

Bad-Honnef, September 24-30, 2017

QCD-inspired model predictions of forward observables at the LHC and beyond

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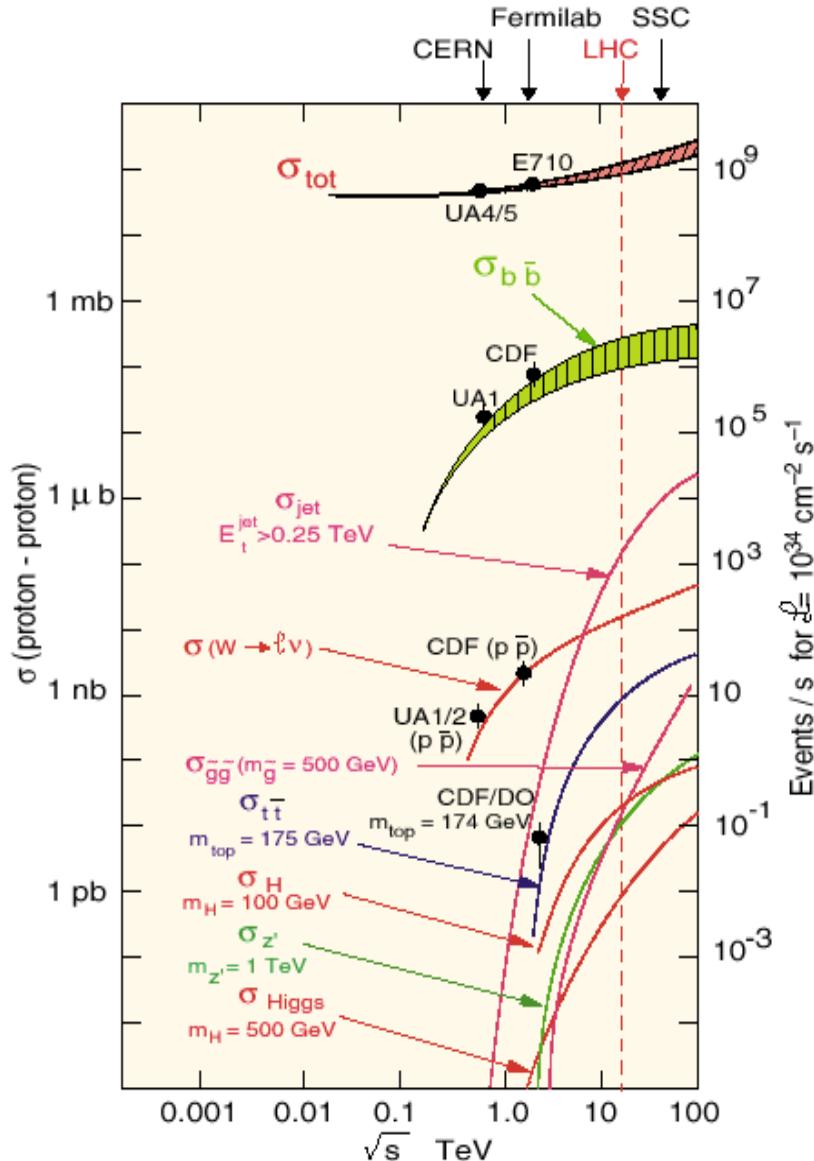
jenk@bitp.kiev.ua

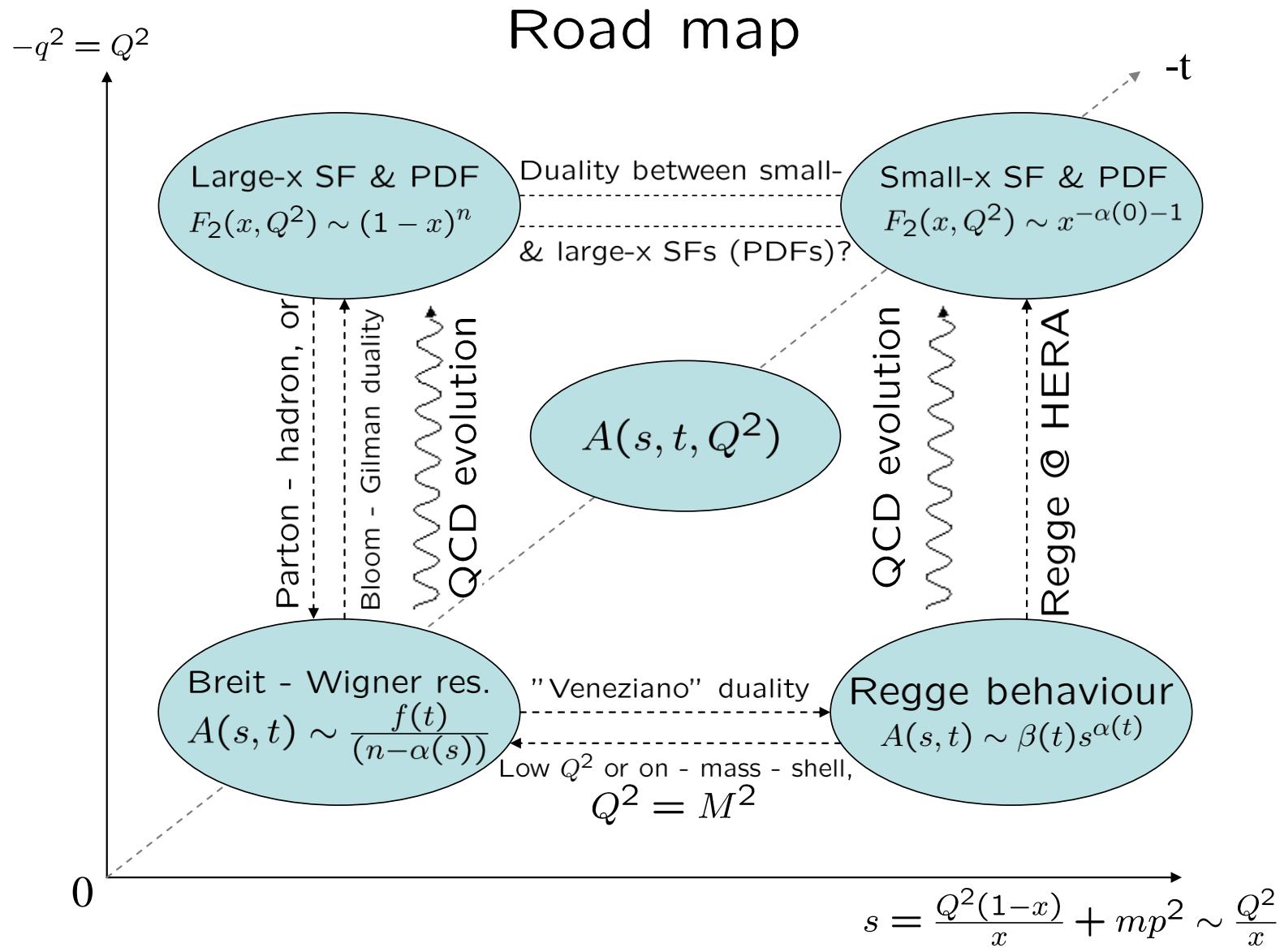
- *My thanks go to the Wilhelm and Else Heraeus Foundation for its hospitality and to Norbert Bence, Francesco Celiberto, Oleg Kuprash, Mikael Mieskolainen, Rainer Schicker, István Szanyi for their collaboration*

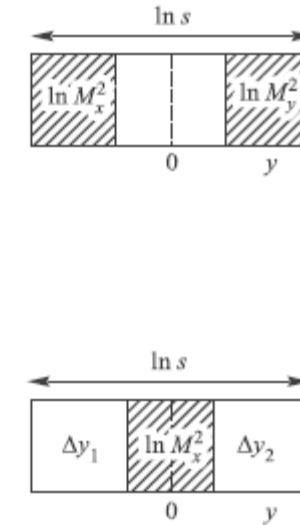
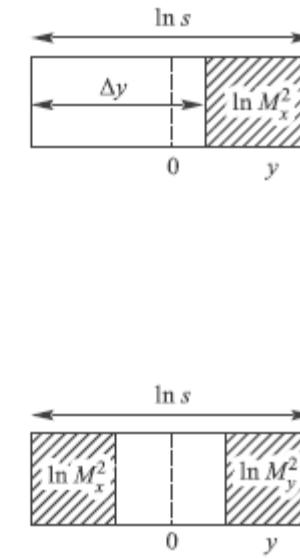
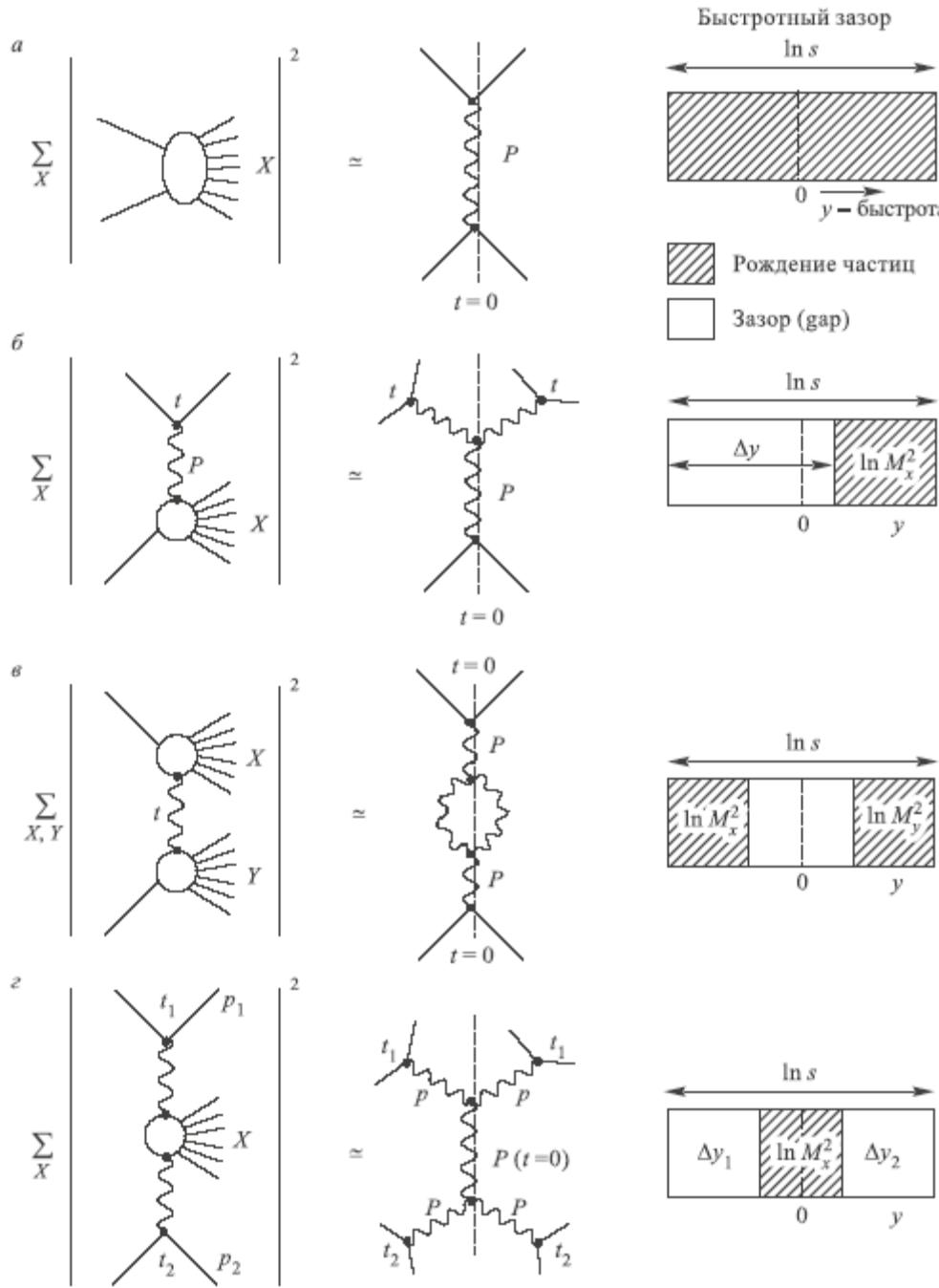
ITEMS:

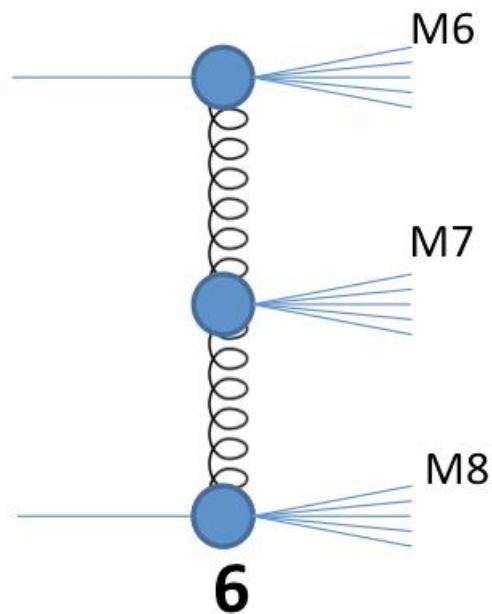
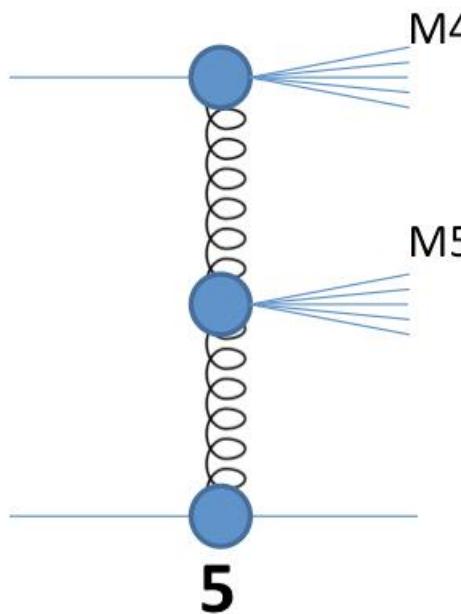
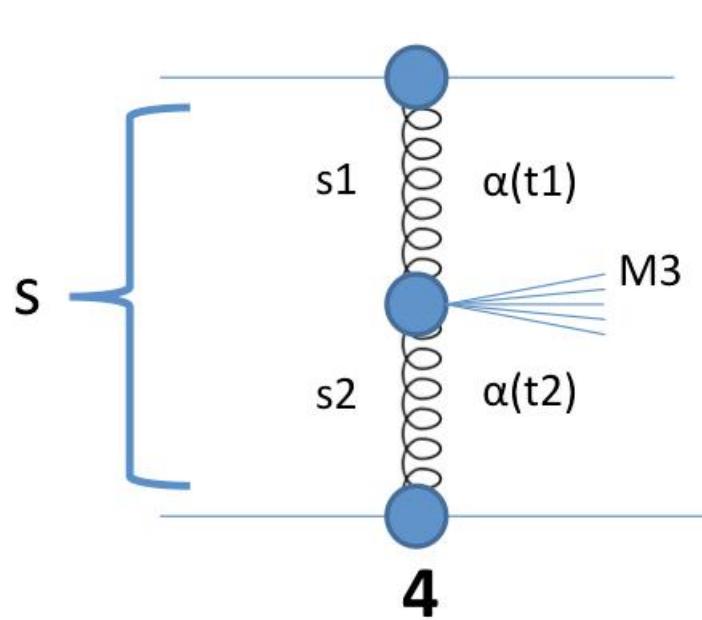
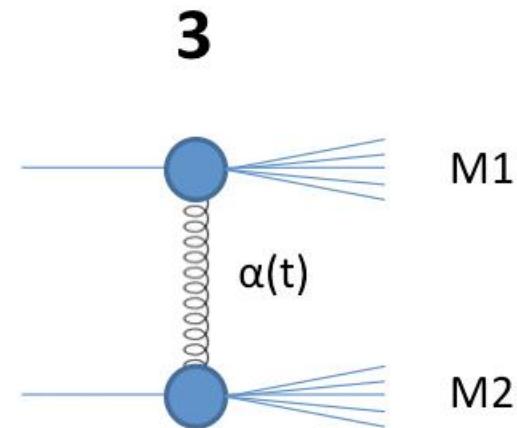
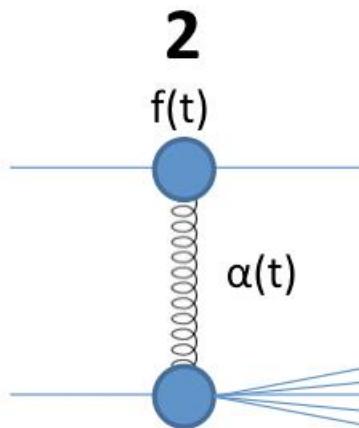
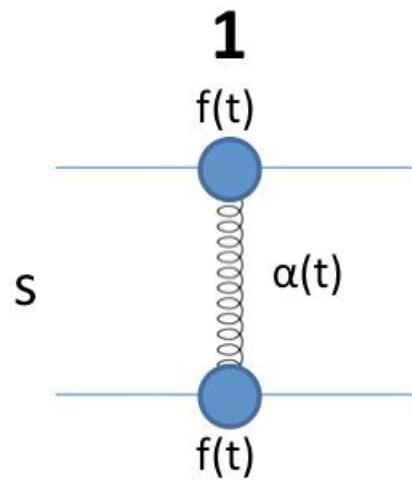
- Why is forward physics important (however difficult and expensive!);
- Road map (in s , t and Q^2); NN and ep ;
- Basic ingredients, tools, measurables;
- General recipe for the construction of a scattering amplitude: input + unitarity;
- Elastic NN is a “building block”, however elastic low- $|t|$ scattering is not rewarding (no black holes, no supersym., no QGP, no dark matter, no sc. fiction);
- How many pomerons; “hard” vs “soft”, QCD vs. “DL”, extremism (Landau);
- History with the “hard” pomeron intercept;
- The odderon: \bar{pp} - pp ;
- Local qft, QCD, analytic S-matrix theory, res.-Regge duality; Regge trajectory;
- How many resonances (infinite or finite: melting and boiling?);
- Inelastic (SD, DD and central diffraction);
- “Reggeometry”; balancing between “soft” (NN) and “hard” (VMP);
- Theory and phenomenology (history of the A-bomb).

- Total cross section at LHC
 $\sigma(pp \rightarrow \text{anything}) \sim 0.1 \text{ barn}$
- So a 1 pb Higgs cross section corresponds to one being *produced* every 10^{11} interactions!
(further reduced by $\text{BR} \times \text{efficiency}$)
- Experiments have to be designed so that they can separate such a rare signal process from the background
- Rate = $L \cdot \sigma$
where luminosity L (units $\text{cm}^{-2}\text{s}^{-1}$) is a measure of how intense the beams are
LHC design luminosity = $10^{34} \text{ cm}^{-2}\text{s}^{-1}$









Factorization (nearly perfect at the LHC!)

$$(g_1 g_2)^2 = \frac{(g_1 f_1)^2 (f_1 g_2)^2}{(f_1 f_2)^2}.$$

Hence

$$\frac{d^3 \sigma}{dt dM_1^2 dM_2^2} = \frac{d^2 \sigma_1}{dt dM_1^2} \frac{d^2 \sigma_2}{dt dM_2^2} \frac{d\sigma_{el}}{dt}.$$

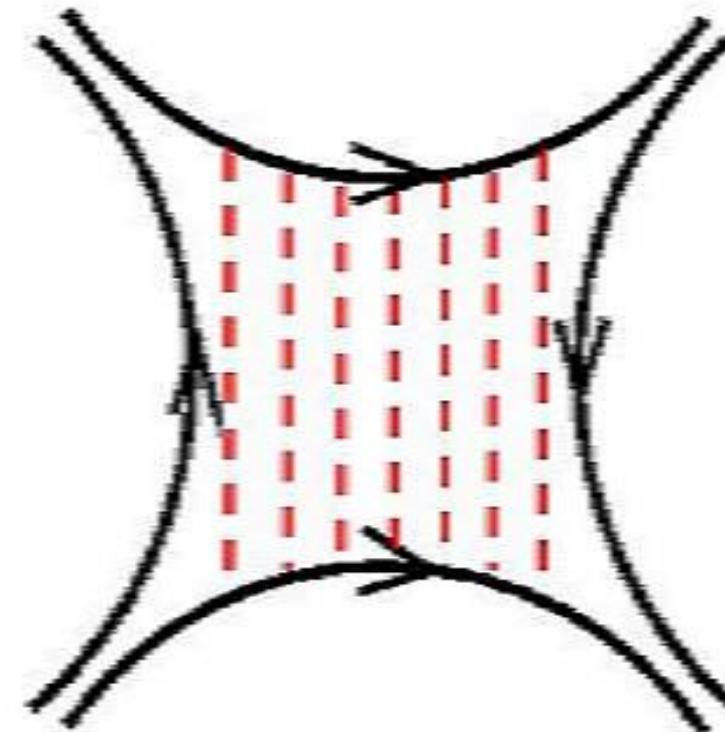
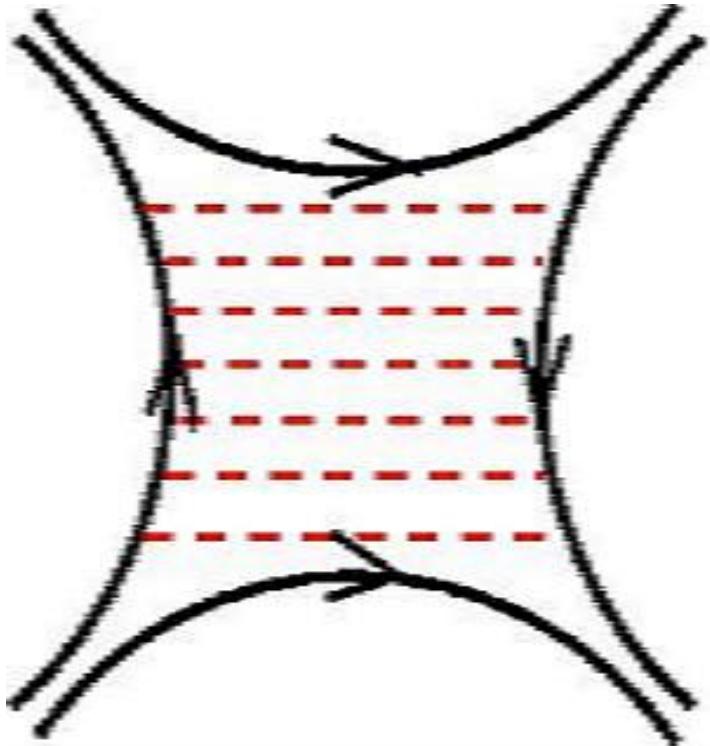
Assuming exponential cone, t^{bt} and integrating in t , one gets

$$\frac{d^2 \sigma_{DD}}{dM_1^2 dM_2^2} = k \frac{1}{\sigma_{el}} \frac{d\sigma_1}{dM_1^2} \frac{d\sigma_2}{dM_2^2},$$

where $k = r^2/(2r - 1)$, $r = b_{SD}/b_{el}$.

Further integration in M^2 yields $\sigma_{DD} = k \frac{\sigma_{SD}^2}{\sigma_{el}}$.

DUALITY:



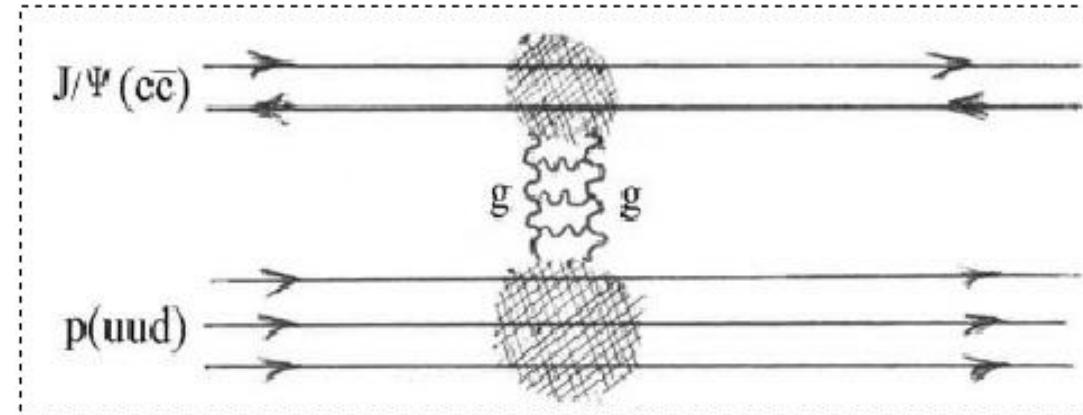
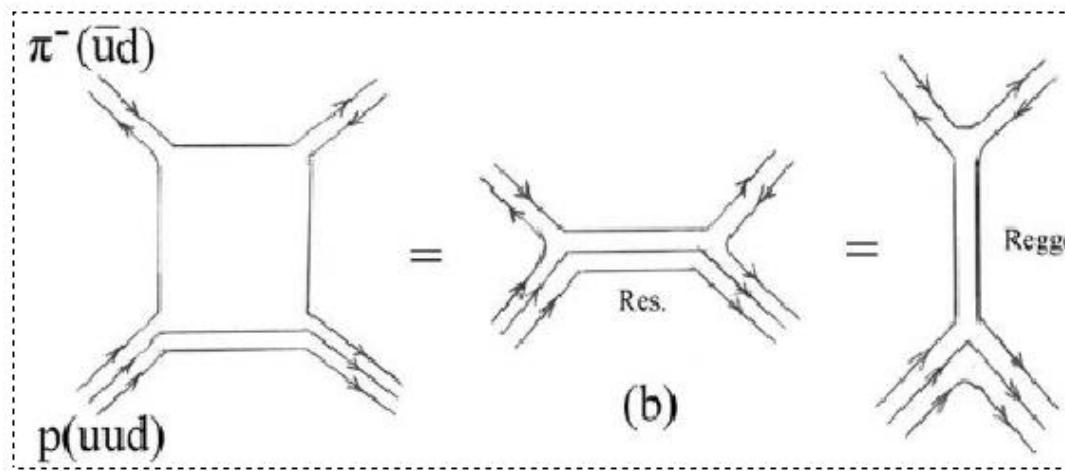
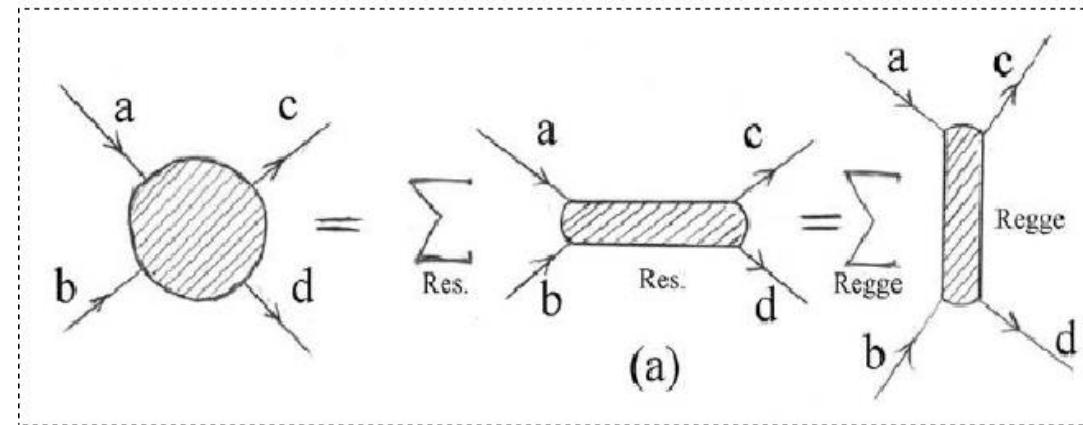
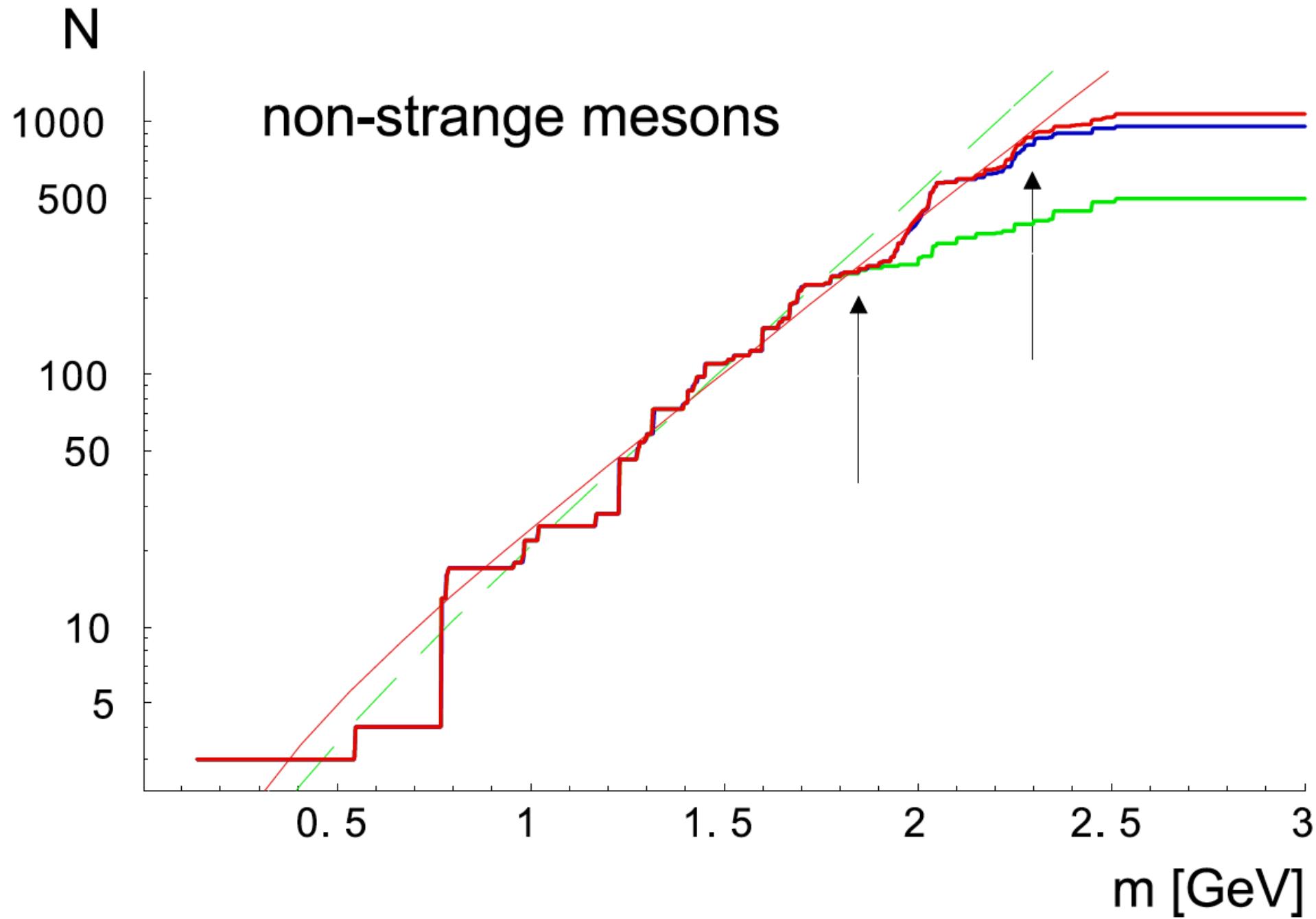


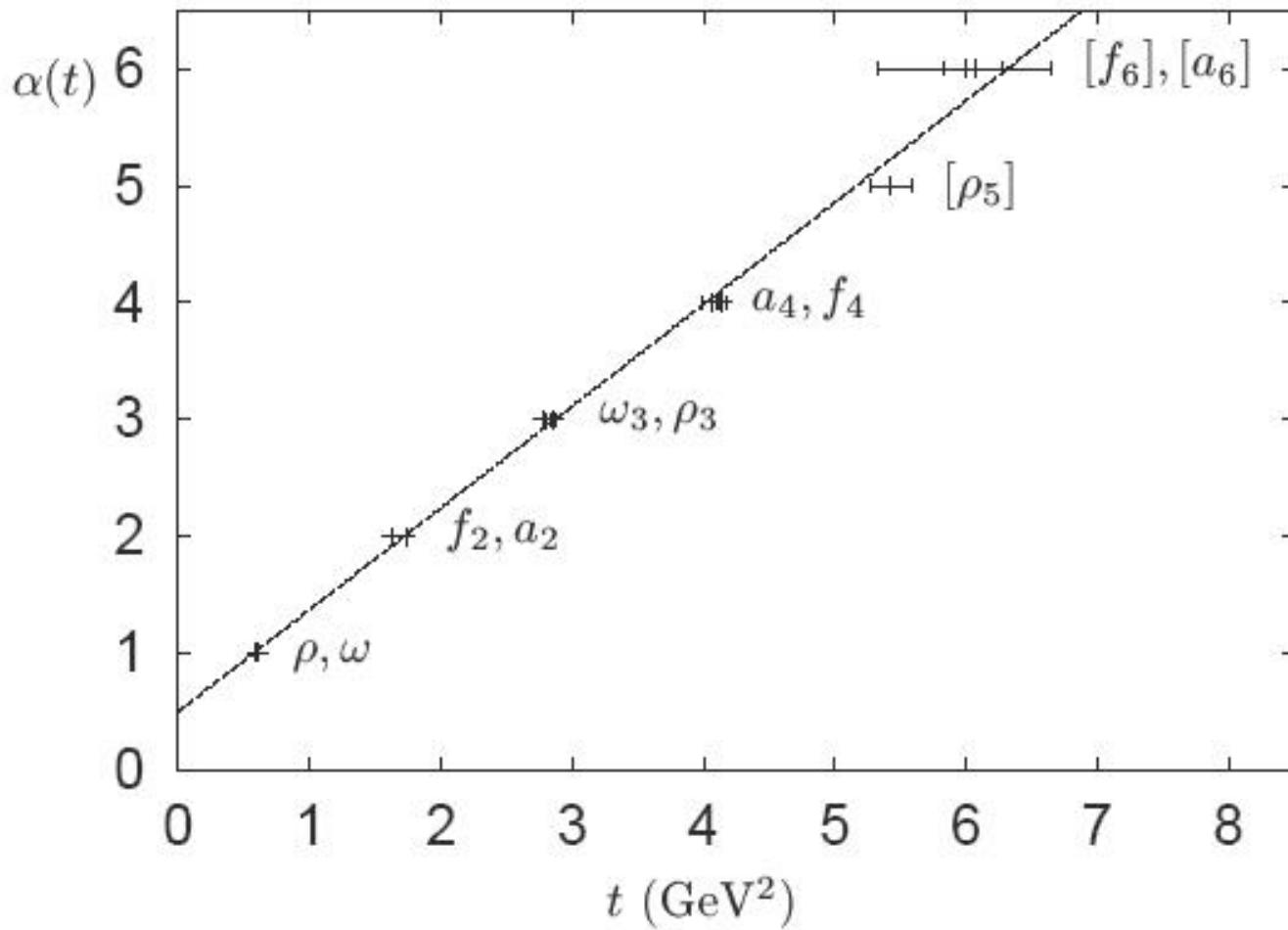
TABLE I: Two-component duality

$\text{Im}A(a + b \rightarrow c + d) =$	R	Pomeron
s -channel	$\sum A_{Res}$	Non-resonant background
t -channel	$\sum A_{Regge}$	Pomeron ($I = S = B = 0; C = +1$)
Duality quark diagram	Fig. 1b	Fig. 2
High energy dependence	$s^{\alpha-1}, \alpha < 1$	$s^{\alpha-1}, \alpha \geq 1$



