Flavour physics and CP violation

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European School of High Energy Physics
Evora, September 6–19, 2017

- Lecture I – Flavour physics and CP violation in the Standard Model; basic observables
- Lecture II – Hadronic matrix elements; theoretical tools: effective Lagrangians; important phenomena and examples: B_s mixing and $B \to K^{(*)}\ell^+\ell^-$
- Lecture III – Flavour physics beyond the SM: SMEFT, MFV, more Higgs doublets, MSSM, warped extra dimensions

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not lecture notes.
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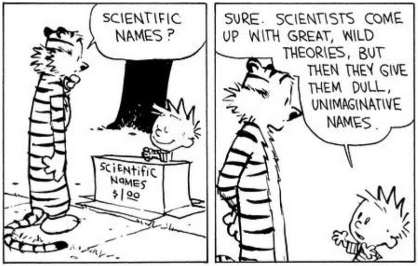
Lecture III
Flavour beyond the SM

"There is a theory in physics that explains, at the deepest level, nearly all of the phenomena that rule our daily lives [...] It surpasses in precision, in universality, in its range of applicability from the very small to the astronomically large, every scientific theory that has ever existed. This theory bears the unassuming name 'The Standard Model of Elementary Particles' [...] It deserves to be better known, and it deserves a better name. I call it 'The Theory of Almost Everything'."

(Robert Oerter, The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics, 2006)



Recap: Flavour and CP violation in the SM

$SU(3) \times SU(2) \times U(1)_{\mathrm{Y}}$
Field content and gauge charges

$$-\lambda_I^D (\bar{Q}_L V_{CKM})_I \phi d_{RI} - \lambda_I^U \bar{Q}_{LI} \tilde{\phi} u_{RI} + \text{h.c.}$$

Flavour violation only charged current.
No Z^0 and Higgs FCNC at tree level.

Empirically, the CKM matrix is apparently the dominant source of flavour and CP violation
Flavour violation in the SM is natural and predictive (especially CPV), but ...

- Why is $\lambda_I^{U,D}$, V_{CKM} what it is?
  (Origin of flavour hierarchies)
- Is this all there is? If not, what is it? Why didn't we see it already?
  (The other flavour problem)
- Baryogenesis? Leptogenesis? Strong CP problem, absence of EDMs.

The “flavour problem”

Two essentially different flavour problems:

New Physics/TeV scale flavour problem

Generic extensions of the SM at the TeV scale (SUSY, 2HDM, ...) have too much flavour and CP violation.

What suppresses it?

Man made problem [wait for a discovery]?

Origin of flavour

Why are the quark masses and the CKM matrix hierarchical?

[Or even: Why are there *three* generations?]

The gauge hierarchy-flavour problem

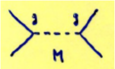
SM presumably valid only below some scale Λ

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm dim\,4} - \frac{\Lambda^2}{2}\Phi^\dagger\Phi + \sum_i \frac{1}{\Lambda^2}\,(\bar{q}q\bar{q}q)_i + \ldots$$

- Scalar mass term is the only dimensionful parameter in the renormalizable part of the Lagrangian.
  Sets the electroweak scale.

- Scalar mass term receives large quantum corrections if there is another scale Λ.
  Electroweak physics requires $\Lambda \leq 4\pi M_W/g \approx$ few hundred GeV to TeV.
- But flavour physics restricts the scale of dimension-6 operators to

$$\Lambda \geq 10^{4-5}\,\text{TeV} \quad (\bar{s}d)(\bar{s}d) \qquad \Lambda \geq 10^3\,\text{TeV} \quad (\bar{b}d)(\bar{b}d)$$



unless it is special (weak coupling, loop suppression, CKM-like suppressions).
Generic scale far beyond reach of LHC!

Difficult to construct natural models.
But the argument may simply be wrong because nature may not care about naturalness ...

Standard Model Effective Theory

General parameterization if no new light degrees of freedom below some scale Λ. [Change of notation: not the QCD scale!]

Same construction principles (same field content, $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry), but add non-renormalizable operators. Theory *cannot* be valid above scale Λ

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\dim \leq 4} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \ldots$$

- $Q_{IJ}^{(5)} = (\tilde{\phi}^\dagger L_I)_i i\tau^2_{ij} (\tilde{\phi}^\dagger L_J)_j$

  Single operator
  Lepton number violation, neutrino-flavour violation (oscillations), Majorana neutrino masses

- $Q_k^{(6)}$ – 59 operators

  Due to three generations/flavour indices there are 1350 CP-even and 1149 CP-odd new couplings constants ...

In the absence of explicit new degrees of freedom, SM EFT is the new SM.

