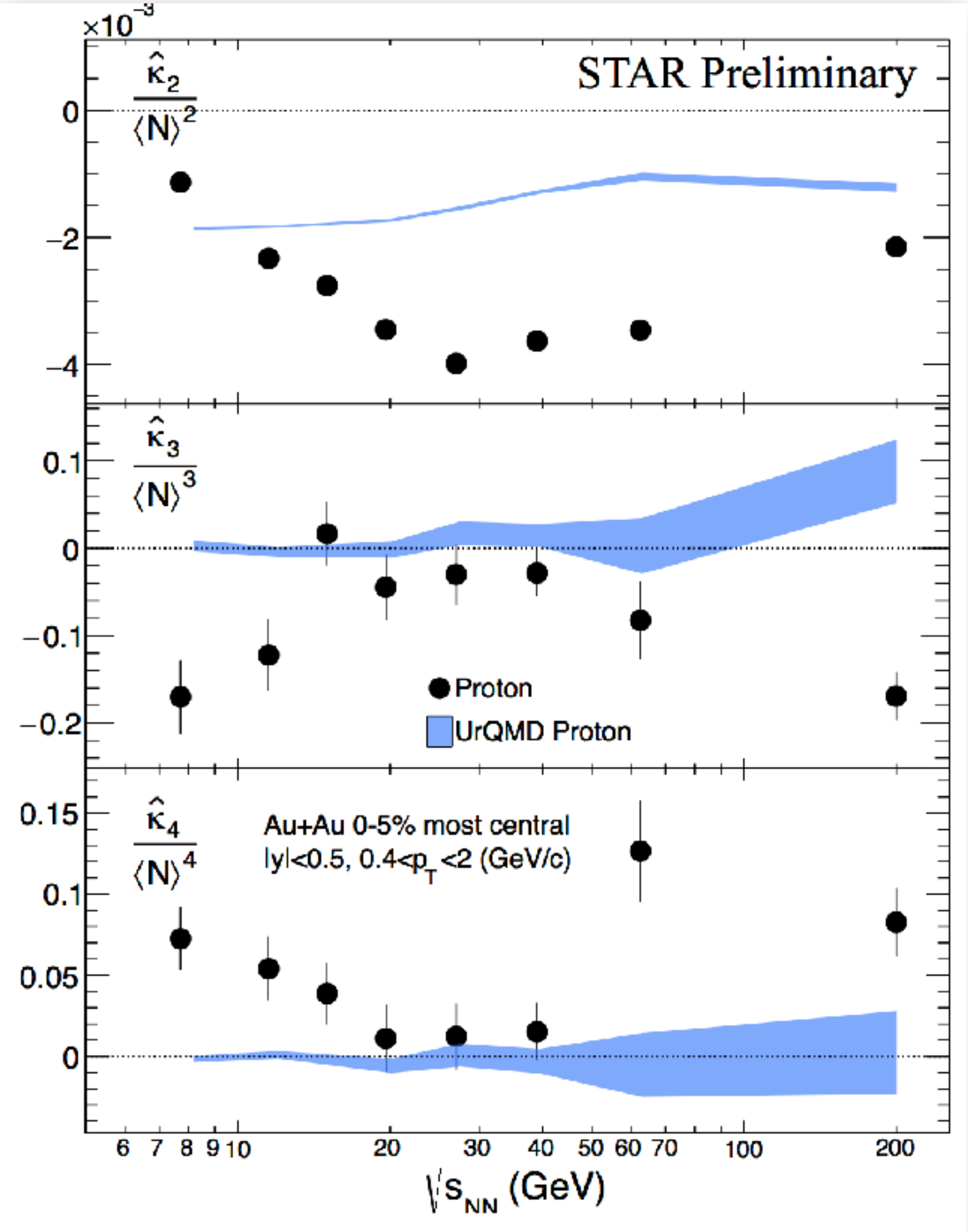
$$K_2 = \langle N \rangle + \langle N \rangle^2 c_2$$

$$K_3 = \langle N \rangle + 3\langle N \rangle^2 c_2 + \langle N \rangle^3 c_3$$

$$K_4 = \langle N \rangle + 7\langle N \rangle^2 c_2 + 6\langle N \rangle^3 c_3 + \langle N \rangle^4 c_4$$

