Cosmology II

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IPPP, Durham

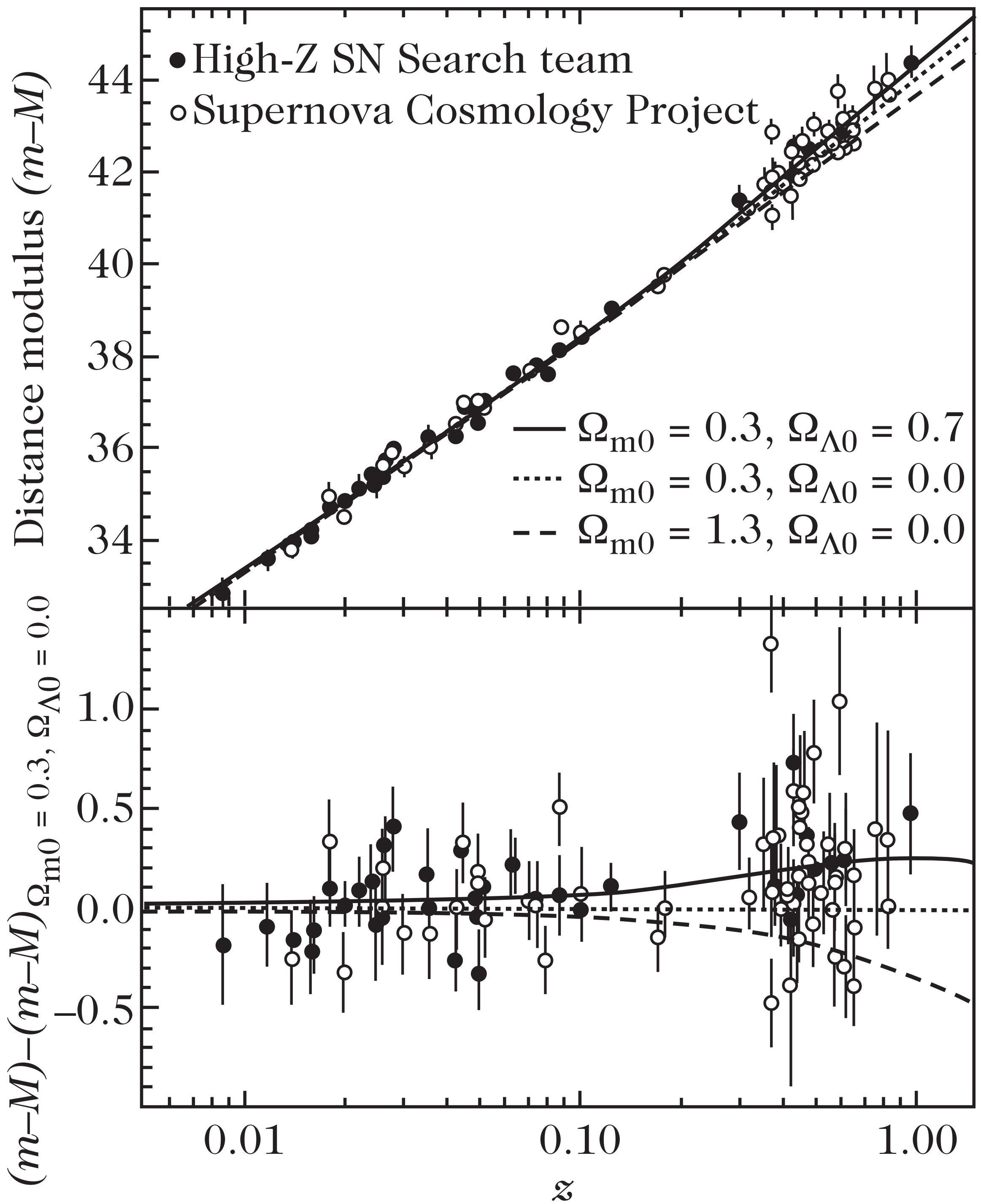
LAPTH, Annecy

Evora, CERN School
2017

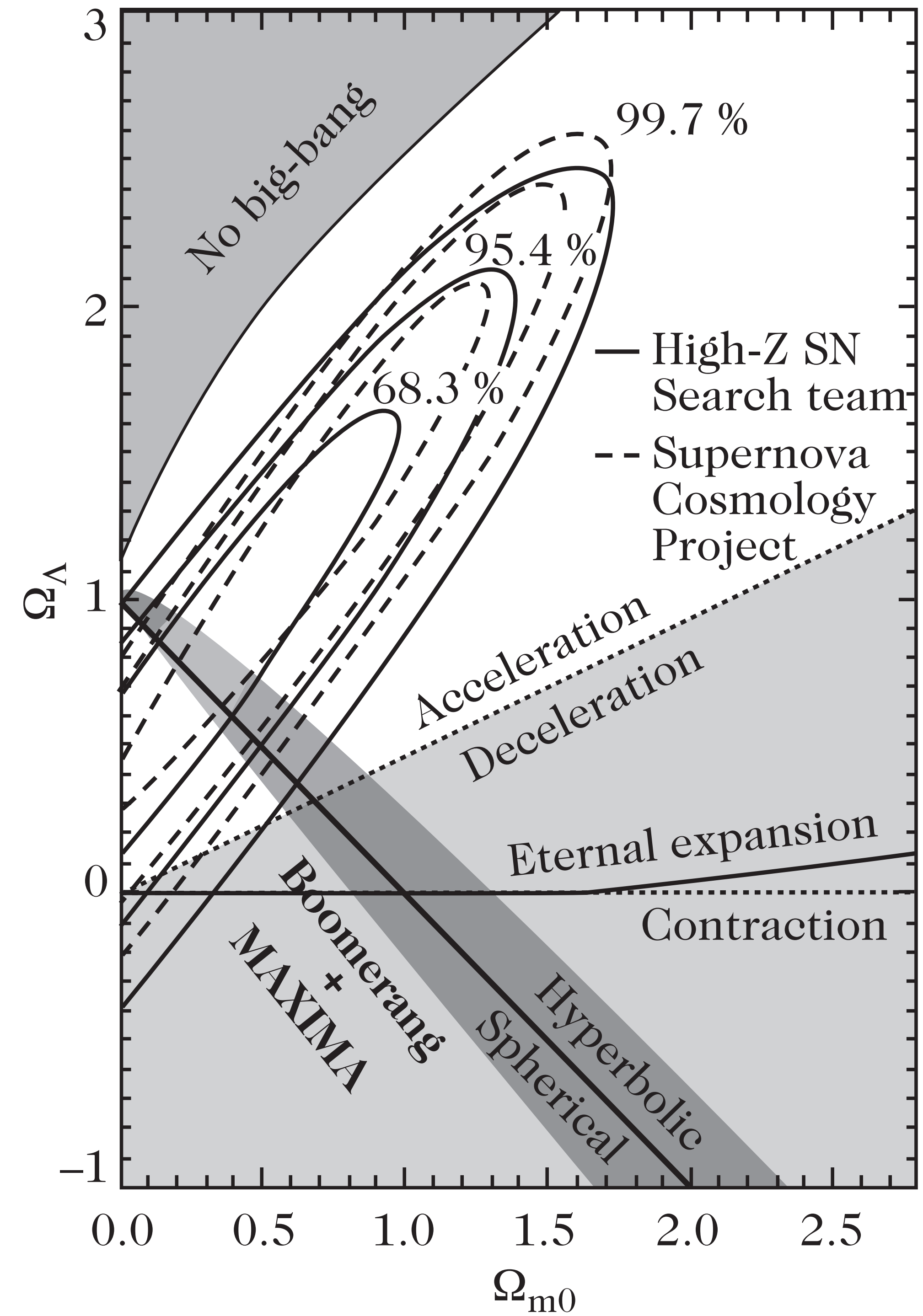
Recap

$$m - M = -2.5 \log \left[\phi(z)/\phi(10\,\mathrm{pc}) \right]$$

$$m - M = 25 + 5 \log \left| 3000 \frac{D_\mathrm{L}}{D_{\mathrm{H}_0}} \right| - 5 \log h$$



Courtesy Peter&Uzan textbook



The Universe is flat. How weird!!

$$\Omega_K = \frac{H_0^2}{H^2}\frac{\Omega_{K,0}}{a^2} \qquad K = \frac{(\Omega_0 - 1)H_0^2}{c^2}$$

$|\Omega - 1|$ thus gives the fractional contribution of the curva ture term in the Friedmann equation. If the universe is *spatially flat*, $K = 0$, then $RHS = 0$ in the above equation, so $\Omega = 1$ at all times.

If $|\frac{Kc^2}{a^2}| \ll \frac{8\pi G\rho}{3}$, then $\Omega \approx 1$, and the universe behaves like a spatially flat universe around that time. Since $\rho_m \propto a^{-3}$ and $\rho_r \propto a^{-4}$, the total density term $\frac{8\pi G\rho}{3}$ grows more rapidly than the curvature term $|\frac{Kc^2}{a^2}|$ as $a \to 0$, so $\Omega \to 1$ at early times even if $\Omega_0 \neq 1$.

The Universe is dominated by DE
How weird!!

$$H^2 = \frac{8\,\pi\,G\,\rho}{3} + \frac{\Lambda}{3} - \frac{K}{a^2}$$

$$H^2 \simeq \frac{\Lambda}{3} \simeq cst$$

$$H \simeq \sqrt{\frac{\Lambda}{3}} \simeq cst$$

$$\frac{\dot{a}}{a} \simeq \sqrt{\frac{\Lambda}{3}} \simeq cst$$

$$a \propto e^{\sqrt{\frac{\Lambda}{3}}\,t}$$

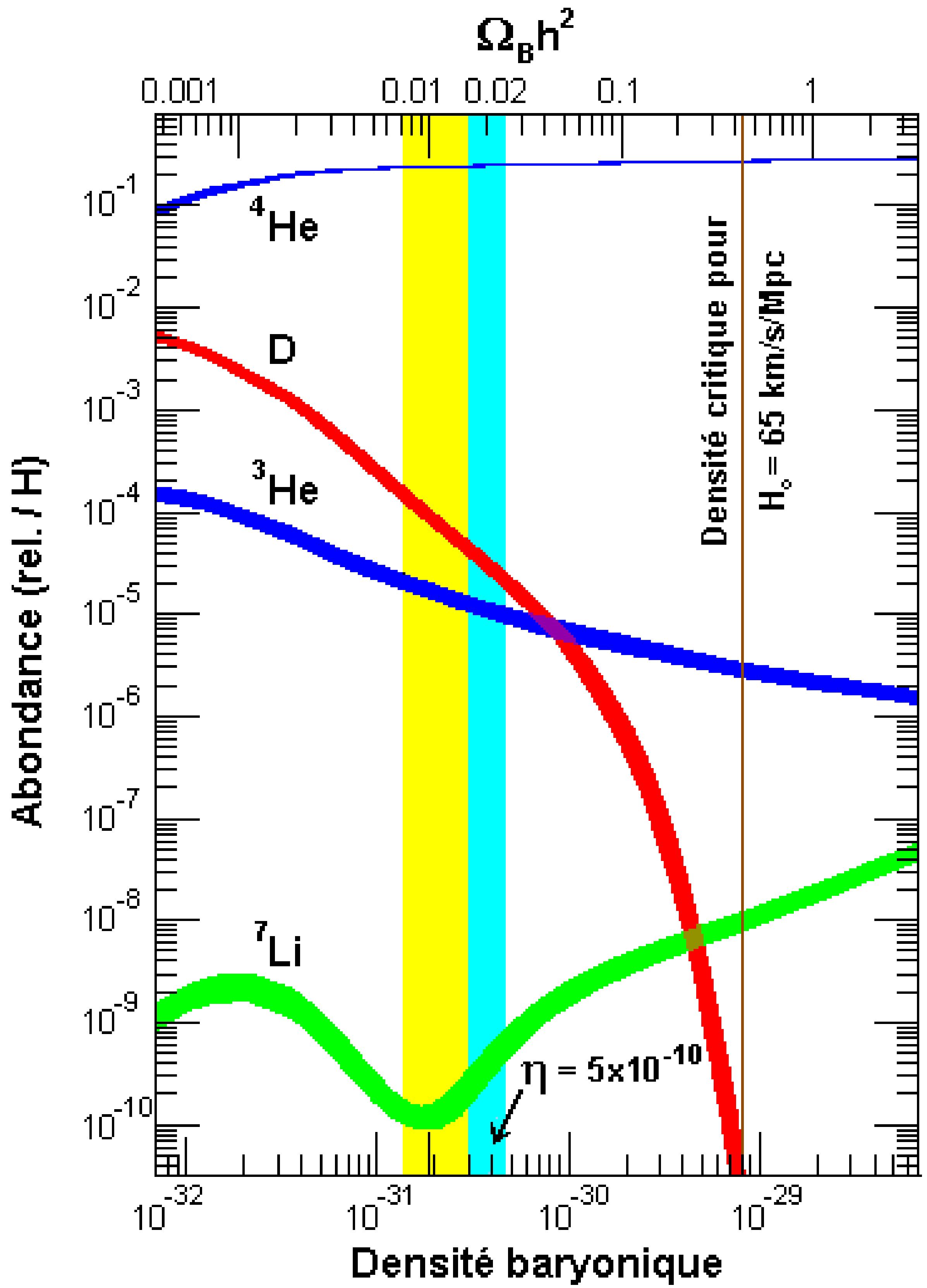
Constant energy density leads to an accelerated expansion of the Universe

5% matter; 70% dark energy

what is the rest?

IV. Towards modern Cosmology

Baryons in the Universe



Baryons can be dark but cannot be (all) the dark matter.

Only **~5% max** of baryons

Consistent with CMB!

Where are the baryons?

5% matter; 70% dark energy

what is the rest?

IV. Towards modern Cosmology

J. Oort, 1932 — Doppler redshift values of stars moving near the galactic plane;
The Galaxy needs to be twice as massive to prevent stars to escaping

F. Zwicky 1933

1937 ApJ 86, 217

ON THE MASSES OF NEBULAE AND OF CLUSTERS OF NEBULAE

F. ZWICKY

The Coma cluster contains about one thousand nebulae. The average mass of one of these nebulae is therefore

$$\overline{M} > 9 \times 10^{43}\ \text{gr} = 4.5 \times 10^{10} M_{\odot}\,. \qquad (36)$$

Inasmuch as we have introduced at every step of our argument inequalities which tend to depress the final value of the mass $\mathcal{M}$, the foregoing value (36) should be considered as the lowest estimate for the average mass of nebulae in the Coma cluster. This result is somewhat unexpected, in view of the fact that the luminosity of an average nebula is equal to that of about 8.5×10^{7} suns. According to (36), the conversion factor γ from luminosity to mass for nebulae in the Coma cluster would be of the order

$$\text{Mass/Light} = \quad \gamma = 500\,, \qquad (37)$$

as compared with about $\gamma' = 3$ for the local Kapteyn stellar system.

more mass in the Coma Cluster than is visible

based on 21 radial velocities of galaxies in the Coma cluster

IV. Towards modern Cosmology

The rotation of galaxies was discovered in 1914 — Slipher (1914)

(many people contributed!)

Freeman (1970) for M33 and NGC 300:

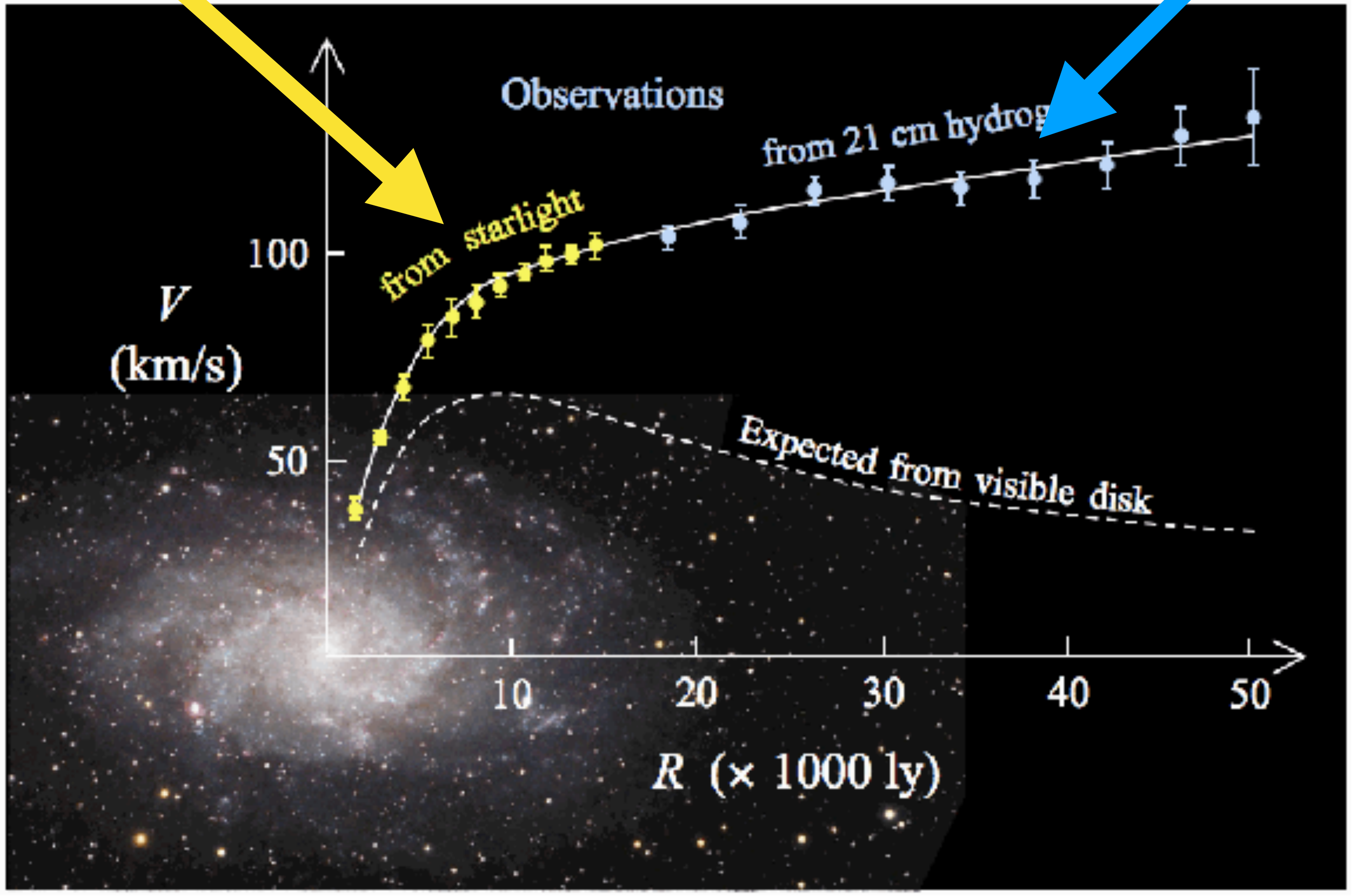
rotation curve peaks at the edge of the optical disk

so ~ 1/3 of the mass outside the optical radius.

