Flavour physics and CP violation

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- Lecture I – Flavour physics and CP violation in the Standard Model; basic observables
- Lecture II – Hadronic matrix elements; theoretical tools: effective Lagrangians; important phenomena and examples: B_s mixing and $B \to K^{(*)}\ell^+\ell^-$
- Lecture III – Flavour physics beyond the SM: SMEFT, MFV, more Higgs doublets, MSSM, warped extra dimensions

Disclaimer: these are presentation slides, not lecture notes.

No proof-reading has been done.

Please report errors.

Lecture I
Flavour in the SM, basic observables

Principles of the SM

- Symmetries

  Lorentz symmetry and translation invariance

  $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry

- Fields and their symmetry transformations

$$Q\,(3,2,\frac{1}{6}) = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad U\,(3,1,\frac{2}{3}) = u_R \qquad D\,(3,1,-\frac{1}{3}) = d_R$$

$$L\,(1,2,-\frac{1}{2}) = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad E\,(1,1,-1) = e_R \qquad \phi\,(1,2,\frac{1}{2}) = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

  + gauge fields

- Action principle

$$S = \int d^4x\,\mathcal{L}, \qquad \delta S \stackrel{!}{=} 0$$

- Renormalizability as an effective field theory — S contains all field products compatible with symmetries and field content

SM Lagrangian

$$\mathcal{L} = \sum_{G=SU(3),\,SU(2),\,U(1)} -\frac{1}{4} F^A_{G\mu\nu} F^{A\mu\nu}_G + \sum_{\psi=Q,U,D,L,E} \bar{\psi}\, i \not{D}\, \psi$$

$$+ (D_\mu \phi)^\dagger (D^\mu \psi) + \mu^2 \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2$$

$$- \lambda^E \bar{L}\phi E - \lambda^D \bar{Q}\phi D - \lambda^U \bar{Q}\tilde{\phi} U + \text{h.c.} \qquad (\tilde{\phi} = i\sigma^2 \phi)$$

$$+ \mathcal{L}_{d>4}$$ (Lepton number violation ($d = 5$), come back to $d = 6$ in lecture III)

SM Lagrangian with flavour

$$\mathcal{L} = \sum_{G=SU(3),\,SU(2),\,U(1)} -\frac{1}{4}\,F^A_{G\mu\nu}F^{A\mu\nu}_G + \sum_{\psi=Q,U,D,L,E} \bar\psi\, i\not{D}\,\psi$$

$$+\,(D_\mu\phi)^\dagger(D^\mu\psi) + \mu^2\phi^\dagger\phi - \frac{\lambda}{4}\,(\phi^\dagger\phi)^2$$

$$-\,\lambda^E_{IJ}\,\bar L_I\phi E_J - \lambda^D_{IJ}\,\bar Q_I\phi D_J - \lambda^U_{IJ}\,\bar Q_I\tilde\phi U_J + \text{h.c.} \qquad (\tilde\phi = i\sigma^2\phi)$$

$$+\,\mathcal{L}_{d>4} \quad \text{(Lepton number violation } (d=5)\text{, come back to } d=6 \text{ in lecture III)}$$

Flavour is about indices ...
For all fermion fields (Why all? Anomalies!)

$$\psi \to \psi_I \quad I = 1,2,3 \text{ generations}, \qquad \lambda^X \to \lambda^X_{IJ}$$

- If $\lambda^X_{IJ} = \lambda^X\delta_{IJ}$, no experiment would allow us to distinguish the different generations. However, none of the principles above forces $\lambda^X_{IJ} \propto \delta_{IJ}$ – unless we add $U(3)^5$ flavour symmetry to the postulated symmetries.
- $\hookrightarrow$ Flavour physics is the physics of non-trivial Yukawa coupling matrices.

Why should this be interesting?

