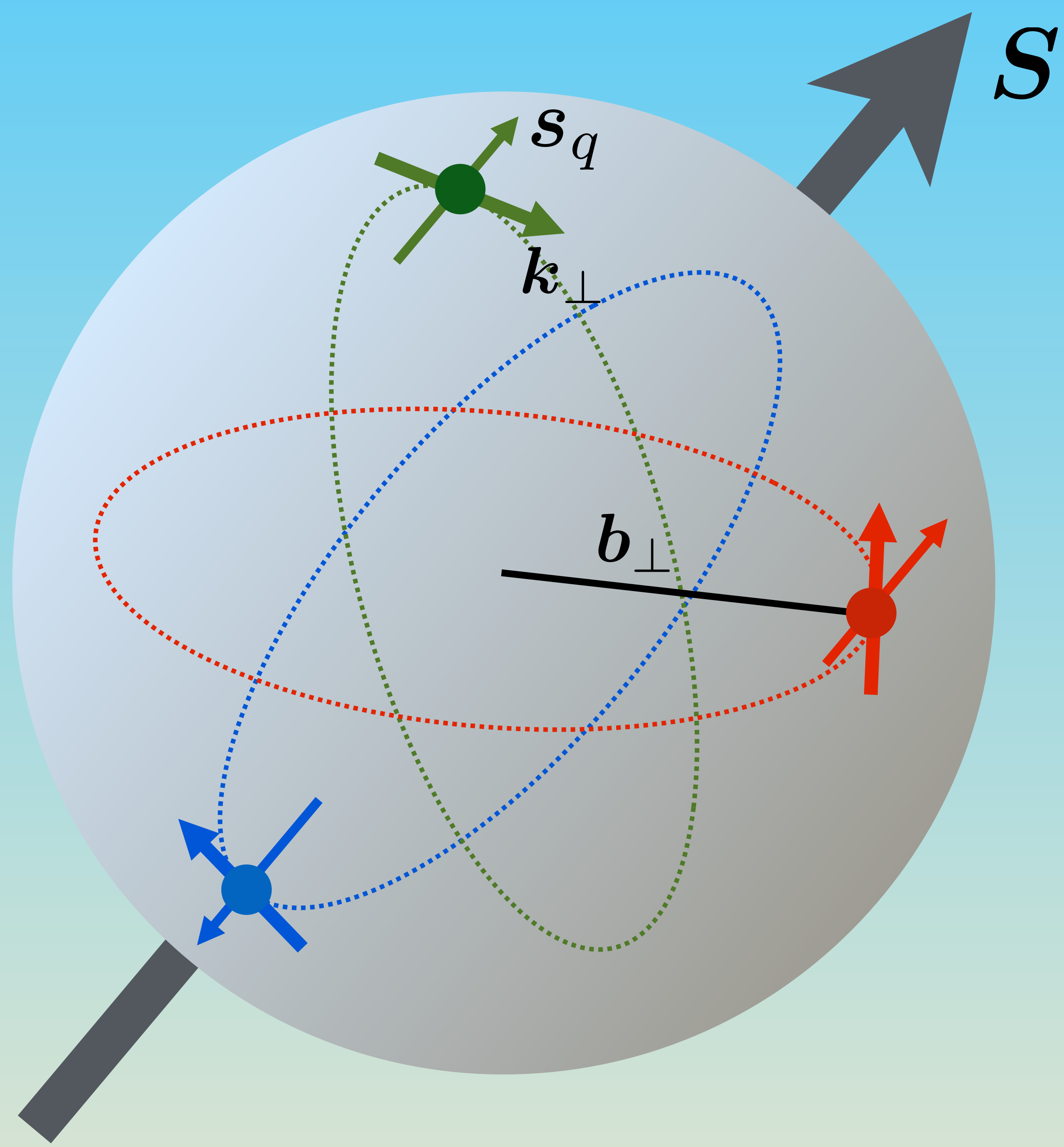
The 3-D nucleon structure in QCD



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WE-Heraeus Physics School

QCD – Old Challenges and New Opportunities

Bad Honnef, Sept 24–30, 2017

The 1-D nucleon picture, successes and some failures

Beyond the 1-D picture, quark intrinsic motion

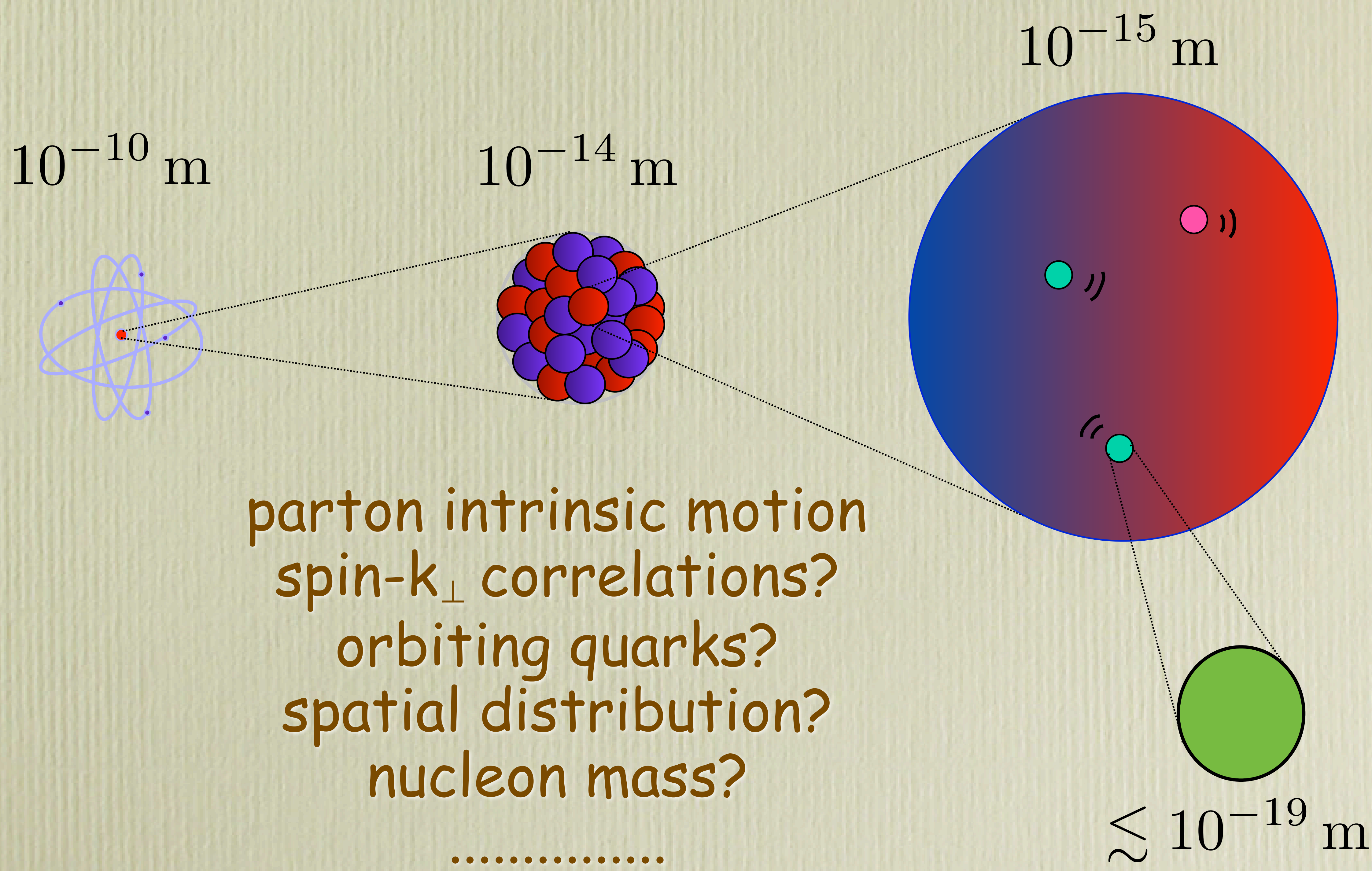
New concepts, TMDs and GPDs, 3-D momentum and spatial distributions of quarks and gluons

How and what do we learn from data about the 3-D nucleon structure? (mainly in momentum space)

Open problems and future experiments

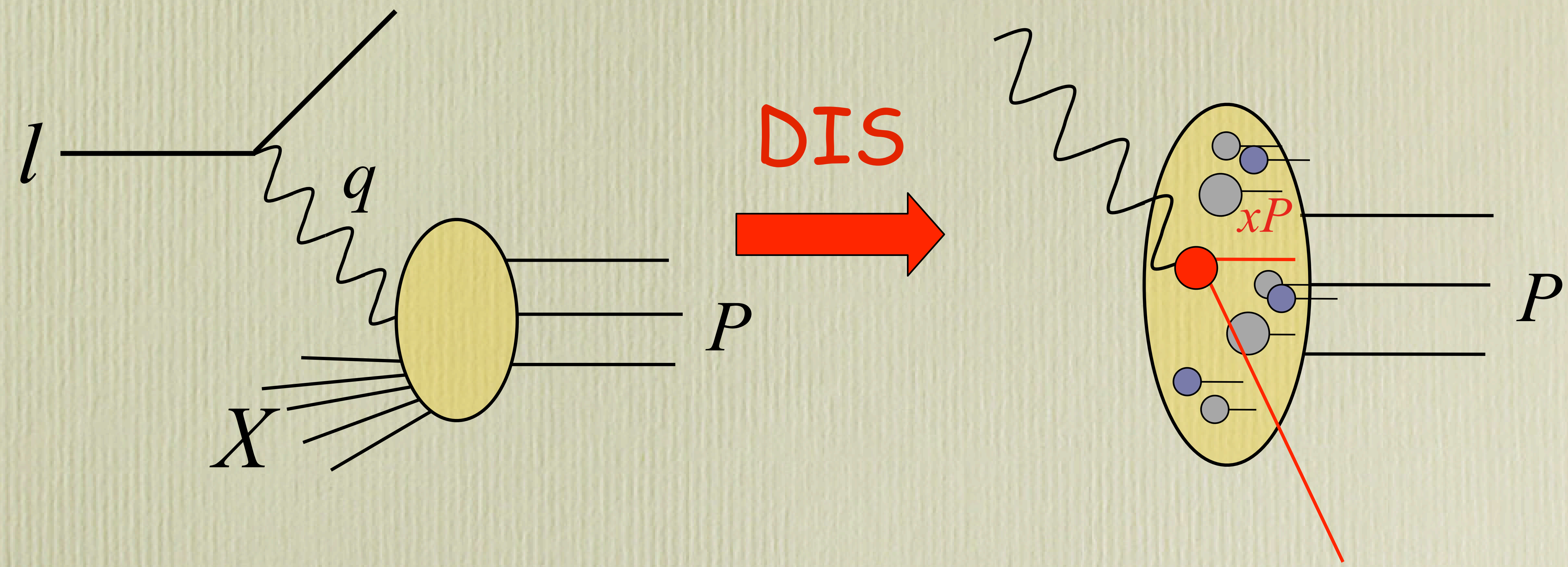
special issue of EPJA dedicated to the 3D nucleon structure,
EPJ A52, 2016, n.6 - 15 contributions, (Edts. M.A., P. Rossi, M. Guidal)

despite 50 years of studies the nucleon is still a very mysterious object, yet the most abundant piece of matter in the visible Universe



parton intrinsic motion
spin-$k_\perp$ correlations?
orbiting quarks?
spatial distribution?
nucleon mass?
....

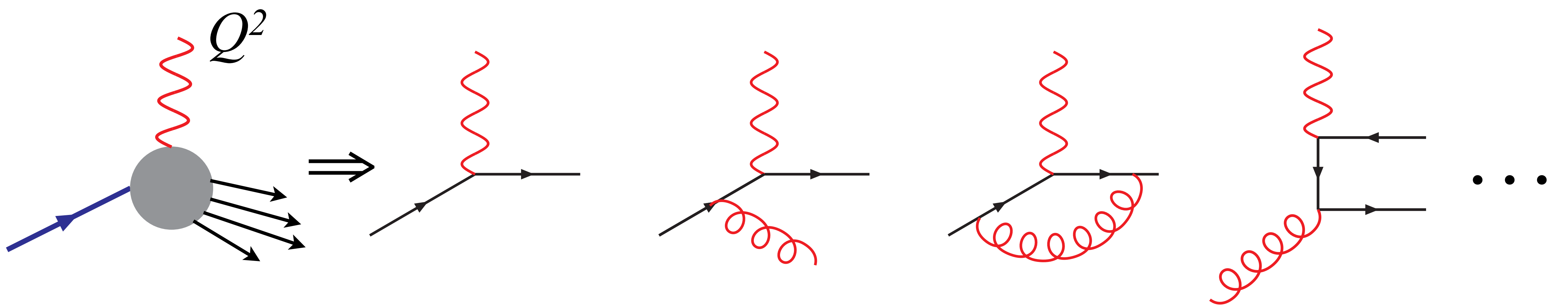
usual (successful) way of exploring the proton structure (collinear parton model)



$$\mathrm{DIS}:\ \ell\, p \to \ell\, X \qquad Q^2 = -q^2 \qquad x = \frac{Q^2}{2P\cdot q} \qquad y = \frac{P\cdot \ell}{P\cdot q}$$

Naive parton model: $$\frac{\mathrm{d}\sigma^{\ell p\to \ell X}}{\mathrm{d}x\,\mathrm{d}Q^2} = \sum_q q(x)\,\frac{\mathrm{d}\hat{\sigma}^{\ell q\to \ell q}}{\mathrm{d}Q^2}$$

QCD interactions induce a well known Q^2 dependence



$$\mathrm{DIS - pQCD:} \quad q(x) \Rightarrow \underbrace{q(x,Q^2)}_{\text{PDFs}}$$

DGLAP evolution equations

factorization:

$$\frac{d\sigma}{dx\, dQ^2} = \sum_q q(x,Q^2) \otimes \frac{d\hat{\sigma}_q}{dQ^2}$$

universality: same $q(x,Q^2)$ measured in DIS can be used in other processes

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equation

$$\frac{d}{d(lnQ^2)}\,q_i(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\left[q_i \otimes P_{qq} + g \otimes P_{qg}\right]$$

$$\frac{d}{d(lnQ^2)}\,g(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\left[\sum_i (q_i + \bar{q}_i) \otimes P_{gq} + g \otimes P_{gg}\right]$$

$$q \otimes P = P \otimes q \equiv \int_x^1 dy\,\frac{q(y,Q^2)}{y}\,P\left(\frac{x}{y}\right)$$

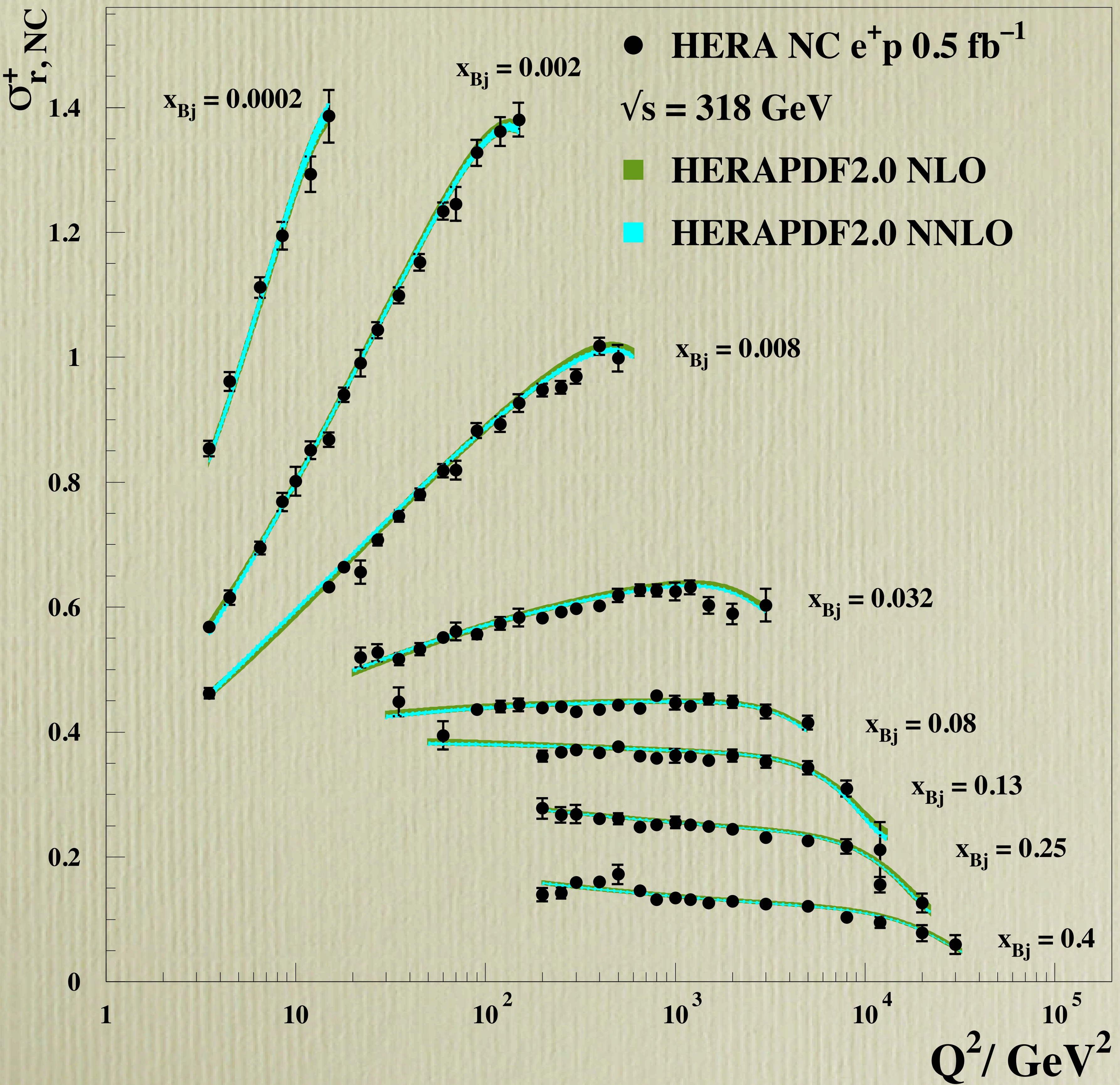
$$P_{qq} = \frac{4}{3}\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\,\delta(1-x)\right] + \mathcal{O}(\alpha_s)$$

$$P_{qg} = \frac{1}{2}\,\left[x^2 + (1-x)^2\right] + \mathcal{O}(\alpha_s) \qquad P_{qg} = \frac{4}{3}\,\frac{1+(1-x)^2}{x} + \mathcal{O}(\alpha_s)$$

$$P_{gg} = 6\,\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \frac{33-2n_f}{6}\,\delta(1-x) + \mathcal{O}(\alpha_s)$$

$$\int_0^1 dx\,\frac{f(x)}{(1-x)_+} = \int_0^1 dx\,\frac{f(x)-f(1)}{(1-x)}$$

H1 and ZEUS



$$\sigma^{\pm}_{r,\mathrm{NC}} = \frac{d^2\sigma^{e^\pm p}_{\mathrm{NC}}}{dx_{\mathrm{Bj}}dQ^2} \cdot \frac{Q^4 x_{\mathrm{Bj}}}{2\pi\alpha^2 Y_+}$$

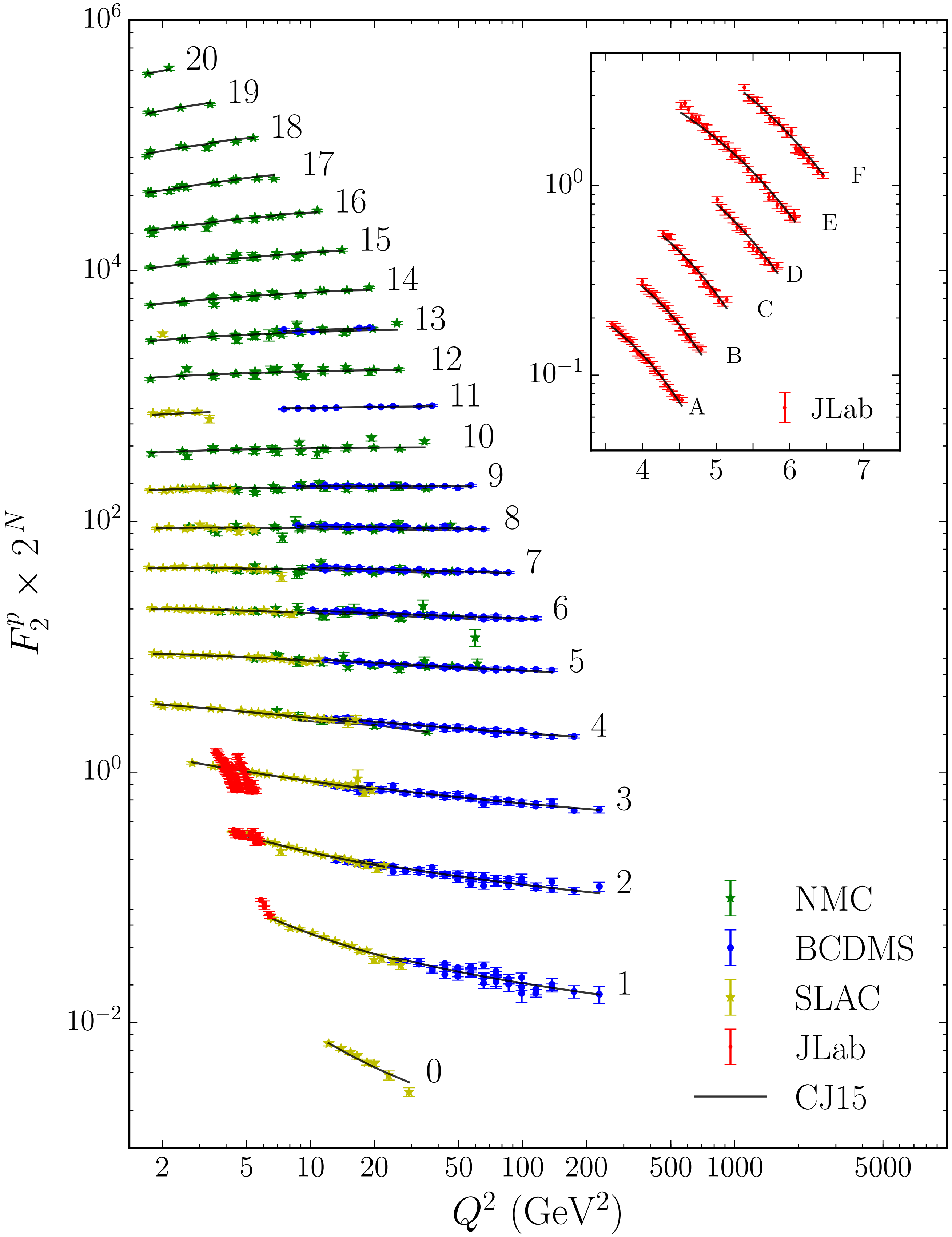
$$Y_\pm = 1 \pm (1-y)^2$$

Eur. Phys. J. C75 (2015) 580

remember beautiful plots by K. Wichmann

$$F_2 = \sum_q x\, q(x, Q^2)$$

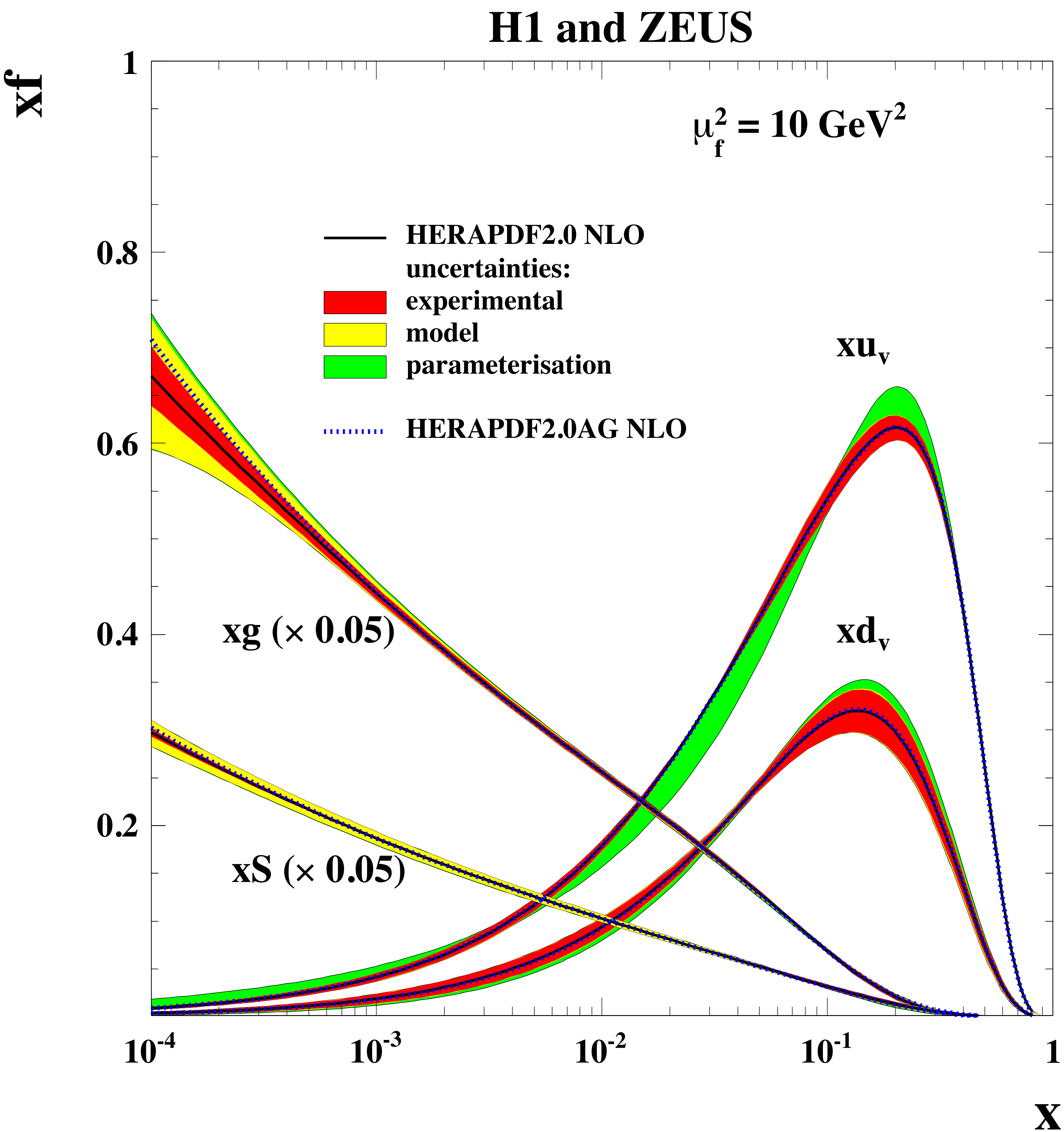
from M. Pennington, arXiv:1604.01441



N	x
0	0.85
1	0.74
2	0.65
3	0.55
4	0.45
5	0.34
6	0.28
7	0.23
8	0.18
9	0.14
10	0.11
11	0.10
12	0.09
13	0.07
14	0.05
15	0.04
16	0,026
17	0,018
18	0,013
19	0,008
20	0,005

JLab insert

I	°	N
A	38°	0
B	41°	1
C	45°	2
D	55°	3
E	60°	4
F	70°	5



unpolarized distribution

$$xf_a(x, Q^2)$$

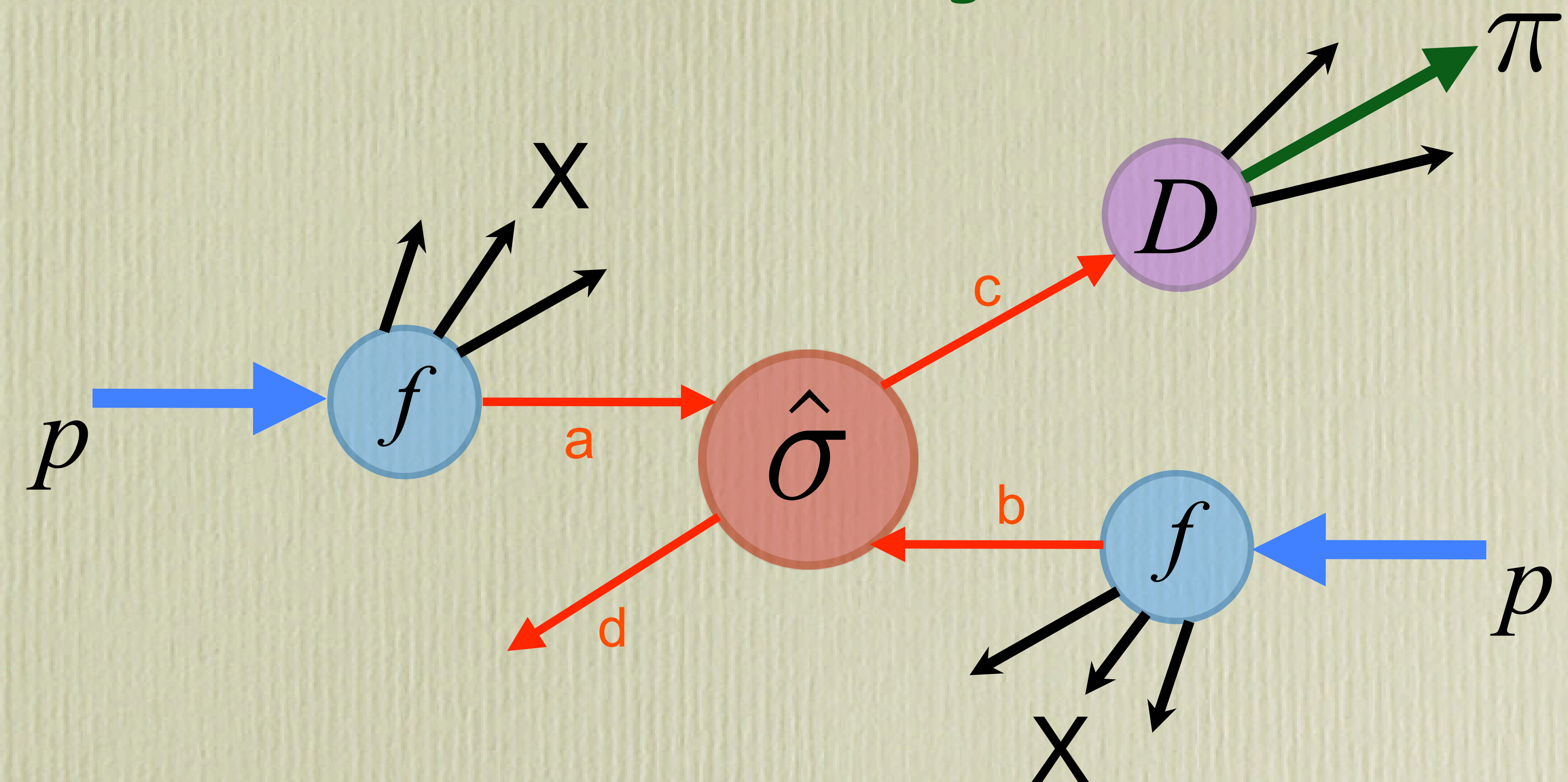
H. Abramowicz et al., Eur. Phys. J. C75 (2015) 580

PDFs are very useful, they can be used to predict cross sections for several processes ….

Cross section for $p\,p \to \pi\,X$ in pQCD

based on factorization theorem

(in collinear configuration)

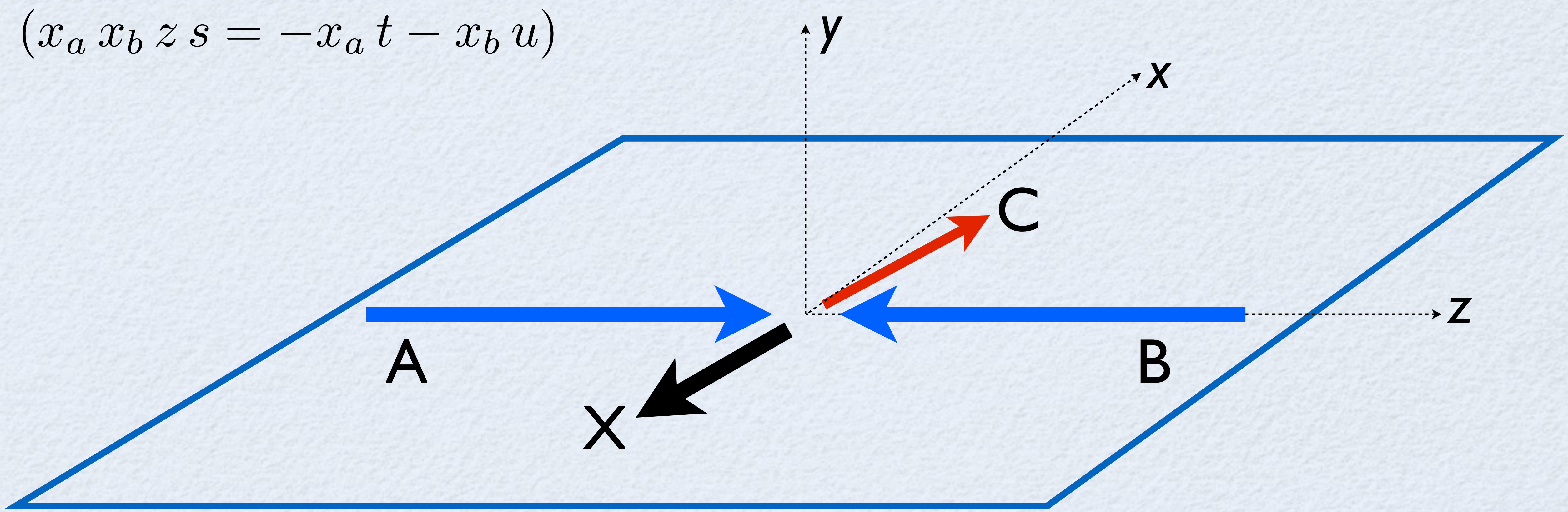


$$\mathrm{d}\sigma = \sum_{a,b,c,d=q,\bar{q},g} \underbrace{f_{a/p}(x_a) \otimes f_{b/p}(x_b)}_{\text{PDF}} \otimes \mathrm{d}\hat{\sigma}^{ab\to cd} \otimes \underbrace{D_{\pi/c}(z)}_{\text{FF}}$$

pQCD elementary interactions

$$
\begin{aligned}
\frac{E_C\, d\sigma^{AB\to CX}}{d^3\boldsymbol{p}_C} &= \sum_{a,b,c,d} \int dx_a\, dx_b\, dz\, f_{a/A}(x_a, Q^2)\, f_{b/B}(x_b, Q^2) \\
&\times \frac{\hat{s}}{\pi z^2}\, \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b)\, \delta(\hat{s}+\hat{t}+\hat{u})\, D_{C/c}(z, Q^2) \\
&= \sum_{a,b,c,d} \int dx_a\, dx_b\, f_{a/A}(x_a, Q^2)\, f_{b/B}(x_b, Q^2) \\
&\times \frac{1}{\pi z}\, \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b)\, D_{C/c}(z, Q^2)
\end{aligned}
$$

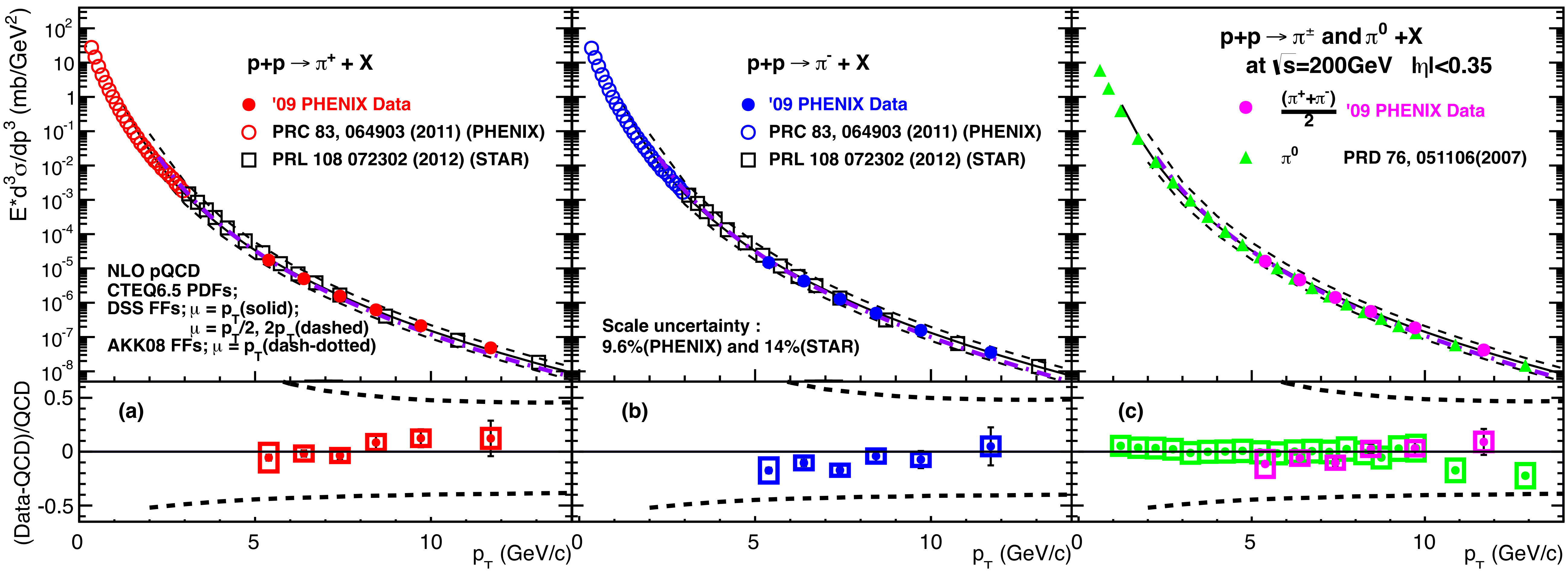
$(x_a\, x_b\, z\, s = -x_a\, t - x_b\, u)$



mid-rapidity RHIC data, unpolarised cross sections

(arXiv:1409.1907 [hep-ex], Phys. Rev. D91 (2015) 3, 032001)

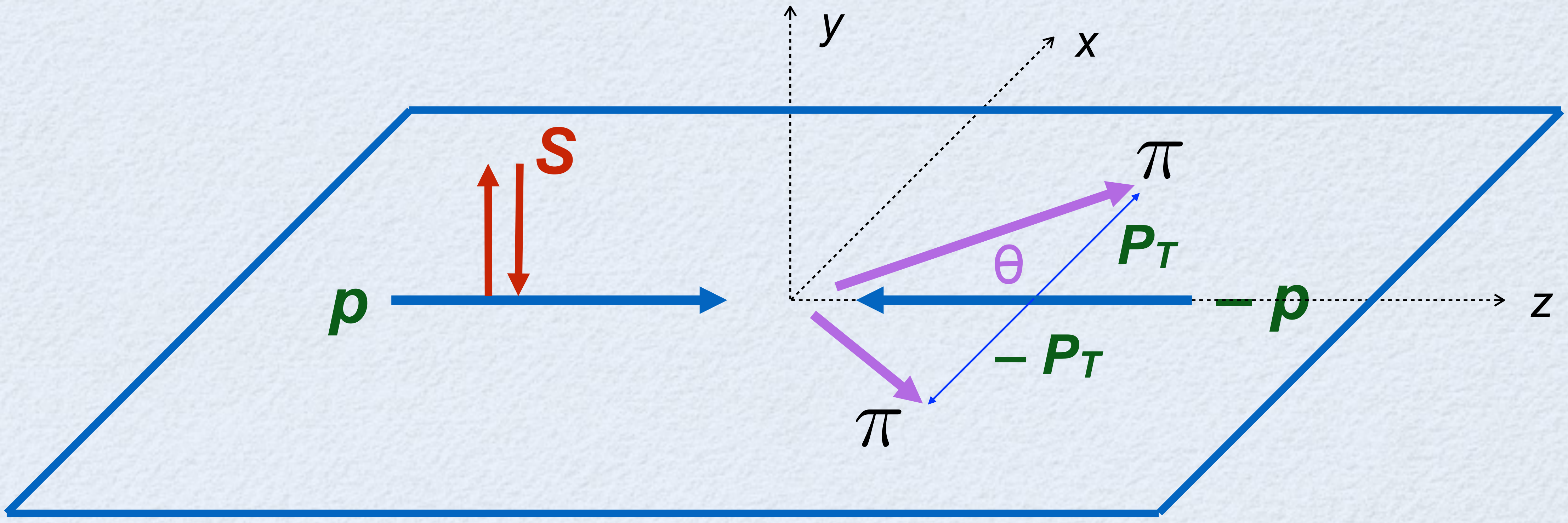
large P_T single pion production $\quad p\,p \to \pi\,X$



good agreement between RHIC data and collinear pQCD calculations but there are problems with spin dependent data …

