Neutrino Physics

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1. *neutrino interactions* $\left\{ \begin{array}{c} \text{weak} \\ \text{gravity} \end{array} \right.$
2. *neutrino masses in the Lagrangian*
   - Majorana, Dirac, phases, 2s and all that
3. *neutrinos oscillate* $\Leftrightarrow$ *have mass*
   - atmospheric oscillations
   - usual 2 flavour oscillations
   - why can one use a Schrodinger Eqn ?
4. oscillations in matter
   - solar (2-flav) oscillations
   - no supernovae :(
5. ...but there are (at least) 3 generations ?
6. Leptonic New Physics beyond 3 m_ν...

(hypothetical /known) history of neutrinos (shy in the lab, relevant in cosmo)

- ...
- inflation (gives large scale CMB fluctuations) (?driven by sneutrino ?)
- baryogenesis (excess of matter over anti-matter)via leptogenesis ?
- relic density of (cold) Dark Matter (?heavy neutrinos ?)Shaposhnikov
- Big Bang Nucleosynthesis ($H, D,^3 He,^4 He,^7 Li$ at $T \sim$ MeV))
  $\Leftrightarrow$ 3 species of relativistic ν in the thermal soup
- decoupling of photons — $e + p \to H$ (CMB spectrum today)
  cares about radiation density $\leftrightarrow N_\nu, m_\nu$
- for 10^{10} yrs —stars are born, radiate (γ, ν), and die
- supernovae explode (?thanks to ν ?) spreading heavy elements

- 1930 : Pauli hypothesises the "neutrino", to conserve E in $n \to p + e(+\nu)$
- 1953 Reines and Cowan : neutrino CC interactions in detector near a reactor
- invention of the Standard Model (SM) : massless ν
-
- **neutrinos have mass ! There is more in the Lagrangian than the SM...**

References...

Giunti website "neutrino unbound" : http ://www.nu.to.infn.it/

fits : http ://www.nu-fit.org/

Raffelt talks (astropart) :http ://wwwth.mpp.mpg.de/members/raffelt/

Plots thanks to Strumia + Vissani : hep-ph/0606054

simple 3-gen probabilities for LBL :Cervera etal 0002108 (+ later versions)

current state of oscillation measurements : Gonzalez-Garcia @ CERN ν plafform kickoff : https ://indico.cern.ch/event/572831/

neutrino cosmology : Lesgourgues at CERN ν plafform kickoff : https ://indico.cern.ch/event/572831/

Definitions and such...

I use Dirac spinors, with 4 degrees of freedom(dof) labelled by $\{\pm E, \pm s\}$, in *chiral* decomposition

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad , \ \{\gamma^\alpha\} = \left\{ \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} \right\}$$

$$\{\sigma_i\} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\psi_L = \ P_L\,\psi \quad \text{avec} \quad P_L = \frac{(1-\gamma_5)}{2} \quad , \ \psi_R = \ P_R\,\psi$$

chirality is *not* an observable ($\rightarrow$ helicity $= \pm\hat{s}\cdot\hat{k} = \pm 1/2$ in relativistic limit), but $P_{L,R}$ simple to calculate with :)

notation : $\overline{(\psi_R)} = (P_R\psi)^\dagger\gamma_0 = \psi^\dagger P_R\gamma_0 = \psi^\dagger\gamma_0 P_L = (\overline{\psi})_L$

$$(\psi^c)_L = P_L(-i\gamma_0\gamma_2\gamma_0\psi^*) = -i\gamma_0\gamma_2\gamma_0\psi^*_R$$

Leptons in the Standard Model

- 3 generations of lepton doublets, and charged singlets :

$$\ell_{\alpha L} \in \left\{ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} , \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} , \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \right\} \qquad e_{\alpha R} \in \{ e_R,\ \mu_R,\ \tau_R \}$$

in charged lepton mass basis (greek index, eg α).

- No ν_R in SM because

1. data did not require m_ν when SM was defined (ν are shy in the lab...)

2. ν_R an SU(2) singlet $\Leftrightarrow$ no gauge interactions

$\Rightarrow$ not need ν_R for anomaly cancellation

$\Rightarrow$ if its there, its hard to see

Lagrangian that reproduces leptons interactions(but not ν masses)

$$\mathcal{L} = i\overline{\ell_{L_\alpha}}^T \gamma^\mu \mathrm{D}_\mu \ell_{L\alpha} + i\overline{e_{R\alpha}} \gamma^\mu D_\mu e_{R\alpha} - \left[(\overline{\nu_{\sigma L}}, \overline{e_{\sigma L}}) y_\sigma \begin{pmatrix} -H^+ \\ H^{0*} \end{pmatrix} e_{\sigma R} + \text{h.c.} \right]$$

$$\mathrm{D}_\mu = \partial_\mu + i\frac{g}{2}\sigma^a W^a_\mu + ig' Y(\ell_L) B_\mu \ , \ D_\mu = \partial_\mu + ig' Y(e_R) B_\mu$$

B^μ hypercharge gauge boson, $Y(f) = T_3 + Q_{em}$

$\widetilde{H} = \begin{pmatrix} -H^+ \\ H^{0*} \end{pmatrix}, m_\alpha = y_\alpha \langle H^0 \rangle$

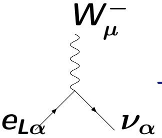
greek index = generation in charged lepton mass basis

Neutrino weak interaction in doublet kinetic terms

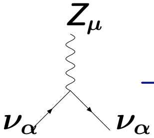
First term $\overline{\ell_{L\alpha}}^T \gamma^\mu \mathbf{D}_\mu \ell_{L\alpha}$ gives :

$$\left(\overline{\nu_L} \quad \overline{e_L} \right) \gamma^\mu \begin{pmatrix} \frac{g}{2\cos\theta_W} Z_\mu & \frac{g}{\sqrt{2}} W^+_\mu \\ \frac{g}{\sqrt{2}} W^-_\mu & eA_\mu - \ldots Z_\mu \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad , \quad s_W \equiv \sin\theta_W$$

$\tan\theta_W = g'/g$, $A_\mu \equiv c_W B_\mu + s_W W^3_\mu$, $Z_\mu \equiv -s_W B_\mu + c_W W^3_\mu$ Get :



$$-i \frac{g}{\sqrt{2}} \gamma_\mu \left(\frac{1-\gamma^5}{2} \right)$$



$$-i \frac{g}{2\cos\theta_W} \gamma_\mu \left(\frac{1-\gamma^5}{2} \right)$$

CC production of νs (Pauli)
NO flavour change (ν or e) in lepton sector for massless ν !
Universal Z cpling to 3 ν (Γ_{inv} says 2.994 ± 0.012)

(parenthese —diagonal/canonical kinetic terms ?)

