

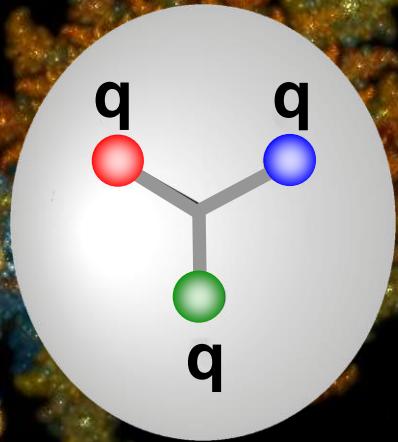
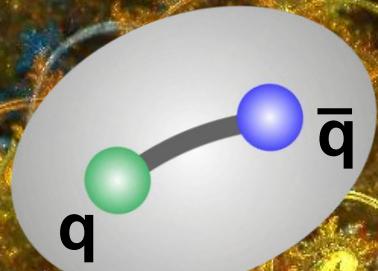


WE-Heraeus Physics School

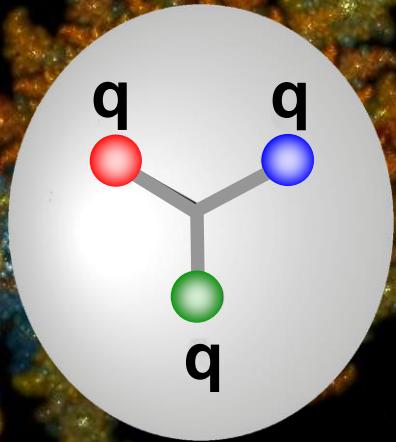
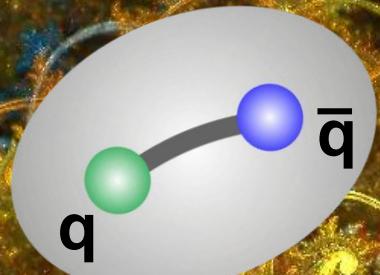


Introduction to the SDE/BSE approach to QCD

Michael Pennington
September 2017



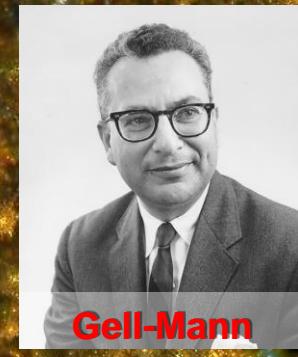
$$\mathcal{L}_{QCD} = \sum_{q=u,d,s,c,b,t} \bar{q} (i \gamma_\mu D^\mu - m_q) q - \frac{1}{4} G^{\mu\nu} G_{\mu\nu}$$



Fritzsch



Leutwyler

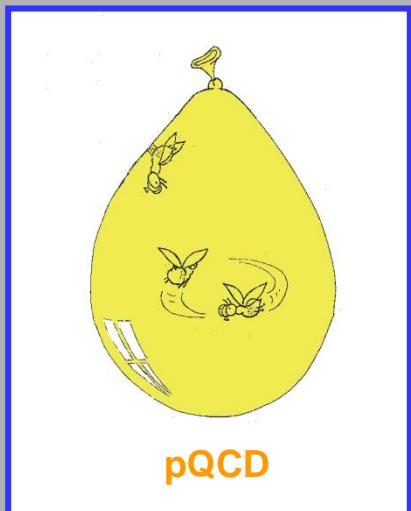


Gell-Mann

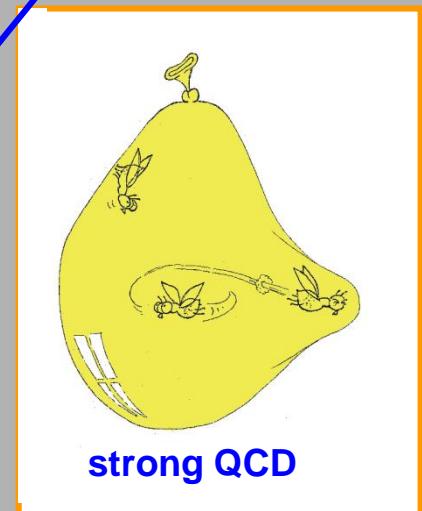
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QCD

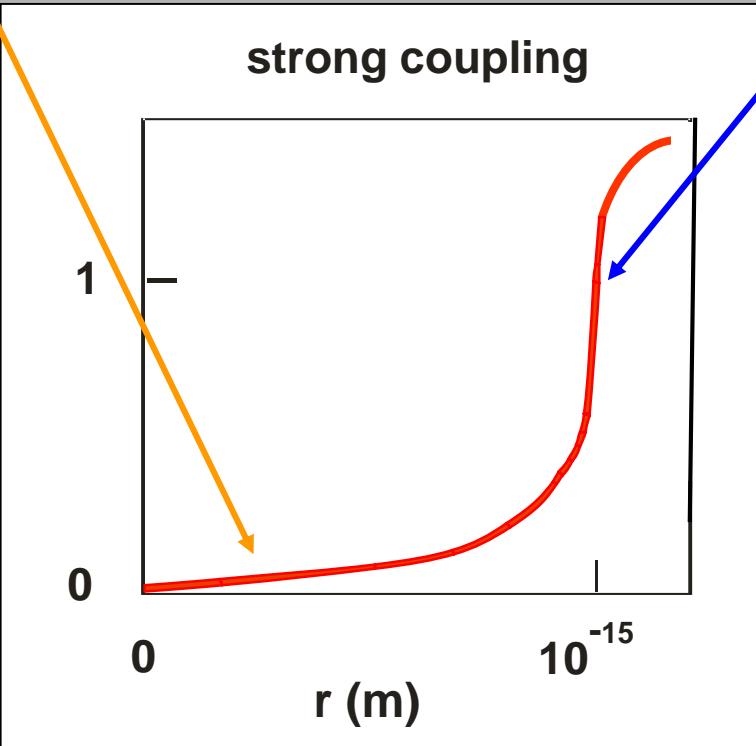
asymptotic freedom



confinement



strong coupling

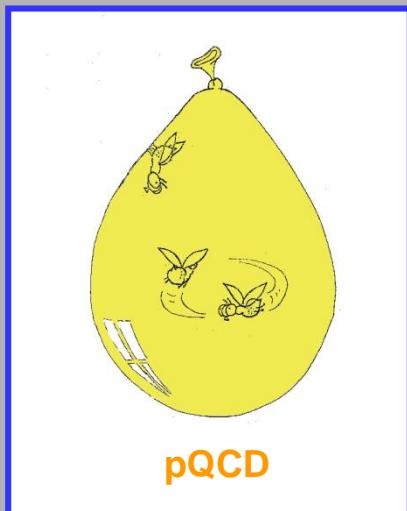




Politzer

QCD

asymptotic freedom

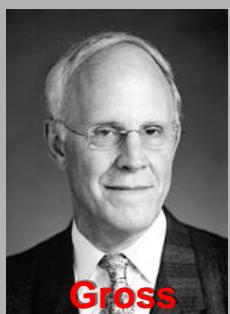


pQCD

confinement



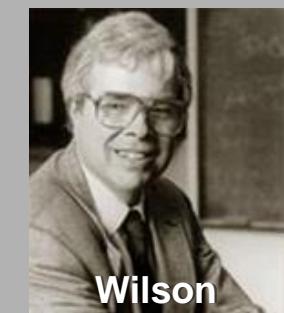
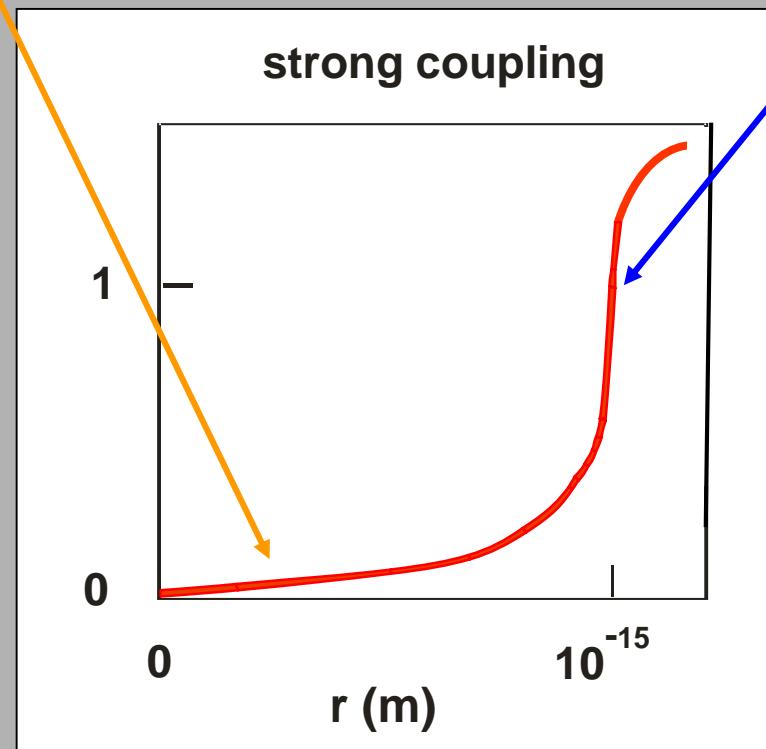
strong QCD



Gross

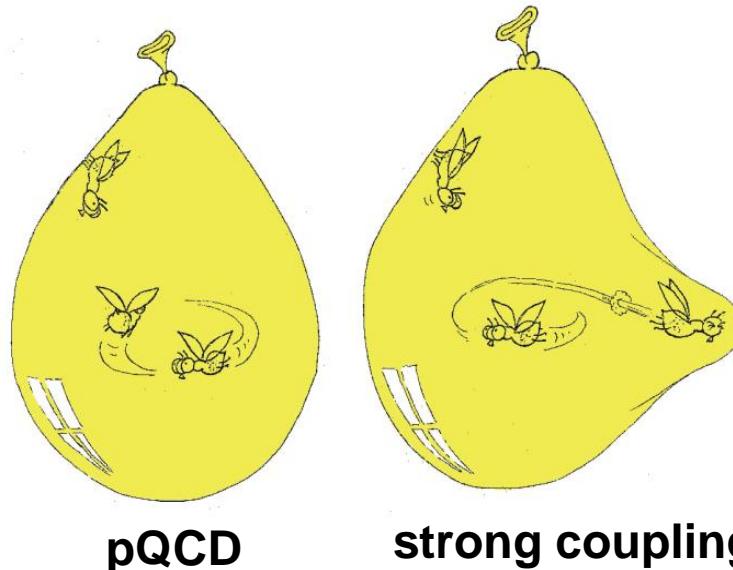


Wilczek

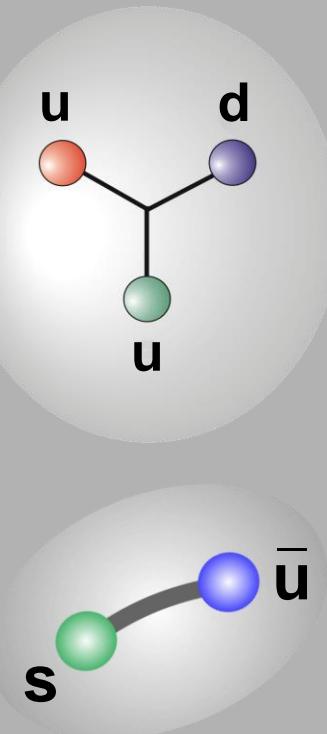
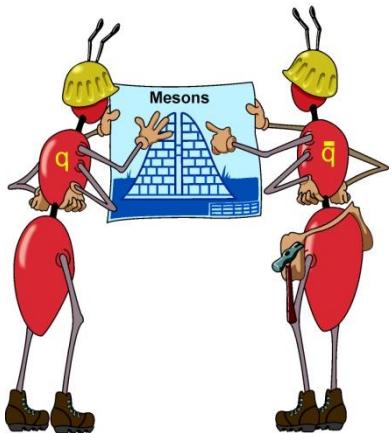


