# Search and Discovery Statistics in HEP

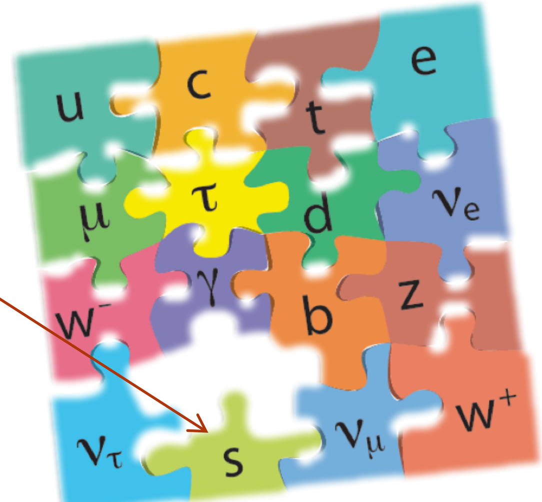
**Eilam Gross**, Weizmann Institute of Science

This presentation would have not been possible without the tremendous help of the following people throughout many years

Louis Lyons, Alex Read, Glen Cowan ,Kyle Cranmer
Ofer Vitells & Bob Cousins

# What is the statistical challenge in HEP?

- High Energy Physicists (HEP) have an hypothesis: The Standard Model.
- This model relies on the existence of the 2012 discovery of the Higgs Boson
- The minimal content of the Standard Model includes the Higgs Boson , but extensions of the Model include other particles which are yet to be discovered




- The challenge of HEP is to generate tons of data and to develop powerful analyses to tell if the data indeed contains evidence for the new particle, and confirm if it is the expected Higgs Boson (Mass, Spin, CP) or a member of a family of Scalar Bosons

# The Large Hadron Collider (LHC)

The LHC is a very powerful accelerator which managed to hunt a Higgs with a $10^{-12}$ production probability

This is statistics of rare events!

# Higgs Hunter's Independence Day
# July 4th 2012

*From Wikipedia: On 4 July 2012, the discovery of a new particle with a mass between 125 and 127 GeV/c2 was announced; physicists suspected that it was the Higgs boson. Since then, the particle has been shown to behave, interact, and decay in many of the ways predicted by the Standard Model.*

# The Charge of the Lectures

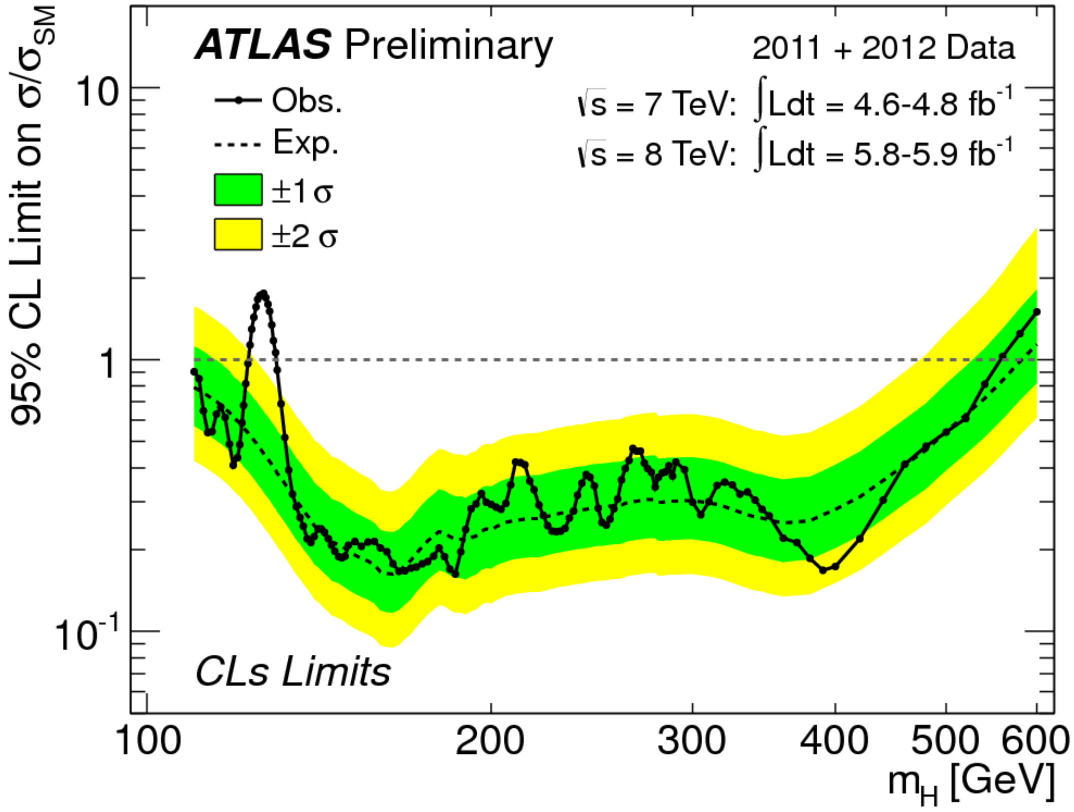
# The Brazil Plot, what does it mean?

## How do we exclude theory parameters?

Observed Limit

Bands

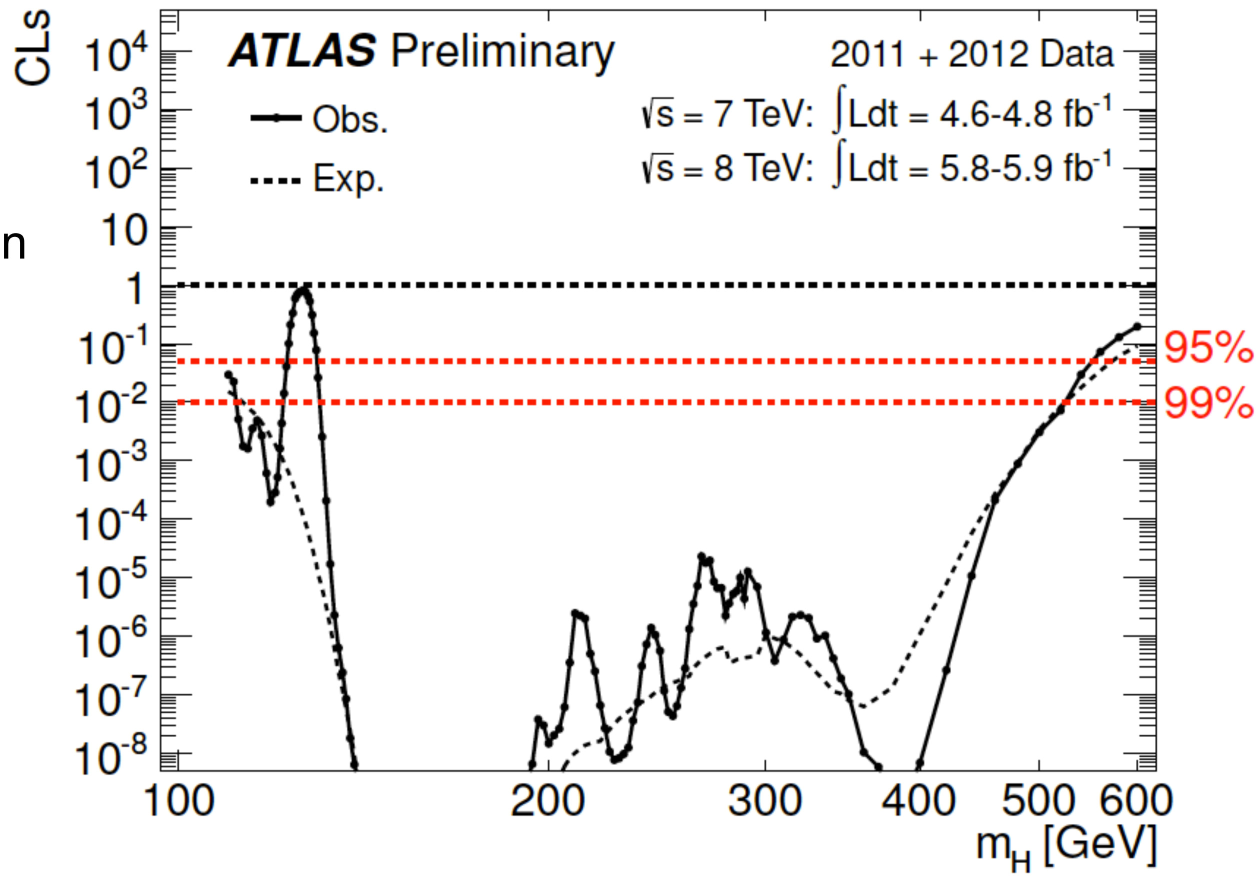
Expected Limit



# What the ---- CLs?

What is exclusion at the 95% CL?

99% CL?



# How do we make discoveries?

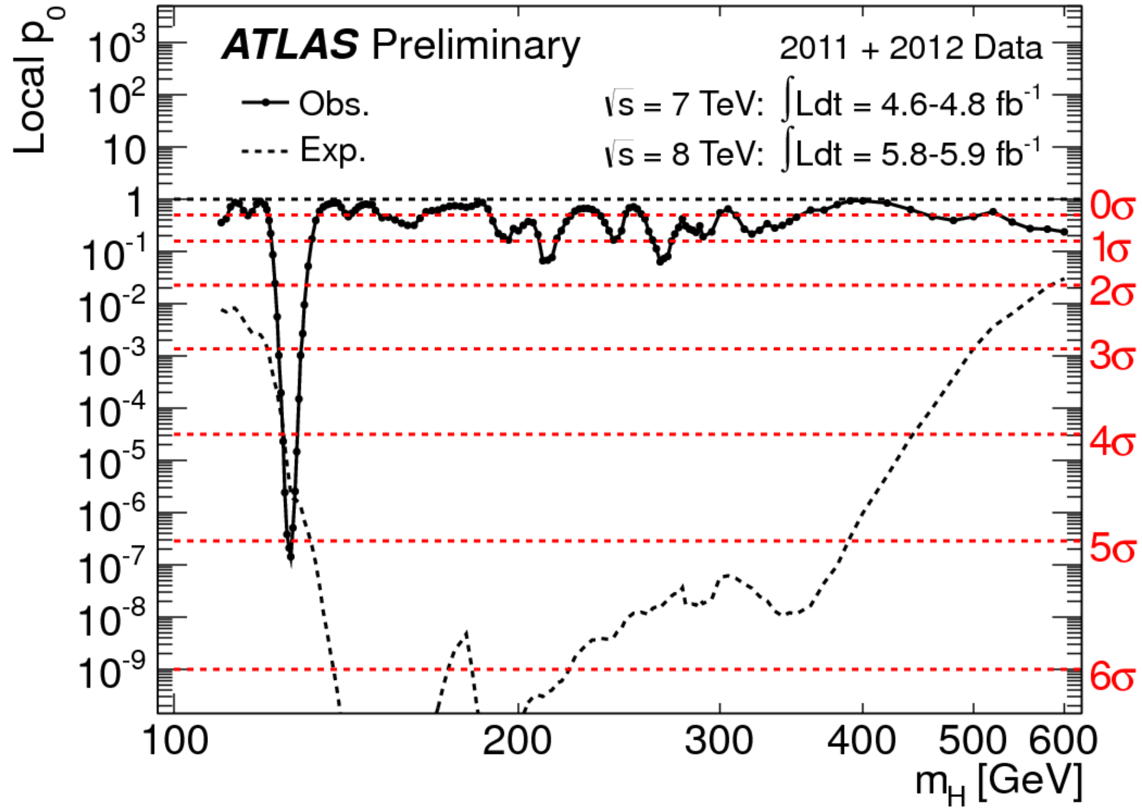
## The p0 discovery plot, how to read it?