Flavour is associated with/leads to new/interesting fundamental physical phenomena

- Particle–anti-particle mixing
- CP violation $\equiv$ microscopic time-reversal non-invariance
- Matter and anti-matter have different physical properties

In the past, flavour has often given the first hints of new particles/New Physics

- $\dfrac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K^- \to \mu^-\bar{\nu}_\mu)} \ll 1 \quad \rightsquigarrow \quad$ charm quark
- $m_{K_L} - m_{K_S} \quad \rightsquigarrow \quad$ mass of the charm quark
- CP violation $\rightsquigarrow$ third generation
- $m_{B_H^0} - m_{B_L^0} \quad \rightsquigarrow \quad$ top quark must be heavy

Flavour may do it again!

Flavour parameters of the SM

- Flavour is responsible for most of the parameters of the SM

  g_s, g, g' – gauge couplings (3)
  μ^2, λ – Higgs potential (2)
  $\lambda^E, \lambda^D, \lambda^U$ complex matrices $\rightsquigarrow$ $3 \times 9 \times 2 = 54$ real parameters

- Note quite – can redefine fields $\psi_I \to U_{\psi,IJ}\psi_J$. U_ψ must be unitary [Exercise: Why?]

$$\lambda^E \to U_L^\dagger \lambda^E U_E \qquad \lambda^D \to U_Q^\dagger \lambda^D U_D \qquad \lambda^U \to U_Q^\dagger \lambda^U U_U$$

  Any complex matrix A can be diagonalized with non-negative diagonal elements by a bi-unitary transformation $A \to V^\dagger A U =$ diagonal, non-negative.

- We're missing one unitary matrix to make both quark Yukawa couplings diagonal. Convention: make λ^U diagonal and write

$$\lambda^D = V_{\rm CKM} \times \text{diagonal}$$

$\hookrightarrow$

$$\mathcal{L}_{\rm Yukawa} = -\sum_I \lambda_I^E \, \bar{L}_I \phi E_I - \sum_{I,J} \lambda_J^D \, \bar{Q}_I V_{IJ,\rm CKM} \phi D_J - \sum_I \lambda_I^U \, \bar{Q}_I \tilde{\phi} U_I + \text{h.c.}$$

Flavour parameters of the SM (II)

$$\mathcal{L}_{\text{Yukawa}} = -\sum_I \lambda_I^E \,\bar{L}_I \phi E_I - \sum_{I,J} \lambda_J^D \,\bar{Q}_I V_{IJ,\text{CKM}} \phi D_J - \sum_I \lambda_I^U \,\bar{Q}_I \tilde{\phi} U_I + \text{h.c.}$$

- Nine diagonal elements [$\rightsquigarrow$ quark and lepton masses]
- One unitary quark flavour mixing matrix. A unitary $n \times n$ matrix has

$$n^2 = \frac{n(n-1)}{2} \;\; \text{moduli/angles} \; + \frac{n(n+1)}{2} \;\; \text{phases}$$

- Can still perform quark field phase rotations $\psi_I \to e^{i\varphi_{\psi_I}} \psi_I$ provided $\varphi_{Q_I} = \varphi_{U_I}$

$\hookrightarrow$ Can remove $2n-1$ phase differences [Exercise: Why not $2n$?], and there are

$$\frac{n(n-1)}{2} \;\; \text{moduli/angles} \; + \frac{(n-1)(n-2)}{2} \;\; \text{phases} \overset{n=3}{=} 3+1$$

quark flavour mixing parameters.

$\hookrightarrow$ For $n \geq 3$ the Yukawa coupling parameters are not in general real

[Exercise: Note that the lepton Yukawa coupling can be made diagonal. Add the term $-f_{IJ}\,[L_I^T \epsilon\, i\sigma^2 \Phi]\,[\Phi^T i\sigma^2 L_J] + \text{h.c.}$ to the Lagrangian of the SM and repeat the above analysis. How many flavour mixing parameters (angles and phases) do you now find for the lepton sector? Which physical phenomena does this (does this not) entail?]