Standard Model Effective Theory – dim-6 Operators

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

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$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$			
$Q_{\varphi G}$	$\varphi^\dagger \varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	
$Q_{\varphi\widetilde G}$	$\varphi^\dagger \varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	
$Q_{\varphi W}$	$\varphi^\dagger \varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	
$Q_{\varphi\widetilde W}$	$\varphi^\dagger \varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	
$Q_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	
$Q_{\varphi\widetilde B}$	$\varphi^\dagger \varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu \tau^I q_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu \tau^I l_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

Up-type quark and Higgs flavour physics

Top

- Low-energy phenomenology determined by $V_{ts}, V_{td} \ll 1$: Meson-mixing, down-type quark FCNC
- FCNC in the up-type quark sector strongly GIM suppressed

$$\mathrm{Br}(t \to c\gamma) \sim |V_{tb}^* V_{cb}|^2 \times \frac{\alpha}{16\pi^3} \times \left(\frac{m_b^2}{m_t^2}\right)^2 \sim 10^{-13}$$

- Collider phenomenology (top decay, single top production) dominated by $V_{tb} \sim 1$.

SMEFT contains $c_{IJ}\, \bar{Q}_{LI}\sigma_{\mu\nu}\tilde{\phi}U_{RJ}B^{\mu\nu}$, where c_{IJ} does not become diagonal in the basis that makes the SM Yukawa matrix λ^U diagonal $\rightsquigarrow$ $\mathrm{Br}(t \to c\gamma)$ can be strongly enhanced.

Higgs

- Higgs FCNC in the SM loop and CKM suppressed.
- Unobservable in present collider experiments and irrelevant for low-energy phenomenology due to Yukawa coupling suppression.

SMEFT leads to misalignment of mass and Yukawa coupling matrices.

Tree-level Higgs FCNC from mass–Yukawa misalignment

Dim-6 SMEFT contains further fermion-Higgs couplings:

$$\mathcal{L}_{\mathrm{dim}-6} \supset -\lambda^U_{IJ}\,\bar{Q}_{LI}\tilde{\phi}U_{RJ} - \frac{\lambda'^U_{IJ}}{\Lambda^2}\,\bar{Q}_{LI}\tilde{\phi}U_{RJ}(\phi^\dagger\phi) + \phi^\dagger i\overleftrightarrow{D}_\mu\phi(\bar{\psi}\gamma^\mu\psi) \ \text{operators}$$

$$+\,D_{RI} \ \text{quark operators}$$

$$\overset{\text{EWSB}}{\hookrightarrow} \qquad \bar{u}_{LI}u_{RJ}: \quad \frac{\sqrt{2}m^U}{v} = \lambda^U + \frac{v^2}{2\Lambda^2}\lambda'^U \qquad \bar{u}_{LI}u_{RJ}H: \quad \sqrt{2}Y' = \lambda^U + \frac{3v^2}{2\Lambda^2}\lambda'^U$$

Breaks the mass–Yukawa coupling relation.
Misalignment generates Higgs FCNC
Diagonalize mass matrix by $u_{LI} \to V_{L,IJ}u_{LJ}$, $u_{RI} \to V_{R,IJ}u_{RJ}$

$$\hookrightarrow \qquad \mathcal{L}_{H,\mathrm{FCNC}} = -Y_{IJ}\bar{u}_{LI}u_{RJ}H \,+\, \text{h.c.} + \ldots \qquad Y_{IJ} = \frac{m_I}{v}\delta_{IJ} + \frac{v^2}{\sqrt{2}\Lambda^2}\,[V_L^\dagger\lambda'^U V_R]_{IJ}$$

- No tuning between λ^U and λ'^U to obtain the observed CKM hierarchy, if $|Y_{tc}Y_{ct}| \lesssim \frac{m_c m_t}{v^2}$ etc.
- Present in many NP models: multi-Higgs, RS (see below), ...

Low- and high-energy flavour physics

Complementary: Higgs and top produced now in large numbers

Low energy

- Quark couplings: meson (K, D, B) mixing
- Neutron EDM
- Lepton couplings: radiative penguins
- $\ell_i \to \ell_j \gamma$, $\ell_i \to \ell_j \ell\ell$, μe conversion, $(g-2)_\ell$, ℓEDM

Direct observation in FV Higgs decay?
$H \to \ell_i \ell_j$,
$H \to t^*(\to bW)q$

High energy

- Single top production
- Same-sign tt production
- (LEP)
- $t \to h + \text{jet}$

Suppression of flavour violation

SM flavour violation is suppressed by small CKM factors and GIM mechanism. SMEFT as well explicit models with $\Lambda = \mathcal{O}(\mathrm{TeV})$ generically produce too much flavour and CP violation in conflict with observation.

