

The tensor-pomeron concept for soft and hard reactions in QCD

O. Nachtmann , Univ. Heidelberg

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A Unitarity and DIS

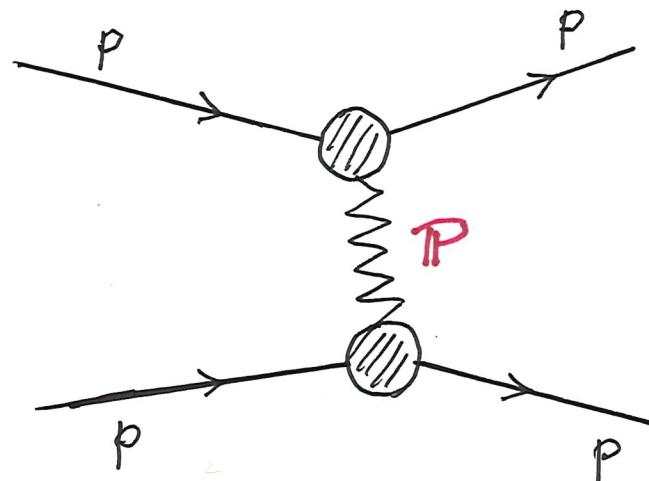
B Low x physics as a critical phenomenon?

1 Introduction

In this talk we shall discuss the pomeron, mainly the soft one, whose exchange governes many high-energy reactions.

Examples:

- $p\ p$ elastic scattering

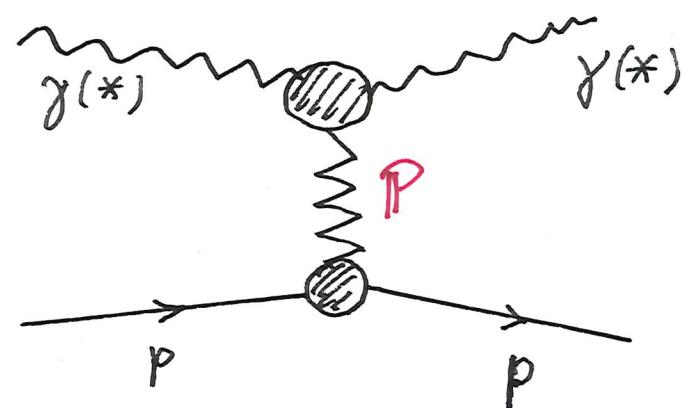


- real and virtual Compton scattering,

$$\gamma^{(*)} + p \rightarrow \gamma^{(*)} + p$$

$$\rightarrow \sigma_{\text{tot}}(\gamma, p) \text{ and}$$

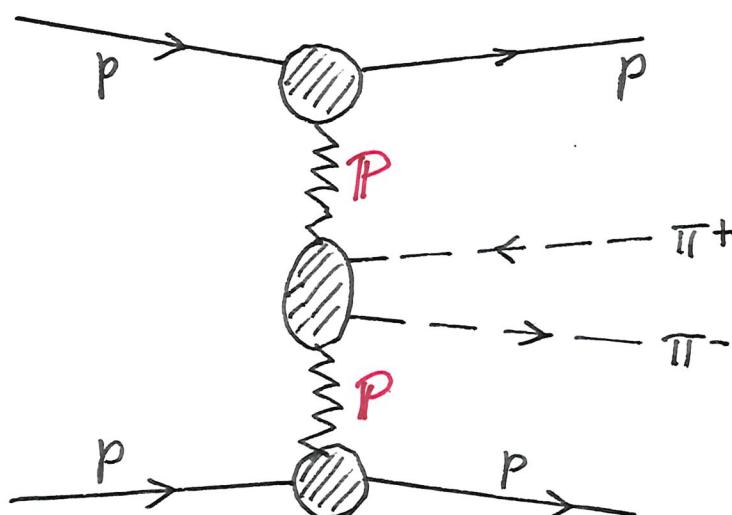
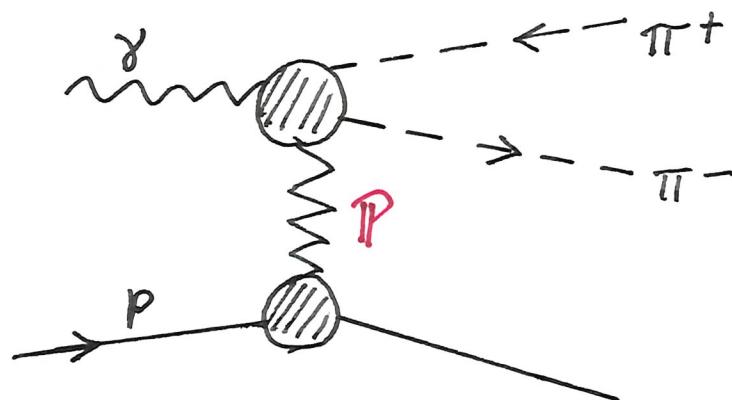
structure functions of DIS



exclusive reactions



Everybody draws such diagrams but the interpretation given to them can differ widely.



Properties of the pomeron:

vacuum internal quantum numbers:

charge $Q = 0$

colour charge $Q_c = 0$

isospin $I = 0$

charge conjugation $C = 1$

spin structure? We believe that the pomeron is best described as the effective exchange of a symmetric rank 2 tensor object, the tensor pomeron.

We shall show that the STAR experiment on pp elastic scattering with spin gives decisive evidence for this view.

References:

Ewerz, Maniatis, O.N., Ann. Phys. 342 (2014) 31

Lebiedowicz, O.N., Szczurek , Ann. Phys. 344 (2014) 301

Bolz, Ewerz, Maniatis, O.N., Sauter, Schöning, JHEP 1501 (2015) 151

Lebiedowicz, O.N., Szczurek, PR D91 (2015) 074023

PR D93 (2016) 054015

PR D94 (2016) 034017

PR D95 (2017) 034036

Ewerz, Lebiedowicz, O.N., Szczurek, PL B763 (2016) 382

Britzger, Ewerz, Glazov, O.N., Schmitt, (in preparation)

2 Helicity amplitudes in pp elastic scattering

$$p(p_1, s_1) + p(p_2, s_2) \rightarrow p(p_3, s_3) + p(p_4, s_4)$$

$s_j \in \{+1/2, -1/2\}$, helicity indices

kinematic variables:

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$

$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

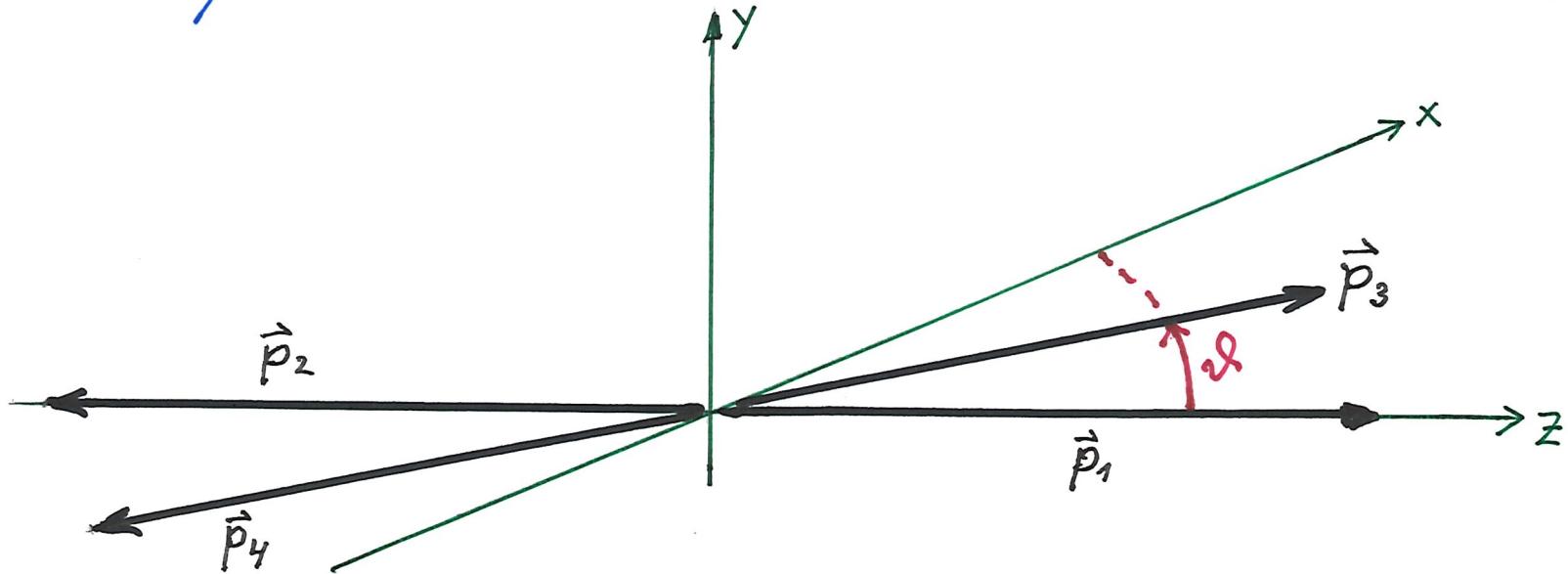
There are $2^4 = 16$ helicity amplitudes.

General analysis of the reaction:

Goldberger et al., PR 120 (1960) 2250

Buttimore et al., PR D18 (1978) 694; PR D59 (1999) 114010

c.m. Coordinate system



Definition of the helicity states $|p(p_1, s_1)\rangle$, $|p(\vec{p}_2, s_2)\rangle$ and corresponding Dirac spinors:

$$u_{sj}(p_j) = \sqrt{p_j^0 + m_p} \begin{pmatrix} \chi_{sj}^{(j)} \\ \frac{\vec{\sigma} \cdot \vec{p}_j}{p_j^0 + m_p} \chi_{sj}^{(j)} \end{pmatrix}, \quad \begin{aligned} \chi_{1/2}^{(1)} &= \chi_{-1/2}^{(2)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \chi_{-1/2}^{(1)} &= \chi_{1/2}^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned}$$

$$j = 1, 2$$

Consider $U_2(\vartheta)$, a rotation by ϑ around the positive y-axis.

We define the helicity states $|p(p_3, s_3)\rangle$, $|p(p_4, s_4)\rangle$ by

$$|p(p_3, s_3)\rangle = U_2(\vartheta) |p(p_1, s_3)\rangle,$$

$$|p(p_4, s_4)\rangle = U_2(\vartheta) |p(p_2, s_4)\rangle.$$

This fixes all phases of the states.

Notation:

$$\langle p(p_3, s_3), p(p_4, s_4) | T | p(p_1, s_1), p(p_2, s_2) \rangle$$

$$\equiv \langle 2s_3, 2s_4 | T | 2s_1, 2s_2 \rangle$$

Symmetries of the reaction:

$U_2(\pi)$ rotation by π around the positive y axis

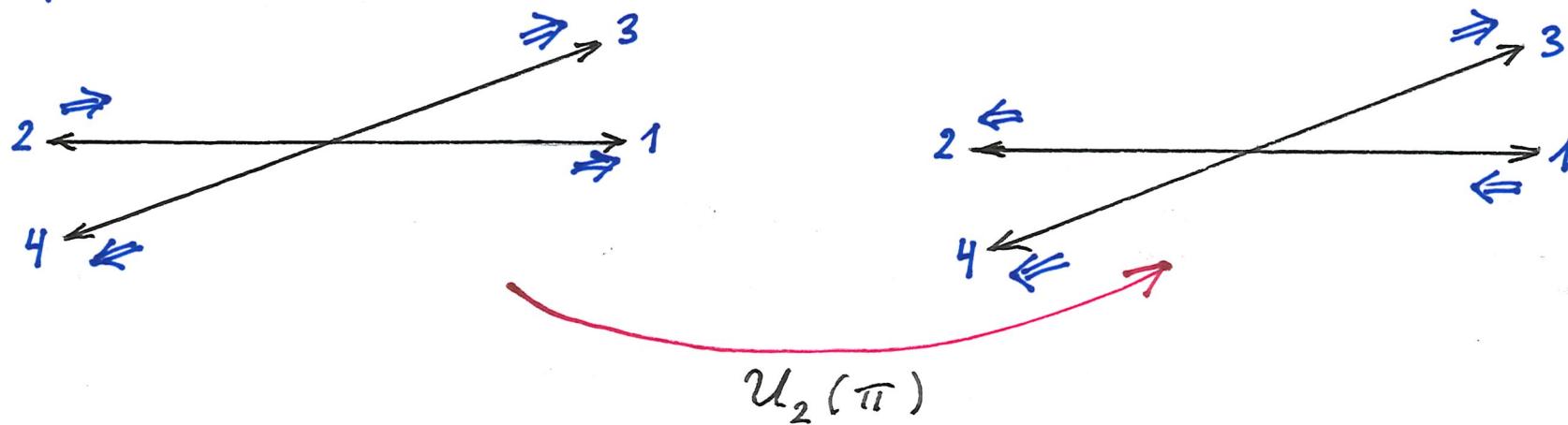
$U(P)$ parity transformation

$U_3(\pi) U_2(-\vartheta) V(T)$ time reversal (antiunitary)

followed by rotation by $-\vartheta$ around pos. y axis

followed by rotation by π around pos. z axis

Example:



