

Finite-volume scaling of Polyakov loop susceptibilities

Michał Szymański

Collaborators: P. M. Lo, K. Redlich, C. Sasaki

Institute of Theoretical Physics, University of Wrocław

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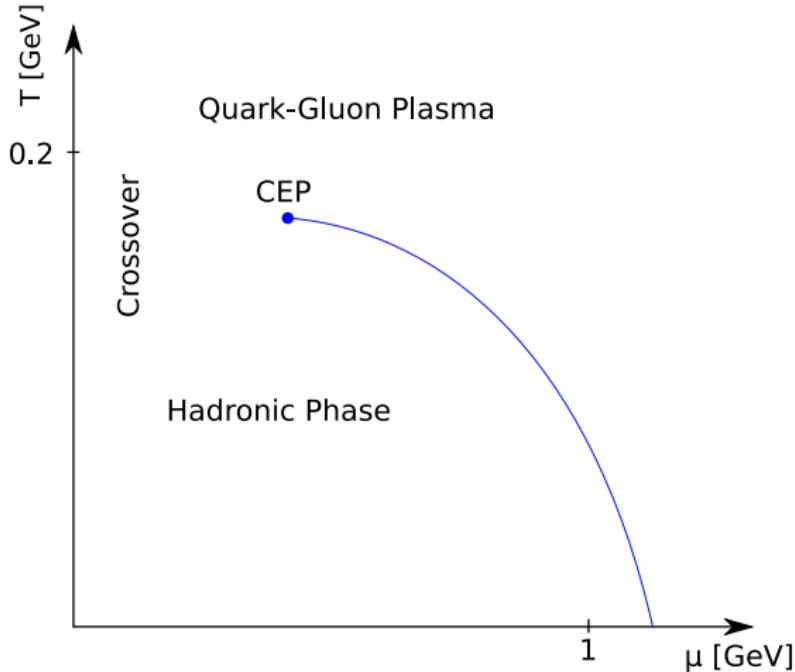


Figure: Conjectured phase diagram of QCD.

Polyakov loop → probes the free energy of a static quark

$$|L| = \exp(-F_Q/T)$$

Pure gauge: deconfinement → 1st order

$T < 270 \text{ MeV} \Rightarrow L = 0, F_Q = \infty$

$T > 270 \text{ MeV} \Rightarrow L > 0, F_Q \text{ finite}$

QCD: deconfinement → crossover

- ▶ Polyakov loop → approximate order parameter

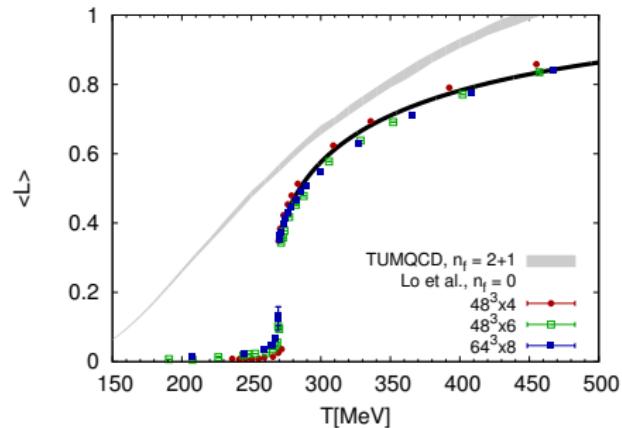


Figure: Polyakov loop in pure gauge¹ (colored points+black line) and in 2+1 QCD² (gray band)

¹P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D **88**, 014506 (2013)

²A. Bazavov, N. Brambilla, H.-T Ding, P. Petreczky, H. -P. Schadler, A. Vairo and J. H. Weber, Phys. Rev. D **93**, 114502 (2016).