p-value

Local p0

Expected p0

Observed local p0

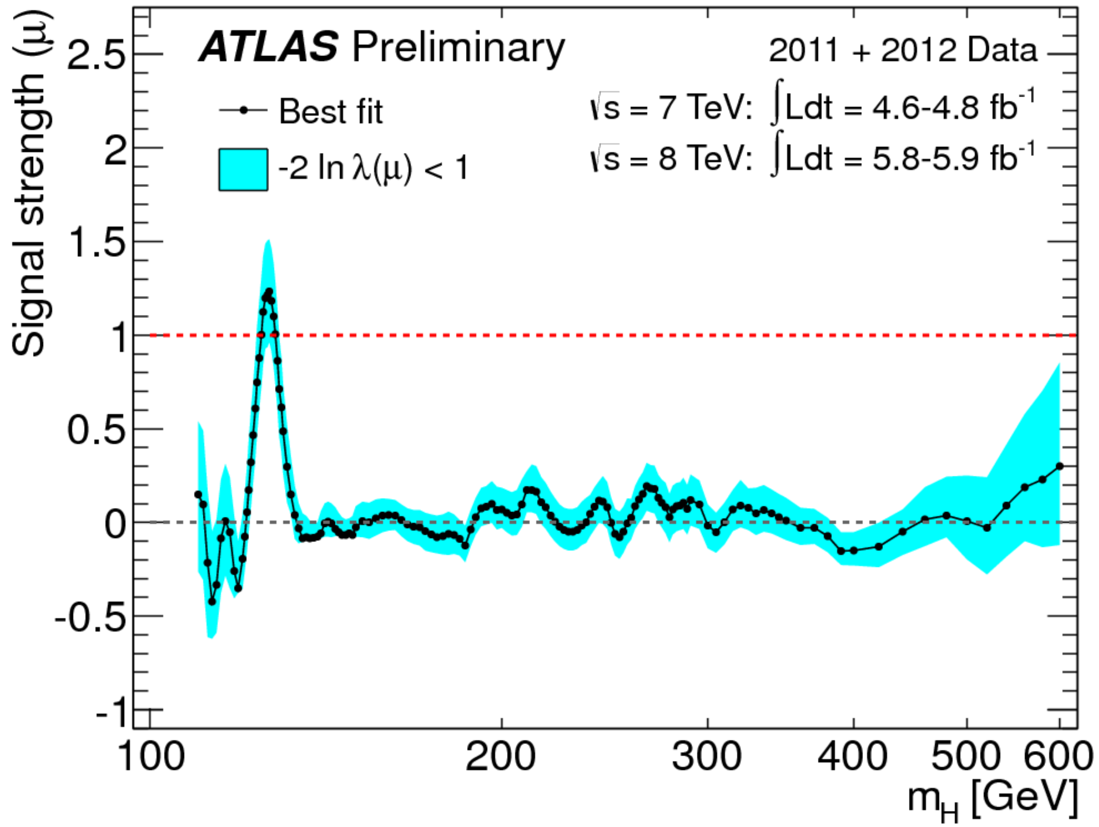



Global p0 and the Look Elsewhere Effect

# How do we make discoveries?

## The cyan band plot, what is it?

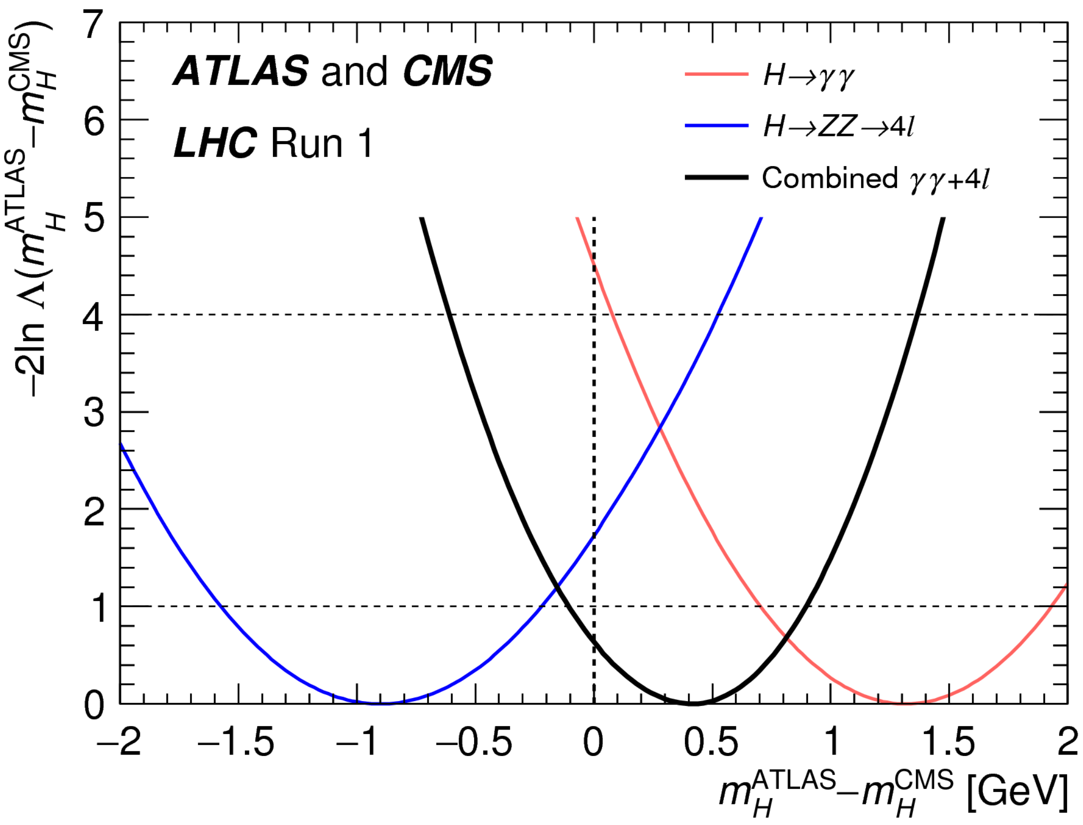
What is mu hat? $\hat{\mu}$



# How do we estimate parameters?

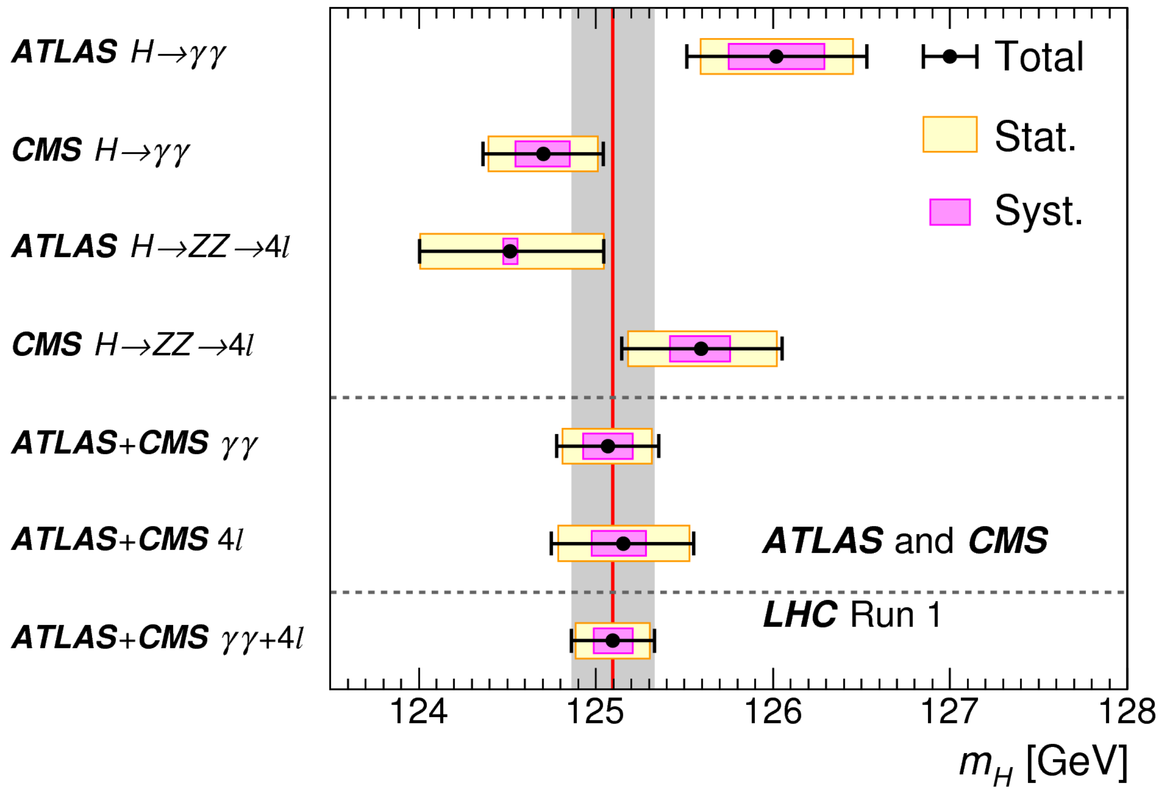
# Towards a measurement

Likelihoods Scans



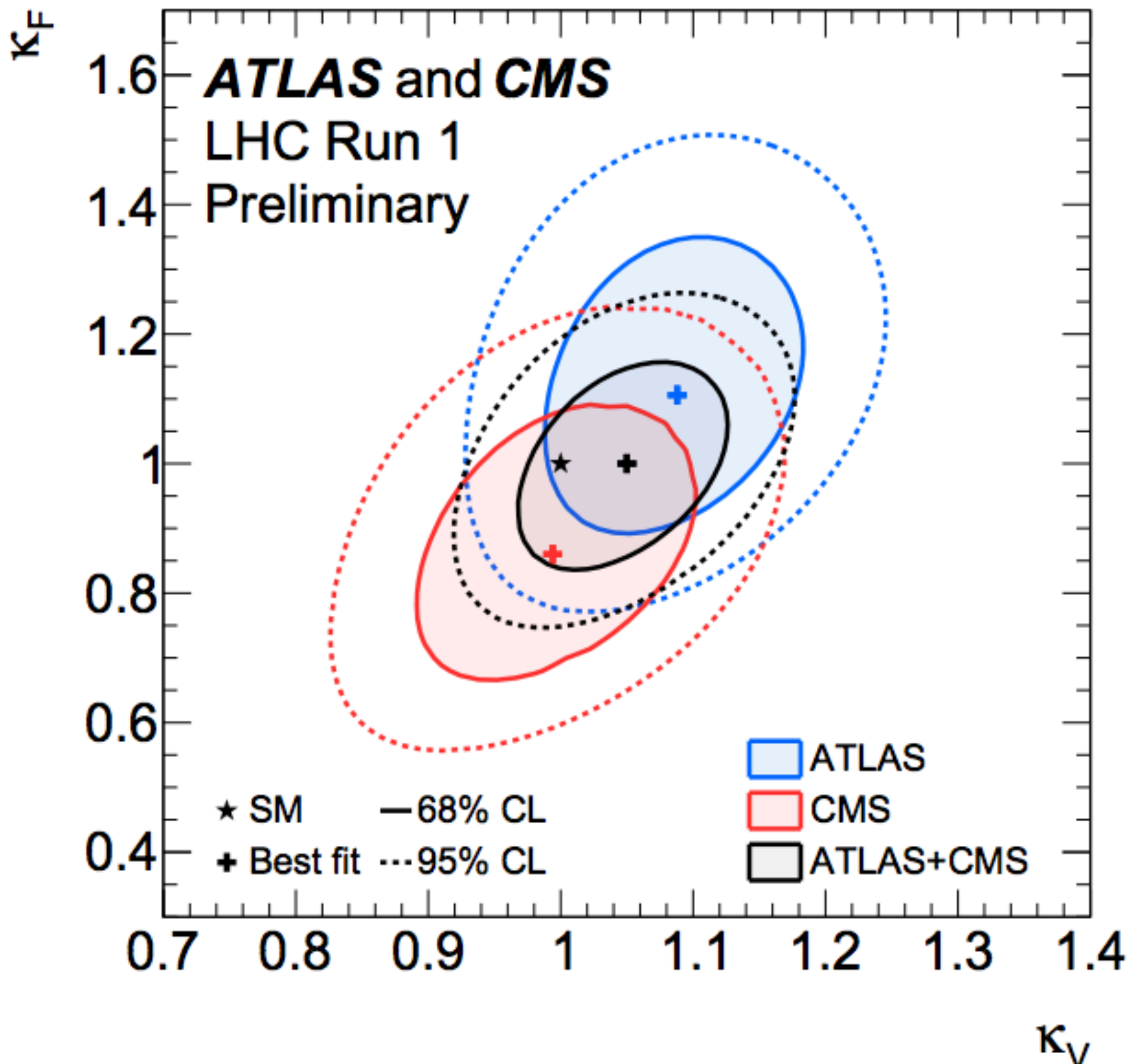
# Towards a measurement

Measurements & Systematics vs Stat errors



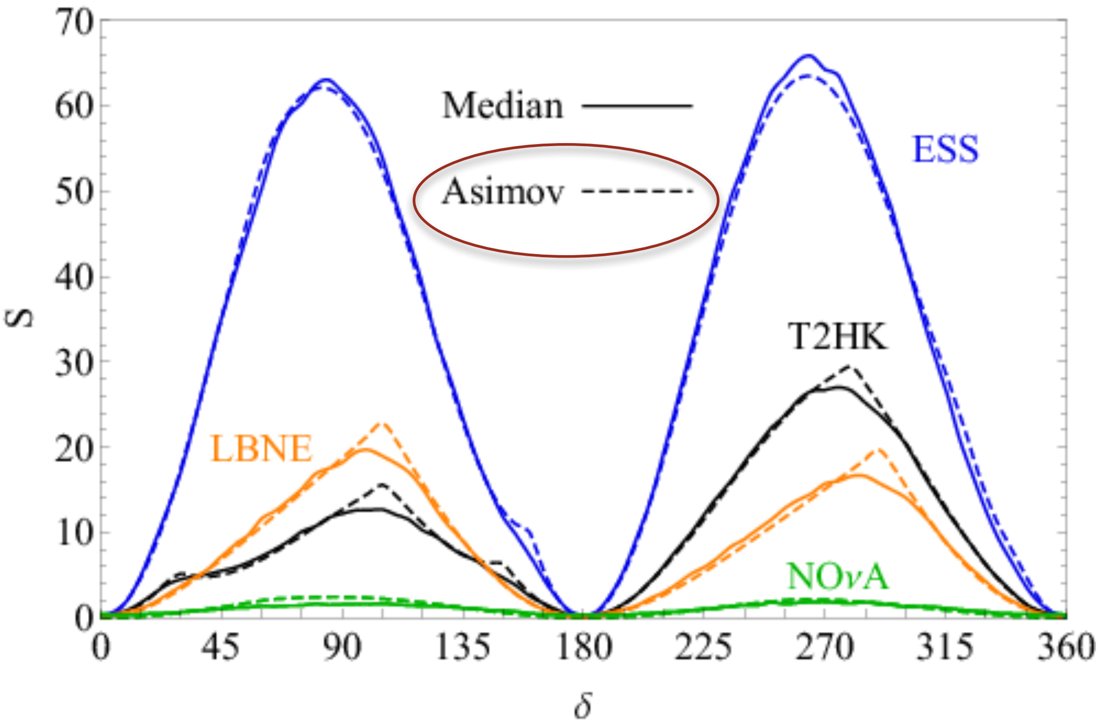
# Towards a measurement

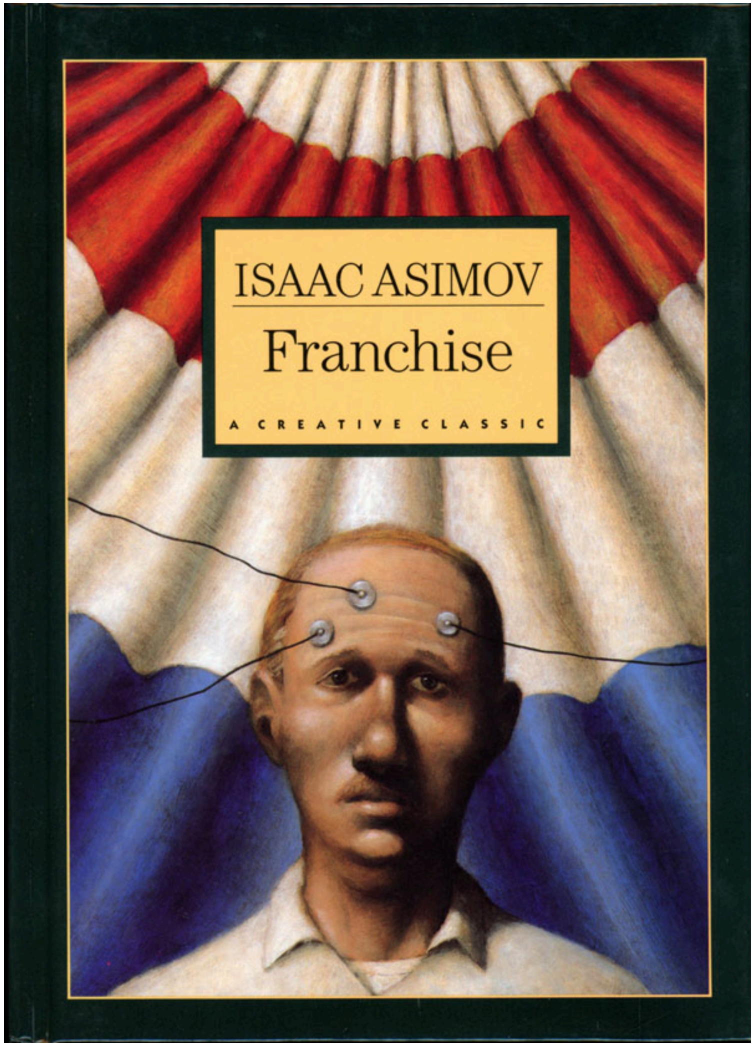
2-D Likelihoods



# How do we estimate the expected sensitivity?

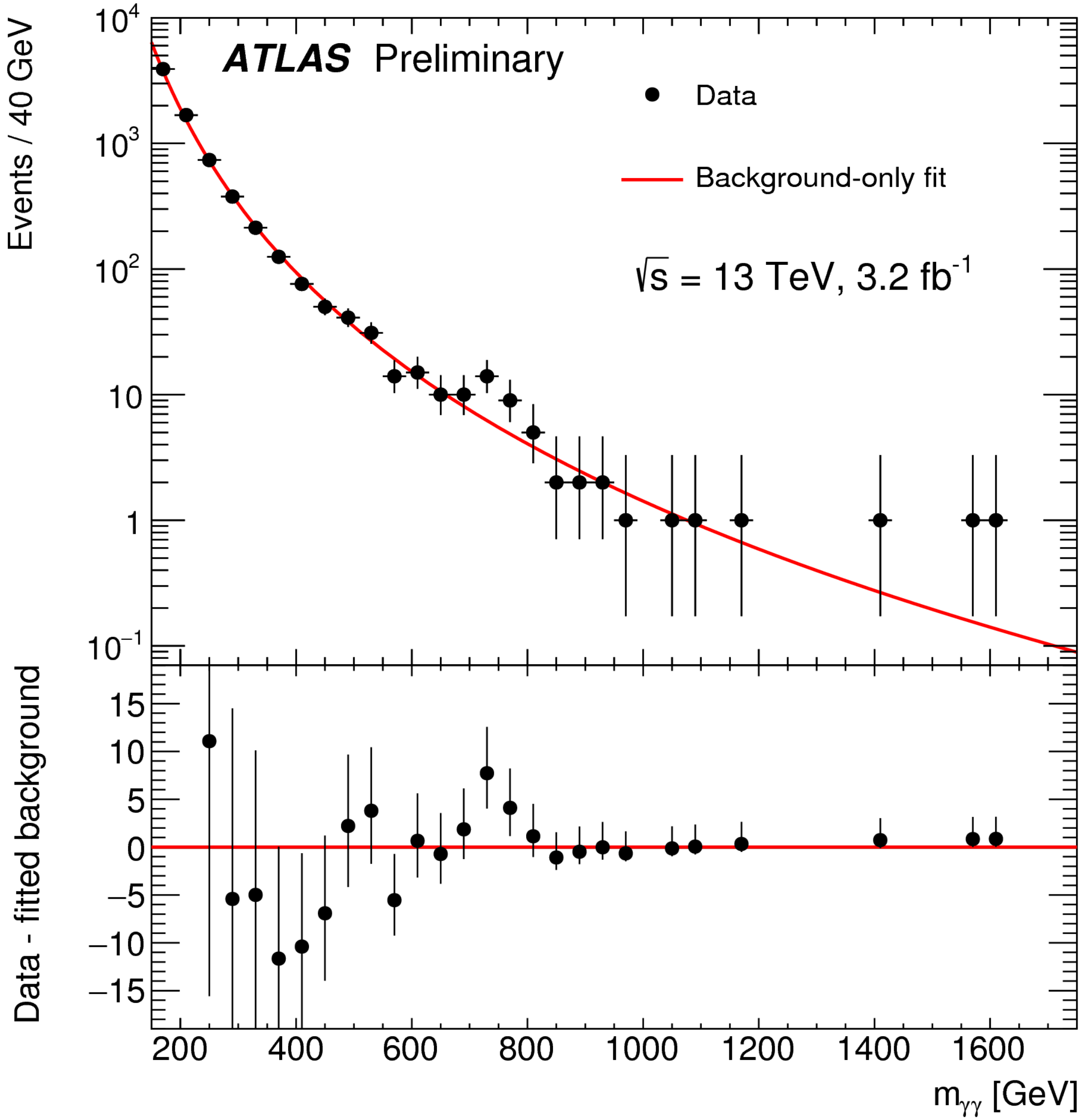
## The Asimov Data Set



# The Look Elsewhere Effect in 1 and 2D

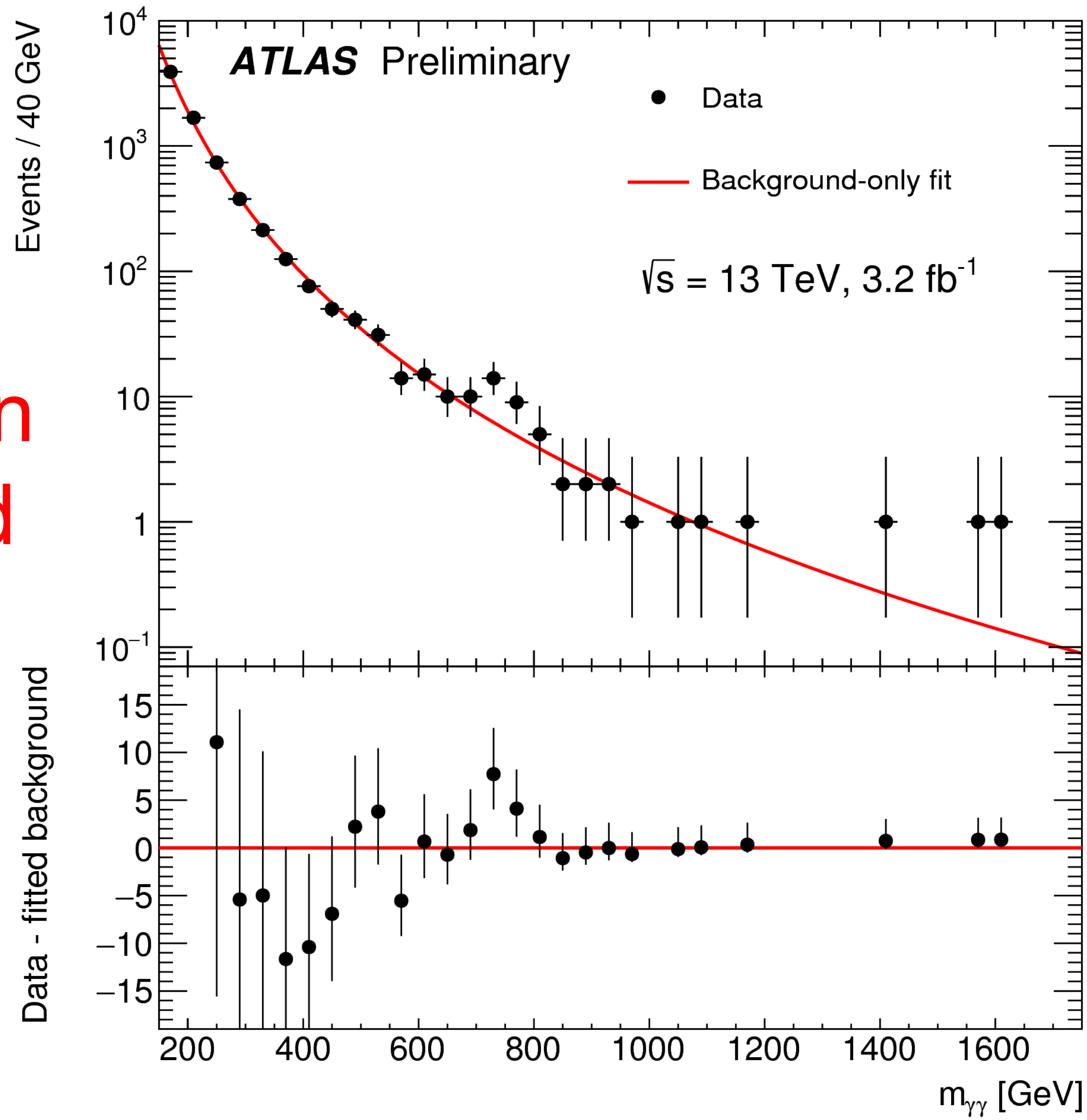
What is the probability for a random fluctuation of the background at a mass of 750 GeV with a width of 45 GeV ?



# The Look Elsewhere Effect in 1 and 2D

LEE

What is the probability for a random fluctuation of the background at SOME mass with SOME width anywhere in the search range?



# References in the Discovery Papers

ATLAS


PL [26] G. Cowan, K. Cranmer, E. Gross and O. Vitells, *Asymptotic formulae for likelihood-based tests of new physics*, Eur. Phys. J. **C71** (2011) 1554. CCGV

CLs [27] A. L. Read, *Presentation of search results: The CL(s) technique*, J. Phys. **G28** (2002) 2693–2704.

LEE [28] E. Gross and O. Vitells, *Trial factors for the look elsewhere effect in high energy physics*, Eur. Phys. J. **C70** (2010) 525–530.


CMS


PL [90] G. Cowan et al., "Asymptotic formulae for likelihood-based tests of new physics", *Eur. Phys. J. C* **71** (2011) 1–19, doi:10.1140/epjc/s10052-011-1554-0, arXiv:1007.1727. CCGV

RooStats [91] Moneta, L. et al., "The RooStats Project", in *13th International Workshop on Advanced Computing and Analysis Techniques in Physics Research (ACAT2010)*. SISSA, 2010. arXiv:1009.1003. PoS(ACAT2010)057.

CLs [92] T. Junk, "Confidence level computation for combining searches with small statistics", *Nucl. Instrum. Meth. A* **434** (1999) 435–443, doi:10.1016/S0168-9002(99)00498-2.

CLs [93] A. L. Read, "Presentation of search results: the CLs technique", *J. Phys. G: Nucl. Part. Phys.* **28** (2002) 2693, doi:10.1088/0954-3899/28/10/313.

LEE [94] Gross, E. and Vitells, O., "Trial factors for the look elsewhere effect in high energy physics", *Eur. Phys. J. C* **70** (2010) 525–530, doi:10.1140/epjc/s10052-010-1470-8, arXiv:1005.1891.

# More Refs (taken from CMS legacy Run 1 Paper)

Wilks Approximation

[186] S. S. Wilks, "The Large-Sample Distribution of the Likelihood Ratio for Testing Composite Hypotheses", *Ann. Math. Statist.* **9** (1938) 60, doi:10.1214/aoms/1177732360.

Wald Approximation

[187] A. Wald, "Tests of statistical hypotheses concerning several parameters when the number of observations is large", *Trans. Amer. Math. Soc.* **54** (1943) 426, doi:10.1090/S0002-9947-1943-0012401-3.

Wald Approximation

[188] R. F. Engle, "Chapter 13 Wald, likelihood ratio, and Lagrange multiplier tests in econometrics", in *Handbook of Econometrics*, Z. Griliches and M. D. Intriligator, eds., volume 2, p. 775. Elsevier, 1984. doi:10.1016/S1573-4412(84)02005-5.

[189] G. J. Feldman and R. D. Cousins, "Unified approach to the classical statistical analysis of small signals", *Phys. Rev. D* **57** (1998) 3873, doi:10.1103/PhysRevD.57.3873, arXiv:physics/9711021.

Feldman-Cousins

# From ATLAS di-photon 750 GeV "resonance"

[41] O. Vitells and E. Gross, *Estimating the significance of a signal in a multi-dimensional search*, Astropart. Phys. **35** (2011) 230, arXiv: 1105.4355 [astro-ph.IM].

LEE 2D

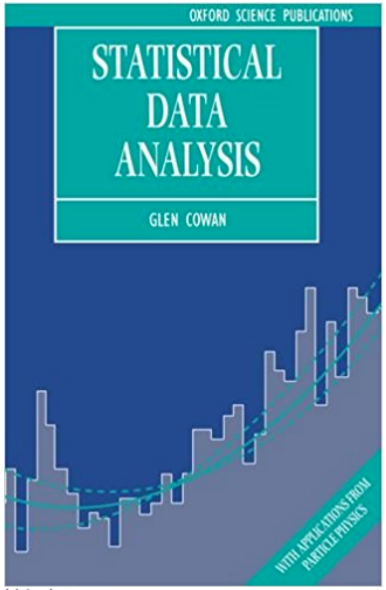
# Books

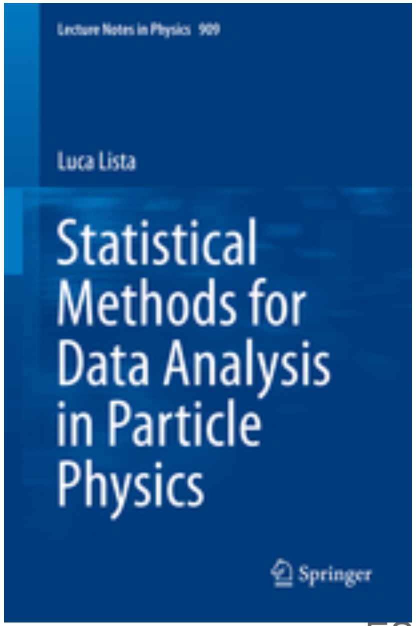
G. Cowan, Statistical Data Analysis, Clarendon Press, Oxford, 1998.