By the way, that was most general renormalisable, $SU(2) \times U(1)$-invariant $\mathcal{L}$ for those particles...*What about*

$$\left(\overline{\nu_{eL}}, \overline{e_L}\right) \not{D} \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad , \quad \left(\overline{\nu_{eL}}, \overline{e_L}\right) \widetilde{H} \tau_R$$

are gauge invariant ⇔ why not the Lagrangien :

$$i \overline{\ell_L}^{b'} Z_{bc} \gamma^\mu \mathbf{D}_\mu \ell_L^{c'} + i \overline{e_R}^f \gamma^\mu D_\mu e_R^f - \overline{\ell}_L'^b [\tilde{Y}_e]_{bd} \widetilde{H} e_R^d + h.c.$$

Because its equivalent ! To recover to canonical $\mathcal{L}$, diagonalise Z (hermitian pcq $\mathcal{L} \in \Re$)

$$\overline{\ell_L'^b} \mathbf{Z}_{bc} \not{D} \ell_L'^c = \overline{\ell_L'^b} [V_Z^\dagger \mathbf{D_Z} V_Z]_{bc} \not{D} \ell_L'^c = \overline{\ell_L^{b''}} \mathbf{D_Z}_{bb} \not{D} \ell_L^{b''} = \overline{\ell_L}^b \not{D} \ell_L^b$$

where absorb the eigenvalues of Z in field defns : $\ell_L^b = \sqrt{z^b} \ell_L^{b''}$.

Then redefine Yukawas : $\mathbf{Y_e} = \mathbf{D_Z^{-1/2} V_Z \widetilde{Y}_e}$, to get canonical kinetic terms :

$$\mathcal{L} \;=\; i \overline{\ell_L^b}^T \not{D} \ell_L^b + i \overline{e_R^a} \not{D} e_R^a - \{ (\overline{\ell_L^b} [\mathbf{Y_e}]_{bc} \widetilde{H}) e_R^c + \text{h.c.} \}$$

Now diagonalise charged lepton mass matrix...

$$i\,\overline{\ell_L}^{\beta T}\gamma^\mu \mathbf{D}_\mu \ell_L^\beta + i\,\overline{e_R}^\alpha \gamma^\mu \mathbf{D}_\mu e_R^\alpha + \overline{\ell}_L^\beta [Y_e]_{\beta\alpha} \widetilde{H}\, e_{\alpha R} + h.c.)$$

$[Y_e]_{\beta\alpha}$ arbitrary 3×3 matrix
To obtain diagonal charged-lepton mass matrix, use different unitary transformations on left and right of Yukawa (makes sense : different fields on either side) :

$$V_L[\mathbf{Y_e}]V_R^\dagger = D_e$$

- order of my indices is LR
- obtain V_L, V_R by diagonalising hermitian matrices $[\mathbf{Y_e}][\mathbf{Y_e}]^\dagger = V_L^\dagger D_e^2 V_L$ $[\mathbf{Y_e}]^\dagger[\mathbf{Y_e}] = V_R^\dagger D_e^2 V_R$
- only basis choice in flavour space from Yukawas ; there is no such thing as an "interaction basis"

Neutrinos have gravitational interactions

1. expected from equivalence principle : carry 4-momentum
2. Big Bang Nucleosynthesis ($\tau_U \sim$ few minutes) :
   - $n_n/n_p \propto exp\{-(m_n - m_p)/T\}$ in thermal equil
   - *frozen* when $\Gamma(n + \nu \to p + e) \lesssim H$
   - $H^2 \simeq 3\rho_{rad}/m_{pl}^2$; $\rho_{rad} \supset \{\gamma, 3\nu\}$
   - n_n/n_p controls ^{4}He ratio to H...
3. Cosmic Microwave Background : (is a fit to a multi-parameter model), and U is mat-dim at recombination. But sensitivity for similar reasons to # of relativistic species present...

Lesgourgues

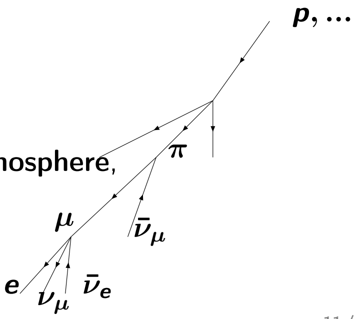
Historical problems...

BUT...historical "problems" : neutrinos disappear...

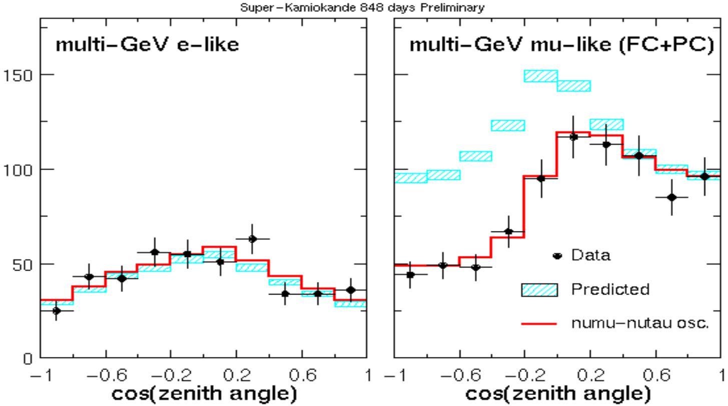
2. the sun produces energy by a network of nuclear reactions, should produce ν_e (lines and continuum) which escape. The photons diffuse to the surface. Observed ν_e flux $\sim .3 \to .5$ expected from solar energy output. Flux in $\sum$ flavours $\sim$ expected (SNO).
$\Rightarrow$ new ν physics (BSM !), that changes ν flavour on way out of sun :

- magnetic moments ?
- wierd new interactions ?
- masses (and mixing angles) in matter
- ...