Wilson

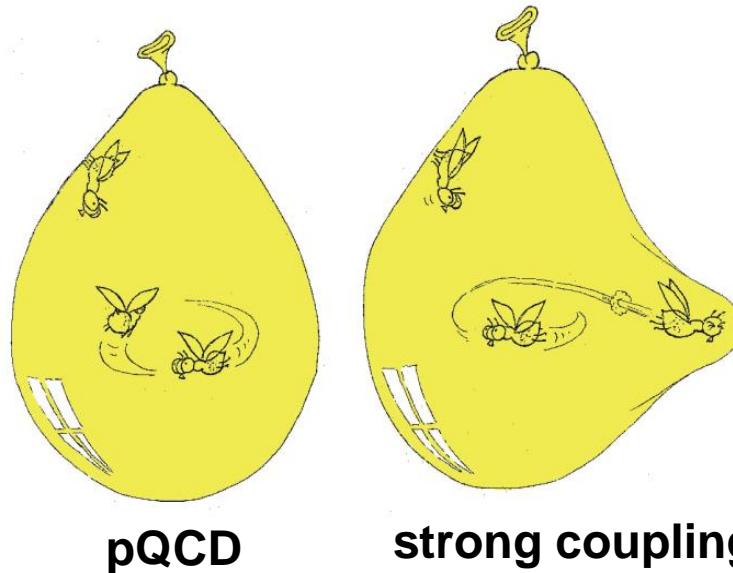
Strong physics problems



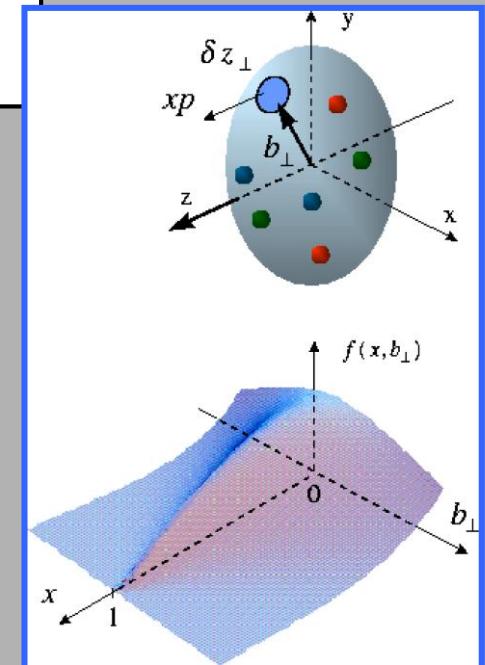
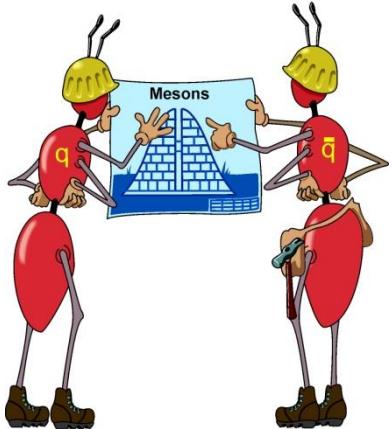
bound states



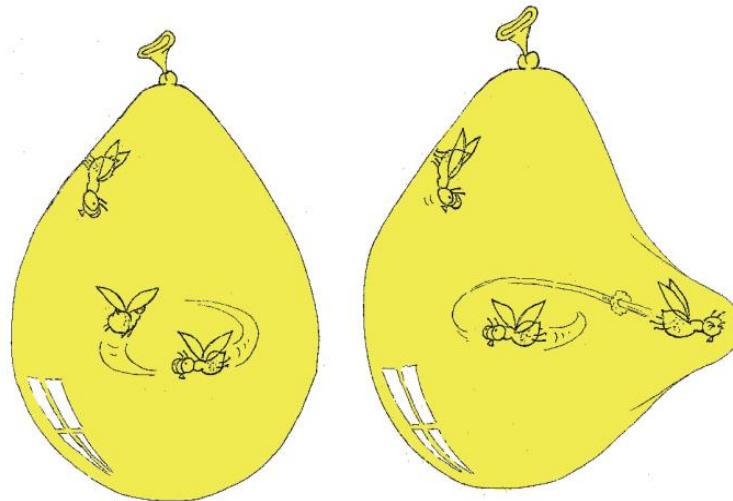
Strong physics problems



bound states



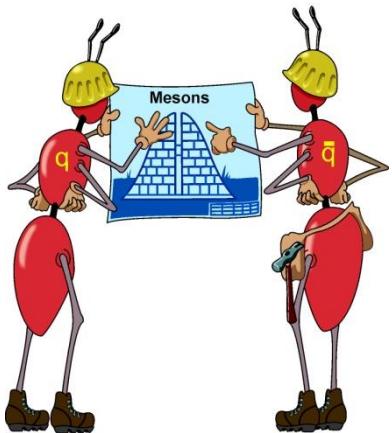
Strong physics problems



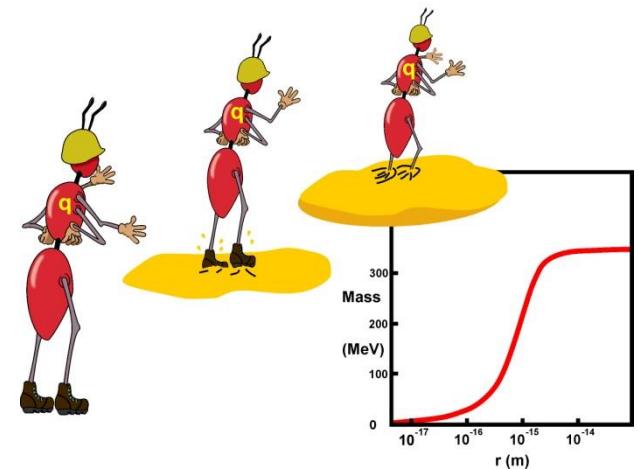
pQCD

strong coupling

bound states



mass generation



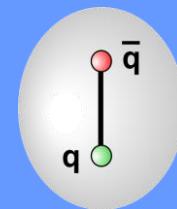
Hadron masses ²

Mass²
(GeV²)

1.0

0.5

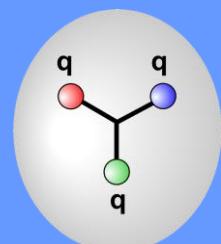
0



ρ



N



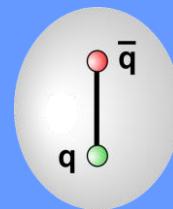
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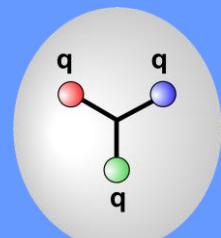
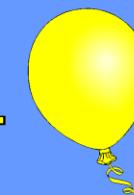
0



ρ

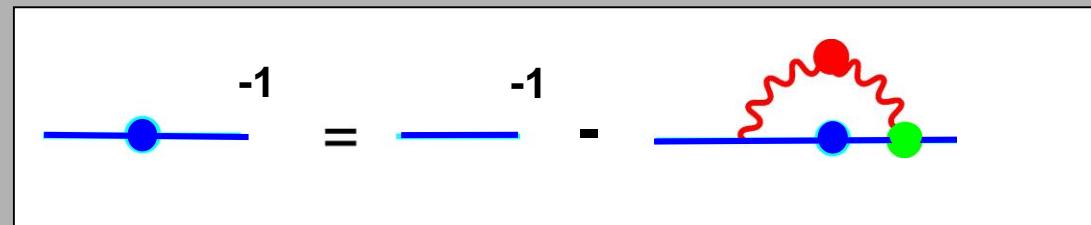


N



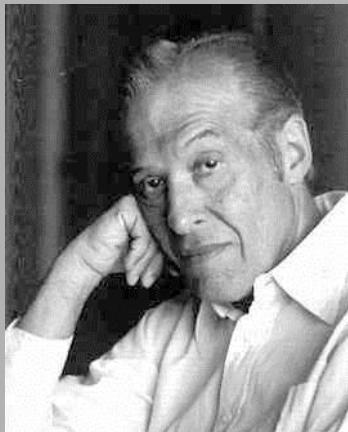
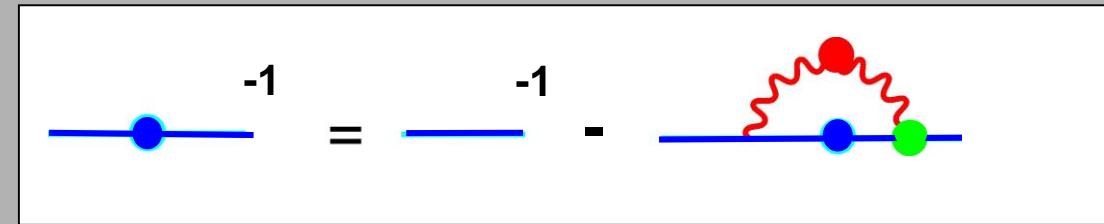


Schwinger-Dyson Equations



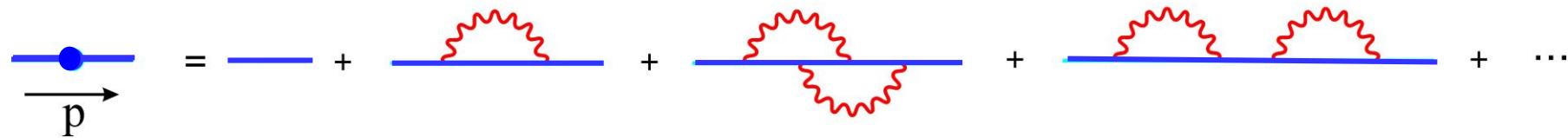


Schwinger-Dyson Equations



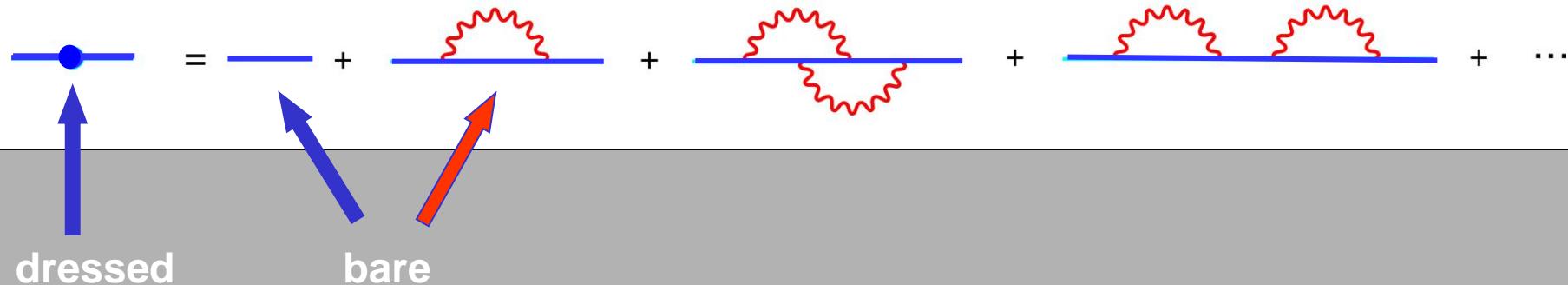
Schwinger-Dyson Equations

$$S_F(p) = S_F^0(p) + \dots$$



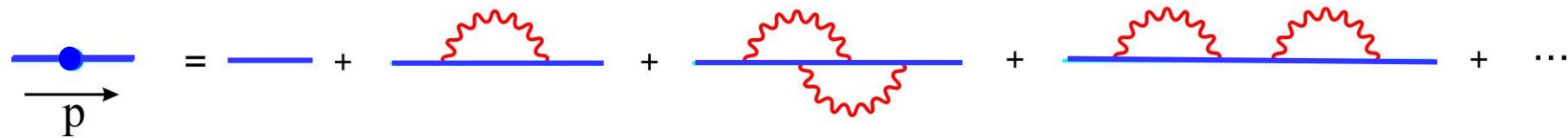
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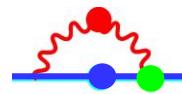


Schwinger-Dyson Equations

$$S_F(p) = S_F^0(p) + \dots$$

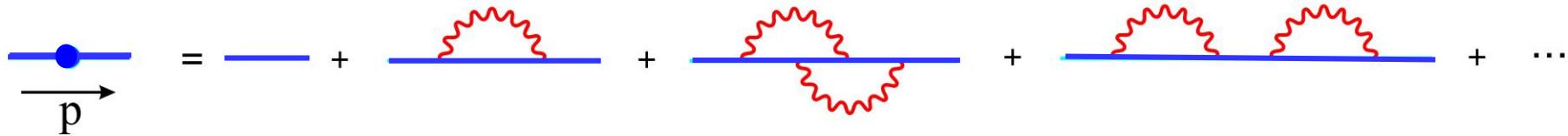


$$\Sigma(p) \equiv$$



Schwinger-Dyson Equations

$$S_F(p) = S_F^0(p) + \dots$$



$$\Sigma(p) \equiv$$

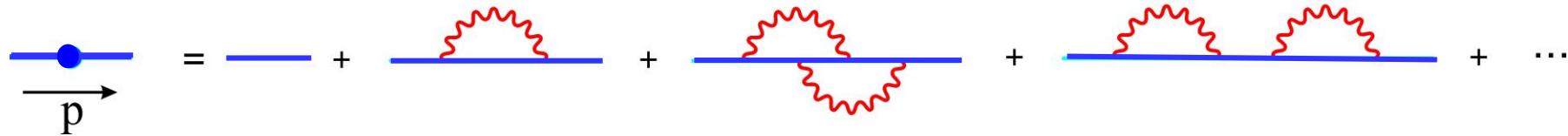
A horizontal blue line with a blue dot and a green dot attached to it, with a red wavy gluon line connecting them, representing the self-energy $\Sigma(p)$.

