

S-MATRIX APPROACH TO HADRON GAS

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WE-HERAEUS PHYSICS SCHOOL
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IN COLLABORATION WITH

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SUMMARY

- change in density of state / time delay

$$2 \frac{d\delta}{dE}$$

- S-matrix approach to thermodynamics

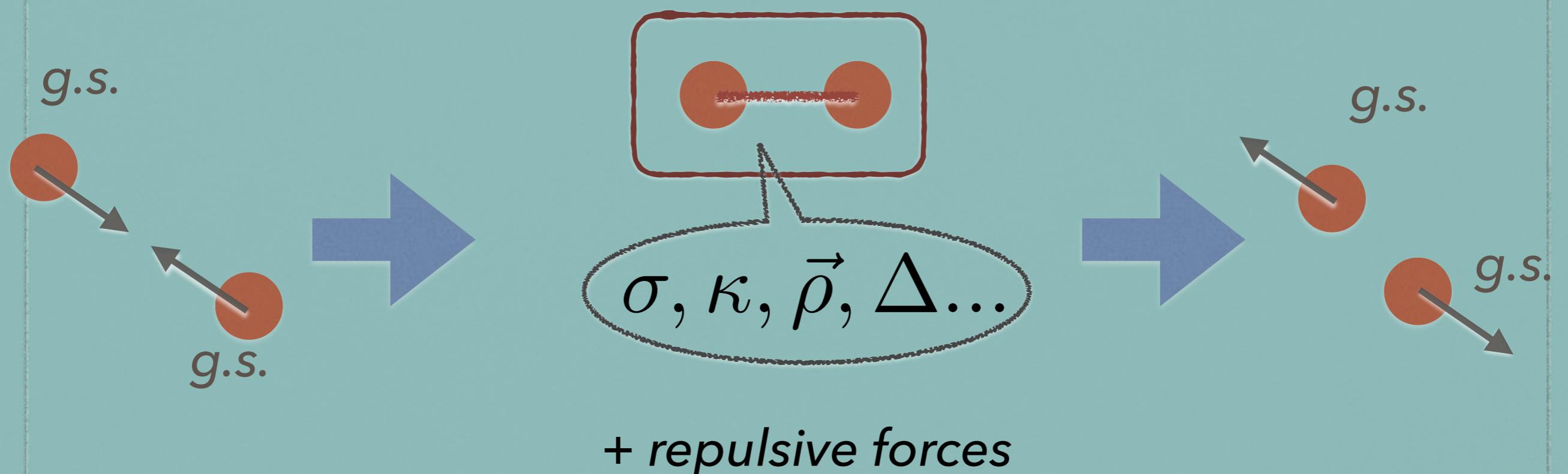
Broad resonances

Repulsive channels

S-MATRIX APPROACH

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187 (1969) 345.

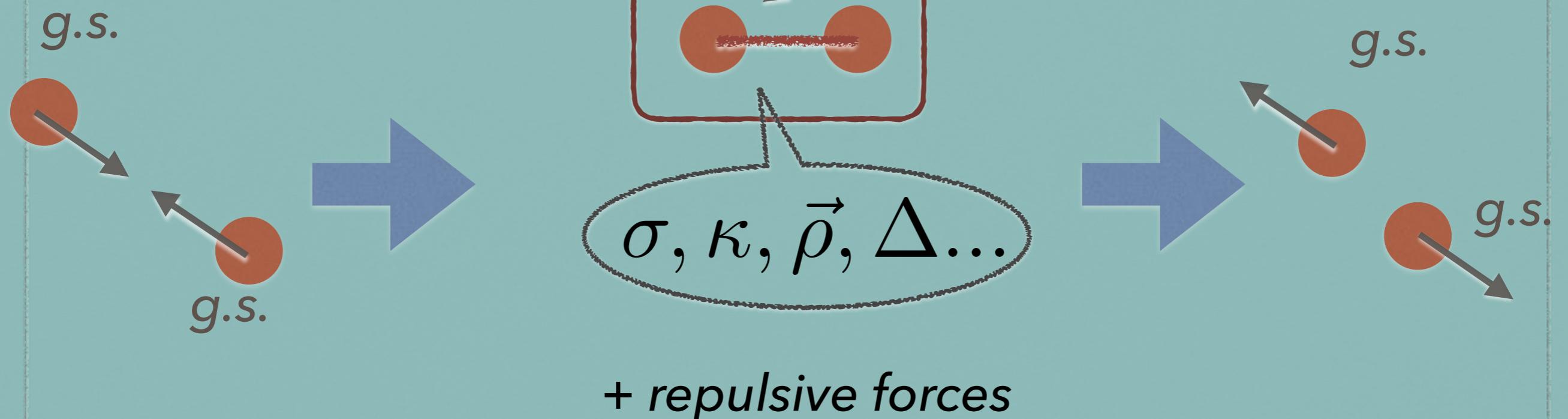
S-MATRIX APPROACH



consistent treatment of both
attractive and repulsive forces

S-MATRIX APPROACH

$$\rho_E \sim 2 \frac{d\delta}{dE}$$



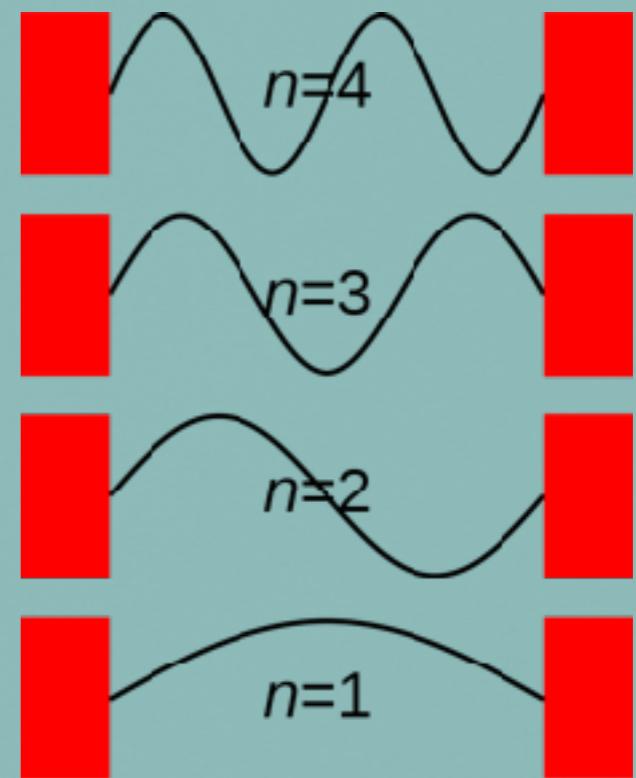
consistent treatment of both
attractive and repulsive forces

PHASE SHIFT AND DENSITY OF STATES

particle in a box

$$\psi \sim \sin(k^{(0)}x)$$

$$k^{(0)} = \frac{n\pi}{L}$$



PHASE SHIFT AND DENSITY OF STATES

particle in a box

$$\psi \sim \sin(k^{(0)}x) \quad k^{(0)} = \frac{n\pi}{L}$$

in the presence of a scattering potential

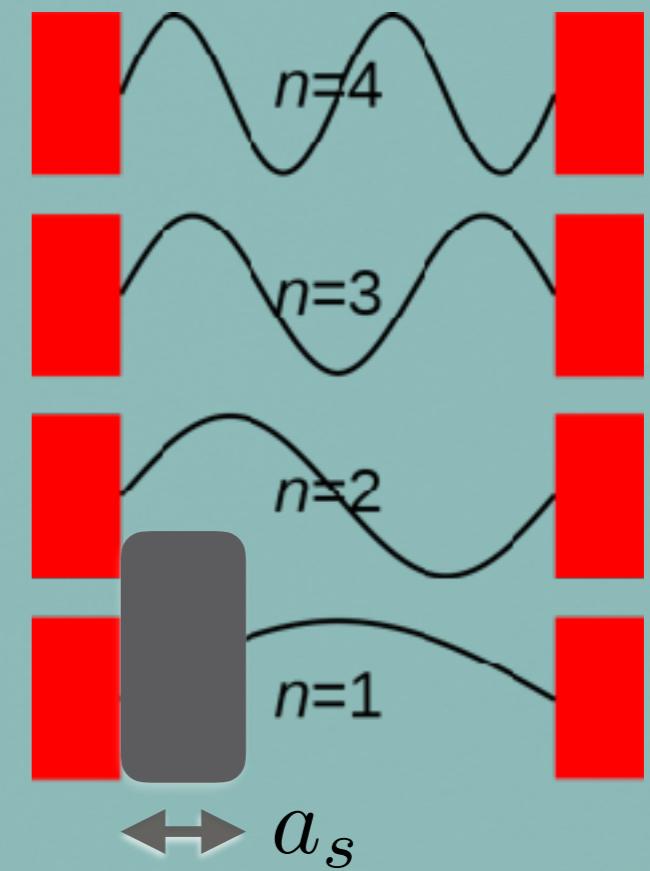
$$\psi \sim \sin(kx + \delta(k))$$

density of states

$$kL + \delta(k) = n\pi$$



$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi} \frac{d\delta}{dk}$$

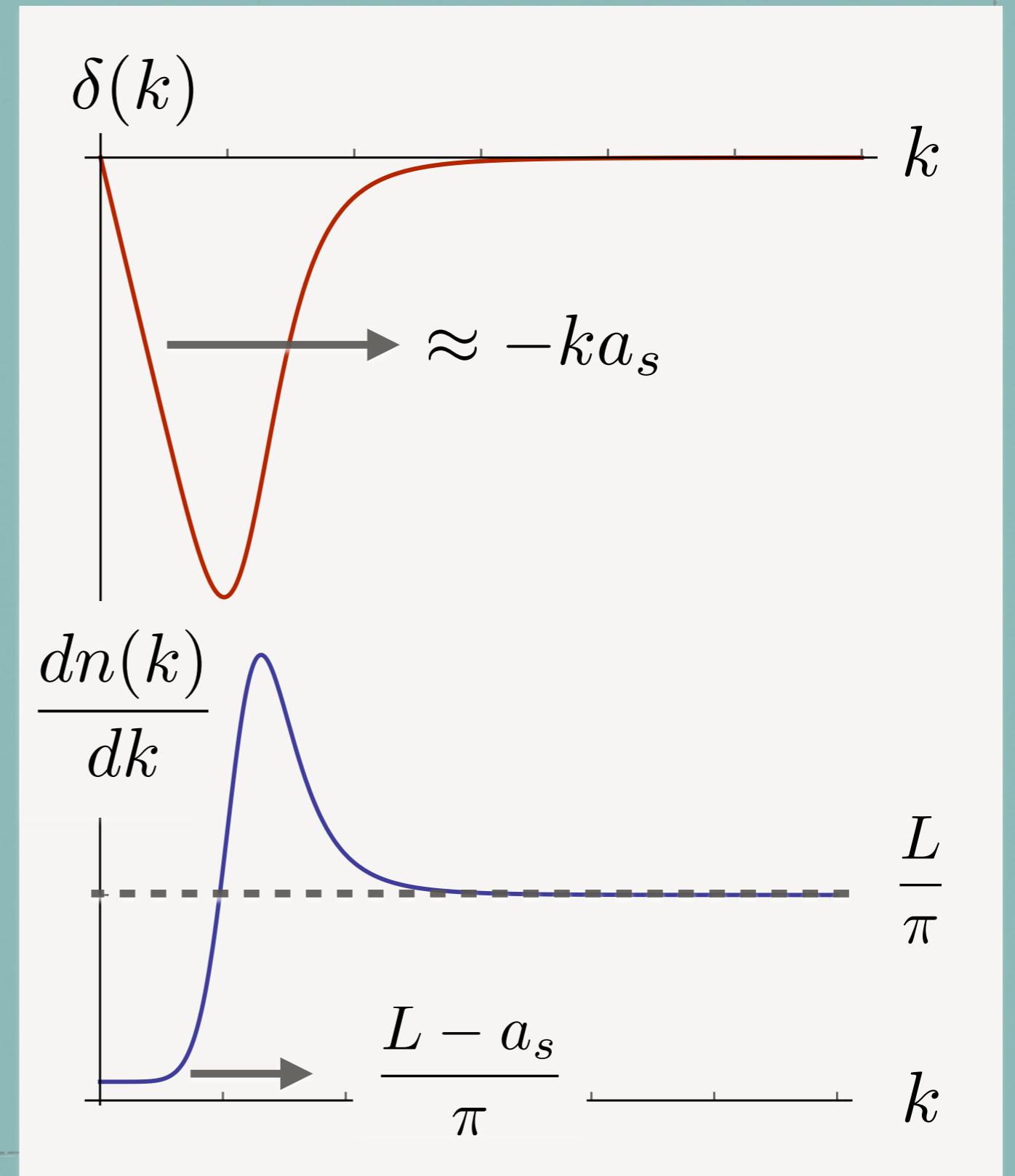


PHASE SHIFT AND DENSITY OF STATES

$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi} \frac{d\delta}{dk}$$

*change in d.o.s.
due to int.*

Effect of repulsive
interaction:
pushing states from low k
to high k



phase shift and d.o.s. (schematics)