1. deficit of ν_μ arriving from **earth's atmosphere**, produced in cosmic ray interactions :
expect $N(\nu_\mu + \bar{\nu}_\mu) \simeq 2N(\nu_e + \bar{\nu}_e)$
see deficit of $\nu_\mu, \bar{\nu}_\mu$ from below.



SK-98 : $\nu_\mu + H_2 0 \to \mu + ..$, deficit in ν_μ from below



upwards $\leftrightarrow$ cos= -1 ; down $\leftrightarrow$ cos= + 1.

L : 20 km $\leftrightarrow$ 10 000 km.

SNO : solar ν_e deficit, but expected $\sum \nu_\alpha$ flux

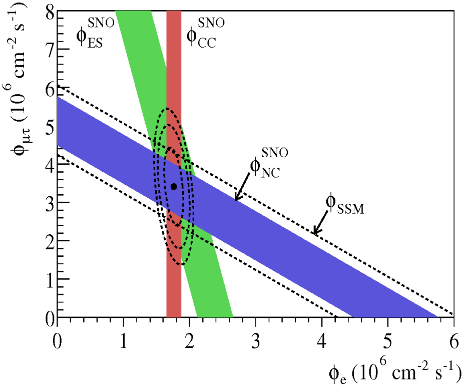


FIG. 3: Flux of ^{8}B solar neutrinos which are μ or τ flavor vs flux of electron neutrinos deduced from the three neutrino reactions in SNO. The diagonal bands show the total ^{8}B flux as predicted by the SSM [11] (dashed lines) and that measured

To write a neutrino mass

To write a mass for ν_L...Lorentz Invariance

Before discussing oscillations and the kinematics of m_ν, think about how to write a mass term for neutrinos in $\mathcal{L}$...
Cosmology says : $\sum m_i \lesssim$ eV. Oscillations say :

(global fits of www.nu-fit.org)

$$
\begin{aligned}
|\Delta^2_{31j}| &= |m_3^2 - m_j^2| = 2.52 \pm 0.04 \times 10^{-3}\ \mathrm{eV}^2 \\
&\gg \Delta m^2_{21} = 7.50 \pm 0.2 \times 10^{-5}\ \mathrm{eV}^2 \\
&\sqrt{\Delta m^2_{31}} \simeq 0.05\ \mathrm{eV} \qquad \sqrt{\Delta m^2_{21}} \simeq 0.008\ \mathrm{eV}
\end{aligned}
$$

Low scale so work in effective theory of SM below m_W (neglect SU(2) invariance). Mass must be Lorentz invariant. Only possibility for a four-component fermion ψ :

$$
m\overline{\psi}\,\psi \ = \ m\overline{\psi_L}\,\psi_R \ + m\overline{\psi_R}\,\psi_L
$$

To write a Dirac mass for ν_L

Work in effective theory below m_W. Neutrino mass must be Lorentz invariant. For four-component fermion ψ :

$$m\overline{\psi}\,\psi \;=\; m\overline{\psi_L}\,\psi_R \;+ m\overline{\psi_R}\,\psi_L$$

1. Dirac masss term : SM has only ν_L, 2-component chiral fermion $\Rightarrow$ introduce chiral gauge singlet fermion ν_R
Construct fermion number conserving mass term like for other SM fermions :

$$m\overline{\nu_L}\,\nu_R \;+ m\overline{\nu_R}\,\nu_L$$

In full SM : $\lambda(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} H_0 \\ H_- \end{pmatrix} \nu_R \equiv \lambda(\overline{\ell} H) e_R \to \; m = \lambda\langle H_0\rangle$

Diagonalising Dirac mass matrix in flavour space

Like for charged leptons and quarks : $[\lambda]_{\sigma I}$ arbitrary 3×3 matrix in flavour space

Diagonalise with different unitary transformations on left and right of Yukawa :

$$U[\boldsymbol{\lambda}]U^{\dagger}_{R\nu} = D_{\nu}$$

- $U = 3{\times}3$ leptonic version of CKM called PMNS matrix (Pontecorvo, Maki, Nakagawa and Sakata) : U_{PMNS}.

To write a Majorana mass for ν_L

Lorentz-invar mass term for a four-component fermion ψ :

$$m\overline{\psi}\,\psi \;=\; m\overline{\psi_L}\,\psi_R \;+ m\overline{\psi_R}\,\psi_L$$

2. Majorana mass term : the charge conjugate of ν_L is right-handed ! check !

$\Rightarrow$ write a mass term with ν_L ; *no new fields*, but lepton number violating mass :

$$\begin{aligned}\frac{m}{2}[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L] &= \frac{m}{2}[(\nu_L)^\dagger\gamma_0(\nu_L)^c + ((\nu_L)^c)^\dagger\gamma_0\nu_L] \\ &= -i\frac{m}{2}[\nu_L^\dagger\sigma_2\nu_L^* + \nu_L^T\sigma_2\nu_L] \equiv \frac{m}{2}\nu_L\nu_L + h.c.\end{aligned}$$

(2nd line = 2 comp notn) *Non*-renormalisable in full SM :

$$\mathcal{L} = ... + \frac{K}{4M}(\ell H)(\ell H) + h.c. \rightarrow \frac{m}{2}\nu_L\nu_L + h.c. \;\;,\;\; m = \frac{K}{2M}\langle H_0\rangle^2$$

$\Rightarrow$ *requires New Particles*

Majorana mass term : $(\nu_L)^c$ is right-handed

Recall that :

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad , \ \{\gamma^\alpha\} = \left\{ \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} \right\}$$

$$\{\sigma_i\} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Majorana mass term : the charge conjugate of ν_L is right-handed

$$
\begin{aligned}
\psi^c &= -i\gamma_0\gamma_2\bar{\psi}^T = -i\gamma_0\gamma_2\gamma_0\psi^* = i\gamma_2^*\psi^* \\
&= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \begin{pmatrix} \psi_L^* \end{pmatrix} \\ \begin{pmatrix} \psi_R^* \end{pmatrix} \end{pmatrix}
\end{aligned}
$$

$$
(\nu_L)^c = \begin{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} \nu_L^* \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0 \end{pmatrix} \\ \begin{pmatrix} -i\sigma_2\nu_L^* \end{pmatrix} \end{pmatrix}
$$

Diagonalising Majorana mass matrix

With multiple generations, Majorana

$$\frac{1}{2}\overline{\nu_L}^{\alpha}[m]_{\alpha\beta}(\nu_L)^c_{\beta}$$

is *symmetric*, diagonalise as :

$$U^T m U = D_m$$

• $m^\dagger m$ hermitian, for non-degen eigenvalues can obtain U from $U^\dagger m^\dagger m U = D_m^2$.