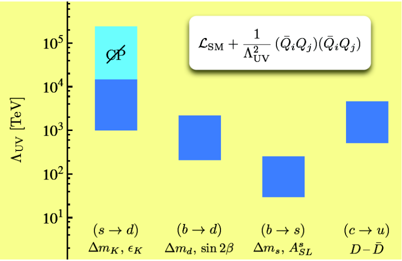
Example: B_d mixing, Δm_{B_d}

$$\mathcal{L}_{\mathrm{SM}}^{\Delta B=2} = -\frac{G_F^2 m_W^2}{16\pi^2} (V_{tb}V_{td}^*)^2 \, S(x_t) \, Q_{\mathrm{V-A}}, \qquad Q_{\mathrm{V-A}} = (\bar{d}\gamma_\mu(1-\gamma_5)b)^2$$

$$\mathcal{L}_{\mathrm{dim-6}} \supset \frac{c}{\Lambda^2} (\bar{Q}_I\gamma_\mu Q_J)(\bar{Q}_K\gamma_\mu Q_L) + \ldots$$

$\hookrightarrow$ comparable for $\Lambda \approx 1300\,\mathrm{TeV} \times \sqrt{c}$

Similar for the other flavour transitions and $\Delta F = 1$ FCNC decays.



[Figure from Neubert, 2011]

Any TeV scale extension of the Standard Model that affects the quark sector must contain a mechanism/principle that enforces a non-generic flavour structure.

Minimal flavour violation

- Standard model Yukawa couplings $-\lambda^D_{IJ}\,\bar{Q}_{LI}\phi D_{RJ} - \lambda^U_{IJ}\,\bar{Q}_{LI}\tilde{\phi}U_{RJ} + \text{h.c.}$ have a $U(3)_{Q_L}\otimes U(3)_{u_R}\otimes U(3)_{d_R}$ flavour symmetry when $Q_L\to U_q Q_L$, $U_R\to U_u U_R$, $D_R\to U_d D_R$, if one (formally) transforms the Yukawa matrices

$$\lambda^U \to U_q\lambda^U U_u^\dagger \qquad \lambda^D \to U_q\lambda^D U_d^\dagger$$

  i.e. the flavour symmetry is broken only by the fixed numerical values of the Yukawa couplings ("spurion fields").

- Minimal Flavour Violation (MFV) makes the following assumptions for (TeV scale) extensions of the SM
  - The SM Yukawa matrices are the only sources of flavour and CP violation, that is, any new coupling

$$c_{IJ\ldots}(\lambda^U,\lambda^D)\,Q_{IJ\ldots}$$

    depends on the generation index only through the Yukawa matrices of the SM.
  - The Lagrangian is invariant under the $U(3)_{Q_L}\otimes U(3)_{u_R}\otimes U(3)_{d_R}$ flavour symmetry, when the Yukawa matrices transform as above.

- In the end, set Yukawa matrices to fixed values in $\mathcal{L}$. After field rotations

$$\lambda^U \sim (3,\bar{3},1) \to V^\dagger_{\text{CKM}}\text{diag}(y_{u_i}) \qquad \lambda^D \sim (3,1,\bar{3}) \to \text{diag}(y_{d_i})$$

Minimal flavour violation (II)

Example: B_d mixing, Δm_{B_d} again

LL $\quad \dfrac{C_{ij}}{\Lambda^2}(\bar{Q}_i\gamma_\mu Q_j)(\bar{Q}_i\gamma_\mu Q_j)$ [no sum over $i, j, i \neq j$]

$$\begin{aligned} C_{ij} &= (a\delta_{ij} + b[\lambda^D\lambda^{D\dagger}]_{ij} + c[\lambda^U\lambda^{U\dagger}]_{ij} + \ldots)^2 \\ &= (cV^\dagger_{\mathrm{CKM}}\mathrm{diag}(y_u^2, y_c^2, y_t^2)V_{\mathrm{CKM}})^2_{ij} \approx c^2 y_t^4 (V^*_{ti}V_{tj})^2 \end{aligned}$$

Same CKM suppression as in the SM

$$\frac{G_F^2 m_W^2}{16\pi^2} S(x_t) \approx \frac{c^2 y_t^4}{\Lambda^2} \quad \Rightarrow \quad \Lambda \approx 4.7\,\mathrm{TeV} \times c$$

LR $\quad \dfrac{C_{ij}}{\Lambda^2}(\bar{D}_i\gamma_\mu Q_j)(\bar{Q}_i\gamma_\mu D_j)$ [no sum over $i, j, i \neq j$]

$$\begin{aligned} C_{ij} &= \Big[\lambda^{D\dagger}(a + b\,\lambda^D\lambda^{D\dagger} + c\,\lambda^U\lambda^{U\dagger} + \ldots)\Big]_{ij} \times \Big[(\tilde{a} + \tilde{b}\,\lambda^D\lambda^{D\dagger} + \tilde{c}\,\lambda^U\lambda^{U\dagger} + \ldots)\lambda^D\Big]_{ij} \\ &\approx c y_i y_t^2 (V^*_{ti}V_{tj}) \times \tilde{c} y_j y_t^2 (V^*_{ti}V_{tj}) \end{aligned}$$

Additional suppression down-type quark masses $m_i m_j/v^2$, provided the standard relation $m_i \sim y_i v$ holds (not always true).