A horizontal blue line with a blue dot at its left end and a right-pointing arrow below it labeled 'p' represents the full propagator $S_F(p)$. This is followed by an equals sign and a sum of terms. The first term is a bare propagator (blue line with no vertices). Subsequent terms are represented by blue lines with red wavy gluon lines attached to them, indicating the loop corrections to the propagator.

$$S_F(p) = S_F^0(p) + S_F^0(p)\Sigma(p)S_F^0(p) + S_F^0(p)\Sigma(p)S_F^0(p)\Sigma(p)S_F^0(p) + \dots$$

Schwinger-Dyson Equations

$$S_F(p) = S_F^0(p) + \dots$$



$$\Sigma(p) \equiv$$

A horizontal blue line with a blue dot at its left end and a green dot at its right end, with a red wavy gluon line connecting them, represents the self-energy $\Sigma(p)$.

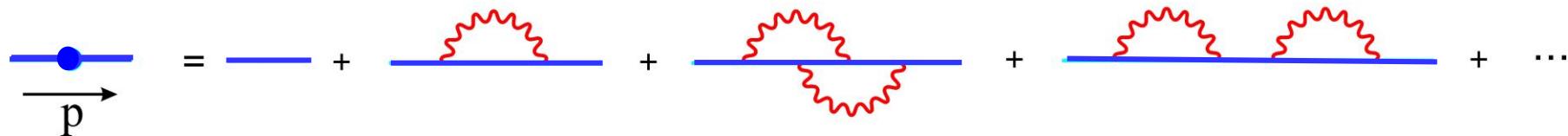
A horizontal blue line with a blue dot at its left end, labeled \vec{p} below it, represents the full propagator $S_F(p)$. This is followed by an equals sign and a sum symbol ($= +$). The first term in the sum is a bare propagator (blue line with no vertices). Subsequent terms show the propagator $S_F(p)$ with increasing numbers of red wavy gluon lines attached to its vertices, representing the loop corrections.

$$S_F(p) = S_F^0(p) + S_F^0(p)\Sigma(p)S_F^0(p) + S_F^0(p)\Sigma(p)S_F^0(p)\Sigma(p)S_F^0(p) + \dots$$

$$S_F(p) = \frac{S_F^0(p)}{1 - \Sigma(p)S_F^0(p)}$$

Schwinger-Dyson Equations

$$S_F(p) = S_F^0(p) + \dots$$



$$\Sigma(p) \equiv$$

A blue horizontal line with a dot at p is followed by a red wavy line ending in a red dot, which is then connected to a green dot on the blue line.

A blue horizontal line with a dot at p is shown. An equals sign follows, followed by a series of terms separated by plus signs. Each term consists of a blue horizontal line with a red wavy loop attached to it.

$$S_F(p) = S_F^0(p) + S_F^0(p)\Sigma(p)S_F^0(p) + S_F^0(p)\Sigma(p)S_F^0(p)\Sigma(p)S_F^0(p) + \dots$$

$$S_F(p) = \frac{S_F^0(p)}{1 - \Sigma(p)S_F^0(p)} = S_F^0(p) + \frac{S_F^0(p)\Sigma(p)S_F^0(p)}{1 - \Sigma(p)S_F^0(p)}$$

Schwinger-Dyson Equations

$$S_F(p) = \frac{S_F^0(p)}{1 - \Sigma(p) S_F^0(p)} = S_F^0(p) + \frac{S_F^0(p) \Sigma(p) S_F^0(p)}{1 - \Sigma(p) S_F^0(p)}$$

Schwinger-Dyson Equations

$$S_F(p) = \frac{S_F^0(p)}{1 - \Sigma(p) S_F^0(p)} = S_F^0(p) + \frac{S_F^0(p) \Sigma(p) S_F^0(p)}{1 - \Sigma(p) S_F^0(p)}$$

$$S_F(p) = S_F^0(p) + S_F^0(p) \Sigma(p) S_F(p)$$



Schwinger-Dyson Equations

$$S_F(p) = \frac{S_F^0(p)}{1 - \Sigma(p) S_F^0(p)} = S_F^0(p) + \frac{S_F^0(p) \Sigma(p) S_F^0(p)}{1 - \Sigma(p) S_F^0(p)}$$

$$S_F(p) = S_F^0(p) + S_F^0(p) \Sigma(p) S_F(p)$$



1PI

$$S_F(p)^{-1} = S_F^0(p)^{-1} - \Sigma(p)$$

Schwinger-Dyson Equations

QED

$$\begin{aligned} -1 &= \text{---} \xrightarrow{\quad p \quad} -1 - \text{---} \xrightarrow{\quad p \quad} \text{---} \xrightarrow{\quad k \quad} \text{---} \xrightarrow{\quad q=p \quad} \\ -1 &= \text{---} \xrightarrow{\quad q \quad} -1 - N_F \text{---} \xrightarrow{\quad q \quad} \text{---} \xrightarrow{\quad k \quad} \text{---} \xrightarrow{\quad p=k-q \quad} \\ \text{---} \xrightarrow{\quad q \quad} \text{---} \xrightarrow{\quad k \quad} &= \text{---} \xrightarrow{\quad q \quad} \text{---} \xrightarrow{\quad k \quad} + \text{---} \xrightarrow{\quad q \quad} \text{---} \xrightarrow{\quad k \quad} \text{---} \xrightarrow{\quad p \quad} \end{aligned}$$

.....

Schwinger-Dyson Equations

QED

$$\begin{aligned} -1 &= \text{---} \xrightarrow{\quad p \quad} -1 - \text{---} \xrightarrow{\quad p \quad} \text{---} \xrightarrow{\quad k \quad} \text{---} \xrightarrow{\quad q=k-p \quad} \\ -1 &= \text{---} \xrightarrow{\quad q \quad} -1 - N_F \text{---} \xrightarrow{\quad q \quad} \text{---} \xrightarrow{\quad p=k-q \quad} \\ \text{---} \xrightarrow{\quad q \quad} \text{---} \xrightarrow{\quad k \quad} &= \text{---} \xrightarrow{\quad q \quad} \text{---} \xrightarrow{\quad k \quad} + \text{---} \xrightarrow{\quad q \quad} \text{---} \xrightarrow{\quad k \quad} \text{---} \xrightarrow{\quad p \quad} \end{aligned}$$

2 equations

2 equations

12 equations

Masses from Nothing

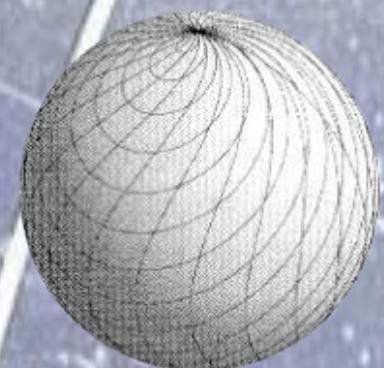
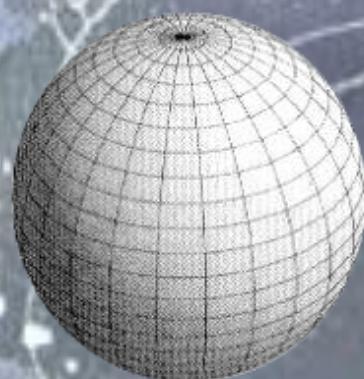




$$G = M = \dots$$

(e ϵ iθ.0)