R.J.Barlow, A Guide to the Use of Statistical Methods in the Physical Sciences, John Wiley, 1989;

L.Lyons, Statistics for Nuclear and Particle Physics, CUP, 1986

L. Lista Statistical methods for Data Analysis, Springer, 2015



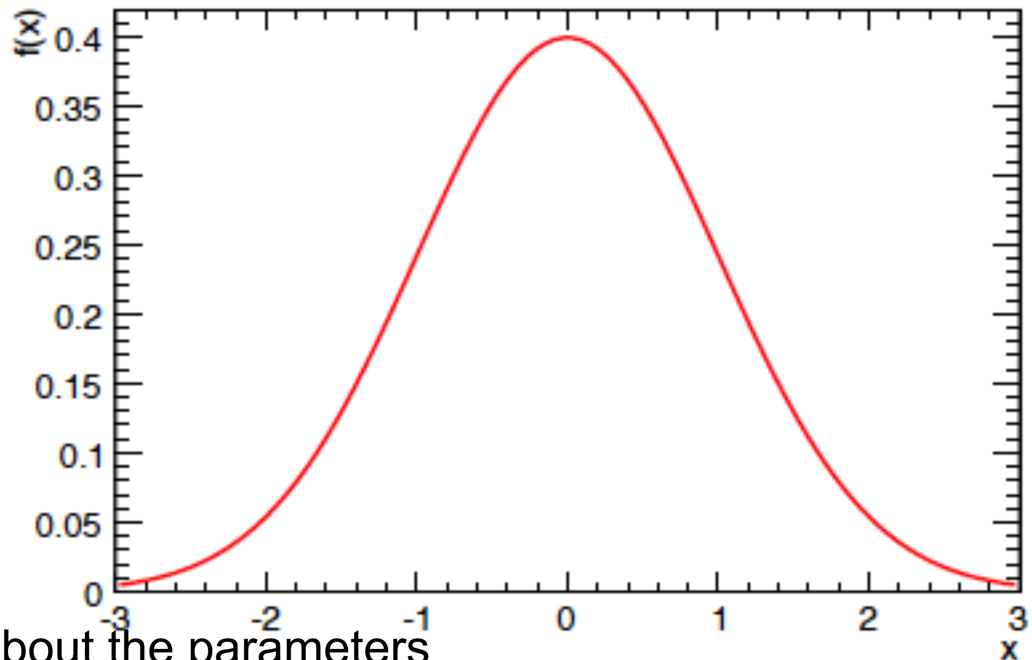



# Preliminaries

X is a random variable
Probability Distribution Function
PDF

$$P(x \in [x, x+dx]) = f(x)dx$$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$




f(x) is not a probability
f(x)dx is a probability

$$G(x|\mu, \sigma)$$

Is a pdf parametrized by $(\mu, \sigma)$

We would like to make inference about the parameters

# Likelihood is NOT a PDF

A Poisson distribution describes a discrete event count $n$ for a real valued Mean $\mu$.

$$Pois(n|\mu) = \mu^n \frac{e^{-\mu}}{n!}$$

Say, we observe $n_o$ events

What is the likelihood of $\mu$?

The likelihood of $\mu$ is given by

$L(\mu) = Pois(n_o \mid \mu)$

It is a continues function of $\mu$ but it is NOT a PDF

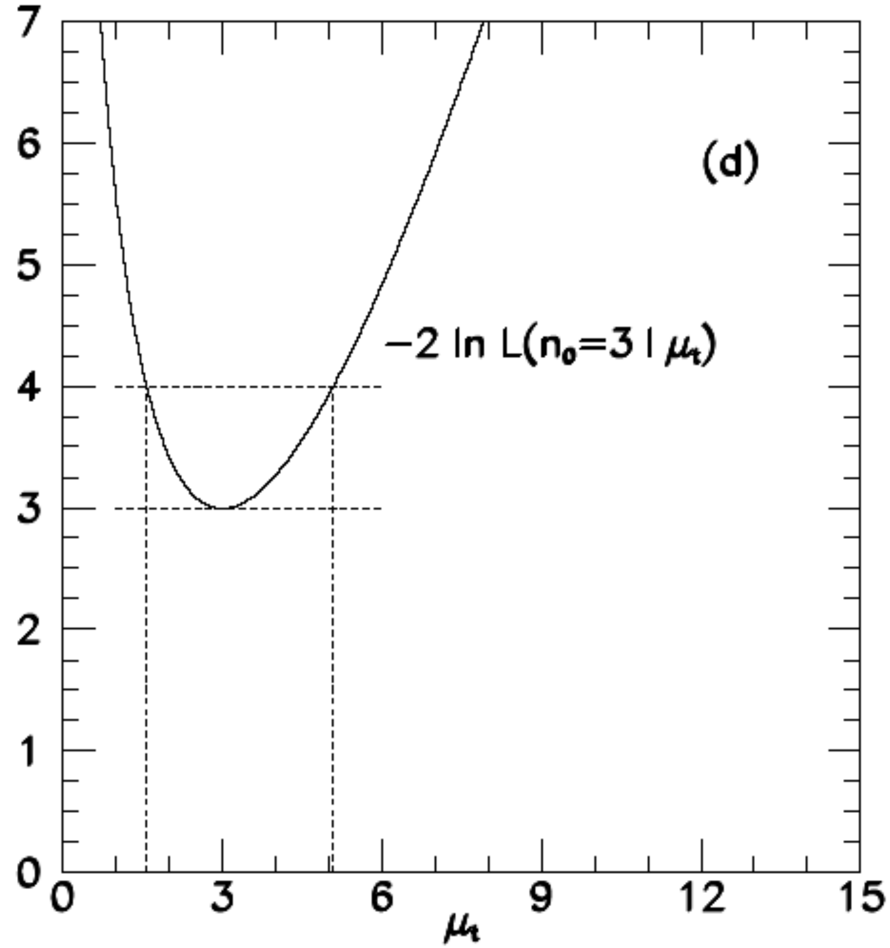



Figure from R. Cousins, Am. J. Phys. 63 398 (1995)

# Use of Likelihood Ratio

$x \to y(x)$

$p(y(x)|\theta) = p(x|\theta) / \left| dy/dx \right| \qquad (prove)$

*probabilities* are invariant under change of variable

$P(y(x_1) < y < y(x_2)) = P(x_1 < x < x_2)$

$p(y(x)|\theta) \neq p(x|\theta)$

*But* Likelihood ratio is invariant under change of variable x

(cancellation of Jacobian)

*Likelihood* is not a PDF in $\theta$

$L(\theta) = L(u(\theta))$

# Uniform PDF

$$x \in (a,b)$$

$$y(x) = \int_a^x p(x')dx'$$

*then*

$$y \in (0,1) \; and \; p(y) \; is \; uniform \; for \; all \; y$$

---

$$p\text{-}value \equiv \int_{q_{obs}}^{\infty} f(q \mid H_{null})dq$$

In statistics, when a p-value is used to test a simple null hypothesis, and the distribution of the test statistic, *q,* is continuous, then the p-value is uniformly distributed between 0 and 1 if the null hypothesis is true.

# Bayesian vs Frequentist

Is there a Higgs Boson?

Frequentist

$\Pr ob(Data \mid Theory)$

Run simulations of the theory and calculate the probability on the limit of long term frequency

*Bayesia* will try to answer

$\mathrm{Prob}(Theory \mid Data)$

$i.e.\,\mathrm{Prob}(Higgs \mid Data)$

# Bayesian vs Frequentist

- Is there a Higgs Boson? What do you mean?
  Given the data , is there a Higgs Boson?
- Can you really answer that without any a priori knowledge of the Higgs Boson?
  Change your question: What is your degree of belief in the Higgs Boson given the data… Need a prior degree of belief regarding the Higgs Boson itself…

$$P(Higgs\,|\,Data) = \frac{P(Datas\,|\,Higgs)P(Higgs)}{P(Data)} = \frac{L(Data\,|\,Higgs)\pi(Higgs)}{\int L(Data\,|\,Higgs)\pi(Higgs)d(Higgs)}$$

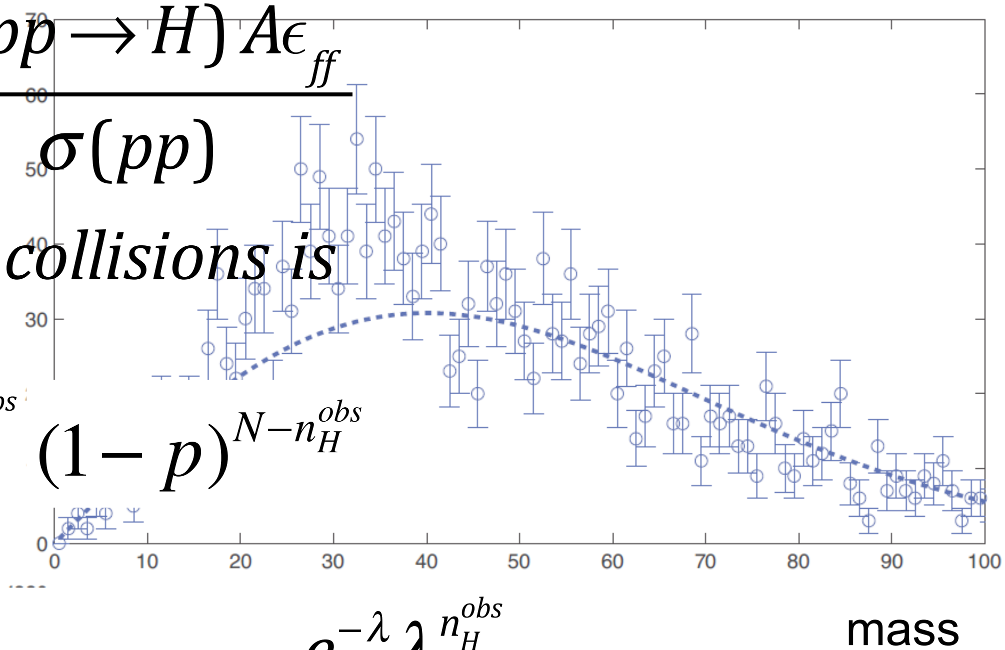
- If not, make sure that when you quote your answer you also quote your prior assumption!
- The most refined question is:
  - Assuming there is a Higgs Boson with some mass $m_H$, how well the data agrees with that? $P(Data\,|\,Higgs)$
  - But even then the answer relies on the way you measured the data (i.e. measurement uncertainties), and that might include some pre-assumptions, priors!

# Histograms

$N$ *collisions*

$$p(Higgs\ event) = \frac{\sigma(pp \to H)\,A\epsilon_{ff}}{\sigma(pp)}$$

Prob to see $n_H^{obs}$ in $N$ collisions is




$$P(n_H^{obs}) = \binom{N}{n_H^{obs}} p^{n_H^{obs}} (1-p)^{N-n_H^{obs}}$$

$$\ell im_{N\to\infty} P(n_H^{obs}) = Poiss(n_H^{obs}, \lambda) = \frac{e^{-\lambda}\lambda^{n_H^{obs}}}{n_H^{obs}!}$$

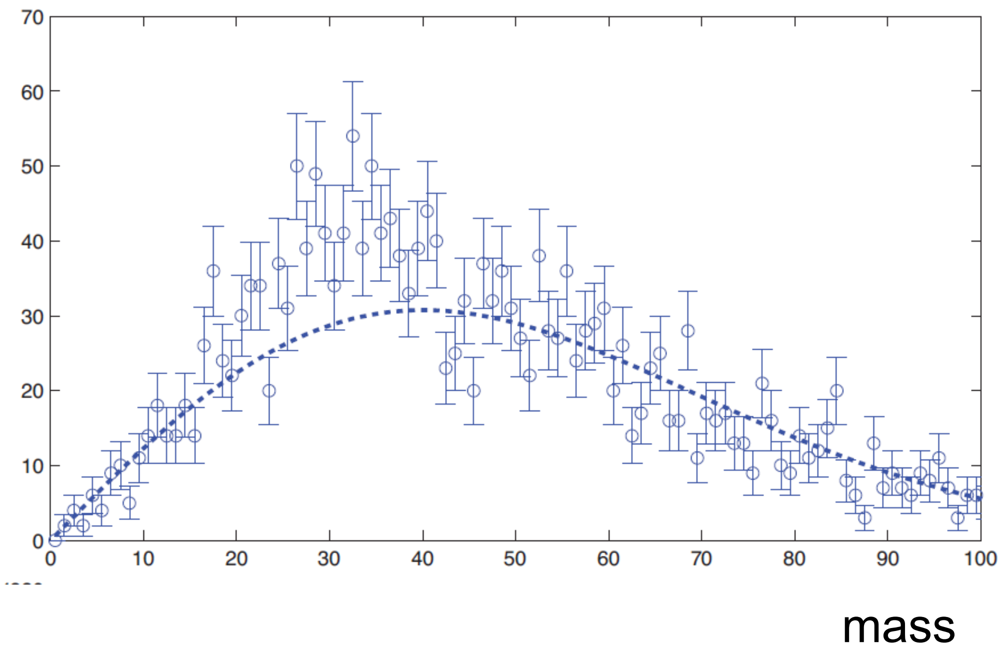
$$\lambda = Np = n_H^{exp}$$

# The Statistical Challenge of HEP

The DATA: Billions of Proton-Proton collisions
which could be visualized with histograms

The searched particle mass is unknown (for the sake of this lecture)

In this TOY example, we ask if the expected background (e.g. the Standard Model WITHOUT the Higgs Boson) contains a Higgs Boson, which would manifest itself as a peak in the distribution



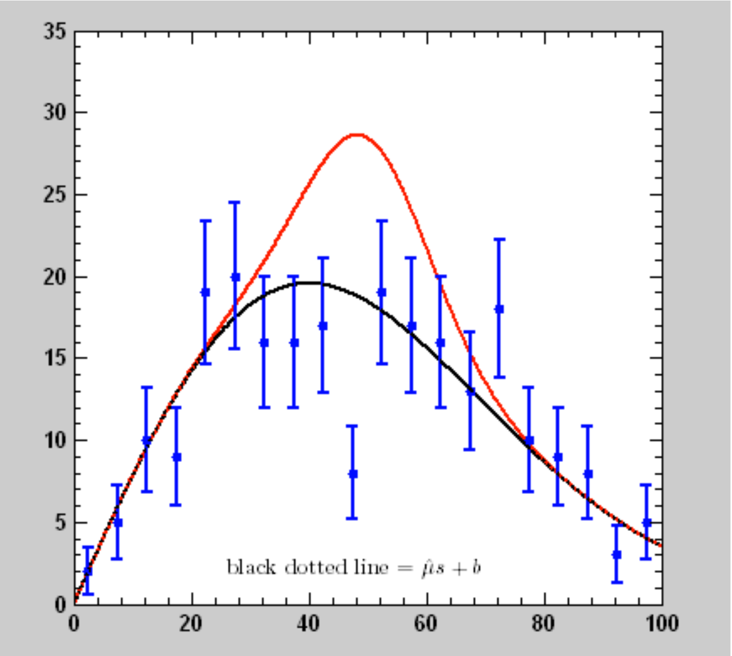

WE NEED TO KNOW WHAT WE SEARCH FOR…..

We need to have a model

We need to have two hypotheses if we want a powerful test

# What is the statistical challenge?

- The black line represents the Standard Model (**SM**) expectation (Background only),
- How compatible is the **data** (**blue**) with the **SM expectation** (**black**)?
- Is there a signal hidden in this data?
- What is its statistical significance?
- What is the most powerful test statistic that can tell the SM (**black**) from an **hypothesized signal (red)**?



# The Model

- The Higgs hypothesis is that of signal $s(m_H)$

$$s(m_H) = L \cdot \sigma_{SM}(m_H) \cdot A \cdot eff$$

For simplicity unless otherwise noted $s(m_H) = L \cdot \sigma_{SM}(m_H)$

- In a counting experiment

$$n = \mu \cdot s(m_H) + b$$

$$\mu = \frac{L \cdot \sigma(m_H)}{L \cdot \sigma_{SM}(m_H)} = \frac{\sigma(m_H)}{\sigma_{SM}(m_H)}$$

- $\mu$ is the strength of the signal (with respect to the expected Standard Model one
- The hypotheses are therefore denoted by $H_\mu$
- $H_1$ is the SM with a Higgs, $H_0$ is the background only model

# A Frequentist Tale of Two Hypotheses

NULL

ALTERNATE

- Test the Null hypothesis and try to reject it
- Fail to reject it OR reject it in favor of the Alternate hypothesis

# The Null Hypothesis

- The Standard Model without the Higgs is an hypothesis, (BG only hypothesis) many times referred to as the null hypothesis and is denoted by $H_0$ (remember that it is the null hypothesis ONLY if we aim at a discovery)
- In the absence of an alternate hypothesis, one would like to test the compatibility of the data with $H_0$
- This is actually a **goodness of fit test**, NOT an hypothesis test

# A Tale of Two Hypotheses

NULL

ALTERNATE

$H_0$- SM w/o Higgs

$H_1$- SM with Higgs

- Test the Null hypothesis and try to reject it
- Fail to reject it OR reject it in favor of the Alternate hypothesis

# The Alternate Hypothesis?

- Let's zoom on

## $H_1$ - SM with Higgs

- Higgs with a specific mass $m_H$

  OR

- Higgs anywhere in a specific mass-range
  - The look elsewhere effect

# A Tale of Two Hypotheses

**NULL**

$H_0$- SM w/o Higgs

**ALTERNATE**

$H_1$- SM with Higgs

- Reject $H_0$ in favor of $H_1$ – A DISCOVERY

# Swapping Hypotheses→exclusion

NULL

$H_0$- SM w/o Higgs

ALTERNATE

$H_1$- SM with Higgs

- Reject $H_1$ in favor of $H_0$

Excluding $H_1$ ($m_H$)→Excluding the Higgs with a mass $m_H$

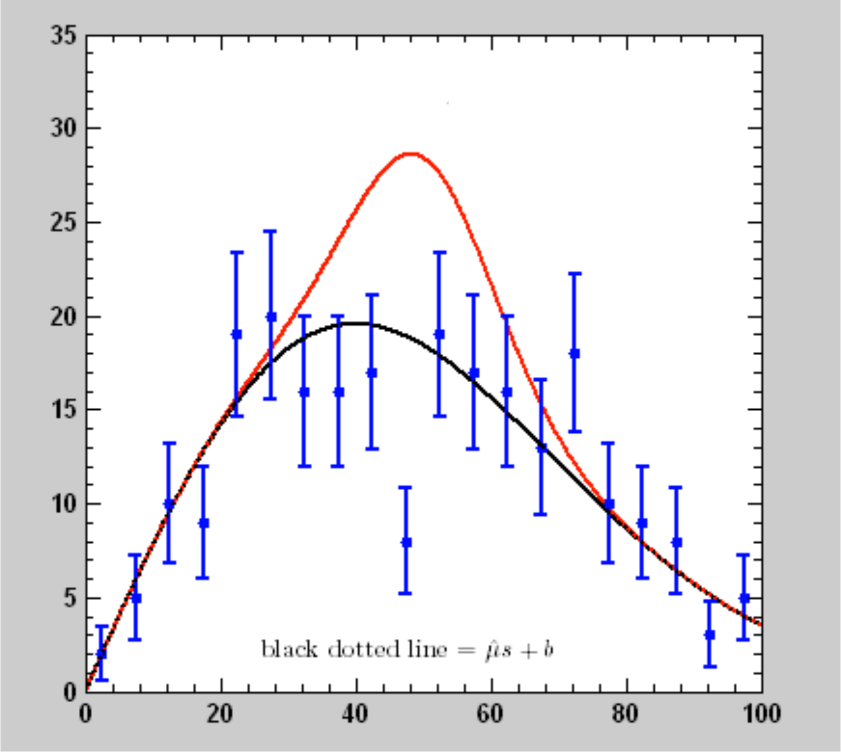
# Testing an Hypothesis (wikipedia...)

- The first step in any hypothesis testing is to state the relevant **null,** $H_0$ and **alternative hypotheses,** say, $H_1$
- The next step is to define a test statistic, q, under the null hypothesis
- Compute from the observations the observed value $q_{obs}$ *of the test statistic* q.
- Decide (based on $q_{obs}$ ) to either
  **fail to reject the null hypothesis** or
  **reject it in favor of an alternative hypothesis**

