Cosmology II

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\[ m - M = -2.5 \log \left( \frac{\phi(z)}{\phi(10 \text{ pc})} \right) \]

\[ m - M = 25 + 5 \log \left( \frac{3000 \frac{D_L}{D_{H_0}}} {3} \right) - 5 \log h \]

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**I. Introduction**

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**Recap**

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**Courtesty Peter&Uzan textbook**

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|Ω − 1| thus gives the fractional contribution of the curvature term in the Friedmann equation. If the universe is spatially flat, \( K = 0 \), then RHS = 0 in the above equation, so Ω = 1 at all times.

If \( \left| \frac{Kc^2}{a^2} \right| \ll \frac{8\pi G\rho}{3} \), then Ω ≈ 1, and the universe behaves like a spatially flat universe around that time. Since \( \rho_m \propto a^{-3} \) and \( \rho_r \propto a^{-4} \), the total density term \( \frac{8\pi G\rho}{3} \) grows more rapidly than the curvature term \( \left| \frac{Kc^2}{a^2} \right| \) as \( a \to 0 \), so Ω → 1 at early times even if \( \Omega_0 \neq 1 \).
I. Introduction

The Universe is dominated by DE

How weird!!

\[ H^2 = \frac{8 \pi G \rho}{3} + \frac{\Lambda}{3} - \frac{K}{a^2} \]

\[ H^2 \approx \frac{\Lambda}{3} \approx \text{cst} \]

\[ H \approx \sqrt{\frac{\Lambda}{3}} \approx \text{cst} \]

\[ \dot{a} \approx \sqrt{\frac{\Lambda}{3}} \approx \text{cst} \]

\[ a \propto e^{\sqrt{\frac{\Lambda}{3}} t} \]

Constant energy density leads to an accelerated expansion of the Universe
5% matter; 70% dark energy

what is the rest?
Baryons can be dark but cannot be (all) the dark matter.

Only ~5% max of baryons

Consistent with CMB!

Where are the baryons?
5% matter; 70% dark energy

what is the rest?
I. Introduction

J. Oort, 1932

Doppler redshift values of stars moving near the galactic plane;
The Galaxy needs to be twice as massive to prevent stars to escaping.

F. Zwicky 1933

more mass in the Coma Cluster than is visible

ON THE MASSES OF NEBULAE AND OF CLUSTERS OF NEBULAE

F. ZWICKY

The Coma cluster contains about one thousand nebulae. The average mass of one of these nebulae is therefore

\[ M > 9 \times 10^{41} \text{ gr} = 4.5 \times 10^{50} M_\odot. \]

(36)

Inasmuch as we have introduced at every step of our argument inequalities which tend to depress the final value of the mass \( M \), the foregoing value (36) should be considered as the lowest estimate for the average mass of nebulae in the Coma cluster. This result is somewhat unexpected, in view of the fact that the luminosity of an average nebula is equal to that of about 8.5 \( \times \) 10^7 suns. According to (36), the conversion factor \( \gamma \) from luminosity to mass for nebulae in the Coma cluster would be of the order

\[ \frac{\text{Mass}}{\text{Light}} = \gamma = 500, \]

(37)

as compared with about \( \gamma' = 3 \) for the local Kapteyn stellar system.

based on 21 radial velocities of galaxies in the Coma cluster
I. Introduction

The rotation of galaxies was discovered in 1914 — Slipher (1914)

(many people contributed!)

Freeman (1970) for M33 and NGC 300:
rotation curve peaks at the edge of the optical disk
so ~ 1/3 of the mass outside the optical radius.

Shostak & Rogstad (1973),
Seielstad & Wright (1973).
M31: (Roberts 1975a,
Roberts & Whitehurst 1975);
Final straw: Bosma (1978)

IV. Towards modern Cosmology

Rotation curves of galaxies

1970ApJ...159..379R
We need DM to explain the flat rotation curves far from the GC

\[ v_c^2 = \frac{G M(r)}{r} \quad M(r) = \int 4\pi^2 \rho(r) \, dr \]

But the highest mass density would be in the inner part of the galaxy...
I. Introduction

EROS and MACHO

(La Silla vs Mount Stromlo Observatory, Australia)

IV. Towards modern Cosmology

Fig. 3.— Halo fraction upper limit (95% c.l.) versus lens mass for the five EROS models (top) and the eight MACHO models (bottom). The line coding is the same as in Figure 2.