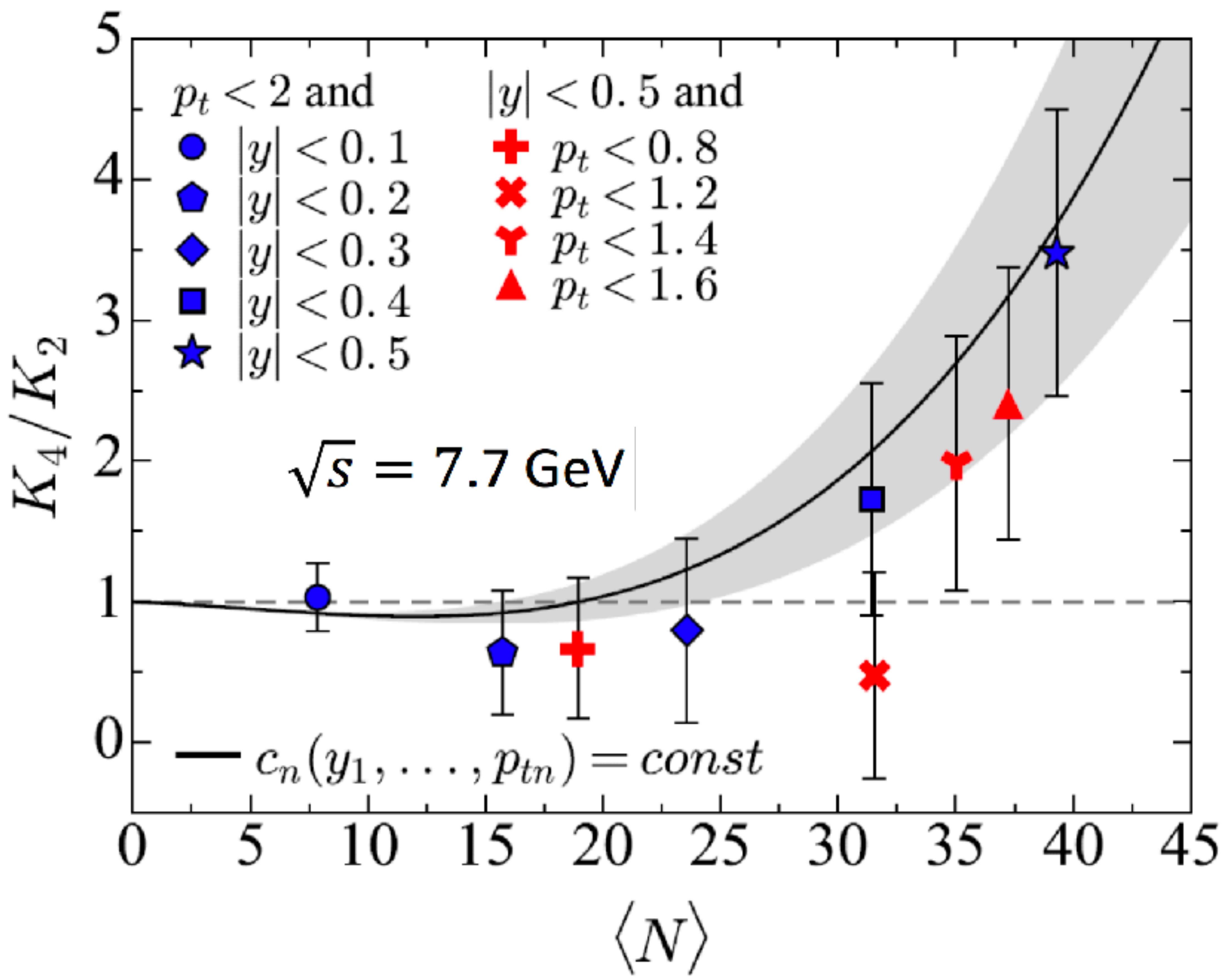



Ling, Stephanov: PRC 93 (2016) no.3, 034915
Bzdak, Koch, Strodthoff: PRC 95 (2017) no.5, 054906


×10^-3
STAR Preliminary
$\hat{\kappa}_2/\langle N\rangle^2$
$\hat{\kappa}_3/\langle N\rangle^3$
$\hat{\kappa}_4/\langle N\rangle^4$
Proton
UrQMD Proton
Au+Au 0-5% most central
|y|<0.5, 0.4<$p_T$<2 (GeV/c)
$\sqrt{s_{NN}}$ (GeV)