- **next: How to construct a test statistic, how to decide?**

# Test statistic and p-value

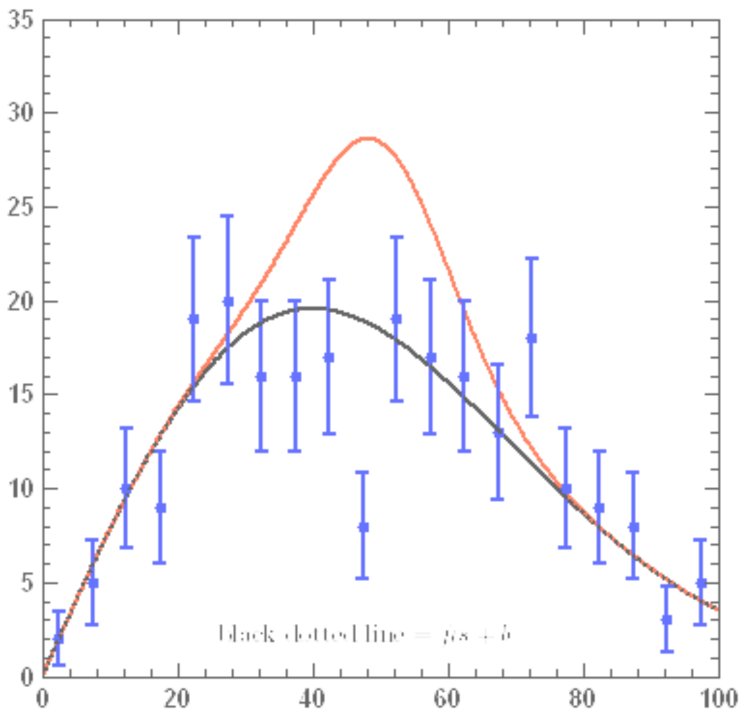
# The Physics Model

- SM without Higgs Background
  No signal $\langle n \rangle = b$
- 

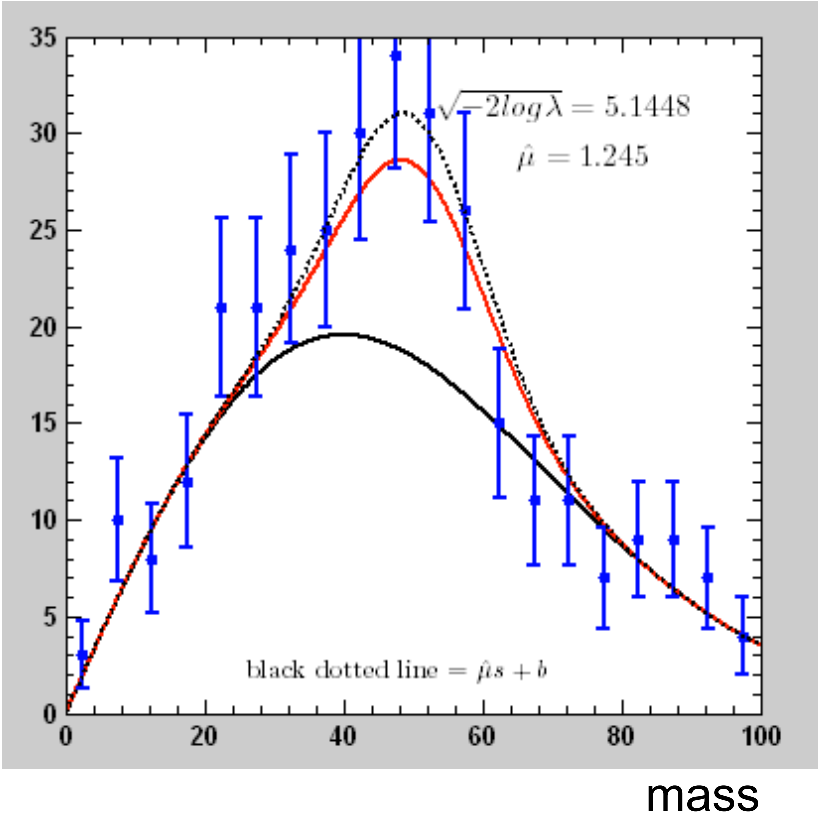

mass

# The Physics Model

- SM without Higgs Background Only $\langle n \rangle = b$
- 



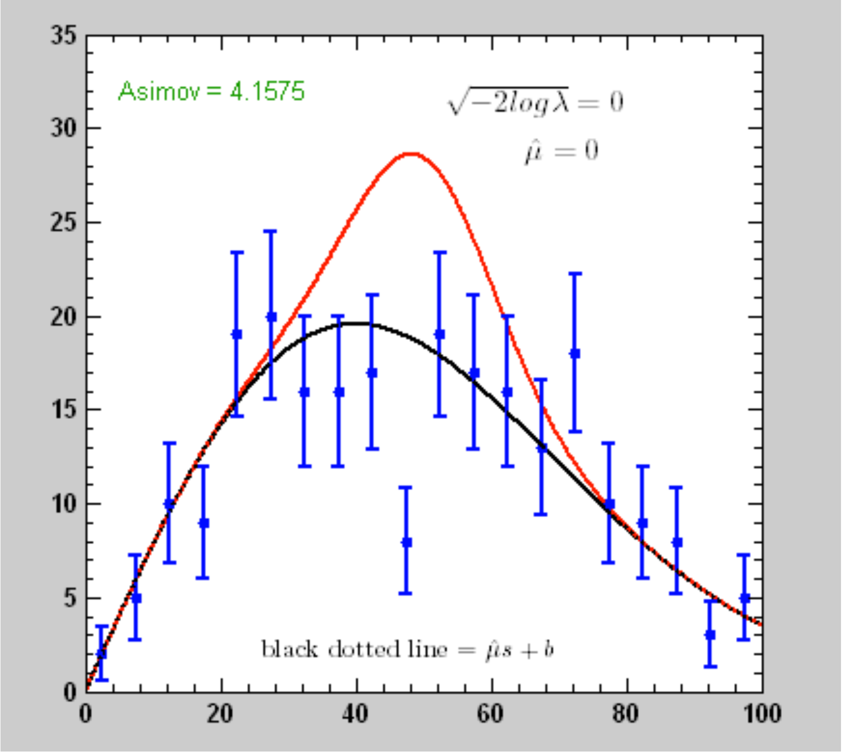

- SM with a Higgs Boson with a mass $m_H$ $\langle n \rangle = s(m_H) + b$



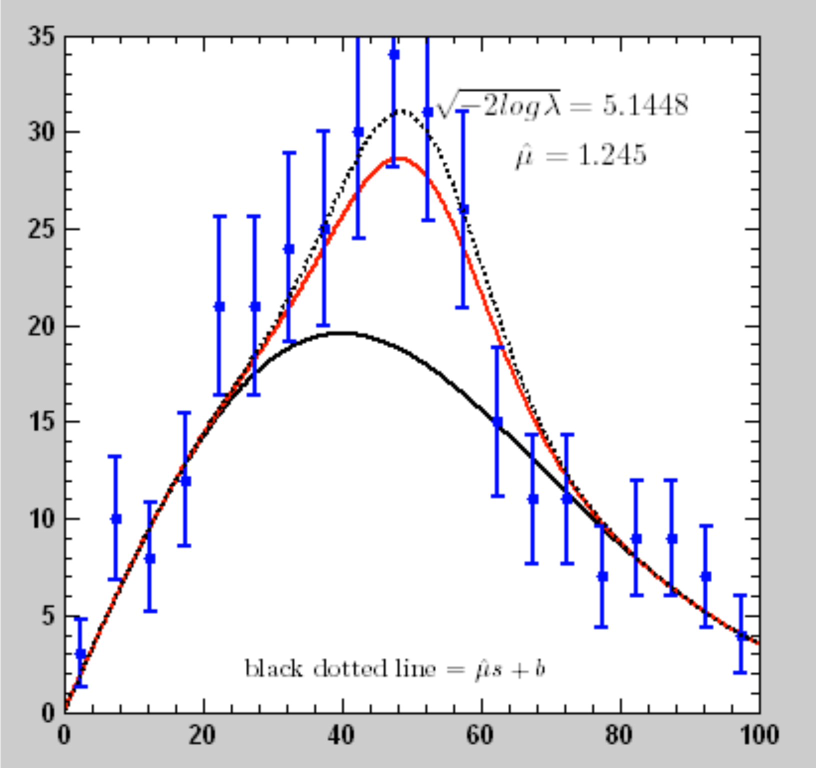
# The Physics Model

$$n = \mu s + b \qquad MLE \quad \hat{\mu}$$

$$\langle \hat{\mu} \rangle = 0 \;\; under \;\; H_0 \qquad \langle \hat{\mu} \rangle = 1 \;\; under \;\; H_1$$




mass




mass

# The Neyman-Pearson Test Statistic

- NP **test statistic**

$$\boxed{Q = -2\ln\frac{L(H_0)}{L(H_1)}}$$

-

$$n = \mu s + b$$

$$H_0;\ \hat{\mu} = 0, \langle n_{obs} \rangle = b$$

$$H_1;\ \hat{\mu} = 1, \langle n_{obs} \rangle = s + b$$

$$L\left(H_0\right) = prob(x \mid H_0) = prob(x \mid b)$$

$$L(H_1) = prob(x \mid H_1) = prob(x \mid s + b)$$

$$L = f(x),\ L(H_\mu) = prob(x \mid H_\mu) = f(\hat{\mu})$$

# Likelihood

- Likelihood is a function of the data

$$L(H) = L(H \mid x) = f(x)$$

$$L(H \mid x) = P(x \mid H)$$

$$q_{NP} = q_{NP}(x) = -2\ln\frac{L(H_1 \mid x)}{L(H_0 \mid x)}$$

- Likelihood is not the probability of the hypothesis given the data

Bayes Theorem

$$P(H \mid x) = \frac{P(x \mid H)\cdot P(H)}{\Sigma_H P(x \mid H) P(H)}$$

$$P(H \mid x) \approx P(x \mid H)\cdot P(H)$$

Prior

# Frequentist vs Bayesian

- The Bayesian infers from the data using priors

posterior $P(H\,|\,x) \approx P(x\,|\,H)\cdot P(H)$

- Priors is a science on its own.
  Are they objective? Are they subjective?
- The Frequentist calculates the probability of an hypothesis to be inferred from the data based on a large set of hypothetical experiments
  Ideally, the frequentist does not need priors, or any degree of belief while the Baseian posterior based inference **is** a "Degree of Belief".

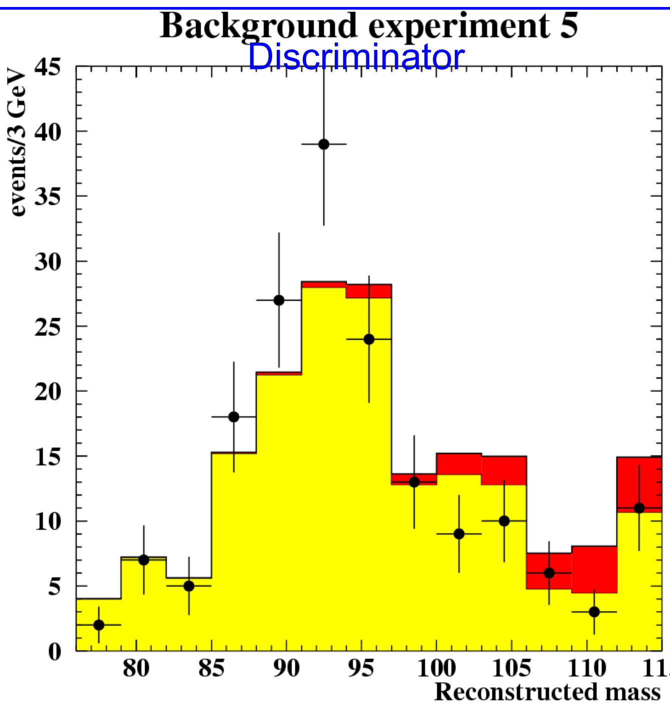

- However, NPs inject a Bayesian flavour to any Frequentist analysis

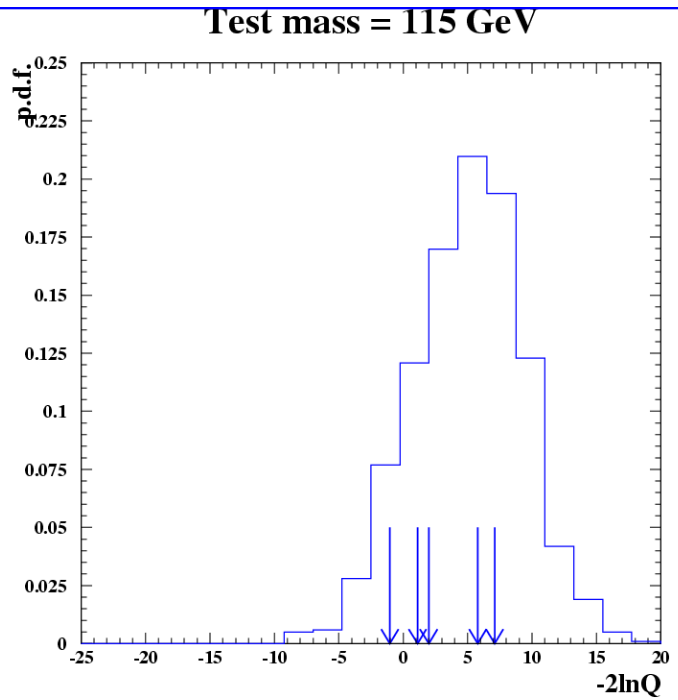
# Example:
# Simulating BG Only Experiments

$$q_{NP}(m)=\frac{L(H_1)}{L(H_0)}=\frac{L(s(m)+b)}{L(b)}$$

- The likelihood ratio, $-2\ln q(m_H)$ tells us how much the outcome of an experiment is signal-like
- NOTE, here the s+b pdf is plotted to the left (it's the null hypothesis)!
- Test the s+b hypothesis

$$f(q^{NP}\mid b)$$




-2lnQ

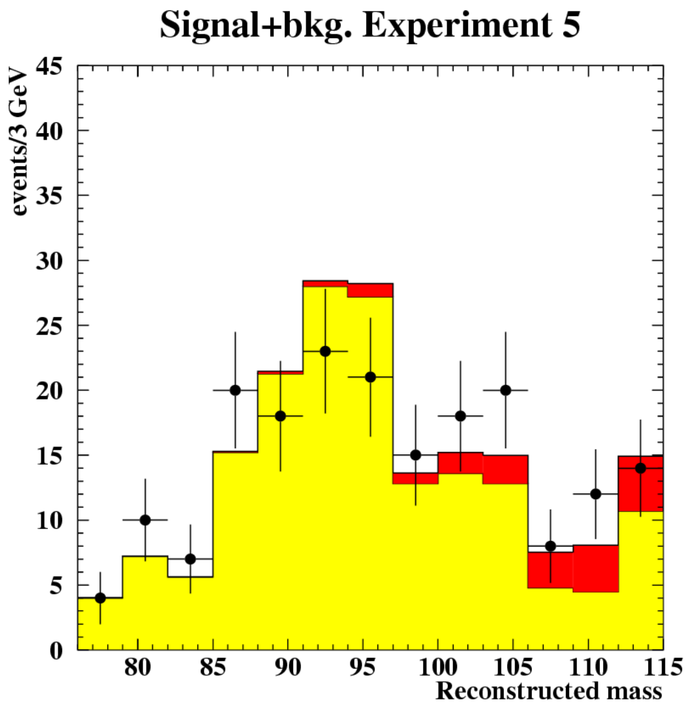
s+b like

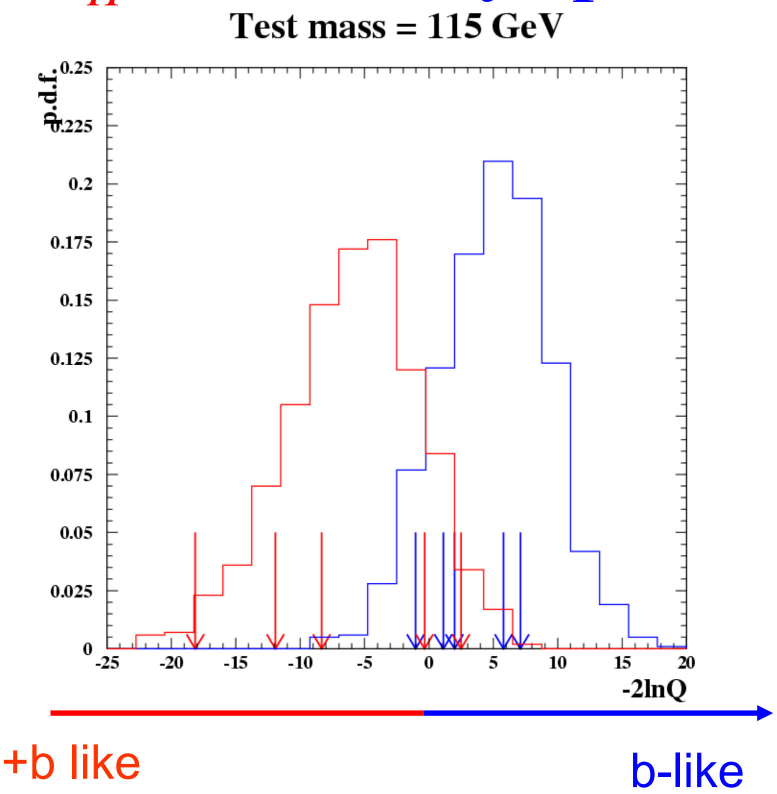
b-like

Example:

Simulating $S(m_H)+b$ Experiments

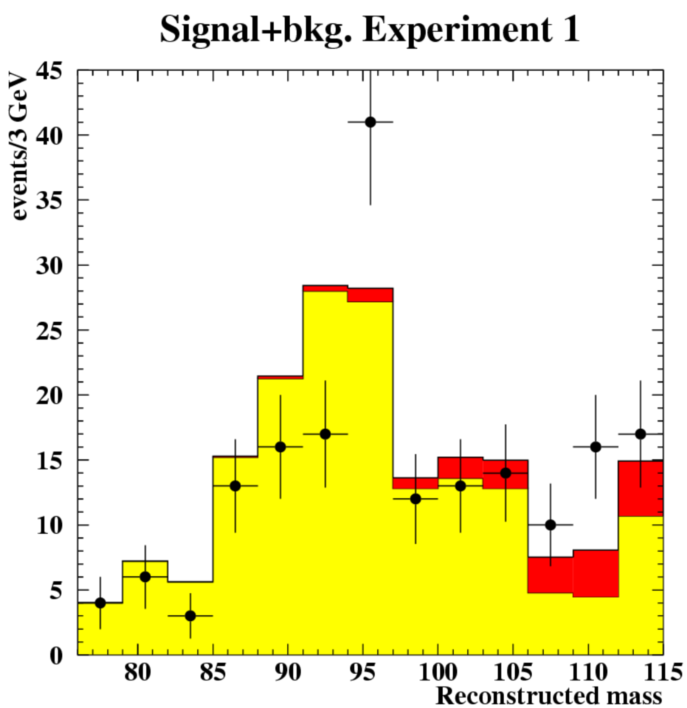
$$f(q^{NP} \mid s(m_H)+b) \qquad f(q^{NP} \mid b)$$