Linear particle trajectories

Plot of spins of families of particles against their squared masses:



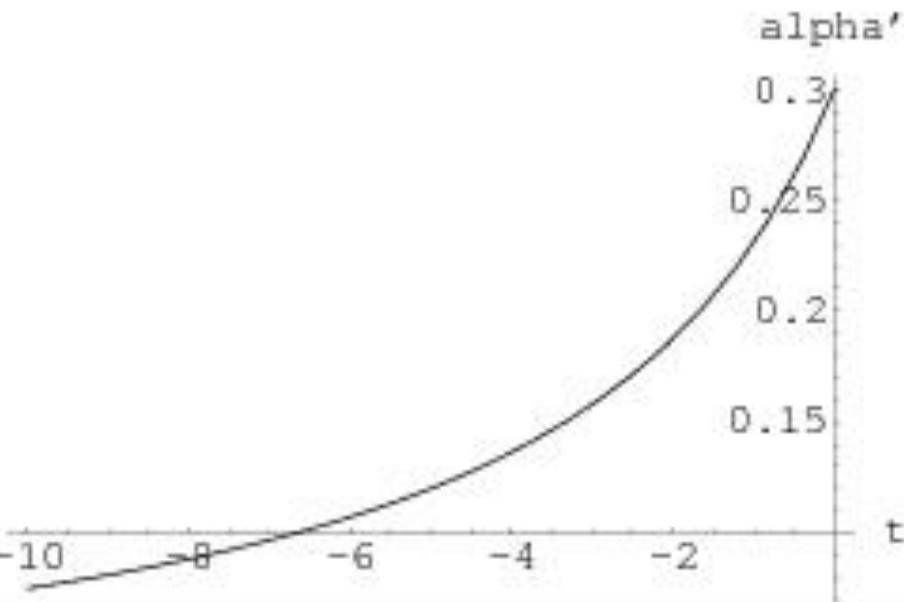
The Pomeron trajectory

The Pomeron trajectory has threshold singularities, the lowest one being due to the two-pion exchange, required by the t -channel unitarity. There is a constrain (Barut, Zwanziger; Gribov) from the t - channel unitarity, by which

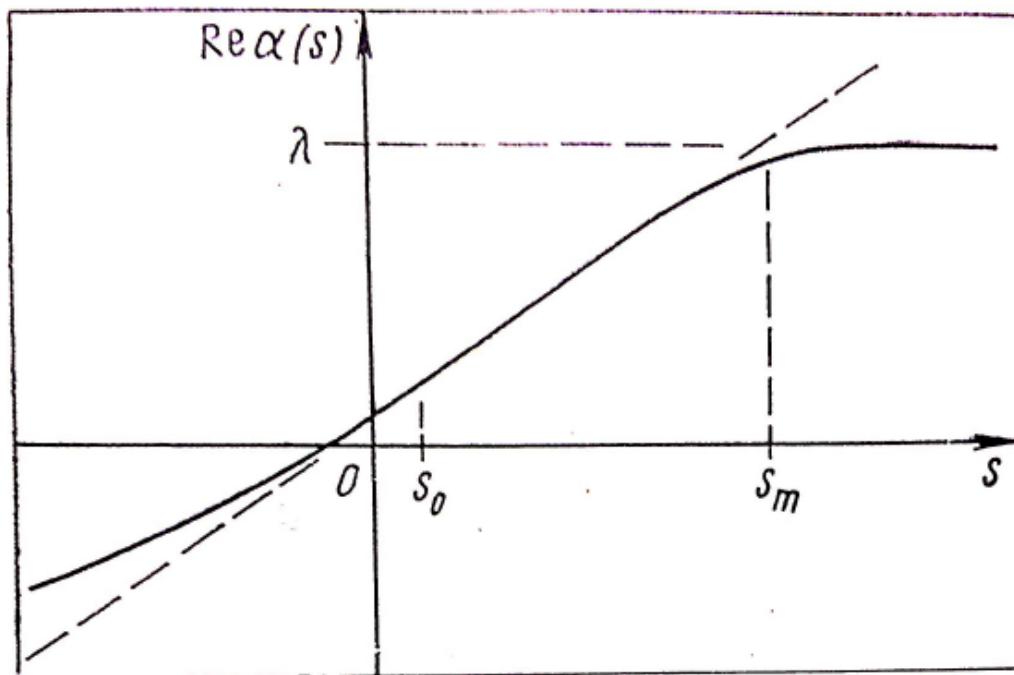
$$\Im\alpha(t) \sim (t - t_0)^{\Re\alpha(t_0) + 1/2}, \quad t \rightarrow t_0,$$

where t_0 is the lightest threshold. For the Pomeron trajectory it is $t_0 = 4m_\pi^2$, and near the threshold:

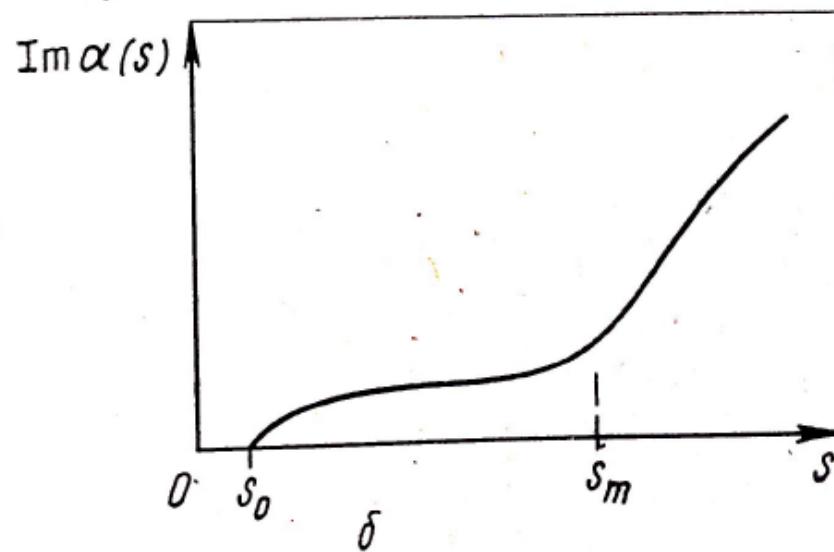
$$\alpha(t) \sim \sqrt{4m_\pi^2 - t}. \tag{1}$$



The slope of the cone for a single pole is:
 $B(s, t) \sim \alpha'(t) \ln s$. The Regge residue $e^{b\alpha(t)}$ with a logarithmic trajectory $\alpha(t) = \alpha(0) - \gamma \ln(1 - \beta t)$, is identical to a form factor (geometrical model).



α



$$\Re e \alpha(s) = \alpha(0) + \frac{s}{\pi} PV \int_0^\infty ds' \frac{\Im m \alpha(s')}{s'(s'-s)}. \quad (7)$$

In Eq. (7), PV denotes the Cauchy Principal Value of the integral. The imaginary part is related to the decay width by

$$\Gamma(M_R) = \frac{\Im m \alpha(M_R^2)}{\alpha' M_R}. \quad (8)$$

The quantity α' in Eq. (8) denotes the derivative of the real part, $\alpha' = \frac{d\Re e \alpha(s)}{ds}$. The relation between $\Gamma(M)$ and $\Im m \alpha(s)$ requires $\Im m \alpha(s) > 0$. In a simple analytical model, the imaginary part is chosen as a sum of single threshold terms [25]

$$\Im m \alpha(s) = \sum_n c_n (s - s_n)^{1/2} \left(\frac{s - s_n}{s}\right)^{|\Re e \alpha(s_n)|} \theta(s - s_n). \quad (9)$$

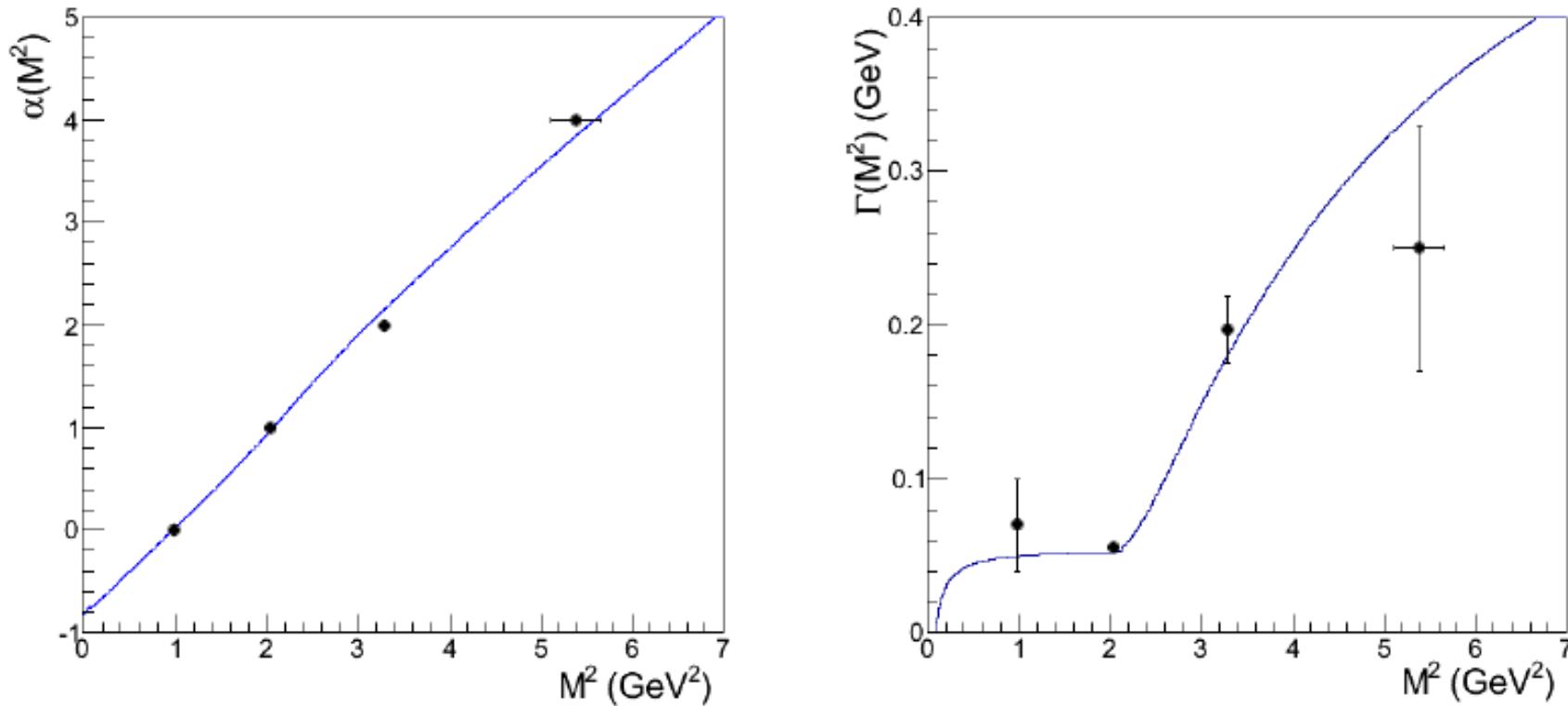


FIG. 6: Real part of f_1 trajectory on the left, width function $\Gamma(M^2)$ on the right.

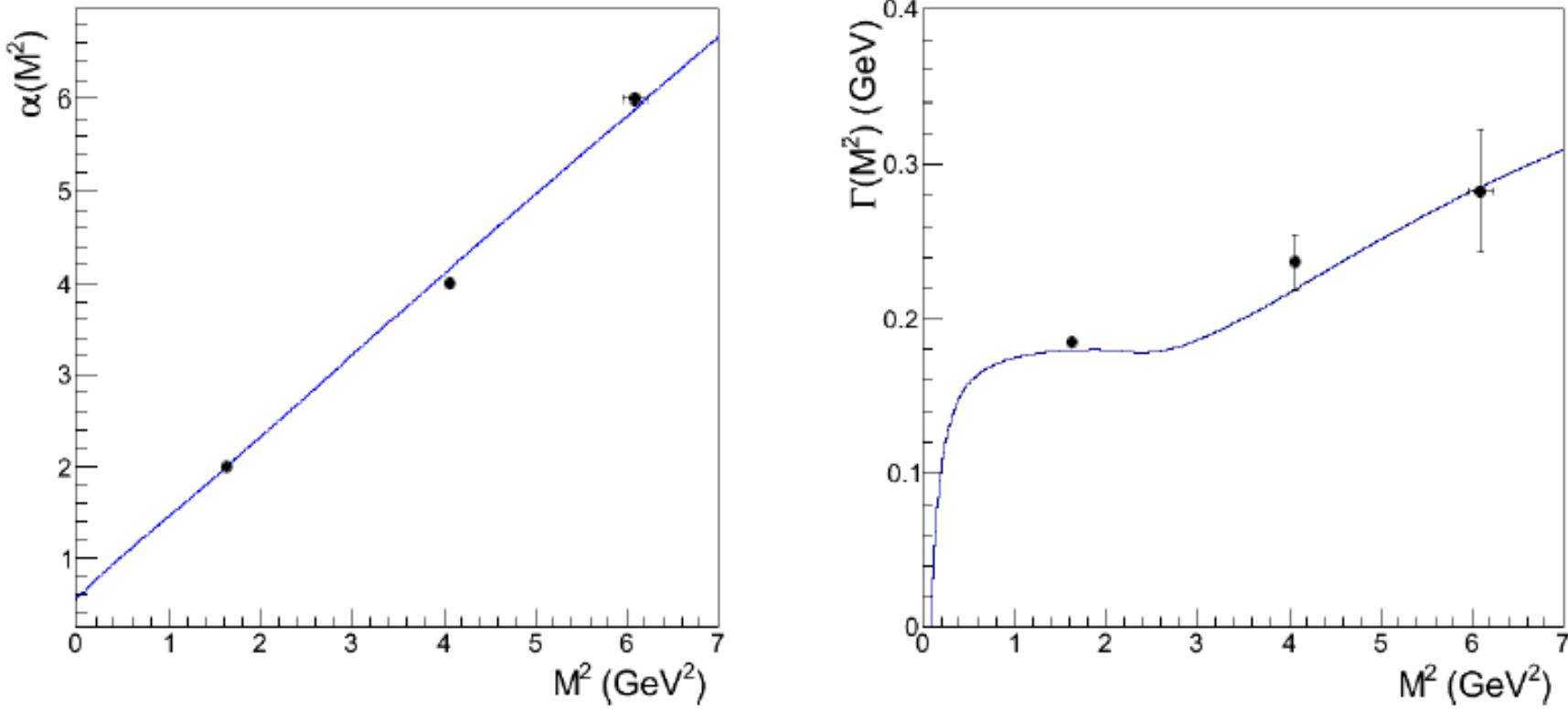
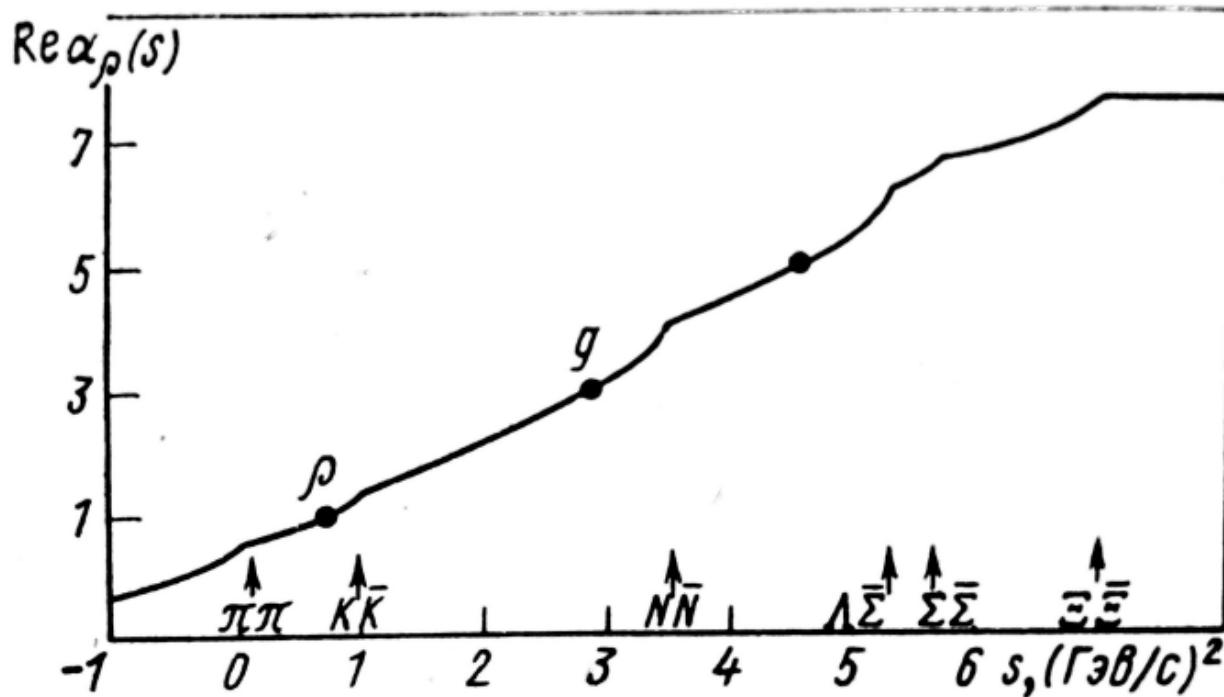


FIG. 7: Real part of f_2 trajectory on the left, width function $\Gamma(M^2)$ on the right.

Let us start with a toy model of a non-linear trajectory. Following [18, 31], we write a simple trajectory in which the (additive) thresholds are those made of stable particles allowed by quantum numbers. For the ρ trajectory these are: $\pi\pi$, $K\bar{K}$, $N\bar{N}$, $\Lambda\bar{\Sigma}$, $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$. The relevant trajectory is:

$$\alpha_\rho(m) = 7.64 - 0.127\sqrt{m - 0.28} - 0.093\sqrt{m - 0.988} - 0.761\sqrt{m - 1.88} - " \Lambda\bar{\Sigma}, \Sigma\bar{\Sigma}, \Xi\bar{\Xi}" , \quad (10)$$

with the parameters of higher threshold quoted in Ref. [18].



$$N_{theor} = \int_0^m \rho_{theor}(m') dm',$$

where

$$\rho_{theor}(m) = f(m) \exp(m/T)$$

and $f(m) \approx A/(m^2 + (500\text{MeV})^2)^{5/4}$ (alternative choices for this slowly varying function are possible).

According to Hagedorn's conjecture, confirmed by subsequent studies, the density of hadronic resonances increases exponentially, modulus a slowly varying function of mass, $f(m)$,

$$\rho(m) = f(m) \exp(m/T) \quad (1)$$

up to about $m = 2 \div 2.5$ MeV, whereupon the exponential rise slows down

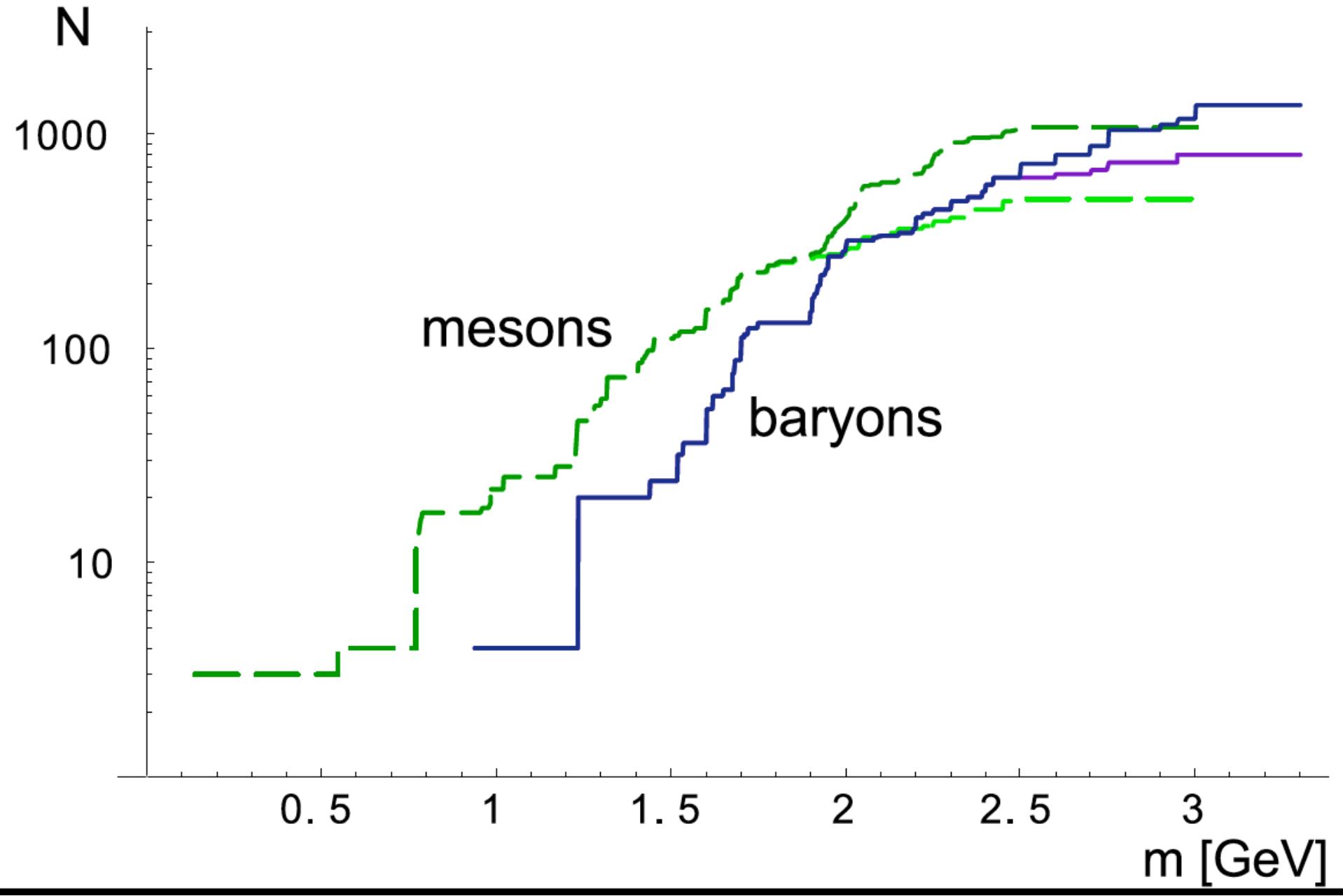
We extend the Hagedorn formula by introducing in the slope of relevant non-linear Regge trajectories. Anticipating a detailed quantitative analyses, one may observe immediately that flattening of $\Re\alpha(s = m^2)$ results in a drastic decrease of the relevant slope $\alpha'(m)$ and a corresponding change of the Hagedorn spectrum, which we parametrize as

$$\rho(m) \sim (\Re\alpha(m))' \exp(m/T). \quad (3)$$

Usually, one compares the cumulants of the spectrum, defined as the number of states with mass lower than m_i . The experimental curve is

$$N_{exp}(m) = \sum_i g_i \Theta(m - m_i), \quad (4)$$

where $g_i = (2J_i + 1)(2I_i + 1)$ is the spin-isospin degeneracy of the i -th state and m_i is its mass. The theoretical curve



The (s, t) term of a dual amplitude is

$$D(s, t) = c \int_0^1 dx \left(\frac{x}{g_1}\right)^{-\alpha(s')-1} \left(\frac{1-x}{g_2}\right)^{-\alpha(t')-1},$$

where s and t are the Mandelstam variables, and g_1, g_2 are parameters, $g_1, g_2 > 1$. For simplicity, we set $g_1 = g_2 = g_0$.

1. Regge behavior, $s \rightarrow \infty$, $t = \text{const}$: $D(s, t) \sim s^{\alpha(t)-1}$;
2. Thereshold behavior, $s \rightarrow s_0$: $D(s, t) \sim \sqrt{s_0 - s} [\text{const} + \ln(1 - s_0/s)]$;

3. Direct-channel poles:

$$D(s, t) = \sum_{n=0}^{\infty} g^{n+1} \sum_{l=o}^n \frac{[-s\alpha'(s)]^l C_{n-l}(t)}{[n - \alpha(s)]^{l+1}}.$$

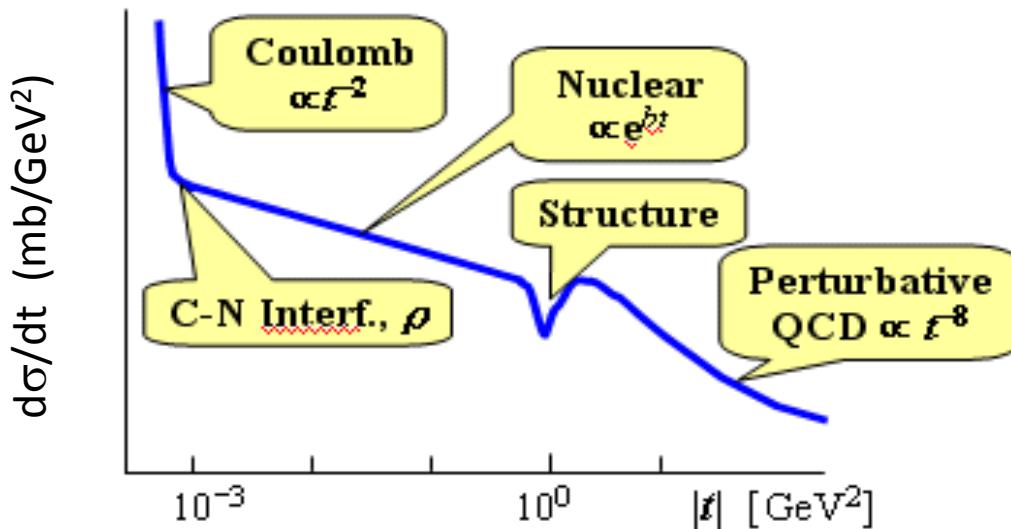
Exotic direct-channel trajectory: $\alpha(s) = \alpha(0) + \alpha_1(\sqrt{s_0} - \sqrt{s_0 - s})$.

"GOLDEN" diffraction reaction: $J/\Psi p -$ scattering: By VMD, photoproduction is reduced to elastic hadron scattering:

$$D(\gamma p - Vp) = \sum \frac{e}{f_V} D(Vp - Vp).$$

Elastic Scattering

$\sqrt{s} = 14 \text{ TeV}$ prediction of BSW model



momentum transfer $-t \sim (p\theta)^2$

θ = beam scattering angle
 p = beam momentum

$$\rho = \frac{\text{Re}(f_{el}(t))}{\text{Im}(f_{el}(t))}_{t \rightarrow 0}$$

$$\left. \frac{dN}{dt} \right|_{t=CNI} = L\pi |f_C + f_N|^2 \approx L\pi \left| -\frac{2\alpha_{\text{EM}}}{|t|} + \frac{\sigma_{\text{tot}}}{4\pi} (i + \rho) e^{-\frac{b|t|}{2}} \right|^2$$

L , σ_{tot} , b , and ρ
from FIT in CNI
region (UA4)

CNI region: $|f_C| \sim |f_N| \rightarrow @ \text{LHC: } -t \sim 6.5 \cdot 10^{-4} \text{ GeV}^2; \theta_{\min} \sim 3.4 \mu\text{rad}$
 $(\theta_{\min} \sim 120 \mu\text{rad} @ \text{SPS})$

$$\sigma_t(s) = \frac{4\pi}{s} \text{Im} A(s, t=0); \quad \frac{d\sigma}{dt} = \frac{\pi}{s^2} |A(s, t)|^2; \quad n(s);$$

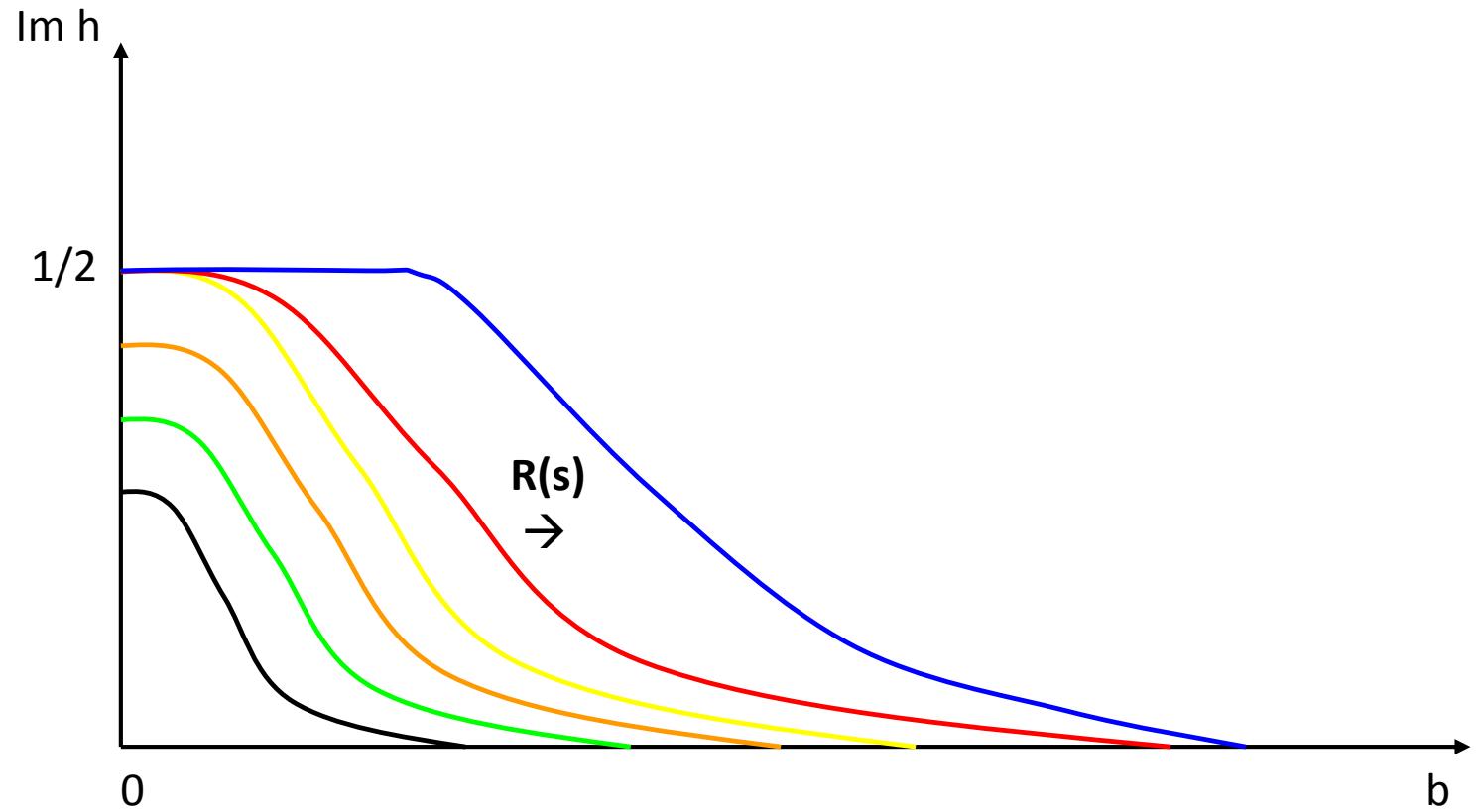
$$\sigma_{el} = \int_{t_{min} \approx -s/2 \approx \infty}^{t_{thr. \approx 0}} \frac{d\sigma}{dt} dt; \quad \sigma_{in} = \sigma_t - \sigma_{el}; \quad B(s, t) = \frac{d}{dt} \ln\left(\frac{d\sigma}{dt}\right);$$

$$A_{pp}^{p\bar{p}}(s, t) = P(s, t) \pm O(s, t) + f(s, t) \pm \omega(s, t) \rightarrow_{LHC} \approx P(s, t) \pm O(s, t),$$

where P , O , f . ω are the Pomeron, odderon and non-leading Reggeon contributions.