$$\partial A/\partial x = -\bar{V}N^* D^* \psi + G$$



Phil D

Masses from Nothing

perturbative

$$m(p) = m_0 [1 + C \alpha \ln p^2/\mu^2 + O(\alpha^2)]$$

$$m(p) \neq 0 \quad \text{when} \quad m_0 \rightarrow 0 \quad ?$$

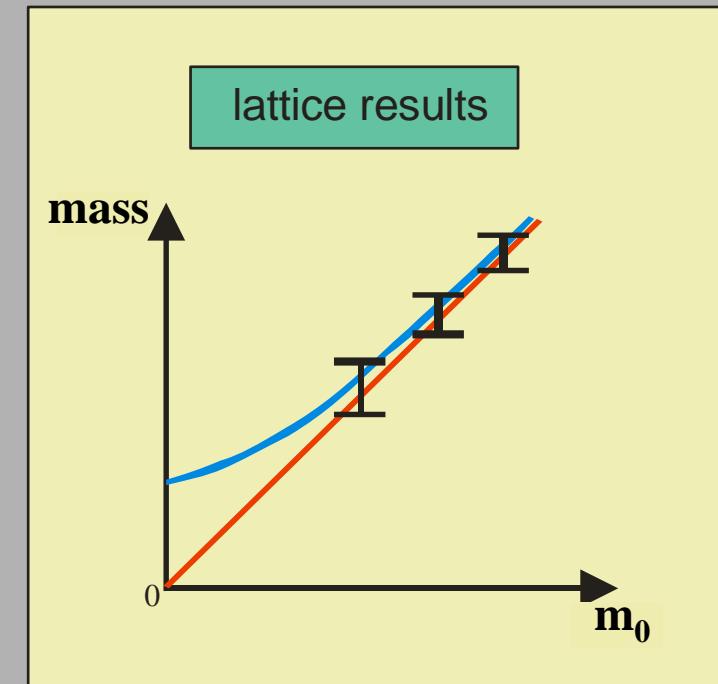


Masses from Nothing

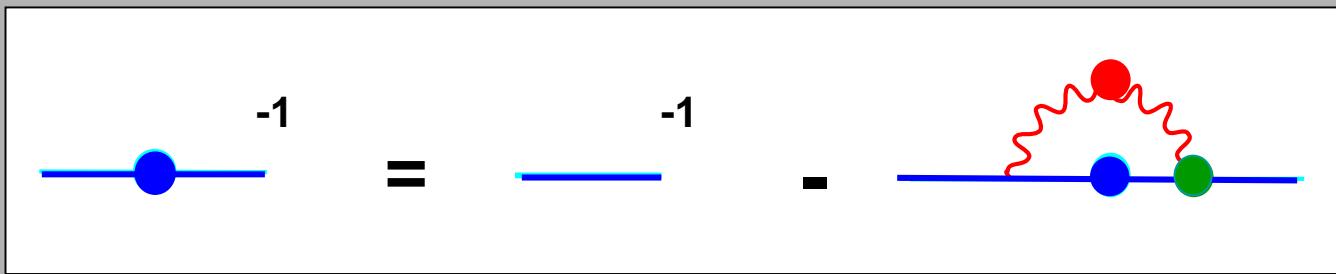
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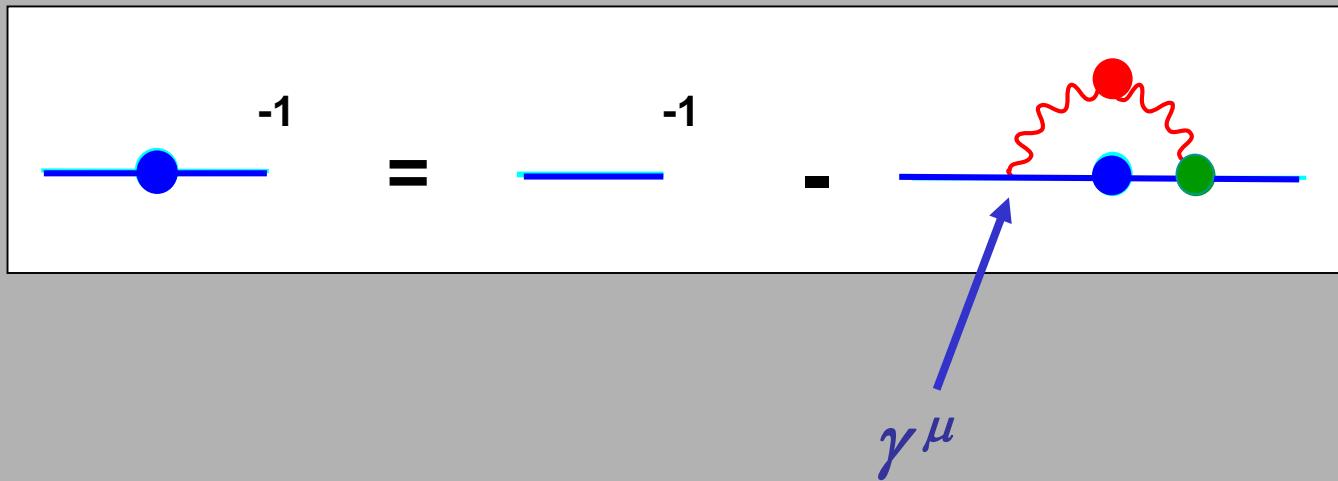
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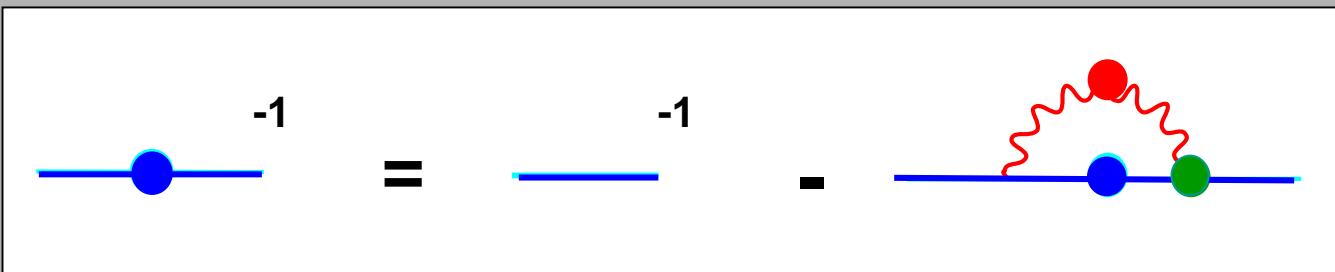
Fermion mass generation



Fermion mass generation

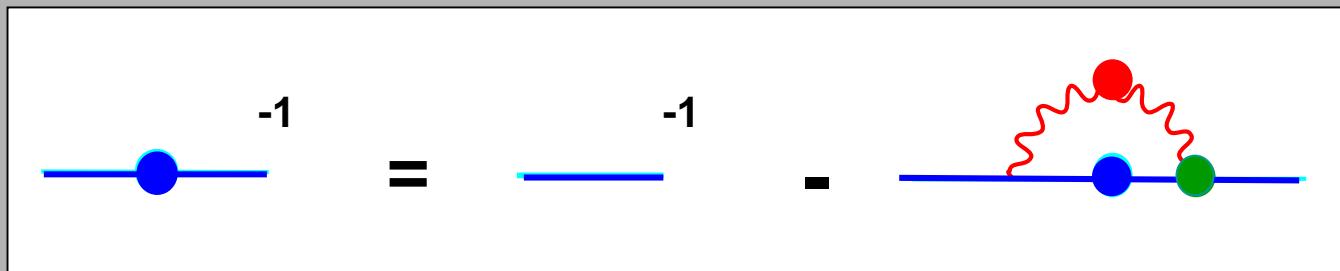


Fermion mass generation



$$S_F(p) = \frac{\mathcal{F}(p)}{p - \mathcal{M}(p)}$$

Fermion mass generation



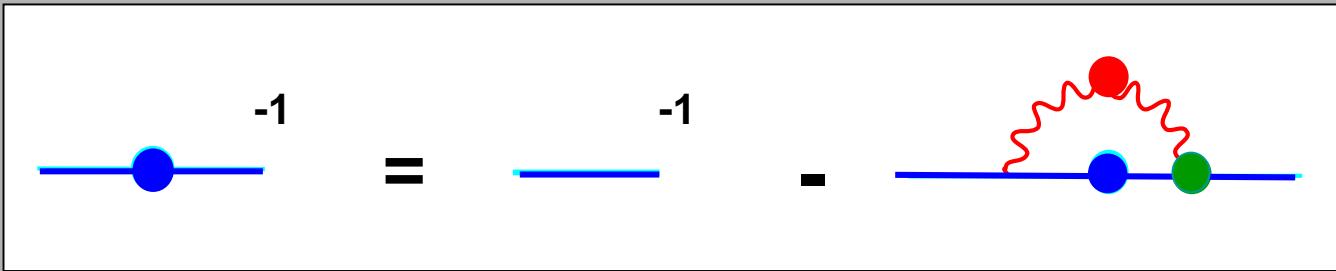
$$S_F(p) = \frac{\mathcal{F}(p)}{p - \mathcal{M}(p)}$$

wavefunction renormalisation

mass function

Bare : $\mathcal{F}(p) = 1$, $\mathcal{M}(p) = m_0$

Fermion mass generation



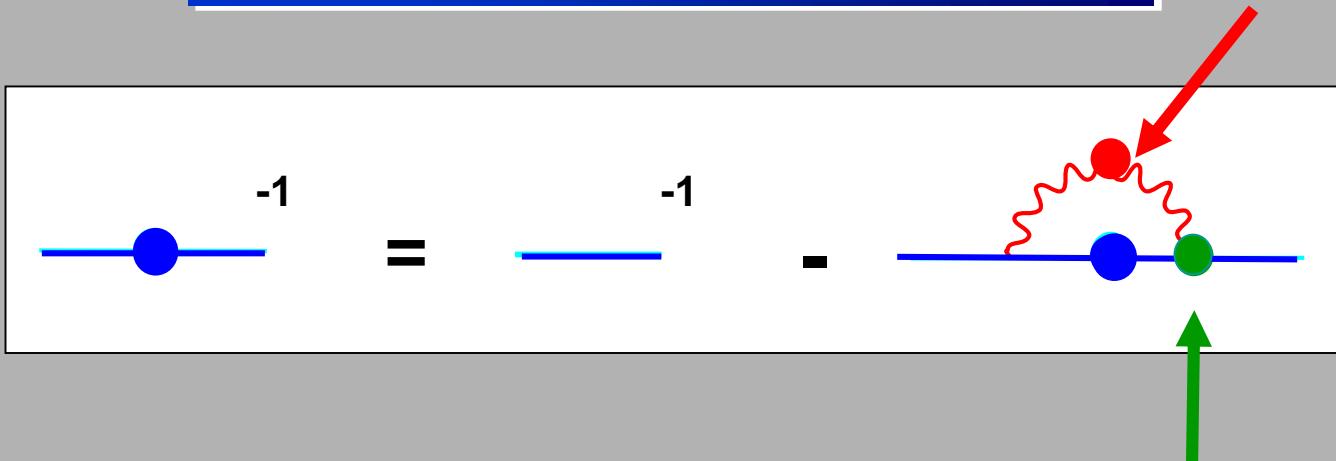
$$S_F(p) = \frac{\mathcal{F}(p)}{p - \mathcal{M}(p)}$$

wavefunction renormalisation

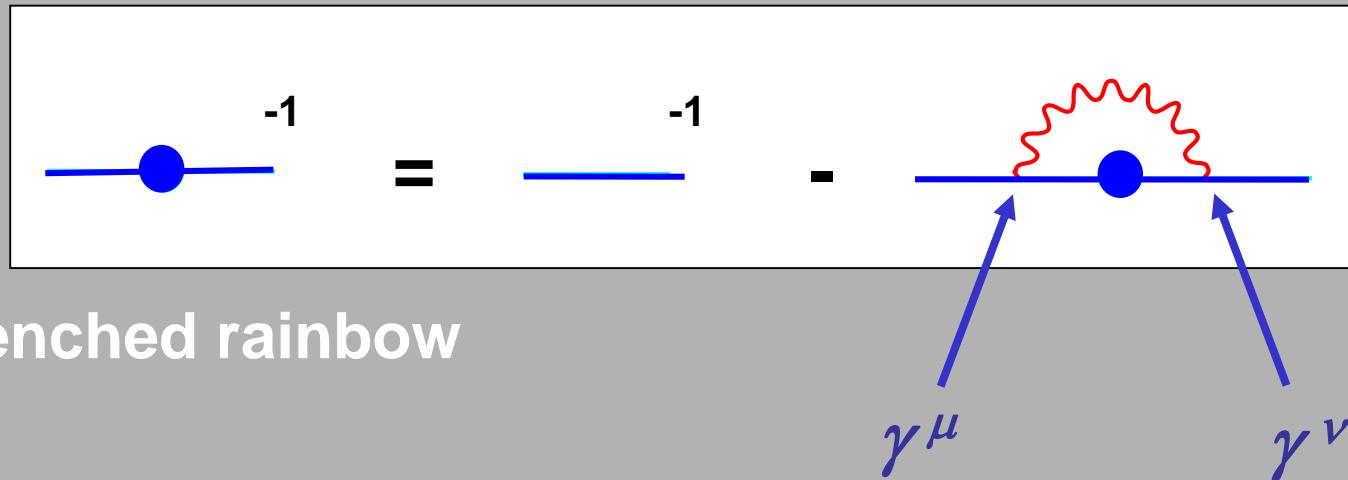
mass function

$$S_F(p)^{-1} = p - m_0 - \frac{\alpha}{4\pi} \int d^4k \gamma_\mu S_F(k) \Gamma_\nu(k, p) \Delta^{\mu\nu}(q)$$

Fermion mass generation

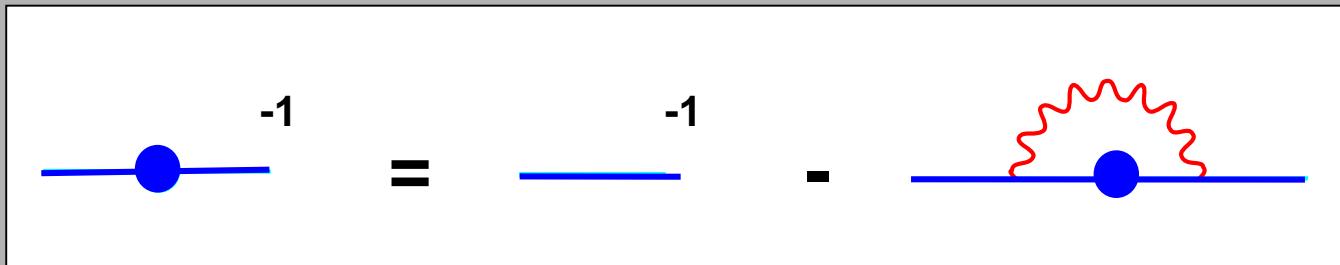


Fermion mass generation

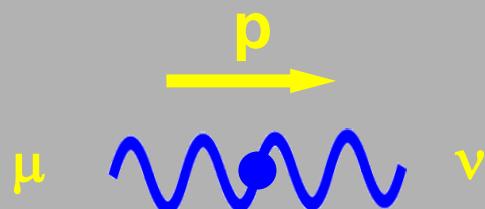


quenched rainbow

Fermion mass generation



quenched rainbow

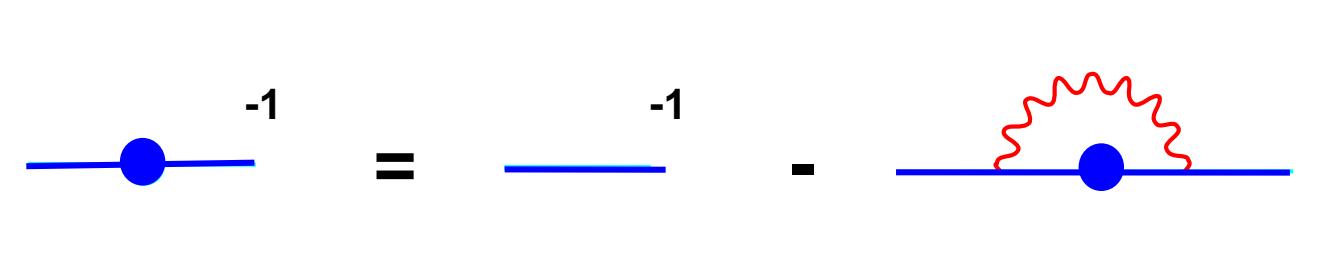


$$\Delta^{\mu\nu}(p) = \Delta(p^2) \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) + \xi \frac{p^\mu p^\nu}{p^4}$$