Polyakov loop susceptibility → sensitive to deconfinement and crossover

$$\chi_A = V \langle |L|^2 \rangle_c = V (\langle |L|^2 \rangle - \langle |L| \rangle^2)$$

$N_c = 3 \rightarrow$ Polyakov loop complex: $L = L_L + i L_T$

- ▶ Longitudinal (real) susceptibility

$$\chi_L = V \langle (L_L)^2 \rangle_c = V (\langle (L_L)^2 \rangle - \langle L_L \rangle^2)$$

- ▶ Transverse (imaginary) susceptibility

$$\chi_T = V \langle (L_T)^2 \rangle_c = V (\langle (L_T)^2 \rangle - \langle L_T \rangle^2)$$

Ratios of Polyakov loop susceptibilities → alternative probes of deconfinement¹

$$R_A = \frac{\chi_A}{\chi_L} \qquad \qquad R_T = \frac{\chi_T}{\chi_L}$$

¹P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D **88**, 014506 (2013)

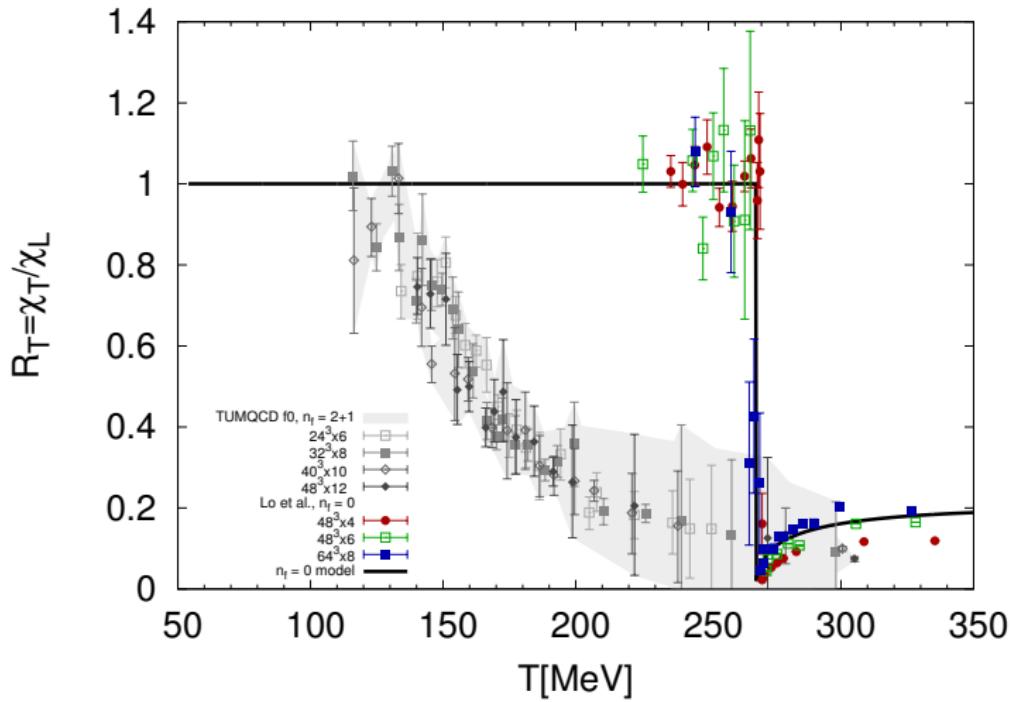


Figure: Lattice data on the R_T ratio in pure gauge¹ (colored points+black line) and 2+1 QCD² (gray band).

¹P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D **88**, 014506 (2013)

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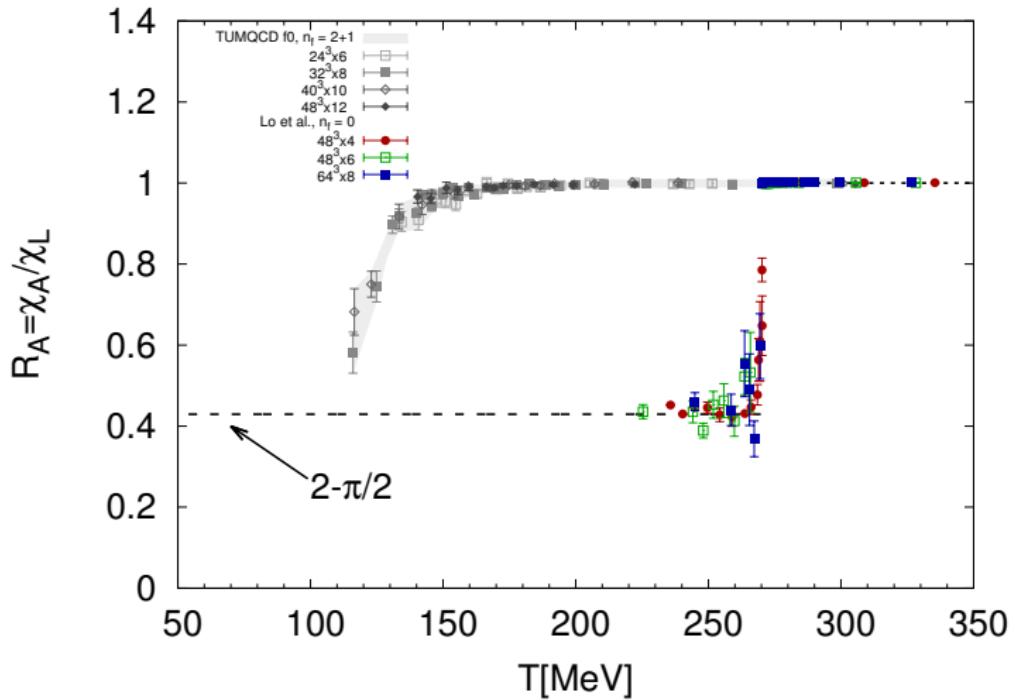


