Search and Discovery Statistics in HEP

Eilam Gross, Weizmann Institute of Science

This presentation would have not been possible without the tremendous help of the following people throughout many years:

Louis Lyons, Alex Read, Glen Cowan, Kyle Cranmer, Ofer Vitells & Bob Cousins
What is the statistical challenge in HEP?

- High Energy Physicists (HEP) have an hypothesis: The Standard Model.
- This model relies on the existence of the 2012 discovery of the Higgs Boson.
- The minimal content of the Standard Model includes the Higgs Boson, but extensions of the Model include other particles which are yet to be discovered.
- The challenge of HEP is to generate tons of data and to develop powerful analyses to tell if the data indeed contains evidence for the new particle, and confirm if it is the expected Higgs Boson (Mass, Spin, CP) or a member of a family of Scalar Bosons.
The Large Hadron Collider (LHC)

The LHC is a very powerful accelerator which managed to hunt a Higgs with a $10^{-12}$ production probability.

This is statistics of rare events!
Higgs Hunter’s Independence Day
July 4th 2012

From Wikipedia: On 4 July 2012, the discovery of a new particle with a mass between 125 and 127 GeV/c² was announced; physicists suspected that it was the Higgs boson. Since then, the particle has been shown to behave, interact, and decay in many of the ways predicted by the Standard Model.
The Charge of the Lectures
The Brazil Plot, what does it mean?

How do we exclude theory parameters?

Observed Limit

Bands

Expected Limit

<table>
<thead>
<tr>
<th>ATLAS Preliminary</th>
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<tbody>
<tr>
<td><strong>Obs.</strong></td>
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<tr>
<td><strong>Exp.</strong></td>
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<tr>
<td><strong>\pm 1 \sigma</strong></td>
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<td><strong>\pm 2 \sigma</strong></td>
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\[ \sqrt{s} = 7 \text{ TeV}: \int \text{Ldt} = 4.6-4.8 \text{ fb}^{-1} \]

\[ \sqrt{s} = 8 \text{ TeV}: \int \text{Ldt} = 5.8-5.9 \text{ fb}^{-1} \]
What is exclusion at the 95% CL?

99% CL?
How do we make discoveries?

The $p_0$ discovery plot, how to read it?

$p$-value

Local $p_0$

Expected $p_0$

Observed local $p_0$

Global $p_0$ and the Look Elsewhere Effect
How do we make discoveries?

The cyan band plot, what is it?

What is mu hat? \( \hat{\mu} \)
How do we estimate parameters?
Towards a measurement

Likelihoods Scans

ATLAS and CMS

LHC Run 1

$-2\ln \Lambda (m_H^{ATLAS}, m_H^{CMS})$

$H \rightarrow \gamma \gamma$

$H \rightarrow ZZ \rightarrow 4l$

Combined $\gamma \gamma + 4l$

$m_H^{ATLAS} - m_H^{CMS}$ [GeV]
How do we take uncertainties into account?

Towards a measurement

Measurements & Systematics vs Stat errors

- **ATLAS** $H \to \gamma\gamma$
- **CMS** $H \to \gamma\gamma$
- **ATLAS** $H \to ZZ \to 4l$
- **CMS** $H \to ZZ \to 4l$
- **ATLAS + CMS** $\gamma\gamma$
- **ATLAS + CMS** $4l$
- **ATLAS + CMS** $\gamma\gamma + 4l$

**ATLAS and CMS**

**LHC** Run 1

$m_H$ [GeV]
How do we estimate parameters?

Towards a measurement

2-D Likelihoods

\( \kappa_F \)

\( \kappa_V \)

ATLAS and CMS

LHC Run 1

Preliminary

- SM
- 68% CL
- Best fit
- 95% CL

ATLAS

CMS

ATLAS+CMS
How do we estimate the expected sensitivity?