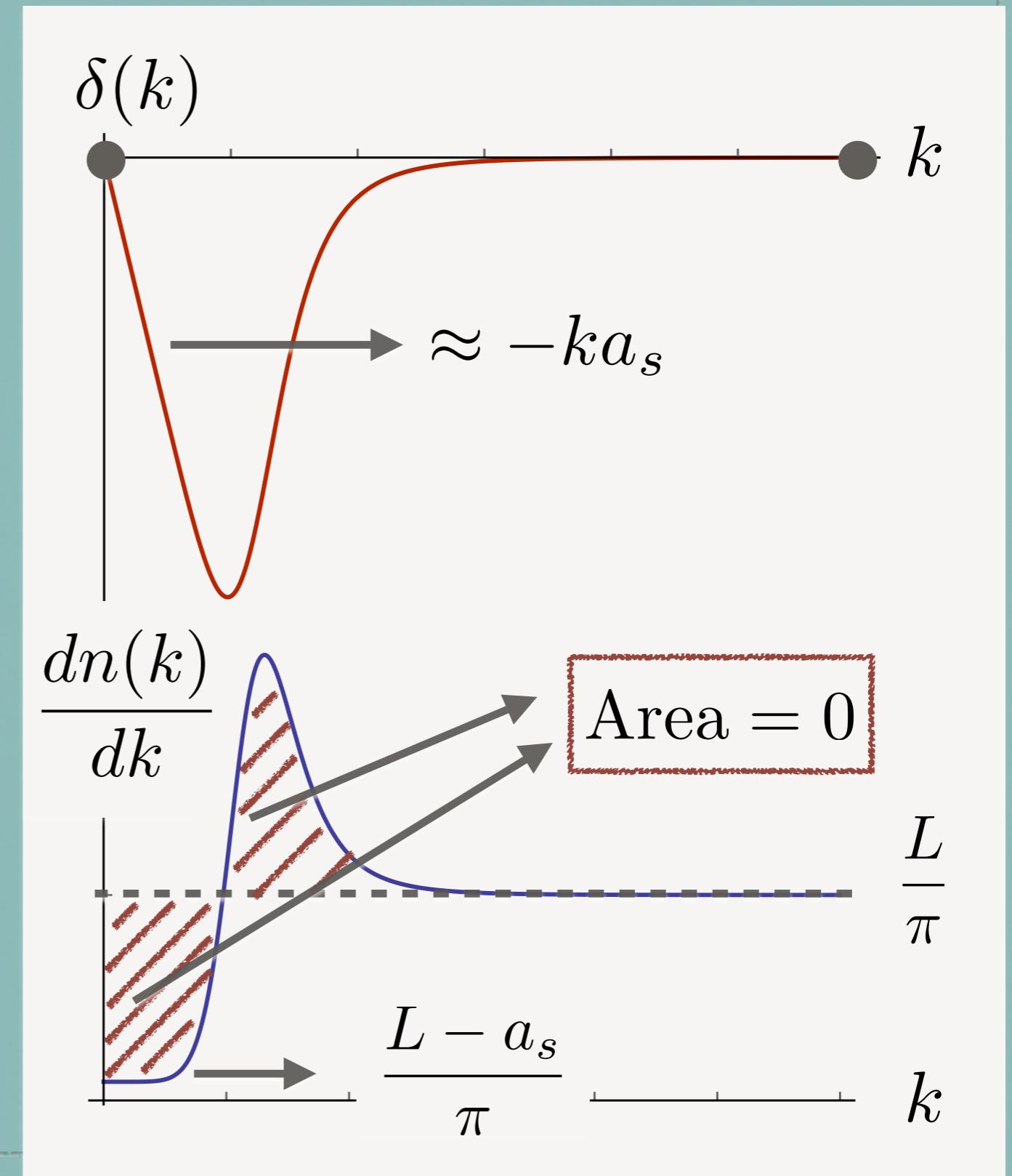
PHASE SHIFT AND DENSITY OF STATES

$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi} \frac{d\delta}{dk}$$

sum rule
(Levinson's theorem)

$$\int_0^\infty dk \frac{1}{\pi} \delta' = \frac{\delta(\infty) - \delta(0)}{\pi}$$

n_{int}



phase shift and d.o.s. (schematics)

S-MATRIX FORMULATION OF THERMODYNAMICS

$$\Delta \ln Z = \int dE e^{-\beta E} \frac{1}{4\pi i} \text{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187 (1969) 345.

A SIMPLE TRICK

$$\frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

$$= \frac{1}{2\pi} \times 2 \frac{\partial}{\partial E} \frac{1}{2} \operatorname{Im} \operatorname{tr} \{ \ln S_E \}$$

$$\Delta \ln Z = \int dE e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \operatorname{tr} (\delta_E).$$

E. Beth and G. Uhlenbeck,
Physica (Amsterdam) 4, 915 (1937).

A SIMPLE TRICK

$$\frac{1}{4\pi i} \text{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c = \frac{1}{2\pi} \times 2 \frac{\partial}{\partial E} \frac{1}{2} \text{Im} \text{tr} \{ \ln S_E \}$$

$S_E = e^{2i\delta_E}$

$$\mathcal{Q}(E) \longrightarrow$$

*Generalised
phase shift*

$$B = 2 \frac{\partial}{\partial E} \mathcal{Q}(E) \longrightarrow$$

*Generalised
spectral function*

FORMULATION

given the exact phase shift δ_l

from theory

or

from experiment



thermodynamics

$$B_l = 2 \frac{d}{dq} \delta_l$$

eff. spectral function

$$P = P^{(0)} + \Delta P^{B.U.}$$

free gas + interaction

FORMULATION

dynamical

$$\Delta P^{\text{B.U.}} = (2l + 1) \int \frac{dq}{2\pi} B_l(q)$$

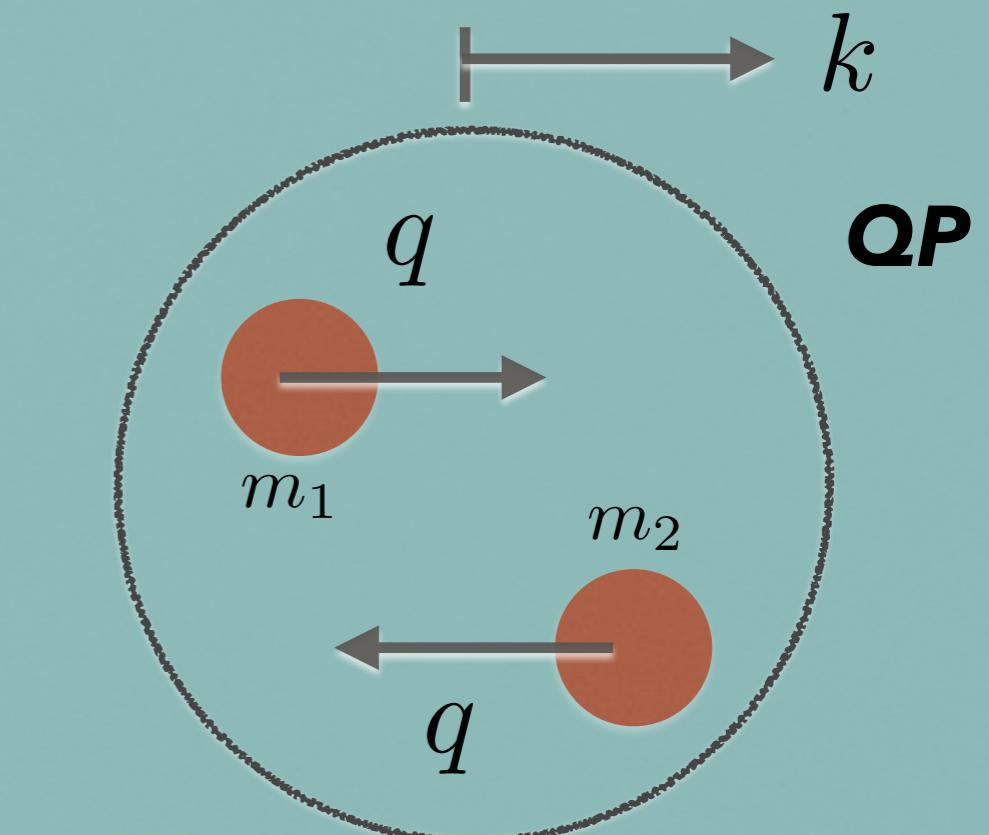
statistical (thermal weight)

$$\int \frac{d^3 k}{(2\pi)^3} T \ln(1 + e^{-\beta E(k, q, m_i)})$$

$$E = \sqrt{k^2 + M(q)^2}$$

$$M(q) = \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2}$$

$$B_l = 2 \frac{d}{dq} \delta_l$$



$$M(q) = \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2}$$

WHAT'S IN A NAME? THAT WHICH WE CALL A RESONANCES?

- A resonance is MORE than a MASS and a WIDTH

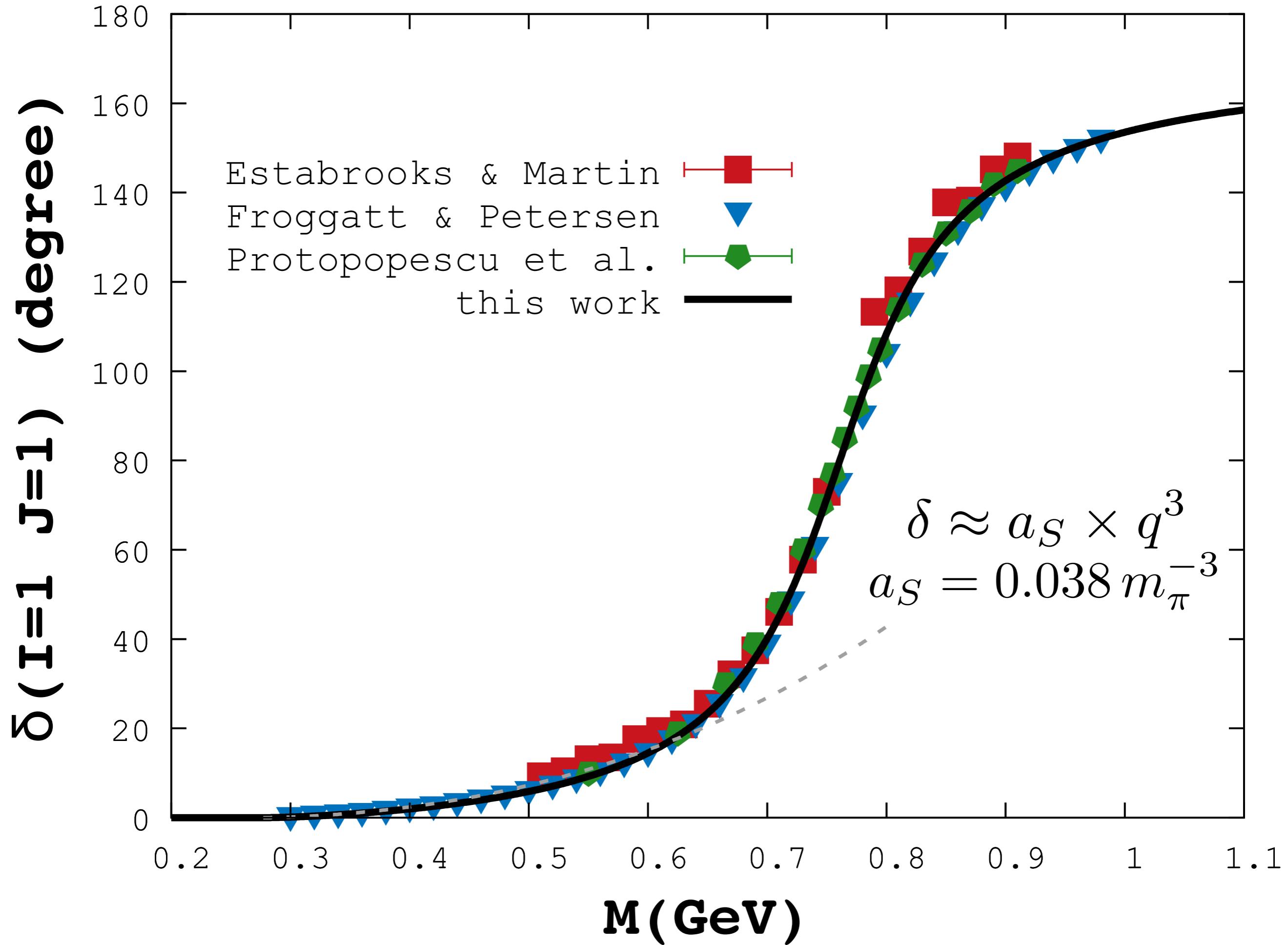
$\rho(770)$ [^h]