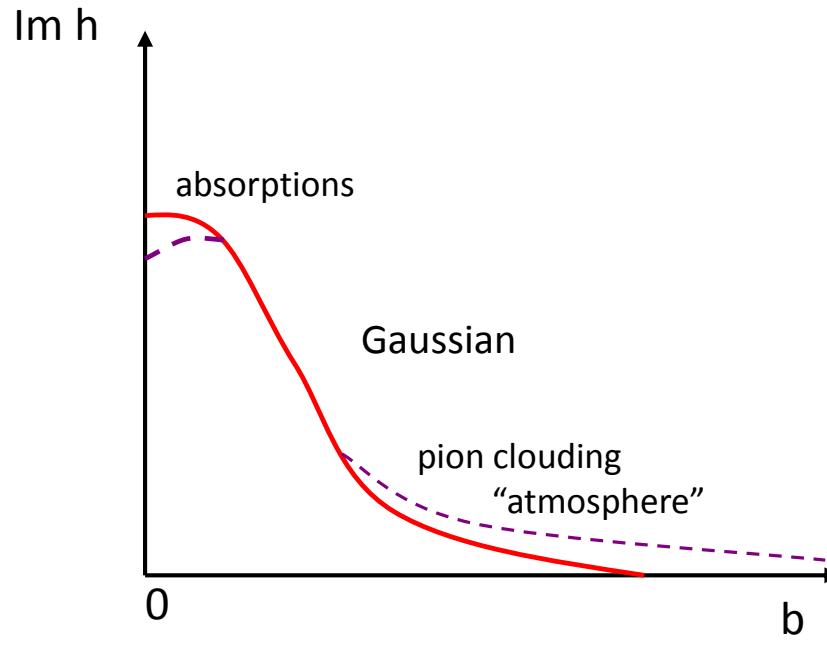
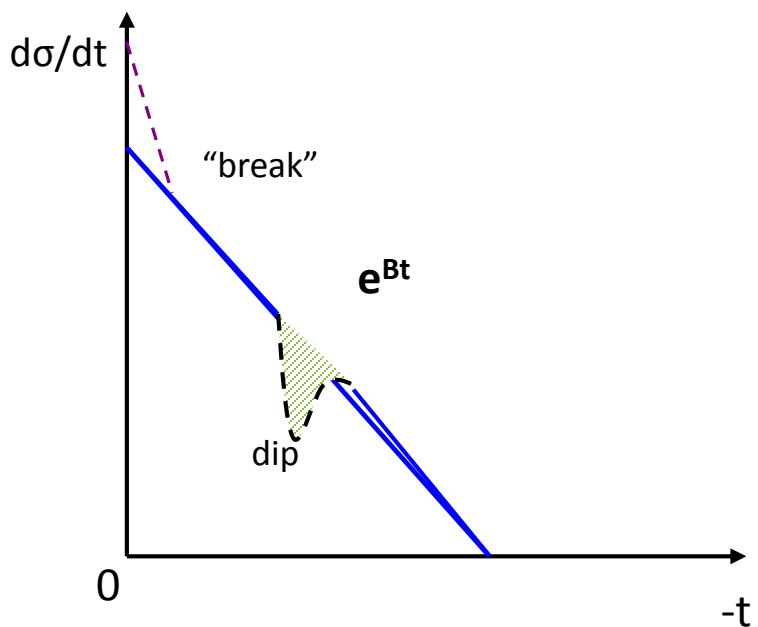
a(0)\C	+	-
1	P	O
1/2	f	ω

NB: The S-matrix theory (including Regge pole) is applicable to asymptotically free states only (not to quarks and gluons)!



1. On-shell (hadronic) reactions ($s, t, Q^2 = m^2$);
 $t \leftrightarrow b$ transformation dictionary:

$$h(s, b) = \int_0^\infty d\sqrt{-t} \sqrt{-t} A(s, t)$$



The Pomeron is a dipole in the j -plane

$$A_P(s, t) = \frac{d}{d\alpha_P} \left[e^{-i\pi\alpha_P/2} G(\alpha_P) \left(s/s_0 \right)^{\alpha_P} \right] = \\ e^{-i\pi\alpha_P(t)/2} \left(s/s_0 \right)^{\alpha_P(t)} \left[G'(\alpha_P) + (L - i\pi/2) G(\alpha_P) \right]. \quad (6)$$

Since the first term in squared brackets determines the shape of the cone, one fixes

$$G'(\alpha_P) = -a_P e^{b_P[\alpha_P-1]}, \quad (7)$$

where $G(\alpha_P)$ is recovered by integration, and, as a consequence, the Pomeron amplitude Eq. (??) can be rewritten in the following “geometrical” form (for the details of the calculations see [] and references therein)

$$A_P(s, t) = i \frac{a_P}{b_P} \frac{s}{s_0} [r_1^2(s) e^{r_1^2(s)[\alpha_P-1]} - \varepsilon_P r_2^2(s) e^{r_2^2(s)[\alpha_P-1]}], \quad (8)$$

where $r_1^2(s) = b_P + L - i\pi/2$, $r_2^2(s) = L - i\pi/2$, $L \equiv \ln(s/s_0)$ and the Pomeron trajectory:

$$\alpha_P \equiv \alpha_P(t) = 1 + \delta_P + \alpha_{1P} t - \alpha_{2P} (\sqrt{4m_\pi^2 - t} - 2m_\pi) \quad (9)$$

Dipole model

- Scattering amplitude:

$$A(s, t)_{pp}^{\bar{p}p} = A_P(s, t) + A_f(s, t) \pm A_\omega(s, t)$$

Pomeron term:

$$A_P(s, t) = i \frac{a_P s}{b_P s_{0P}} [r_1^2(s) e^{r_1^2(s)[\alpha_P - 1]} - \varepsilon_P r_2^2(s) e^{r_2^2(s)[\alpha_P - 1]}]$$

where

$$r_1^2(s) = b_P + L - i\pi/2$$

$$L \equiv \ln(s/s_{0P})$$

$$r_2^2(s) = L - i\pi/2$$

Pomeron trajectory:

$$\alpha_P \equiv \alpha_P(t) = 1 + \delta_P + \alpha_{1P}t - \alpha_{2P} \left(\sqrt{4m_\pi^2 - t} - 2m_\pi \right)$$

Reggeon terms:

$$A_f(s, t) = a_f e^{b_f t} e^{-\frac{i\pi\alpha_f(t)}{2}} (s/s_0)^{\alpha_f(t)}$$

$$A_\omega(s, t) = i a_\omega e^{b_\omega t} e^{-\frac{i\pi\alpha_\omega(t)}{2}} (s/s_0)^{\alpha_\omega(t)}$$

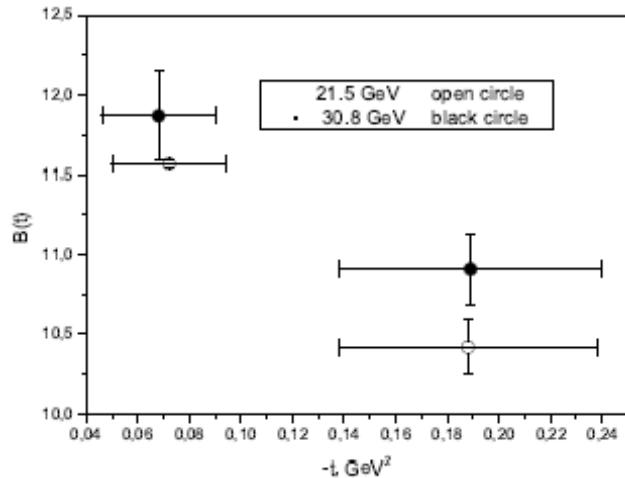
Reggeon trajectories:

$$\alpha_f(t) = 0.703 + 0.84t$$

$$\alpha_\omega(t) = 0.435 + 0.93t$$

arXiv:1206.5837

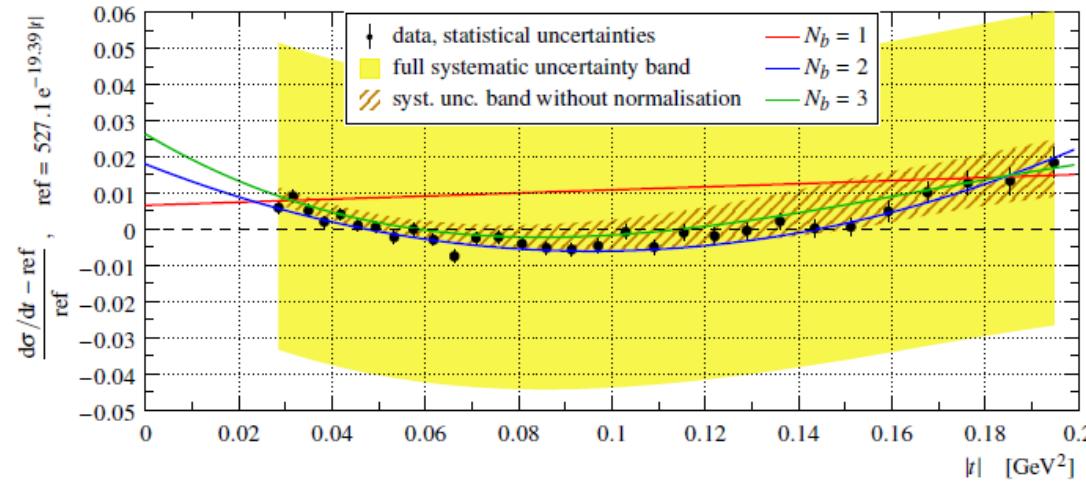
“break”



Local slopes $B(t)$ calculated for low- $|t|$ ISR

$$B(s, t) = \frac{d}{dt} \ln \frac{d\sigma(s, t)}{dt}$$

arXiv:1410.4106
G. Barbiellini et al., Phys. Lett. B 39 (1972) 663



$R(t)$ calculated for low- $|t|$ 8 TeV data.

$$R(t) = \frac{d\sigma(t)/dt - ref}{ref}$$

$$ref = Ae^{Bt}$$

arXiv:1503.08111

Increase

Elastic Scattering: Coulomb-Nuclear Interference Region (2)

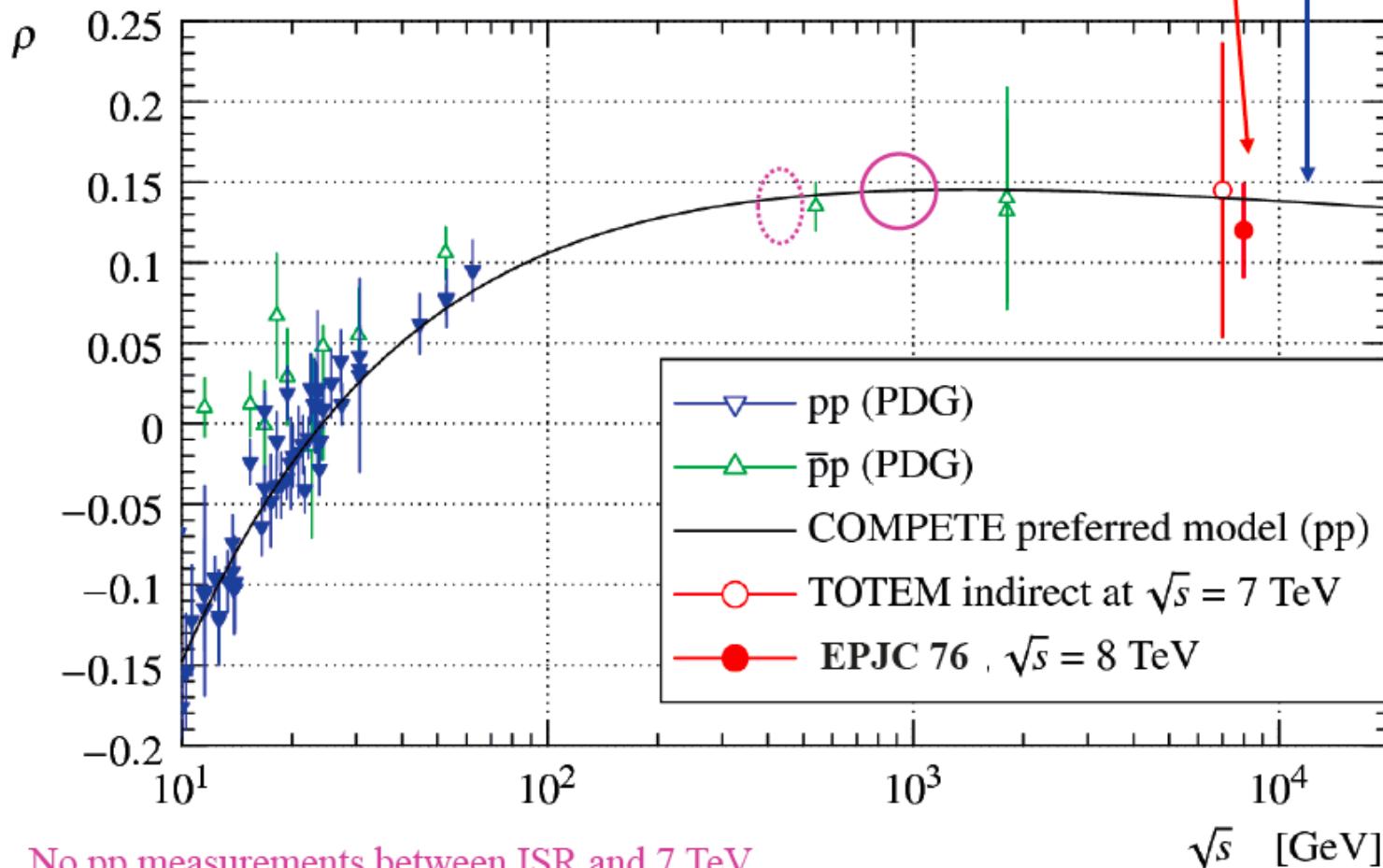


$$\rho = \frac{\Re F^H(0)}{\Im F^H(0)} = \cot \arg F^H(0)$$

TOTEM measurement @ $\sqrt{s} = 8$ TeV:

$\rho = 0.12 \pm 0.03$ [EPJC 76 (2016) 661]

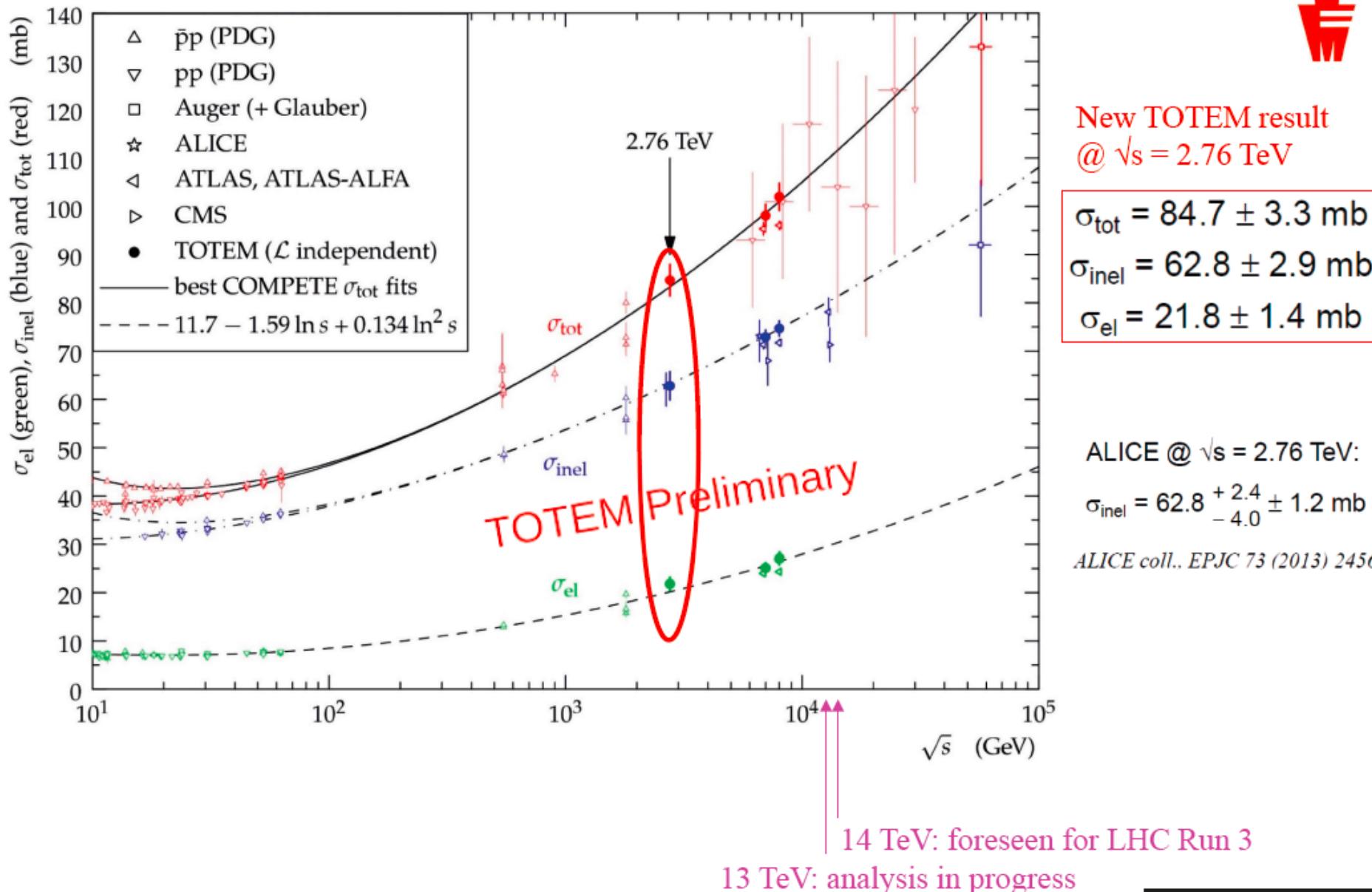
$\beta^* = 2.5$ km @ $\sqrt{s} = 13$ TeV
 $\Rightarrow \sigma(\rho) = 0.01$



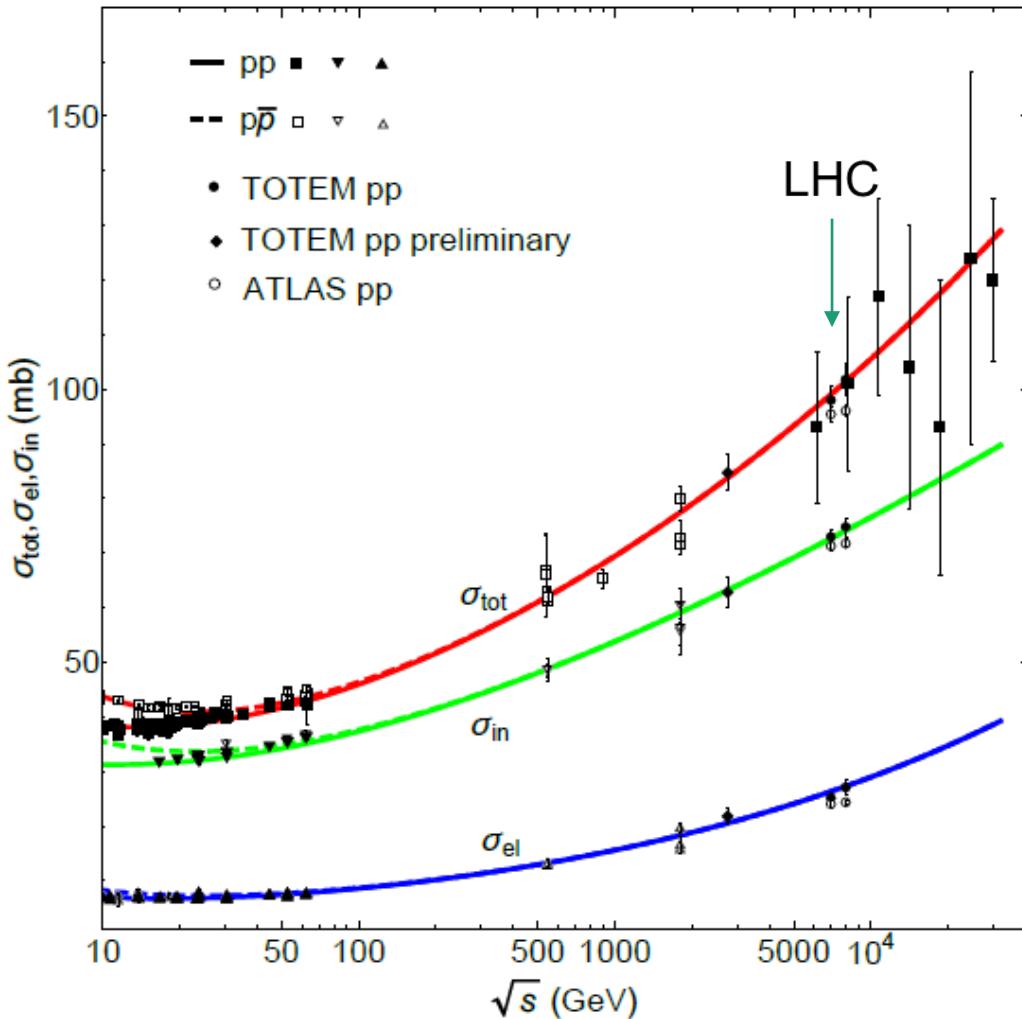
No pp measurements between ISR and 7 TeV

→ TOTEM request for 2018: special run @ $\sqrt{s} = 900$ GeV, later perhaps at 450 GeV (½ injection energy)

pp Cross-Section Measurements



Elastic, inelastic and total cross sections



Fitted pp and $p\bar{p}$ total cross sections and calculated elastic and inelastic cross sections.

$$\sigma_{tot}(s) = \frac{4\pi}{s} Im A(s, t=0)$$

$$\frac{d\sigma}{dt}(s, t) = \frac{\pi}{s^2} |A(s, t)|^2$$

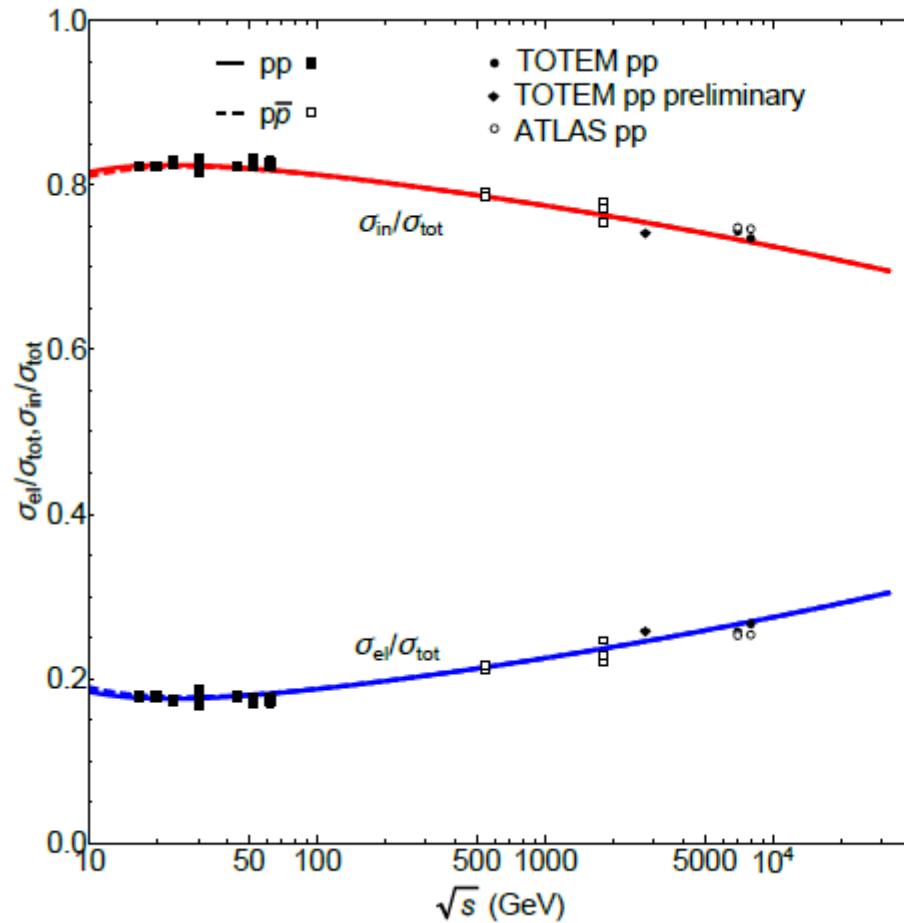
$$\sigma_{el}(s) = \int_{t_{min}}^{t_{max}} \frac{d\sigma(s, t)}{dt} dt$$

$$\sigma_{in}(s) = \sigma_{tot}(s) - \sigma_{el}(s)$$

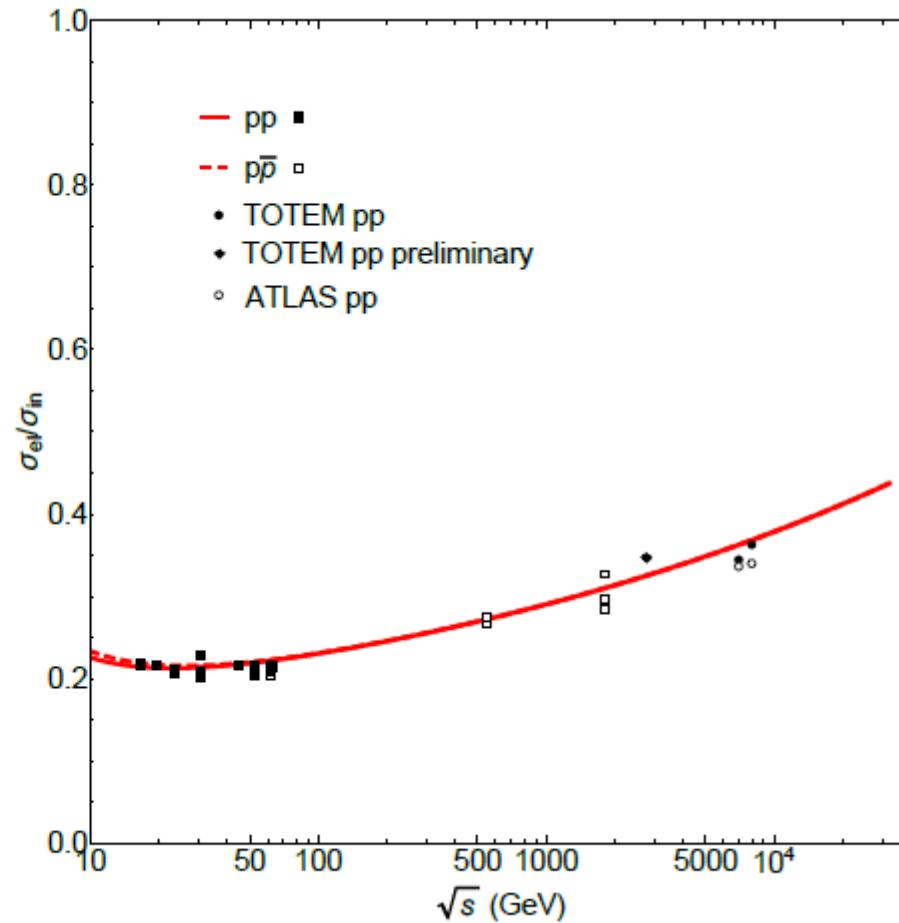
a_P	307.15	a_f	-17.0151
b_P	8.9767	b_f	4.54423
δ_P	0.04489	a_ω	9.78393
α_{1P}	0.42955	b_ω	8.21191
α_{2P}	0.0063817	s_0	1 (fixed)
ε_P	0 (fixed)	s_{0P}	100 (fixed)

Values of fitted parameters.

Ratios of σ_{el}/σ_{tot} , σ_{in}/σ_{tot} and σ_{el}/σ_{in}

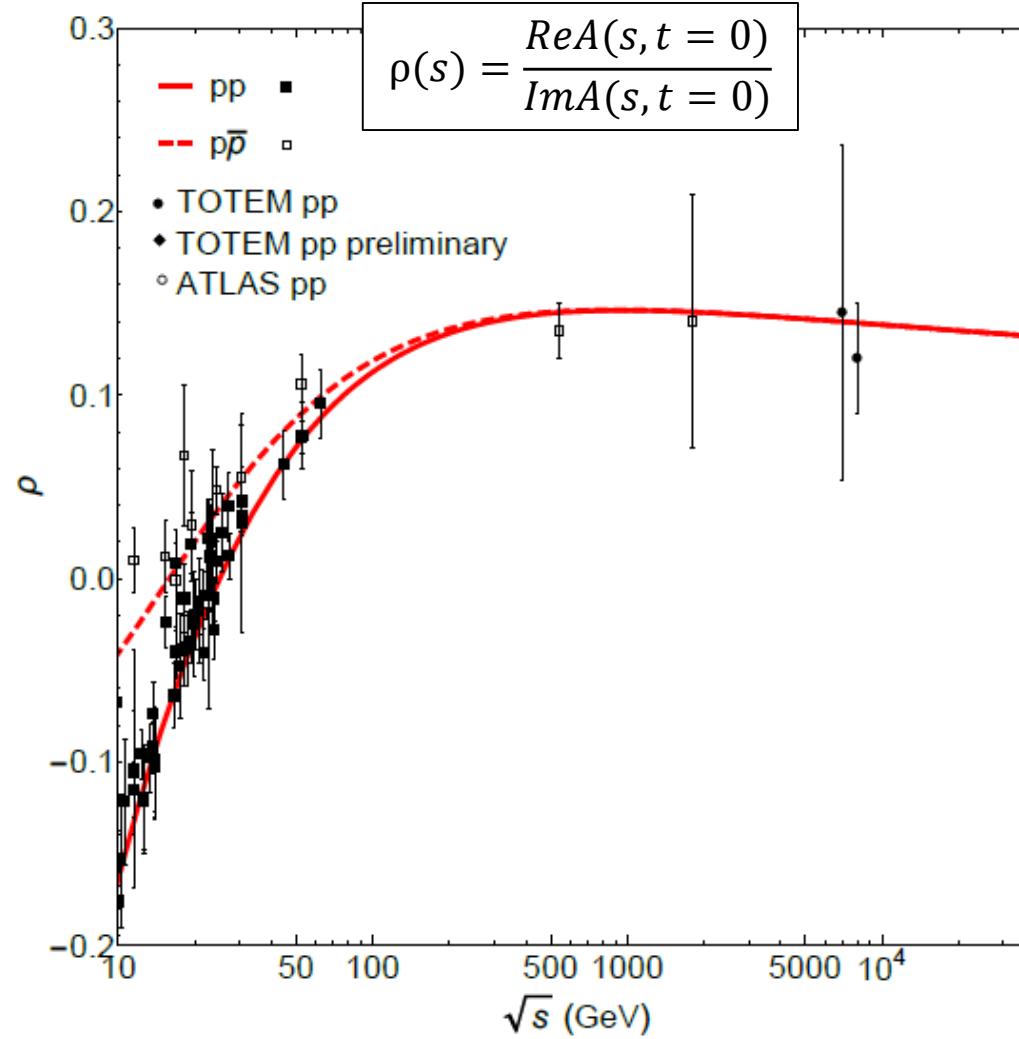


Calculated σ_{el}/σ_{tot} and σ_{in}/σ_{tot} ratios to pp and $p\bar{p}$ scattering.

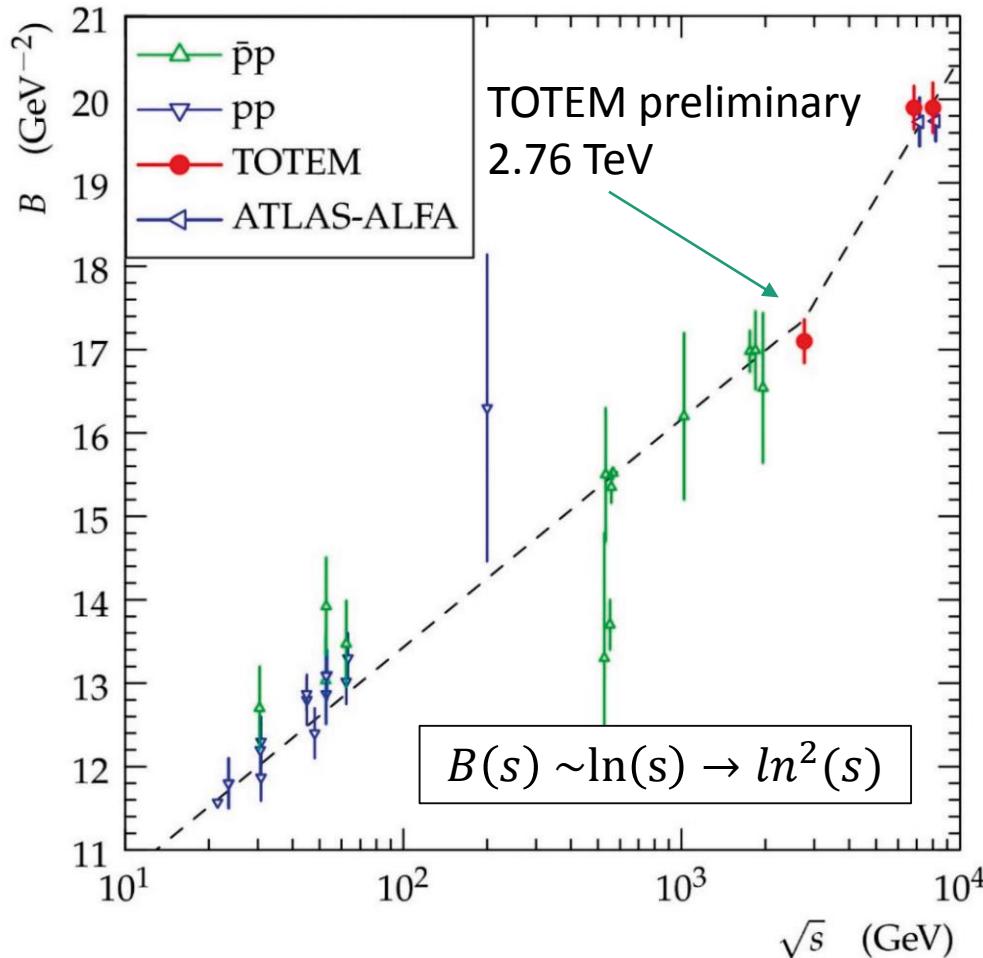


Calculated σ_{el}/σ_{in} ratio to pp and $p\bar{p}$ scattering.