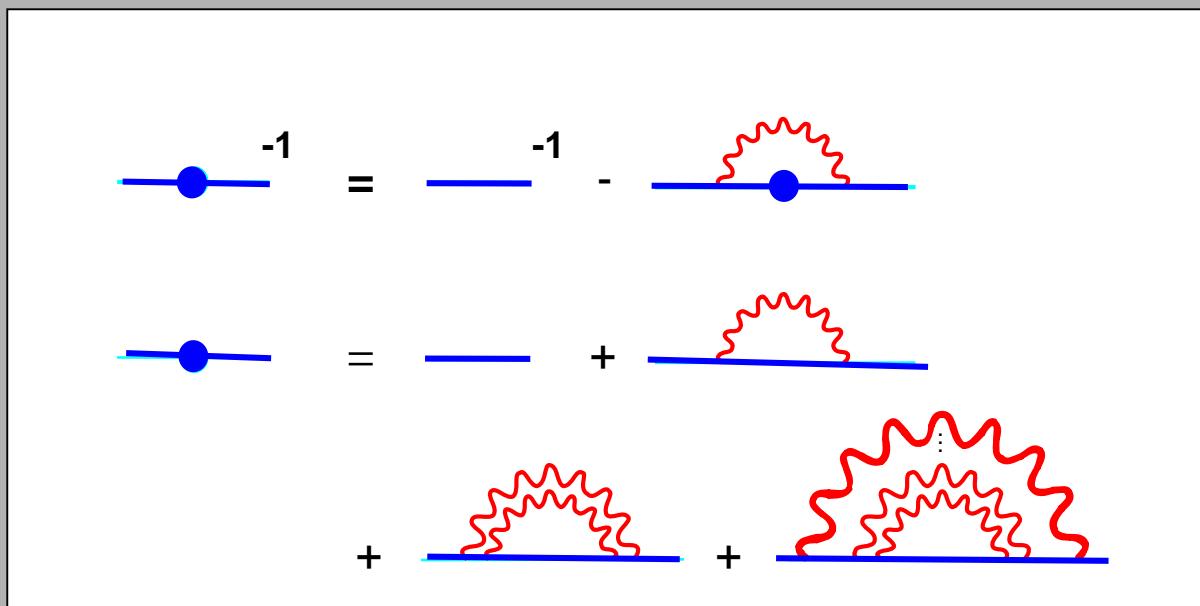
quenched

$$\Delta(p^2) = 1 / p^2$$

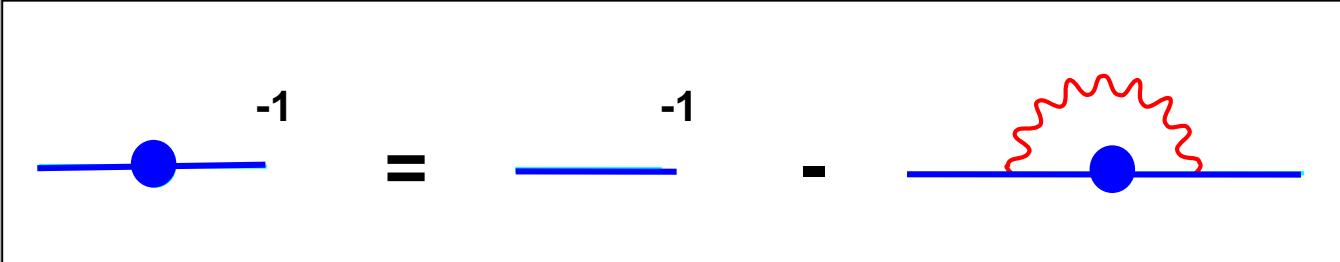
Fermion mass generation



quenched rainbow



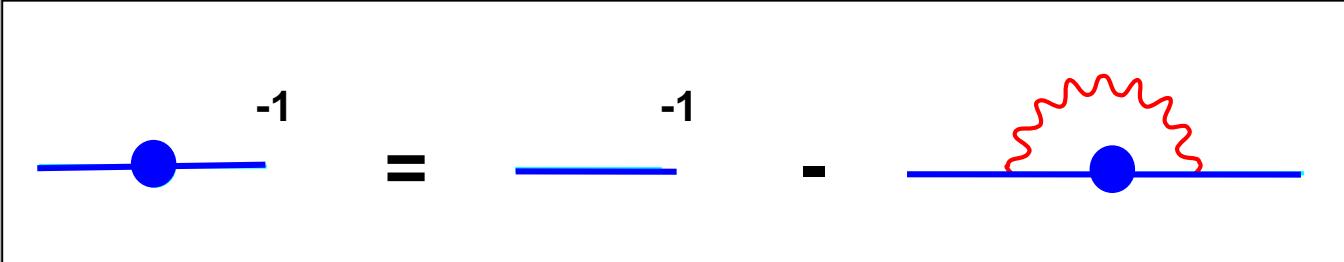
Fermion mass generation



quenched rainbow

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Fermion mass generation



quenched rainbow

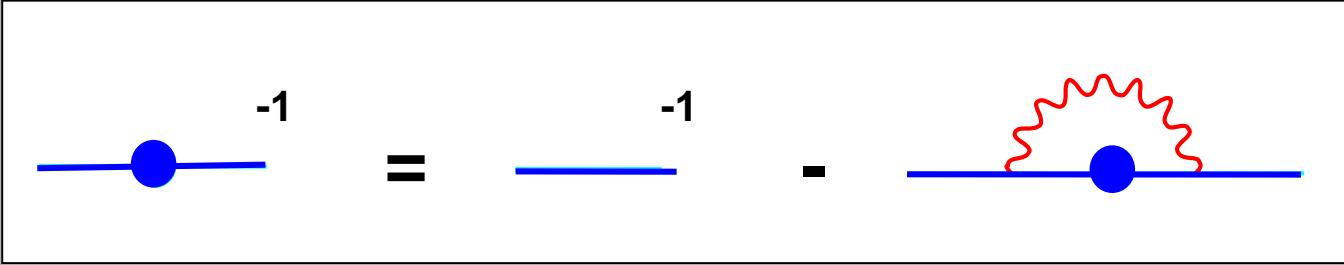
$$\frac{\mathcal{M}(p)}{\mathcal{F}(p)} = m_0 + \frac{\alpha_0}{4\pi} (3 + \xi) \int_0^{\kappa^2} dk^2 \frac{\mathcal{F}(k)\mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} \left[\theta_+ \frac{k^2}{p^2} + \theta_- \right]$$

$$\frac{1}{\mathcal{F}(p)} = 1 + \frac{\alpha_0 \xi}{4\pi} \int_0^{\kappa^2} dk^2 \frac{\mathcal{F}(k)}{k^2 + \mathcal{M}(k)^2} \left[\theta_+ \frac{k^4}{p^4} + \theta_- \right]$$

$$\Delta^{\mu\nu}(p) = \frac{1}{p^2} \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) + \xi \frac{p^\mu p^\nu}{p^4}$$

$$\theta_{\pm} = \begin{cases} 1 & \pm (p^2 - k^2) > 0 \\ 0 & \pm (p^2 - k^2) < 0 \end{cases}$$

Fermion mass generation



quenched rainbow

$\kappa = \text{cutoff}$

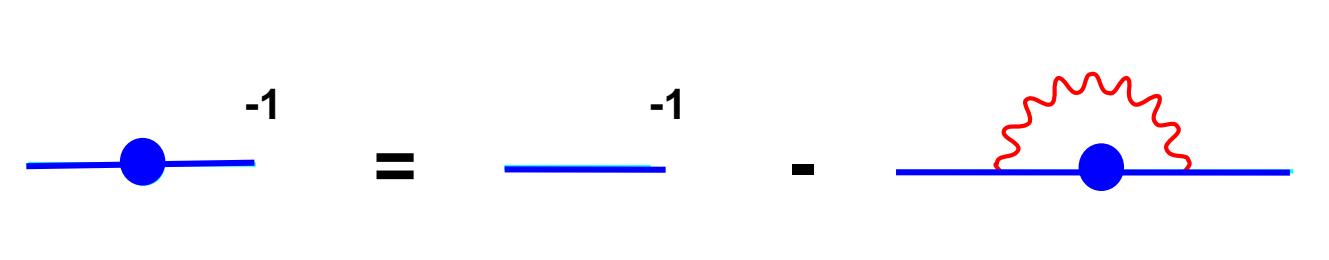
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Fermion mass generation



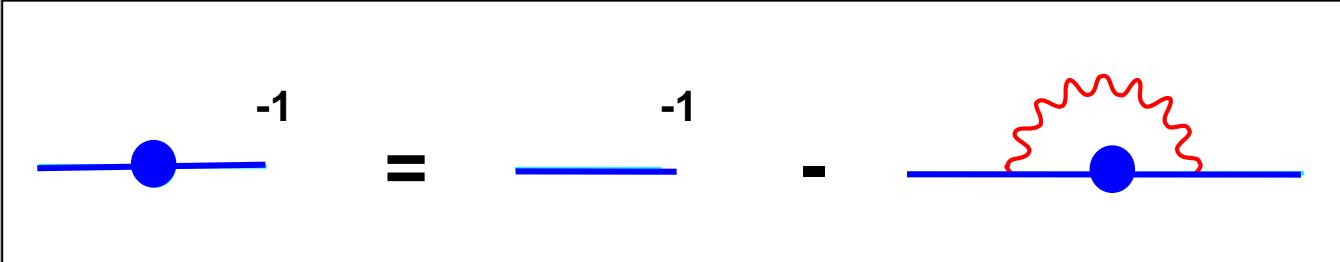
quenched rainbow

Landau gauge: $\xi = 0$

$$\frac{\mathcal{M}(p)}{\mathcal{F}(p)} = m_0 + \frac{\alpha_0}{4\pi} \quad 3 \quad \int_0^{\kappa^2} dk^2 \frac{\mathcal{F}(k)\mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} \left[\theta_+ \frac{k^2}{p^2} + \theta_- \right]$$

$$\frac{1}{\mathcal{F}(p)} = 1$$

Fermion mass generation



quenched rainbow

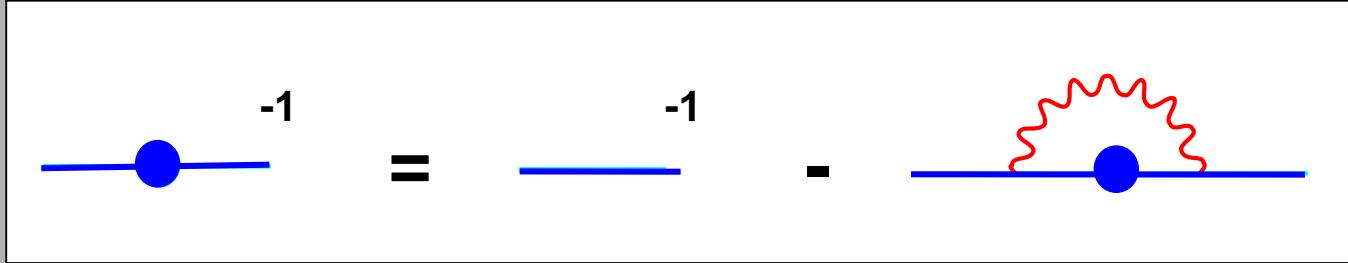
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$$m_0 = 0 \quad \rightarrow \quad M(p) \neq 0$$