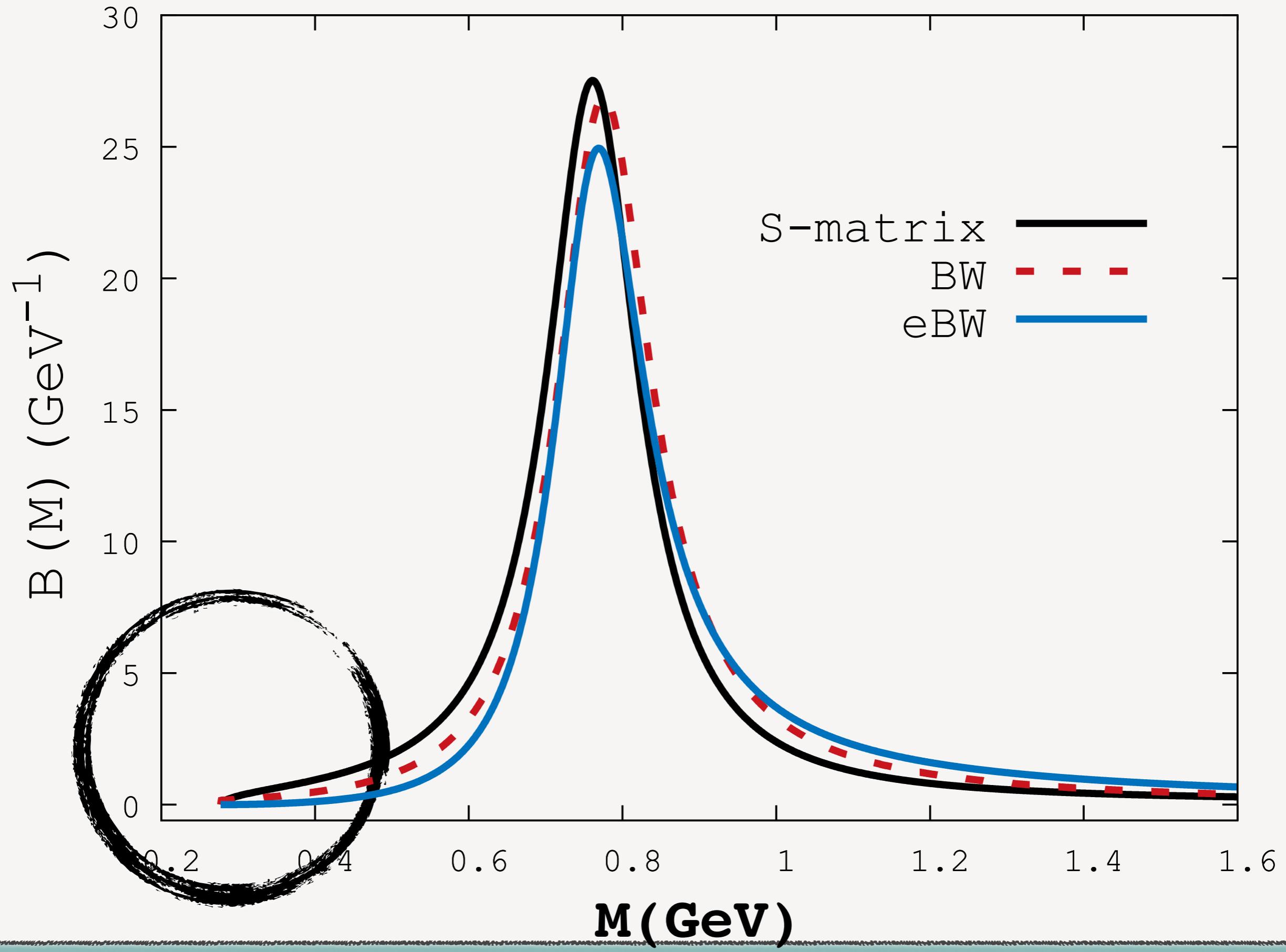
$I^G(J^{PC}) = 1^+(1^{--})$

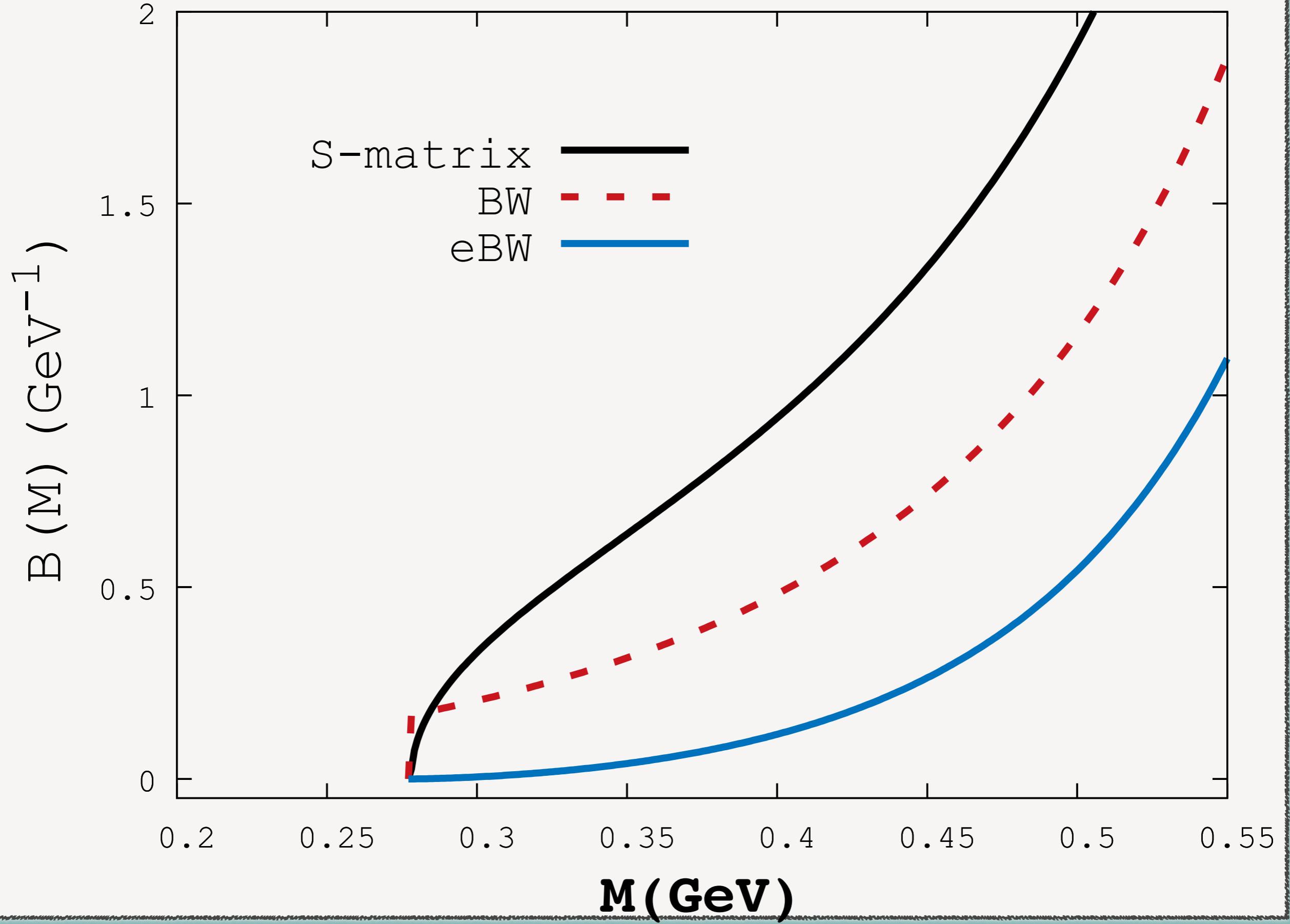
Mass $m = 775.26 \pm 0.25$ MeV

Full width $\Gamma = 149.1 \pm 0.8$ MeV

$\Gamma_{ee} = 7.04 \pm 0.06$ keV







BETH-UHLENBECK APPROXIMATION

$$\delta = -\text{Im} \text{Tr} \ln G_\rho^{-1}$$

physical interpretation:

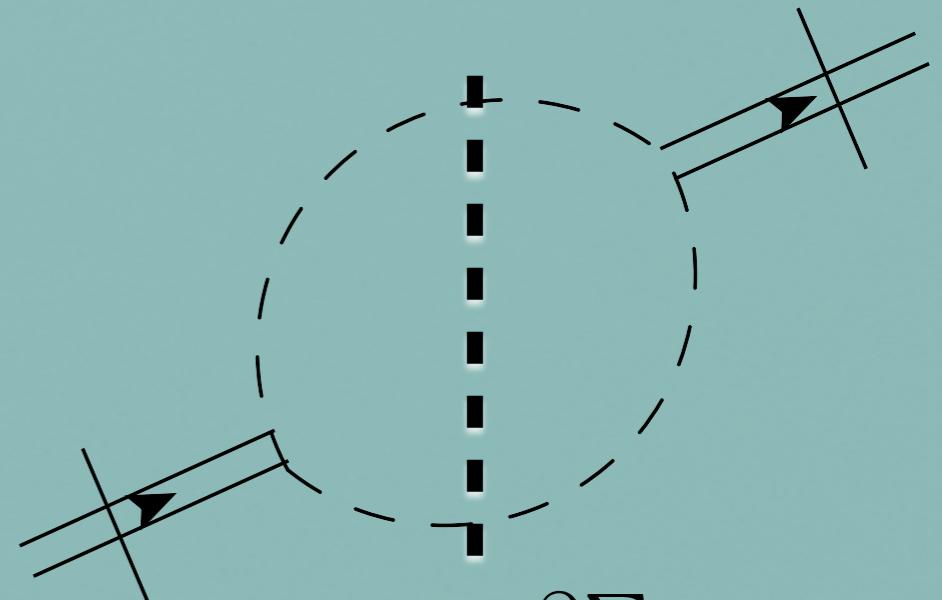
$$B = 2 \frac{\partial}{\partial E} \delta$$

$$= -2 \text{Im} \frac{\partial}{\partial E} \ln G_\rho^{-1}$$

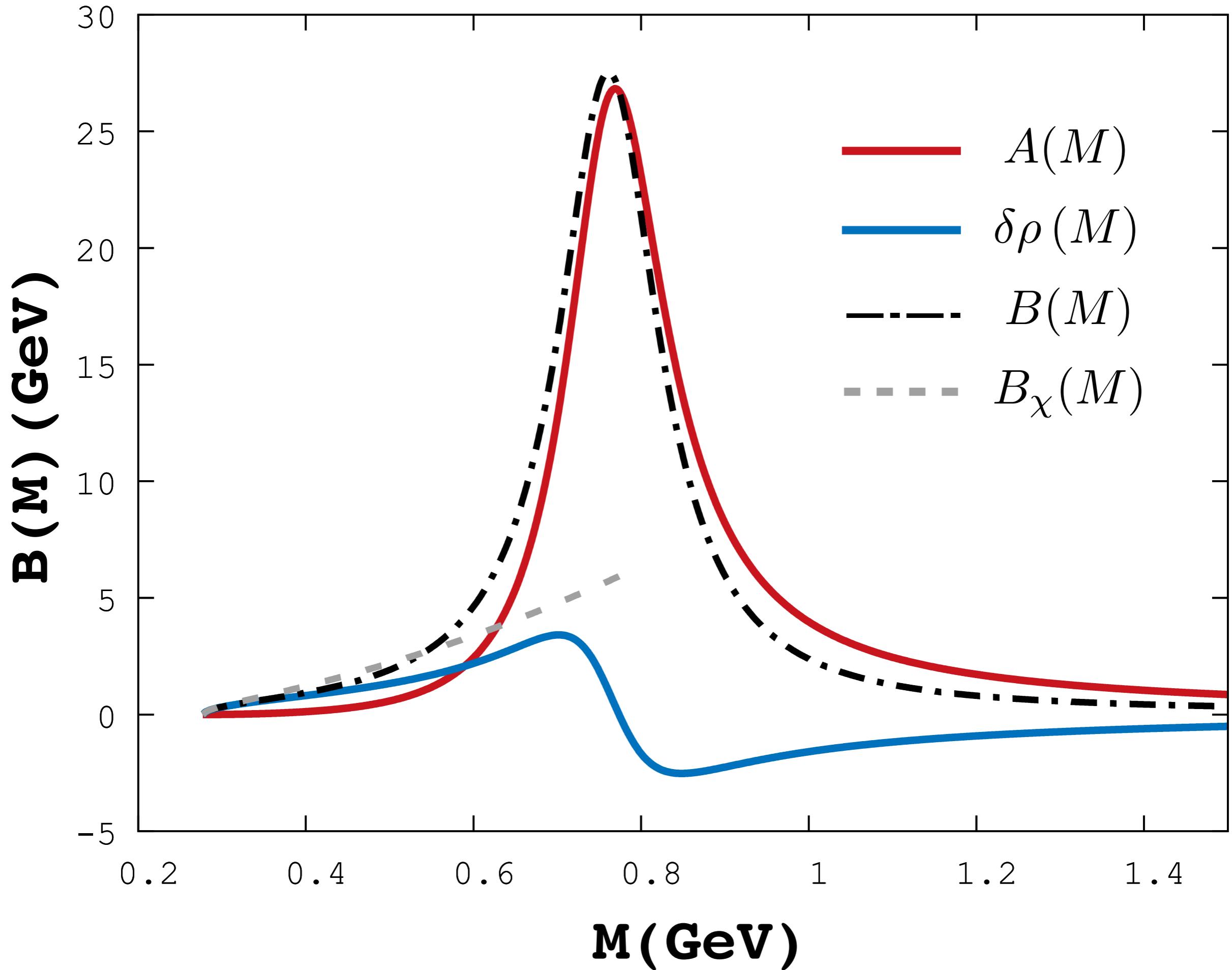
$$= -2 \text{Im}[G_\rho](2E) + 2 \text{Im}\left[\frac{\partial \Sigma_\rho}{\partial E} G_\rho\right]$$

