Multi-particle production and thermalization in hadron-hadron collisions
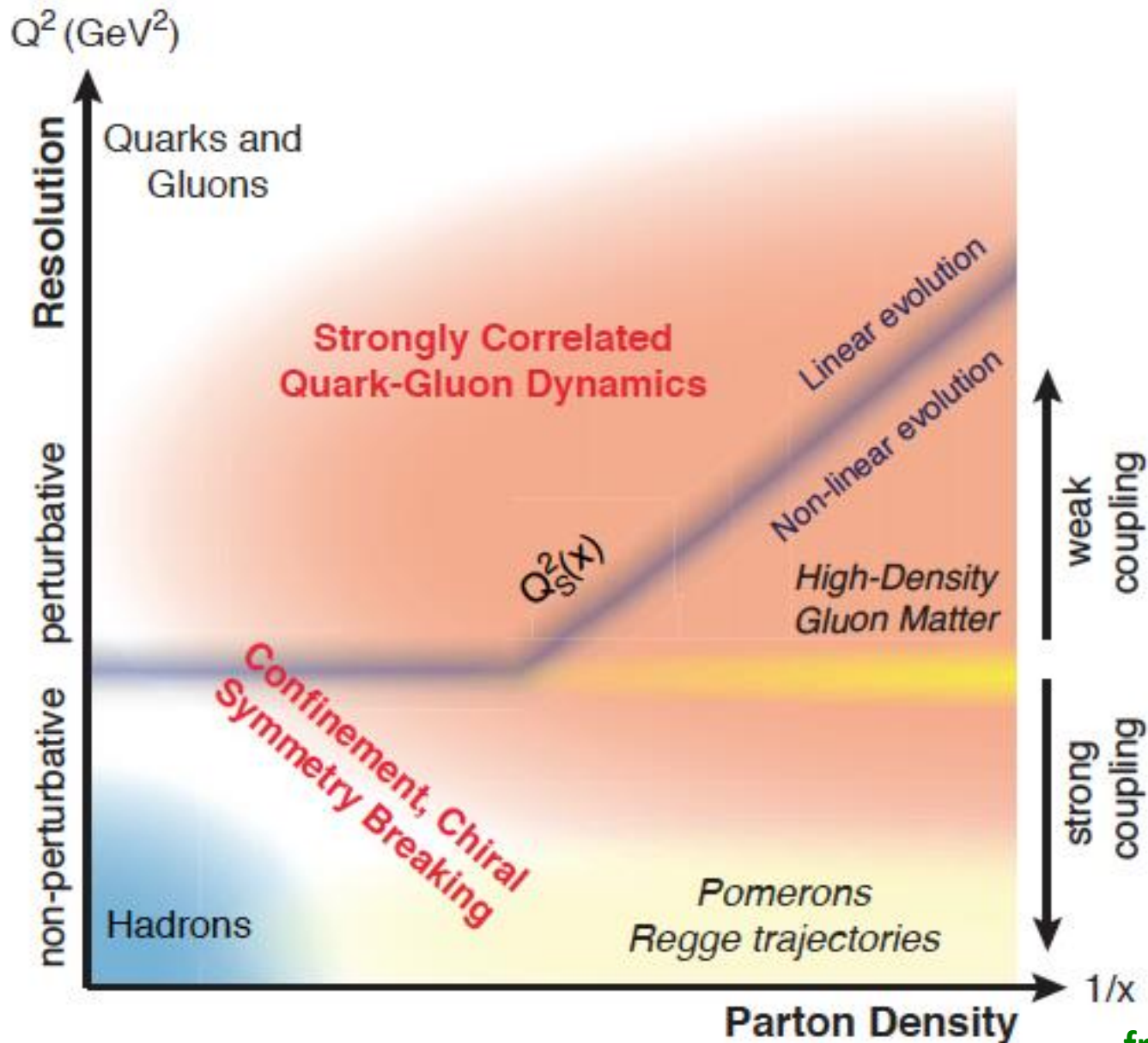
Raju Venugopalan
Brookhaven National Laboratory

Bad Honnef QCD School, September 24, 2017

QCD: Known-Knowns and Known-Unknowns

from arXiv:1708.01527

QCD: Known-Knowns and Known-Unknowns

Known-knowns in QCD:

◆ **Perturbative QCD: precision physics for large Q^2 – rare processes (also weak coupling techniques in finite T and μ_B QFT)**

◆ **Lattice QCD: Quantitative description of (mostly) hadron ground state properties. See Prof. Alexandrou's talk on Thursday**

◆ **Chiral perturbation theory: low energy meson and baryon interactions**

Prof. Colangelo's talk on Friday)

# QCD: Known-Knowns and Known-Unknowns

Known-unknowns in QCD:

◆ **The bulk of elastic, inelastic and diffractive cross-sections in QCD (sometimes called ``soft" physics – though includes scales of a few GeV).**

◆ **Fragmentation/hadronization is not understood— though useful and successful parametrizations exist.**

◆ **Stringy models (PYTHIA,DPM,AMPT,EPOS) successfully parametrize a lot of data and loosely capture features of the underlying theory.**

◆ **However, they *cannot be derived* in any limit from QCD, and require further ad hoc assumptions and parameters when applied In extreme environments**

Discussed in several talks at this Heraeus School

What we need

➢ **An effective theory to describe the varied phenomena of multi-particle production in high energy collisions**

➢ **Smoothly matches to QCD in appropriate kinematic limits**

➢ **The rest of my talk will briefly outline the elements of such a theory.**

➢ **The theory has much predictive power—however, it is least effective when the physics is sensitive to the infrared scales that govern chiral symmetry breaking and confinement.**
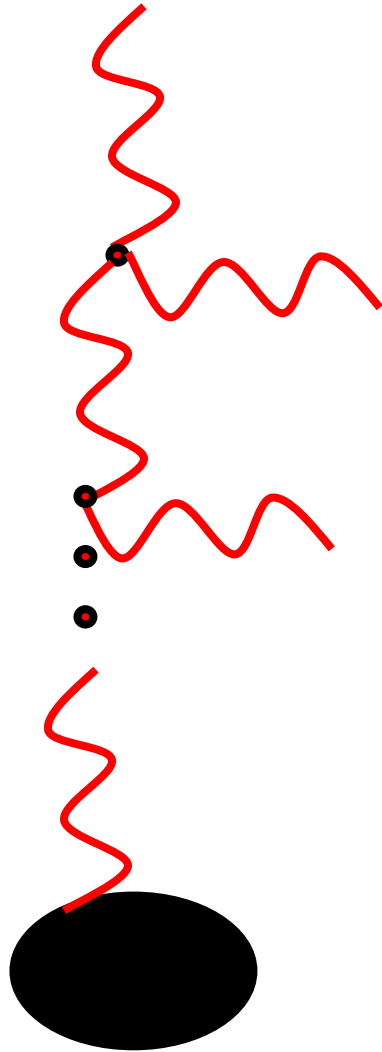
The proton at high energies

γ^*

u

u

d

Parton model

x f(x)

1/3

x

γ^*

u

g

u

g

d

QCD -log corrections

x f(x)

~1/7

x
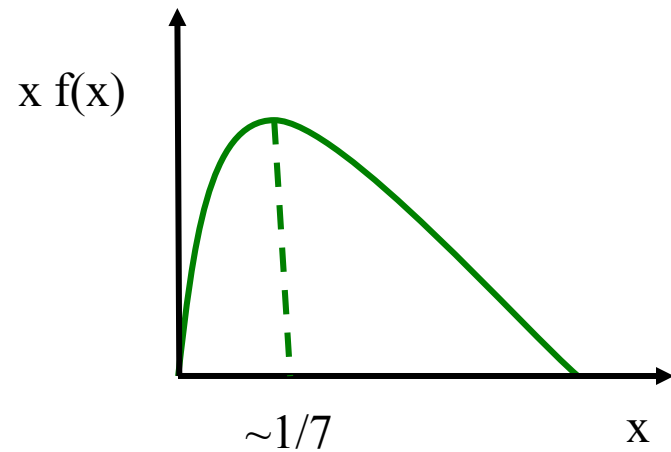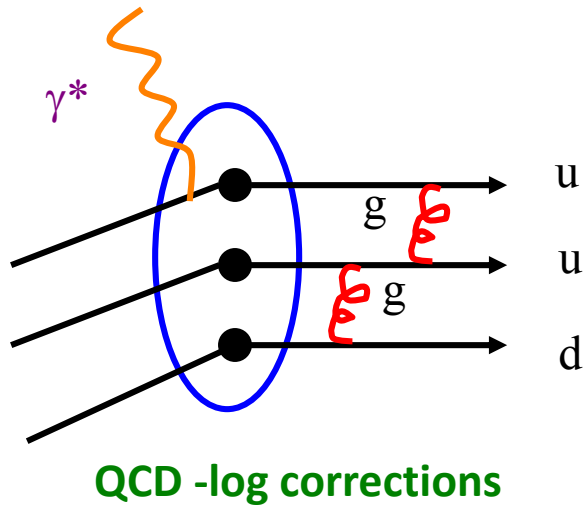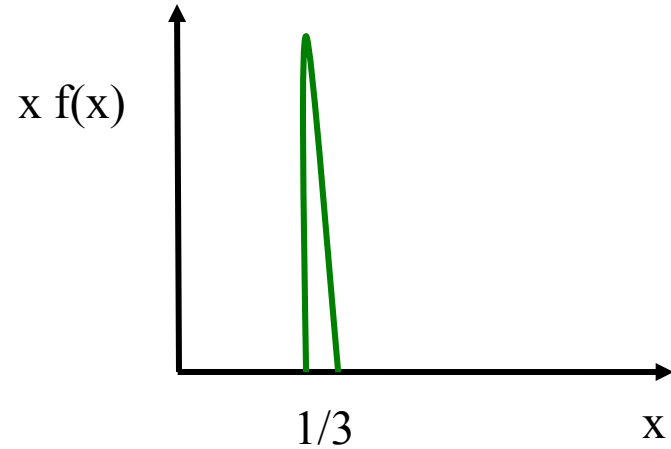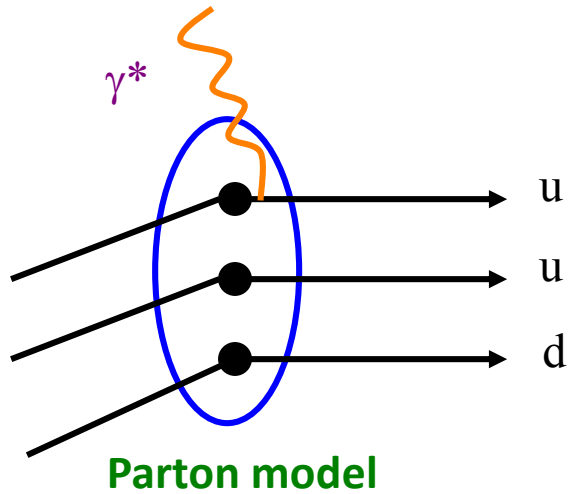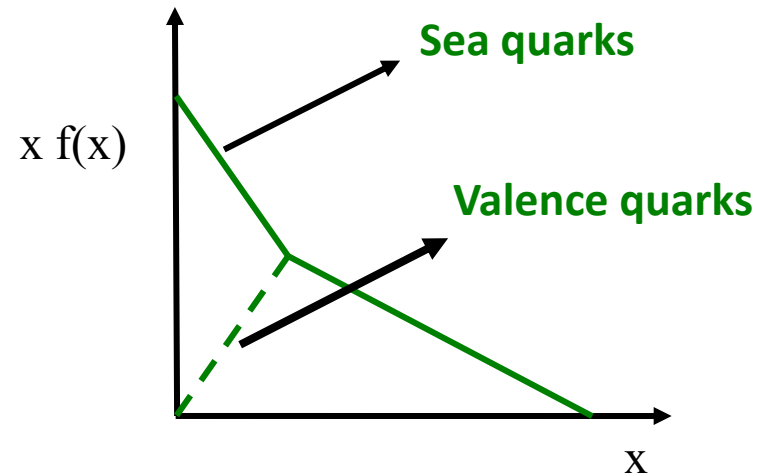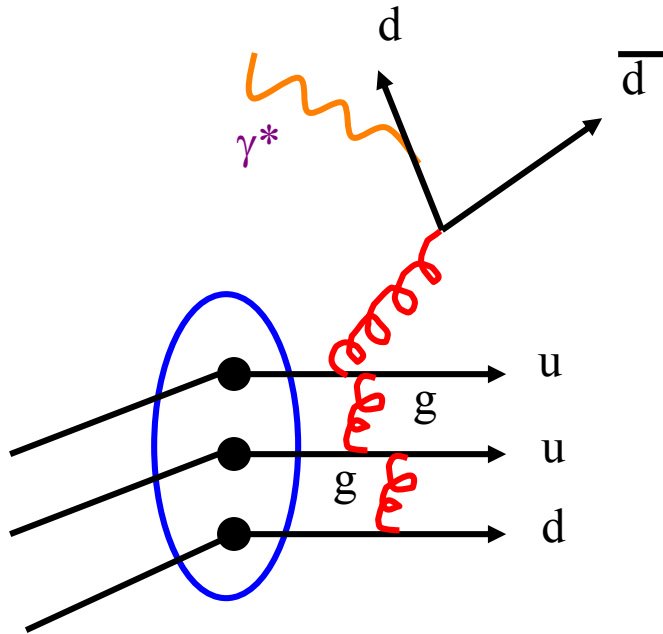
"x-QCD"- small x evolution

$$\int_0^1 \frac{dx}{x}\left(xq(x) - x\bar{q}(x)\right) = 3 \longrightarrow \text{# of valence quarks}$$

$$\int_0^1 \frac{dx}{x}\left(xq(x) + x\bar{q}(x)\right) \to \infty \longrightarrow \text{# of quarks}$$

Bremsstrahlung-linear QCD evolution

Each rung of the ladder gives

$$\alpha_S \int \frac{dk_t^2}{k_t^2} \int \frac{dx}{x} \equiv \alpha_S \ln\left(\frac{x_0}{x}\right) \ln\left(\frac{Q^2}{Q_0^2}\right)$$

If only transverse momenta are ordered from target to projectile:

$$k_{T1}^2 << k_{T2}^2 << \cdots Q^2$$
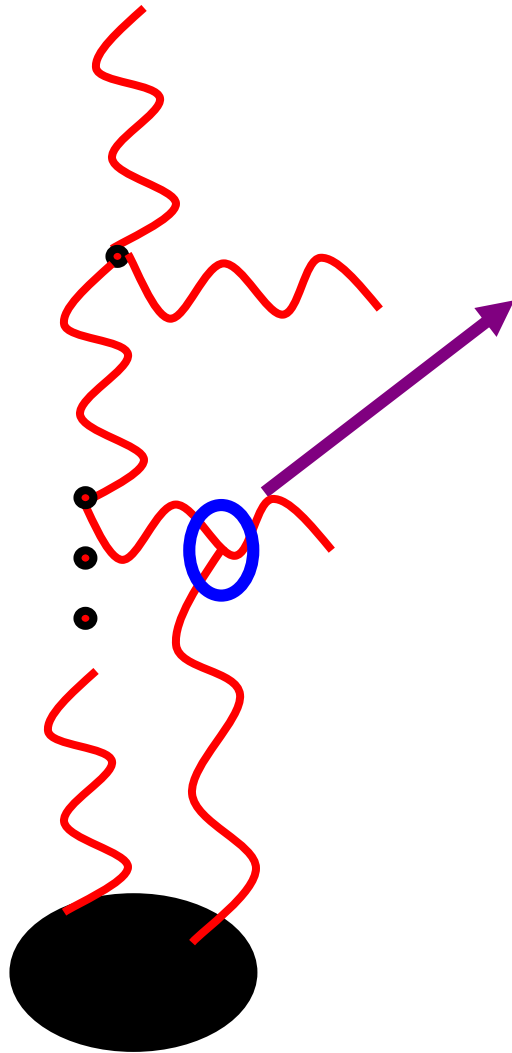
Sum leading logs in Q² (DGLAP evolution)

Conversely, $\quad x_0 >> x_1 \cdots >> x$
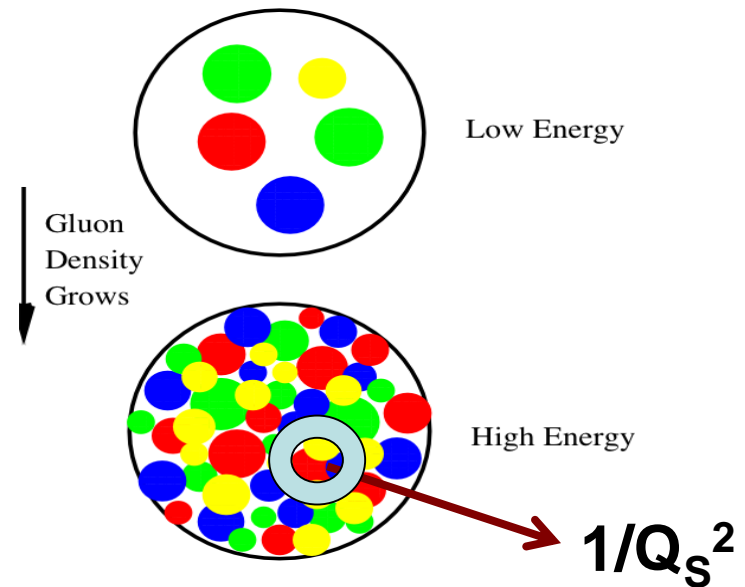
Sum leading logs in x (BFKL evolution)

Both DGLAP and BFKL give rapid growth of gluon density at small x

More about BFKL in Prof. Bartels' talk

Bremsstrahlung-linear QCD evolution

Gluon recombination and screening -non-linear QCD evolution

Gluon Density Grows

Low Energy

High Energy

$1/Q_S^2$

Proton becomes a dense many body system at high energies

Parton Saturation

Gribov,Levin,Ryskin (1983)
Mueller,Qiu (1986)

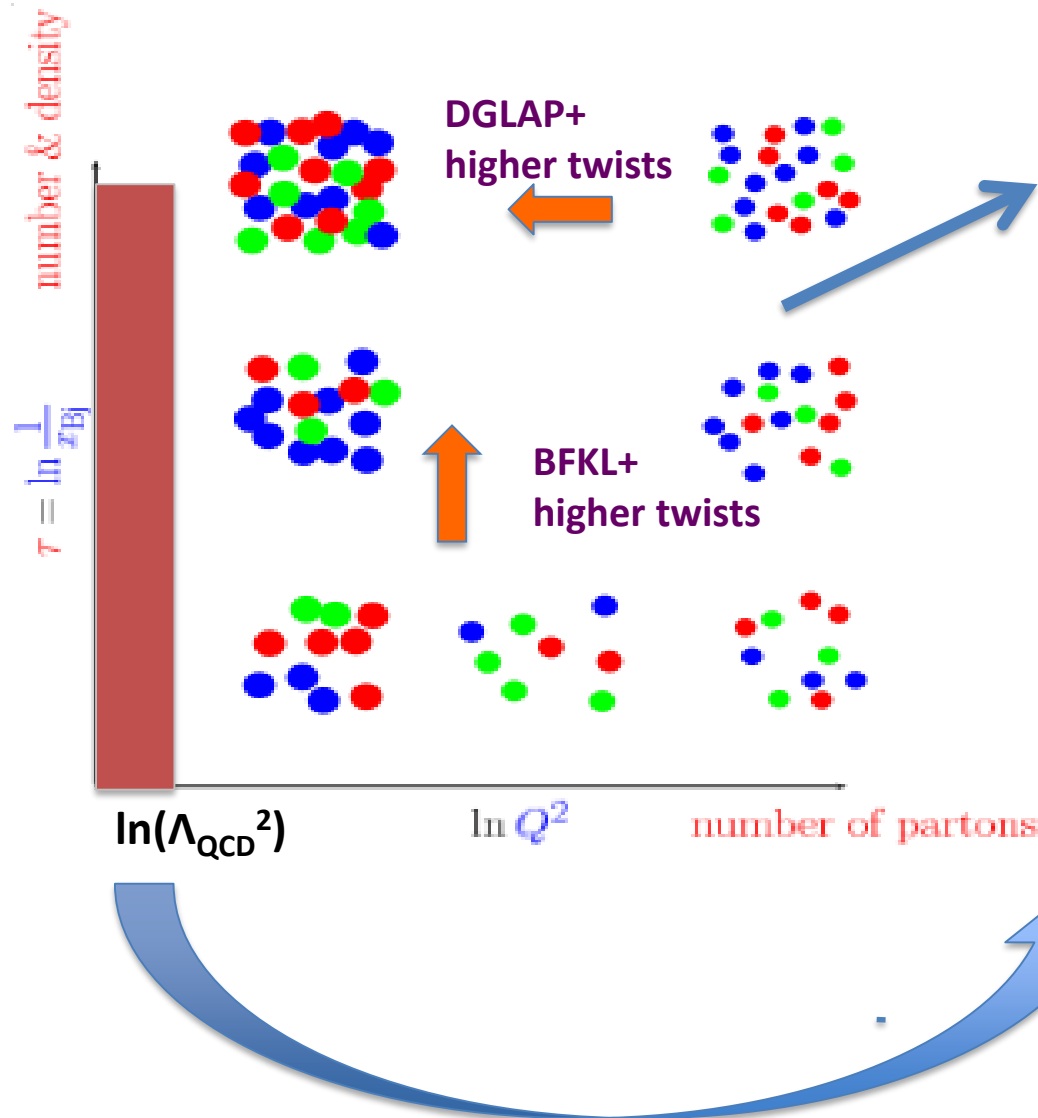**Competition between attractive bremsstrahlung and repulsive recombination and screening effects**

Maximum phase space density (f = $1/\alpha_S$) =>

$$\frac{1}{2(N_c^2 - 1)} \frac{x\, G(x, Q^2)}{\pi R^2\, Q^2} = \frac{1}{\alpha_S(Q^2)}$$

This relation is saturated for

$$Q = Q_s(x) >> \Lambda_{\text{QCD}} \approx 0.2\ \text{GeV}$$

Many-body dynamics of universal gluonic matter

number & density

$\tau = \ln \frac{1}{x_{Bj}}$

DGLAP+
higher twists

BFKL+
higher twists

$\ln(\Lambda_{QCD}{}^2)$

$\ln Q^2$

number of partons

How does this happen ? What are the right degrees of freedom ?
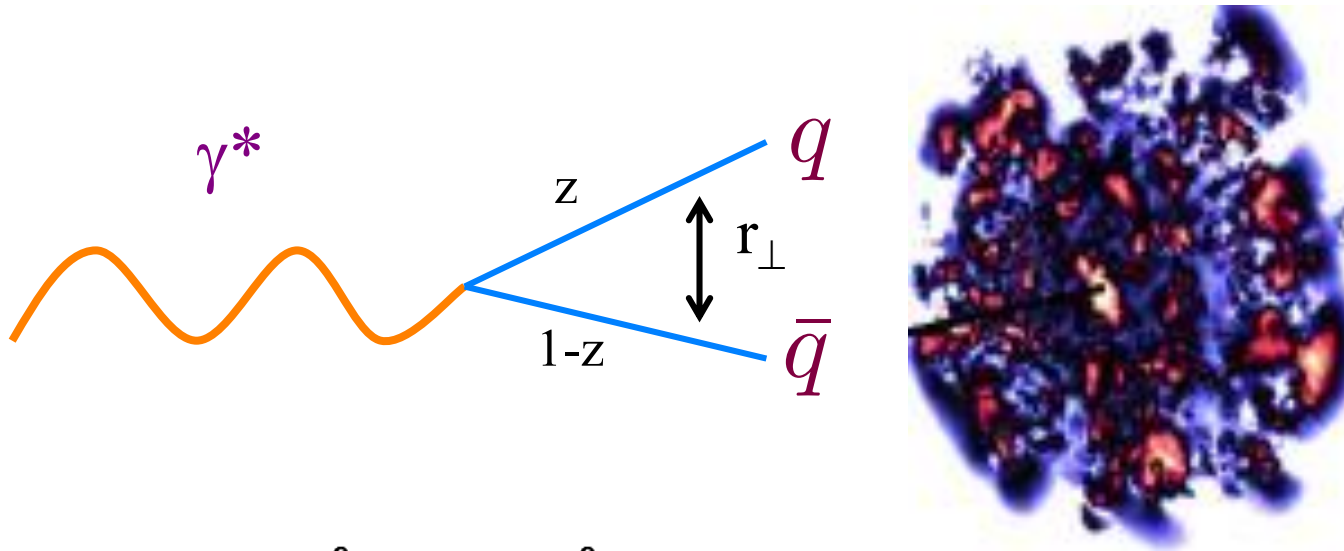
How do correlation functions of these evolve ?

Is there a universal fixed point for the RG evolution of d.o.f

Does the coupling run with $Q_s{}^2$?

How does saturation transition to chiral symmetry breaking and confinement

Parton Saturation: Golec-Biernat &Wusthoff's dipole model

$$\sigma_{\mathrm{T,L}}^{\gamma^*,P} = \int d^2 r_\perp \int dz \, |\psi_{\mathrm{T,L}}(r_\perp, z, Q^2)|^2 \, \sigma_{q,\bar{q},P}(r_\perp, x)$$
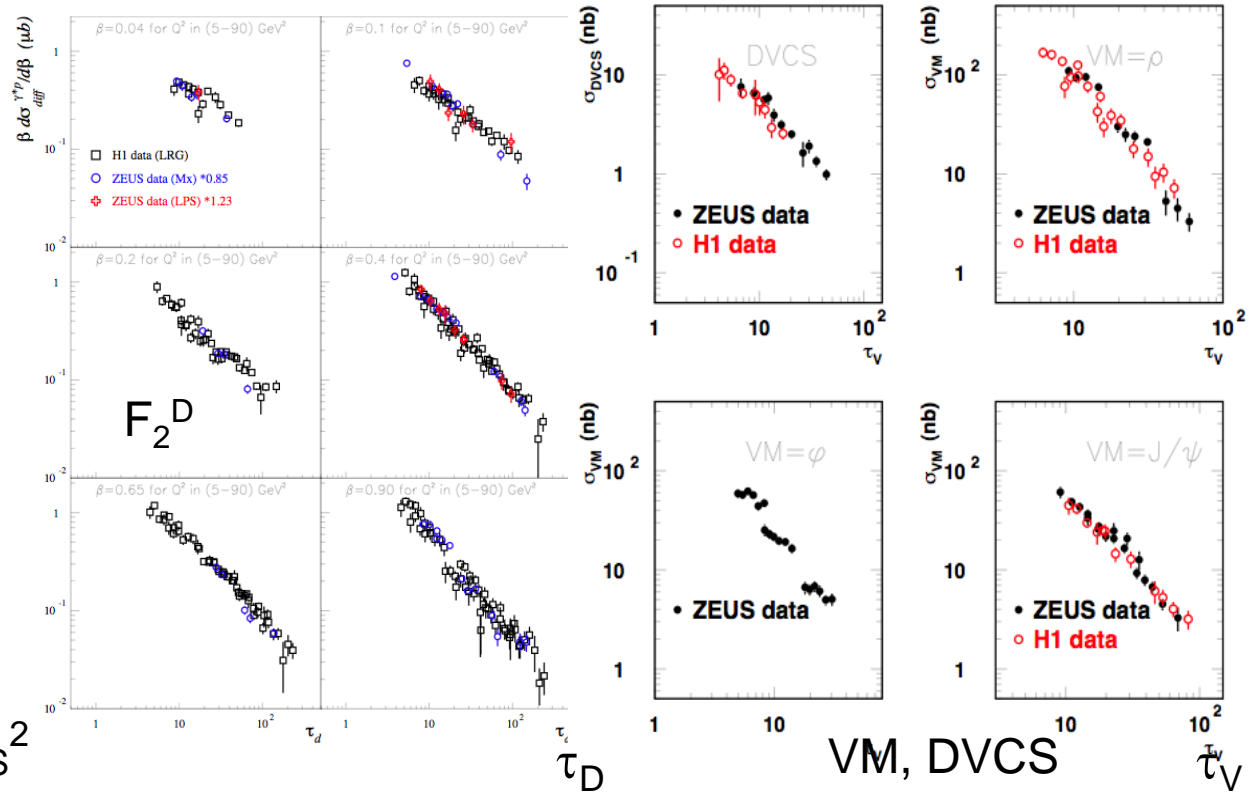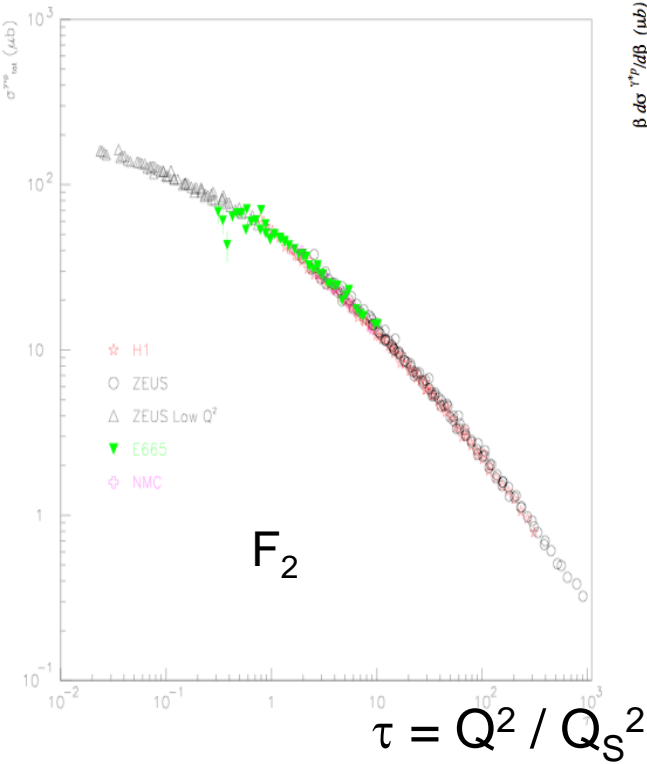
$$\sigma_{q\bar{q}P}(r_\perp, x) = \sigma_0 \left[1 - \exp\left(-r_\perp^2 \, Q_s^2(x) \right) \right] \qquad \boxed{Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x} \right)^\lambda}$$

Parameters: Q_0 = 1 GeV; λ = 0.3; x_0 = 3* 10^{-4} ; σ_0 = 23 mb

Sophisticated dipole models give excellent fits to all HERA small x data

Evidence from HERA for geometrical scaling
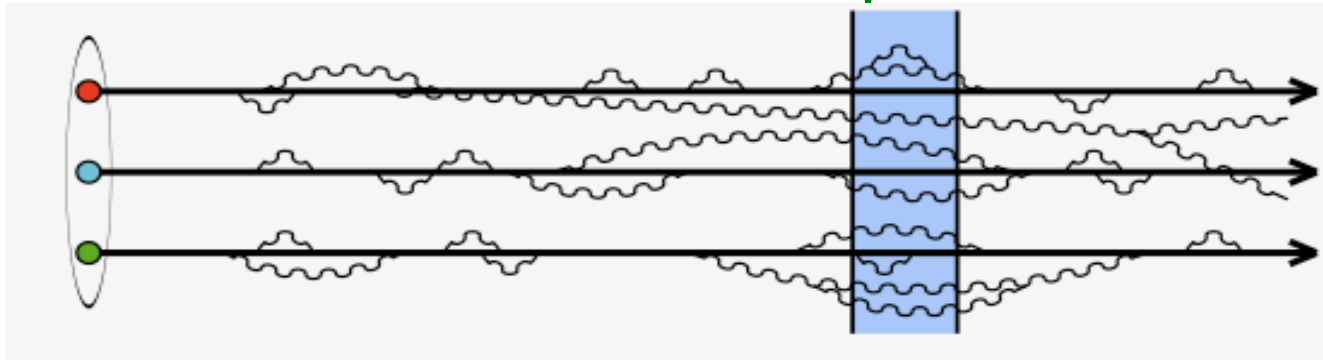
Golec-Biernat, Stasto,Kwiecinski