$$\langle + + | T | + - \rangle = - \langle + + | T | - + \rangle$$

$U_2(\pi)$	$U(P)$	$U_{32}(\pi_1, -\pi_2) V(T)$	
$\langle + + T + + \rangle$	$\langle + + T + + \rangle$	$\langle - - T - \rangle$	$\langle + + J + + \rangle$
$\langle + + T + - \rangle$	$- \langle + + T - + \rangle$	$\langle - - T + - \rangle$	$\langle - + J + + \rangle$
$\langle + + T - + \rangle$	$- \langle + + T + - \rangle$	$\langle - - T - + \rangle$	$\langle + - J + + \rangle$
$\langle + + T - - \rangle$	$\langle + + T - - \rangle$	$\langle - - T + + \rangle$	$\langle - - J + + \rangle$
$\langle + - T + + \rangle$	$- \langle - + T + + \rangle$	$\langle + - T - - \rangle$	$\langle + + J - + \rangle$
$\langle + - T + - \rangle$	$\langle - + T - + \rangle$	$\langle + - T + - \rangle$	$\langle - + J - + \rangle$
$\langle + - T - + \rangle$	$\langle - + T + - \rangle$	$\langle + - T - + \rangle$	$\langle + - J - + \rangle$
$\langle + - T - - \rangle$	$- \langle - + T - - \rangle$	$\langle + - T + + \rangle$	$\langle - - J - + \rangle$
$\langle - + T + + \rangle$	$- \langle + - T + + \rangle$	$\langle - + T - - \rangle$	$\langle + + J + - \rangle$
$\langle - + T + - \rangle$	$\cdot \langle + - T - + \rangle$	$\langle - + T + - \rangle$	$\langle - + J + - \rangle$
$\langle - + T - + \rangle$	$\langle + - T + - \rangle$	$\langle - + T - + \rangle$	$\langle + - J + - \rangle$
$\langle - + T - - \rangle$	$- \langle + - T - - \rangle$	$\langle - + T + + \rangle$	$\langle - - J + - \rangle$
$\langle - - T + + \rangle$	$\langle - - T + + \rangle$	$\langle + + T - - \rangle$	$\langle + + J + - \rangle$
$\langle - - T + - \rangle$	$- \langle - - T - + \rangle$	$\langle + + T + - \rangle$	$\langle - + J - - \rangle$
$\langle - - T - + \rangle$	$- \langle - - T + - \rangle$	$\langle + + T - + \rangle$	$\langle + - J - - \rangle$
$\langle - - T - - \rangle$	$\langle - - T - - \rangle$	$\langle + + T + + \rangle$	$\langle - - J - - \rangle$

five independent amplitudes: $\langle 2s_3, 2s_4 | T | 2s_1, 2s_2 \rangle$

helicity:

$$\phi_1(s, t) = \langle + + | T | + + \rangle$$

non flip

$$\phi_2(s, t) = \langle + + | T | - \rightarrow \rangle$$

double flip

$$\phi_3(s, t) = \langle + - | T | + \rightarrow \rangle$$

non flip

$$\phi_4(s, t) = \langle + - | T | - + \rangle$$

double flip

$$\phi_5(s, t) = \langle + + | T | + - \rangle$$

single flip

$$\sigma_{\text{tot}}(p, p) = \frac{1}{4\sqrt{s}(s - 4m_p^2)} \sum_{s_1, s_2} \text{Im} \langle 2s_1, 2s_2 | T | 2s_1, 2s_2 \rangle \Big|_{t=0}$$

$$= \frac{1}{2\sqrt{s}(s - 4m_p^2)} \text{Im} [\phi_1(s, 0) + \phi_3(s, 0)]$$

3 Tensor, vector, or scalar pomeron?

These are the three hypotheses we want to test.

We choose our ansätze for the effective \bar{P} propagators and $\bar{P}pp$ couplings such that at high energies the non-flip amplitudes ϕ_1 and ϕ_3 are the same for all three cases. This gives the same $\sigma_{\text{tot}}(p, p)$.

The Donnachie - Landshoff (DL) model treats the pomeron as effective vector exchange and gives a phenomenologically successful fit to $\sigma_{\text{tot}}(p, p)$ and $d\sigma/dt$.

Our ansätze are chosen such that ϕ_1 and ϕ_3 are as in the DL model.

Tensor pomeron

Here we describe the pomeron by a symmetric, traceless, tensor field of rank 2

$$P_T^{\mu\nu}(x) = P_T^{\nu\mu}(x) , \quad P_T^{\mu\nu}(x) g_{\mu\nu} = 0$$

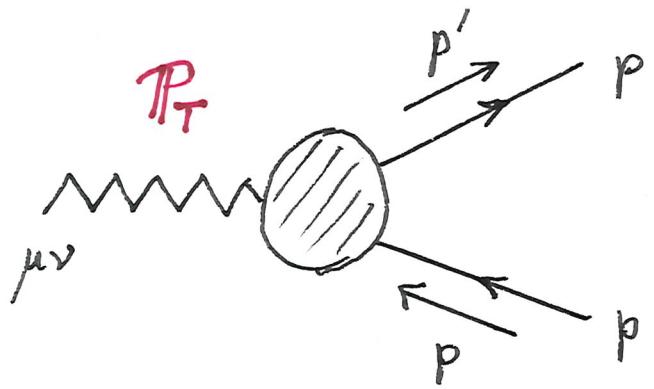
P p p coupling:

$$\mathcal{L}'_T(x) = J_{T\mu\nu}(x) P_T^{\mu\nu}(x),$$

$$J_{T\mu\nu}(x) = -3\beta_{PNN} \frac{i}{2} \bar{\psi}_p(x) \left[\gamma_\mu \overset{\leftrightarrow}{\partial}_\nu + \gamma_\nu \overset{\leftrightarrow}{\partial}_\mu - \frac{1}{2} g_{\mu\nu} \gamma^\lambda \overset{\leftrightarrow}{\partial}_\lambda \right] \psi_p(x)$$

$$3\beta_{PNN} = 3 \times 1.87 \text{ GeV}^{-1}$$

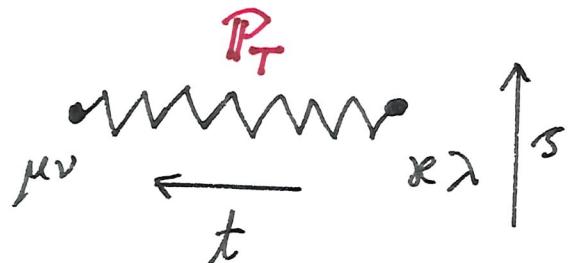
$\psi_p(x)$ proton field operator



$$i \int_{\mu\nu}^{(P_T pp)} (p', p) = -i 3 \beta_{PNN} F_1 [(p' - p)^2] \left\{ \frac{1}{2} g_{\mu} (p' + p),_{\nu} + (\mu \leftrightarrow \nu) - \frac{1}{4} g_{\mu\nu} (p' + p) \right\}$$

$F_1(t)$: form factor, $F_1(0) = 1$.

P_T propagator:



$$i \Delta_{\mu\nu, x\lambda}^{(P_T)}(s, t) = \frac{1}{4s} \left(g_{\mu x} g_{\nu \lambda} + g_{\mu \lambda} g_{\nu x} - \frac{1}{2} g_{\mu \nu} g_{x \lambda} \right) (-is\alpha'_P)^{\alpha_P(t)-1}$$

$$\alpha_P(t) = 1 + \varepsilon_P + \alpha'_P t \quad \text{linear pomeron trajectory}$$

$$\varepsilon_P = 0.0808 ,$$

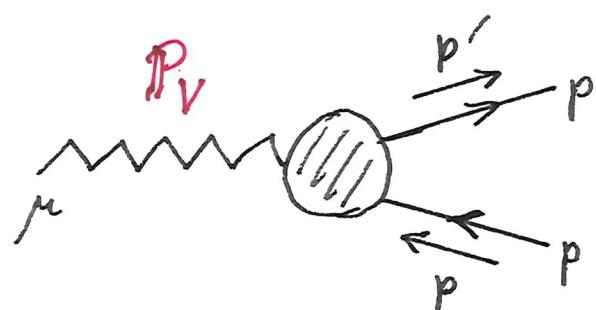
$$\alpha'_P = 0.25 \text{ GeV}^{-2}$$

Vector pomeron (DL pomeron)

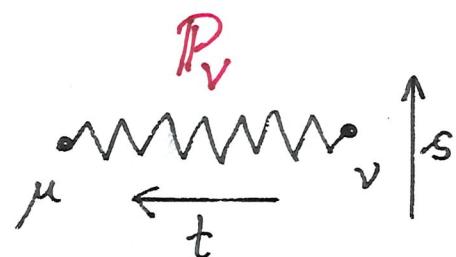
Here the pomeron is described by a vector field $P_V^\mu(x)$.

$$\mathcal{L}'_V(x) = \bar{J}_{V\mu}(x) P_V^\mu(x),$$

$$\bar{J}_{V\mu}(x) = -3\beta_{PNN} M_0 \bar{\Psi}_p(x) \gamma_\mu \Psi_p(x), M_0 \equiv 1 \text{ GeV}$$



$$i\Gamma_\mu^{(P_V pp)}(p', p) = -i3\beta_{PNN} M_0 F_1[(p'-p)^2] \gamma_\mu$$



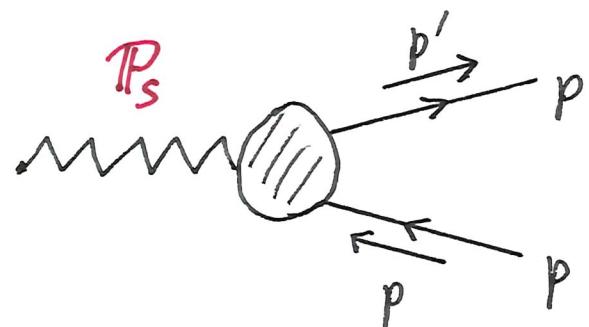
$$i\Delta_{\mu\nu}^{(P_V)}(s, t) = \frac{1}{M_0^2} g_{\mu\nu} (-is\alpha'_p)^{\alpha_p(t)-1}$$

Scalar pomeron

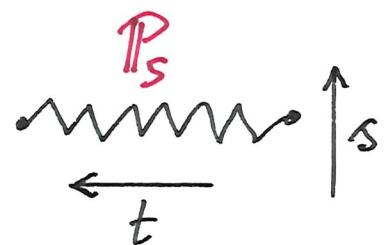
Here the pomeron is described by a scalar field $P_s(x)$.

$$\mathcal{L}'_s(x) = \mathcal{J}_s(x) P_s(x)$$

$$\mathcal{J}_s(x) = -3\beta_{PNN} M_0 \bar{\psi}_p(x) \psi_p(x)$$

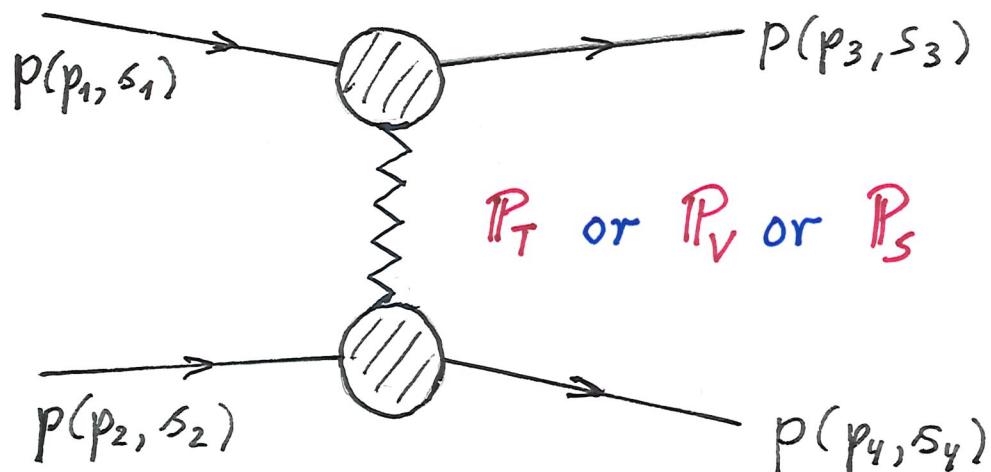


$$i \Gamma^{(P_s pp)}(p', p) = -i 3\beta_{PNN} M_0 F_1 [(p' - p)^2]$$



$$i \Delta^{(P_s)}(s, t) = \frac{s}{2m_p^2 M_0^2} (-is\alpha'_p)^{\alpha_p(t)-1}$$

Calculation of the amplitudes $\phi_j(s, t)$:



reduced amplitudes:

$$\hat{\phi}_j(s, t) = \phi_j(s, t) / \mathcal{F}(s, t) \quad j=1, \dots, 5$$

$$\mathcal{F}(s, t) = i [3\beta_{PNN} F_1(t)]^2 \frac{1}{4s} (-is\alpha'_P)^{\alpha_P(t)-1}$$

pomeron ansatz

	tensor	vector	scalar
$\hat{\phi}_1(s, t)$	$8s^2$	$8s^2$	$8s^2$
$\hat{\phi}_2(s, t)$	$10m_p^2 t$	$16m_p^2 t$	$2s^2 t / m_p^2$
$\hat{\phi}_3(s, t)$	$8s^2$	$8s^2$	$8s^2$
$\hat{\phi}_4(s, t)$	$-10m_p^2 t$	$-16m_p^2 t$	$-2s^2 t / m_p^2$
$\hat{\phi}_5(s, t)$	$-8s m_p \sqrt{-t}$	$-8s m_p \sqrt{-t}$	$-4s^2 \sqrt{-t} / m_p$

$$s \gg m_p^2, |t|$$

4 Comparison with experiment

total cross section:

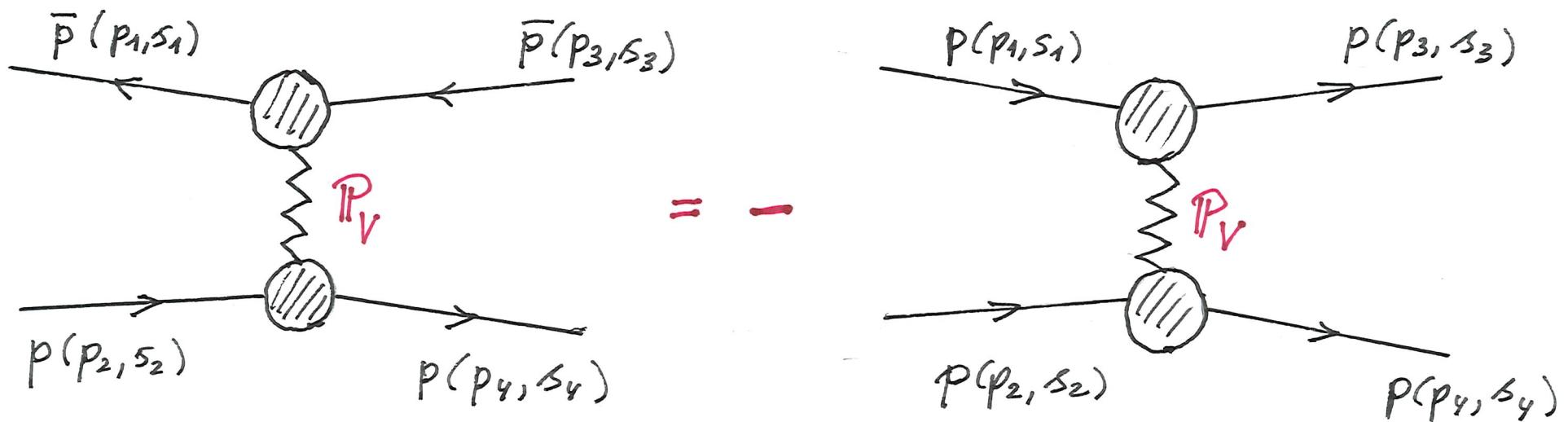
$$\begin{aligned}\sigma_{\text{tot}}(p,p) &= \frac{1}{2s} \operatorname{Im} [\phi_1(s,0) + \phi_3(s,0)] \\ &= 2 \left(3\beta_{PNN}\right)^2 \cos\left(\frac{\pi}{2}\varepsilon_p\right) (\simeq \alpha'_p)^{\varepsilon_p}\end{aligned}$$

This is the standard result of the DL model:

σ_{tot} rises with a small power, $\varepsilon_p = 0.0808$, of s .