$$\Rightarrow \rho_\rho(E) + \delta\rho_\rho(E)$$

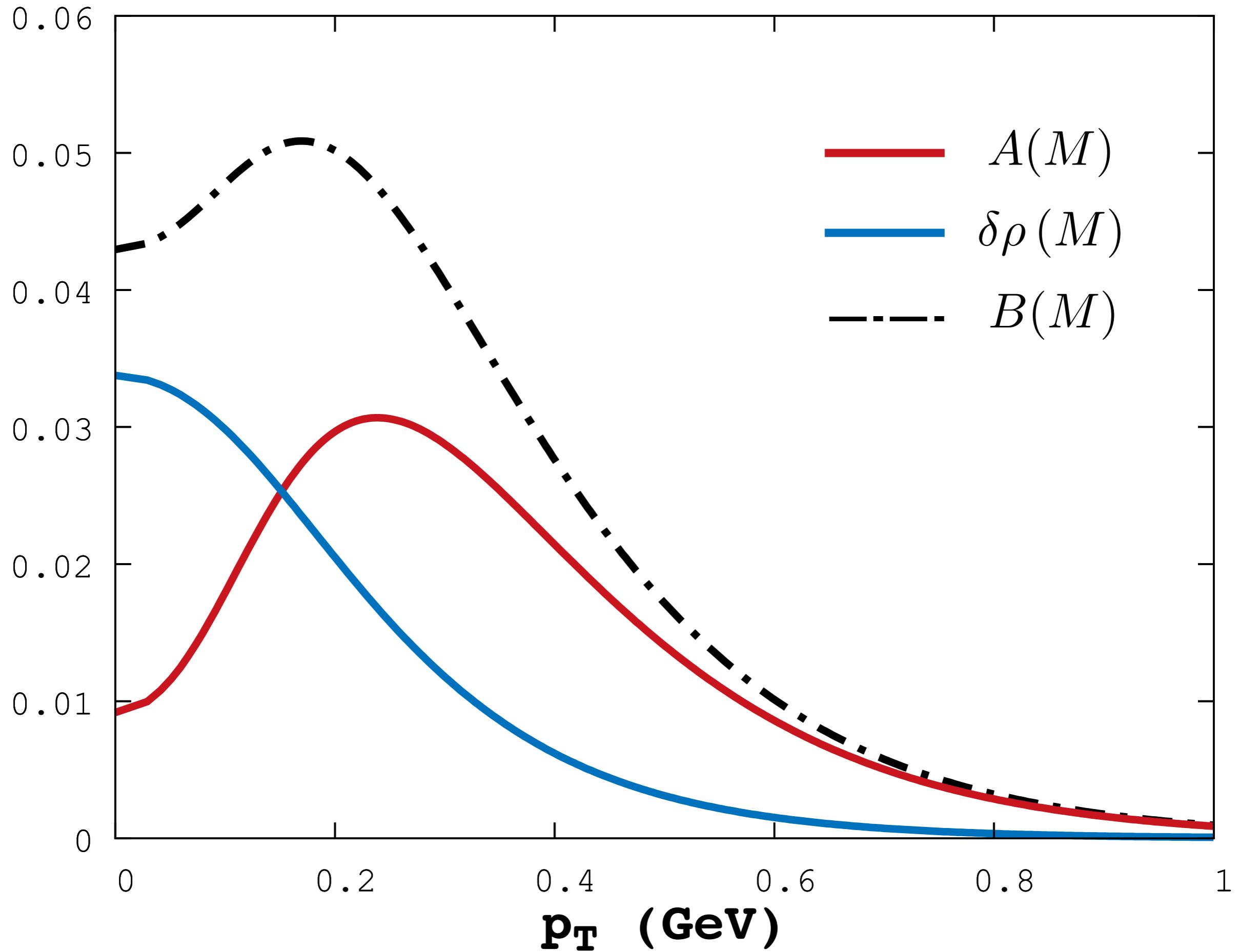
contribution from correlated pi pi pair

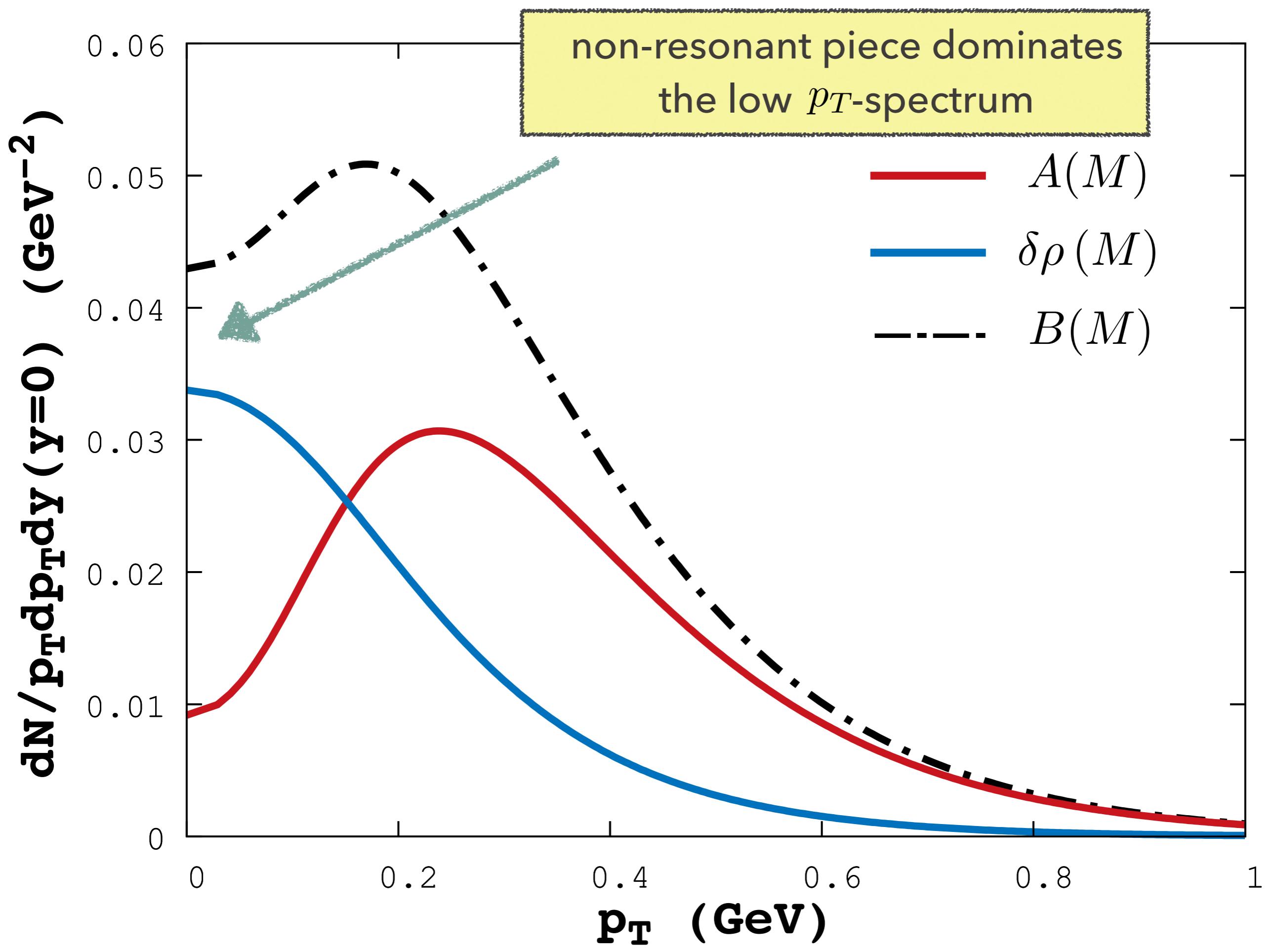


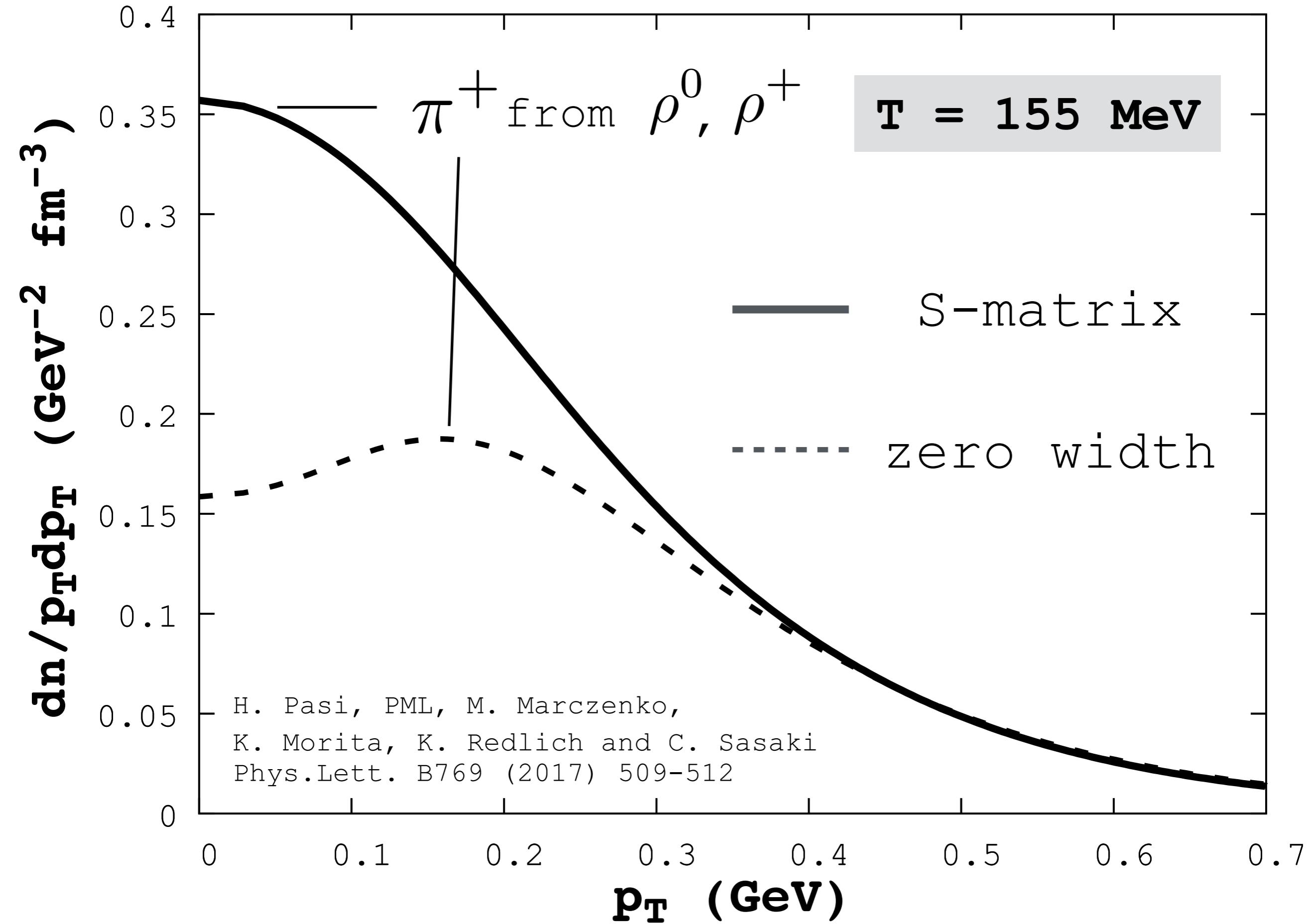
$$\frac{\partial \Sigma_\rho}{\partial E}$$

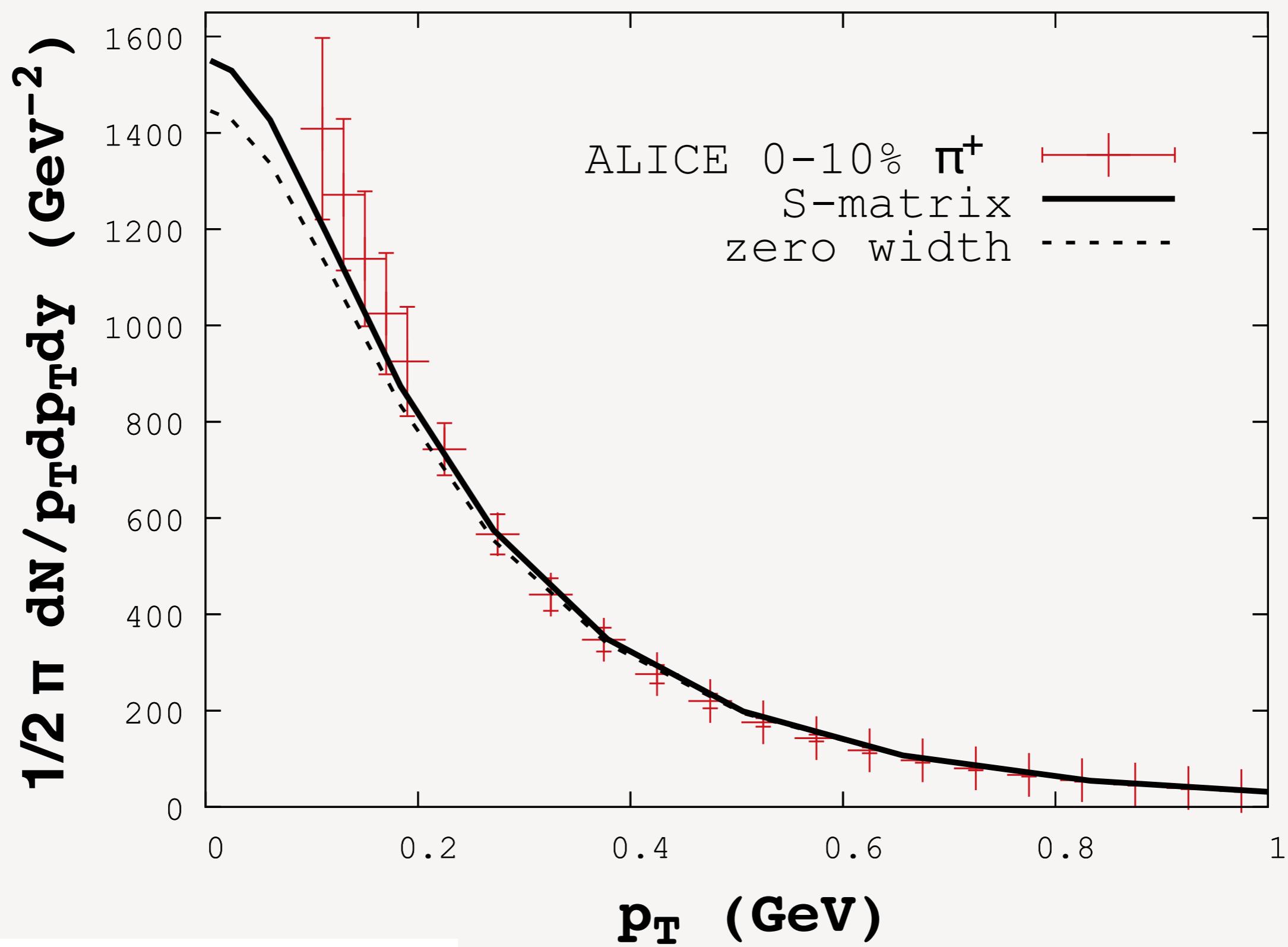


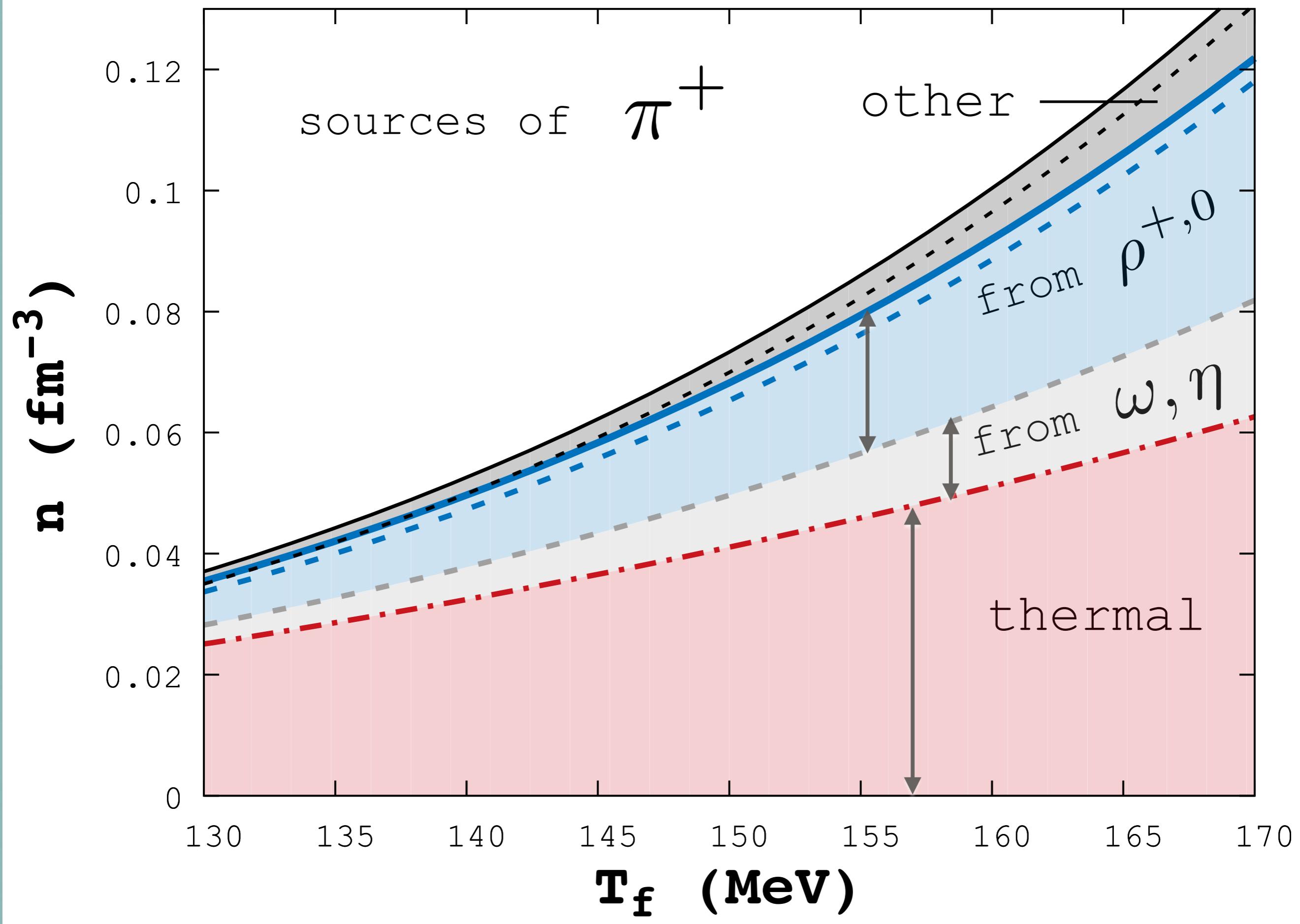
$dN/dp_T dp_T dy (y=0)$ (GeV $^{-2}$)