$\tau = Q^2 / Q_S^2$

F_2

F_2^D

τ_D

VM, DVCS

τ_V

Marquet, Schoeffel hep-ph/0606079

❖ **Scaling seen for F_2^D and VM,DVCS for same Q_S as F_2**

Gelis et al., hep-ph/0610435

The high energy nuclear wavefunction in QFT

$$|A> = |qqq...q> + ... + |qqq...q\textbf{gg...g}>$$

❖ **At high energies, interaction time scales of fluctuations
   are dilated well beyond typical hadronic time scales**

❖ **Lots of short lived (gluon) fluctuations now seen by probe
   -- proton/nucleus -- dense many body system of (primarily) gluons**

❖ **Fluctuations with lifetimes much longer than interaction
   time for the probe function as static color sources for more
   short lived fluctuations**

Nuclear wavefunction at high energies is a Color Glass Condensate

The nuclear wavefunction at high energies
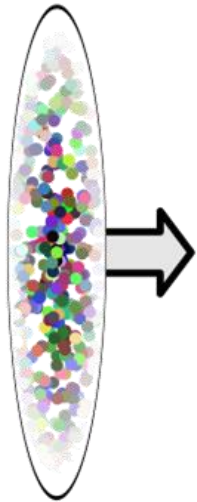
$$|A> = |qqq...q> + ... + |qqq...q\textbf{gg...gg}>$$

Higher Fock components dominate multiparticle production - construct Effective Field Theory
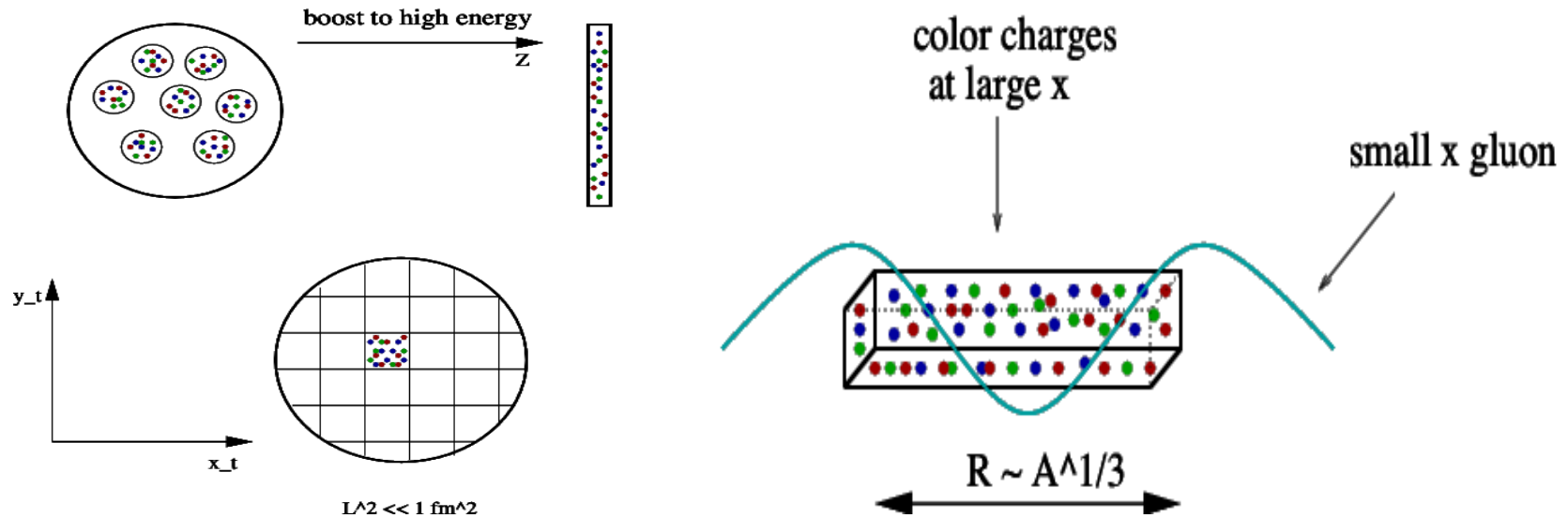

Dynamical wee modes
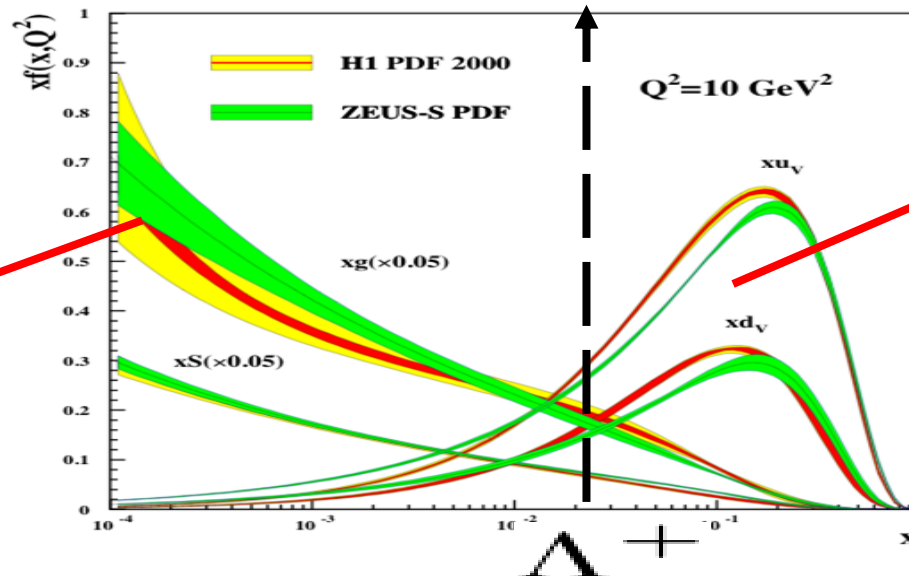
Valence modes-are static sources for wee modes

Born-Oppenheimer light cone separation natural for EFT

RG eqns describe evolution of wavefunction with energy

What do sources look like in the IMF ?

$$\lambda_{\text{wee}} \approx \frac{1}{k^+} \equiv \frac{1}{xP^+} >> \lambda_{\text{val.}} \equiv \frac{Rm_p}{P^+} => x << A^{-1/3}$$

Wee partons "see" a large density of color sources at small transverse resolutions

Effective Field Theory on Light Front

Poincare group on LF ⟷ **Galilean sub-group of 2D Quantum Mechanics**

isomorphism

Eg., LF dispersion relation

$$P^- = \frac{P_\perp^2}{2P^+}$$

→ **Momentum**

↘ **Energy**

↘ **Mass**
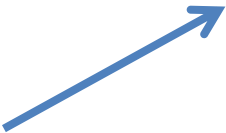
Large x (P⁺) modes: static LF (color) sources ρ^a

Small x (k⁺ <<P⁺) modes: dynamical fields A_μ^a

Susskind
Bardacki-Halpern

Effective Field Theory on Light Front

CGC: Coarse grained many body EFT on LF   **McLerran, RV**

$$< P|\mathcal{O}|P > \longrightarrow \int [d\rho^a][dA^{\mu,a}] \, W_{\Lambda^+}[\rho] \, e^{iS_{\Lambda^+}[\rho,A]} \, \mathcal{O}[\rho,A]$$

**Non-pert. gauge invariant "density matrix"
defined at initial scale Λ_0^+**

RG equations describe evolution of W with x   **JIMWLK, BK**

Classical Weizsäcker-Williams field of a large nucleus

$$< AA >_\rho = \int [d\rho] A_{cl.}(\rho) A_{cl.}(\rho) W_{\Lambda^+}[\rho]$$

For a large nucleus, A >>1,

"Pomeron" excitations

"Odderon" excitations

$$W_{\Lambda^+} = \exp\left(-\int d^2 x_\perp \left[\frac{\rho^a \rho^a}{2\,\mu_A^2} - \frac{d_{abc}\,\rho^a \rho^b \rho^c}{\kappa_A}\right]\right)$$

McLerran,RV
Kovchegov
Jeon, RV

A$_{cl}$ from $\longrightarrow$ $(D_\mu F^{\mu\nu})^a = J^{\nu,a} \equiv \delta^{\nu+}\,\delta(x^-)\,\rho^a(x_\perp)$

1/Λ_{QCD}   **2R / γ**
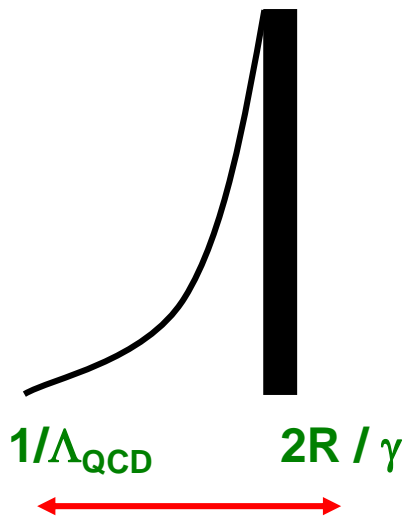
Wee parton distribution $\dfrac{1}{\Lambda_{QCD}} e^{-\lambda \Delta Y/2}$

determined from RG
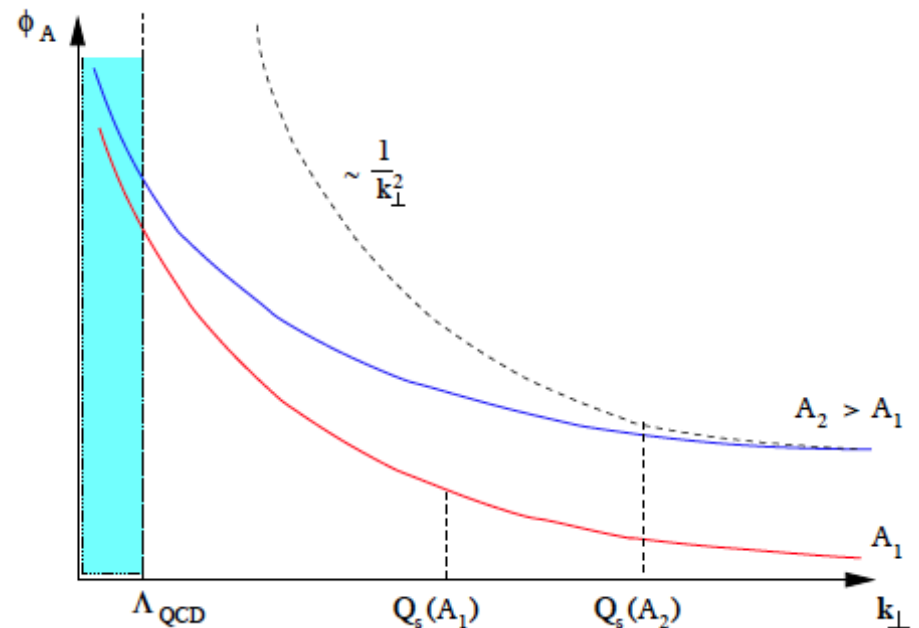
Quantum evolution of classical theory: Wilson RG

Fields   **Sources**

ρ

A^i

$1/\Lambda^+$ x^-

x'
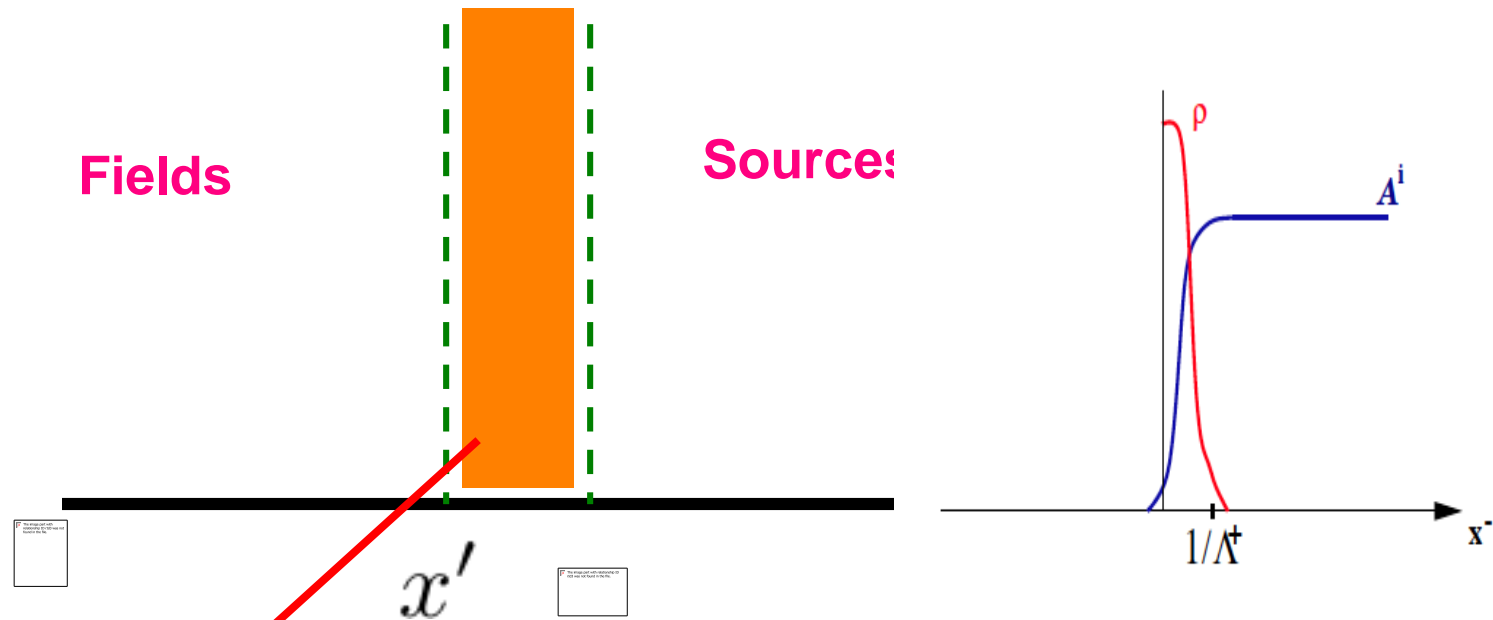
Integrate out small fluctuations => Increase color charge of sources

**Wilsonian RG equations describe evolution of all N-point correlation functions with energy**

JIMWLK   **Jalilian-marian, Iancu, McLerran,Weigert, Leonidov,Kovner**

Saturation scale grows with energy

Typical gluon momenta are large

Typical gluon k_T in hadron/nuclear wave function

$$k_\perp \frac{dN}{k_\perp dk_\perp}$$

$$\Lambda_{\text{QCD}} \quad Q_s(x) \quad Q_s(x')$$
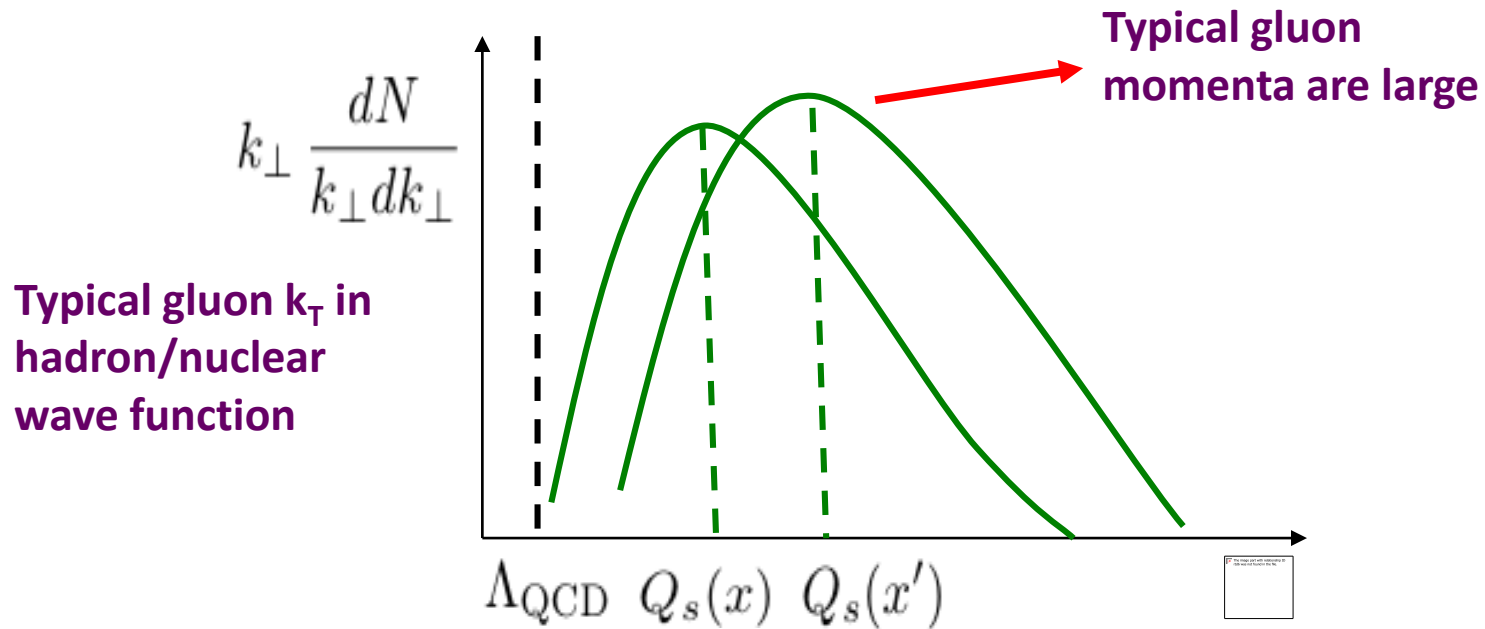
Bulk of high energy cross-sections:
a)   obey dynamics of novel non-linear QCD regime
b)   Can be **computed systematically** in weak coupling

Many-body high energy QCD:
The Color Glass Condensate
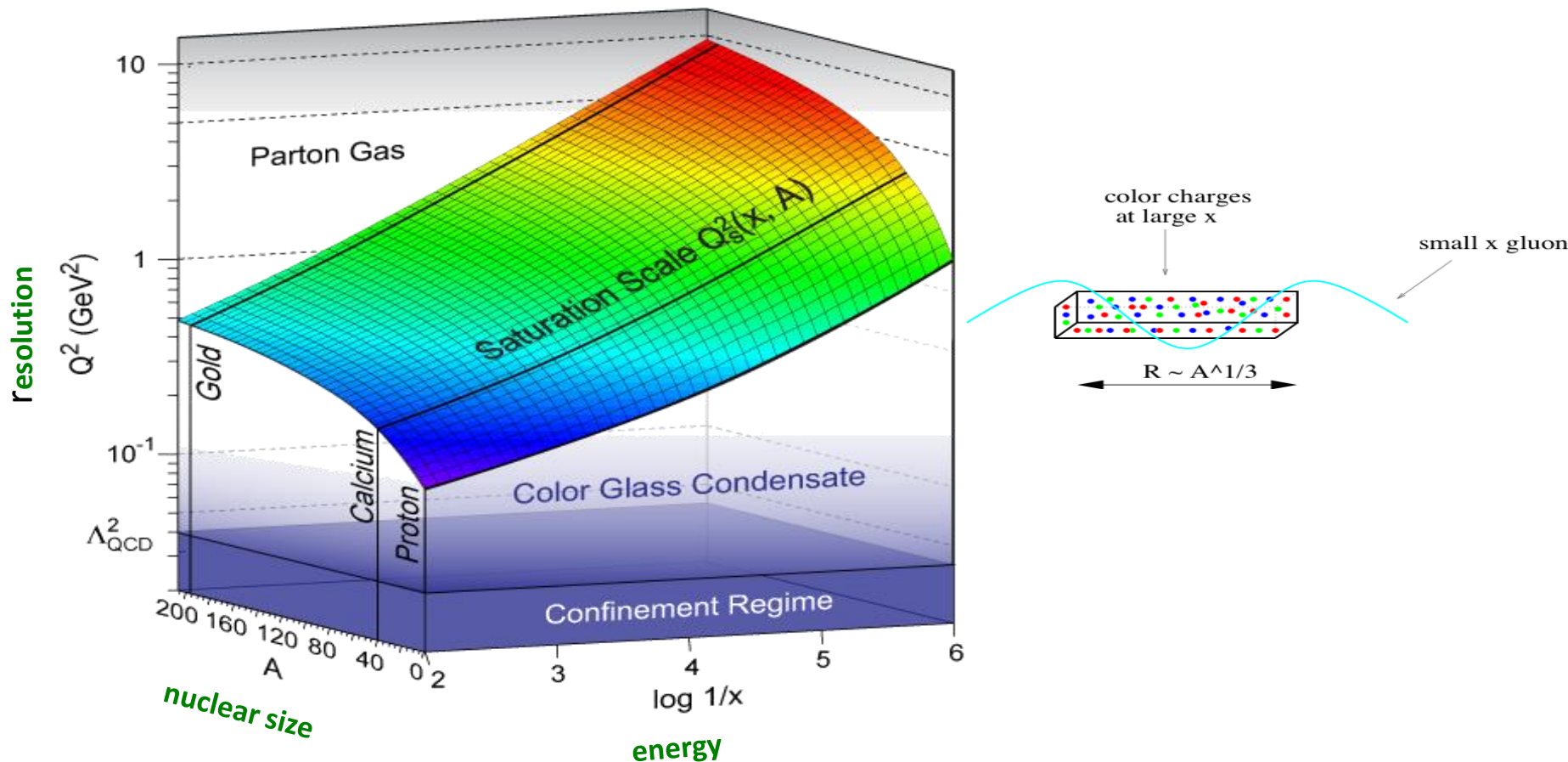
Dynamically generated semi-hard "saturation scale" opens window for weak coupling study of nonperturbative dynamics

JIMWLK RG evolution for a single nucleus

$$\mathcal{O}_{\mathrm{NLO}} = \left(\chi(x_\perp, y_\perp) \right) \mathcal{O}_{\mathrm{LO}}$$

$$= \ln \left(\frac{\Lambda^+}{p^+} \right) \mathcal{H} \, \mathcal{O}_{\mathrm{LO}}$$ **(keeping leading log divergences)**

$$\langle \mathcal{O}_{\mathrm{LO}} + \mathcal{O}_{\mathrm{NLO}} \rangle = \int [d\tilde{\rho}] W[\tilde{\rho}] \, [\mathcal{O}_{\mathrm{LO}} + \mathcal{O}_{\mathrm{NLO}}]$$

$$= \int [d\tilde{\rho}] \left\{ \left[1 + \ln \left(\frac{\Lambda^+}{p^+} \right) \mathcal{H} \right] W_{\Lambda^+} \right\} \mathcal{O}_{\mathrm{LO}}$$

LHS independent of Λ^+ =>

$$\boxed{\frac{\partial W[\tilde{\rho}]}{\partial Y} = \mathcal{H} \, W[\tilde{\rho}]}$$

23

Inclusive DIS: dipole evolution
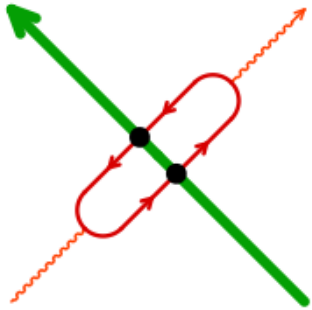
$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2 r_\perp \, |\psi(z, r_\perp)|^2 \sigma_{\mathrm{dipole}}(x, r_\perp)$$

$$\sigma_{\mathrm{dipole}}(x, r_\perp) = 2 \int d^2 b \int [D\rho] \, W_{\Lambda^+}[\rho] \, T(b + \frac{r_\perp}{2}, b - \frac{r_\perp}{2})$$

$$1 - \frac{1}{N_c} \mathrm{Tr}\left(V\left(b + \frac{r_\perp}{2} \right) V^\dagger \left(b - \frac{r_\perp}{2} \right) \right)$$

Inclusive DIS: dipole evolution
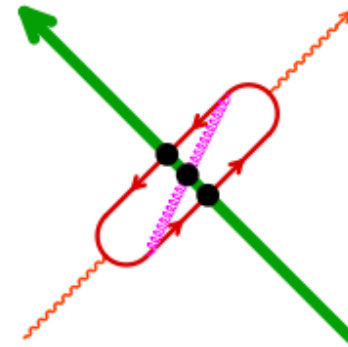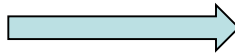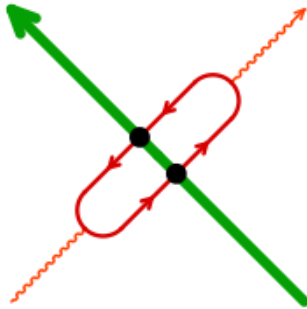
B-JIMWLK eqn. for dipole correlator

$$\frac{\partial}{\partial Y} \langle \mathrm{Tr}(V_x V_y^\dagger) \rangle_Y = -\frac{\alpha_S N_c}{2\pi^2} \int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2} \langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c} \mathrm{Tr}(V_x V_z^\dagger) \, \mathrm{Tr}(V_z V_y^\dagger) \rangle_Y$$

Dipole factorization:

$$\langle \mathrm{Tr}(V_x V_z^\dagger) \, \mathrm{Tr}(V_z V_y^\dagger) \rangle_Y \longrightarrow \langle \mathrm{Tr}(V_x V_z^\dagger) \rangle_Y \langle \mathrm{Tr}(V_z V_y^\dagger) \rangle_Y \qquad N_c \rightarrow \infty$$

Resulting closed form equation is the Balitsky-Kovchegov equation.

Reduces in the "low density" limit to the BFKL equation I discussed previously

"Photon impact factor" and "kernel" now known to NLO accuracy

**Balitsky, Chirill
Beuf**

CGC Effective Theory: B-JIMWLK hierarchy of correlators

$$\frac{\partial}{\partial Y} <O[\alpha]>_Y = <\frac{1}{2}\int_{x,y} \frac{\delta}{\delta\alpha_Y^a(x)}\chi_{x,y}^{ab}\frac{\delta}{\delta\alpha_Y^b(y)}O[\alpha]>_Y$$

"time"   "diffusion coefficient"

Fast hadron   fast partons (ρ)

small-x gluons $(\mathbf{A}[\rho])$

Dipole

$\gamma *$
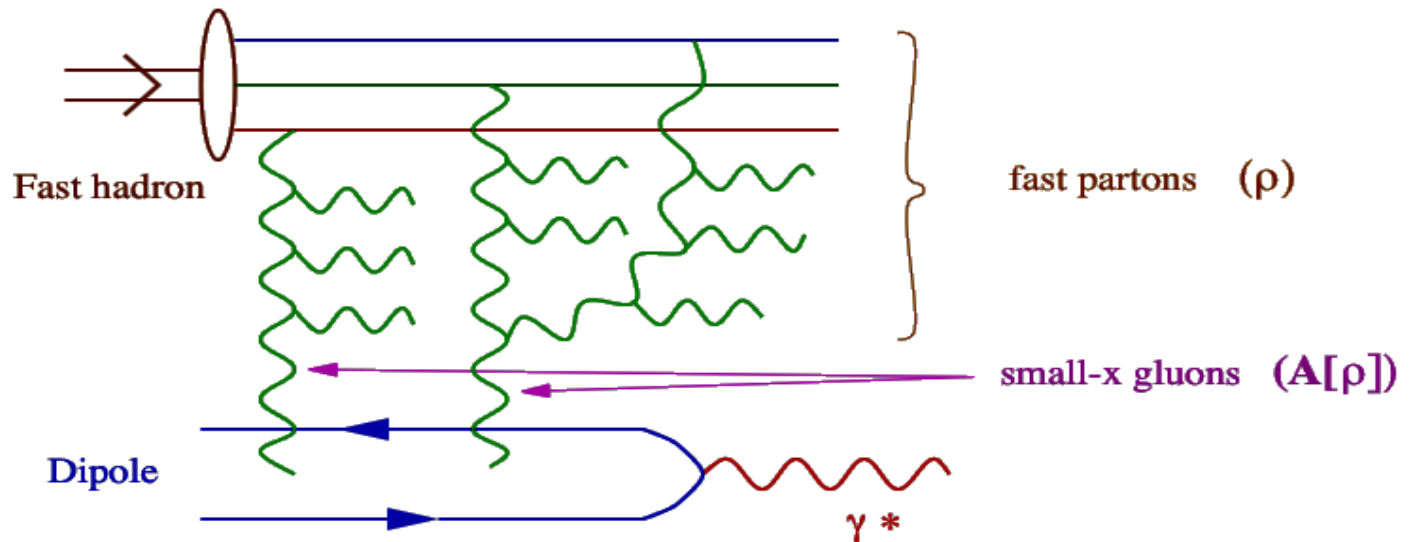
At high energies, the d.o.f that describe the frozen many-body gluon configurations are novel objects: dipoles, quadrupoles, …

Universal – appear in a number of processes in p+A and e+A; how do these evolve with energy?

Solving the B-JIMWLK hierarchy

❑ **JIMWLK includes multiple scatterings & leading log evolution in x**

❑ **Expectation values of Wilson line correlators at small x satisfy a Fokker-Planck eqn. in functional space**   *Weigert (2000)*

❑ **This translates into a hierarchy of equations for n-point Wilson line correlators**

❑ **As is generally the case, Fokker-Planck equations can be re-expressed as Langevin equations – in this case for Wilson lines**

Blaizot,Iancu,Weigert
Rummukainen,Weigert

B-JIMWLK hierarchy: Langevin realization

Numerical evaluation of Wilson line correlators on 2+1-D lattices:

$$\left\langle \mathcal{O}[U] \right\rangle_Y = \int D[U] \, W_Y[U] \, \mathcal{O}[U] \longrightarrow \frac{1}{N} \sum_{U \epsilon W} \mathcal{O}[U]$$

Langevin eqn:

Gaussian random variable

$$\partial_Y [V_x]_{ij} = [V_x i t^a]_{ij} \left[\int d^2 y \, [\mathcal{E}_{xy}^{ab}]_k \, [\xi_y^b]_k + \sigma_x^a \right]$$

$$\mathcal{E}_{xy}^{ab} = \left(\frac{\alpha_S}{\pi^2} \right)^{1/2} \frac{(x-y)_k}{(x-y)^2} \left[1 - U_x^\dagger U_y \right]^{ab}$$

$$\sigma_x^a = -i \left(\frac{\alpha_S}{2\pi^2} \int d^2 z \frac{1}{(x-z)^2} \mathrm{Tr}(T^a \, U_x^\dagger U_z) \right)$$

"square root" of JIMWLK kernel

"drag"

❑ **Initial conditions for V's from the MV model**

❑ **Daughter dipole prescription for running coupling**

Functional Langevin solutions of JIMWLK hierarchy

Rummukainen,Weigert (2003)

Dumitru,Jalilian-Marian,Lappi,Schenke,RV, PLB706 (2011)219

We are now able to compute all n-point correlations of a theory of strongly correlated gluons and study their evolution with energy!