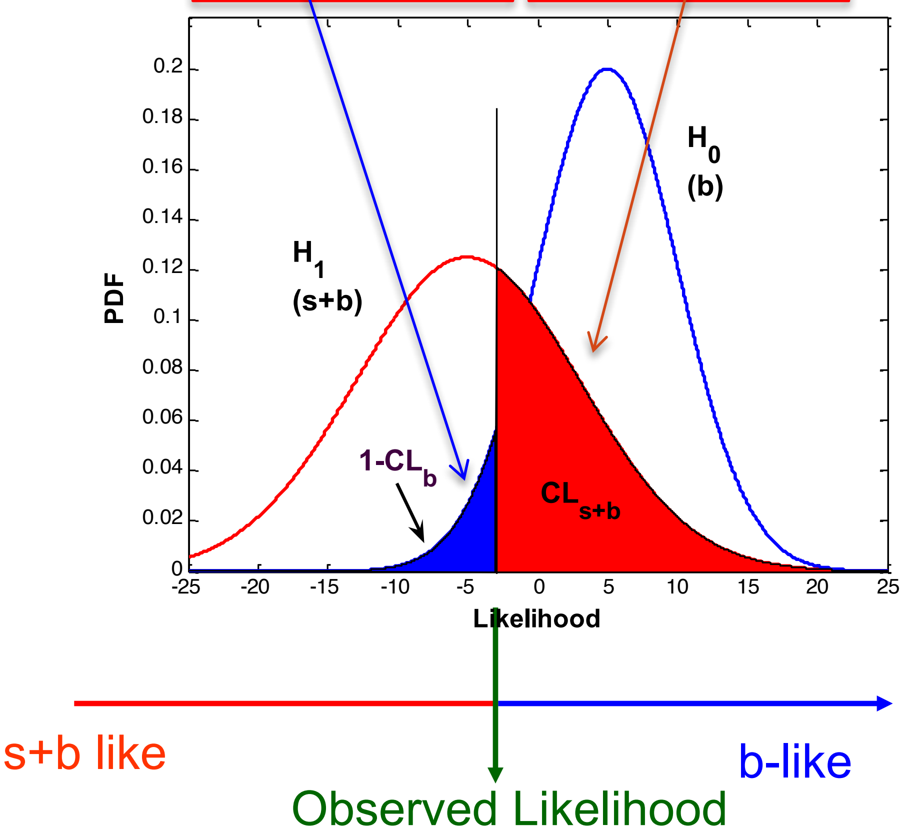



# Example:

## Simulating $S(m_H)+b$ Experiments



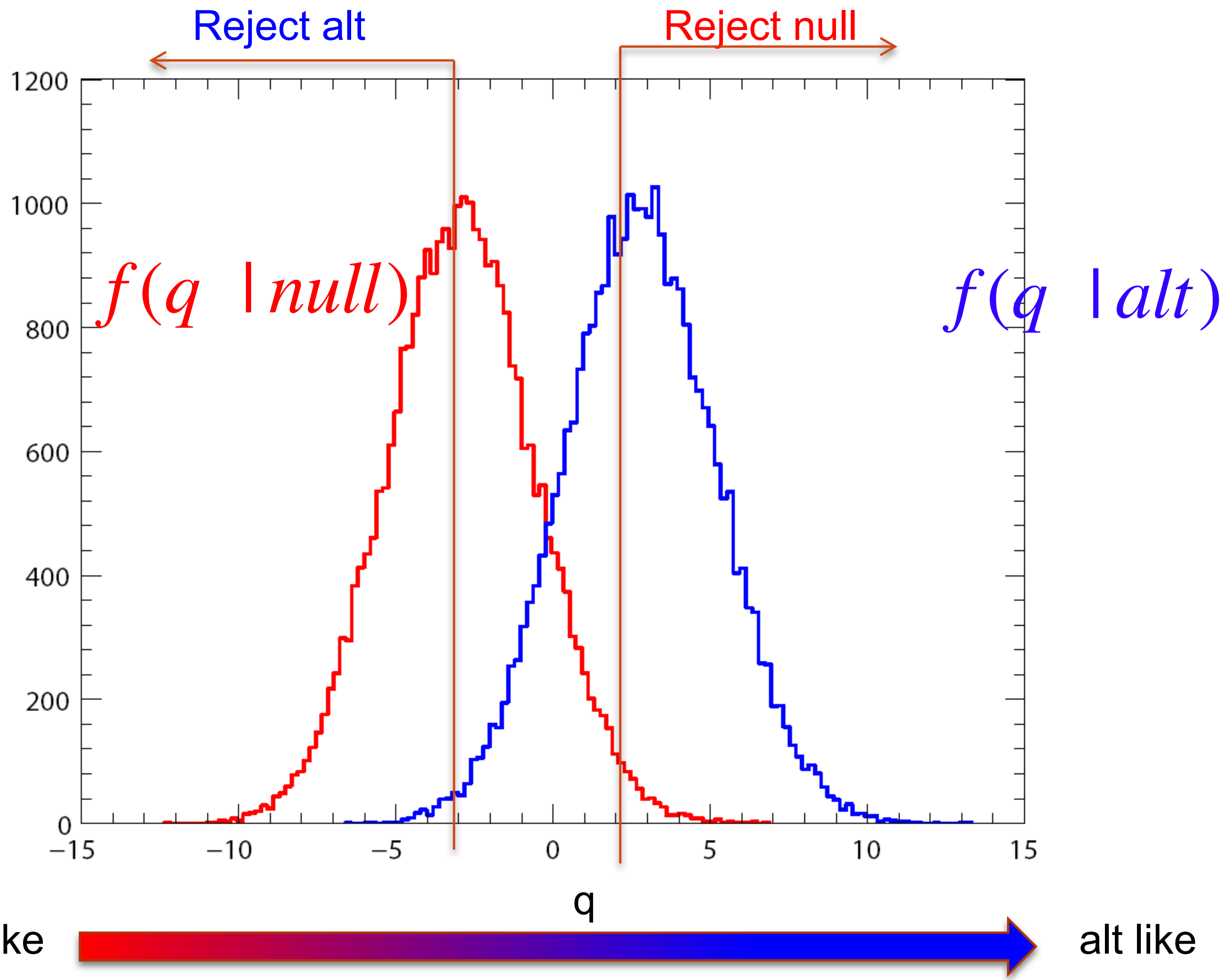

$$p_b \sim 1 - CL_b \qquad p_{s+b} \sim CL_{s+b}$$



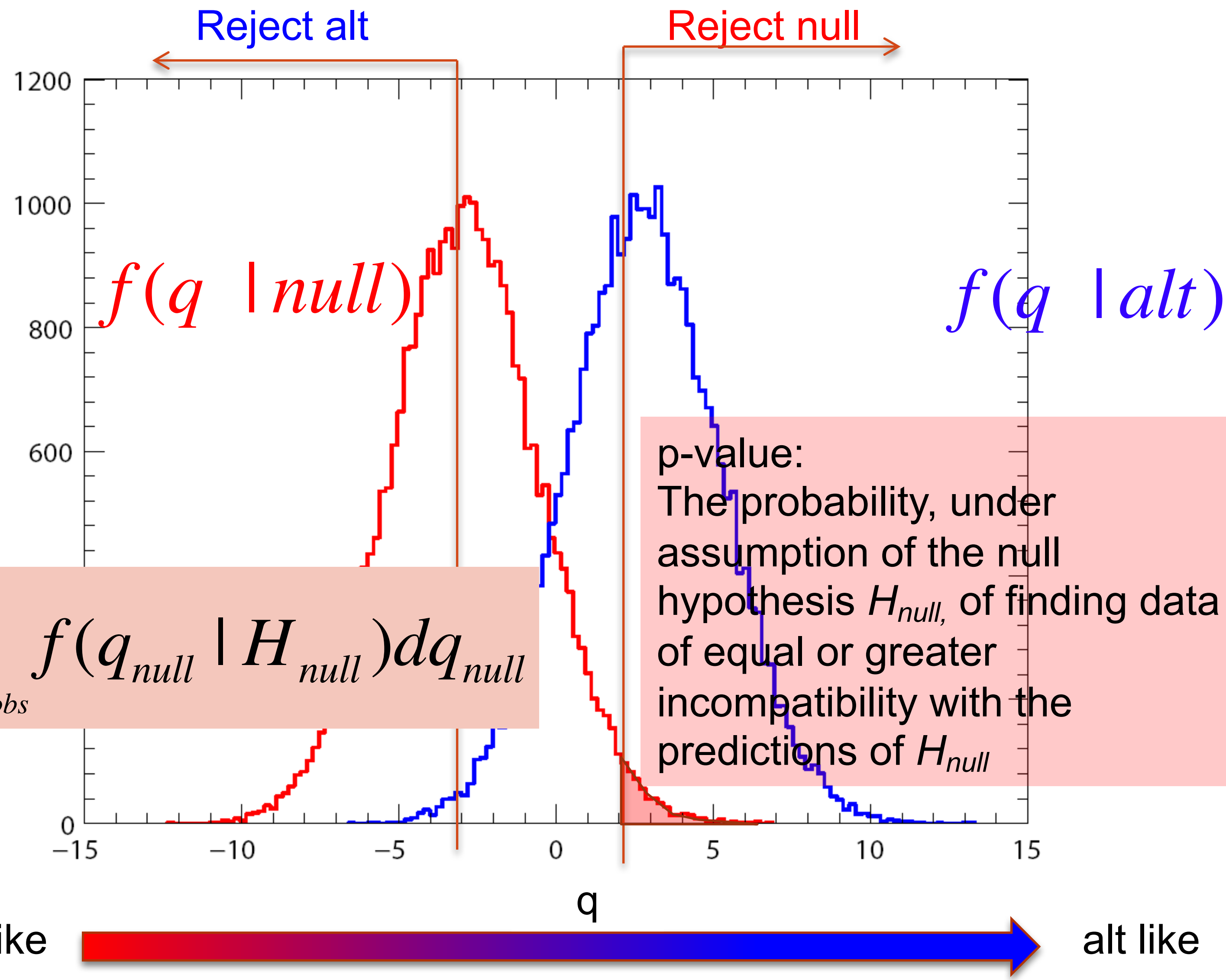
# p-Value

- The observed *p-value* is a measure of the incompatibility of the data with the tested hypothesis.
- It is the probability, under assumption of the null hypothesis $H_{null,}$ of finding data of equal or greater incompatibility with the predictions of $H_{null}$
- Wikipedia: *An important property of a test statistic is that its sampling distribution under the null hypothesis must be calculable, either exactly or approximately, which allows p-values to be calculated.*

# PDF of a test statistic



# PDF of a test statistic




$$p = \int_{q_{obs}}^{\infty} f(q_{null} \mid H_{null})\, dq_{null}$$

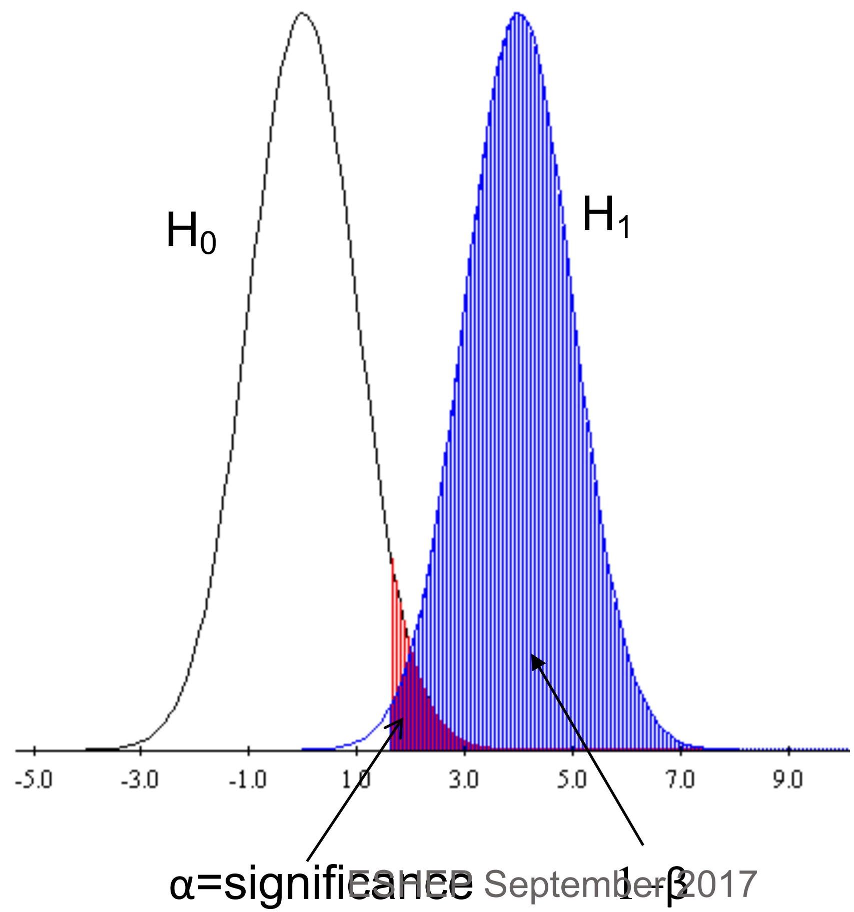
p-value:
The probability, under assumption of the null hypothesis $H_{null}$, of finding data of equal or greater incompatibility with the predictions of $H_{null}$

# Basic Definitions: type I-II errors

- By defining α you determine your tolerance towards mistakes… (accepted mistakes frequency)
- **type-I error**: the probability to reject the tested (null) hypothesis ($H_0$) when it is true
- $$\alpha = \Pr ob(reject\ H_0 \mid H_0)$$
  $$\alpha = typeI\ error$$
- **Type II**: The probability to accept the null hypothesis when it is wrong

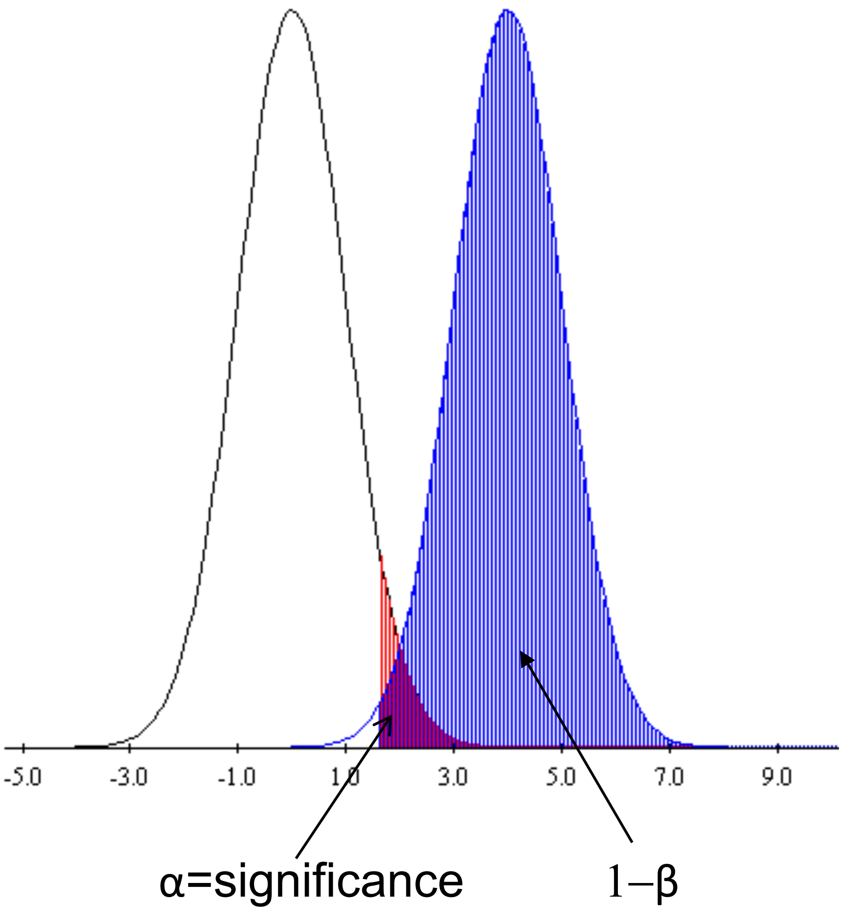
$$\beta = \Pr ob(accept\ H_0 \mid \bar{H}_0)$$
$$= \Pr ob(reject\ H_1 \mid H_1)$$
$$\beta = typeII\ \ error$$

- The pdf of q….



# Basic Definitions: POWER

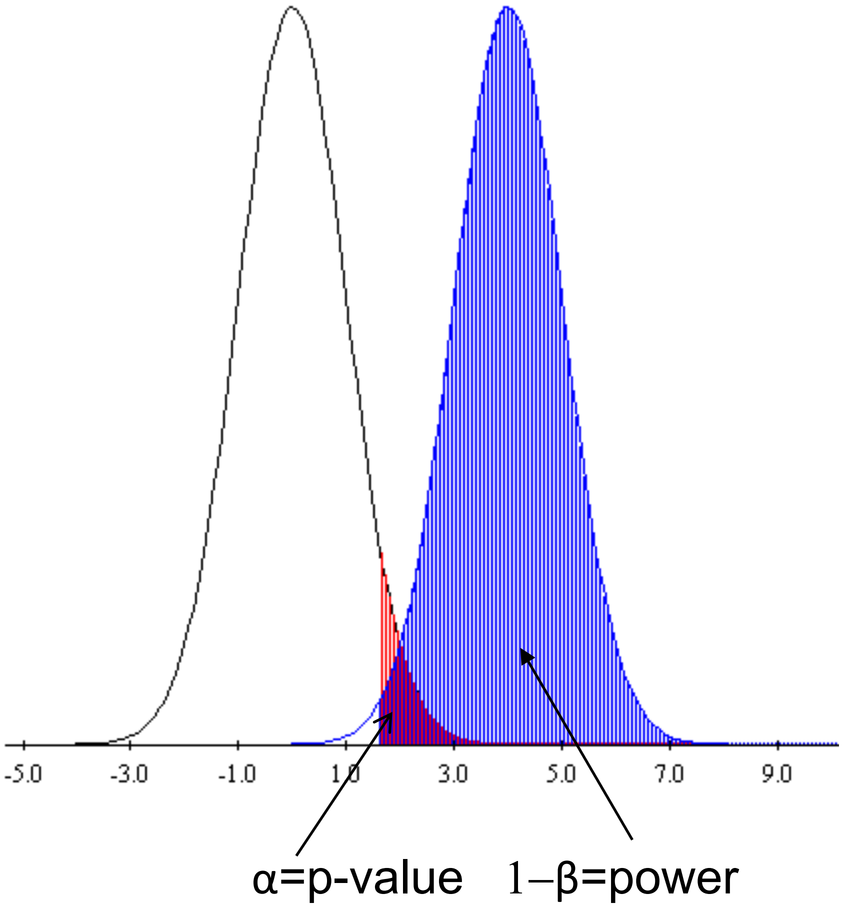
- $\alpha = \Pr ob(reject\ H_0 \mid H_0)$
- The POWER of an hypothesis test is the probability to reject the null hypothesis when the alternate analysis is true!
- $$POWER = \text{Prob}(reject\ H_0 \mid H_1)$$
  $$\beta = \Pr ob(reject\ H_1 \mid H_1) \Rightarrow$$
  $$1-\beta = \Pr ob(accept\ H_1 \mid H_1) =$$
  $$1-\beta = \Pr ob(reject\ H_0 \mid H_1) \Rightarrow$$
  $$POWER = 1-\beta$$
- The power of a test increases as the rate of type II error decreases



# Which Analysis is Better

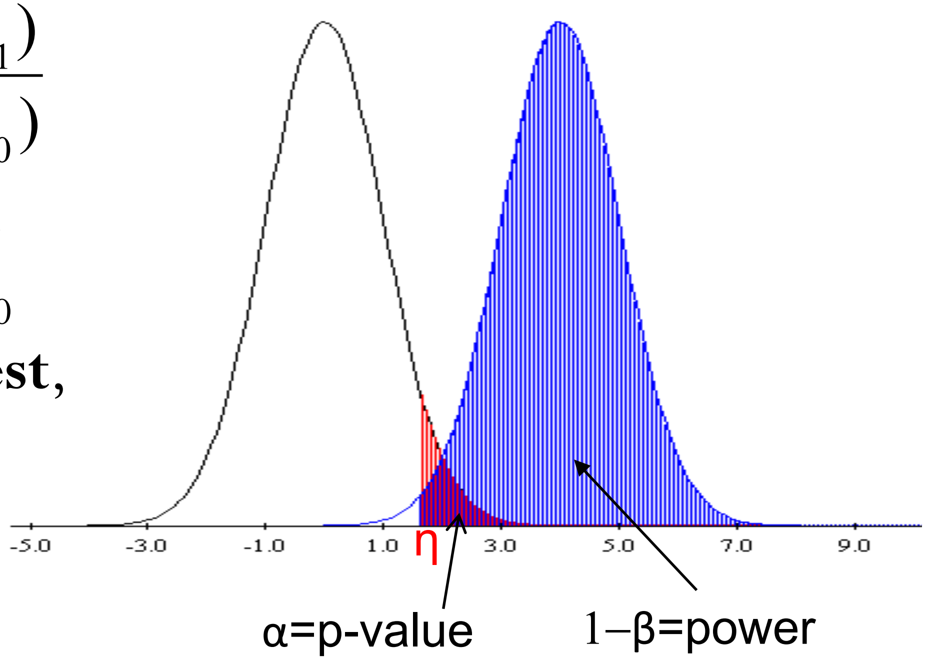
- To find out which of two methods is better plot the p-value vs the power for each analysis method
- Given the p-value, the one with the higher power is better
- p-value~significance

Different Test Statistics
Differ by
POWER
and
Asymptotic Properties



# The Neyman-Pearson Lemma

- Define a **test statistic** $\lambda = \frac{L(H_1)}{L(H_0)}$
- When performing a hypothesis test between two simple hypotheses, $H_0$ and $H_1$, **the Likelihood Ratio test**, which rejects $H_0$ in favor of $H_1$, **is the most powerful test** of size α for a threshold η
- **Note:** Likelihoods are functions of the data,
even though we often not specify it explicitly



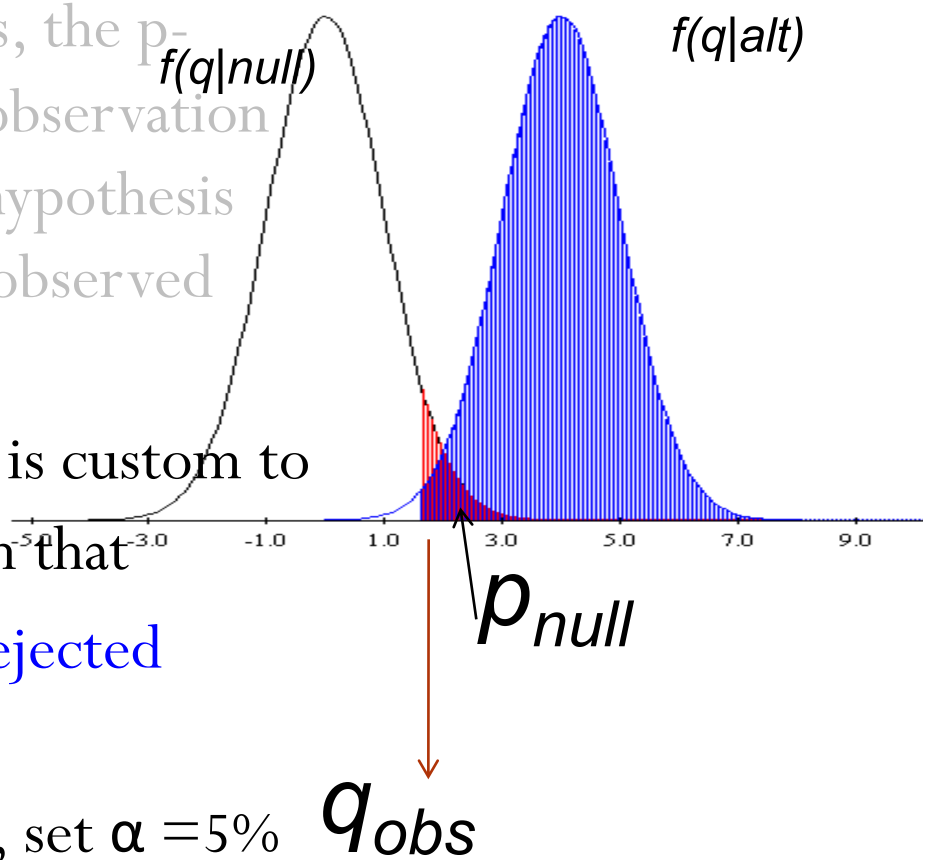
# p-value – testing the null hypothesis

- When testing the null hypothesis, the p-value is the probability that the observation is less compatible with the null hypothesis (more alternative like) than the observed one
- When testing the b hypotheis, it is custom to set α = $2.9\ 10^{-7}$ and it is custom that

if $p_b < 2.9\ 10^{-7}$ the b hypothesis is rejected →discovery

- When testing the s+b hypotheis, set α =5%

if $p_{s+b} < 5\%$ the signal hypothesis is rejectedat the 95% Confidence Level (CL) → Exclusion

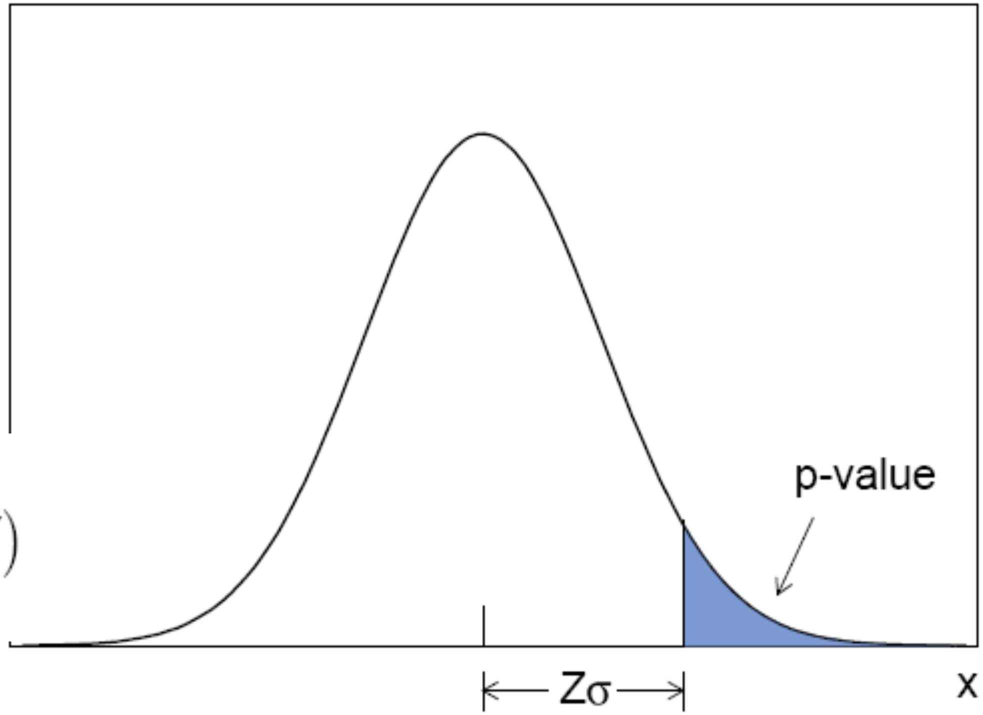


# From p-values to Gaussian significance

It is a custom to express the p-value as the significance associated to it, had the pdf were Gaussians

$$p = \int_Z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx = 1 - \Phi(Z)$$

$$Z = \Phi^{-1}(1 - p)$$

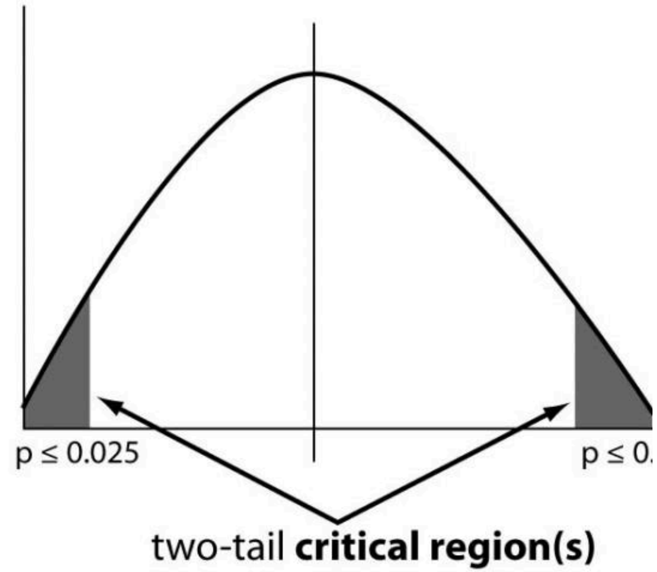



$\boxed{\text{A significance of } Z = 5 \text{ corresponds to } p = 2.87 \times 10^{-7}}$

Beware of 1 vs 2-sided definitions!