By construction we have the same result for our tensor, vector, and scalar pomeron.

Problems with the vector pomeron:



The minus sign for $\bar{p}p$ versus pp has the same origin as for e^+e^- versus e^-e^- scattering in the one-photon exchange approximation. Vector exchange has $C = -1$.
It follows

$$\sigma_{\text{tot}}(\bar{p}, p)^{\text{P}_V} = - \sigma_{\text{tot}}(p, p)^{\text{P}_V}$$

In our opinion a vector pomeron P_V is not a viable option.

We are left with P_T and P_S . Both correspond to charge conjugation $C = +1$ exchanges:

$$\langle \bar{p}, p | T | \bar{p}, p \rangle^{P_T} = \langle p, p | T | p, p \rangle^{P_T}$$

$$\langle \bar{p}, p | T | \bar{p}, p \rangle^{P_S} = \langle p, p | T | p, p \rangle^{P_S}$$

Remember gravity, also a tensor exchange, which gives the same attraction for $\bar{p}p$ and pp .

To decide between P_T and P_S we turn to the STAR experiment (PL B 719 (2013) 62) which measured the single spin asymmetry A_N in polarised pp elastic scattering at $\sqrt{s} = 200$ GeV.

The experiment is done at very small $|t|$,

$$0.003 \leq |t| \leq 0.035 \text{ GeV}^2,$$

that is, in the Coulomb-nuclear interference region. This allows to extract real and imaginary part of the single-flip amplitude $\phi_5(s, t)$. Quoted is

$$\pi_5(s, t) = \frac{2m_p \phi_5(s, t)}{\sqrt{-t} \operatorname{Im} [\phi_1(s, t) + \phi_3(s, t)]}$$

We get for the tensor pomeron

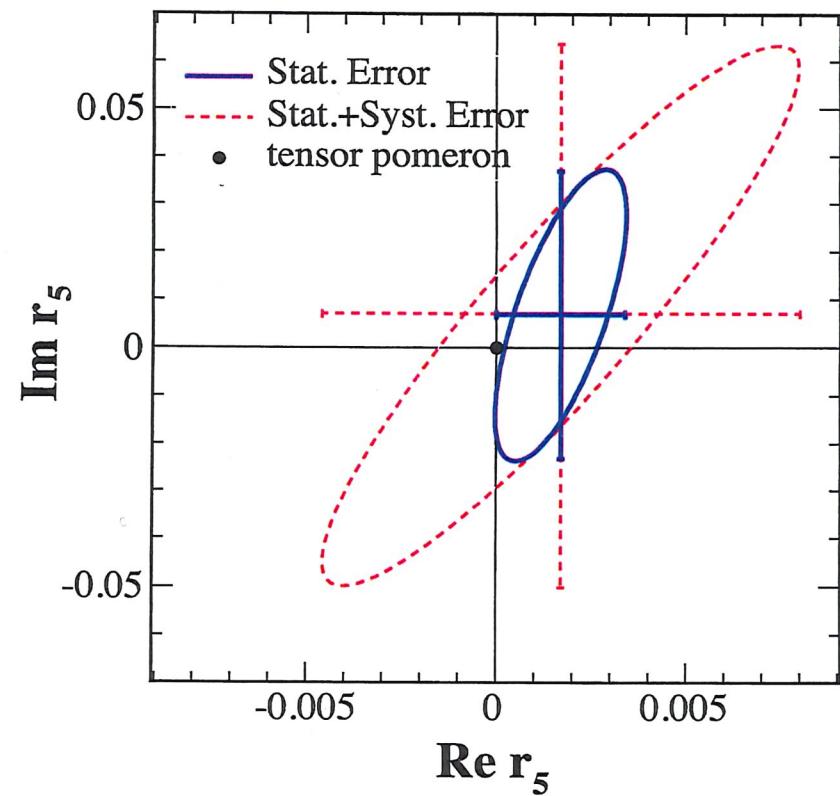
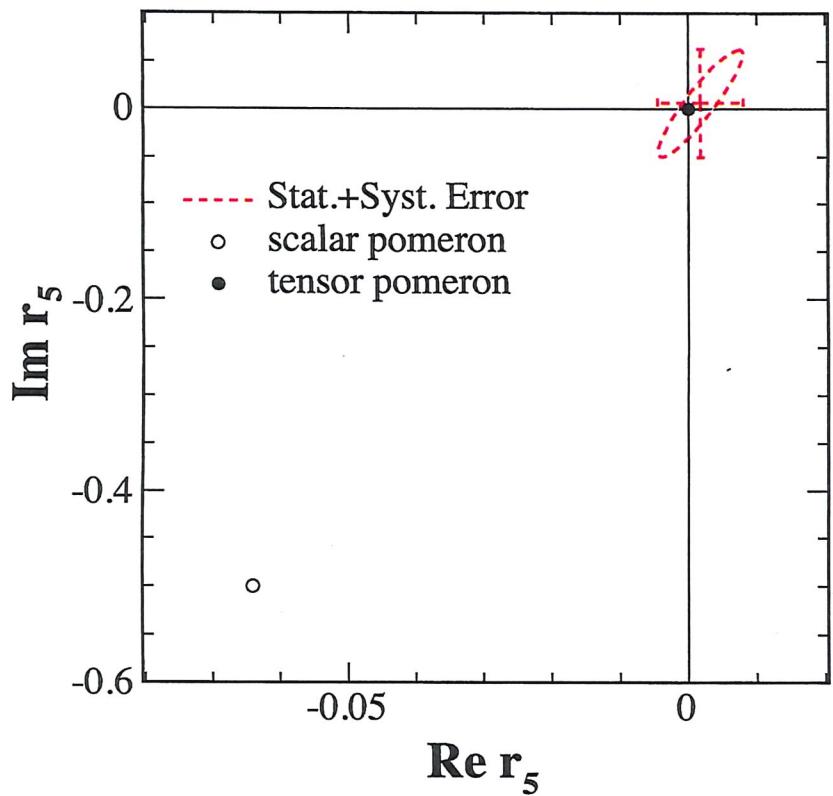
$$r_5^{P_T}(s, t) = -\frac{m_P^2}{s} \left[i + \tan \left(\frac{\pi}{2} (\alpha_P(t) - 1) \right) \right]$$

$$r_5^{P_T}(s, 0) \Big|_{s=(200 \text{ GeV})^2} = (-0.28 - i 2.20) \times 10^{-5}$$

and for the scalar pomeron

$$r_5^{P_S}(s, t) = -\frac{1}{2} \left[i + \tan \left(\frac{\pi}{2} (\alpha_P(t) - 1) \right) \right]$$

$$r_5^{P_S}(s, 0) \Big|_{s=(200 \text{ GeV})^2} = -0.064 - i 0.500$$



Experiment : STAR

Theory : Ewerz, Lebiedowicz, O.N., Szczurek

5 Conclusions and historical remarks for part I

From our three ansätze for the pomeron, tensor, vector, or scalar, only the tensor pomeron is compatible with the general rules of QFT and the STAR experimental result.

History: attempts to relate the pomeron to tensors were already made in the 1960s and 1970s

Freund, PL 2 (1962) 136, NCA 5 (1971) 9

Carlitz et al. PRL 26 (1971) 1515

But these attempts were abandoned with the advent of QCD

The pomeron as a gluonic object in QCD perturbation theory:

Low, PR D 12 (1975) 163

Nussinov PRL 34 (1975) 1286

Kuraev, Lipatov, Fadin Zh. Eksp. Teor. Fiz. 72 (1977) 377

Balitsky, Lipatov, Yad. Fiz. 28 (1978) 1597

⋮

Phenomenological vector pomeron:

Donnachie, Landshoff, NP B 231 (1984) 189, B 267 (1986) 690

⋮

Pomeron in soft reactions and nonperturbative QCD:

Landshoff, O.N., Z. Phys. C 35 (1987) 405: abelian toy model

O.N. Ann. Phys. 209 (1991) 436: functional integral
techniques

In the latter paper we could show that the pomeron could be understood as the coherent sum of elementary exchanges of spin $2 + 4 + 6 + \dots$

The same applies to our tensor pomeron (Ewerz et al. (2014)), thus giving it good backing in QCD.

To see this explicitly we write the tensor-pomeron propagator in terms of the variable

$$v = \frac{1}{4} (p_1 + p_3, p_2 + p_4) = \frac{1}{4} (s - u) \approx \frac{1}{2} s \text{ for small } |t|.$$

$$i\Delta_{\mu\nu, \alpha\lambda}^{(P_T)}(v, t) = -i(g_{\mu\alpha}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\alpha} - \frac{1}{2}g_{\mu\nu}g_{\alpha\lambda}) \\ \frac{1}{4}\alpha_P' (-4v^2\alpha_P'^2)^{\frac{1}{2}}(\alpha_P(t)-2).$$

This is the form how the P_T propagator comes out when considering analyticity and crossing relations for the $p p \rightarrow p p$ amplitude. Using this and for $s \gg |t|, m_p^2$

$$\bar{u}_{s_3}(p_3) \gamma^\mu u_{s_1}(p_1) \cong (p_3 + p_1)^\mu \delta_{s_3 s_1}, \quad \bar{u}_{s_4}(p_4) \gamma^\mu u_{s_2}(p_2) \cong (p_4 + p_2)^\mu \delta_{s_4 s_2},$$

we get the following:

$$\langle p(p_3, s_3), p(p_4, s_4) | T | p(p_1, s_1), p(p_2, s_2) \rangle^{P_T} =$$

$$\tilde{f}(t) \Gamma\left(1 - \frac{1}{2}\alpha_P(t)\right) \left(4\nu^2 \alpha'_P\right)^2 \frac{1}{2}(\alpha_P(t)-2)$$

$$(p_3 + p_1)^{\mu_1} (p_3 + p_1)^{\mu_2} (p_4 + p_2)_{\mu_1} (p_4 + p_2)_{\mu_2} \delta_{s_3 s_1} \delta_{s_4 s_2},$$

$$\tilde{f}(t) = \left[3\beta_{PNN} F_1(t) \right]^2 \frac{\alpha'_P}{2} e^{-i \frac{\pi}{2} (\alpha_P(t)-2)} \Gamma^{-1}\left(1 - \frac{1}{2}\alpha_P(t)\right).$$

Now we use an integral representation for the regge term:

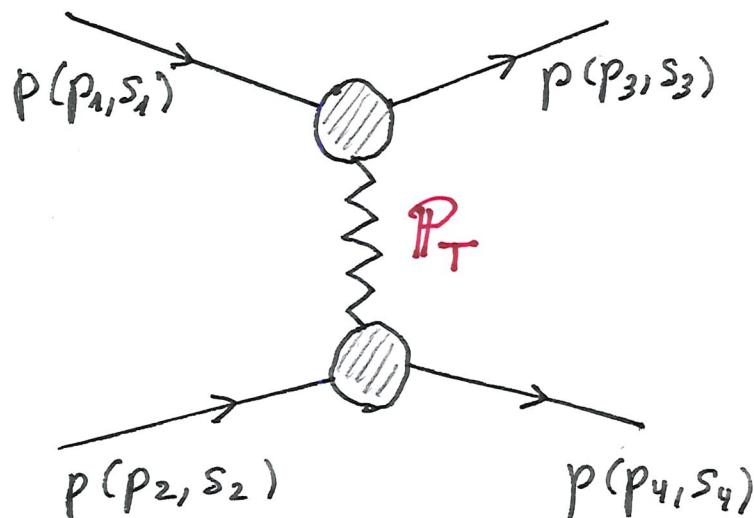
$$\begin{aligned} (4\alpha'^2_P \nu^2)^{\frac{1}{2}(\alpha_P(t)-2)} &= \Gamma^{-1}\left(1 - \frac{1}{2}\alpha_P(t)\right) \int_0^\infty d\tau \tau^{-\frac{1}{2}\alpha_P(t)} \exp\left[-4\alpha'^2_P \nu^2 \tau\right] \\ &= \Gamma^{-1}\left(1 - \frac{1}{2}\alpha_P(t)\right) \int_0^\infty d\tau \tau^{-\frac{1}{2}\alpha_P(t)} \sum_{n=1}^\infty \frac{1}{(n-1)!} \left(-\frac{\alpha'^2_P}{4} \tau\right)^{n-1} (p_3 + p_1, p_4 + p_2)^{2n-2}. \end{aligned}$$

Inserting this we get for the amplitude

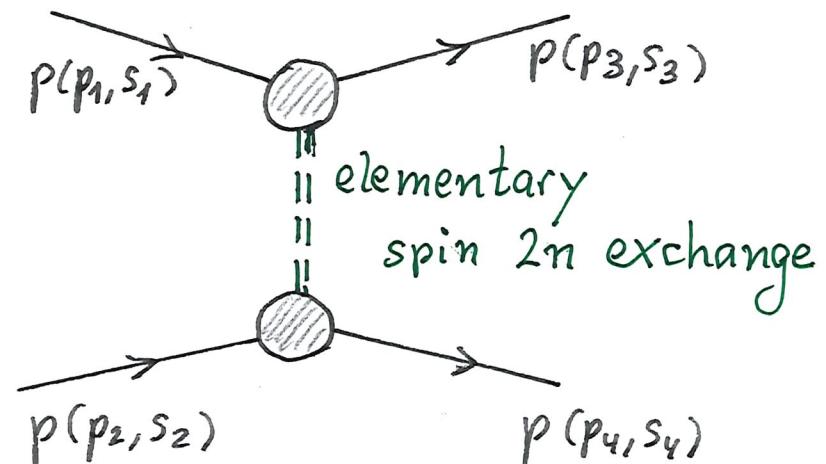
$$\langle p(p_3, s_3), p(p_4, s_4) | T | p(p_1, s_1), p(p_2, s_2) \rangle^{P_T} =$$

$$\tilde{f}(t) \int_0^\infty d\tau \tau^{-\frac{1}{2}} \alpha_P(t) \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \left(-\frac{\alpha'_P}{4} \tau \right)^{n-1}$$

$$(p_3 + p_1)^{\mu_1} \dots (p_3 + p_1)^{\mu_{2n}} (p_4 + p_2)_{\mu_1} \dots (p_4 + p_2)_{\mu_{2n}} \delta_{s_3 s_1} \delta_{s_4 s_2} .$$



$$= \sum_{n=1}^{\infty}$$



Thus, we have indeed written the tensor - pomeron exchange as coherent sum of elementary spin $2 + 4 + 6 + \dots$ exchanges. Note that spin 0 exchange is missing.