Nothing found (so far but ...) they can’t be all the missing mass well not in that range ...
Courtesy: EROS experiment. They were looking for "brown dwarfs" or "MACHOs" which belong gravitationally to our Galaxy. This was made possible by their gravitational microlensing effects on stars in the Magellanic Clouds (two dwarf galaxies, Milky Way satellites).
I. Introduction

IV. Towards modern Cosmology

Dark Matter is everywhere

But what is the DM?
Gravitational lensing evidence...

Reconstruction (lower left) of the brightest galaxy whose image has been distorted by the gravity of a distant galaxy cluster.

The small rectangle in the center shows the location of the background galaxy on the sky if the intervening galaxy cluster were not there. The rounded outlines show distinct, distorted images of the background galaxy resulting from lensing by the mass in the cluster. The image at lower left is a reconstruction of what the lensed galaxy would look like in the absence of the cluster, based on a model of the cluster's mass distribution derived from studying the distorted galaxy images.
+ Weak lensing...

Courtesy Michael Sachs
More lensing evidence...

X-ray emitted by gas  
(Thomson interactions, Bremsstrahlung,...)  
But the gravitational potential is dominant in the blue region where no light is emitted
5% matter; 70% dark energy

what is the rest?

well seems a sort of dark matter
Now that we know the ingredients in the Universe, how do we form objects like this?
It is argued that the expanding universe is unstable against the growth of gravitational perturbations. The argument is directed toward two problems, the physical conditions in the early, highly contracted phase of the expanding universe, and the formation of the galaxies.


Primordial fluctuations in the Early Universe grow under gravity (Peebles, 66)

\[ I(x) \propto \frac{x^3}{e^x - 1} \quad x = \frac{h \nu}{\kappa T} \quad \frac{\Delta T}{T} \simeq 10^{-5} \]
more matter; will accrete and clump under gravity

less matter; will become even emptier with gravity
baryonic fluctuations do not survive the baryon scattering off the photon background. (Question first asked by Misner for neutrinos)

letters to nature

Nature 215, 1155 - 1156 (09 September 1967); doi:10.1038/2151155a0

Fluctuations in the Primordial Fireball

JOSEPH SILK

Harvard College Observatory, Cambridge, Massachusetts.

ONE of the overwhelming difficulties of realistic cosmological models is the inadequacy of Einstein’s gravitational theory to explain the process of galaxy formation1-6. A means of evading this problem has been to postulate an initial spectrum of primordial fluctuations7. The interpretation of the recently discovered 3° K microwave background as being of cosmological origin8,9 implies that fluctuations may not condense out of the expanding universe until an epoch when matter and radiation have decoupled4, at a temperature $T_D$ of the order of 4,000° K. The question may then be posed: would fluctuations in the primordial fireball survive to an epoch when galaxy formation is possible?
V. Primordial Fluctuations

Was Peebles right?

![Image of cosmic microwave background spectrum from COBE]

Theory and observation agree

<table>
<thead>
<tr>
<th>Waves / centimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 1 2 3 4 5 6 7 8</td>
</tr>
<tr>
<td>1.2 1.0 0.8 0.6 0.4</td>
</tr>
</tbody>
</table>

COBE    WMAP    Planck

courtesy wikipedia!

YES!

so either Silk wrong or more matter than baryons
I. Introduction

V. Primordial Fluctuations

All regions of the sky have a temperature around 2.7k! How come?

\[ d(t) = \frac{c}{H(t)} \]

hot spot

cold spot

>> galaxy scale (1 Mpc ~ 10^27 cm)

All regions of the sky have a temperature around 2.7k!
I. Introduction

All regions of the sky have a temperature around $2.7k \pm 10^{-5}$.

How come such a tiny difference on such gigantic scales?

V. Primordial Fluctuations

All regions of the sky have a temperature around $2.7k + 10^{-5}$.