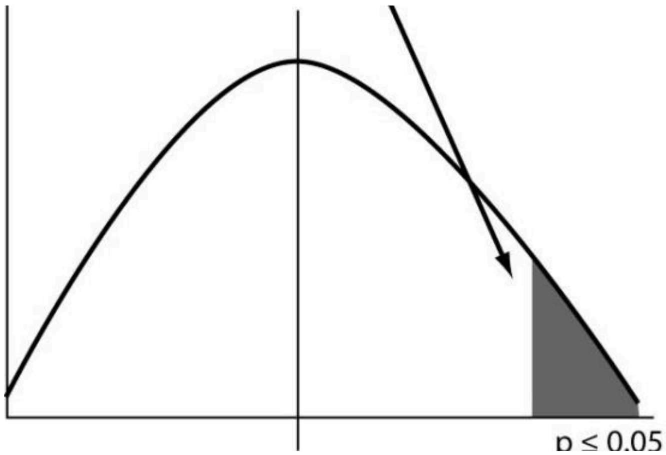
# 2-Sided p-value

- When performing a measurement, any deviation above or below the mean is drawing our attention and might serve an indication of some anomaly or new physics.
- Here we use a 2-sided p-value



# 1-Sided p-value

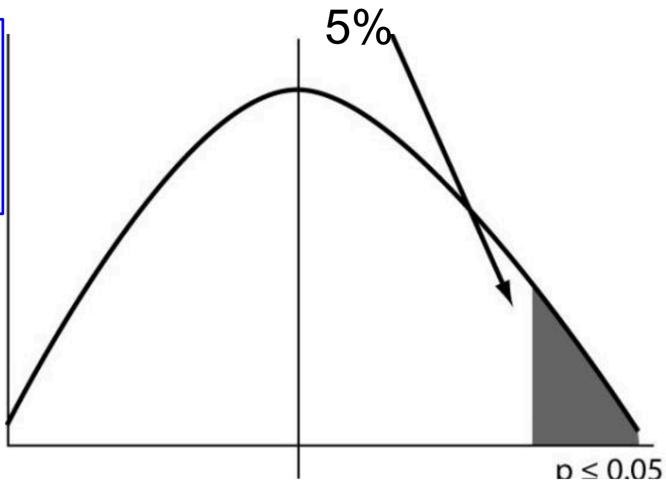
- When trying to reject an hypothesis while performing searches, one usually considers only one-sided tail probabilities.

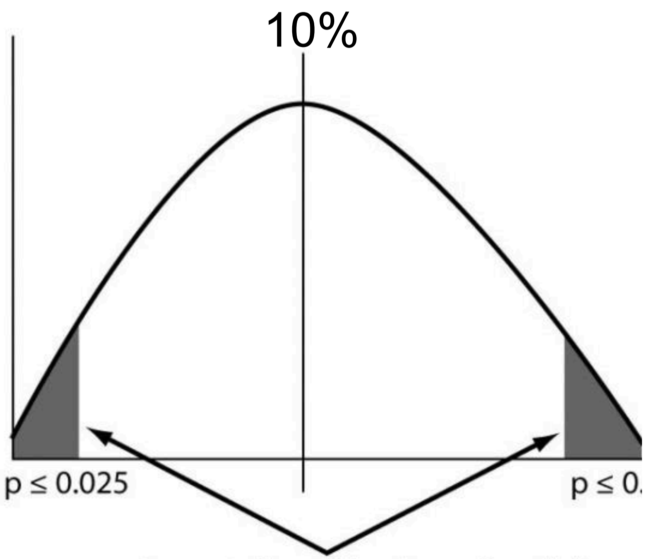



- Downward fluctuations of the background will not serve as an evidence against the background
- Upward fluctuations of the signal will not be considered as an evidence against the signal

# 1-sided 2-sided

- To determine a 1 sided 95% CL,
  we sometimes need to set the critical region to 10% 2 sided



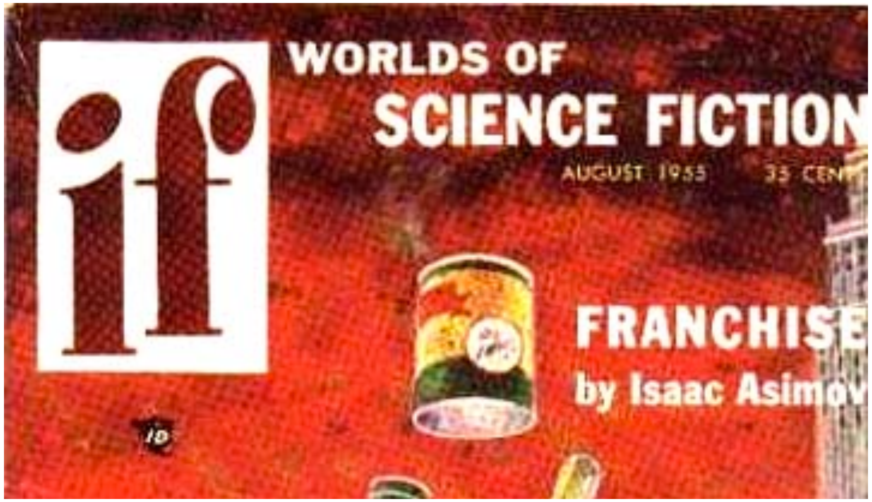



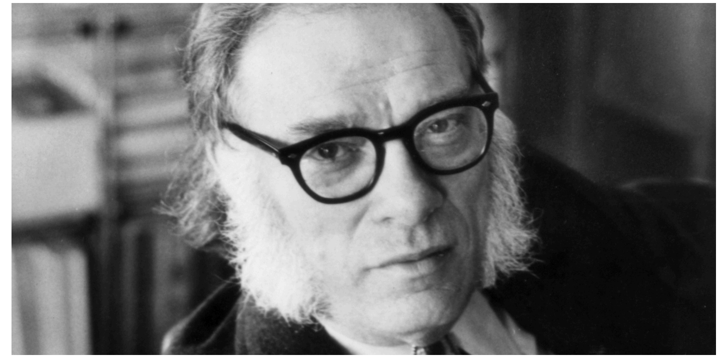
# Estimating the Sensitivity of an Experiment

- Estimate the expected significance one could achieve (for discovering the Higgs Boson) with a given analysis, a given Luminosity and CM energy..
- Option 1:
  - Toss,say, 1000000 BG only events (null) and derive the BG-only pdf of q, $f(q_{null}|BG)$.
    Toss 1000000 S+BG (alt) events and find the significance for each one of them Find the median significance….
- This may take ages…, is there a shortcut?

←→Option2: Asimov Data Set

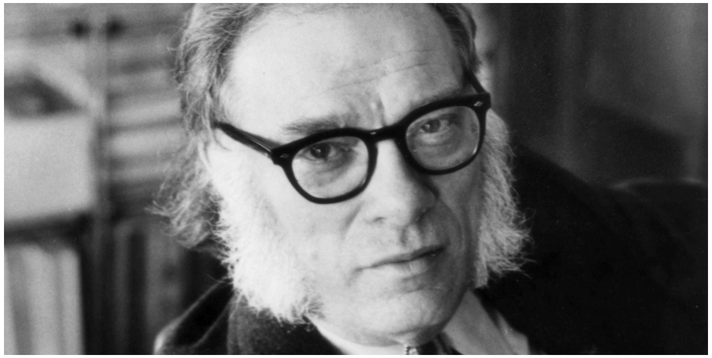
# The Asimov Data Set



In the future, the United States has converted to an "electronic democracy" where the computer Multivac selects a single person to answer a number of questions. Multivac will then use the answers and other data to determine what the results of an election would be, avoiding the need for an actual election to be held.

# The Asimov Data Set



- *The use of a single representative individual to stand in for the entire population can help in evaluating the sensitivity of a statistical method.*
- *The "**Asimov data set**":*
  ***an ensemble of simulated experiments can be replaced by a single representative one.***

# Estimating the Sensitivity of an Experiment

- Option 2: the Asimov Data Set
  - one can replace each ensemble of the alternate-hypothesis experiments with one data set that represents the typical experiment. This "Asimov" data set delivers the desired median sensitivity. Hence, one is exempted from the need to perform an ensemble of experiments for each set of parameters.
- The Asimov data set is constructed such that when one uses it to evaluate the estimators for all parameters, one obtains the true parameter values.
- the Asimov data set can trivially be constructed from the true parameters values. For example, a set corresponding to the $H_1$ hypothesis is $n_A = s + b$. and the one correspond to the $H_0$ hypothesis is $n_A = b$.
- As strange as it reads, the Asimov data set is not necessarily an integer.

# The Profile Likelihood

## The choice of the LHC for hypothesis inference in Higgs search

# The Profile Likelihood ("PL")

The best signal $\hat{\mu} = 0.3 \rightarrow 1.27\sigma$

For discovery we test the $H_0$ null hypothesis

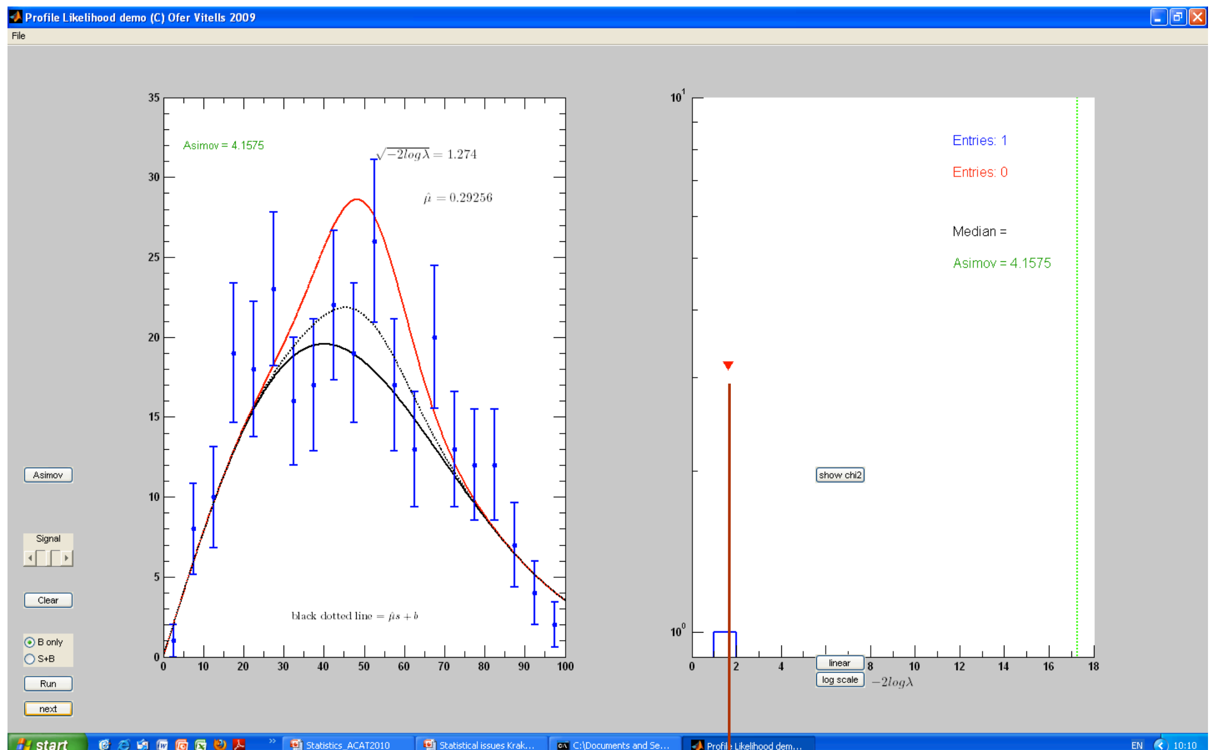
For $q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s + b)}$

$\hat{\mu} \sim 0,\ q_0$ small

$\hat{\mu} \sim 1,\ q_0$ large

In general: testing the $H_\mu$ hypothesis i.e., a SM with a signal of strength $\mu$,

$$q_\mu = -2\ln\frac{L(\mu)}{L(\hat{\mu})}$$



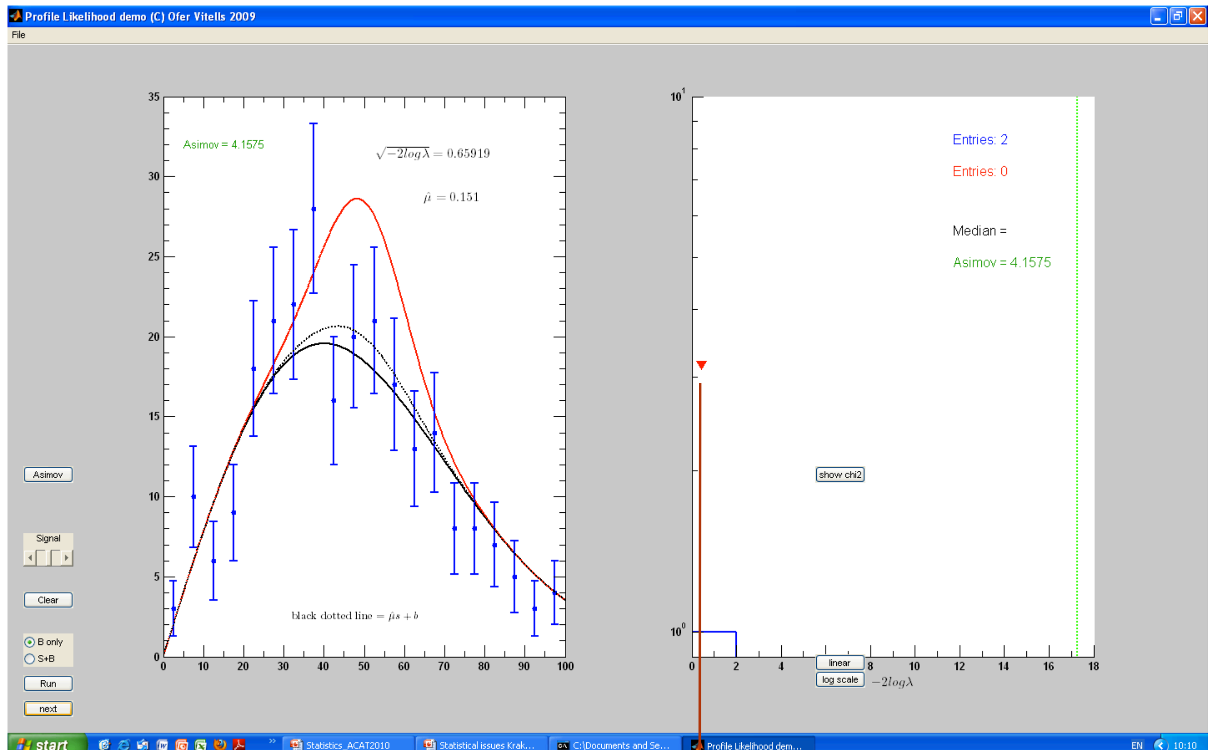

$$Z_{obs} = \sqrt{q_{0,obs}}$$

$q_0 = 1.6 \rightarrow Z = \sqrt{1.6} = 1.27$

# PL: test $q_0$ under BG only ; $f(q_0|H_0)$

$$\hat{\mu} = 0.15 \rightarrow 0.6\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$