Fermion mass generation



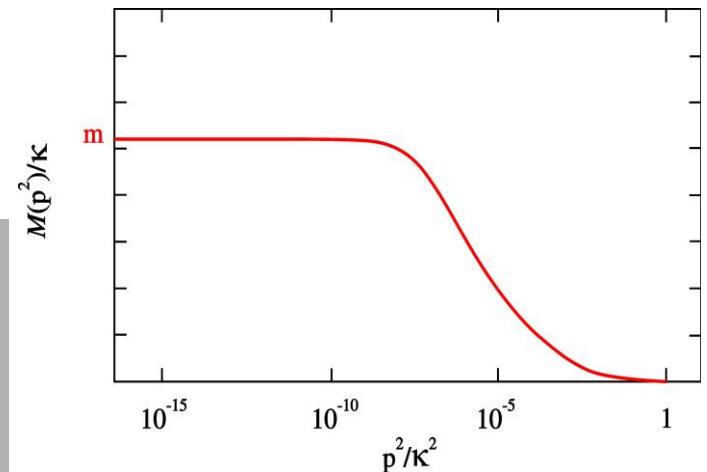
quenched rainbow

Landau gauge: $\xi = 0$

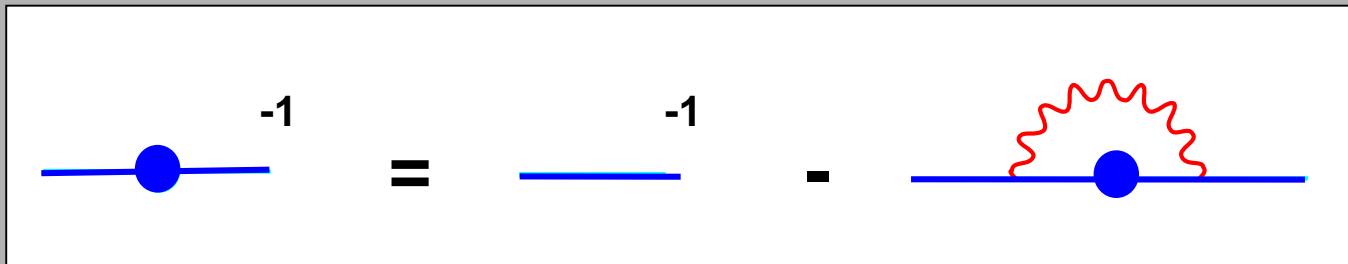
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Fermion mass generation



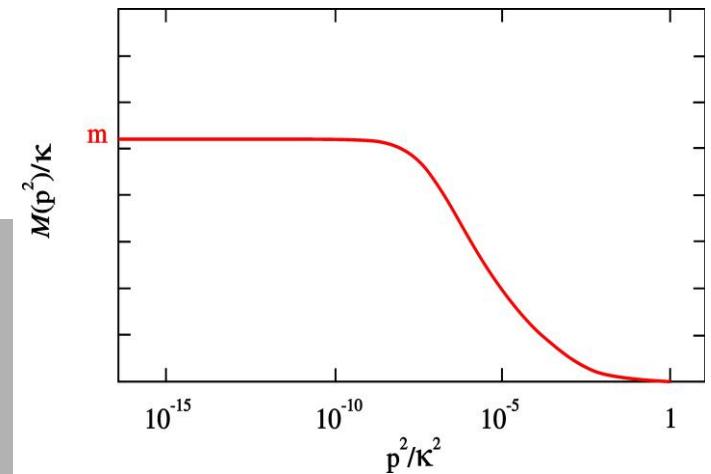
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$$\frac{1}{\mathcal{F}(p)} = 1$$

$$\alpha_0 \geq \pi/3$$



Fermion mass generation

$$\mathcal{M}(p) = m_0 + \frac{3\alpha_0}{4\pi} \left[\frac{1}{p^2} \int_0^{p^2} dk^2 \frac{k^2 \mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} + \int_{p^2}^{\kappa^2} dk^2 \frac{\mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} \right]$$

Fermion mass generation

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$$\frac{d}{dp^2} M(p) = -\frac{3\alpha_0}{4\pi p^4} \int_0^{p^2} dk^2 \frac{k^2 M(k)}{k^2 + M(k)^2}$$

Fermion mass generation

$$\mathcal{M}(p) = m_0 + \frac{3\alpha_0}{4\pi} \left[\frac{1}{p^2} \int_0^{p^2} dk^2 \frac{k^2 \mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} + \int_{p^2}^{\kappa^2} dk^2 \frac{\mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} \right]$$

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$$\frac{d}{dp^2} \left[p^4 \frac{d}{dp^2} M(p) \right] = -\frac{3\alpha_0}{4\pi} \frac{p^2 M(p)}{p^2 + M(p)^2}$$

Fermion mass generation

$$\mathcal{M}(p) = m_0 + \frac{3\alpha_0}{4\pi} \left[\frac{1}{p^2} \int_0^{p^2} dk^2 \frac{k^2 \mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} + \int_{p^2}^{\kappa^2} dk^2 \frac{\mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} \right]$$

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$$p^2 \gg M(p)^2 \rightarrow M(p) \sim p^s$$

Fermion mass generation

$$\mathcal{M}(p) = m_0 + \frac{3\alpha_0}{4\pi} \left[\frac{1}{p^2} \int_0^{p^2} dk^2 \frac{k^2 \mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} + \int_{p^2}^{\kappa^2} dk^2 \frac{\mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} \right]$$

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$$s = -1 \pm \sqrt{1 - 3\alpha_0/\pi}$$

Fermion mass generation

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$p^2 \gg M(p)^2 \rightarrow M(p) \sim p^s$

$$s = -1 \pm \sqrt{1 - 3\alpha_0/\pi}$$

Boundary conditions:

$$\alpha_0 \geq \pi/3$$

Fermion mass generation



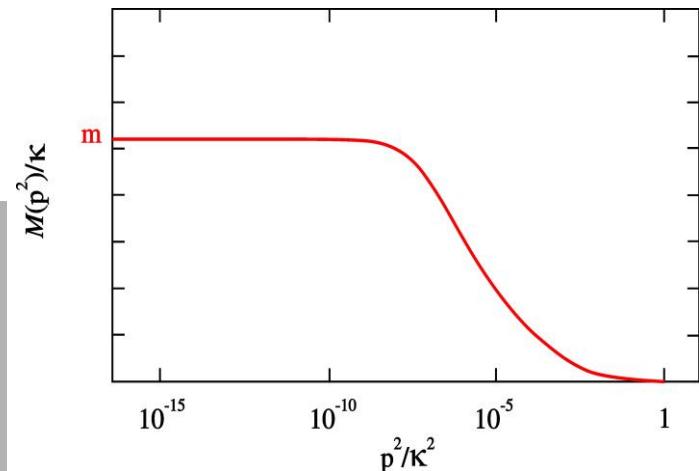
quenched rainbow

Landau gauge: $\xi = 0$

$$\mathcal{M}(p) = m_0 + \frac{3\alpha_0}{4\pi} \left[\frac{1}{p^2} \int_0^{p^2} dk^2 \frac{k^2 \mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} + \int_{p^2}^{\kappa^2} dk^2 \frac{\mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} \right]$$

$$\frac{1}{\mathcal{F}(p)} = 1$$

$$\alpha_0 \geq \pi/3$$



Fermion mass generation



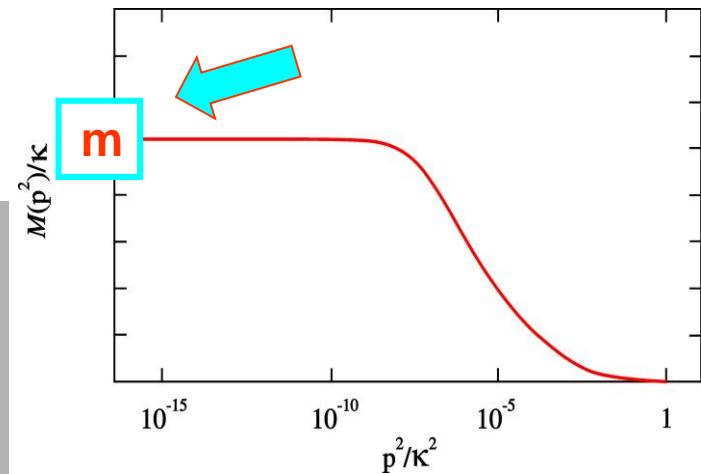
quenched rainbow

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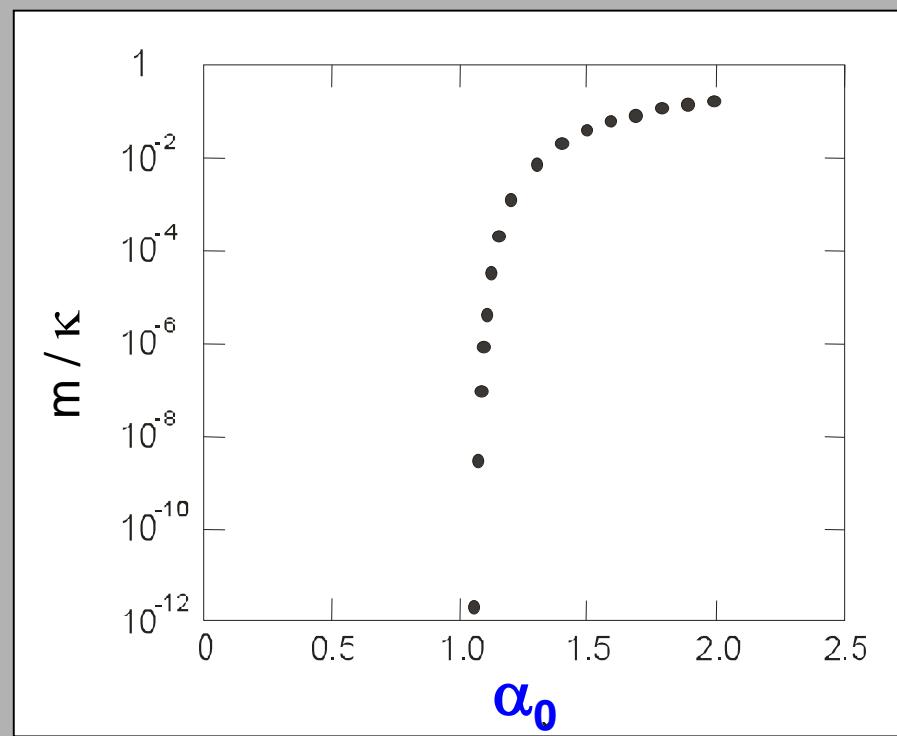
$$\frac{1}{\mathcal{F}(p)} = 1$$

$$\alpha_0 \geq \pi/3$$



Fermion mass generation

$$\alpha_0 \geq \pi/3$$



BUT in other gauges

$$\Delta^{\mu\nu}(p) = \Delta(p^2) \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) + \xi \frac{p^\mu p^\nu}{p^4}$$

BUT in other gauges



quenched rainbow

$$\frac{\mathcal{M}(p)}{\mathcal{F}(p)} = m_0 + \frac{\alpha_0}{4\pi} (3 + \xi) \int_0^{\kappa^2} dk^2 \frac{\mathcal{F}(k)\mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} \left[\theta_+ \frac{k^2}{p^2} + \theta_- \right]$$

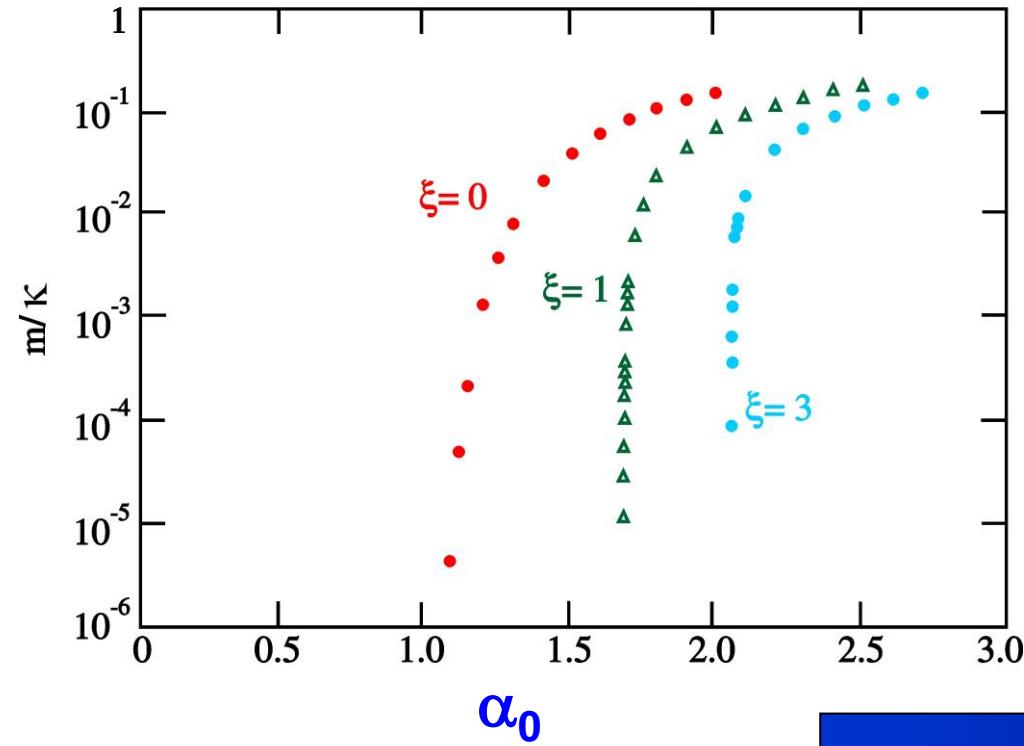
$$\frac{1}{\mathcal{F}(p)} = 1 + \frac{\alpha_0 \xi}{4\pi} \int_0^{\kappa^2} dk^2 \frac{\mathcal{F}(k)}{k^2 + \mathcal{M}(k)^2} \left[\theta_+ \frac{k^4}{p^4} + \theta_- \right]$$

$$\Delta^{\mu\nu}(p) = \Delta(p^2) \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) + \xi \frac{p^\mu p^\nu}{p^4}$$