The Asimov Data Set
What is the probability for a random fluctuation of the background at a mass of 750 GeV with a width of 45 GeV?
LEE
What is the probability for a random fluctuation of the background at SOME mass with SOME width anywhere in the search range?
References in the Discovery Papers

**ATLAS**


**CMS**


**RooStats**


More Refs (taken from CMS legacy Run 1 Paper)

Wilks Approximation

Wald Approximation

Wald Approximation


Feldman-Cousins
From ATLAS di-photon 750 GeV “resonance”


LEE 2D
Books


L. Lyons, Statistics for Nuclear and Particle Physics, CUP, 1986

Preliminaries

X is a random variable

Probability Distribution Function

PDF

\[ P(x \in [x, x + dx]) = f(x) \, dx \]

\[ \int_{-\infty}^{\infty} f(x) \, dx = 1 \]

f(x) is not a probability
f(x)dx is a probability

\[ G(x|\mu, \sigma) \]

Is a pdf parametrized by \((\mu, \sigma)\)

We would like to make inference about the parameters
Likelihood is NOT a PDF

A Poisson distribution describes a discrete event count \( n \) for a real valued Mean \( \mu \).

\[
Pois(n|\mu) = \mu^n \frac{e^{-\mu}}{n!}
\]

Say, we observe \( n_o \) events. What is the likelihood of \( \mu \)? The likelihood of \( \mu \) is given by

\[
L(\mu) = Pois(n_o | \mu)
\]

It is a continues function of \( \mu \) but it is NOT a PDF.
Use of Likelihood Ratio

\[ x \to y(x) \]

\[ p(y(x)|\theta) = \frac{p(x|\theta)}{|dy/dx|} \quad (prove) \]

*probabilities* are invariant under change of variable

\[ P(y(x_1) < y < y(x_2)) = P(x_1 < x < x_2) \]

\[ p(y(x)|\theta) \neq p(x|\theta) \]

*But* Likelihood ratio is invariant under change of variable \( x \) (cancellation of Jacobian)

*Likelihood* is not a PDF in \( \theta \)

\[ L(\theta) = L(u(\theta)) \]
Uniform PDF

\[ x \in (a, b) \]

\[ y(x) = \int_{a}^{x} p(x')dx' \]

then

\[ y \in (0, 1) \text{ and } p(y) \text{ is uniform for all } y \]

\[ p\text{-value} \equiv \int_{q_{obs}}^{\infty} f(q \mid H_{null})dq \]

In **statistics**, when a **p-value** is used to test a simple **null hypothesis**, and the distribution of the test statistic, \( q \), is continuous, then the p-value is uniformly distributed between 0 and 1 if the null hypothesis is true.
Bayesian vs Frequentist

Is there a Higgs Boson?

Frequentist

\[ \text{Prob}(Data | Theory) \]

Run simulations of the theory and calculate the probability on the limit of long term frequency

*Bayesia* will try to answer

\[ \text{Prob}(Theory | Data) \]

\[ i.e. \text{Prob}(Higgs | Data) \]
Bayesian vs Frequentist

• Is there a Higgs Boson? What do you mean? Given the data, is there a Higgs Boson?

• Can you really answer that without any a priori knowledge of the Higgs Boson? Change your question: What is your degree of belief in the Higgs Boson given the data… Need a prior degree of belief regarding the Higgs Boson itself…

\[
P(Higgs \mid Data) = \frac{P(Data \mid Higgs)P(Higgs)}{P(Data)} = \frac{L(Data \mid Higgs)\pi(Higgs)}{\int L(Data \mid Higgs)\pi(Higgs)d(Higgs)}
\]

• If not, make sure that when you quote your answer you also quote your prior assumption!

• The most refined question is:
  • Assuming there is a Higgs Boson with some mass \(m_H\), how well the data agrees with that? \(P(Data \mid Higgs)\)
  • But even then the answer relies on the way you measured the data (i.e. measurement uncertainties), and that might include some pre-assumptions, priors!
Histograms

$N$ collisions

$p(\text{Higgs event}) = \frac{\sigma(pp \rightarrow H) \alpha_{\text{ff}}}{\sigma(pp)}$

Prob to see $n^\text{obs}_H$ in $N$ collisions is

$P(n^\text{obs}_H) = \binom{N}{n^\text{obs}_H} \left( p^{n^\text{obs}_H} (1 - p)^{N-n^\text{obs}_H} \right)$

$\lim_{N \to \infty} P(n^\text{obs}_H) = \text{Poiss}(n^\text{obs}_H, \lambda) = \frac{e^{-\lambda} \lambda^{n^\text{obs}_H}}{n^\text{obs}_H!}$

$\lambda = Np = n^\text{exp}_H$
The Statistical Challenge of HEP

The DATA: Billions of Proton-Proton collisions which could be visualized with histograms
The searched particle mass is unknown (for the sake of this lecture)
In this TOY example, we ask if the expected background (e.g. the Standard Model WITHOUT the Higgs Boson) contains a Higgs Boson, which would manifest itself as a peak in the distribution

WE NEED TO KNOW WHAT WE SEARCH FOR…..
We need to have a model
We need to have two hypotheses if we want a powerful test
What is the statistical challenge?