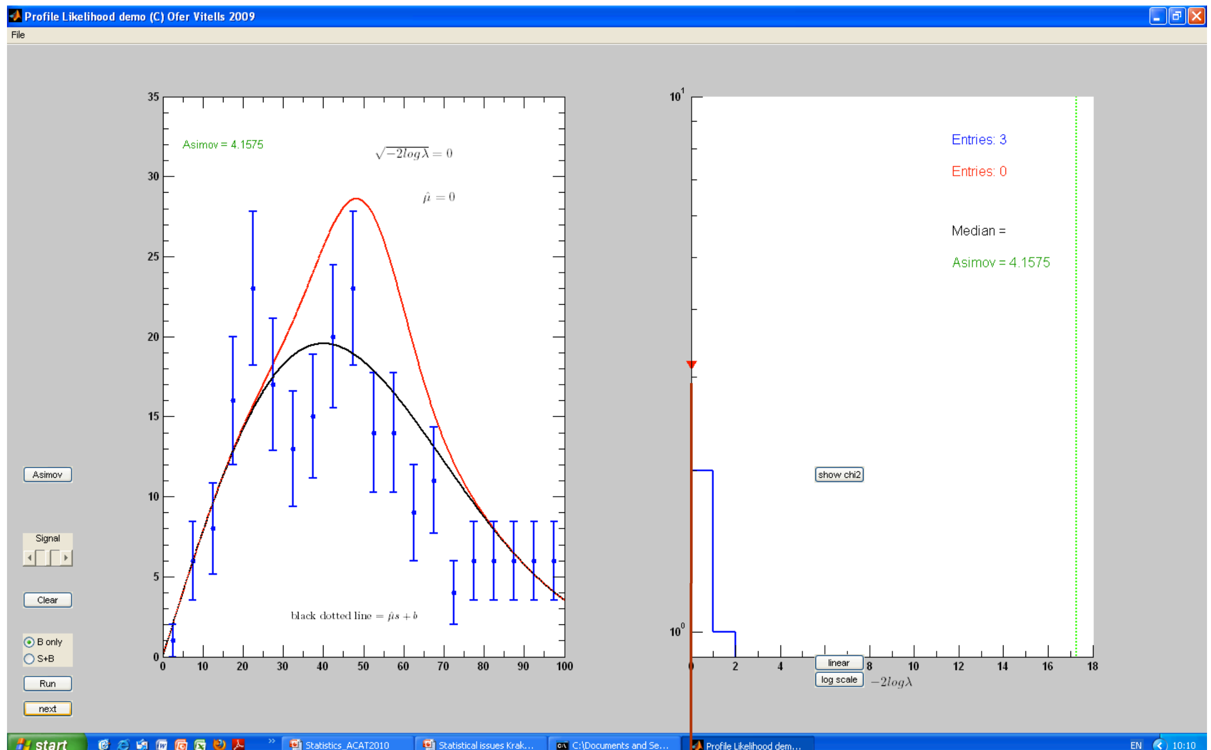

$$Z_{obs} = \sqrt{q_{0,obs}}$$

$$q_0 = 0.43 \rightarrow Z = 0.66\sigma$$

# PL: test $q_0$ under BG only ; $f(q_0|H_0)$

$$\hat{\mu} = 0$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s + b)}$$



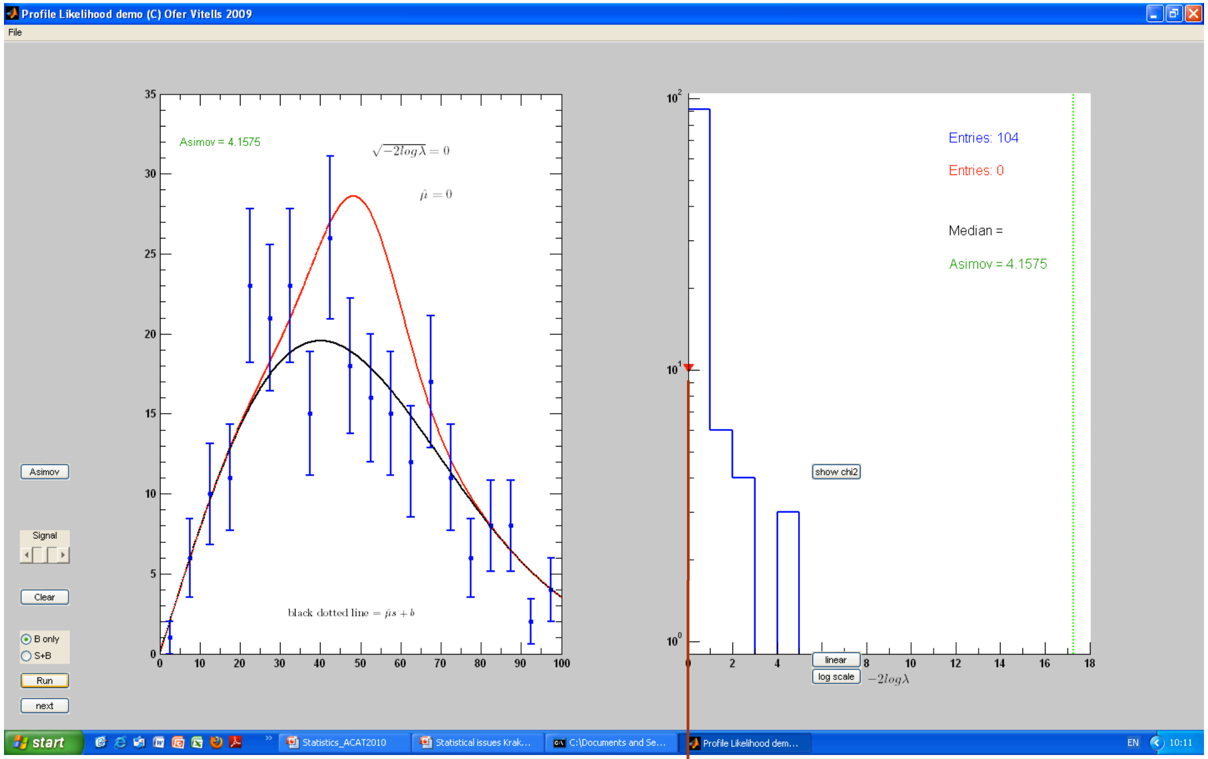

$$Z_{obs} = \sqrt{q_{0,obs}}$$

$$q_0 = 0$$

# PL: test $q_0$ under BG only ; $f(q_0|H_0)$

$$\hat{\mu}=0$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$



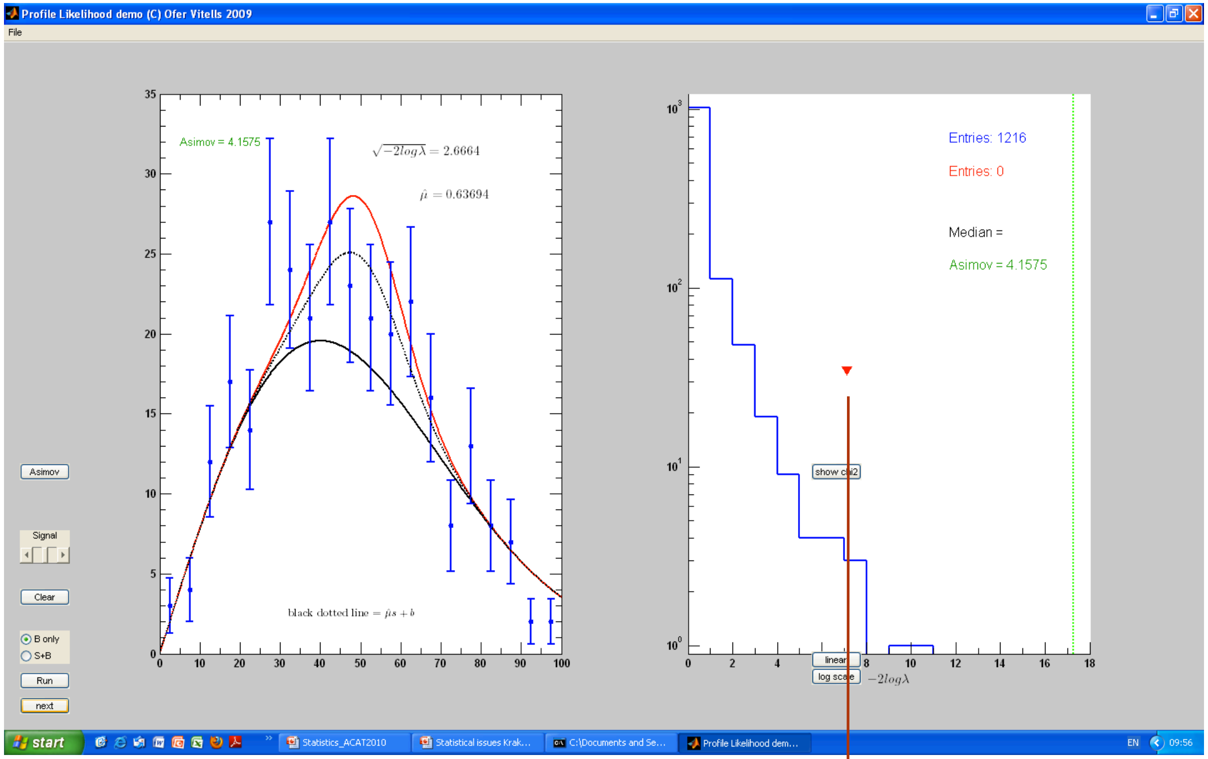

$$Z_{obs} = \sqrt{q_{0,obs}}$$

$$q_0 = 0$$

# PL: test $q_0$ under BG only ; $f(q_0|H_0)$

$$\hat{\mu} = 0.6 \rightarrow 2.6\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$



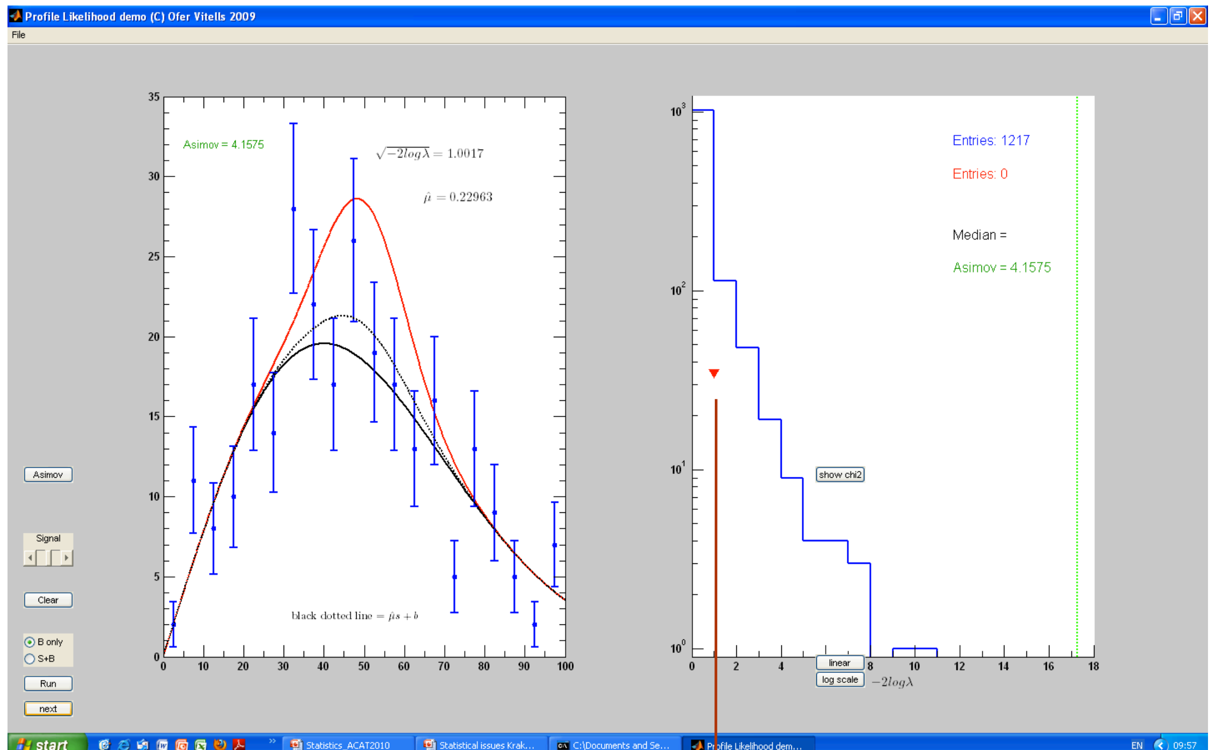

$$q_0 = 6.76 \rightarrow Z = 2.6\sigma$$

$$Z_{obs} = \sqrt{q_{0,obs}}$$

# PL: test $q_0$ under BG only ; $f(q_0|H_0)$

$$\hat{\mu} = 0.22 \rightarrow 1.1\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$



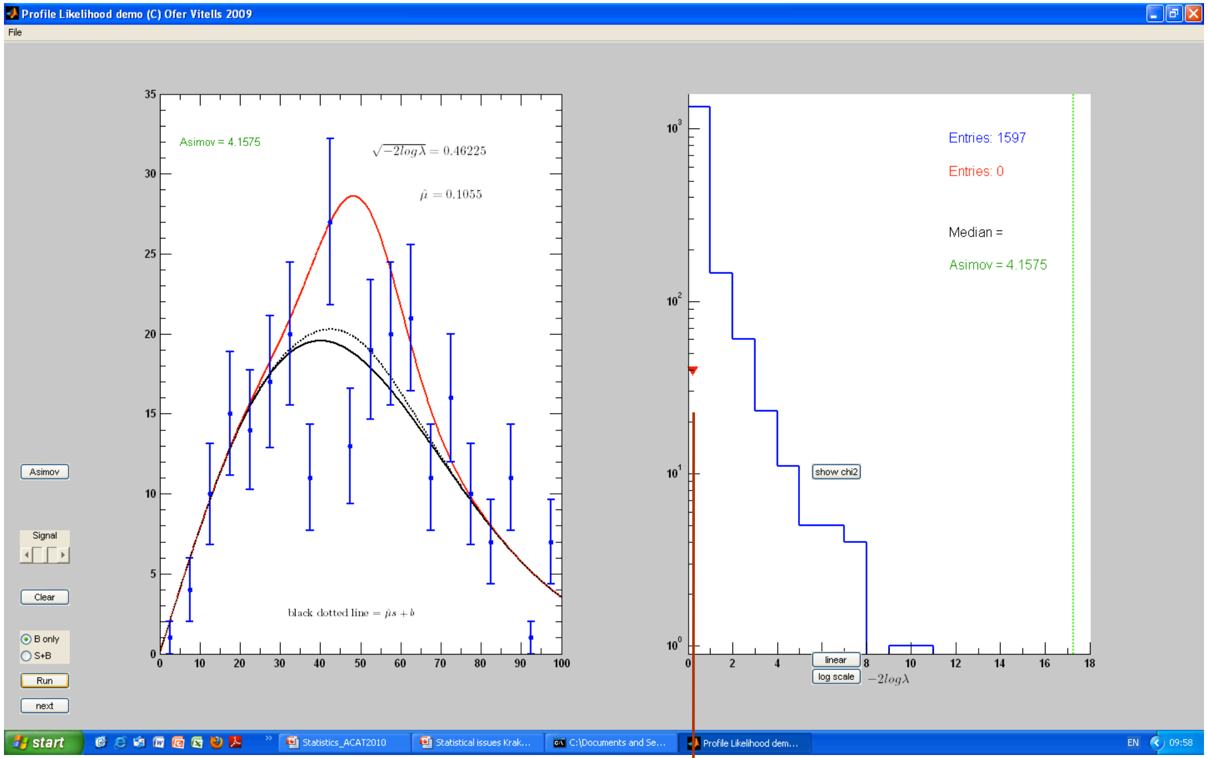

$$Z_{obs} = \sqrt{q_{0,obs}}$$

$$q_0 = 1.2 \rightarrow Z = 1.1\sigma$$

# PL: test $q_0$ under BG only ; $f(q_0|H_0)$

$$\hat{\mu} = 0.11 \rightarrow 0.4\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$



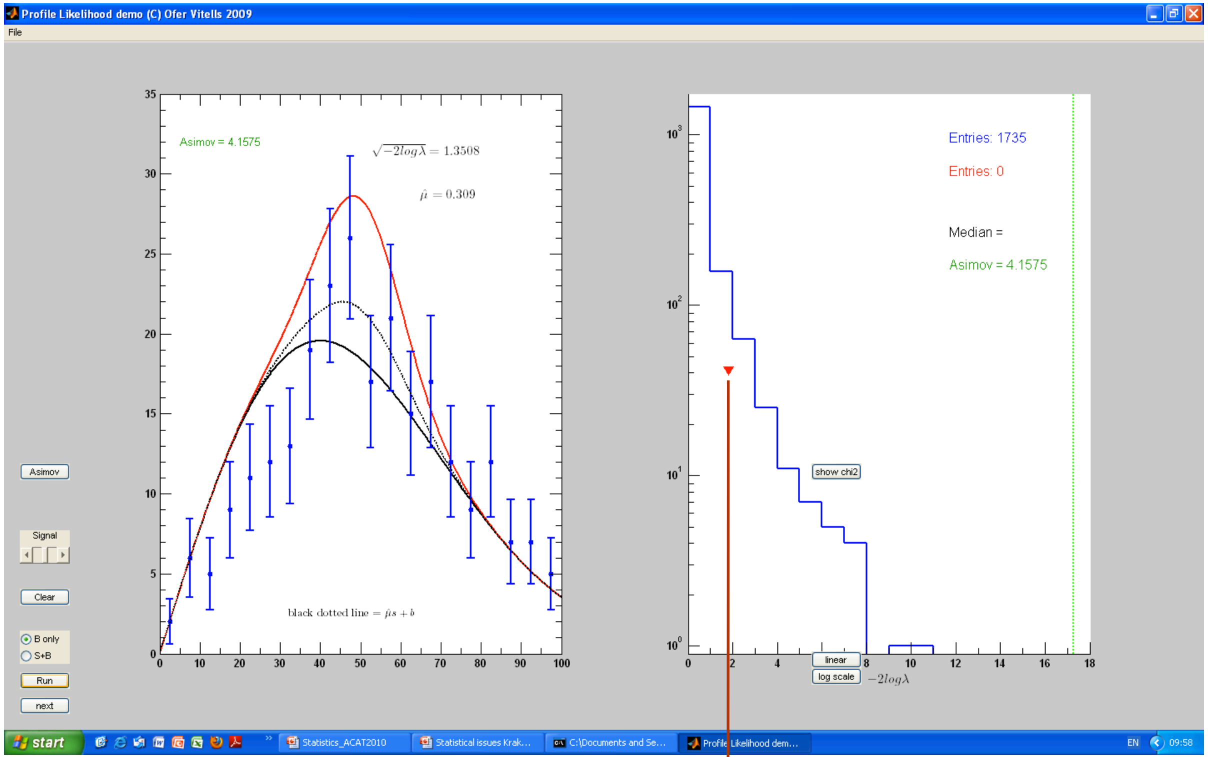

$$Z_{obs} = \sqrt{q_{0,obs}}$$

$$q_0 = 0.16 \rightarrow Z = 0.4\sigma$$

# PL: test $q_0$ under BG only ; $f(q_0|H_0)$

$$\hat{\mu} = 0.31 \rightarrow 1.35\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$



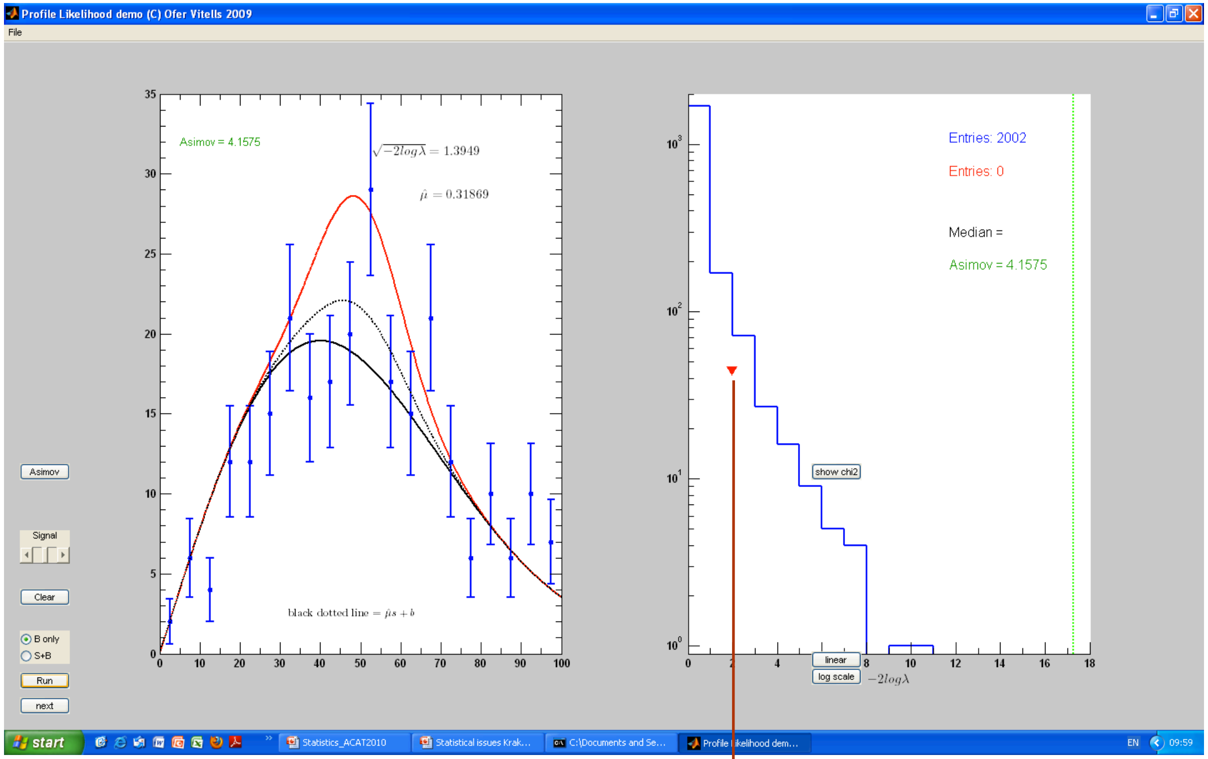

$$Z_{obs} = \sqrt{q_{0,obs}}$$

$$q_0 = 1.8 \rightarrow Z = 1.35\sigma$$

# PL: test $q_0$ under BG only ; $f(q_0|H_0)$

$$\hat{\mu} = 0.32 \rightarrow 1.39\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$



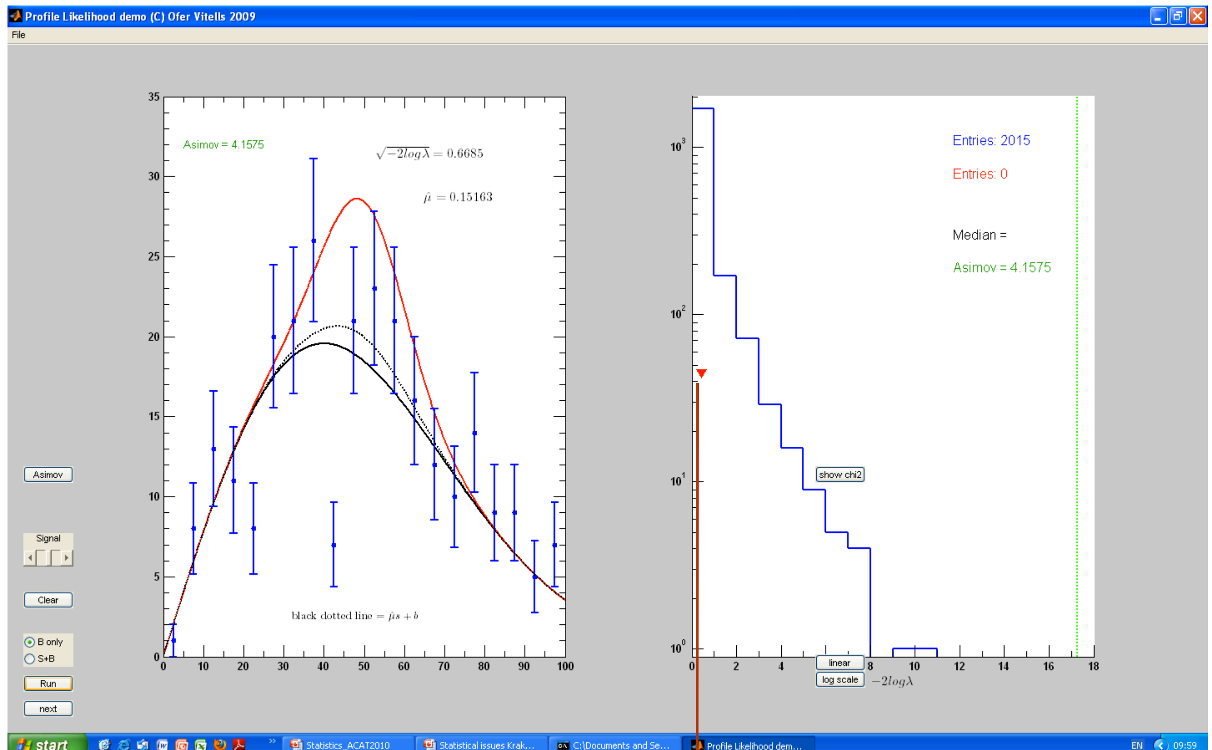

$$Z_{obs} = \sqrt{q_{0,obs}}$$

$$q_0 = 1.9 \rightarrow Z = 1.39\sigma$$

# PL: test $q_0$ under BG only ; $f(q_0|H_0)$

$$\hat{\mu} = 0.15 \rightarrow 0.66\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$