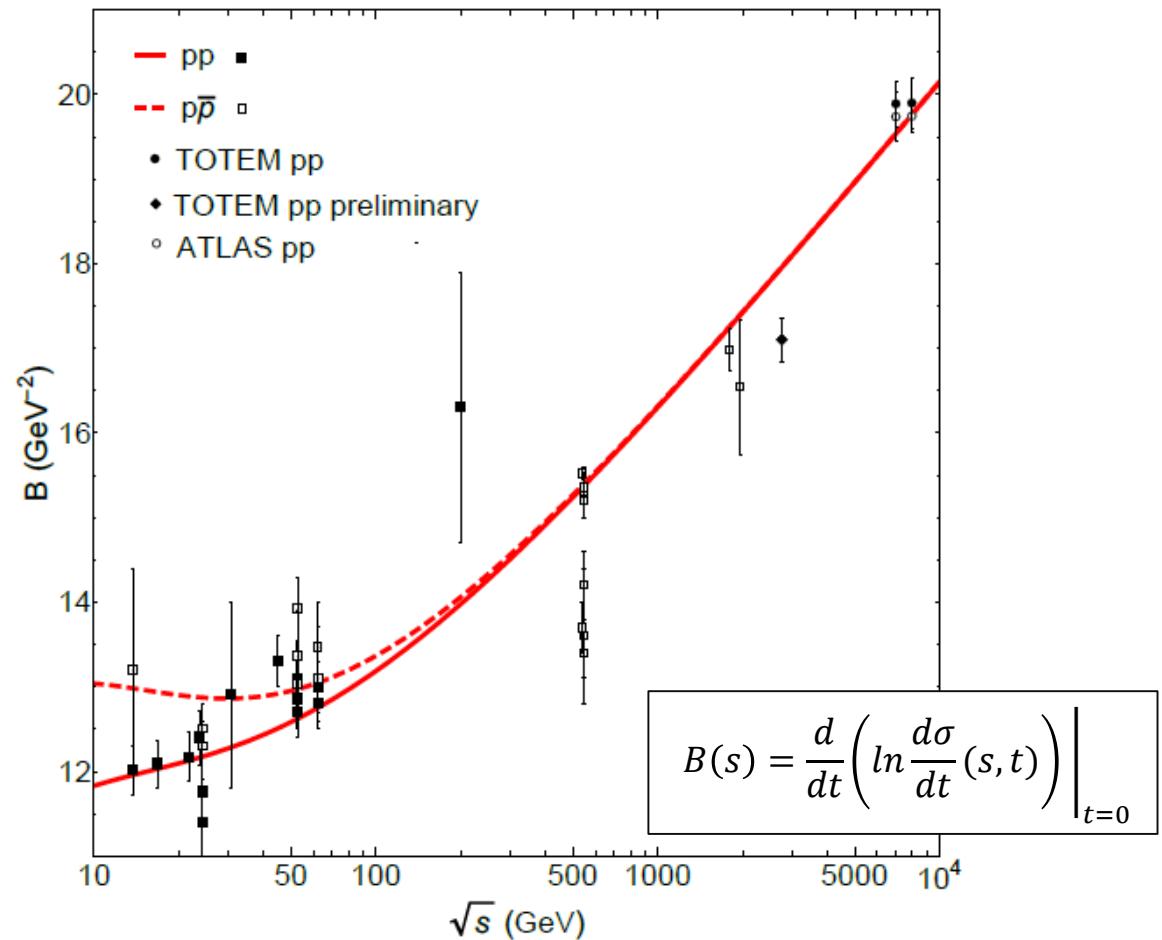
ρ -paramater



New elastic slope measurements



The elastic slope data with preliminary
TOTEM results.



Fitted pp and $p\bar{p}$ elastic slope.

- [1] R. Hagedorn, Nuovo Cim. Suppl. **3** (1965) 147.
- [2] Wojciech Broniowski, Wojciech Florkowski, and Leonid Gózman, hep-ph/0407290.
- [3] Wojciech Broniowski and Wojciech Florkowski, hep-ph/0004104; Wojciech Broniowski, Enrique Ruiz Arriola, hep-ph/1008.2317; Enrique Ruiz Arriola and Wojciech Broniowski, hep-ph/1210.7153; Wojciech Broniowski, nucl-th/1610.0967.
- [4] Wojciech Broniowski, hep-ph/0008112.
- [5] K.A. Olive et al. (Particle Data Group) Chinese Physics C **38** (2014) 090001, <http://pdg.lbl.gov/>.
- [6] Thomas D. Cohen and Vojtech Krejcirik, hep-ph/1107.2130.
- [7] S.Z. Belenky and L.D. Landau, Sov. Phys. Uspekhi **56** (1955) 309.
- [8] E.V. Shuryak, Sov. J. Nucl. Phys. **16** (1973) 220.
- [9] L. Burakovsky, *Hadron spectroscopy in Regge Phenomenology*, hep-ph/9805286.
- [10] M.M. Brisudova, L. Burakovsky, T. Goldman and A. Szczepaniak, *Nonlinear Regge trajectories and glueballs*, nucl-th/030303012.

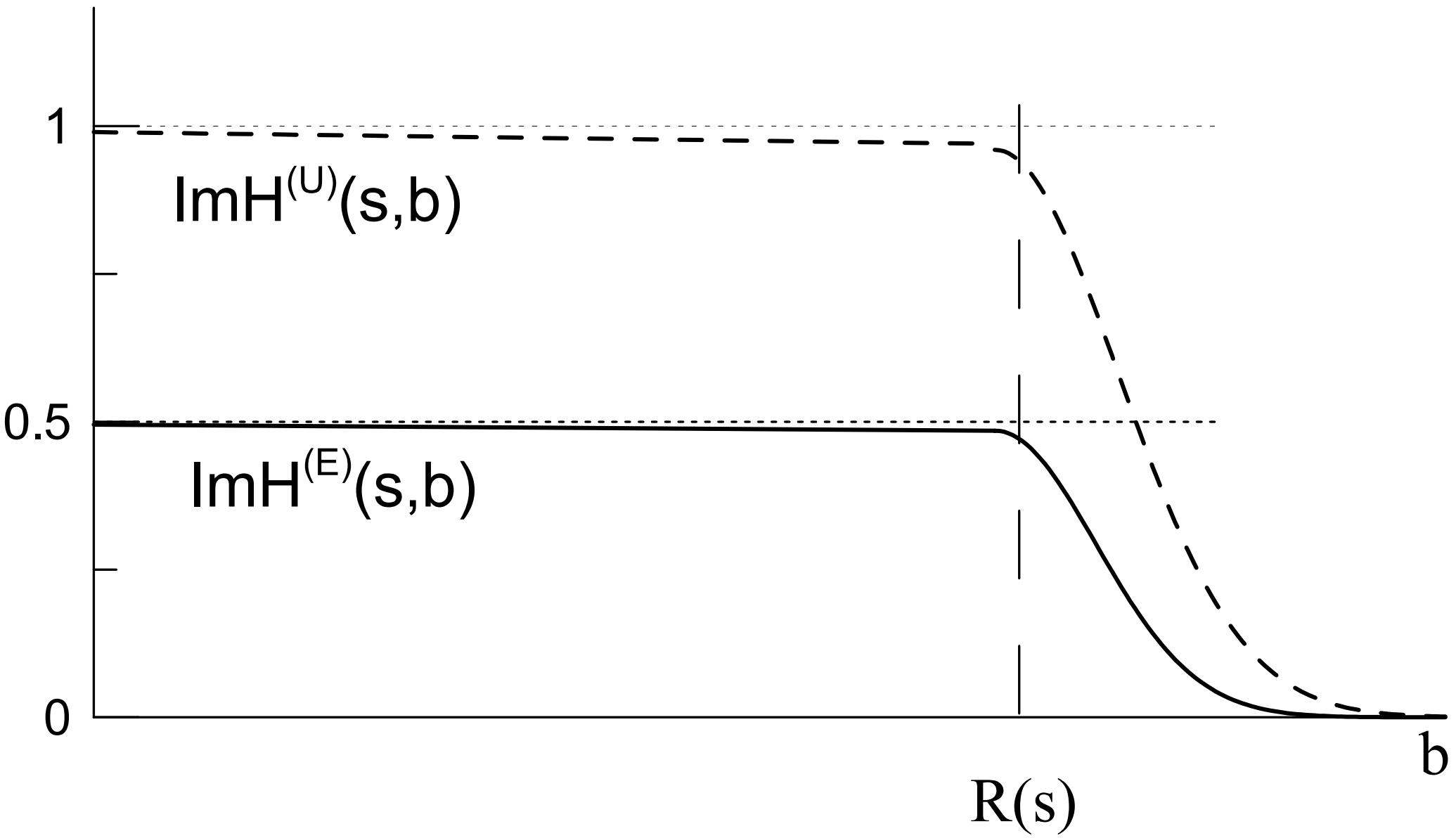
$ImH(s, b) = |h(s, b)|^2 + G_{in}(s, b)$, (h is associated with the "opacity"), Here from: $0 \leq |h(s, b)|^2 \leq \Im h(s, b) \leq 1$. The Black Disc Limit (BDL) corresponds to $\Im h(s, b) = 1/2$, provided $h(s, b) = i(1 - \exp[i\omega(s, b)])/2$, with an imaginary eikonal $\omega(s, b) = i\Omega(s, b)$.

There is an alternative solution, that with the "minus" sign in $h(s, b) = [1 \pm \sqrt{1 - 4G_{in}(s, b)}]/2$, giving (S.Troshin and N.Tyurin (Protvino)): $h(s, b) = \Im u(s, b)/[1 - iu(s, b)]$,

$$H(s,b)=\frac{1}{8\pi s}\int\limits_0^\infty dq\,qJ_0(qb)A(s,t=-\vec q^2),\qquad \sigma_t=\frac{1}{s}{\rm Im} A(s,0),\quad \frac{d\sigma}{dt}=\frac{1}{16\pi s^2}|A(s,t)|^2.$$

$$2iH^{(E)}(s,b)=e^{2ih(s,b)}-1\qquad\qquad H^{(U)}(s,b)=\frac{h(s,b)}{1-ih(s,b)}.$$

$${\rm Im} H(s,b)=|H(s,b)|^2+G_{inel}(s,b)>0$$



We use a unitarization procedure, different from the familiar eikonalization, according to which, in the impact parameter representation the unitarized amplitude in terms of the (Regge-pole) input $u(\rho, s)$ is

$$T(\rho, s) = \frac{u(\rho, s)}{1 - iu(\rho, s)}. \quad (20)$$

The advantage of if this unitarization procedure (called, misleadingly also "u-matrix") is its simplicity compared to the eikonalization (rational functions compared to exponentials).

For $t = 0$ calculations can be done analytically as a series in $1/L$. By keeping terms and introducing the variable $x = \rho^2/(4\alpha'L)$, one gets [16] up to $O(1/L)$

$$u(x, s) = i g e^{-\chi} (1 + \chi x), \quad (21)$$

where

$$g = \frac{\sigma_0 \lambda}{16\pi\alpha'}, \quad \chi = \left(1 + i\pi\lambda/2\right)/\lambda L, \quad (22)$$

whereupon the profile function assumes the form

$$\frac{1}{1 - iu} = \frac{ig e^{-\chi}}{1 + g e^{-\chi}} \left(1 + \chi \frac{x}{1 + g e^{-\chi}}\right). \quad (23)$$

$$g = \frac{\sigma_0 \lambda}{16\pi\alpha'}, \quad \lambda = (1 - \epsilon)/b. \quad (24)$$

by substituting Eq. into Eq.we obtain

After a Fourier-Bessel transform over the DP amplitude Eq. (??), Sec. II, one gets for forward measurables, in the $O(1/L)$ approximation ($L \equiv \ln s$):

$$\sigma_{tot} = \frac{16\pi\alpha'}{\lambda} \ln(1 + g)(1 + \lambda L). \quad (25)$$

Unitarization

- Unitarized scattering amplitude:

$$T(\rho, s) = \frac{u(\rho, s)}{1 - iu(\rho, s)}$$



$$T(s, t) = q^2 \int_0^\infty \frac{u(\rho, s)}{1 - iu(\rho, s)} J_0(\rho\sqrt{-t}) d\rho^2$$

(ρ – impact parameter; q – momentum in center-of-mass frame)

- The unitarized formulas for the forward measurables:

$$\sigma_{tot} = \frac{4\pi\alpha_{1P}}{\lambda} \ln(1 + g)(1 + \lambda L)$$

$$\sigma_{el} = \frac{4\pi\alpha_{1P}}{\lambda} \frac{g}{1 + g}(1 + \lambda L)$$

$$\sigma_{tot} = \frac{4\pi\alpha_1}{\lambda} \left(\ln(1 + g) - \frac{g}{1 + g} \right) (1 + \lambda L)$$

$$\rho = \frac{ReT(s, 0)}{ImT(s, 0)} = \frac{\pi\lambda}{2(1 + \lambda L)}$$

$$\frac{\sigma_{el}}{\sigma_{tot}} = 1 - \frac{g}{(1 + g)\ln(1 + g)}$$

$$B = \frac{2\alpha_{1P}}{\lambda} \frac{\Sigma}{\ln(1 + g)} (1 + \lambda L)$$

$$\lambda = (1 - \epsilon_P)/b_P$$

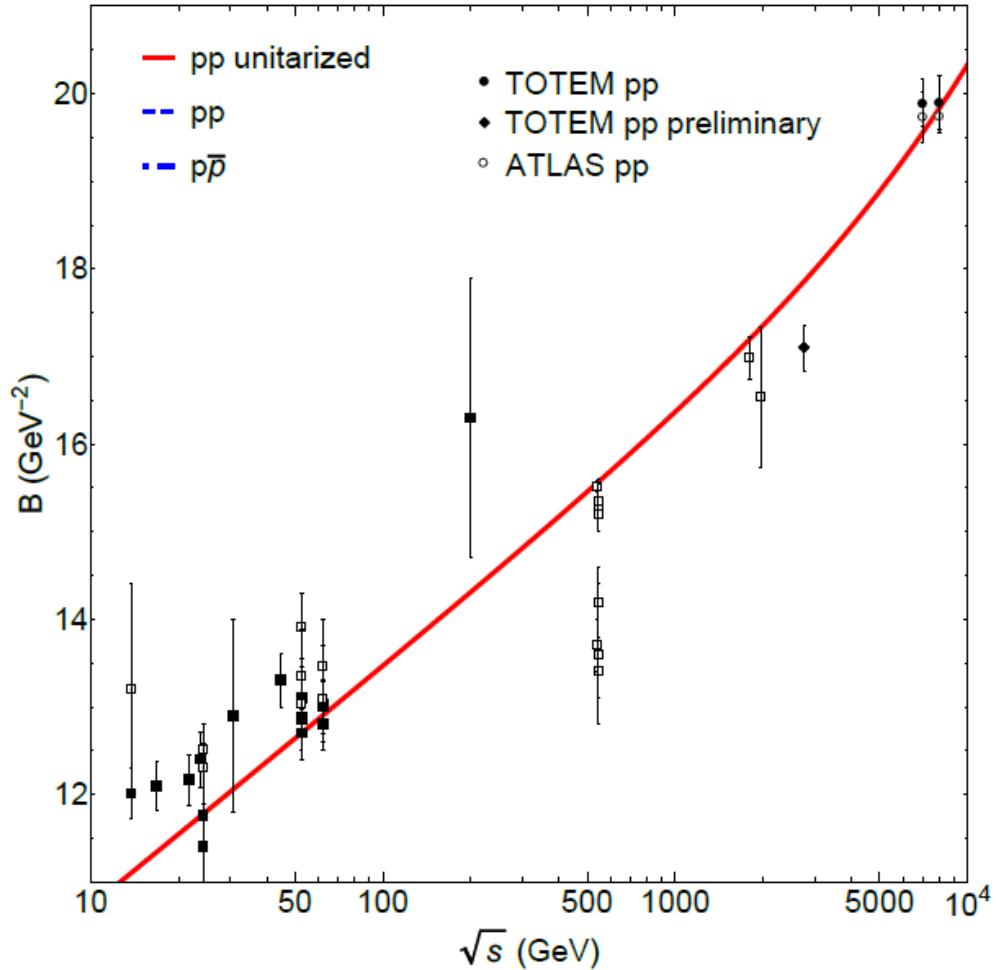
$$L = \ln(s/s_{0P})$$

$$g(s) = g_{01}(s/s_{01})^{\varepsilon_1} + g_{02}(s/s_{02})^{\varepsilon_2}$$

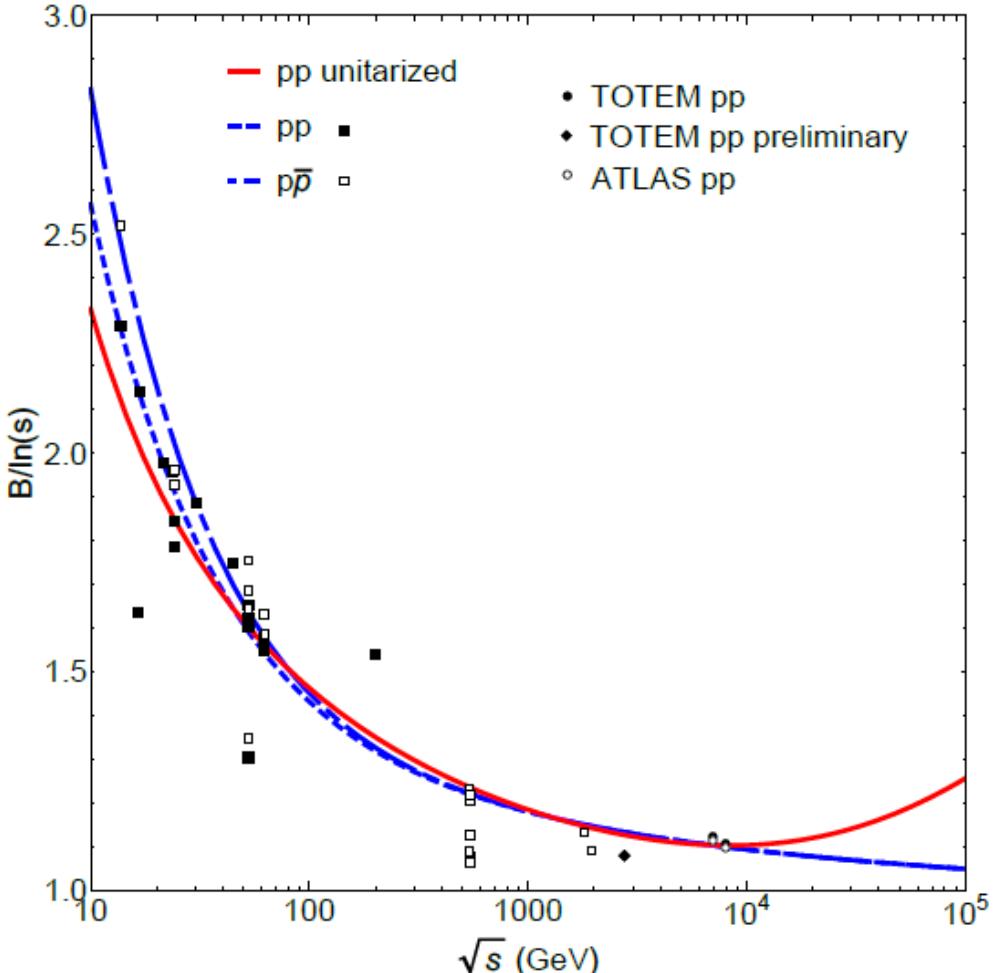
$$\Sigma = \int_0^\infty \frac{ge^{-x}x dx}{1 + ge^{-x}}$$

A.N. Wall, L.L. Jenkovszky, B.V. Struminsky, Sov. J. Particles and Nuclei, 19 (1988)

Application of the unitarized slope



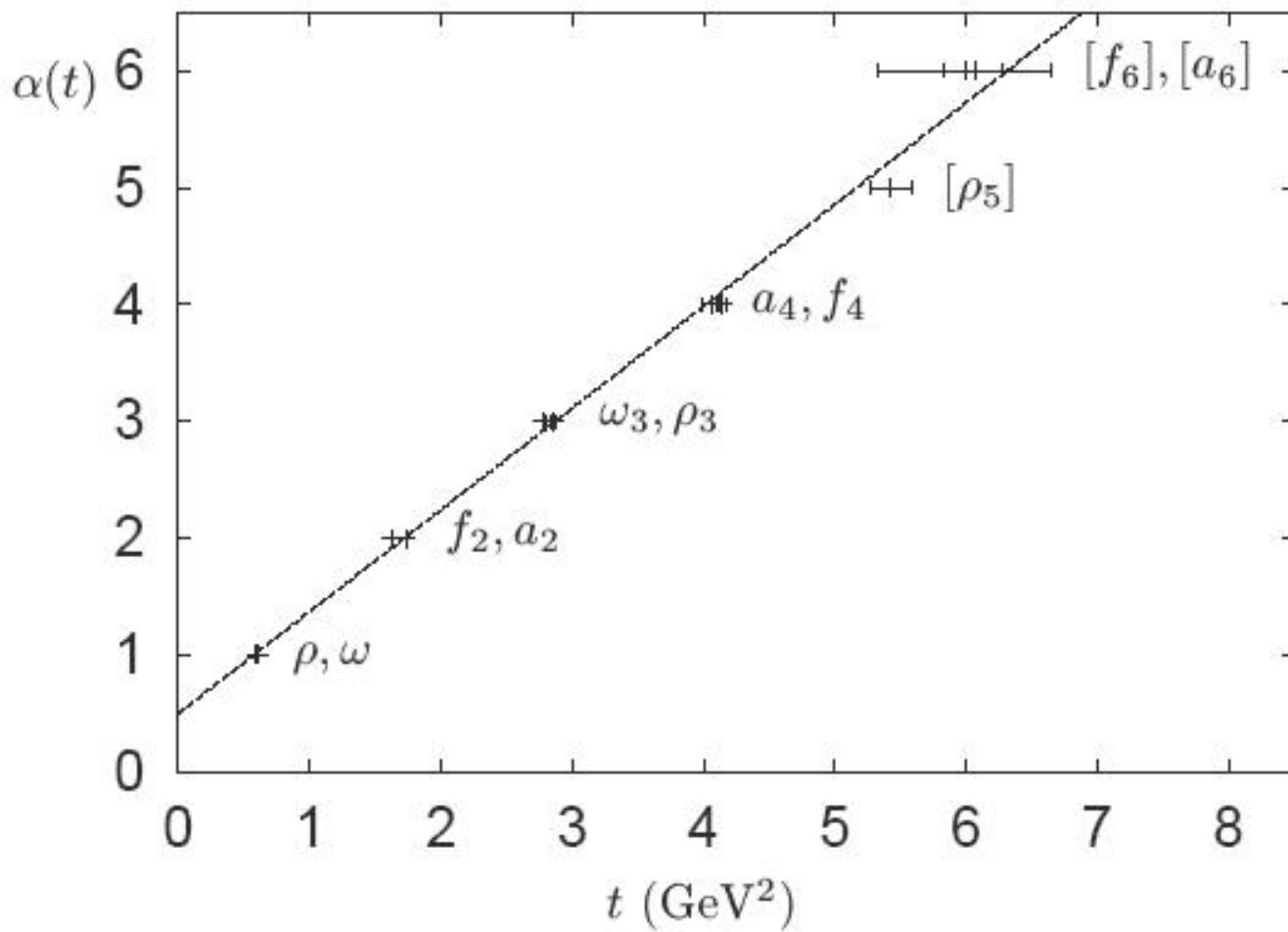
Description for the elastic slope data using
unitarization procedure.



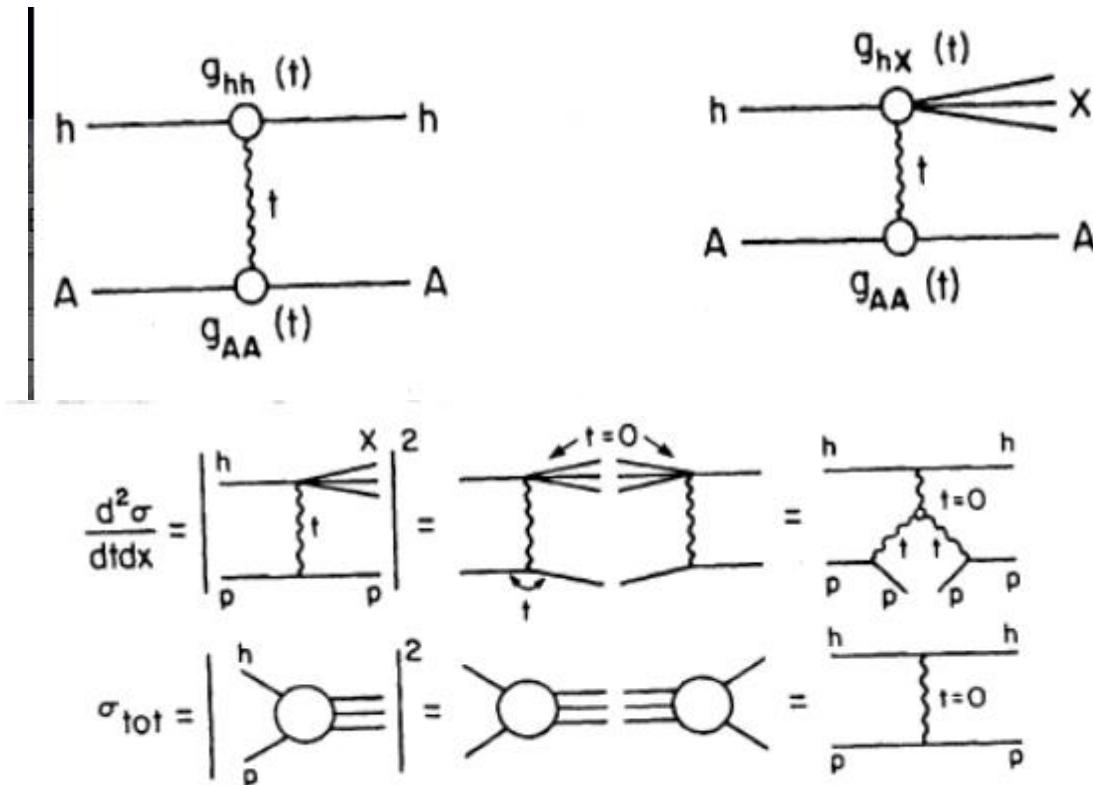
$B/\ln(s)$ ratio.

Linear particle trajectories

Plot of spins of families of particles against their squared masses:



The optical (generalised optical (Müller) theorem and triple-Regge limit (for high M only!)



The differential cross section for $1 + 2 \rightarrow X$ is

$$\frac{d^2\sigma}{dt dM^2} = \frac{G(t)}{16\pi^2 s_0^3} \left(\frac{s}{s_0}\right)^{2\alpha(t)-2} \left(\frac{M^2}{s_0}\right)^{\alpha(0)-2\alpha(t)}, \quad (1)$$

where $G(t)$ is the triple Pomeron vertex, $G(t) = Ge^{at}$ for simplicity, and $\alpha(t) = \alpha^0 + \alpha'(t)$ is the (linear for the moment) Pomeron trajectory.

For a critical Pomeron, $\alpha^0 = 1$, one can use the formula

$$\int \frac{dx}{x \ln x} = \ln(\ln x)) \quad (2)$$

to get

$$\sigma^{SD}(s) \sim (2\alpha')^{-1} \ln\left(1 + \frac{2\alpha'}{a} \ln s\right) \sim \ln(\ln s)), \quad (3)$$

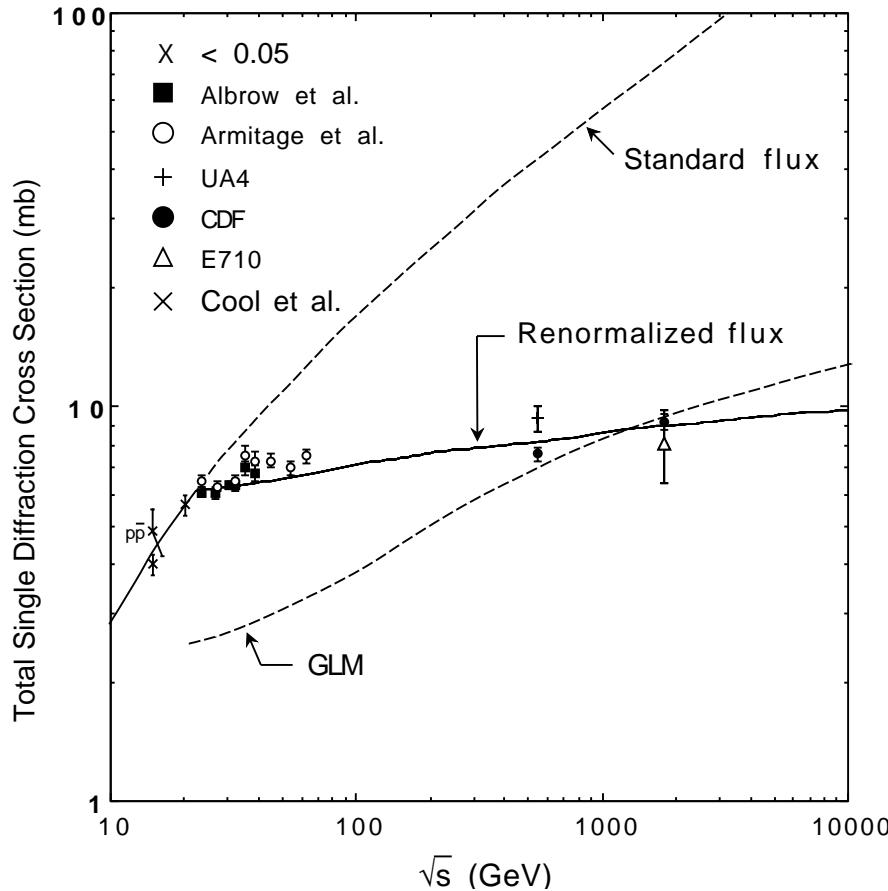
while the total cross section

$$\sigma^{tot}(s) \rightarrow \text{const.} \quad (4)$$

It contradicts unitarity since e.g. for critical Pomeron, $\alpha^0 = 1$, the partial (SD) cross section overshoots the total cross section $\sigma^{SD} > \sigma^{tot}$.

A trivial trick to avoid violation of unitarity is to assume the triple Pomeron vertex $G(t)$ vanishing at $t = 0$. Huge literature (Kaidalov, Brower, Ganguli, Kopeliovich,...) exists reflecting the efforts along this direction. The main conclusion is that decoupling (vanishing of the triple Pomeron vertex at $t = 0$) is incompatible with the data.

To remedy this difficulty, Dino Goulianos suggested a renormalization procedure, by which the Pomeron flux is multiplied by a factor $N(s)$ moderating the rise of inelastic diffraction starting from a certain threshold. The appearance of a threshold, however may violate analyticity.