A_N = simple left-right asymmetry

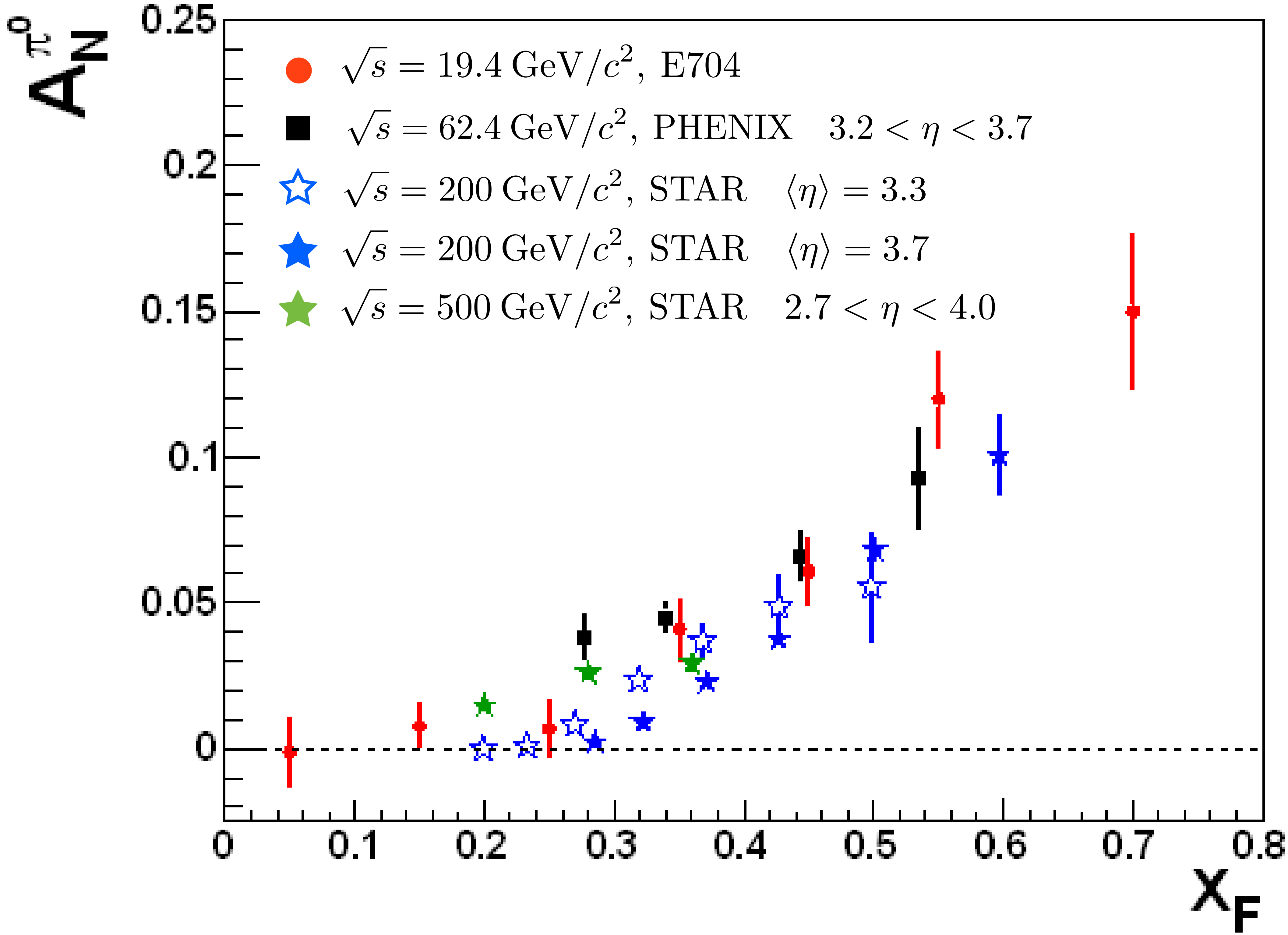
$$A_N = \frac{d\sigma^{\uparrow}(\boldsymbol{P}_T) - d\sigma^{\downarrow}(\boldsymbol{P}_T)}{d\sigma^{\uparrow}(\boldsymbol{P}_T) + d\sigma^{\downarrow}(\boldsymbol{P}_T)} = \frac{d\sigma^{\uparrow}(\boldsymbol{P}_T) - d\sigma^{\uparrow}(-\boldsymbol{P}_T)}{2\, d\sigma^{\mathrm{unp}}(P_T)}$$



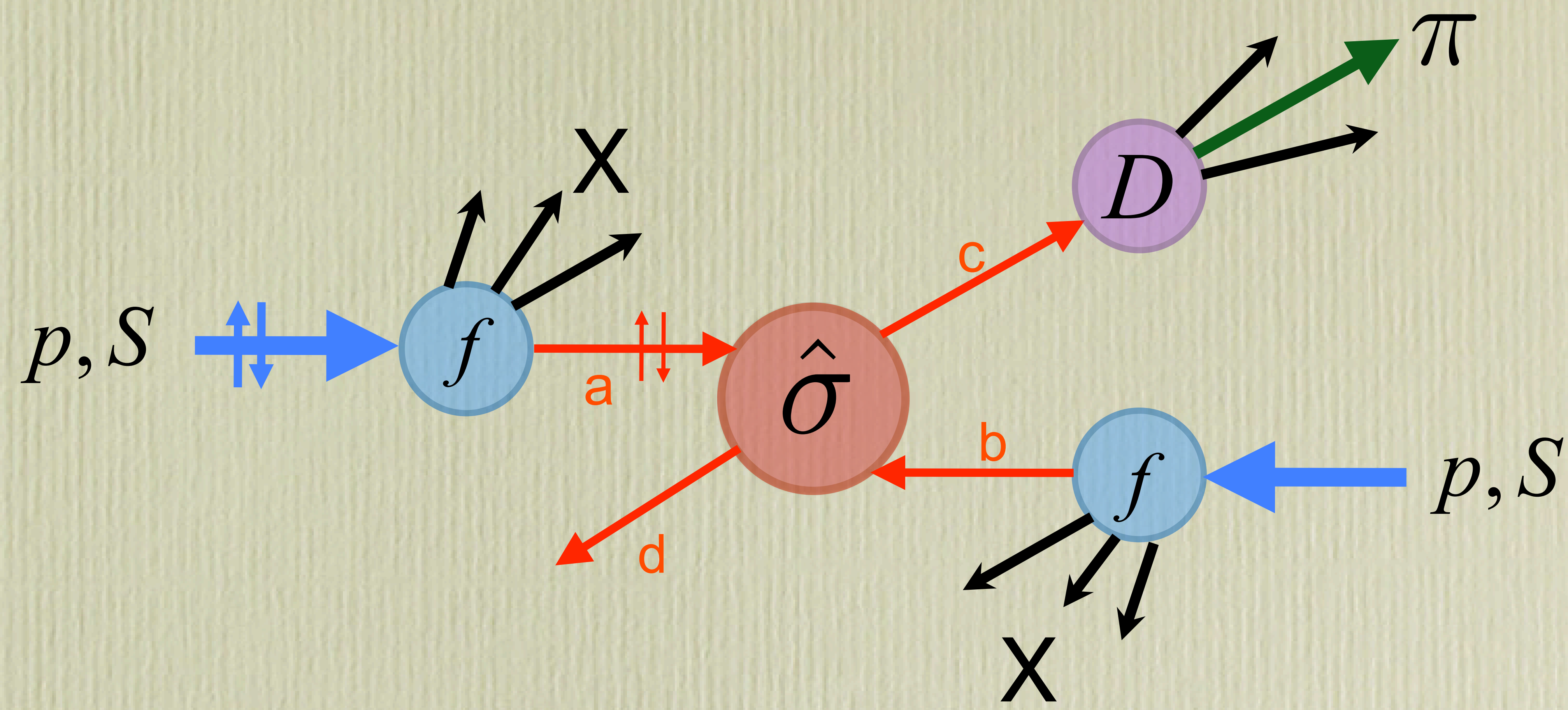
$$A_N \equiv \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} \propto \boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{P}_T) \propto \sin\theta$$

transverse Single Spin Asymmetry (SSA)

A_N large and persistent at high energies ....



SSA in $p\,p \to \pi\,X$?

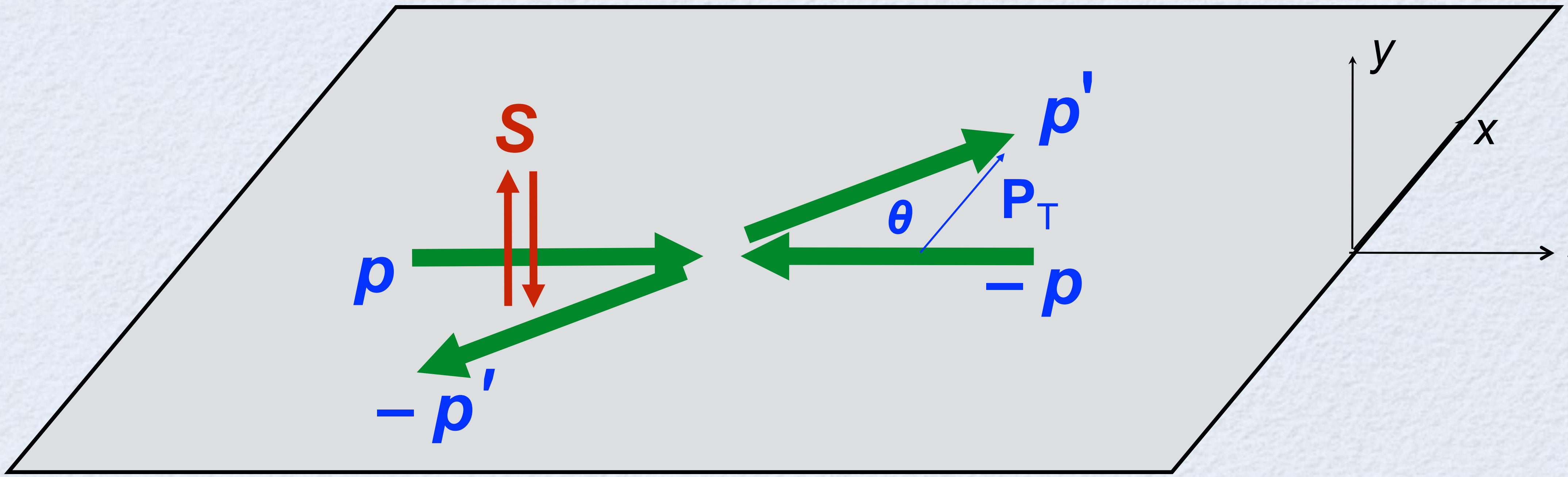


$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} = \sum_{a,b,c,d=q,\bar{q},g} \underbrace{\Delta_T f_a}_{\text{transversity}} \otimes f_b \otimes \underbrace{[\mathrm{d}\hat{\sigma}^{\uparrow} - \mathrm{d}\hat{\sigma}^{\downarrow}]}_{\text{pQCD elementary SSA}} \otimes \underbrace{D_{\pi/c}}_{\text{FF}}$$

$$A_N = \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} \propto \hat{a}_N \propto \frac{m_q}{E_q}\,\alpha_s$$

was considered almost a theorem

Transverse single spin asymmetries in elastic scattering



$$A_N \equiv \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} \propto \boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{P}_T) \propto \sin\theta$$

Example: $p\,p \longrightarrow p\,p$ $\Longrightarrow$

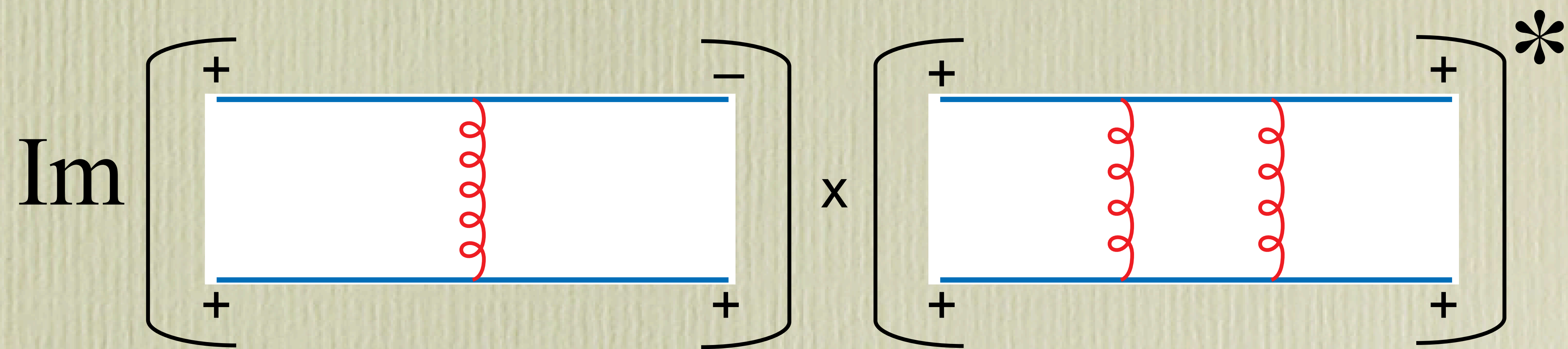
$$\left\{ \begin{array}{l} H_{++;++} \equiv \Phi_1 \\ H_{--;++} \equiv \Phi_2 \\ H_{+-;+-} \equiv \Phi_3 \\ H_{-+;+-} \equiv \Phi_4 \\ H_{-+;++} \equiv \Phi_5 \end{array} \right.$$

5 independent helicity amplitudes

$$A_N \propto \mathrm{Im}\left[\Phi_5 \left(\Phi_1 + \Phi_2 + \Phi_3 - \Phi_4\right)^*\right]$$

Single spin asymmetries at partonic level. Example: $q\,q' \to q\,q'$

$A_N \neq 0$ needs helicity flip + relative phase



QED and QCD interactions conserve helicity, up to corrections $\mathcal{O}\left(\frac{m_q}{E_q}\right)$

$$\Longrightarrow \quad A_N \propto \frac{m_q}{E_q}\,\alpha_s \quad \text{at quark level}$$

(Kane, Pumplin, Repko)

but large SSA observed at hadron level!

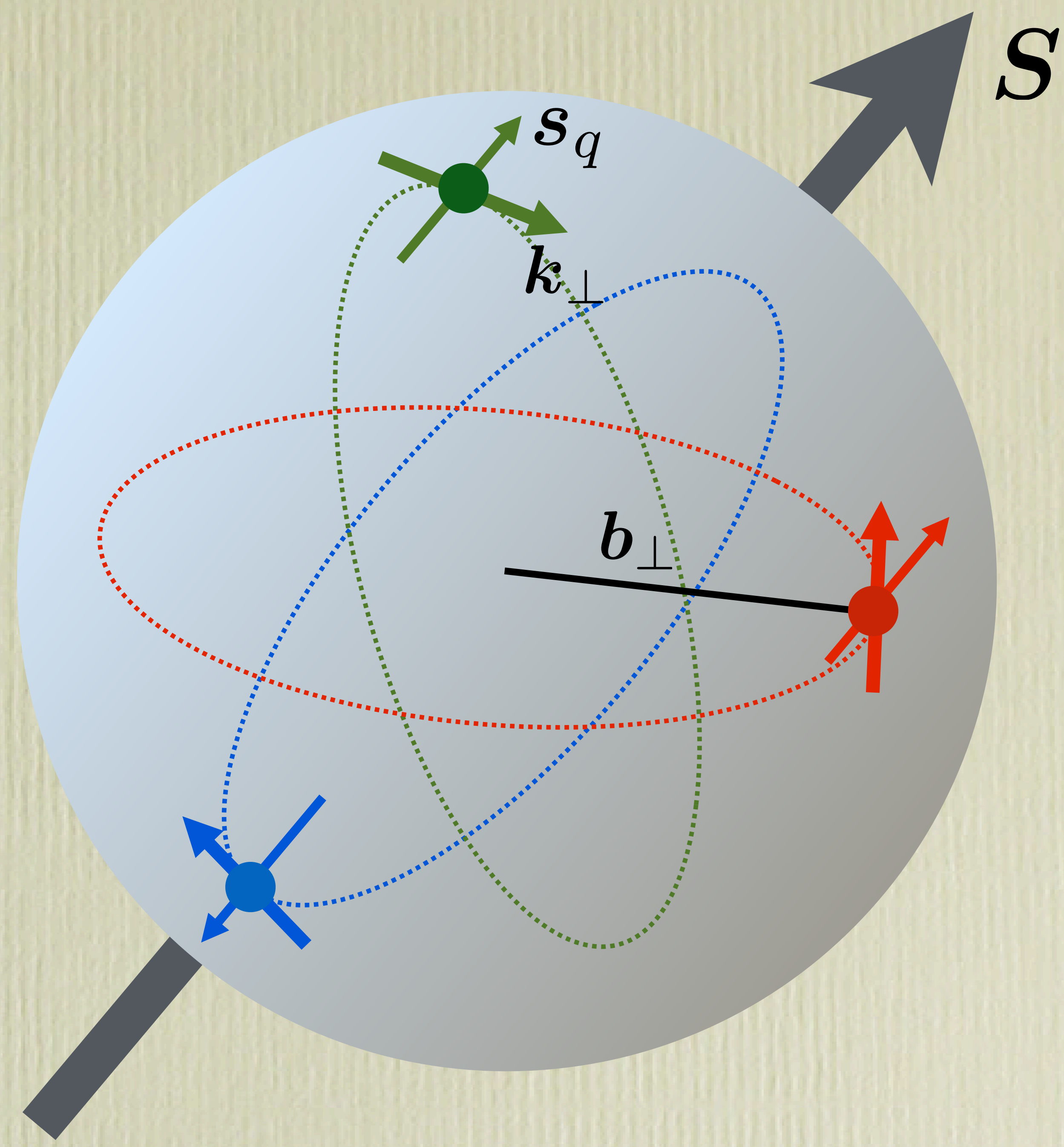
the (longstanding) proton spin puzzle

$$\frac{1}{2} = \underbrace{\frac{1}{2}\Sigma_q}_{\text{total spin carried by quarks}} + \underbrace{\Sigma_g}_{\text{total spin carried by gluons}} + \underbrace{L_q + L_g}_{\text{orbital angular momentum of quarks and gluons}}$$

the total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3-D description

many other (spin) effects in high energy interactions cannot be understood in the collinear configuration

.... we cannot state that we know the full partonic nucleon structure ....



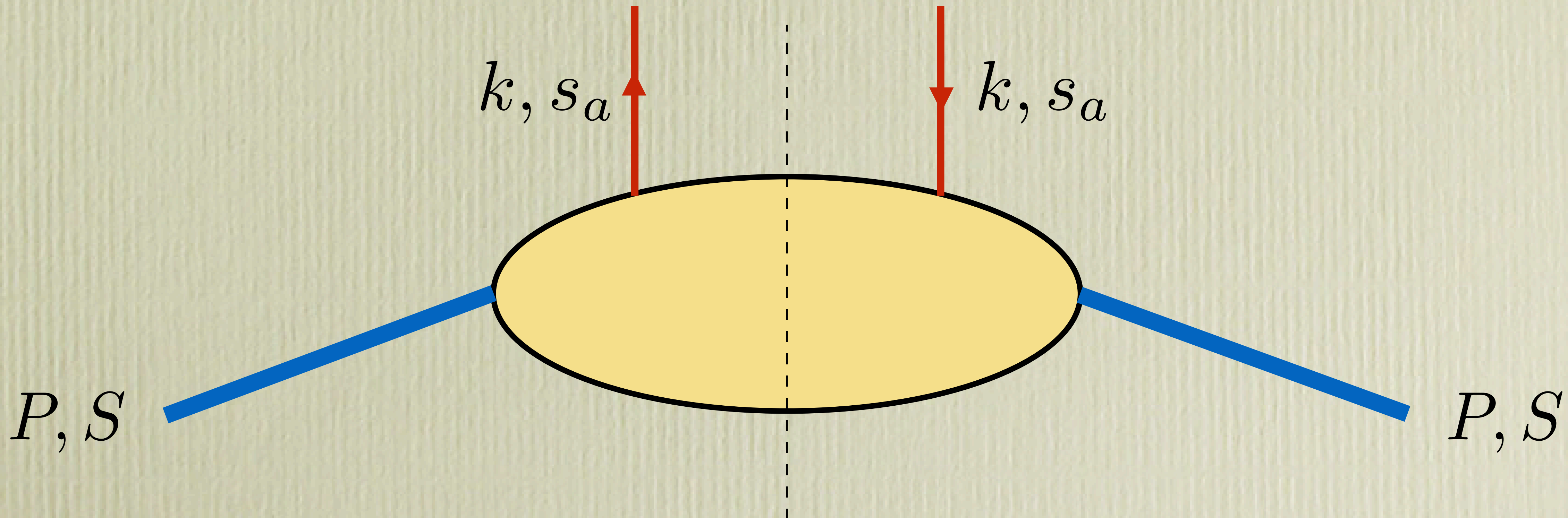
parton intrinsic motion
spin-$k_\perp$ correlations?
orbiting quarks?
spatial distribution?
nucleon mass?
....

new concepts: Transverse Momentum Dependent distribution and fragmentation functions - TMDs
Generalized Partonic Distributions - GPDs

new probes and concepts to explore the nucleon structure

TMDs - Transverse Momentum Dependent (distribution and fragmentation functions)

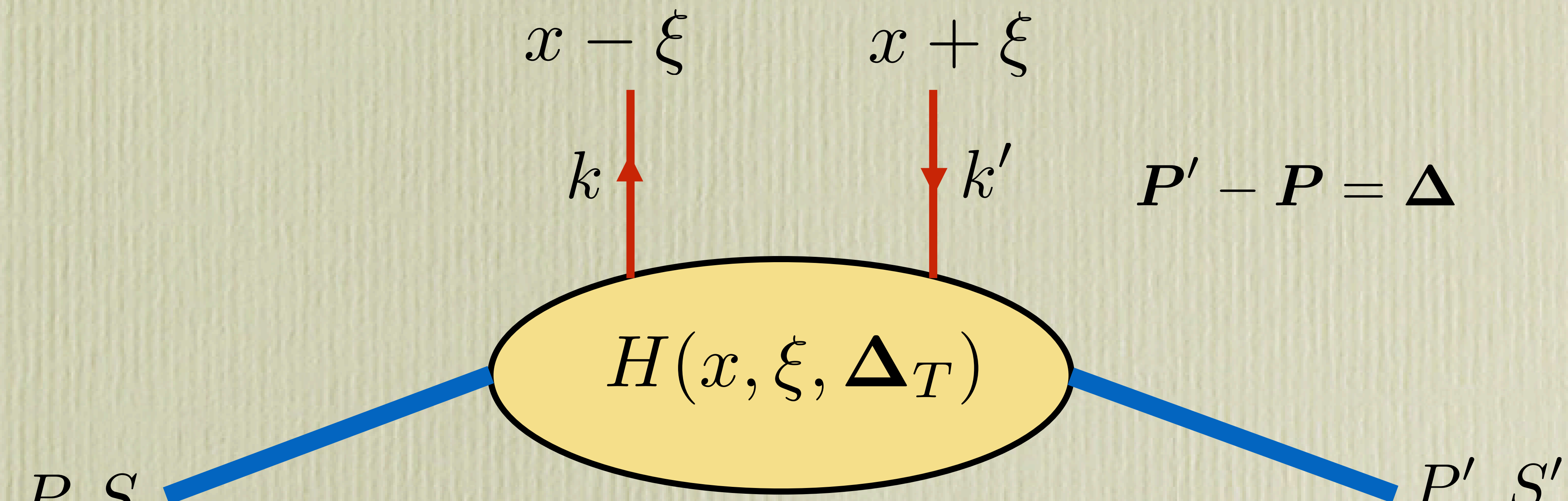
(polarized) SIDIS and Drell-Yan, spin asymmetries in inclusive (large p_T) NN processes



$$f_{a/p}(x, \boldsymbol{k}_\perp; \boldsymbol{s}_a, \boldsymbol{S})$$

GPDs - Generalized Partonic Distributions

exclusive processes in leptonic and hadronic interactions

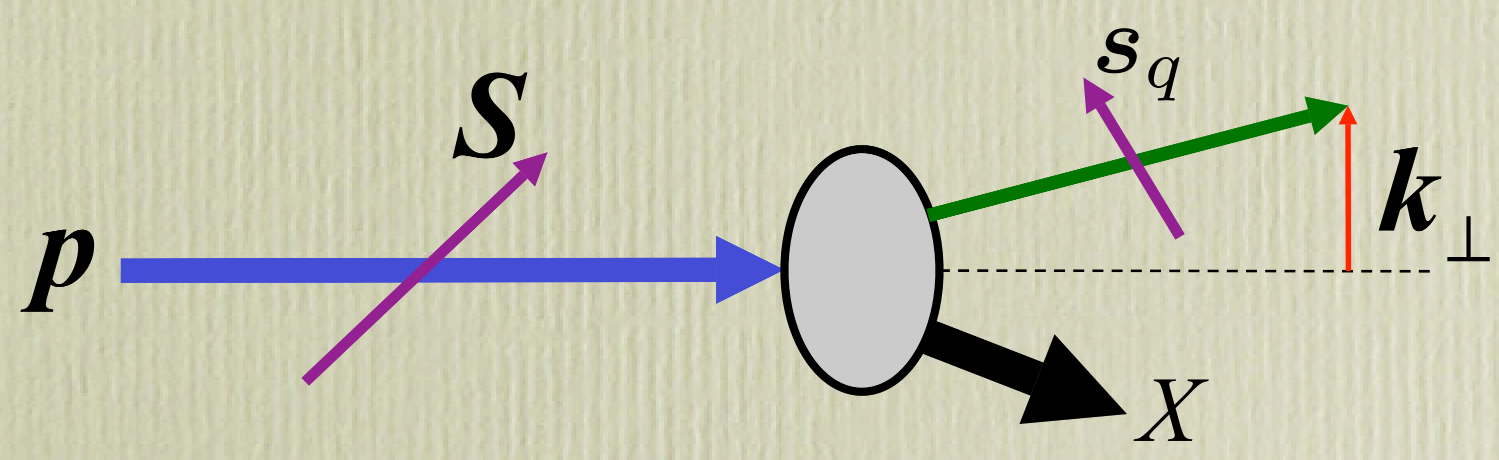


$$q(x, \boldsymbol{b}_T) = \int \frac{d^2\boldsymbol{\Delta}_T}{(2\pi)^2} H_q(x, 0, -\boldsymbol{\Delta}_T^2) e^{-i\, \boldsymbol{b}_T \cdot \boldsymbol{\Delta}_T}$$

TMDs in simple parton model

TMDs = Transverse Momentum Dependent Parton Distribution Functions (TMD-PDF) or Transverse Momentum Dependent Fragmentation Functions (TMD-FF)

TMD-PDFs give the number density of partons, with their intrinsic motion and spin, inside a fast moving proton, with its spin.



$\boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{k}_\perp)$ "Sivers effect"

$\boldsymbol{s}_q \cdot (\boldsymbol{p} \times \boldsymbol{k}_\perp)$ "Boer-Mulders effect"

$\boldsymbol{S} \cdot \boldsymbol{s}_q$

$\cdots$

there are 8 independent TMD-PDFs

$f_1^q(x, \boldsymbol{k}_\perp^2)$ unpolarized quarks in unpolarized protons
unintegrated unpolarized distribution

$g_{1L}^q(x, \boldsymbol{k}_\perp^2)$ correlate s_L of quark with S_L of proton
unintegrated helicity distribution

$h_{1T}^q(x, \boldsymbol{k}_\perp^2)$ correlate s_T of quark with S_T of proton
unintegrated transversity distribution

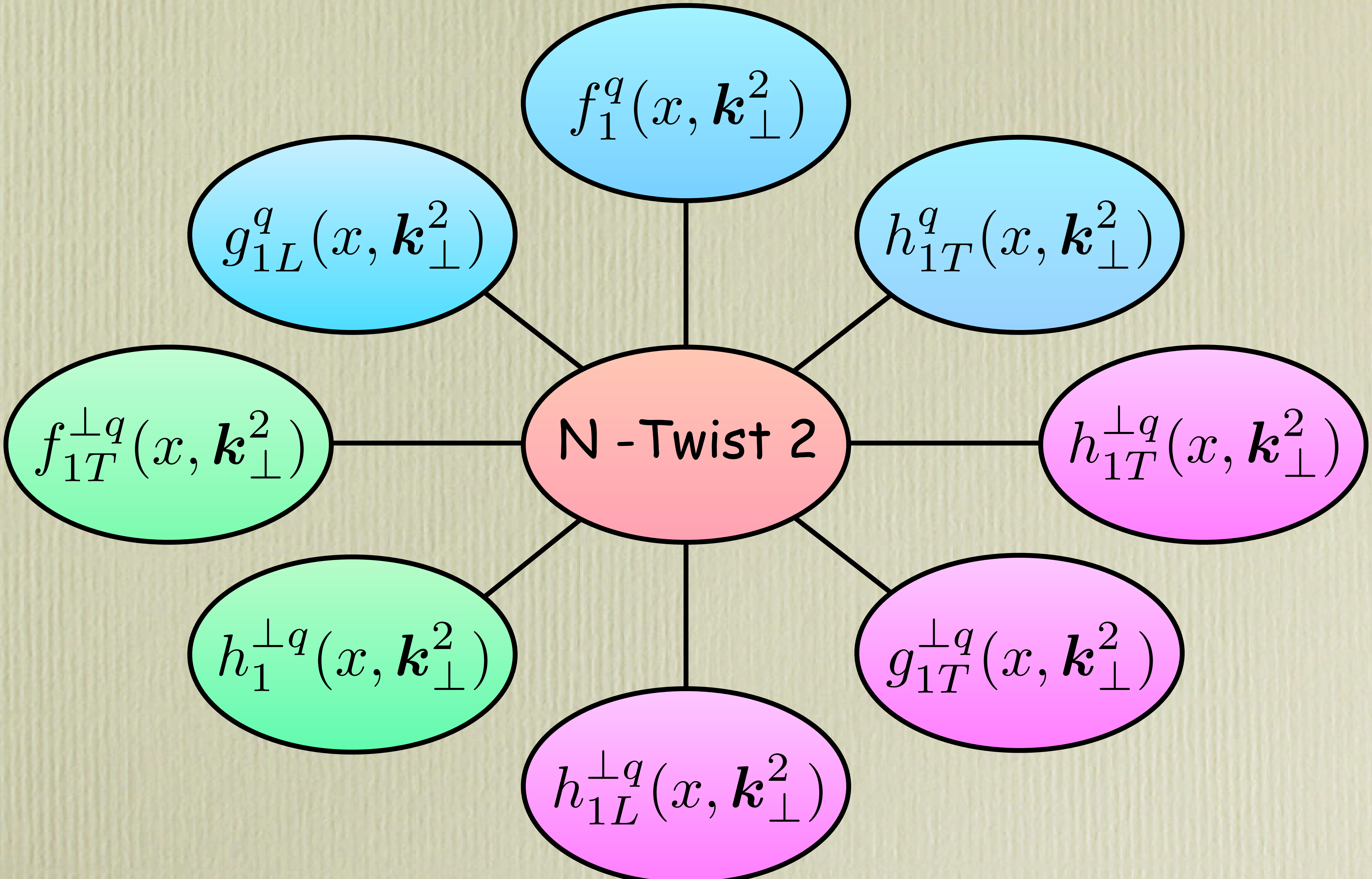
only these survive in the collinear limit

$f_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$ correlate $k_\perp$ of quark with S_T of proton (Sivers)

$h_1^{\perp q}(x, \boldsymbol{k}_\perp^2)$ correlate $k_\perp$ and s_T of quark (Boer-Mulders)

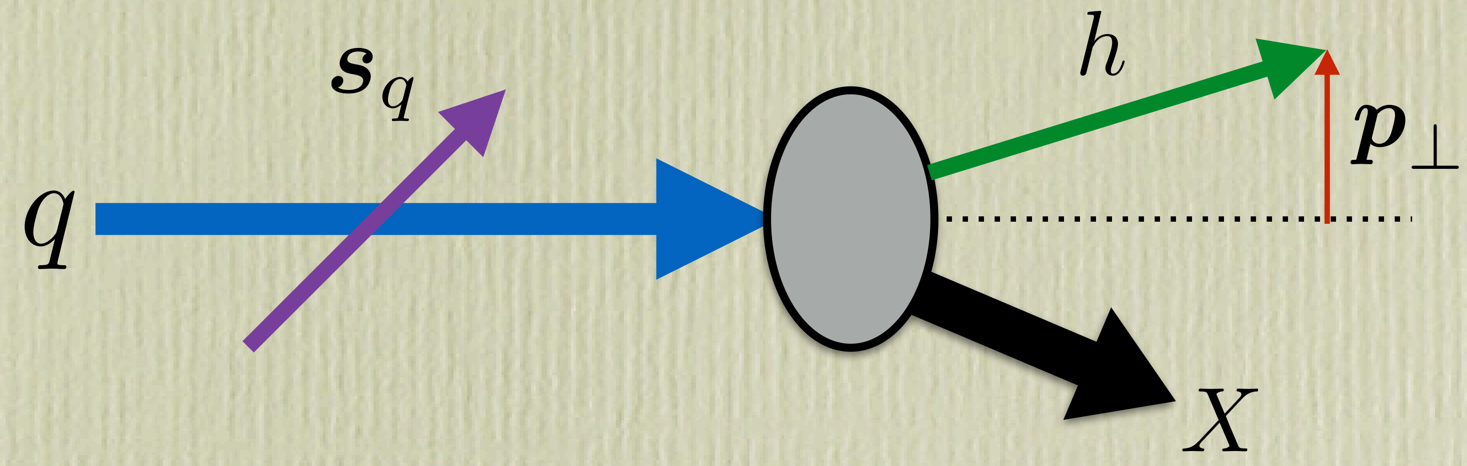
$$\underbrace{g_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2) \qquad h_{1L}^{\perp q}(x, \boldsymbol{k}_\perp^2)}_{\text{worm-gears}} \qquad \underbrace{h_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)}_{\text{pretzelosity}}$$

The nucleon at twist-2,



courtesy of A. Kotzinian

TMD-FFs give the number density of hadrons, with their momentum, originated in the fragmentation of a fast moving parton, with its spin.