Correlator of Light-like Wilson lines Tr(V(0,0)V^dagger (x,y))
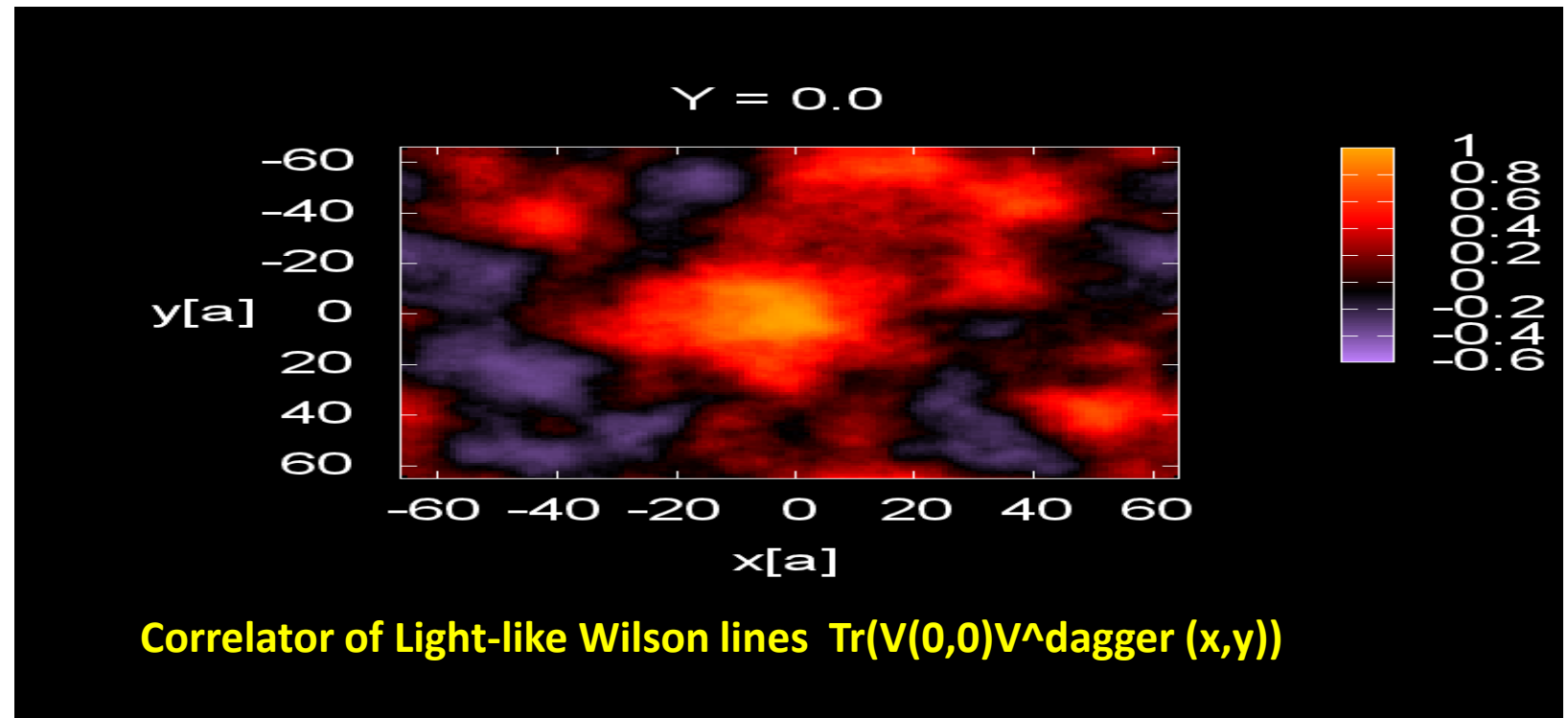
Semi-inclusive DIS: quadrupole evolution
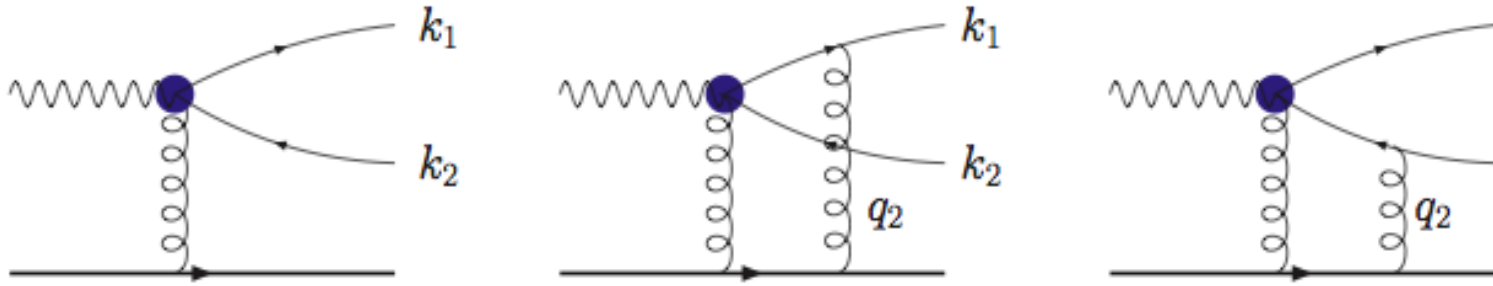
Dominguez,Marquet,Xiao,Yuan (2011)

$$\frac{d\sigma^{\gamma^*_{T,L}A \to q\bar{q}X}}{d^3k_1 d^3k_2} \propto \int_{x,y,\bar{x}\bar{y}} e^{ik_{1\perp}\cdot(x-\bar{x})} e^{ik_{2\perp}\cdot(y-\bar{y})} \left[1 + Q(x,y;\bar{y},\bar{x}) - D(x,y) - D(\bar{y},\bar{x})\right]$$

Semi-inclusive DIS: quadrupole evolution
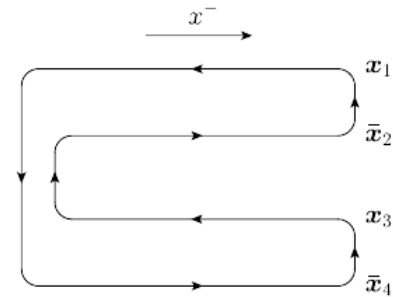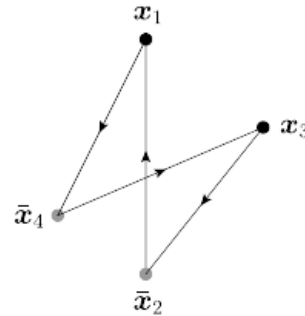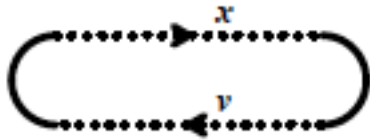
$$D(x,y) = \frac{1}{N_c}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y$$

$$Q(x,y;\bar{y},\bar{x}) = \frac{1}{N_c}\langle \mathrm{Tr}(V_x V_{\bar{x}}^\dagger V_{\bar{y}} V_y^\dagger)\rangle_Y$$

RG evolution provides fresh insight into multi-parton correlations

Dumitru,Jalilian-Marian,Lappi,Schenke,RV: arXiv:1108.1764

Quadrupoles, like Dipoles, exhibit geometrical Scaling

Iancu,Triantafyllopolous,arXiv:1112.1104
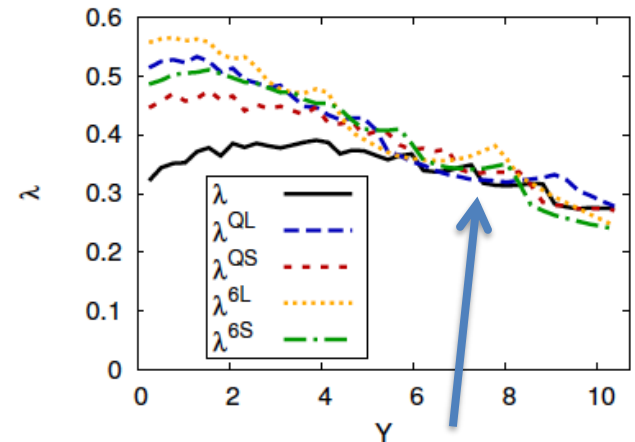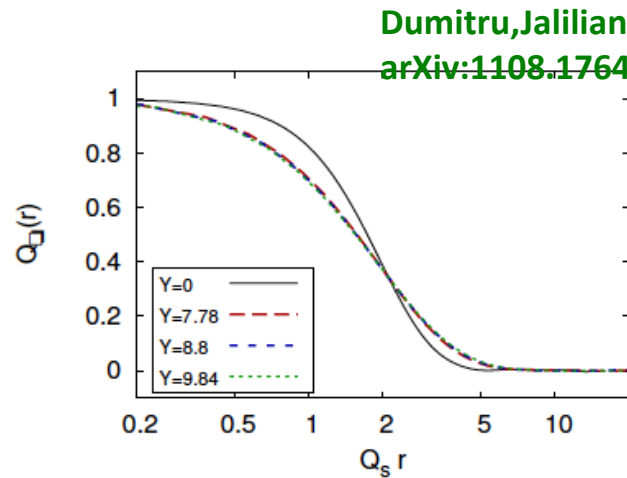
Rate of energy evolution of dipole and quadrupole saturation scales

Universality: Di-hadrons in p/d-A collisions

Jalilian-Marian, Kovchegov (2004)
Marquet (2007), Tuchin (2010)
Dominguez,Marquet,Xiao,Yuan (2011)
Strikman,Vogelsang (2010)
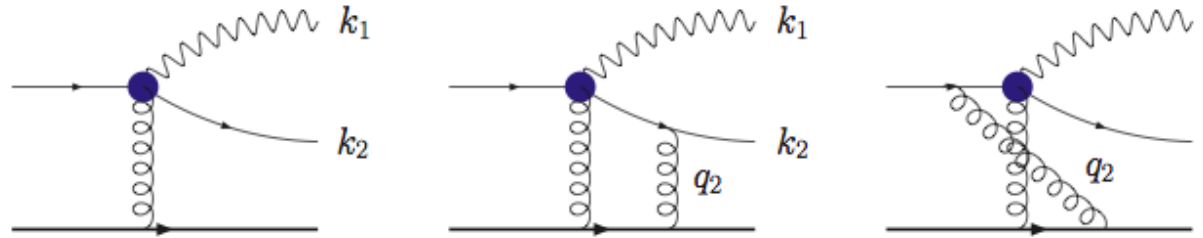
$$\frac{d\sigma^{qA \to qgX}}{d^3k_1 d^3k_2} \propto \int_{x,y,\bar{x},\bar{y}} e^{ik_{1\perp} \cdot (x-\bar{x})} e^{ik_{2\perp} \cdot (y-\bar{y})} \left[S_6(x,y,\bar{x},\bar{y}) - S_4(x,y,v) - ... \right]$$

$$\frac{N_c}{2C_F} \left\langle Q(x,y\,\bar{y},\bar{x}) D(y,\bar{y}) - \frac{D(x,\bar{x})}{N_c} \right\rangle \quad \frac{N_c}{2C_F} \left\langle D(x,y) D(\bar{y},\bar{x}) - \frac{D(x,\bar{x})}{N_c} \right\rangle$$

Forward-forward di-hadrons sensitive to both dipole and quadrupole correlators
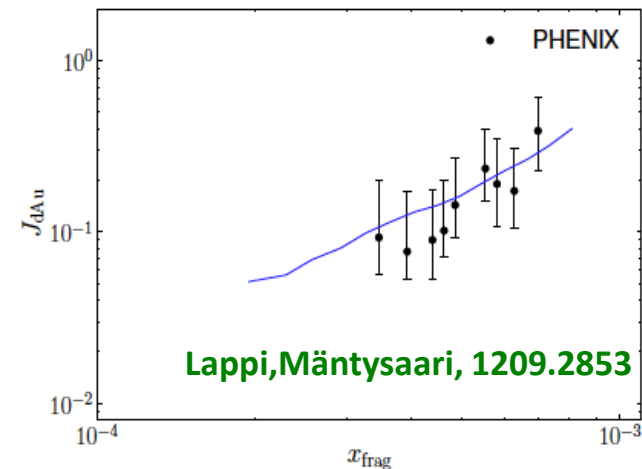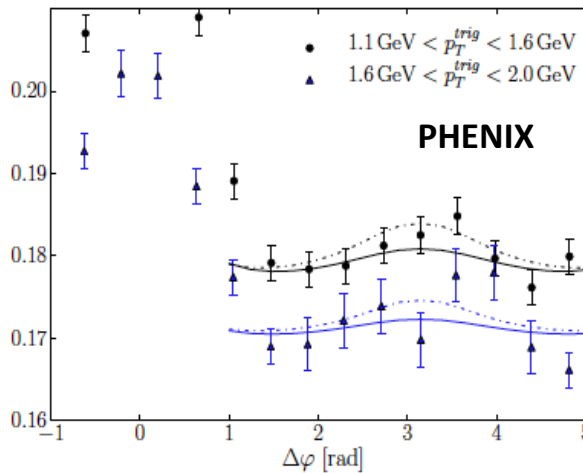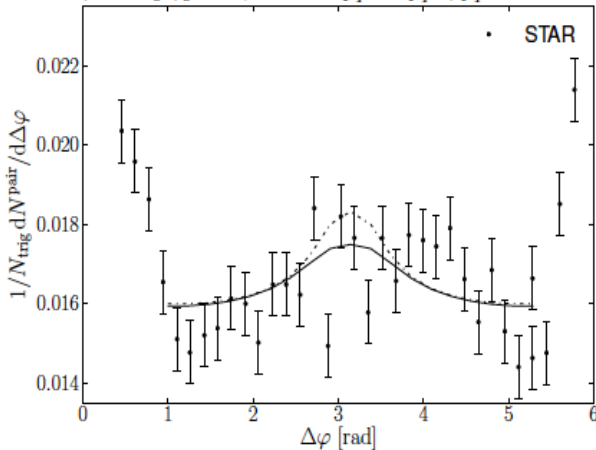
Recent computations (Stasto,Xiao,Yuan + Lappi,Mäntysaari) include Pedestal, Shadowing (color screening) and Broadening (multiple scattering) effects in CGC

What is the evidence for gluon saturation?

DIS off the proton

Inclusive distributions
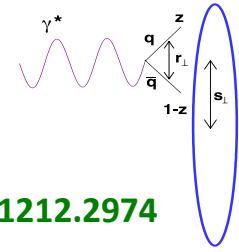
Rezaiean, Siddikov, Van der Klundert, RV:1212.2974
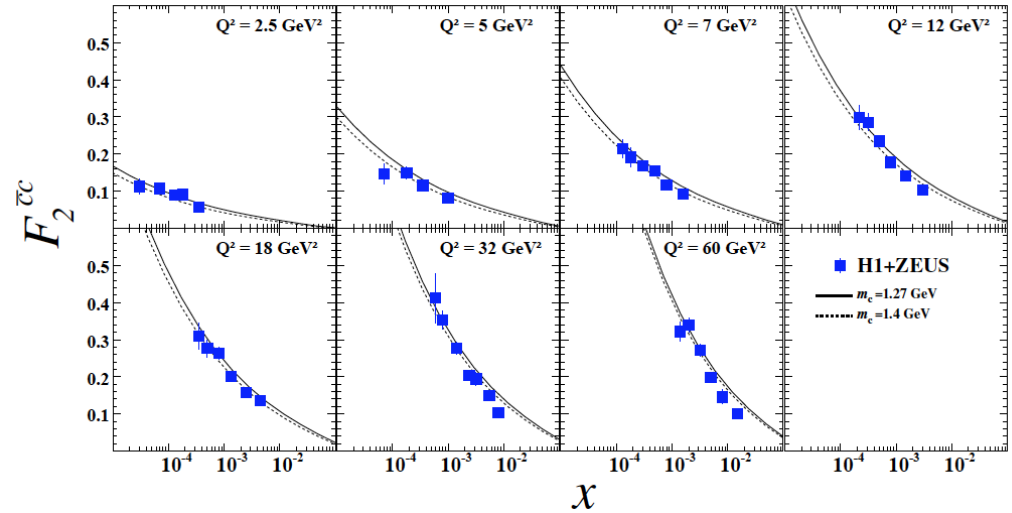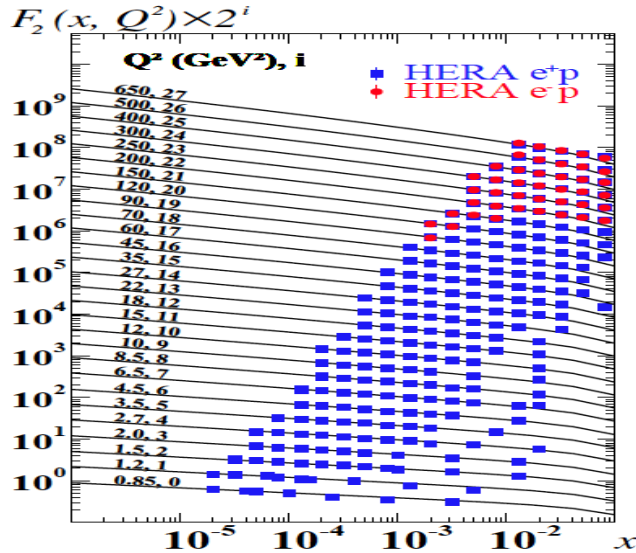
Exclusive distributions
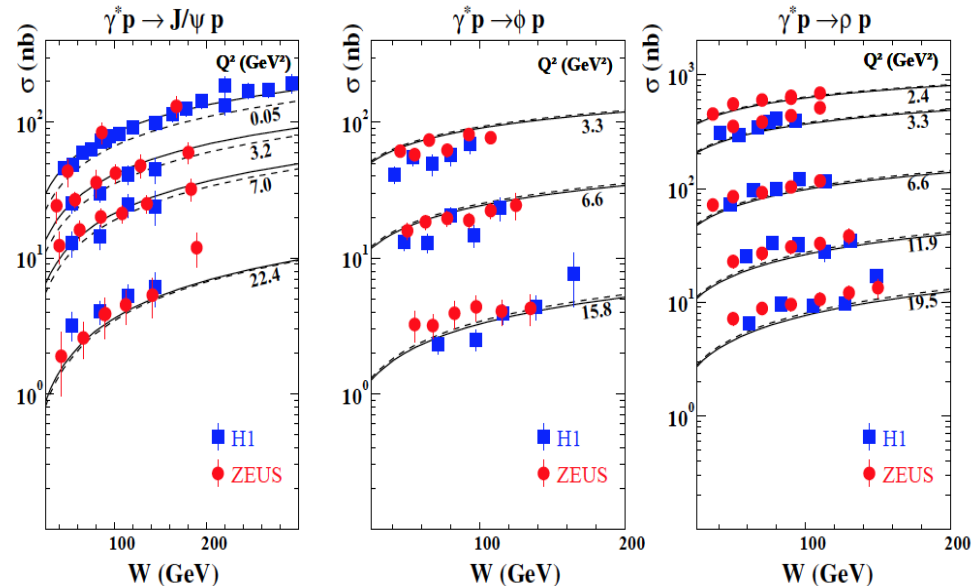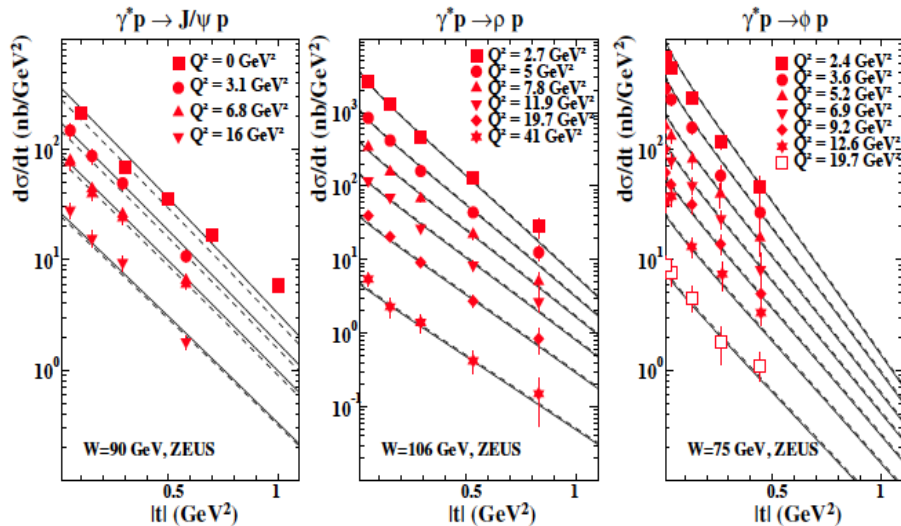
DIS off nuclei
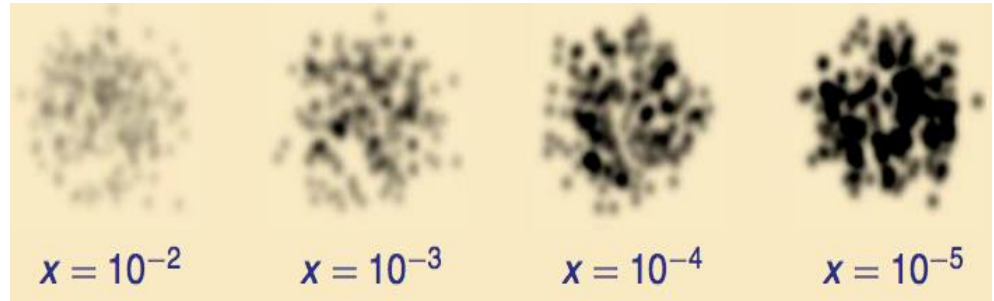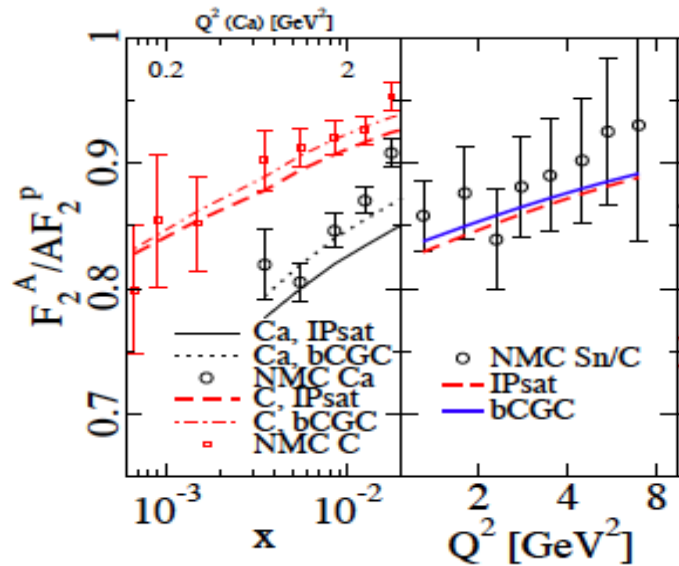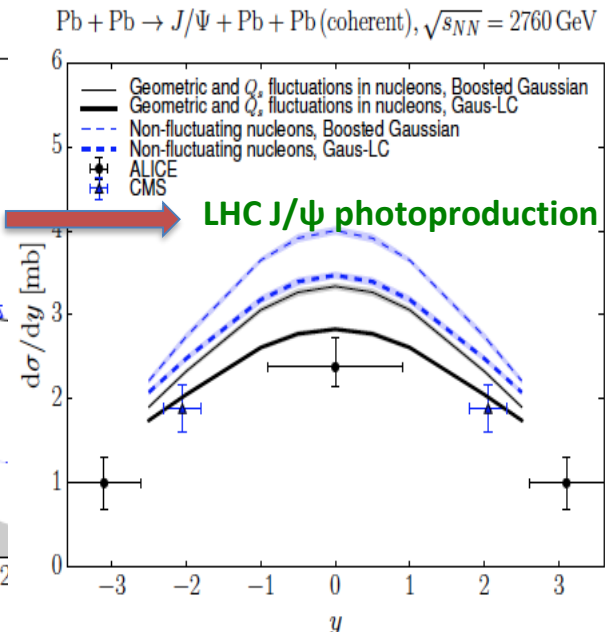

Kowalski,Lappi,RV:0705.3047

Consistent, within limited available data, with shadowing obseved in e+A collisions
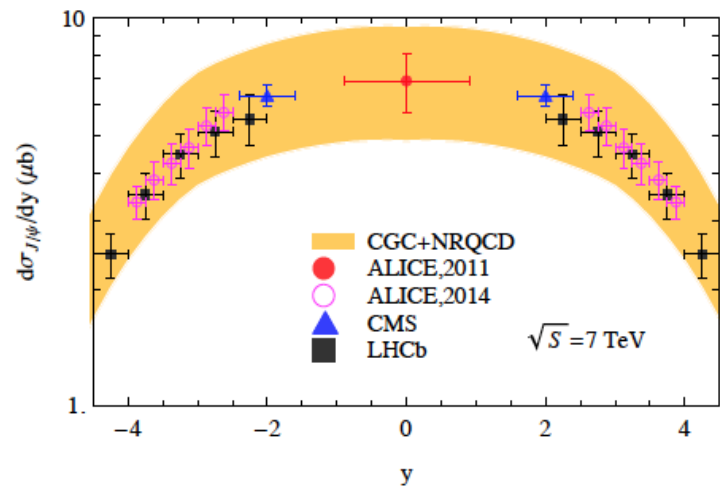
Role of gluon shape fluctuations in the proton
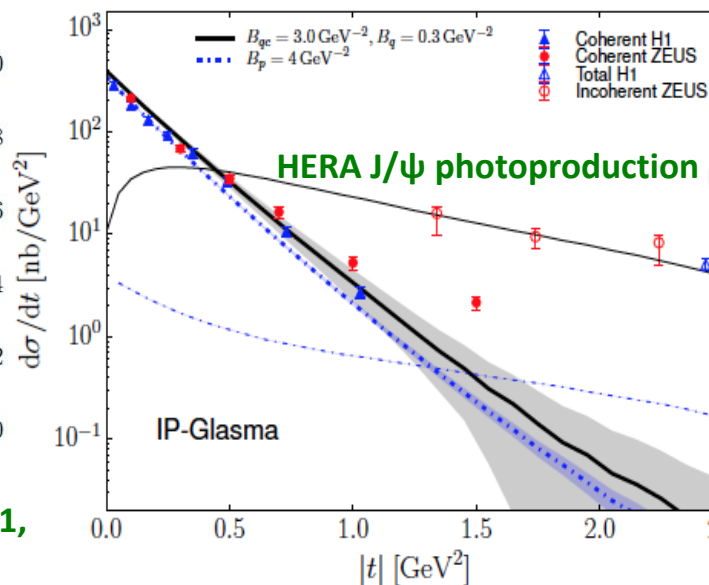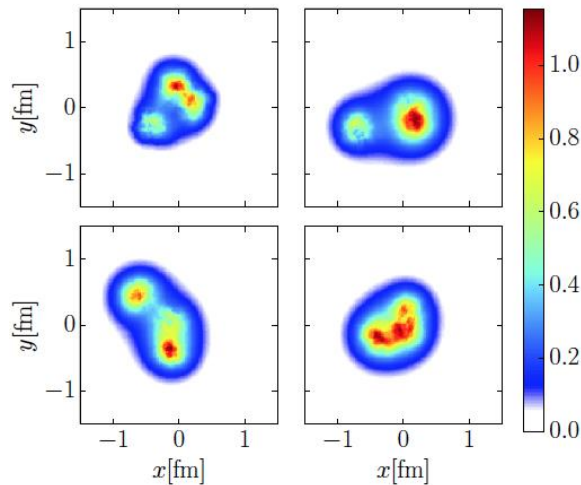
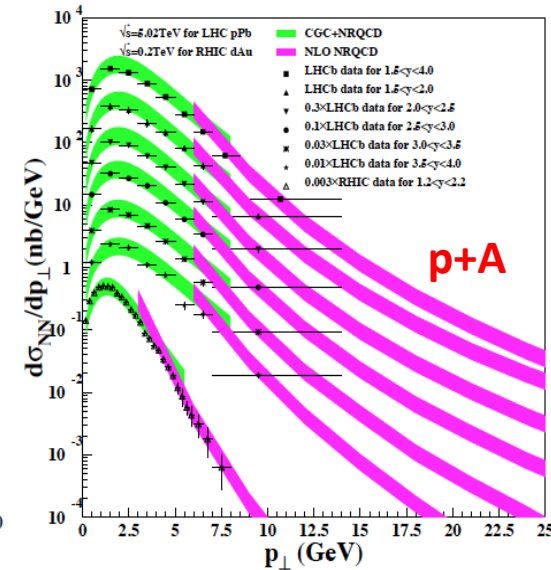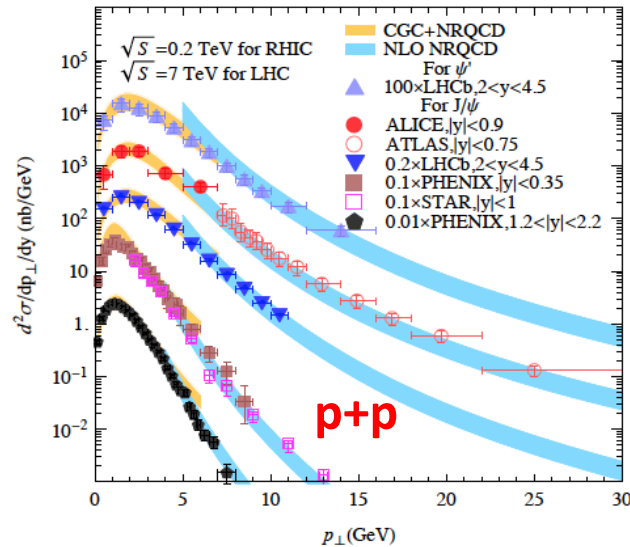HERA J/ψ photoproduction → LHC J/ψ photoproduction