How come such a tiny difference on such gigantic scales?
I. Introduction

V. Primordial Fluctuations

Horizon problem

start with small and dense Universe phase of extremely rapid expansion then normal phase of expansion (H)

What is driving the expansion? Potentially a scalar field (like Higgs)

\[ p = -\rho c^2 = -V(\phi) \]

if \( V \sim \text{constant} \)

\[ H^2(a) \approx \frac{8\pi G}{3} \frac{V(\phi)}{c^2} \equiv H_\phi^2 \approx \text{const} \]

\[ H(a) = \frac{\dot{a}}{a} = H_\phi \]

now integrate

\[ a(t) \propto e^{H_\phi t} \]
II CMB & structure formation
I. Decoupling

moment when photons are free
(elastic scattering rate with photons < Hubble rate)

High energy = Compton interactions
Low energy = Thomson interactions

\[ \Gamma_{\gamma-e,p} \approx \sigma_T c n_e = H \quad \Rightarrow \quad \Gamma_{\gamma-e,p} \approx \sigma_T c \times 10^{-9} n_{\gamma} = H \]

\[ \sigma_T c \times 10^{-9} n_\gamma(T_0) a^{-3} = H_\alpha \quad a^{-1/\alpha} \]

\[ a = \left( \frac{\sigma_T c \times 10^{-9} n_\gamma(T_0)}{H_\alpha} \right)^{1/(3-1/\alpha)} \]

\[ T_{\text{dec}(\gamma)} \approx T_0 \left( \frac{7 \times 10^{-21} \text{ s}^{-1}}{H_\alpha} \right)^{-2/3} \approx 100 \text{ K.} \]
II. Recombination

\[ e^- + p \leftrightarrow H + \gamma \quad 2e^- + H_{e}^{++} \leftrightarrow He + \gamma. \]

The one-arrow process \( e^- + p \rightarrow H + \gamma \) is referred to recombination. The neutral Hydrogen formed at BBN is indeed destroyed by the reverse reaction

\[ H + \gamma \rightarrow e^- + p \]

till the photon energy becomes too low to break the neutral Hydrogen bound state. At that stage the electrons can recombine with protons to form again neutral Hydrogen. The binding energy of the Hydrogen atom is about 13.6 eV so a crude estimate shows that electrons must be free until \( E_{\gamma} = 13.6 \) eV, which translates into a temperature of

\[ 3K T = E = 13.6 \text{ eV} \Rightarrow T \sim 5 \times 10^4 \text{ K.} \]

This is too crude however because this assumes only one transition from the fundamental to fully ionised state. In reality one will pass through all energy levels which are given by

\[ E_n = -\frac{E_B}{n^2} \]

with \( E_B = 13.6 \) eV. The transition from 1 to 2 requires a photon energy of about \( E_2 - E_1 = -\frac{E_B}{2^2} + \frac{E_B}{1^2} = 10.2 \text{ eV} \)
II. Recombination 2.0

But one should also remember that there are many more photons than protons and electrons. Hence dissociation is very efficient. Typically the number of Hydrogens ions which are expected to get sufficiently excited to not recombine is given by

\[ n_{p^+} = n_\gamma (E > 10.2 \text{ eV}) \]

However this also assumes that one photon of such energy causes the Hydrogen to reach the first excited state and causes immediate ionisation. In reality this is again too crude and one needs to follow the next steps to get a better estimate of the moment at which all the electrons and protons would ”recombine” to form neutral Hydrogen that will not be dissociated any longer.

Let us start by describing the distribution of charges in the medium. The number of free electrons is governed by

\[ \frac{dn_e}{dt} = -3 \, H \, n - \langle \sigma v \rangle \left( n_e \, n_p - n_{e,eq} \, n_{p,eq} \right) \]

so

\[ \frac{dn_e}{dt} = -3 \, H \, n - \langle \sigma v \rangle \left( n_e \, n_p - n_{H} \right) \]
The departure from equilibrium happens when \( n_H > N_{H,eq} \) or equivalently \( n_e n_p > n_{e,eq} n_{p,eq} \) while, before departure, the relation

\[
\frac{n_e n_p}{n_H} = \frac{n_{e,eq} n_{p,eq}}{n_{H,eq}}
\]

is satisfied. The latter can be rewritten using the equilibrium number densities

\[
n_{i,eq} = \left( \frac{m_i T_i}{2\pi} \right)^{3/2} e^{-\frac{m_i}{T_i}}
\]

as

\[
\frac{n_e n_p}{n_H} = \frac{n_{e,eq} n_{p,eq}}{n_{H,eq}} = \left( \frac{m_e T}{2\pi} \right)^{3/2} e^{-\frac{(m_e + m_p - m_H)}{T}} \quad (4.1)
\]