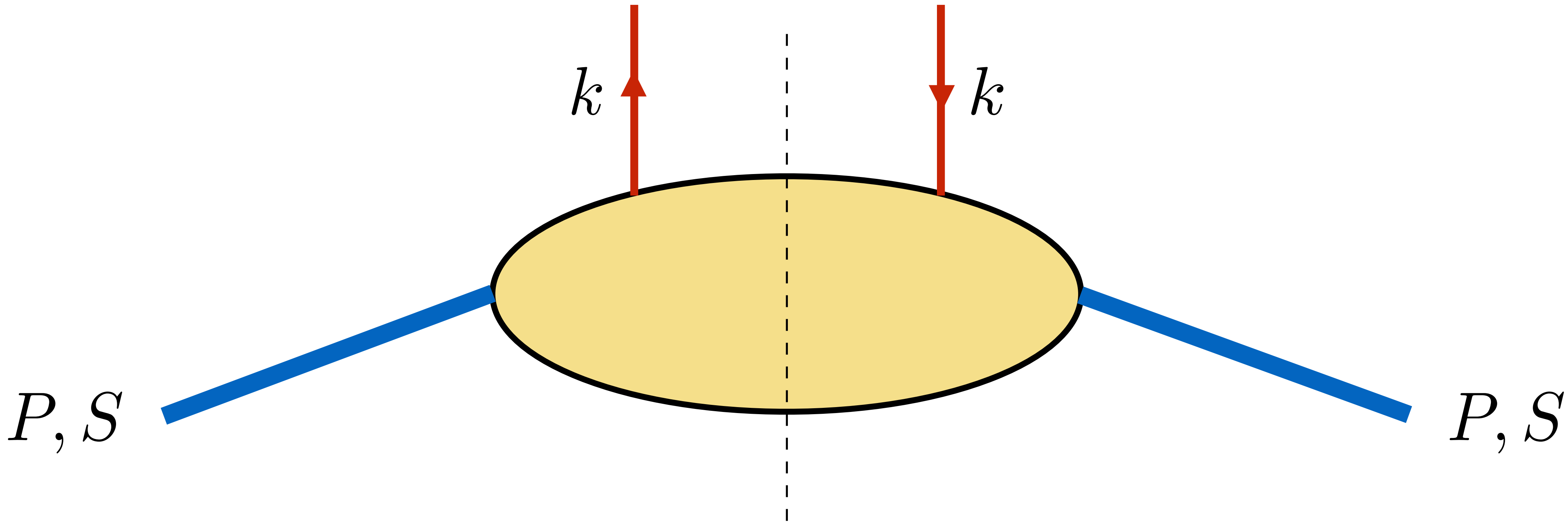
$\boldsymbol{s}_q \cdot (\boldsymbol{p}_q \times \boldsymbol{p}_\perp)$ "Collins effect"

there are 2 independent TMD-FFs for spinless hadrons

$D_1^q(z, \boldsymbol{p}_\perp^2)$ unpolarized hadrons in unpolarized quarks unintegrated fragmentation function

$H_1^{\perp q}(z, \boldsymbol{p}_\perp^2)$ correlate $p_\perp$ of hadron with s_T of quark (Collins)

TMD formalism - The nucleon correlator, in collinear configuration: 3 distribution functions



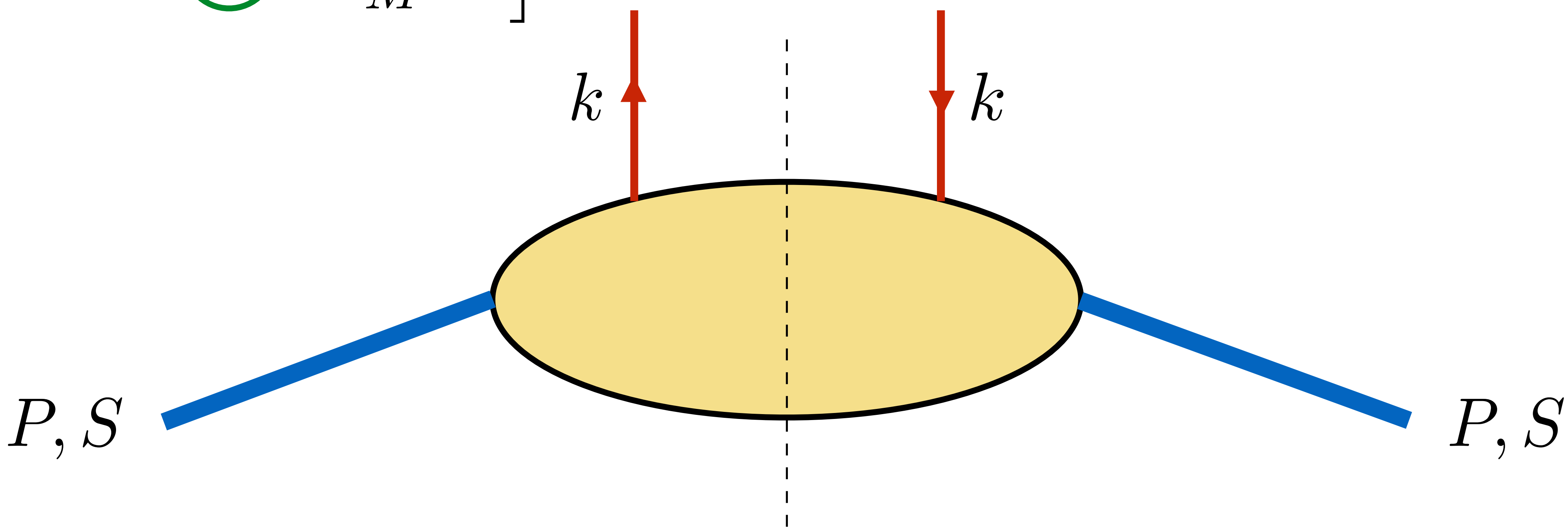
$$\Phi_{ij}(k;P,S) = \sum_X \int \frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3\,2E_X}\,(2\pi)^4\,\delta^4(P-k-P_X)\langle PS|\overline{\Psi}_j(0)|X\rangle\langle X|\Psi_i(0)|PS\rangle$$

$$= \int \mathrm{d}^4\,\xi\,e^{ik\cdot\xi}\langle PS|\overline{\Psi}_j(0)\Psi_i(\xi)|PS\rangle$$

$$\Phi(x,S) = \frac{1}{2}\left[\underbrace{f_1(x)}_{q}\,\not{n}_+ + S_L\,\underbrace{g_{1L}(x)}_{\Delta q}\,\gamma^5\,\not{n}_+ + \underbrace{h_{1T}}_{\Delta_T q}\,i\sigma_{\mu\nu}\gamma^5 n_+^{\mu} S_T^{\nu}\right]$$

TMD-PDFs: the leading-twist correlator, with intrinsic $k_\perp$, contains 8 independent functions

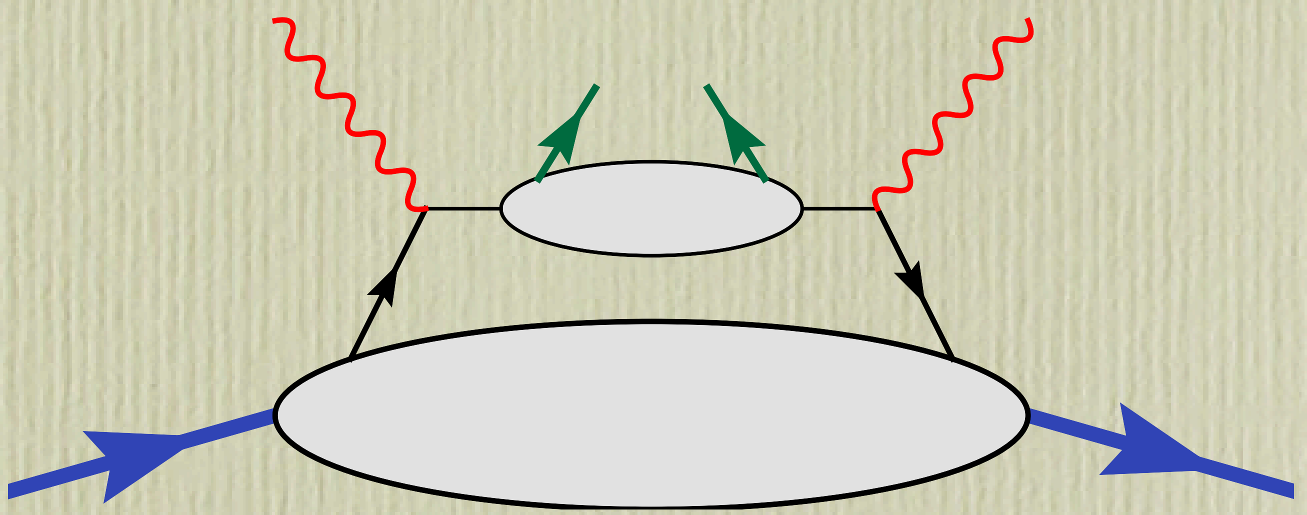
$$
\begin{aligned}
\Phi(x, \boldsymbol{k}_\perp) \;=\; & \frac{1}{2}\Bigg[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \left(S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp \right) \gamma^5 \not{n}_+ \\
& + h_{1T} i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \left(S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp \right) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M} \\
& + h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \Bigg]
\end{aligned}
$$



with partonic interpretation

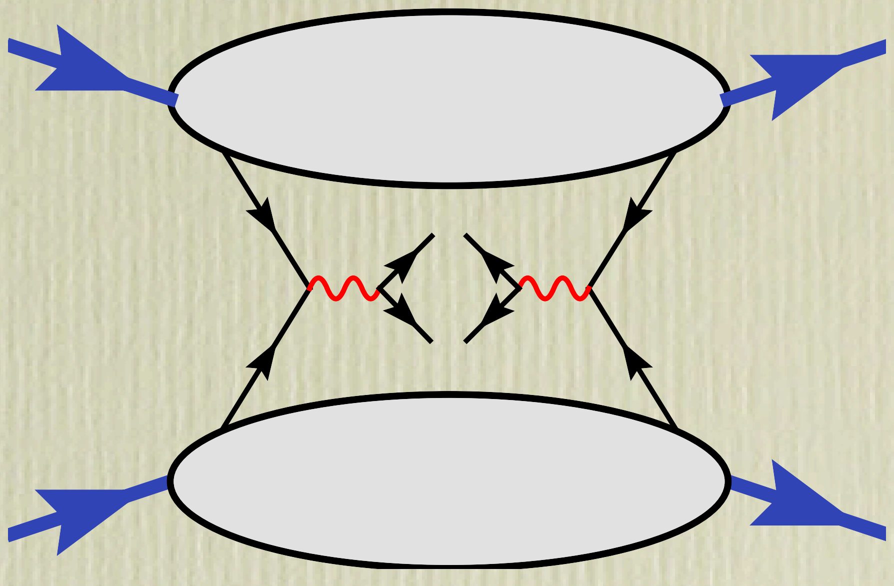
how to "measure" TMDs?

need processes which relate physical observables
to parton intrinsic motion



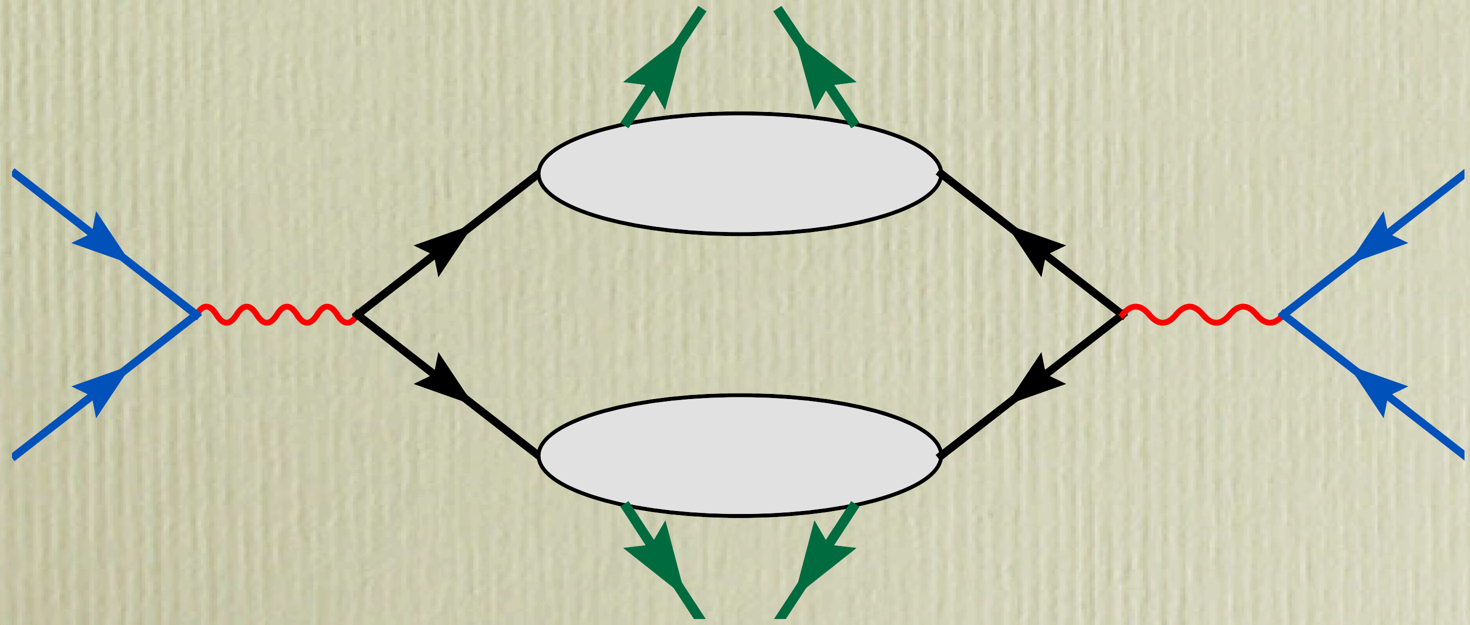
SIDIS

$\ell\, N \to \ell\, h\, X$



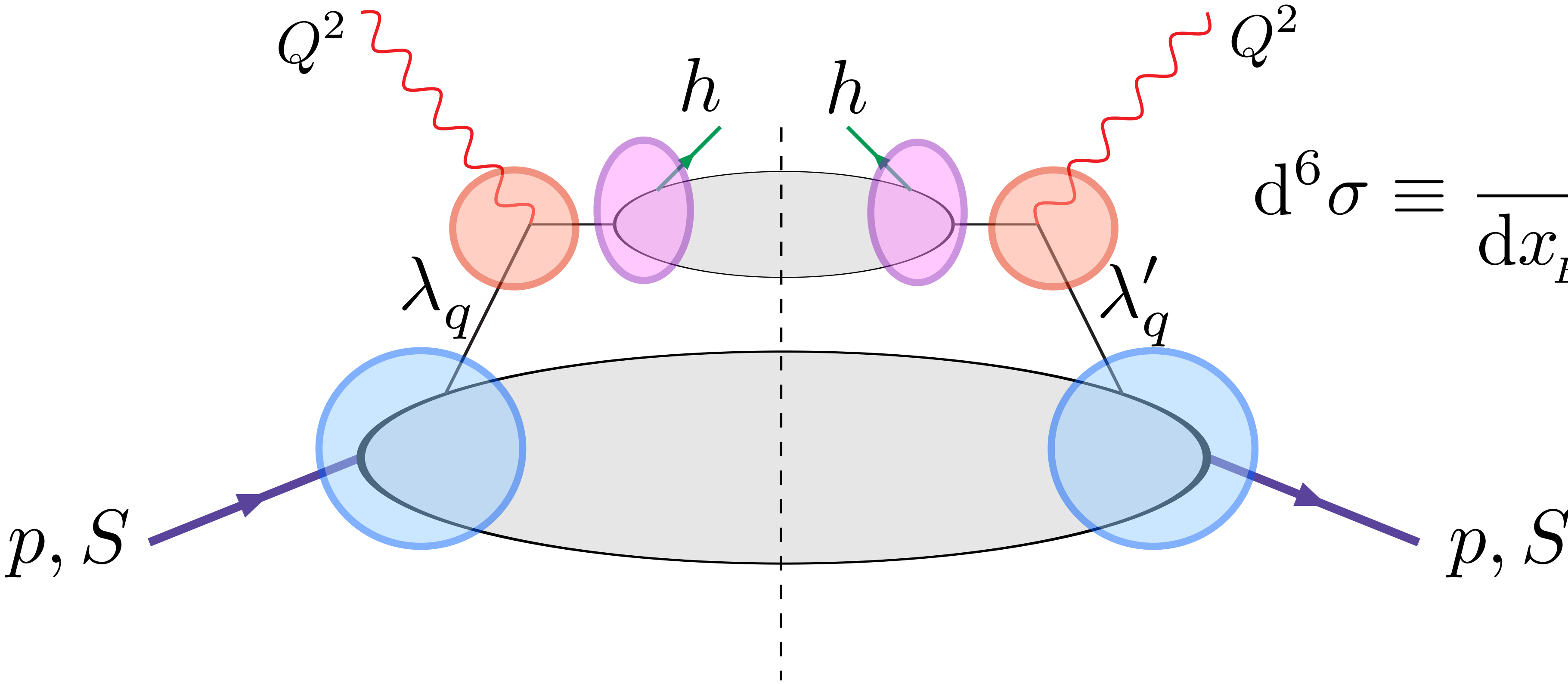
Drell-Yan processes

$p\, N \to \ell^+ \ell^- X$



$e^+ e^- \to h_1\, h_2\, X$

TMDs in SIDIS



$$\mathrm{d}^6\sigma \equiv \frac{\mathrm{d}^6\sigma^{\ell p^\uparrow \to \ell h X}}{\mathrm{d}x_B\, \mathrm{d}Q^2\, \mathrm{d}z_h\, \mathrm{d}^2\boldsymbol{P}_T\, \mathrm{d}\phi_S}$$

$$\boldsymbol{P}_T \simeq \boldsymbol{p}_\perp + z\,\boldsymbol{k}_\perp$$

$$x_B \simeq x \qquad z_h \simeq z$$

TMD factorization holds at large Q^2, and $P_T \approx k_\perp \approx \Lambda_{\rm QCD}$

Two scales: $P_T \ll Q^2$

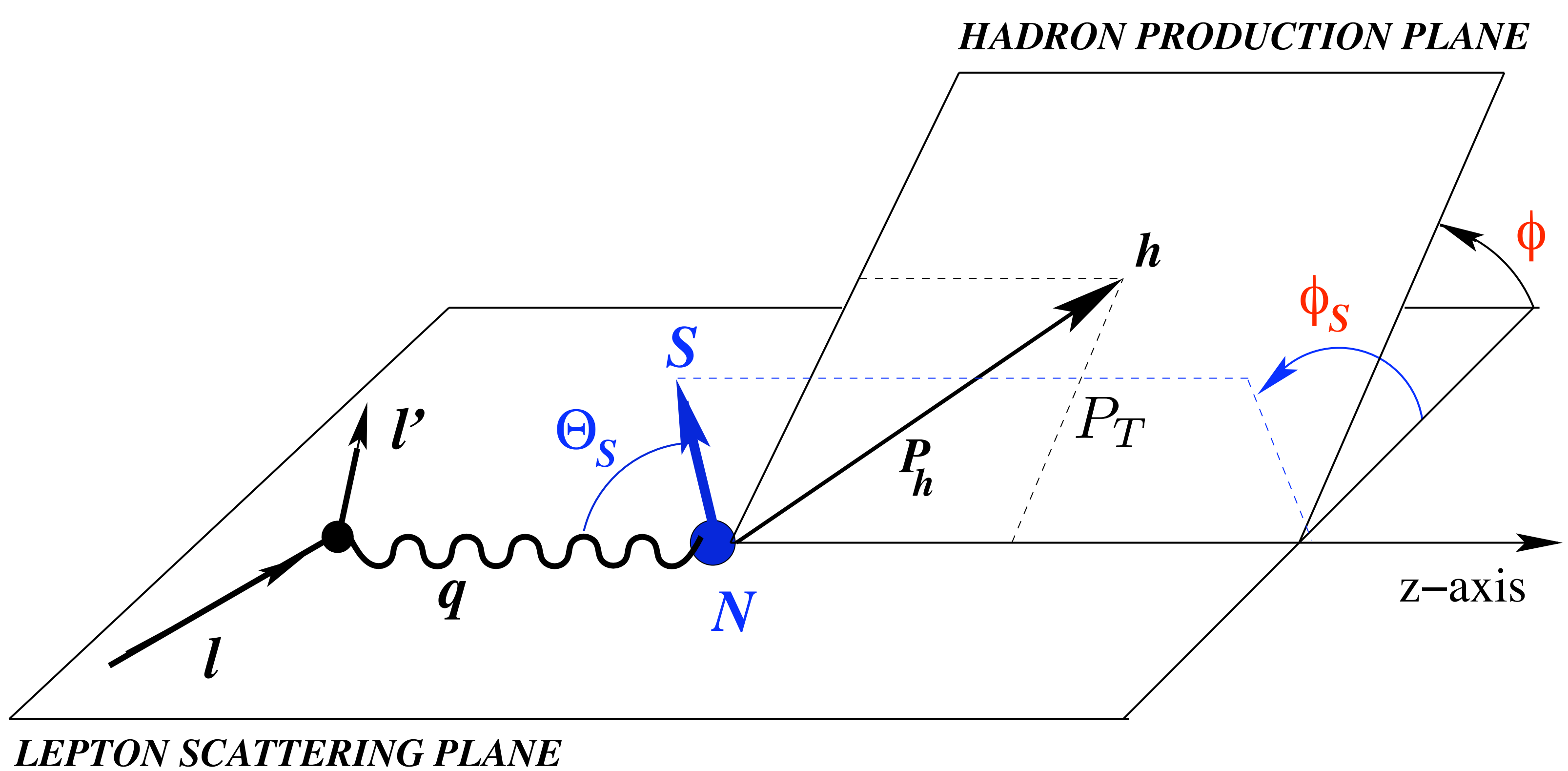
TMD-PDFs hard scattering TMD-FFs

$$\mathrm{d}\sigma^{\ell p \to \ell h X} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes \mathrm{d}\hat{\sigma}^{\ell q \to \ell q}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)$$

(Collins, Soper, Ji, J.P. Ma, Yuan, Qiu, Vogelsang, Collins, Metz...)

$$\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} &= F_{UU} + \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\cos\phi\, F_{UU}^{\cos\phi} + \lambda \frac{1}{Q}\sin\phi\, F_{LU}^{\sin\phi} \\
&+ S_L \left\{ \sin(2\phi)\, F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\sin\phi\, F_{UL}^{\sin\phi} + \lambda \left[F_{LL} + \frac{1}{Q}\cos\phi\, F_{LL}^{\cos\phi} \right] \right\} \\
&+ S_T \Big\{ \underbrace{\sin(\phi-\phi_S)\, F_{UT}^{\sin(\phi-\phi_S)}}_{\text{Sivers}} + \underbrace{\sin(\phi+\phi_S)\, F_{UT}^{\sin(\phi+\phi_S)}}_{\text{Collins}} + \sin(3\phi-\phi_S)\, F_{UT}^{\sin(3\phi-\phi_S)} \\
&+ \frac{1}{Q}\left[\sin(2\phi-\phi_S)\, F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\, F_{UT}^{\sin\phi_S} \right] \\
&+ \lambda \left[\cos(\phi-\phi_S)\, F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q}\left(\cos\phi_S\, F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S)\, F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \Big\}
\end{aligned}$$

the $F_{S_B\,S_T}^{(\cdots)}$ cont the TMDs; plen of Spin Asymmetries



at leading twist there are 8 independent azimuthal modulations, with different combinations of TMDs.

$$F_{UU} \sim \sum_a e_a^2 f_1^a \otimes D_1^a$$

$$F_{LL} \sim \sum_a e_a^2 g_{1L}^a \otimes D_1^a$$

$$F_{UU}^{\cos(2\phi)} \sim \sum_a e_a^2 h_1^{\perp a} \otimes H_1^{\perp a}$$

$$F_{UL}^{\sin(2\phi)} \sim \sum_a e_a^2 h_{1L}^{\perp a} \otimes H_1^{\perp a}$$

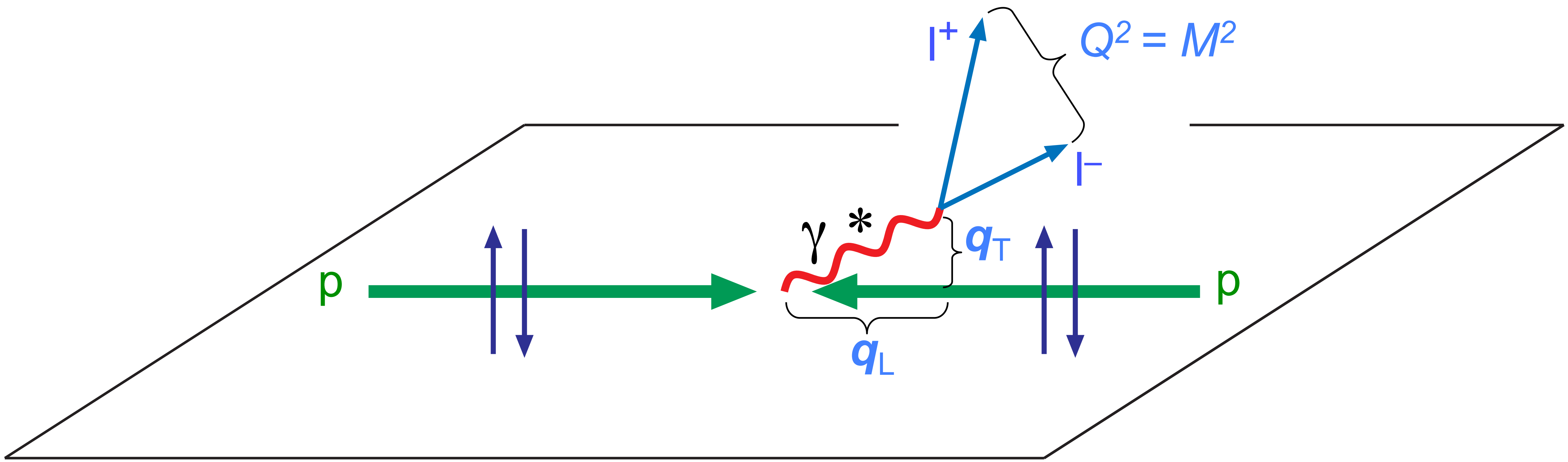
$$\left.\begin{aligned} F_{LT}^{\cos(\phi-\phi_S)} &\sim \sum_a e_a^2 g_{1T}^{\perp a} \otimes D_1^a \\ F_{UT}^{\sin(\phi-\phi_S)} &\sim \sum_a e_a^2 f_{1T}^{\perp a} \otimes D_1^a \end{aligned}\right\} \text{chiral-even TMDs}$$

$$\left.\begin{aligned} F_{UT}^{\sin(\phi+\phi_S)} &\sim \sum_a e_a^2 h_{1T}^{a} \otimes H_1^{\perp a} \\ F_{UT}^{\sin(3\phi-\phi_S)} &\sim \sum_a e_a^2 h_{1T}^{\perp a} \otimes H_1^{\perp a} \end{aligned}\right\} \text{chiral-odd TMDs}$$

integrated $f_1^q(x)$ and $g_{1L}^q(x)$ can be measured in usual DIS

TMDs in Drell-Yan processes

COMPASS, RHIC, Fermilab, NICA, AFTER...



factorization holds, two scales, M^2, and $q_T \ll M$

$$\mathrm{d}\sigma^{D-Y} = \sum_a f_q(x_1, \boldsymbol{k}_{\perp 1}; Q^2) \otimes f_{\bar{q}}(x_2, \boldsymbol{k}_{\perp 2}; Q^2)\, \mathrm{d}\hat{\sigma}^{q\bar{q}\to \ell^+\ell^-}$$

direct product of TMDs, no fragmentation process

Case of one polarized nucleon only

$$\frac{d\sigma}{d^4q\, d\Omega} = \frac{\alpha^2}{\Phi\, q^2}\Big\{(1+\cos^2\theta)\, F_U^1 + (1-\cos^2\theta)\, F_U^2 + \sin 2\theta\cos\phi\, F_U^{\cos\phi} + \sin^2\theta\cos 2\phi\, F_U^{\cos 2\phi}$$

B-M ⊗ B-M

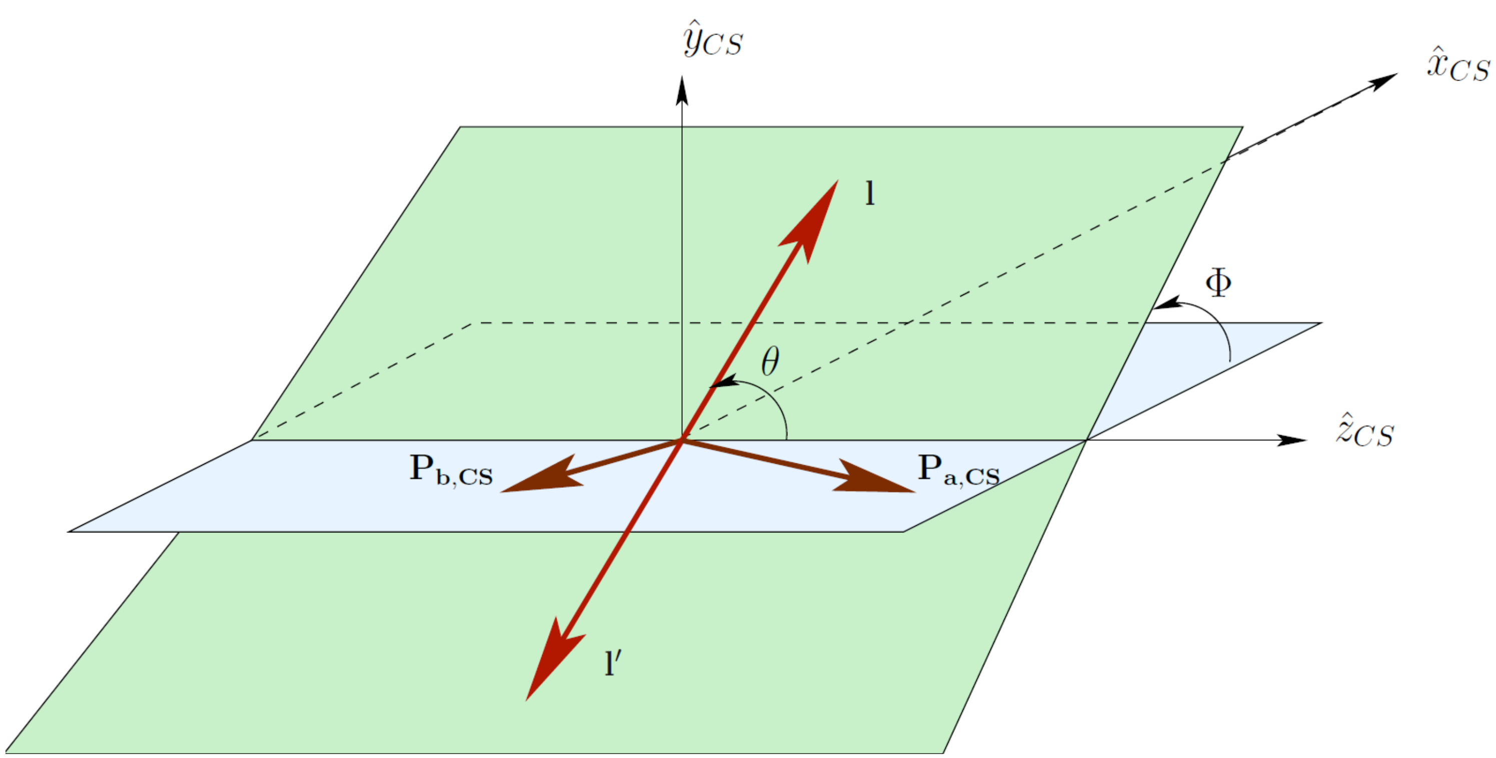
$$+\ S_L\left(\sin 2\theta\sin\phi\, F_L^{\sin\phi} + \sin^2\theta\sin 2\phi\, F_L^{\sin 2\phi}\right)$$

$$+\ S_T\Big[\left(F_T^{\sin\phi_S} + cos^2\theta\, \tilde{F}_T^{\sin\phi_S}\right)\sin\phi_S + \sin 2\theta\Big(\sin(\phi+\phi_S)\, F_T^{\sin(\phi+\phi_S)}$$

$$+\ \sin(\phi-\phi_S)\, F_T^{\sin(\phi-\phi_S)}\Big)$$

Sivers

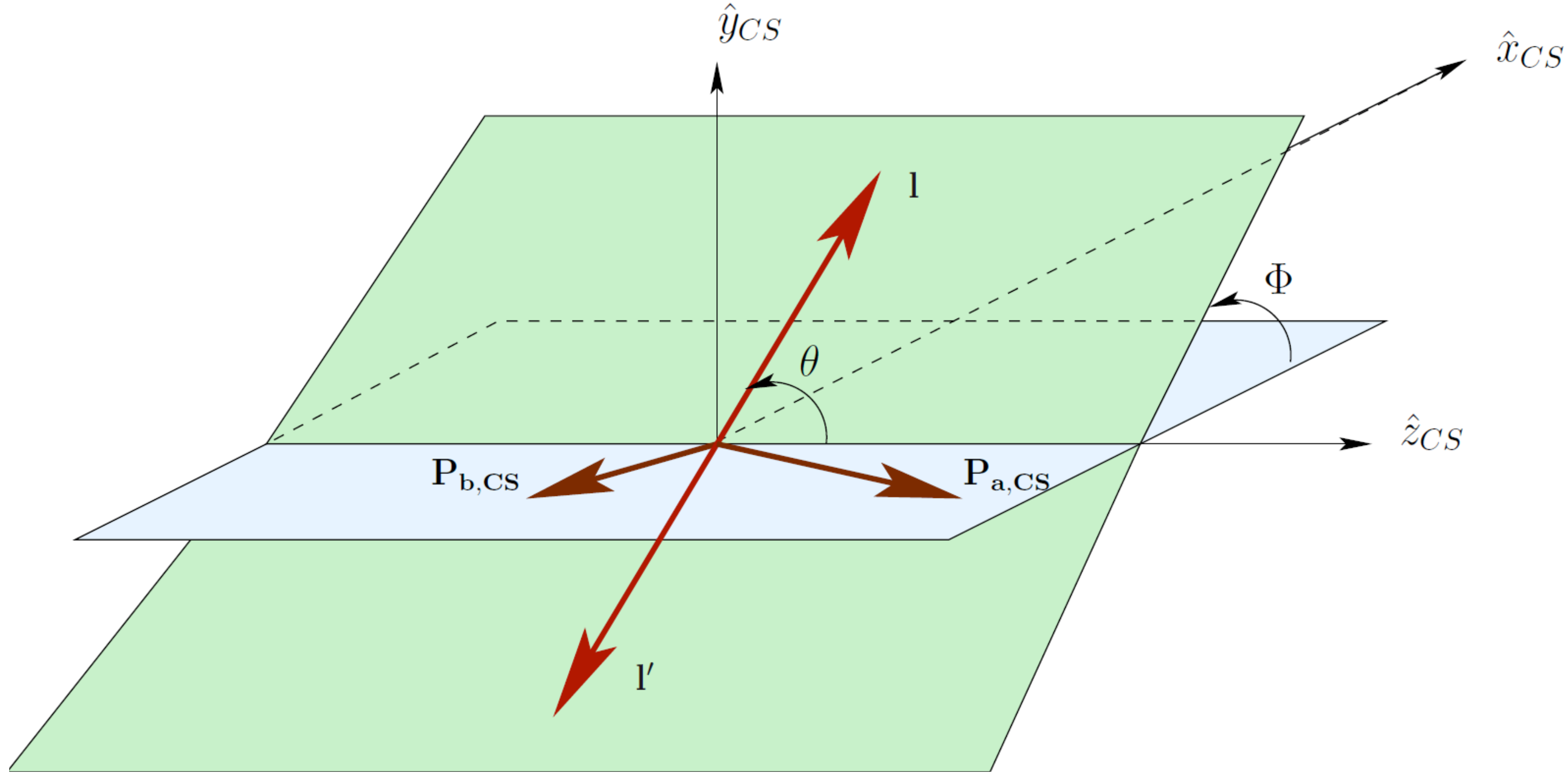
$$+\ \sin^2\theta\left(\sin(2\phi+\phi_S)\, F_T^{\sin(2\phi+\phi_S)} + \sin(2\phi-\phi_S)\, F_T^{\sin(2\phi-\phi_S)}\right)\Big]\Big\}$$



Collins-Soper frame

Unpolarized cross section already very interesting

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1+\lambda\cos^2\theta+\mu\,\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$

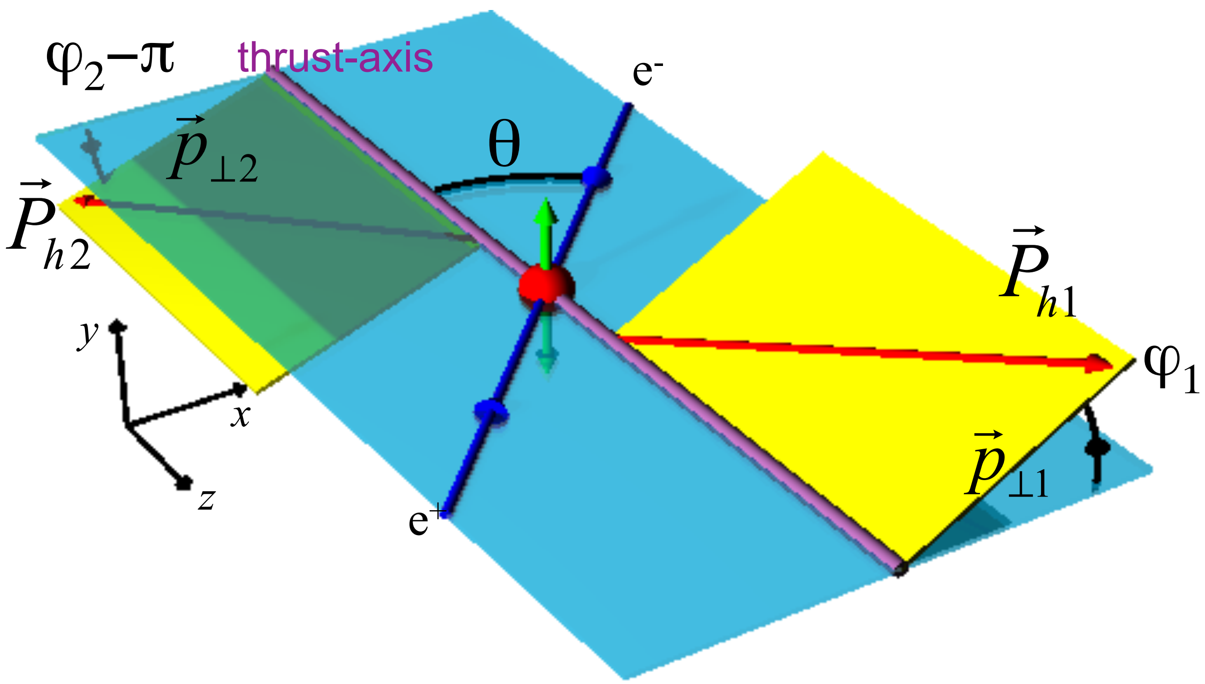


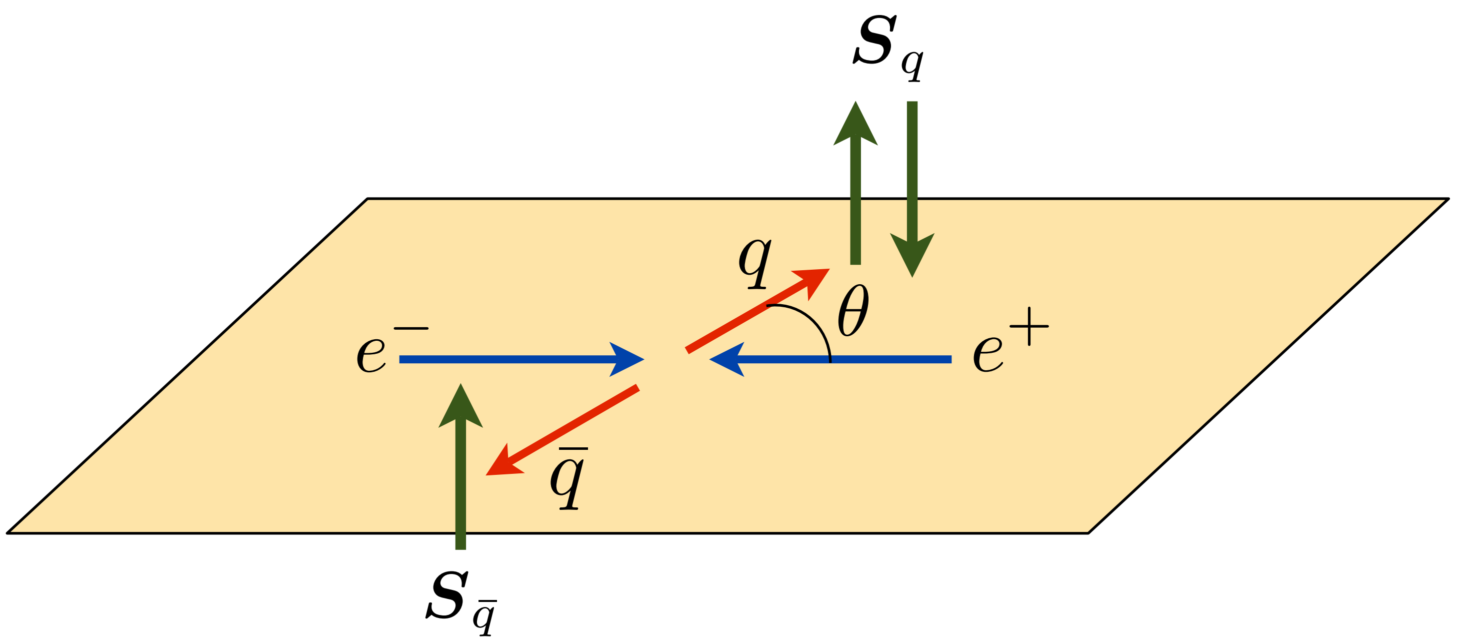
Collins-Soper frame

naive collinear parton model: $\lambda = 1 \quad \mu = \nu = 0$

$$\lambda \neq 1 \quad \mu, \nu \neq 0 \quad 1 - \lambda - 2\nu \neq 0$$

Collins function from e^+e^- processes (Belle, BaBar, BES-III)





$$\frac{\mathrm{d}\sigma^{e^+e^-\to q^\uparrow \bar{q}^\uparrow}}{\mathrm{d}\cos\theta} = \frac{3\pi\alpha^2}{4s}\, e_q^2\, \cos^2\theta \qquad \frac{\mathrm{d}\sigma^{e^+e^-\to q^\downarrow \bar{q}^\uparrow}}{\mathrm{d}\cos\theta} = \frac{3\pi\alpha^2}{4s}\, e_q^2$$

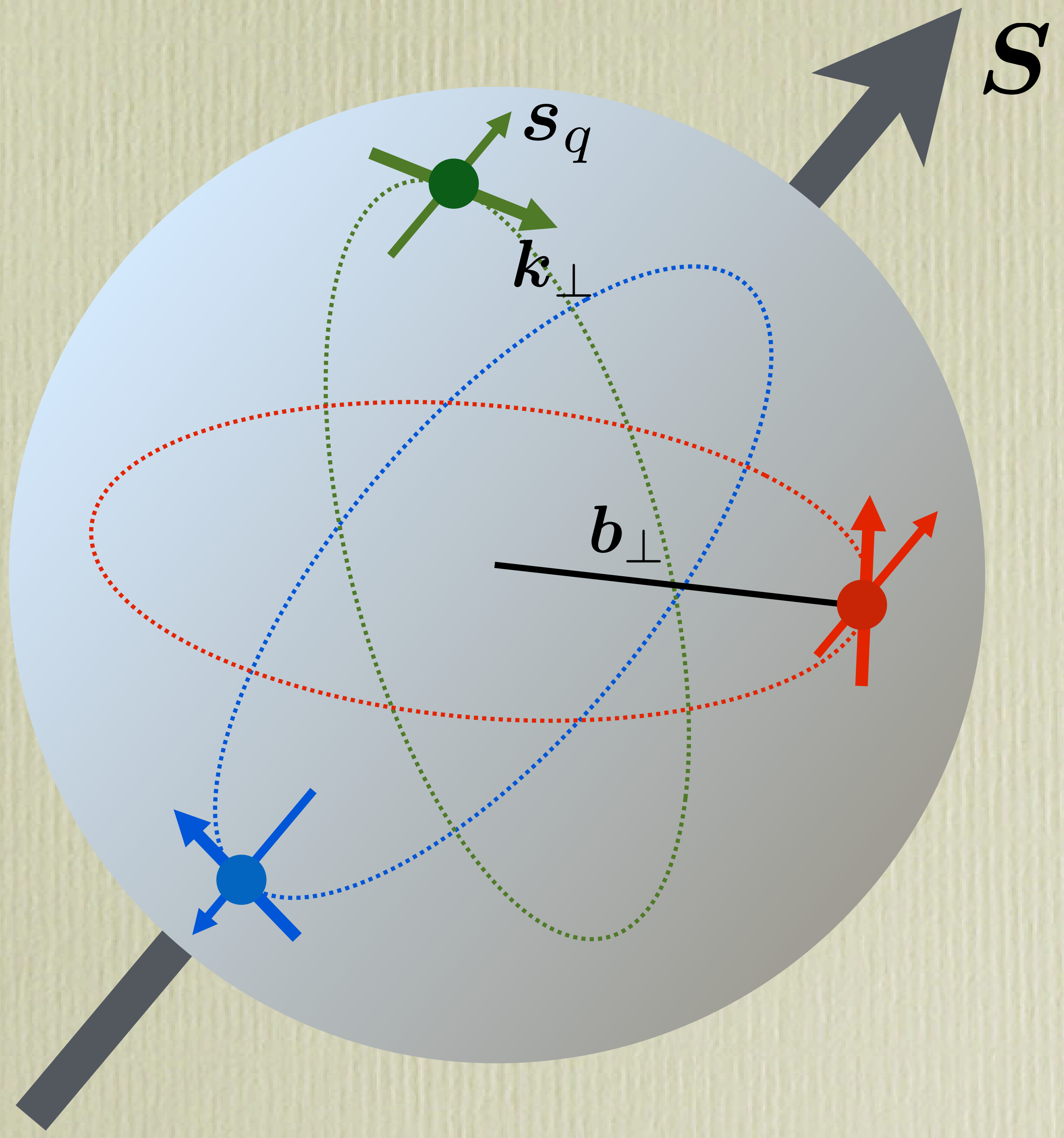
$$A_{12}(z_1, z_2, \theta, \varphi_1+\varphi_2) \equiv \frac{1}{\langle d\sigma\rangle} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1+\varphi_2)}$$

$$= 1 + \frac{1}{4}\frac{\sin^2\theta}{1+\cos^2\theta}\cos(\varphi_1+\varphi_2) \times \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

$$\Delta^N D_{h/q^\uparrow} = \frac{2\, p_\perp}{z M_h} H_1^{\perp q}$$

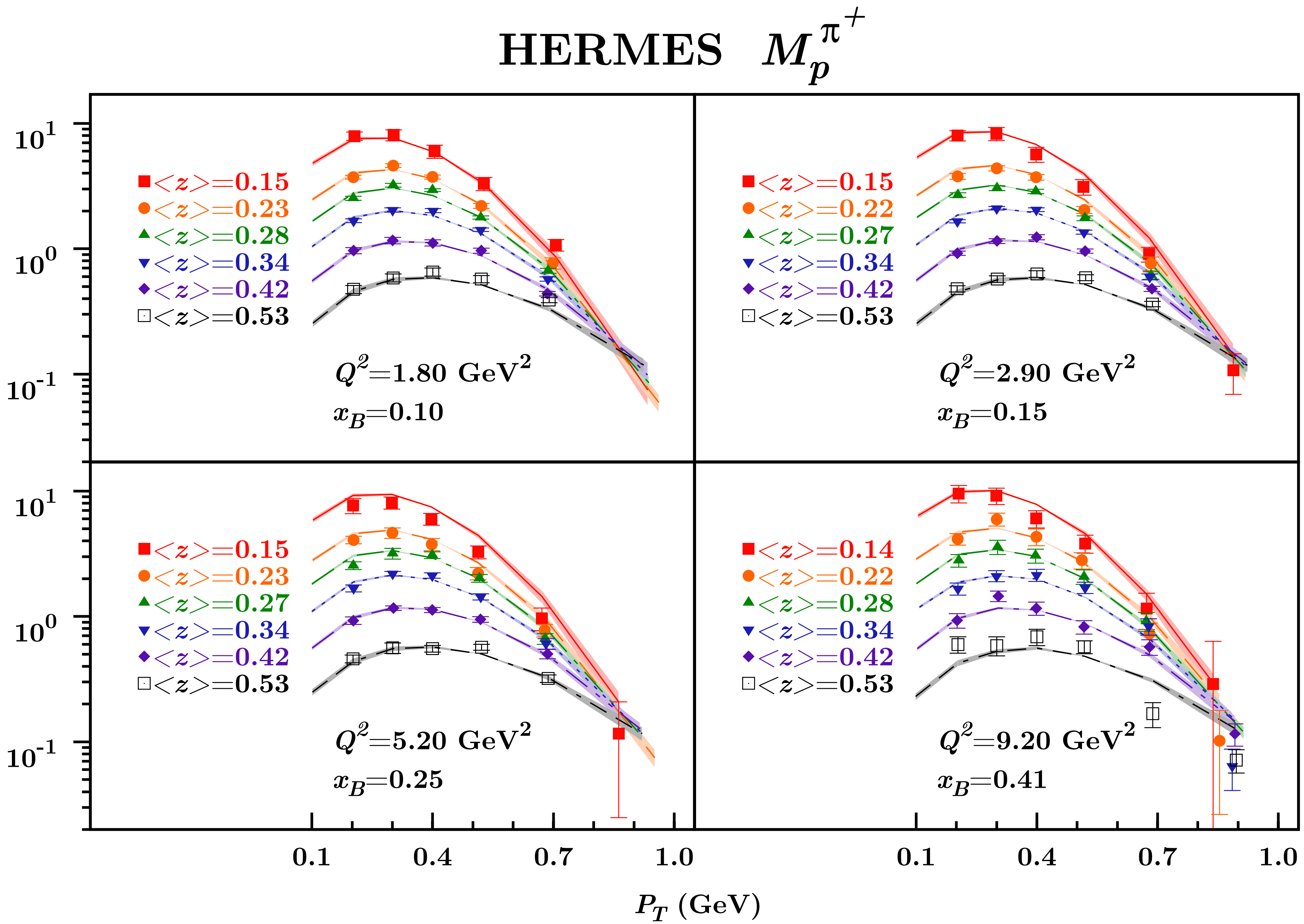
(another similar asymmetry can be measured, A_0)

Do we have experimental evidence of TMD effects?