Mantysaari,Schenke,arXiv:1607.01711, arXiv:1703.09257

A sampling of results from p+p & p+A collisions

Ma et al., arXiv 1408.4075 & 1503.0772

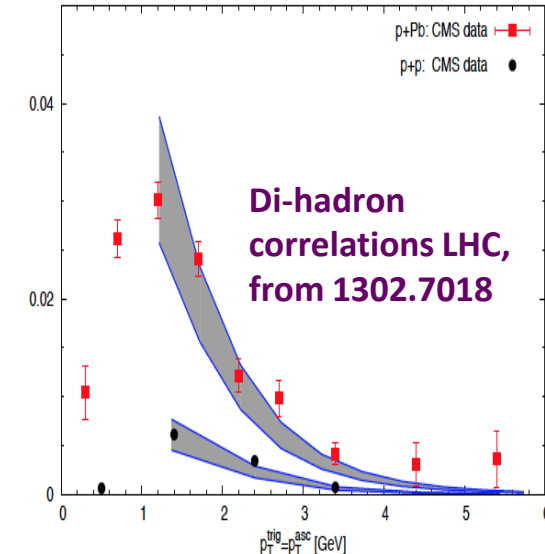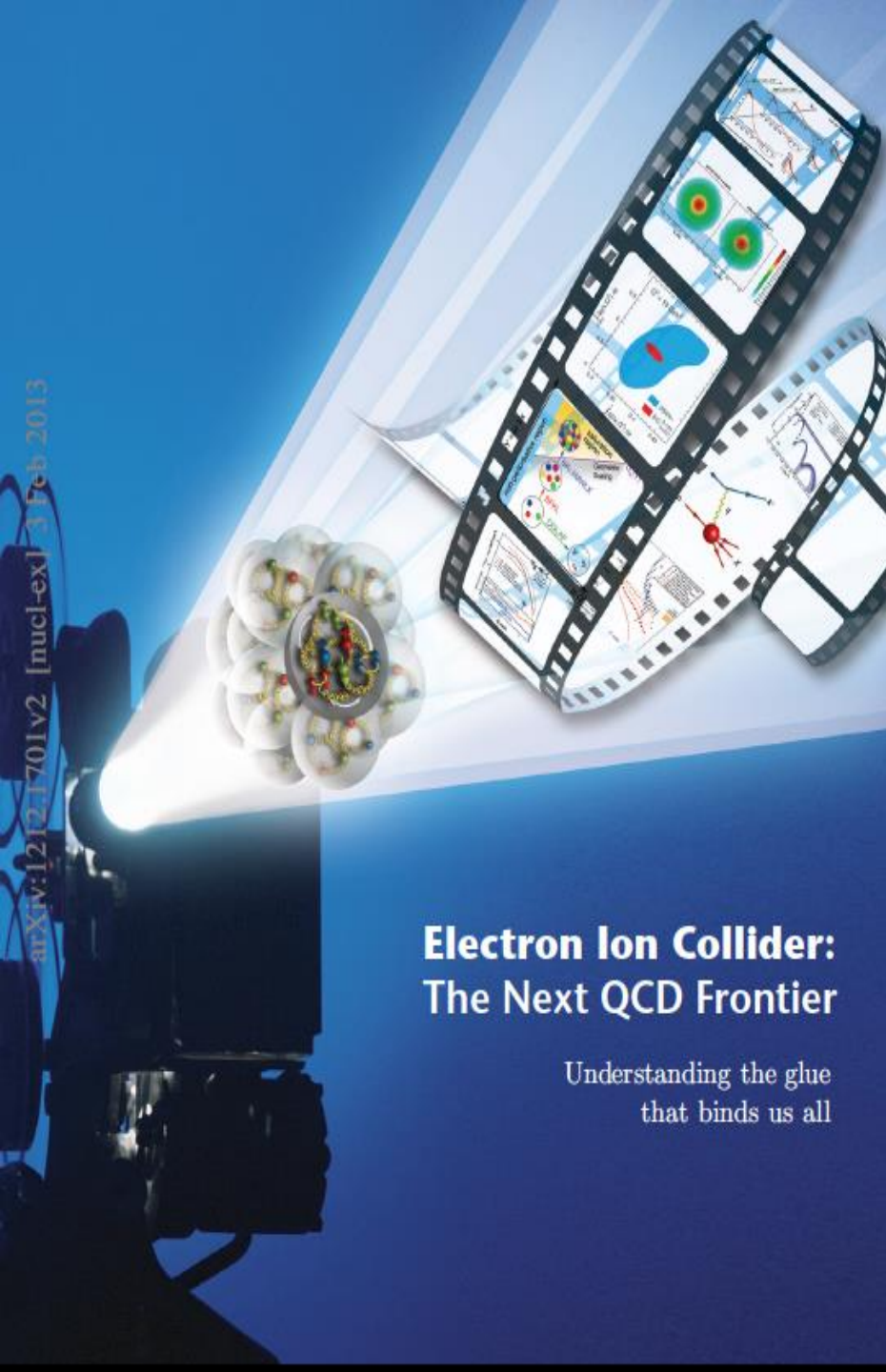**J/ψ rapidity and p_T rapidity distributions in p+p and p+A at RHIC & the LHC**

Di-hadron azimuthal correlations in p+p

PHENIX

Ratio of d+Au/p+p di-hadron yield as function of x

Curves from Lappi, Mantysaari, arXiv:1209.2853

Di-hadron correlations LHC, from 1302.7018
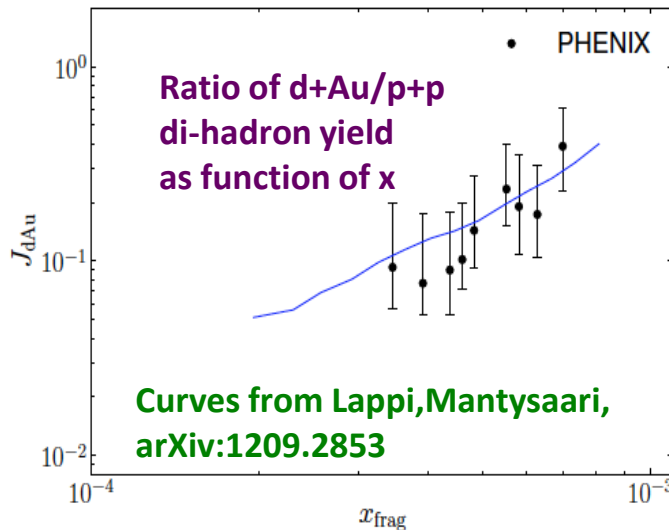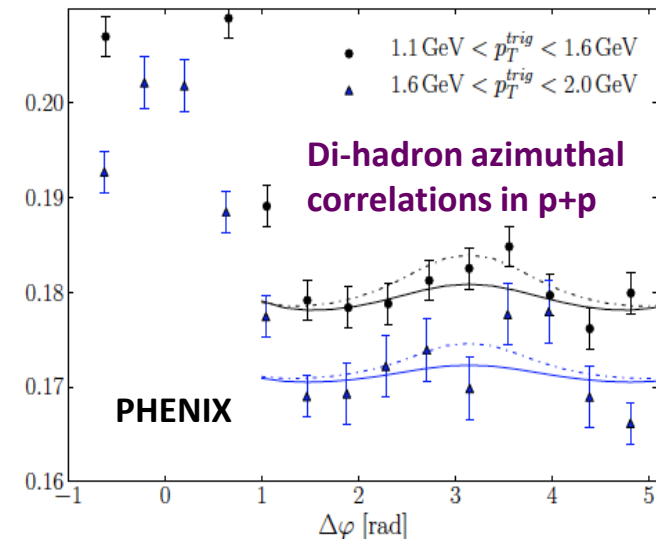
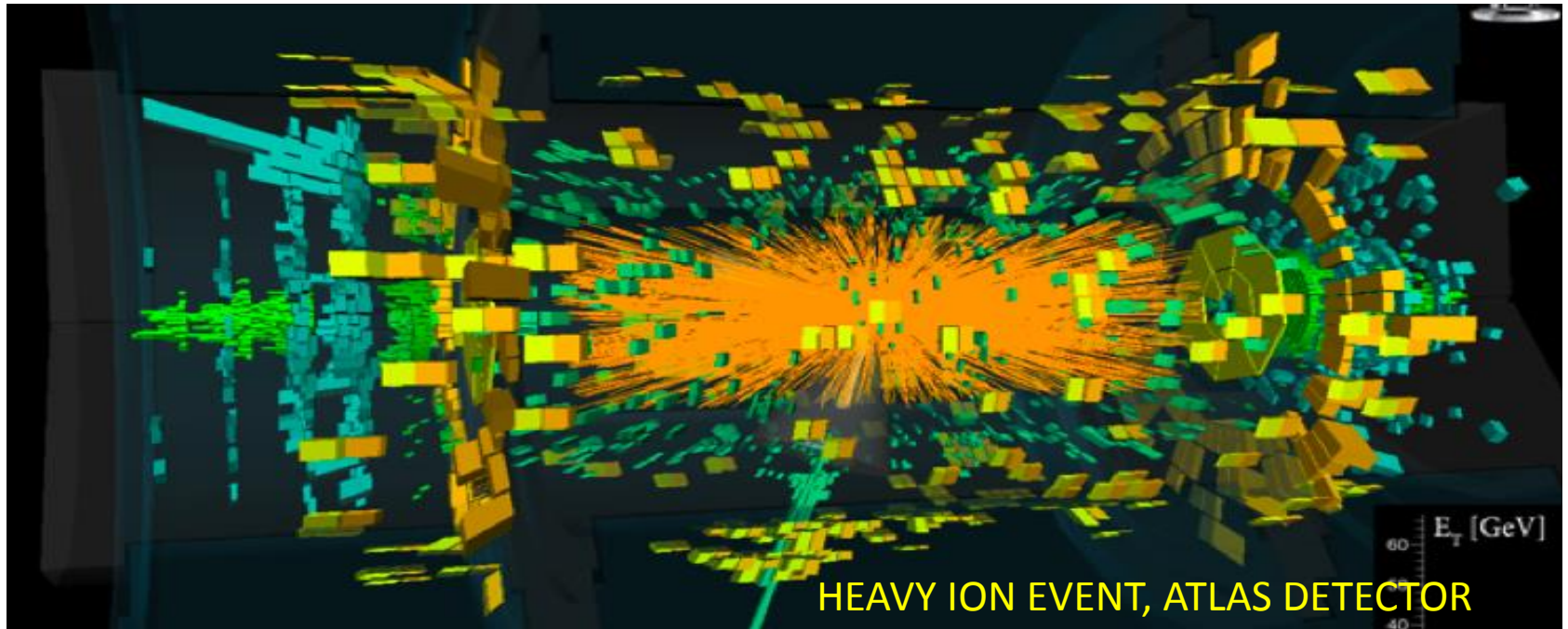**Electron Ion Collider:
The Next QCD Frontier**

Understanding the glue
that binds us all

The Electron-Ion Collider

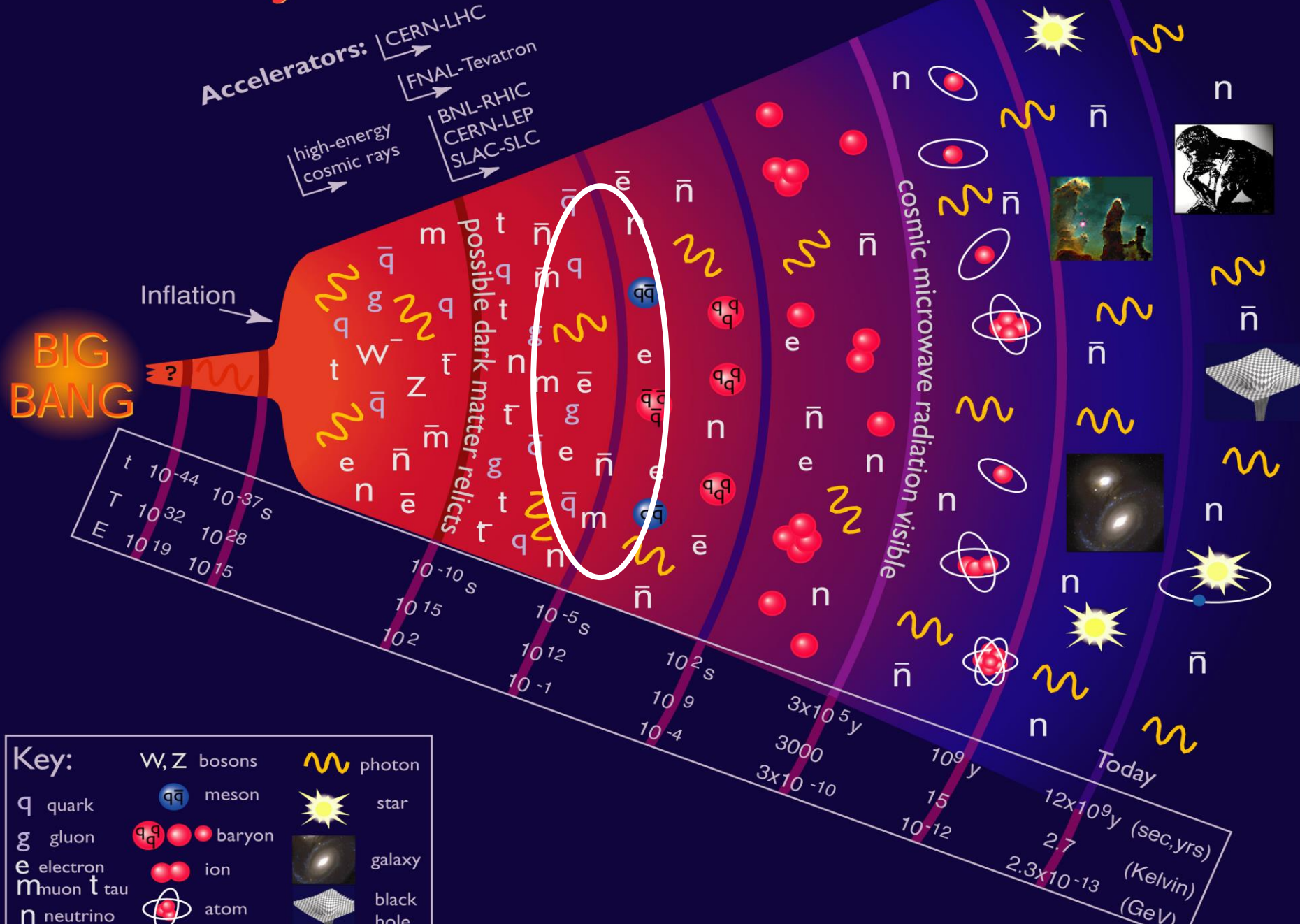## Assessing the Energy Dependence
of Key Measurements
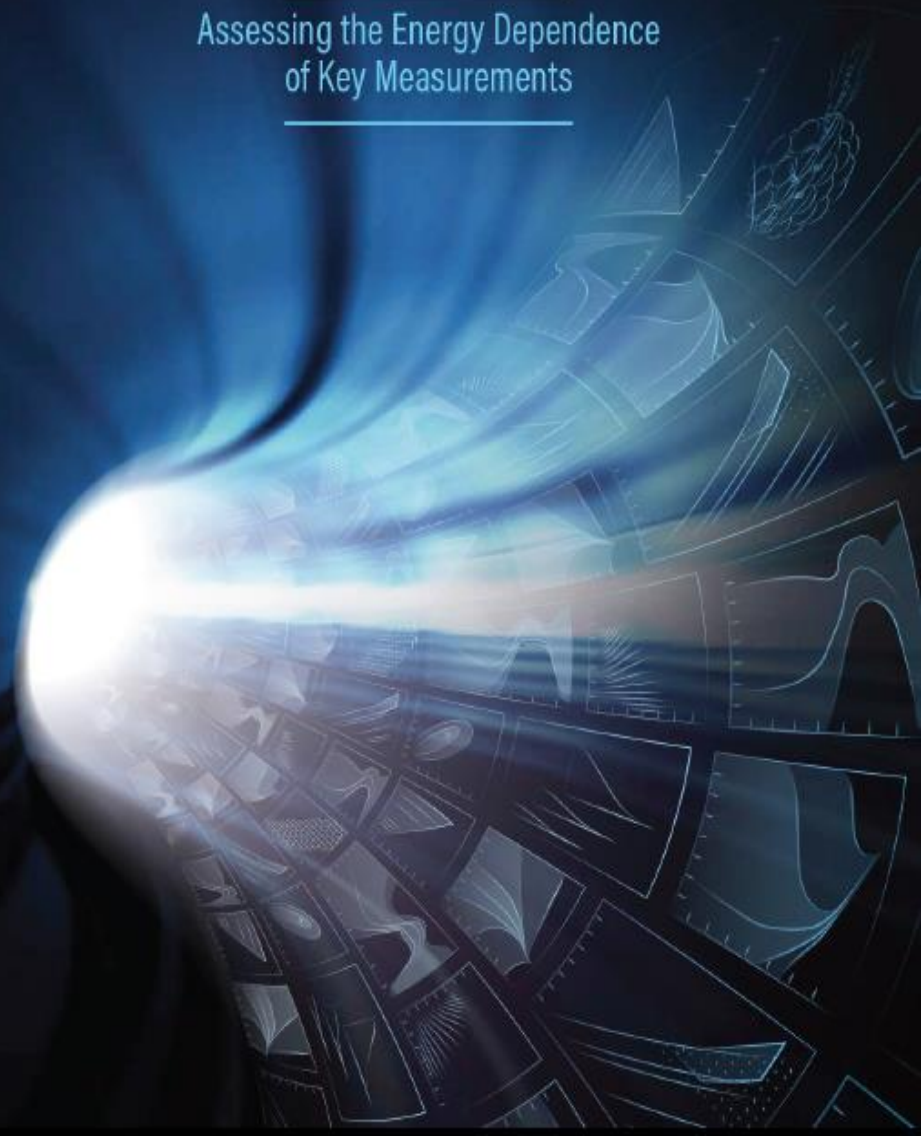
Universal dynamics in the Quark-Gluon Plasma

HEAVY ION EVENT, ATLAS DETECTOR

Studying the real time dynamics of a strongly correlated non-Abelian gauge theory

"The early universe was "liquid-like"

Perfect fluidity across energy scales

"Bjorken Hydrodynamics"

$$\frac{d\varepsilon}{d\tau} = -\frac{\left(\varepsilon + P - \frac{4}{3}\frac{\eta}{\tau}\right)}{\tau}$$

Viscous term smaller than ideal term for

$$\frac{\eta}{\varepsilon + P}\frac{1}{\tau} \equiv \frac{\eta}{s}\frac{1}{\tau T} << 1$$

From kinetic theory

$$\frac{\eta}{s} \sim \frac{\hbar}{k_B}\frac{\tau_{\text{relax.}}}{\tau_{\text{quant.}}}$$

Fluid	$T\ [K]$	$\eta\ [Pa \cdot s]$	$\eta/n\ [\hbar]$	$\eta/s\ [\hbar/k_B]$
H_2O	370	2.9×10^{-4}	85	8.2
^4He	2	1.2×10^{-6}	0.5	1.9
$^6Li\ (|a_s| \simeq \infty)$	23×10^{-6}	$\leq 1.7 \times 10^{-15}$	≤ 1	≤ 0.5
QGP	2×10^{12}	$\leq 5 \times 10^{11}$	-	≤ 0.4

Perfect fluidity across energy scales

"Bjorken Hydrodynamics"

$$\frac{d\varepsilon}{d\tau} = -\frac{\left(\varepsilon + P - \frac{4}{3}\frac{\eta}{\tau}\right)}{\tau}$$

Viscous term smaller than ideal term for

$$\frac{\eta}{\varepsilon + P}\frac{1}{\tau} \equiv \frac{\eta}{s}\frac{1}{\tau T} << 1$$

From kinetic theory

$$\frac{\eta}{s} \sim \frac{\hbar}{k_B}\frac{\tau_{\text{relax.}}}{\tau_{\text{quant.}}}$$

QGP is ~ 10^4 times more viscous than pitch tar...

Perfect fluidity across energy scales

"Bjorken Hydrodynamics"

$$\frac{d\varepsilon}{d\tau} = -\frac{\left(\varepsilon + P - \frac{4}{3}\frac{\eta}{\tau}\right)}{\tau}$$

Viscous term smaller than ideal term for

$$\frac{\eta}{\varepsilon + P}\frac{1}{\tau} \equiv \frac{\eta}{s}\frac{1}{\tau T} << 1$$
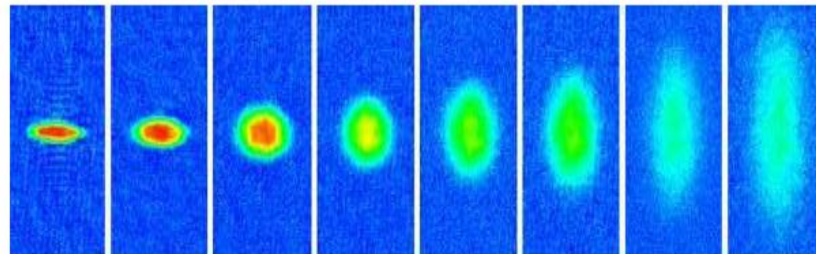
From kinetic theory

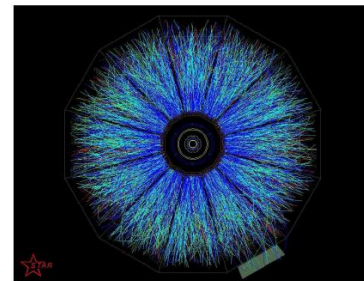$$\frac{\eta}{s} \sim \frac{\hbar}{k_B}\frac{\tau_{\text{relax.}}}{\tau_{\text{quant.}}}$$

H$_2$O

^{4}He

^{6}Li

sQGP

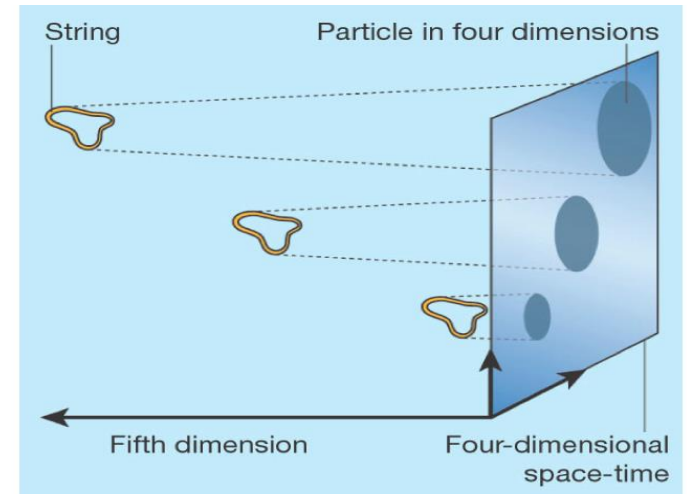Schafer,Teaney, Rep.Prog.Phys.72 (2009) 126001

Viscosity of strongly coupled relativistic fluids

AdS/CFT conjecture:

Duality between strongly coupled
N=4 supersymmetric Yang-Mills theory
at large coupling and Nc
& classical 10 dimensional gravity in the
background of D3 branes

J.Maldacena, Nature 2003

KSS bound:
**Conjectured lower bound for
a ``perfect fluid''**

$$\frac{\eta}{s} = \frac{\hbar}{k_B} \frac{1}{4\pi}$$

Kovtun,Son,Starinets (2006)

**Derived using classical absorption cross-section of a graviton
with energy ω on a black brane and Bekenstein's formula relating its
Entropy to its area**

$$\sigma(\omega) = \frac{8\pi G}{\omega} \int dt d\mathbf{x}\, e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, 0)] \rangle$$

Deconstructing lumpiness
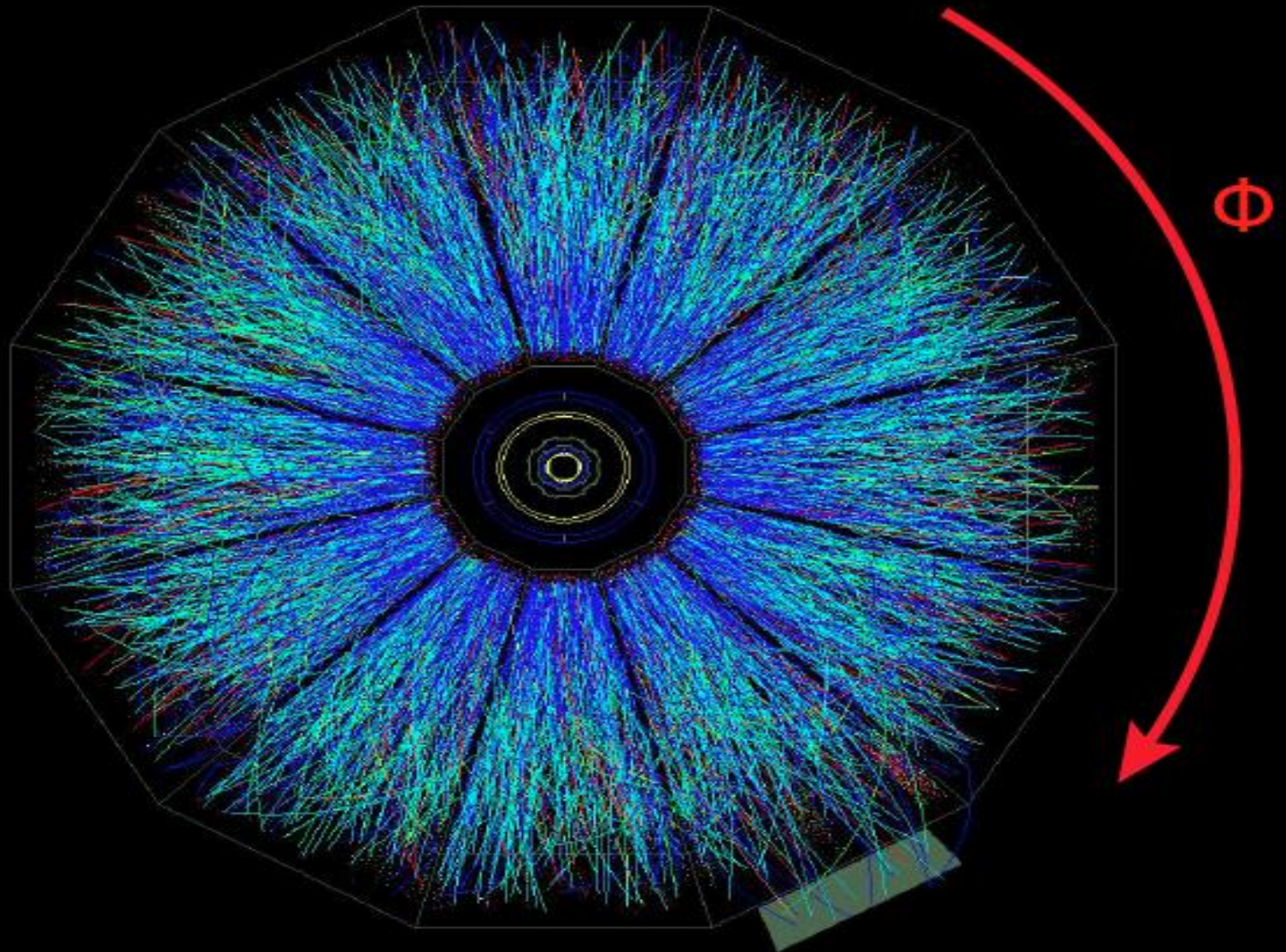
ANGULAR PARTICLE DISTRIBUTION
EXPERIMENTAL DATA: ATLAS COLLABORATION
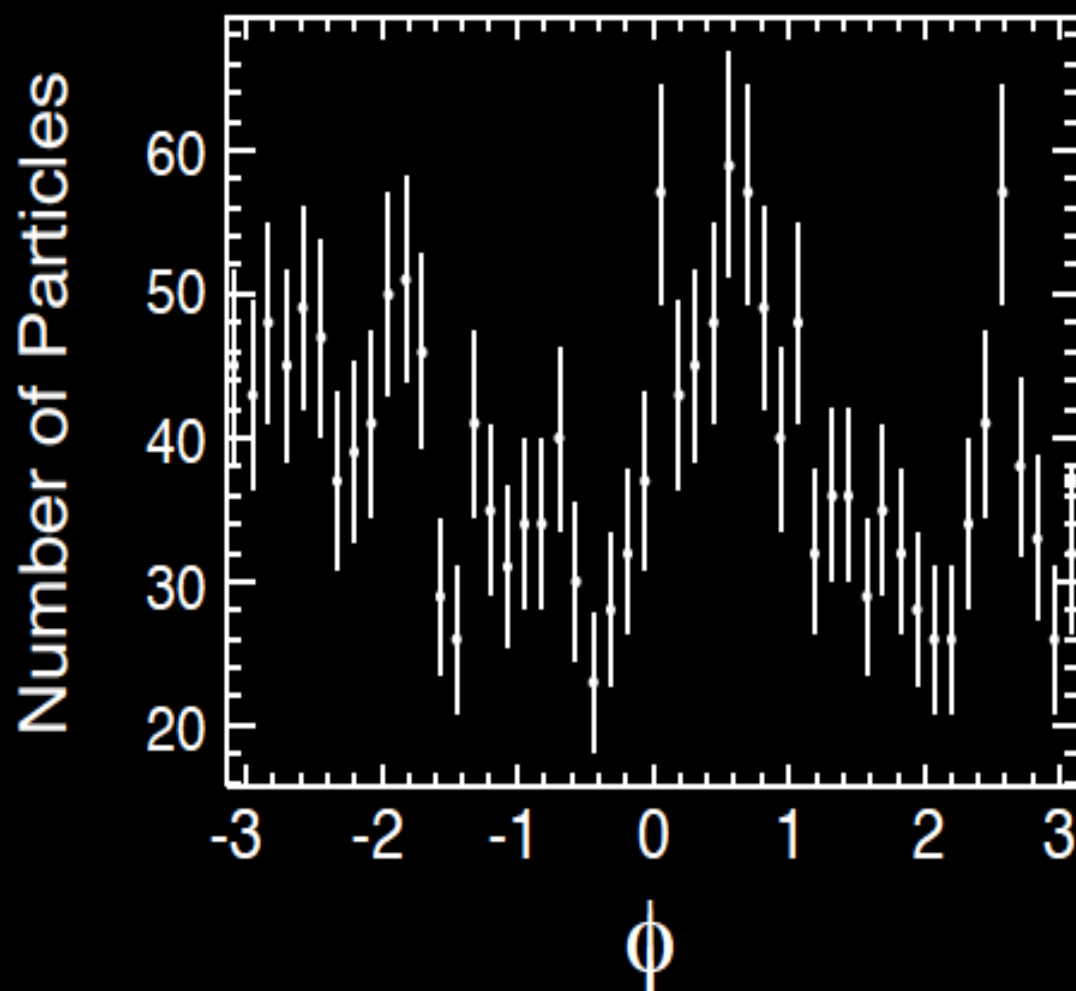
ANGULAR PARTICLE DISTRIBUTION
EXPERIMENTAL DATA: ATLAS COLLABORATION
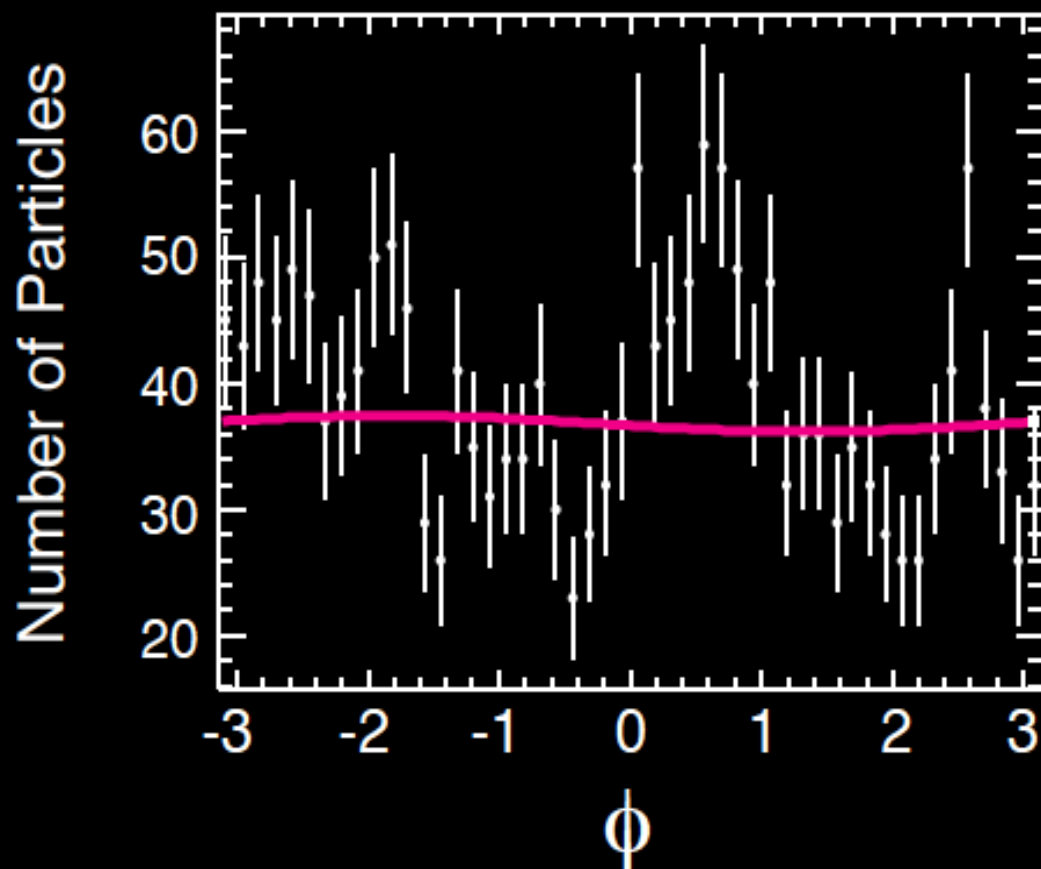
$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + 2(v_1 \cos(\phi))\right)$$