- The black line represents the Standard Model (SM) expectation (Background only),
- How compatible is the data (blue) with the SM expectation (black)?
- Is there a signal hidden in this data?
- What is its statistical significance?
- What is the most powerful test statistic that can tell the SM (black) from an hypothesized signal (red)?
The Model

- The Higgs hypothesis is that of signal $s(m_H)$

$$s(m_H) = L \cdot \sigma_{SM}(m_H) \cdot A \cdot \text{eff}$$

For simplicity unless otherwise noted

- In a counting experiment

$$n = \mu \cdot s(m_H) + b$$

$$\mu = \frac{L \cdot \sigma(m_H)}{L \cdot \sigma_{SM}(m_H)} = \frac{\sigma(m_H)}{\sigma_{SM}(m_H)}$$

- $\mu$ is the strength of the signal (with respect to the expected Standard Model one)

- The hypotheses are therefore denoted by $H_\mu$

- $H_1$ is the SM with a Higgs, $H_0$ is the background only model
A Frequentist Tale of Two Hypotheses

- Test the Null hypothesis and try to reject it
- Fail to reject it OR reject it in favor of the Alternate hypothesis
The Null Hypothesis

- The Standard Model without the Higgs is an hypothesis, (BG only hypothesis) many times referred to as the null hypothesis and is denoted by $H_0$ (remember that it is the null hypothesis ONLY if we aim at a discovery)

- In the absence of an alternate hypothesis, one would like to test the compatibility of the data with $H_0$

- This is actually a **goodness of fit test**, NOT an hypothesis test
A Tale of Two Hypotheses

- Test the Null hypothesis and try to reject it
- Fail to reject it OR reject it in favor of the Alternate hypothesis
The Alternate Hypothesis?

- Let’s zoom on

\[ H_1 - \text{SM with Higgs} \]

- Higgs with a specific mass \( m_H \)
  OR
- Higgs anywhere in a specific mass-range
  - The look elsewhere effect
A Tale of Two Hypotheses

- Reject $H_0$ in favor of $H_1$ — A DISCOVERY

$H_0$: SM w/o Higgs

$H_1$: SM with Higgs
Swapping Hypotheses $\rightarrow$ exclusion

- **NULL**
  - $H_0$: SM w/o Higgs

- **ALTERNATE**
  - $H_1$: SM with Higgs

- Reject $H_1$ in favor of $H_0$

Excluding $H_1 (m_H)$ $\rightarrow$ Excluding the Higgs with a mass $m_H$
Testing an Hypothesis (wikipedia...)

- The first step in any hypothesis testing is to state the relevant null, $H_0$, and alternative hypotheses, say, $H_1$.
- The next step is to define a test statistic, $q$, under the null hypothesis.
- Compute from the observations the observed value $q_{obs}$ of the test statistic $q$.
- Decide (based on $q_{obs}$) to either fail to reject the null hypothesis or reject it in favor of an alternative hypothesis.

- next: How to construct a test statistic, how to decide?
Test statistic and p-value
The Physics Model

- SM without Higgs Background

No signal $\langle n \rangle = b$
The Physics Model

- SM without Higgs Background Only

\[ \langle n \rangle = b \]

- SM with a Higgs Boson with a mass \( m_H \)

\[ \langle n \rangle = s(m_H) + b \]
The Physics Model

\[ n = \mu s + b \]

\[ \langle \hat{\mu} \rangle = 0 \text{ under } H_0 \]

\[ \langle \hat{\mu} \rangle = 1 \text{ under } H_1 \]
The Neyman-Pearson Test Statistic