[Shostak & Rogstad (1973)](), [Seielstad & Wright (1973)](). [M31](): ([Roberts 1975a](), [Roberts & Whitehurst 1975]()); Final straw: [Bosma (1978)]()

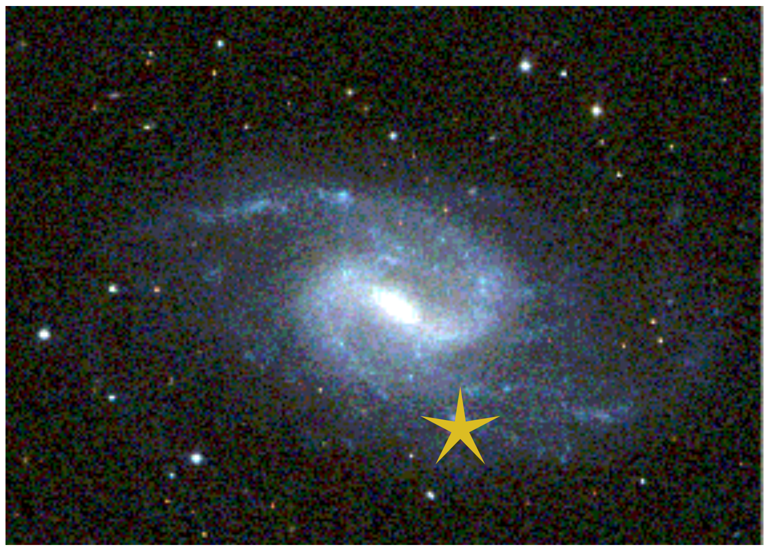
1970ApJ...159..379R

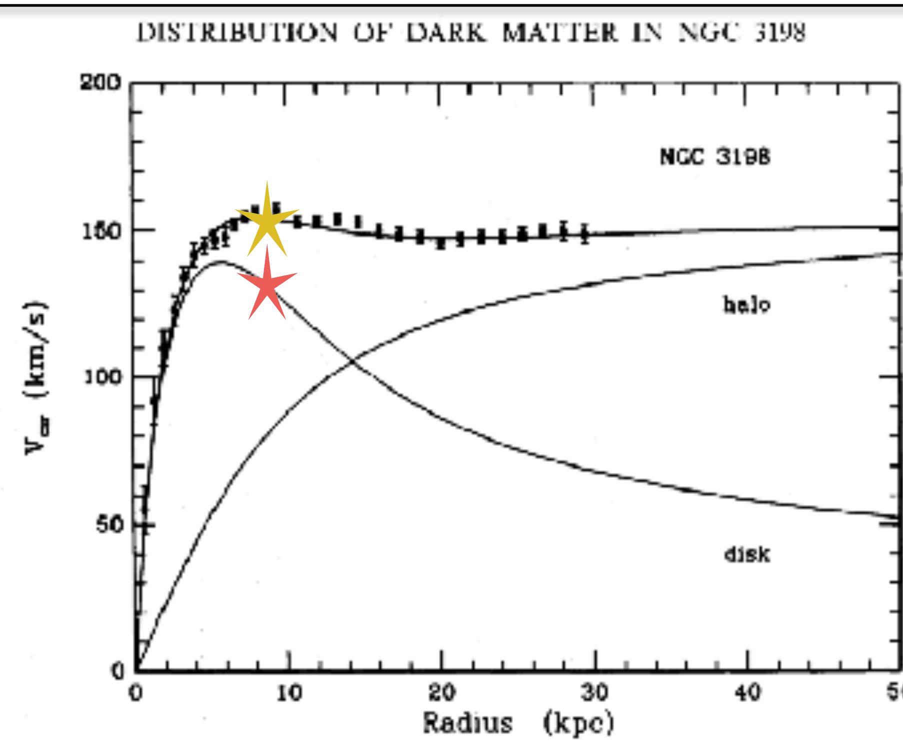
Rotation curves of galaxies



IV. Towards modern Cosmology

Famous evidence for non standard physics



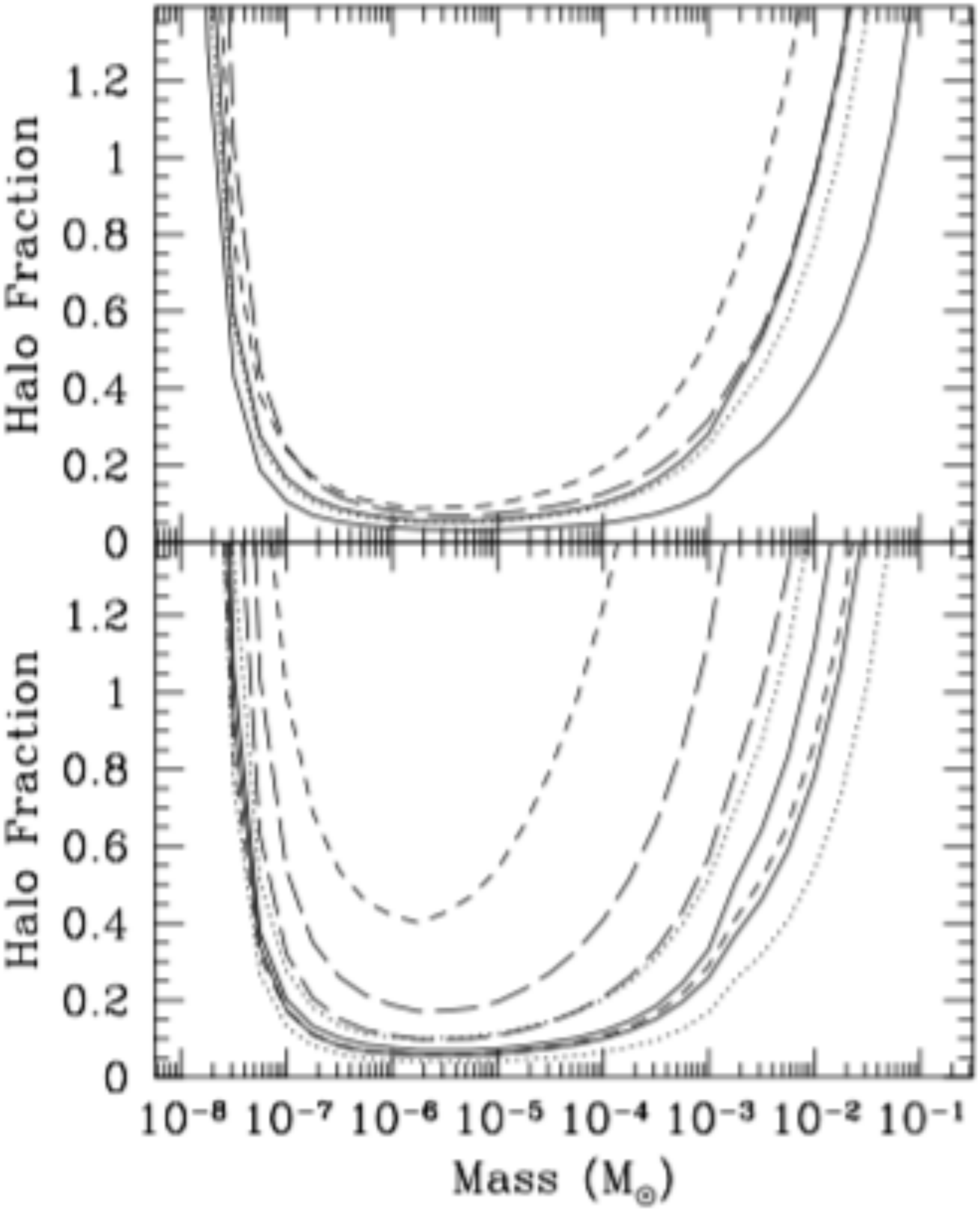


We need DM to explain the flat rotation curves **far from the GC**

$$v_c^2 = \frac{G\,M(r)}{r} \qquad M(r) = \int 4\pi^2 \rho(r)\, d^3r$$

But the highest mass density would be in the inner part of the galaxy...

IV. Towards modern Cosmology



EROS and MACHO

(La Silla vs Mount Stromlo Observatory, Australia)

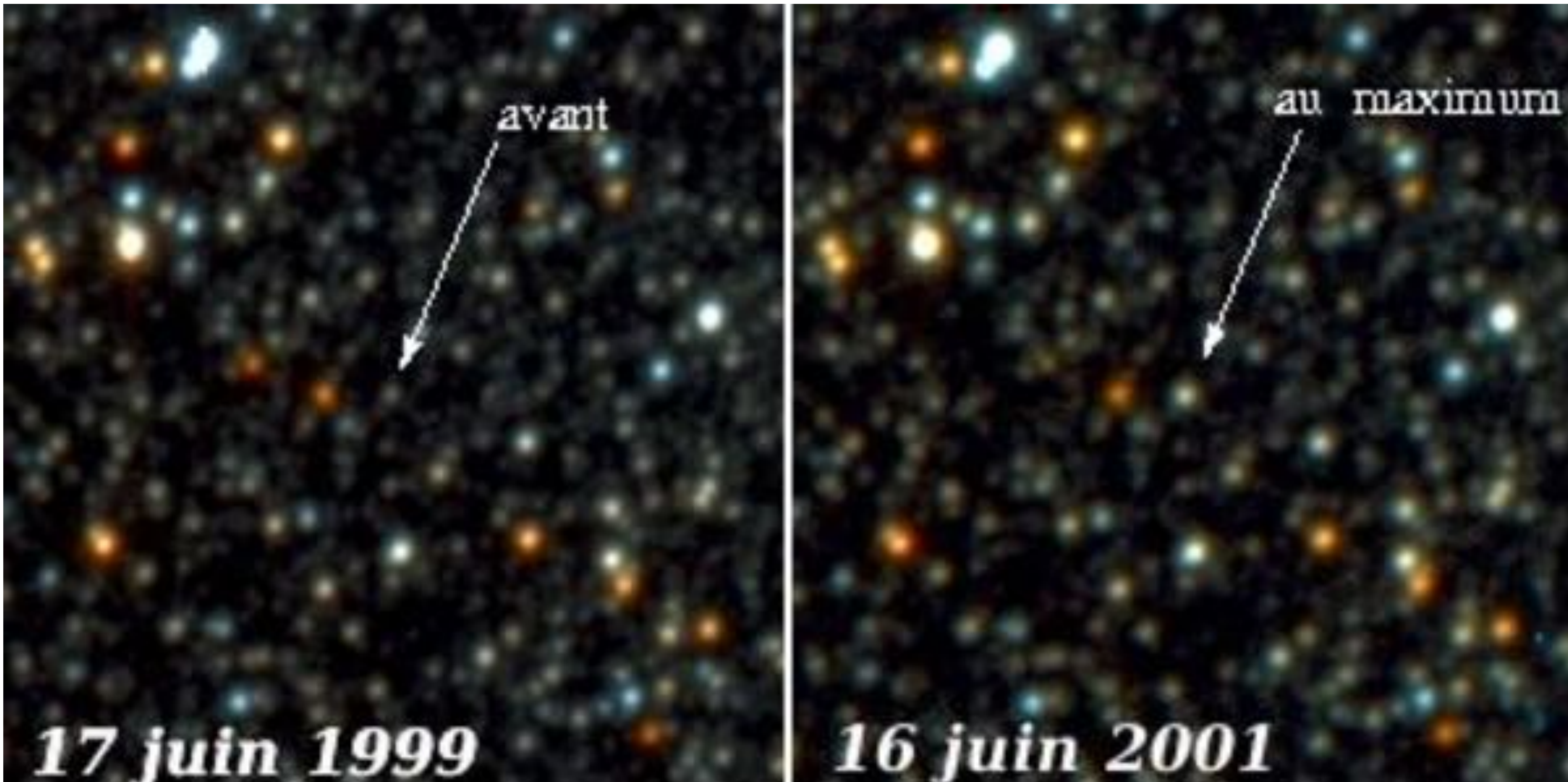
Fig. 3.— Halo fraction upper limit (95% c.l.) versus lens mass for the five EROS models (top) and the eight MACHO models (bottom). The line coding is the same as in Figure 2.

Nothing found (so far but ...)
they can't be all the missing mass
well not in that range ...

IV. Towards modern Cosmology

Microlensing effect...

Before and two years after (during the maximum of amplification)



Courtesy: EROS experiment. They were looking for "brown dwarfs" or "MACHOs" which belong gravitationally to our Galaxy. This was made possible by their gravitational microlensing effects on stars in the Magellanic Clouds (two dwarf galaxies, Milky Way satellites).

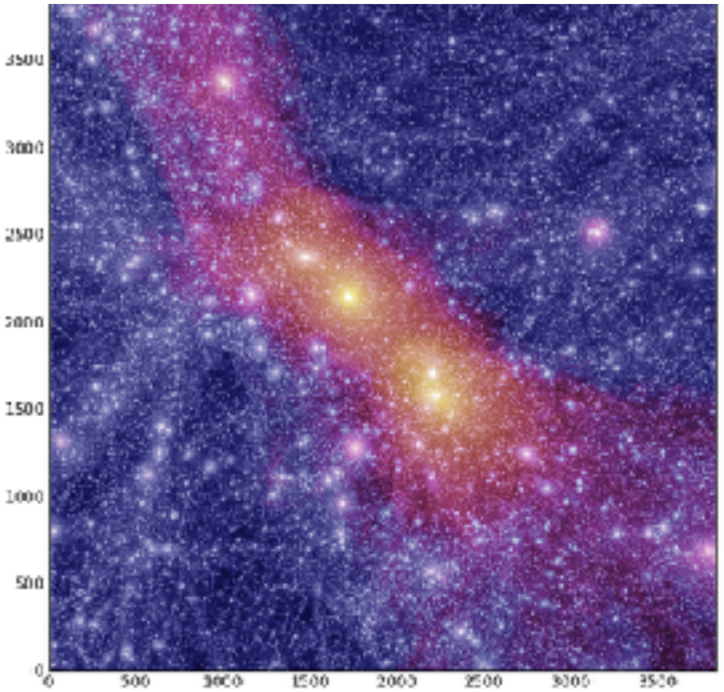
IV. Towards modern Cosmology

Dark Matter is every where

Cluster of galaxies

NGC 6814 Credit: NASA

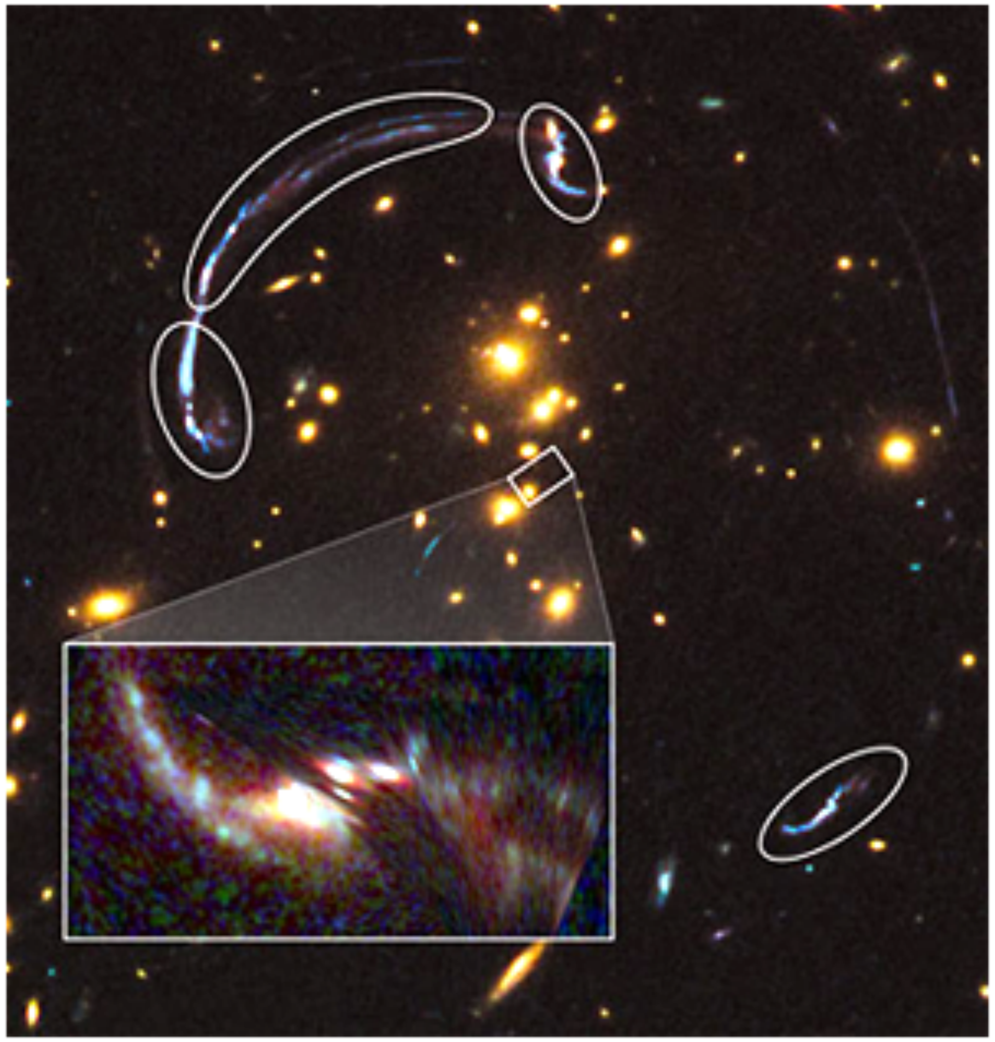
NGC 4621 Credit: WikiSky/SDSS



But what is the DM?