(in 2-comp notn :

$$\frac{1}{2}\nu_{L\alpha}[m]_{\alpha\beta}\nu_{L\beta} + h.c. = \frac{1}{2}\nu_{L\alpha}[U^* U^T m U U^\dagger]_{\alpha\beta}\nu_{L\beta} + h.c. = \frac{1}{2}\nu_{Li} m_i \nu_{Li} + h.c.$$

fermion fields anti-commute. But for ρ, σ 2-comp spinor indices,

$\nu^{\rho}_{Li}\varepsilon_{\rho\sigma}\nu^{\sigma}_{Lj} = -\nu^{\sigma}_{Lj}\varepsilon_{\rho\sigma}\nu^{\rho}_{Li} = \nu^{\sigma}_{Lj}\varepsilon_{\sigma\rho}\nu^{\rho}_{Li}$)

Muddle for theorists : Majorana 2s in the Lagrangian

Recall : usual to distribute $\frac{1}{2}$s for identical fields in $\mathcal{L}$, in order that F-rules and physical parameters not contain 2s : $\frac{m}{2}\nu_L\nu_L + h.c.$, $\frac{\kappa}{4M}(\ell H)\ell H) + h.c.$ (like for real scalar masses) because get F-rules as $\delta^n\mathcal{L}/\delta\nu^n$...
A majorana mass m appears in $\mathcal{L}$ as (4-comp notn on left, 2-comp notn on right)

$$\frac{m}{2}[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L] \equiv \frac{m}{2}\nu_L\nu_L + h.c.$$

A dirac mass m appears in $\mathcal{L}$ as

$$m\overline{\psi}\psi + h.c.$$

2.8 eigenvectors of a Majorana mass matrix

eigenvectors $\vec{v}_i$ of a hermitian matrix A, with eigenvalues $\{a_i\}$ from

$$A\vec{v}_i = a_i\vec{v}_i$$

because hermitian : $V^\dagger AV = D_A = diag\{a_1, ...a_n\}$ (V unitary)

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} \begin{pmatrix} \\ \vec{v}_1 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{v}_2 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{v}_3 \\ \\ \end{pmatrix} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} \\ \vec{v}_1 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{v}_2 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{v}_3 \\ \\ \end{pmatrix} \end{bmatrix} \begin{bmatrix} a_1 & & \\ & ... & \\ & & a_n \end{bmatrix}$$

Whereas for majorana : $U^T AU = D_A \Rightarrow AU = U^* D_A$ (U unitary $UU^\dagger = 1$)

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} \begin{pmatrix} \\ \vec{u}_1 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{u}_2 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{u}_3 \\ \\ \end{pmatrix} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} \\ \vec{u}_1^* \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{u}_2^* \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{u}_3^* \\ \\ \end{pmatrix} \end{bmatrix} \begin{bmatrix} a_1 & & \\ & ... & \\ & & a_n \end{bmatrix}$$

For Majorana matrix :

$$A\vec{u}_i = a_i\vec{u}_i^*$$

2.9 But... how to get majorana eigenvalues ?

For hermitian matrices (like $MM^\dagger$), have "characteristic equation" :

$$\mathbf{M}\mathbf{M}^\dagger \vec{v}_i - |m_i|^2 \mathbf{I} \vec{v}_i = 0$$

allows to obtain eigenvals from $\det[\mathbf{M}\mathbf{M}^\dagger - |m_i|^2\ \mathbf{I}\ ] = 0$.
Naively, this reasoning does not work when you start from

$$\mathbf{M}\vec{v}_i - m_i \mathbf{I} \vec{v}_i^* = 0$$

so ... get absolute values of eigenvals from $\mathbf{M}\mathbf{M}^\dagger$.

For degen eigenvals of $\mathbf{M}\mathbf{M}^\dagger$: get eigenvectors using $\mathbf{M}$ rather than $\mathbf{M}\mathbf{M}^\dagger$; extra phases can matter.
Ex : its not the same to diagonalise $M^\dagger M = V^\dagger D_M^2 V$, or $M = U^T D_M U$

$$M = \begin{bmatrix} 0 & M_1 e^{i\phi} \\ M_1 e^{i\phi} & 0 \end{bmatrix} , \qquad M^\dagger M = \begin{bmatrix} M_1^2 & 0 \\ 0 & M_1^2 \end{bmatrix} \quad M_1 \in \Re$$

Exercise1

For $m_1, m_D, m_2 \in \mathbf{Re}$, and $\neq 0$, show that the phases α and β can be removed from the Majorana mass matrix

$$M = \left[\begin{array}{cc} m_1 e^{i\alpha} & m_D e^{i\phi} \\ m_D e^{i\phi} & m_2 e^{i\beta} \end{array} \right]$$

by a phase redefn on the fields. Show that the combination $2\phi - \alpha - \beta$ is not removeable.

Exercise2

Obtain the eigenvalues and eigenvectors of

$$M = \begin{bmatrix} m_1 & m_D e^{i\phi} \\ m_D e^{i\phi} & m_1 \end{bmatrix}$$

1. for $m_1 = m_D, \phi \neq \pi/2$
(Hint : obtain eigenvals and eigenvectors of $MM^\dagger$, then check whether the eigenvectors work for M. What eigenvaluess are they associated to ?)

2. for $m_1 = 0, \phi = 0$
This is a "dirac" fermion mass matrix. Conclude that a Dirac fermion is two mass-degen Majorana fermions.

3. for $m_1 = m_D, \phi = \pi/2$
(degenerate eigenvals... recall that the familiar eqn for the eigenvector $\vec{v}_i$ of a hermitian matrix : $H\vec{v}_i = h_i\vec{v}_i$, can be obtained from the diagonalisation of H using unitary matrices : $VHV^\dagger = diag\{h_i\}$. Obtain the corresponding eigenvector eqn for a symmetric matrix from $UMU^T = diag\{m_i\}$, then use it to get the eigenvectors of M.)