- **NP test statistic**
  \[ Q = -2 \ln \frac{L(H_0)}{L(H_1)} \]

  \[ n = \mu s + b \]

  \[ H_0; \hat{\mu} = 0, \langle n_{obs} \rangle = b \]

  \[ H_1; \hat{\mu} = 1, \langle n_{obs} \rangle = s + b \]

  \[ L(H_0) = \text{prob}(x \mid H_0) = \text{prob}(x \mid b) \]

  \[ L(H_1) = \text{prob}(x \mid H_1) = \text{prob}(x \mid s + b) \]

  \[ L = f(x), \quad L(H_{\mu}) = \text{prob}(x \mid H_{\mu}) = f(\hat{\mu}) \]
Likelihood

- Likelihood is a function of the data

\[ L(H) = L(H \, | \, x) = f(x) \]
\[ L(H \, | \, x) = P(x \, | \, H) \]
\[ q_{NP} = q_{NP}(x) = -2 \ln \frac{L(H_1 \, | \, x)}{L(H_0 \, | \, x)} \]

Bayes Theorem

- Likelihood is not the probability of the hypothesis given the data

\[ P(H \, | \, x) = \frac{P(x \, | \, H) \cdot P(H)}{\sum_H P(x \, | \, H)P(H)} \]
\[ P(H \, | \, x) \approx P(x \, | \, H) \cdot P(H) \quad \text{Prior} \]
Frequentist vs Bayesian

• The Bayesian infers from the data using priors.

    $P(H | x) \approx P(x | H) \cdot P(H)$

• Priors is a science on its own. Are they objective? Are they subjective?

• The Frequentist calculates the probability of an hypothesis to be inferred from the data based on a large set of hypothetical experiments. Ideally, the frequentist does not need priors, or any degree of belief while the Bayesian posterior based inference is a “Degree of Belief”.

• However, NPs inject a Bayesian flavour to any Frequentist analysis.
Example: Simulating BG Only Experiments

- The likelihood ratio, $-2\ln q(m_H)$ tells us how much the outcome of an experiment is signal-like.
- NOTE, here the s+b pdf is plotted to the left (it’s the null hypothesis).
- Test the s+b hypothesis $f(q^{NP} \mid b)$

\[ q_{NP}(m) = \frac{L(H_1)}{L(H_0)} = \frac{L(s(m) + b)}{L(b)} \]
Example:
Simulating $S(m_H) + b$ Experiments

$$f(q^{NP} \mid s(m_H) + b)$$

Test mass = 115 GeV

Signal+bkg. Experiment 5

<table>
<thead>
<tr>
<th>Events/3 GeV</th>
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<td>45</td>
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Reconstructed mass

s+b like

b-like

ESHEP September 2017
Example:
Simulating $S(m_{H}) + b$ Experiments

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Eilam Gross Statistics in PP

ESHEP September 2017
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p-Value

- The observed $p$-value is a measure of the incompatibility of the data with the tested hypothesis.
- It is the probability, under assumption of the null hypothesis $H_{null}$, of finding data of equal or greater incompatibility with the predictions of $H_{null}$.
- Wikipedia: An important property of a test statistic is that its sampling distribution under the null hypothesis must be calculable, either exactly or approximately, which allows $p$-values to be calculated.
PDF of a test statistic

\[ f(q \mid \text{null}) \quad f(q \mid \text{alt}) \]

Reject alt

Reject null

Null like

alt like
PDF of a test statistic

\[ f(q | \text{null}) \]

\[ f(q | \text{alt}) \]

\[ p = \int_{q_{obs}}^{\infty} f(q_{null} \mid H_{null}) \, dq_{null} \]

**p-value:**
The probability, under assumption of the null hypothesis \( H_{null} \), of finding data of equal or greater incompatibility with the predictions of \( H_{null} \).
Basic Definitions: type I-II errors

- By defining $\alpha$ you determine your tolerance towards mistakes… (accepted mistakes frequency)
- **type-I error**: the probability to reject the tested (null) hypothesis ($H_0$) when it is true

$$\alpha = \Pr(\text{reject } H_0 \mid H_0)$$

$\alpha$ = type I error

- Type II: The probability to accept the null hypothesis when it is wrong

$$\beta = \Pr(\text{accept } H_0 \mid \bar{H}_0) = \Pr(\text{reject } H_1 \mid H_1)$$

$\beta$ = type II error

- The pdf of $q$….
Basic Definitions: POWER

- \( \alpha = \text{Prob}(\text{reject } H_0 \mid H_0) \)
- The POWER of an hypothesis test is the probability to reject the null hypothesis when the alternate analysis is true!