Gravitational lensing evidence…

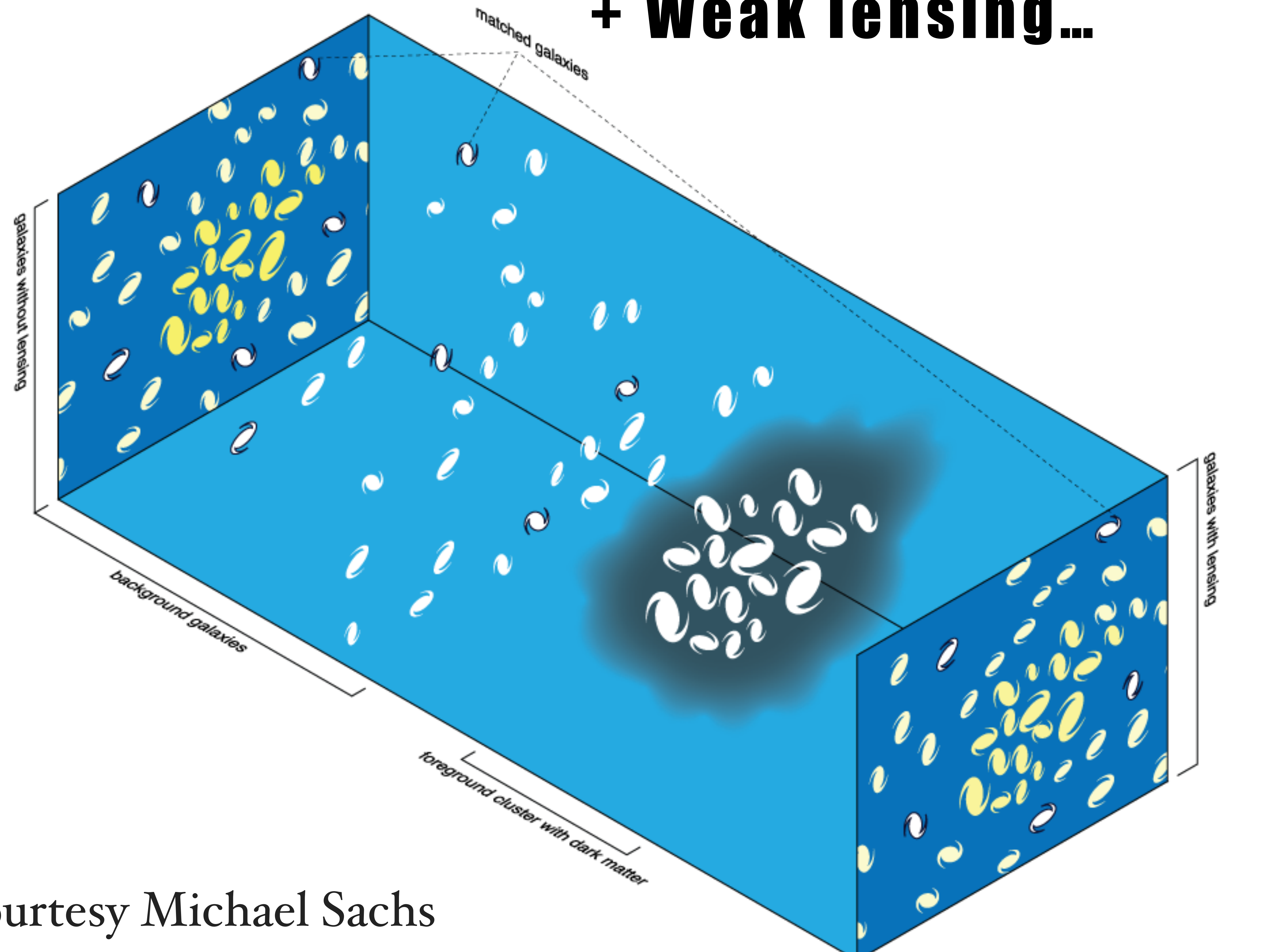
Illustration Credit: NASA, ESA, and Z. Levay (STScI)
Science Credit: NASA, ESA, J. Rigby (NASA Goddard Space Flight Center), K. Sharon (Kavli Institute for Cosmological Physics, University of Chicago), and M. Gladders and E. Wuyts (University of Chicago)



Reconstruction (lower left) of the brightest galaxy whose image has been distorted by the gravity of a distant galaxy cluster.

The small rectangle in the center shows the location of the background galaxy on the sky if the intervening galaxy cluster were not there.
The rounded outlines show distinct, distorted images of the background galaxy resulting from lensing by the mass in the cluster. The image at lower left is a reconstruction of what the lensed galaxy would look like in the absence of the cluster, based on a model of the cluster's mass distribution derived from studying the distorted galaxy images.

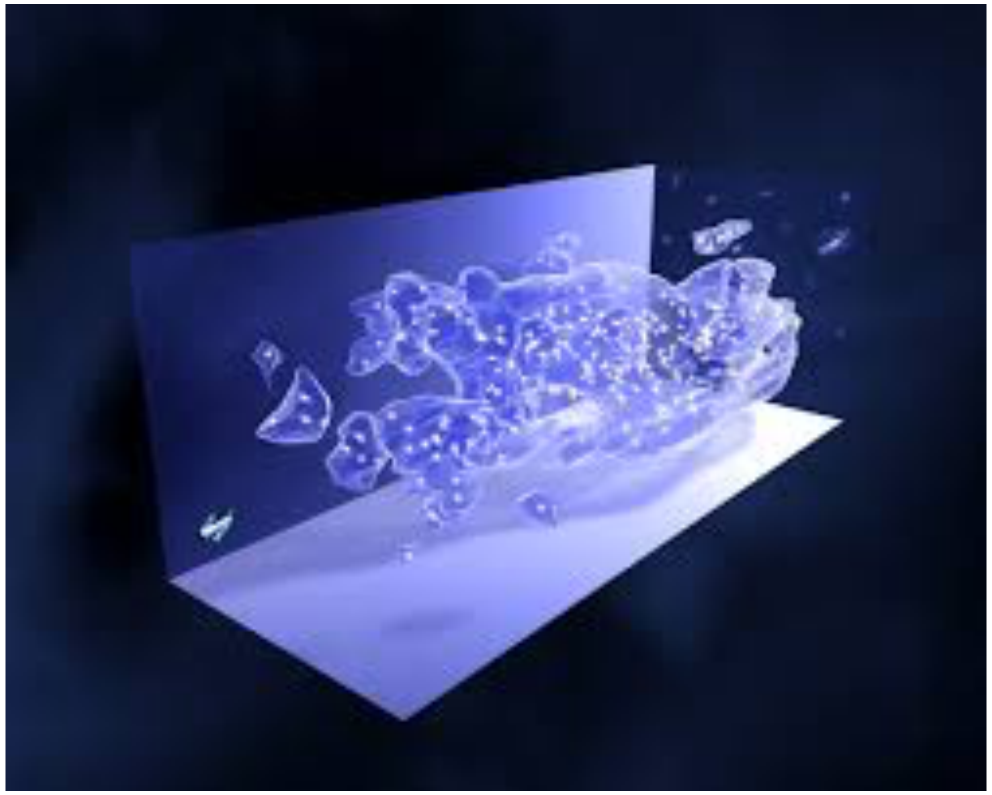
+ Weak lensing…

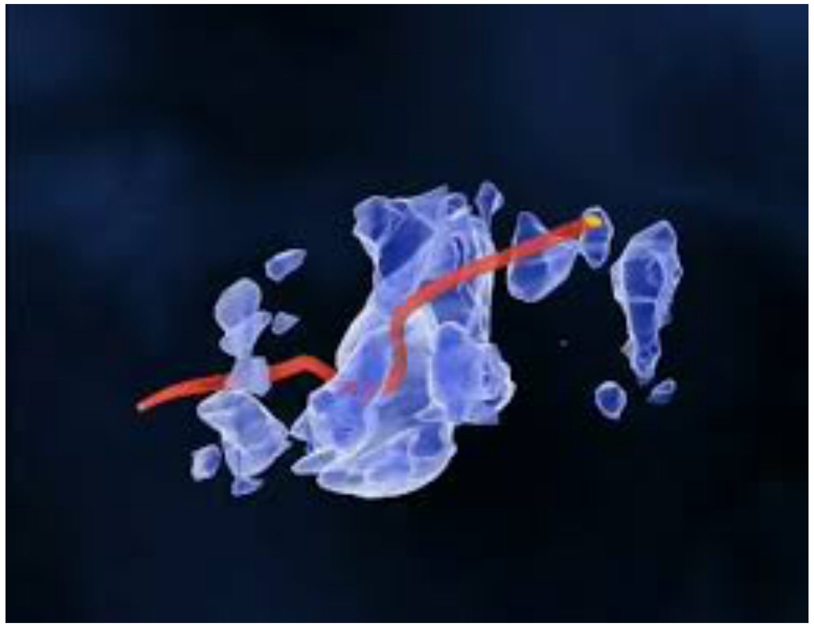


Courtesy Michael Sachs

More lensing evidence…

X-ray emitted by gas
(Thomson interactions, Bremsstrahlung,…)
But the gravitational potential is dominant
in the blue region where no light is emitted





5% matter; 70% dark energy

what is the rest?

well seems a sort of dark matter

Now that we know the ingredients in the Universe, how do we form objects like this?

V. Primordial Fluctuations

J. Peebles

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Title:	The Gravitational Instability of the Universe
Authors:	Peebles, P. J. E.
Publication:	Astrophysical Journal, vol. 147, p.859 (ApJ Homepage)
Publication Date:	03/1967
Origin:	ADS
DOI:	10.1086/149077
Bibliographic Code:	1967ApJ...147..859P

Abstract

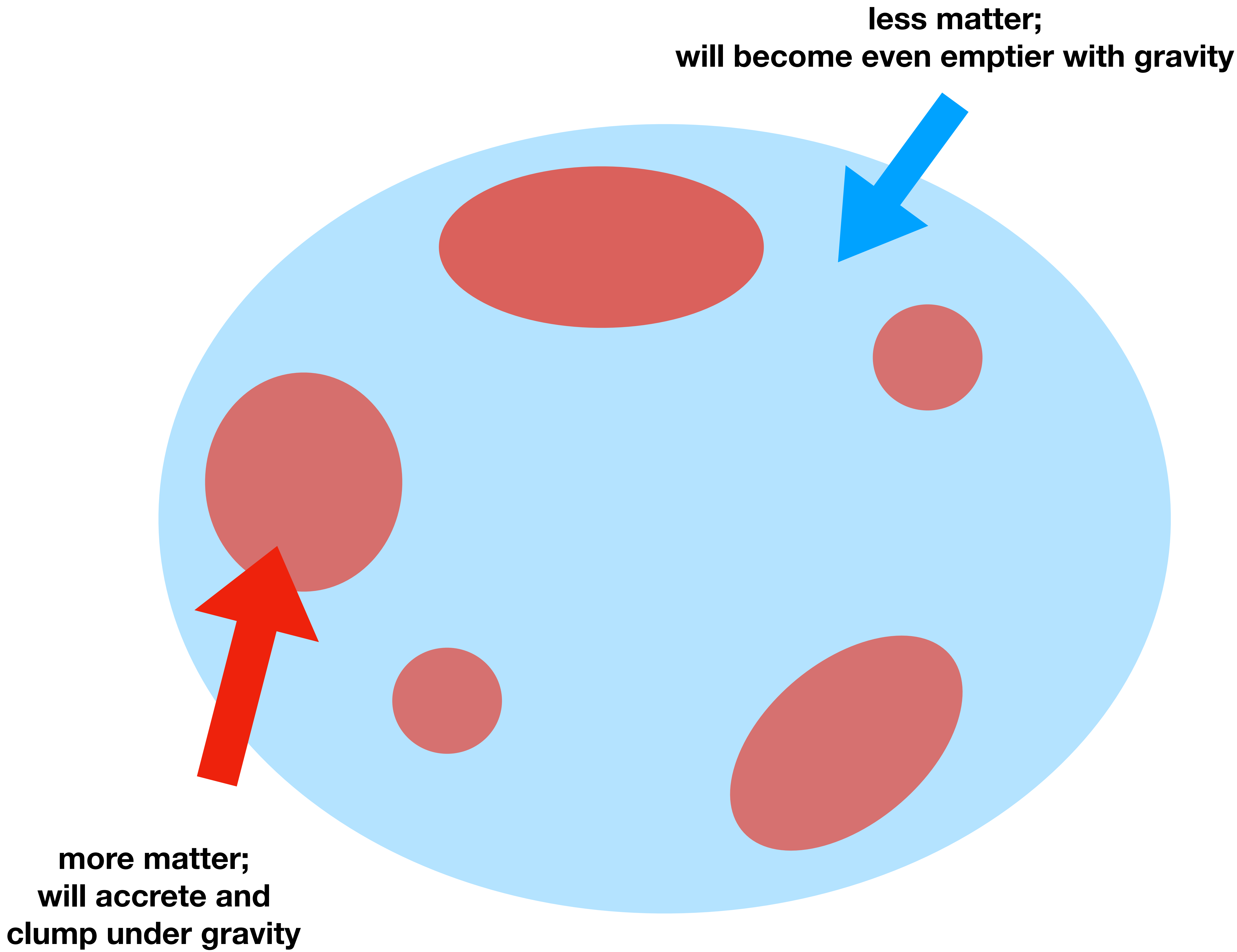
It is argued that the expanding universe is unstable against the growth of gravitational perturbations. The argument is directed toward two problems, the physical conditions in the early, highly contracted phase of the expanding universe, and the formation of the galaxies.