Flavour-changing charged but no neutral currents

- Spontaneous electroweak symmetry breaking
  $SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{\rm em}$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \qquad \tilde{\phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H \\ 0 \end{pmatrix}$$

$\hookrightarrow$ (focus on quarks and EW gauge interaction)

$$\begin{aligned} \mathcal{L} = & \sum_{\psi=u,d} \sum_I \frac{g s_W^2}{c_W} Q_\psi \bar{\psi}_{RI} \gamma^\mu \psi_{RI} Z^0_\mu - \sum_{\psi=u,d} \sum_I \frac{g}{c_W} \left(T^3_\psi - s_W^2 Q_\psi \right) \bar{\psi}_{LI} \gamma^\mu \psi_{LI} Z^0_\mu \\ & - \frac{g}{\sqrt{2}} \sum_I \left(\bar{u}_{LI} \gamma^\mu d_{LI} W^+_\mu + \bar{d}_{LI} \gamma^\mu u_{LI} W^-_\mu \right) \\ & - \sum_{I,J} \lambda^D_J (v+H) \bar{d}_{LI} V_{IJ,\rm CKM} d_{RJ} - \sum_I \lambda^U_I (v+H) \bar{u}_{LI} u_{RI} + \text{h.c.} \end{aligned}$$

- Gauge interactions don't mix generations but the down-type quark mass matrix is not diagonal. Go to fermion mass basis by field rotation $d_{LI} \to V_{IK,\rm CKM} d_{LK}$:

$$\begin{aligned} \mathcal{L} = & \sum_{\psi=u,d} \sum_I \frac{g s_W^2}{c_W} Q_\psi \bar{\psi}_{RI} \gamma^\mu \psi_{RI} Z^0_\mu - \sum_{\psi=u,d} \sum_I \frac{g}{c_W} \left(T^3_\psi - s_W^2 Q_\psi \right) \bar{\psi}_{LI} \gamma^\mu \psi_{LI} Z^0_\mu \\ & - \frac{g}{\sqrt{2}} \sum_{I,J} \left(\bar{u}_{LI} \gamma^\mu V_{IJ,\rm CKM} d_{LJ} W^+_\mu + \bar{d}_{LI} \gamma^\mu V^\dagger_{IJ,\rm CKM} u_{LJ} W^-_\mu \right) \\ & - \sum_I \lambda^D_J (v+H) \bar{d}_{LJ} d_{RJ} - \sum_I \lambda^U_I (v+H) \bar{u}_{LI} u_{RI} + \text{h.c.} \end{aligned}$$

Flavour-changing charged but no neutral currents (II)

$$\mathcal{L} = \sum_{\psi=u,d} \sum_I \frac{g s_W^2}{c_W} Q_\psi \, \bar{\psi}_{RI} \gamma^\mu \psi_{RI} \, Z_\mu^0 - \sum_{\psi=u,d} \sum_I \frac{g}{c_W} \left(T_\psi^3 - s_W^2 Q_\psi \right) \bar{\psi}_{LI} \gamma^\mu \psi_{LI} \, Z_\mu^0$$
$$- \frac{g}{\sqrt{2}} \sum_{I,J} \left(\bar{u}_{LI} \gamma^\mu V_{IJ,\mathrm{CKM}}^\dagger d_{LJ} \, W_\mu^+ + \bar{d}_{LI} \gamma^\mu V_{IJ,\mathrm{CKM}} u_{LJ} \, W_\mu^- \right)$$
$$- \sum_I \lambda_J^D \, (v + H) \bar{d}_{LJ} d_{RJ} - \sum_I \lambda_I^U \, (v + H) \bar{u}_{LI} u_{RI} + \text{h.c.}$$

- Flavour- (generation-) changing phenomena in the SM result from the interplay of the Higgs-Yukawa and the electroweak interactions.

- Strong, electromagnetic, Z^0 and Higgs interactions with fermions do not change generations and not even flavour.

- The absence of flavour-changing neutral currents is not based on any particular symmetry, but a consequence of the gauge symmetry and the particular field content of the SM.