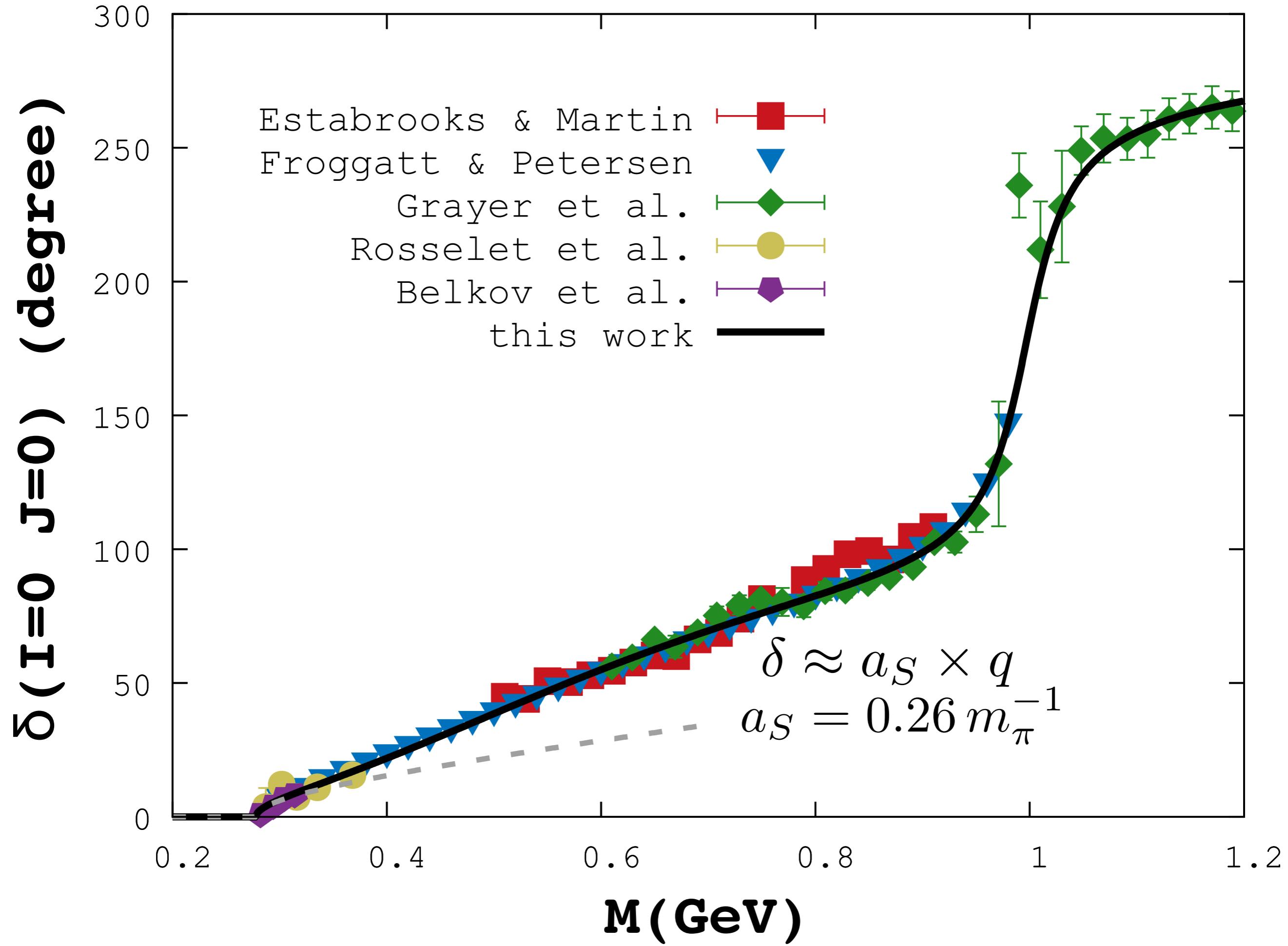


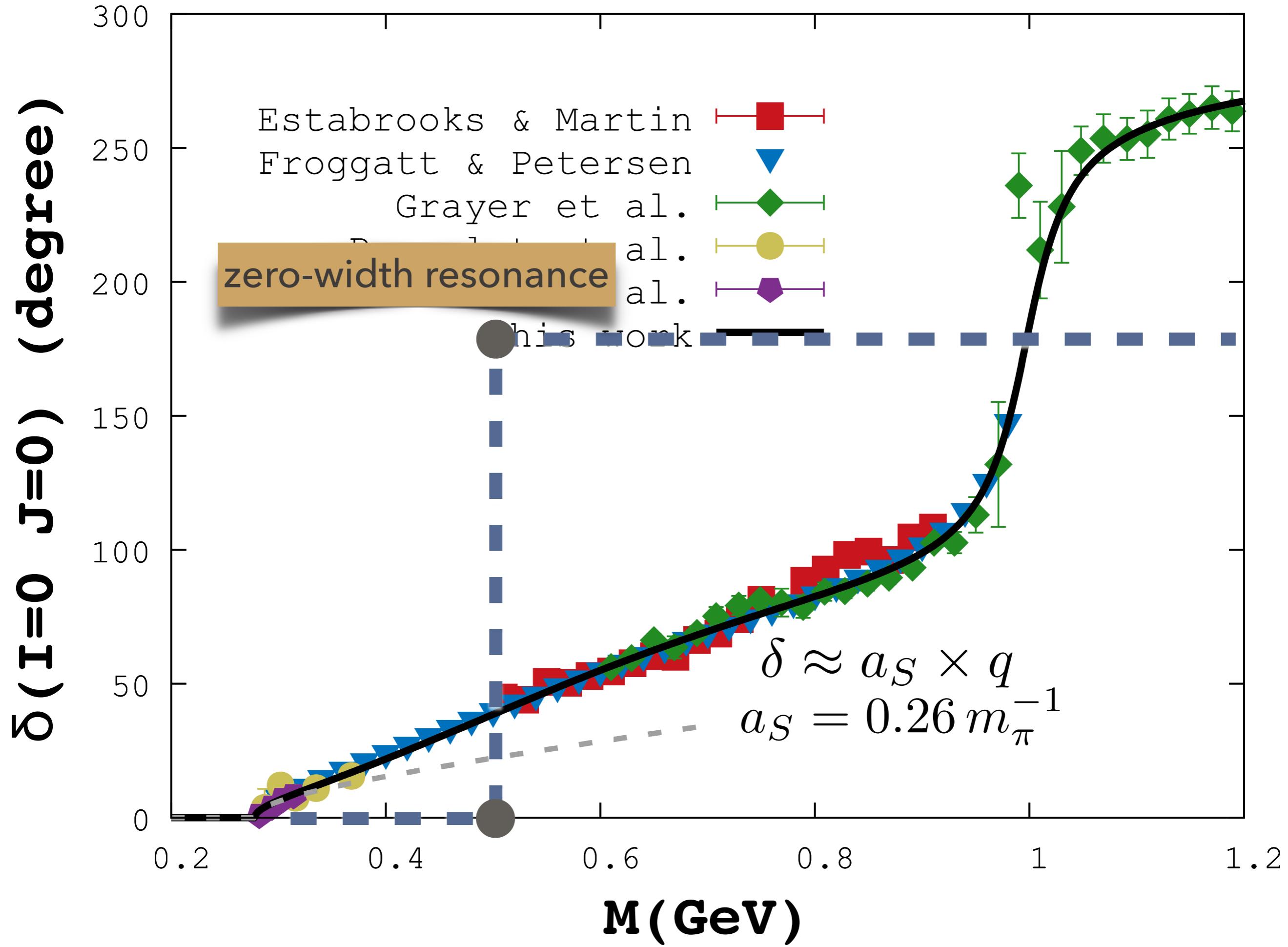


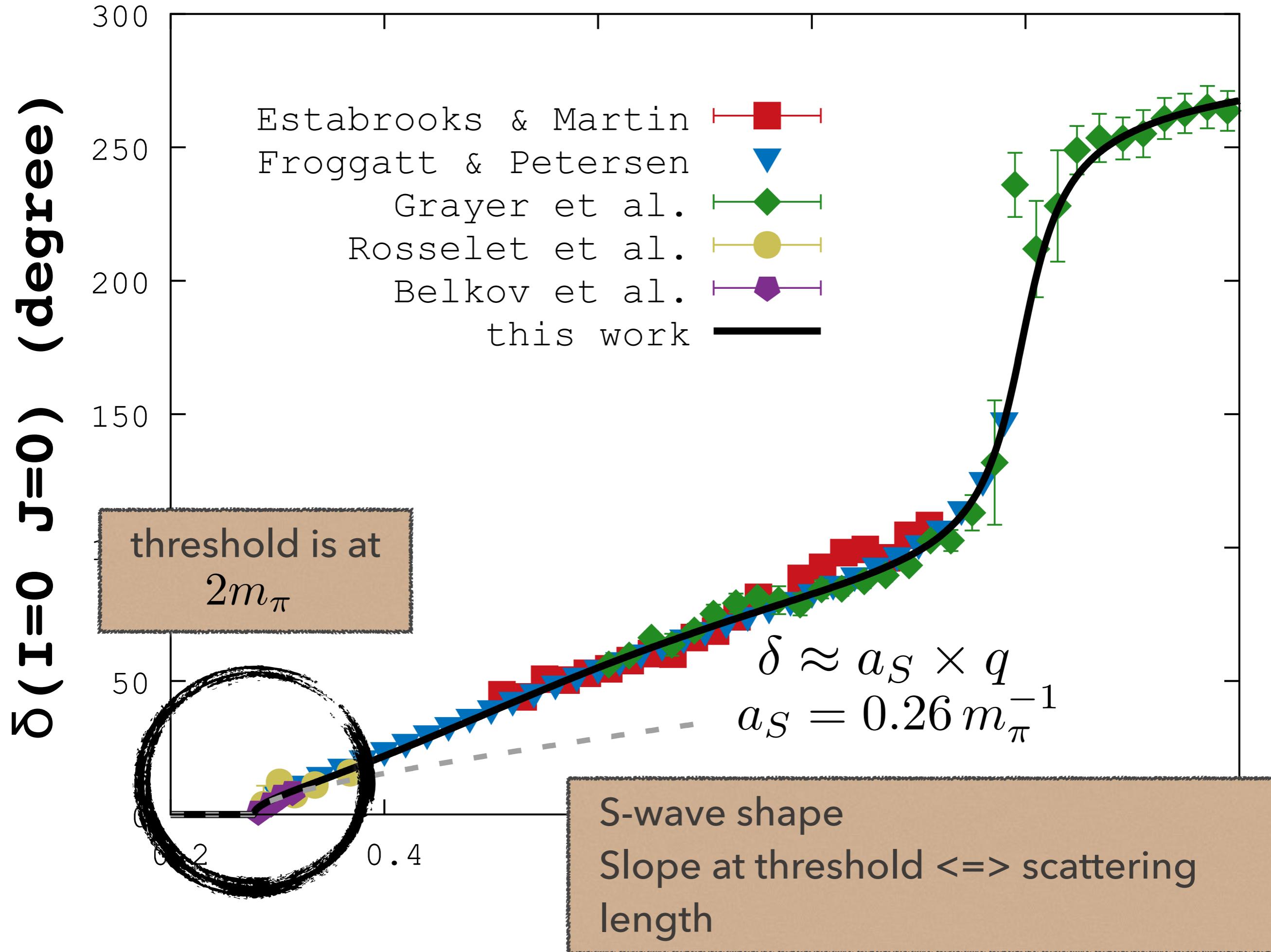


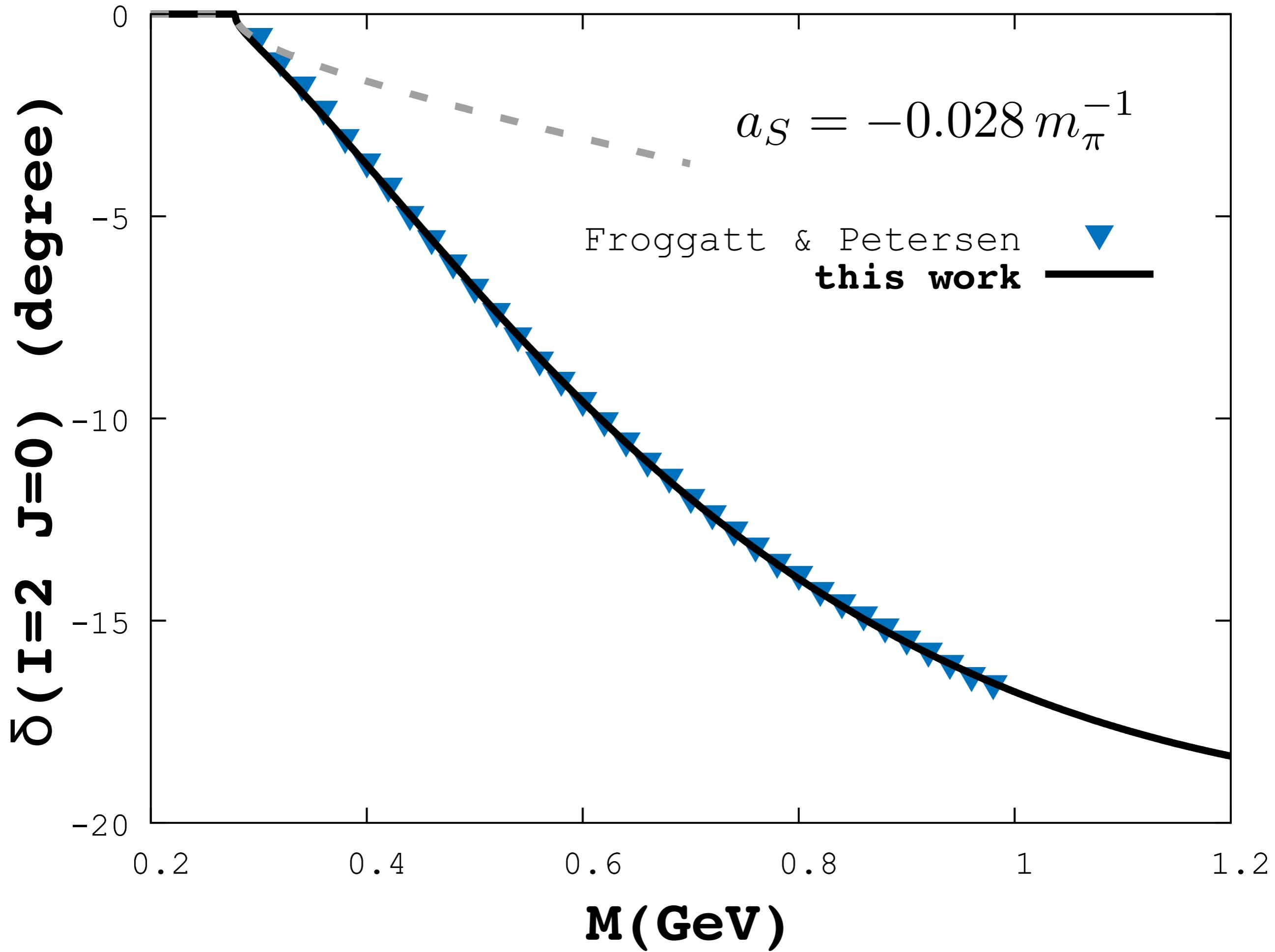


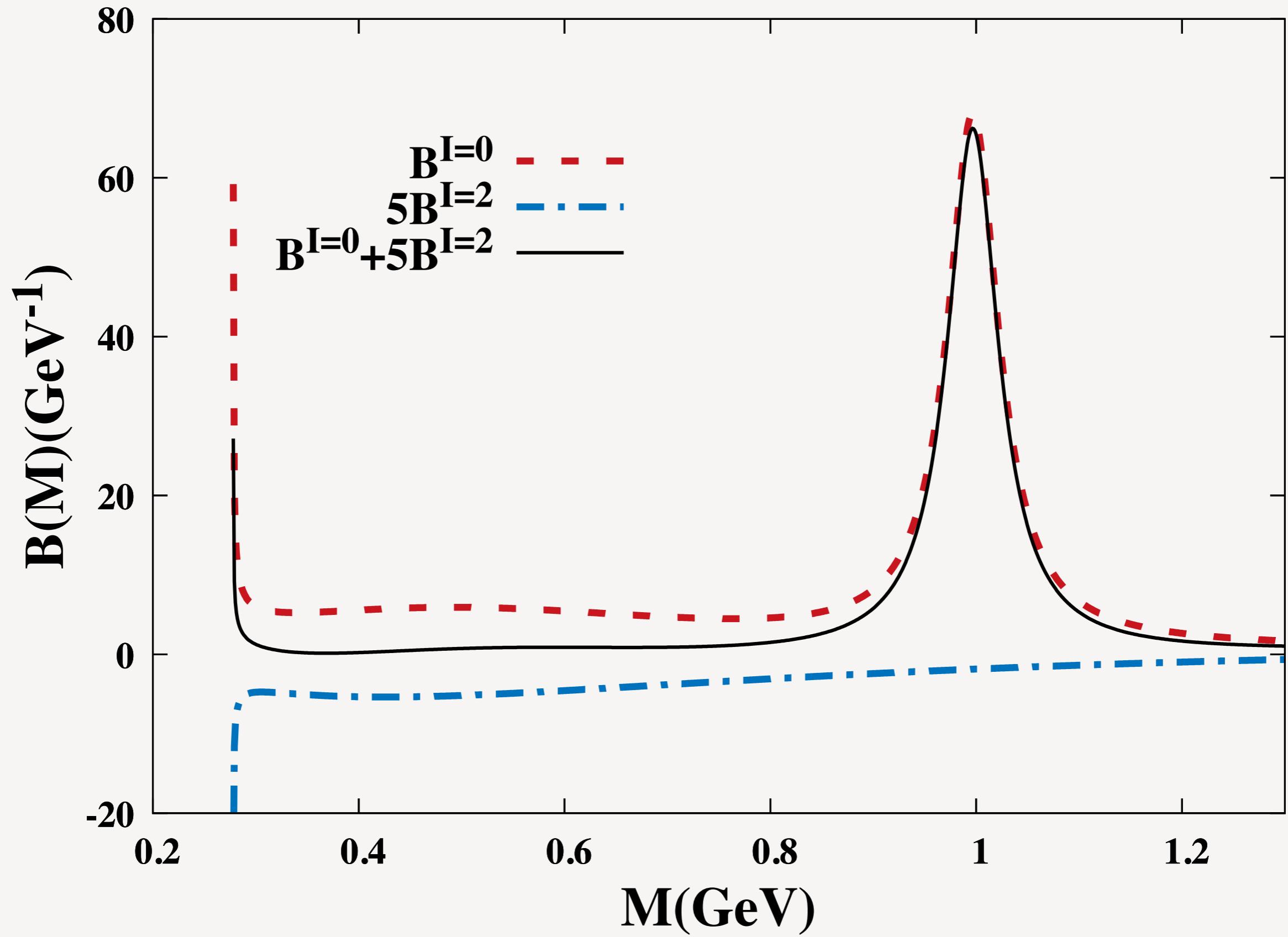
PI PI SCATTERING (S-WAVE)

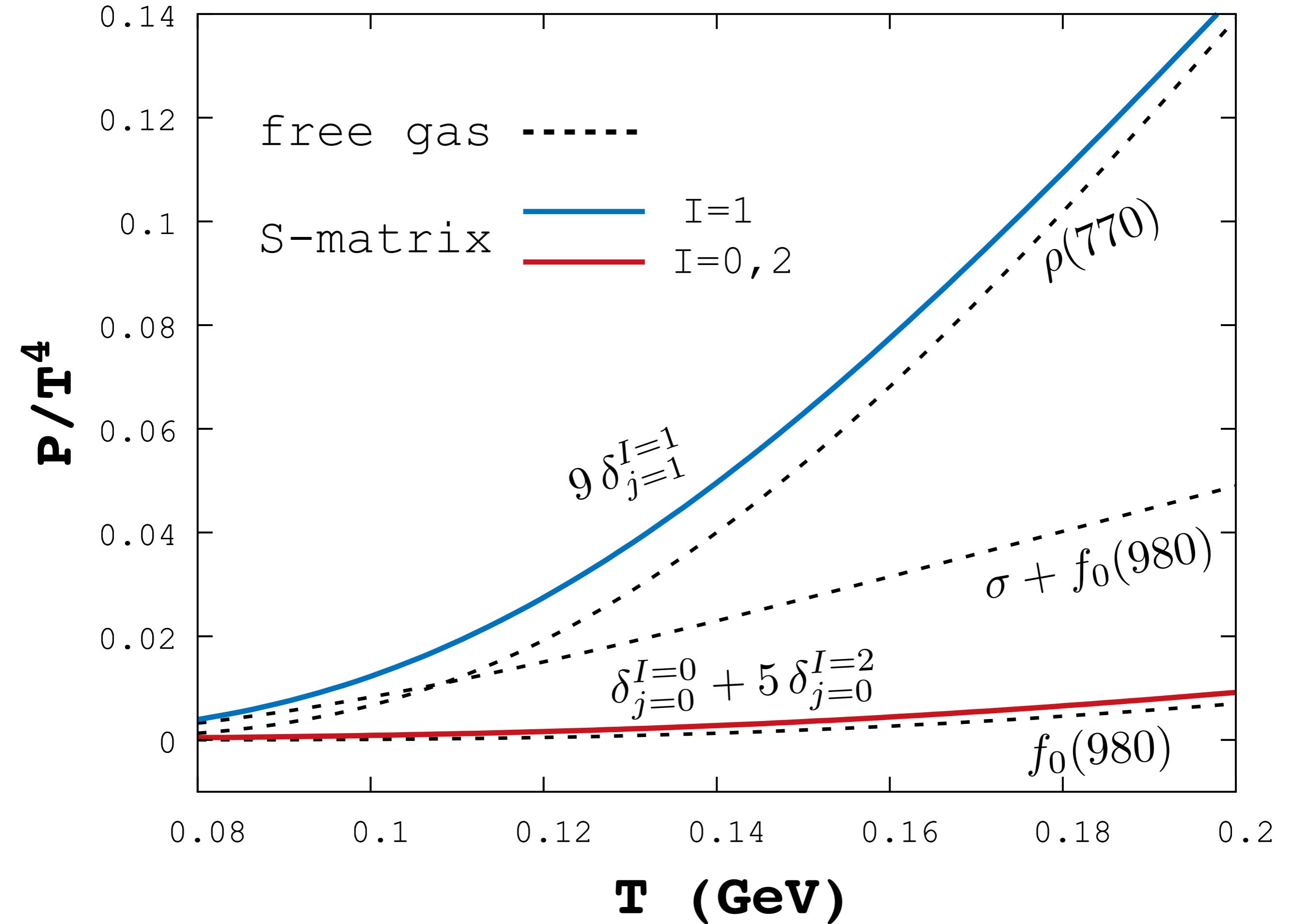












K-N SYSTEM

FLUCTUATIONS

- studying the system by linear response



$$\mu = \mu_B B + \mu_Q Q + \mu_S S$$

$$\chi_{B,S,\dots} = \frac{1}{\beta V} \frac{\partial^2}{\partial \bar{\mu}_B \partial \bar{\mu}_S \dots} \ln Z$$

 μ_B  μ_Q  μ_S  m_q

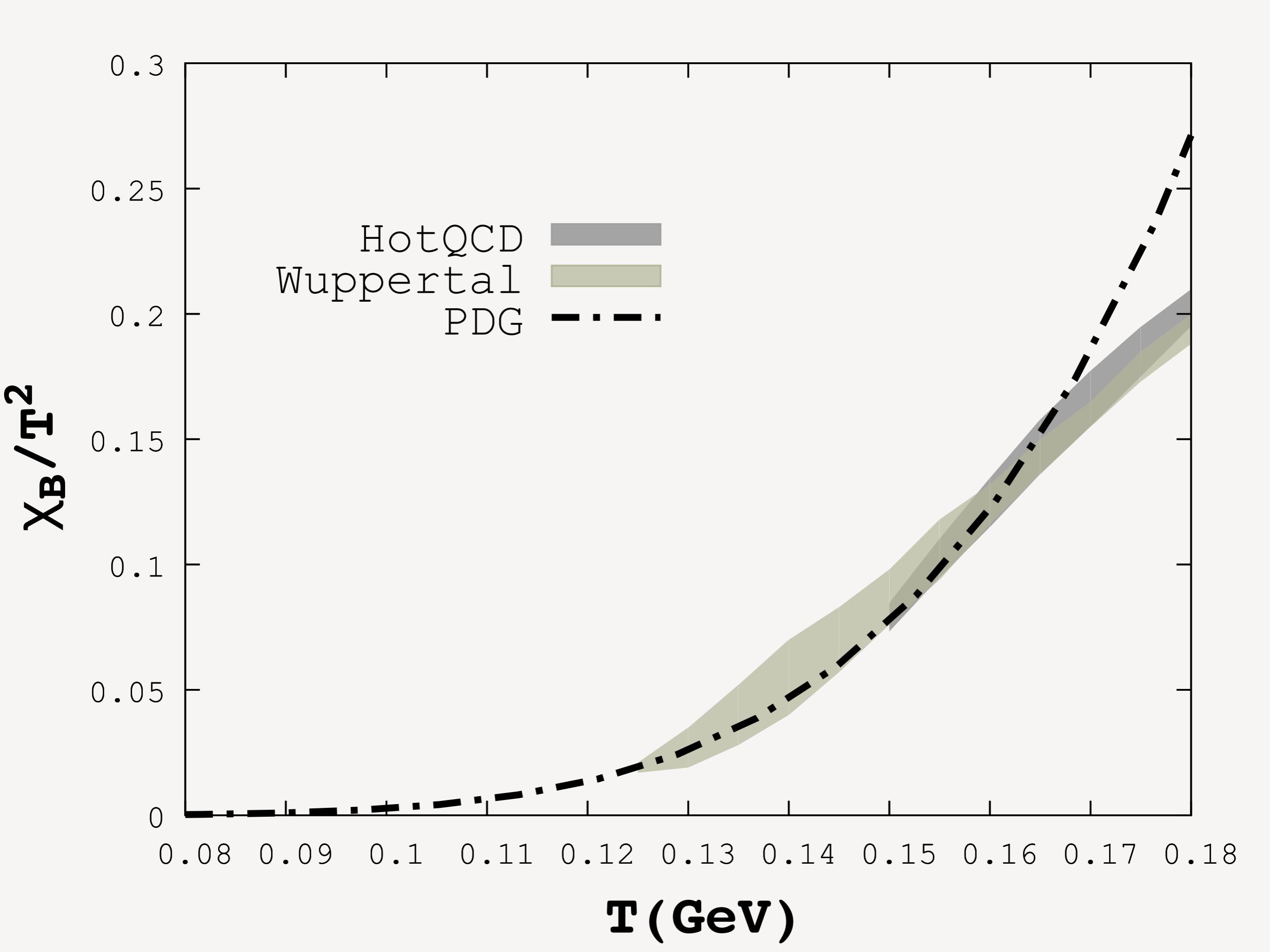
FLUCTUATIONS

- taking derivative

$$\chi_B = \frac{\partial^2}{\partial \bar{\mu}_B \partial \bar{\mu}_B} P \quad \text{at the limit} \quad \mu_B \rightarrow 0$$

probes fluctuations

$$\begin{aligned} \chi_B &= \frac{1}{\beta V} \frac{\partial^2}{\partial \bar{\mu}_B \partial \bar{\mu}_B} \ln Z \\ &= T^2 \langle\langle \int d^4x \bar{\psi}(x) \gamma^0 \psi(x) \bar{\psi}(0) \gamma^0 \psi(0) \rangle\rangle_c \end{aligned}$$