How to write a regge exchange as a coherent sum of elementary exchanges was first shown by L. van Hove , PL 24B(1967) 183.

Specific tests for the spin structure of the pomeron have been proposed in

Arens, Diehl, Landshoff, D.N. Z. Phys. C74 (1997) 651 for
diffractive deep inelastic lepton-nucleon scattering;

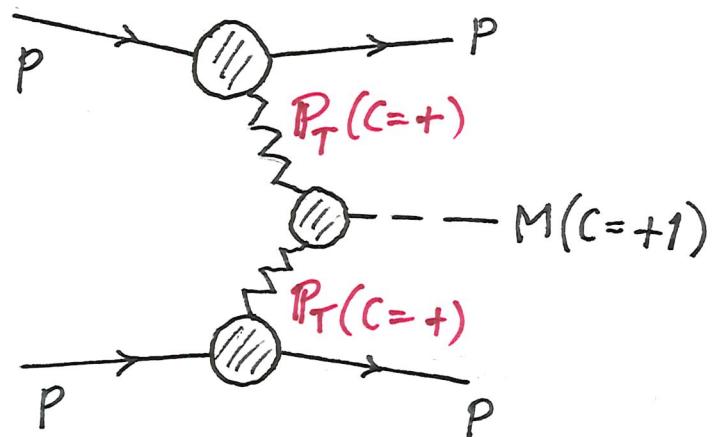
Close, Schuler, PL B458 (1999) 127, B464 (1999) 279
for central exclusive meson production:



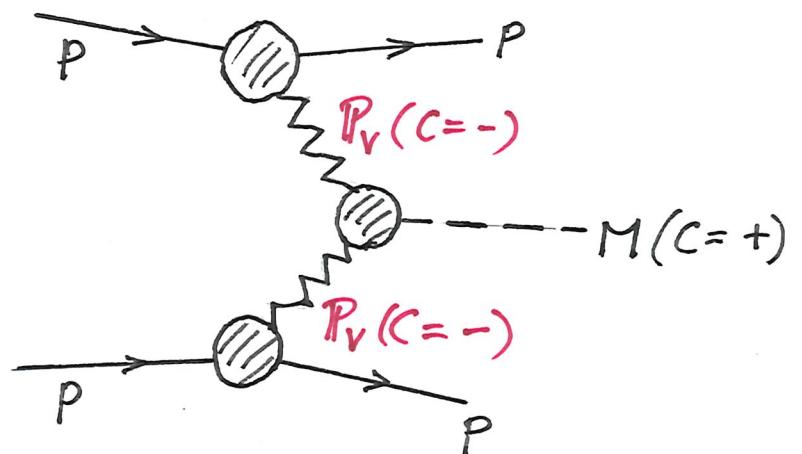
Close and Schuler claim to find evidence that the pomeron transforms as a non-conserved vector current. As we have seen we cannot support such a picture. What is the explanation of this discrepancy?

For central exclusive meson production with double pomeron exchange we have

with the tensor pomeron



with a vector pomeron
(Close, Schuler)

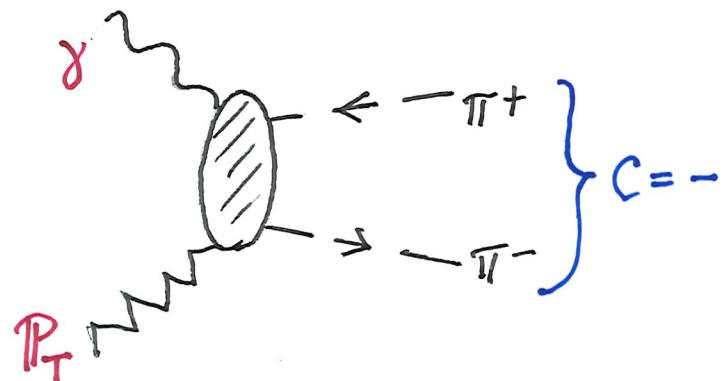


Here we find no drastic differences between P_T and P_V (Lebiedowic, O.N., Szczurek, Ann. Phys. (2014)).

But drastic differences turn up if in the middle we have, e.g., gamma-pomeron fusion.

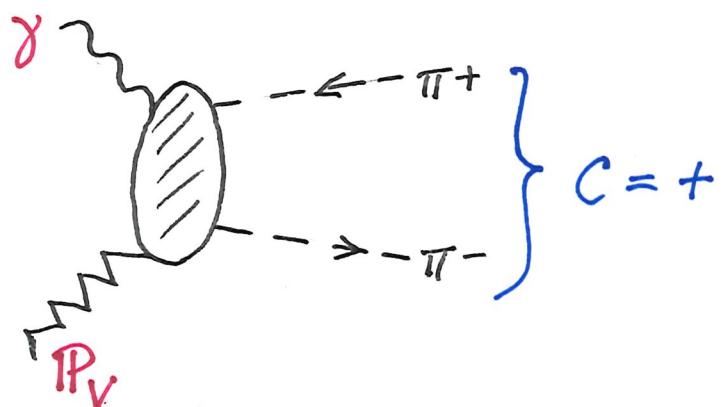
$$\text{Example : } \gamma + P \rightarrow \pi^+ + \pi^-$$

tensor pomeron



$\pi^+ \pi^-$ in antisymmetric state

vector pomeron



$\pi^+ \pi^-$ in symmetric state

This gives a further clear evidence against a vector nature of the pomeron.

Investigations of the pomeron using the methods
of AdS/CFT correspondence also prefer a tensor
nature of the pomeron:

Domokos, Harvey, Mann, PRD 80 (2009) 126015

Iatrakis, Ramamurti, Shuryak, PRD 94 (2016) 045005

Here is a partial list of soft reactions which were investigated using the tensor-pomeron concept.

- $\gamma + p \rightarrow \pi^+ + \pi^- + p$ allows also search for the odderon.
- $p + p \rightarrow p + M + p$ central production of mesons,
 $M = \eta, \eta', f_0(980), f_0(1370),$
 $f_0(1500).$
- $p + p \rightarrow p + \pi^+ + \pi^- + p$ central production of $\pi^+ \pi^-$ non-resonant, from g^0 , scalar and tensor resonances.
- $p + p \rightarrow p + g^0 + n + \pi^+$ central production of g^0 with diffractive excitation of one proton.

All these studies could only be published due to
the excellent work of

A. Bolz,

C. Ewerz,

P. Lebiedowicz,

M. Maniatis,

M. Sauter,

A. Schöning,

A. Szczurek

My thanks go to them.

Part II From the soft to the hard pomeron

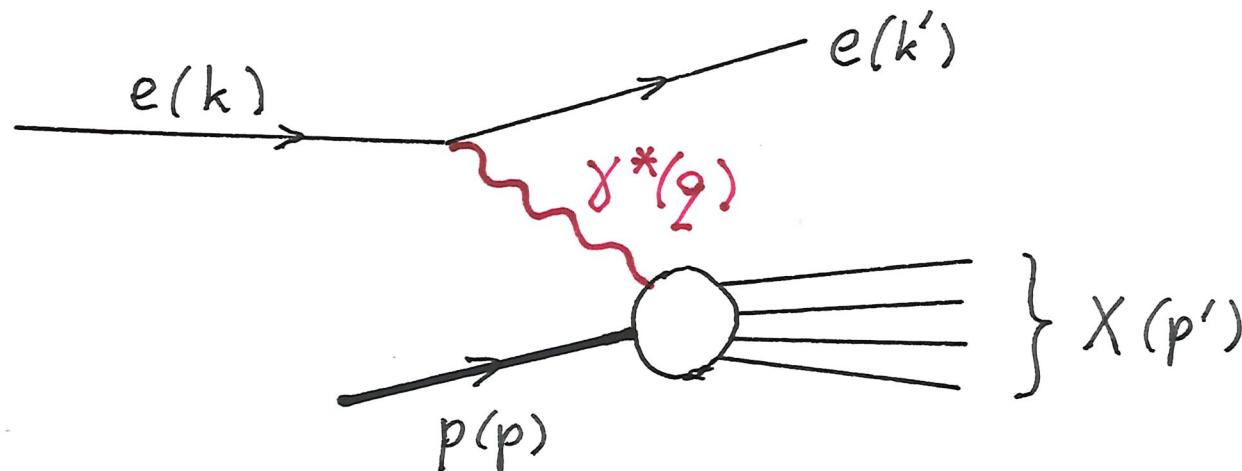
6 Introductory remarks

We hope to have convinced you that the tensor-pomeron concept makes sense for high-energy soft reactions.

At least we have a reasonable model where the pomeron satisfies all standard relations of QFT, in particular, it has automatically $C = +1$. The pomeron couplings are described by effective Lagrangians. This makes the inclusion of photon couplings easy via the minimal substitution.

What can we say in the tensor-pomeron approach when going from soft to hard reactions?

We investigate this for DIS:



kinematic variables:

$$s = (p+k)^2, \quad q = k - k', \quad Q^2 = -q^2,$$

$$W^2 = p'^2 = (p+q)^2, \quad v = p \cdot q / m_p = \frac{W^2 + Q^2 - m_p^2}{2m_p}$$

$$x = \frac{Q^2}{2m_p v} = \frac{Q^2}{W^2 + Q^2 - m_p^2}, \quad y = \frac{p \cdot q}{p \cdot k}$$

We follow in our work Donnachie & Landshoff,
PL B 437 (1998) 408, and assume that there are two
pomerons, but of tensor type:

hard pomeron \mathcal{P}_0

soft pomeron \mathcal{P}_1

Our aim is to develop a model describing in one
framework

- the hard regime of low x DIS, that is, HERA
data for $x \leq 0.01$, $Q^2 \leq 50 \text{ GeV}^2$;
- the soft regime, $Q^2 \leq 1 \text{ GeV}^2$, in particular
photoproduction, $Q^2 = 0$.

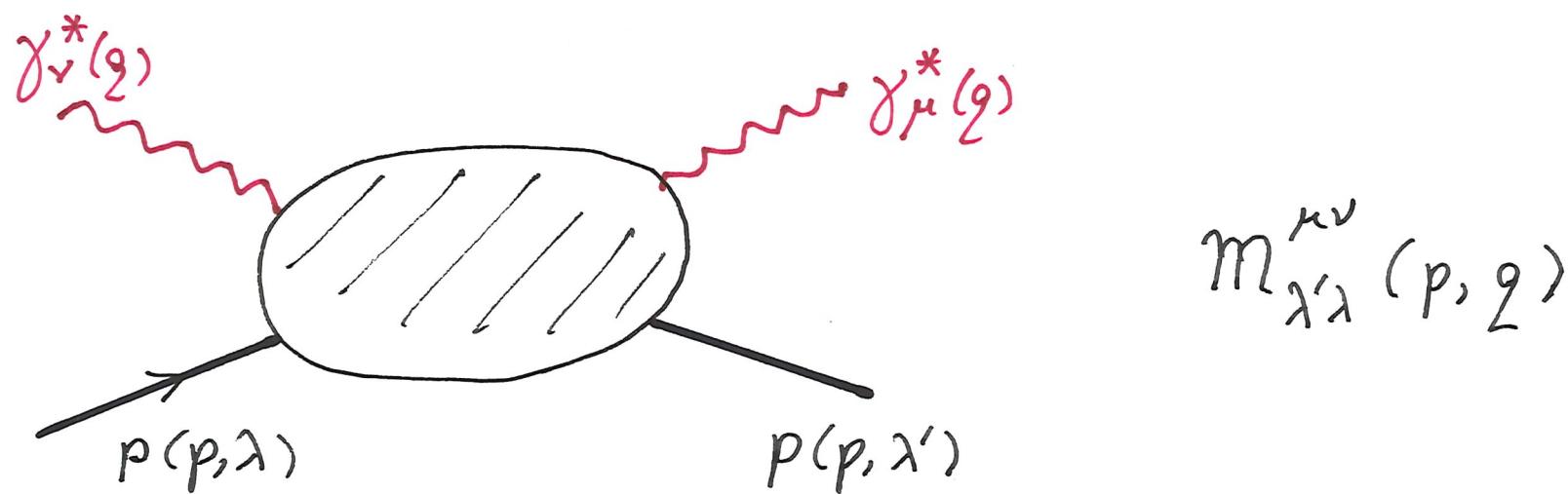
D. Britzger, C. Ewerz, S. Glazov, O.N., S. Schmitt, work in preparation

7 The two-tensor-pomeron model

The reaction we study as theorists is forward real and virtual Compton scattering

$$\gamma_v^{(*)}(q) + p(p, \lambda) \rightarrow \gamma_\mu^{(*)}(q) + p(p, \lambda')$$

$$\lambda, \lambda' \in \{ \frac{1}{2}, -\frac{1}{2} \}$$



Hadronic tensor of DIS:

$$W^{\mu\nu}(p, q) = \sum_{\lambda', \lambda} \frac{1}{2} \delta_{\lambda\lambda'} \frac{1}{2i} \left[M_{\lambda'\lambda}^{\mu\nu}(p, q) - (M_{\lambda\lambda'}^{\nu\mu}(p, q))^* \right]$$