P, C, T

- Parity, Charge conjugation, Time reversal
  Not required as symmetries by any fundamental principle, but strong and electromagnetic interactions are invariant under all of them.
- Transformation of fields and states of the Hilbert space, unitary or anti-unitary operator such that
$$U_X \psi(x) U_X^{-1} = \text{linear in } \psi \text{ or } \psi^\dagger$$
  (up to phase)

	P	C	CP	T	CPT
$\phi(x)$	$\phi(Px)$	$\phi^\dagger(x)$	$\phi^\dagger(Px)$	$\phi(-Px)$	$\phi^\dagger(-x)$
$\psi(x)$	$\gamma^0\psi(Px)$	$-i\gamma^2\psi^\dagger(x)$	$-i\gamma^2\gamma^0\psi^\dagger(Px)$	$\gamma^1\gamma^3\psi(-Px)$	$-\psi^\dagger(-x)$
$\psi_L(x)$	0	0	$-i\sigma^2\psi_L^\dagger(Px)$	$-i\sigma^2\psi_L(-Px)$	$-\psi_L^\dagger(-x)$
$A^\mu(x)$	$-P^\mu{}_\nu A^\nu(Px)$	$-A^\mu(x)$	$P^\mu{}_\nu A^\nu(Px)$	$P^\mu{}_\nu A^\nu(-Px)$	$P^\mu{}_\nu A^\nu(-x)$

($P = \text{diag}\,(1,-1,-1,-1)$, $Px = (x^0, -\vec{x})$)

- P, C are violated by the weak interaction, since right- and left-handed fields have different gauge quantum numbers. The charge-conjugate of the left-handed neutrino would be a left-handed anti-neutrino. However, CP is consistent with the field content.

CP, *CPT*

- Kinetic terms (gauge interactions) and Higgs potential are CP symmetric [Exercise: check this, trivial for the Higgs potential]

- Yukawa interactions

$$\bar{Q}_I \phi D_J \stackrel{CP}{\to} \eta_\phi \eta_{D_J} \eta^*_{Q_I} Q_I(-i\sigma^2)\phi^\dagger i\sigma_2 \bar{D}_J = -\eta_\phi \eta_{D_J} \eta^*_{Q_I} \bar{D}_J \phi^\dagger Q_I$$

$\hookrightarrow$

$$-(\eta_\phi \eta_{D_J} \eta^*_{Q_I} \lambda^D_{IJ}) \bar{D}_J \phi^\dagger Q_I \qquad \text{vs.} \qquad \lambda^{D*}_{IJ} \bar{D}_J \phi^\dagger Q_I \quad \text{from + h.c.}$$

Corresponding equation for λ^U is easy to satisfy, because λ^U can be made diagonal and positive. But for $\lambda^D = V^\dagger_{\text{CKM}} \times$ diagonal need

$$V^*_{\text{CKM}} = D_{\eta_D} V_{\text{CKM}} D^\dagger_{\eta_Q} \qquad D_{\eta_D} = \text{diag}\,(\eta_{D_1}, \eta_{D_2}, \eta_{D_3}) \text{ etc.}$$

i.e. it must be possible to make V_{CKM} real by a re-phasing of the D- and Q-fields. We already know that this is not possible for $n \geq 3$ generations.

- CPT is not violated

$\hookrightarrow$ CP and hence T is violated in the SM unless V_{CKM} is real accidentally or by some additional principle

[Exercise: Show that CPT is a symmetry. Discuss the concept of an anti-unitary operator and why time-reversal must be represented by an anti-unitary operator. Hint: the fundamental Heisenberg relation]

The Cabibbo-Kobayashi-Maskawa (CKM) matrix

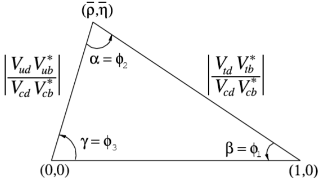
- Standard parameterizations (three angles, one phase)

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$\lambda = 0.225, A = 0.82, \bar\rho = 0.15, \bar\eta = 0.35 \quad \rightsquigarrow \quad$ hierarchical structure!