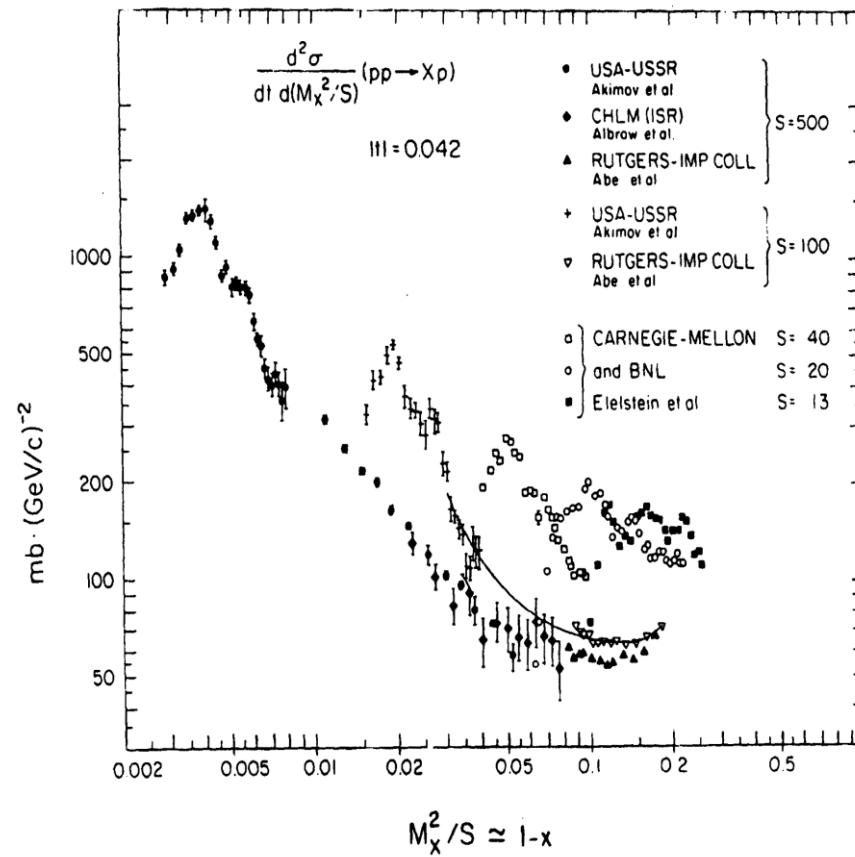
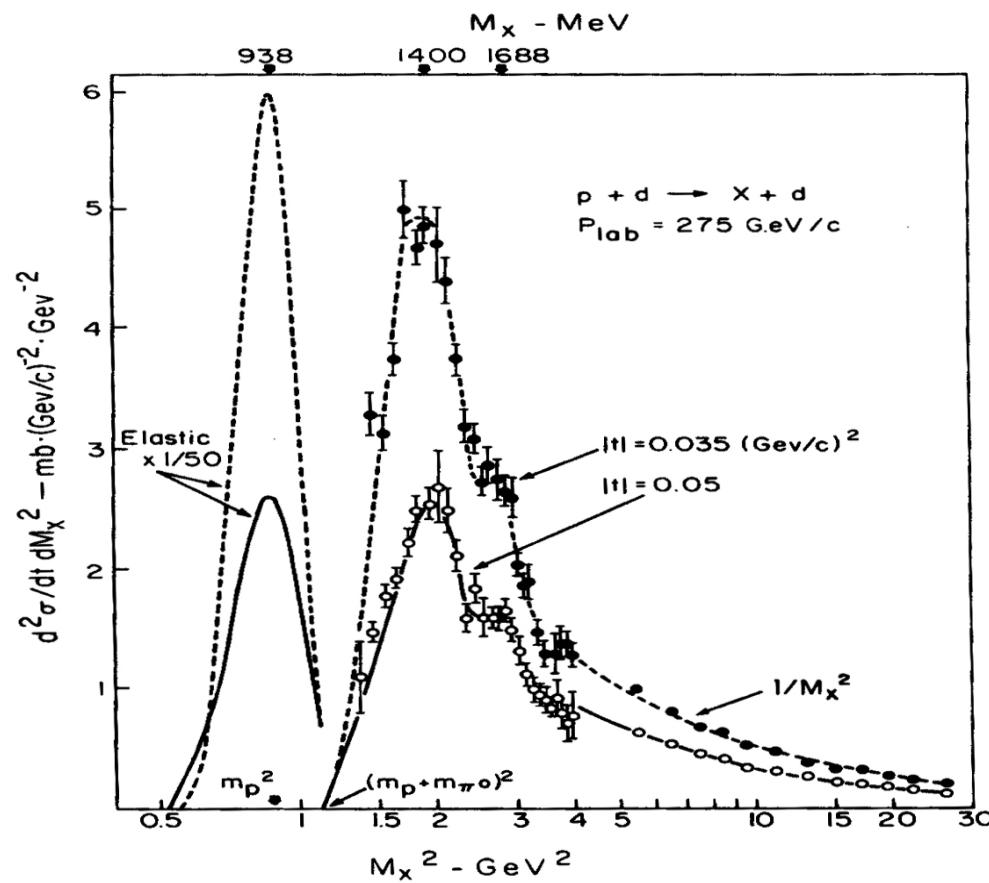


The function $N(s)$ is the so-called renormalization factor, introduced by K. Goulianatos,

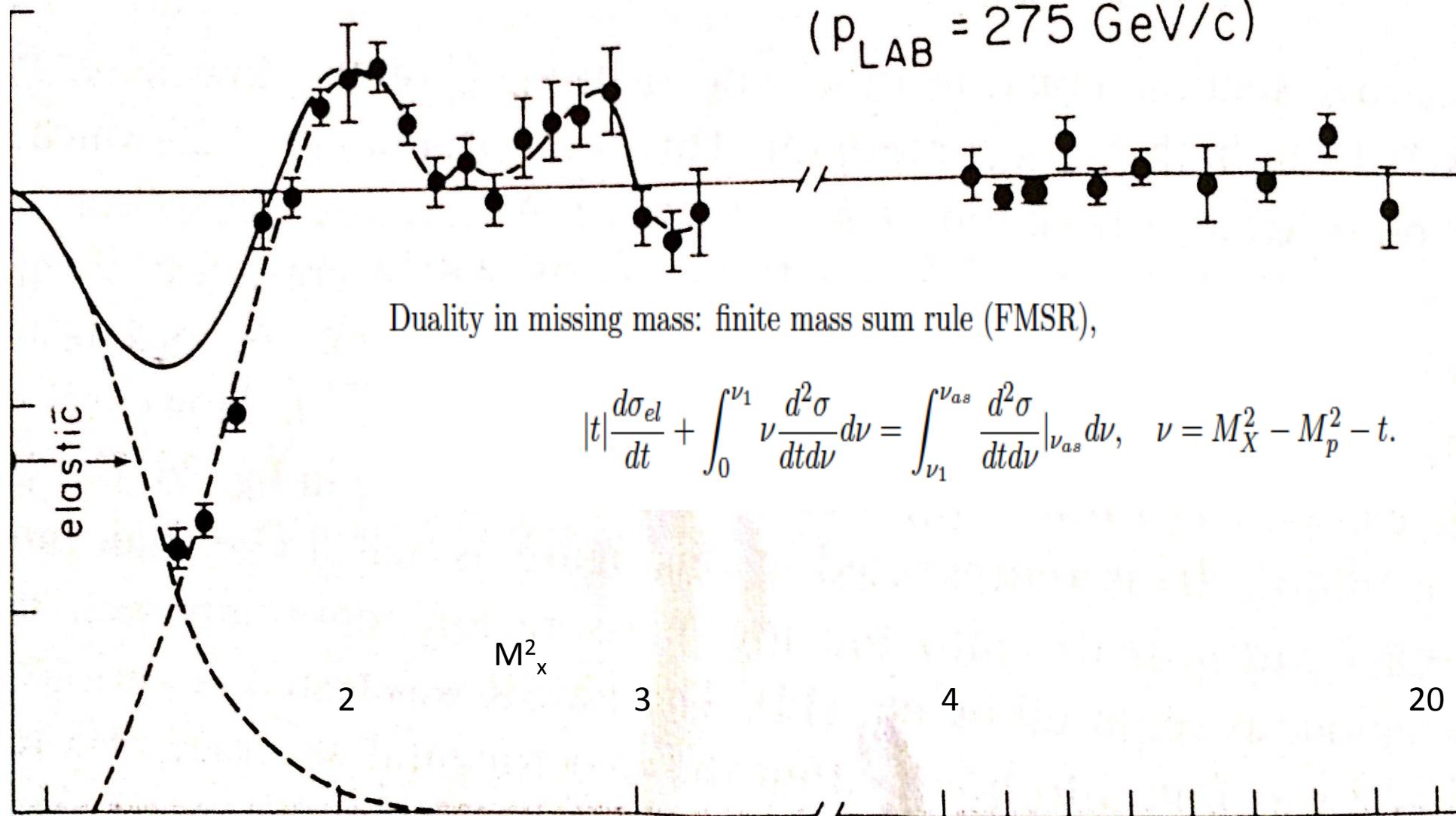
$$N_s \equiv \int_{\xi(\min)}^{\xi(\max)} \int_{t=0}^{-\infty} dt f_{P/p}(\xi, t) \sim s^{2\epsilon},$$

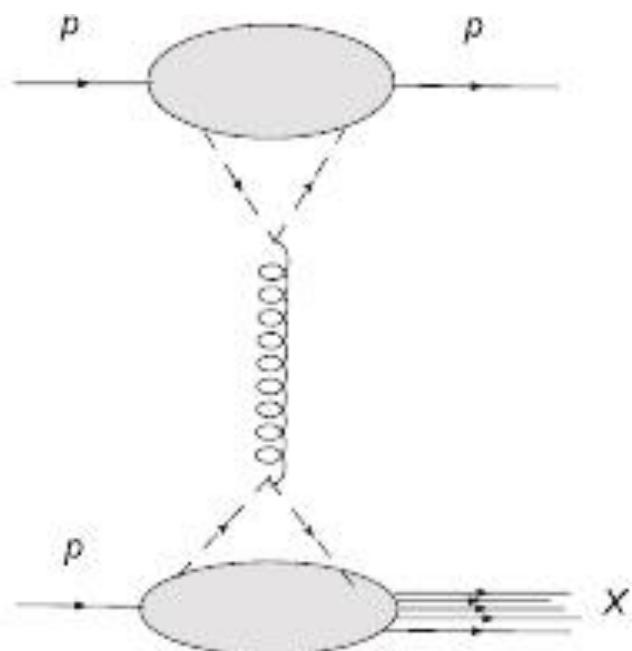
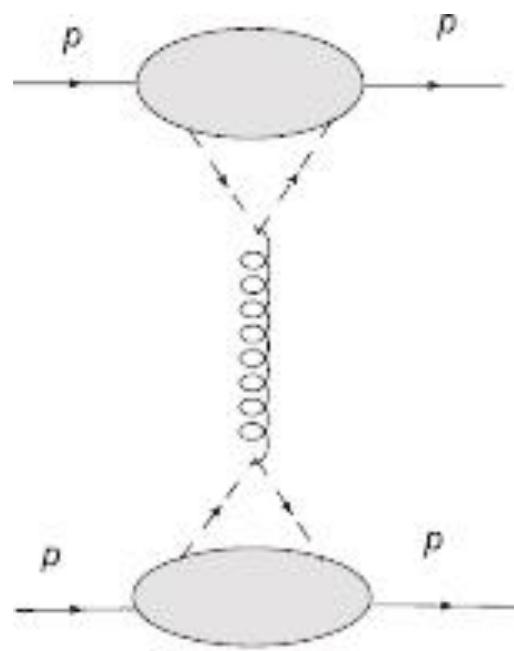
where $\xi(\min) = 1.4/s$ and $\xi(\max) = 0.1$. This factor secures unitarity.

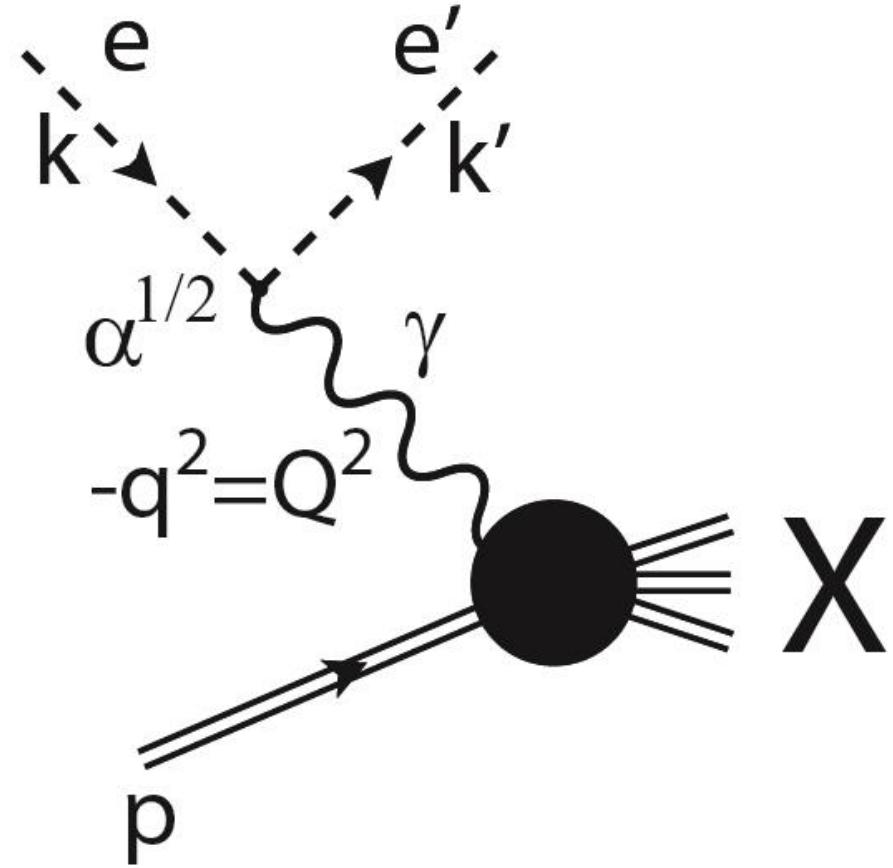
FNAL



$$\nu \frac{d^2\sigma}{dt dM_X^2} \Big|_{|t|=0.035} (p+d \rightarrow X+d) / F_d$$







$$\left| \text{Diagram} \right|^2 = \sum_{X} \text{Diagram}_X = \sum_{R} \text{Diagram}_R = \sum_{\text{Res}} \text{Diagram}_{\text{Res}}$$

Diagrammatic representation of the scattering amplitude $\left| \text{Diagram} \right|^2$ as a sum of contributions from different intermediate states X and R , and residues Res . The diagram shows a particle p (double line) interacting with a particle X (black circle) which then decays into two virtual photons (wavy lines). The label "Unitarity $t=0$ " is shown below the diagram.

Veneziano duality

Similar to the case of elastic scattering, the double differential cross section for the SDD reaction, by Regge factorization, can be written as

$$\frac{d^2\sigma}{dt dM_X^2} = \frac{9\beta^4 [F^p(t)]^2}{4\pi \sin^2[\pi\alpha_P(t)/2]} (s/M_X^2)^{2\alpha_P(t)-2} \times \\ \left[\frac{W_2}{2m} \left(1 - M_X^2/s \right) - m W_1 (t + 2m^2)/s^2 \right], \quad (1)$$

where W_i , $i = 1, 2$ are related to the structure functions of the nucleon and $W_2 \gg W_1$. For high M_X^2 , the $W_{1,2}$ are Regge-behaved, while for small M_X^2 their behavior is dominated by nucleon resonances. The knowledge of the inelastic form factors (or transition amplitudes) is crucial for the calculation of low-mass diffraction dissociation.

The pp scattering amplitude

$$A(s, t)_P = -\beta^2 [f^u(t) + f^d(t)]^2 \left(\frac{s}{s_0}\right)^{\alpha_P(t)-1} \frac{1 + e^{-i\pi\alpha_P(t)}}{\sin \pi\alpha_P(t)}, \quad (1)$$

where $f^u(t)$ and $f^d(t)$ are the amplitudes for the emission of u and d valence quarks by the nucleon, β is the quark-Pomeron coupling, to be determined below; $\alpha_P(t)$ is a vacuum Regge trajectory. It is assumed that the Pomeron couples to the proton via quarks like a scalar photon.

A single-Pomeron exchange is valid at the LHC energies, however at lower energies (e.g. those of the ISR or the SPS) the contribution of non-leading Regge exchanges should be accounted for as well.

Thus, the unpolarized elastic pp differential cross section is

$$\frac{d\sigma}{dt} = \frac{[3\beta F^p(t)]^4}{4\pi \sin^2[\pi\alpha_P(t)/2]} (s/s_0)^{2\alpha_P(t)-2}. \quad (2)$$

The final expression for the double differential cross section reads:

$$\begin{aligned}
 \frac{d^2\sigma}{dt dM_X^2} = & \\
 A_0 \left(\frac{s}{M_X^2} \right)^{2\alpha_P(t)-2} & \frac{x(1-x)^2 [F^p(t)]^2}{(M_x^2 - m^2) \left(1 + \frac{4m^2x^2}{-t} \right)^{3/2}} \times & (1) \\
 \sum_{n=1,3} & \frac{[f(t)]^{2(n+1)} \operatorname{Im} \alpha(M_X^2)}{(2n + 0.5 - \operatorname{Re} \alpha(M_X^2))^2 + (\operatorname{Im} \alpha(M_X^2))^2}.
 \end{aligned}$$

SD and DD cross sections

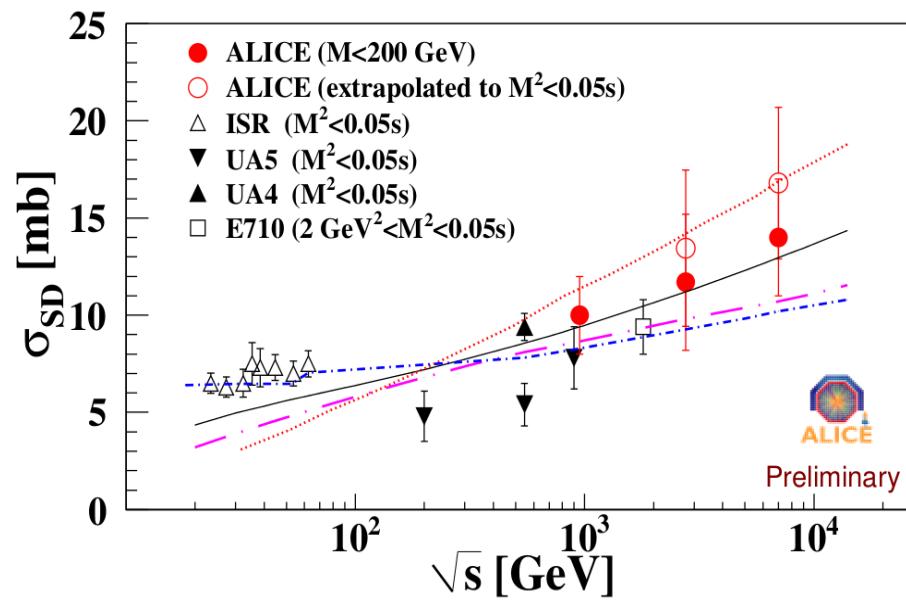
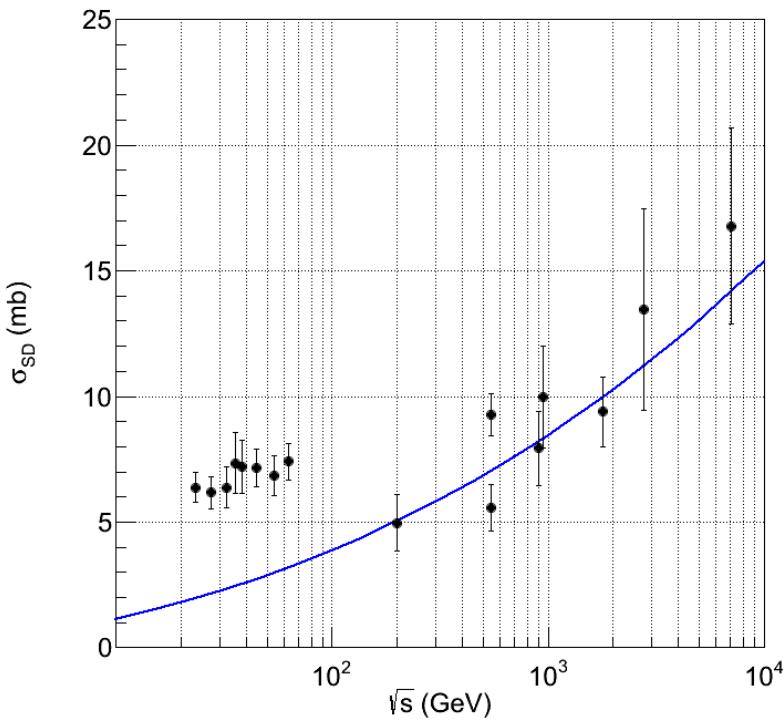
$$\frac{d^2\sigma_{SD}}{dt dM_x^2} = F_p^2(t) F(x_B, t) \frac{\sigma_T^{Pp}(M_x^2, t)}{2m_p} \left(\frac{s}{M_x^2} \right)^{2(\alpha(t)-1)} \ln \left(\frac{s}{M_x^2} \right)$$

$$\begin{aligned} \frac{d^3\sigma_{DD}}{dt dM_1^2 dM_2^2} &= C_n F^2(x_B, t) \frac{\sigma_T^{Pp}(M_1^2, t)}{2m_p} \frac{\sigma_T^{Pp}(M_2^2, t)}{2m_p} \\ &\quad \times \left(\frac{s}{(M_1 + M_2)^2} \right)^{2(\alpha(t)-1)} \ln \left(\frac{s}{(M_1 + M_2)^2} \right) \end{aligned}$$

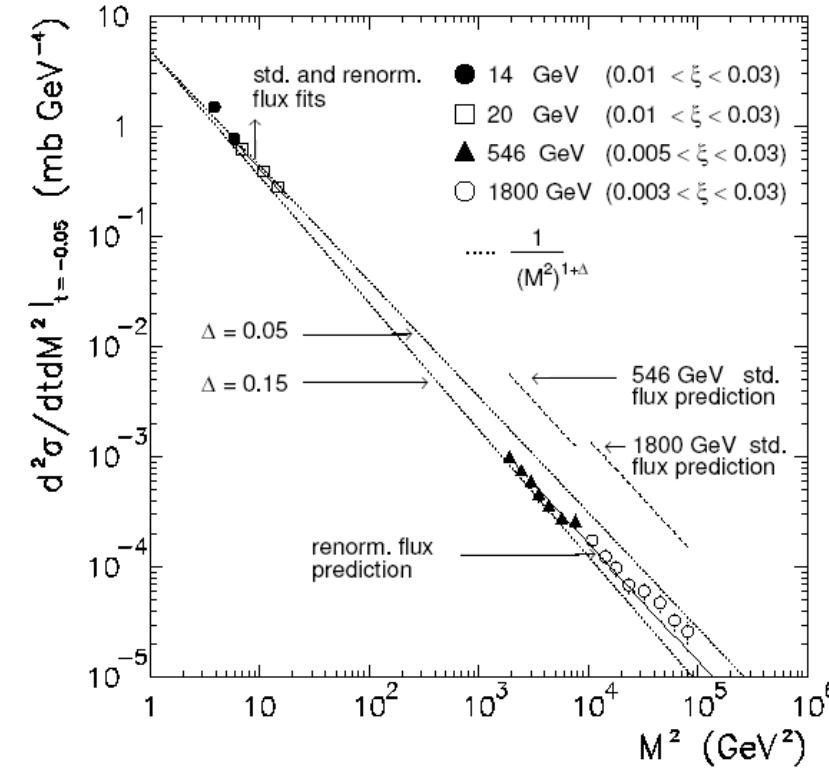
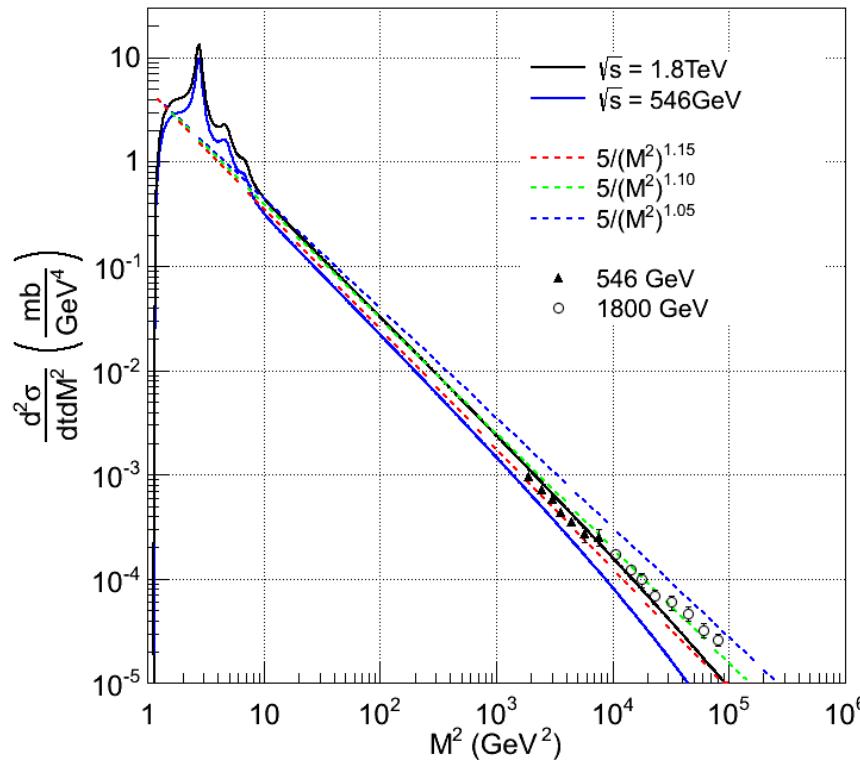
“Reggeized (dual) Breit-Wigner” formula:

$$\begin{aligned}
\sigma_T^{Pp}(M_x^2, t) &= \text{Im } A(M_x^2, t) = \frac{A_{N^*}}{\sum_n n - \alpha_{N^*}(M_x^2)} + Bg(t, M_x^2) = \\
&= A_n \sum_{n=0,1,\dots} \frac{[f(t)]^{2(n+1)} \text{Im } \alpha(M_x^2)}{(2n + 0.5 - \text{Re } \alpha(M_x^2))^2 + (\text{Im } \alpha(M_x^2))^2} + B_n e^{b_{in}^{bg} t} (M_x^2 - M_{p+\pi}^2)^\epsilon \\
F(x_B, t) &= \frac{x_B(1-x_B)}{(M_x^2 - m_p^2) (1 + 4m_p^2 x_B^2 / (-t))^{3/2}}, \quad x_B = \frac{-t}{M_x^2 - m_p^2 - t} \\
F_p(t) &= \frac{1}{1 - \frac{t}{0.71}}, \quad f(t) = e^{b_{in} t} \\
\alpha(t) &= \alpha(0) + \alpha' t = 1.04 + 0.25t
\end{aligned}$$

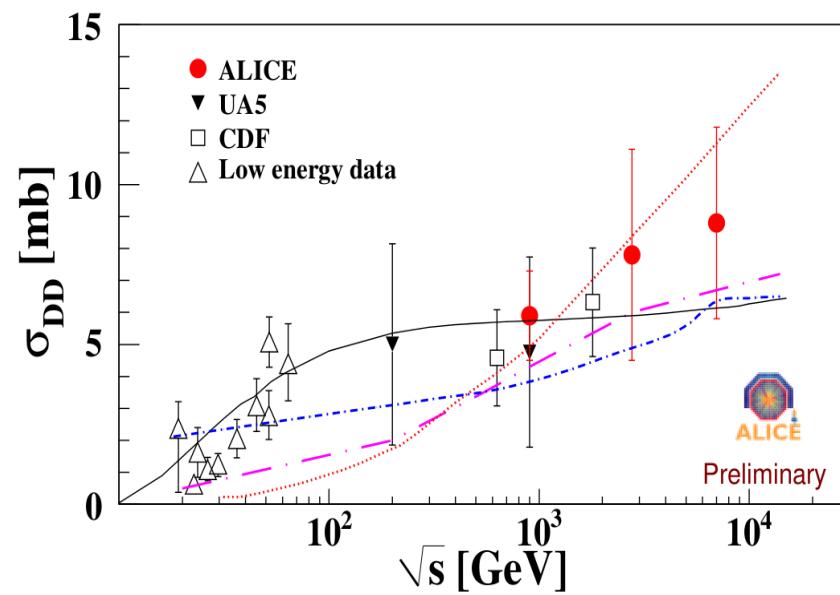
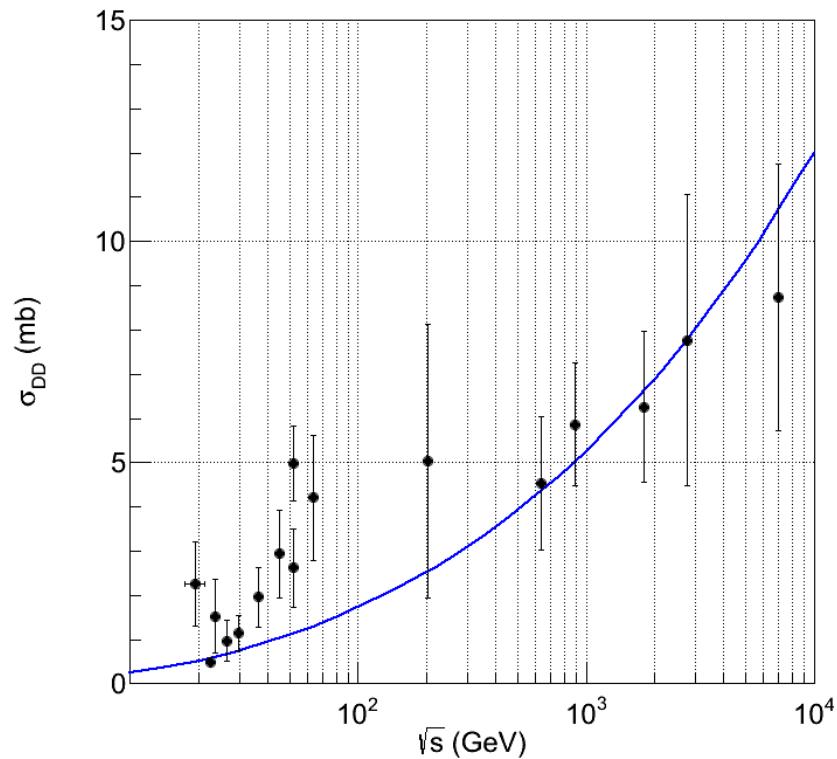
SDD cross sections vs. energy.



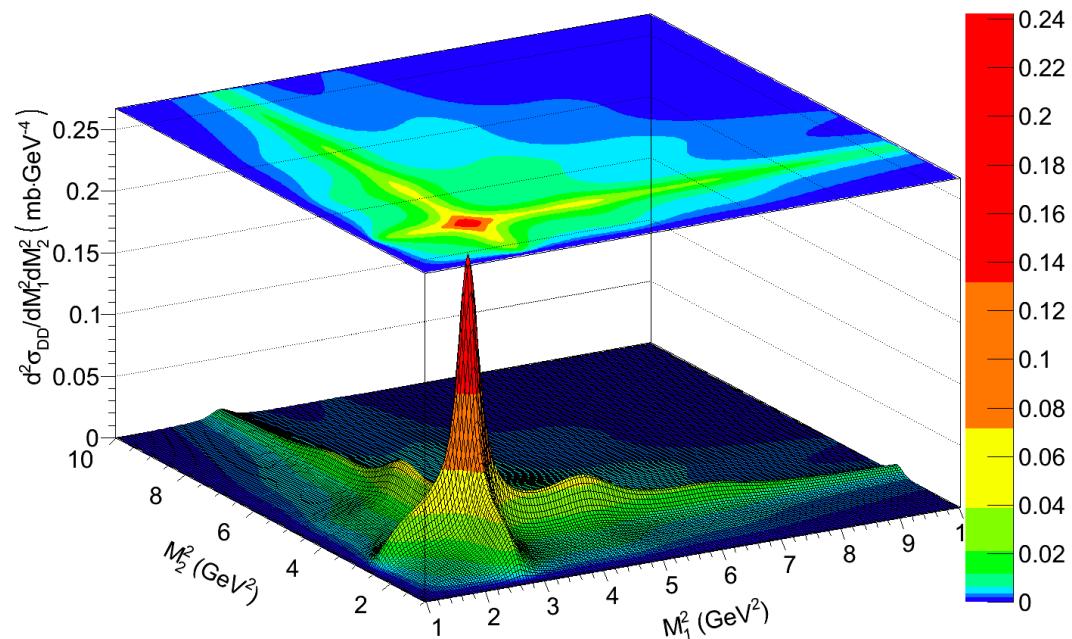
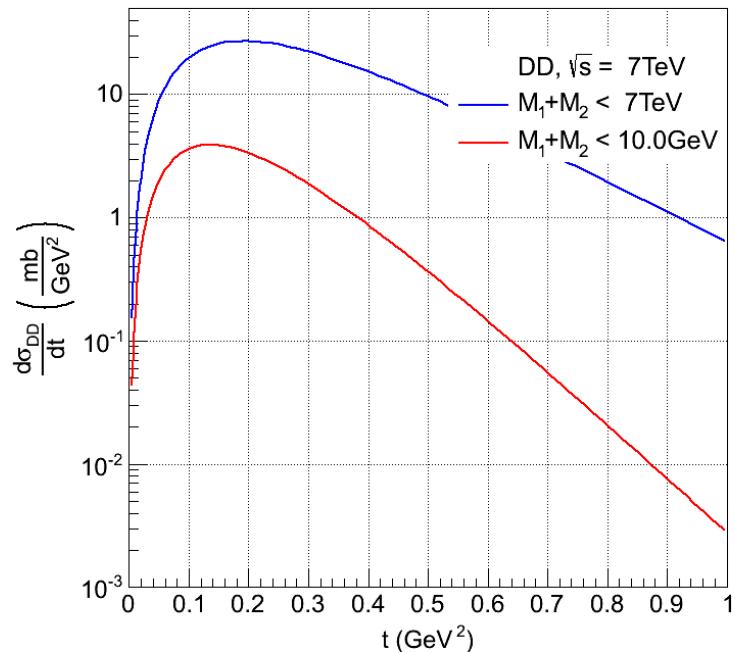
Approximation of background to reference points (t=-0.05)



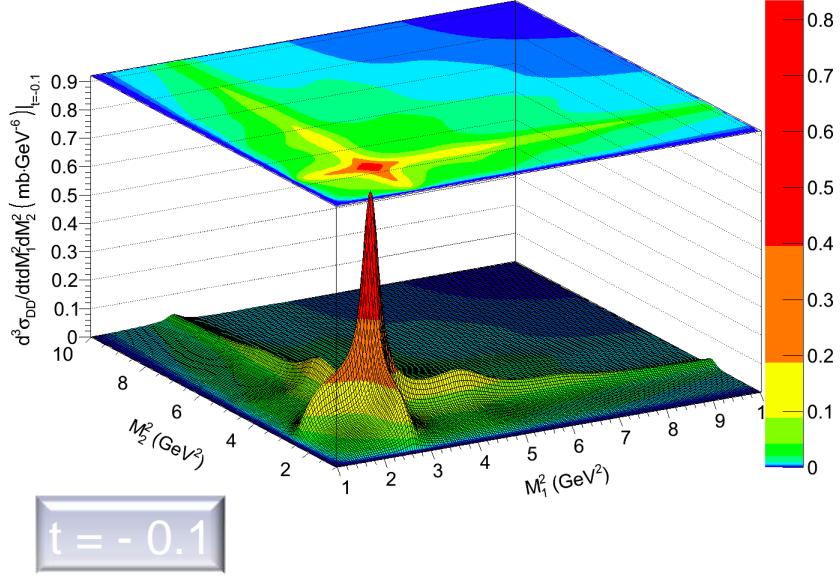
DDD cross sections vs. energy.



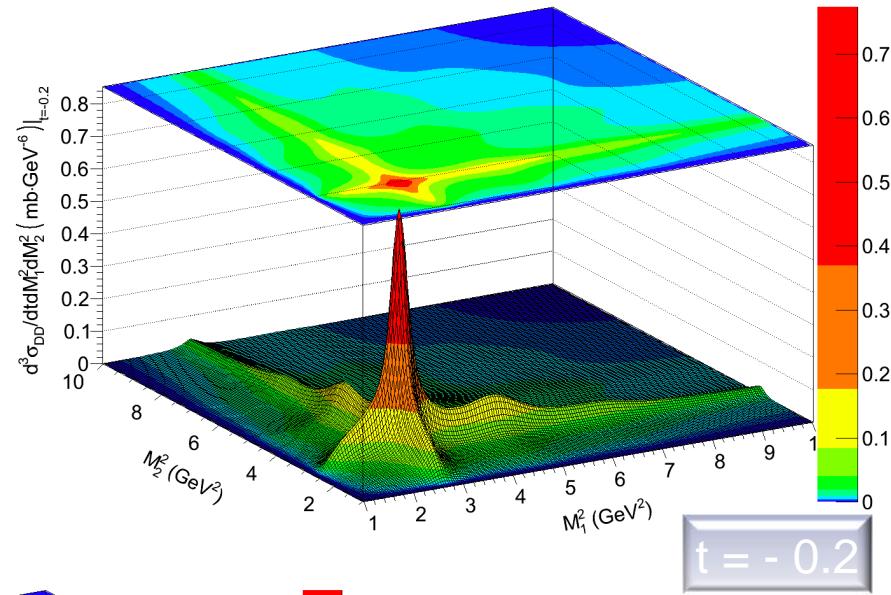
Integrated DD cross sections



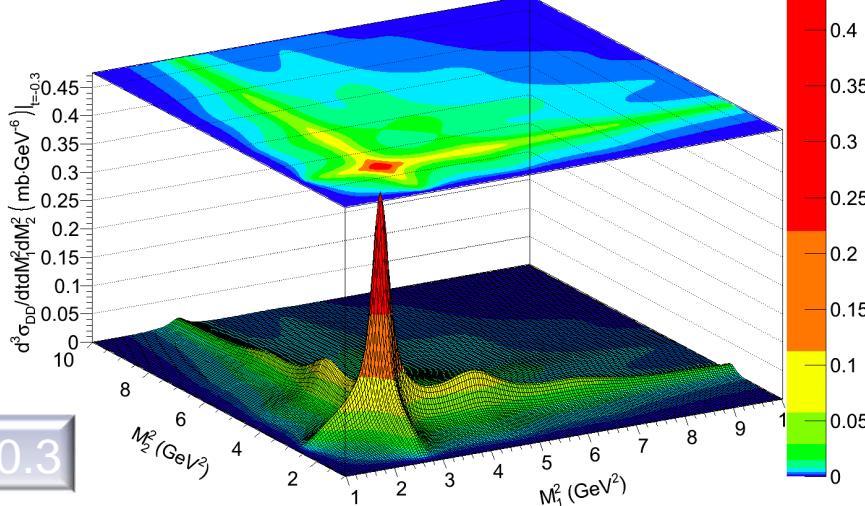
Triple differential DD cross sections



$t = -0.1$



$t = -0.2$



$t = -0.3$

NN & ep

Basic ideas

Reggeometry=Regge+geometry (play on words, or pun)

How to combine s, t and Q^2 dependencies in a binary reaction?