From this we get in the usual way the cross sections for absorption of transverse and longitudinal virtual photons on the proton

$$\sigma_T(w^2, Q^2), \quad \sigma_L(w^2, Q^2).$$

The photoabsorption cross section for real photons is

$$\sigma_{\text{tot}, \gamma p}(w^2) = \sigma_T(w^2, 0).$$

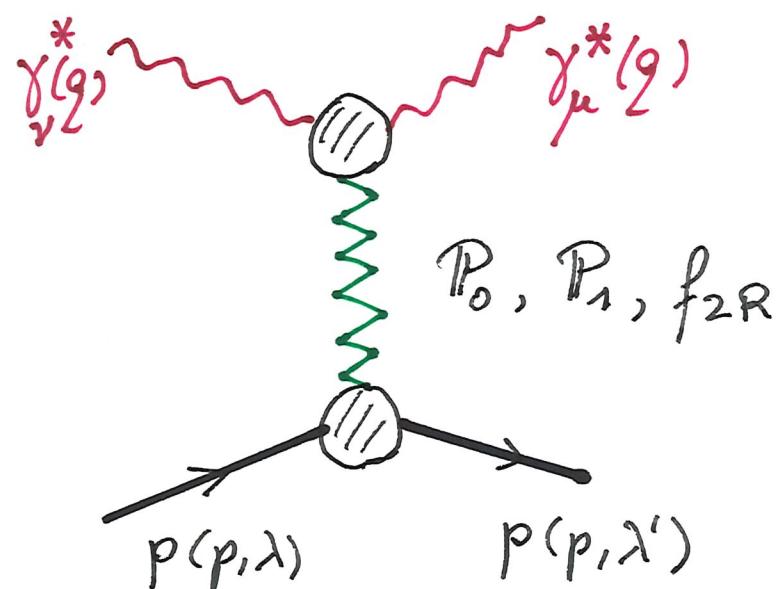
$$W^{\mu\nu}(p, q) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) W_1(v, Q^2)$$

$$+ \frac{1}{m_p^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) W_2(v, Q^2)$$

$$\sigma_T(w^2, Q^2) = \frac{2\pi m_p}{w^2 - m_p^2} e^2 W_1(v, Q^2)$$

$$\sigma_L(w^2, Q^2) = \frac{2\pi m_p}{w^2 - m_p^2} e^2 \left[\frac{v^2 + Q^2}{Q^2} W_2(v, Q^2) - W_1(v, Q^2) \right]$$

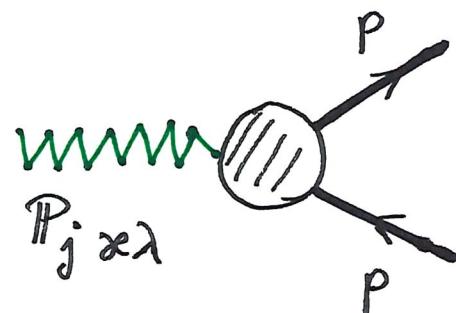
In our model we describe the virtual Compton amplitude at high energies by exchange of hard and soft pomeron. In addition we consider f_{2R} reggeon exchange, relevant for lower W .



Exchange diagrams for

$$M_{\lambda\lambda'}^{\mu\nu}(p, q)$$

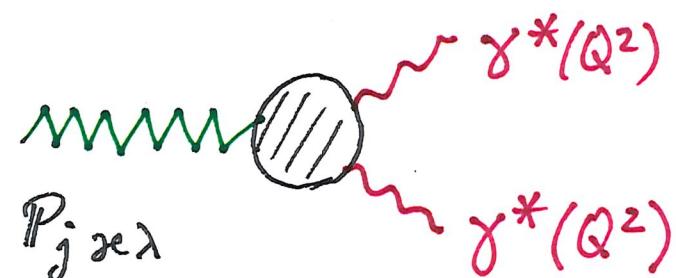
Vertices and propagators



$$\mathcal{L}'_{P_j pp}(x) = -3\beta_j P_j x_\lambda(x)$$

$$\frac{i}{2} \bar{\psi}_p(x) \left[\gamma^\mu \overset{\leftrightarrow}{\partial}^\lambda + \gamma^\lambda \overset{\leftrightarrow}{\partial}^\mu - \frac{1}{2} g^{\mu\lambda} \gamma^\rho \overset{\leftrightarrow}{\partial}_\rho \right] \psi_p(x),$$

$$j=0,1, \quad \beta_j = 1.87 \text{ GeV}^{-1},$$



$$\mathcal{L}'_{P_j \gamma^* \gamma^*}(x) = \left(g^{\mu\lambda} g^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} g^{\lambda\lambda} \right) P_j x_\lambda(x)$$

$$e^2 \left[\hat{a}_j(Q^2) \left(\partial_\mu F_{\mu\nu}(x) \right) \left(\partial_\lambda F^{\lambda\nu}(x) \right) + \hat{b}_j(Q^2) F_{\mu\nu}(x) F^{\mu\nu}(x) \right],$$

$$j=0,1, \quad F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x),$$

$$i \Delta_{\mu\nu, \nu\lambda}^{(P_j)} (W^2, t) = \frac{(-i W^2 \tilde{\alpha}'_j)^{\alpha_{P_j}(t)-1}}{4 W^2} \\ \left(g_{\mu\nu} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\nu} - \frac{1}{2} g_{\mu\nu} g_{\nu\lambda} \right),$$

$\alpha_{P_j}(t)$ P_j trajectory ($j=0,1$),

$\alpha_{P_j}(0) = 1 + \varepsilon_j$ intercept,

$\tilde{\alpha}'_j = 0.25 \text{ GeV}^{-2}$ scale parameter

Our ansatz for the f_{2R} reggeon is similar.

Theoretical results, neglecting for this presentation terms of order m_p^2/W^2 and Q^2/W^2 :

$$\tilde{\sigma}_{\text{tot}, \gamma p}(W^2) = \tilde{\sigma}_T(W^2, 0) = 4\pi\alpha \sum_{j=0,1} 3\beta_j (W^2 \tilde{\alpha}_j')^{\varepsilon_j} \cos\left(\frac{\pi}{2}\varepsilon_j\right) \hat{b}_j(0)$$

+ f_2 R \text{ reggeon term,}

$$\tilde{\sigma}_T(W^2, Q^2) + \tilde{\sigma}_L(W^2, Q^2) = 4\pi\alpha \sum_{j=0,1} 3\beta_j (W^2 \tilde{\alpha}_j')^{\varepsilon_j} \cos\left(\frac{\pi}{2}\varepsilon_j\right) \hat{b}_j(Q^2),$$

$$\tilde{\sigma}_L(W^2, Q^2) = 4\pi\alpha Q^2 \sum_{j=0,1} 3\beta_j (W^2 \tilde{\alpha}_j')^{\varepsilon_j} \cos\left(\frac{\pi}{2}\varepsilon_j\right) 2 \hat{a}_j(Q^2).$$

All gauge invariance relations are satisfied, in particular,

$$\tilde{\sigma}_L(W^2, Q^2) \propto Q^2 \quad \text{for } Q^2 \rightarrow 0.$$

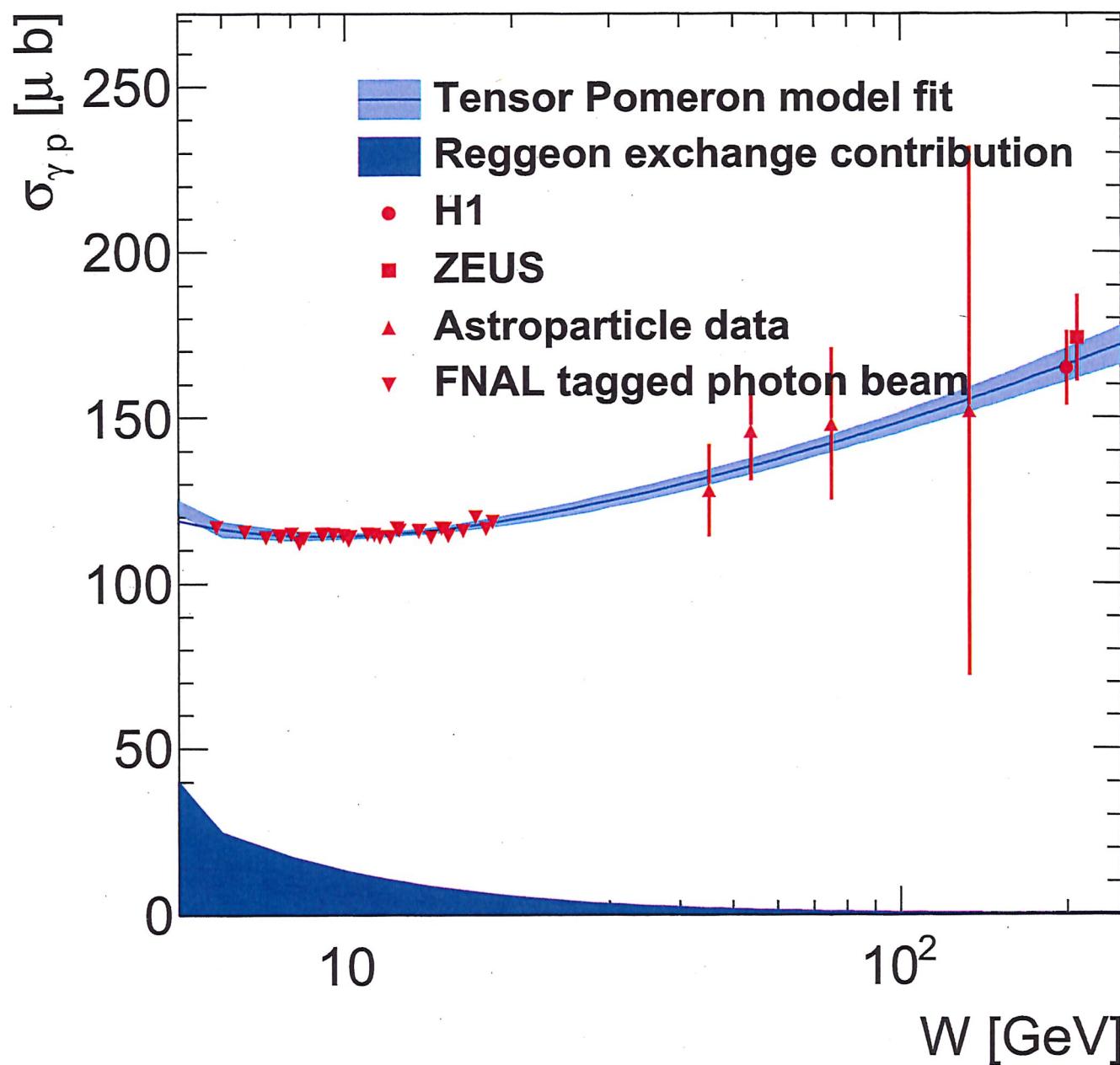
8 Results

Important: all results referring to data shown in the following are **preliminary!**

We make a global fit to the following data set:

- HERA inclusive data for $x \leq 0.01$ and $Q^2 \leq 50 \text{ GeV}^2$
(Abramowicz et al., H1 and ZEUS Coll., EPJ C75 (2015) 580)
- Photo production data from
H1 at $W = 200 \text{ GeV}$ (Aid et al. Z.Phys. C69 (1995) 27)
ZEUS at $W = 209 \text{ GeV}$ (Chekanov et al. NP B 627 (2002), 3)
astroparticle obs. at $W = 40$ to 150 GeV
(Vereshkov et al. Phys. Atom. Nucl. 66 (2003) 565)
- FNAL at $W \approx 6$ to 19 GeV
(Caldwell et al. PRL 40 (1978) 1222)

Fit result for photo production



For real photoabsorption, a soft reaction, we find dominance of the soft pomeron P_1 . The hard pomeron, P_0 , contribution is compatible with zero. At $W = 200 \text{ GeV}$, for instance, we find the following contributions to $\sigma_{\text{tot}, \gamma p}$:

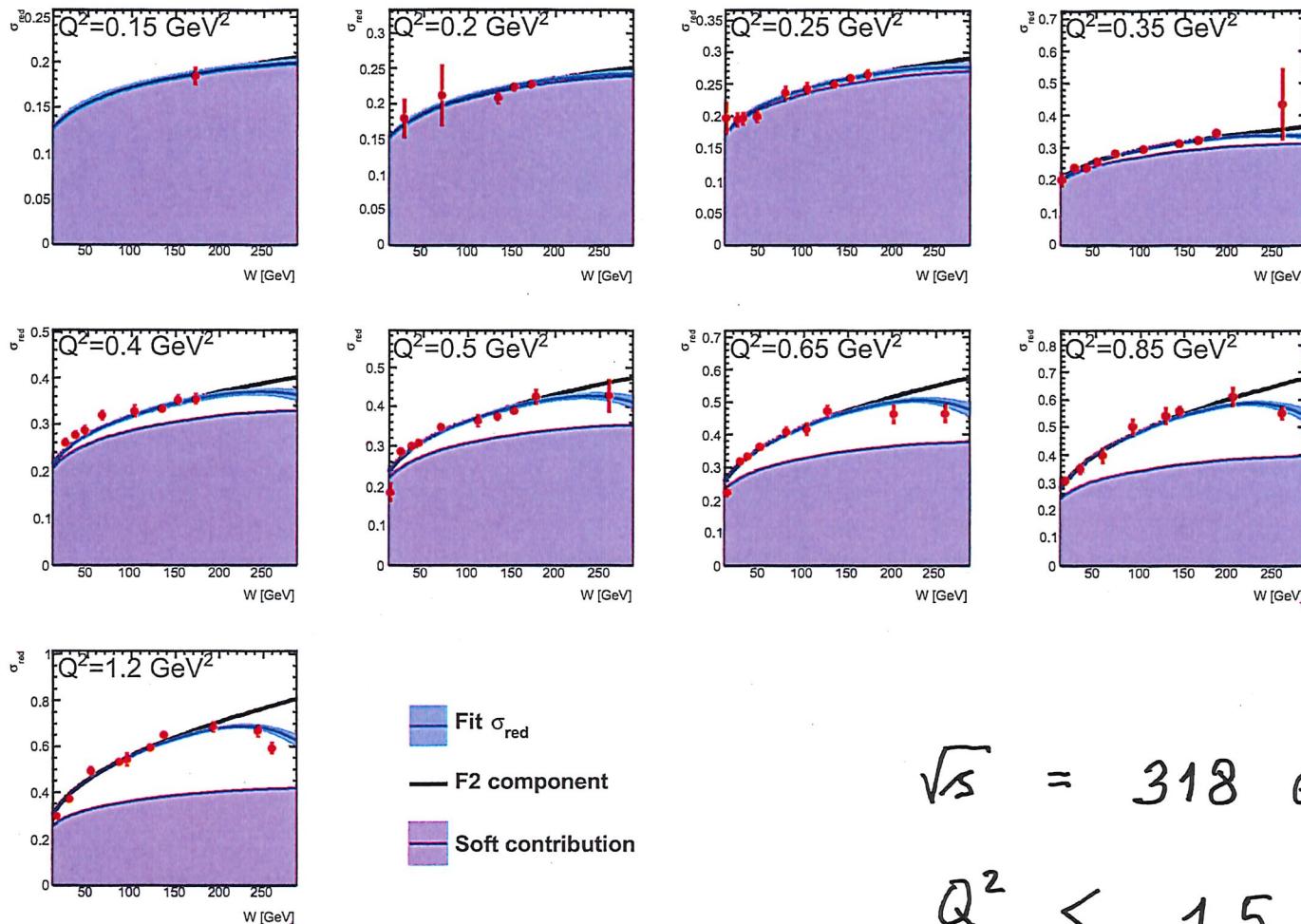
soft pomeron	P_1	$165.9 \pm 5.0 \mu b,$
hard pomeron	P_0	$0.04 \pm 0.17 \mu b,$
reggeon	f_{2R}	$0.33 \pm 1.14 \mu b.$

For DIS the directly measured quantity is the reduced cross section :

$$\begin{aligned}\tilde{\sigma}_T(W^2, Q^2, y) &= \frac{Q^4 x}{2\pi \alpha^2 [1 + (1-y)^2]} \frac{d^2\sigma(ep \rightarrow eX)}{dx dQ^2} \\ &= \frac{Q^2}{4\pi^2 \alpha} (1-x) \left[\tilde{\sigma}_T(W^2, Q^2) + \tilde{\sigma}_L(W^2, Q^2) - \frac{y^2}{1 + (1-y)^2} \tilde{\sigma}_L(W^2, Q^2) \right]\end{aligned}$$

As an example we show our fit to the HERA data at $\sqrt{s} = 318$ GeV. The fits at the other values of $\sqrt{s} = 225, 251$, and 300 GeV are similar.