Minimal flavour violation (III)

- All flavour-changing neutral currents transitions in the down-type quark sector suppressed by the small quantity $y_t^2 V_{ti}^{\mathrm{CKM}*} V_{tj}^{\mathrm{CKM}}$ unless y_b is also large.

  [Exercise: Apply the MFV ansatz to a) the other four-quark operators that contribute to meson-mixing; b) the top-quark electromagnetic dipole operator. Estimate the rate for $t \to c\gamma$ when $\Lambda = 1$ TeV]

- Reduces sensitivity of flavour physics observables to scales $4\pi/g \times m_W \times$ few (unless y_b is large; or SM helicity suppression).

- Useful working hypothesis given observations but ...
  ... not a theory.

  MFV does not explain why a given extension of the SM should be MFV.

  Example: MSSM – MFV must be explained by a dynamical mechanism of SUSY breaking (unbroken SUSY is MFV in contrast to a general 2HDM)

"IT'S UNIFIED AND IT'S A THEORY, BUT IT'S NOT THE UNIFIED THEORY WE'VE ALL BEEN LOOKING FOR."

Two-Higgs Doublet Models

One Higgs exists, why not two?
Multi-Higgs Doublet models do not solve any problem of the Standard Model, in particular the hierarchy problem.
Nice example how a simple generalization of the SM leads to a wealth of new phenomena, in particular for flavour physics and CP violation.
Why doublets?

$$\frac{m_W^2}{m_Z^2} = \frac{g^2}{g^2 + g'^2} \quad \text{at tree level} \quad \rightsquigarrow$$

SU(2) doublets or singlets or else
any other Higgs must have vev $w \ll v$.

General 2HDM
[Higgs FCNC, flavour-diagonal CPV]

Natural Flavour Conservation
(NFC)
[No Higgs FCNC, flavour-diagonal CPV]

Spontaneous CP violation
(SCPV)

NFC+SCPV
[excluded]

Two-Higgs Doublet Models – Yukawa couplings and NFC

$$\mathcal{L}_{\text{Yukawa}} = -\lambda^D_{1,IJ}\,\bar{Q}_I\phi_1 D_J - \lambda^D_{2,IJ}\,\bar{Q}_I\phi_2 D_J - \lambda^U_{1,IJ}\,\bar{Q}_I\tilde{\phi}_1 U_J - \lambda^U_{2,IJ}\,\bar{Q}_I\tilde{\phi}_2 U_J + \text{h.c.}$$

$$\xhookrightarrow{\text{EWSB},\,\phi_i = v_i + H_i} \qquad \text{Mass matrices:} \quad \sqrt{2}m^{U,D} = \lambda_1^{U,D} v_1 + \lambda_2^{U,D} v_2$$

$$\text{Higgs coupling } \;\bar{\psi}_{LI}\psi_{RJ}H_i: \quad \lambda_1^{U,D} + \lambda_2^{U,D}$$

Available four unitary field rotations can be used to diagonalize the mass matrix, for generally different vevs of the Higgs fields $v_1 \neq v_2$, the Higgs couplings cannot be simultaneously diagonalized $\rightsquigarrow$ Higgs FCNC. Tree-level contributions to $\Delta F = 2$ processes too large, unless Higgs bosons are very heavy ($\propto 1/m_H^2$).

Natural Flavour Conservation – impose the discrete symmetry

$$\phi_1 \to -\phi_1, \quad \phi_2 \to \phi_2, \quad Q_I \to Q_I, \quad D_I \to -D_I, \quad U_I \to U_I$$

$\hookrightarrow$ Each doublet couples only to one right-handed field, $\lambda_1^U = \lambda_2^D = 0$.

$\hookrightarrow$ Mass matrix and Higgs coupling matrix are proportional to each other as in the SM and can be simultaneously diagonalized.

$\rightsquigarrow$ Flavour physics SM-like, with neutral/charged Higgs exchange in addition to Z^0 and $W^\pm$.

CP violation in the Higgs potential & spontaneous CPV

$$
\begin{aligned}
V_{\text{Higgs}} &= \mu_1 \phi_1^\dagger \phi_1 + \mu_2 \phi_2^\dagger \phi_2 + (\mu_{12} \phi_1^\dagger \phi_2 + \text{h.c.}) \\
&\quad + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \\
&\quad + \left[(\lambda_5 \phi_1^\dagger \phi_2 + \lambda_6 \phi_1^\dagger \phi_1 + \lambda_7 \phi_2^\dagger \phi_2)(\phi_1^\dagger \phi_2) + \text{h.c.} \right]
\end{aligned}
$$

μ_{12}, λ_{5-7} can be complex. In general, this implies CP violation (though not always)

$\rightsquigarrow$ Higgs boson mass eigenstates are not CP eigenstates, Higgs interactions CP-violating, CP violation without flavour change.