P_T dependence of unpolarised SIDIS multiplicities

$$M_n^h(x_B, Q^2, z_h, P_T) \equiv \frac{1}{\dfrac{d^2\sigma^{DIS}(x_B, Q^2)}{dx_B\, dQ^2}} \frac{d^4\sigma(x_B, Q^2, z_h, P_T)}{dx_B\, dQ^2\, dz_h\, dP_T}$$



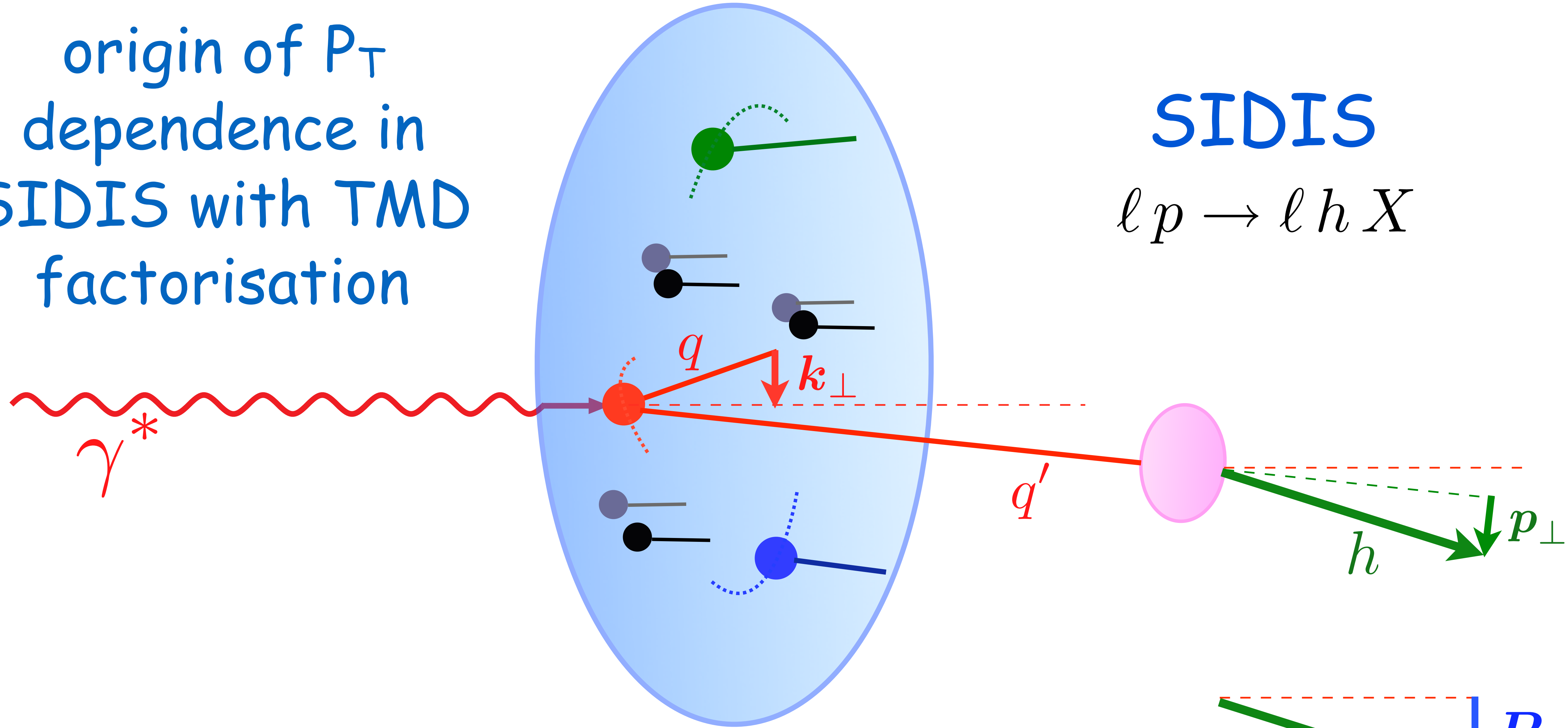
A. Airapetian et al. (HERMES Collaboration), Phys. Rev. D87, 074029 (2013)

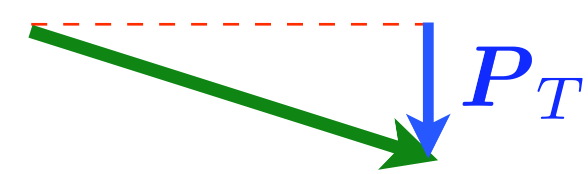
$$\boldsymbol{P}_T \simeq \boldsymbol{p}_\perp + z\,\boldsymbol{k}_\perp$$

origin of P_T dependence in SIDIS with TMD factorisation

SIDIS

$$\ell\, p \to \ell\, h\, X$$

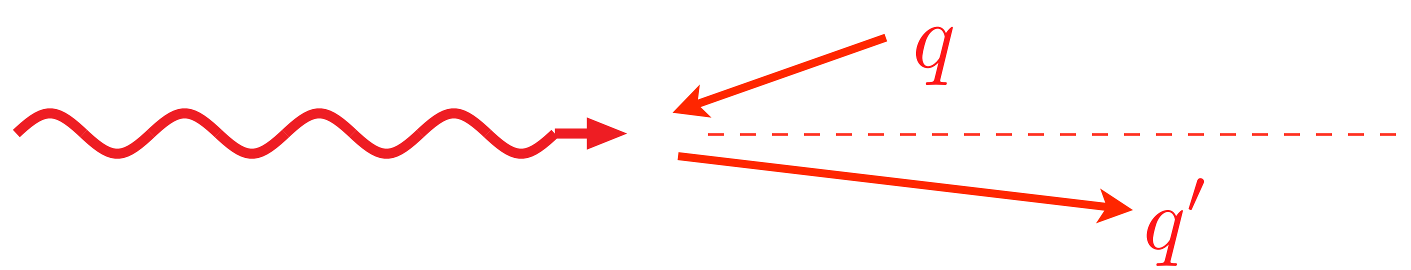




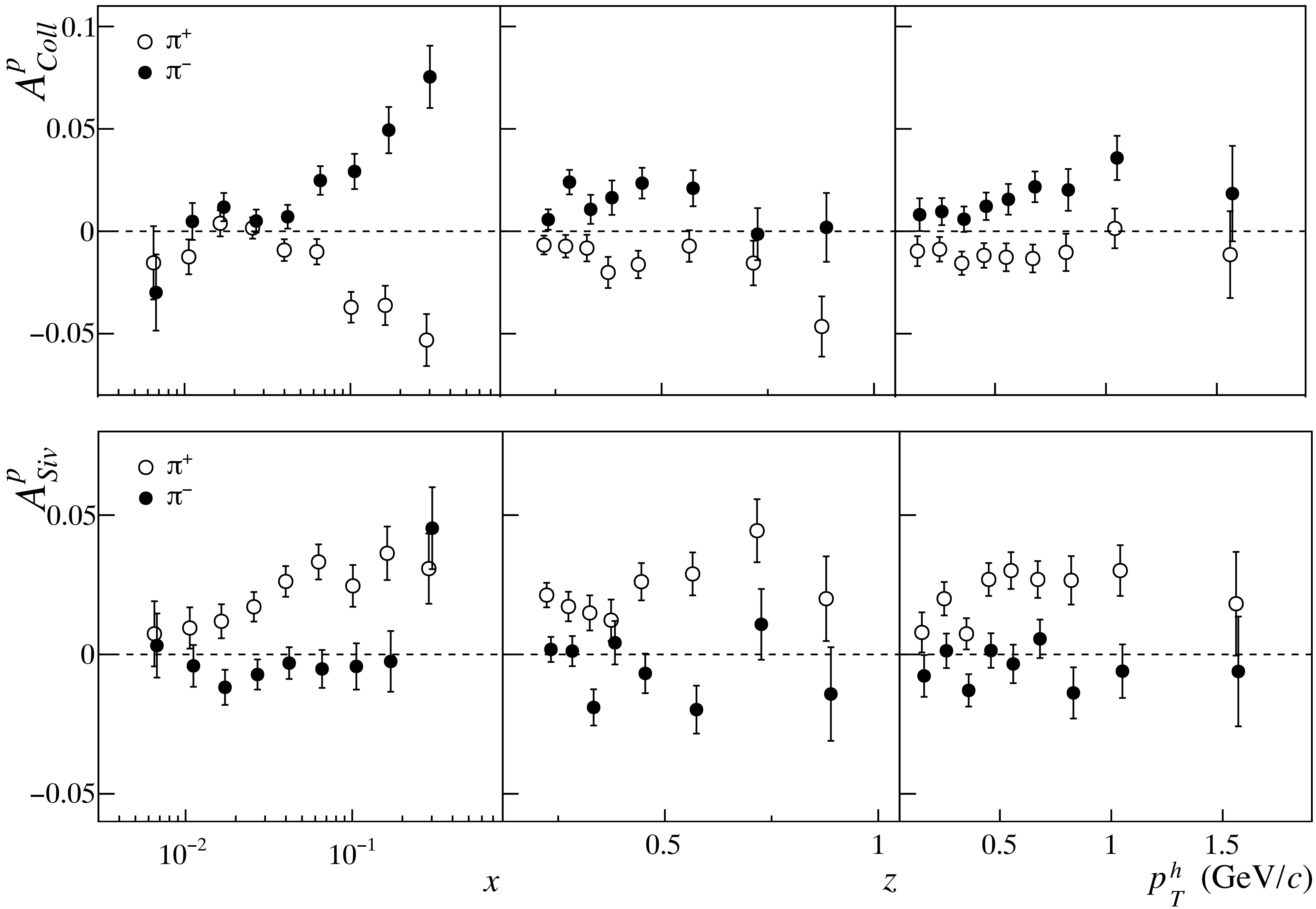
$$\Lambda_{\rm QCD} \simeq k_\perp \simeq P_T \ll Q$$

$$\boldsymbol{P}_T \simeq \boldsymbol{p}_\perp + z_h\, \boldsymbol{k}_\perp$$

elementary interaction: $\gamma^*\, q \to q'$



Clear evidence for Sivers and Collins effects from SIDIS data (HERMES, COMPASS, JLab)



COMPASS data, Phys. Lett. B744 (2015) 250

origin of Sivers effect in SIDIS - $F_{UT}^{\sin(\phi-\phi_S)}$

$$\mathrm{d}\sigma^{\uparrow,\downarrow} = \sum_q f_{q/p^{\uparrow,\downarrow}}(x, \boldsymbol{k}_\perp; Q^2) \otimes \mathrm{d}\hat{\sigma}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_{h/q}(z, \boldsymbol{p}_\perp; Q^2)$$

$$f_{q/p^{\uparrow,\downarrow}}(x, \boldsymbol{k}_\perp) = f_{q/p}(x, k_\perp) \pm \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, k_\perp)\, \boldsymbol{S} \cdot (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}_\perp)$$

$$\left(\Delta^N f_{q/p^\uparrow} = -\frac{2k_\perp}{M} f_{1T}^{\perp q} \right)$$

$$\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow = \sum_q \Delta^N f_{q/p^\uparrow}(x, k_\perp) \underbrace{\boldsymbol{S} \cdot (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}_\perp)}_{\sin(\varphi - \phi_S)} \otimes \mathrm{d}\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes D_{h/q}(z, \boldsymbol{p}_\perp)$$

no SSA if $\mathbf{k}_\perp = 0$!

measured quantity

$$2\langle \sin(\phi - \phi_S) \rangle = A_{UT}^{\sin(\phi-\phi_S)}$$

$$\equiv 2 \frac{\int \mathrm{d}\phi\, \mathrm{d}\phi_S \, [\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow] \, \sin(\phi - \phi_S)}{\int \mathrm{d}\phi\, \mathrm{d}\phi_S \, [\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow]}$$

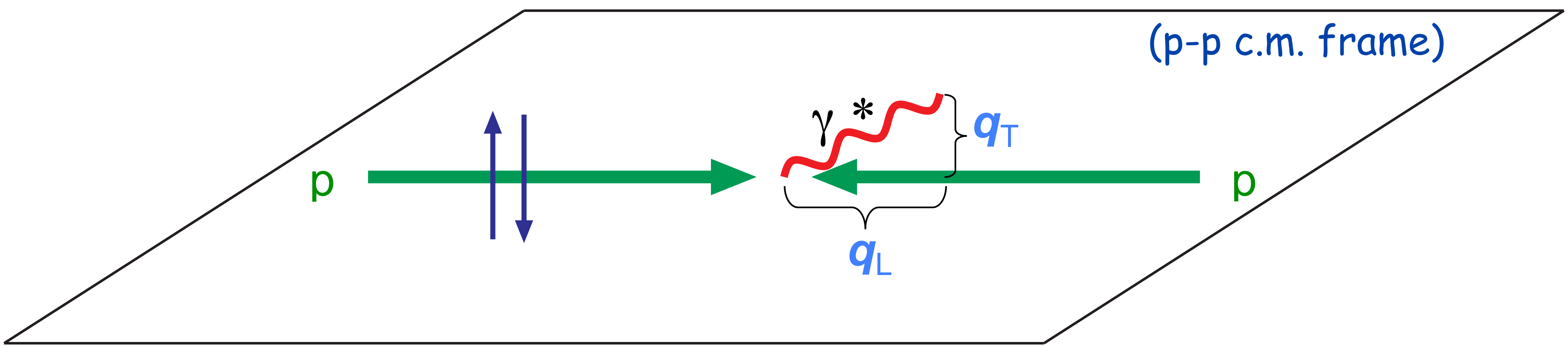
origin of Sivers effect in DY processes

By looking at the $d^4\sigma/d^4q$ cross section one can single out the Sivers effect in D-Y processes

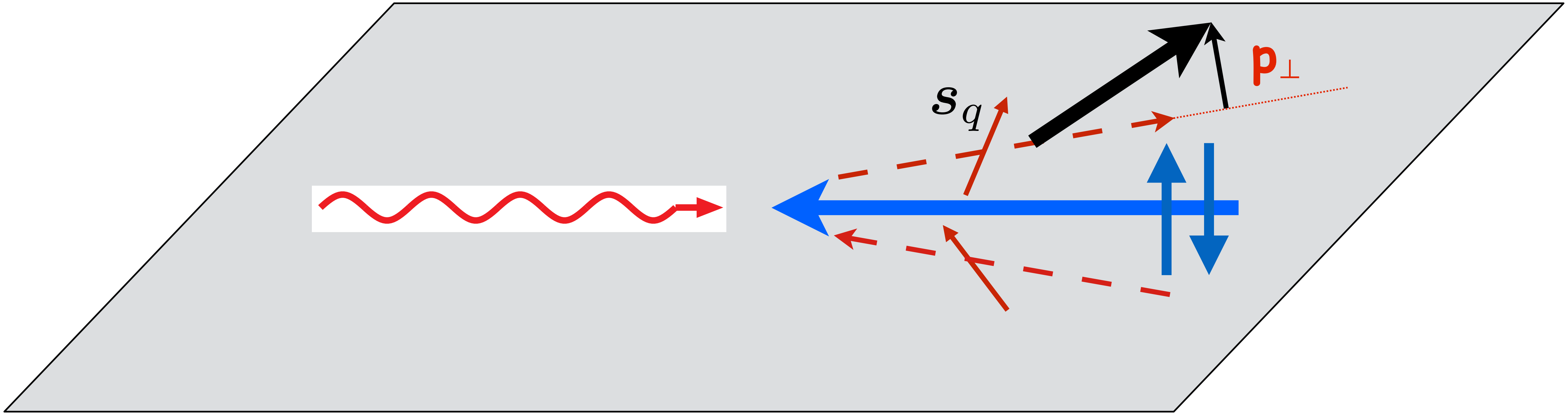
$$\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow \propto \sum_q \Delta^N f_{q/p^\uparrow}(x_1, \boldsymbol{k}_{\perp 1}) \otimes f_{\bar{q}/p}(x_2, k_{\perp 2}) \otimes \mathrm{d}\hat{\sigma}$$

$q = u, \bar{u}, d, \bar{d}, s, \bar{s}$

$$A_N^{\sin(\phi_S - \phi_\gamma)} \equiv \frac{2\int_0^{2\pi} \mathrm{d}\phi_\gamma \, [\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow] \, \sin(\phi_S - \phi_\gamma)}{\int_0^{2\pi} \mathrm{d}\phi_\gamma \, [\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow]}$$



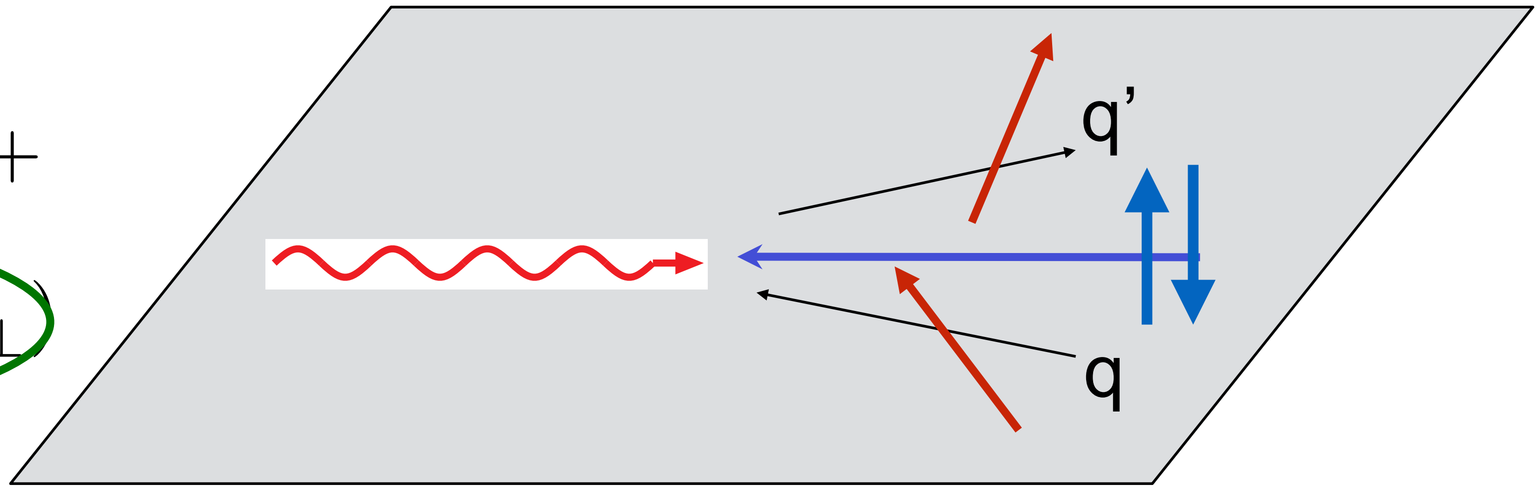
Collins effect in SIDIS



$$
\begin{aligned}
D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) &= D_{h/q}(z,p_\perp) + \frac{1}{2}\,\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp) \\
&= D_{h/q}(z,p_\perp) + \frac{p_\perp}{zM_h}\,H_1^{\perp q}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)
\end{aligned}
$$

origin of Collins asymmetry in SIDIS - $F_{UT}^{\sin(\phi+\phi_S)}$

$$D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) = D_{h/p}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$



$$d\sigma^\uparrow - d\sigma^\downarrow = \sum_q h_{1q}(x,k_\perp)\otimes d\Delta\hat{\sigma}(y,\boldsymbol{k}_\perp)\otimes\Delta^N D_{h/q^\uparrow}(z,\boldsymbol{p}_\perp)$$

no SSA if $\mathbf{p}_\perp = 0$!

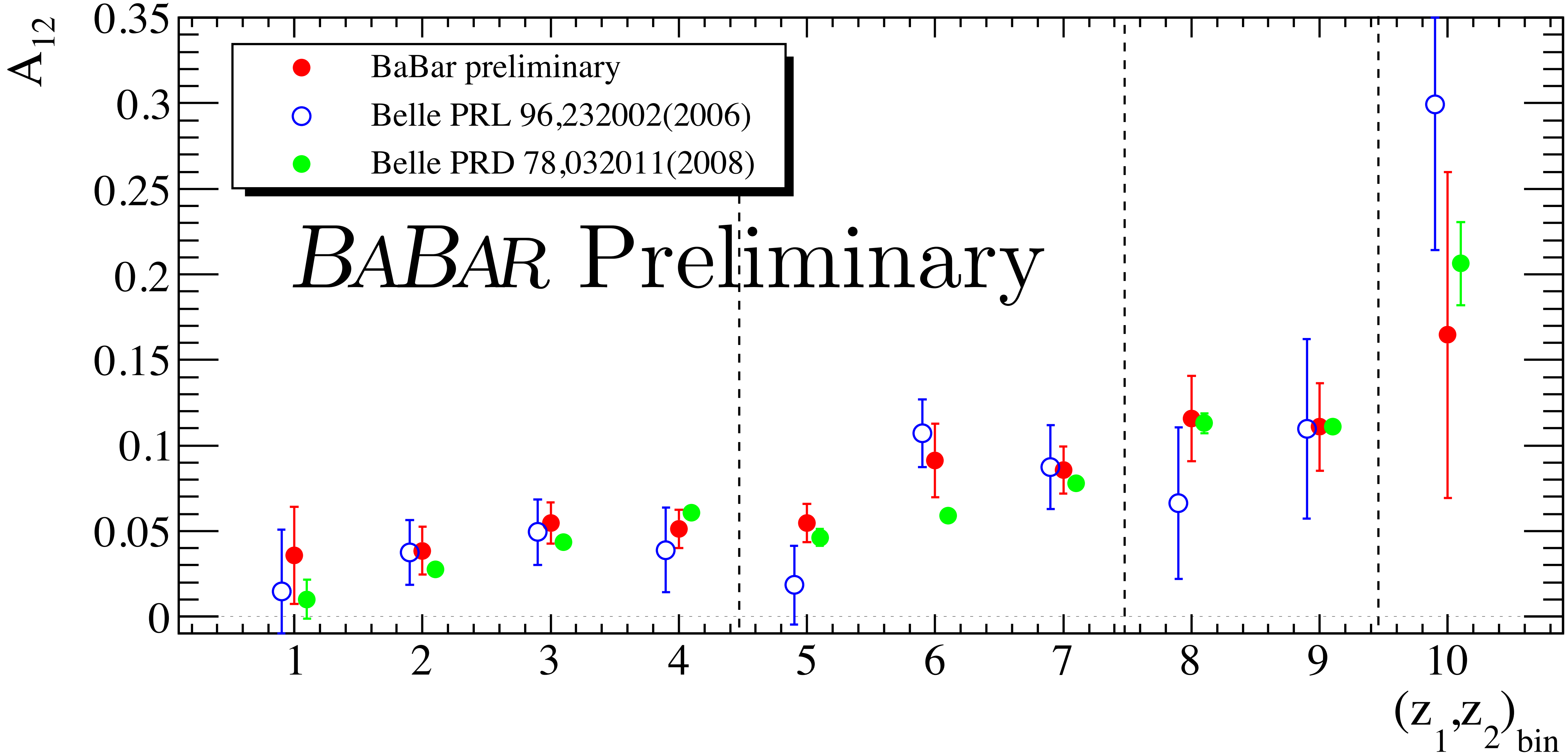
$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2\,\frac{\int d\phi\, d\phi_S\,[d\sigma^\uparrow - d\sigma^\downarrow]\,\sin(\phi+\phi_S)}{\int d\phi\, d\phi_S\,[d\sigma^\uparrow + d\sigma^\downarrow]}$$

$$d\Delta\hat{\sigma} = d\hat{\sigma}^{\ell q^\uparrow\to\ell q^\uparrow} - d\hat{\sigma}^{\ell q^\uparrow\to\ell q^\downarrow}$$

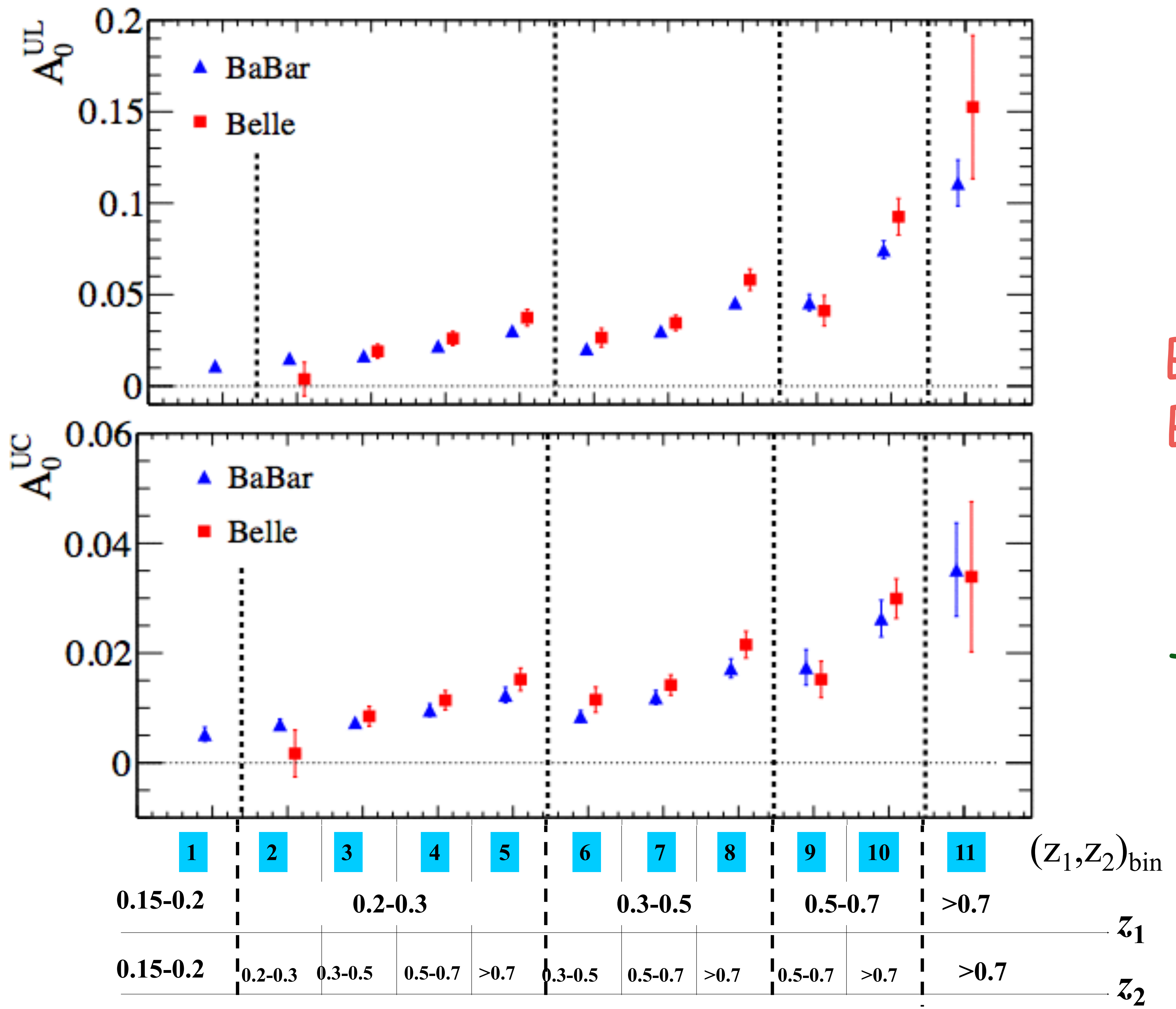
Collins effect in SIDIS couples to transversity

independent evidence for Collins effect from e^+e^- data at Belle, BaBar and BES-III

$$A_{12}(z_1, z_2) \sim \Delta^N D_{h_1/q^\uparrow}(z_1) \otimes \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)$$



I. Garzia, arXiv:1201.4678

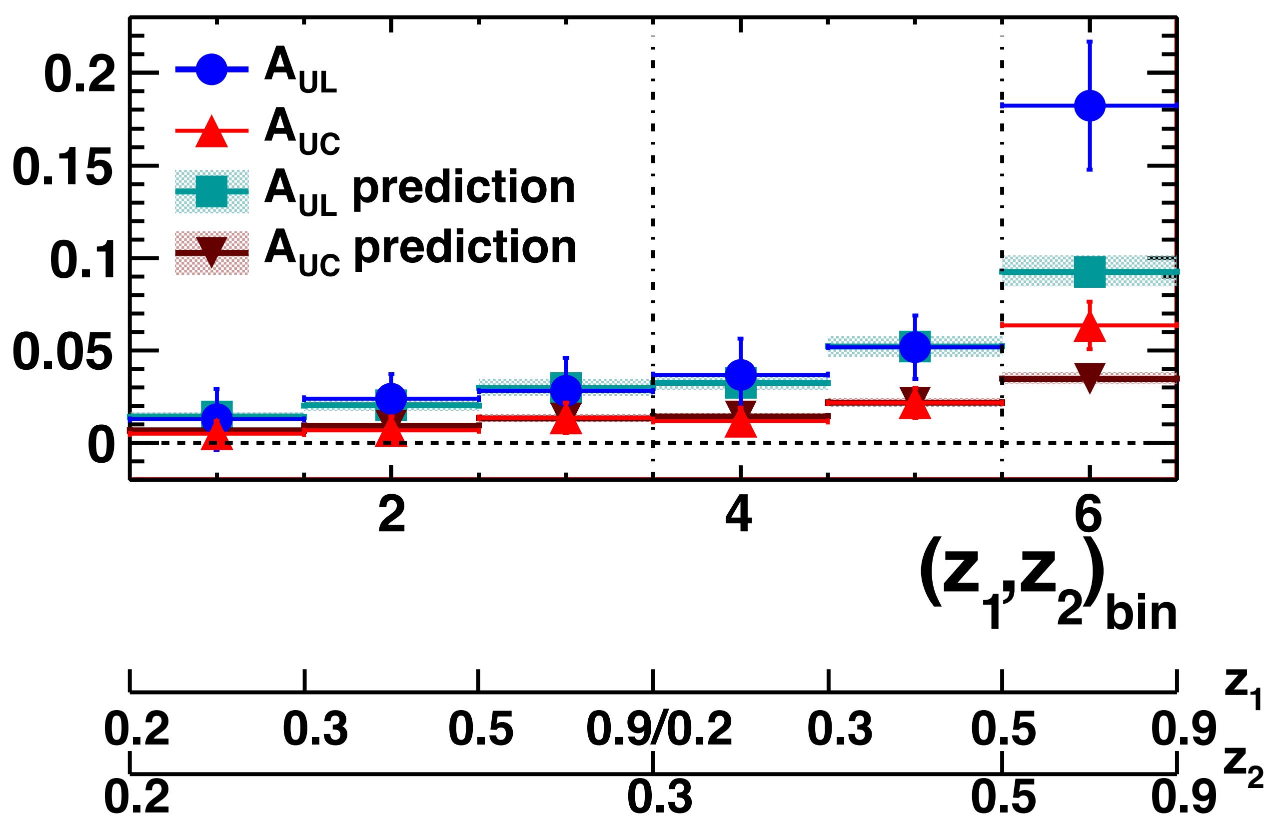


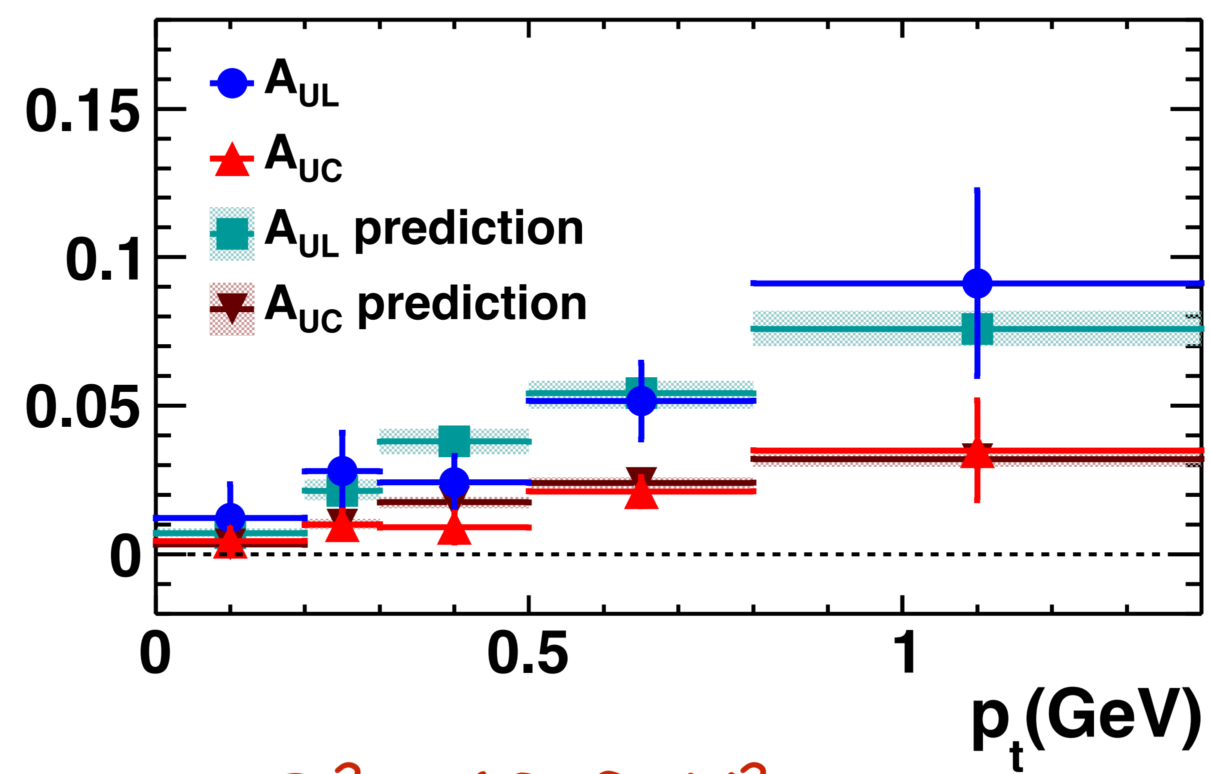
BaBar and Belle data on A_0

(I. Garzia talk at TMDe2015)

a similar asymmetry just measured by BES-III

(arXiv 1507:06824)



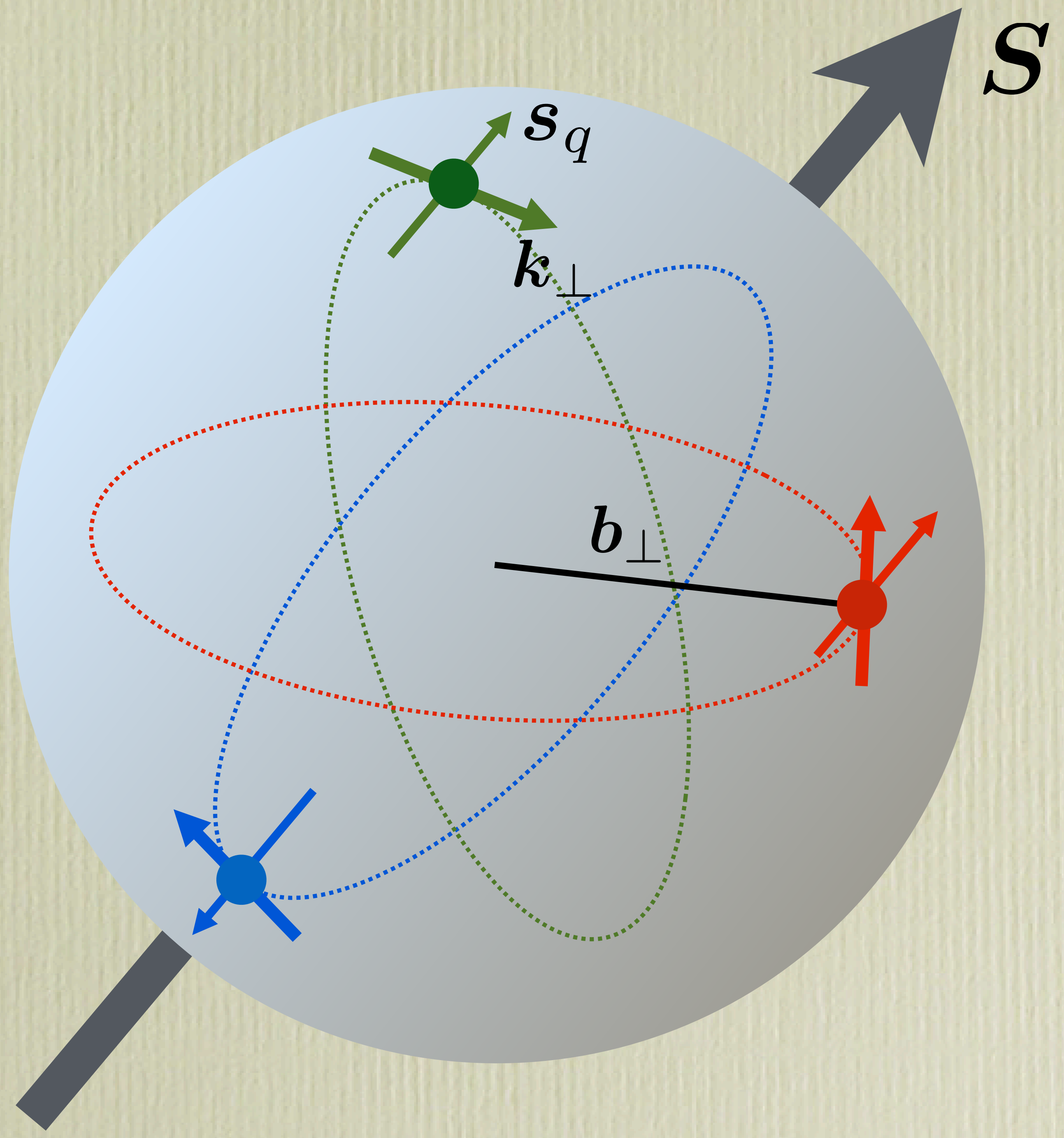


$Q^2 = 13\ GeV^2$

Collins effect clearly observed both in SIDIS and e+e- processes, by several Collaborations

In general clear evidence for quark intrinsic motion; how do we extract information on TMDs from data?