U

leptonic mixing matrix (lives in generation space ; rotates from charged lepton α to neutrino i) with three angles :

$$U_{\alpha i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} P$$

$$\theta_{23} \simeq \pi/4 \quad \theta_{12} \simeq \pi/6 \quad \theta_{13} \simeq 0.15, 8^o \quad \delta \sim 1.4\pi$$

(global fits of www.nu-fit.org)

$P = \text{diag}\{e^{-i\phi_1/2}, e^{-i\phi_2/2}, 1\}$ for Majorana, $\text{diag}\{1, 1, 1\}$ for Dirac

$$= \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{bmatrix} P$$

for comparaison, in CKM :

$$\theta_{23} \simeq V_{cb} \simeq 0.04 \quad \theta_{12} \simeq V_{us} \simeq 0.225 \quad \theta_{13} \simeq V_{ub} \simeq 0.004$$

more CPV in U, m_ν if Majorana

• suppose that all parameters in $\mathcal{L}$ that can be complex (U and $m_{\nu i}$), are complex
• 3 angles and 6 phases in generic unitary matrix U (18 real parameters in arbitrary 3×3 complex matrix. Unitarity $UU^\dagger = 1$ reduces this to 9.)
• five relative phases between the fields $e_L, \mu_L, \tau_L, \nu_1, \nu_2$, ν_3
...so can choose the 5 relative phases among LH fermions, to remove all but one phase in the mixing matrix.
• now check if can make the masses real : if dirac masses, absorb phase of mass with ν_{RI}. If ν_{L3} has Majorana mass, between self and anti-self, choose absolute phase of ν_{L3} to make the mass real. Now all LH fermion phases are fixed, and cannot remove phases from $m_{\nu 1}, m_{\nu 2}$.
$\Rightarrow$ extra CPV in processes where Majorana mass appears linearly (not as mm^*, not in kinematics = not in oscillations)

Dirac vs Majorana — what differences ?

1. *discrete* difference : number of light degrees of freedom
   massive $\left\{ \begin{array}{c} \text{majorana} \\ \text{dirac} \end{array} \right\}$fermion $\leftrightarrow \left\{ \begin{array}{c} 1 \\ 2 \end{array} \right\}$ chiral fermions
   ? ? but how to count degrees of freedom ?
2. *continuous* difference : majorana mass is Lepton Number Violating $\Rightarrow$ look for $\Delta L = 2$ processes *e.g.* $0\nu 2\beta$.
3. more CPV (but in LNV processes ?) : all but one of majorana ν masses are complex

Dinner topic (?) : is Majorana vs Dirac a boolean question?

• Can ask a boolean "model discrimination" question : are there three light majorana ν with LNV masses, or three light dirac ν with LN conserving masses.
But maybe its neither of those models?
• I think the phenomenological question is the LNV rate, because can't measure number of light chiral fermions

? ... if add an undetectably small LNV mass to a Dirac mass matrix; does that make the neutrinos Majorana?
(have 6 chiral fermions as for Dirac, and no observed LNV...)

Where do mixing matrices appear ?

Only one mass eigenstate basis for $\{e_R^\alpha\}$, $\{\nu_R^I\} = (\nu_R^2, \nu_R^1)$....so sit there (means $U_{R\nu}$ unphysical). What to do for ℓ^a ? ? Take mass basis of charged leptons :

$$\ell_L^e \equiv \begin{pmatrix} U_{ei}\nu_L^i \\ e_L \end{pmatrix} \ , \ \ell_L^\mu \equiv \begin{pmatrix} U_{\mu j}\nu_L^j \\ \mu_L \end{pmatrix} \ , \ell_L^\tau \equiv \begin{pmatrix} U_{\tau k}\nu_L^k \\ \tau_L \end{pmatrix}$$

and Lagrangian becomes

$$i\,(U_{ej}^*\overline{\nu_L^j} \ \ \overline{e_L})\,\gamma^\mu \mathbf{D}_\mu \begin{pmatrix} U_{ek}\nu_L^k \\ e_L \end{pmatrix} + i\,(U_{\mu j}^*\overline{\nu_L^j} \ \ \overline{\mu_L})\,\gamma^\mu \mathbf{D}_\mu \begin{pmatrix} U_{\mu k}\nu_L^k \\ \mu_L \end{pmatrix} + ..$$

3×3 mixing matrix $U_{\alpha,i}$ appears at $W^\pm$ vertices (like CKM)

$$\rightarrow \quad -i\,\frac{gU_{ej}^*}{\sqrt{2}}\overline{\nu_L^j}\gamma^\mu W_\mu^+ e_L + ...$$

but flavour-diagonal Z vertex :

$$\propto \sum_\alpha -i\,\frac{g}{2}U_{\alpha j}^*\overline{\nu_L^j}\gamma^\mu Z_\mu^+ U_{\alpha k}\nu_L^k = \delta_{jk}\frac{g}{2}\overline{\nu_L^j}\gamma^\mu Z_\mu^+\nu_L^k$$

Oscillations

neutrinos "oscillate"(QM version : easy to rederive)

A relativistic neutrino, with momentum $\vec{k}$, is produced in muon decay at $t = 0$ (at Tokai/edge atmosphere). Describe as a quantum mechanical state :

$$|\nu(t=0)\rangle = |\nu_\mu\rangle$$

It travels a distance L in time t to the detector (SuperK)

$$|\nu(t)\rangle$$

where it produces an μ in CC scattering. With what probability ?

$$\mathcal{P}_{\mu\to\mu}(t) = |\langle\nu_\mu|\nu(t)\rangle|^2 \quad = \quad ?$$

1. Suppose massive neutrinos (two generations for simplicity).
   Flavour and mass eigenstates related by : $\nu_\alpha = U_{\alpha i}\nu_i$

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}.$$

2. Suppose time evolution in the mass basis described by

$$i\frac{d}{dt}\begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} = \begin{bmatrix} E_2 & 0 \\ 0 & E_3 \end{bmatrix}\begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} \qquad , \quad E_i^2 = k^2 + m_i^2$$