Followed Peebles, P. J. E., *Astrophys. J.*, **142**, 1317 (1965)

http://adsabs.harvard.edu/abs/1970ApJ...162..815P

Primordial fluctuations in the Early Universe grow under gravity (Peebles, 66)

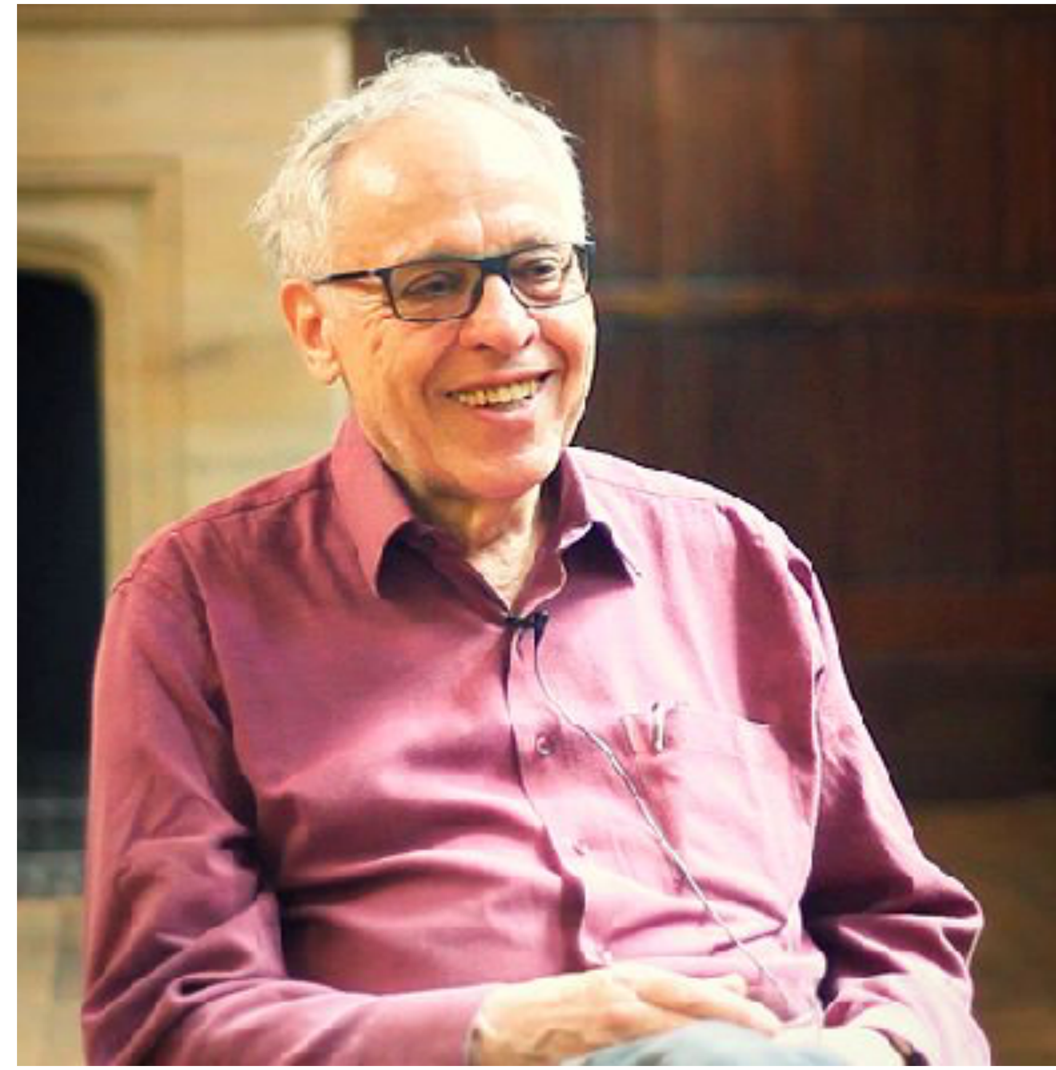
$$I(x) \propto \frac{x^3}{e^x - 1} \qquad x = \frac{h\nu}{\kappa T} \quad \Rightarrow \quad \frac{\Delta T}{T} \simeq 10^{-5}$$


less matter;
will become even emptier with gravity
more matter;
will accrete and
clump under gravity

V. Primordial Fluctuations

baryonic fluctuations

baryonic fluctuations do not survive the baryon scattering off the photon background.
(Question first asked by Misner for neutrinos)



J. Silk

letters to nature

Nature **215**, 1155 - 1156 (09 September 1967); doi:10.1038/2151155a0

Fluctuations in the Primordial Fireball

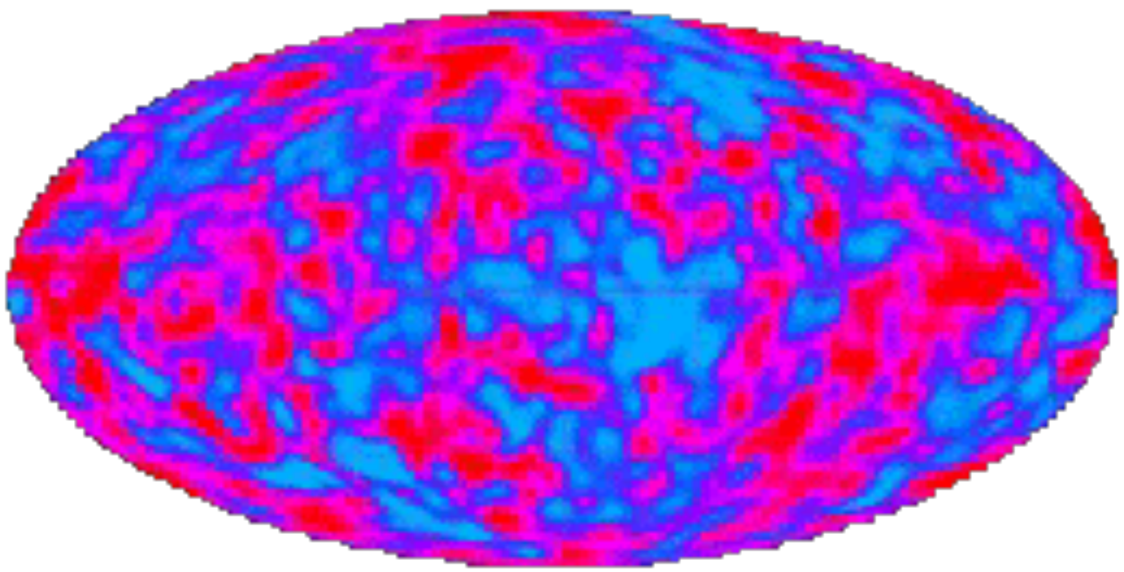
JOSEPH SILK

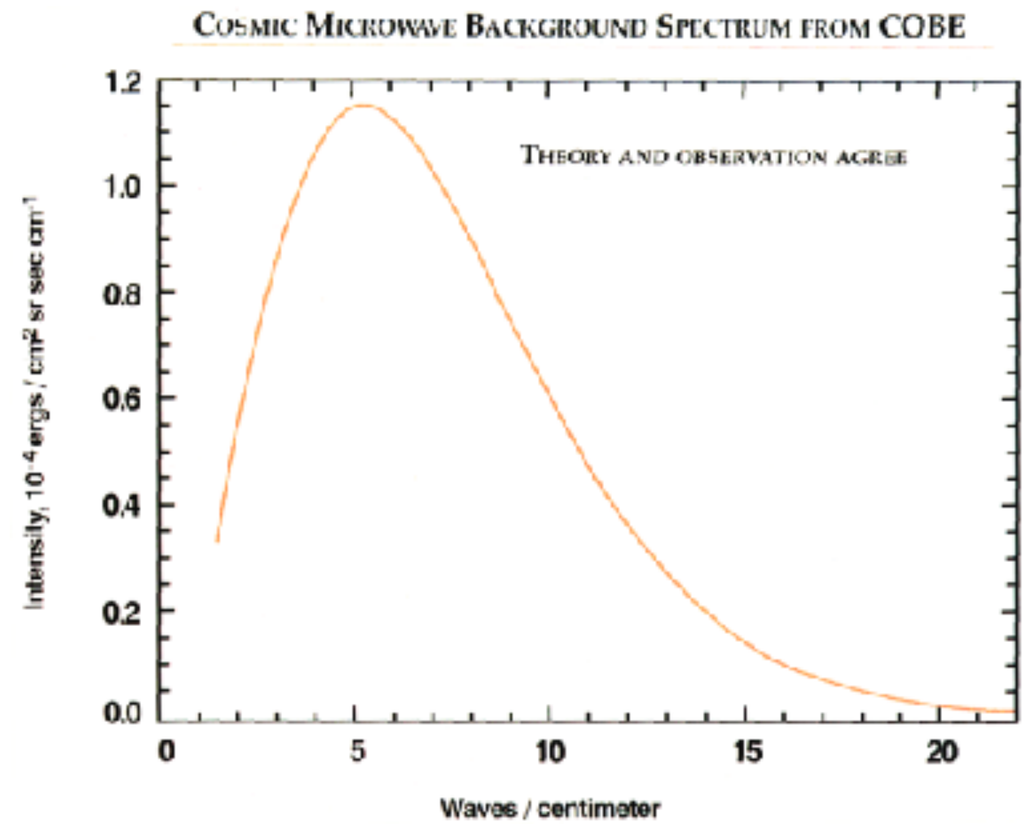
Harvard College Observatory, Cambridge, Massachusetts.

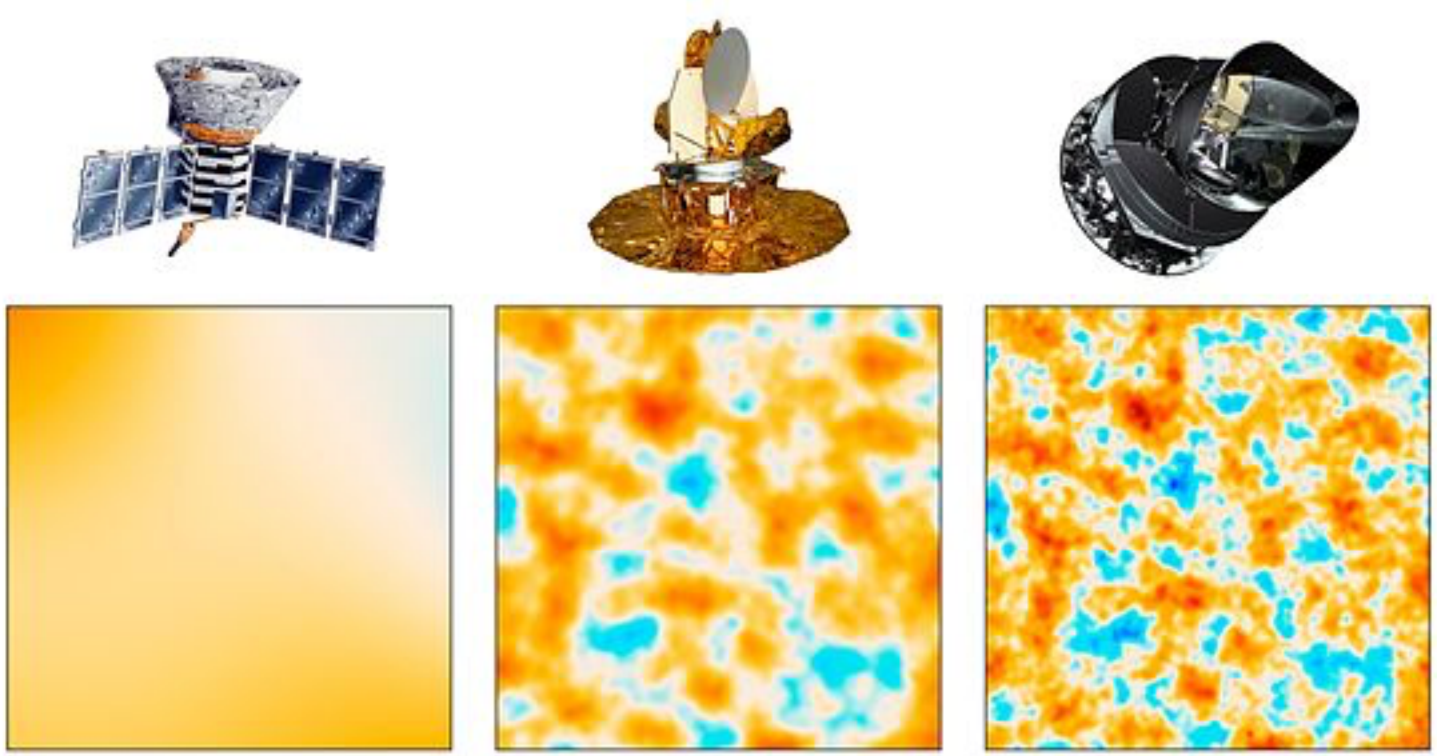
ONE of the overwhelming difficulties of realistic cosmological models is the inadequacy of Einstein's gravitational theory to explain the process of galaxy formation[1–6]. A means of evading this problem has been to postulate an initial spectrum of primordial fluctuations[7]. The interpretation of the recently discovered 3° K microwave background as being of cosmological origin[8,9] implies that fluctuations may not condense out of the expanding universe until an epoch when matter and radiation have decoupled[4], at a temperature T_D of the order of 4,000° K. The question may then be posed: would fluctuations in the primordial fireball survive to an epoch when galaxy formation is possible ?

V. Primordial Fluctuations

Was Peebles right?







COBE WMAP Planck

courtesy wikipedia!

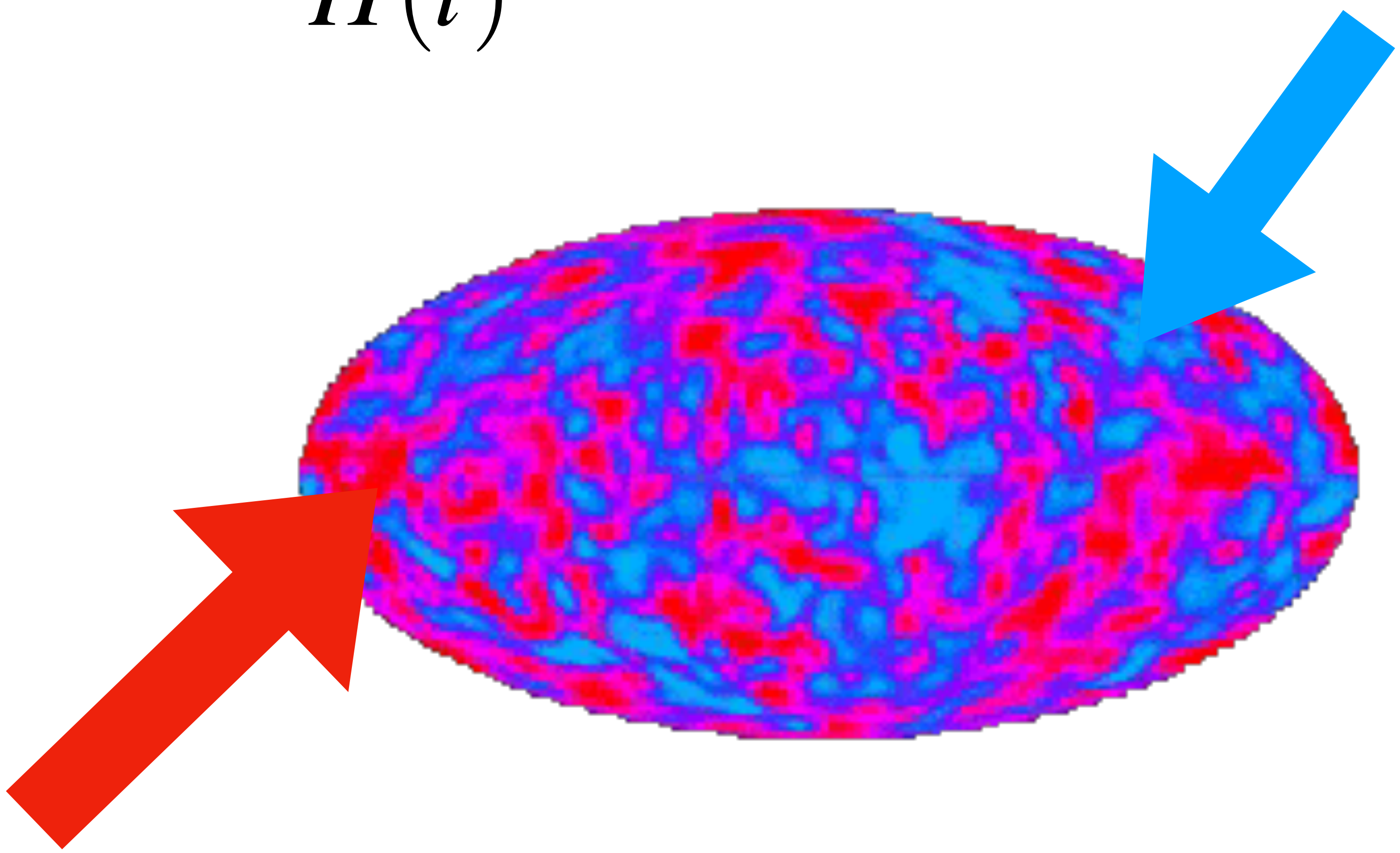
YES!

so either Silk wrong or
more matter than baryons

V. Primordial Fluctuations

$$d(t) = \frac{c}{H(t)}$$

cold spot



hot spot

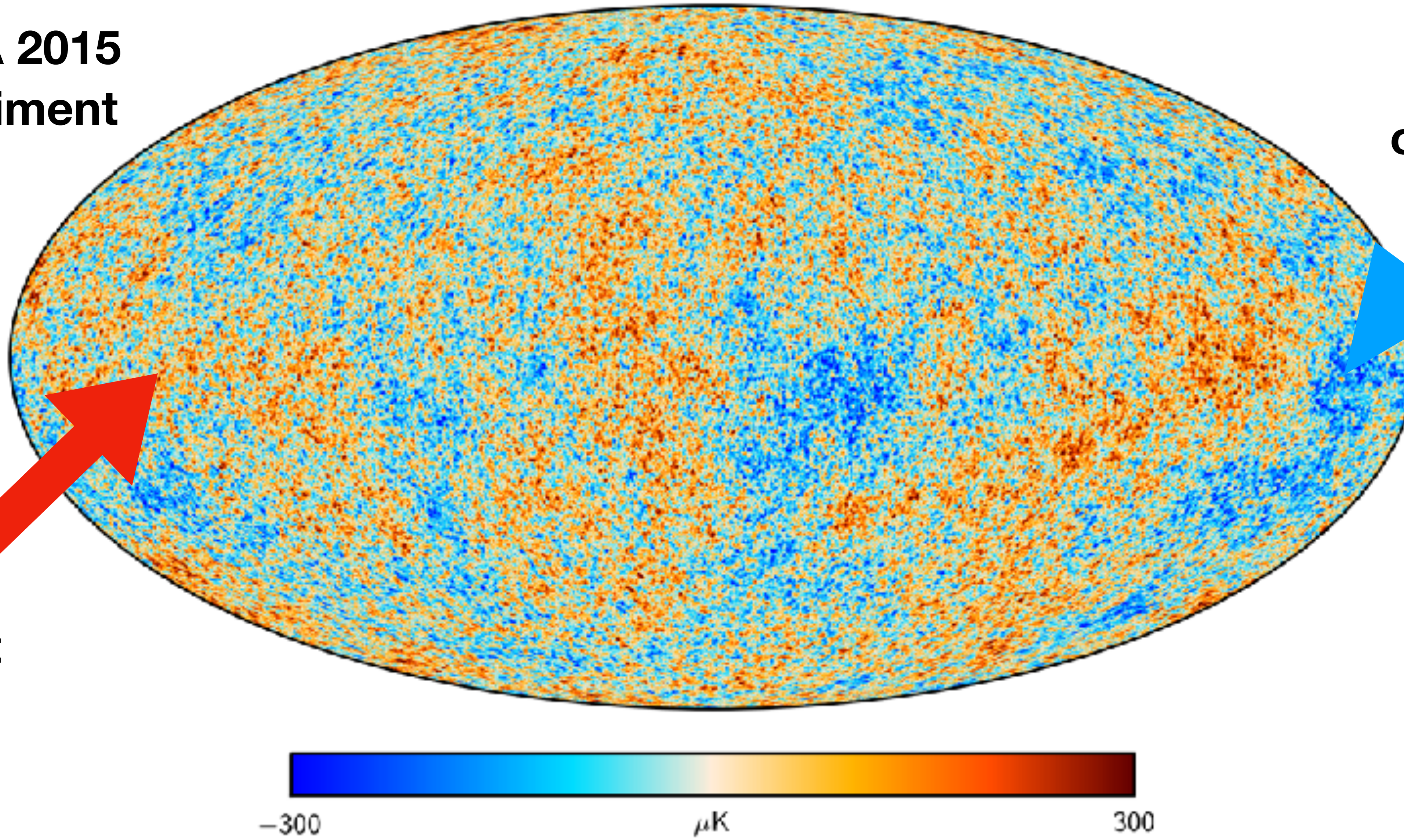
>> galaxy scale (1 Mpc ~ 10^27 cm)

All regions of the sky have a temperature around 2.7k!
How come?