What do we learn from data?

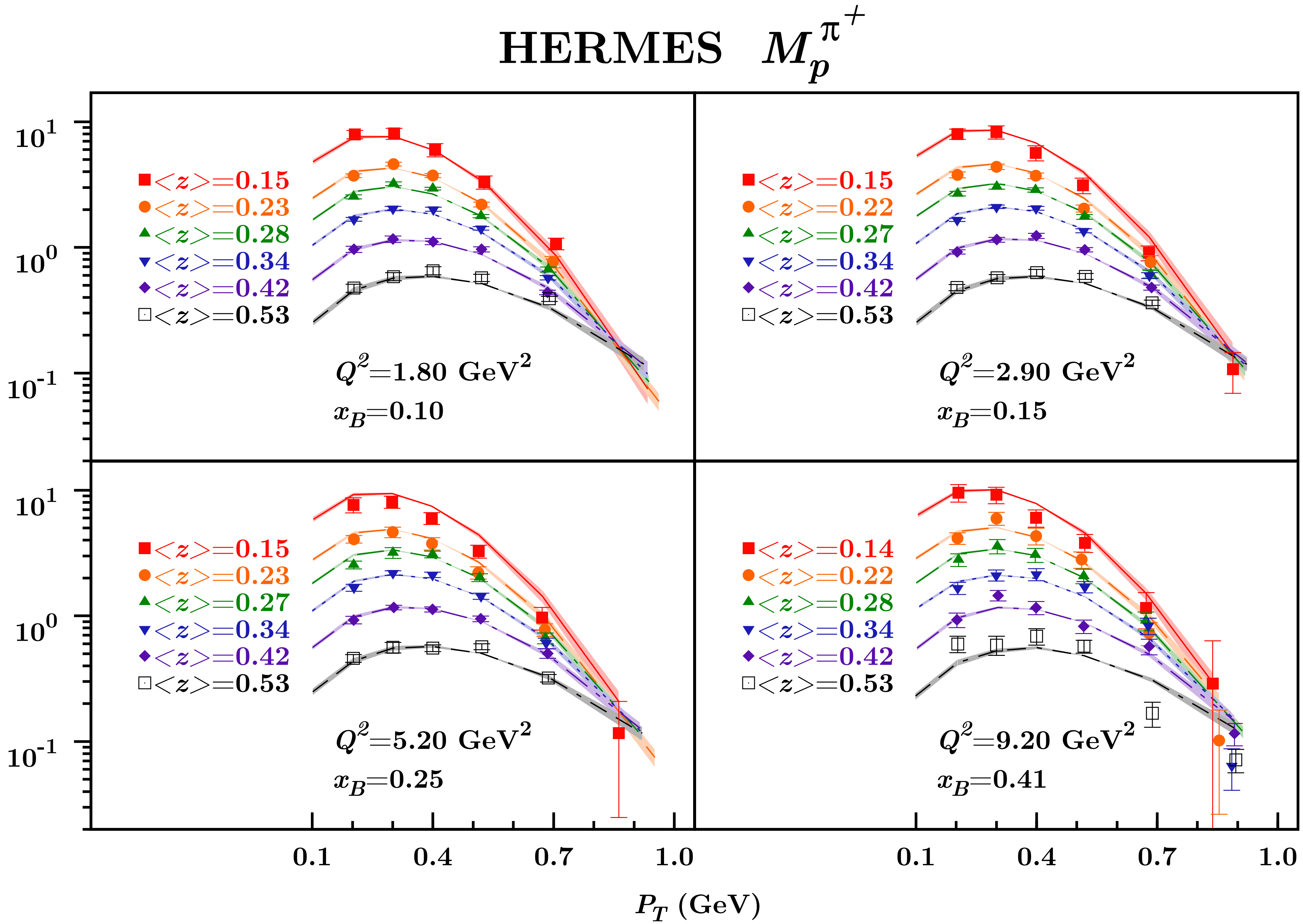


TMD extraction from data - first phase

(simple parameterisation, no TMD evolution, limited number of parameters, ...)

unpolarised TMDs - fit of SIDIS multiplicities

(M.A, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (2014) 005)



measured quantity

$$M_n^h(x_{_B}, Q^2, z_h, P_T) \equiv \frac{1}{\dfrac{d^2\sigma^{DIS}(x_{_B}, Q^2)}{dx_{_B}\, dQ^2}} \, \frac{d^4\sigma(x_{_B}, Q^2, z_h, P_T)}{dx_{_B}\, dQ^2\, dz_h\, dP_T}$$

in TMD factorisation at order ($k_\perp$/Q)

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_{_B}\, dQ^2\, dz_h\, dP_T^2} = \frac{2\,\pi^2\alpha^2}{(x_{_B}\, s)^2}\, \frac{[1+(1-y)^2]}{y^2}$$

elementary interaction, lq $\to$lq

$$\times \sum_q e_q^2 \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{p}_\perp\, \delta^{(2)}\Big(\boldsymbol{P}_T - z_h\boldsymbol{k}_\perp - \boldsymbol{p}_\perp\Big)\, f_{q/p}(x, k_\perp)\, D_{h/q}(z, p_\perp)$$

$$\equiv \frac{2\,\pi^2\alpha^2}{(x_{_B}\, s)^2}\, \frac{[1+(1-y)^2]}{y^2}\, F_{UU}\,.$$

assume simple x and $k_\perp$ factorization and a gaussian $k_\perp$ dependence

$$f_{q/p}(x, k_\perp) = f_{q/p}(x)\, \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z)\, \frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

then $$F_{UU} = \sum_q e_q^2\, f_{q/p}(x_{_B})\, D_{h/q}(z_h)\, \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$$

the good fit of the data shows a clear support for a gaussian distribution

$$\frac{d^2 n^h(x_{\scriptscriptstyle B}, Q^2, z_h, P_T)}{dz_h\, dP_T^2} = \frac{1}{2P_T} M_n^h(x_{\scriptscriptstyle B}, Q^2, z_h, P_T) = \frac{\pi \sum_q e_q^2 \, f_{q/p}(x_{\scriptscriptstyle B})\, D_{h/q}(z_h)}{\sum_q e_q^2 \, f_{q/p}(x_{\scriptscriptstyle B})} \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi \langle P_T^2\rangle}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2 \, \langle k_\perp^2\rangle$$

two correlated parameters

$$\langle k_\perp^2\rangle = 0.57 \pm 0.08 \ \mathrm{GeV}^2 \qquad \langle p_\perp^2\rangle = 0.12 \pm 0.01 \ \mathrm{GeV}^2$$

(M.A, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (2014) 005)

a similar analysis performed by Signori, Bacchetta, Radici, Schnell, JHEP 1311 (2013) 194; it also assumes gaussian behaviour

extraction of u and d Sivers functions - first phase

measured quantity

$$A_{UT}^{\sin(\phi_h-\phi_S)} = 2\,\frac{\int d\phi_S\, d\phi_h\, [d\sigma^\uparrow - d\sigma^\downarrow]\, \sin(\phi_h-\phi_S)}{\int d\phi_S\, d\phi_h\, [d\sigma^\uparrow + d\sigma^\downarrow]}$$

TMD factorization at $\mathcal{O}(k_\perp/Q)$

$$\frac{d\sigma^{\ell p^\uparrow \to \ell h X}}{dx_{_B}\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T} = \sum_q \boxed{e_q^2} \int d^2\boldsymbol{k}_\perp\, f_{q/p^\uparrow}(x, k_\perp) \boxed{\frac{2\pi\alpha^2}{x^2 s^2}\, \frac{\hat{s}^2+\hat{u}^2}{Q^4}} D_{h/q}(z, p_\perp)$$

$$f_{q/p^\uparrow}(x, \boldsymbol{k}_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2} \boxed{\Delta^N f_{q/p^\uparrow}(x, k_\perp)\, \boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)}$$

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp \boxed{\Delta^N f_{q/p^\uparrow}(x, k_\perp) \sin(\varphi-\phi_S)} \boxed{\frac{d\hat{\sigma}^{\ell q\to \ell q}}{dQ^2}} D_q^h(z, p_\perp) \sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, f_{q/p}(x, k_\perp)\, \frac{d\hat{\sigma}^{\ell q\to \ell q}}{dQ^2}\, D_q^h(z, p_\perp)}$$

two different notations $\quad \Delta^N f_{q/p^\uparrow} = -\frac{2\,k_\perp}{M_p}\, f_{1T}^{\perp q}$

simple parameterisations

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp, Q) = 2\,\mathcal{N}(x)\,h(k_\perp)\,\underbrace{f_{q/p}(x, Q)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}}_{f_{q/p}(x, k_\perp)}$$

$$\mathcal{N}_q(x) = N_q\,x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\,e^{-k_\perp^2/M_1^2}$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

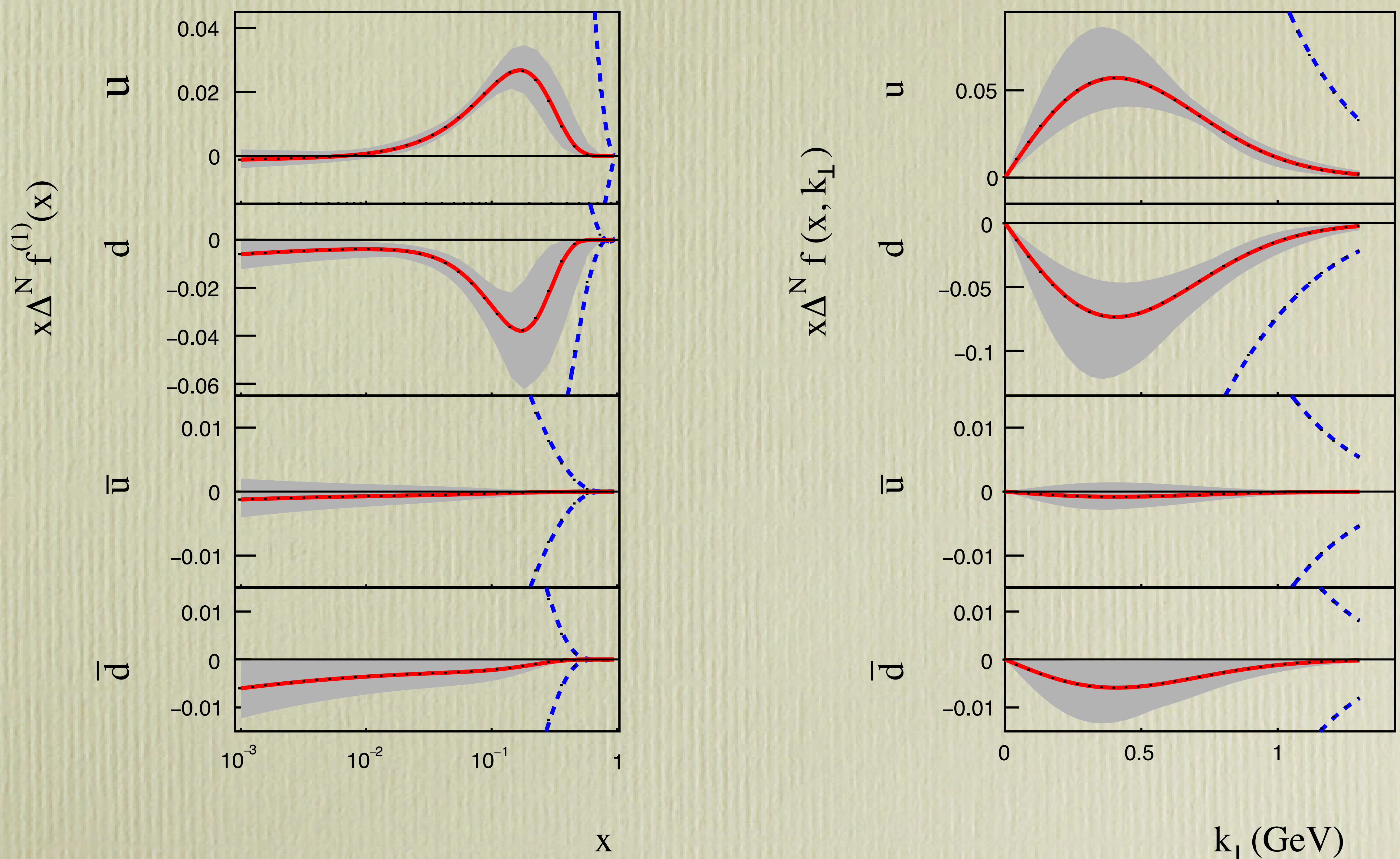
Q^2 evolution only taken into account in the collinear part (usual DGLAP PDF evolution)

M.A, M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, Phys. Rev. D71 (2005) 074006; Eur. Phys. J. A39 (2009) 89

(results in agreement with those of several other groups)

most recent extraction of the Sivers functions

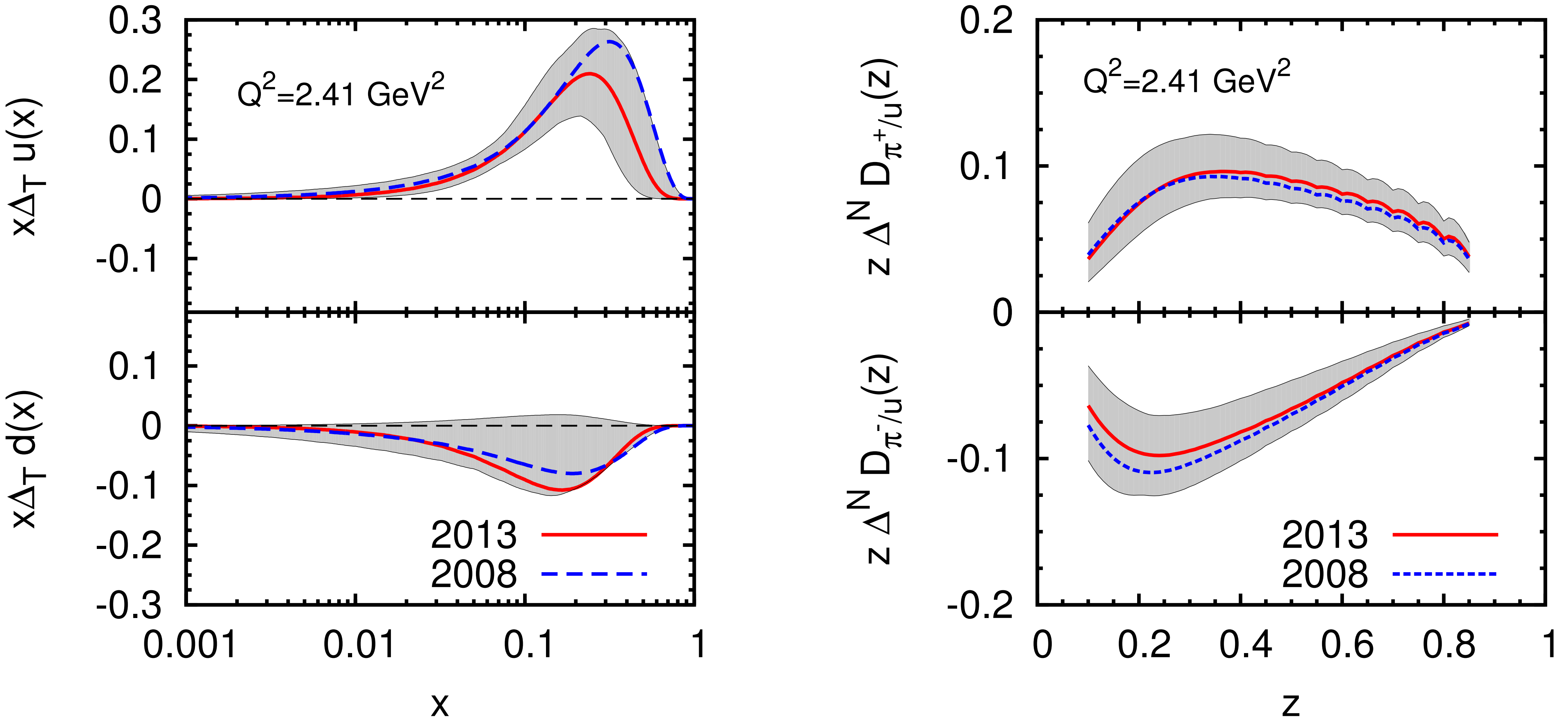
M.A, M. Boglione, U. D'Alesio, F. Murgia, A. Prokudin, JHEP 1704 (2017) 046



$$\Delta^N f_q^{(1)}(x,Q) = \int d^2\boldsymbol{k}_\perp \, \frac{k_\perp}{4M_p} \, \Delta^N f_{q/p^\uparrow}(x,k_\perp;Q) = -f_{1T}^{\perp(1)}(x,Q)$$

TMD extraction: transversity and Collins functions - first phase

M. A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, PRD 87 (2013) 094019

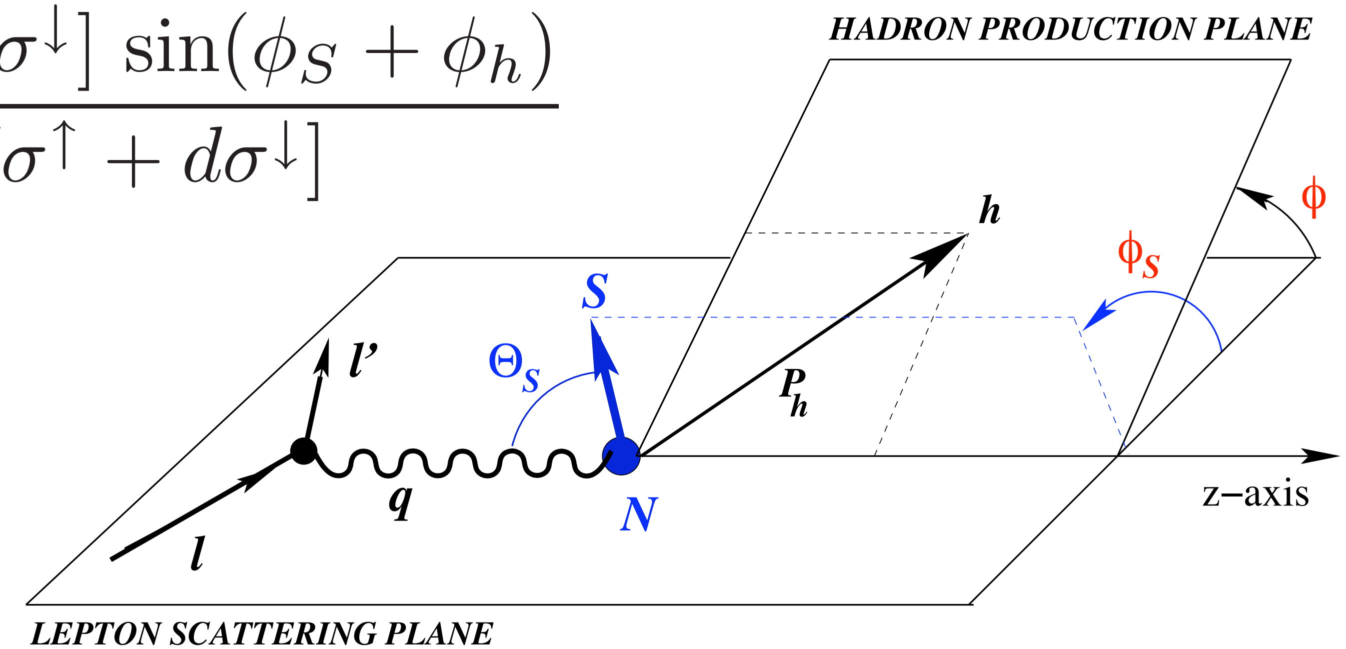


SIDIS and e+e- data, simple parameterization, no TMD evolution, agreement with extraction using di-hadron FF

(recent papers by Bacchetta, Courtoy, Guagnelli, Radici, JHEP 1505 (2015) 123; Kang, Prokudin, Sun, Yuan, Phys. Rev. D91 (2015) 071501; Phys. Rev. D93 (2016) 014009)

measured quantities

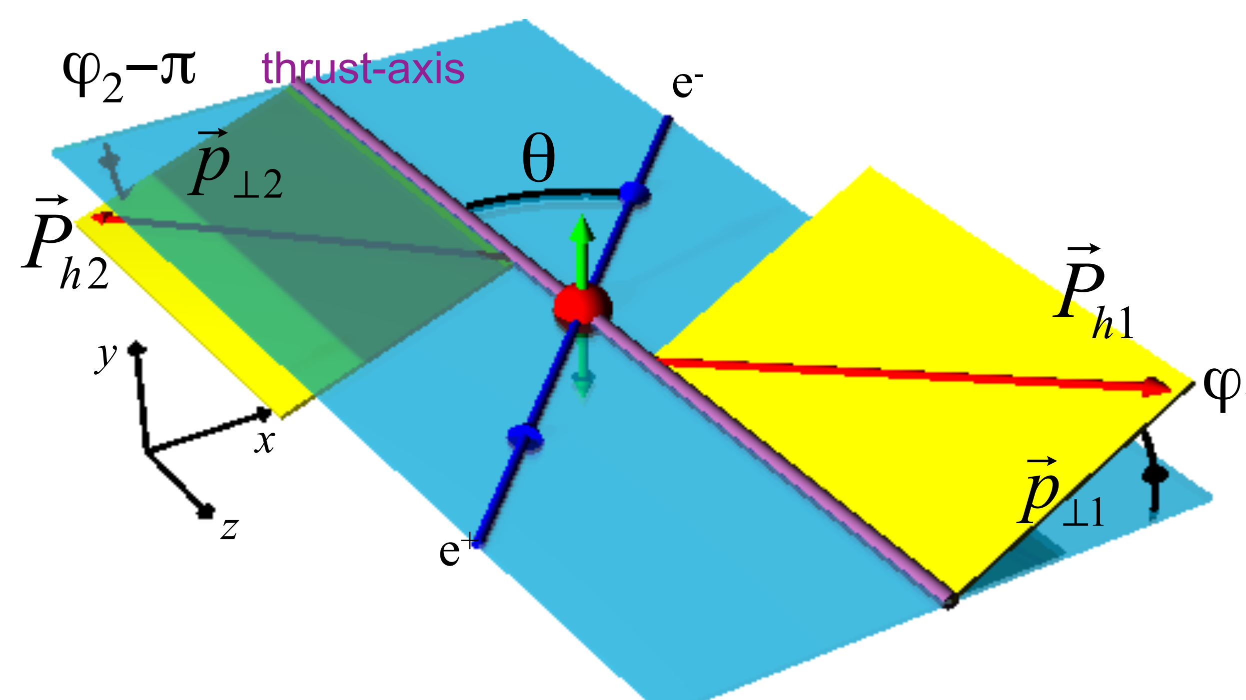
$$A_{UT}^{\sin(\phi_S+\phi_h)} = 2\,\frac{\int d\phi_S\, d\phi_h\, [d\sigma^{\uparrow} - d\sigma^{\downarrow}]\, \sin(\phi_S+\phi_h)}{\int d\phi_S\, d\phi_h\, [d\sigma^{\uparrow} + d\sigma^{\downarrow}]}$$



$$\mathrm{d}^6\sigma \equiv \frac{\mathrm{d}^6\sigma^{\ell p^{\uparrow}\to\ell h X}}{\mathrm{d}x_B\,\mathrm{d}Q^2\,\mathrm{d}z_h\,\mathrm{d}^2\boldsymbol{P}_T\,\mathrm{d}\phi_S}$$

$$D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) = D_{h/p}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^{\uparrow}}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\,dz_2\,d\cos\theta\,d(\varphi_1+\varphi_2)}$$



actual measurement is a ratio of such cross sections

with simple parameterization, TMD factorisation gives

$$\Delta_T q(x, k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)\ \left[f_{q/p}(x) + \Delta q(x)\right]\ \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T}$$

$$\Delta_T\, q = h_{1T}^q$$

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = 2\,\mathcal{N}_q^C(z)\ D_{h/q}(z)\ h(p_\perp)\ \frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

$$\Delta^N D_{h/q^\uparrow} = \frac{2\,p_\perp}{zM_h} H_1^{\perp q}$$

$$A_{UT}^{\sin(\phi_S+\phi_h)} = \frac{\sum_q e_q^2 \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, \Delta_T q(x,k_\perp)\, \frac{d(\Delta\hat\sigma)}{dy}\, \Delta^N D_{h/q^\uparrow}(z,p_\perp) \sin(\phi_S+\varphi+\phi_q^h)\sin(\phi_S+\phi_h)}{\sum_q e_q^2 \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\ f_{q/p}(x,k_\perp)\ \frac{d\hat\sigma}{dy}\ \ D_{h/q}(z,p_\perp)}$$

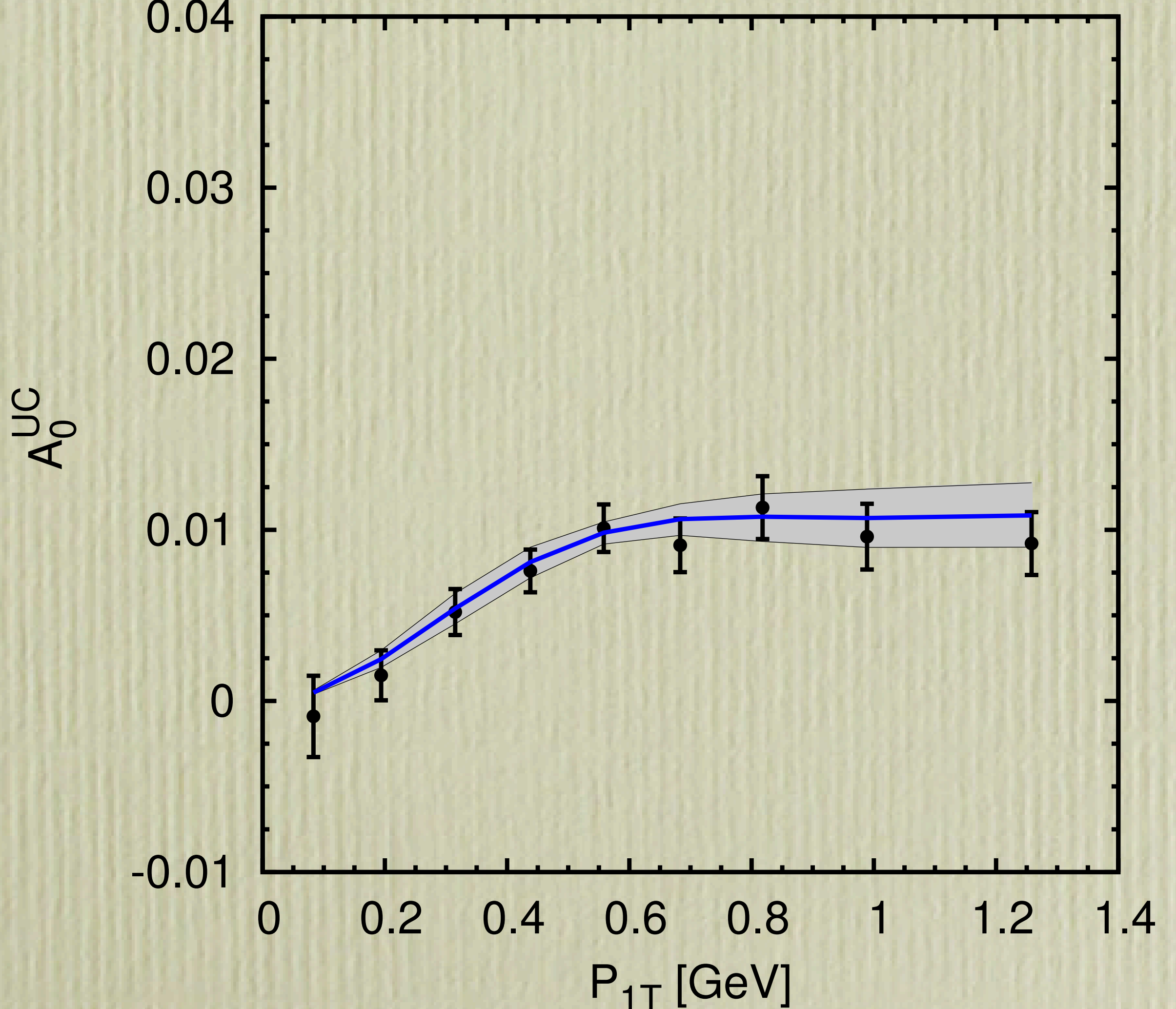
where $$\frac{d(\Delta\hat\sigma)}{dy} = \frac{d\hat\sigma^{\ell q^\uparrow\to\ell q^\uparrow}}{dy} - \frac{d\hat\sigma^{\ell q^\uparrow\to\ell q^\downarrow}}{dy} = \frac{4\pi\alpha^2}{sxy^2}(1-y)$$

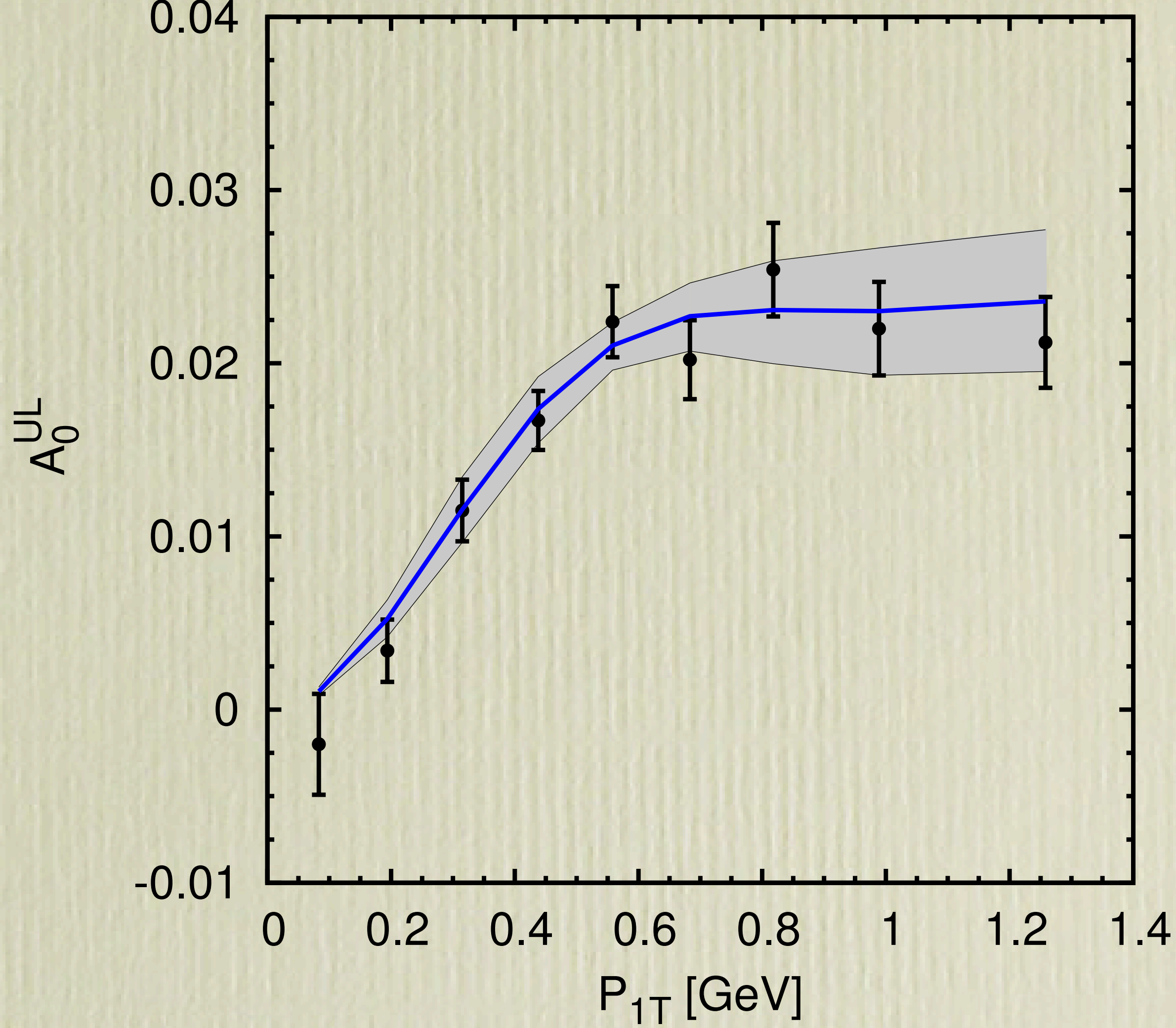
$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1+\varphi_2)} = \frac{3\alpha^2}{4s}\sum_q e_q^2 \Big\{(1+\cos^2\theta)\, D_{h_1/q}(z_1)\, D_{h_2/\bar q}(z_2) + \frac{1}{4}\sin^2\theta\,\cos(\varphi_1+\varphi_2)\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar q^\uparrow}(z_2)\Big\}$$

$$\int d^2\boldsymbol{p}_\perp \Delta^N D_{h/q^\uparrow}(z,p_\perp) \equiv \Delta^N D_{h/q^\uparrow}(z)$$

recent BaBar data on the $p_\perp$ dependence of the Collins function (first direct measurement)

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\, dz_2\, p_{\perp 1}\, dp_{\perp 1}\, p_{\perp 2}\, dp_{\perp 2}\, d\cos\theta\, d(\varphi_1+\varphi_2)}$$

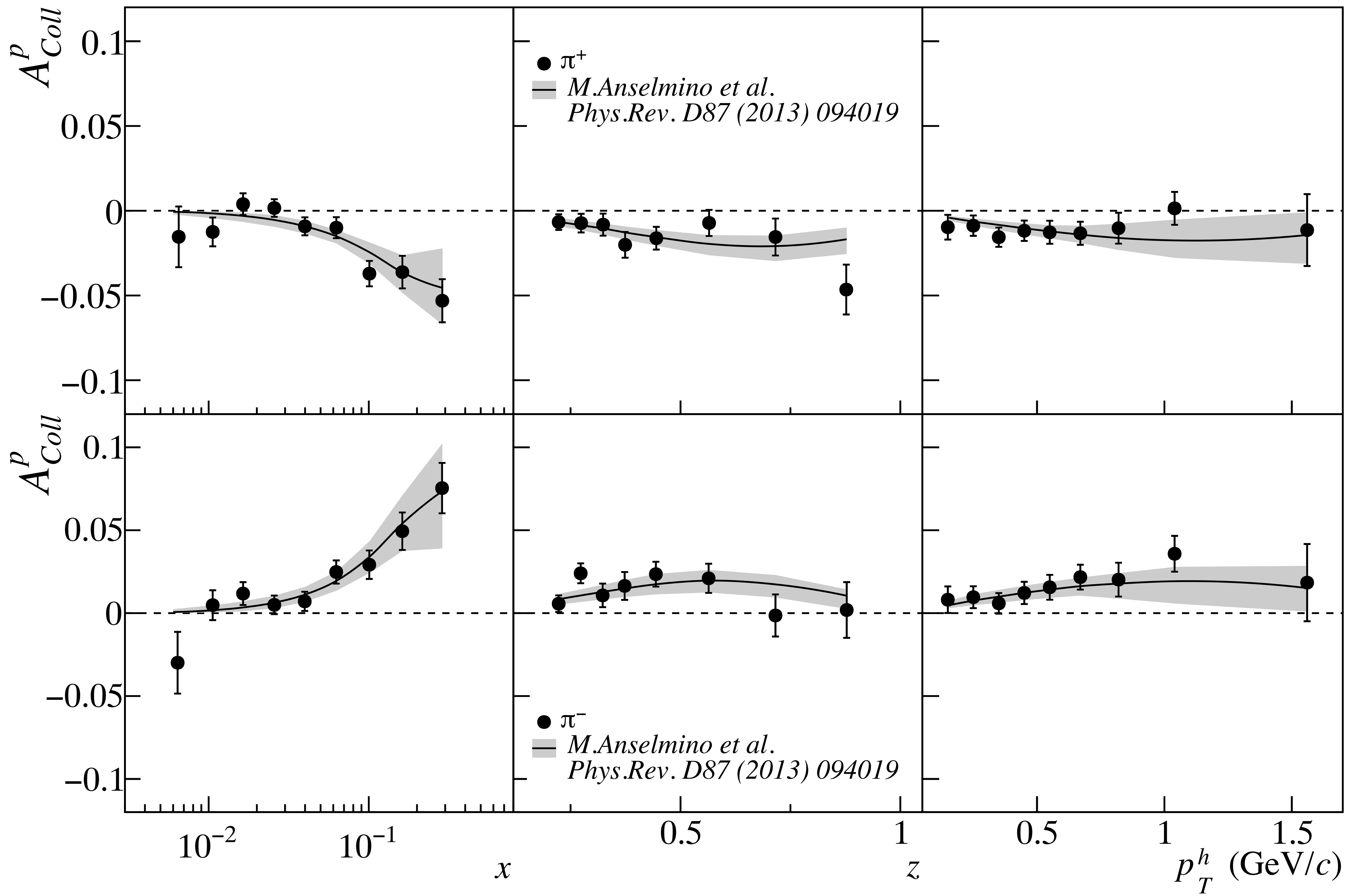




gaussian $p_\perp$ dependence of Collins functions

(M.A., Boglione, D'Alesio, Gonzalez, Melis, Murgia, Prokudin, Phys. Rev. D92 (2015) 114023)

recent results from COMPASS and a previous combined fit of SIDIS (HERMES and COMPASS) and e⁺e⁺ asymmetries