- \( \beta = \text{Prob}(\text{reject } H_1 \mid H_1) \Rightarrow 
\quad 1 - \beta = \text{Prob}(\text{accept } H_1 \mid H_1) = 
\quad 1 - \beta = \text{Prob}(\text{reject } H_0 \mid H_1) \Rightarrow \)
\[
\text{POWER} = 1 - \beta
\]
- The power of a test increases as the rate of type II error decreases
Which Analysis is Better

- To find out which of two methods is better, plot the p-value vs the power for each analysis method.

- Given the p-value, the one with the higher power is better.

- p-value ~ significance

Different Test Statistics Differ by POWER and Asymptotic Properties

\[ \alpha = \text{p-value} \quad 1 - \beta = \text{power} \]
The Neyman-Pearson Lemma

- Define a test statistic \( \lambda = \frac{L(H_1)}{L(H_0)} \)

- When performing a hypothesis test between two simple hypotheses, \( H_0 \) and \( H_1 \), the Likelihood Ratio test, which rejects \( H_0 \) in favor of \( H_1 \), is the most powerful test of size \( \alpha \) for a threshold \( \eta \)

- Note: Likelihoods are functions of the data, even though we often not specify it explicitly
p-value – testing the null hypothesis

- When testing the null hypothesis, the p-value is the probability that the observation is less compatible with the null hypothesis (more alternative like) than the observed one.

- When testing the b hypothesis, it is custom to set $\alpha = 2.9 \times 10^{-7}$ and it is custom that if $p_b < 2.9 \times 10^{-7}$ the b hypothesis is rejected.

- When testing the s+b hypothesis, set $\alpha = 5\%$ if $p_{s+b} < 5\%$ the signal hypothesis is rejected at the 95% Confidence Level (CL) → Exclusion.
From p-values to Gaussian significance

It is a custom to express the p-value as the significance associated to it, had the pdf were Gaussians

\[ p = \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = 1 - \Phi(Z) \]

\[ Z = \Phi^{-1}(1 - p) \]

A significance of \( Z = 5 \) corresponds to \( p = 2.87 \times 10^{-7} \)

Beware of 1 vs 2-sided definitions!
2-Sided p-value

- When performing a measurement, any deviation above or below the mean is drawing our attention and might serve an indication of some anomaly or new physics.
- Here we use a 2-sided p-value
1-Sided p-value

- When trying to reject an hypothesis while performing searches, one usually considers only one-sided tail probabilities.
- Downward fluctuations of the background will not serve as an evidence against the background.
- Upward fluctuations of the signal will not be considered as an evidence against the signal.
1-sided 2-sided

- To determine a 1 sided 95% CL, we sometimes need to set the critical region to 10% 2 sided
Estimating the Sensitivity of an Experiment

- Estimate the expected significance one could achieve (for discovering the Higgs Boson) with a given analysis, a given Luminosity and CM energy.

- Option 1:
  - Toss, say, 1000000 BG only events (null) and derive the BG-only pdf of q, \( f(q_{null} \mid BG) \).
  - Toss 1000000 S+BG (alt) events and find the significance for each one of them. Find the median significance.

- This may take ages..., is there a shortcut?

\[\leftrightarrow\rightarrow\text{Option 2: Asimov Data Set}\]
In the future, the United States has converted to an "electronic democracy" where the computer Multivac selects a single person to answer a number of questions. Multivac will then use the answers and other data to determine what the results of an election would be, avoiding the need for an actual election to be held.
The Asimov Data Set

- The use of a single representative individual to stand in for the entire population can help in evaluating the sensitivity of a statistical method.

- The "Asimov data set": an ensemble of simulated experiments can be replaced by a single representative one.
Option 2: the Asimov Data Set

- one can replace each ensemble of the alternate-hypothesis experiments with one data set that represents the typical experiment. This “Asimov” data set delivers the desired median sensitivity. Hence, one is exempted from the need to perform an ensemble of experiments for each set of parameters.