- Unitarity implies relations when multiplying columns and rows. THE unitarity relation/triangle is



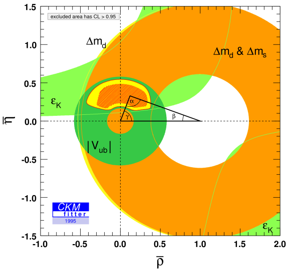
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

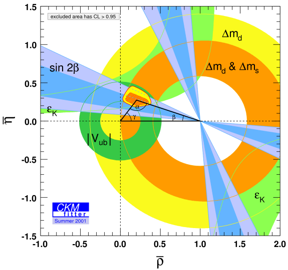
$$V_{ub} = |V_{ub}|\, e^{-i\gamma} = A\lambda^3(\rho - i\eta)$$
$$V_{td} = |V_{td}|\, e^{-i\beta} = A\lambda^3(1-\rho - i\eta)$$

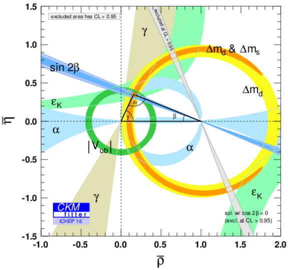
[standard phase convention]

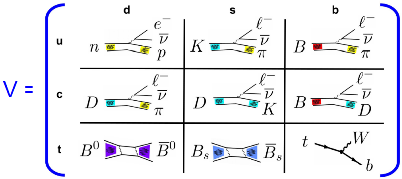
The Unitarity Triangle

1995 (top discovery) | 2001 (B factory turn-on) | 2016 (Precision flavour physics)









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- Angles mostly from semileptonic
- V_{td}, V_{ts} indirectly from effect of top loops on B and K decays, mixing
- Phase from CP violation or through unitarity

CP violation in the SM

- Phase convention independent, dimensionless measure of CP violation

$$J = \underbrace{\mathrm{Im}\,[V_{us}V_{cb}V_{ub}^*V_{cs}^*]}_{\lambda^6 A^2\eta\approx 3\cdot 10^{-5}}(m_t^2-m_c^2)(m_t^2-m_u^2)(m_c^2-m_u^2)$$
$$\times(m_b^2-m_s^2)(m_b^2-m_d^2)(m_s^2-m_d^2)/v^{12}$$

CP violation in the SM not only vanishes if the CKM matrix is real, but also if any of the up-type or any of the down-type quarks are mass-degenerate.

[Exercise: Degenerate quarks can be unitarily rotated. The extra phase of this rotation can be used to set the CKM phase to zero. Prove this.]

$\hookrightarrow$ CP violation is a rare effect not because the CP phase is small, but because flavour mixing and quark masses are small and involves all three generations.

- Although CP violation was discovered as a small effect in 1964 in kaon decay/mixing, we know the above only since 2001, when a large CP asymmetry was observed in B decay/mixing
- The CKM phase is the only known source of CP violation. CP violation in the lepton sector has not yet been seen. The strong interactions could be CP-violating due to the so-called θ-term, but this has not been observed either.

Basic observables — Meson mixing

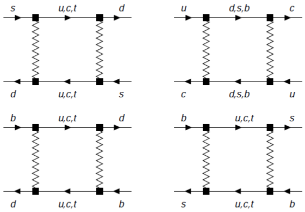
- $K(d\bar{s})$ and $\bar{K}(s\bar{d})$, $B_d^0(d\bar{b})$ and $\bar{B}_d^0(b\bar{d})$, $B_s^0(s\bar{b})$ and $\bar{B}_s^0(b\bar{s})$, and $D^0(c\bar{u})$ and $\bar{D}^0(u\bar{c})$
  Neutral mesons, particle–anti-particle, eigenstates of the strong interaction Hamiltonian with equal mass, stable particles.
- Turn on the weak interaction

$$H = H_S + H_{\text{weak}}$$

  No longer exact mass eigenstates, H_{weak} can change strangeness, bottom, charm by $|\Delta F| = 2$. Mesons decay.
- Time evolution

$$|M^0(t=0)\rangle = |M^0\rangle \quad \rightarrow \quad |M^0(t)\rangle = a(t)|M^0\rangle + b(t)|\bar{M}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \ldots$$

  can be approximated by the action of a non-hermitian Hamiltonian on the $M^0, \bar{M}^0$ two-particle system without the decay states f_i (Wigner-Weisskopf approximation)