Spontaneous CP violation – $\mathcal{L}$ is CP conserving. But CP is violated by the ground state/vacuum

$$\phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix}$$

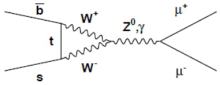
such that minimum of the potential has $\theta \neq 0, \pi$.

- Can be realized in the 2HDM [Lee, 1972]. Potential and all Yukawa matrices are real.
- Not completely excluded experimentally. CP violation transmitted to flavour sector after field rotations.
- When combined with natural flavour violation, however, one gets a real CKM matrix, in conflict with observations.

Example: $B_s \to \mu^+\mu^-$

$$\mathrm{Br}(B_s \to \mu^+\mu^-) = \frac{G_F^2\alpha^2}{64\pi^3} \times f_{B_s}^2 \times \tau_{B_s} m_{B_s}^3 |V_{tb}V_{ts}^*|^2 \sqrt{1-\frac{4m_\mu^2}{m_{B_s}^2}}\,\frac{4m_\mu^2}{m_{B_s}^2} \times |C_{10}|^2$$

- Single hadronic matrix element/parameter $\langle 0|\bar{s}\gamma^\mu b|\bar{B}_s\rangle$: $f_{B_s} = (227.7 \pm 4.5)$ MeV
- Single short-distance coefficient: $\mathcal{L}_{\text{weak}} \propto C_{10} \times [\bar{s}b]_{V-A}[\bar{\ell}\ell]_A$
- LHCb [1703.05747]: $(3.0^{+0.7}_{-0.6}) \times 10^{-9}$ vs.
  Theory [1708.09152]: $(3.57 \pm 0.17) \times 10^{-9}$
- SM only $C_{10} \Rightarrow$ helicity suppression
  Sensitive to scalar couplings due to Higgs exchange.



$$\mathrm{Br}(B_s \to \mu^+\mu^-) = \frac{G_F^2\alpha^2}{64\pi^3} f_{B_s}^2 \tau_{B_s} m_{B_s}^3 |V_{tb}V_{ts}^*|^2 \sqrt{1-\frac{4m_\mu^2}{m_{B_s}^2}}$$
$$\times \left\{ \left(1-\frac{4m_\mu^2}{m_{B_s}^2}\right) |C_S - C_S'|^2 + \left|(C_P - C_P') + \frac{2m_\mu}{m_{B_s}}(C_{10}-C_{10}')\right|^2 \right\}$$

Minimal Supersymmetric SM

Supersymmetric SM is motivated by a) hierarchy problem, b) gauge coupling unification, c) natural dark matter candidate.
Flavour and CP violation are, however, problematic.

- Yukawa couplings: 2HDM with NFC because $\tilde{\phi}_i$ cannot be used for theoretical reasons (holomorphy of the superpotential). Higgs potential constrained (light Higgs!), no SCPV
- y_b can be large

$$\langle H_d \rangle = \frac{v_d}{\sqrt{2}} = \frac{1}{\sqrt{2}} v \cos\beta \qquad \langle H_u \rangle = \frac{v_u}{\sqrt{2}} = \frac{1}{\sqrt{2}} v \sin\beta \qquad v = \sqrt{v_u^2 + v_d^2}$$

$$\hookrightarrow \qquad \frac{y_b}{y_t} = \frac{m_b}{m_t} \tan\beta$$

Supersymmetric part of the Lagrangian is unproblematic. But SUSY must be explicitly (but softly, only dim ≤ 3 operators) broken.

$$\begin{aligned} \mathcal{L}_{\rm soft} = & \frac{1}{2} M_1 \lambda_B \lambda_B + \frac{1}{2} M_2 \lambda_W \lambda_W + \frac{1}{2} M_3 \lambda_g \lambda_g - m_{H_d}^2 |H_d|^2 - m_{H_u}^2 |H_u|^2 \\ & - \tilde{m}_Q^2 \tilde{Q}_L^* \tilde{Q}_L - \tilde{m}_D^2 \tilde{d}_R^* \tilde{d}_R - \tilde{m}_U^2 \tilde{u}_R^* \tilde{u}_R - \tilde{m}_L^2 \tilde{\ell}_L^* \tilde{\ell}_L - \tilde{m}_E^2 \tilde{e}_R^* \tilde{e}_R \\ & + B\mu H_u H_d + \hat{A}_\ell \tilde{\ell} H_d \tilde{e}_R^* + \hat{A}_D \tilde{q} H_d \tilde{d}_R^* - \hat{A}_U \tilde{q} H_u \tilde{u}_R^* + \text{h.c.} \end{aligned}$$

Minimal Supersymmetric SM (II)