PHASE SHIFT FROM PWA

Coupled Channels partial wave calculator for KN scattering
by the Joint Physics Analysis Center (JPAC)
Version: September 1, 2015

Authors:

Cesar Fernandez-Ramirez (Jefferson Lab)
Igor V. Danilkin (Jefferson Lab)
Vincent Mathieu (Indiana University)
Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)
This version: Cesar Fernandez-Ramirez (Jefferson Lab)

Contact: cefera@gmail.com (Cesar Fernandez-Ramirez)

Disclaimers:

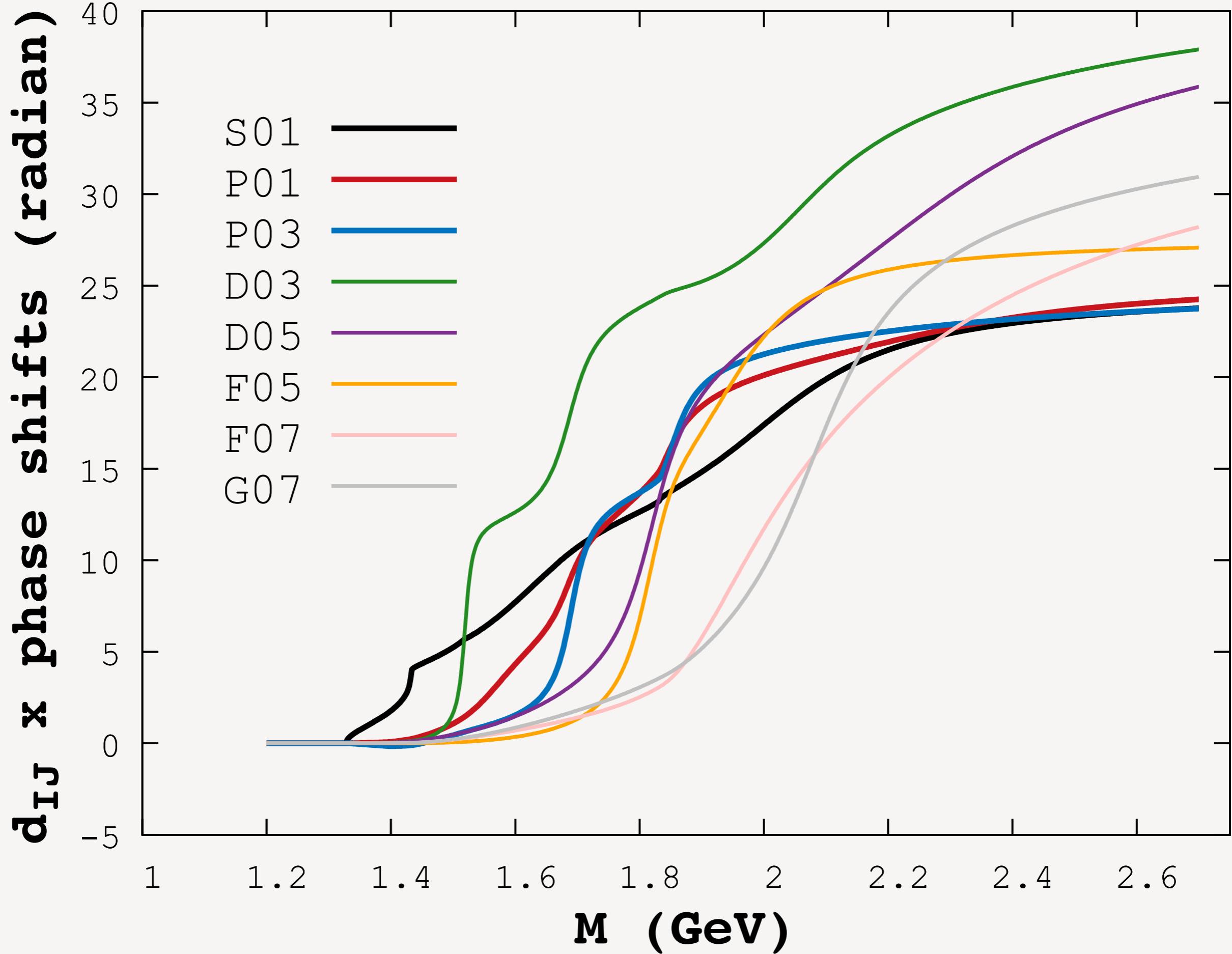
- 1 - This code follows the 'garbage in, garbage out' philosophy. If your parameters do not make sense, the output will not make sense either.
 - 2 - You can use, share and modify this code under your own responsibility.
 - 3 - This code is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
 - 4 - No PhD students or postdocs were severely damaged during the development of this project.
-