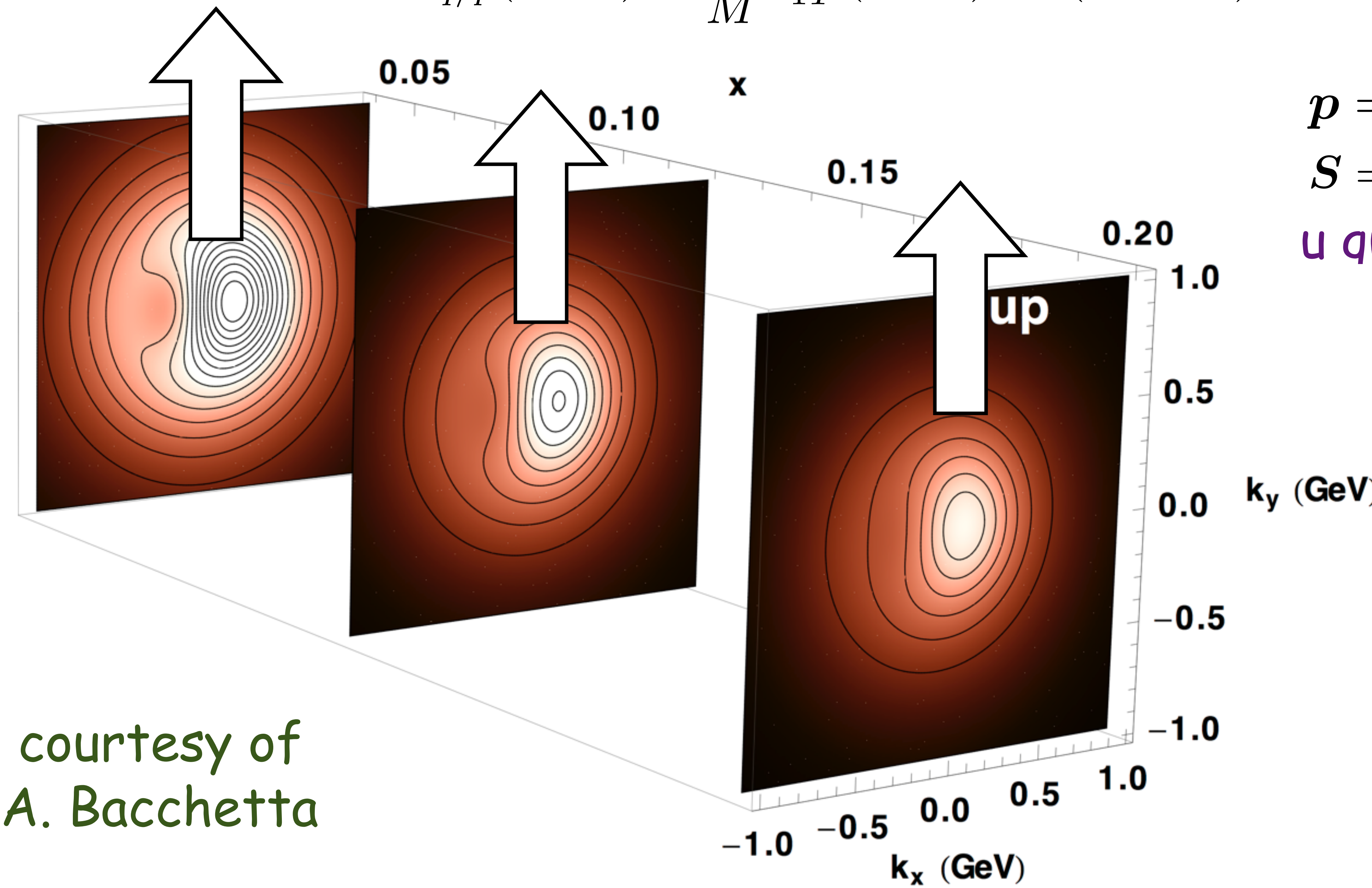
COMPASS Collaboration, Phys. Lett. B744 (2015) 250

more on the Sivers effect, what does it teach us?

it induces distortions in the parton distributions

$$f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$



$\boldsymbol{p} = p\hat{\boldsymbol{z}}$

$\boldsymbol{S} = S\hat{\boldsymbol{y}}$

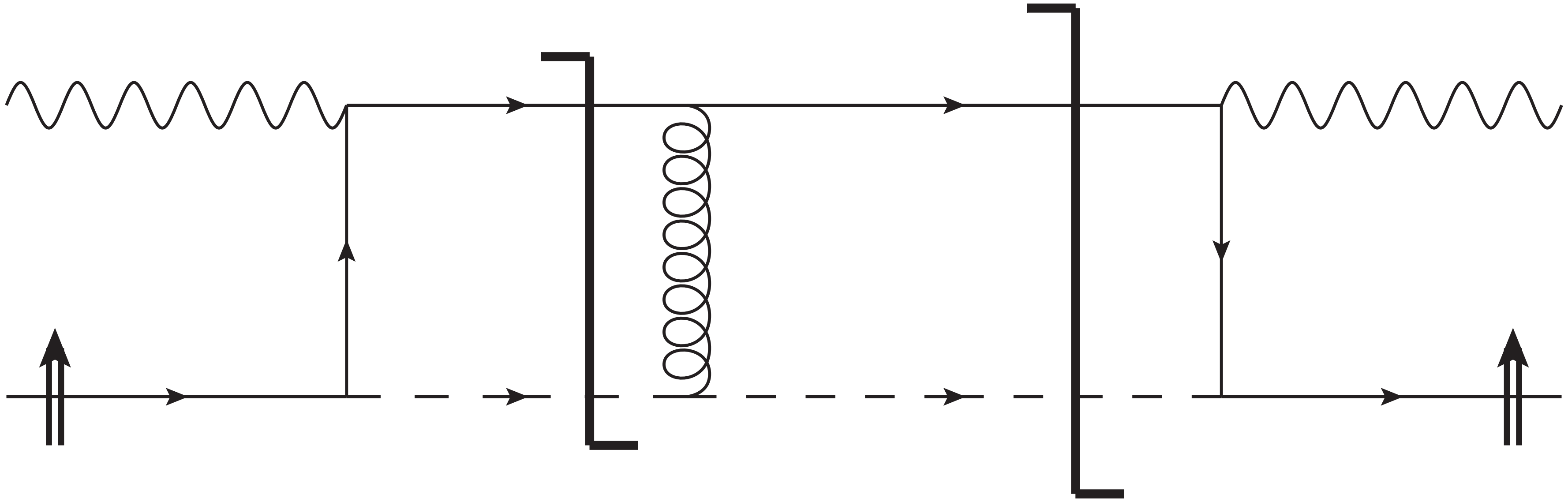
u quark

courtesy of
A. Bacchetta

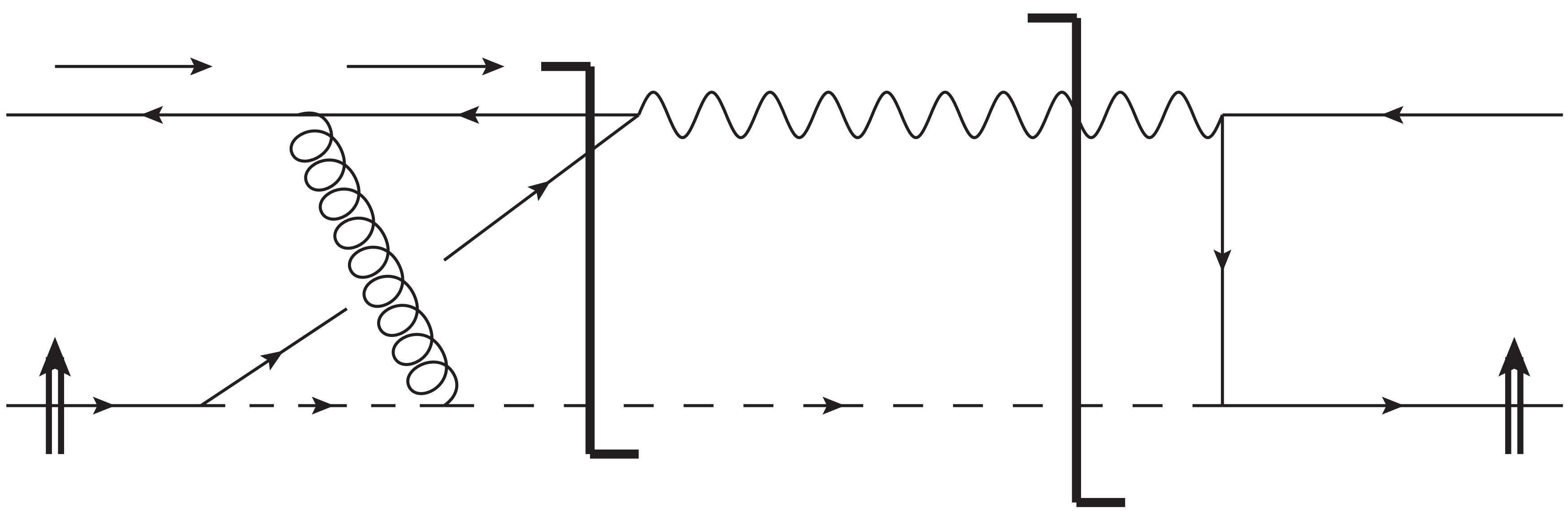
models of Sivers function and gauge links, process dependence

$$\boxed{[f_{1T}^{q\perp}]_{\mathrm{SIDIS}} = -[f_{1T}^{q\perp}]_{\mathrm{DY}}}$$

SIDIS final state interactions ($\Rightarrow A_N$)

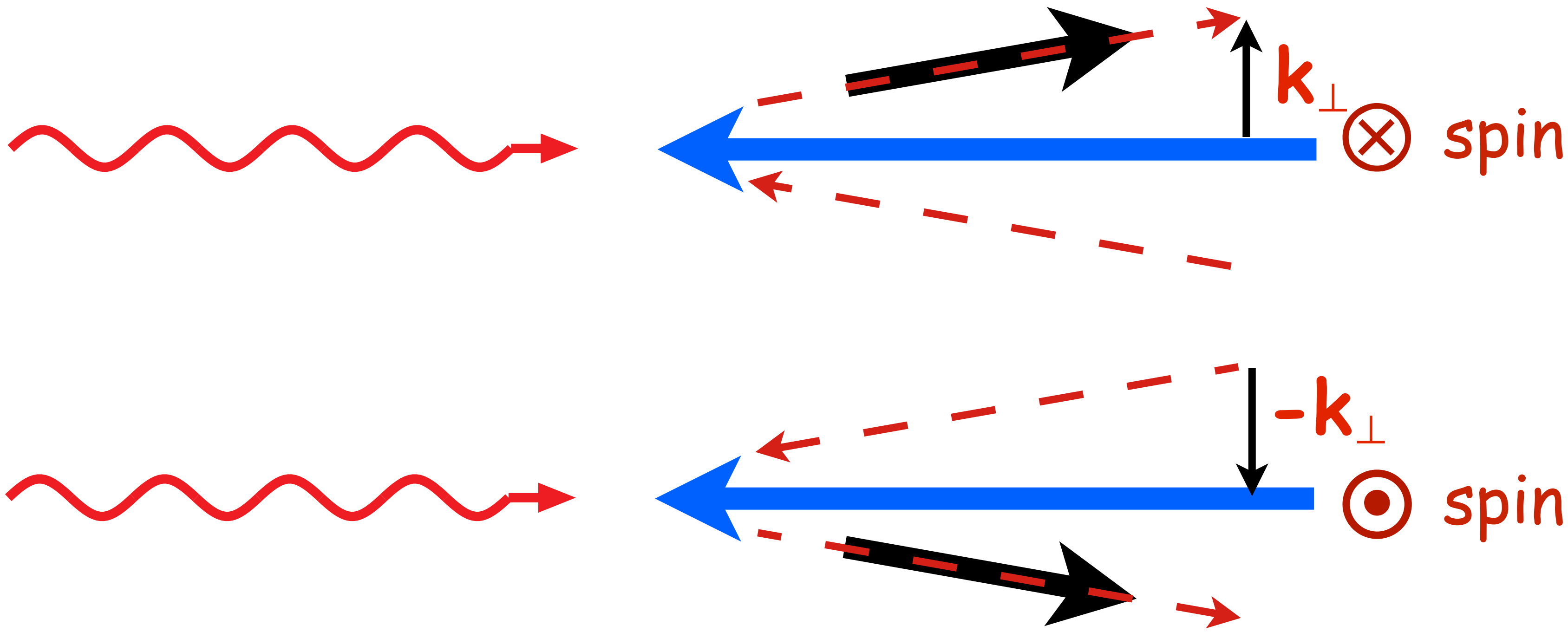


D-Y initial state interactions ($\Rightarrow -A_N$)



Brodsky, Hwang, Schmidt, PL B530 (2002) 99; NP B642 (2002) 344
Brodsky, Hwang, Kovchegov, Schmidt, Sievert, PR D88 (2013) 014032

but the the Sivers effect has a simple physical picture...



$$P_T \sim k_\perp$$

$$f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

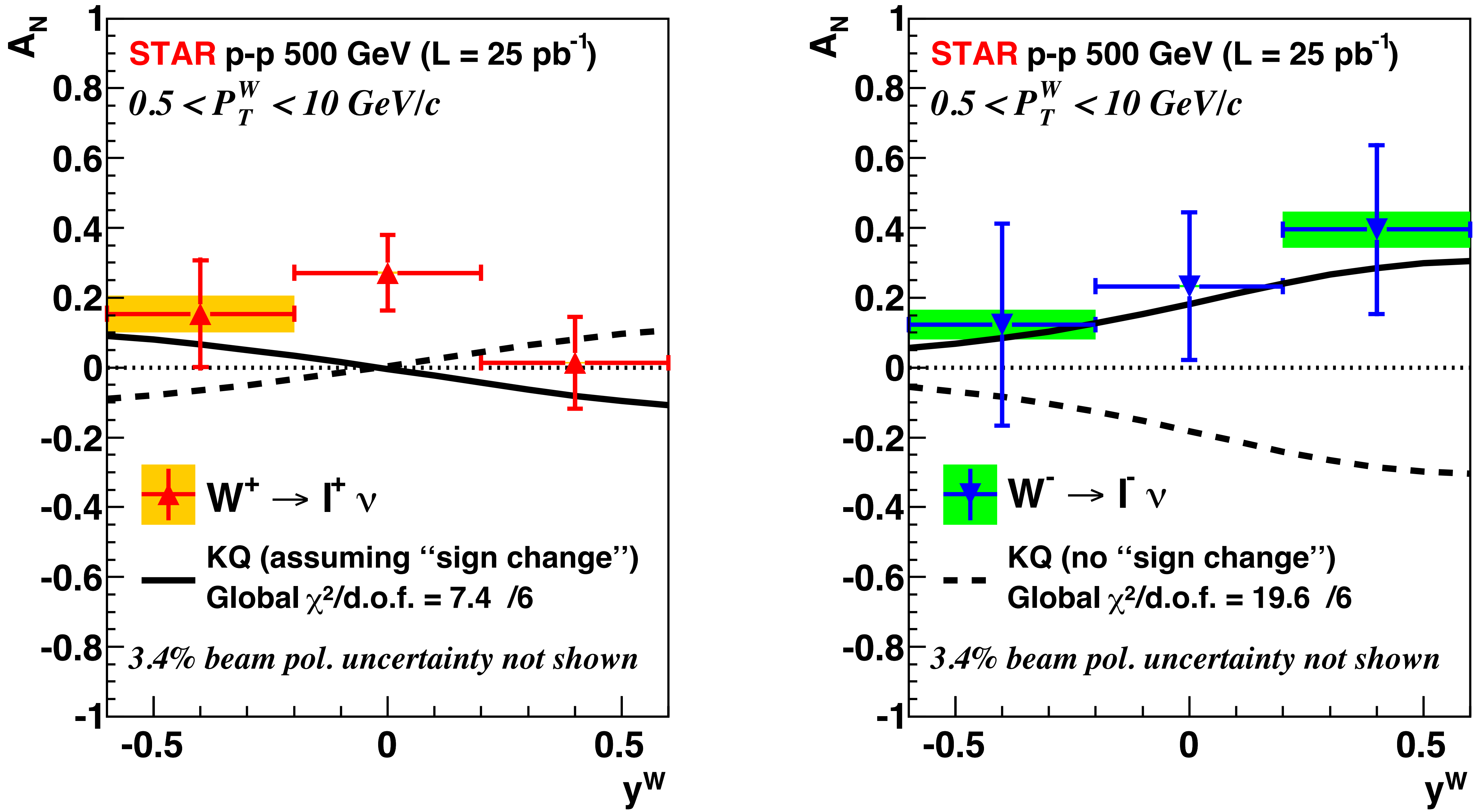
$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

left-right spin asymmetry for the process $\gamma^* q \to q$

the spin-**k**$_\perp$ correlation is an intrinsic property of the nucleon; it should be related to the parton orbital motion

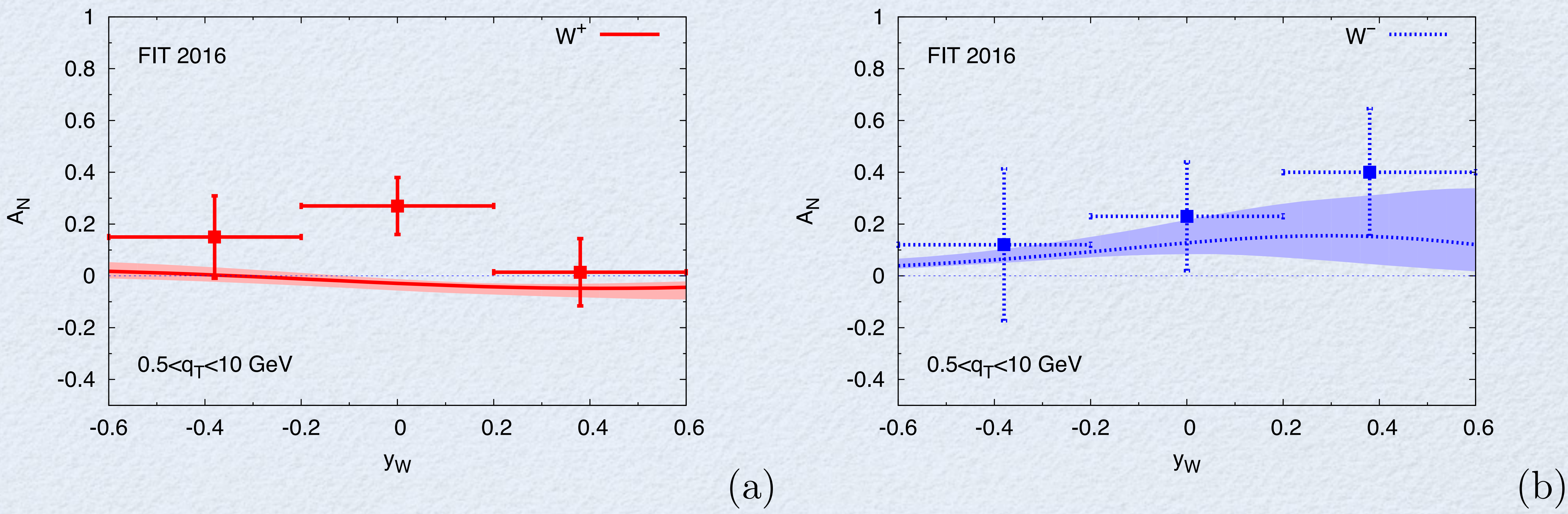
First results from RHIC, $p^{\uparrow}p \to W^{\pm}X$

STAR Collaboration, PRL 116 (2016) 132301



some hints at sign change of Sivers function.....
(new results from COMPASS expected soon)

M.A, M. Boglione, U. D'Alesio, F. Murgia, A. Prokudin,
JHEP 1704 (2017) 046



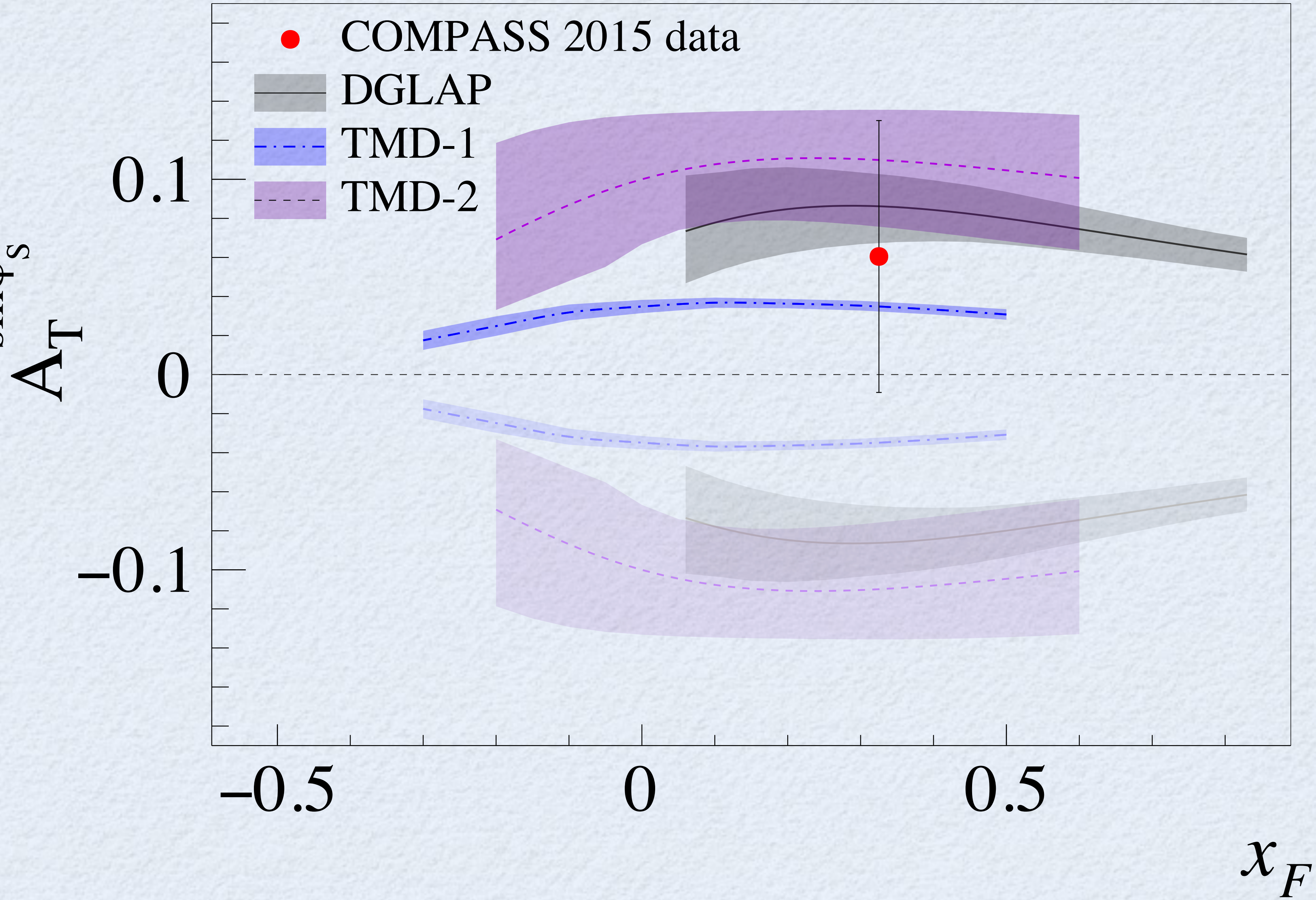
estimates of the Sivers asymmetry A_N for W^+(a) and W^-(b) production, assuming a sign change of the SIDIS Sivers functions, compared with the experimental data as function of y_W

$\langle\chi^2/\mathrm{n.o.d.}\rangle = 1.63$ with sign change

$\langle\chi^2/\mathrm{n.o.d.}\rangle = 2.35$ with no sign change

Sivers asymmetry in DY at COMPASS

arXiv:1704.00488



sign change

no sign change

Sivers function and orbital angular momentum

Ji's sum rule

forward limit of GPDs

$$J^q = \frac{1}{2}\int_0^1 dx\, x\, [H^q(x,0,0) + E^q(x,0,0)]$$

usual PDF $q(x)$

cannot be measured directly

anomalous magnetic moments

$$\kappa^p = \int_0^1 \frac{dx}{3}\left[2E^{u_v}(x,0,0) - E^{d_v}(x,0,0) - E^{s_v}(x,0,0)\right]$$

$$\kappa^n = \int_0^1 \frac{dx}{3}\left[2E^{d_v}(x,0,0) - E^{u_v}(x,0,0) - E^{s_v}(x,0,0)\right]$$

$$(E^{q_v} = E^q - E^{\bar{q}})$$

Sivers function and orbital angular momentum

assume

$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)E^a(x,0,0;Q_L^2)$$

$$f_{1T}^{\perp(0)a}(x,Q) = \int d^2\boldsymbol{k}_\perp\, \widehat{f}_{1T}^{\perp a}(x,k_\perp;Q)$$

L(x) = lensing function
(unknown, can be computed in models)

parameterize Sivers and lensing functions

fit SIDIS and magnetic moment data

obtain E^q and estimate orbital angular momentum

results at $Q^2 = 4$ GeV^2: $J^u \approx 0.23$, $J^{q\neq u} \approx 0$

Bacchetta, Radici, PRL 107 (2011) 212001

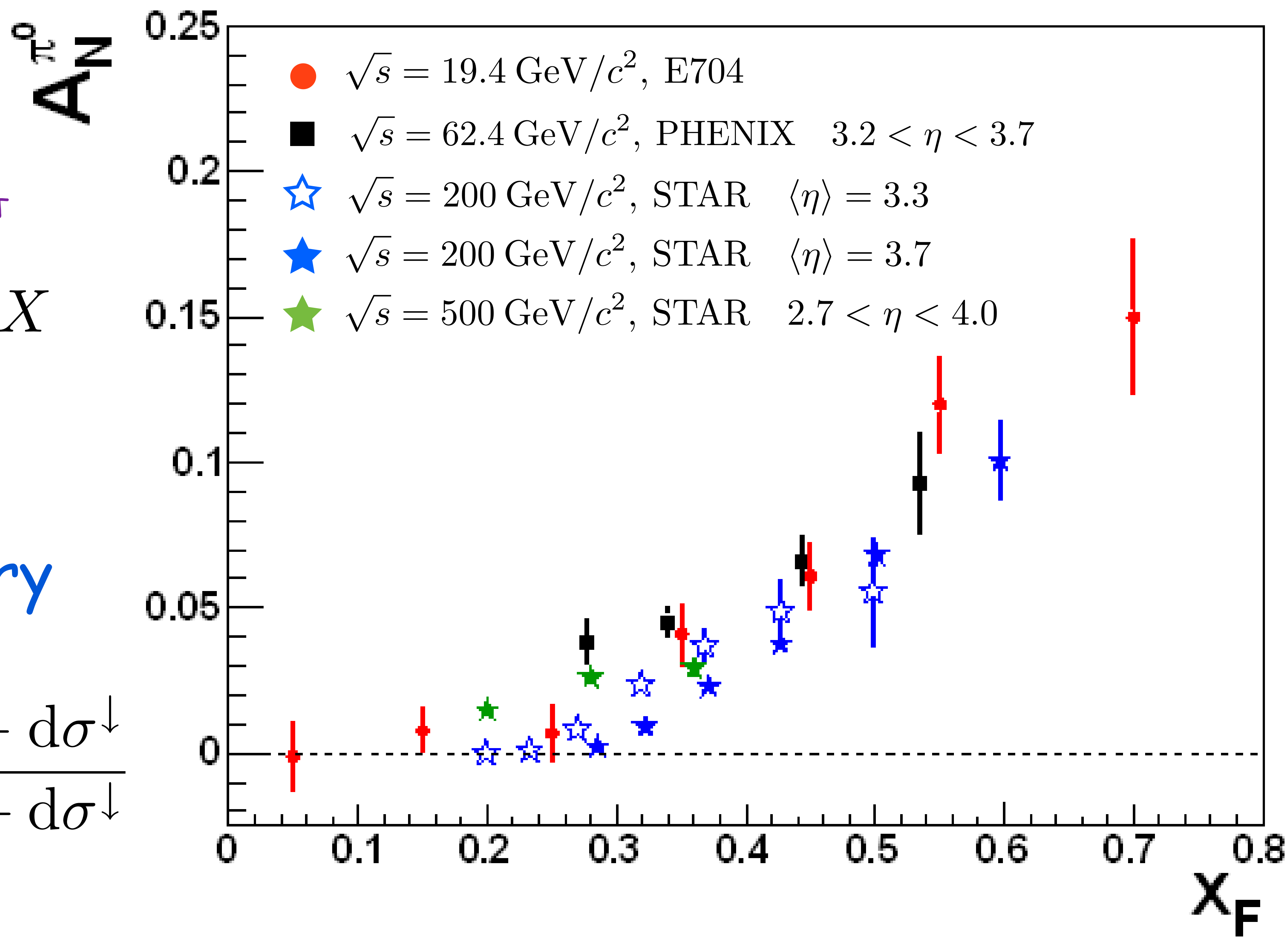
other experimental evidence of the Sivers and Collins effects

large P_T

$$p^{\uparrow}\, p \to \pi\, X$$

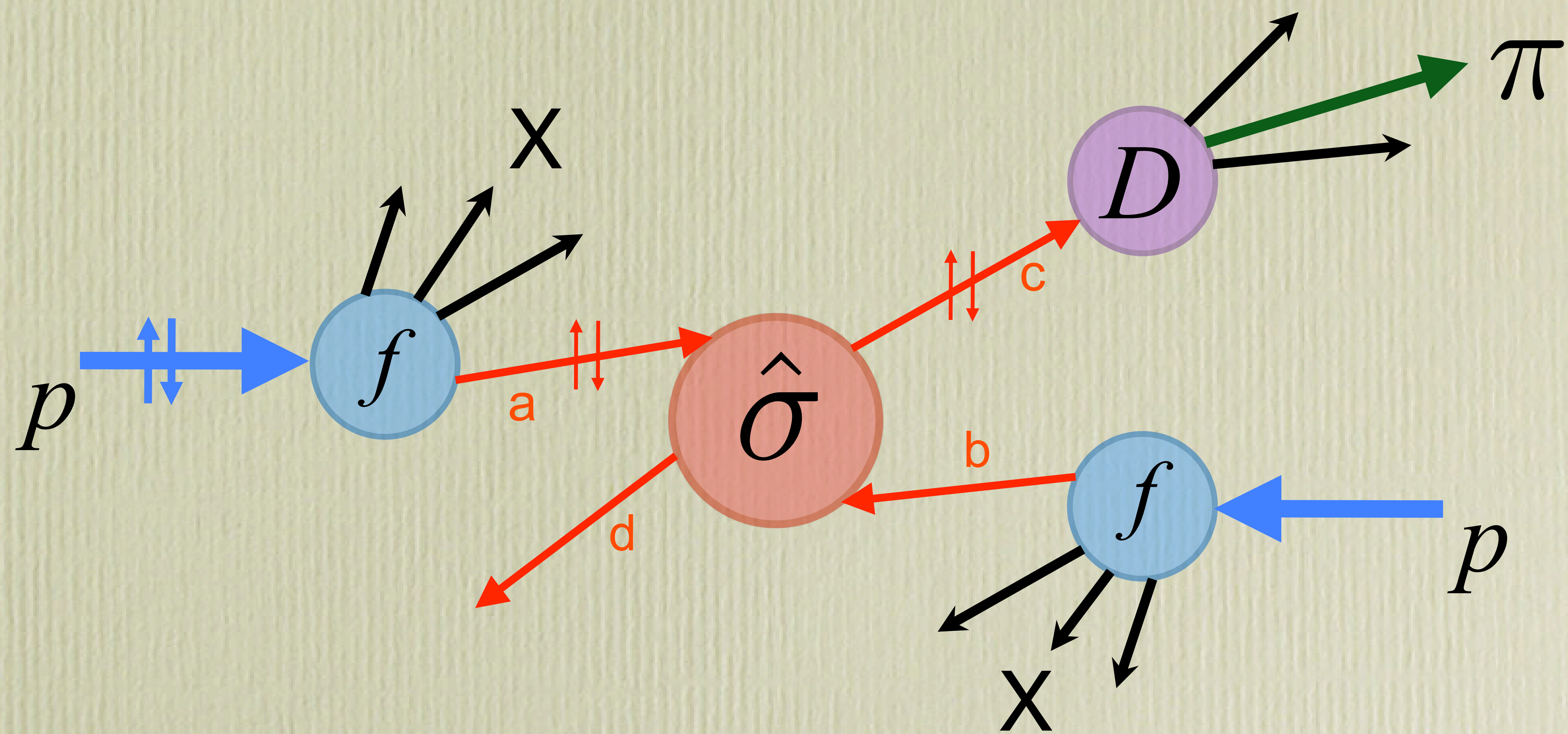
Single Spin Asymmetry

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$



SSA in hadronic processes: TMDs, a possible explanation

Generalization of collinear scheme (GPM)
(assuming factorization)



$$d\sigma^{\uparrow} = \sum_{a,b,c=q,\bar{q},g} \underbrace{f_{a/p^{\uparrow}}(x_a, \boldsymbol{k}_{\perp a})} \otimes \underbrace{f_{b/p}(x_b, \boldsymbol{k}_{\perp b})} \otimes d\hat{\sigma}^{ab\to cd}(\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}) \otimes \underbrace{D_{\pi/c}(z, \boldsymbol{p}_{\perp \pi})}$$

single spin effects in TMDs

TMDs and QCD - TMD evolution

how does gluon emission affect the parton transverse motion?

TMD phenomenology - phase 2

Different TMD evolution schemes and different implementations within the same scheme

it needs non perturbative inputs

dedicated workshops, QCD Evolution
2011, 2012, 2013, 2014, 2015, 2016, 2017

dedicated tools:

TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions

study of the QCD evolution of TMDs and TMD factorisation in rapid development

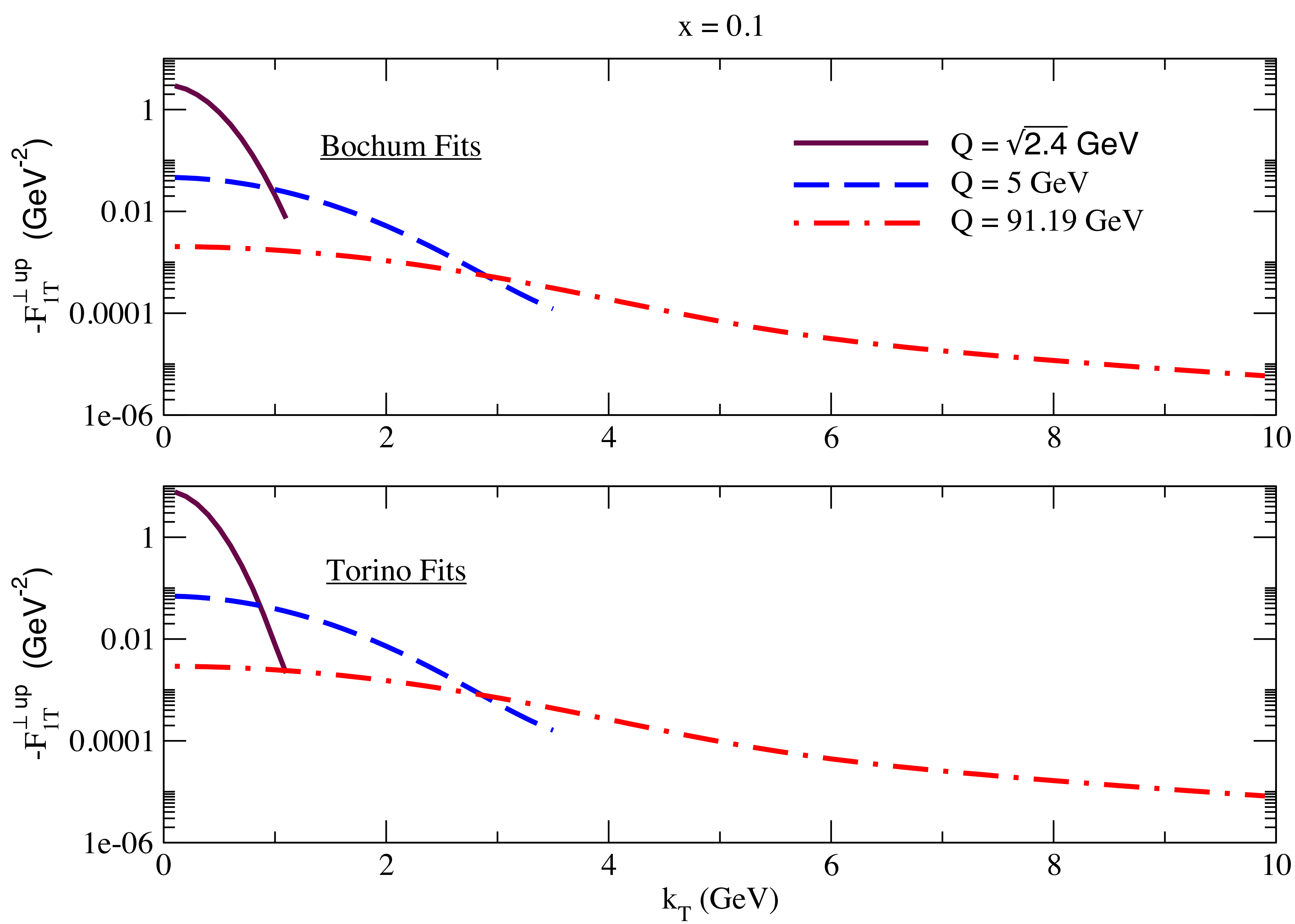
Collins, "Foundations of perturbative QCD", Cambridge University Press (2011)

TMD phenomenology - phase 2

how does gluon emission affect the transverse motion?