BUT in other gauges

mass

κ = cutoff



gauge dependent

$$\Delta^{\mu\nu}(p) = \Delta(p^2) \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) + \xi \frac{p^\mu p^\nu}{p^4}$$

Schwinger-Dyson Equations

QED

$$-1 = \text{---} - \text{---} - \text{---}$$

2 equations

$$-1 = \text{---} - N_F \text{---} - \text{---}$$

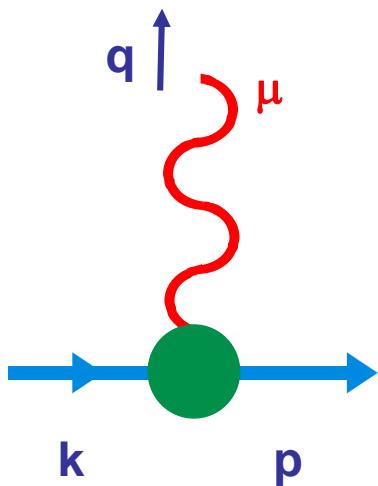
2 equations

$$= \text{---} + \text{---}$$

12 equations

Ward identity

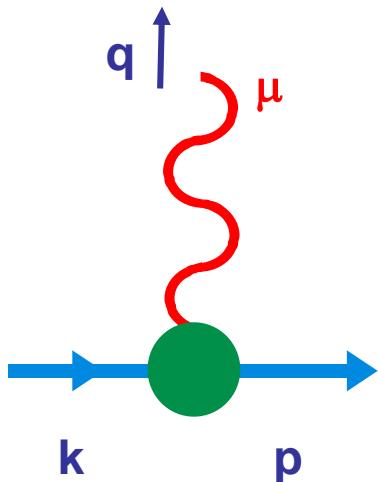
Gauge Invariance



$$\Gamma^\mu(p,p) = \frac{d}{dp_\mu} S_F^{-1}(p)$$

Ward identity

Gauge Invariance

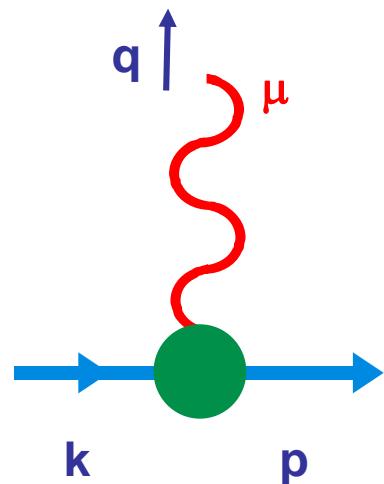


lowest order

$$\frac{d}{dp_\mu} S_F^{0^{-1}}(p) = \frac{d}{dp_\mu} (\gamma^\mu p_\mu - m_0) = \gamma^\mu = \Gamma_0^\mu$$

$$\Gamma^\mu(p, p) = \frac{d}{dp_\mu} S_F^{-1}(p)$$

Ward identity



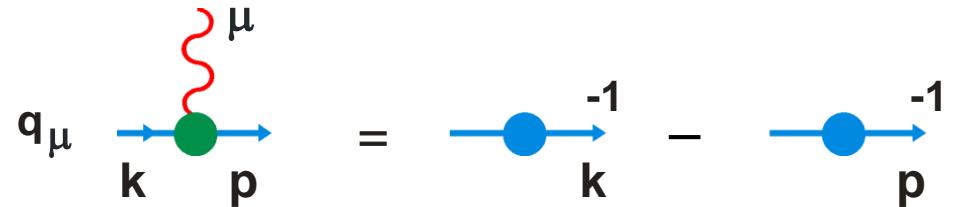
Gauge Invariance

$$q = k - p \longrightarrow 0$$

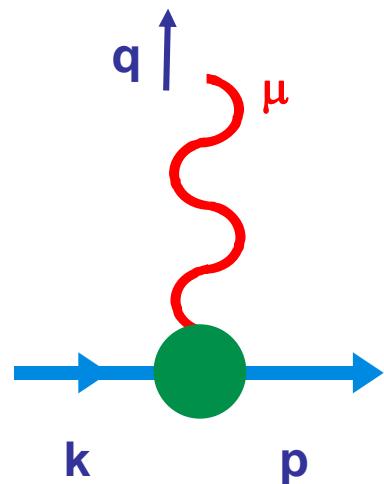
$$\Gamma^\mu(p, p) = \frac{d}{dp_\mu} S_F^{-1}(p)$$

Ward – Green –Takahashi

$$q_\mu \Gamma^\mu(k, p) = S_F^{-1}(k) - S_F^{-1}(p)$$



Ward identity



Gauge Invariance

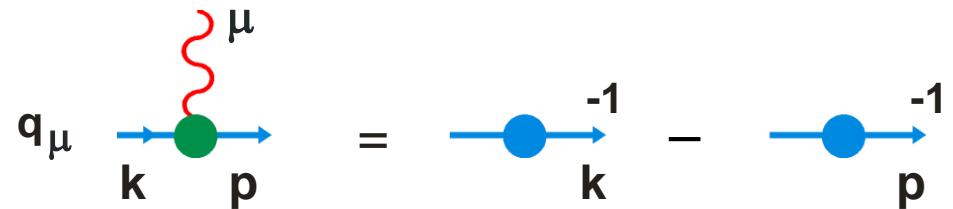
$$q = k - p \longrightarrow 0$$

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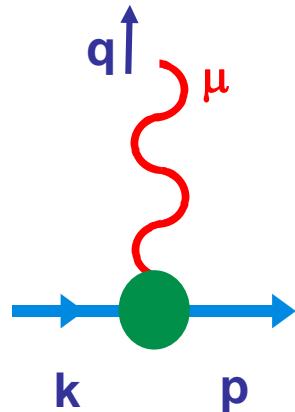
Ward – Green –Takahashi

$$Z_1 = Z_2$$

$$q_\mu \Gamma^\mu(k, p) = S_F^{-1}(k) - S_F^{-1}(p)$$



Ward – Green –Takahashi



$$q_\mu \Gamma^\mu(k, p) = S_F^{-1}(k) - S_F^{-1}(p)$$

$$q_\mu \Gamma^\mu(k, p) = \text{---} \overset{-1}{\bullet} \text{---} - \text{---} \overset{-1}{\bullet} \text{---}$$

$$\Gamma_L^\mu(p, k, q) \equiv \Gamma_{BC}^\mu(p, k, q) = \sum_{i=1}^4 \lambda_i(p^2, k^2, q^2) L_i^\mu(p, k; q)$$

Ball &
Chiu

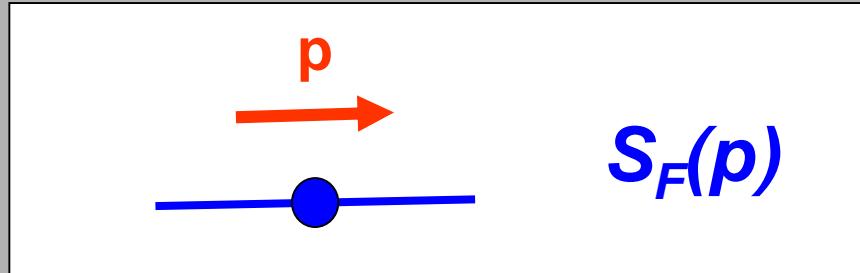
$$q_\mu \Gamma_T^\mu(p, k; q) = 0$$

$$Z_1 = Z_2$$

$$\Gamma_T^\mu(p, k; q) = \sum_{i=1,2,\dots,8} \tau_i(p^2, k^2, q^2) T_i^\mu(p, k; q)$$

Fermion propagator

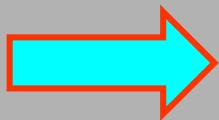
Recall



$$S_F(p) = \frac{\mathcal{F}(p)}{p - \mathcal{M}(p)}$$

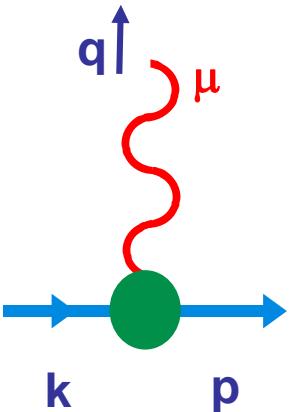
wavefunction renormalisation

mass function



$$\Gamma_{BC}^\mu \sim \frac{1}{\mathcal{F}}, \frac{\mathcal{M}}{\mathcal{F}}$$

Ward – Green –Takahashi



$$q_\mu \Gamma^\mu(k, p) = S_F^{-1}(k) - S_F^{-1}(p)$$

$$\Gamma_L^\mu(p, k, q) \equiv \Gamma_{BC}^\mu(p, k, q) = \sum_{i=1}^4 \lambda_i(p^2, k^2, q^2) L_i^\mu(p, k; q)$$

Ball &
Chiu

$$\lambda_1(p^2, k^2, q^2) = \frac{1}{2} \left(\frac{1}{F(k^2)} + \frac{1}{F(p^2)} \right)$$

$$L_1^\mu(p, k; q) = \gamma^\mu$$

$$\lambda_2(p^2, k^2, q^2) = \frac{1}{2(k^2 - p^2)} \left(\frac{1}{F(k^2)} - \frac{1}{F(p^2)} \right)$$

$$L_2^\mu(p, k; q) = (k^\mu + p^\mu)(k + p)$$

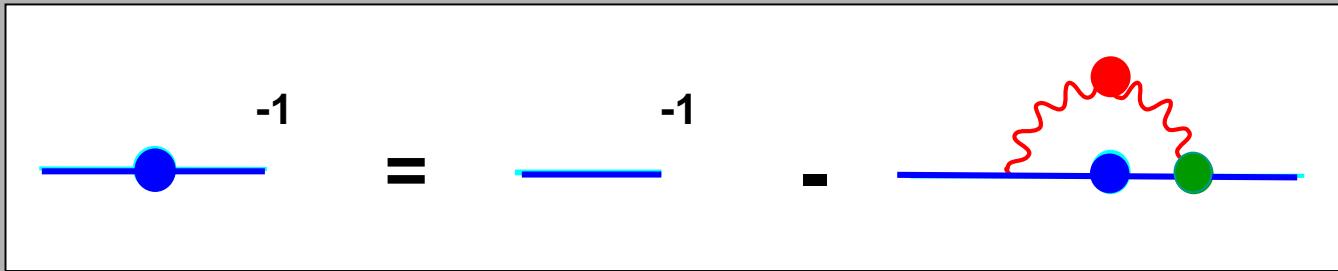
$$\lambda_3(p^2, k^2, q^2) = -\frac{1}{(k^2 - p^2)} \left(\frac{M(k^2)}{F(k^2)} - \frac{M(p^2)}{F(p^2)} \right)$$

$$L_3^\mu(p, k; q) = (k^\mu + p^\mu)$$

$$\lambda_4(p^2, k^2, q^2) = 0$$

$$L_4^\mu(p, k; q) = \sigma^{\mu\nu} (k_\nu + p_\nu)$$

Fermion mass generation



$$S_F(p) = \frac{\mathcal{F}(p)}{p - \mathcal{M}(p)}$$

wavefunction renormalisation

mass function

$$S_F(p)^{-1} = p - m_0 - \frac{\alpha}{4\pi} \int d^4k \gamma_\mu S_F(k) \Gamma_\nu(k, p) \Delta^{\mu\nu}(q)$$