(where we neglect the electron mass in the ratio \( m_p/m_H \) and therefore took \( m_p/m_H = 1 \) leading to

\[
\frac{n_e n_p}{n_H} = \left( \frac{m_e T}{2\pi} \right)^{3/2} e^{-\frac{11.6 \text{ eV}}{T}}
\]

if we consider the fully ionised Hydrogen. The fraction of ionised Hydrogen is given by the number of free electrons

\[n_e = X_e n_B\]

with \( n_B \) the number density of Baryons. Therefore we can define this fraction as

\[X_e = \frac{n_e}{n_e + n_H} = \frac{n_p}{n_p + n_H},\]

which is such that

\[1 - X_e = \frac{n_H}{n_e + n_H},\]

and therefore implies

\[n_e = n_p.\]

The number density of electrons after partial or complete recombination is then given by

\[n_{e,p} = n_H \frac{X_e}{(1 - X_e)}\]

\[
\frac{n_e n_p}{n_H} = \left( \frac{m_e T}{2\pi} \right)^{3/2} e^{-\frac{\epsilon_H}{T}}
\]

(where we now account for a partially ionised Hydrogen, i.e. the first excited state of Hydrogen, with \( \epsilon_H = 10.2 \text{ eV} \) is in fact

\[
\frac{X_e n_p}{(1 - X_e)} = \left( \frac{m_e T}{2\pi} \right)^{3/2} e^{-\frac{\epsilon_H}{T}}
\]

or similarly

\[
\frac{X_e n_e}{(1 - X_e)} = \left( \frac{m_e T}{2\pi} \right)^{3/2} e^{-\frac{\epsilon_H}{T}}.
\]

This expression can be finally written, using \( X_e = \frac{n_e}{n_e + n_H} \) \( \Rightarrow n_e = X_e (n_e + n_H) \), as

\[
\frac{X_e^2}{(1 - X_e)} = \frac{1}{(n_e + n_H)} \left( \frac{m_e T}{2\pi} \right)^{3/2} e^{-\frac{\epsilon_H}{T}}
\]

which is known as the Saha equation.

\[
\frac{dn_e}{dt} = -3 H n_e - \langle \sigma v \rangle (n_e n_p - n_H)
\]

\[
\frac{dX_e}{dt} = \langle \sigma v \rangle \left( (1 - X_e) \left( \frac{m_e T}{2\pi} \right)^{3/2} e^{-\frac{\epsilon_H}{T}} - X_e^2 n_B \right)
\]
I. Decoupling 2.0

moment when photons are free
(elastic scattering rate with photons < Hubble rate)

\[
\frac{dn_e}{dt} = n_B h s v (1-X_e) \sqrt{\frac{m_e}{T}} \left( \frac{2}{p} \right)^{3/2} e H_T^2 n_B!
\]

Calculations then lead to

\[X_e \rightarrow 0 \quad \text{when } z \approx 1100,\]

corresponding to a temperature of

\[T = T_0 (1+z) \approx 3 \times 10^3 \text{ K}.\]

Since the recombination temperature is much higher than the photon and electron decoupling temperature, the decoupling of both species with each other happens at recombination.

We have seen that photons and electrons decouple when the electrons recombine with proton to form neutral Hydrogen atoms, that is at recombination. Consequently, before \(T \approx 3000 \text{ K}\), the medium is opaque while after \(T \approx 3000 \text{ K}\), photons can freely propagate and reach us.
photon from Early Universe

Last scattering surface

\[ T_{\gamma_{\text{CMB}}} \sim 2.710^{-4} \text{ eV} \]
I. Decoupling 2.0

$$D_{A*} = \frac{1}{(1 + z_*)} f_k \left( \int_{z=0}^{\infty} \frac{dz}{H_0 \sqrt{\Omega_r + \Omega_m + \Omega_\Lambda + \Omega_k (1 + z)^2}} \right)$$

$$D_{A*}^{\text{obs}} = \frac{l_*}{\Theta_*} = 17 \text{ Gpc.}$$

$$D_A(z) = \frac{D_L}{(1 + z)^2}$$
Our vision of the Universe
Radiation  Matter  Energy

BBN  
\[ T \sim \text{MeV} \]
\[ \nu e^- \xrightarrow{\checkmark} \nu e^- \]

Equality  matter-radiation

Decoupling  
\[ \gamma e^- \xrightarrow{\checkmark} \gamma e^- \]

structures form

\[ a \sim 10^{-10} \]
\[ a \sim 10^{-4} \]
\[ a \sim 10^{-3} \]
Anisotropies of temperature