Figure: Lattice data on the R_A ratio in pure gauge¹ (colored points) and 2+1 QCD² (gray band).

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Pure SU(3): $T < 270 \text{ MeV} \rightarrow R_A \approx 0.43$

Can be understood in terms of Gaussian distribution function:

$$Z = \int dL_L dL_T e^{-V\mathcal{U}/T}, \quad \mathcal{U} = \alpha T^4 (L_L^2 + L_T^2)$$

with averages

$$\langle O \rangle = \frac{1}{Z} \int dL_L dL_T O(L_L, L_T) e^{-V\mathcal{U}/T}$$

With this approximation one finds

$$\langle |L|^2 \rangle_c = \left(2 - \frac{\pi}{2}\right) \frac{1}{2\alpha VT^3} \quad \langle (L_{L,T})^2 \rangle_c = \frac{1}{2\alpha VT^3}$$

Which gives $R_A = 2 - \pi/2 \approx 0.43$ and $R_T = 1$.

Generalization: to capture details of phase transition

$$\mathcal{U} \rightarrow \mathcal{U} = \mathcal{U}_{Gluon} + \mathcal{U}_{Int}$$

\mathcal{U}_{Gluon} : Pure gauge potential

$$\frac{\mathcal{U}_G}{T^4} = -\frac{1}{2}a(T)L\bar{L} + b(T)\ln M_H(L, \bar{L}) + \frac{1}{2}c(T)(L^3 + \bar{L}^3) + d(T)(L\bar{L})^2$$

- ▶ Underlying Z_3 symmetry
- ▶ $M_H(L, \bar{L}) = 1 - 6L\bar{L} + 4(L^3 + \bar{L}^3) - 3(L\bar{L})^3$
- ▶ Reproduces pure gauge P , L , χ_L and χ_T

\mathcal{U}_{Int} : Quark-gluon interaction:

$$\frac{\mathcal{U}_{Int}}{T^4} = -\underbrace{[2h(T, \textcolor{red}{m}_l) + h(T, \textcolor{blue}{m}_s)]}_{h_0} L_L, \quad h(T, m_q) = \frac{6m_q^2}{(\pi T)^2} K_2\left(\frac{m_q}{T}\right).$$

- ▶ External breaking field coupled to Polyakov loop
- ▶ $m_l = 6$ MeV, $m_s = 20m_l = 120$ MeV