$\sigma_n(w^2, Q^2, y)$ for fixed Q^2 as function of W [GeV].



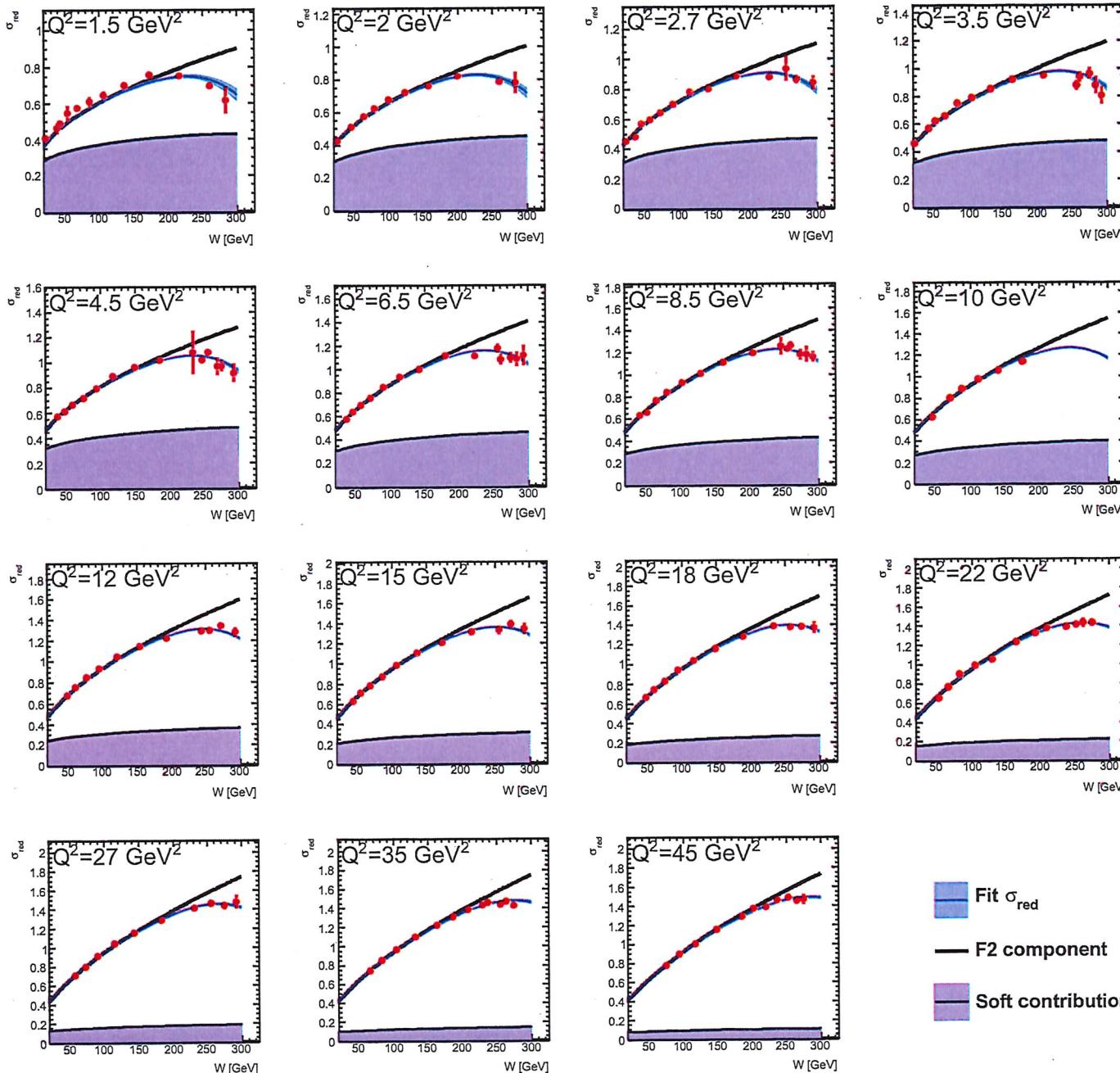
$$\sqrt{s} = 318 \text{ GeV},$$

$$Q^2 < 1.5 \text{ GeV}^2,$$

$$y = \frac{w^2 + Q^2 - m_p^2}{s - m_p^2}.$$

$$\sqrt{s} = 318 \text{ GeV}, \quad 1.5 \leq Q^2 \leq 45 \text{ GeV}^2$$

Note
the
decrease
of the
soft
contribution
and
the
need
for
 $\sigma_L \neq 0$.



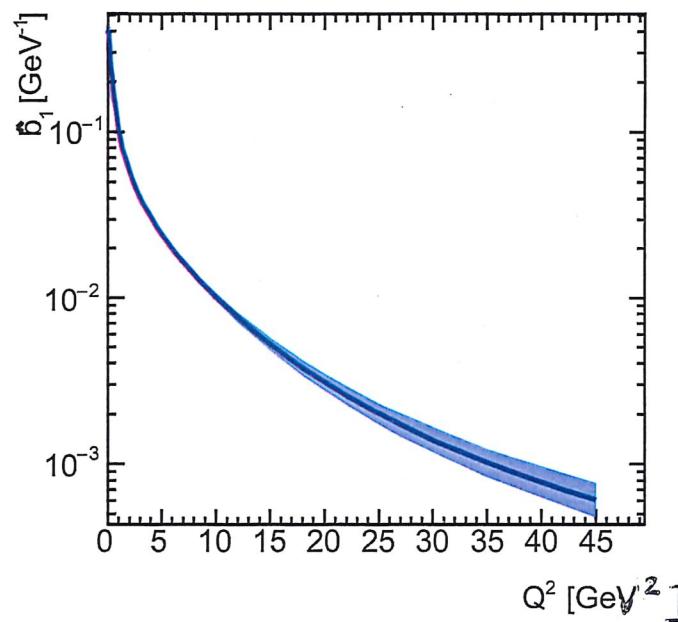
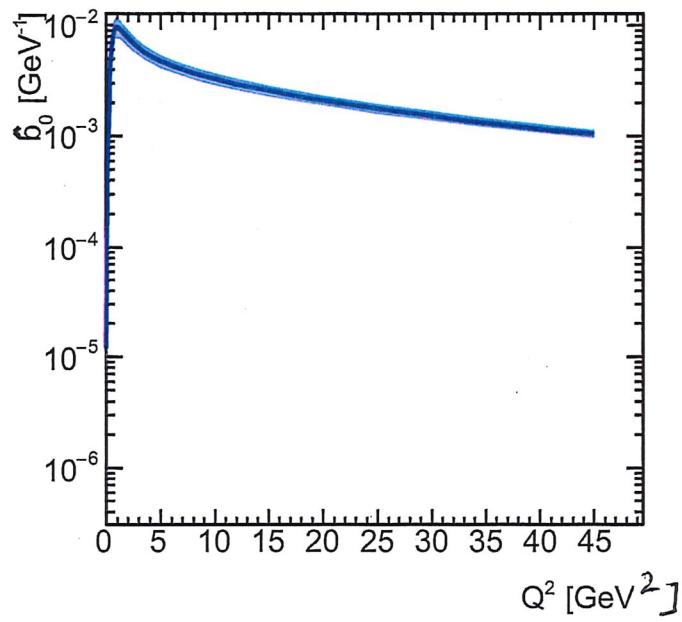
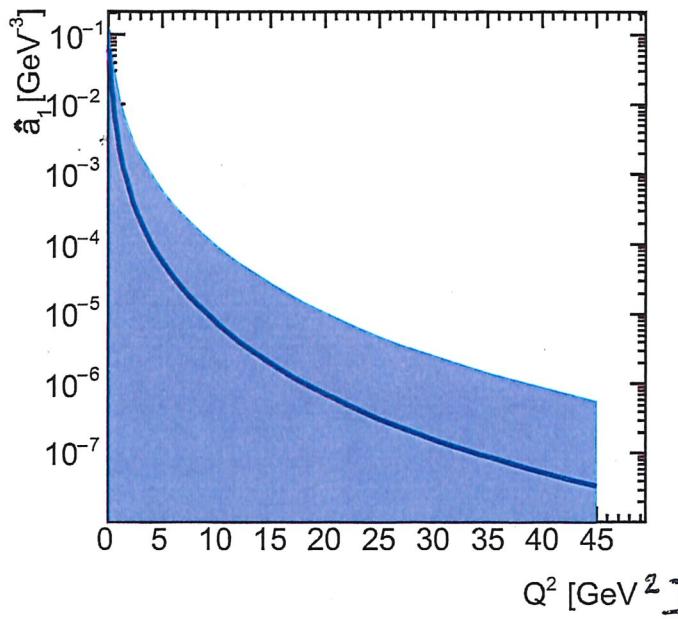
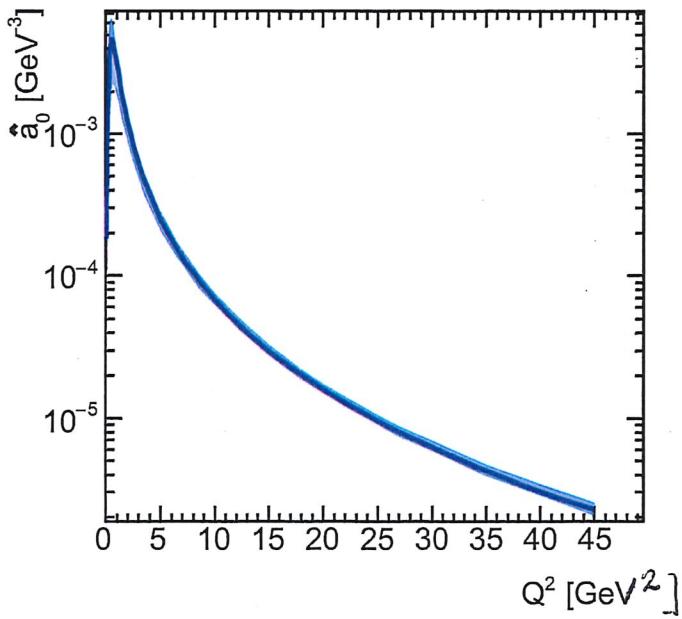
- Fit σ_{red}
- F2 component
- Soft contribution

Fit results:

- intercepts:
 - hard pomeron $\alpha_{P_0}(0) = 1 + \varepsilon_0$, $\varepsilon_0 = 0.3070$ (83)
 - soft pomeron $\alpha_{P_1}(0) = 1 + \varepsilon_1$, $\varepsilon_1 = 0.082$ (14)
 - reggeon $\alpha_{f_{2R}}(0) = 0.38$ (26).
- coupling functions $\hat{a}_j(Q^2)$, $\hat{b}_j(Q^2)$.
- the fit seems to prefer rather large values for the ratio

$$R(W^2, Q^2) = \frac{\sigma_L(W^2, Q^2)}{\sigma_T(W^2, Q^2)}.$$

This is currently under further investigations.



The functions

$\hat{a}_j(Q^2)$ and $\hat{b}_j(Q^2)$, logarithmic scale for the amplitude.

9 Conclusions for part II

We have developed a two-tensor pomeron model describing low x DIS data from photo production, $Q^2 = 0$, up to $Q^2 = 50 \text{ GeV}^2$. We find cross sections rising like powers of W in the region explored ($W \lesssim 300 \text{ GeV}$).

- Photo production: $\sigma_{\gamma p}(W^2) \propto (W^2)^{\varepsilon_1}$, $\varepsilon_1 = 0.082(14)$,
This rise is as for hadron-hadron scattering.
- DIS, $Q^2 > 0$: $\sigma_{T,L}(W^2, Q^2) \propto (W^2)^{\varepsilon_0}$, $\varepsilon_0 = 0.3070(83)$.
In the energy range explored we find no sign of saturation.
There is no unitarity, Froissart-like, bound for $\sigma_{T,L}$.
The power-like rise of $\sigma_{T,L}$ could go on, as suggested e.g. by
QCD arguments relating low x DIS to a critical phenomenon.
(O.N., EPJ C 26 (2003) 579).