ANGULAR PARTICLE DISTRIBUTION
EXPERIMENTAL DATA: ATLAS COLLABORATION
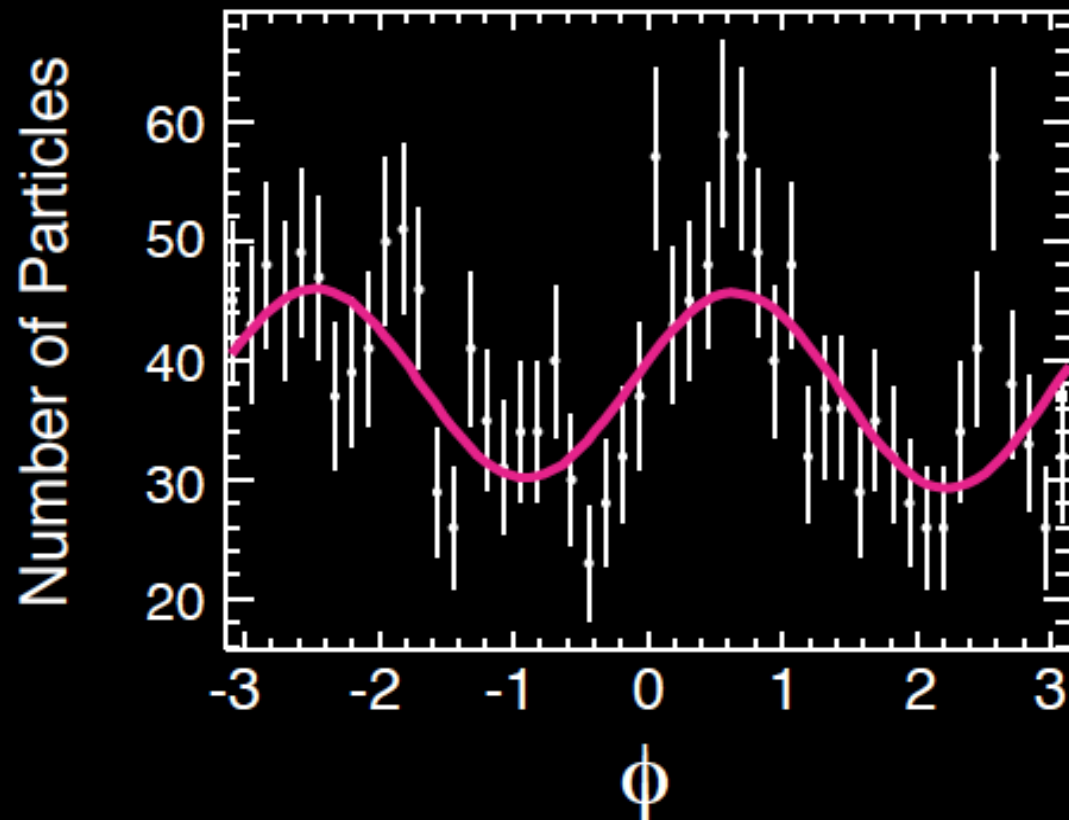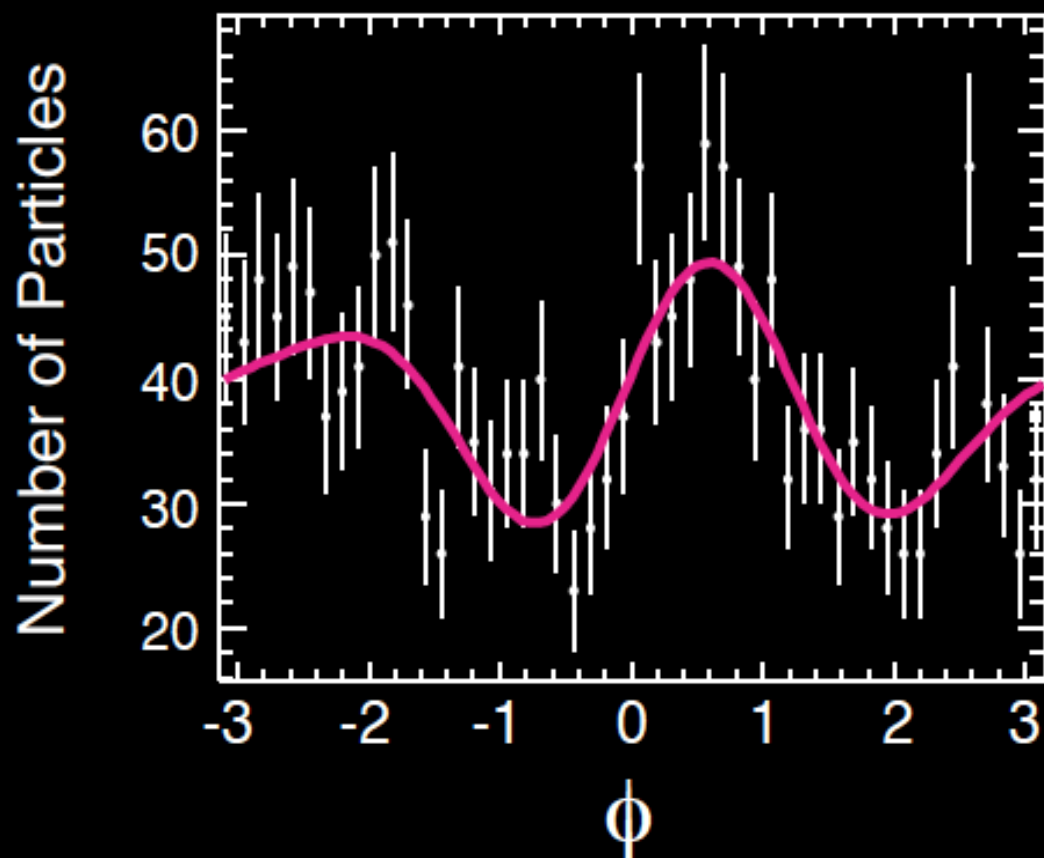
$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + 2\left(v_1 \cos(\phi) + v_2 \cos(2\phi)\right)\right)$$

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + 2(v_1\cos(\phi) + v_2\cos(2\phi) + v_3\cos(3\phi))\right)$$

ANGULAR PARTICLE DISTRIBUTION
EXPERIMENTAL DATA: ATLAS COLLABORATION
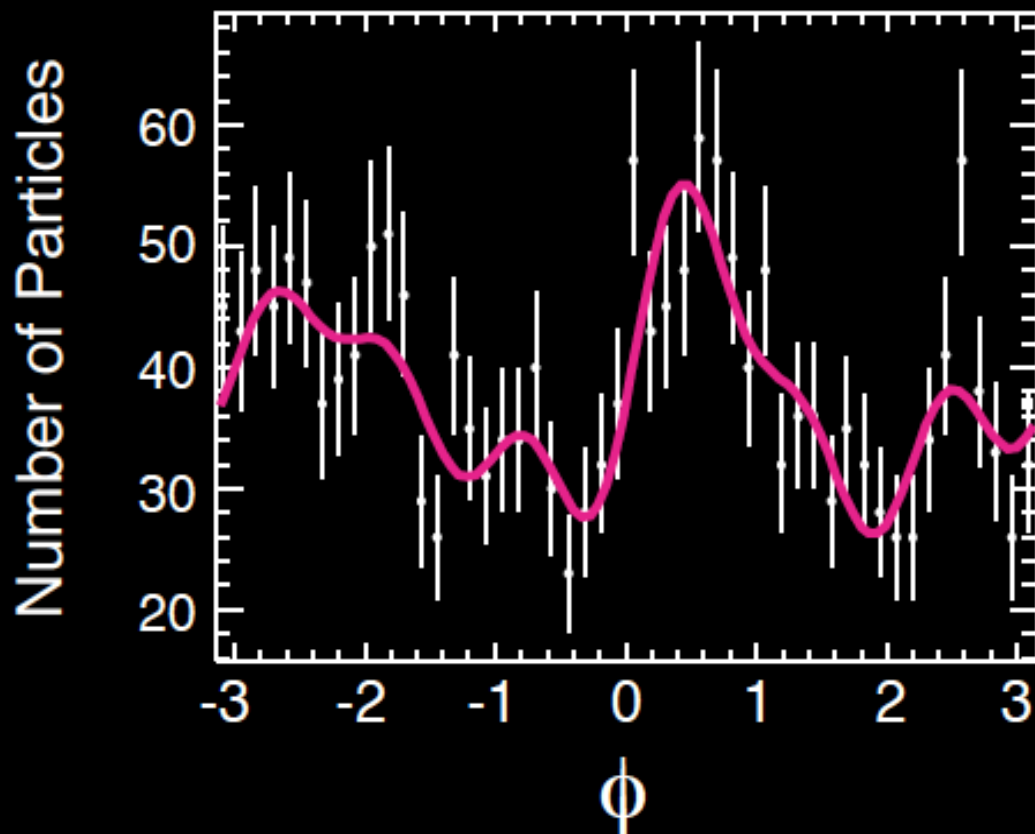
$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + 2(v_1\cos(\phi) + v_2\cos(2\phi) + v_3\cos(3\phi) + v_4\cos(4\phi) + \ldots)\right)$$

Flow moments: analogy with the Early Universe

Mishra et al; Mocsy- Sorensen; Floerchinger, Wiedemann
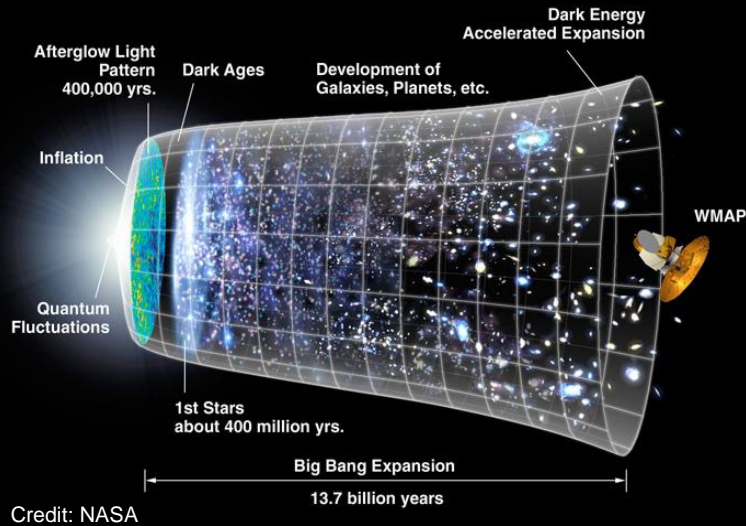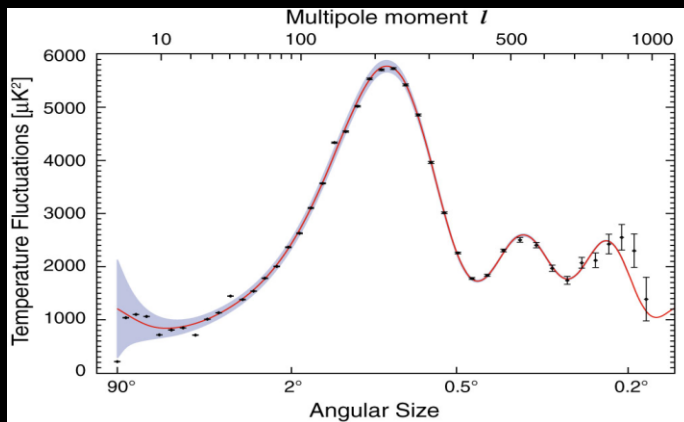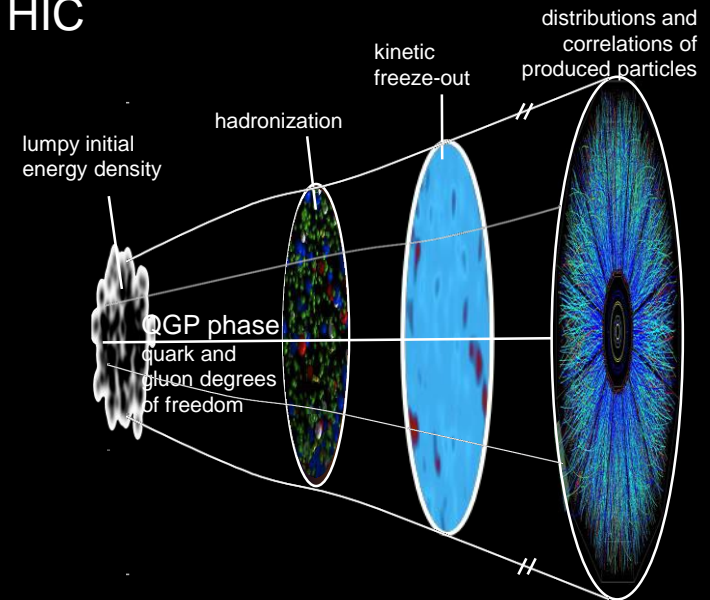
WMAP
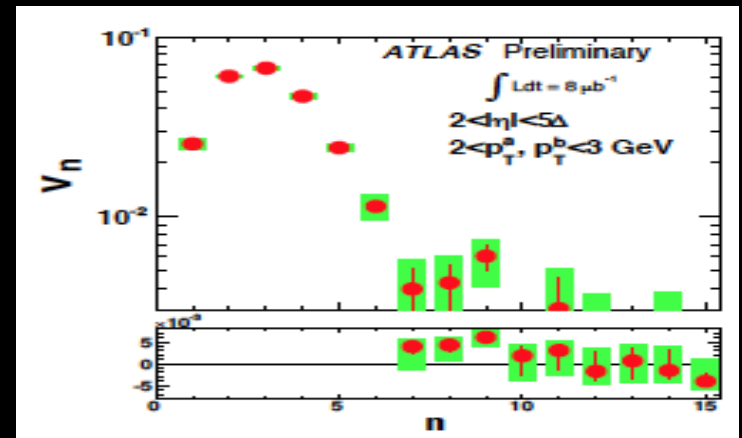
Anisotropic flow driven by initial geometry: relativistic viscous hydrodynamics
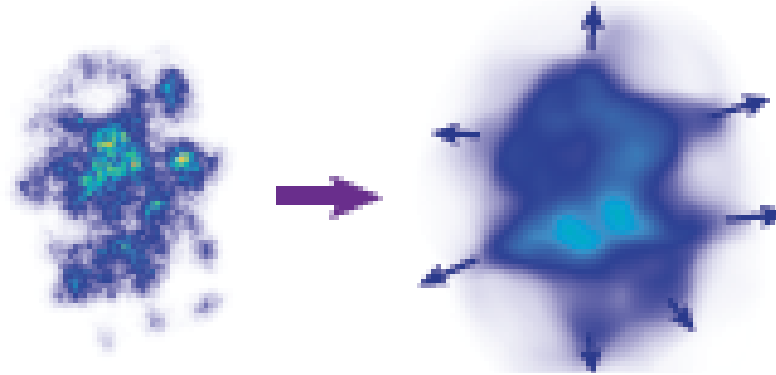
viscous to ideal results
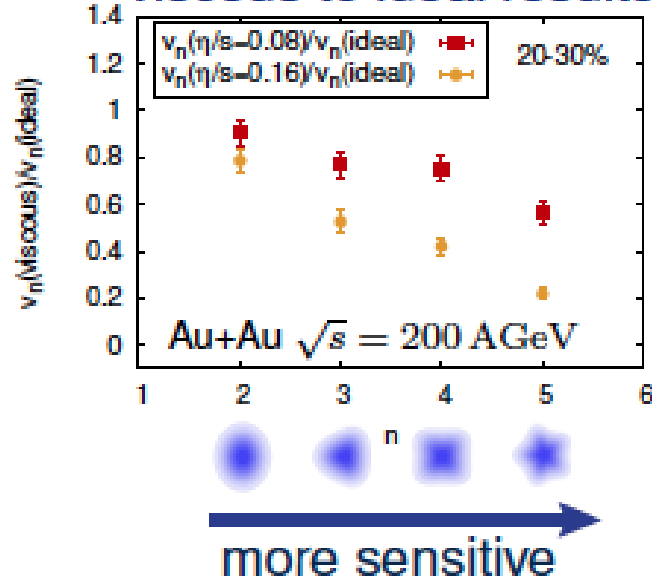
smoother to more granular results
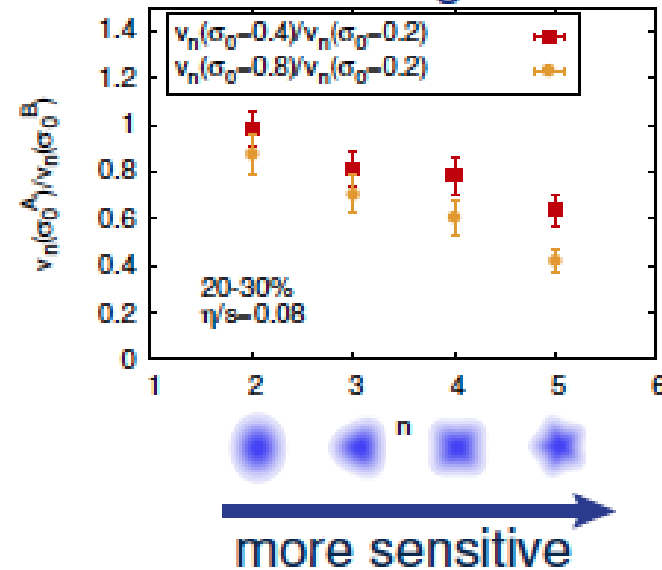
B. Schenke, S. Jeon, C. Gale, Phys.Rev.C85, 024901 (2012)

**High harmonics of angular distribution very sensitive to viscosity
... *and to details of the initial state***

VISCOUS FLOW AT LHC
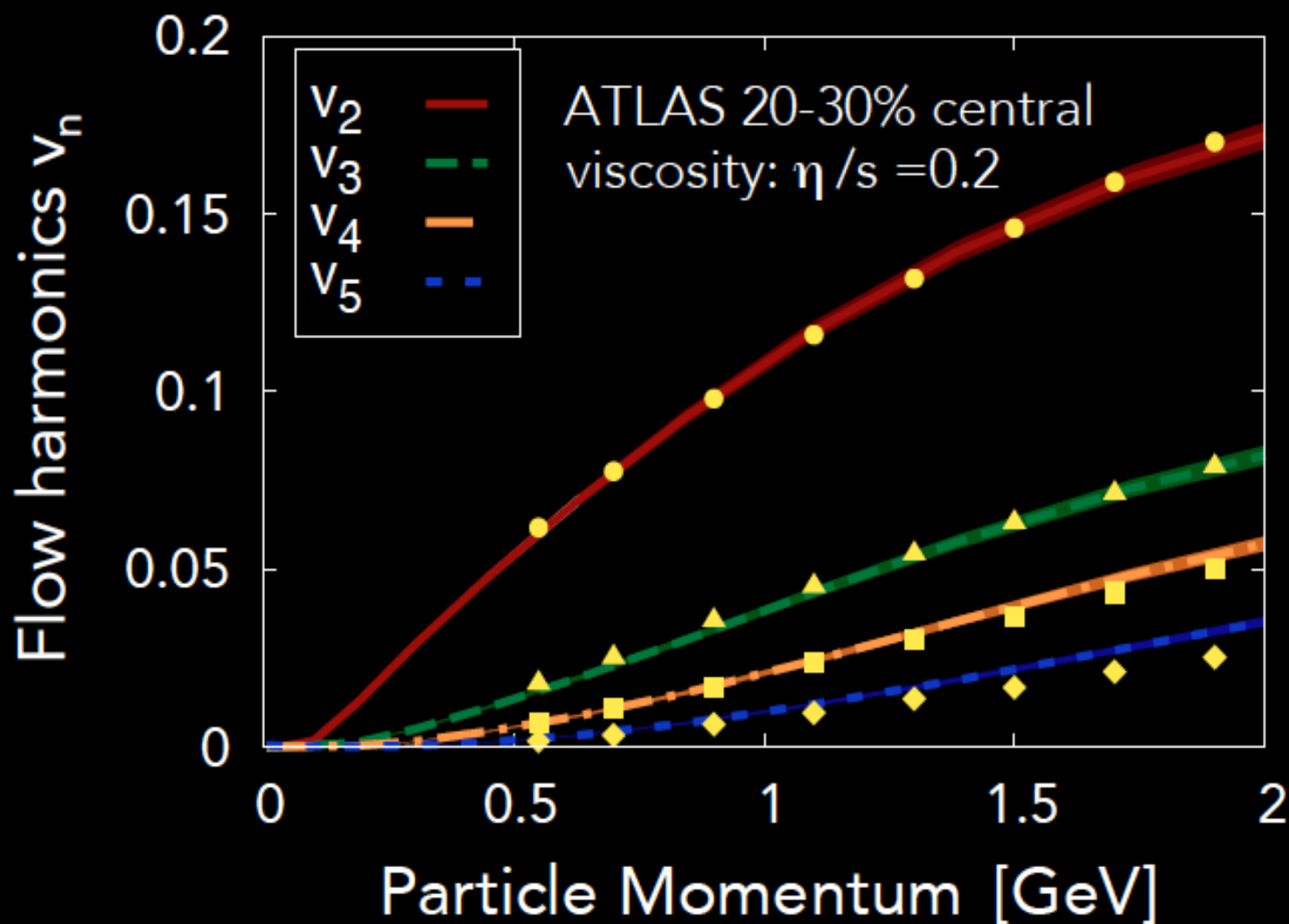
GEOMETRY AND FLUCTUATIONS DRIVE FLOW
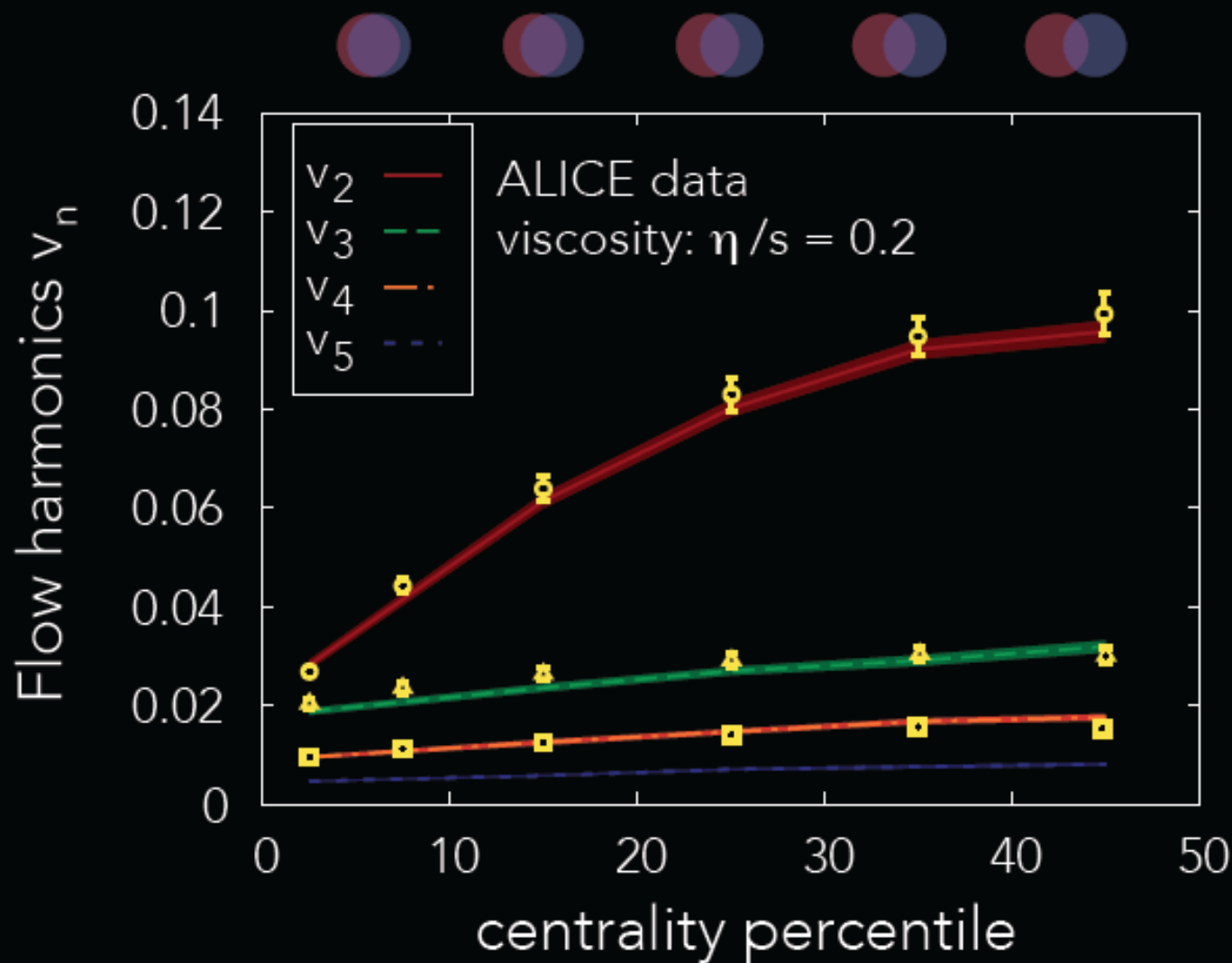
The temperature dependence of η/s

VISCOSITY AT RHIC AND LHC

RHIC
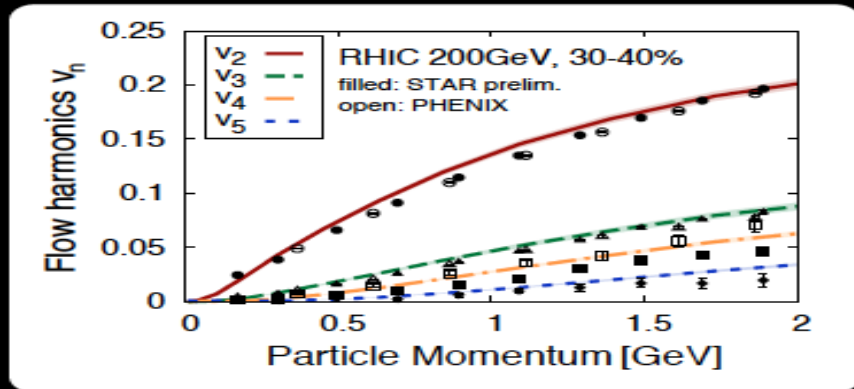
LHC ~14 x higher energy
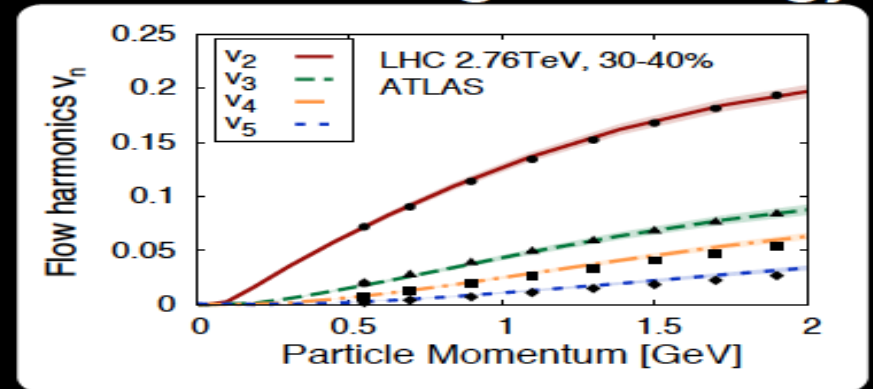
RHIC viscosity η/s = 0.12   LHC viscosity η/s = 0.2

Hints at increasing viscosity η/s with increasing temperature
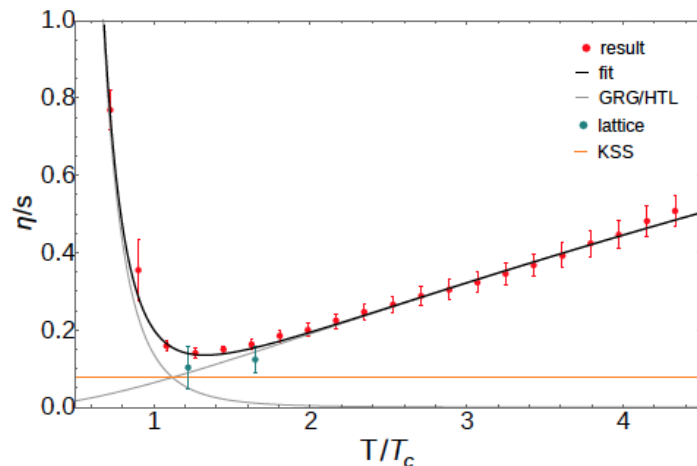
Data ⟷ Theory comparisons of temperature dependence of transport coefficients provides insight into the microscopic strongly coupled dynamics of the QGP

Christiansen,Haas,Pawlowski,Strodthoff,PRL115 (2015)112002

From the violence of a nuclear collision ...to the calm of a quark-gluon fluid

Initial state:
Far from equilibrium

Non-equilibrium
dynamics

Final state:
Thermal equilibrium

How is thermal equilibrium achieved?