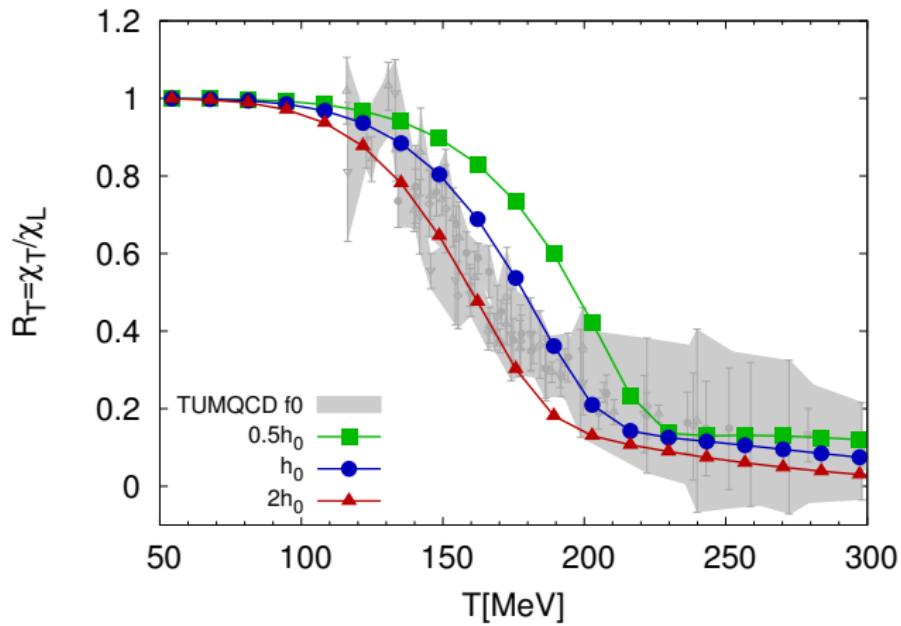


Figure: Model R_T ratio for different values of breaking field versus the lattice one.

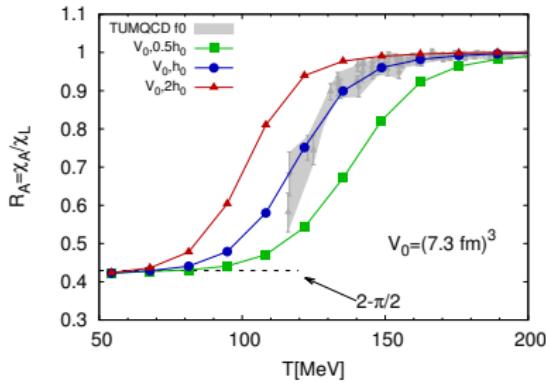
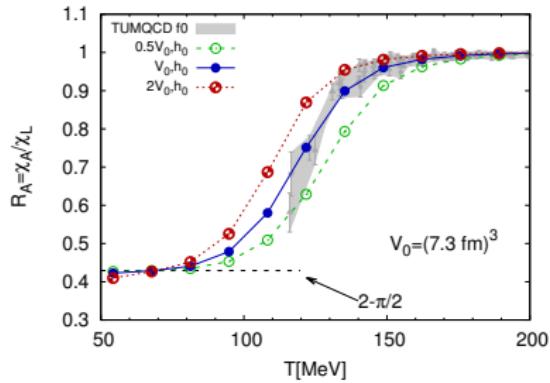


Figure: Model R_A ratio for different values of volume (left panel) and breaking field (right panel) versus the lattice one.

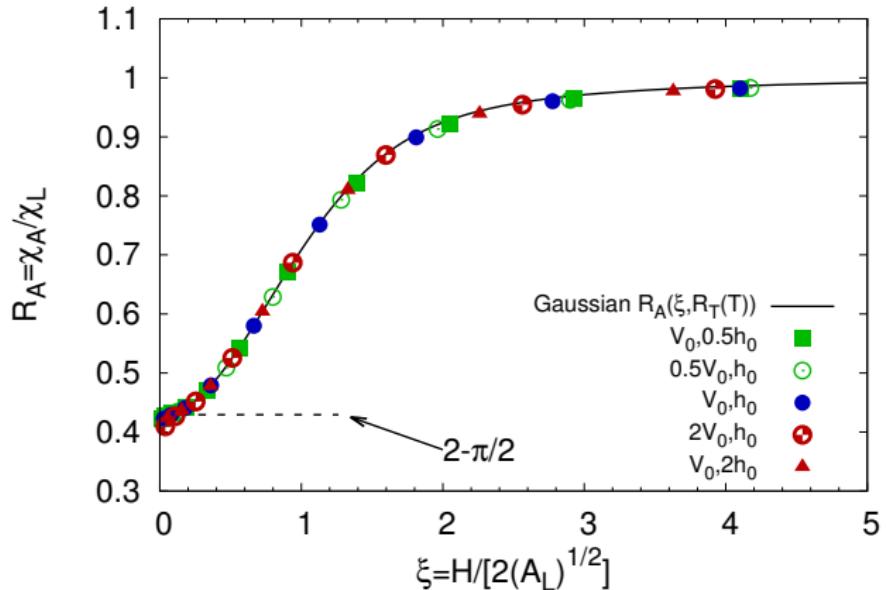


Figure: R_A ratio in function of effective scaling variable. \mathcal{H} , \mathcal{A}_L – determined from effective potential.