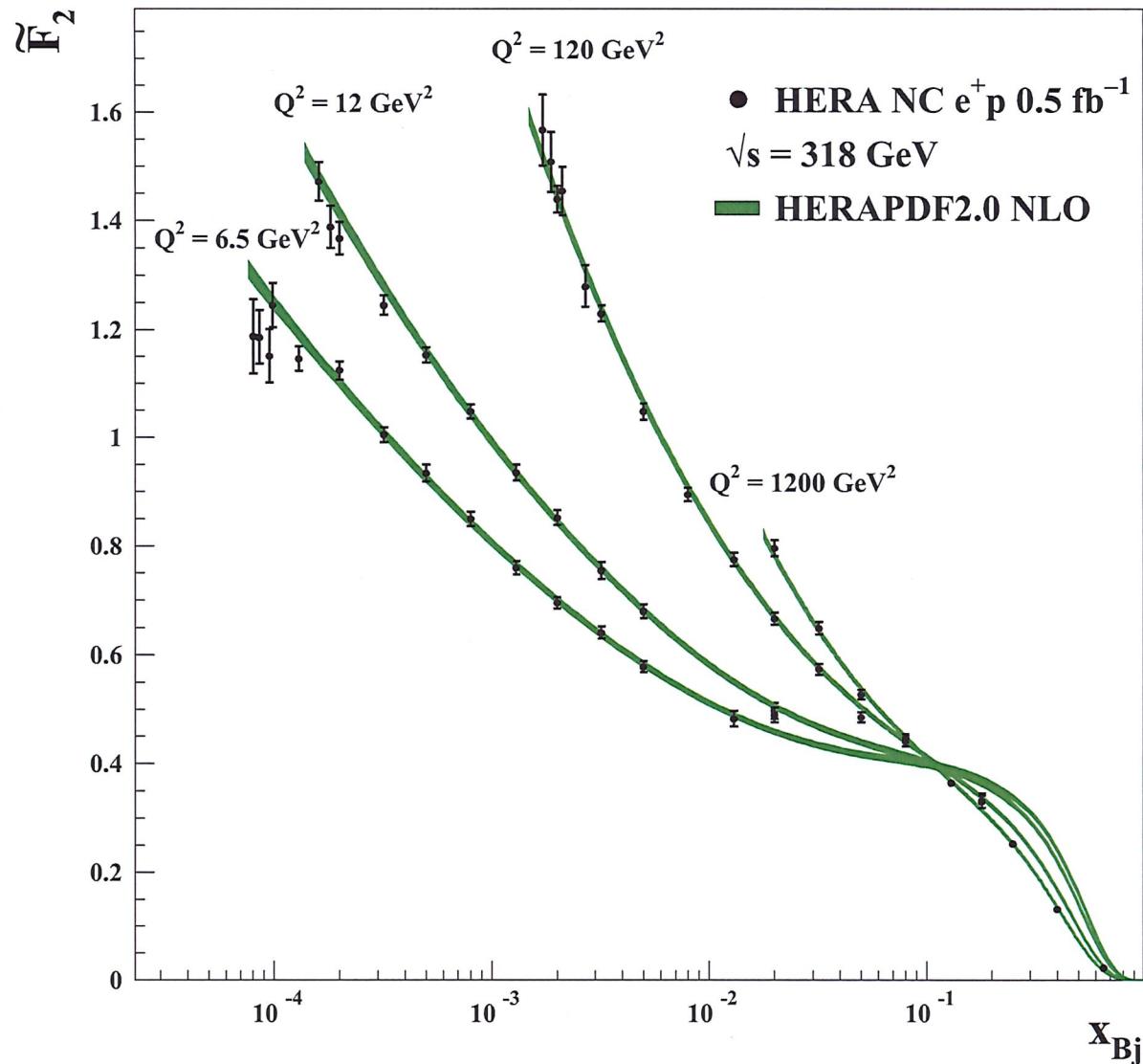
Appendix A : Unitarity and DIS

The HERA data for $F_2(x, Q^2)$ show a strong rise for $x \rightarrow 0$ at fixed Q^2 for larger Q^2 .

Can this rise go on forever, or is there a "unitarity limit"?

Answer from a rigorous theoretical point of view:

There is no unitarity limit!



$$\tilde{F}_2 \approx F_2 \approx$$

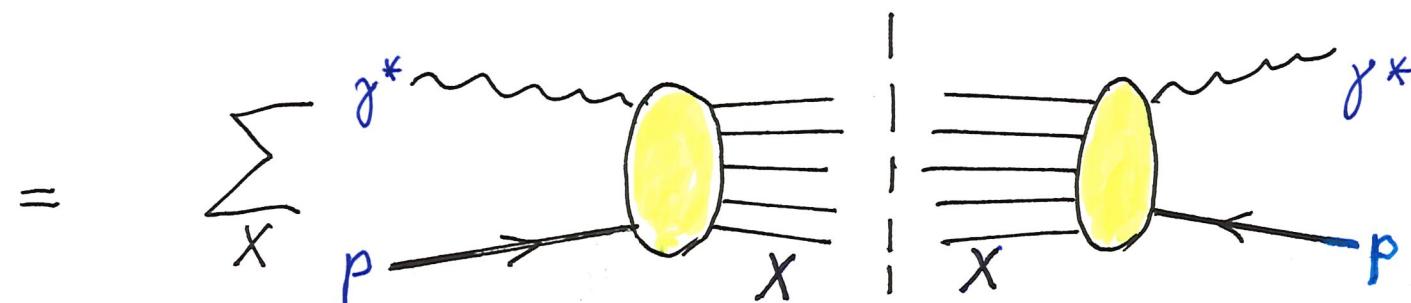
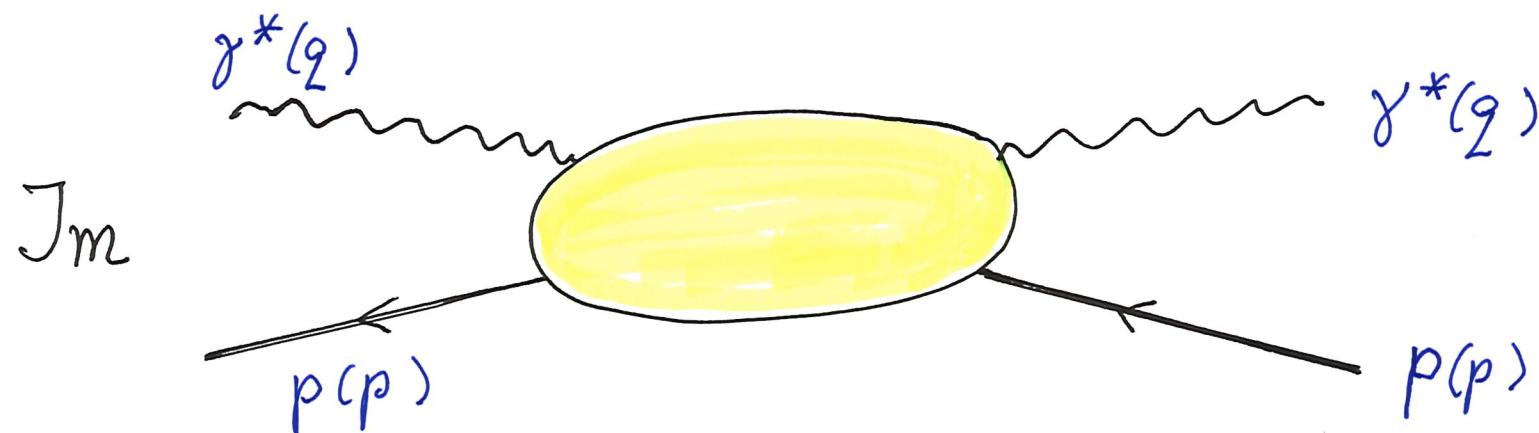
$$\frac{Q^2}{4\pi^2 \alpha} (1 - x)$$

$$(\sigma_T + \sigma_L)$$

Figure 83: The structure function \tilde{F}_2 as extracted from the measured reduced cross sections for four values of Q^2 together with the predictions of HERAPDF2.0 NLO. The bands represent the total uncertainty on the predictions.

Forward virtual Compton scattering amplitude:

$$\gamma^*(q) + p(p) \rightarrow \gamma^*(q) + p(p)$$



sum over all
hadronic states

$$\propto \sigma_{T, L} \quad \text{resp.} \quad W_{1, 2}.$$

- The S matrix is a unitary operator

$$S^\dagger S = \mathbb{1}$$

This gives relations / restrictions for the scattering of asymptotic particles, that is, asymptotic states. The virtual photon γ^* is not an asymptotic state.

- The relation of the imaginary part of the forward virtual Compton amplitude to the structure functions follows from the definition of the T-product and is not a unitarity relation.

In a unitarity relation \sum_X would have to include an asymptotic hadronic state $|X\rangle = |\gamma^* p\rangle$. But such a state does not exist!

- Can we get unitarity restrictions for the structure functions by considering $e p$ scattering?

Again no! In a "Gedanken experiment" we can make α and thus the $e p$ amplitude as small as we wish without changing the structure functions.

- If for the physical value of α the lowest order $e p$ cross section containing the structure functions becomes very very large, then higher order effects in α must come into play "taming" the cross section.

Conclusion

- Unitarity and, thus, the Froissart bound have nothing to say on $F_2(x, Q^2)$ for $x \rightarrow 0$.
- In our view $F_2(x, Q^2)$ may continue to rise for $x \rightarrow 0$. It may also saturate, but then, not because of unitarity.

B Low x physics as a critical phenomenon?

How can we understand the behaviour of $F_2(x, Q^2)$ with x and Q^2 found at HERA?

- Evolution with Q^2 for larger x is given by the DGLAP equation (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi)
Unreliable for very small x , since higher order terms in α_s get out of control. Remember also the correlation terms, that is, the higher twists.
- BFKL equation? (Balitsky, Fadin, Kuraev, Lipatov)
- Fan diagrams, BK equation (Balitsky, Kovchegov)....?

- Our guess (Hebecker, Meggiolaro, O.N., NP B571, 26 (2000);
O.N., EPJC 26, 579 (2003)):

Low x physics can be viewed as a
critical phenomenon.

Starting point: virtual forward Compton scattering amplitude

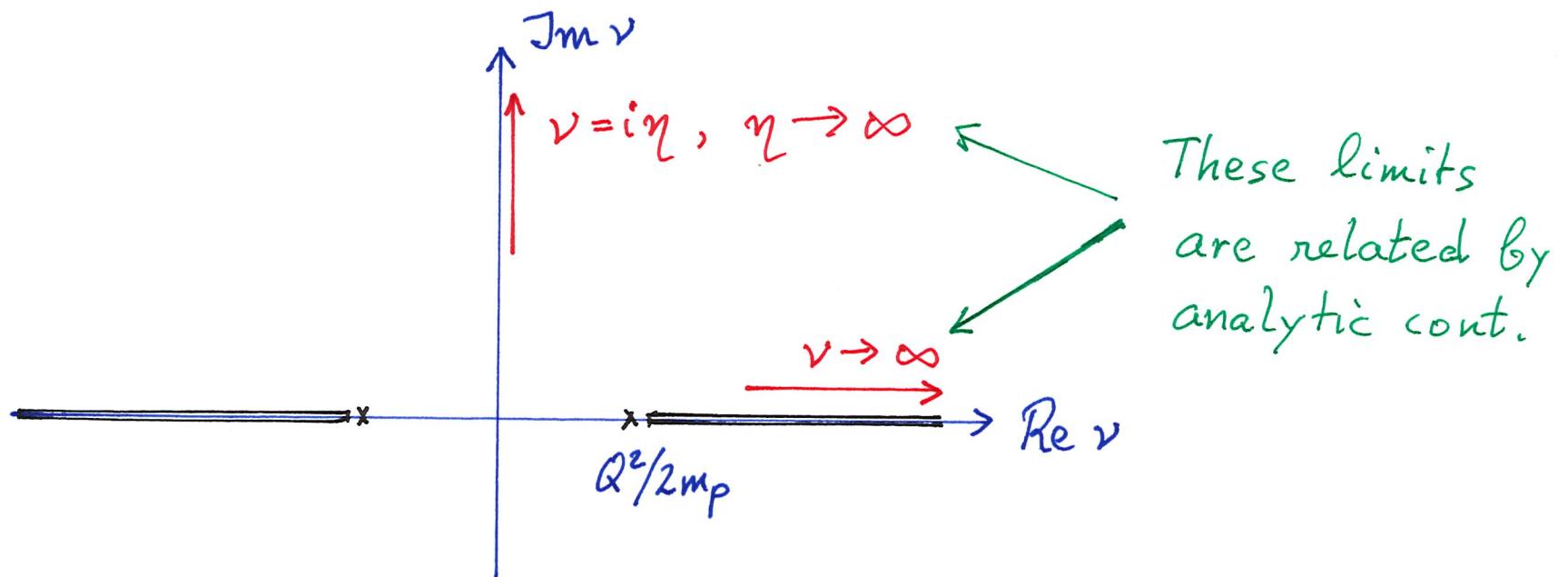
$$\text{Amplitude } (\gamma^* p \rightarrow \gamma^* p) = T_{\mu\nu}(p, q) =$$

$$\frac{i}{2\pi m_p} \int d^4x e^{iq \cdot x} \theta(x^0) \langle p(p) | [j_\mu(x), j_\nu(0)] | p(p) \rangle$$

We want to study the limit Q^2 fixed, $v \rightarrow \infty$

$$T_{\mu\nu}(p, q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2} \right) T_1(v, Q^2) + \frac{1}{m_p^2} \left(p_\mu - \frac{(pq) q_\mu}{Q^2} \right) \left(p_\nu - \frac{(pq) q_\nu}{Q^2} \right) T_2(v, Q^2)$$

Analyticity properties of $T_j(v, Q^2)$ for fixed Q^2 :



We use the Phragmén - Lindelöf theorem!