V. Primordial Fluctuations

Courtesy ESA 2015
Planck experiment

cold spot



hot spot

>> galaxy scale (1 Mpc ~ 10^27 cm)

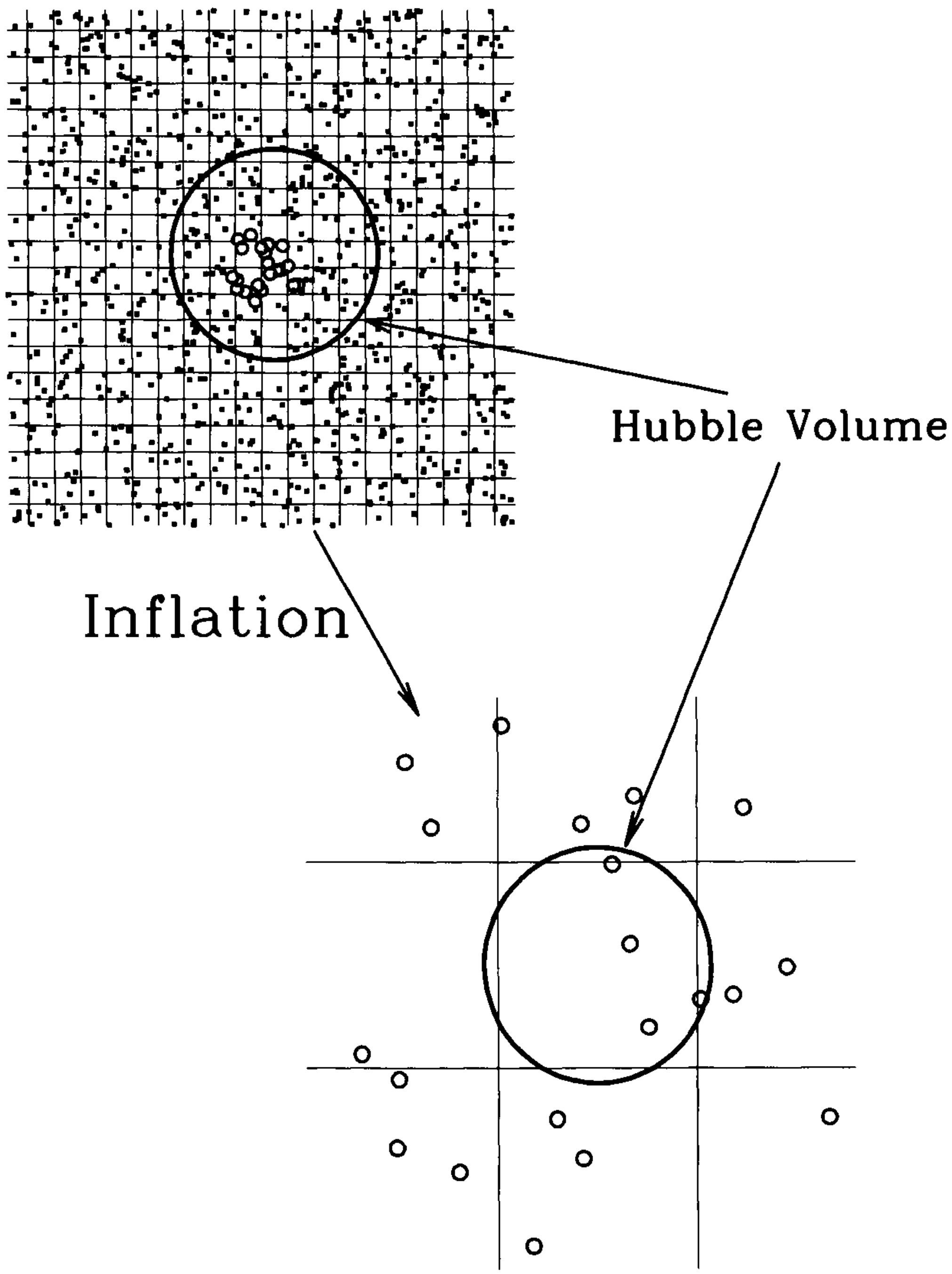
All regions of the sky have a temperature around 2.7k + 10^-5

How come such a tiny difference on such gigantic scales?

V. Primordial Fluctuations

Horizon problem

start with small and dense Universe
phase of extremely rapid expansion
then normal phase of expansion (H)



Textbook Dodelson

What is driving the expansion?
Potentially a scalar field (like Higgs)

$$p = -\rho c^2 = -V(\phi)$$

if V ~ constant

$$H^2(a) \approx \frac{8\pi G}{3}\frac{V(\phi)}{c^2} \equiv H_\phi^2 \approx const$$

$$H(a) = \frac{\dot{a}}{a} = H_\phi$$ now integrate

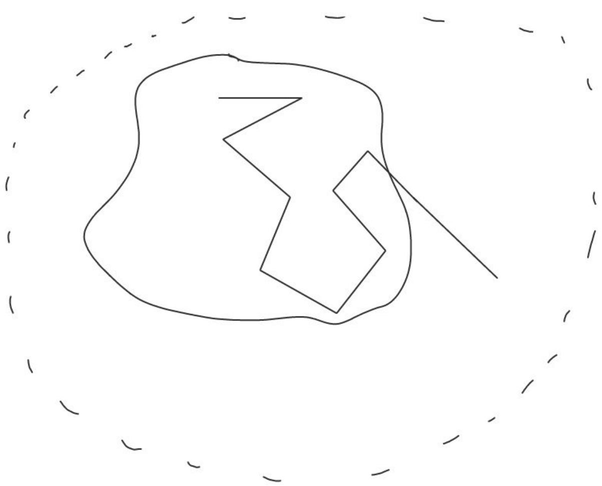
$$a(t) \propto e^{H_\phi t}$$

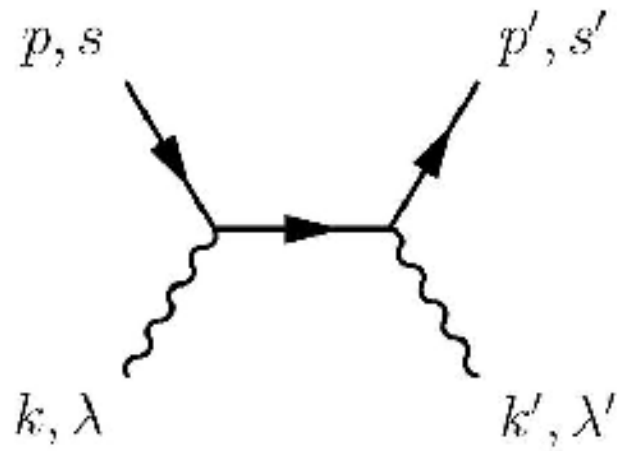
II CMB & structure formation

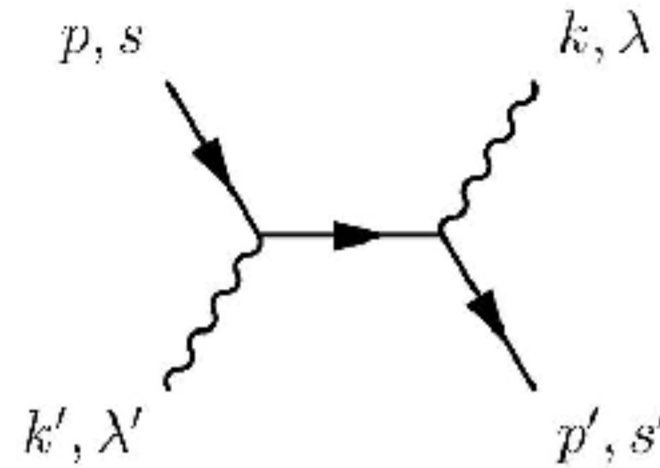
I. Decoupling

moment when photons are free
(elastic scattering rate with photons < Hubble rate)

High energy = Compton interactions
Low energy = Thomson interactions







$$\Gamma_{\gamma-e,p} \simeq \sigma_T \, c \, n_e = H \quad \Rightarrow \quad \Gamma_{\gamma-e,p} \simeq \sigma_T \, c \, 10^{-9} \, n_\gamma = H$$

$$\sigma_T \, c \, 10^{-9} \, n_\gamma(T_0) \, a^{-3} = H_\alpha \, a^{-1/\alpha}$$

$$a = \left(\frac{\sigma_T \, c \, 10^{-9} \, n_\gamma(T_0)}{H_\alpha} \right)^{1/(3-1/\alpha)}$$

$$\boxed{T_{dec(\gamma)} \simeq T_0 \left(\frac{7 \, 10^{-21} \, \mathrm{s}^{-1}}{H_\alpha} \right)^{-2/3} \simeq 100 \, K.}$$

II. Recombination

$$e^- + p \leftrightarrow H + \gamma \qquad 2e^- + H_e^{++} \leftrightarrow H_e + \gamma.$$

The one-arrow process $e^- + p \rightarrow H + \gamma$ is referred to recombination. The neutral Hydrogen formed at BBN is indeed destroyed by the reverse reaction

$$H + \gamma \rightarrow e^- + p$$

till the photon energy becomes too low to break the neutral Hydrogen bound state. At that stage the electrons can recombine with protons to form again neutral Hydrogen. The binding energy of the Hydrogen atom is about 13.6eV so a crude estimate shows that electrons must be free until $E_\gamma = 13.6$ eV, which translates into a temperature of

$$3\,K\,T \;=\; E \;=\; 13.6\ \mathrm{eV} \Rightarrow \mathrm{T} \sim 5\ 10^4\ \mathrm{K}.$$

This is too crude however because this assumes only one transition from the fundamental to fully ionised state. In reality one will pass through all energy levels which are given by

$$E_n = -\frac{E_B}{n^2}$$

with $E_B = 13.6$ eV. The transition from 1 to 2 requires a photon energy of about $E_2 - E_1 = -\frac{E_B}{2^2} + \frac{E_B}{1^2} = 10.2$ eV

II. Recombination 2.0

But one should also remember that there are many more photons than protons and electrons. Hence dissociation is very efficient. Typically the number of Hydrogens ions which are expected to get sufficiently excited to not recombine is given by

$$n_{p^+} = n_\gamma(E > 10.2\ \mathrm{eV})$$

However this also assumes that one photon of such energy causes the Hydrogen to reach the first excited state and causes immediate ionisation. In reality this is again too crude and one needs to follow the next steps to get a better estimate of the moment at which all the electrons and protons would "recombine" to form neutral Hydrogen that will not be dissociated any longer.

Let us start by describing the distribution of charges in the medium. The number of free electrons is governed by

$$\frac{dn_e}{dt} = -3\,H\,n \;-\; \langle\sigma v\rangle\,(n_e\,n_p - n_{e,eq}\,n_{p,eq})$$

so

$$\frac{dn_e}{dt} = -3\,H\,n \;-\; \langle\sigma v\rangle\;(n_e\,n_p - n_H)$$

The departure from equilibrium happens when $n_H > N_{H,eq}$ or equivalently $n_e\, n_p > n_{e,eq}\, n_{p,eq}$ while, before departure, the relation

$$\frac{n_e n_p}{n_H} = \frac{n_{e,eq}\, n_{p,eq}}{n_{H,eq}}$$

is satisfied. The latter can be rewritten using the equilibrium number densities

$$n_{i,eq} = \left(\frac{m_i T_i}{(2\pi)}\right)^{3/2} e^{-\frac{m_i}{T_i}}$$

as

$$\frac{n_e n_p}{n_H} = \frac{n_{e,eq}\, n_{p,eq}}{n_{H,eq}} \equiv \left(\frac{m_e T}{(2\pi)}\right)^{3/2} e^{-\frac{(m_e+m_p-m_H)}{T}} \tag{4.1}$$

(where we neglect the electron mass in the ratio m_p/m_H and therefore took $m_p/m_H = 1$) leading to

$$\frac{n_e\, n_p}{n_H} = \left(\frac{m_e T}{(2\pi)}\right)^{3/2} e^{-\frac{13.6\ \mathrm{eV}}{T}}$$

if we consider the fully ionised Hydrogen.
The fraction of ionised Hydrogen is given by the number of free electrons

$$n_e = X_e\, n_B$$

with n_B the number density of Baryons. Therefore we can define this fraction as

$$X_e = \frac{n_e}{n_e + n_H} = \frac{n_p}{n_p + n_H},$$

which is such that

$$1 - X_e = \frac{n_H}{n_e + n_H}.$$

and therefore implies

$$n_e = n_p.$$

The number density of electrons after partial or complete recombination is then given by

$$n_{e,p} = n_H \frac{X_e}{(1 - X_e)}$$

$$\frac{n_e\, n_p}{n_H} = \left(\frac{m_e T}{(2\pi)}\right)^{3/2} e^{-\frac{\varepsilon_H}{T}}$$

(where we now account for a partially ionised Hydrogen, i.e. the first excited state of Hydrogen, with $\varepsilon_H = 10.2$ eV) is in fact

$$\frac{X_e\, n_p}{(1 - X_e)} = \left(\frac{m_e T}{(2\pi)}\right)^{3/2} e^{-\frac{\varepsilon_H}{T}}$$

or similarly

$$\frac{X_e\, n_e}{(1 - X_e)} = \left(\frac{m_e T}{(2\pi)}\right)^{3/2} e^{-\frac{\varepsilon_H}{T}}.$$

This expression can be finally written, using $X_e = \frac{n_e}{n_e + n_H} \Rightarrow n_e = X_e\,(n_e + n_H)$, as

$$\boxed{\frac{X_e^2}{(1 - X_e)} = \frac{1}{(n_e + n_H)} \left(\frac{m_e T}{(2\pi)}\right)^{3/2} e^{-\frac{\varepsilon_H}{T}}}$$

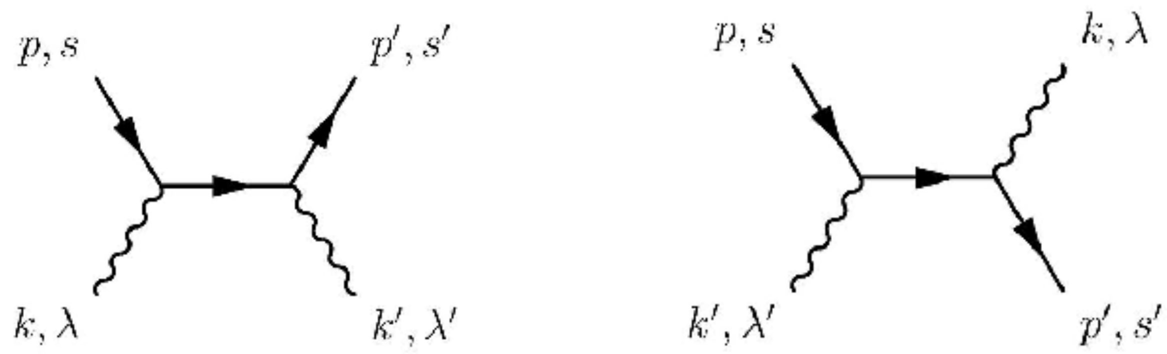
which is known as the **Saha equation**.

$$\frac{dn_e}{dt} = -3\, H\, n_e - \langle \sigma v \rangle\, (n_e\, n_p - n_H)$$

$$\boxed{\frac{dX_e}{dt} = \langle \sigma v \rangle \left((1 - X_e) \left(\frac{m_e T}{(2\pi)}\right)^{3/2} e^{-\frac{\varepsilon_H}{T}} - X_e^2\, n_B \right)}$$

I. Decoupling 2.0

moment when photons are free
(elastic scattering rate with photons < Hubble rate)