A bit less particles

A bit more particles
Anisotropies of temperature

Not perfect isotropy \( \rightarrow \) anisotropies

Theory of perturbations

\[
T_{\gamma_{\text{CMB}}} \sim 2.710^{-4} \text{ eV} + \delta T
\]

Distribution of matter: over densities and under densities

\[
\rho_r \propto T^4 \quad \rho_m \propto T^3 \quad \frac{\delta \rho_r}{\rho_r} \propto \frac{\delta \rho_m}{\rho_m} \propto \frac{\delta T}{T}
\]

Black Body spectrum

\[
B(v) = \frac{2h v^3}{c^2} \frac{1}{e^{h v/(kT)} - 1}
\]
$$f(E_{\text{CMB}}) = \left( e^{\frac{E_{\text{CMB}}}{T}} - 1 \right)^{-1}$$

with

$$E = \vec{E} \left( 1 + \frac{\vec{v}.\vec{n}}{c} \right)$$

so

$$T = \bar{T} \left( 1 - \frac{\vec{v}.\vec{n}}{c} \right)$$

$$\Delta T = \frac{T - \bar{T}}{\bar{T}} = \frac{\vec{v}.\vec{n}}{c} \simeq 10^{-3}$$

$$\frac{\vec{v}.\vec{n}}{c} = \frac{v}{c} \cos \theta$$

$$Y_0^0 = \text{cst} \quad Y_1^0 = \cos \theta$$
Anisotropies of temperature

\[ Y_0^0 = \text{cst} \]

\[ Y_1^0 = \cos \theta \]

\[ Y_l^m \]

The higher \( l \), the more details you get

\( l > 2000 \)
What is the Physics of the anisotropies?
Anisotropies of temperature

BBN
T \sim \text{MeV}
\nu e^- \rightarrow \nu e^-

Equality
matter-radiation

Decoupling
\gamma e^- \rightarrow \gamma e^-

a \sim 10^{-10}

a \sim 10^{-4}

a \sim 10^{-3}

e-, photons, ...

pressure from interactions
versus gravitation
in a matter dominated Universe

W. Hu

CMB & LSS
Anisotropies of temperature

pressure from interactions versus gravitation in a matter dominated Universe

empty regions are not growing by gravity and they get destroyed by the interactions but denied regions too…

(Silk damping)
Anisotropies of temperature

\[ T(\vec{n}) = \bar{T} (1 + \Theta(\vec{n})) \]
\[ \Theta(\vec{n}) = \frac{(T(\vec{n}) - \bar{T})}{\bar{T}} \equiv \frac{\Delta T}{\bar{T}} \]

\[ \Theta_{lm} = \int \frac{d^3 n}{(2\pi)^3} \int \frac{d^3 k}{(2\pi)^3} e^{i \vec{k} \cdot \vec{n} D_{a*}} \Theta(\vec{k}) Y_{lm}(\vec{n}) \]

\[ e^{i \vec{k} \cdot \vec{n} D_{a*}} = 4\pi \sum_{l=0}^{\infty} \frac{a_l}{(2l+1)} j_l(k D_{a*}) \sum_m (-1)^m Y_l^m(\vec{k}) Y_l^{-m}(\vec{n}) \]

\[ \Theta_{lm} = 4\pi \int \frac{d^3 k}{(2\pi)^3} i^l j_l(k D_{a*}) \Theta(\vec{k}) Y_l^m(\vec{k}) \]

Bessel functions — damped oscillations

\[ \langle \Theta_{l' m'}^* \Theta_{lm} \rangle = 4\pi \delta_{l'l} \delta_{m'm} \int d\ln k \ j_l(k D_{a*})^2 \ \Delta_T^2(k) \]
\[ = C_l^T \delta_{l'l} \delta_{m'm} \]
\[ C_l^T \approx 4\pi \frac{\Delta_T^2(k)}{2\ l \ (l+1)} \]

the coefficients of the Ylm decomposition
Anisotropies of temperature

\[ \langle \Theta_{l'm'} \Theta_{lm} \rangle = 4\pi \delta_{l'l} \delta_{m'm} \int d\ln k \ j_l(kD_{\alpha'})^2 \Delta_T^2(k) \]

\[ = C_l^T \delta_{l'l} \delta_{m'm} \]

Planck Collaboration: The Planck mission 2015

Planck satellite
the red curve = baryons only

the suppression of small-scales is indicative of the presence of baryons

Therefore there must be more than the baryons
END of LECTURE 2