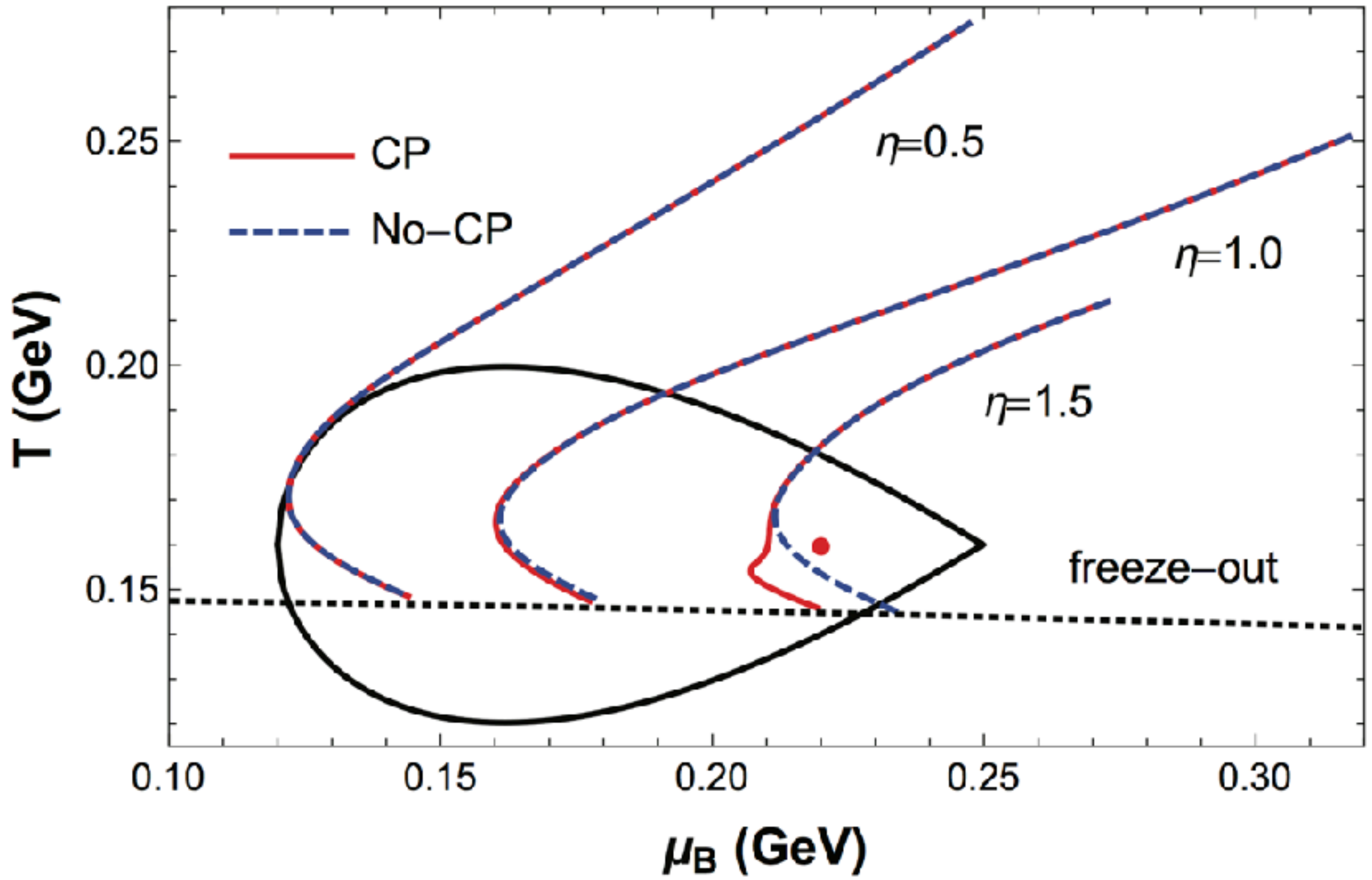
# constant correlation length ?