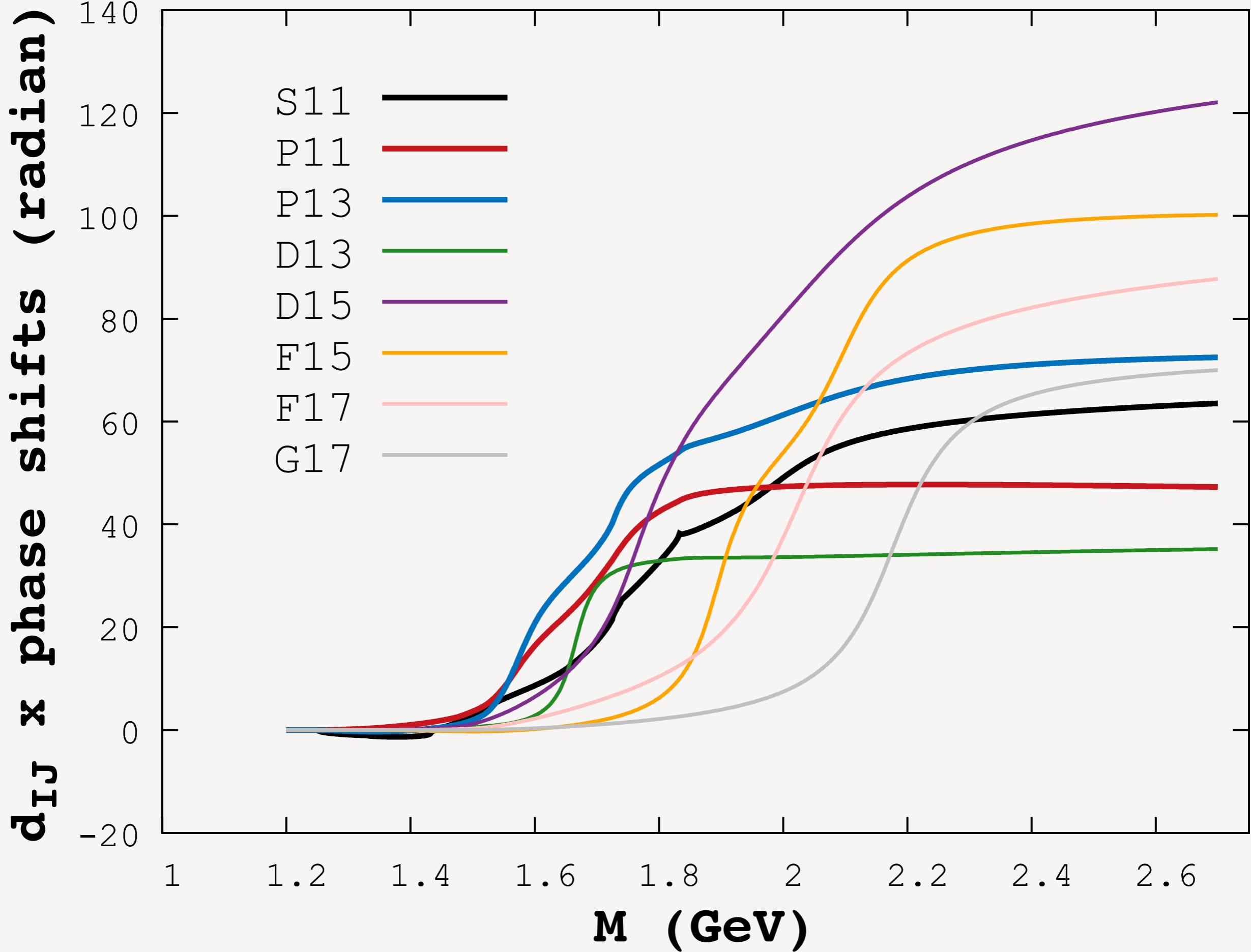
STRANGENESS CONTENT IN A HADRON GAS

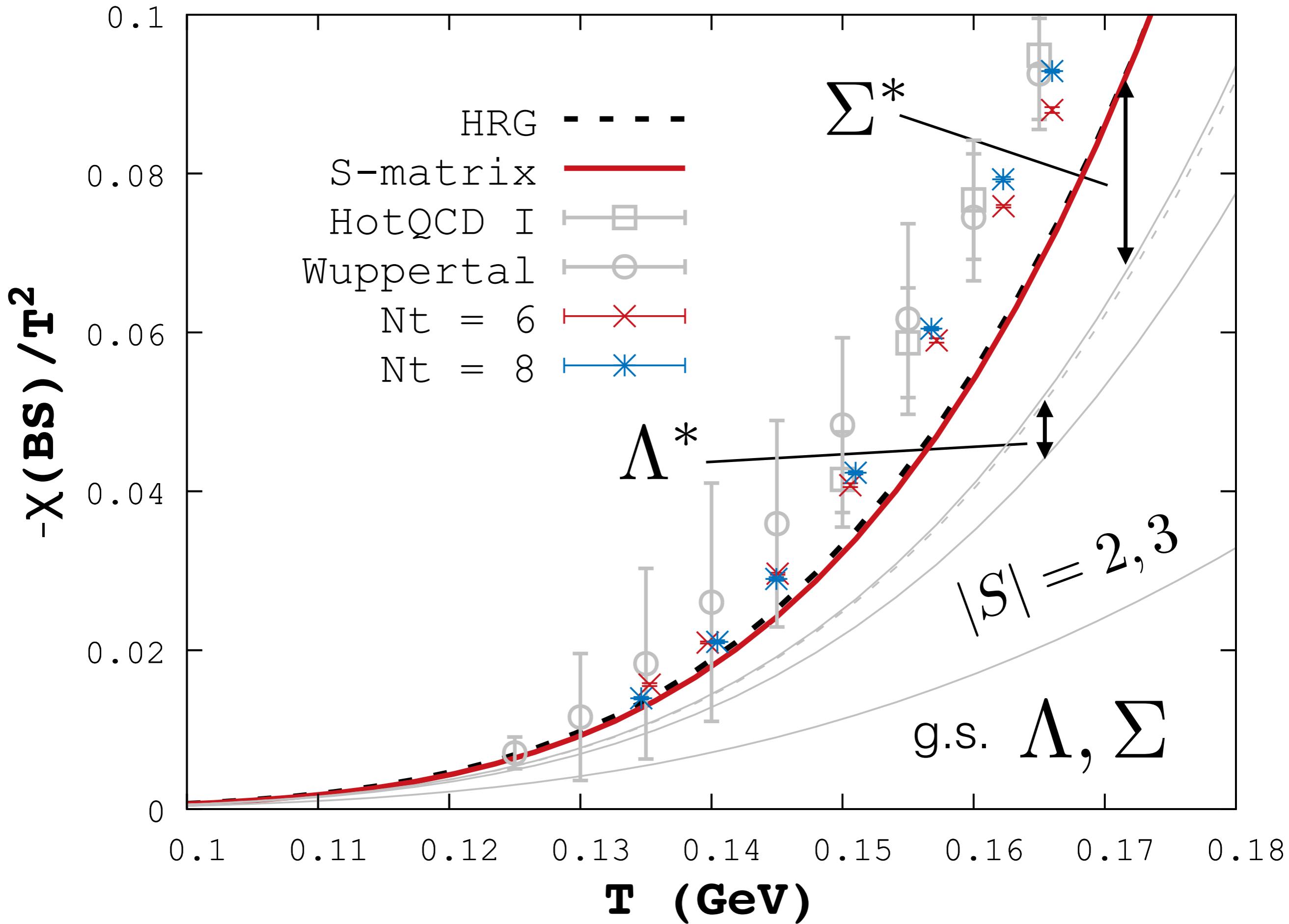
- K-N system requires a coupled channel analysis

$|\bar{K}N\rangle, |\pi\Sigma\rangle, |\pi\Lambda\rangle, |\eta\Lambda\rangle, \dots$ *16 basis states*

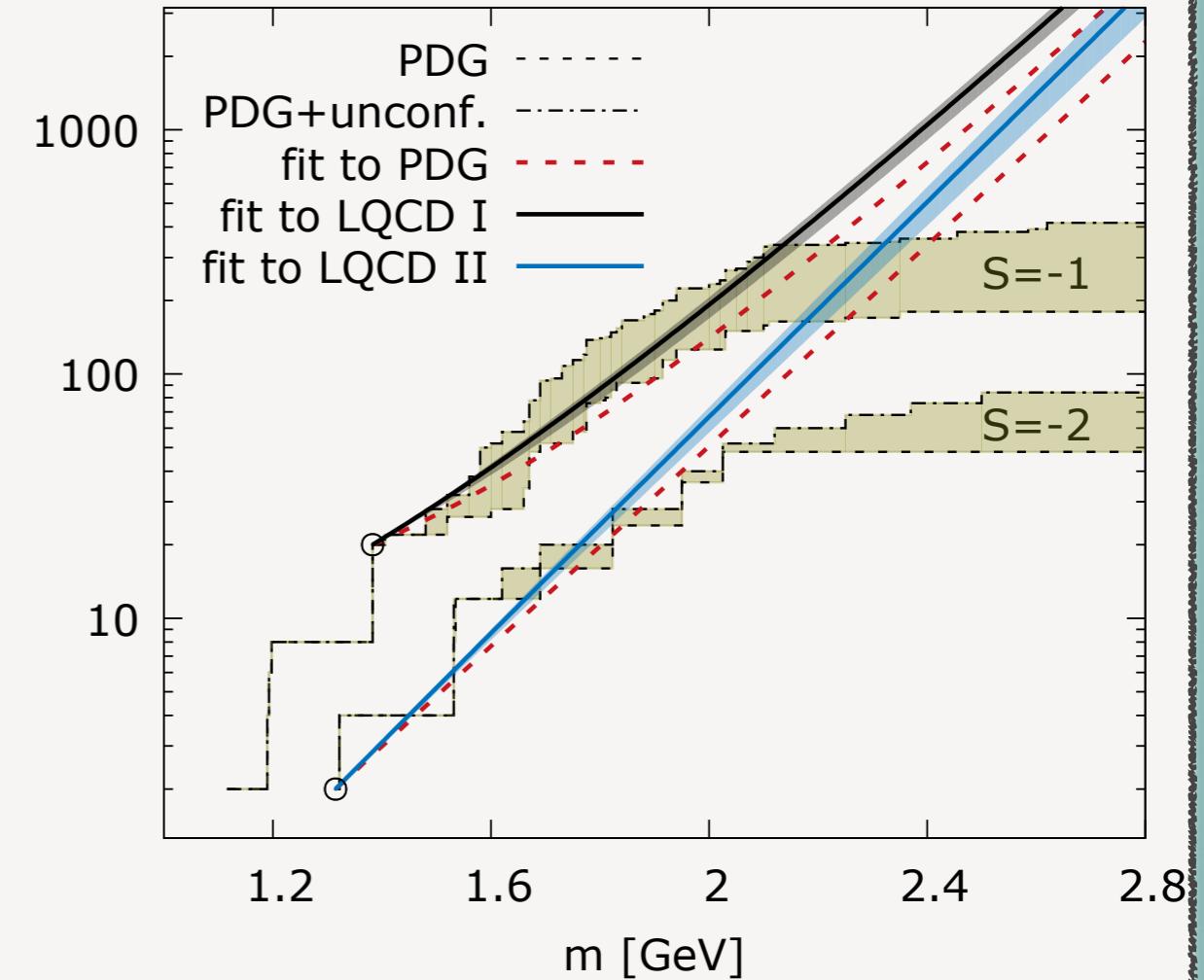
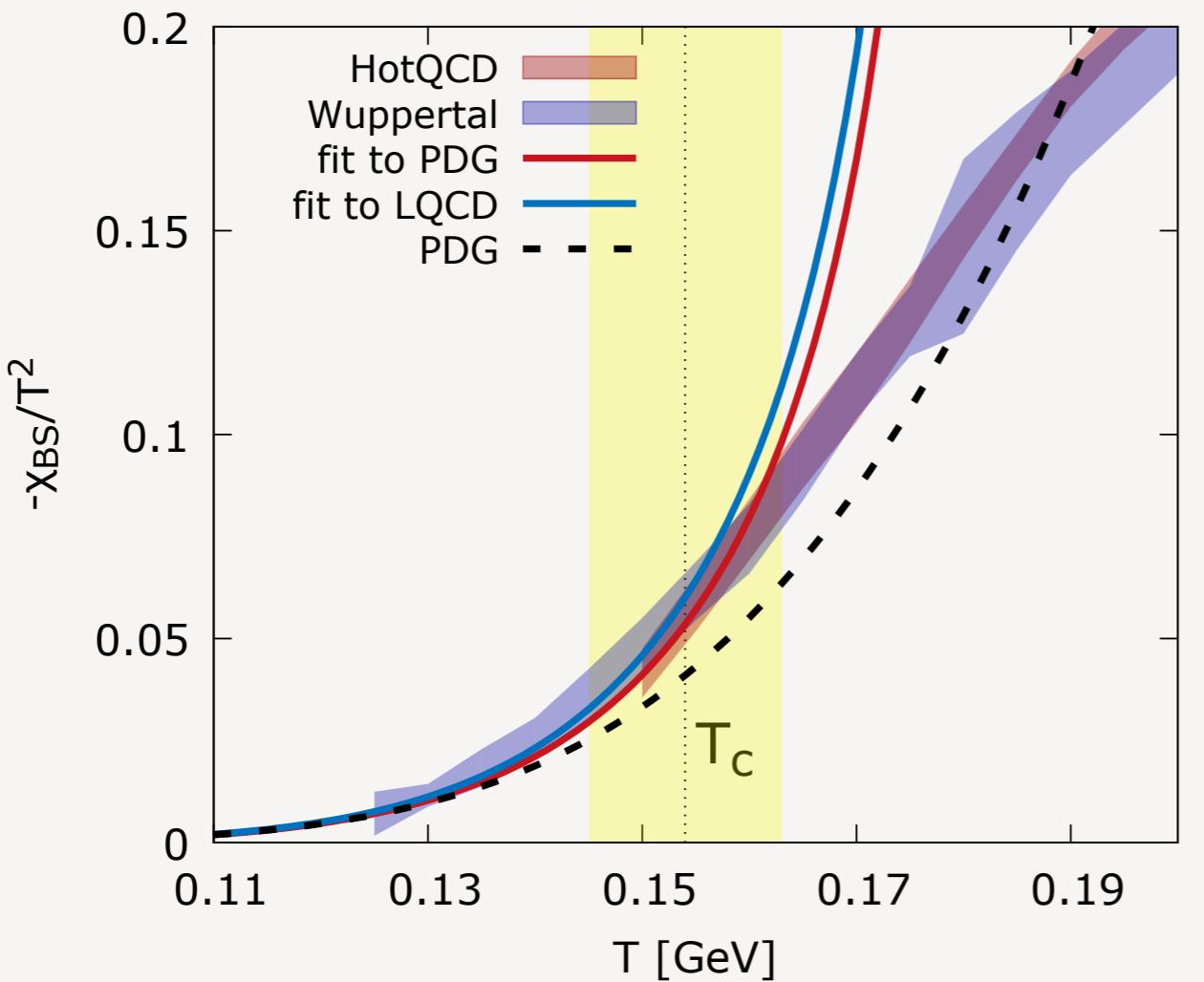
$$\begin{aligned}\mathcal{Q}(M) &\equiv \frac{1}{2} \operatorname{Im} (\operatorname{tr} \ln S) \\ &= \frac{1}{2} \operatorname{Im} (\ln \det [S]) \\ &= \delta_{KN} + \delta_{\pi\Sigma} + \delta_{\pi\Lambda} + \dots\end{aligned}$$