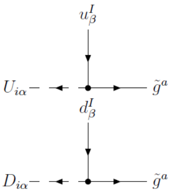
$$
\begin{aligned}
\mathcal{L}_{\text{soft}} \;=\; & \frac{1}{2}M_1\lambda_B\lambda_B + \frac{1}{2}M_2\lambda_W\lambda_W + \frac{1}{2}M_3\lambda_g\lambda_g - m_{H_d}^2|H_d|^2 - m_{H_u}^2|H_u|^2 \\
& - \tilde{m}_Q^2\tilde{Q}_L^*\tilde{Q}_L - \tilde{m}_D^2\tilde{d}_R^*\tilde{d}_R - \tilde{m}_U^2\tilde{u}_R^*\tilde{u}_R - \tilde{m}_L^2\tilde{\ell}_L^*\tilde{\ell}_L - \tilde{m}_E^2\tilde{e}_R^*\tilde{e}_R \\
& + B\mu H_u H_d + \hat{A}_\ell\tilde{\ell}H_d\tilde{e}_R^* + \hat{A}_D\tilde{q}H_d\tilde{d}_R^* - \hat{A}_U\tilde{q}H_u\tilde{u}_R^* \;+\; \text{h.c.}
\end{aligned}
$$

- 43 new CP phases from sfermion mass matrices, tri-linear scalar interactions and Majorana phases.
- CP violation without flavour change from complex mass and scalar potential terms, generates too large electric dipole moments (neutron, nuclear, atomic).
- Available unitary field rotations for the sfermion fields already used up for the Yukawa interaction. Cannot make the mass matrices flavour diagonal simultaneously

  $\hookrightarrow$ FCNC from the strong interaction $\tilde{g}\tilde{q}q$ vertex.

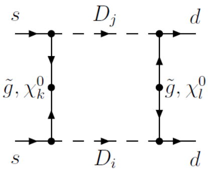
  [Tri-linear squark interaction $\hat{A}$ couplings often assumed to be proportional to the Yukawa matrices.]

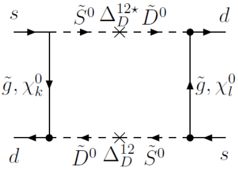
  Loop-suppressed in low-energy processes



$\hookrightarrow$ Flavour-changing off-diagonal terms must be small to avoid conflict with observations.

Minimal Supersymmetric SM (III)





Gluino box contribution to $\Delta F = 2$ transitions w/o mass insertion approximation

$$H^{\Delta F=2} \propto \frac{\delta \tilde{m}^2_{Q,IJ}}{\tilde{m}^2_Q} \frac{\delta \tilde{m}^2_{D,IJ}}{\tilde{m}^2_D}$$

Flavour protection mechanisms

- Gauge mediation, "split supersymmetry" (or, at least, the first two squark generations very heavy), flavour symmetries that enforce squark mass matrix "alignment".
- Note: even with flavour universality at a high scale, RG evolution introduces new flavour and CP violation (in gluino interactions, as above). But involves the SM Yukawa matrices.
- More generally: MFV ansatz

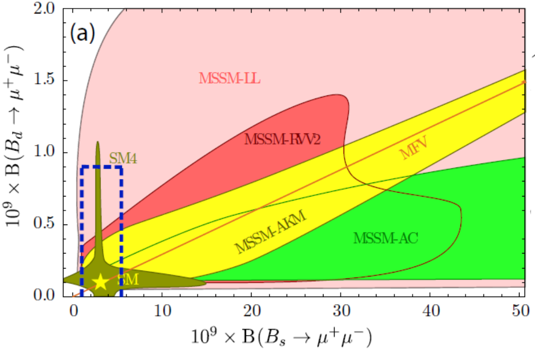
$$\tilde{m}^2_Q = \tilde{m}^2 \left(a_1 \mathbf{1} + b_1 Y_u Y_u^\dagger + b_2 Y_d Y_d^\dagger + c_1 Y_d Y_d^\dagger Y_u Y_u^\dagger + \ldots \right) ,$$

$$\tilde{m}^2_U = \tilde{m}^2 \left(a_2 \mathbf{1} + b_3 Y_u^\dagger Y_u + \ldots \right) , \qquad \tilde{m}^2_D = \tilde{m}^2 \left(a_3 \mathbf{1} + b_4 Y_d^\dagger Y_d + \ldots \right) ,$$

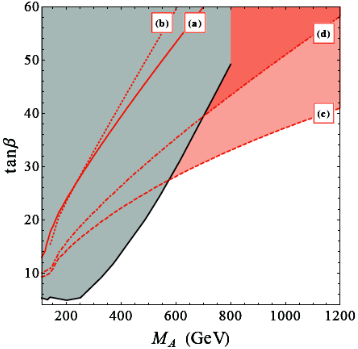
$$\hat{A}_U = \tilde{A} \left(a_4 \mathbf{1} + b_5 Y_d Y_d^\dagger + \ldots \right) Y_u , \qquad \hat{A}_D = \tilde{A} \left(a_5 \mathbf{1} + b_6 Y_u Y_u^\dagger + \ldots \right) Y_d ,$$

$B_s \to \mu^+\mu^-$ in 2HDM and MSSM

[Straub, 1205.6094]



[Altmannshofer, 1306.0022]



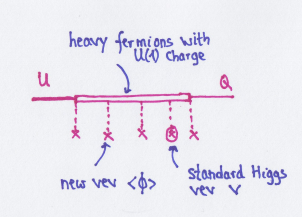
- Scalar FCNC cannot play an important role in non-helicity-suppressed amplitudes.
- Suppression relative to SM possible for pseudoscalar Higgs interfering with SM axial-vector contribution.