Standard model of heavy ion collisions

Color Glass Condensates — **Wavefunctions overlap** — **Glasma** — **sQGP - perfect fluid** — **Hadron Gas**

t
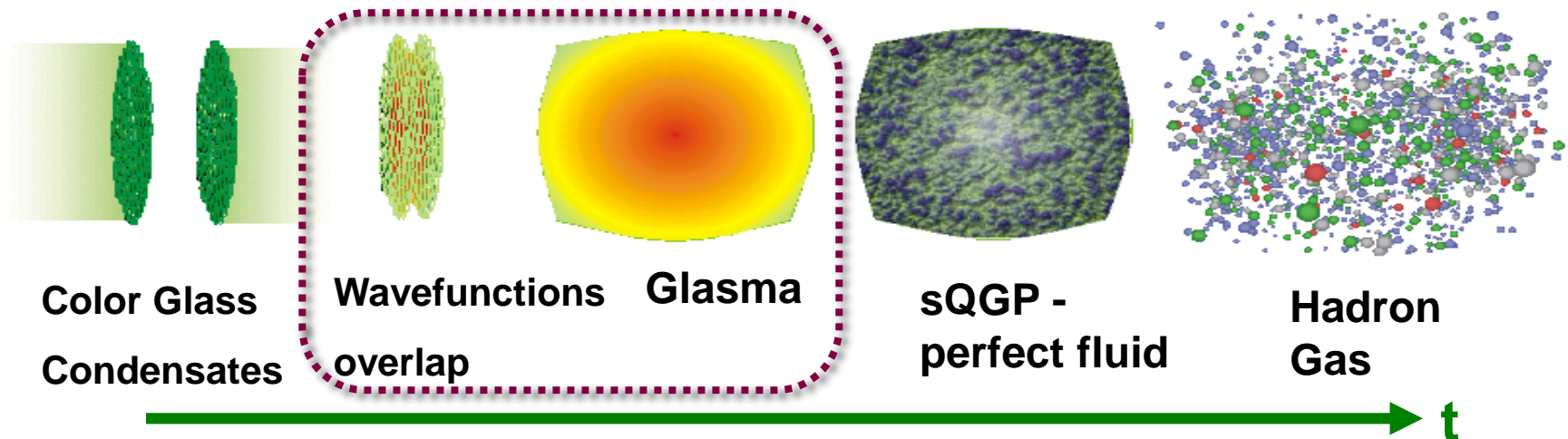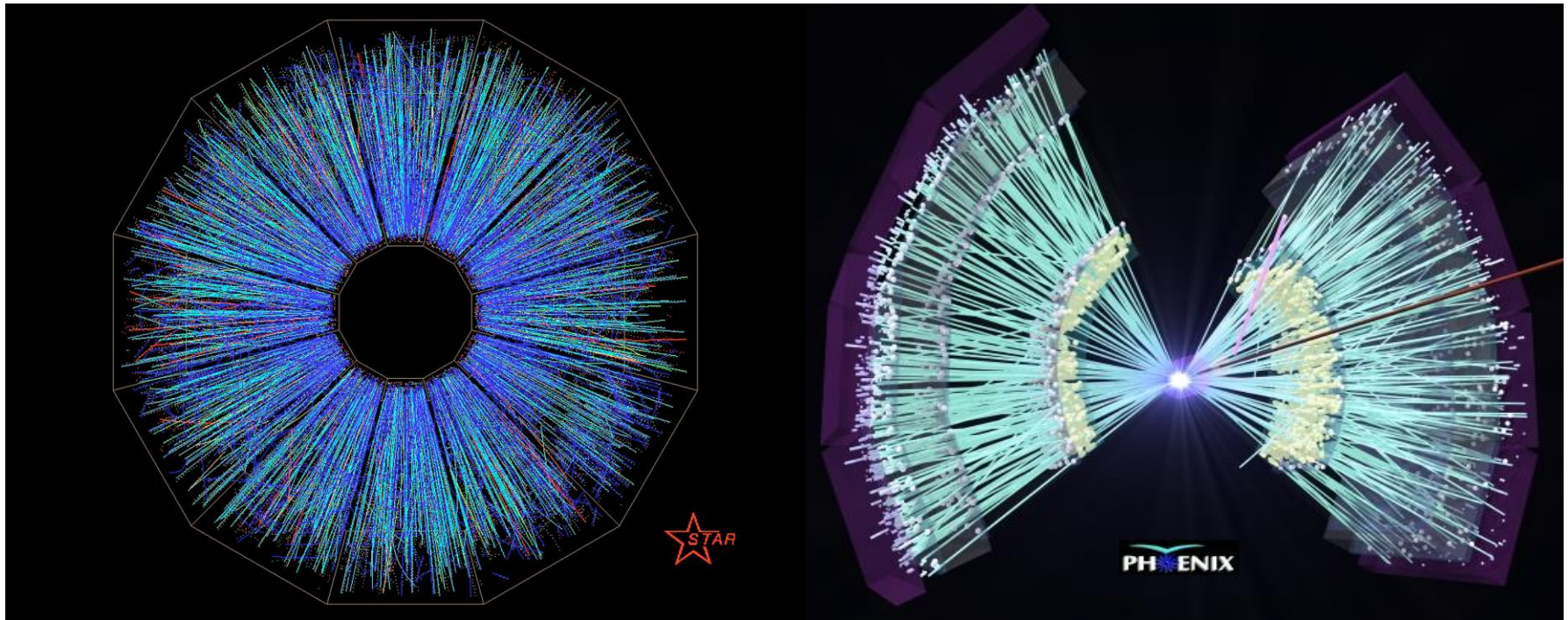
RG evolution for 2 nuclei

Gelis,Lappi,RV (2008)

Log divergent contributions crossing nucleus 1 or 2:
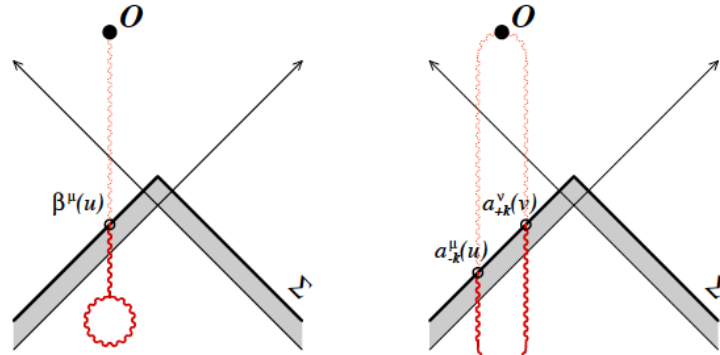
$$\mathcal{O}_{\text{NLO}} = \left[\frac{1}{2} \int_{\vec{u},\vec{v}} \mathcal{G}(\vec{u},\vec{v}) \, \mathcal{T}_u \mathcal{T}_v + \int_{\vec{u}} \beta(\vec{u}) \, \mathcal{T}_u \right] \mathcal{O}_{\text{LO}}$$

$\mathcal{G}(\vec{u},\vec{v})$ **and** $\beta(\vec{u})$ **can be computed on the initial Cauchy surface**

$$\mathcal{T}_u = \frac{\delta}{\delta A(\vec{u})}$$ **linear operator on initial surface**

Contributions across both nuclei are finite-no log divergences => factorization

$$\mathcal{O}_{\text{NLO}} = \left[\ln\left(\frac{\Lambda^+}{p^+}\right) \mathcal{H}_1 + \ln\left(\frac{\Lambda^-}{p^-}\right) \mathcal{H}_2 \right] \mathcal{O}_{\text{LO}}$$
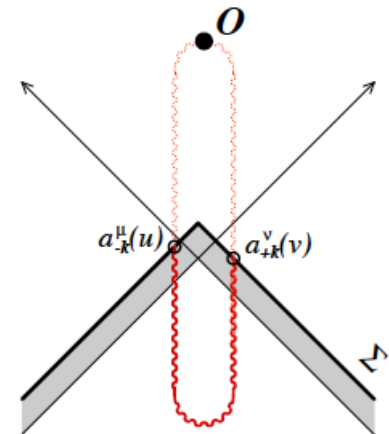
Single inclusive gluon production

Initial configuration of sources for nucleus 1

Gelis,Lappi,RV
arXiv:0804.2630 [hep-ph];
arXiv:0807.1306 [hep-ph]
arXiv:0810.4829 [hep-ph]

BK/JIMWLK evolution for nucleus 1

$$\frac{dN_{\rm LO}}{d^3 p}$$

Computed from Yang-Mills Glasma gluon fields

Krasnitz, Nara, RV; Lappi

Identical evol. for nucleus 2
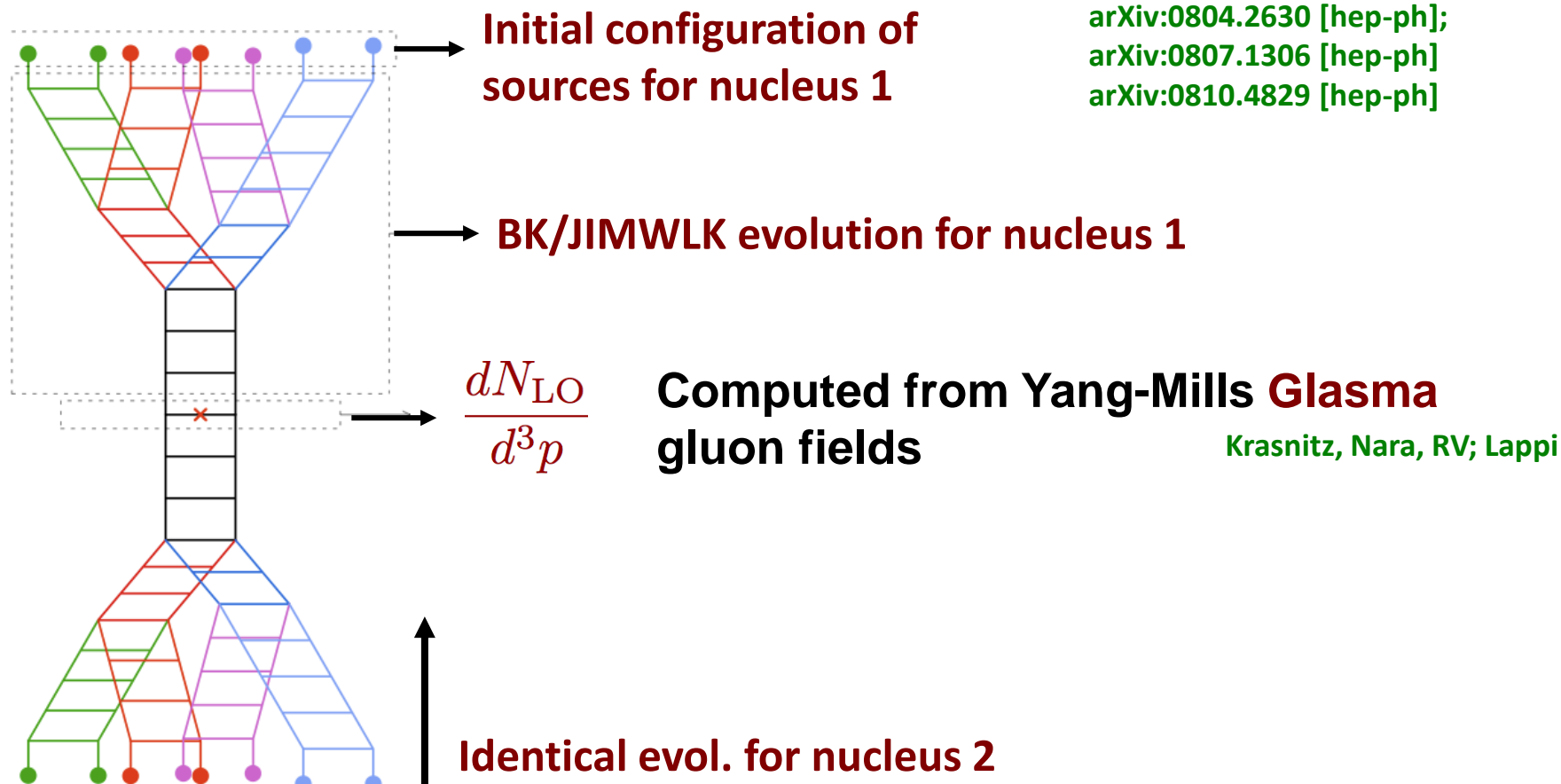
◆ **Full JIMWLK+YM evolution feasible** **Lappi, PLB 703 (2011)209**

◆ **In practice: approximations of varying rigor**

Heavy ion phenomenology in weak coupling

Collisions of lumpy gluon ``shock" waves
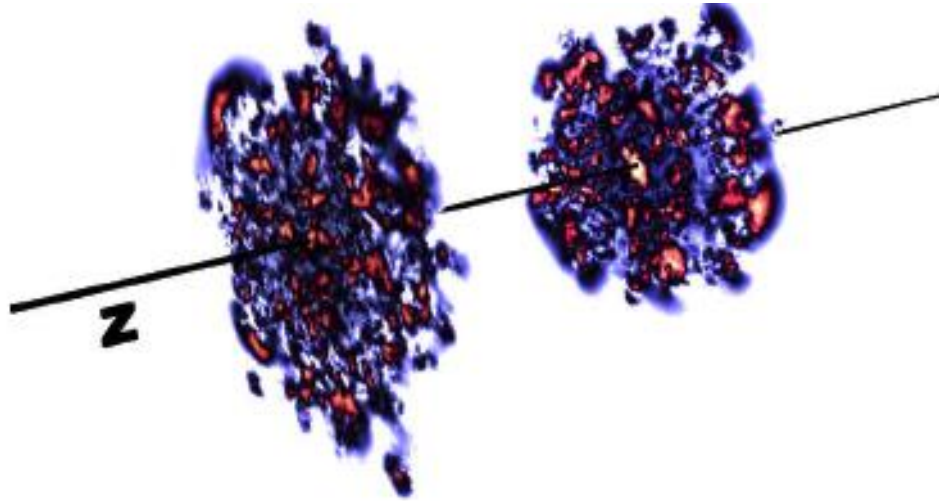
Systematic framework: Quantum field theory in presence of strong time dependent color sources.

For inclusive quantities, *initial value problem in the Schwinger-Keldysh formalism.*

In QCD, important and subtle issues: factorization, renormalization, universality

Gelis, Venugopalan (2006)
Gelis, Lappi, Venugopalan (2008,2009)
Jeon (2014)

Heavy ion phenomenology in weak coupling

Collisions of lumpy gluon ``shock" waves
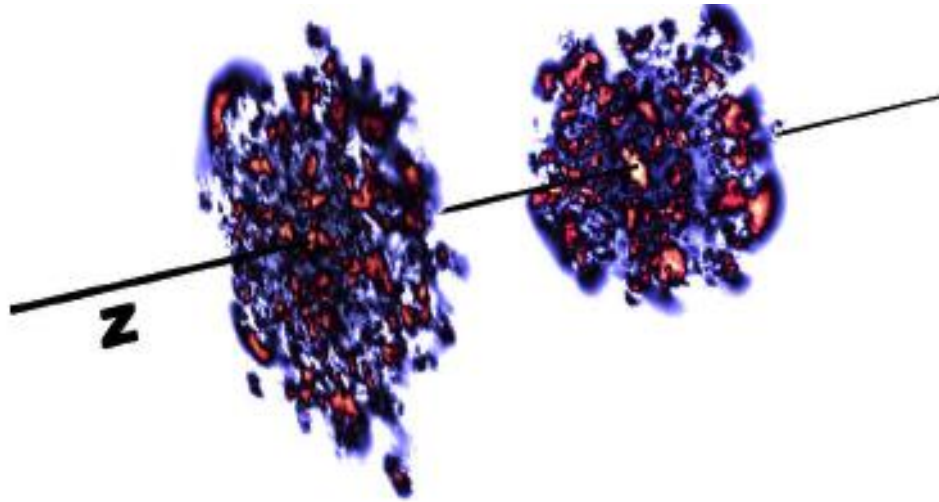
Leading order solution: Solution of QCD Yang-Mills equations

$$D_\mu F^{\mu\nu,a} = \delta^{\nu+}\rho_A^a(x_\perp)\delta(x^-) + \delta^{\nu-}\rho_B^a(x_\perp)\delta(x^+)$$

$$x^\pm = t \pm z \qquad\qquad F^{\mu\nu,a} = \partial_\mu A^{\nu,a} - \partial_\nu A^{\nu,a} + gf^{abc}A^{\mu,b}A^{\nu,c}$$

**The gauge field solutions are boost invariant;
independent of spacetime rapidity η=0.5 × Ln(x⁺/x⁻)**

Imaging the force fields of QCD
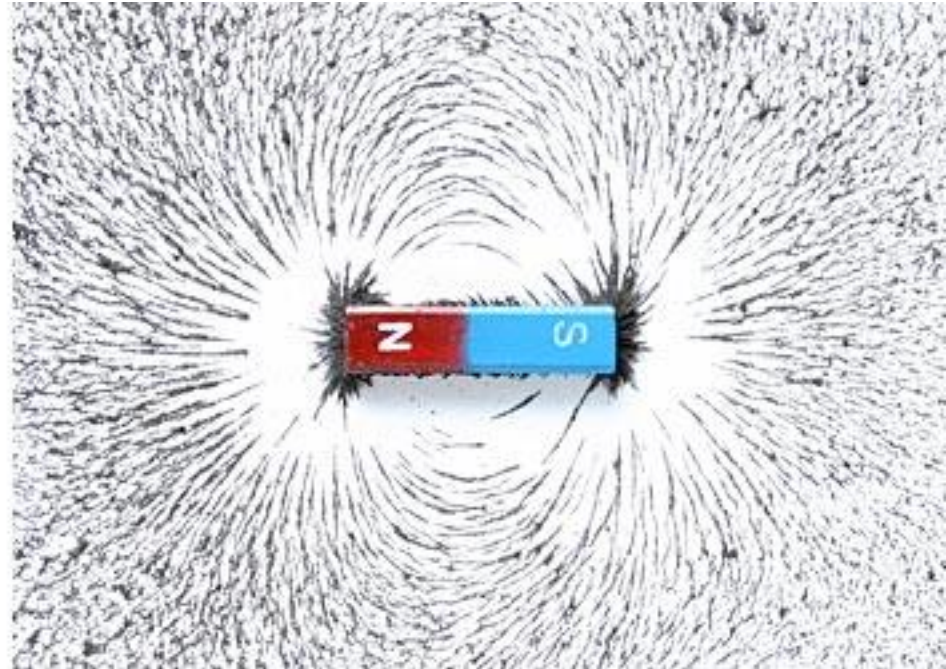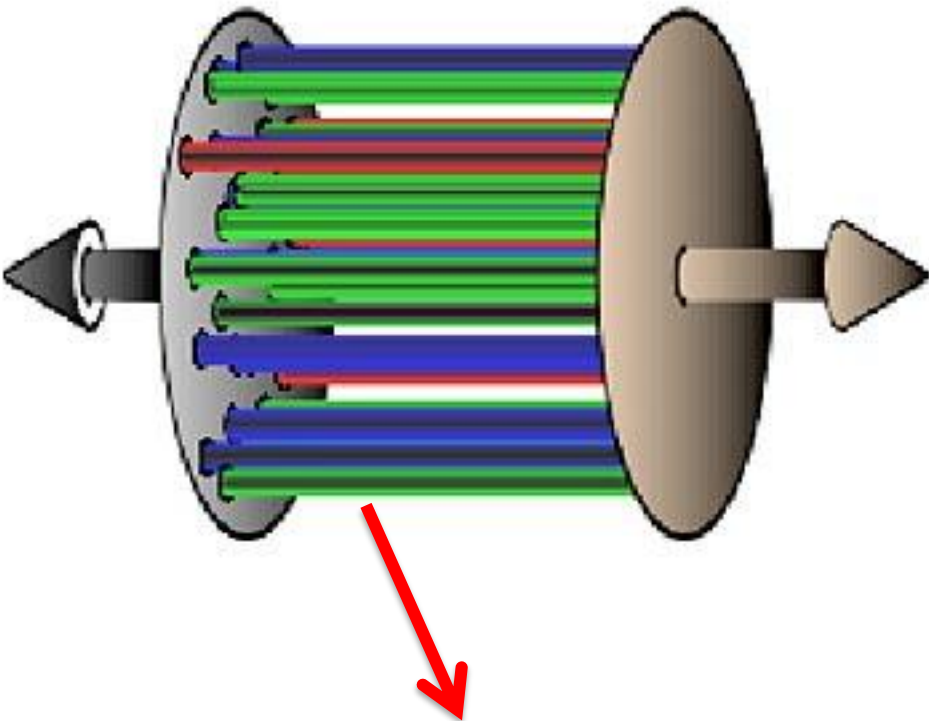
Solutions of QCD Yang-Mills equations demonstrate that each of these color "flux tubes" stretching out in rapidity is of transverse size $1/Q_S \ll 1$ fm

T$^{\mu\nu}$ from Yang-Mills dynamics
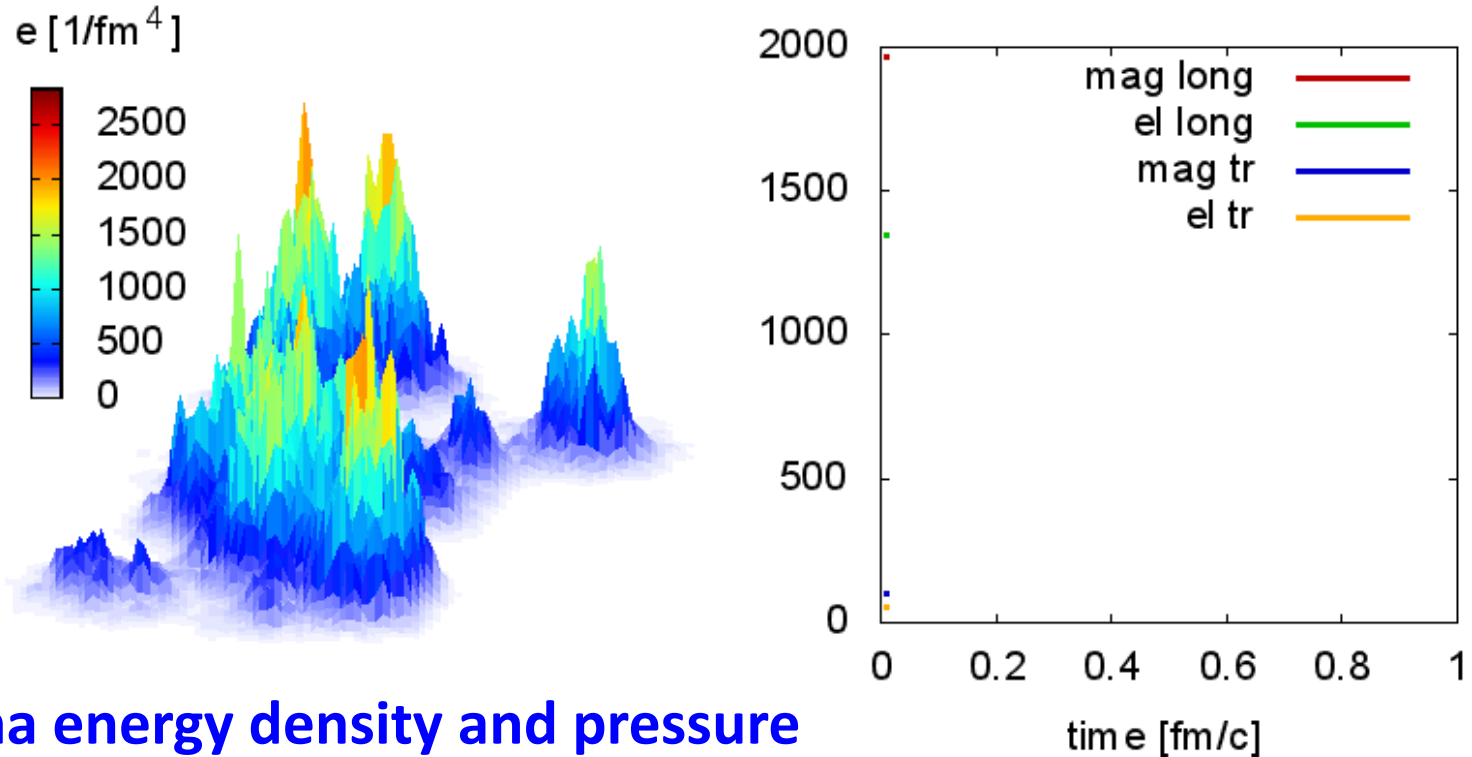
Glasma energy density and pressure

$$T_{\mu\nu}(\tau = 0) = \frac{1}{2}(B_z^2 + E_z^2) \times \text{diag}(1, 1, 1, -1)$$

Initial longitudinal pressure is negative:
Goes to P$_L$ =0 from below with time evolution

The Glasma: colliding gluon shock waves
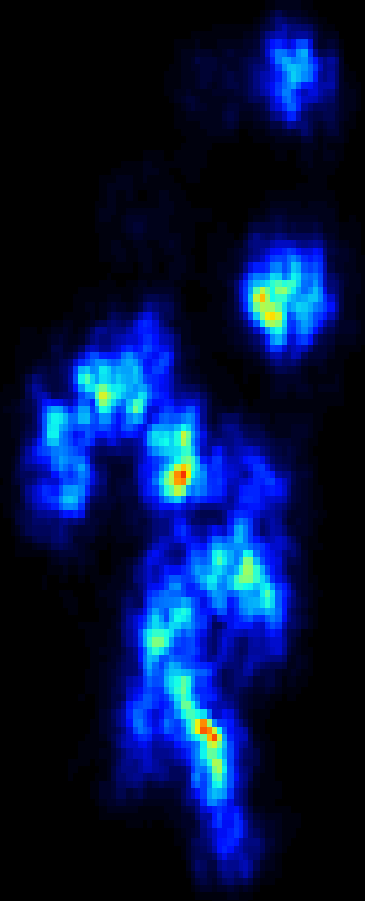
Glasma color fields
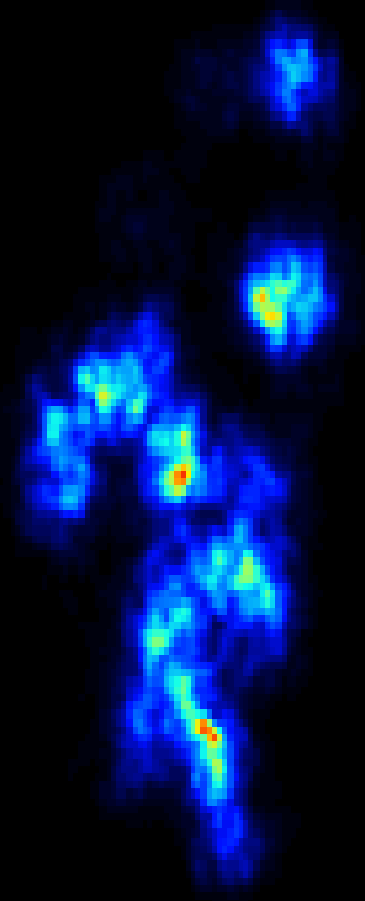
**Glasma color fields matched
to viscous hydrodynamics**

Krasnitz,Venugopalan, Nucl.Phys.B557 (1999)

Lappi, Phys.Rev. C67 (2003)

Schenke,Tribedy,Venugopalan,PRL108 (2012)

t = 0.0 fm/c

Note: 1 fm/c = 3*10^{-24} seconds!

The Glasma at NLO: plasma instabilities

Romatschke,Venugopalan (2006)
Dusling,Gelis,Venugopalan (2011)
Gelis, Epelbaum (2013)

At LO: boost invariant gauge fields $A_{cl}^{\mu,a}(x_T,\tau) \sim 1/g$

NLO: $A^{\mu,a}(xT,\tau,\eta) = A_{cl}^{\mu,a}(x_T,\tau) + a^{\mu,a}(\eta)$

$a^{\mu,a}(\eta) = O(1)$

increasing seed size
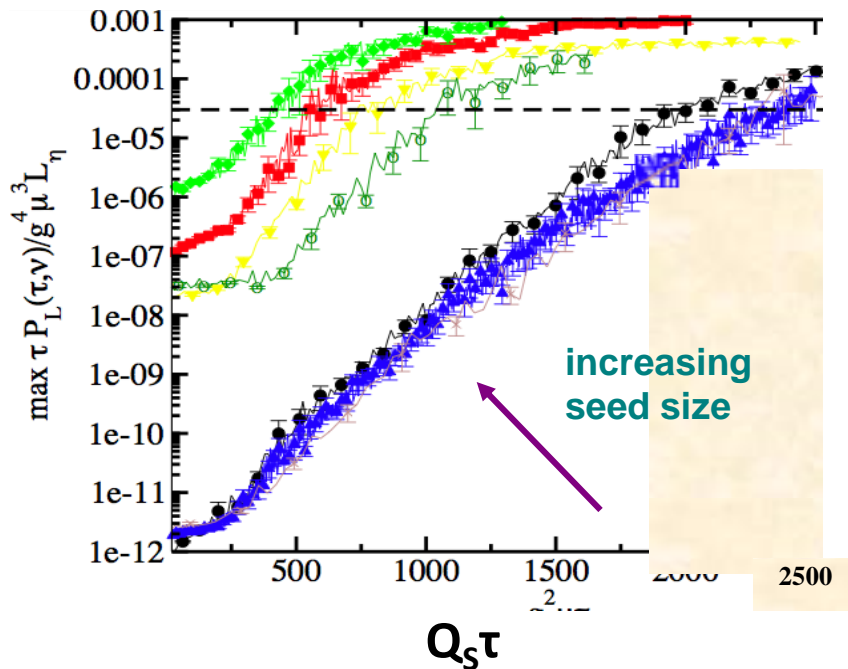
➤ **Small fluctuations grow exponentially as ~**

$$e^{\sqrt{Q_S\tau}}$$

➤ **Same order of classical field at**

$$\tau = \frac{1}{Q_S}\ln^2\frac{1}{\alpha_S}$$

➤ **Resum such contributions to all orders**

$$(g\,e^{\sqrt{Q_S\tau}})^n$$

$$T_{\text{resum}}^{\mu\nu} = \int_{\tau=0^+}[da]\,F_{\text{init.}}[a]\,T_{\text{LO}}[A_{cl}+a]$$

Initial conditions in the overpopulated Glasma

Choose for the initial classical-statistic ensemble of gauge fields

$$A_\nu(\tau,\eta,\boldsymbol{x}_\perp) = \sum_\lambda \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d\nu}{2\pi} \sqrt{f_{\boldsymbol{k}_\perp \nu} + \frac{1}{2}} \left[c^{(\lambda)\boldsymbol{k}_\perp \nu} \xi_\mu^{(\lambda)\boldsymbol{k}_\perp \nu +}(\tau) e^{i\boldsymbol{k}_\perp \boldsymbol{x}_\perp} e^{i\nu\eta} + c^{*(\lambda)\boldsymbol{k}_\perp \nu} \xi_\mu^{(\lambda)\boldsymbol{k}_\perp \nu +*}(\tau) e^{-i\boldsymbol{k}_\perp \boldsymbol{x}_\perp} e^{-i\nu\eta} \right]$$

Stochastic random variables

$$\langle c^{(\lambda)\boldsymbol{k}_\perp \nu} c^{(\lambda')\boldsymbol{k'}_\perp \nu'} \rangle = 0,$$

$$\langle c^{(\lambda)\boldsymbol{k}_\perp \nu} c^{*(\lambda')\boldsymbol{k'}_\perp \nu'} \rangle = (2\pi)^3 \delta^{\lambda\lambda'} \delta(\boldsymbol{k} - \boldsymbol{k'}) \delta(\nu - \nu')$$

$$\langle c^{*(\lambda)\boldsymbol{k}_\perp \nu} c^{*(\lambda')\boldsymbol{k'}_\perp \nu'} \rangle = 0.$$

Polarization vectors ξ expressed in terms of Hankel functions in Fock-Schwinger gauge $A^\tau = 0$

Occupation #

$$f(\mathbf{P}_\perp, \mathbf{P}_z, \tau) = \frac{\tau^2}{N_g V_\perp L_\eta} \sum_{a=1}^{N_c^2 - 1} \sum_{\lambda=1,2}$$

$$\left\langle \left| g^{\mu\nu} \left[\left(\xi_\mu^{(\lambda)\mathbf{P}_\perp \nu +}(\tau) \right)^* \overleftrightarrow{\partial_\tau} A_\nu^a(\tau, \mathbf{P}_\perp, \nu) \right] \right|^2 \right\rangle_{\text{Coul.gauge}}$$

$$f(p_\perp, p_z, t_0) = \frac{n_0}{\alpha_S} \Theta\left(Q - \sqrt{p_\perp^2 + (\xi_0 p_z)^2} \right)$$