Conclusions

- ▶ Ratios of Polyakov loop are sensitive to deconfinement and size of external symmetry breaking field
- ▶ The model discussed here reproduces lattice trends of R_A and R_T ratios
- ▶ Lattice data on ratios can be used to estimate the size of external breaking field
 - ▶ Useful for constraining effective models
- ▶ The scaling properties of R_A ratio could be tested on the lattice

Appendix

Effective gluon potential \mathcal{U}_G

Parametrization of the effective potential:

- ▶ $a(T)$, $c(T)$ and $d(T)$ of the form:

$$x(t \equiv T/T_c) = \frac{x_1 + x_2/t + x_3/t^2}{1 + x_4/t + x_5/t^2}$$

- ▶ $b(T)$ of the form:

$$b(t \equiv T/T_c) = x_1 t^{-x_4} (1 - e^{x_2/t^{x_3}})$$

Parameters x_n can be found in: *P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D* **88**, 074502 (2013)

Effective gluon potential \mathcal{U}_G

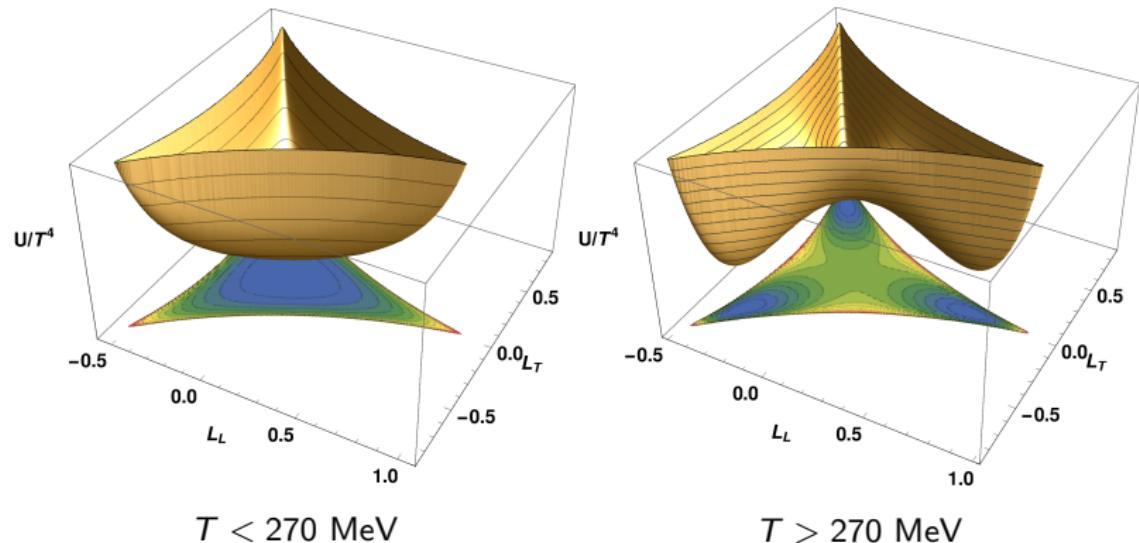


Figure: Shape of effective potential used in model calculations in confined (left panel) and deconfined (right panel) phases.