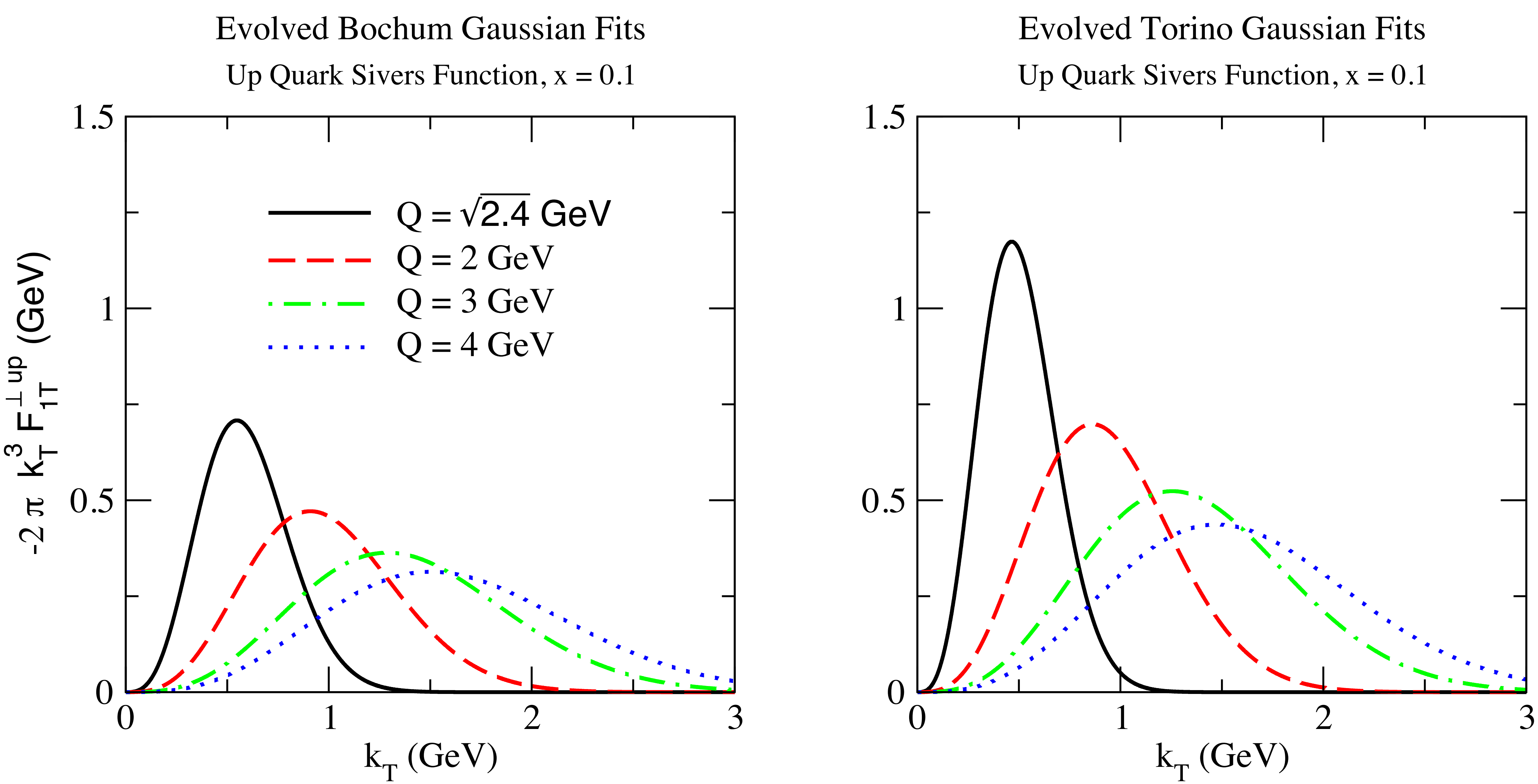
a few selected results, examples

TMD evolution of up quark Sivers function



Aybat, Collins, Qiu, Rogers, Phys. Rev. D85 (2012) 034043

TMD evolution of up quark Sivers function



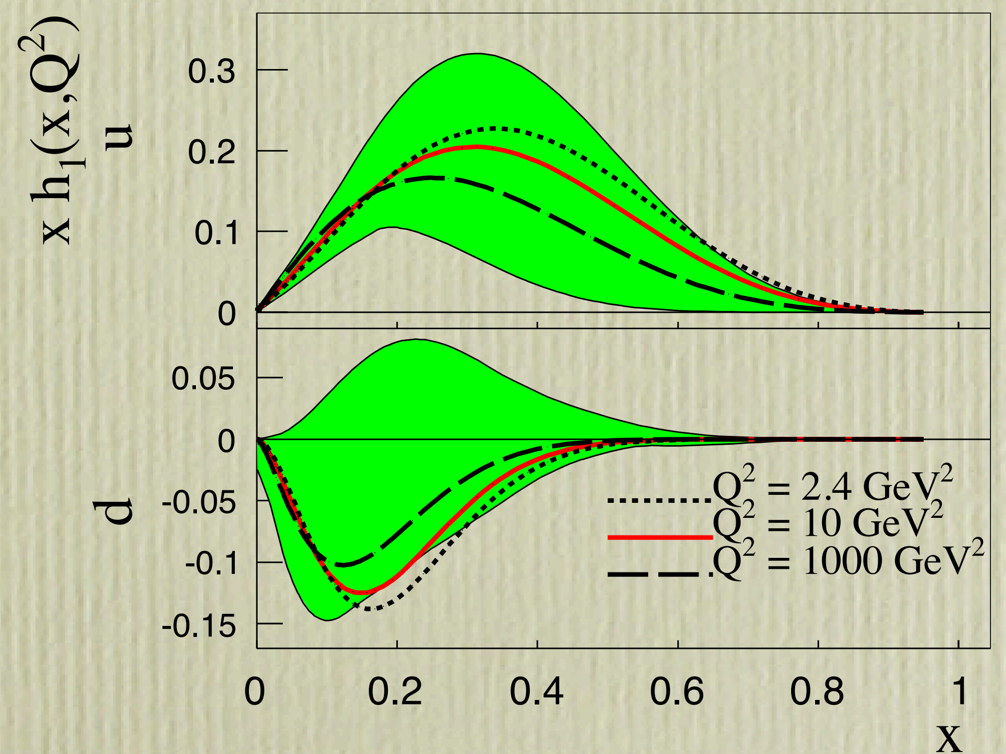
Aybat, Collins, Qiu, Rogers, Phys.Rev. D85 (2012) 034043

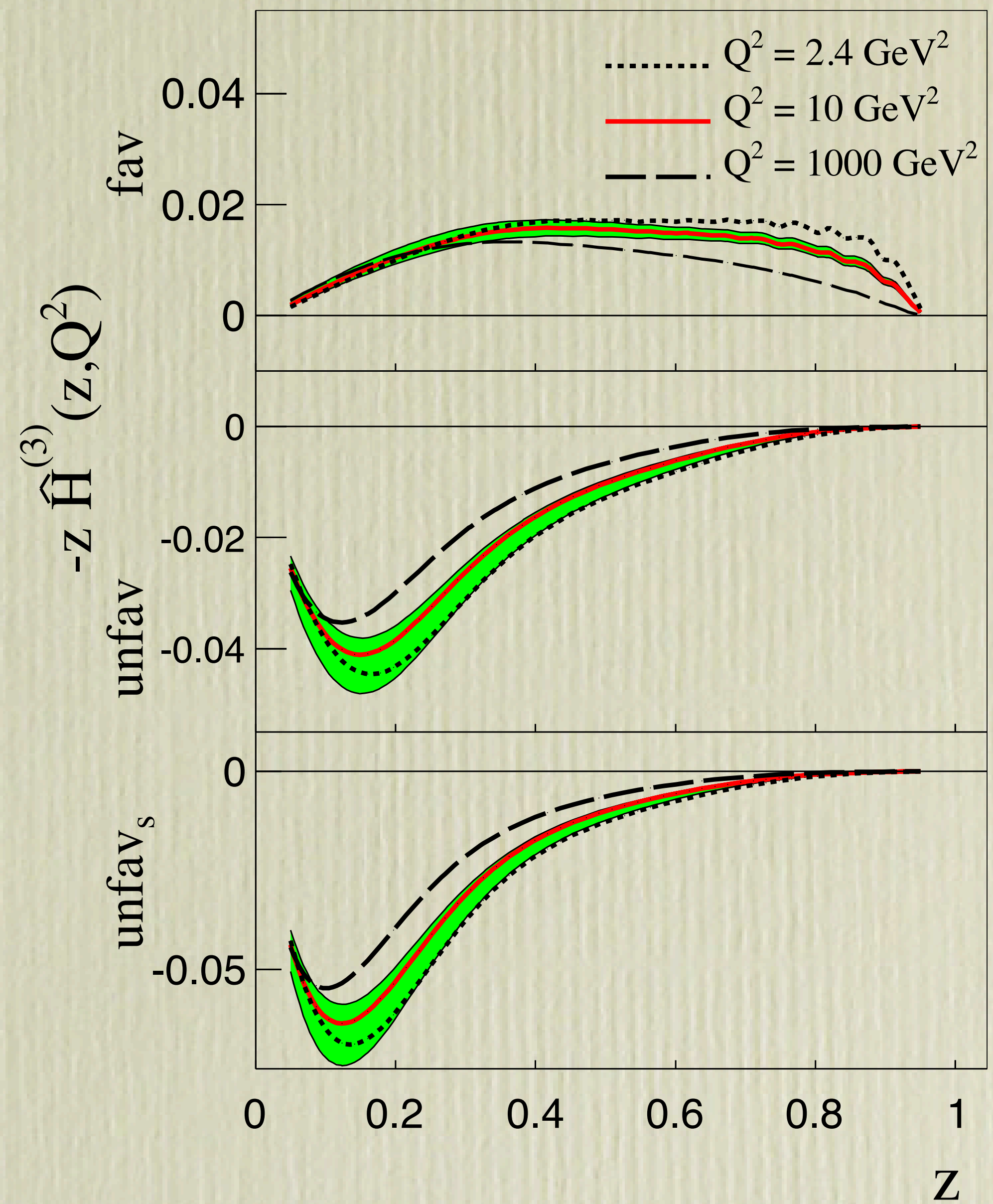
TMD evolution of Sivers function studied also by
Echevarria, Idilbi, Kang, Vitev, Phys. Rev. D89 (2014) 074013

Extraction of transversity and Collins functions with TMD evolution

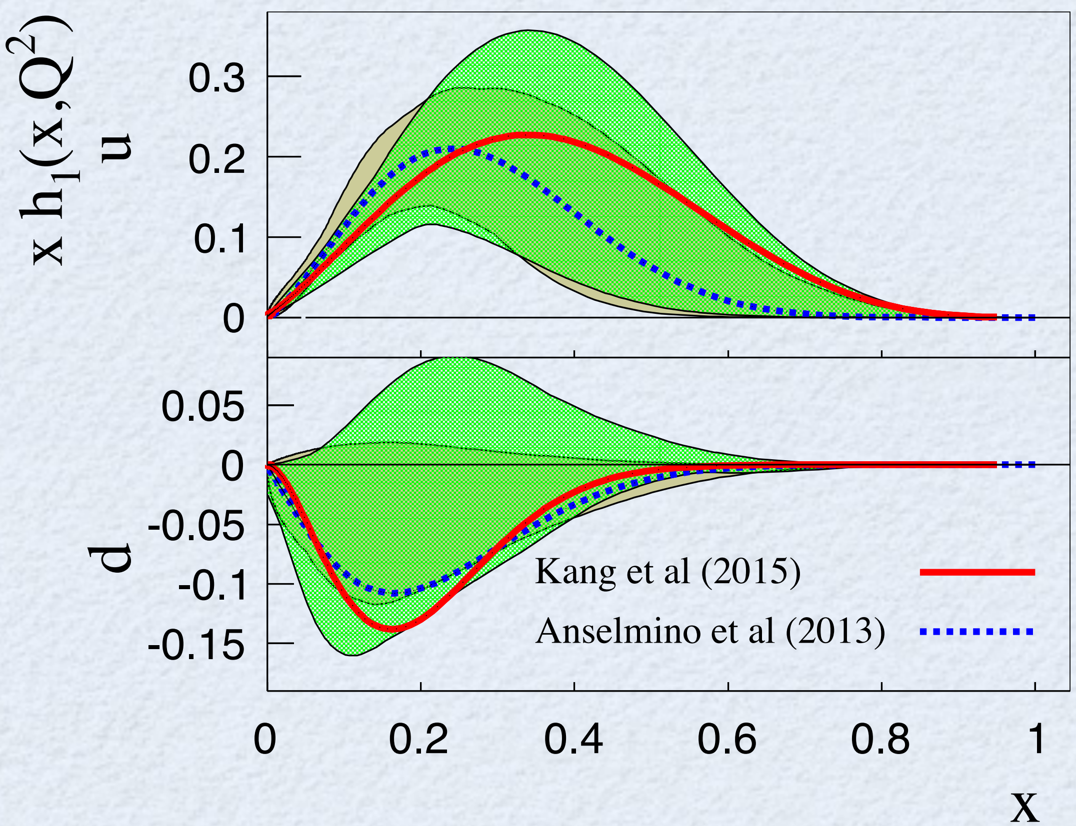
(Kang, Prokudin, Sun, Yuan, Phys. Rev. D93 (2016) 014009)

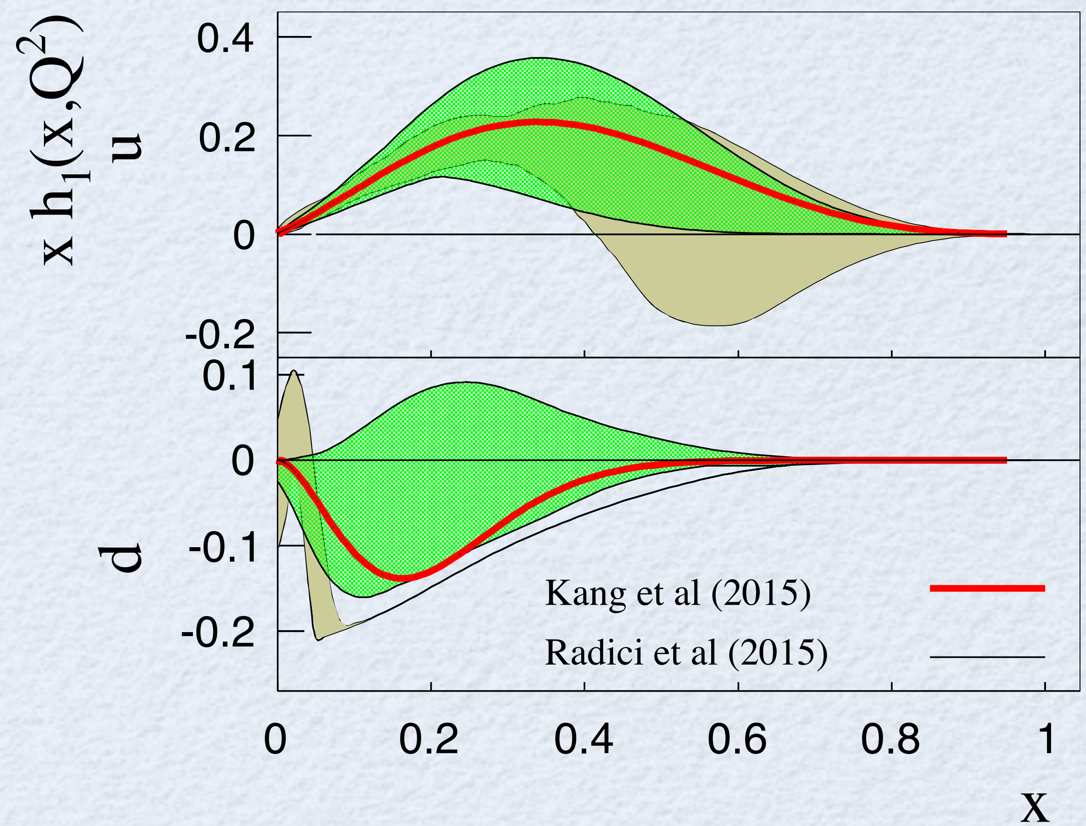
transversity distributions

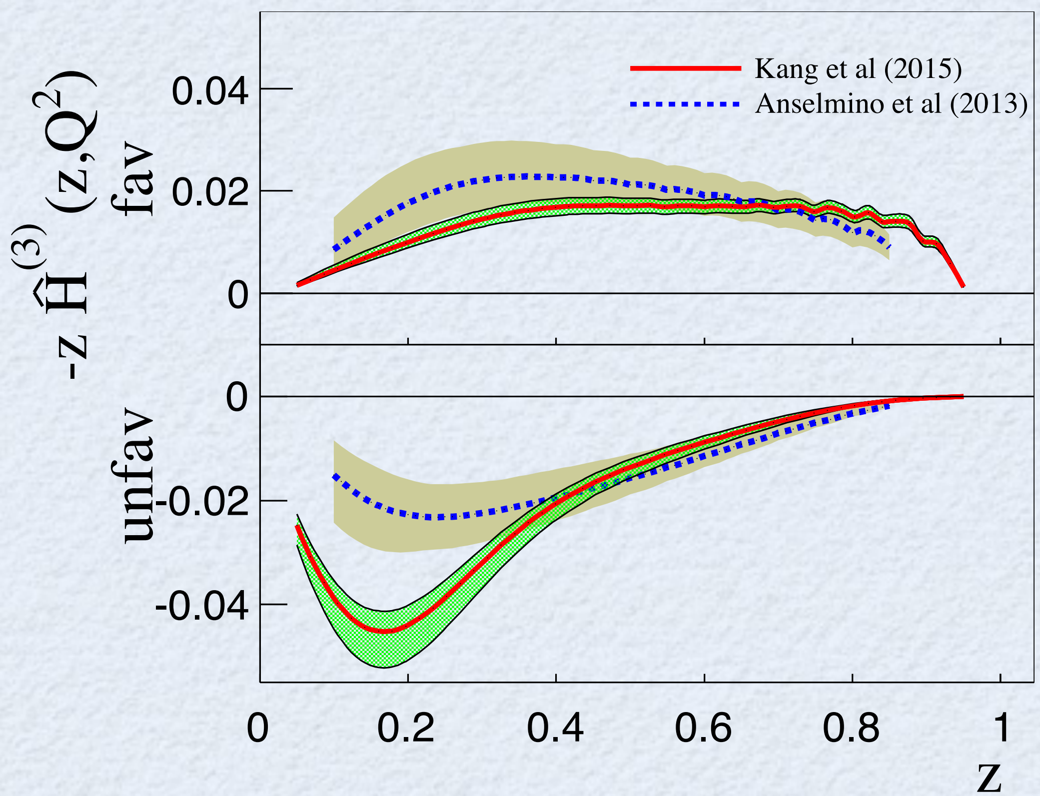




moment of Collins functions





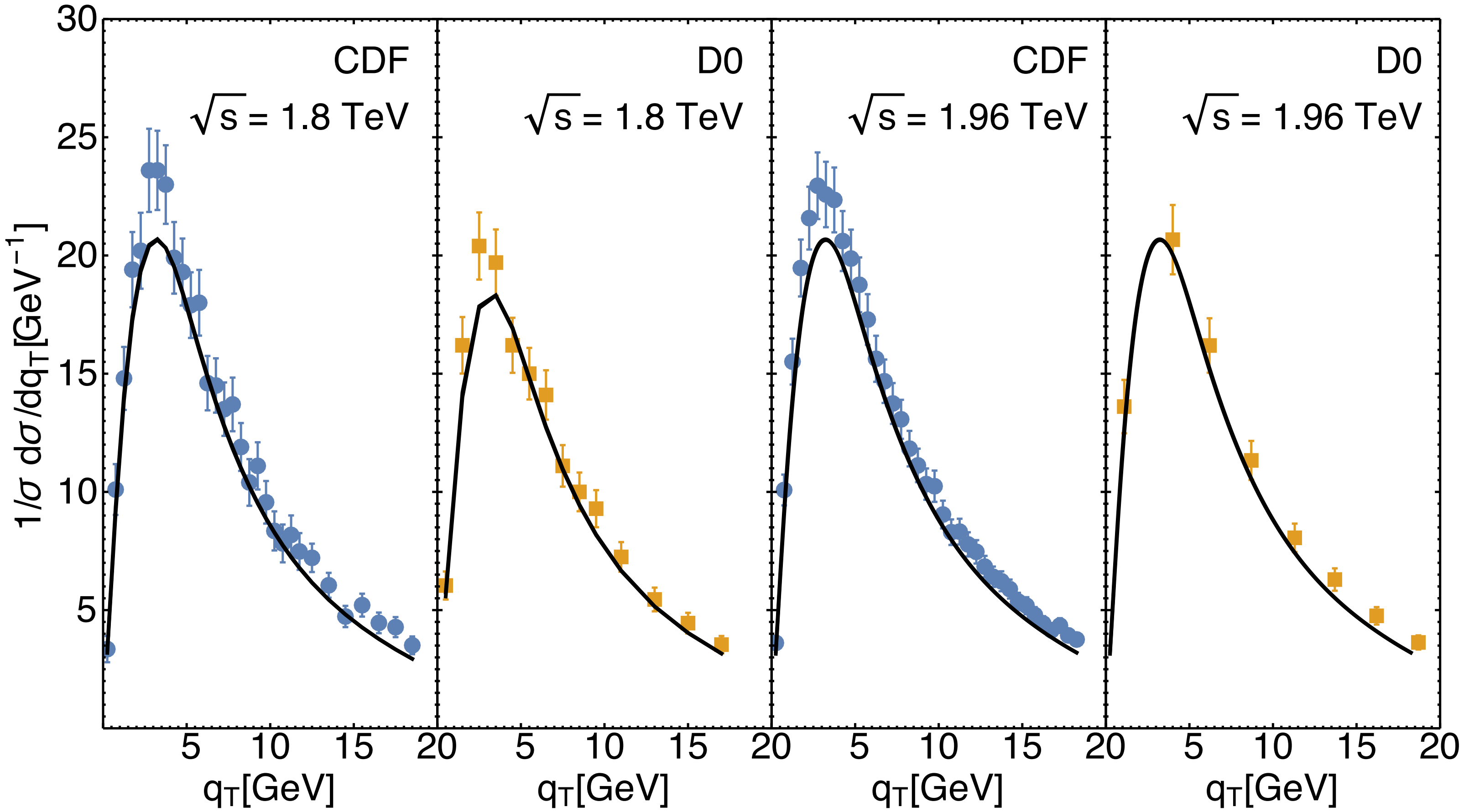


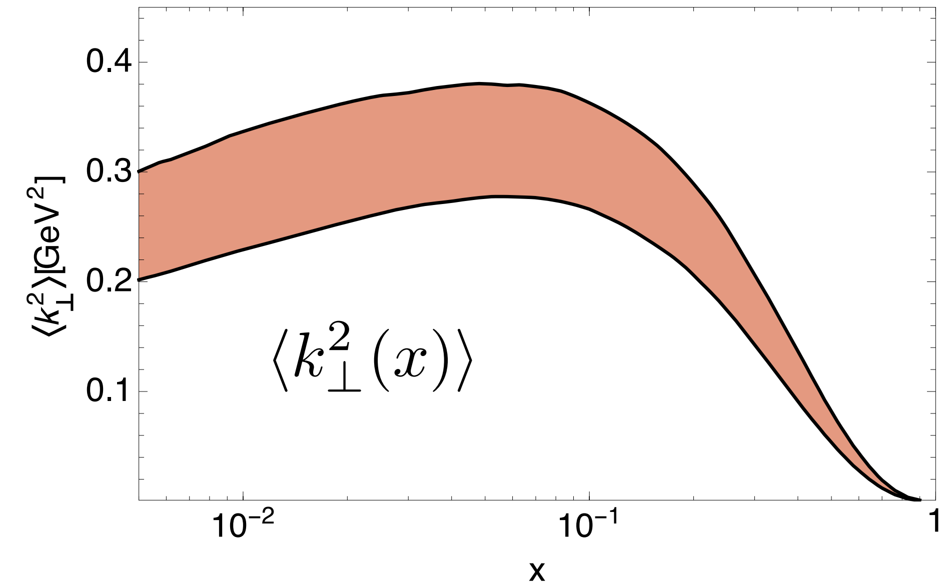
comparison with phase 1 extraction, $Q^2 = 2.4\ \mathrm{GeV}^2$

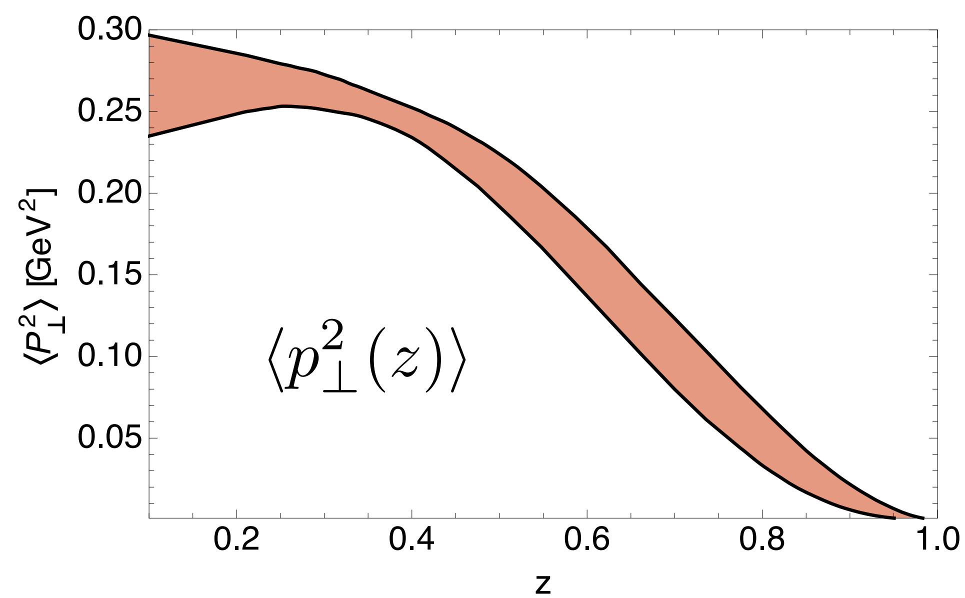
(Kang, Prokudin, Sun, Yuan, Phys. Rev. D93 (2016) 014009

no compelling evidence of TMD evolution yet

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori,
JHEP 1706 (2017) 081



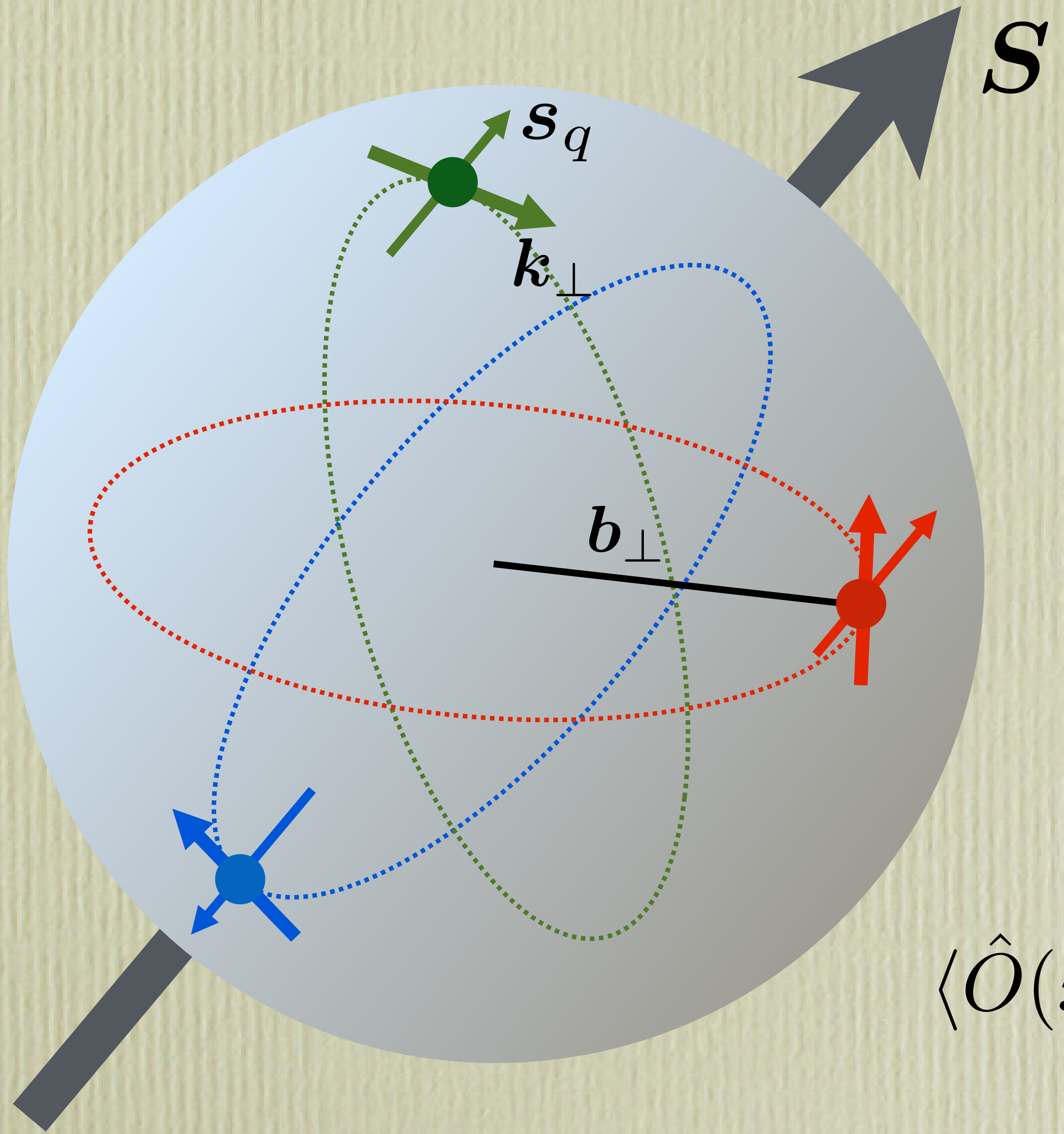




q_T dependence of cross-section for Z_0 production at TEVATRON

first global fit of unpolarized TMD-PDFs and TMD-FFs from SIDIS, DY and Z_0 production data. Based on TMD factorisation with TMD evolution

but TMD-PDFs are not the whole story ……



Ideally: obtain a quantum phase-space distribution (like the Wigner function)

in 1-dimensional QM:

$$\int dp\, W(x,p) = |\psi(x)|^2$$

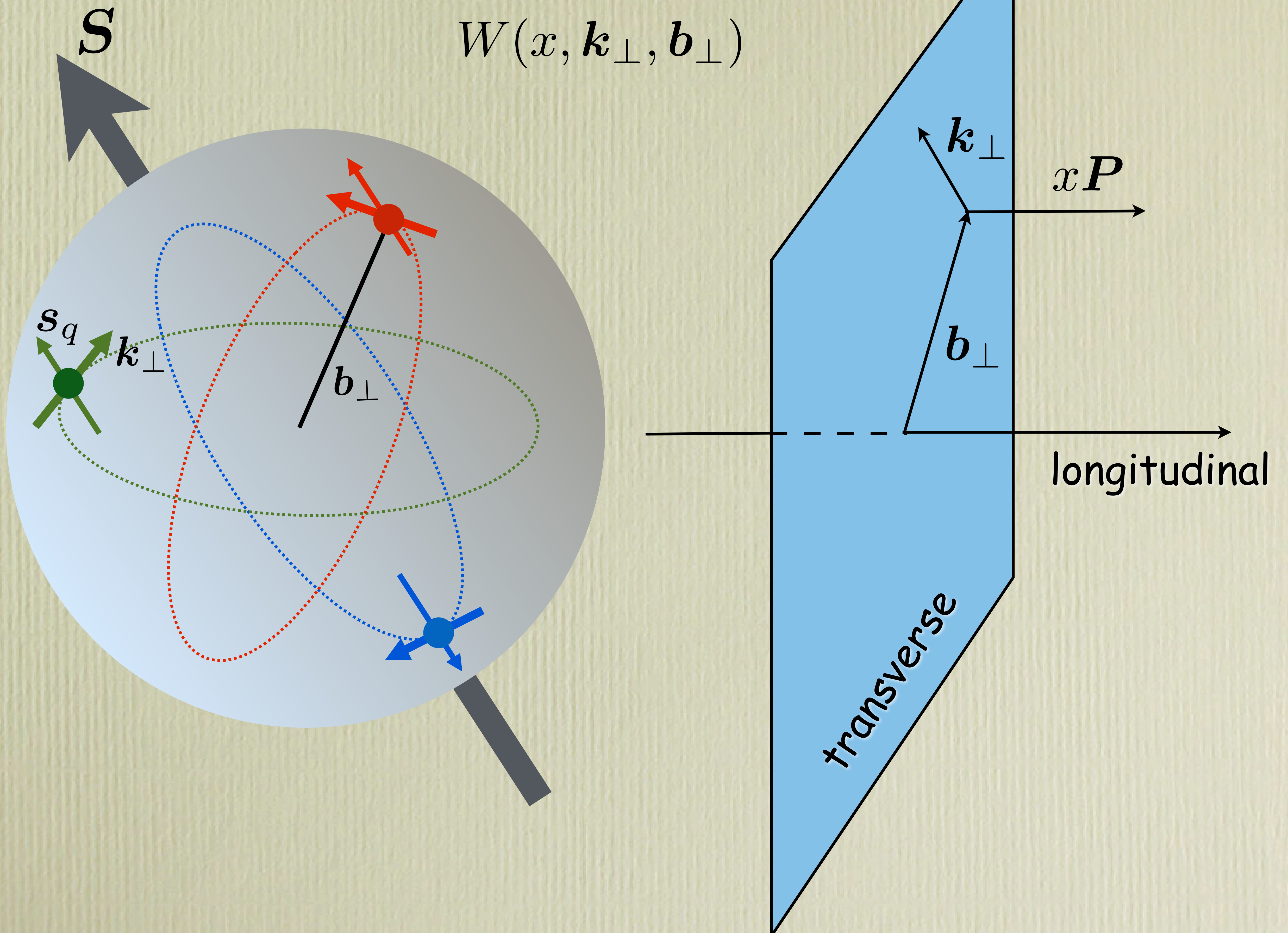
$$\int dx\, W(x,p) = |\phi(p)|^2$$

$$\langle \hat{O}(x,p)\rangle = \int dx\, dp\, W(x,p)\, O(x,p)$$

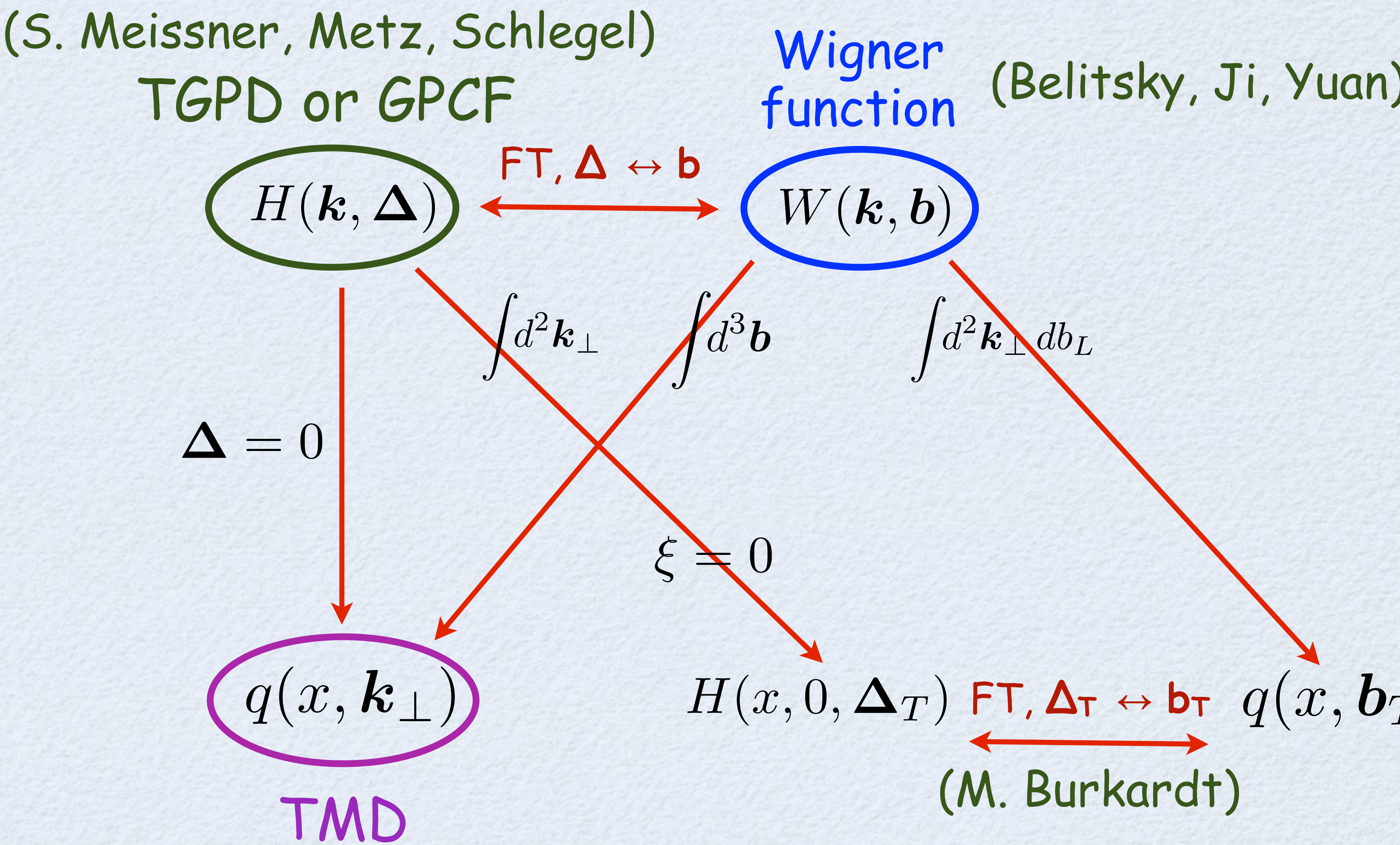
(A. Belitsky, X. Ji, F. Yuan, Phys. Rev. D69 (2004) 074014)

Exploring the 3-dimensional phase-space structure of the nucleon

$$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$



phase-space parton distribution, $W(\boldsymbol{k},\boldsymbol{b})$

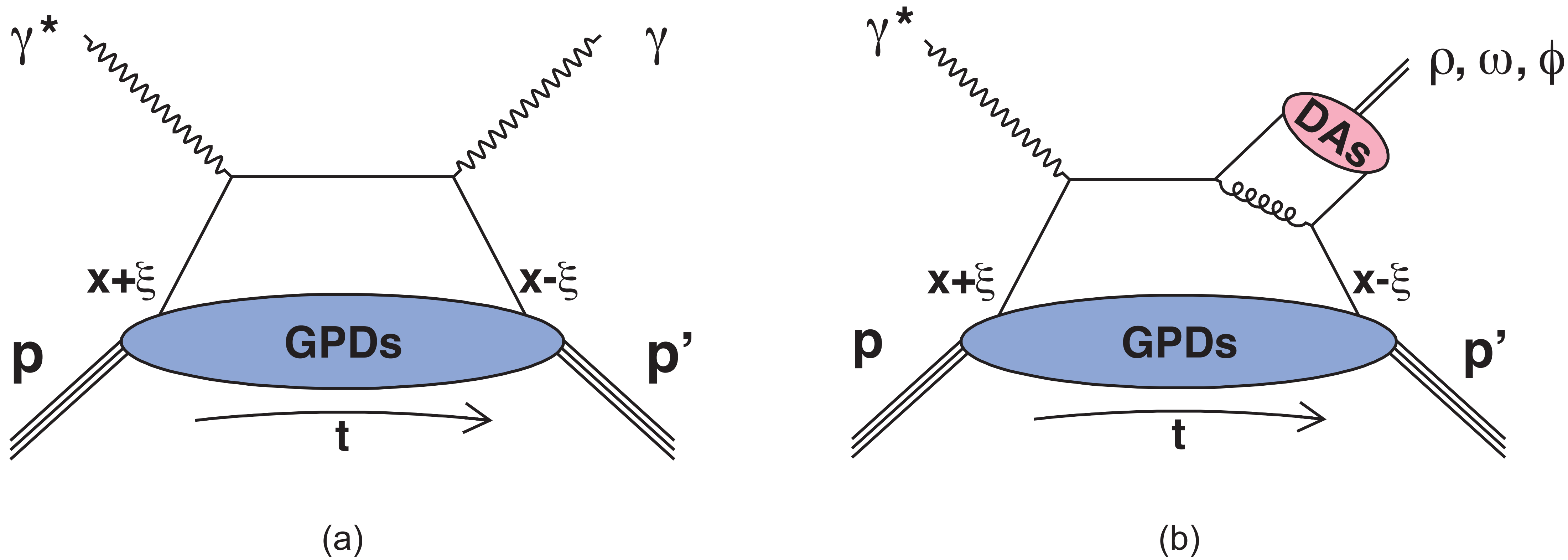


$$\int d^2\boldsymbol{k}_\perp\, H(\boldsymbol{k},\boldsymbol{\Delta}) = H(x,\xi,\boldsymbol{\Delta}_T)$$ GPDs

GPDs (8 independent ones)

(recover partonic distributions in the forward limit)

$$H, E, \tilde{H}, \tilde{E}; H_T, E_T, \tilde{H}_T, \tilde{E}_T(x, \xi, t)$$

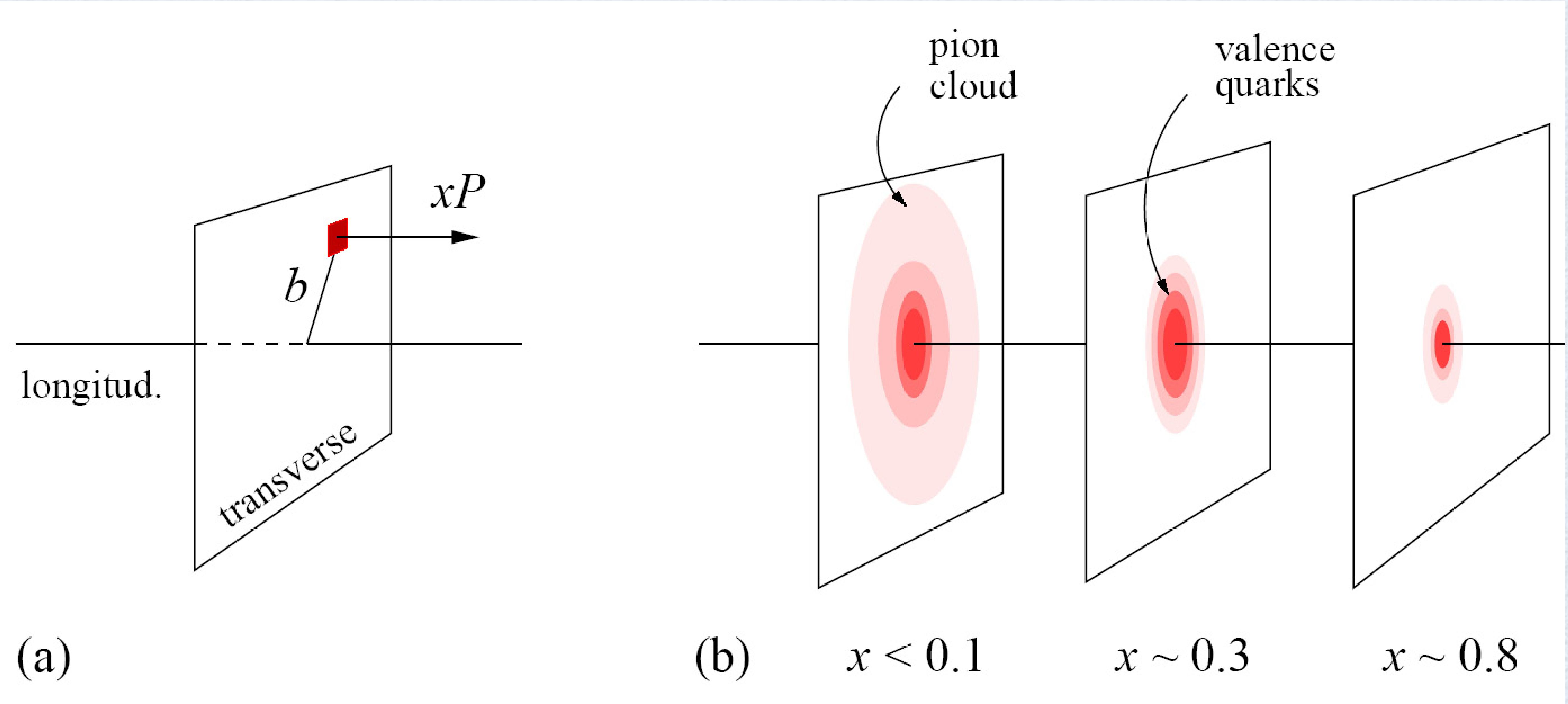


DVCS

hard meson production

exclusive leptonic processes.