Controls "prolateness" or "oblateness" of initial momentum distribution

Temporal evolution in the overpopulated QGP

**Solve Hamilton's equation for 3+1-D SU(2) gauge theory
in Fock-Schwinger gauge**
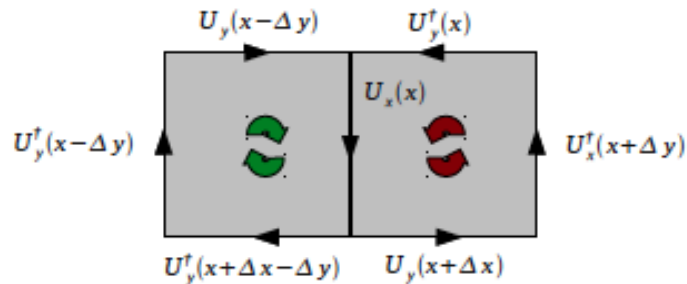
**Fix residual gauge freedom
imposing Coloumb gauge at
each readout time**

$$\partial_i A_i + t^{-2}\partial_\eta A_\eta = 0$$
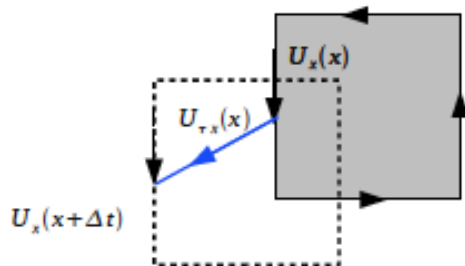
◆ **Largest classical-statistical numerical simulations of
expanding Yang-Mills to date: $256^2 \times 4096$ lattices**

From Glasma to Quark Gluon Plasma

Glasma fields produced in the shock wave collision are unstable to quantum fluctuations…

This instability leads to rapid overpopulation of all momentum modes
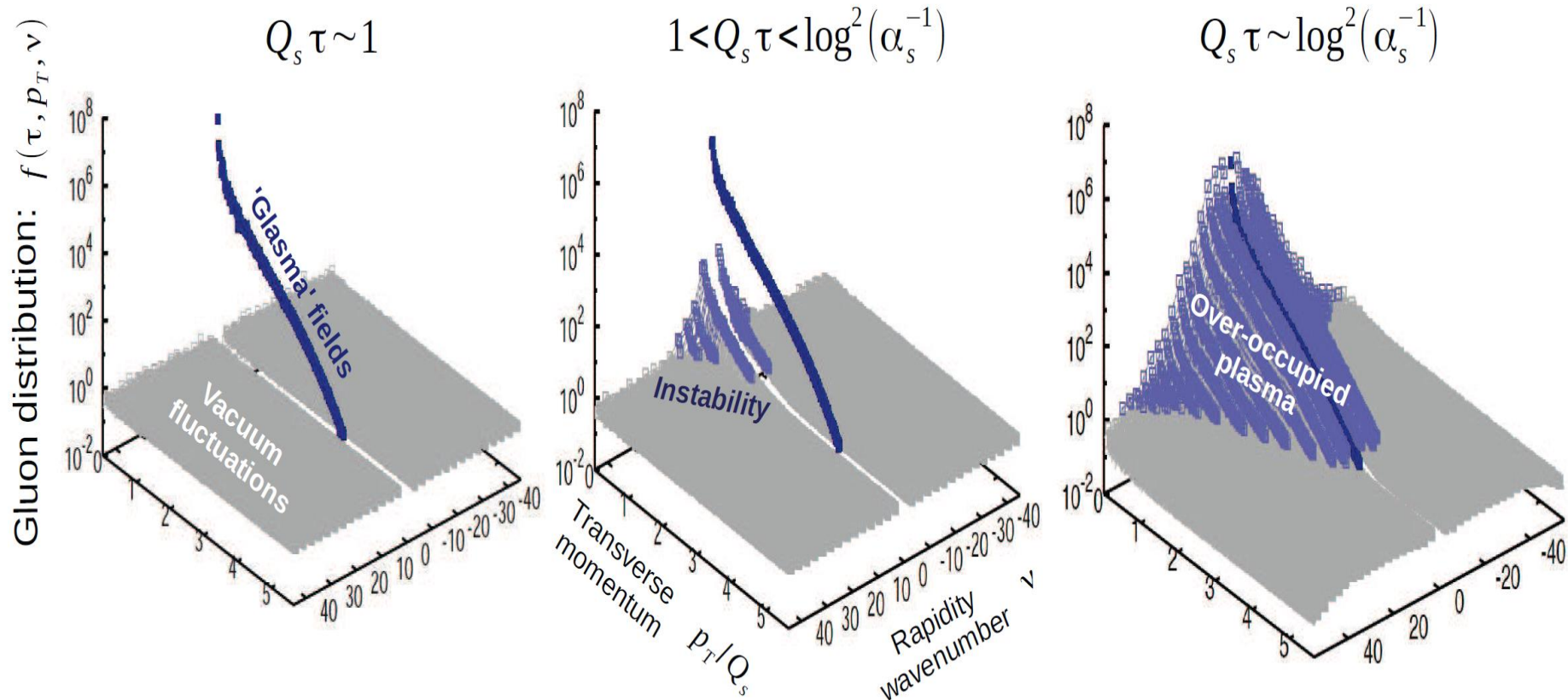
Classical-statistical QFT numerical lattice simulations of gluon fields exploding into the vacuum

Berges,Schenke,Schlichting,RV, NPA 931 (2014) 348

From Glasma to Quark Gluon Plasma

Glasma + vacuum fluctuations

$$\tau = \frac{1}{Q_S}$$

Plasma Instabilities

$$\tau = \frac{1}{Q_S} \ln^2 \frac{1}{\alpha_S}$$

f(p)

Initial over-occupation

???
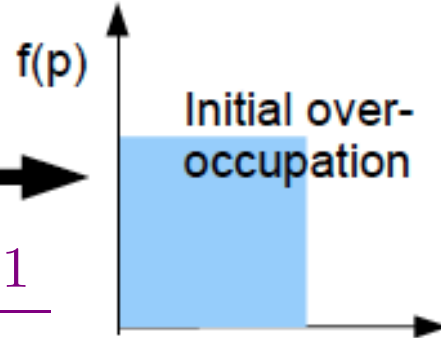
$$\tau \gg \frac{1}{Q_S} \ln^2 \frac{1}{\alpha_S}$$

From Glasma to Quark Gluon Plasma

- There is a natural *competition* between *interactions* and the *longitudinal expansion* which renders the system *anisotropic* on large time scales

Longitudinal Expansion:
- Red-shift of longitudinal momenta p_z
  $\rightarrow$ increase of anisotropy
- Dilution of the system

Interactions:
- Isotropize the system

Overoccupied expanding Glasma: particles or fields?

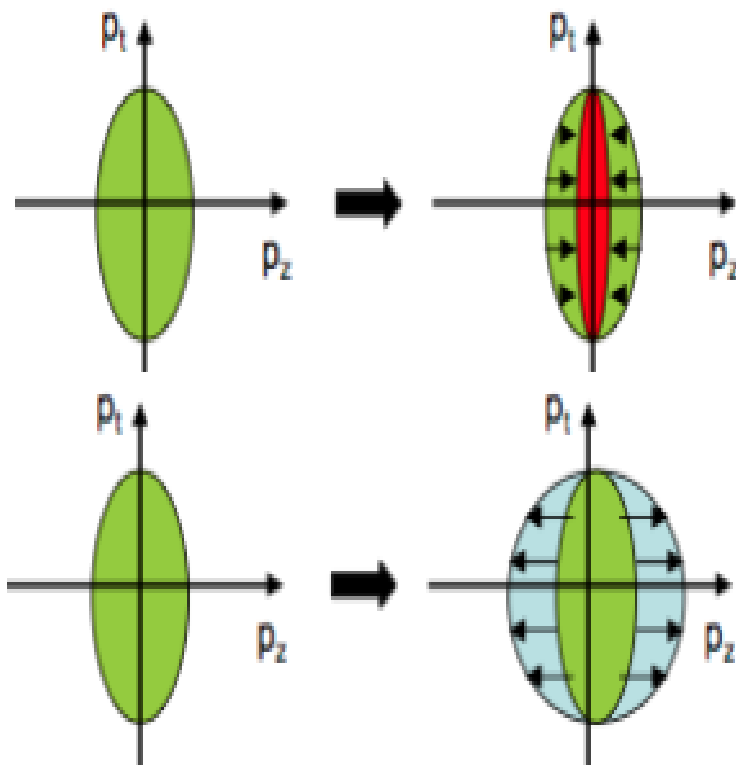For **1 < f < 1/α_S** a dual description is feasible either in terms of kinetic theory or classical-statistical dynamics …

Mueller,Son (2002)
Jeon (2005)

Nonthermal fixed point

Properties independent of initial conditions

Far from equilibrium

Universality

Self-similar evolution characterized by universal scaling exponents

Initial conditions

Close to equilibrium

Thermal equilibrium
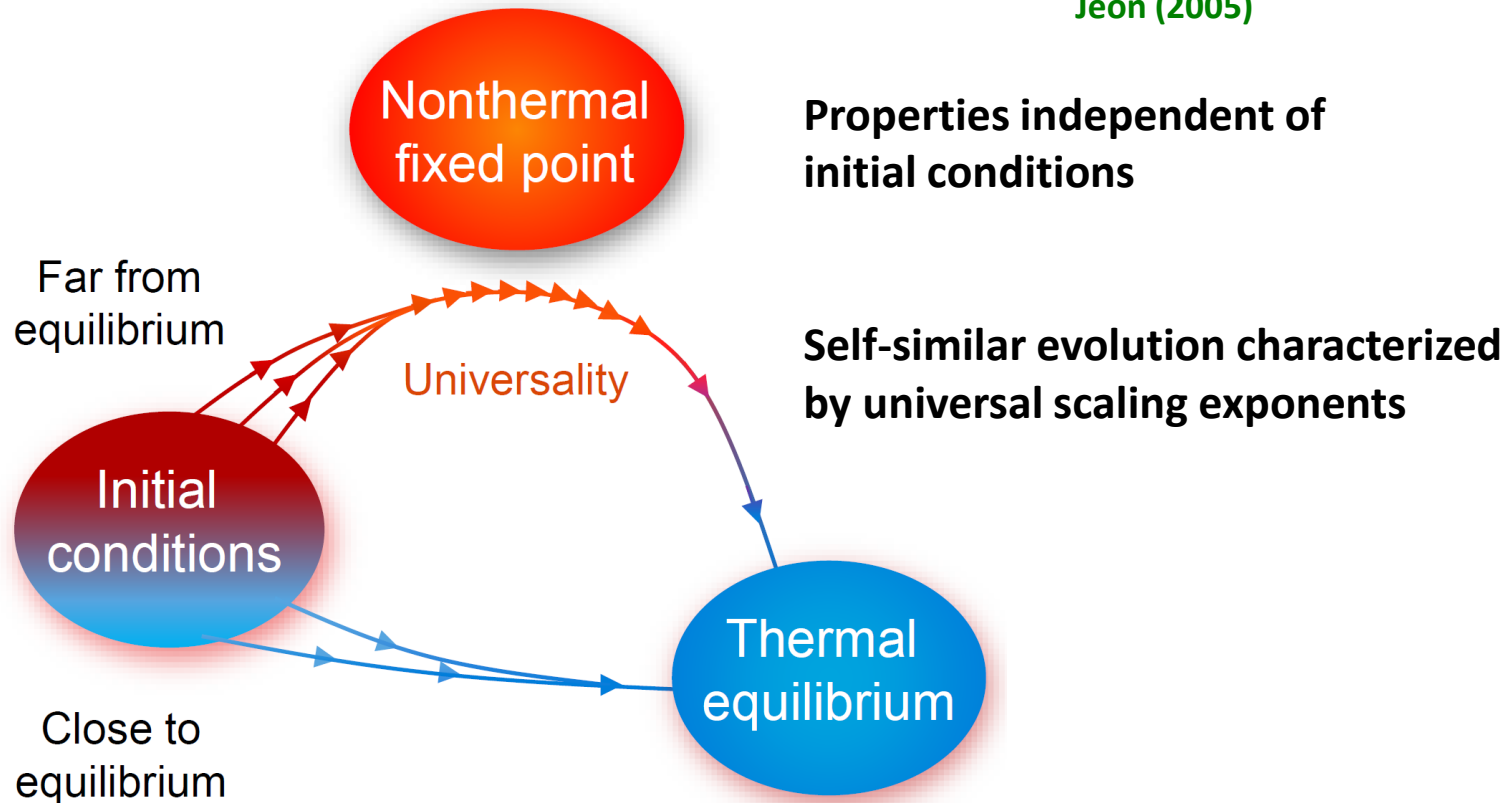
$$f(p_T, p_z, \tau) = (Q\tau)^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Non-thermal fixed point in overpopulated QGP

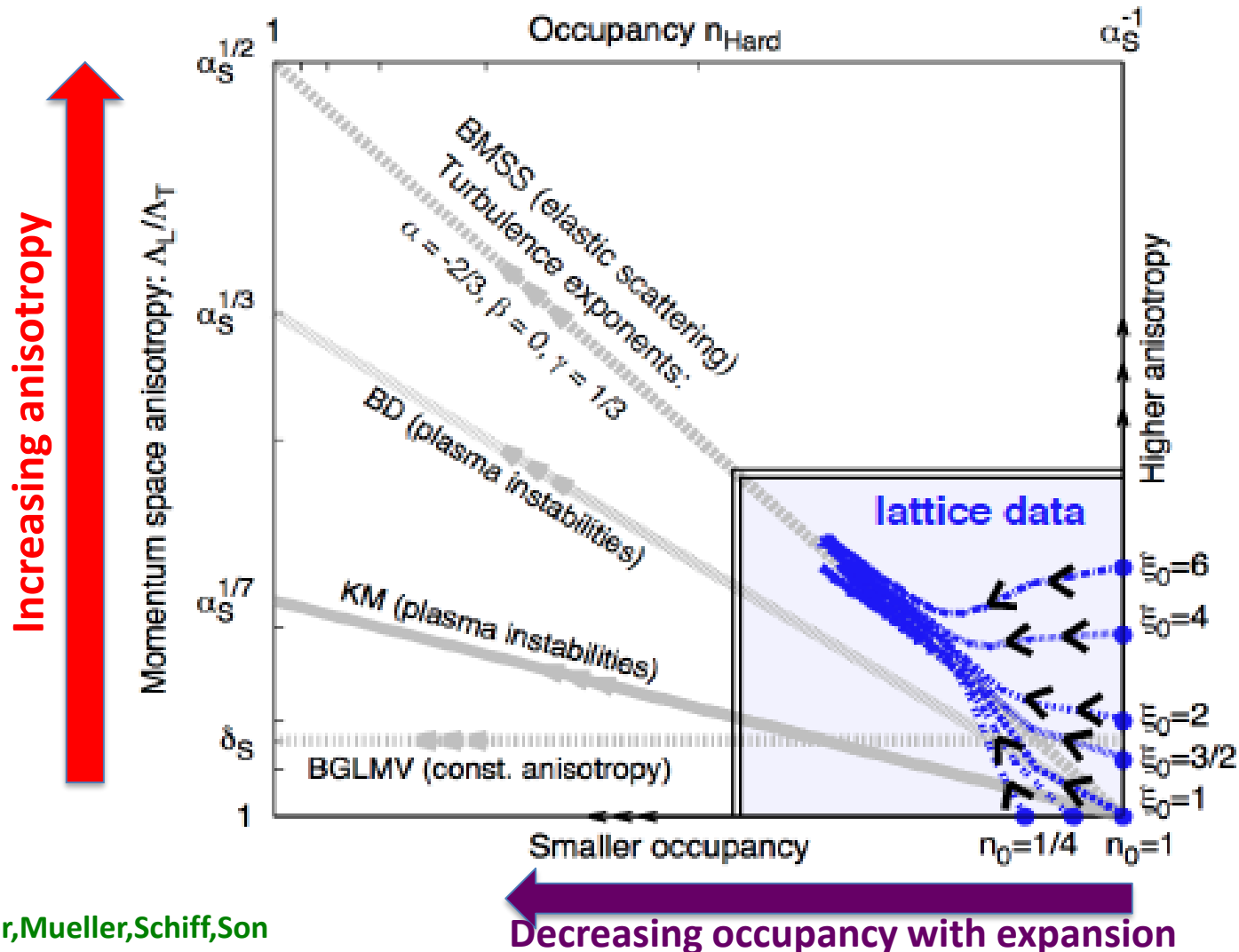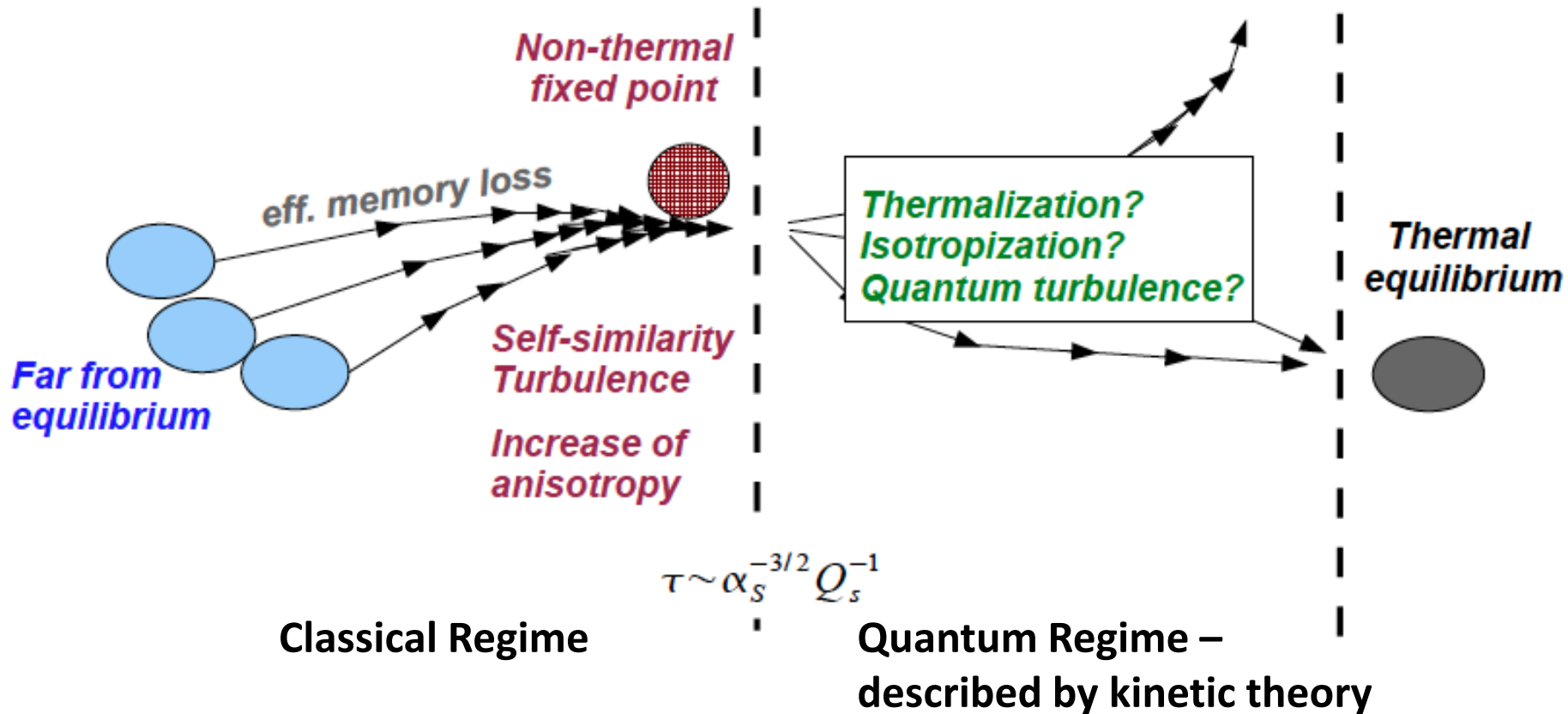BMSS: Baier,Mueller,Schiff,Son
BD: Bodeker
KM: Kurkela, Moore
BGLMV: Blaizot,Gelis,Liao,McLerran,Venugopalan

Quo vadis, thermal QGP?

Non-thermal fixed point

eff. memory loss

Far from equilibrium

Self-similarity Turbulence

Increase of anisotropy

$$\tau \sim \alpha_S^{-3/2} Q_s^{-1}$$

Thermalization?
Isotropization?
Quantum turbulence?

Thermal equilibrium

Classical Regime

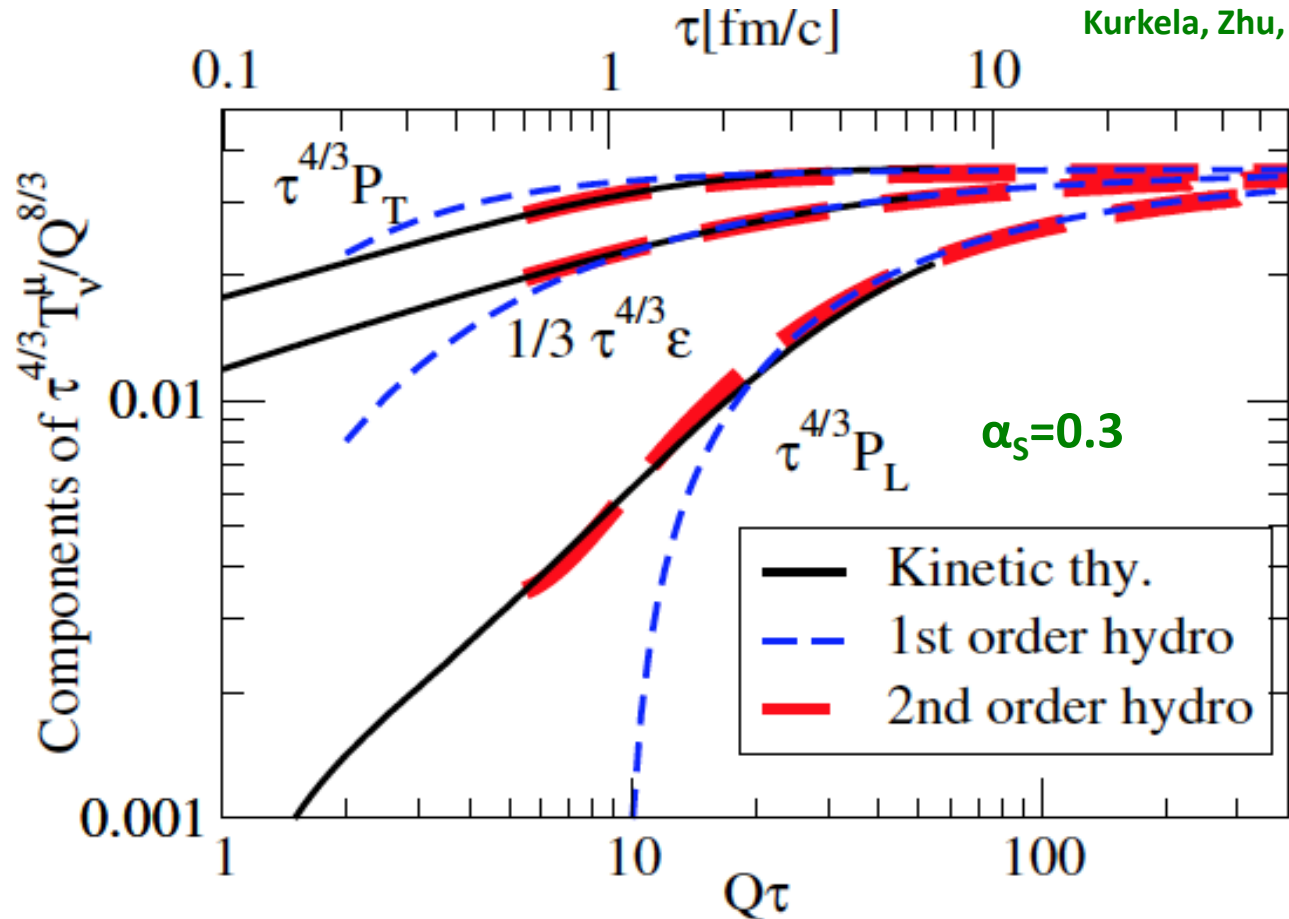**Quantum Regime – described by kinetic theory**

Thermalized soft bath of gluons for $\tau > \dfrac{1}{\alpha_S^{5/2}} \dfrac{1}{Q_S}$

Thermalization temperature of $T_i = \alpha_S^{2/5} Q_S$

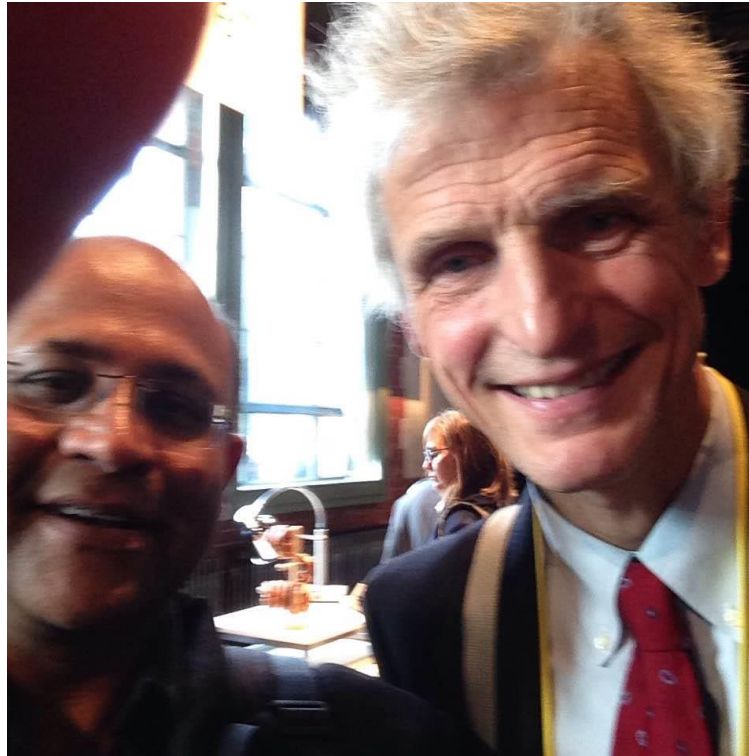# Matching the Glasma to viscous hydrodynamics

Kurkela, Zhu, arXiv: 1506.06647

Good matching of quantitative implementation of kinetic theory to hydrodynamics at times ~ 1 fm

... when extrapolated to realistic couplings (many caveats remain)

Universality: hotness is also cool

Wolfgang Ketterle, Nobel Prize (2001)

For the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates
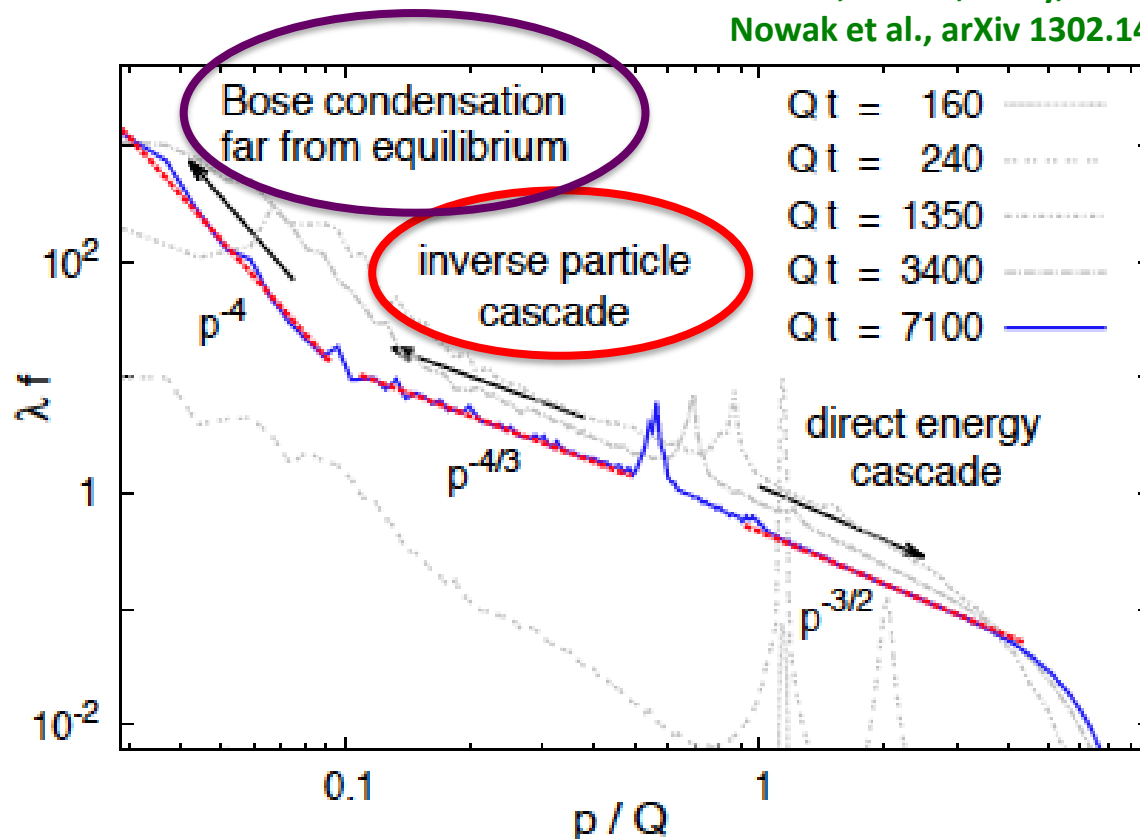
Non.Equil. dynamics of Overoccupied scalar fields

$$S = \int d\tau d^2 x_T d\eta \, \tau \left(\frac{g^{\mu\nu}}{2} (\partial_\mu \varphi_a)(\partial_\nu \varphi_a) - \frac{\lambda}{4!N} (\varphi_a \varphi_a)^2 \right)$$