Calculations then lead to

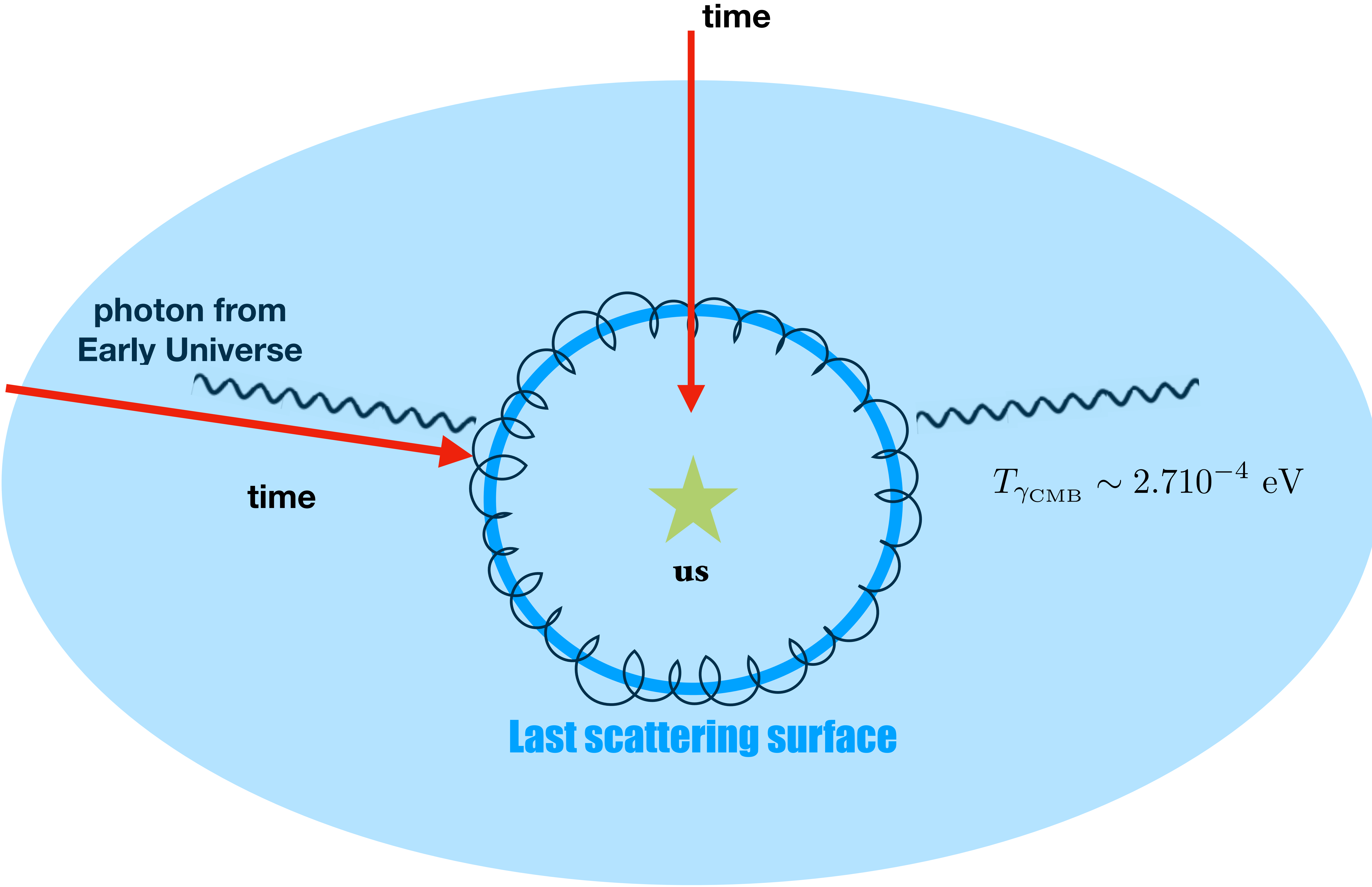
$$X_e \to 0 \quad \text{when} \;\; z \simeq 1100,$$

corresponding to a temperature of

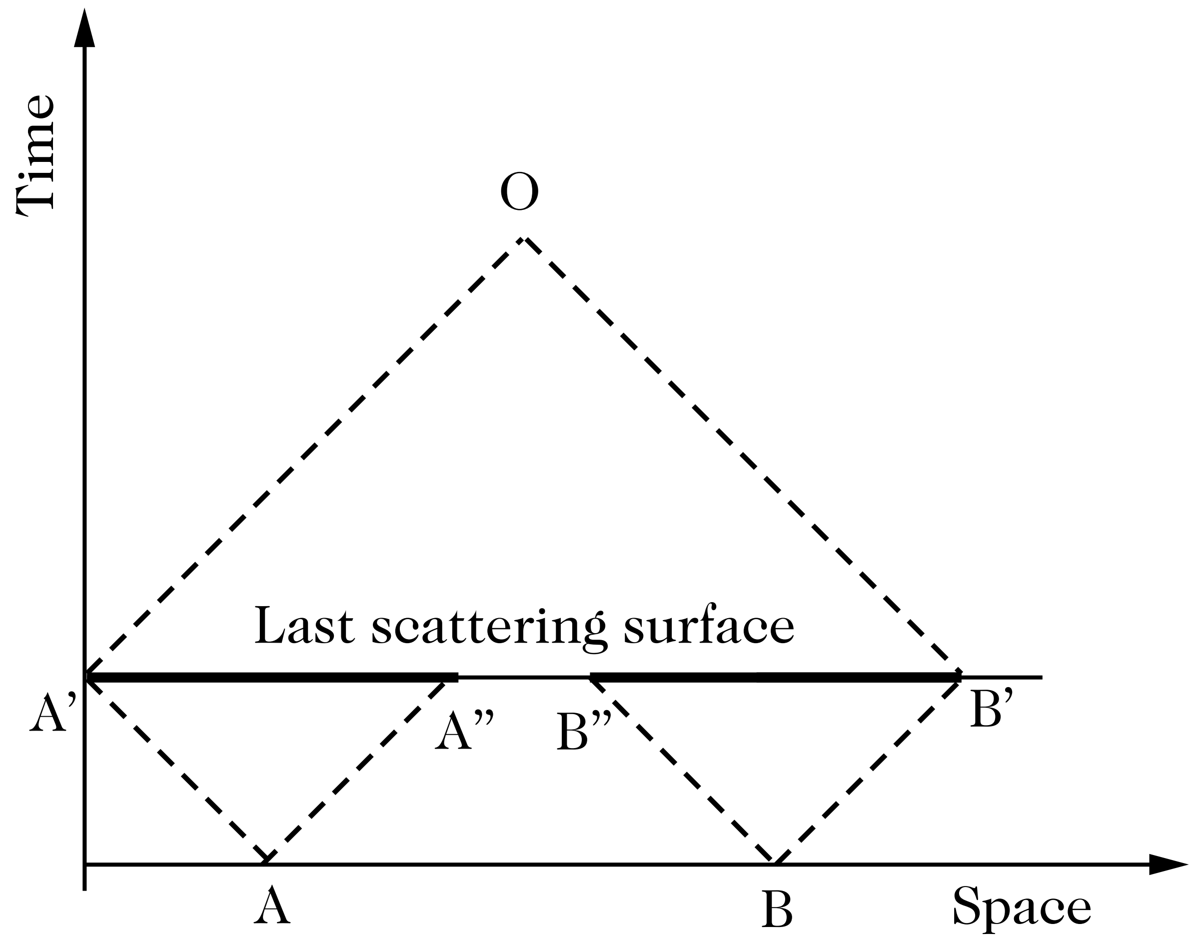
$$T = T_0\,(1+z) \sim 3\;10^3\;K.$$

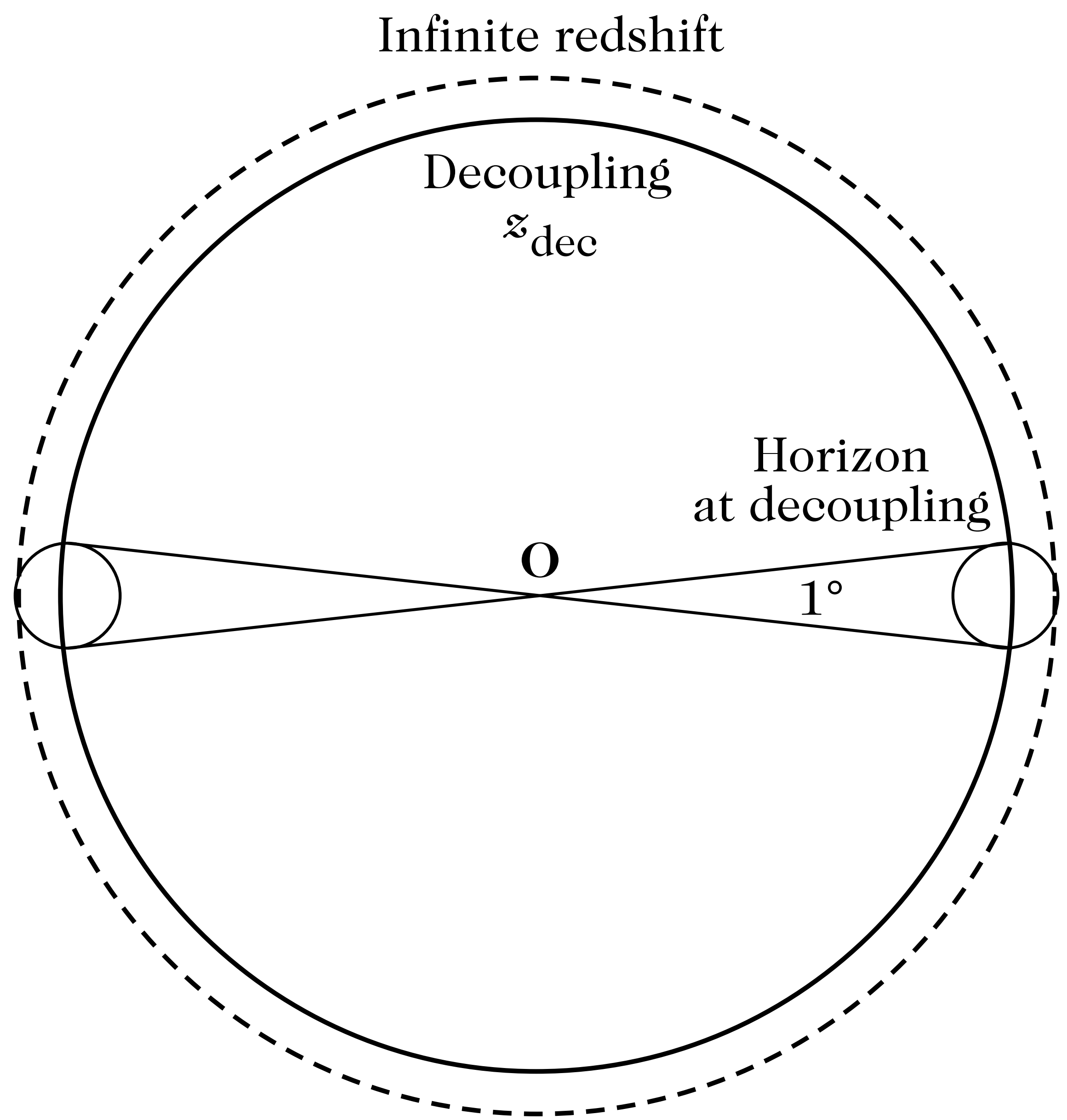
Since the recombination temperature is much higher than the photon and electron decoupling temperature, the decoupling of both species with each other happens at recombination.

We have seen that photons and electrons decouple when the electrons recombine with proton to form neutral Hydrogen atoms, that is at recombination. Consequently, before $T \simeq 3000$ K, the medium is opaque while after $T \simeq 3000$ K, photons can freely propagate and reach us.


time
photon from
Early Universe
time
$T_{\gamma_{\rm CMB}} \sim 2.710^{-4}$ eV
us
Last scattering surface

I. Decoupling 2.0





$$D_{A\star} = \frac{1}{(1+z_\star)} f_k \left(\int_{z=z_\star}^{\infty} \frac{dz}{H_0 \sqrt{\Omega_r + \Omega_m + \Omega_\Lambda + \Omega_k (1+z)^2}} \right)$$

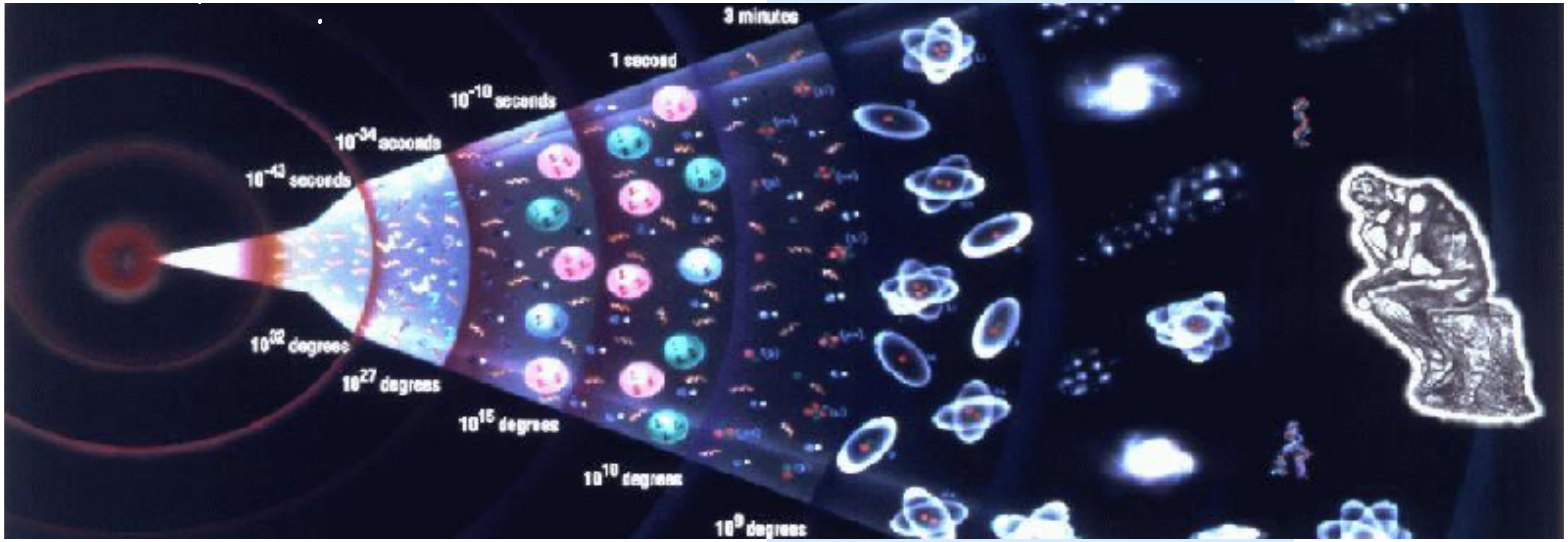
$$D_{A\star}^{obs} = \frac{l_\star}{\Theta_\star} = 17 \text{ Gpc.}$$