3. If produce relativistic ν_μ at $t = 0$, then at t later :

$$|\nu(t)\rangle = \sum_j U_{\mu j}|\nu_j(t)\rangle = \sum_j U_{\mu j} e^{-iE_j t}|\nu_j\rangle$$

Amplitude for neutrino to produce charged lepton α in CC scattering in detector after t :

$$|\langle \nu_\alpha | \nu(t) \rangle| = \left| \sum_j U_{\mu j} e^{-iE_j t} U^*_{\alpha j} \right|$$

So in 2 generation case, using $t = L$, $E_3 - E_2 \simeq \frac{m_3^2 - m_2^2}{2E} \equiv \frac{\Delta^2_{32}}{2E}$:

$$\mathcal{P}_{\mu \to \tau}(t) = \left| \sin\theta \cos\theta \left(e^{i\Delta^2_{32}L/4E} - e^{-i\Delta^2_{32}L/4E} \right) \right|^2$$
$$= \sin^2(2\theta) \sin^2 \left(L \frac{\Delta^2_{32}}{4E} \right)$$
$$\mathcal{P}_{\mu \to \mu}(t) = 1 - \sin^2(2\theta) \sin^2 \left(L \frac{\Delta^2}{4E} \right) = 1 - \sin^2(2\theta) \sin^2 \left(1.27 \frac{L}{km} \frac{\Delta^2}{\mathrm{eV}^2} \frac{\mathrm{GeV}}{4E} \right)$$

$E = \nu$ energy, L source-detector distance, $\Delta^2_{32} \sim 10^{-3}\mathrm{eV}^2$
$E \sim 10$ GeV for atmospheric νs ; $L : 20km \to 10000km$

2nd try at oscillations in vaccuum (Relativistic QM)

Umm : can one use non-rel QM to describe ν propagation ?
Does ν_μ propagate with fixed $\vec{k}$ and variable E_i ?
Suppose relativistic neutrinos, produced in muon decay at $t = 0$. Amplitude to produce mass eigenstate i

$$\propto U_{\mu i}$$

Neutrinos travel distance $L =$ time τ to a detector. Propagator in position space for(scalar) mass eigenstate :

$$G[(0,0);(L,\tau)] \propto \int \frac{d^3p}{(2\pi)^3} e^{i(E\tau - pL)} \theta(\tau)$$

Bjorken+Drell, vol1, 6.26

Describe ν_i by a wave packet peaked at $\sim (E, \vec{k}) \Leftrightarrow \delta^3(\vec{p} - \vec{k})$

Amplitude for initial ν_μ to produce e_α at detector :

$$\mathcal{A}_{\mu\alpha} \propto \sum_j U_{\mu j} \times e^{-i(E_j \tau - k_j L)} \times U^*_{\alpha j}$$

$m_j \ll E, p \Rightarrow L \simeq t,$ so

$$-i(E_j t - p_j L) \simeq -i(E_j - p_j)L = -i\frac{E_j^2 - p_j^2}{E_j + p_j}L \simeq -i\frac{m_j^2}{2E}L$$

$$\mathcal{P}_{\mu\alpha} = |\mathcal{A}_{\mu\alpha}|^2 = |\sum_j U_{\mu j} e^{-i m_j^2 L/(2E)} U^*_{\alpha j}|^2$$

$$\mathcal{P}_{\mu\alpha} = |\mathcal{A}_{\mu\alpha}|^2 = |\sum_j U_{\mu j} e^{-im_j^2 L/(2E)} U^*_{\alpha j}|^2$$

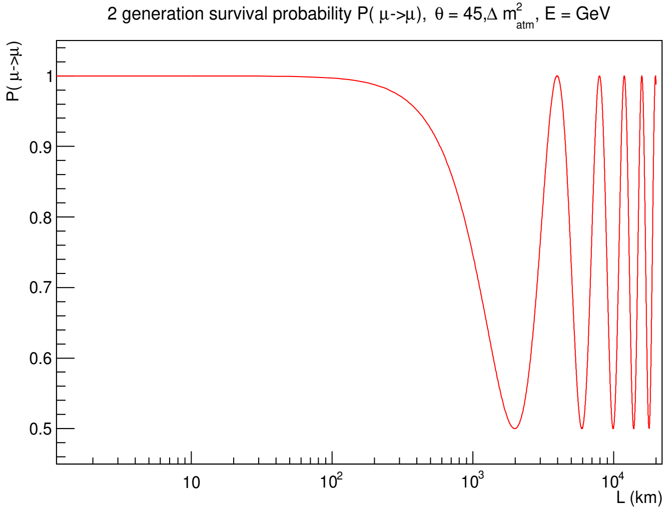
In 2 generation case :

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}.$$

$$\begin{aligned} \mathcal{P}_{\mu\to\tau}(t) = & \left|\sin\theta\ \cos\theta \left(e^{-im_2^2 L/2E} - e^{-im_3^2 L/2E}\right)\right|^2 \\ = & \sin^2(2\theta)\sin^2\left(L\frac{\Delta_{32}^2}{4E}\right) \qquad \Delta_{32}^2 \equiv m_3^2 - m_2^2 \end{aligned}$$

$$\mathcal{P}_{\mu\to\mu}(\tau) = 1 - \sin^2(2\theta)\sin^2\left(L\frac{\Delta^2}{4E}\right) = 1 - \sin^2(2\theta)\sin^2\left(1.27\frac{L\ \ \Delta^2}{km\mathrm{eV}^2}\frac{\mathrm{GeV}}{4E}\right)$$

E is ν energy, $L =$ source-detector distance.
$E \sim 10$ GeV for atmospheric νs ; L : $20km \to 10000km$



$$\mathcal{P}_{\mu\to\mu}(L) = 1 - \sin^2(2\theta)\sin^2\left(1.27\frac{L}{km}\,\frac{\Delta^2}{\mathrm{eV}^2}\,\frac{\mathrm{GeV}}{4E}\right)$$

$$\begin{aligned}
\Delta^2_{32} &= 2.5\times 10^{-3}\mathrm{eV}^2 \\
E &\sim 0.6\mathrm{GeV}(\mathrm{T2K}) \\
&\sim \mathrm{MeV}(\mathrm{reactors}) \\
&\sim 10\mathrm{GeV}(\mathrm{atmospher}
\end{aligned}$$

why don't d quarks oscillate?