Froggatt-Nielsen mechanism [1978]

- Fermions and a new scalar field A charged under a new $U(1)_R$ symmetry. Effective dynamics generates

$$g_{ij}\left(\frac{A}{M}\right)^{a_j+b_i}\bar{Q}_i\tilde{\phi}U_j$$

[b_i = R-charge of Q_i, a_j = R-charge of $\bar{U}_j$, g_{ij} not hierarchical, same for down-type quarks]



- Condensation of A yields Yukawa matrix

$$y^u_{ij} = g_{ij}\epsilon^{a_i+b_j}, \qquad \epsilon \equiv \frac{\langle A\rangle}{M} \ll 1$$

$$\boxed{\frac{m_i}{m_j} \propto \epsilon^{a_i-a_j+b_i-b_j} \qquad V^{CKM}_{ij} \propto \epsilon^{|b_i-b_j|} \qquad \rho,\eta \sim O(1) \ \text{large CP violation}}$$

- Size of M, $\langle A\rangle$ not fixed. No relation to other particle physics problems.

Flavour physics in the Randall-Sundrum model

Addresses the gauge and flavour hierarchy in a unified (extra-dimensional) framework.

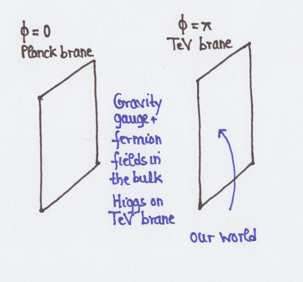
Set-up: 5th dimension a finite interval $[0, \pi]$. All fields except the Higgs in the bulk.

"Warped" space (slice of ADS_5)

$$ds^2 = e^{-2kr_c\phi}\,\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2$$

$$M_P^{4d} \approx \frac{M_P^{5d}}{k}, \qquad \epsilon \equiv e^{-kr_c\pi} \approx 10^{-16} \approx \frac{1\,\mathrm{TeV}}{M_P^{4d}}$$

if $kr_c \approx 12$ ($k, 1/r_c \sim M_P^{5d}$).



On the TeV brane all fundamental mass parameters are rescaled by a factor ϵ:

$$\frac{1}{2}\int d^4x\,\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m_0^2\phi^2\right) \overset{\text{field redef.}}{\longrightarrow} \frac{1}{2}\int d^4x\,\left(\partial_\mu\phi\partial_\nu\phi - m_0^2\epsilon^2\phi^2\right)$$

$\hookrightarrow$ If $m_0 \sim M_P^{4d}$, the Higgs mass $m \equiv m_0\epsilon$ is naturally of order TeV.
KK excitations of all SM fields and gravitons also have TeV masses.

5D fermions

- 5D fermions are vector-like with diagonal bulk (5D) mass term

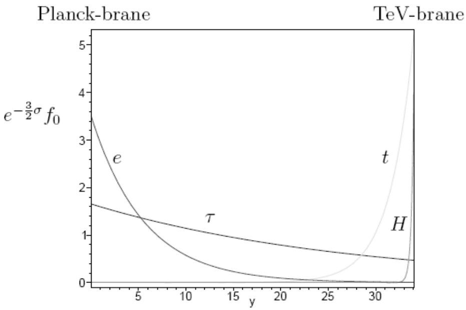
$$-\sum_{\psi=Q,U,D}\sum_{I=1,2,3}\int d^4x\int_0^\pi d\Phi\,\sqrt{-g}\,M_{\psi,I}\,\bar{\psi}_I\psi_I$$

Note diagonal $\neq$ flavour-blind!
Zero modes = SM quarks are chiral by orbifold symmetry.

- Bulk mass determines the shape of the zero mode in the 5th dimension.

$$\Psi^{(0)}(x,\phi) \propto \frac{1}{\sqrt{r_c}}\,\phi^{(0)}(x)\,f(c)\,e^{(2-c)kr_c\phi}$$

$$f(c) \propto \begin{cases} \text{const} & c < \frac{1}{2} \\ \epsilon^{c-\frac{1}{2}} & c > \frac{1}{2} \end{cases} \qquad (c_\psi \equiv M_\psi/k)$$



[Figure from Huber, Shafi]

For $c > 1/2$ exponential sensitivity of $f(c)$ to c.
Small differences in bulk masses can generate large hierarchies.
KK modes localized near Planck brane and not very sensitive to c.