$$D_A(z) = \frac{D_L}{(1+z)^2}$$

Our vision of the Universe

II CMB & LSS

Radiation | Matter | Energy



BBN
T ~ MeV

$\nu e^- \not\rightarrow \nu e^-$

$a \simeq 10^{-10}$

Equality matter-radiation

$a \simeq 10^{-4}$

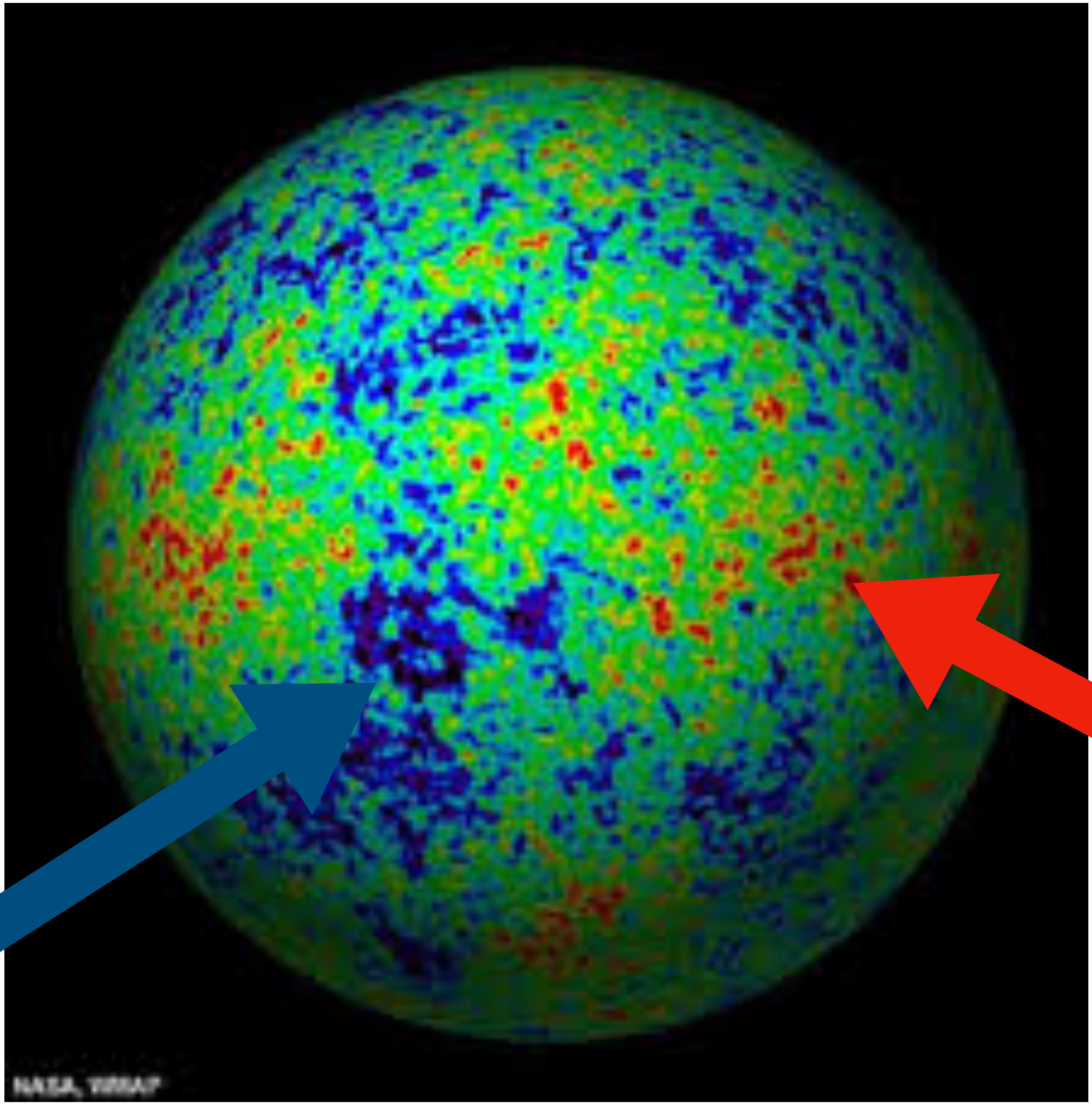
Decoupling

$\gamma e^- \not\rightarrow \gamma e^-$

$a \simeq 10^{-3}$

structures form

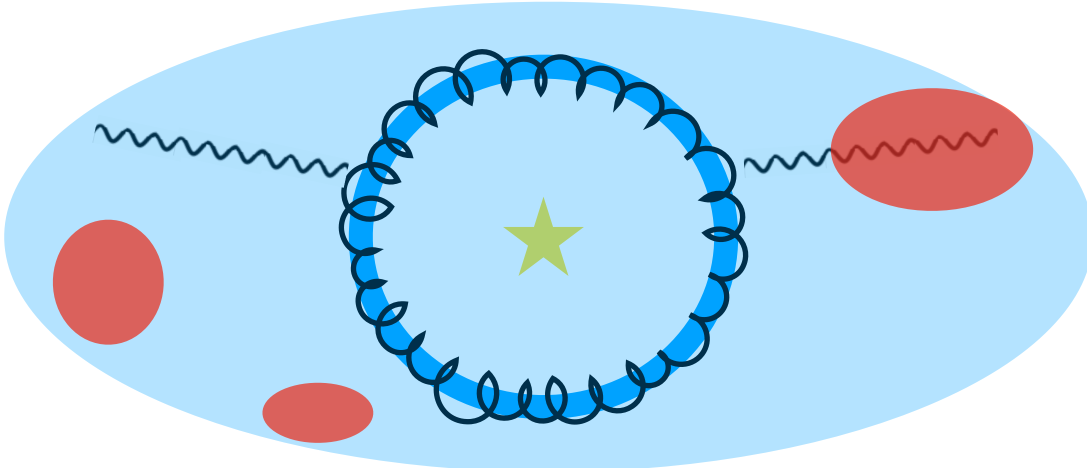
Anisotropies of temperature



A bit more particles

A bit less particles

Anisotropies of temperature



Not perfect isotropy → **anisotropies**

Theory of perturbations

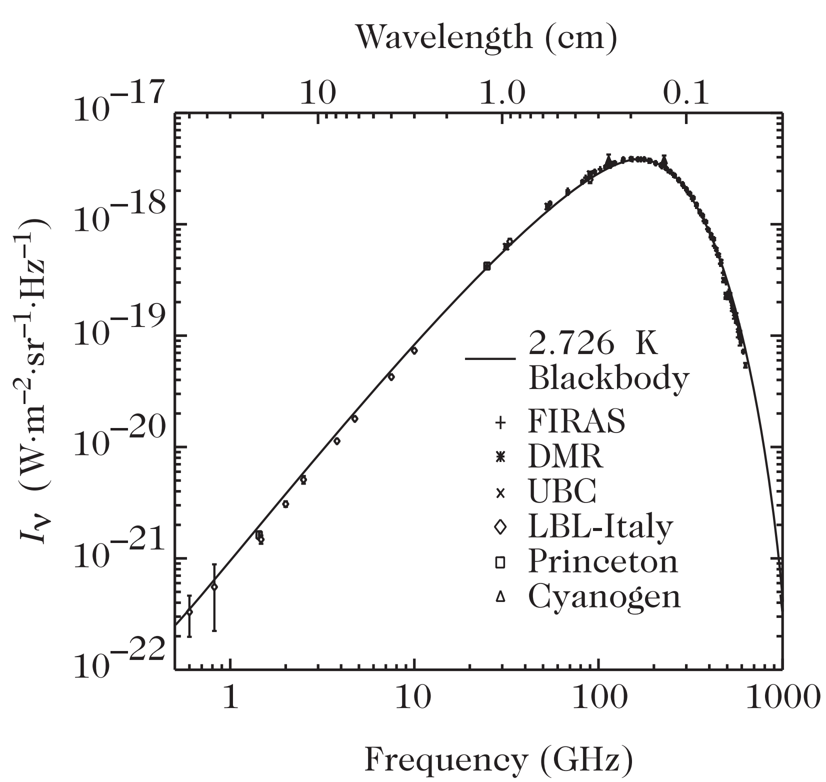
$$T_{\gamma_{\rm CMB}} \sim 2.710^{-4}\ {\rm eV} + \delta {\rm T}$$

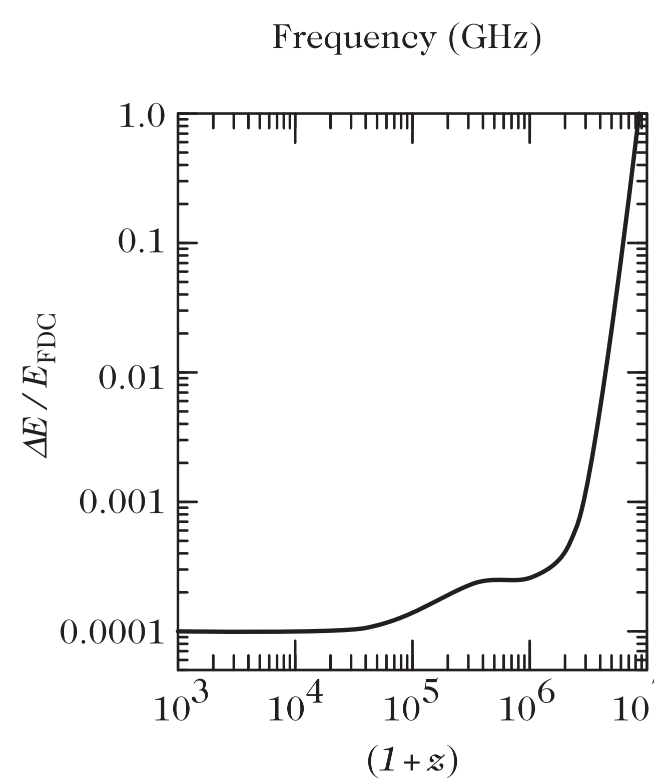
Distribution of matter: over densities and under densities

$$\rho_r \propto T^4 \qquad \rho_m \propto T^3 \qquad \frac{\delta\rho_r}{\rho_r} \propto \frac{\delta\rho_m}{\rho_m} \propto \frac{\delta T}{T}$$

Black Body spectrum

$$B(\nu) = \frac{2h\nu^3}{c^2}\,\frac{1}{e^{\frac{h\nu}{(kT)}} - 1}$$





Anisotropies of temperature

$$f(E_{\mathrm{CMB}}) = \left(e^{\frac{E_{\mathrm{CMB}}}{T}} - 1\right)^{-1}$$ **with** $$E = \bar{E}\left(1 + \frac{\vec{v}.\vec{n}}{c}\right)$$

so $$T = \bar{T}\left(1 - \frac{\vec{v}.\vec{n}}{c}\right)$$

$$\Delta T = \frac{T - \bar{T}}{\bar{T}} = \frac{\vec{v}.\vec{n}}{c} \simeq 10^{-3}$$

$$\frac{\vec{v}.\vec{n}}{c} = \frac{v}{c} \cos\theta$$

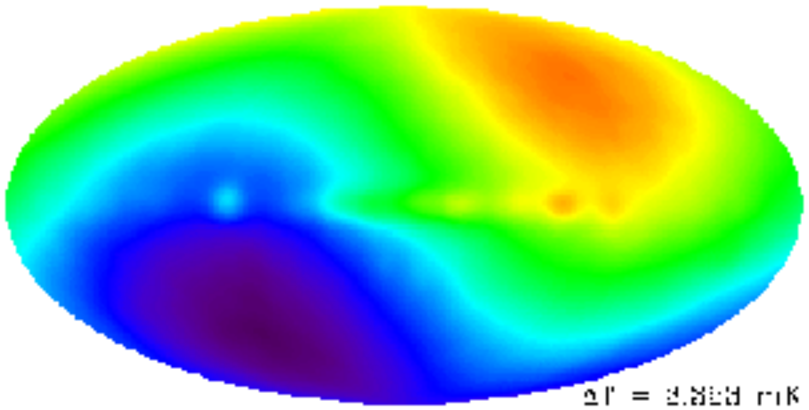
$$Y_0^0 = cst \qquad Y_1^0 = \cos\theta$$

DIPOLE

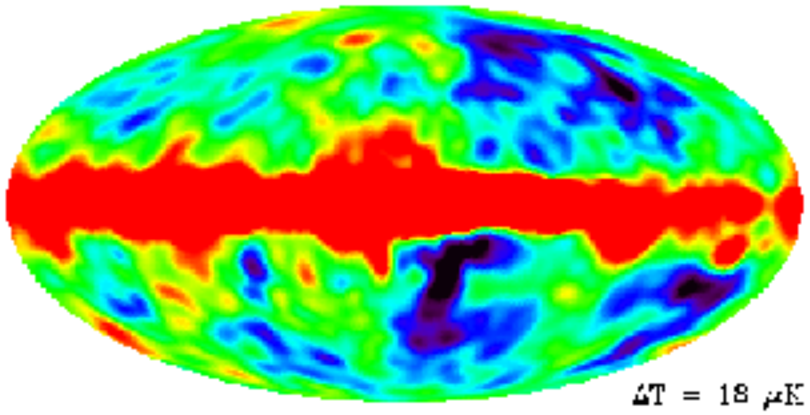
Anisotropies of temperature

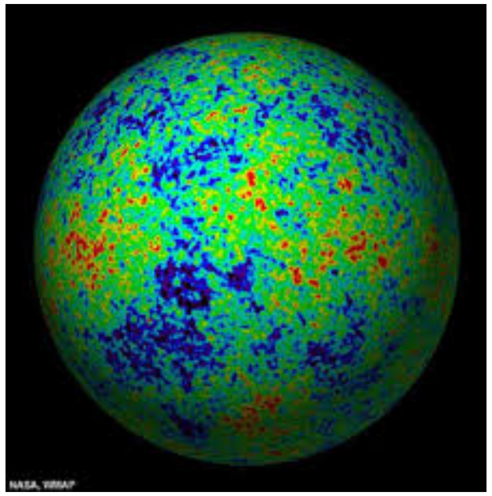
$$Y_0^0 = cst$$

$$Y_1^0 = \cos\theta$$



$$Y_l^m$$





l > 2000

The higher l, the more details you get

What is the Physics of the anisotropies?

Anisotropies of temperature

BBN
T ~ MeV
$\nu e^- \not\to \nu e^-$

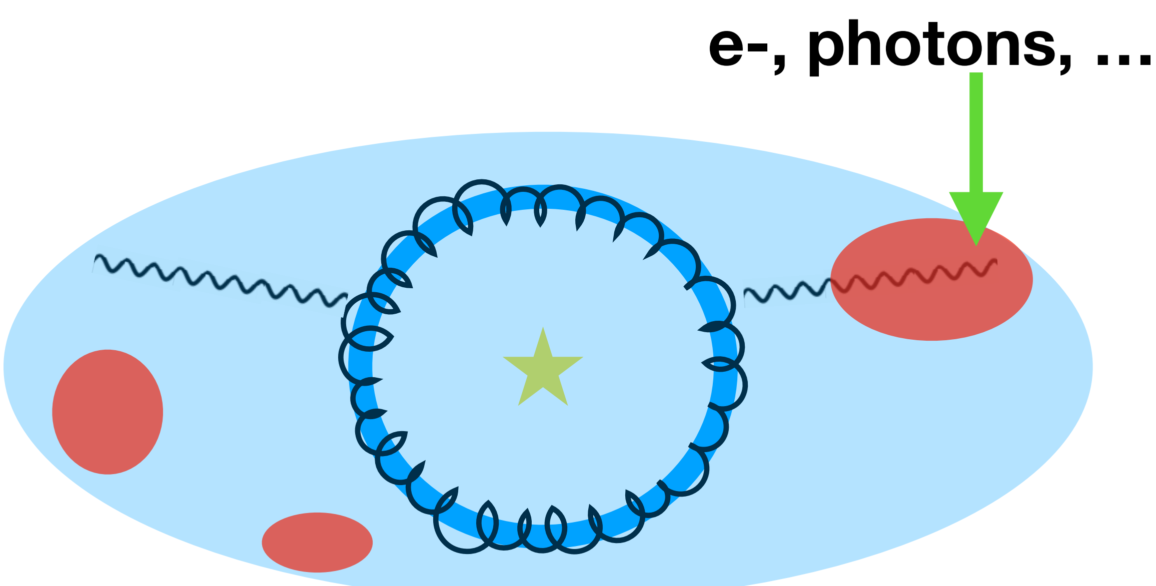
$a \simeq 10^{-10}$

Equality
matter-radiation

$a \simeq 10^{-4}$

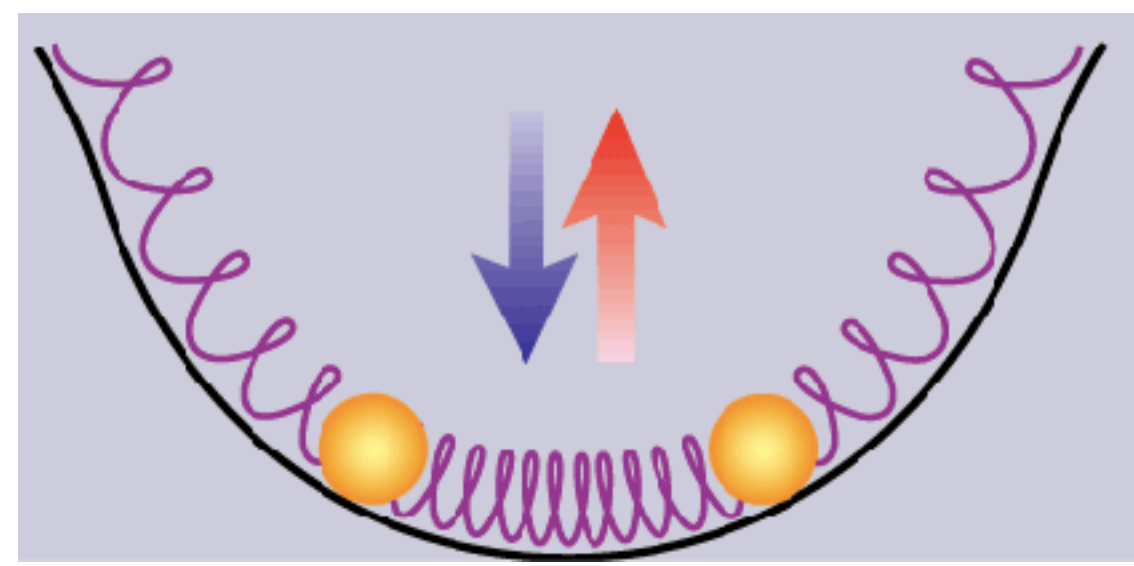
Decoupling
$\gamma e^- \not\to \gamma e^-$