1. The t and \tilde{Q}^2 dependences are combined by "geometry":

A rough estimates (to be fine-tuned!) yields

$$\beta(t, M, Q^2) = \exp\left[4\left(\frac{1}{M_V^2 + Q^2} + \frac{1}{2m_N^2}\right)t\right].$$

2. The s and t behavior are related by the Regge-pole model;
3. There is only one, universal, Pomeron, but it has two components - soft and hard, their relative weights depending on \tilde{Q}^2 .

$$A(s,t,Q^2,{M_v}^2) = \frac{\tilde{A}_s}{\left(1+\frac{\widetilde{Q}^2}{\widetilde{Q}_s^2}\right)^{n_s}} e^{-i\frac{\pi}{2}\alpha_s(t)} \left(\frac{s}{s_{0s}}\right)^{\alpha_s(t)} e^{2\left(\frac{a_s}{\widetilde{Q}^2} + \frac{b_s}{2m_p^2}\right)t}$$

$$+ \frac{\tilde{A}_h\left(\frac{\widetilde{Q}^2}{\widetilde{Q}_h^2}\right)}{\left(1+\frac{\widetilde{Q}^2}{\widetilde{Q}_h^2}\right)^{n_h+1}} e^{-i\frac{\pi}{2}\alpha_h(t)} \left(\frac{s}{s_{0h}}\right)^{\alpha_h(t)} e^{2\left(\frac{a_h}{\widetilde{Q}^2} + \frac{b_h}{2m_p^2}\right)t}$$

	A_s	\widetilde{Q}_s^2	n_s	α_{0s}	α'_s	a_s	b_s	$\tilde{\chi}^2$
pp	5.9 ± 5.7	***	0.00	1.05 ± 0.14	0.276 ± 0.474	2.877 ± 2.837	0.00	1.52
ρ^0	59.5 ± 29.3	1.33	1.35 ± 0.05	1.15 ± 0.06	0.15	-0.22	1.69	6.56
ϕ	31.8 ± 35.3	1.30	1.32 ± 0.10	1.14 ± 0.12	0.15	-0.85 ± 1.60	2.51 ± 2.67	3.81
J/ψ	34.2 ± 19.0	1.4 ± 0.7	1.39 ± 0.13	1.21 ± 0.05	0.09	1.90	1.03	4.50
$\Upsilon(1S)$	37 ± 101	0.9 ± 1.7	1.53 ± 0.55	1.29 ± 0.26	0.01 ± 0.6	1.90	1.03	1.28
DVCS	9.7 ± 9.0	0.45 ± 0.5	0.94 ± 0.24	1.19 ± 0.09	-0.007 ± 0.3	1.94 ± 4.65	1.74 ± 2.28	1.75

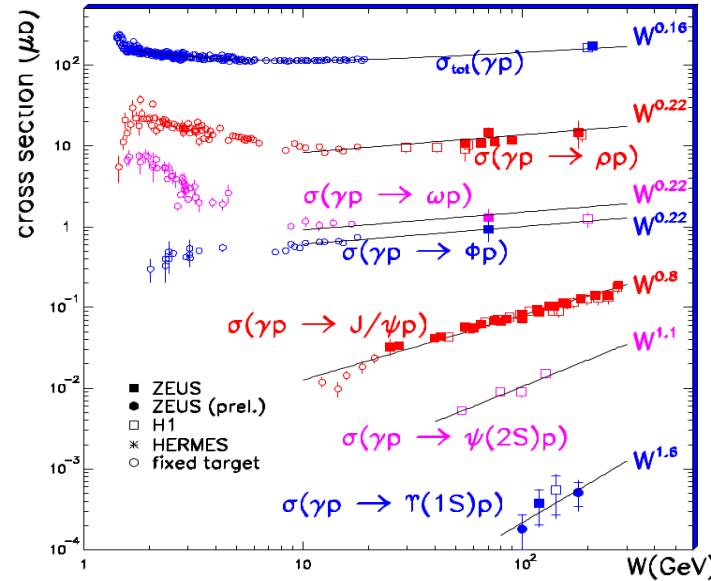
Table 1. Fitting results

	δ	α_{0s}	$\alpha_{0s}(fit)$	α'_s
pp		1.08(DL)	1.05 ± 0.14	0.276 ± 0.474
ρ^0	0.22	1.055	1.15 ± 0.06	0.15
ϕ	0.22	1.055	1.14 ± 0.12	0.15
J/ψ	0.8	1.2	1.21 ± 0.05	0.09
$\Upsilon(1S)$	1.6	1.4	1.29 ± 0.26	0.01 ± 0.6
DVCS	0.54	1.135	1.19 ± 0.09	-0.007 ± 0.3

Table 2. $\alpha(0)$, α'

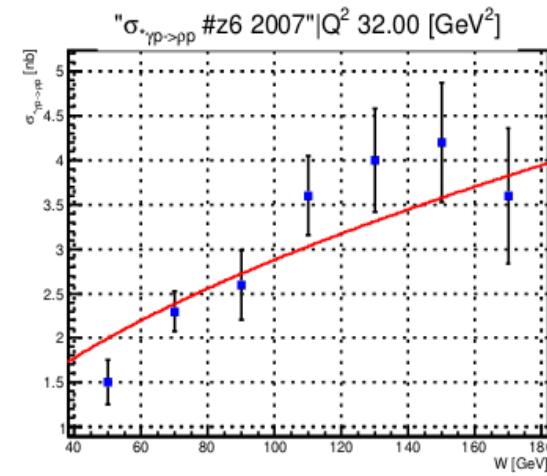
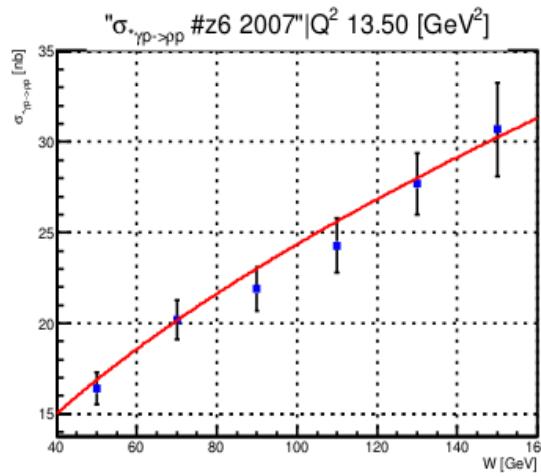
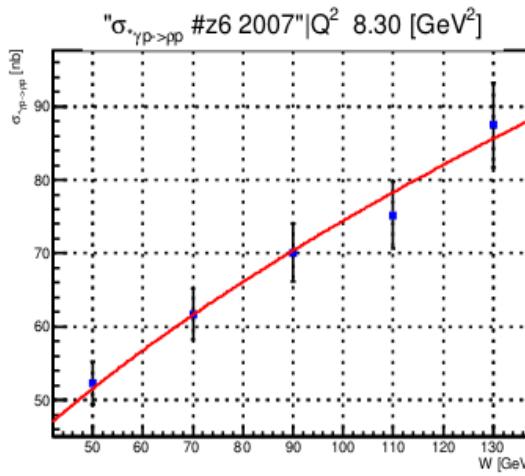
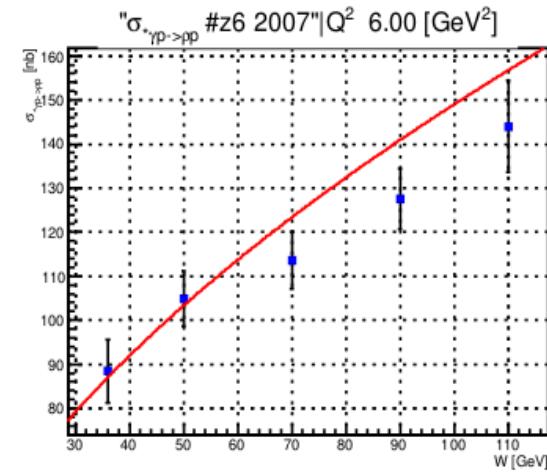
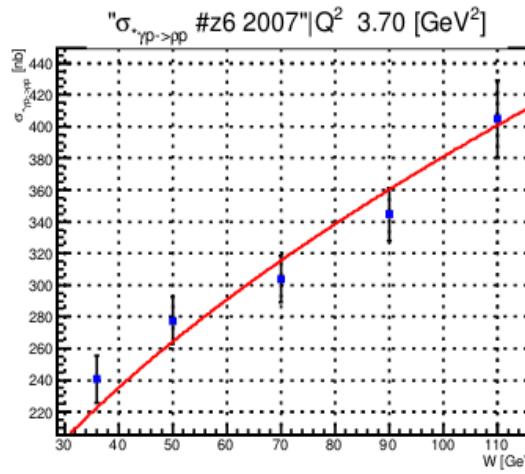
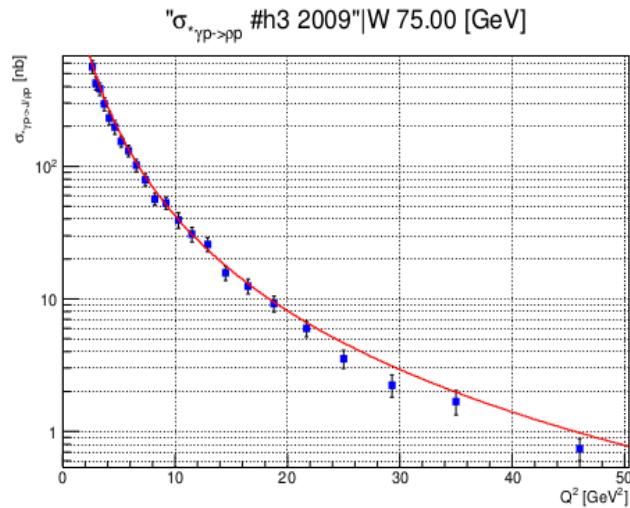
Parameter s_{0s} for simplicity is also fixed $s_{0s} = 1$.

* Parameters that doesn't have errors in table[1] were fixed at fitting stage.



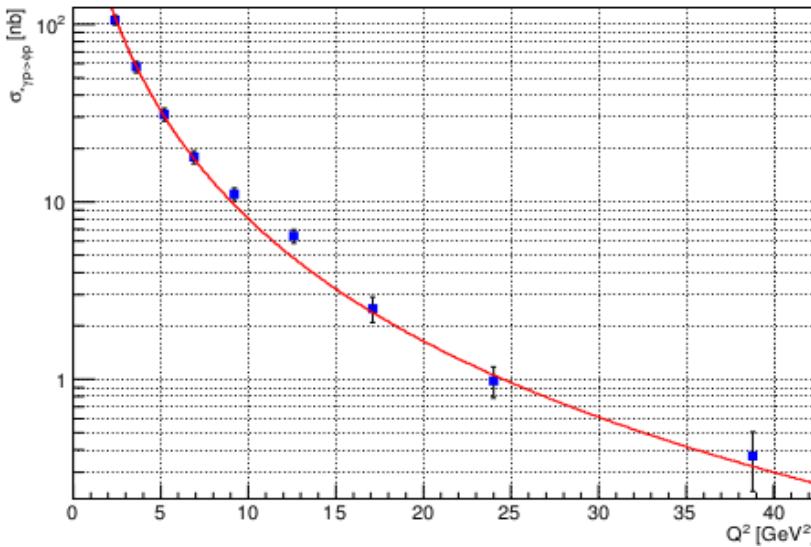
(a) The W dependence of the cross section for exclusive VM photoproduction together with the total photoproduction cross section. Lines are the result of a W^δ fit to the data at high W -energy values.

$\rho(0)$

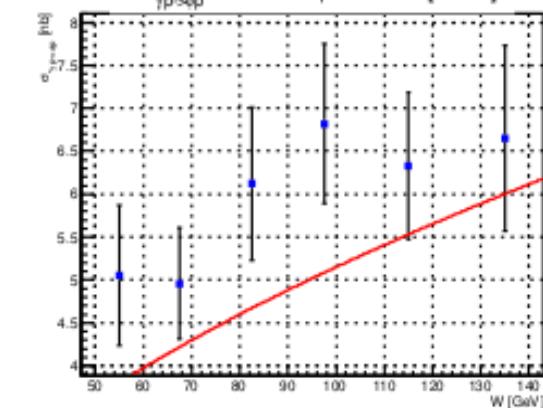


phi (1)

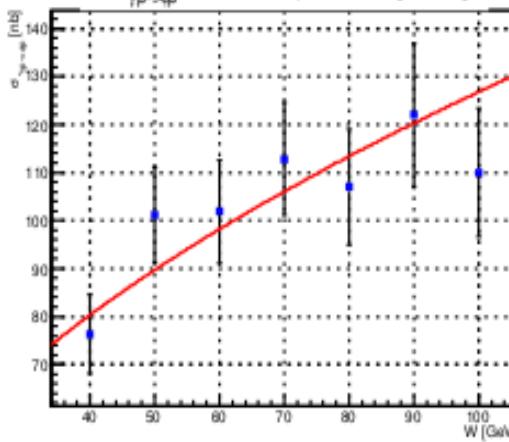
" $\sigma_{\gamma p \rightarrow e p}$ #z8 2005" | W 75.00 [GeV]



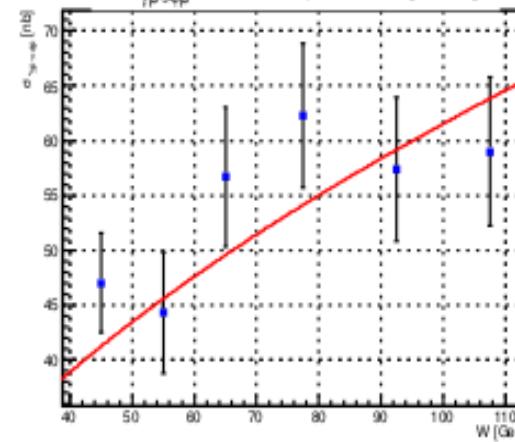
" $\sigma_{\gamma p \rightarrow e p}$ #z8 2005" | Q^2 13.00 [GeV^2]



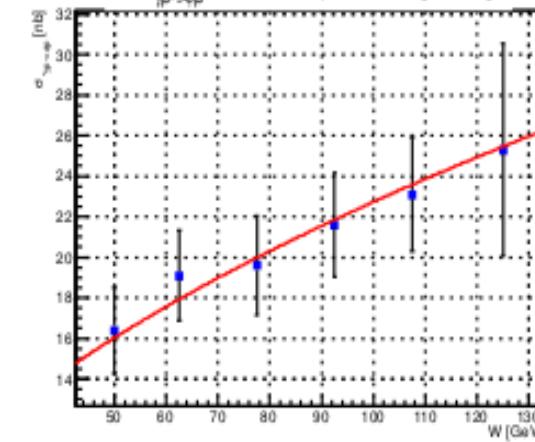
" $\sigma_{\gamma p \rightarrow e p}$ #z8 2005" | Q^2 2.40 [GeV^2]



" $\sigma_{\gamma p \rightarrow e p}$ #z8 2005" | Q^2 3.80 [GeV^2]

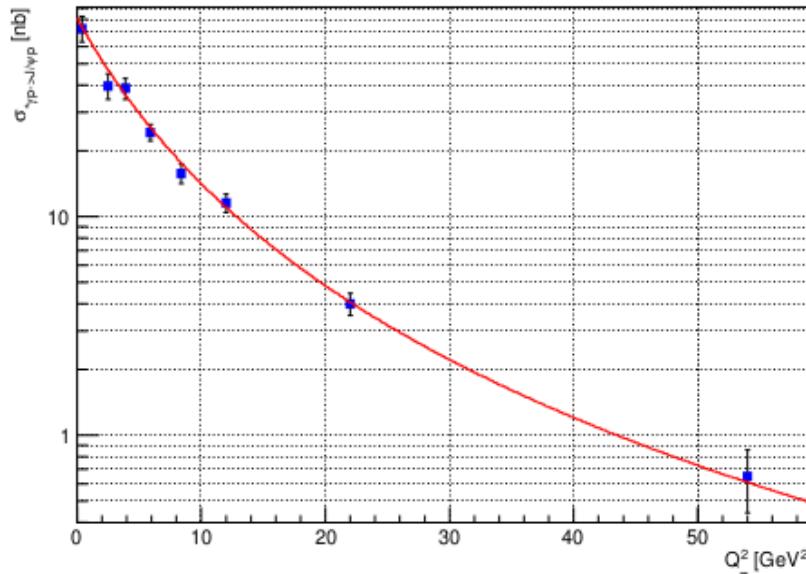


" $\sigma_{\gamma p \rightarrow e p}$ #z8 2005" | Q^2 6.50 [GeV^2]

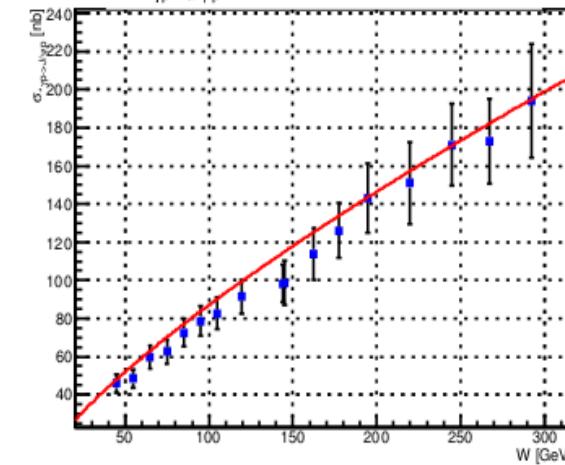


J/psi (1)

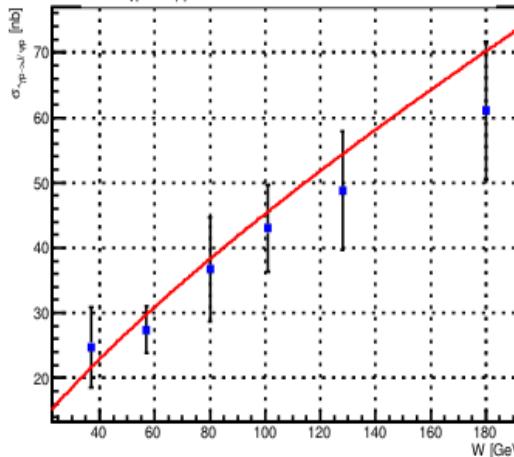
" $\sigma_{\gamma p \rightarrow J/\psi p}$ #z9 2004" | W 90.00 [GeV]



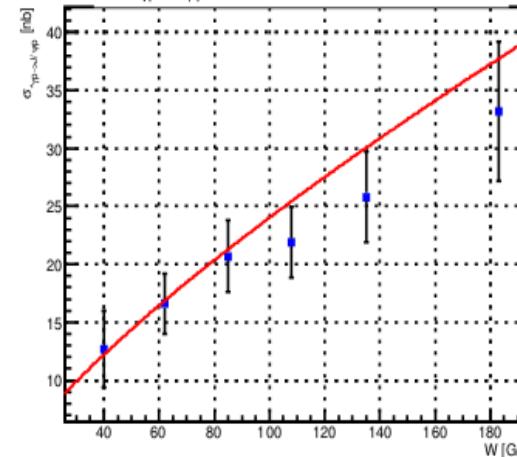
" $\sigma_{\gamma p \rightarrow J/\psi p}$ #h6 2005" | Q^2 0.05 [GeV 2]



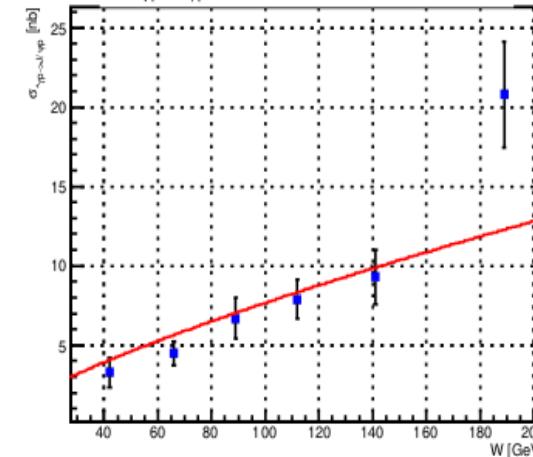
" $\sigma_{\gamma p \rightarrow J/\psi p}$ #z9 2004" | Q^2 3.10 [GeV 2]



" $\sigma_{\gamma p \rightarrow J/\psi p}$ #z9 2004" | Q^2 6.80 [GeV 2]

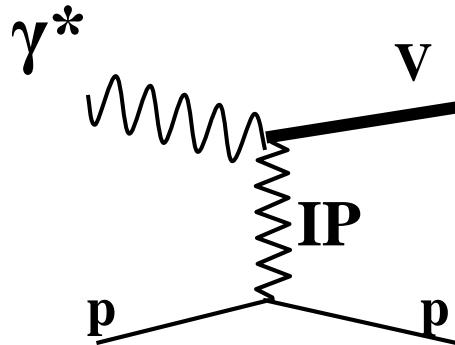


" $\sigma_{\gamma p \rightarrow J/\psi p}$ #z9 2004" | Q^2 16.00 [GeV 2]

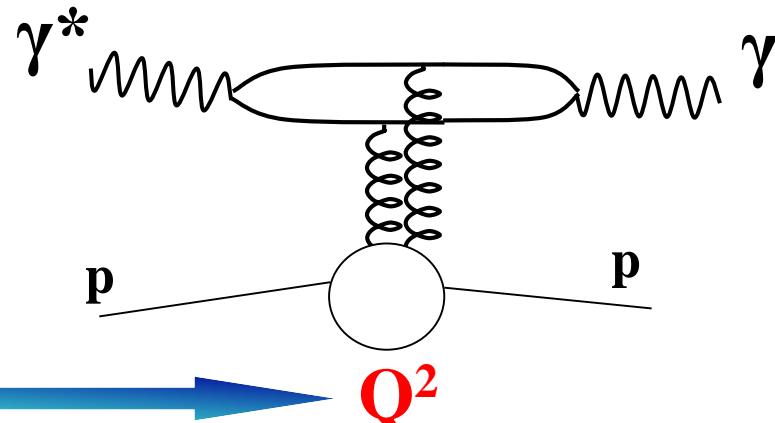


Deeply Virtual Compton Scattering

VM ($\rho, \omega, \phi, J/\psi, Y$)



DVCS (γ)

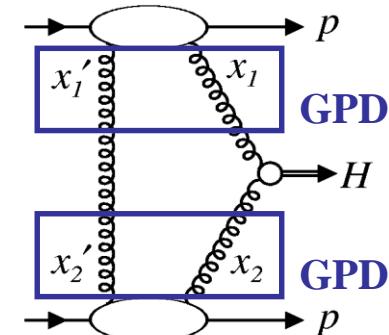


Scale: $Q^2 + M^2$



DVCS properties:

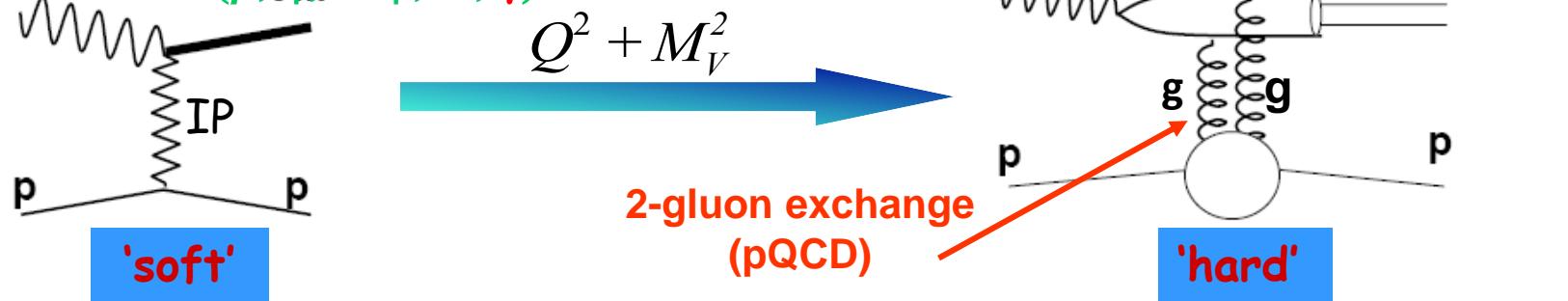
- Similar to VM production, but γ instead of VM in the final state
- No VM wave-function involved
- Important to determine Generalized Parton Distributions sensible to the correlations in the proton
- GPDs are an ingredient for estimating diffractive cross sections at the LHC



Diffraction: soft \rightarrow hard

Vector Meson

production ($\rho, \psi, J/\psi, Y, \gamma$)



Cross section proportional to probability
of finding 2 gluons in the proton

$$\left\{ \begin{array}{l} \sigma \propto [x g(x, \mu^2)]^2 \\ \mu^2 \propto (Q^2 + M_V^2) \end{array} \right.$$

Gluon density in the proton

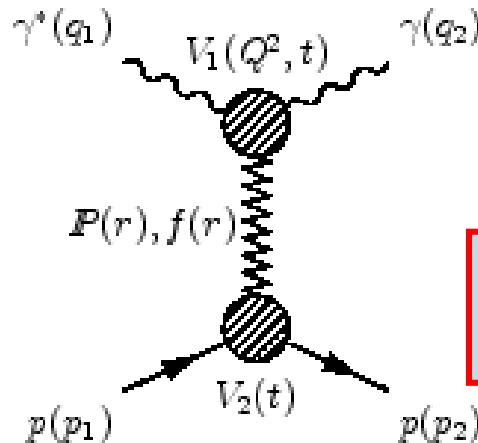
$\sigma(W) \propto W^\delta \rightarrow \delta$ increases from soft (~ 0.2 , "soft Pomeron") to hard (~ 0.8 , "hard Pomeron")

$\frac{d\sigma}{dt} \propto e^{-b|t|} \rightarrow b$ decreases from soft ($\sim 10 \text{ GeV}^{-2}$) to hard ($\sim 4-5 \text{ GeV}^{-2}$)

Regge-type DVCS amplitude

M. Capua, S. F., R. Fiore, L. L. Jenkovszky, and F Paccanoni

Published in: Physics Letters B645 (Feb. 2007) 161-166



$$V_1 = e^{b\beta(z)}$$

$$V_2 = e^{b\alpha(t)}$$

A new variable is introduced: $z = t - Q^2$

Applications for the model can be:

- Study of various regimes of the scattering amplitude vs Q^2, W, t (perturbative \rightarrow unperturbative QCD)
- Study of GPDs

DVCS amplitude: $A(s, t, Q^2)_{\gamma^* p \rightarrow \gamma p} = -A_0 V_1(t, Q^2) V_2(t) (-is/s_0)^{\alpha(t)}$

the t dependence at the vertex $pIPp$ is introduced by: $\alpha(t) = \alpha(0) - \alpha_1 \ln(1 - \alpha_2 t)$

the vertex $\gamma^*IP\gamma$ is introduced by the trajectory: $\beta(z) = \beta(0) - \beta_1 \ln(1 - \beta_2 z)$

indicating with $L = \ln(-is/s_0)$

the DVCS amplitude can be written as:

$$A(s, t, Q^2)_{\gamma^* p \rightarrow \gamma p} = -A_0 e^{b\alpha(t)} e^{b\beta(z)} (-is/s_0)^{\alpha(t)} = -A_0 e^{(b+L)\alpha(t) + b\beta(z)}$$

Basic ideas of the Kiev-Calabria-Padova Collab.: M. Capua, R. Fiore, L. J., F. Paccanoni, A. Papa “A DVCS Amplitude”, Phys. Lett. **B645** (2007) 161-166; hep-ph/0605319; S. Fazio, R. Fiore et al., “Unifying “soft” and “hard...”, PR D90(2014)016007, arXiv 1312.5683.

Reggegeometry=Regge+geometry (play on words = pun)

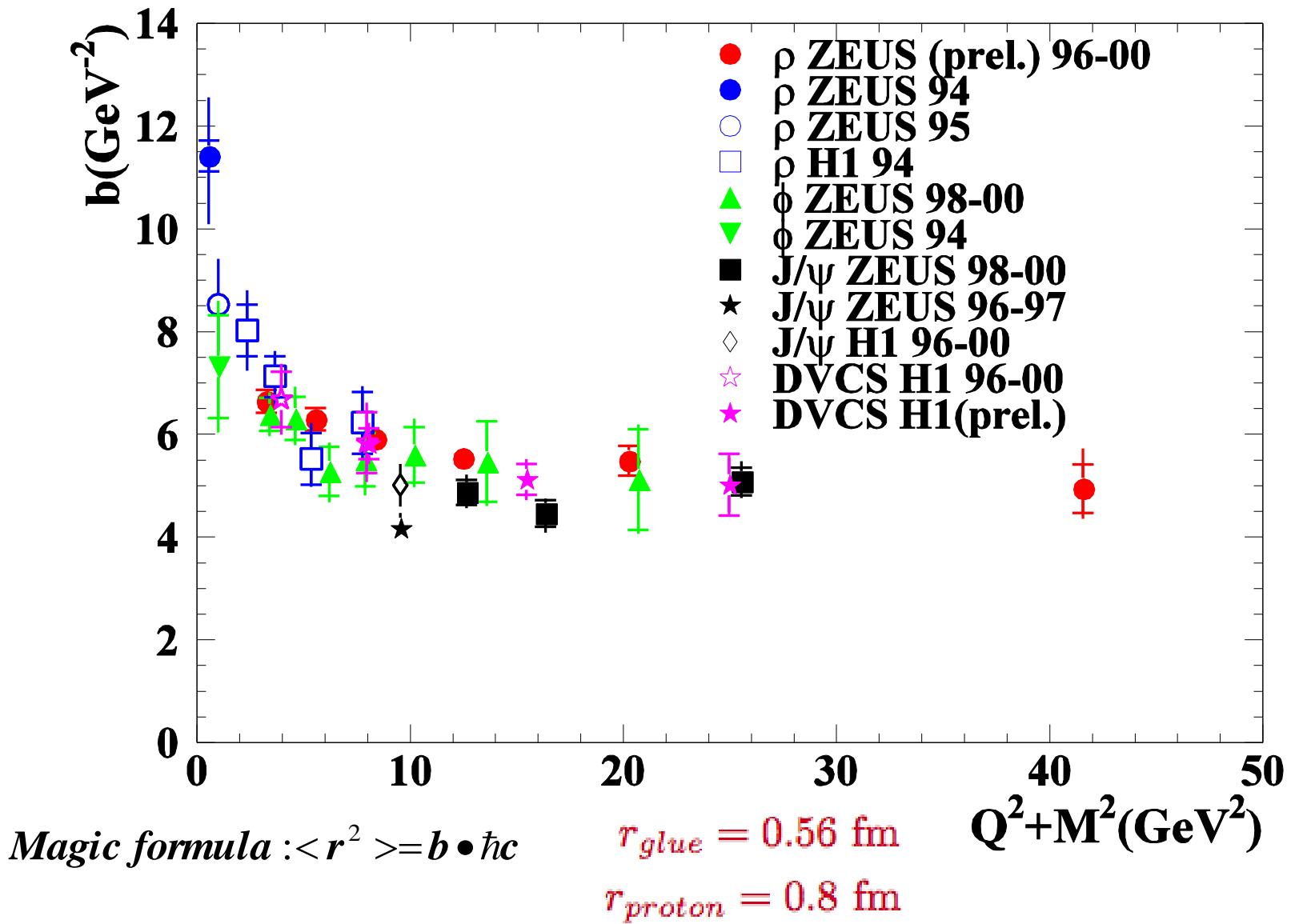
How to combine s, t and Q^2 dependencies in a binary reaction?