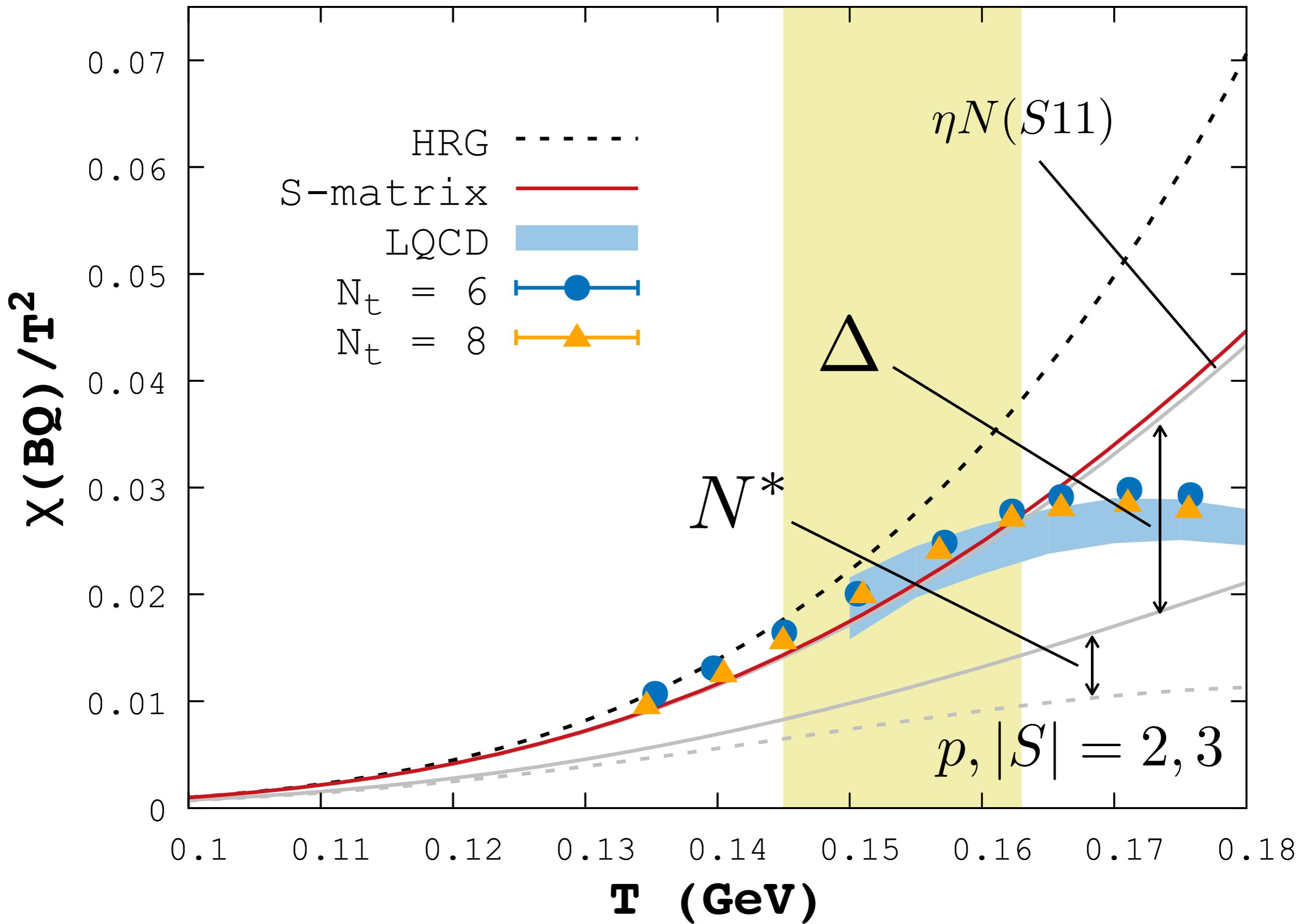




strange mesons to be discovered...

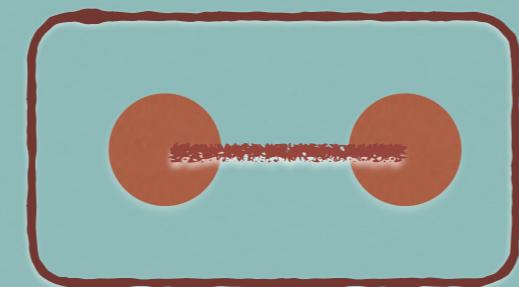
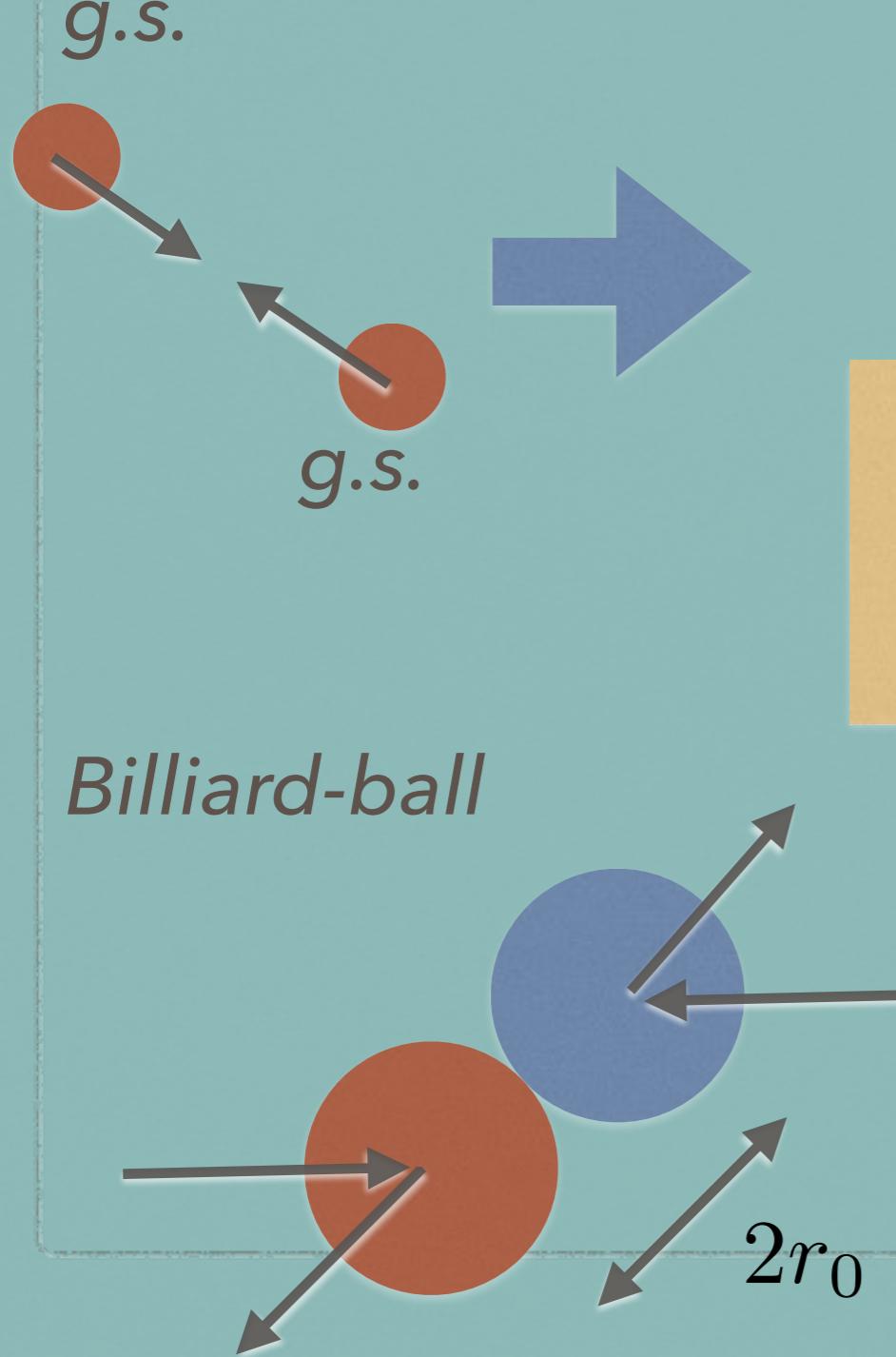


PML, M. Marczenko, K. Redlich and C. Sasaki
Phys. Rev. C92 (2015) no.5, 055206

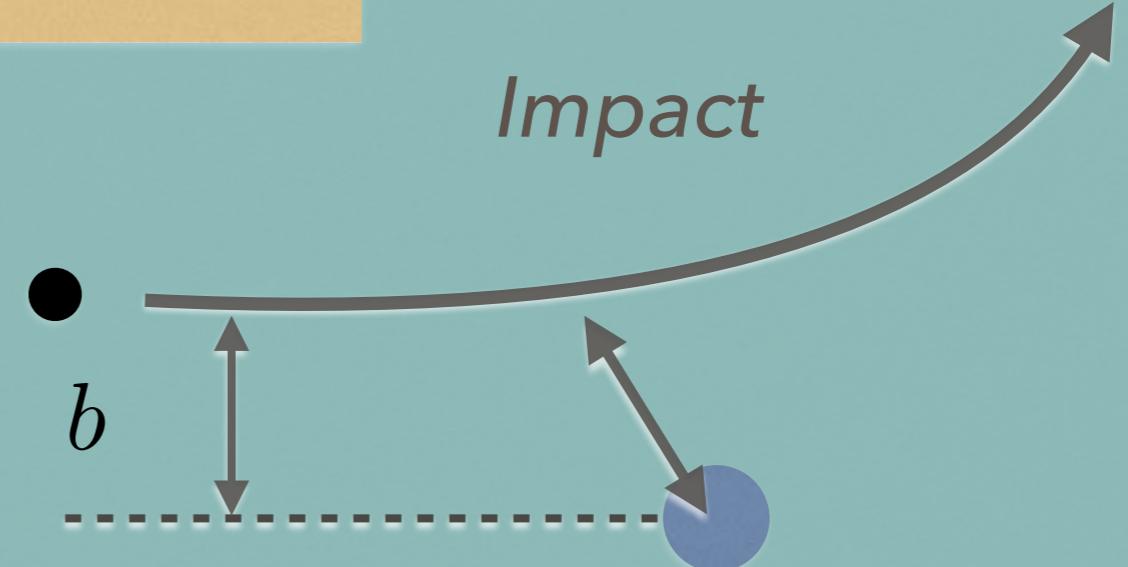
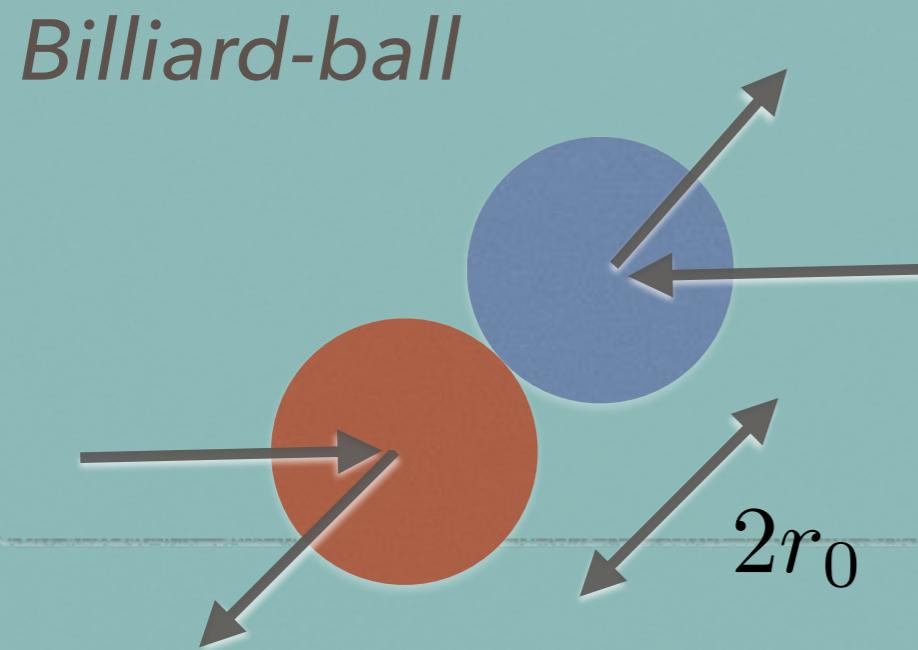


TIME DELAY

P. Danielewicz and S. Pratt
Phys. Rev. C53 (1996) 249–266



$$\tau \sim 2 \frac{d\delta}{dE}$$



TO DO LIST...

- Exotics
- Virial expansion for dense(r) medium

THANK YOU