(over unit rapidity)



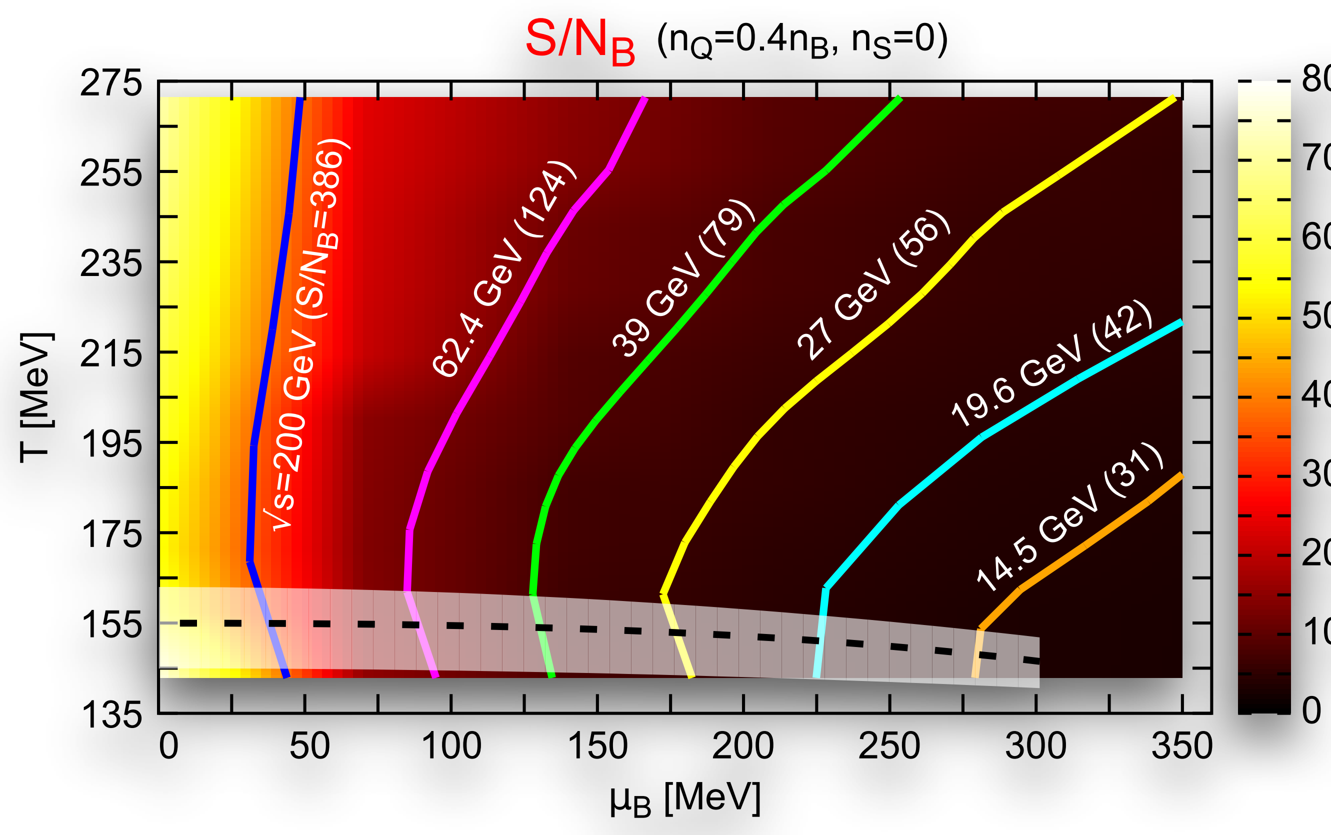
## but baryon chemical potential might also be changing with rapidity …