$a \simeq 10^{-3}$



pressure from interactions
versus gravitation
in a matter dominated Universe

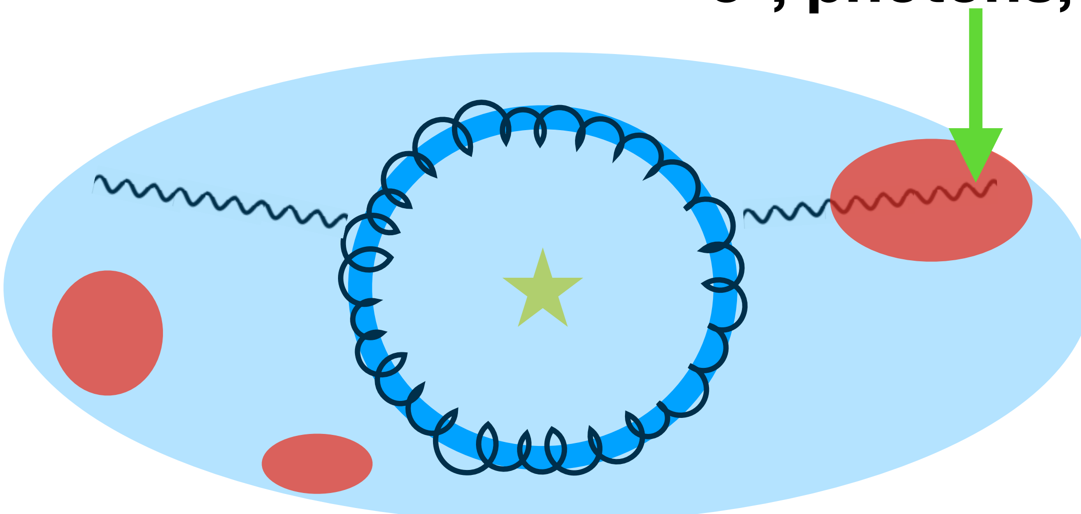
W. Hu

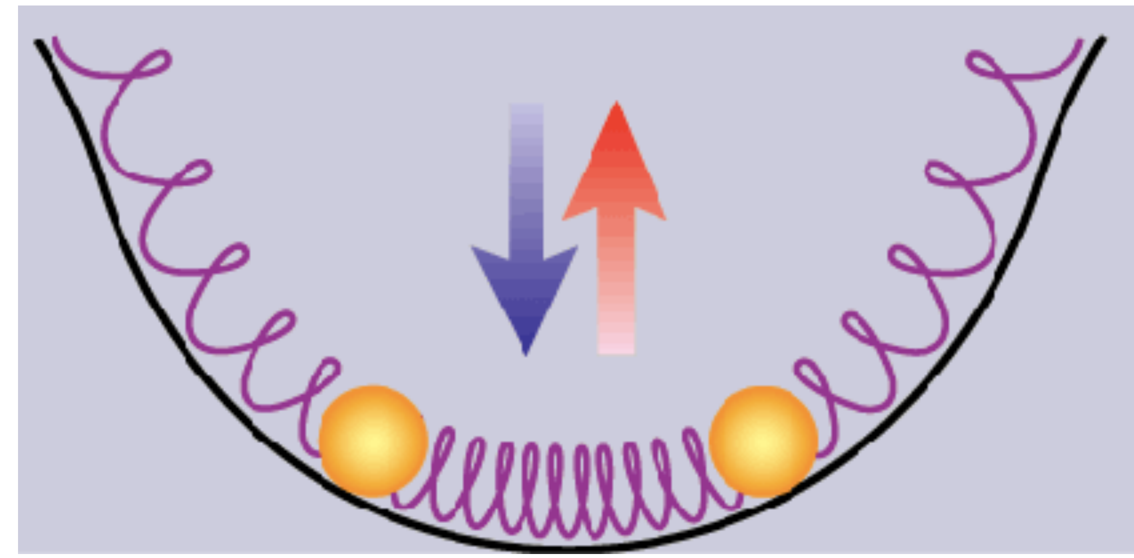


Anisotropies of temperature

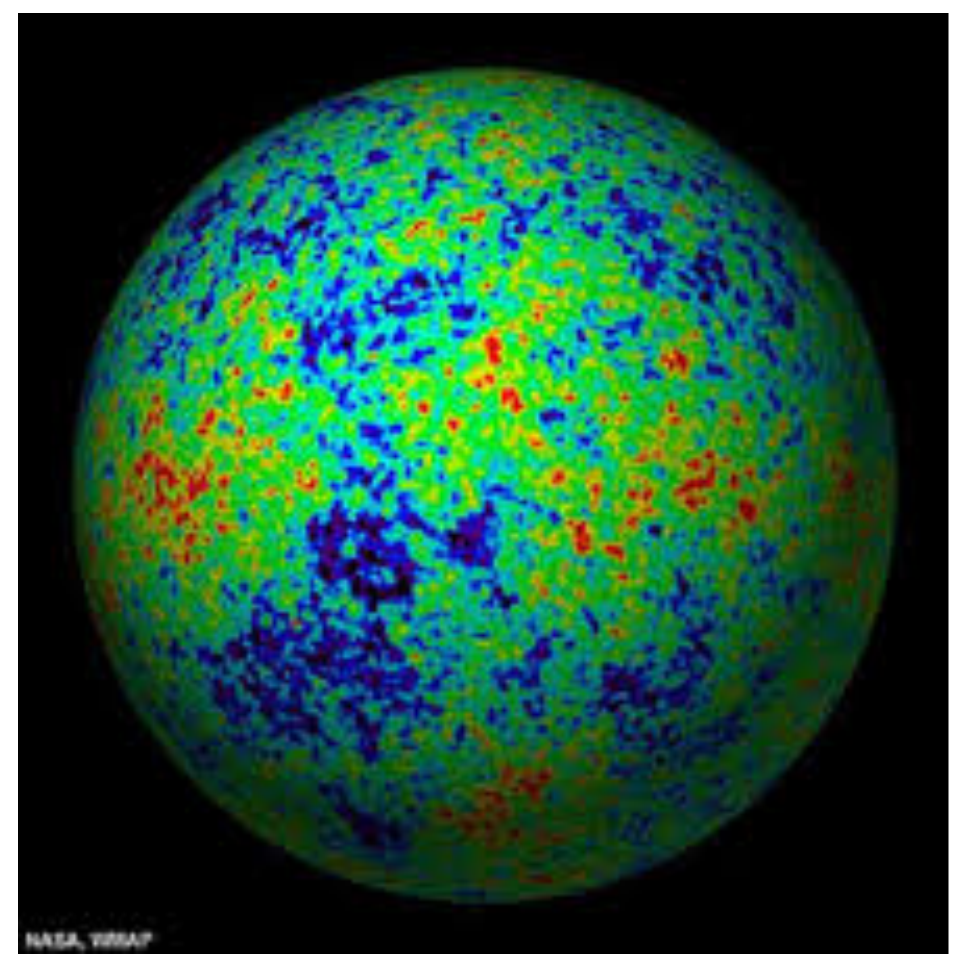
W. Hu

e-, photons, ...





pressure from interactions versus gravitation in a matter dominated Universe



empty regions are not growing by gravity and they get destroyed by the interactions but denied regions too...

(Silk damping)

Anisotropies of temperature

$$T(\vec{n}) = \bar{T}\ \left(1 + \Theta(\vec{n})\right)$$

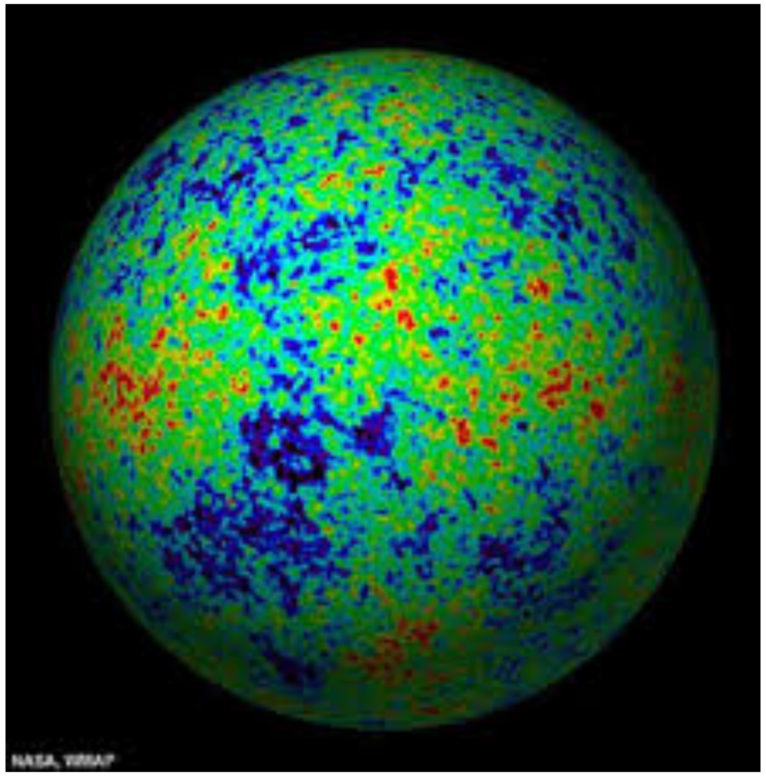
$$\Theta(\vec{n}) = \frac{(T(\vec{n}) - \bar{T})}{\bar{T}} \equiv \frac{\Delta T}{\bar{T}}$$

$$\Theta_{lm} = \int \frac{d^3n}{(2\pi)^3} \int \frac{d^3k}{(2\pi)^3}\, e^{i\,\vec{k}.\vec{n} D_{a\star}}\, \Theta(\vec{k})\, Y_{lm}(\vec{n})$$

$$e^{i\,\vec{k}.\vec{n} D_{a\star}} = 4\pi \sum_{l=0}^{\infty} \frac{a_l}{(2l+1)}\, j_l(kD_{a\star}) \sum_m (-1)^m\, Y_l^m(\vec{k})\, Y_l^{-m}(\vec{n})$$

$$\Theta_{lm} = 4\pi \int \frac{d^3k}{(2\pi)^3}\, i^l\ \ j_l(kD_{a\star})\ \ \Theta(\vec{k})\ \ Y_l^m(\vec{k})$$

Bessel functions — damped oscillations



$$\langle \Theta^\star_{l'm'}\ \Theta_{lm}\rangle = 4\pi\ \ \delta_{l'l}\,\delta_{m'm} \int d\ln k\ \ j_l(kD_{a\star})^2\ \ \Delta_T^2(k)$$

$$= C_l^T\ \ \delta_{l'l}\,\delta_{m'm}$$

$$C_l^T \simeq 4\pi\ \frac{\Delta_T^2(k)}{2\,l\,(l+1)}$$

the coefficients of the Ylm decomposition

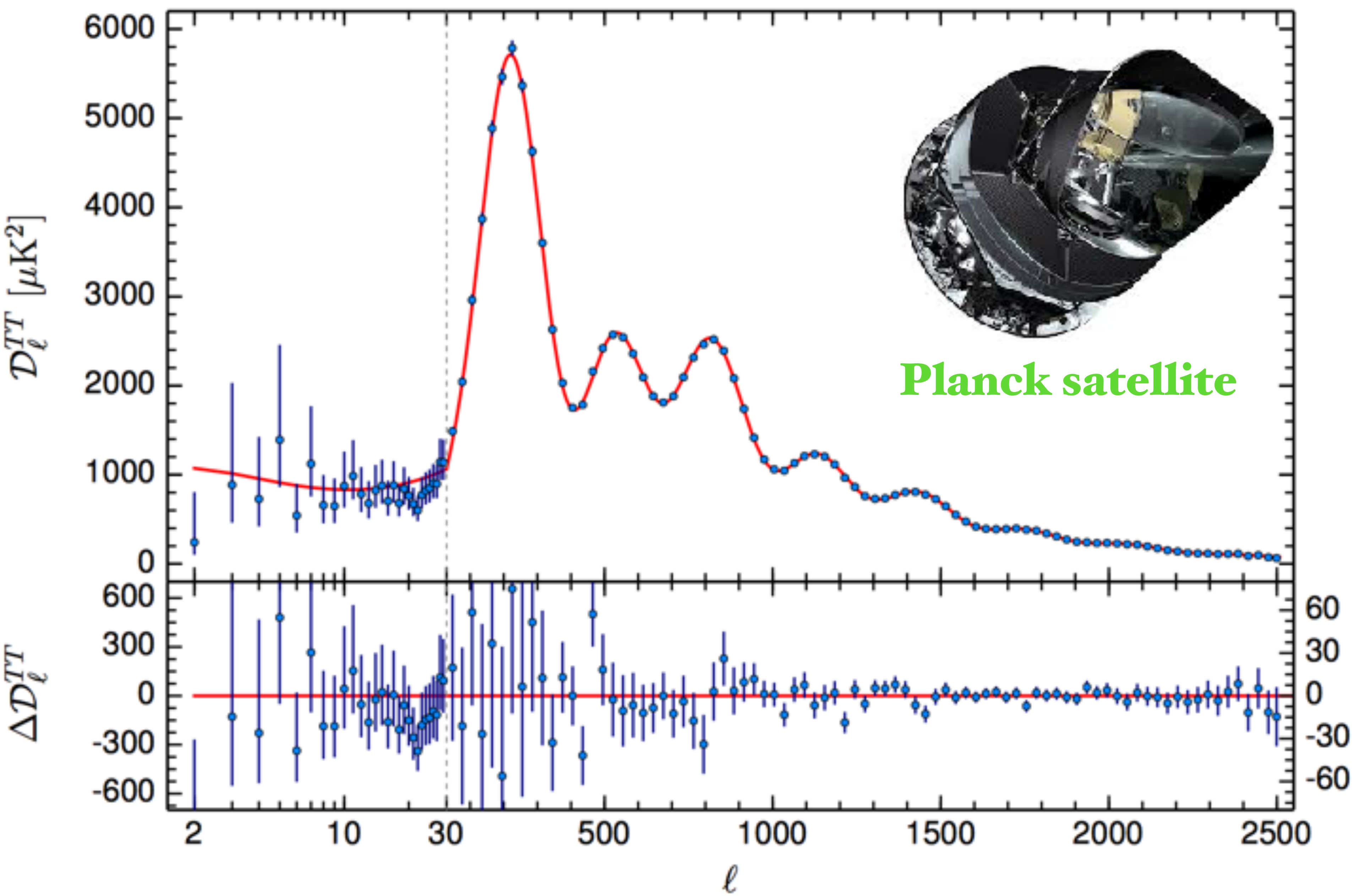
II CMB & LSS

Anisotropies of temperature

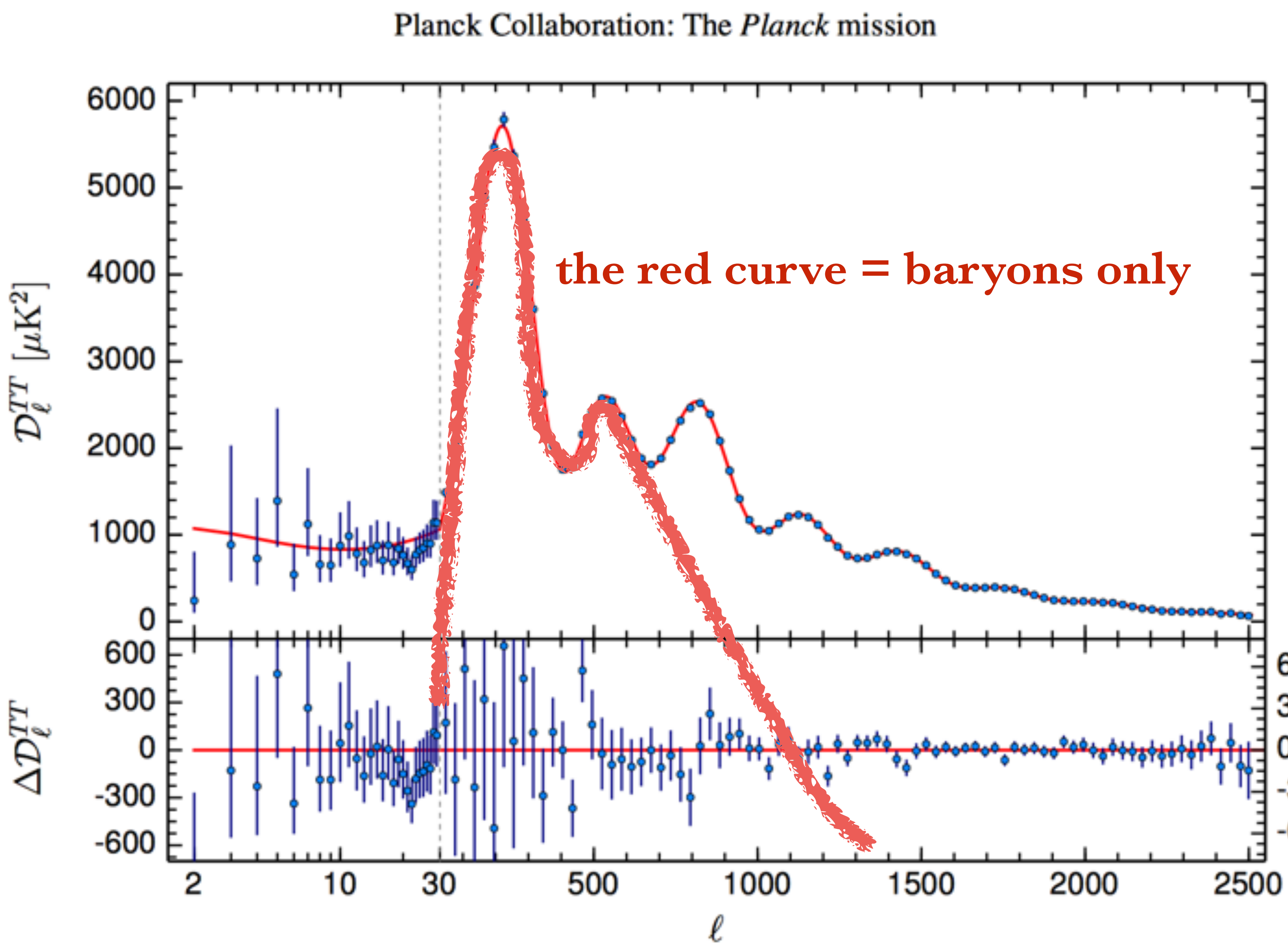
correlations in the sky

$$\langle \Theta^{\star}_{l'm'} \; \Theta_{lm} \rangle = 4\pi \;\; \delta_{l'l}\, \delta_{m'm} \int d\ln k \;\; j_l(kD_{a\star})^2 \;\; \Delta_T^2(k)$$
$$= C_l^T \;\; \delta_{l'l}\, \delta_{m'm}$$

Planck Collaboration: The *Planck* mission **2015**



Anisotropies of temperature



the suppression of small-scales is indicative of the presence of baryons

Therefore there must be more than the baryons

END of LECTURE 2