In a non-relativistic limit, models cold atomic gases

Scheppach,Berges,Gasenzer, PRA 81 (2010) 033611
Nowak, Schole, Sexty, Gasenzer, PRA85 (2012) 043627
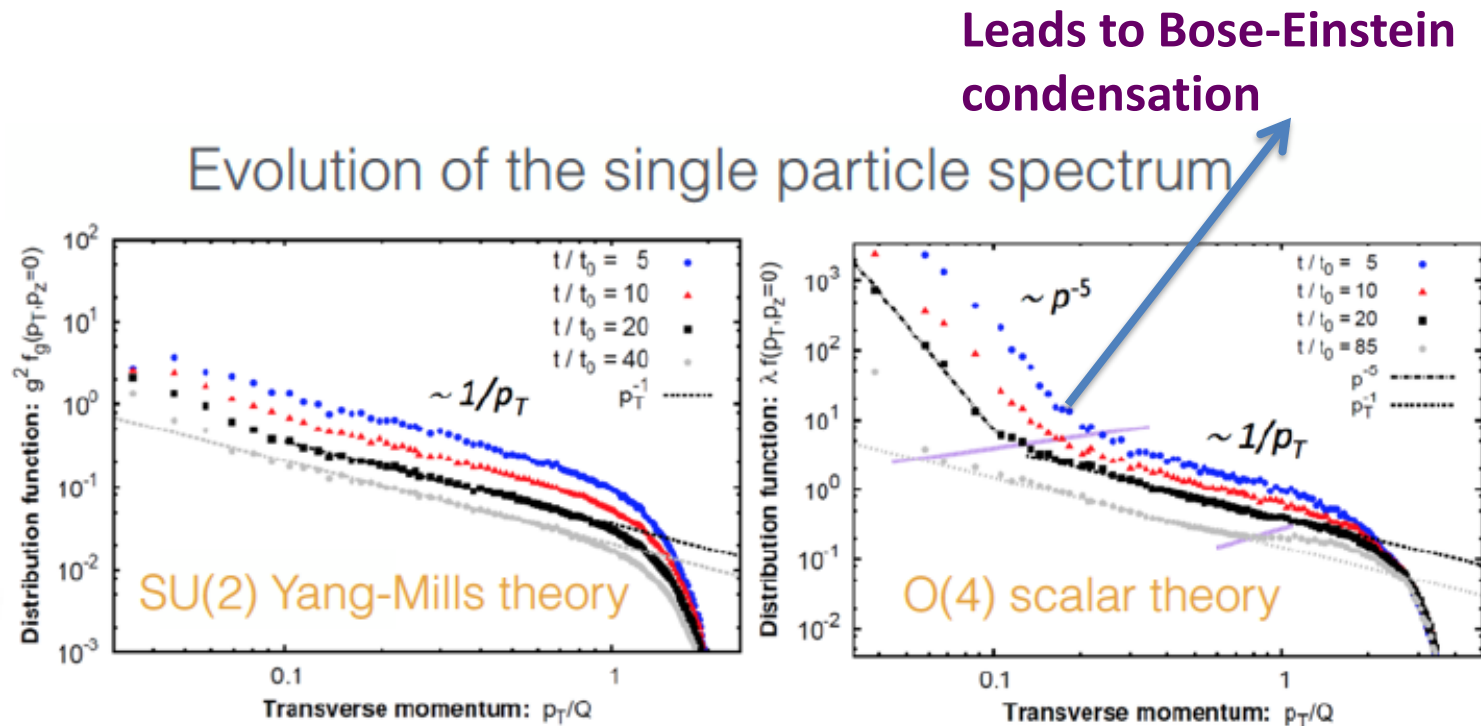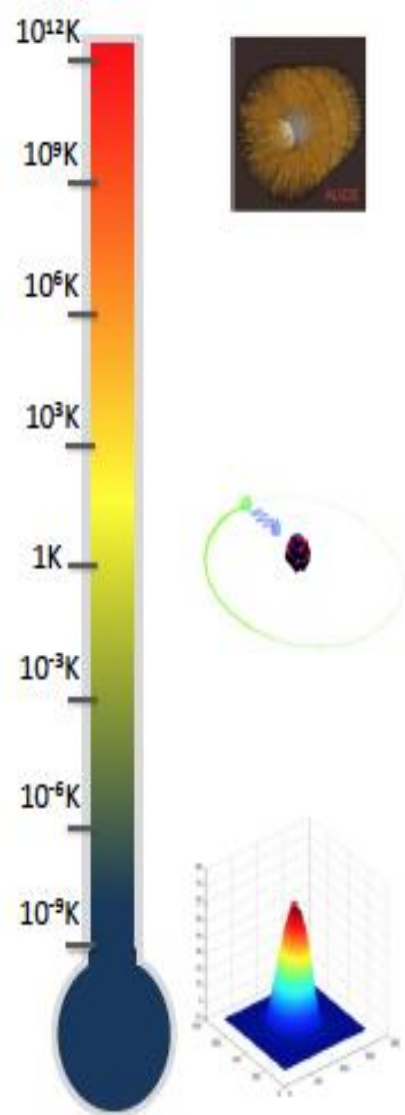Nowak et al., arXiv 1302.1448
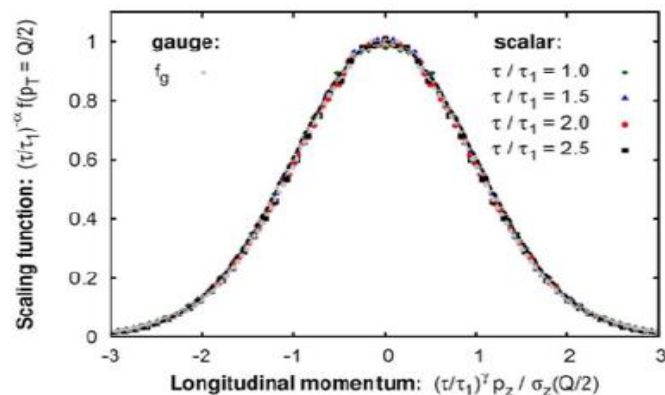
Berges, Sexty PRL 108 (2012) 161601
Berges,Boguslavski,Orioli, PRD 92, 025041 (2015)
Berges,Boguslavskii,Schlichting,Venugopalan, JHEP 1405 (2014) 054

Remarkable universality between world's hottest and coolest fluids

Leads to Bose-Einstein condensation

Evolution of the single particle spectrum
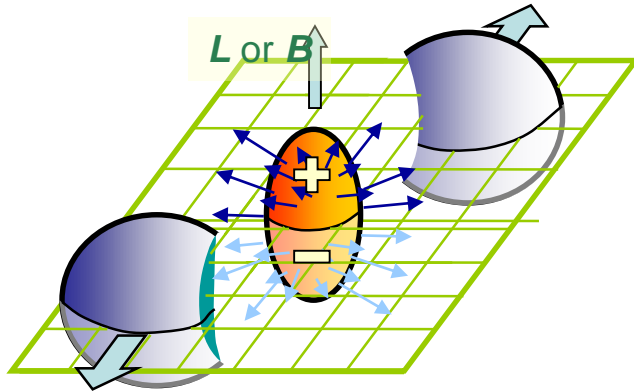

Normalized fixed-point distribution

Berges,Boguslavski,Schlichting,Venugopalan
PRL 114 (2015) 061601, Editor's suggestion
& PRD92 (2015) 096 006

Bonus track:Topological transitions in the Glasma
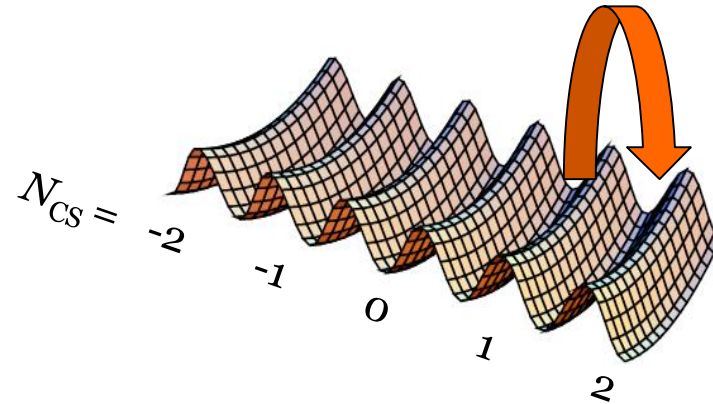## (The Chiral Magnetic Effect)

Topology in heavy-ion collisions:
The Chiral Magnetic Effect

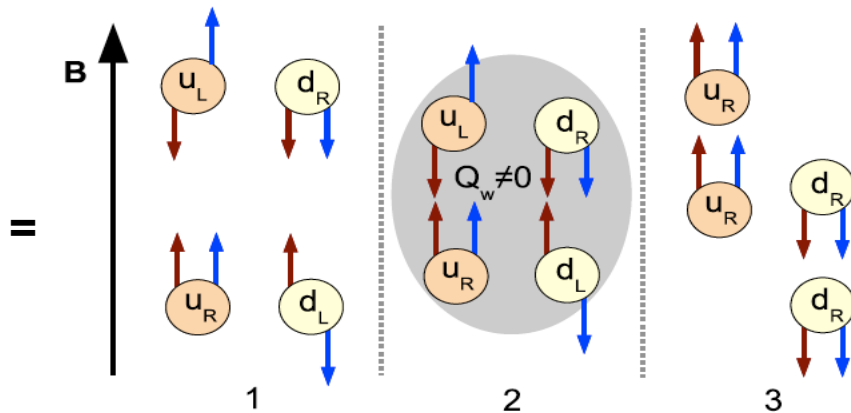**Kharzeev,McLerran,Warringa (2007)**

External (QED) magnetic field
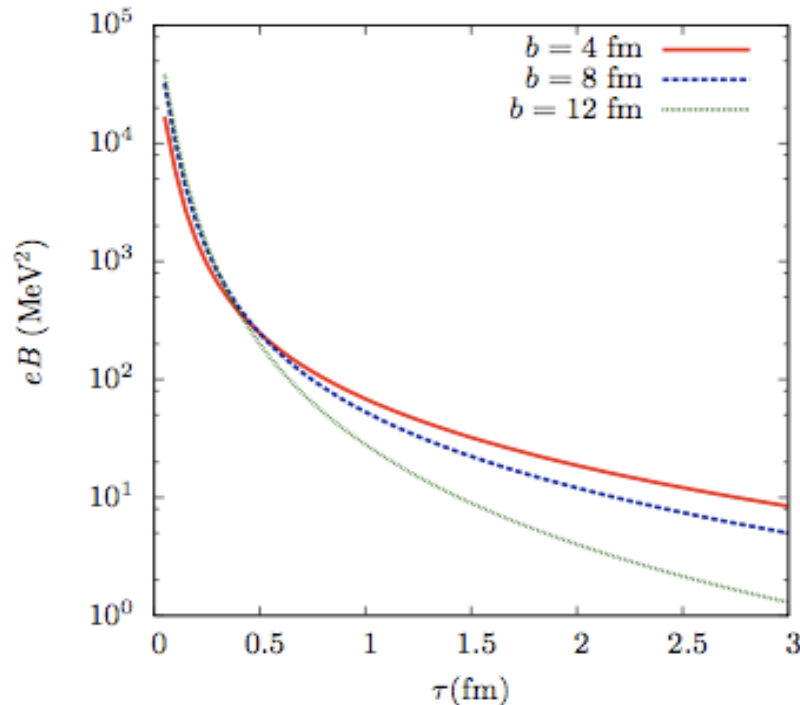- As strong as 10^{18} Gauss – the field of a Magnetar !

Over the barrier topological (sphaleron) transitions ... analogous to proposed mechanism for Electroweak Baryogenesis

$$\vec{J}_{\mathrm{CME}} = \frac{e^2}{2\pi^2}\,\mu_5\,\vec{B}$$

Topology in heavy-ion collisions:
The Chiral Magnetic Effect

External B field dies rapidly…effect most significant, for transitions at early times

Consistent (caveat emptor!) with heavy-ion results from RHIC & LHC
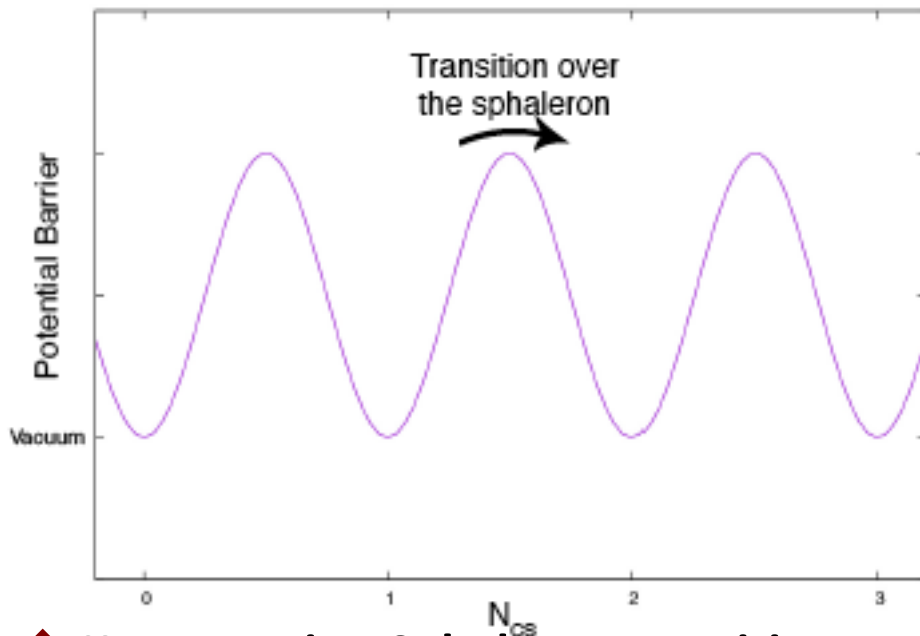
CME seen in condensed matter systems

Q. Li et al., Nature Physics (2015)

Sphaleron transitions in QCD

Sphaleron: spatially localized, unstable finite energy classical solutions

(σφαλερος -``ready to fall'')

EW theory: Klinkhamer, Manton, PRD30 (1984) 2212
QCD: McLerran,Shaposhnikov,Turok,Voloshin, PLB256 (1991) 451

Chiral Anomaly:

$$\partial_\mu J_{5,f}^\mu = 2 m_f \bar{q}\gamma_5 q - \frac{g^2}{16\pi^2} F_{\mu\nu}^a \tilde{F}^{\mu\nu,a}$$

Chern-Simons current:

$$K^\mu = \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \left(F_{\nu\rho}^a A_\sigma^a - \frac{g}{3} f_{abc} A_\nu^b A_\rho^c A_\sigma^c \right)$$

Chern-Simons #:

$$N_{CS}(t) = \int d^3x \, K^0(t, \mathbf{x})$$

Rate of change of CS #

$$\frac{dN_{CS}(t)}{dt} = \frac{g^2}{8\pi^2} \int d^3x \, E_i^a(\mathbf{x}) B_i^a(\mathbf{x})$$

Transition over the sphaleron

◆ **Key quantity: Sphaleron transition rate**

$$\Gamma^{eq} = \lim_{\delta t -> \infty} \frac{\langle (N_{\mathrm{CS}}(t+\delta t) - N_{\mathrm{CS}}(t))^2 \rangle_{\mathrm{eq}}}{V \delta t}$$

Tremendous prior numerical work: Gregoriev,Potter,Rubakov,Shaposhnikov,Ambjorn,Krasnitz,Turok,Moore, Smit,Tranberg,Bödeker,Rummukainen,Tassler,D'Onofrio,…

Topological transitions in the Glasma

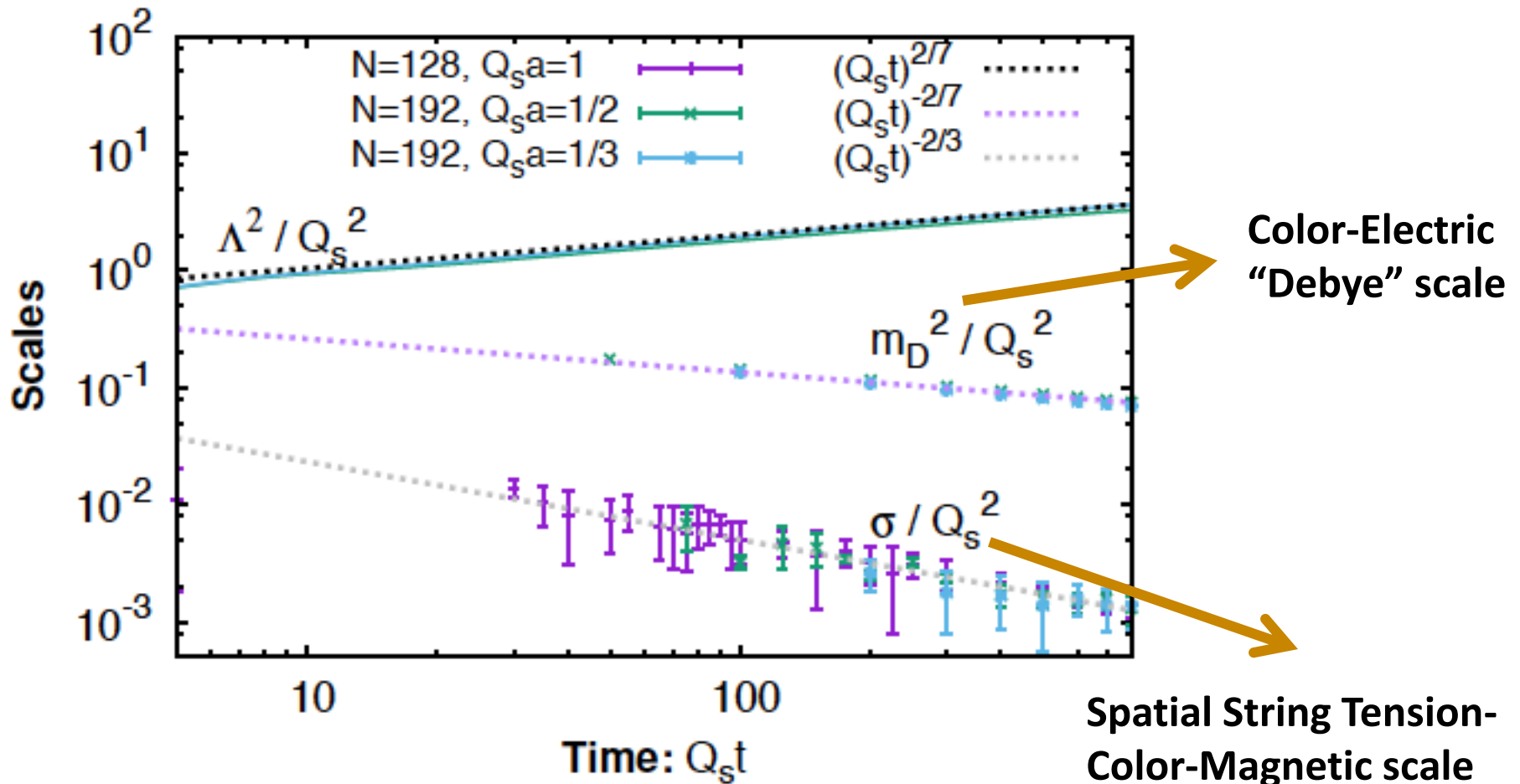**Overoccupied initial conditions in a fixed box:**
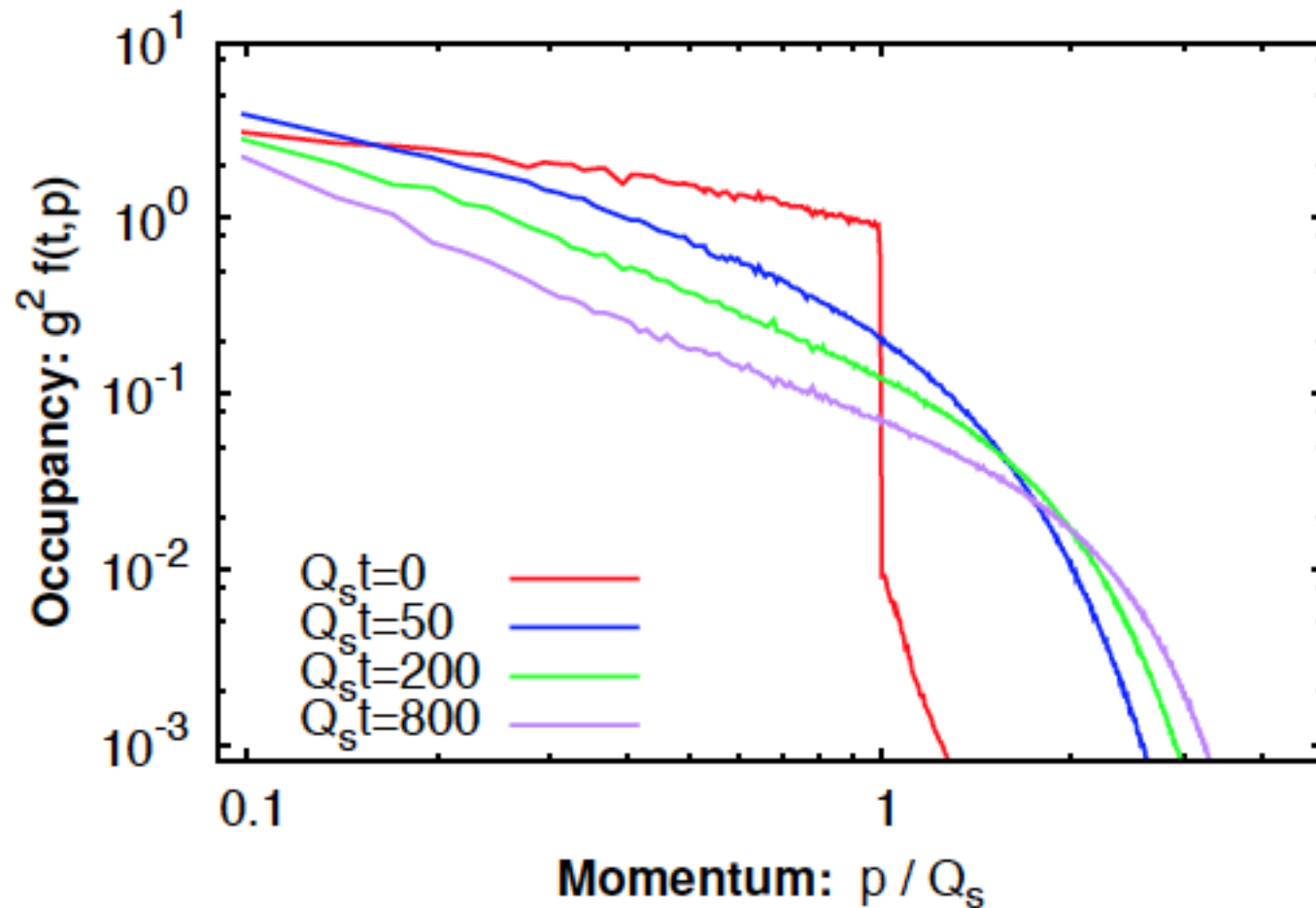
Temporal evolution of Glasma in a box

Soft electric and magnetic scales develop (as in a hot plasma):

Color-Electric "Debye" scale

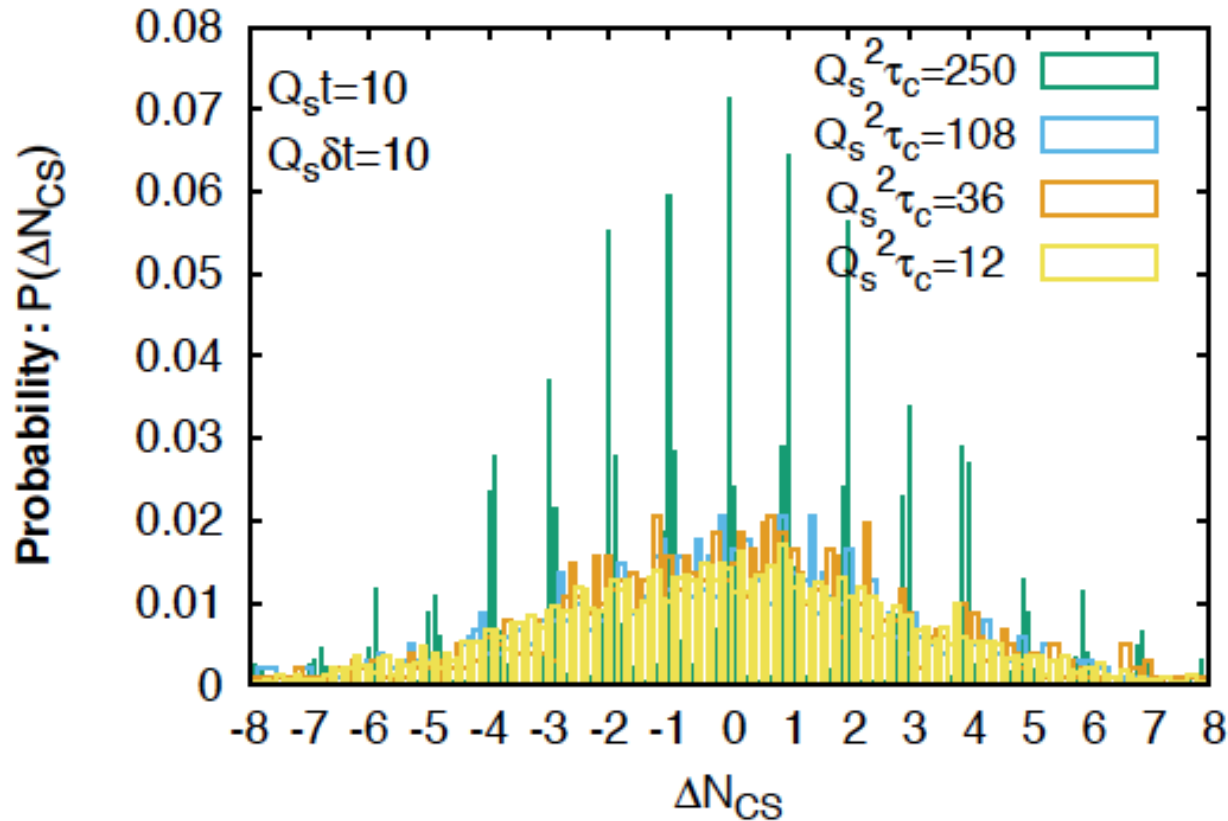**Spatial String Tension-Color-Magnetic scale**

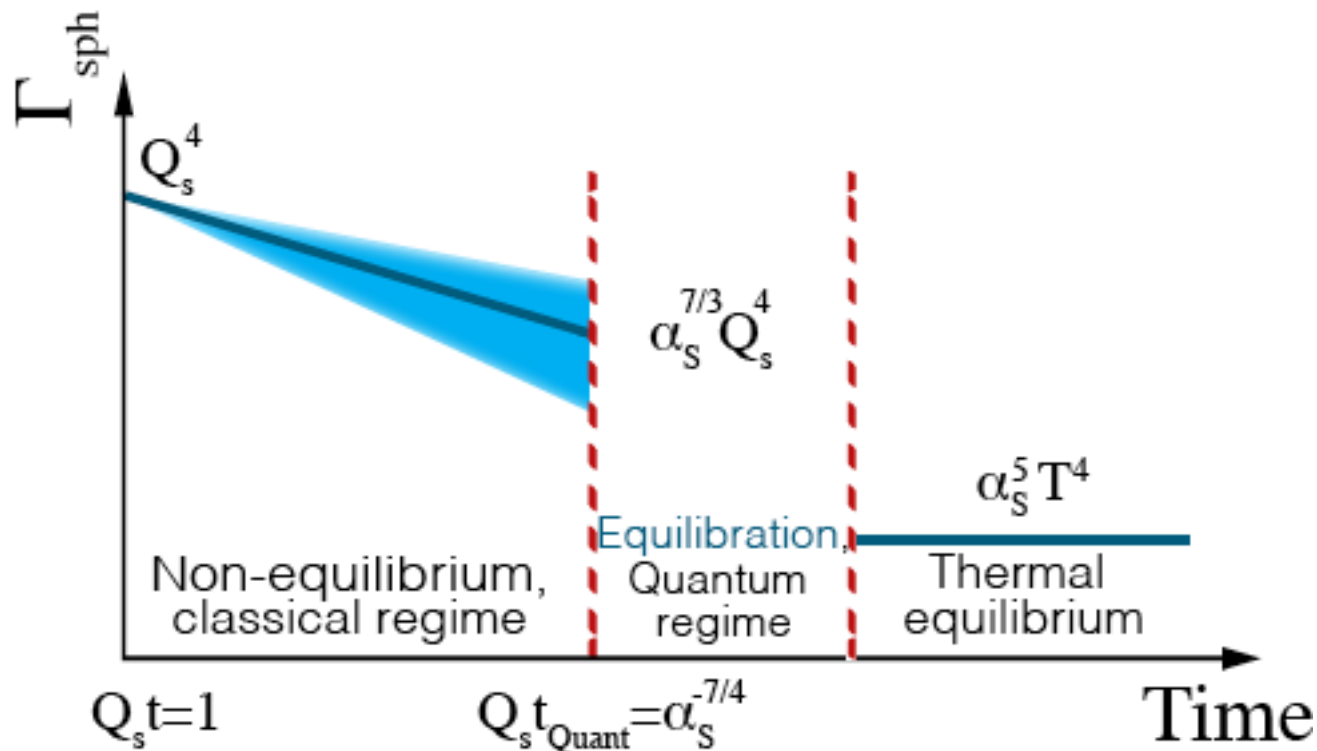# Sphaleron rate controlled by Glasma string tension
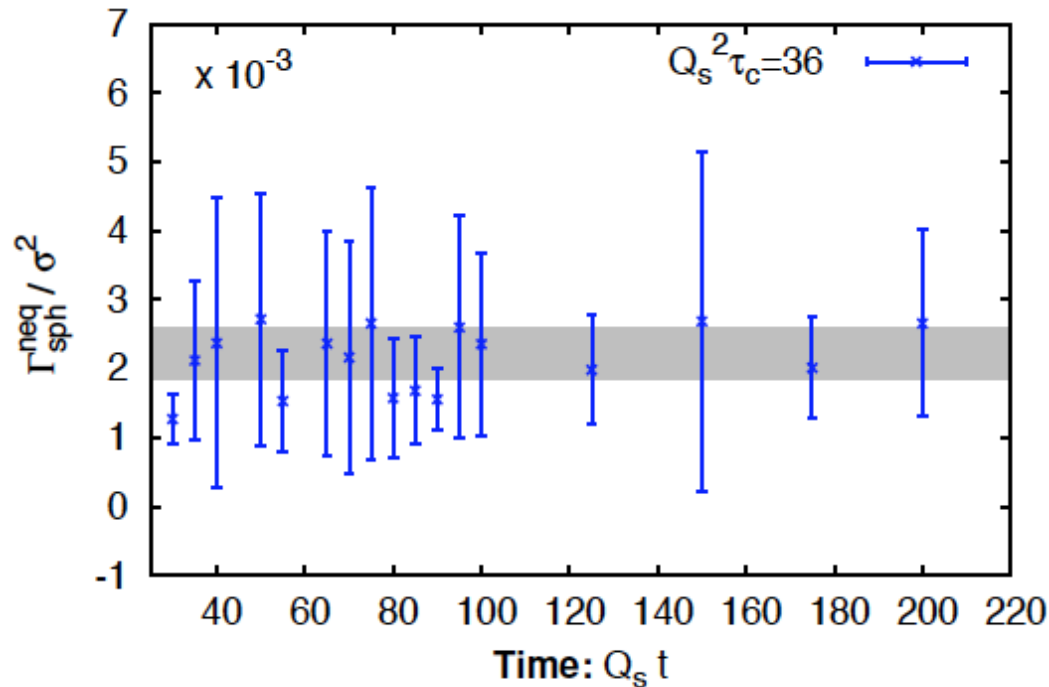
Scaling with string tension precisely as if topological transitions are controlled entirely by the color-magnetic screening scale

Topological transitions in the Glasma

"Cooled" soft Glue configurations in the Glasma are topological!

Topological transitions in the Glasma

Sphaleron transitions in the Glasma...
couple with fermions & external EM fields
 to simulate *ab initio* the Chiral Magnetic Effect!

Gelfand,Hebenstriet,Berges, PRD93 (2016) 085001
Mueller,Tanji,Berges, PRD93 (2016), 074507

Thank you for your attention!