Model Polyakov loop

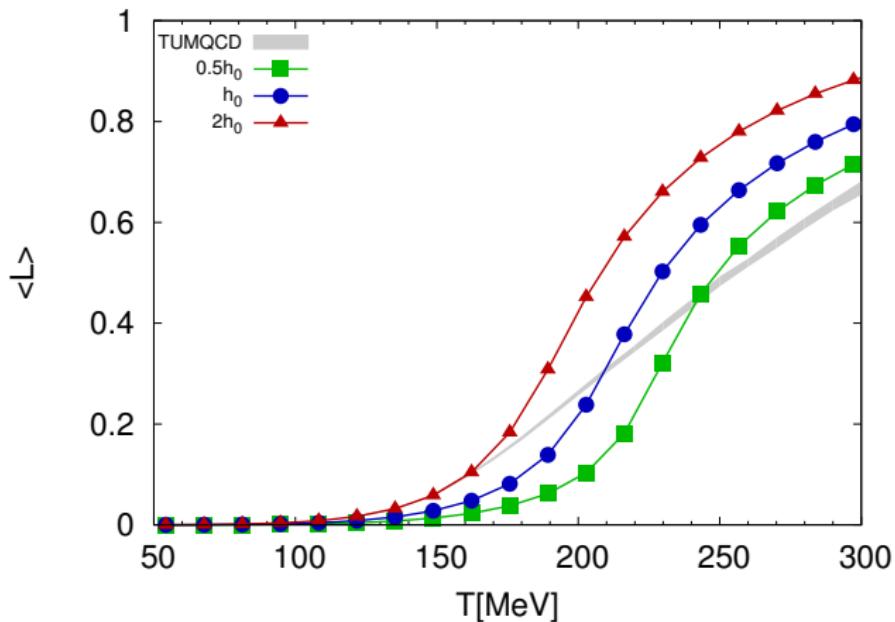


Figure: Model result on Polyakov loop for different values of breaking field versus the lattice one.

Model R_T for different quark mass profiles

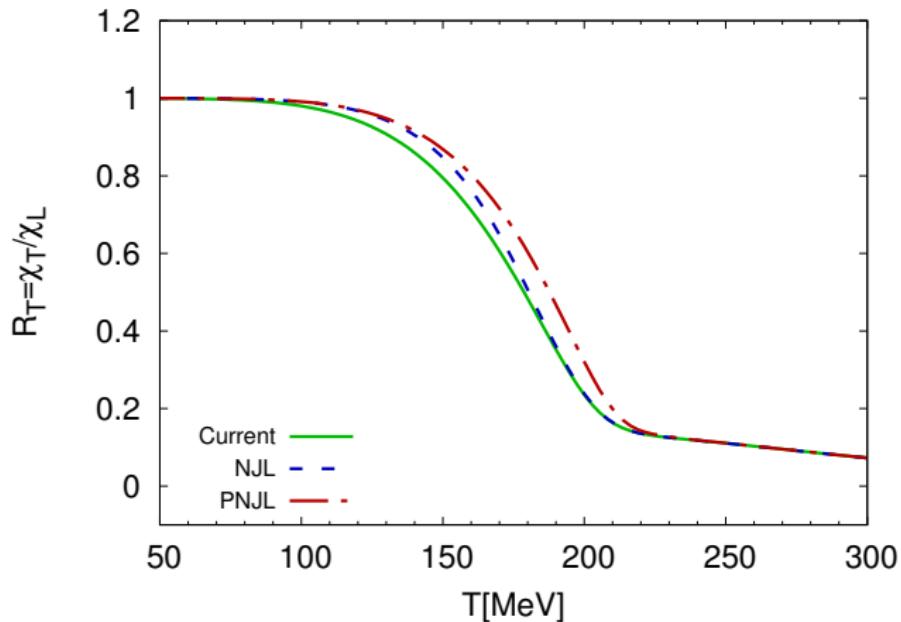


Figure: Comparison between R_T obtained for different temperature profiles of quark mass.

Matching to Gaussian model

The general form of Gaussian distribution function:

$$Z = \int dL_L dL_T \exp \left[-\mathcal{A}_1 L_L^2 - \mathcal{A}_2 L_T^2 + \mathcal{H} L_L \right]$$

- ▶ In this case $R_A(\mathcal{A}_1, \mathcal{A}_2, \mathcal{H}) = R_A(\xi, R_T)$, with

$$\xi = \mathcal{H}/(2\sqrt{\mathcal{A}_1}), \quad R_T = \mathcal{A}_1/\mathcal{A}_2.$$

- ▶ The R_A ratio \rightarrow exact:

$$R_A(\xi, R_T) = 1 + R_T + 2\xi^2 - \frac{2}{\pi^2} R_T e^{-2\xi^2} [\mathcal{I}(\xi, R_T)]^2,$$

$$\mathcal{I}(\xi, R_T) = \int_{-\infty}^{\infty} dx e^{-x^2+2\xi x} \frac{x^2}{2R_T} \left[K_0\left(\frac{x^2}{2R_T}\right) + K_1\left(\frac{x^2}{2R_T}\right) \right]$$

The effective scaling variable $\xi \rightarrow$ Gaussian matching:

$$\mathcal{A}_{1,2} = VT^3 [2\chi_{L,T}(T, h=0)]^{-1}, \quad \mathcal{H} = VT^3 \left[h + \frac{\langle L \rangle(T, h=0)}{\chi_{L,T}(T, h=0)} \right]^{-1}$$