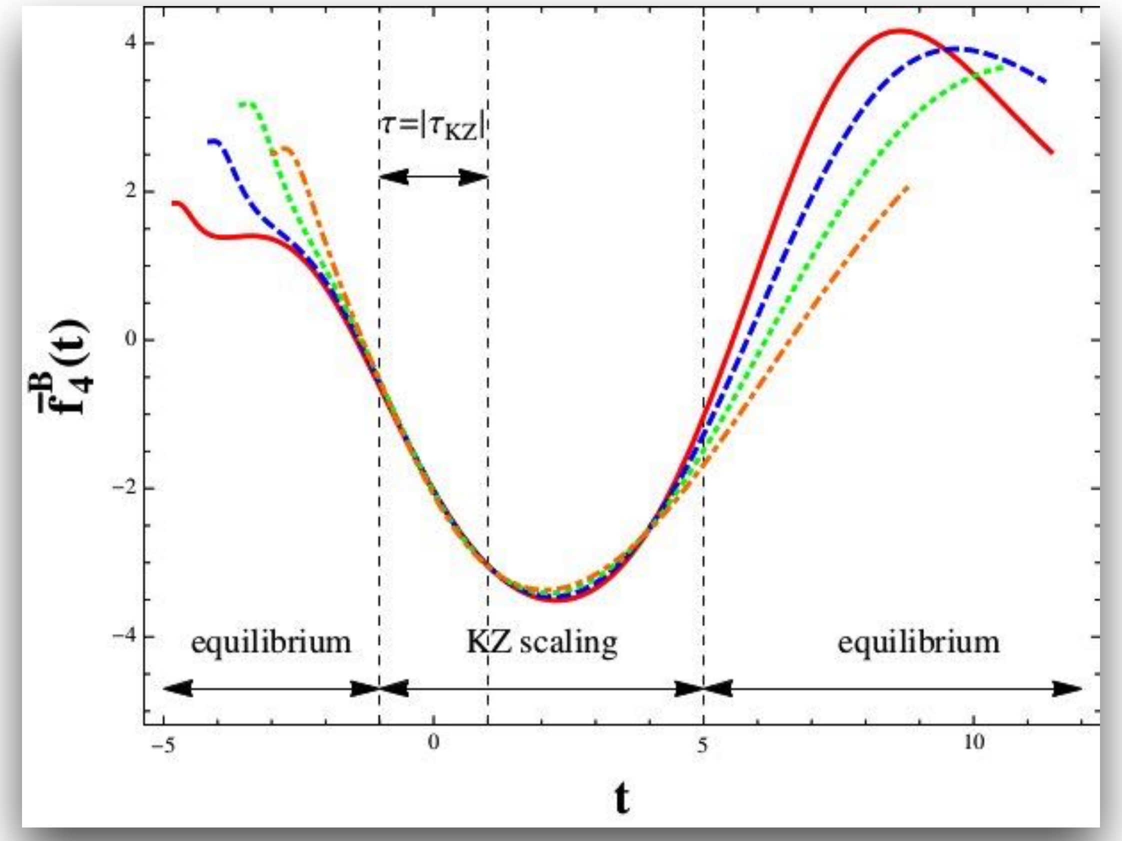
simplified example: (1+1)-d, Israel-Stewart hydro