1. The t and \tilde{Q}^2 dependences are combined by ”geometry”:
A rough estimates (to be fine-tuned!) yields

$$\beta(t, M, Q^2) = \exp \left[4 \left(\frac{1}{M_V^2 + Q^2} + \frac{1}{2m_N^2} \right) t \right].$$

2. The s and t behavior are related by the Regge-pole model;
3. There is only one, universal, Pomeron, but it has two components - soft and hard, their relative weights depending on \tilde{Q}^2 .

$$b(Q^2+M^2) - VM$$



Pomeron Trajectory

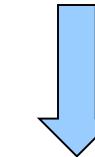
Regge-type: $\frac{d\sigma}{dt}(W) = \exp(b_0 t) W^{[2\alpha_{IP}(t)+2]}$

Linear Pomeron trajectory

$$\alpha(t) = \alpha(0) + \alpha'(t)t$$

$\alpha(0)$ and α' are fundamental parameters to represent the basic features of strong interactions

First measured in h-h scattering



Soft Pomeron values

$$\alpha(0) \approx 1.09$$

$$\alpha' \approx 0.25$$

$\alpha(0)$: determines the energy dependence of the diff. Cross section

$$\frac{d\sigma}{dt} \propto \exp(b_0 t) W^{4\alpha(t)-4} = W^{4\underline{\alpha(0)}-4} \cdot \exp(bt); \quad b = b_0 + 4\underline{\alpha'} \ln(W)$$

α' : determines the energy dependence of the transverse extention system

Unique Pomeron with two (“soft” and “hard”) components

R. Fiore et al. Phys. Rev. PR D90(2014)016007, arXiv 1312.5683

$$A(s, t, Q^2, M_v^2) = \frac{\tilde{A}_s}{\left(1 + \frac{\widetilde{Q}^2}{\widetilde{Q}_s^2}\right)^{n_s}} e^{-i\frac{\pi}{2}\alpha_s(t)} \left(\frac{s}{s_{0s}}\right)^{\alpha_s(t)} e^{2\left(\frac{a_s}{\widetilde{Q}^2} + \frac{b_s}{2m_p^2}\right)t}$$

$$+ \frac{\tilde{A}_h\left(\frac{\widetilde{Q}^2}{\widetilde{Q}_h^2}\right)}{\left(1 + \frac{\widetilde{Q}^2}{\widetilde{Q}_h^2}\right)^{n_h+1}} e^{-i\frac{\pi}{2}\alpha_h(t)} \left(\frac{s}{s_{0h}}\right)^{\alpha_h(t)} e^{2\left(\frac{a_h}{\widetilde{Q}^2} + \frac{b_h}{2m_p^2}\right)t}$$

$$\frac{d\sigma_{el}}{d|t|} = H_s^2 e^{2L_s(\alpha_s(t)-1)+g_st} + H_h^2 e^{2L_h(\alpha_h(t)-1)+g_ht}$$

$$+2H_sH_he^{L_s(\alpha_s(t)-1)+L_h(\alpha_h(t)-1)+(g_s+g_h)t}\cos\left(\frac{\pi}{2}\left(\alpha_s(t)-\alpha_h(t)\right)\right)$$

$$H_s=\frac{A_s}{\left(1+\frac{\widetilde{Q}^2}{Q_s^2}\right)^{n_s}}$$

$$H_h=\frac{A_h\left(\frac{\widetilde{Q}^2}{Q_h^2}\right)}{\left(1+\frac{\widetilde{Q}^2}{Q_h^2}\right)^{n_h+1}}$$

$$L_s = \ln\left(\frac{s}{s_{0s}}\right) \quad L_h = \ln\left(\frac{s}{s_{0h}}\right)$$

$$g_s=2\Biggl(\frac{a_s}{\widetilde{Q}^2}+\frac{b_s}{2m_p^2}\Biggr)\qquad g_h=2\Biggl(\frac{a_h}{\widetilde{Q}^2}+\frac{b_h}{2m_p^2}\Biggr)$$

$$\alpha_s(t)=\alpha_{0s}+\alpha'_s t$$

$$\alpha_h(t)=\alpha_{0h}+\alpha'_h t$$

	A_s	\widetilde{Q}_s^2	n_s	α_{0s}	α'_s	a_s	b_s	$\tilde{\chi}^2$
pp	5.9 ± 5.7	***	0.00	1.05 ± 0.14	0.276 ± 0.474	2.877 ± 2.837	0.00	1.52
ρ^0	59.5 ± 29.3	1.33	1.35 ± 0.05	1.15 ± 0.06	0.15	-0.22	1.69	6.56
ϕ	31.8 ± 35.3	1.30	1.32 ± 0.10	1.14 ± 0.12	0.15	-0.85 ± 1.60	2.51 ± 2.67	3.81
J/ψ	34.2 ± 19.0	1.4 ± 0.7	1.39 ± 0.13	1.21 ± 0.05	0.09	1.90	1.03	4.50
$\Upsilon(1S)$	37 ± 101	0.9 ± 1.7	1.53 ± 0.55	1.29 ± 0.26	0.01 ± 0.6	1.90	1.03	1.28
DVCS	9.7 ± 9.0	0.45 ± 0.5	0.94 ± 0.24	1.19 ± 0.09	-0.007 ± 0.3	1.94 ± 4.65	1.74 ± 2.28	1.75

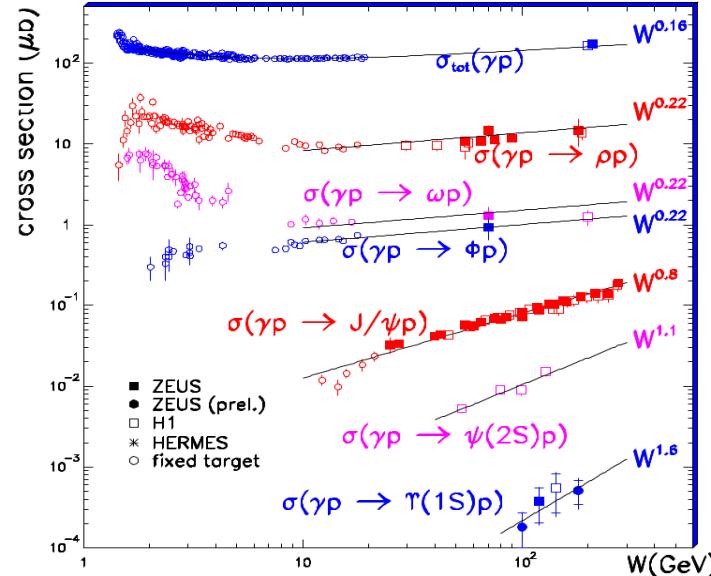
Table 1. Fitting results

	δ	α_{0s}	$\alpha_{0s}(fit)$	α'_s
pp		1.08(DL)	1.05 ± 0.14	0.276 ± 0.474
ρ^0	0.22	1.055	1.15 ± 0.06	0.15
ϕ	0.22	1.055	1.14 ± 0.12	0.15
J/ψ	0.8	1.2	1.21 ± 0.05	0.09
$\Upsilon(1S)$	1.6	1.4	1.29 ± 0.26	0.01 ± 0.6
DVCS	0.54	1.135	1.19 ± 0.09	-0.007 ± 0.3

Table 2. $\alpha(0)$, α'

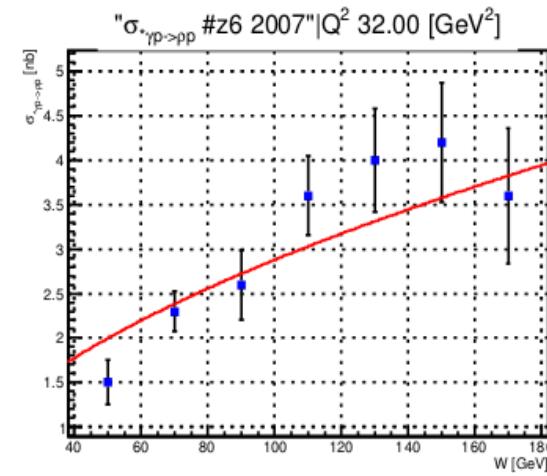
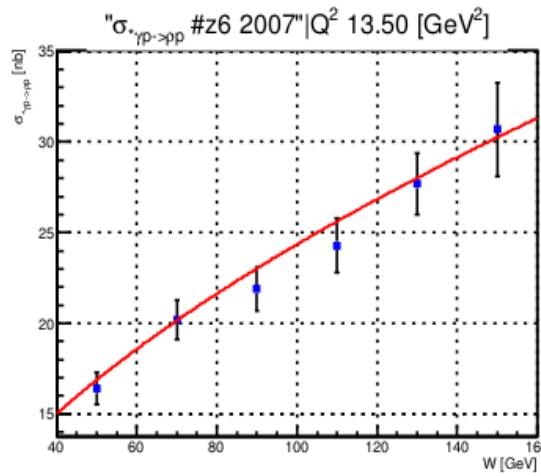
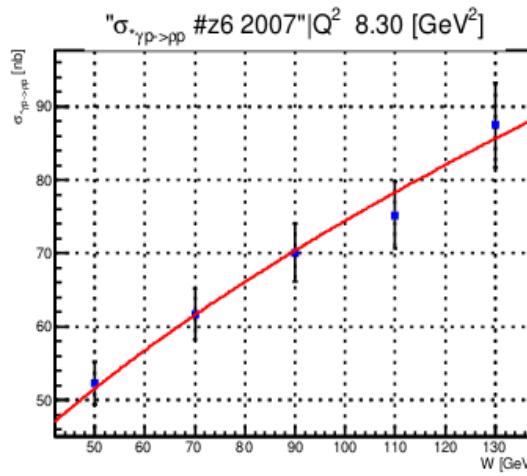
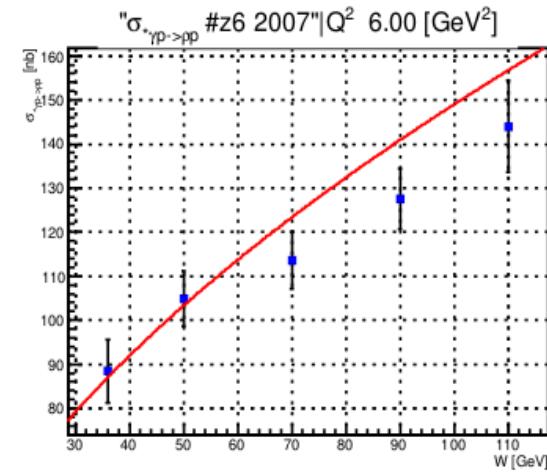
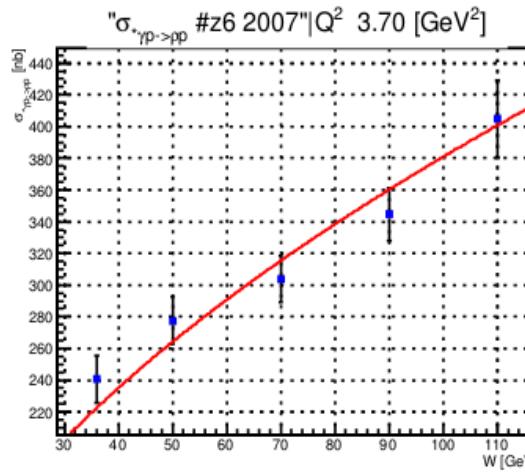
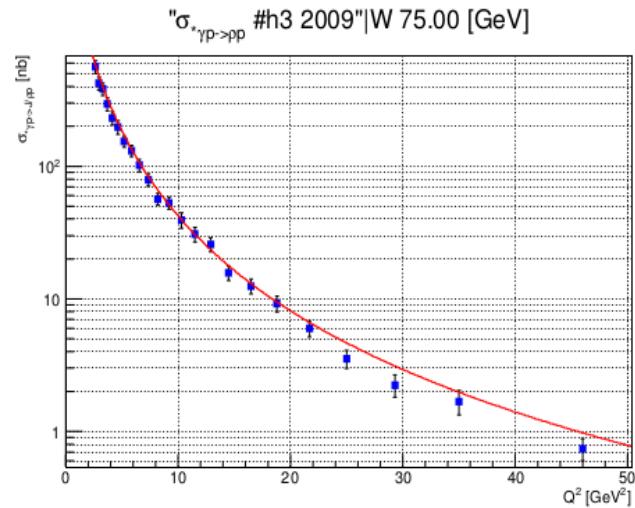
Parameter s_{0s} for simplicity is also fixed $s_{0s} = 1$.

* Parameters that doesn't have errors in table[1] were fixed at fitting stage.



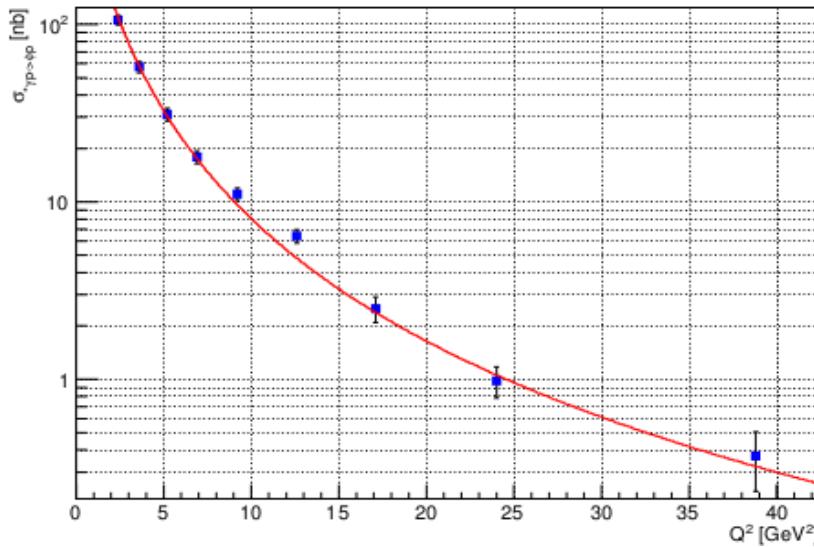
(a) The W dependence of the cross section for exclusive VM photoproduction together with the total photoproduction cross section. Lines are the result of a W^δ fit to the data at high W -energy values.

rho0(1)

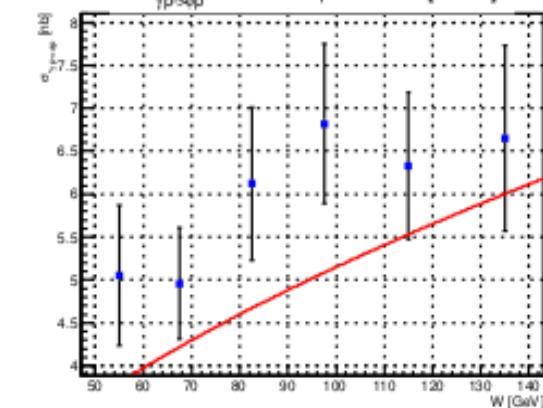


phi (1)

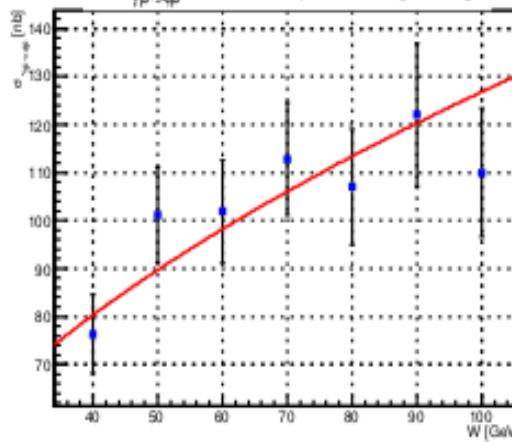
" $\sigma_{\gamma p \rightarrow e p}$ #z8 2005" | W 75.00 [GeV]



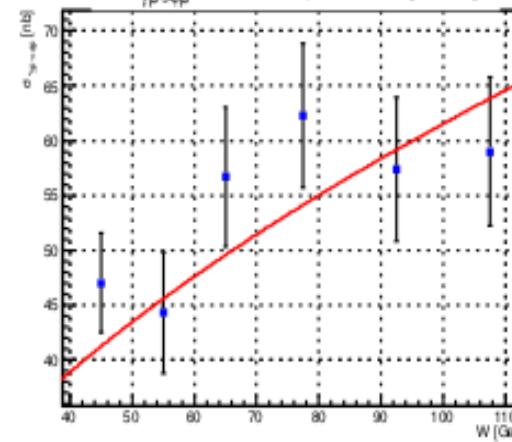
" $\sigma_{\gamma p \rightarrow e p}$ #z8 2005" | Q^2 13.00 [GeV^2]



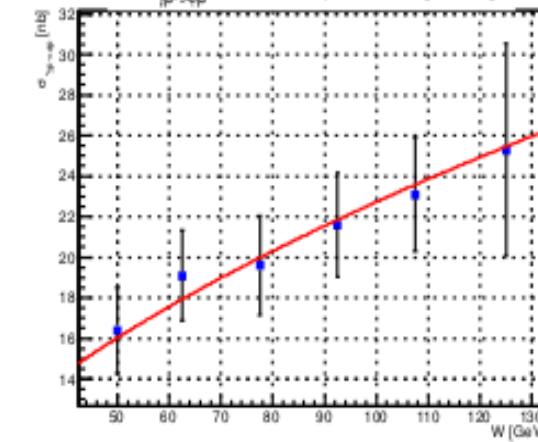
" $\sigma_{\gamma p \rightarrow e p}$ #z8 2005" | Q^2 2.40 [GeV^2]



" $\sigma_{\gamma p \rightarrow e p}$ #z8 2005" | Q^2 3.80 [GeV^2]

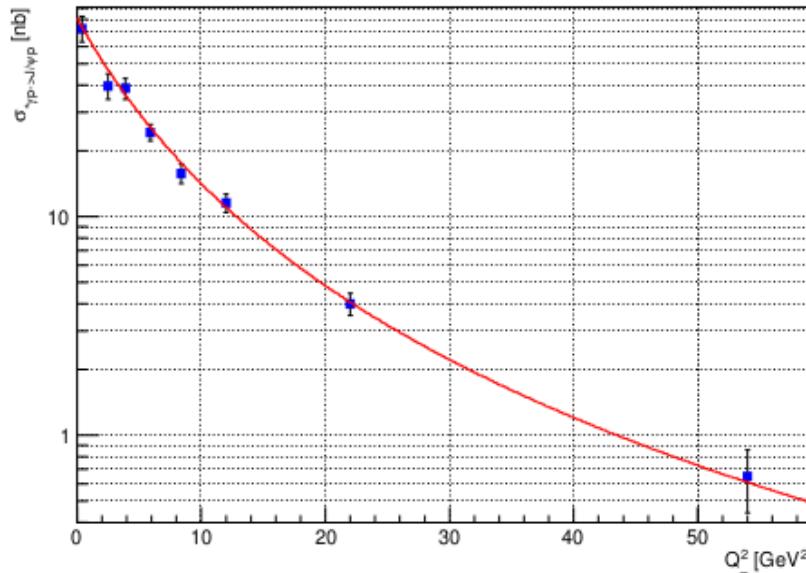


" $\sigma_{\gamma p \rightarrow e p}$ #z8 2005" | Q^2 6.50 [GeV^2]

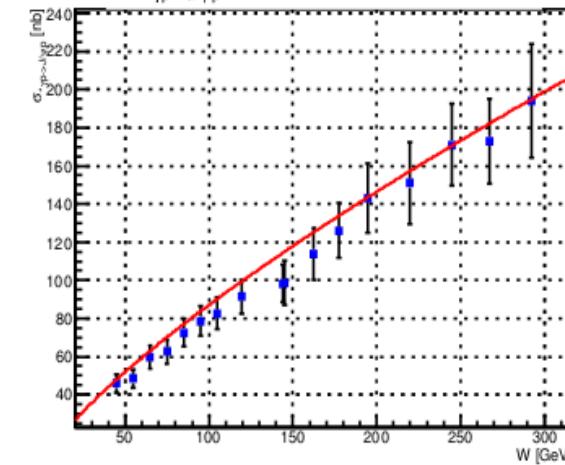


J/psi (1)

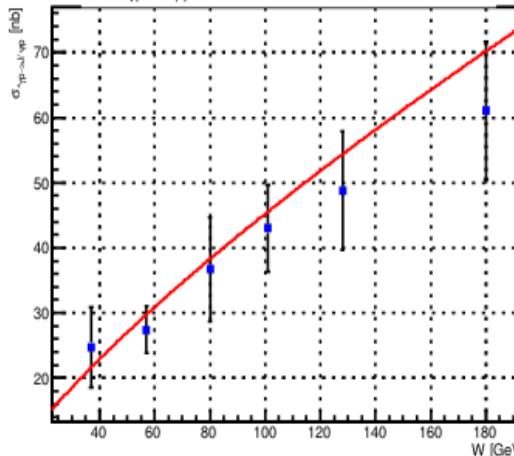
" $\sigma_{\gamma p \rightarrow J/\psi p}$ #z9 2004" | W 90.00 [GeV]



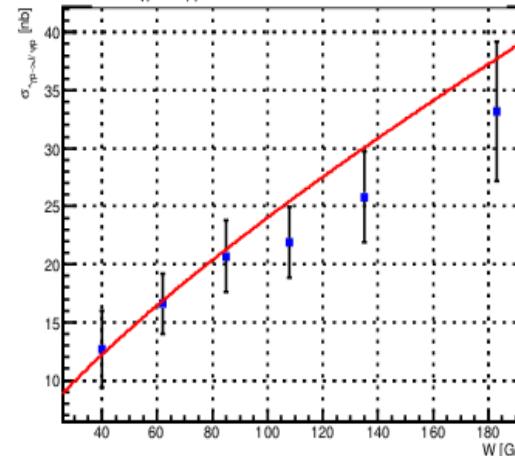
" $\sigma_{\gamma p \rightarrow J/\psi p}$ #h6 2005" | Q^2 0.05 [GeV 2]



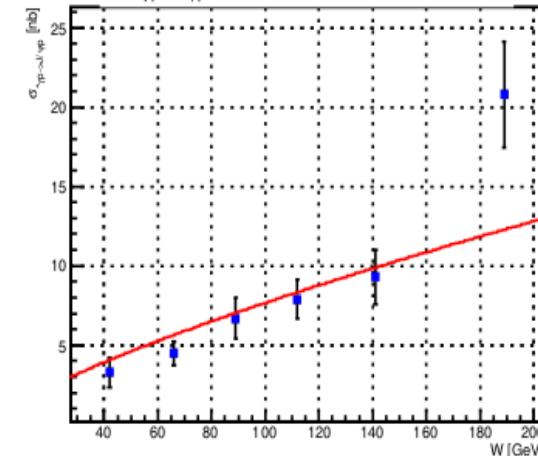
" $\sigma_{\gamma p \rightarrow J/\psi p}$ #z9 2004" | Q^2 3.10 [GeV 2]



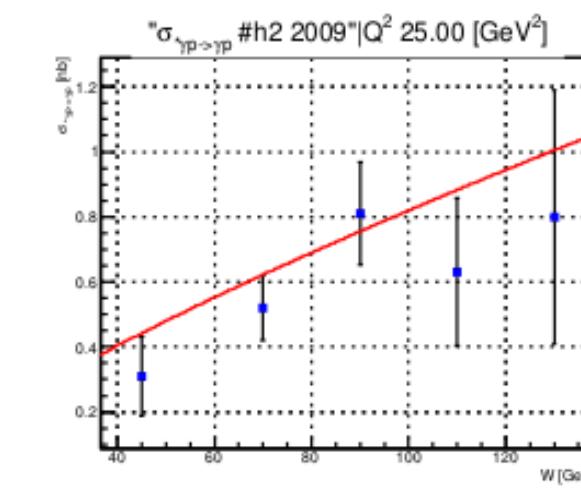
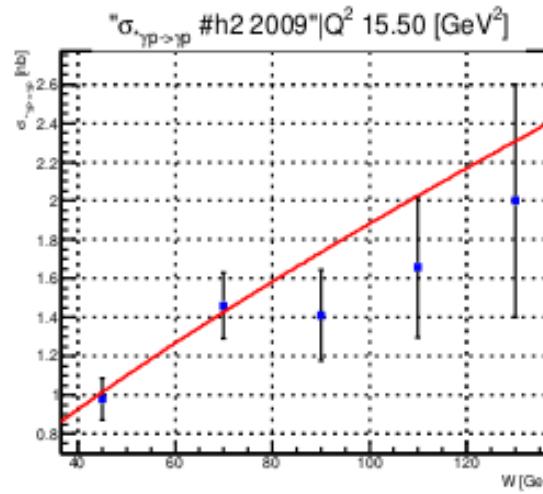
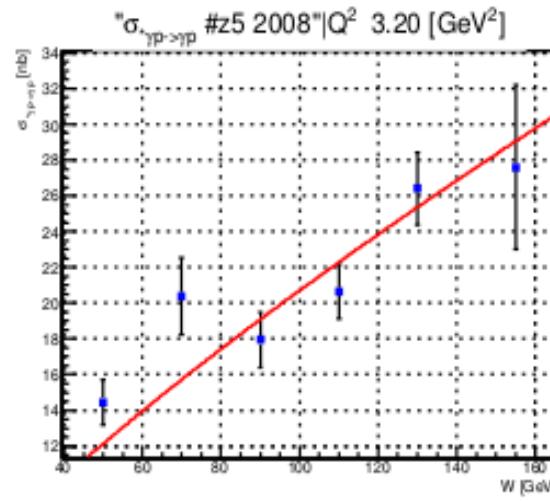
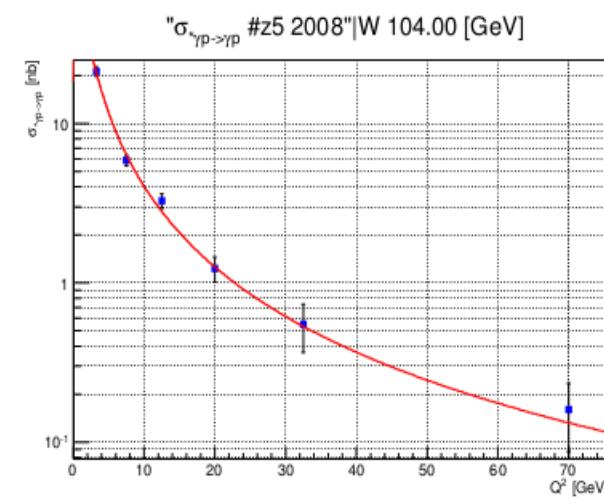
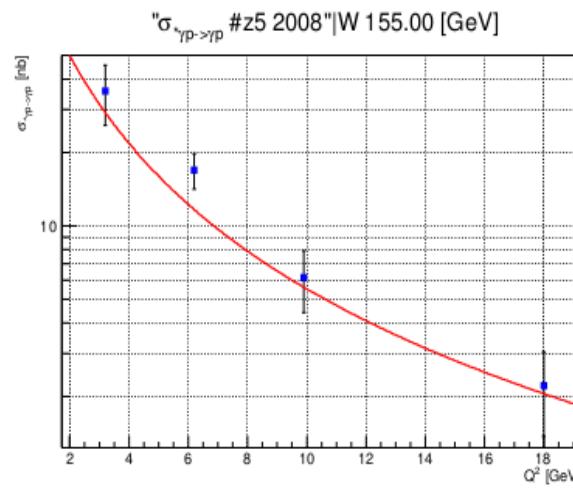
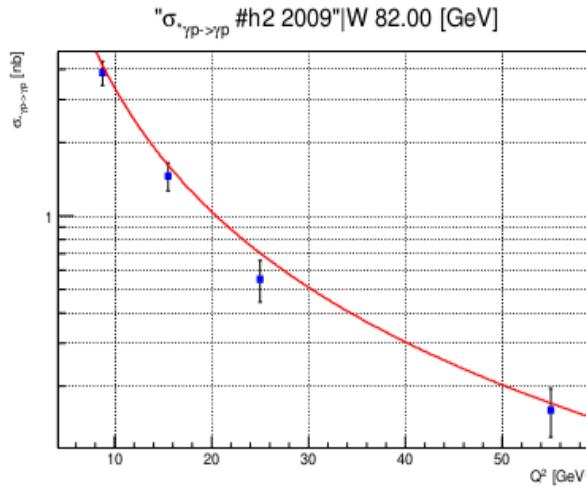
" $\sigma_{\gamma p \rightarrow J/\psi p}$ #z9 2004" | Q^2 6.80 [GeV 2]

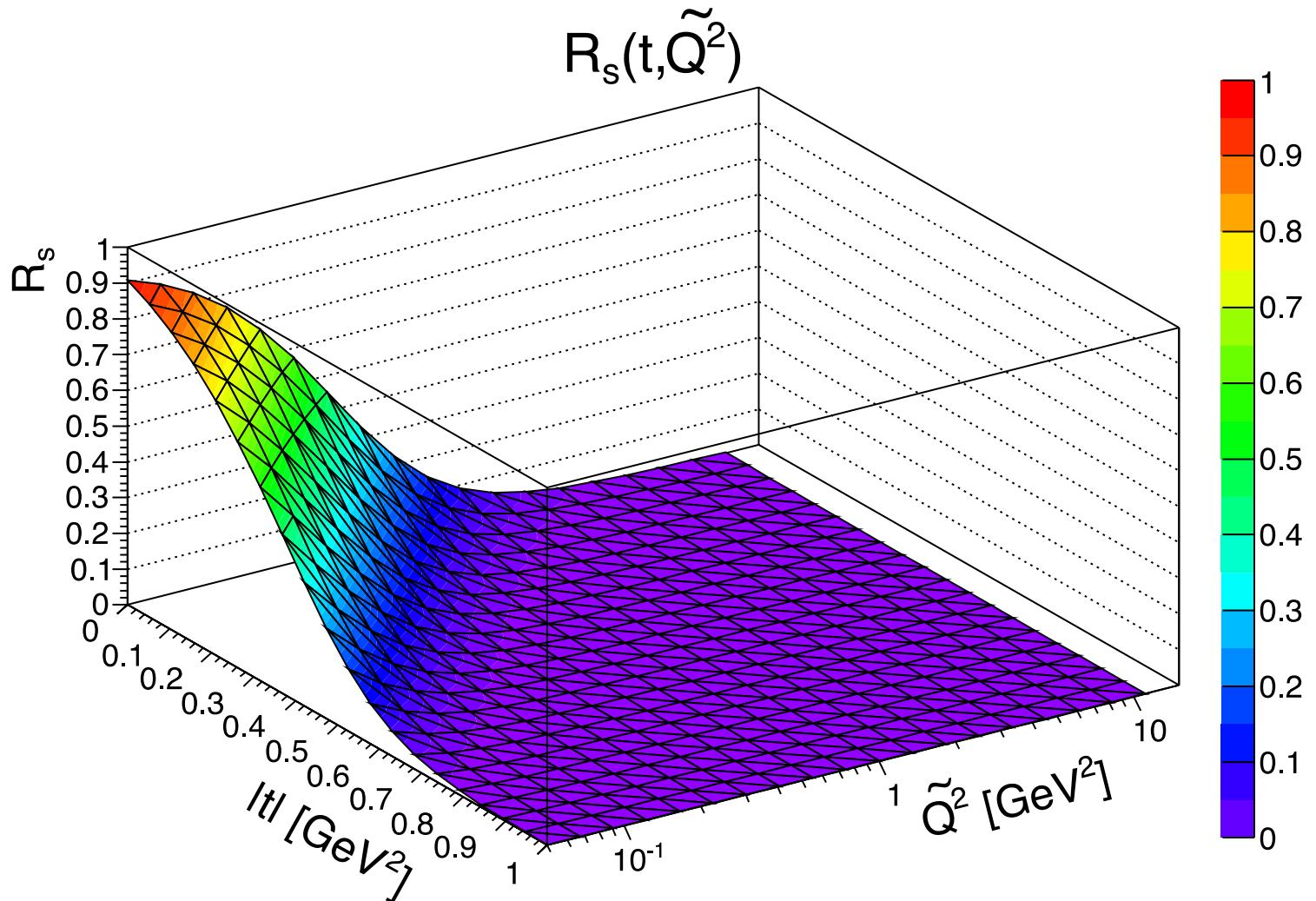


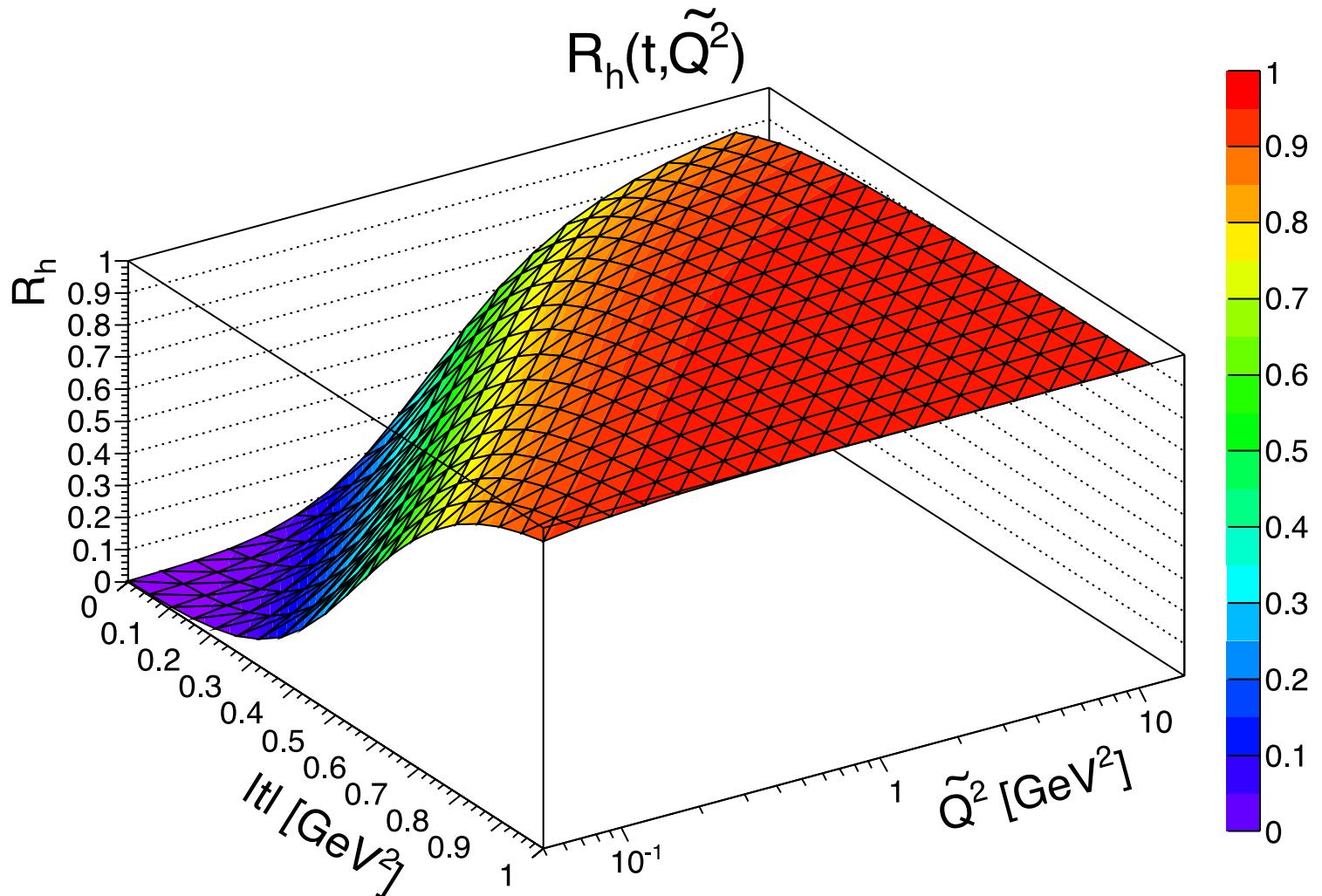
" $\sigma_{\gamma p \rightarrow J/\psi p}$ #z9 2004" | Q^2 16.00 [GeV 2]

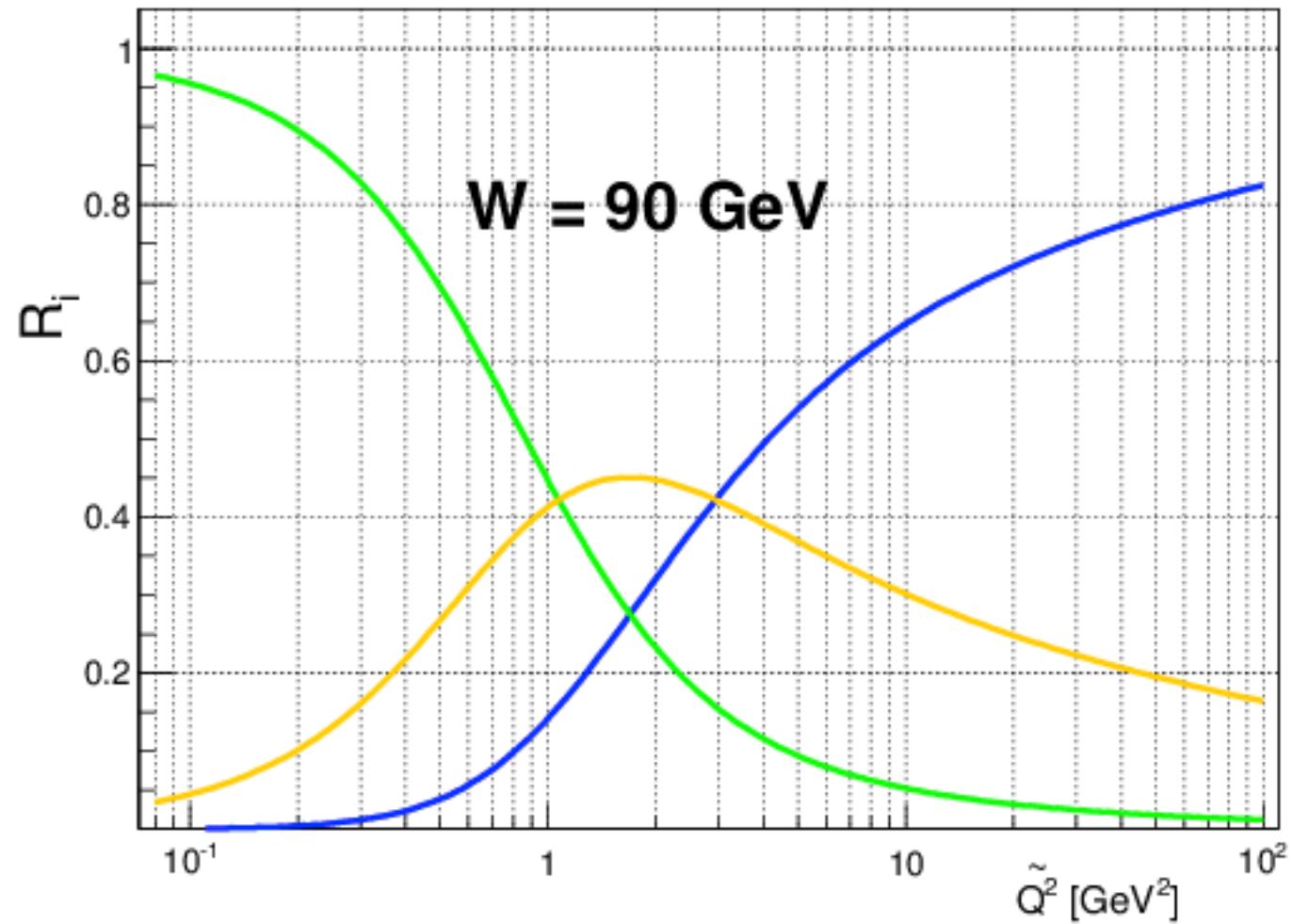


DVCS (1)









Summary and prospects for **ep** collisions:

I. Problems:

- 1) unifying DVCS and VMP (the photon “mass”);
- 2) balancing between “soft” and “hard”, QCD?
- 3) extension to low energies (non-diffractive component, secondary Reggeons) ;
- 4) saturation effects, unitarity (gap survival);
- 5) Is BFKL Regge behaved?
Prospects: application to the new ZEUS data.