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Meson mixing (II)

$$i\frac{d}{dt}\begin{pmatrix}|M(t)\rangle\\ |\bar{M}(t)\rangle\end{pmatrix} = H\begin{pmatrix}|M(t)\rangle\\ |\bar{M}(t)\rangle\end{pmatrix} \qquad H = M - i\,\frac{\Gamma}{2}$$

- Any 2×2 matrix H can be decomposed into its hermitian and anti-hermitian part. M and Γ are the hermitian (not real!) mass and decay matrix, respectively.
- *CPT* symmetry: $M_{11} = M_{22}, \Gamma_{11} = \Gamma_{22}$
- Eigenstates with eigenvalues $M_{L/H} - i\,\Gamma_{L/H}/2$

$$|M_L\rangle = p|M\rangle + q|\bar{M}\rangle \qquad |M_H\rangle = p|M\rangle - q|\bar{M}\rangle \qquad |p|^2+|q|^2 = 1 \quad \text{[CPT!]}$$

$$M = \frac{M_H+M_L}{2} = M_{11} \qquad \Gamma = \frac{\Gamma_L+\Gamma_H}{2} = \Gamma_{11}$$

$$\Delta M = M_H - M_L > 0 \qquad \Delta\Gamma = \Gamma_L - \Gamma_H$$

$$(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2 \qquad \Delta M\Delta\Gamma = -4\,\mathrm{Re}\,(M_{12}\Gamma_{12}^*)$$

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - i\,\Gamma_{12}^*/2}{M_{12} - i\,\Gamma_{12}/2}$$

- CP violation in mixing $\qquad \left|\frac{q}{p}\right| \neq 1 \quad \Leftrightarrow \quad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right) \neq 0, \pi$

Meson Oscillations

- Non-zero off-diagonal elements (due to the weak interaction) lead to meson-anti-meson oscillations of the flavour eigenstates

$$
\begin{aligned}
|M(t=0)\rangle \quad &\to \quad |M(t)\rangle = g_+(t)|M\rangle + \frac{q}{p} g_-(t)|\bar{M}\rangle \\
|\bar{M}(t=0)\rangle \quad &\to \quad |\bar{M}(t)\rangle = \frac{p}{q} g_-(t)|M\rangle + g_+(t)|\bar{M}\rangle
\end{aligned}
$$

with

$$
\begin{aligned}
g_+(t) &= e^{-iMt}\, e^{-\Gamma t/2} \left[\;\; \cosh\frac{\Delta\Gamma\, t}{4}\, \cos\frac{\Delta M\, t}{2} - i \sinh\frac{\Delta\Gamma\, t}{4}\, \sin\frac{\Delta M\, t}{2} \right] \\
g_-(t) &= e^{-iMt}\, e^{-\Gamma t/2} \left[- \sinh\frac{\Delta\Gamma\, t}{4}\, \cos\frac{\Delta M\, t}{2} + i \cosh\frac{\Delta\Gamma\, t}{4}\, \sin\frac{\Delta M\, t}{2} \right].
\end{aligned}
$$

- If $$\left|\frac{q}{p}\right| \neq 1 \quad \Leftrightarrow \quad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right) \neq 0, \pi$$

there is a CP violating phase difference.

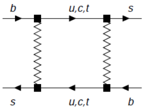
GIM mechanism

- $\Delta F = 2$ transition amplitude (box graphs)

$$H^{\Delta B=2} = \frac{G_F^2 m_W^2}{16\pi^2} \sum_{i,j=u,c,t} \lambda_i^{(D)} \lambda_j^{(D)} F(x_i, x_j)\, (\bar{D}\gamma_\mu(1-\gamma_5)b)^2$$

$$= \frac{G_F^2 m_W^2}{16\pi^2} \left[S(x_t)(\lambda_t^{(D)})^2 + S(x_c)(\lambda_c^{(D)})^2 + 2S(x_c, x_t)\lambda_c^{(D)}\lambda_t^{(D)} \right] (\bar{D}\gamma_\mu(1-\gamma_5)b)^2$$

$$\lambda_i^{(D)} = V_{ib}V_{iD}^*, \qquad \lambda_u^{(D)} + \lambda_c^{(D)} + \lambda_t^{(D)} = 0, \qquad x_i = \frac{m_i^2}{m_W^2}$$



$$S(x_i) \equiv F(x_i, x_i) + F(x_u, x_u) - 2F(x_i, x_u)$$
$$S(x_i, x_j) \equiv F(x_i, x_j) + F(x_u, x_u) - F(x_i, x_u) - F(x_j, x_u)$$

- Similar for kaons ($b \to D, D \to s$).
  For D^0 mesons, sum over internal d, s, b.