4D Yukawa matrices

- Fermion mass terms after EWSB (Higgs field is brane-localized)

$$\mathcal{L}^{5D} = -\bar{Q} M_Q Q - \sum_{q=u,d} \bar{q}^c M_q q_c - \delta(\phi - \pi)\, v \left[\bar{u}_L Y_u^{(5D)} u_R^c + \bar{d}_L Y_d^{(5D)} d_R^c\right]_{\phi=\pi} + \text{h.c.}$$

  The 5D Yukawa matrices are assumed to be "anarchic" (not hierarchical).
  For the zero modes

$$y_{ij}^u = Y_{u,ij}^{(5D)} f(c_{Q_i}) f(c_{u_j}) \qquad y_{ij}^d = Y_{d,ij}^{(5D)} f(c_{Q_i}) f(c_{d_j})$$

  4D quark mass and CKM hierarchy generated by $f(c)$.

- The pattern is exactly of the Froggatt-Nielsen type. $V_{ij} \sim f_{c_{Q_i}} f_{c_{Q_j}}$. Small differences in c_ψ cause large hierarchies [Huber, 2003; Casagrande et al., Blanke et al., 2008]

$$c_{Q_2} - c_{Q1} \to |V_{us}| \qquad c_{Q_3} - c_{Q2} \to |V_{cb}| \qquad \rho, \eta \sim O(1)$$

Potentially a theory of flavour.
Simultaneous solution to the gauge hierarchy problem requires KK resonances at the TeV scale.
What about effects of KK excitations?

KK mode-induced FCNCs [Huber; Agashe, Perez, Soni; Csaki, Falkowski, Weiler; Blanke et al; Bauer et al.]

- Gluon (and other) KK modes generate tree-level FCNC

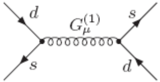
$$g_i^{Lu}\,\bar{u}_{Li}\gamma^\mu G_\mu^{(n)} u_{Li} \quad \stackrel{\text{field rot.}U_{Lu}}{\longrightarrow} \quad g_{ij}^{Lu}\,\bar{u}_{Li}\gamma^\mu G_\mu^{(n)} u_{Lj}$$

$$g_i^{Lu} \approx g_s^{(5d)}\,[\underbrace{\text{const.}}_{\text{for fermion KK modes}} + f(c_i)^2\gamma^{(n)}(c_i)] \quad \rightarrow \quad g_{ij}^{Lu} = \left[U_{Lu}^\dagger \begin{pmatrix} g_1^{Lu} & 0 & 0 \\ 0 & g_2^{Lu} & 0 \\ 0 & 0 & g_3^{Lu} \end{pmatrix} U_{Lu} \right]_{ij}$$

$\hookrightarrow$ g_{ij}^{Lu} is off-diagonal if not all g_i^{Lu} are equal, though $g_{ij}^{Lu} \propto f(c_i)f(c_j)$.
The bulk part is flavour-universal $\rightsquigarrow$ "RS-GIM" mechanism.

- Strongest experimental constraints from CP violation in $K\bar{K}$ mixing (ϵ_K):

$$M_{KK,gluon} \gtrsim 20\,\text{TeV}$$

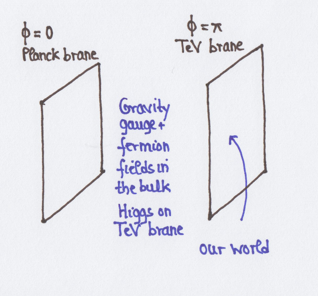


1) Choose somewhat non-generic parameters.
2) Allow $M_K = 20\,\text{TeV}$. Compromises solution to the gauge hierarchy problem.
3) Impose flavour symmetries or textures or MFV. Compromises solution to the flavour hierarchy problem.

FCNC and loop-induced transitions in the RS model

Many studies in recent years.

- Higgs FCNC at tree-level due to mixing with KK excitations [Agashe, Perez, Soni, 2006; Azatov et al., 2009]
- Lepton- and quark penguin transitions $\mu \to e\gamma$, $b \to s\gamma$ [Csaki et al., 2010; Blanke et al., 2012], with WCHC [Delaunay et al., 2012], 5D QFT [MB, Moch, Rohrwild, 2015] (leptons), [Malm, Neubert, Schmell; Moch, Rohrwild, 2015] (quarks)
- Complete 5D calculation of gauge-boson contribution to $g_\mu - 2$, and Higgs-exchange induced lepton-flavour violation. [MB, Dey, Rohrwild, 2012]
- Higgs production and decay [Azatov et al., 2010; Malm et al., 2013; Hahn et al., 2015]



$\hookrightarrow$ KK states too heavy to be observed at LHC

Unfortunately Nature seems to like this model less than theorists do ...