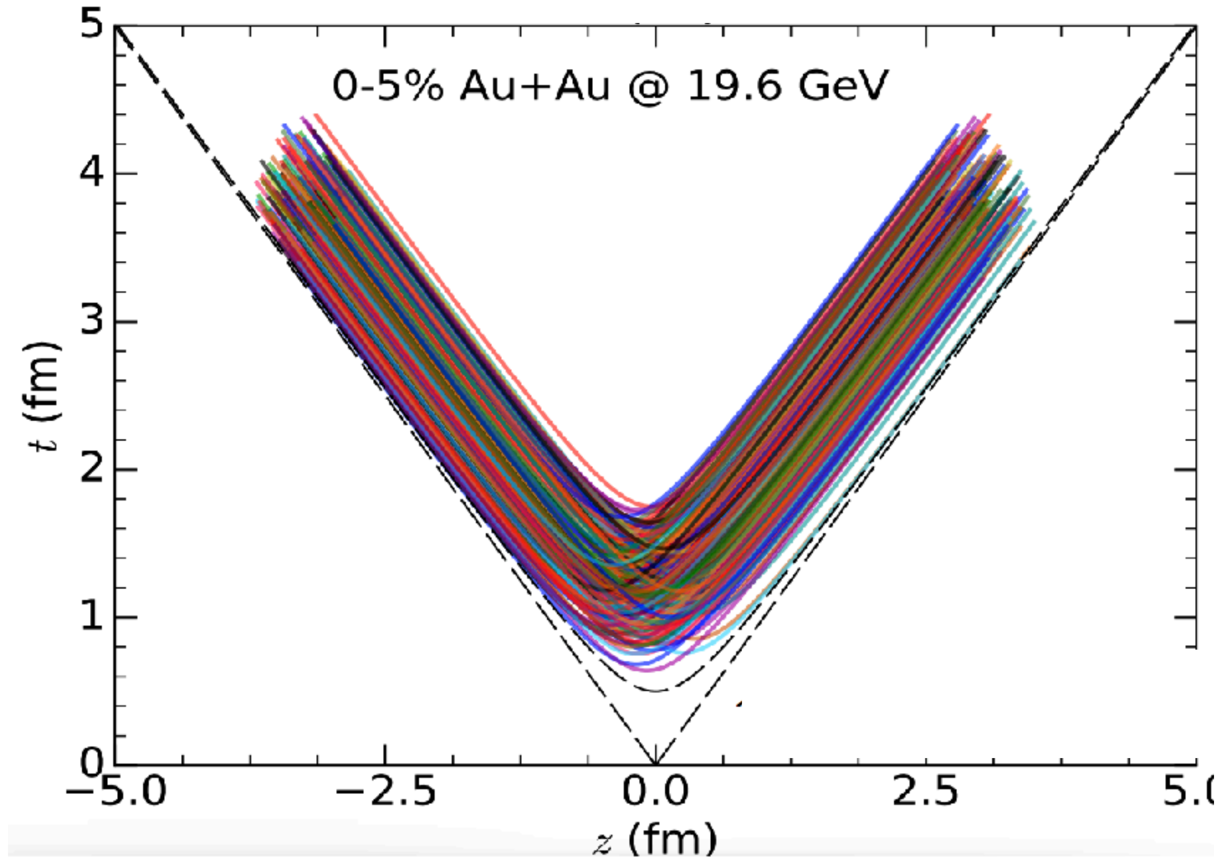


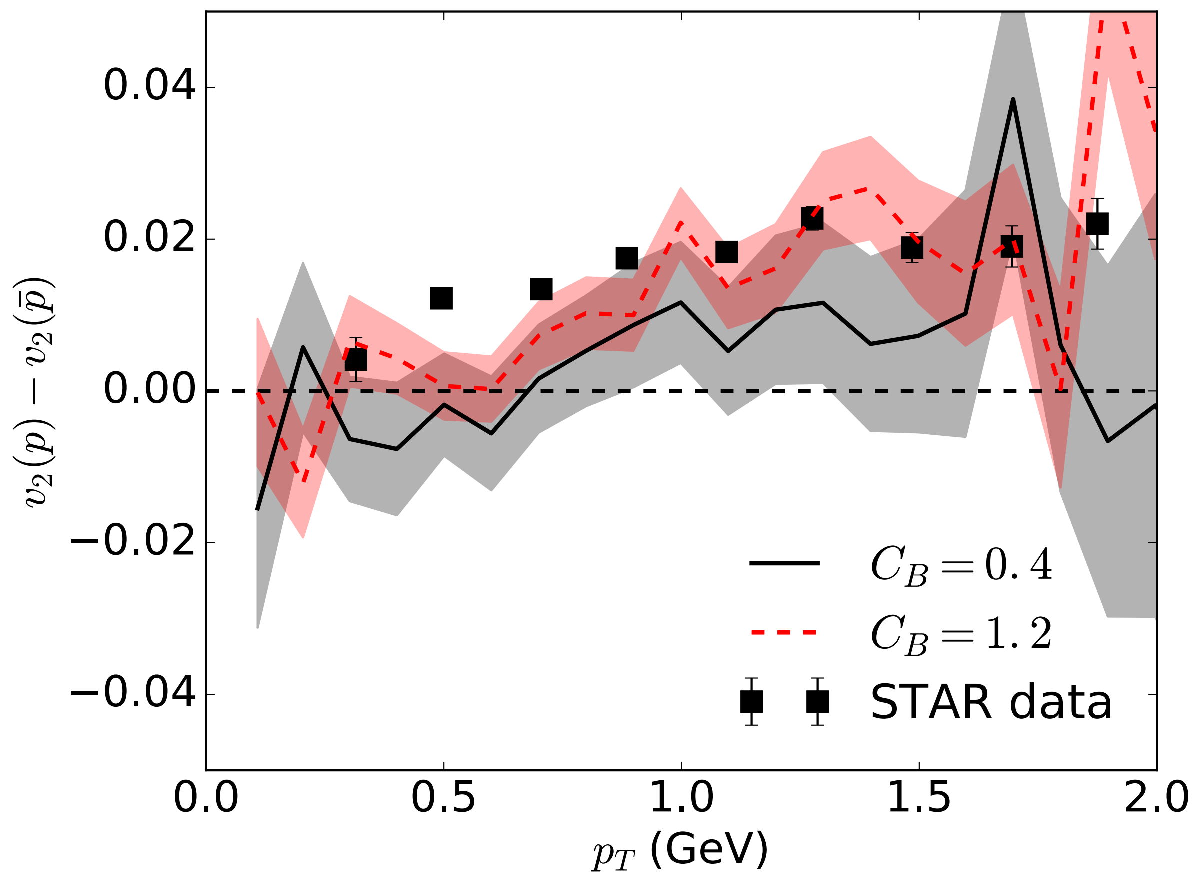

Monnai, Mukherjee, Yin: Phys. Rev. C95 no.3, 034902 (2017)