II. Moving from HERA to the LHC: ultraperipheral **pp, pA and AA** collisions: R. Fiore, L. J., V. Libov, and M. Machado, arXiv, 2014, to be publ. in: Teor. and Math. Physics.

Predecessors: Joakim Nystrand, A. Szczerba et al, L. Motyka, G. Watt; Brazilian group...). Contrary to ep, no Q^2 or t dependence here.

Thank you !

Two ways of relating SF to hadronic total cross sections:

- 1) Through sum rules in Q^2 (see also: Jan Kwieciński, Phys. Letters, **120B** (1983) 418; L.L. Jenkovszky and B.V. Struminsky, Yad. Fizika, 38 (1983) 1568);
- 2) Using the additive quark model, the number of active quark-partons being determined by SFs (preliminary results presented by F. Celiberto, L. J. and V. Myronenko at the *Quarks2016 (St. Petersburg)* and *Low-x (Gyöngyös)* conferences).

Definitions, notation:

$$F_2(x, Q^2) = \frac{Q^2(1-x)}{4\pi\alpha_{em}(1+4m^2x^2/Q^2)} \sigma_t \gamma^* p, \quad s = W^2 = Q^2(1-x)/x + m^2.$$

We use the norm in which $\sigma_t \gamma^* p = 2A(s, Q^2)$.

VMD:

$$\int_{Q_1^2}^{Q_2^2} F_2(x, Q^2) dQ^2 = \frac{1}{4\pi} \sum_V \frac{\sigma_{VM}}{\gamma_V^2} (m_V^2)^2,$$

$$\Gamma(V \rightarrow e^+ e^-) = \frac{\alpha m_V}{2\gamma_V^2}.$$

Sum rule:

$$\int_{Q_1^2}^{Q_2^2} F_2(x, Q^2) dQ^2 = \int_{Q_2^2}^{Q_{as}^2} F_2^{as}(x, Q^2) dQ^2.$$

By setting $\sigma_{\rho N} \approx \sigma_{\omega N} \approx \sigma_{\pi N}$, one gets (optionally)

$$\sigma_{\pi N}^t = 24[1 + 0.1 \ln\left(\frac{s}{Q_0^2}\right)] mbn$$

.

Kinematics:

$$W^2 = s = (p + q)^2 = Q^2(1 - x)/x + m^2 \approx Q^2/s.$$

$$\int_0^1 dx [F_2^V(x, Q^2) + F_2^S(x, Q^2)],$$

with typically logarithmic scaling violation parametrizations known at those times, e.g.

$$F_2^S(x) = F_2(x) \left[1 - \epsilon \ln\left(\frac{Q^2}{Q_0^2}\right) \ln\left(\frac{x}{x_0}\right) \right],$$

or

$$F_2^S(x) = F_2(x) \left[1 + \epsilon \left(\frac{Q^2}{Q_0^2}\right)^{f(x)} \right], \quad f(x) = x_0 - x,$$

with the following values of the parameters: $a = 0.25$, $b = 1.35$, $c = 0.2$, $\epsilon = 0.05$ and $q_0^2 = 3 \text{ GeV}^2$,

Sum rules in Q^2 : Jan Kwieciński, Phys. Letters, **120B** (1983) 418;
L.L. Jenkovszky and B.V. Struminsky, Yad. Fizika, 38 (1983) 1568.

VMD:

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.

Direct, s-channel point of view: additive quark model, single and multiple scattering of partons (Glauber).

Additive quark model relations (Levin-Frankfurt, 70-ies):

$$\sigma_{pp}^t = \sigma_0 n_A n_B. \quad (1)$$

While this simple rule is confirmed *e.g.* by the ratio 2/3 of meson-baryon to baryon-baryon scattering in a fairly wide range of intermediate energies, *e.g.* $\sigma_{\pi p}^t / \sigma_{pp}^t \approx 0.67$ at $\sqrt{s} \approx 10$ GeV, it is progressively violated as the energy increases. It was suggested in Ref. [2, 3] that while the constant components of the cross sections, obeying the above quark rule are determined by constituents quarks their rise comes from the increasing number of sea quarks.

In Refs. [2, 3] the rise of hadronic total cross sections was related the proliferation of sea quarks and gluons in colliding hadron, hence Eq. (1) modifies as

$$\sigma_{pp}^t = (n_v + n_s)^2, \quad (2)$$

where n_v and n_s is the number of valence and sea quarks and gluons in the proton. The number of sea quarks and gluons was related to the logarithmic scaling violation, resulting in

The fraction of momenta carried by quarks can be calculated from the integrals

$$\int_0^1 dx F_2^v(s, Q^2) = 0.423,$$

$$\int_0^1 dx F_2^s(s, Q^2) = 0.01 + 0.001 \ln(s/Q_0^2).$$

Consequently,

$$\sigma_{pp}^{tot} \approx \sigma_0 n_{v_1} n_{v_2} (1 + 0.016 \ln(s/Q_0^2)).$$

We remind that $x \sim Q^2/s$.

P. Desgrolard, L. Jenkovszky and F. Paccanoni: EPJ C 7 (1999) 263; hep-ph/9803286

Interpolating between “soft” (VMD, Pomeron, $\Delta \sim 0.1$) and hard (DGLAP, $\Delta \sim 0.4$) regimes:

1) DGLAP evolution, x fixed, $Q^2 \rightarrow \infty$; 2) Gauge inv.: x fixed, $Q^2 \rightarrow 0$; 3) $x \rightarrow 0$, Q fixed (Regge)

$$F_2^{(S,0)}(x, Q^2) = A \left(\frac{Q^2}{Q^2 + a} \right)^{1 + \tilde{\Delta}(Q^2)} e^{\Delta(x, Q^2)},$$

with “effective power”

$$\tilde{\Delta}(Q^2) = \epsilon + \gamma_1 \ln \left(1 + \gamma_2 \ln \left[1 + \frac{Q^2}{Q_0^2} \right] \right),$$

$$\Delta(x, Q^2) = \left(\tilde{\Delta}(Q^2) \ln \frac{x_0}{x} \right)^{f(Q^2)},$$

$$f(Q^2) = \frac{1}{2} \left(1 + e^{-Q^2/Q_1^2} \right).$$

At small and moderate values of Q^2 (to be specified from the fits, see below), the exponent $\tilde{\Delta}(Q^2)$ (3.2) may be interpreted as a Q^2 -dependent "effective Pomeron intercept".

The function $f(Q^2)$ has been introduced in order to provide for the transition from the Regge behavior, where $f(Q^2) = 1$, to the asymptotic solution of the GLAP evolution equation, where $f(Q^2) = 1/2$.

Large Q^2 , fixed x :

$$F_2^{(S,0)}(x, Q^2 \rightarrow \infty) \rightarrow A \exp^{\sqrt{\gamma_1 \ln \ln \frac{Q^2}{Q_0^2} \ln \frac{x_0}{x}}} ,$$

h is the asymptotic solution of the GLAP evolution equation (see Sec. 1).

Low Q^2 , fixed x :

$$F_2^{(S,0)}(x, Q^2 \rightarrow 0) \rightarrow A e^{\Delta(x, Q^2 \rightarrow 0)} \left(\frac{Q^2}{a}\right)^{1+\tilde{\Delta}(Q^2 \rightarrow 0)}$$

$$\tilde{\Delta}(Q^2 \rightarrow 0) \rightarrow \epsilon + \gamma_1 \gamma_2 \left(\frac{Q^2}{Q_0^2}\right) \rightarrow \epsilon,$$

$$f(Q^2 \rightarrow 0) \rightarrow 1,$$

ice

$$F_2^{(S,0)}(x, Q^2 \rightarrow 0) \rightarrow A \left(\frac{x_0}{x}\right)^\epsilon \left(\frac{Q^2}{a}\right)^{1+\epsilon} \propto (Q^2)^{1+\epsilon} \rightarrow 0 ,$$

quired by gauge invariance.

ired by gauge invariance.

w x , fixed Q^2 :

$$F_2^{(S,0)}(x \rightarrow 0, Q^2) = A \left(\frac{Q^2}{Q^2 + a} \right)^{1+\tilde{\Delta}(Q^2)} e^{\Delta(x \rightarrow 0, Q^2)}.$$

$$f(Q^2) \sim 1 ,$$

en $Q^2 \ll Q_1^2$, we get the standard (Pomeron-dominated) Regge behavior (with a Q^2 dependence in the intercept)

$$F_2^{(S,0)}(x \rightarrow 0, Q^2) \rightarrow A \left(\frac{Q^2}{Q^2 + a} \right)^{1+\tilde{\Delta}(Q^2)} \left(\frac{x_0}{x} \right)^{\tilde{\Delta}(Q^2)} \propto x^{-\tilde{\Delta}(Q^2)}.$$

in this approximation, the total cross-section for (γ, p) scattering as a function of the center of r

$$\sigma_{\gamma,p}^{tot,(0)}(W) = 4\pi^2 \alpha \left[\frac{F_2^{(S,0)}(x, Q^2)}{Q^2} \right]_{Q^2 \rightarrow 0} = 4\pi^2 \alpha A a^{-1-\epsilon} x_0^\epsilon W^{2\epsilon}.$$

[T, we multiply the singlet part of the above structure function $F_2^{(S,0)}$ (c factor to get

$$F_2^{(S)}(x, Q^2) = F_2^{(S,0)}(x, Q^2) (1 - x)^{n(Q^2)},$$

$$n(Q^2) = \frac{3}{2} \left(1 + \frac{Q^2}{Q^2 + c} \right),$$

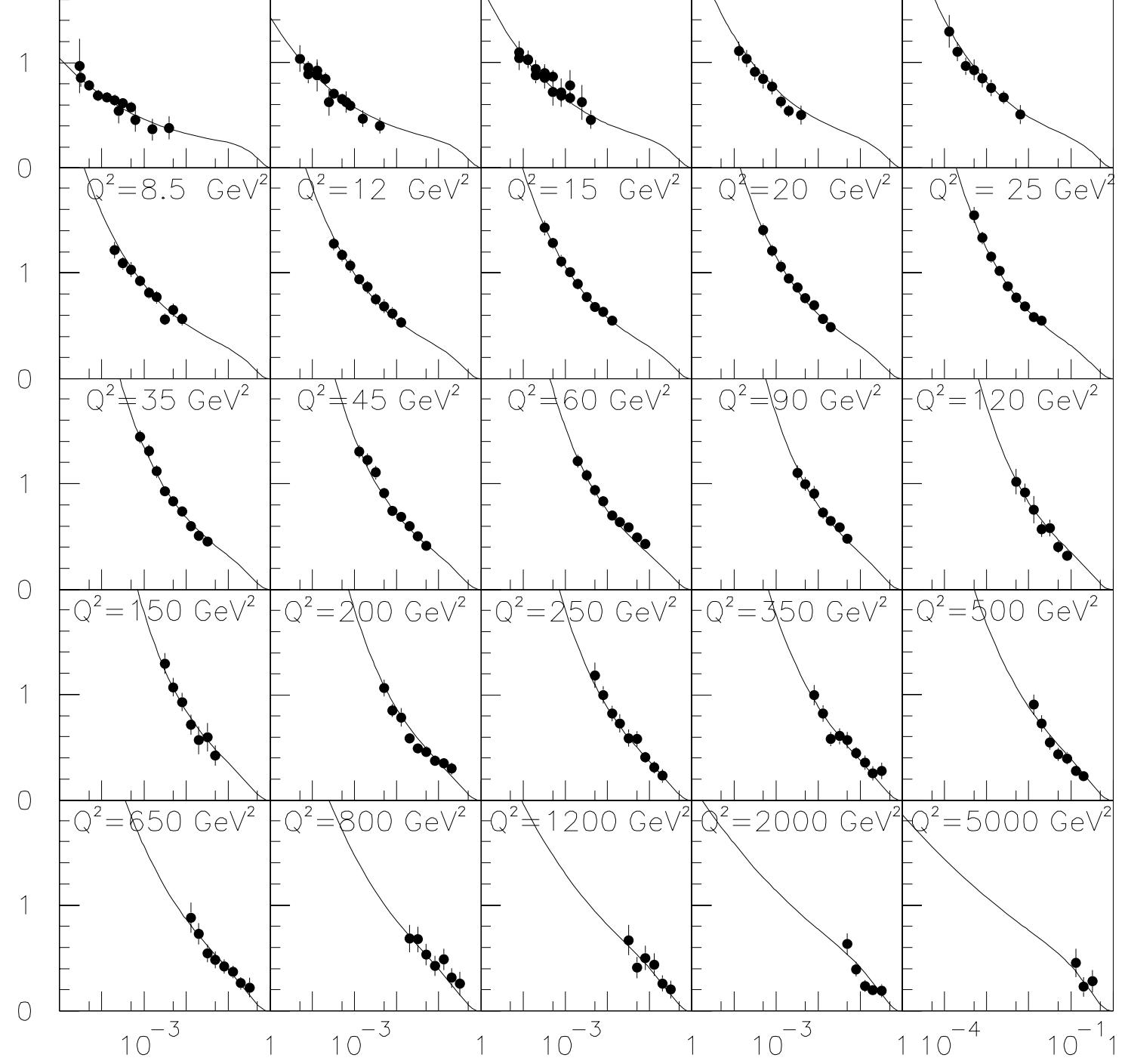
GeV^2 [4a].

The nonsinglet (NS) part of the structure function, also borrowed from CKM

$$F_2^{(NS)}(x, Q^2) = B (1 - x)^{n(Q^2)} x^{1-\alpha_r} \left(\frac{Q^2}{Q^2 + b} \right)^{\alpha_r}.$$

ers that appear with this addendum are c, B, b and α_r . The final and coiunction thus becomes

$$F_2(x, Q^2) \equiv F_2^{(S)}(x, Q^2) + F_2^{(NS)}(x, Q^2).$$

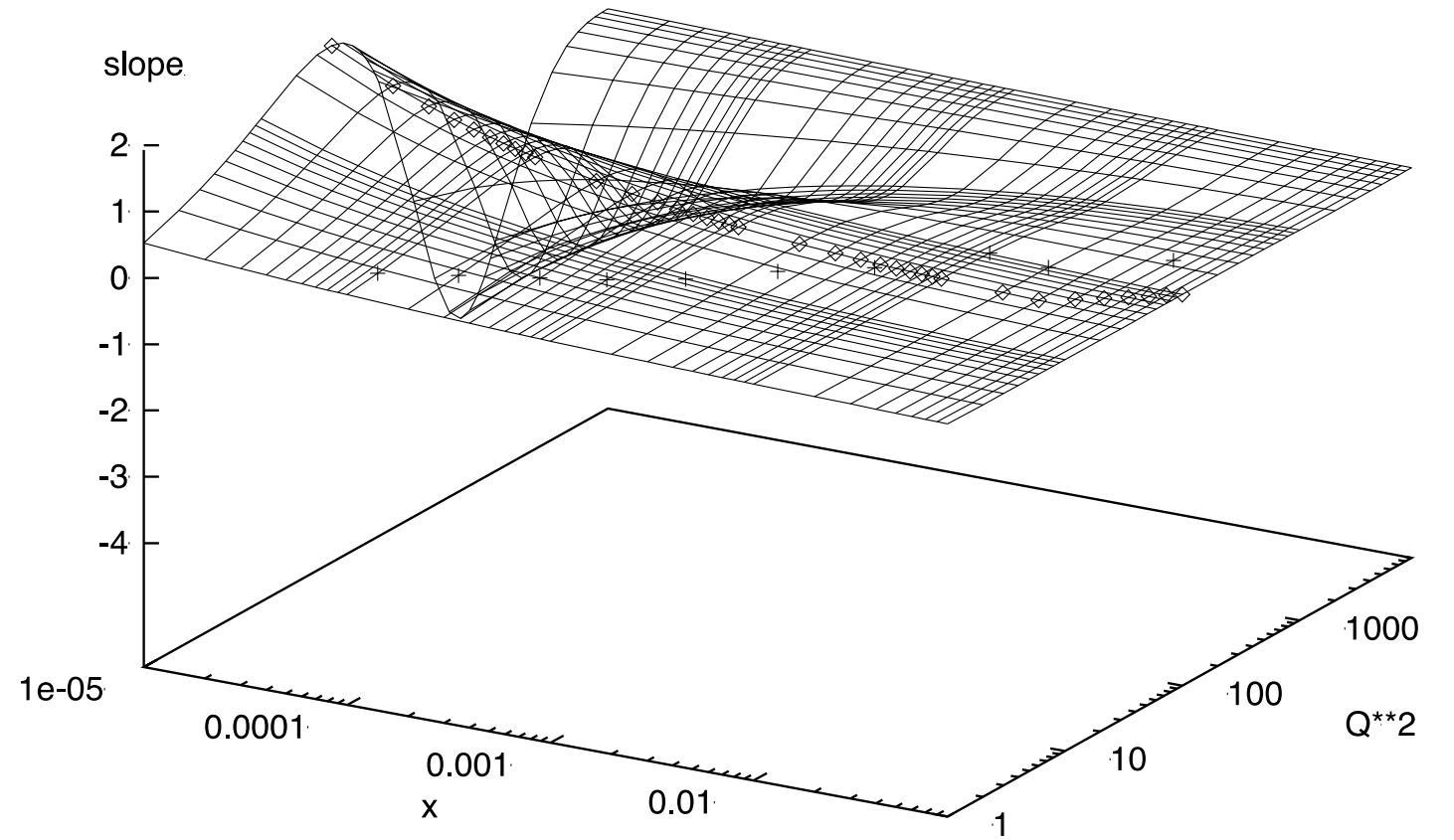


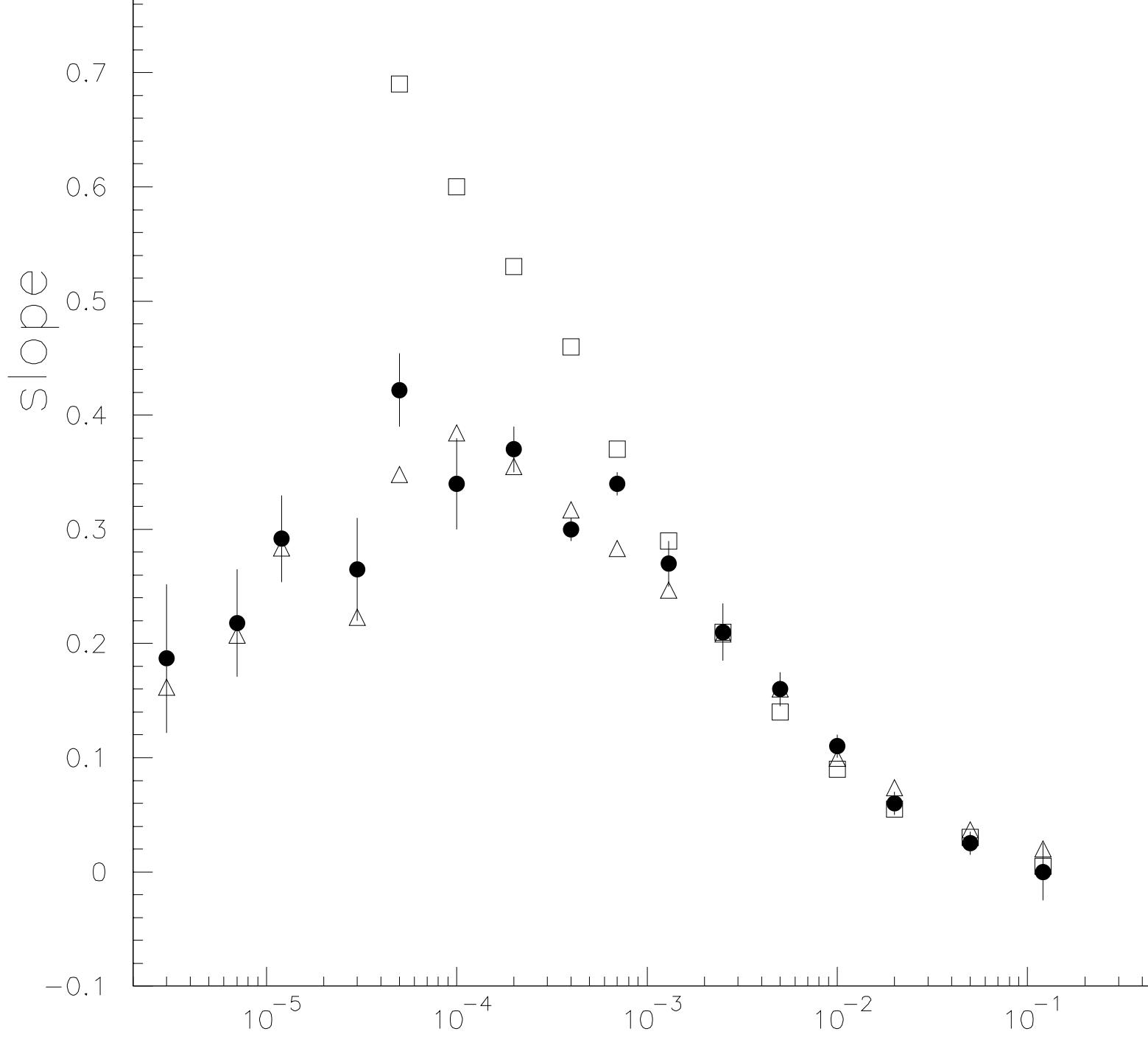
“Saturation” and slopes

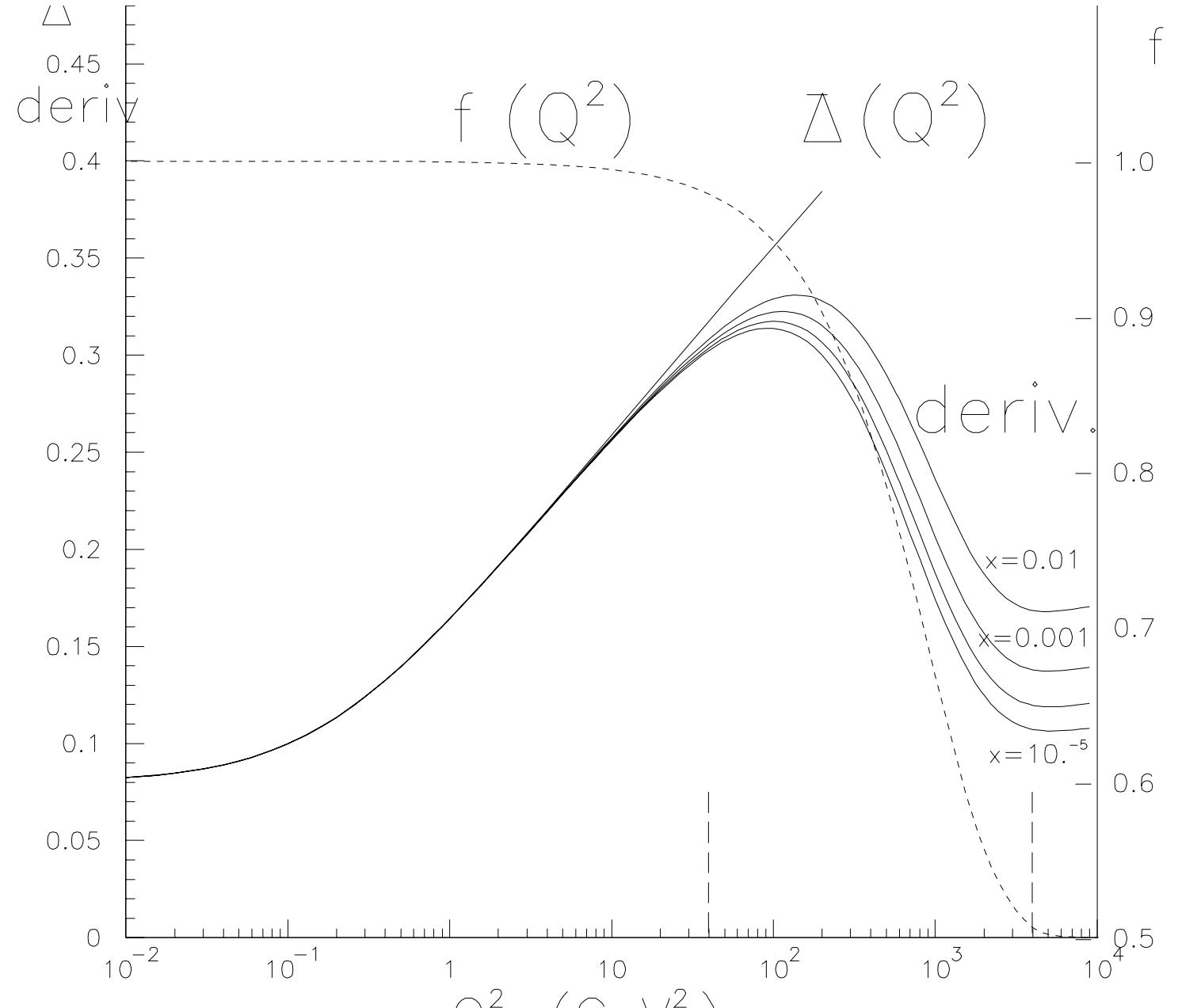
Logarithmic derivatives (slopes) B are sensitive measures of the changing trends/regimes (*unitarity and saturation*):

$$B_Q(x, Q^2) = \frac{\partial F_2(x, Q^2)}{\partial(\ln Q^2)},$$

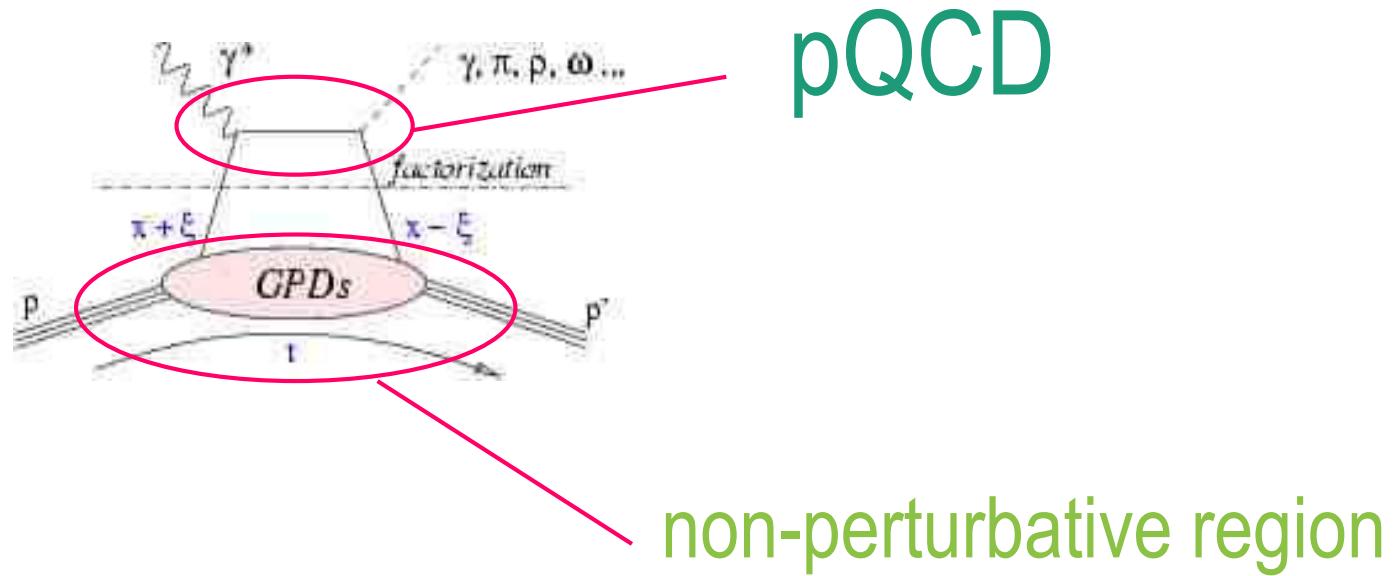
$$B_x(x, Q^2) = \frac{\partial F_2(x, Q^2)}{\partial(\ln 1/x)},$$

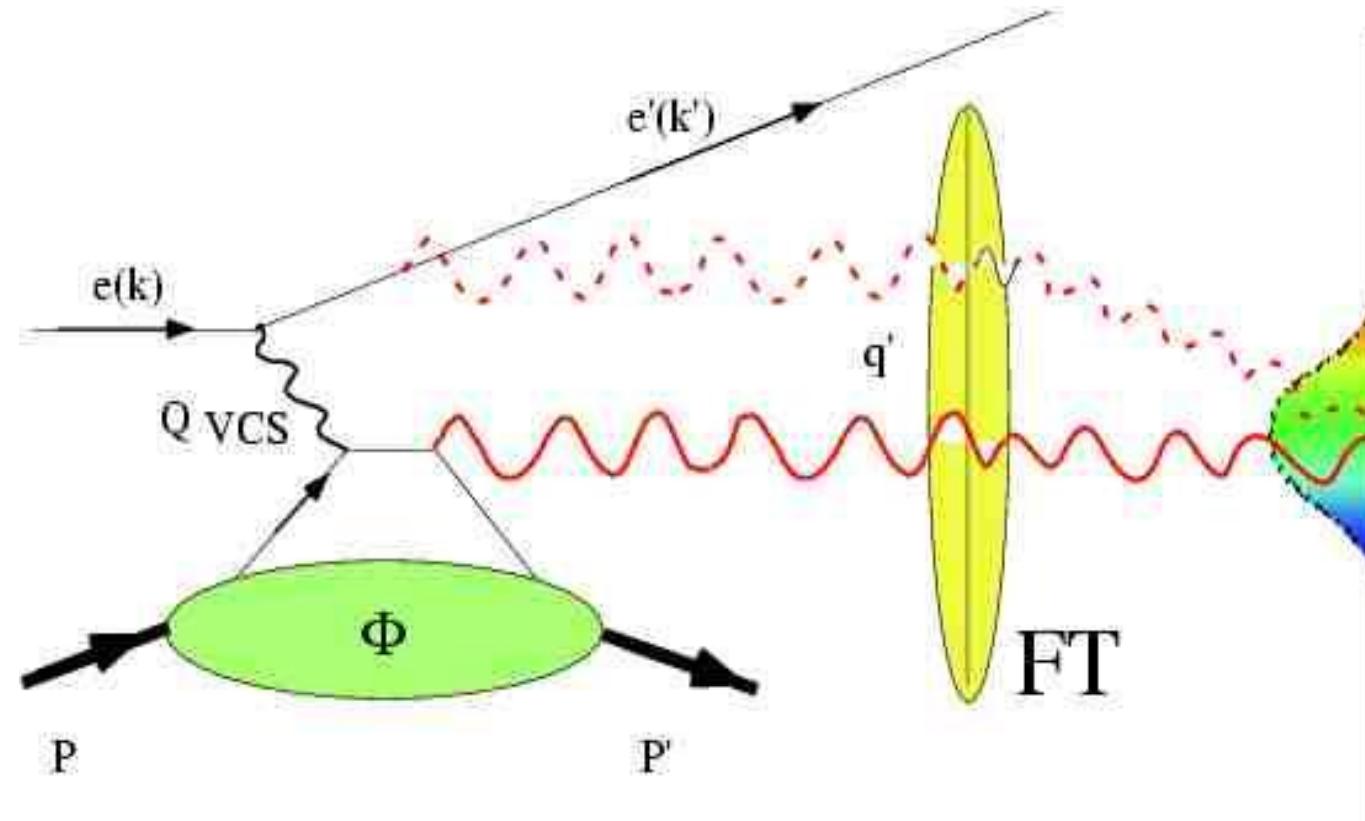






QCD-factorized form of a DVCS scattering amplitude





GPDs cannot be measured directly,
instead they appear as convolution integrals,
difficult to be inverted !

$$A(\xi, \eta, t) \sim \int_{-1}^1 dx \frac{GPD(x, \eta, t)}{x - \xi + i\epsilon}$$

We need clues from
phenomenological models -
Regge behaviour, t -
factorization etc.



$$\sigma_{tot} \sim \Im m A,$$

$$\frac{d\sigma}{dt} \sim |A|^2$$

“Handbag”