1. (in my opinion) always propagate mass eigenstates(because construct free QFT with $E, \vec{p}$ eigenstates)
2. Produce at source a superposition of mass E, and p eigenstates (uncertainty principle : wave packet distributed within $\Delta E, \Delta p$)
   $\Leftrightarrow$ if measure/reconstruct E, p such that can measure $E^2 - p^2 = m^2$, then no oscillations
3. Oscillation distance $L \sim (E/GeV)(eV^2/\Delta m^2)$ km. If mean free path $\lambda \ll L$, reconstruct mass from track so no oscillations?

5. also...neutrino beam has energy spectrum Φ; although oscillation occurs, observe

$$\propto \int dE \frac{d\Phi}{dE} \sigma_{prod}(E) P_{\alpha \to \beta}(E) \sigma_{det}(E)$$

...average over many oscillations of $\sin^2 \to 1/2$ (so see disappearance, but not oscillation, in SK atm spectra)

decoherence of neutrinos for large $L/E \gg 1/\Delta^2$

- at production, 2 superposed wavepackets of masses m_2,m_3.
- group velocity of packets

$$v_i = \frac{\partial E}{\partial p} = \frac{p}{E} \simeq 1 - \frac{m_i^2}{2E^2}$$

- after distance L, packets have separated by

$$(v_2 - v_3)L \simeq \frac{\Delta_{23}^2}{E^2} L \simeq \frac{L}{\ell_{osc}} \frac{1}{E}$$

- no interference if larger than size of packets $\sim 1/(\delta E)$ where packet energy uncertain by δE. so no oscillations once

$$\frac{L}{\ell_{osc}} \gtrsim \frac{E}{\delta E}$$

can make similar estimate doing sum over paths, phases should sum coherently

Oscillations in QFT (skeletal version)

Question = neutrinos are relativistic; should we not do oscillations in QFT?

Here show that QFT is equivalent to "the Schrodinger eqn" (which I will want to use for oscillations in matter later...)
In second quantised field theory, the eqns of motion for the number operator $\hat{n}$ are (Heisenberg rep, t-dep ops)

$$\frac{d}{dt}\hat{n} = +i[\hat{H}, \hat{n}]$$

where the Hamiltonian $\hat{H}$ can be taken as free $= \hat{H}_0 \sim \sum \omega \hat{n}_\omega$. (recall free hamiltonian is sum over all states of number of particles * energy. *Integral* of hamiltonian density.)

1 : Work in second quantised formalism for neutrino field :

$$\hat{\psi}^I(x) = \sum_{s=+,-} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E}} \left(e^{-ip\cdot x} \hat{a}^I_s(\vec{p}) u_s(p) + e^{ip\cdot x} \hat{b}^{I\dagger}_s(\vec{p}) v_s(p) \right)$$

where s is heilicity, I is generation, $\hat{a}^\dagger$ creates particles, et $\hat{b}^\dagger$ creates anti-particules. *Define $\hat{a}$ for energy= mass eigenstates. But formalism is covariant...*

2 : We want to know time/space evolution of a beam neutrinos (no anti), of $+$ helicity and momentum $\vec{p}$. Define number operator :

$$\hat{n}^{IJ}_{sr}(\vec{p}) = \hat{a}^{I\dagger}_+(\vec{p}) \hat{a}^J_+(\vec{p})$$

covariant in flavour space (indices I, J).

3 : The eqns of motion for the number operator $\hat{n}$ are

$$\frac{d}{dt}\hat{n} = +i[\hat{H}, \hat{n}]$$

$\hat{H} = \hat{H}_0$ is the Hamiltonian (free with $_0$), in energy (= mass) eigenstate basis (sum on I, not matrix in generation space) :

$$H_0 = \sum_I \int \frac{d^3p}{(2\pi)^3} \omega_{II}(|\vec{p}|) \left(\hat{n}^{II}_{++}(\vec{p}) + \hat{n}^{II}_{--}(\vec{p}) \right) \ , \ \omega_{II} = \sqrt{\vec{p}^2 + m_I^2}$$

4 : Calculate $\frac{d}{dt}\hat{n}^{IJ}_{++}(\vec{p}) = +i[\hat{H}_0, \hat{n}^{IJ}_{++}(\vec{p})]$

$$= \ i \int \frac{d^3k}{(2\pi)^3} \Big(\omega_2(\vec{k}) \hat{a}^{2\dagger}_+(\vec{k}) \hat{a}^2_+(\vec{k}) + \omega_1(\vec{k}) \hat{a}^{1\dagger}_+(\vec{k}) \hat{a}^1_+(\vec{k}) \Big) \hat{a}^{I\dagger}_+(\vec{p}) \hat{a}^J_+(\vec{p})$$

$$- \hat{a}^{I\dagger}_+(\vec{p}) \hat{a}^J_+(\vec{p}) \Big(\omega_2(\vec{k}) \hat{a}^{2\dagger}_+(\vec{k}) \hat{a}^2_+(\vec{k}) + \omega_1(\vec{k}) \hat{a}^{1\dagger}_+(\vec{k}) \hat{a}^1_+(\vec{k}) \Big)$$

$$= i \langle \left[\begin{array}{cc} 0 & (\omega_1 - \omega_2) \hat{a}^{1\dagger}_+(\vec{p}) \hat{a}^2_+(\vec{p}) \\ (\omega_2 - \omega_1) \hat{a}^{2\dagger}_+(\vec{p}) \hat{a}^1_+(\vec{p}) & 0 \end{array} \right] \rangle$$

...is eqn for neutrino density matrix from Schrodinger Eqn

5 : Take the vacuum-expectation value :
$\langle \hat{n}^{II}_{++}(\vec{p})\rangle \equiv [f_{++}]^{IJ}(\vec{p})$ is the density matrix for the 2-state neutrino system.To check that QFT and QM gives same dynamics, construct QM density matrix for
$|\nu(t)\rangle = s|\nu_1(t)\rangle + c|\nu_2(t)\rangle$:

$$\begin{bmatrix} s^2|\nu_1(t)\rangle\langle\nu_1(t)| & sc|\nu_1(t)\rangle\langle\nu_2(t)| \\ sc|\nu_2(t)\rangle\langle\nu_1(t)| & c^2|\nu_2(t)\rangle\langle\nu_2(t)| \end{bmatrix}$$

The QM Hamiltonian

$$\begin{bmatrix} -\frac{m_2^2-m_1^2}{4\omega} & 0 \\ 0 & \frac{m_2^2-m_1^2}{4\omega} \end{bmatrix}$$

get EoM $\partial_t[f] = i[[H],[f]]$, is same as QFT. NB, now H is matrix in flavour space

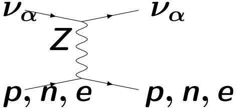
$\Rightarrow$ so the simple quantum mechanical formulae are ok !