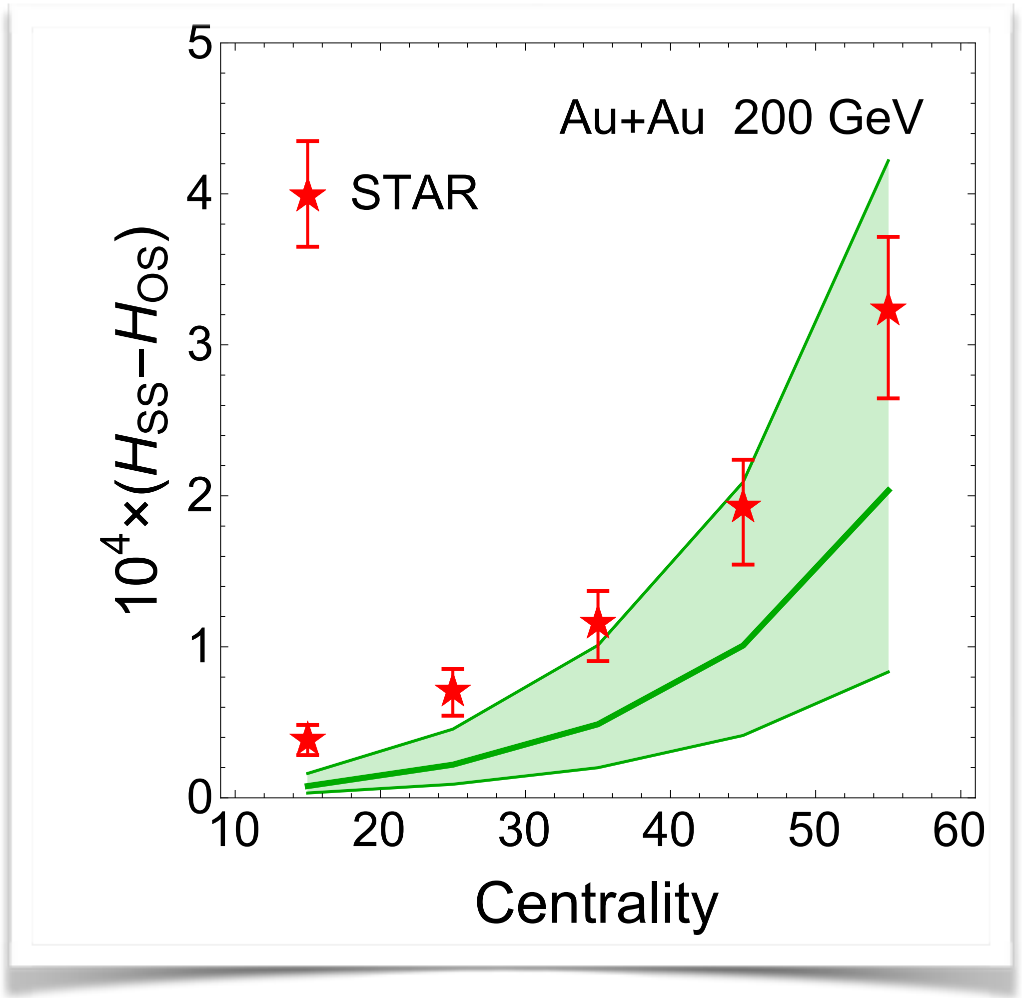
# lot of progress …

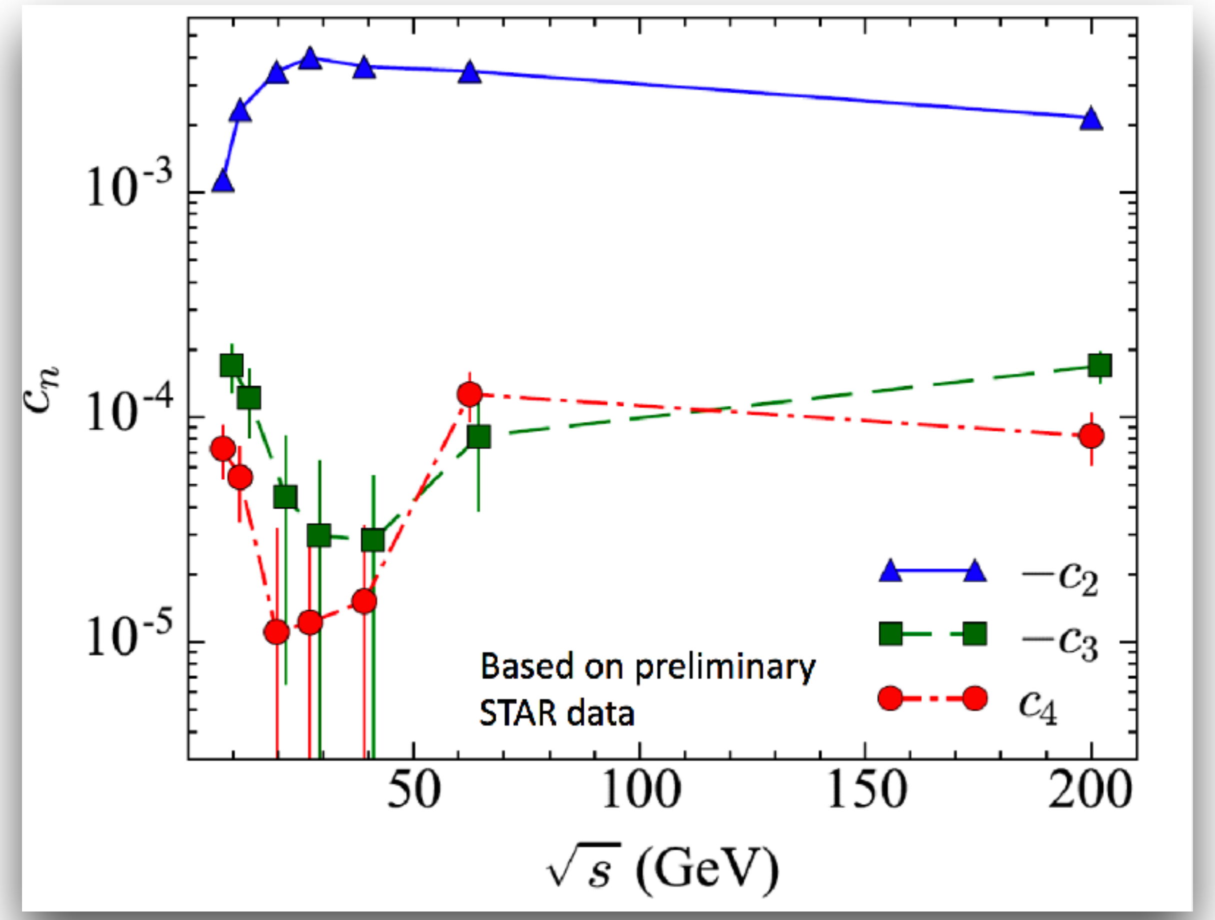


## things needed to be addressed:

- numerical implementation of Hydro+
- improve freeze-out prescription:
  - event-by-event conservation of conserve charges
  - how to freeze-out critical correlations?
- hadronic transport:
  - how to match critical correlations to hadronic transport model
  - more suitable hadronic dynamics for lowe collision energies
- early-time dynamics of chiral anomaly