- The Asimov data set is constructed such that when one uses it to evaluate the estimators for all parameters, one obtains the true parameter values.

- the Asimov data set can trivially be constructed from the true parameters values. For example, a set corresponding to the $H_1$ hypothesis is $n_A = s + b$ and the one correspond to the $H_0$ hypothesis is $n_A = b$.

- As strange as it reads, the Asimov data set is not necessarily an integer.
The Profile Likelihood

The choice of the LHC for hypothesis inference in Higgs search
The Profile Likelihood ("PL")

For discovery we test the $H_0$ null hypothesis

\[ q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu}s + b)} \]

For

- $\hat{\mu} \sim 0, q_0$ small
- $\hat{\mu} \sim 1, q_0$ large

In general: testing the $H_\mu$ hypothesis i.e., a SM with a signal of strength $\mu$,

\[ q_\mu = -2 \ln \frac{L(\mu)}{L(\hat{\mu})} \]

The best signal $\hat{\mu} = 0.3 \rightarrow 1.27 \sigma$

\[ Z_{obs} = \sqrt{q_{0,obs}} \]

$q_0 = 1.6 \rightarrow Z = \sqrt{1.6} = 1.27$
PL: test $q_0$ under BG only; $f(q_0 | H_0)$

$$\hat{\mu} = 0.15 \rightarrow 0.6\sigma$$

$$q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu}s + b)}$$

$$Z_{obs} = \sqrt{q_{0,obs}}$$

$q_0 = 0.43 \rightarrow Z = 0.66\sigma$
PL: test $q_0$ under BG only; $f(q_0 | H_0)$

\[
\hat{\mu} = 0
\]

\[
q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu}s + b)}
\]

\[
Z_{obs} = \sqrt{q_{0,obs}}
\]

$q_0 = 0$
PL: test $q_0$ under BG only; $f(q_0 \mid H_0)$

$$\hat{\mu} = 0$$

$$q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu} s + b)}$$

$$Z_{obs} = \sqrt{q_{0, obs}}$$

$q_0 = 0$
PL: test $q_0$ under BG only; $f(q_0 | H_0)$

$$\hat{\mu} = 0.6 \rightarrow 2.6\sigma$$

$$q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu} s + b)}$$

$$Z_{obs} = \sqrt{q_{0,\text{obs}}}$$

$q_0 = 6.76 \rightarrow Z = 2.6\sigma$
PL: test $q_0$ under BG only; $f(q_0 | H_0)$

$$\hat{\mu} = 0.22 \rightarrow 1.1\sigma$$

$$q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu}s + b)}$$

$$Z_{obs} = \sqrt{q_{0,obs}}$$

$q_0 = 1.2 \rightarrow Z = 1.1\sigma$
PL: test $q_0$ under BG only; $f(q_0 | H_0)$

$$\hat{\mu} = 0.11 \rightarrow 0.4\sigma$$

$$q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu}s + b)}$$

$$Z_{obs} = \sqrt{q_0, obs}$$

$q_0 = 0.16 \rightarrow Z = 0.4\sigma$
PL: test $q_0$ under BG only; $f(q_0 | H_0)$

\[ \hat{\mu} = 0.31 \rightarrow 1.35\sigma \]

\[ q_0 = -2\ln \frac{L(b)}{L(\hat{\mu}s + b)} \]

\[ Z_{obs} = \sqrt{q_{0,obs}} \]

$q_0 = 1.8 \rightarrow Z = 1.35\sigma$
PL: test $q_0$ under BG only; $f(q_0 | H_0)$

$$\mu = 0.32 \rightarrow 1.39\sigma$$

$$q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu}s + b)}$$

$$Z_{\text{obs}} = \sqrt{q_{0,\text{obs}}}$$

$q_0 = 1.9 \rightarrow Z = 1.39\sigma$
PL: test $q_0$ under BG only; $f(q_0 | H_0)$

$$q_0 = -2 \ln \frac{L(b)}{L(\mu s + b)}$$

$\hat{\mu} = 0.15 \rightarrow 0.66\sigma$

$$Z_{obs} = \sqrt{q_0,obs} \quad q_0 = 0.43 \rightarrow Z = 0.66\sigma$$
Confirm Wilks Theorem