- Vanishes if all quarks are degenerate due to the unitarity of the CKM matrix.
  If all $m_i \ll m_W$, then $H^{\Delta B=2} \propto m_i^2/m_W^2 \quad \rightsquigarrow \quad$ Glashow-Iliopoulos-Maiani (GIM) suppression of loop-induced flavour-changing neutral current transitions.

  Doesn't apply when top loops are relevant.

Estimate of mass differences

$$S(x_t)(\lambda_t^{(D)})^2 + S(x_c)(\lambda_c^{(D)})^2 + 2S(x_c,x_t)\lambda_c^{(D)}\lambda_t^{(D)}$$

Strength of $\Delta F = 2$ transition is determined by an interplay of CKM factors and GIM suppression.

	B_d^0	B_s^0	K^0	D^0
	$V_{tb}V_{td}^* \sim \lambda^3$	$V_{tb}V_{ts}^* \sim \lambda^2$	$V_{td}V_{ts}^* \sim \lambda^5$	$V_{cb}V_{ub}^* \sim \lambda^5$
	$V_{cb}V_{cd}^* \sim \lambda^3$	$V_{cb}V_{cs}^* \sim \lambda^2$	$V_{cd}V_{cs}^* \sim \lambda$	$V_{cs}V_{us}^* \sim \lambda$
	top loop dominates	top loop dominates	charm+LD dominates Re SD top+ dominates Im	strong GIM and/or CKM suppression
Re	λ^6	λ^4	$\lambda^2 x_c + \ldots$	$\lambda^2 \times$ GIM
Im	λ^6	λ^6	$\lambda^{10} + \ldots$	$\lambda^{10} \times$ GIM

- $\arg M_{12} \sim O(1)$ only for B_d^0.
- Lifetime difference requires an *absorptive part* of the box graph, corresponding to final states common to M^0 and $\bar{M}^0$ decay $\rightsquigarrow$ top box never contributes (lecture II).

Basic observables — weak decays & CP violation

Decay of M and/or $\bar{M}$ into f and/or CP-conjugate final state $\bar{f}$

$$A_f = \langle f|M\rangle, \quad \bar{A}_f = \langle f|\bar{M}\rangle, \quad A_{\bar{f}} = \langle \bar{f}|M\rangle, \quad \bar{A}_{\bar{f}} = \langle \bar{f}|\bar{M}\rangle$$

$$A_f = a_1 e^{i\delta_1} e^{i\varphi_1} + a_2 e^{i\delta_2} e^{i\varphi_2} + \ldots$$
$$\bar{A}_{\bar{f}} = a_1 e^{i\delta_1} e^{-i\varphi_1} + a_2 e^{i\delta_2} e^{-i\varphi_2} + \ldots$$

- Transition amplitudes are complex numbers even in the absence of CP violation (absorptive parts) $\rightsquigarrow$ "strong phases" δ_i. They are equal for A_f and $\bar{A}_{\bar{f}}$.
- The CKM ("weak") phases φ_i change sign because A_f and $\bar{A}_{\bar{f}}$ are computed with the corresponding complex conjugated interaction terms in the Lagrangian.

Case 0 — Decay $M \to f$ is interesting without CP violation (FCNC, NP, ...)