$$Z_{obs} = \sqrt{q_{0,obs}}$$

$$q_0 = 0.43 \rightarrow Z = 0.66\sigma$$

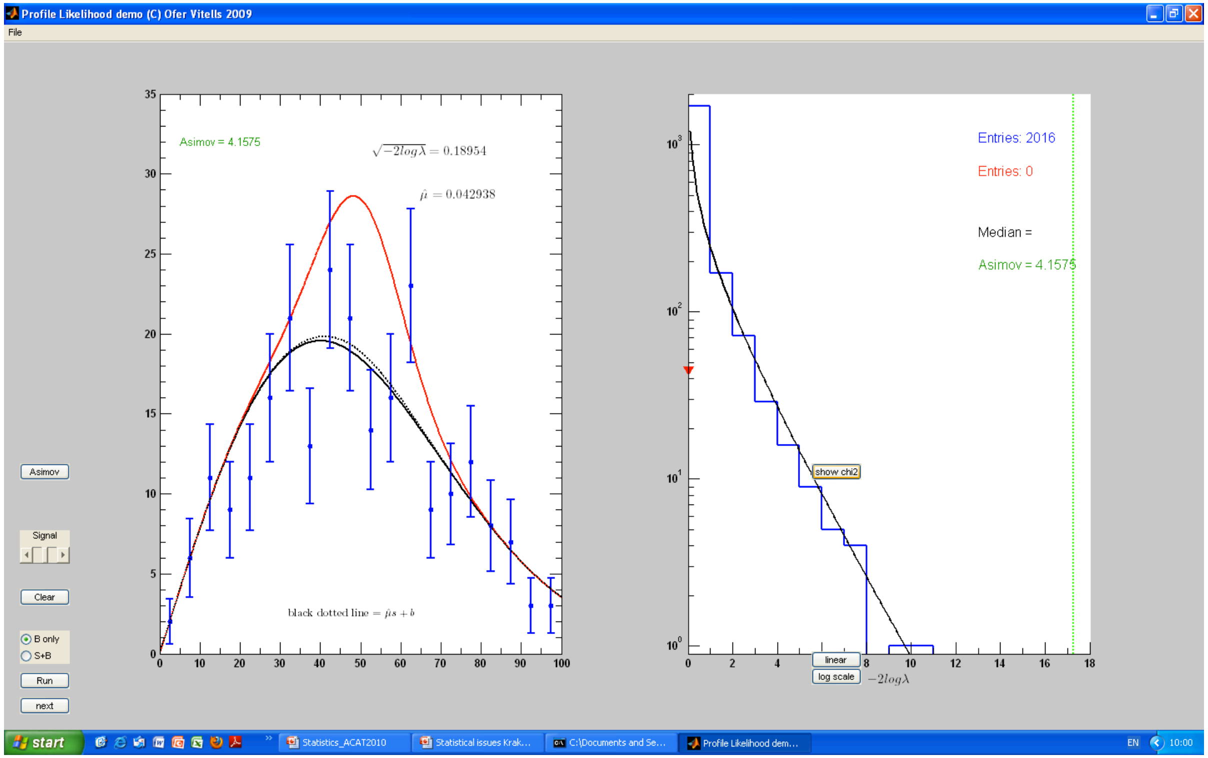
# Confirm Wilks Theorem

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$

For the test statistic

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$

$$f(q_0 \mid H_0) = \chi_1^2$$



# $\chi^2$ *Distribution*
# Chi Squared Distribution

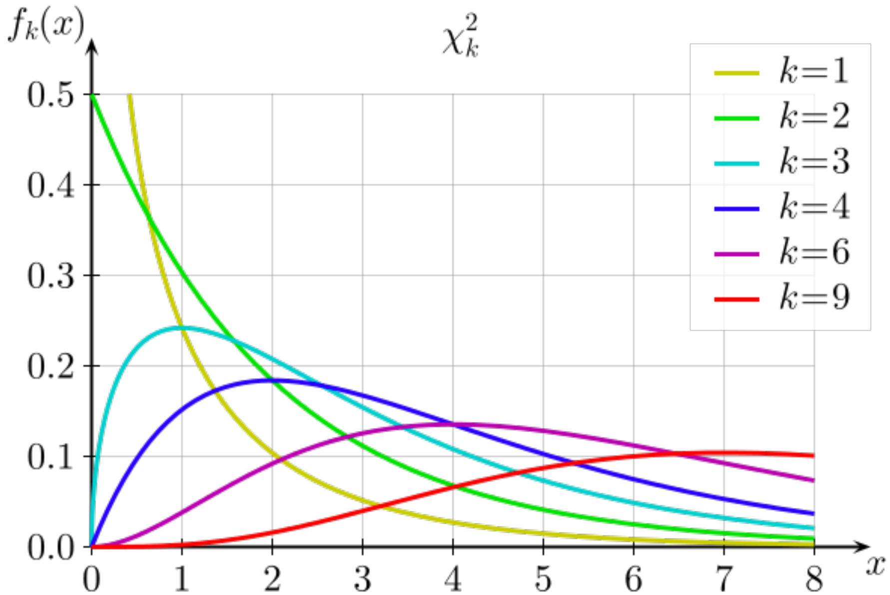
*In probability theory and statistics,*
*the* **chi – squared distribution**(*also* **chi – square** *or* $\chi^2$ – **distribution**)
*with k degrees of freedom is the distribution of a sum of the squares of kindependent standard normal random variables.*

$$\chi_k^2 = \Sigma_i^k \frac{(x_i - \mu_i)^2}{\sigma_i^2}$$

$q_0$ *distributes like a* $\chi^2$

$$\sqrt{q_{0,obs}} = Z$$

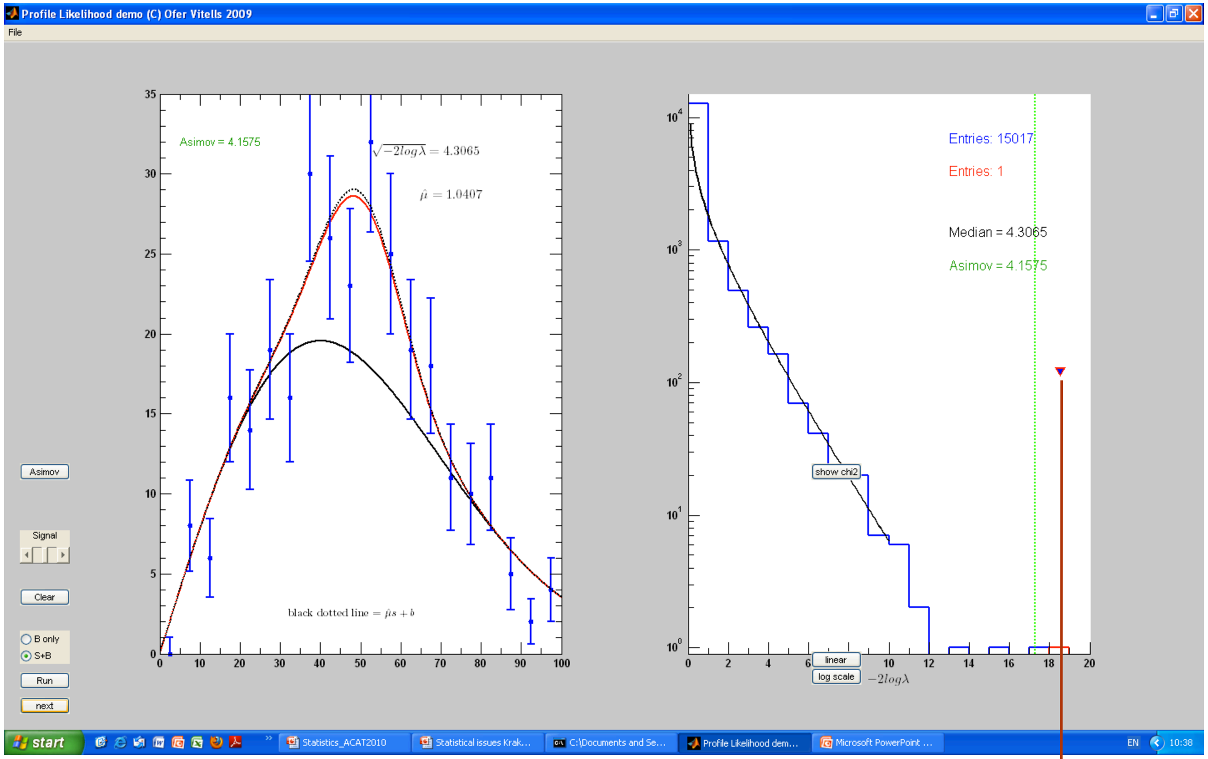
$$\chi^2 = \frac{(x-\mu)^2}{\sigma^2} = Z^2$$



# The PDF of $q_0$ under s+b experiments ($H_1$)

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)} = -2\ln\frac{L(b\,|\,H_1)}{L(\hat{\mu}s+b\,|\,H_1)}$$

$$\hat{\mu} = 1.04 \rightarrow 4.3\sigma$$

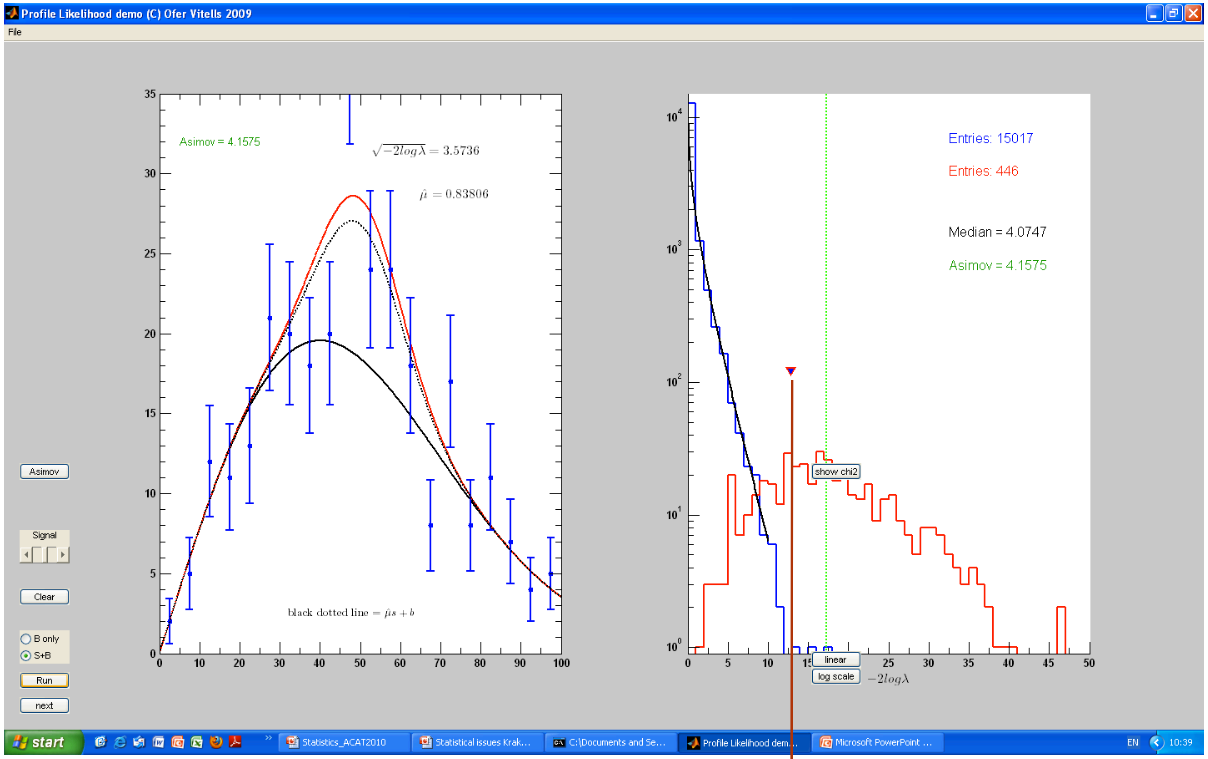



$$q_0 = 18.5 \rightarrow Z = 4.3\sigma$$

# The PDF of $q_0$ under s+b experiments ($H_1$)

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)} = -2\ln\frac{L(b\,|\,H_1)}{L(\hat{\mu}s+b\,|\,H_1)}$$

$$\hat{\mu} = 0.83 \rightarrow 3.6\sigma$$

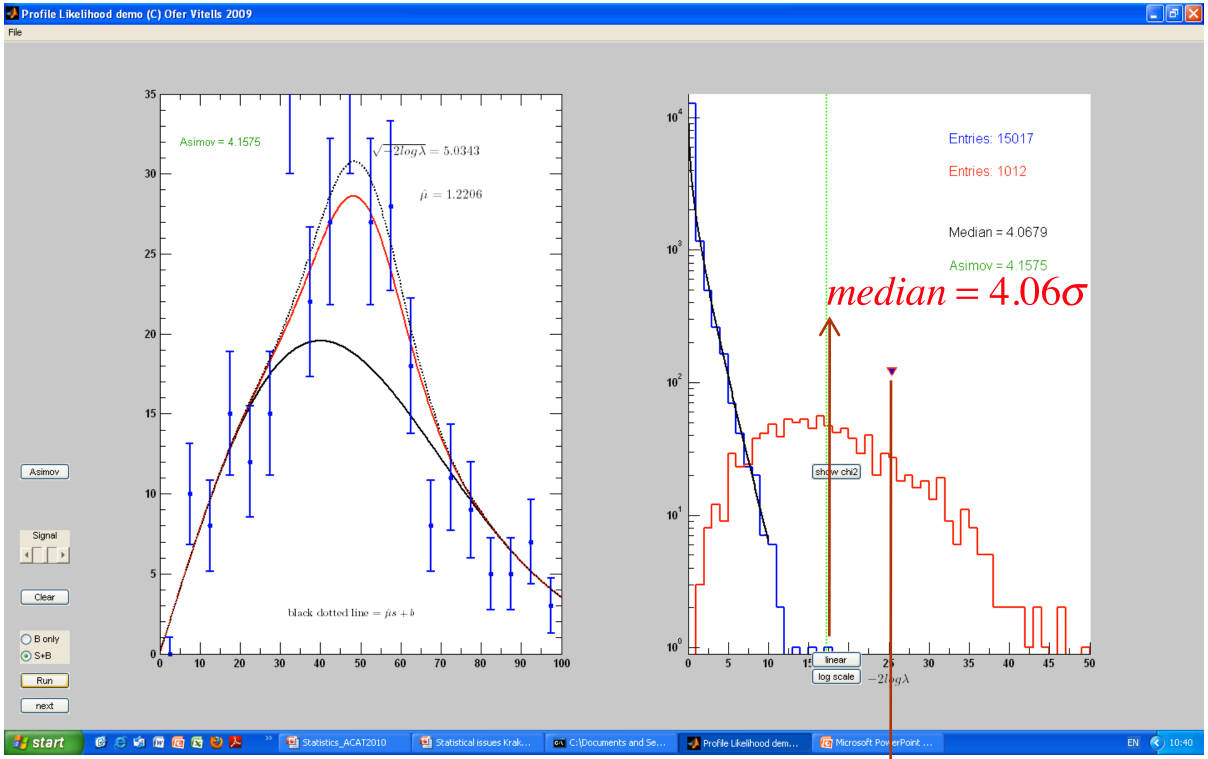



$$q_0 = 12.9 \rightarrow Z = 3.6\sigma$$

# The PDF of $q_0$ under s+b experiments ($H_1$)

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)} = -2\ln\frac{L(b\,|\,H_1)}{L(\hat{\mu}s+b\,|\,H_1)}$$

$$\hat{\mu} = 1.22 \rightarrow 5.0\sigma$$




$median = 4.06\sigma$

$$q_0 = 25 \rightarrow Z = 5.0\sigma$$

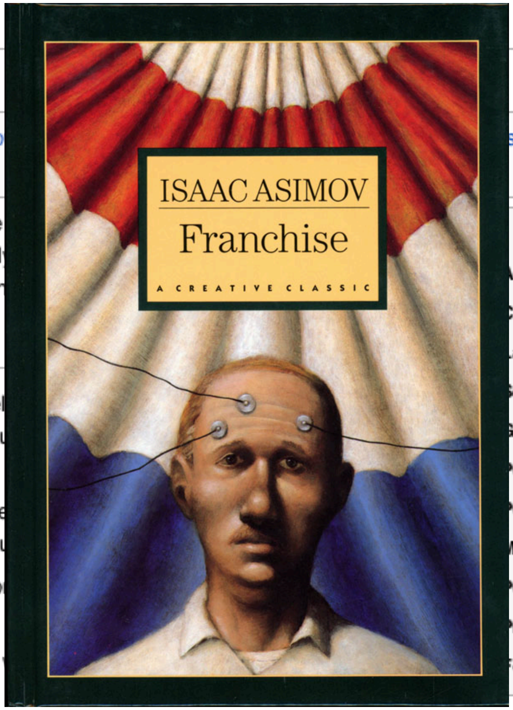
# Median sensitivity in a Click (Asimov)

## Franchise (short story)

From Wikipedia, the free encyclopedia

This article **needs additional citations for verification**. Please help impro ... s. Unsourced material may be challenged and removed. *(December 2009)*

***Franchise*** is a science fiction short story by Isaac Asimov. It first appeared in the August 1955 issue of the ... was reprinted in the collections *Earth Is Room Enough* (1957) and *Robot Dreams* (1986). It is one of a loosel ... fictional computer called Multivac. It is the first story in which Asimov dealt with computers *as computers* ar ...




| "Franchise" | |
|---|---|
| uthor | Isaac Asimo |
| ountry | United State |
| anguage | English |
| eries | Multivac |
| enre(s) | science ficti |
| ublished in | *If* |
| ublisher | Quinn Publis |
| edia type | Magazine |
| ublication date | August 195 |
| receded by | "Question" |
| ollowed by | "The Dead P |

### Plot summary

In the future, the United States has converted to an "electronic democracy" where the computer Multivac sel ... questions. Multivac will then use the answers and other data to determine what the results of an election wo ... to be held.

The story centers around Norman Muller, the man chosen as "Voter of the Year" in 2008. Although the law re ... not sure that he wants the responsibility of representing the entire electorate, worrying that the result will be ...

However, after 'voting', he is very proud that the citizens of the United States had, through him, "exercised o ... a statement that is somewhat ironic as the citizens didn't actually get to vote.

The idea of a computer predicting whom the electorate would vote for instead of actually holding an election ... correct prediction of the result of the 1952 election.

### Influence

The use of a single representative individual to stand in for the entire population can help in evaluating the sensitivity of a statistical method. *Franchise* was cited as the inspiration of the data set", where an ensemble of simulated experiments can be replaced by a single representative one. [1]

### References


1. ^ G. Cowan, K. Cranmer, E. Gross, and O. Vitells (2011). "Asymptotic formulae for likelihood-based tests of new physics". *Eur.Phys.J.* **C71**: 1554. DOI:10.1140/epjc/s10052-011-1554-0.

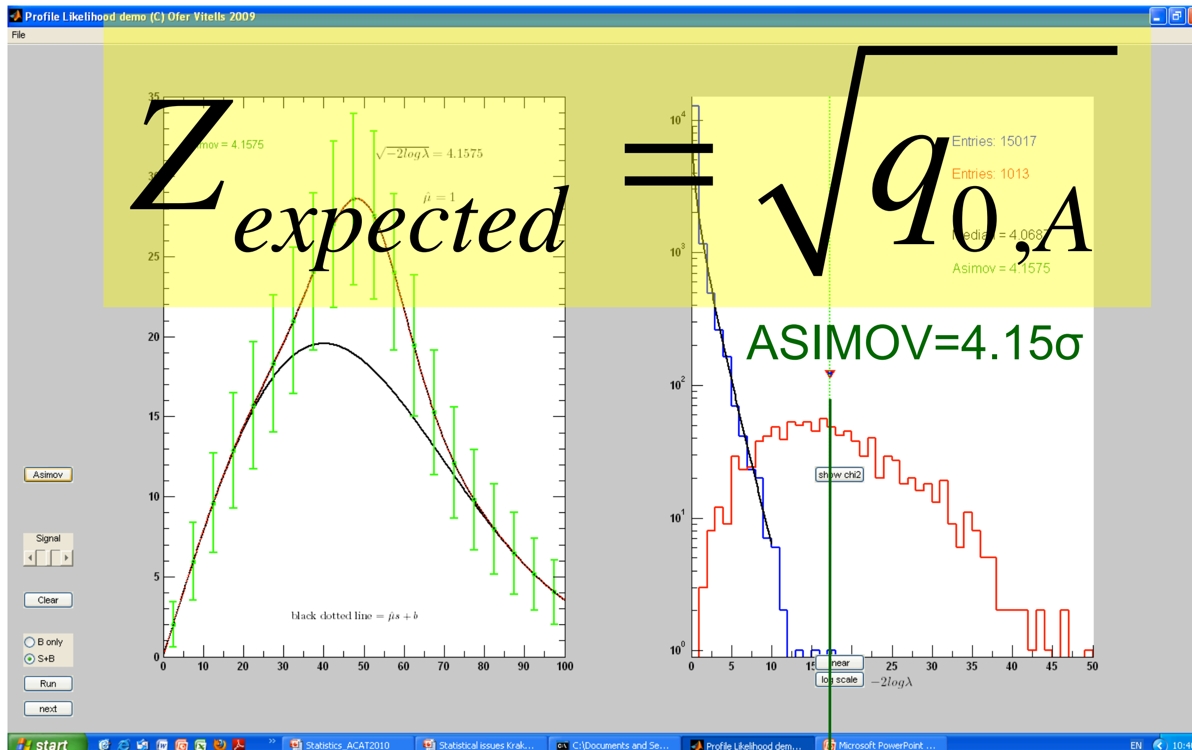

CCGV ref

# The Median Sensitivity (via ASIMOV)

To estimate the median sensitivity of an experiment (**before looking at the data**), one can either perform lots of s+b experiments and estimate the median $t_{0,med}$ or evaluate $t_0$ with respect to a representative data set, the ASIMOV data set with $\mu=1$, i.e. $x=s+b$

$$\hat{\mu} = 1.00 \rightarrow 4.15\sigma$$

$$Z_{expected} = \sqrt{q_{0,A}}$$




$$\boxed{q_A = 17.22 \rightarrow Z_A = 4.15}$$

$$q_{o,med} \simeq q_0(\hat{\mu} = \mu_A = 1) = -2\ln\frac{L(b \mid x = x_A = s+b)}{L(\hat{\mu}s + b \mid x = x_A = s+b)} = -2\ln\frac{L(b)}{L(1 \cdot s + b)}$$

# Asymptotic Distributions – a Teaser

Tossing Monte Carlos to get the test statistic distribution functions (PDF) is sometimes beyong the experiment technical capability.

Knowing both PDF

$$f(q_{null} \mid H_{null})$$
$$f(q_{null} \mid H_{alternate})$$

enables calculating both the observed and expected significance (or exclusion) without a single toy….