For the test statistic

\[ q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu}s + b)} \]

\[ f(q_0 \mid H_0) = \chi^2_1 \]
**Chi Squared Distribution**

In probability theory and statistics, the **chi-squared distribution** (also chi-square or \( \chi^2 \) distribution) with \( k \) degrees of freedom is the distribution of a sum of the squares of \( k \) independent standard normal random variables.

\[
\chi_k^2 = \sum_i^k \frac{(x_i - \mu_i)^2}{\sigma_i^2}
\]

\( q_0 \) distributes like a \( \chi^2 \)

\[
\sqrt{q_{0,\text{obs}}} = Z
\]

\[
\chi^2 = \frac{(x - \mu)^2}{\sigma^2} = Z^2
\]
The PDF of $q_0$ under s+b experiments ($H_1$)

$$q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu}s + b)} = -2 \ln \frac{L(b \mid H_1)}{L(\hat{\mu}s + b \mid H_1)}$$

$\hat{\mu} = 1.04 \rightarrow 4.3\sigma$

$q_0 = 18.5 \rightarrow Z = 4.3\sigma$
The PDF of $q_0$ under s+b experiments ($H_1$)

$$q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu}s + b)} = -2 \ln \frac{L(b | H_1)}{L(\hat{\mu}s + b | H_1)}$$

$$\hat{\mu} = 0.83 \rightarrow 3.6\sigma$$

$$q_0 = 12.9 \rightarrow Z = 3.6\sigma$$
The PDF of $q_0$ under s+b experiments ($H_1$)

$$q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu}s + b)} = -2 \ln \frac{L(b \mid H_1)}{L(\hat{\mu}s + b \mid H_1)}$$

$\hat{\mu} = 1.22 \rightarrow 5.0\sigma$

$$q_0 = 25 \rightarrow Z = 5.0\sigma$$
Median sensitivity in a Click (Asimov)

Franchise (short story)

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Franchise is a science fiction short story by Isaac Asimov. It first appeared in the August 1955 issue of the magazine The magazine was reprinted in the collections Earth is Room Enough (1957) and Robot Dreams (1986). It is one of a loose series of stories about a fictional computer called Multivac. It is the first story in which Asimov dealt with computers as computers are.

Plot summary

In the future, the United States has converted to an "electronic democracy" where the computer Multivac sets the dates on which elections are to be held.

The story centers around Norman Muller, the man chosen as "Voter of the Year" in 2008. Although the law requires him to be sure that he wants the responsibility of representing the entire electorate, worrying that the result will be unrepresentative. However, after 'voting', he is very proud that the citizens of the United States had, through him, "exercised control", a statement that is somewhat ironic as the citizens didn't actually get to vote.

The idea of a computer predicting whom the electorate would vote for instead of actually holding an election was a correct prediction of the result of the 1952 election.

Influence

The use of a single representative individual to stand in for the entire population can help in evaluating the sensitivity of a statistical method. Franchise was cited as the inspiration of the data set", where an ensemble of simulated experiments can be replaced by a single representative one. [1]

References

The Median Sensitivity (via ASIMOV)

To estimate the median sensitivity of an experiment (before looking at the data), one can either perform lots of $s+b$ experiments and estimate the median $t_{o,med}$ or evaluate $t_0$ with respect to a representative data set, the ASIMOV data set with $\mu=1$, i.e. $x=s+b$

$$q_{o,med} \approx q_0 (\hat{\mu} = \mu_A = 1) = -2 \ln \frac{L(b \mid x = x_A = s + b)}{L(\hat{\mu}s + b \mid x = x_A = s + b)} = -2 \ln \frac{L(b)}{L(1 \cdot s + b)}$$

$$\hat{\mu} = 1.00 \rightarrow 4.15\sigma$$

$$\mu = 1 \rightarrow Z_A = 4.15$$

$$\sqrt{q_0, A} = 4.15\sigma$$
Tossing Monte Carlos to get the test statistic distribution functions (PDF) is sometimes beyond the experiment technical capability.

Knowing both PDF

\[ f(q_{null} \mid H_{null}) \]

\[ f(q_{null} \mid H_{alternate}) \]

enables calculating both the observed and expected significance (or exclusion) without a single toy…. 