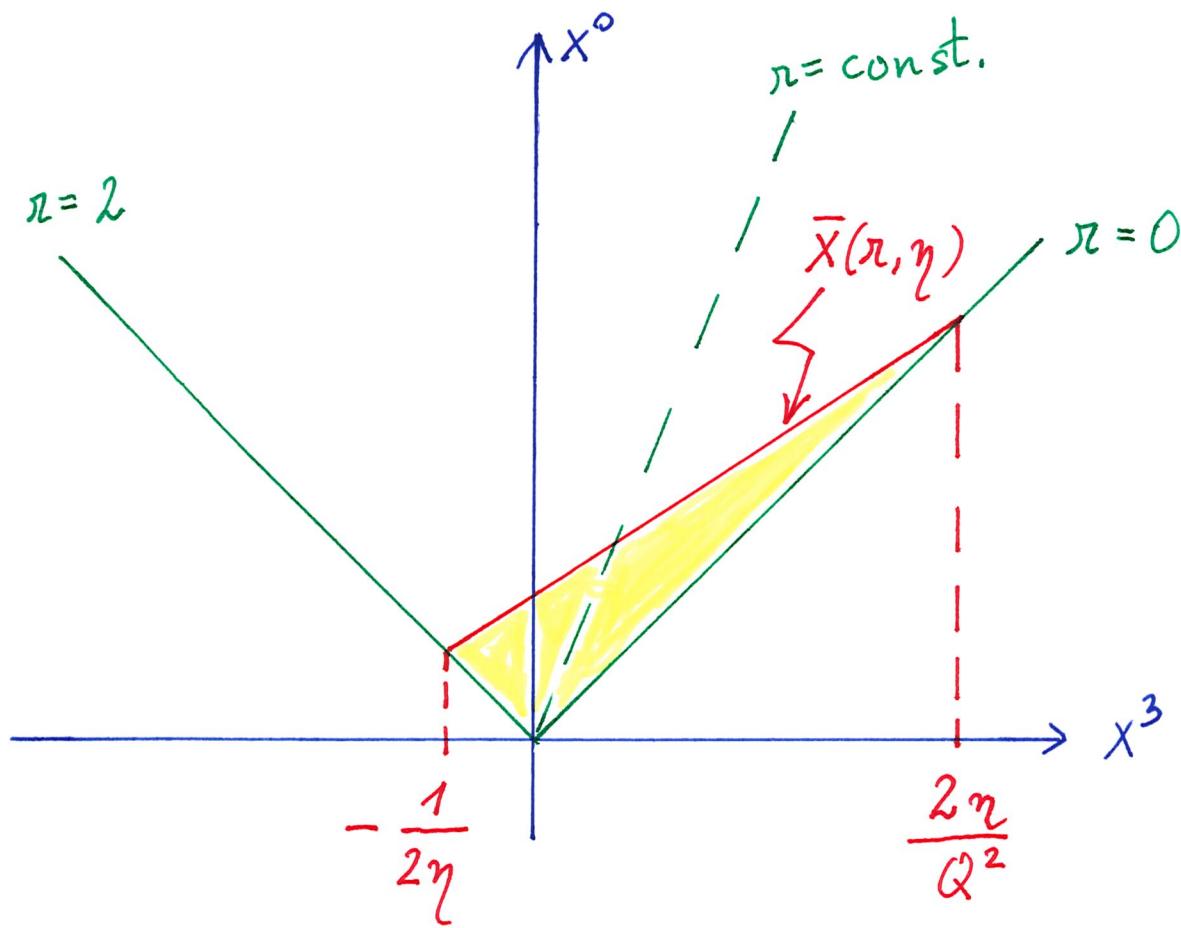
We work in the rest system of the proton and continue in ν to the imaginary axis.

$$p = \begin{pmatrix} m_p \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \nu = i\gamma, \quad \gamma > 0, \quad -iq = \begin{pmatrix} \gamma \\ 0 \\ 0 \\ \sqrt{\gamma^2 - Q^2} \end{pmatrix}$$

$$\exp(iqx) = \exp \left[-x^0 (\gamma - (1-\eta)\sqrt{\gamma^2 - Q^2}) \right] = \exp \left(-\frac{x^0}{\bar{x}(\eta, \gamma)} \right)$$

- $\eta = 1 - x^3/x^0$, important variable in the following!

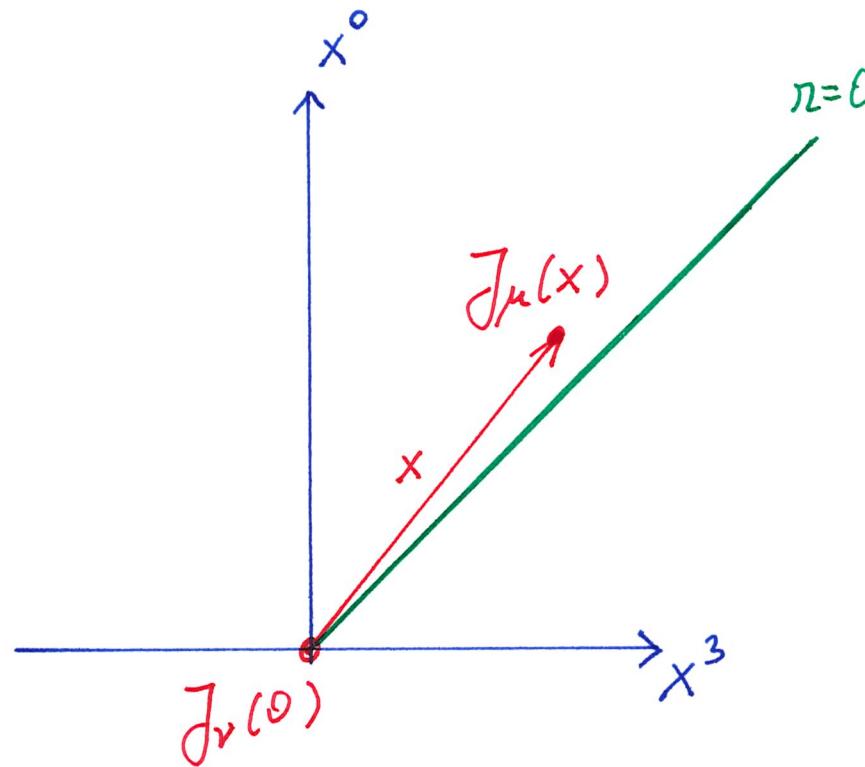
$$\bar{x}(\eta, \gamma) \cong \frac{1}{\gamma \left(\eta + \frac{Q^2}{2\gamma^2} \right)} \quad \text{for large } \gamma$$



As $\eta \rightarrow \infty$ the integration region shrinks towards the line $r=0$ on the forward light cone.

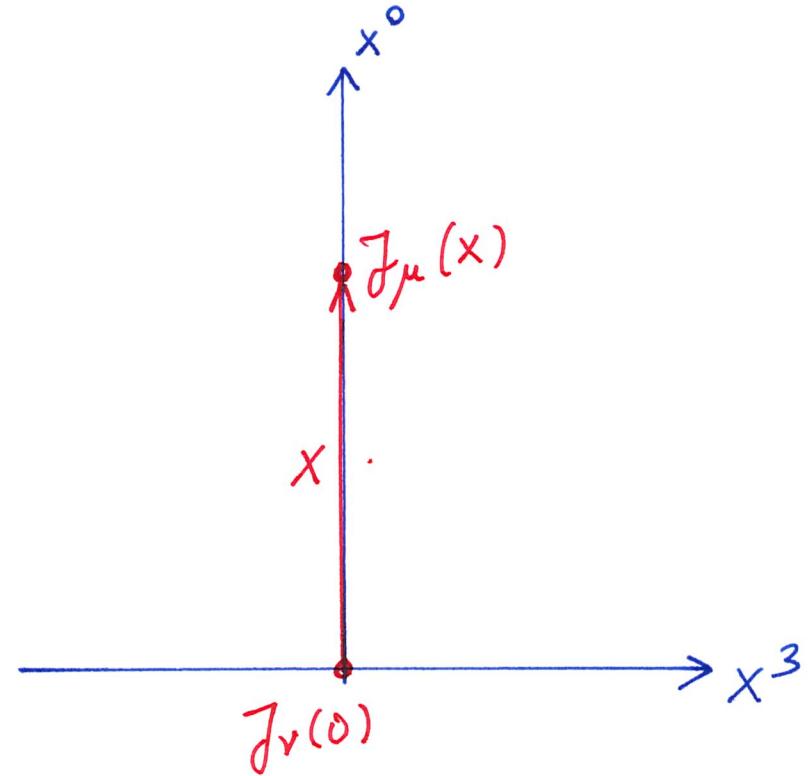
In the original theory we need the behaviour of the current correlation function in the proton near the light cone.

Our idea: transform the theory such that only the correlation function on the time axis is needed.



original theory :

Hamiltonian = H ,
3rd comp. of
momentum op. = P^3



effective theory :

Hamiltonian =
 $H_{\text{eff}}(r) = H - (1-r)P^3$

$$\begin{aligned} & \left\langle p(p) \mid \mathcal{J}_0(x^0, x^3 = (1-\mu)x^0) \mathcal{J}_0(0) \mid p(p) \right\rangle_{\substack{\text{original Lagrangian} \\ \mathcal{L}}} \\ &= M_{00}(x^0, \mu) \\ &= \left\langle p(p) \mid \mathcal{J}_0(x^0, x^3 = 0) \mathcal{J}_0(0) \mid p(p) \right\rangle_{\substack{\text{Lagrangian} \\ \mathcal{L}_{M,\mu}}} \end{aligned}$$

Hamiltonian

Lagrangian

H → L

$$H_{\text{eff}}(n) = H - (1-n)P^3 \longrightarrow L_{M,n}$$

Example of an π -theory: scalar field theory

- Original Lagrangian

$$\mathcal{L}(x) = \frac{1}{2} \partial_\mu \phi(x) \partial^\mu \phi(x) - \frac{1}{2} m^2 \phi^2(x) - \frac{\lambda}{4!} \phi^4(x)$$

- π -theory in Minkowski space

$$\mathcal{L}_{M,\pi}(x) = \frac{1}{2} (\partial_0 \phi - (1-\pi) \partial_3 \phi)^2 - \frac{1}{2} (\vec{\nabla} \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4$$

- π -theory in Euclidean space

$$\mathcal{L}_{E,\pi} = \frac{1}{2} (\partial_0 \phi + i(1-\pi) \partial_3 \phi)^2 + \frac{1}{2} (\vec{\nabla} \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{\lambda}{4!} \phi^4$$

anisotropic propagator.

Correlation length in time direction $\xi(\pi) = \frac{1}{m\sqrt{\pi}}$

$$\xi(\pi) \rightarrow \infty \quad \text{for } \pi \rightarrow 0$$

- From a study of free field theories and of the energy gap we expect that in the effective r -theory there will be a large correlation length $\xi(r)$ in time-like direction for $r \rightarrow 0$.

In free field theory :

$$\xi(r) \propto 1/\sqrt{r}$$

- This indicates a critical phenomenon with r playing the role of $(T - T_c)/T_c$.
- We expect then scaling behaviour of $M_{00}(x^0, r)$ for $0 < x^0 < \xi(r)$

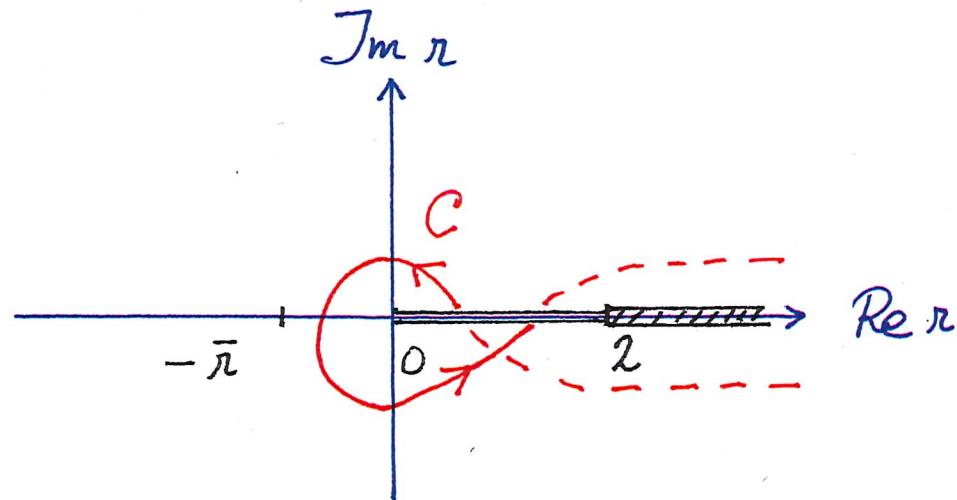
$$M_{\mu\nu}(x, p) = \langle p(p) | J_\mu(x) J_\nu(0) | p(p) \rangle$$

$$= [g_{\mu\nu} \square - \partial_\mu \partial_\nu] M_1(x_p, x^2)$$

$$+ [p_\mu p_\nu \square - (p \partial)(p_\mu \partial_\nu + p_\nu \partial_\mu) + g_{\mu\nu} (p \partial)^2] M_2(x_p, x^2)$$

$$\tilde{M}_j(x^\circ, r) \equiv M_j(x^\circ m_p, (x^\circ)^2 r (2-r))$$

We study the analyticity properties of $\tilde{M}_j(x^\circ, r)$ for fixed x° as fct. of r using the DGS representation.



$$\bar{r} = \frac{Q^2}{2\eta^2}$$

$$T_2(i\eta, Q^2) = \frac{i m_p Q^2}{\sqrt{\eta^2 - Q^2}} \int_C dr (1-r) \int_0^\infty dx^0 (x^0)^2 \tilde{M}_2(x^0, r) \exp\left[-\frac{x^0}{\bar{x}(\eta, r)}\right]$$

$\propto \exp\left[-\frac{x^0}{\xi(r)}\right]$

saddle point of integration: $r = -\bar{r}/2$.

Two competing cutoffs in the x^0 integration: $\xi(-\frac{1}{2}\bar{r})$, $\bar{x}(\eta, -\frac{1}{2}\bar{r})$.

two regimes of the integrations:

soft

$$\xi(-\frac{1}{2}\bar{n}) < \bar{x}(\eta, -\frac{1}{2}\bar{n})$$

hard

$$\xi(-\frac{1}{2}\bar{n}) > \bar{x}(\eta, -\frac{1}{2}\bar{n})$$

if $\xi(n) = 1/(M\sqrt{|n|})$ as in free field theory, with $M = \mathcal{O}(\text{GeV})$

$$Q^2 < 8M^2$$

$$Q^2 > 8M^2,$$

integration is over scaling regime only. We expect power behaviour. Can the hard-pomeron intercept be understood as a critical index?

Let us make a simple power-law ansatz for $\tilde{M}_2(x^*, r)$ in the scaling region, $0 < x^* \ll \xi(r)$,

$$\tilde{M}_2(x^*, r) \propto (x^*)^{\alpha-1} (-r)^{-\frac{1}{2}(\varepsilon_0 - \alpha)}$$

This gives for $Q^2 > 8 M^2$, that is in the hard regime,

$$\sigma_T(w^2, Q^2) + \sigma_L(w^2, Q^2) \propto (Q^2)^{-1 - \frac{\alpha}{2} - \frac{\varepsilon_0}{2}} (w^2)^{\varepsilon_0}$$

for $w^2 \rightarrow \infty$.

In this way the hard pomeron intercept ε_0 may be related to a critical index in the effective π theory.

Thank you for
your attention !