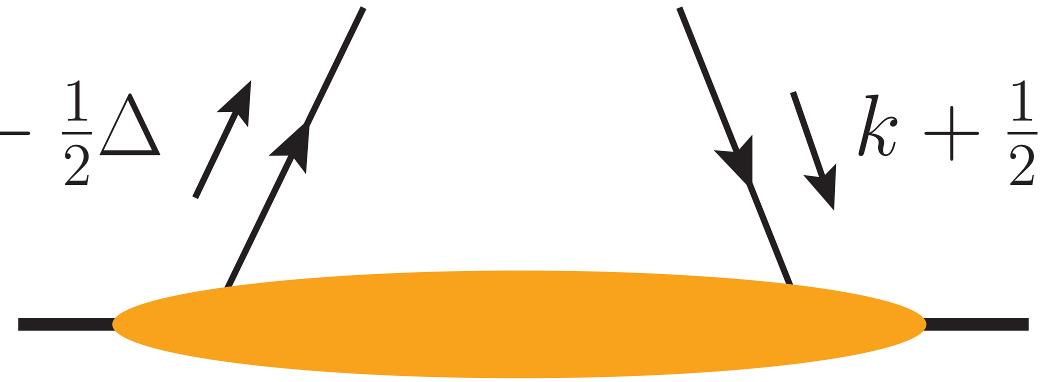
quark spatial transverse distribution $q(x, \boldsymbol{b}_T)$



femtophotography or tomography
of the nucleon

courtesy of C. Weiss

most general correlator (off diagonal)



$$H(k, P, \Delta) = (2\pi)^{-4} \int d^4z \, e^{izk} \times \langle p(P + \tfrac{1}{2}\Delta) | \bar{q}(-\tfrac{1}{2}z) \, \Gamma \, q(\tfrac{1}{2}z) | p(P - \tfrac{1}{2}\Delta) \rangle$$

two-quark correlation function

light-cone variables $\quad v = (v^+, v^-, \boldsymbol{v}) \qquad v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3)$

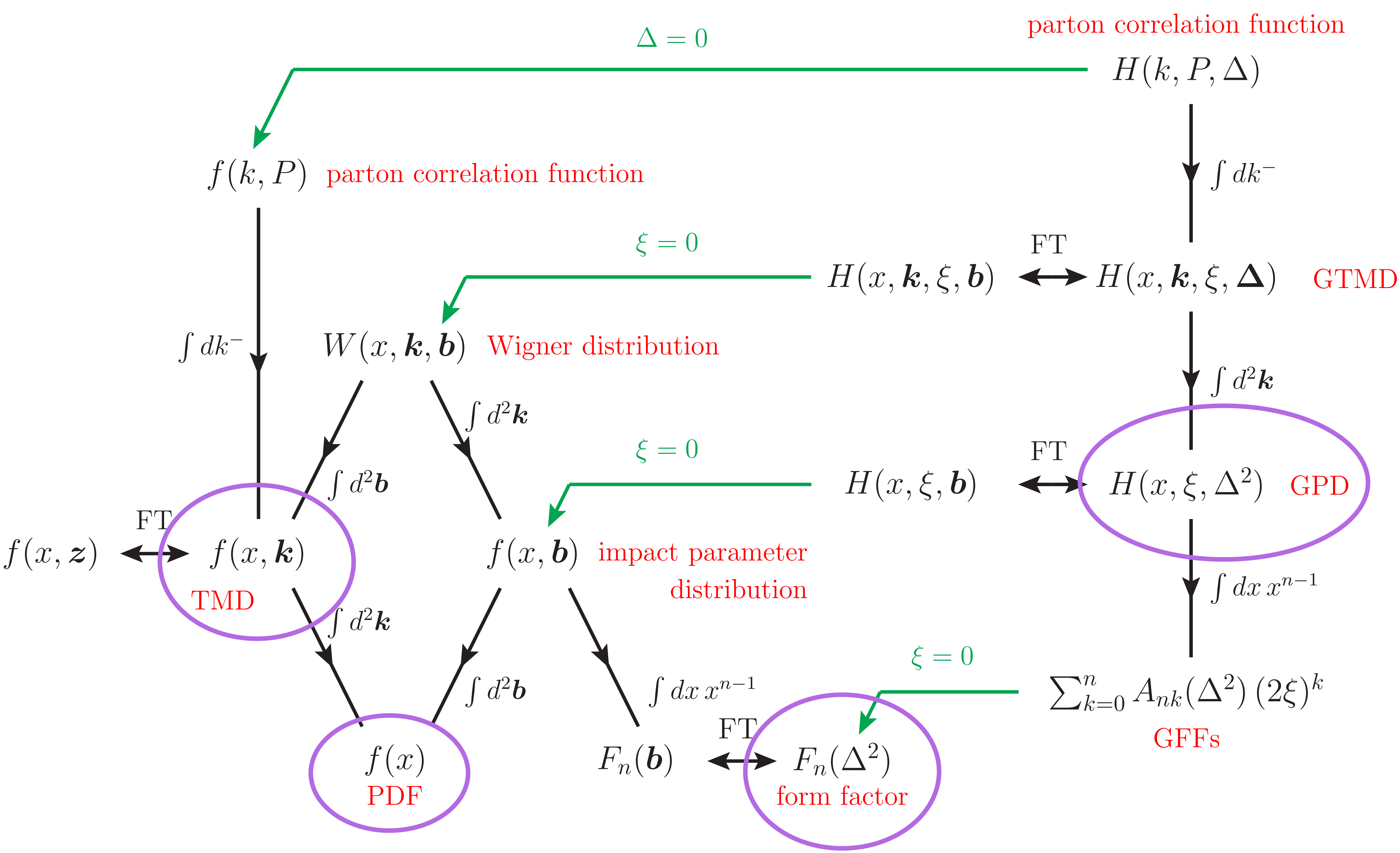
$$x = \frac{k^+}{P^+} \qquad 2\xi = -\frac{\Delta^+}{P^+}$$

$\Delta = 0$ inclusive processes, cross sections

$\Delta \neq 0$ exclusive processes, amplitudes

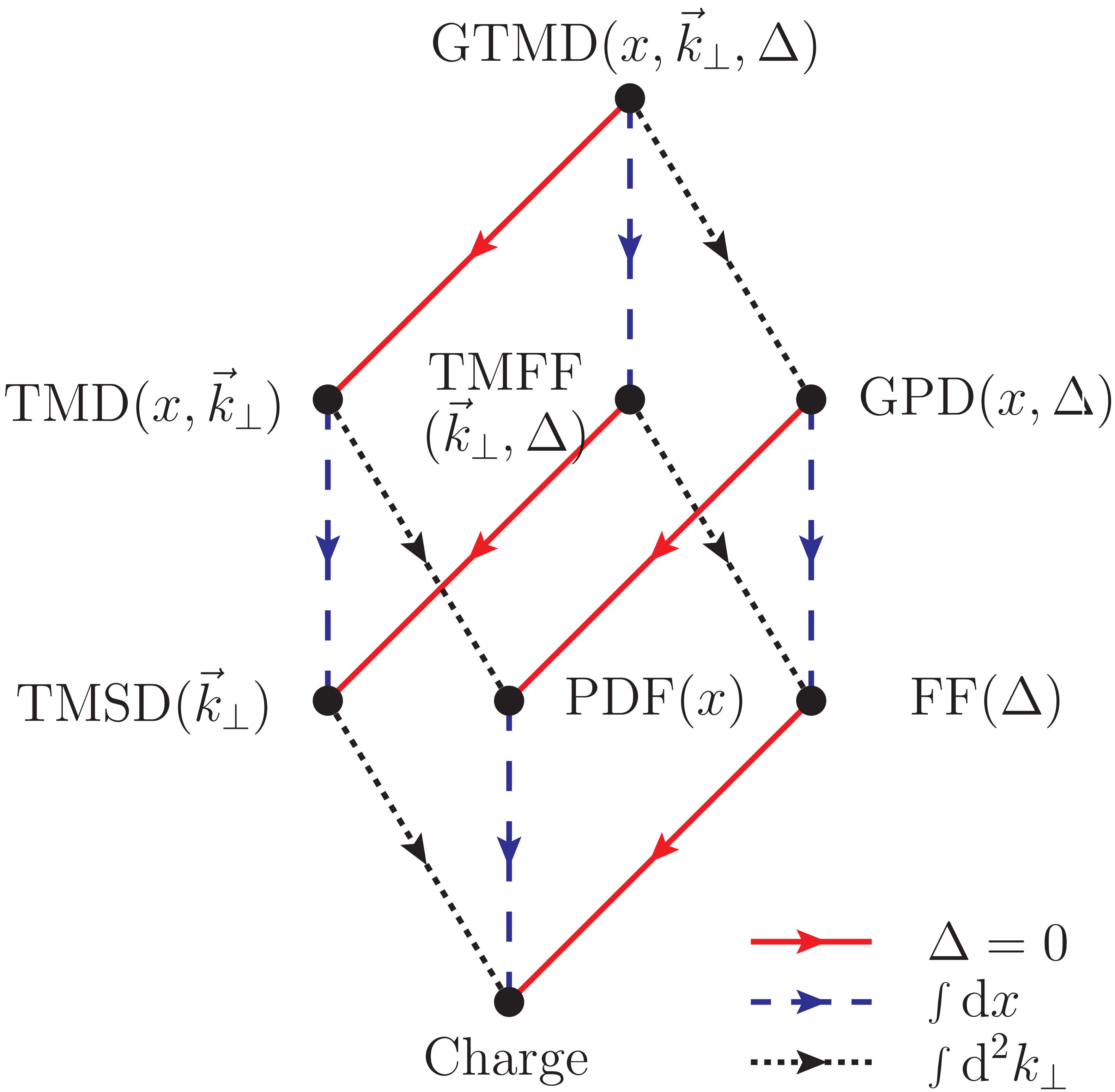
The nucleon landscape

Markus Diehl, Eur. Phys. J. A52 (2016) 149

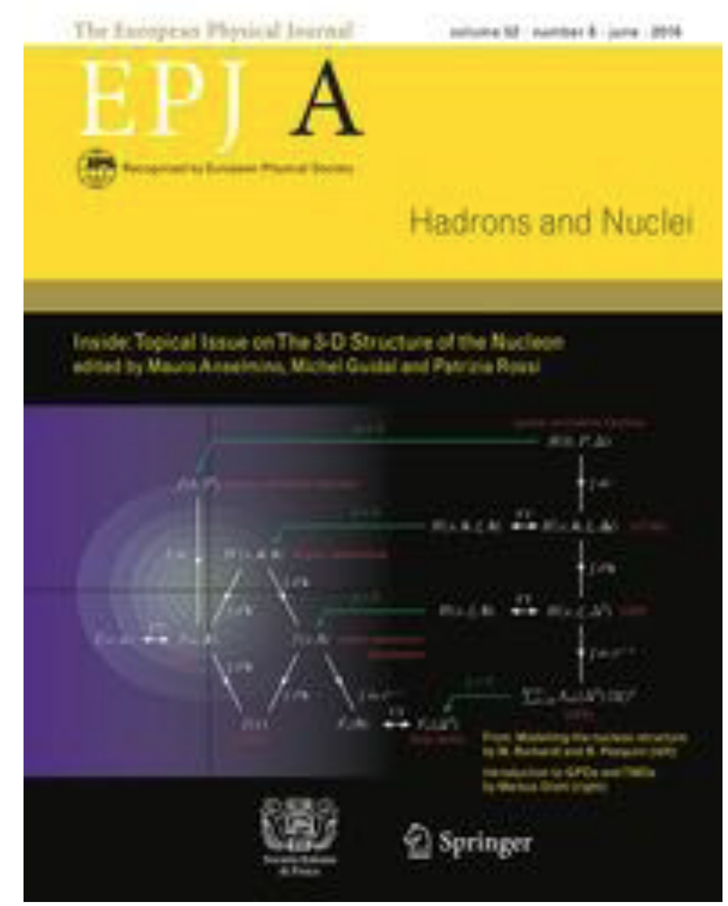


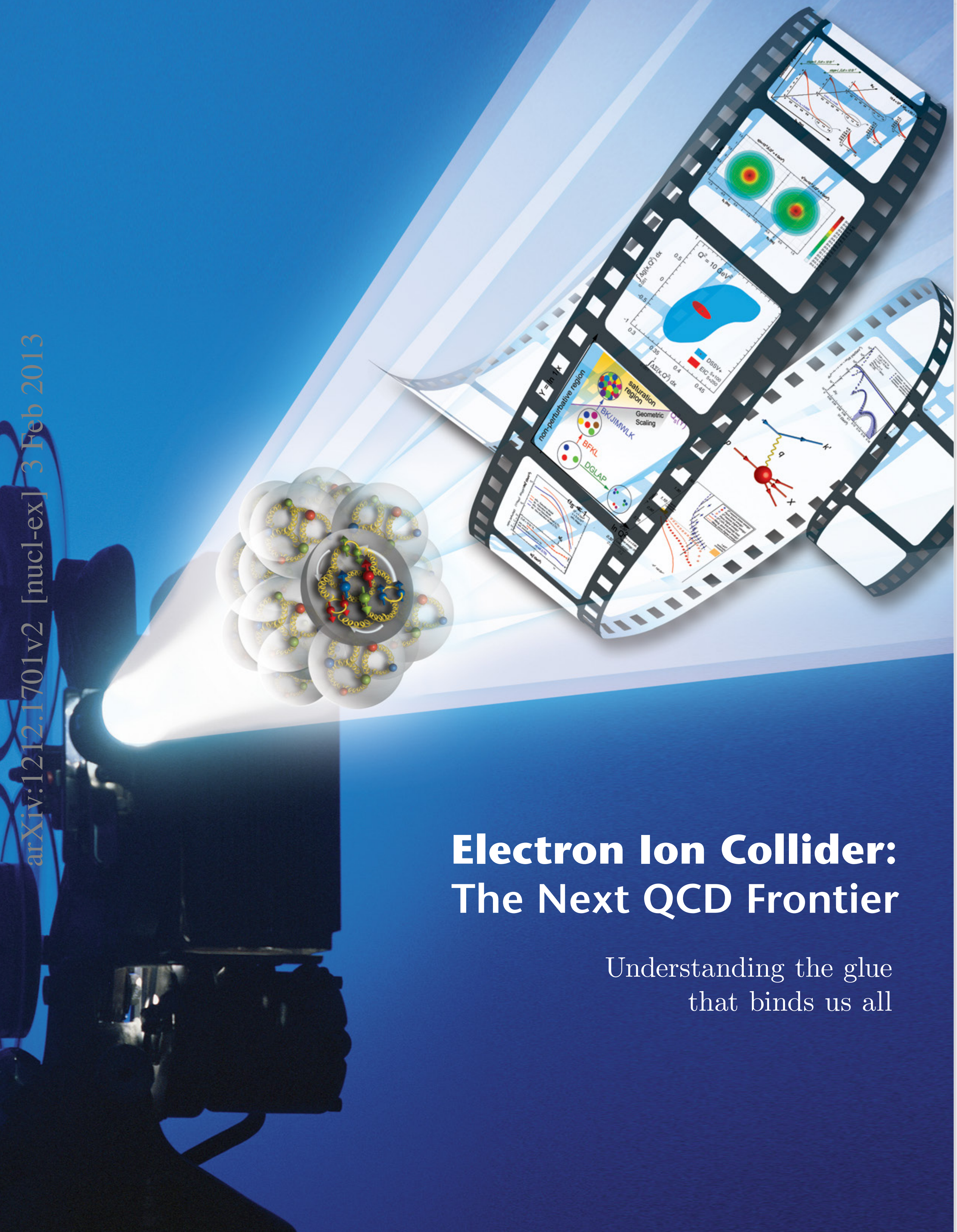
models of the Wigner distribution most welcome....

Burkardt, Pasquini, Eur. Phys. J. A52 (2016) 161



special issue of EPJ A
dedicated to the 3D
nucleon structure,
EPJ A52, 2016, n.6
15 contributions,
(Edts. M.A., P. Rossi, M. Guidal)





future facilities
and experiments:

D-Y @ COMPASS

JLAB 12 GeV

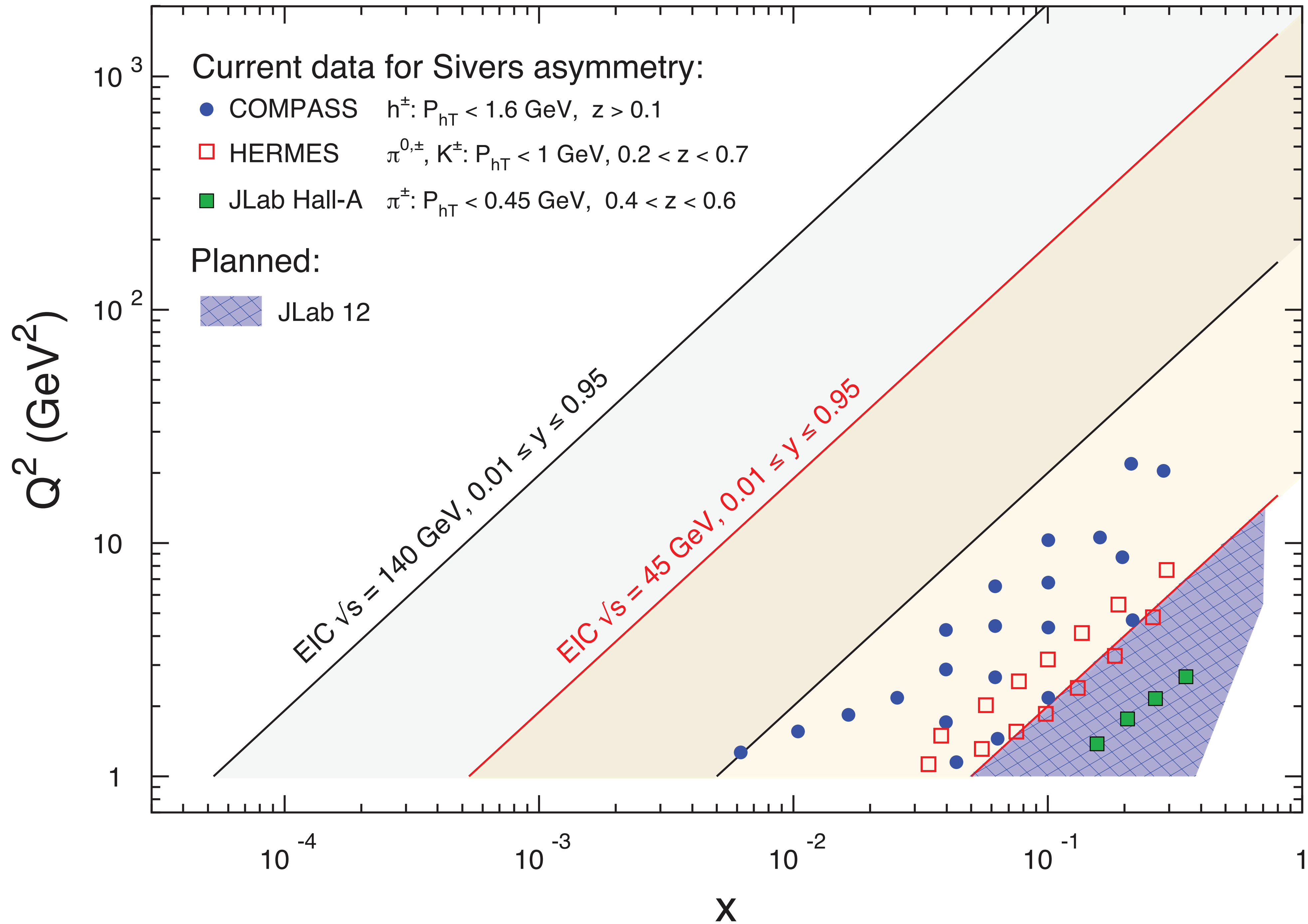
EIC

BESIII

AFTER

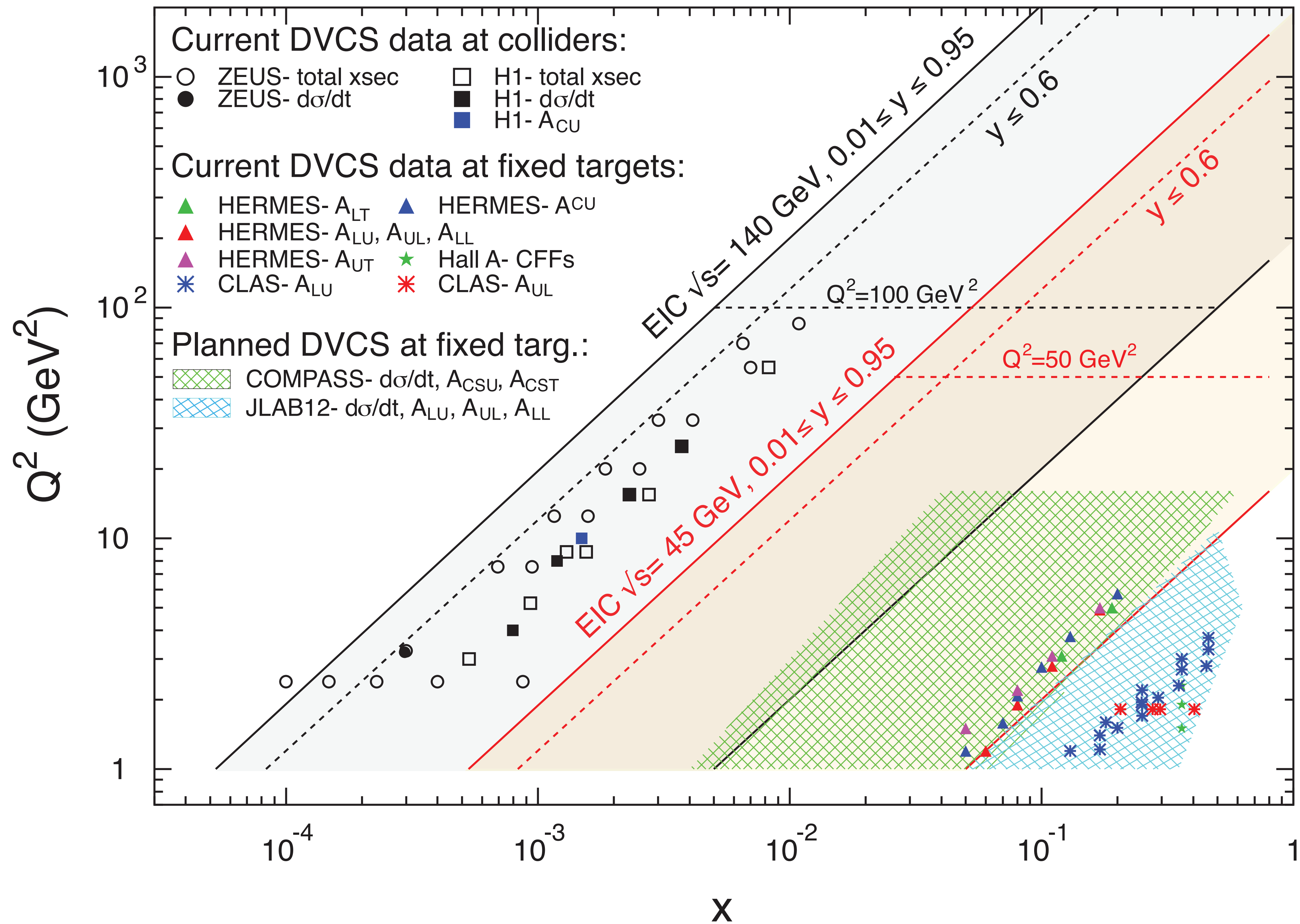
NICA-SPD

....



possible EIC kinematical coverage - SIDIS

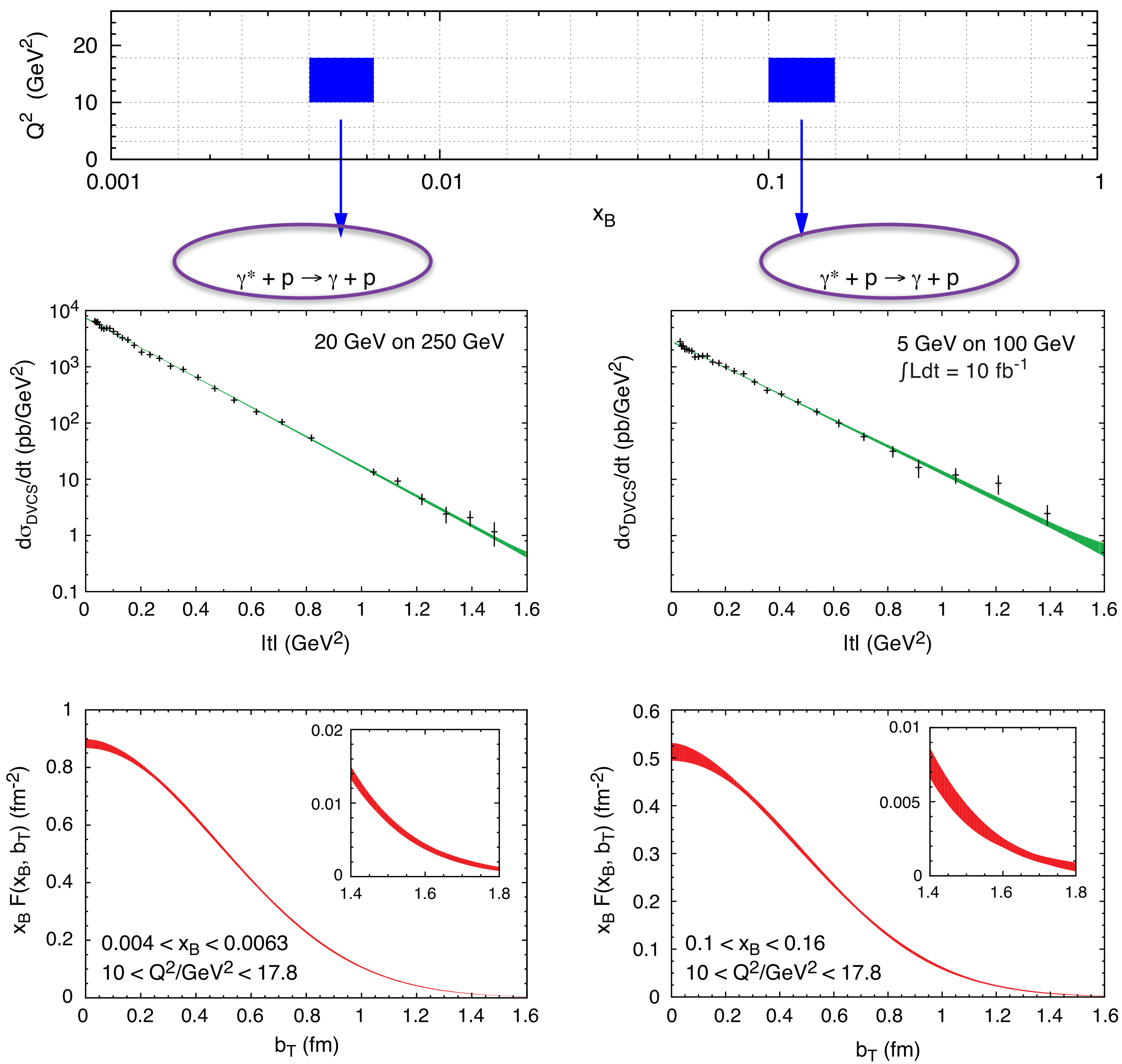
Electron Ion Collider: The Next QCD Frontier - Understanding the glue that binds us all: Eur. Phys. J. A52 (2016) 268

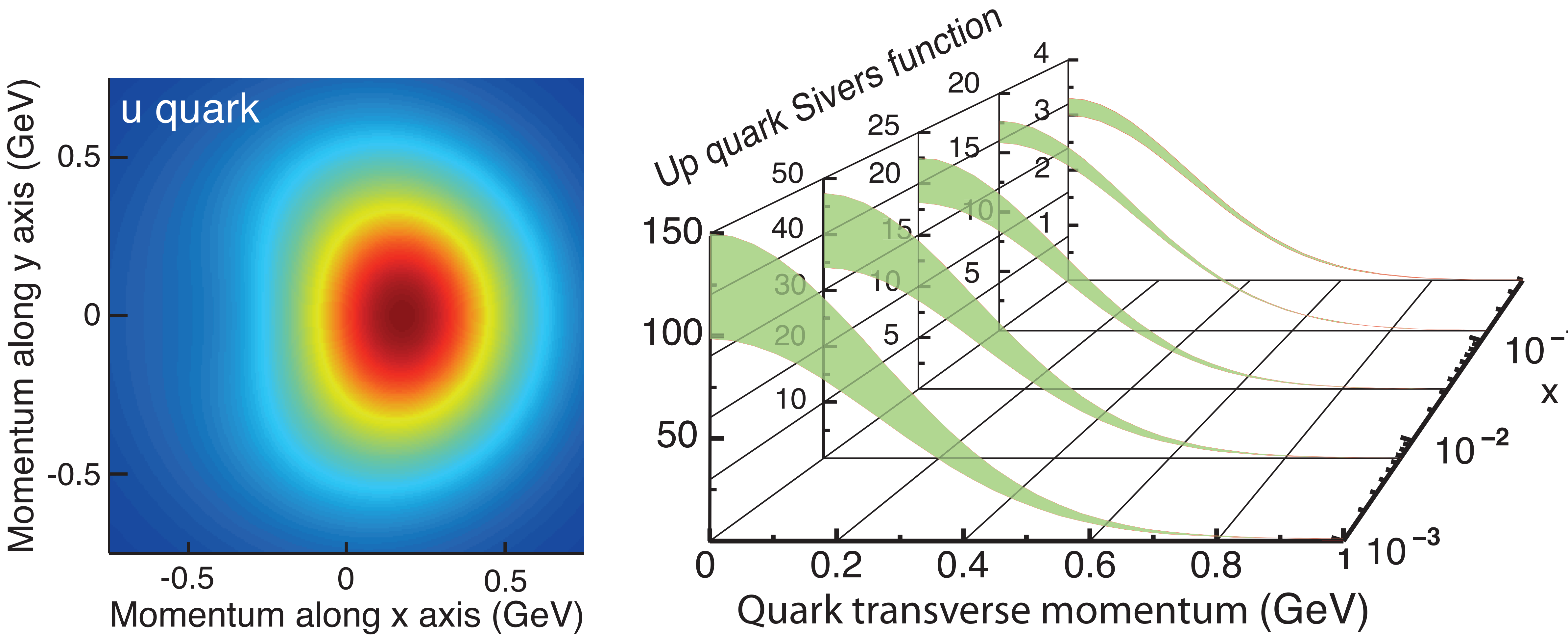


possible EIC kinematical coverage - Deeply Virtual Compton Scattering

expected results at EIC - from DVCS to GPDs to spatial parton distributions

EIC White Paper, arXiv:1212.1701




u quark
Momentum along y axis (GeV)
0.5
0
-0.5
-0.5
0
0.5
Momentum along x axis (GeV)
Up quark Sivers function
150
100
50
50
40
30
20
10
25
20
15
10
5
20
15
10
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1
Quark transverse momentum (GeV)
x
10^{-1}
10^{-2}
10^{-3}

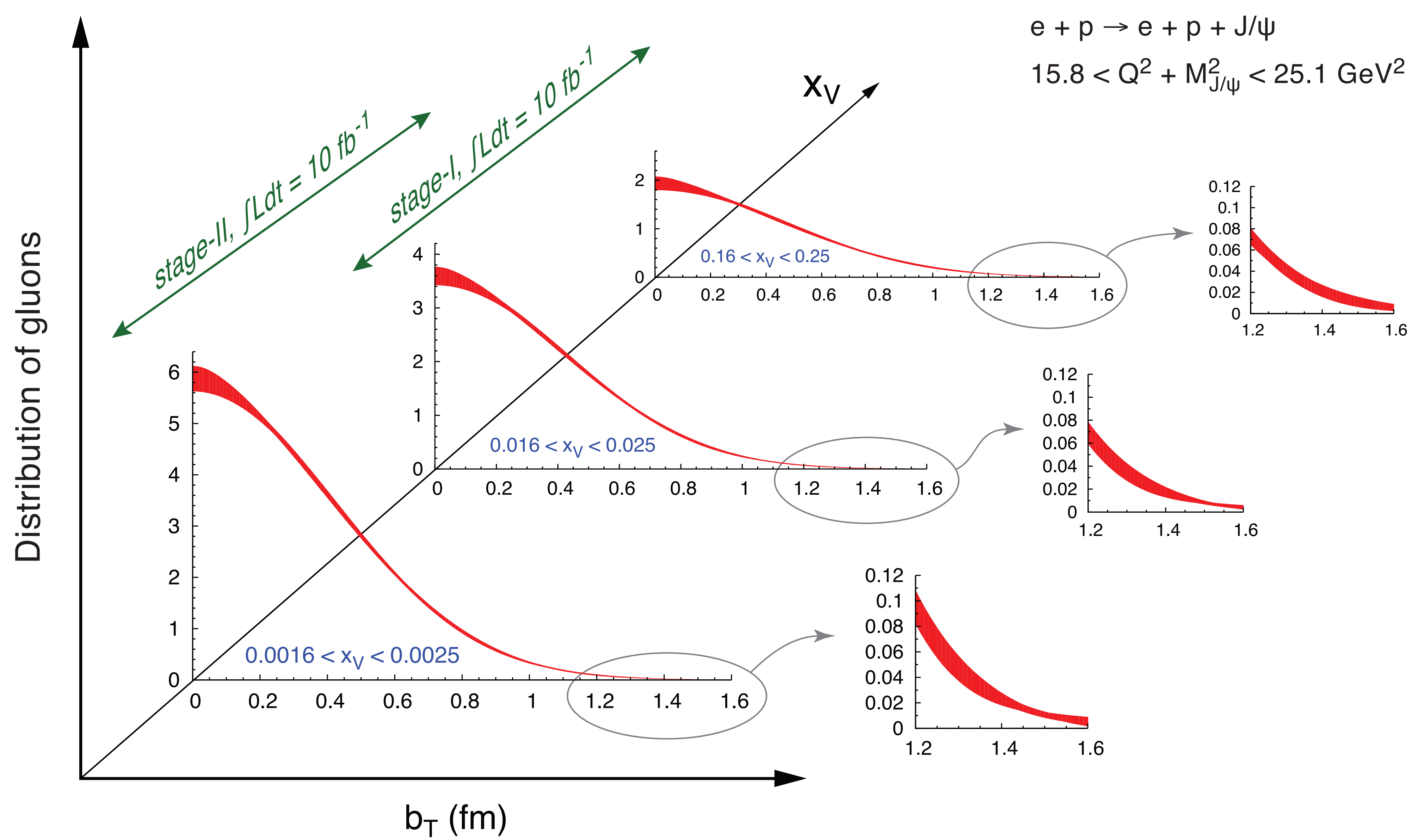
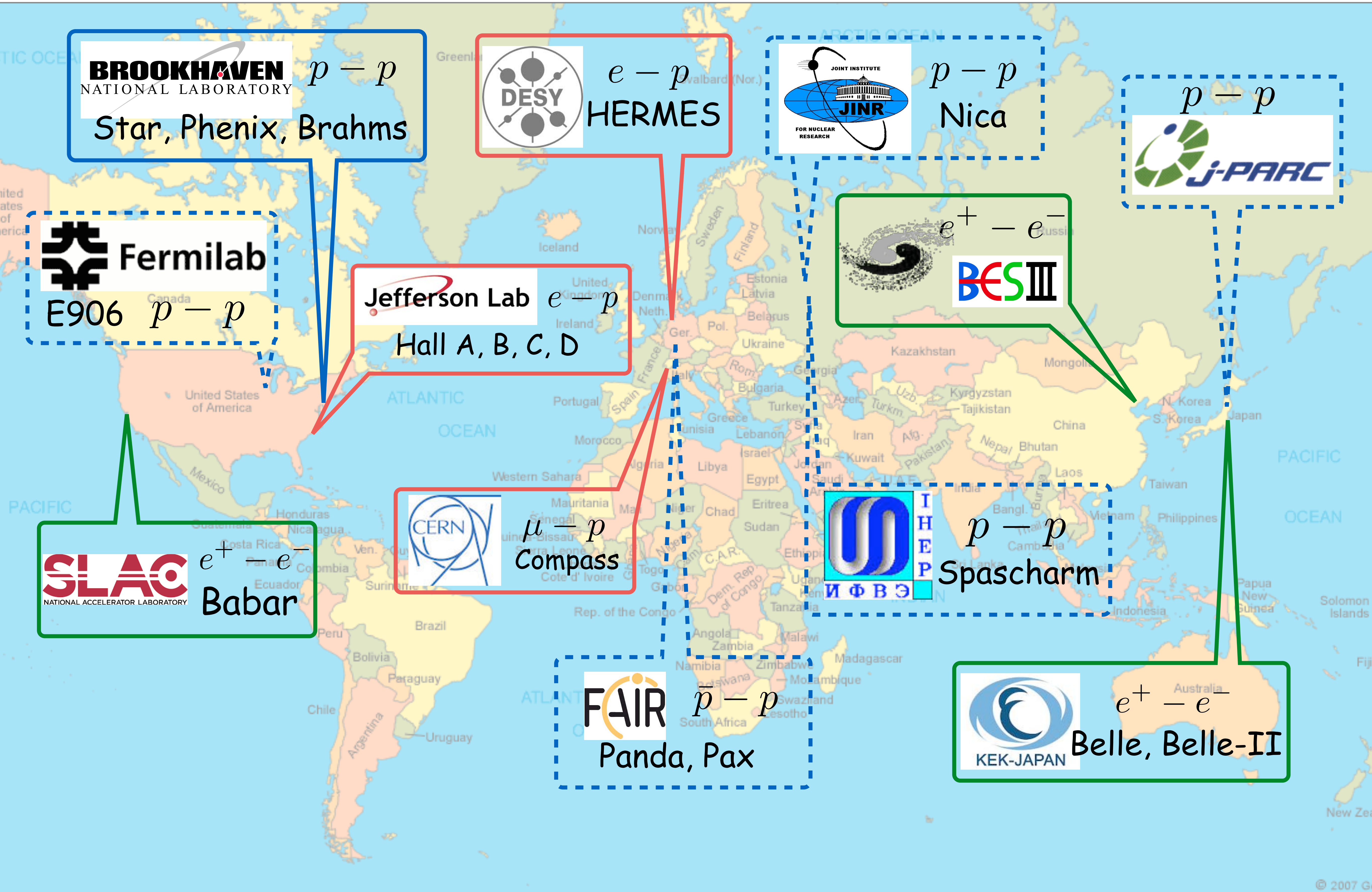


Figure 1.4: The projected precision of the transverse spatial distribution of gluons as obtained from the cross-section of exclusive J/Ψ production. It includes statistical and systematic uncertainties due to extrapolation into the unmeasured region of momentum transfer to the scattered proton. The distance of the gluon from the center of the proton is b_T in femtometers, and the kinematic quantity $x_V = x_B\,(1 + M_{J/\Psi}^2/Q^2)$ determines the gluon's momentum fraction. The collision energies assumed for Stage-I and Stage-II are $E_e = 5, 20$ GeV and $E_p = 100, 250$ GeV, respectively.

some hadron physics in the world



The 3D nucleon structure is mysterious and fascinating. Many experimental results show the necessity to go beyond the simple collinear partonic picture and give new information. Crucial task is interpreting data and building a consistent 3D description of the nucleon.

Sivers and Collins effects are well established, with many transverse spin asymmetries resulting from them. Sivers function, TMDs and orbital angular momentum? QCD analysis of TMDs and GPDs sound and well developed.

Combined data from SIDIS, Drell-Yan, e+e-, with theoretical modelling, should lead to a true 3D imaging of the proton

Waiting for JLab 12, new COMPASS results, and, crucially, for an EIC dedicated facility ....

thank you!