Case 1 — CP violation in decay

Case 2 — CP violation in mixing

Case 3 — CP violation in mixing and decay, time-dependent

Basic observables — weak decays & CP violation (II)

$$A_f = \langle f|M\rangle, \quad \bar{A}_f = \langle f|\bar{M}\rangle, \quad A_{\bar{f}} = \langle \bar{f}|M\rangle, \quad \bar{A}_{\bar{f}} = \langle \bar{f}|\bar{M}\rangle$$

$$A_f = a_1 e^{i\delta_1} e^{i\varphi_1} + a_2 e^{i\delta_2} e^{i\varphi_2} + \ldots$$

$$\bar{A}_{\bar{f}} = a_1 e^{i\delta_1} e^{-i\varphi_1} + a_2 e^{i\delta_2} e^{-i\varphi_2} + \ldots$$

Case 1 CP violation in decay

The relevant quantities are

$$\lambda_f \equiv \frac{\bar{A}_{\bar{f}}}{A_f} \quad \text{and} \quad a_f = \frac{\Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f})}{\Gamma(M \to f) + \Gamma(\bar{M} \to \bar{f})} = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}$$

e.g. $\bar{B}_d^0 \to \pi^- K^+$. CP violation if $|\lambda_f| \neq 1$ requires at least two amplitudes with a non-vanishing strong and weak phase difference, $\delta_1 - \delta_2, \varphi_1 - \varphi_2 \neq 0, \pi$. CP violation sizeable if both amplitudes are of similar size and their phase difference is large.

Case 2 CP violation in mixing

$$\frac{q}{p} \quad \text{and} \quad a_f = \frac{\Gamma(\bar{M}(t) \to f) - \Gamma(M(t) \to \bar{f})}{\Gamma(M(t) \to f) + \Gamma(M(t) \to \bar{f})} = \frac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2}$$

provided f is a flavour-specific final state, i.e. $\bar{M} \to f$ is forbidden, there is no CP violation in the decay to f e.g. $\bar{B}_d^0 \to X\ell^+\nu_l$. CP violation in mixing if $|q/p| \neq 1$.

Basic observables — weak decays & CP violation (III)

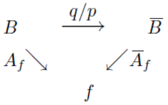
$$A_f = \langle f|M\rangle, \quad \bar{A}_f = \langle f|\bar{M}\rangle, \quad A_{\bar{f}} = \langle \bar{f}|M\rangle, \quad \bar{A}_{\bar{f}} = \langle \bar{f}|\bar{M}\rangle$$

$$A_f = a_1 e^{i\delta_1} e^{i\varphi_1} + a_2 e^{i\delta_2} e^{i\varphi_2} + \ldots$$

$$\bar{A}_{\bar{f}} = a_1 e^{i\delta_1} e^{-i\varphi_1} + a_2 e^{i\delta_2} e^{-i\varphi_2} + \ldots$$

Case 3 CP violation in interference of mixing and decay

The relevant quantities are $\lambda_f \equiv \dfrac{q}{p}\dfrac{\bar{A}_f}{A_f}$ and



$$a_{f_{\rm CP}}(t) = \frac{\Gamma(\bar{M}(t) \to f_{\rm CP}) - \Gamma(M(t) \to f_{\rm CP})}{\Gamma(\bar{M}(t) \to f_{\rm CP}) - \Gamma(M(t) \to f_{\rm CP})}$$

$$= -\frac{A_{CP}^{\rm dir} \cos(\Delta M\, t) + A_{CP}^{\rm mix} \sin(\Delta M\, t)}{\cosh(\Delta\Gamma\, t/2) + A_{\Delta\Gamma} \sinh(\Delta\Gamma\, t/2)} + \mathcal{O}(|q/p| - 1)$$

$$A_{CP}^{\rm dir} = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad A_{CP}^{\rm mix} = -\frac{2\,{\rm Im}\,\lambda_f}{1 + |\lambda_f|^2}, \quad A_{\Delta\Gamma} = -\frac{2\,{\rm Re}\,\lambda_f}{1 + |\lambda_f|^2}$$

$f_{\rm CP}$ a CP eigenstate to which M and $\bar{M}$ can decay e.g. $\bar{B}_d^0 \to J/\psi K_S$. [Exercise: derive this formula]

This becomes particularly simple if $|A_f| = |\bar{A}_f|$, implying $|\lambda_f| = 1$ (in the above approx.), then $\lambda_f = -\eta_{\rm CP} \exp(2\arg(M_{12}^*) - 2\varphi_1)$ $\rightsquigarrow$ "golden mode"