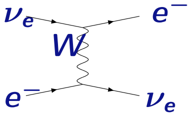
Flavour transition in matter

oscillations and adiabatic

Flavour transitions in matter

Coherent forward scattering of ν in matter give extra contribution to the Hamiltonian :





To see : use $H_{\rm mat} = H_0 + H_{int}$ in QFT oscillation derivation,

$$H_{int} \simeq \pm 2\sqrt{2} G_F \int d^4x (\overline{\nu_e}(x)\gamma^\alpha P_L \nu_e)(\overline{e}\gamma_\alpha P_L e(x)$$

evaluated in a medium with electrons (NC irrelevant ; same for all ν generations = add unit matrix to H. And no μ or τ in the matter. $\pm$ for $\nu, \bar{\nu}$)

$$\langle \mathrm{medium} | \overline{e}\gamma_\alpha P_L e(x) | \mathrm{medium} \rangle \to \delta_{\alpha 0} \frac{n_e}{2}$$

$H_{\rm mat}$ in flavour basis $(\nu_e, (\nu_\mu + \nu_\tau)/\sqrt{2})$, $V_e = \sqrt{2} G_F n_e$:

$$H_{\rm mat} = ... + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & \Delta^2/(2E) \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} V_e & 0 \\ 0 & 0 \end{bmatrix}$$

Oscillations in matter — ctd

$H_{\rm mat}$ in flavour basis $(\nu_e, (\nu_\mu + \nu_\tau)/\sqrt{2})$:

$$H_{\rm mat} \quad = \quad ... + \begin{bmatrix} -\frac{\Delta^2}{4E}\cos 2\theta + V_e & \frac{\Delta^2}{4E}\sin 2\theta \\ \frac{\Delta^2}{4E}\sin 2\theta & \frac{\Delta^2}{4E}\cos 2\theta \end{bmatrix}$$

With $U^T_{mat} H_{mat} U^*_{mat}$ = diagonal :

$$\tan(2\theta_{\rm mat}) \quad = \quad \frac{\Delta^2 \sin(2\theta_{vac})}{2EV_e - \Delta^2\cos(2\theta_{vac})} \xrightarrow{2EV_e \to \Delta^2 c2\theta} \text{large}$$

$$\Delta^2_{\rm mat} \quad = \quad \sqrt{(\Delta^2 c2\theta - 2EV)^2 + (\Delta^2 s2\theta)^2}$$

- for $V_e \ll \frac{\Delta^2}{2E}\cos(2\theta_{vac})$, matter effects negligeable
- $\theta_{mat} \to \pi/4$ ("resonance") at $V_e = \frac{\Delta^2}{2E}\cos(2\theta_{vac})$
- $V \gg \frac{\Delta^2}{2E}\cos(2\theta_{vac})$: $\nu_e \sim$ mass eigenstate

What is V_e ?

$$H_{\mathrm{mat}} = \ldots + \begin{bmatrix} -\frac{\Delta^2}{4E}\cos 2\theta + V_e & \frac{\Delta^2}{4E}\sin 2\theta \\ \frac{\Delta^2}{4E}\sin 2\theta & \frac{\Delta^2}{4E}\cos 2\theta \end{bmatrix}$$

$$\tan(2\theta_{\mathrm{mat}}) = \frac{\Delta^2 \sin(2\theta_{vac})}{2EV_e - \Delta^2\cos(2\theta_{vac})}$$

$$V_e = \sqrt{2}G_F n_e \simeq 8\ \mathrm{eV}\frac{\rho Y_e}{10^{14}g/cm^3}$$

$$Y_e = \frac{n_e}{n_n + n_p}\ ,\ \rho = \begin{cases} 10g/cm^3 & \mathrm{earth} \\ 100g/cm^3 & \mathrm{sun} \\ 10^{14}g/cm^3 & \mathrm{SN} \end{cases}$$

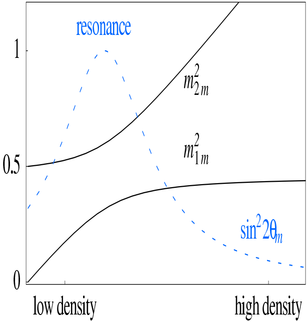
Flavour transitions in matter of varying density

Recall $V_e \simeq 8 Y_e \frac{\rho}{10^{14} g/cm^3}$ eV. For varying $\rho(r)$, have t-dep Hamiltonian :

$$\left[\begin{array}{cc} -\frac{\Delta^2}{4E}\cos 2\theta + V_e(t) & \frac{\Delta^2}{4E}\sin 2\theta \\ \frac{\Delta^2}{4E}\sin 2\theta & \frac{\Delta^2}{4E}\cos 2\theta \end{array} \right]$$

θ_{mat} time dependent...two limits :

1. **adiabatic case** : neglect $\dot{\theta}_{mat}$, instantaneous mass eigenstates ν_i not mix. NB : $\mathcal{P}_{ee} \to 0$ possible if produce ν_e, no oscillations !
2. **non-adiabatic** $\Leftrightarrow$ level hopping



The sun and the bathtub

- produce ν_e at the core of the sun : $.4 \text{ MeV} \lesssim E \lesssim 10 \text{ MeV}$.
- matter oscillation length $<$ vacuum oscillation length $\sim 10 \frac{E}{MeV} \frac{10^{-4} eV^2}{\Delta^2} \ll R_{sun}$. So oscillations decohere $\Leftrightarrow$ propagate mass eigenstates.
- Matter effects negligeable for $E \lesssim$ few MeV :

$$P_{ee} = 1 - \frac{1}{2} \sin^2 2\theta_{vac} > \frac{1}{2}$$

- adiabatic matter effects for $E \gtrsim$ few MeV, allows $P_{ee} < .5$.