how to regularize: $d^4k \rightarrow d^n k$

Multiplicative Renormalizability

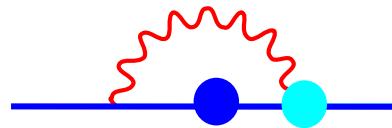
Gauge Invariance and Multiplicative Renormalizability

$$\text{---} \bullet \text{---} -1$$

=

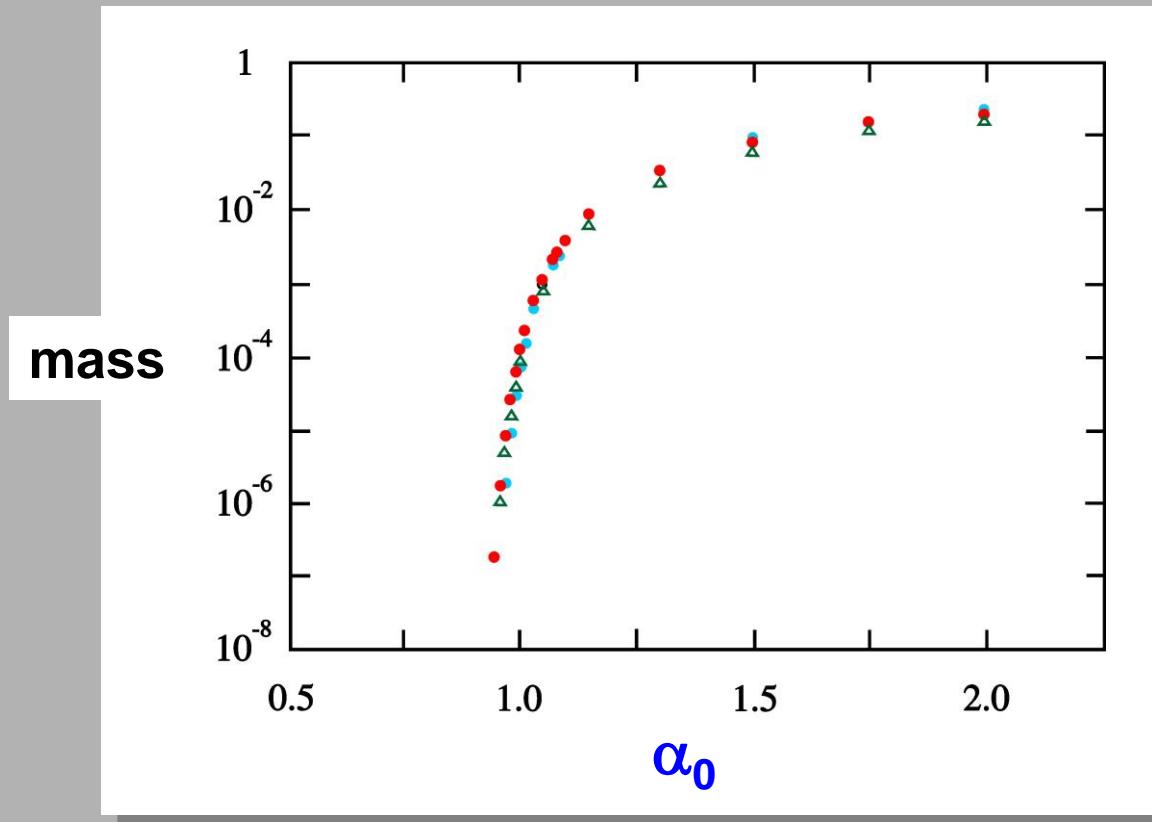
$$\text{---} -1$$

-

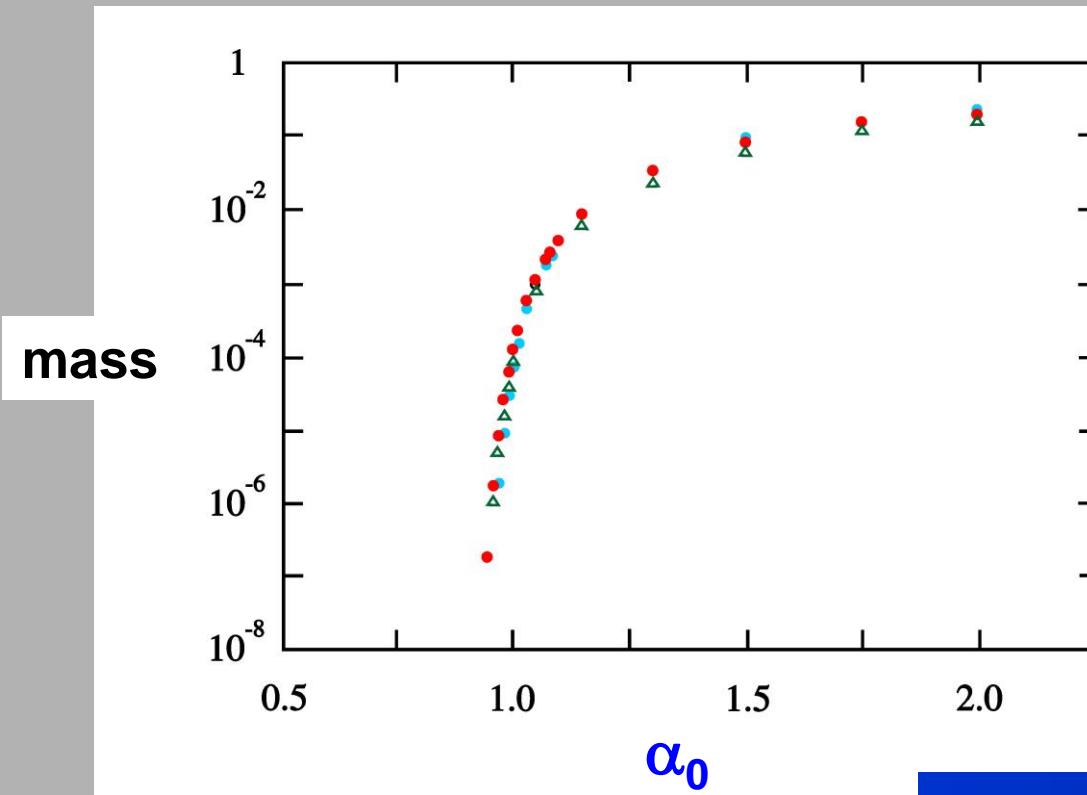


CP vertex

Gauge Invariance and Multiplicative Renormalizability



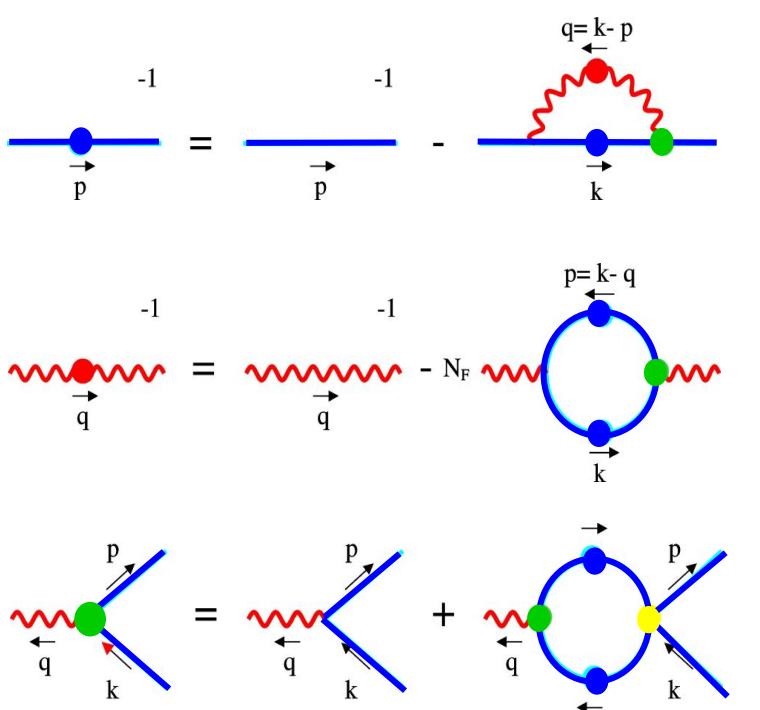
Gauge Invariance and Multiplicative Renormalizability



almost gauge independent

Schwinger-Dyson Equations

Gauge Invariance &
Multiplicative Renormalizability



$$T_2^\mu(p, k; q) = (p^\mu(k \cdot q) - k^\mu(p \cdot q))(\not{k} + \not{p})$$

$$T_3^\mu(p, k; q) = q^2\gamma^\mu - q^\mu\not{q}$$

$$T_6^\mu(p, k; q) = \gamma^\mu(p^2 - k^2) + (p + k)^\mu\not{q}$$

$$T_8^\mu(p, k; q) = -\gamma^\mu k^\lambda p^\nu \sigma_{\lambda\nu} + k^\mu \not{p} - p^\mu \not{k}$$

Kizilersu & P

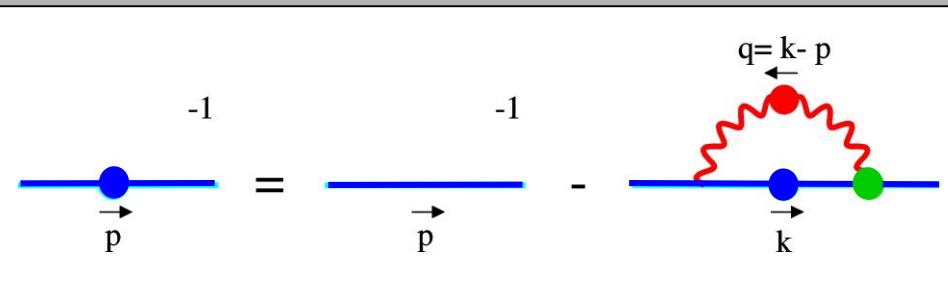
$$\Gamma_T^\mu(p, k; q) = \sum_{i=2,3,6,8} \tau_i(p^2, k^2, q^2) T_i^\mu(p, k; q)$$

Transverse components at $\mathcal{O}(\alpha)$

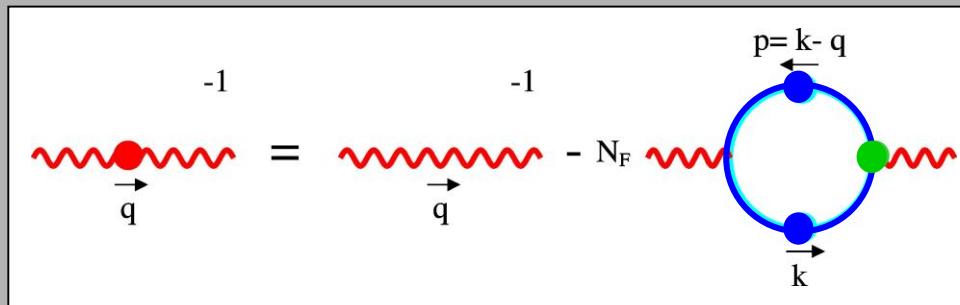
$$\begin{aligned}
\mathcal{T}_6 = & - \frac{3\alpha}{32\pi\Delta^4} (p^2 - k^2) [m^2 + k \cdot p] \left\{ \frac{q^2}{2} [k \cdot p + m^2] J_0 + 2q^2 S + [-p^2 + k \cdot p] L + [-k^2 + k \cdot p] L' \right\} \\
& + \frac{\alpha}{8\pi\Delta^2} (\xi - 1) \left\{ (p^2 - k^2) \left[\frac{q^2}{4} - \frac{3q^2}{8\Delta^2} (m^4 - p^2 k^2) \right] J_0 - \frac{(p^2 - k^2)}{2p^2 k^2} [m^2 k \cdot p - p^2 k^2] \right. \\
& \quad \left. + \frac{1}{\chi} \left[m^2 \Delta^2 \left\{ \left(p^2 - m^2 \frac{k^2}{p^2} \right) L - \left(k^2 - m^2 \frac{p^2}{k^2} \right) L' \right\} + \frac{1}{2} m^2 (k^4 - p^4) [m^2 - k \cdot p] (L + L') - 2m^2 (k \cdot p) (p^4 L - k^4 L') \right. \right. \\
& \quad \left. - \frac{m^6 q^2}{2} \left\{ \left(1 + \frac{k \cdot p}{p^2} \right) L - \left(1 + \frac{k \cdot p}{k^2} \right) L' \right\} - \frac{m^2}{2} k \cdot p (p^2 - k^2) [(m^2 - p^2) L + (m^2 - k^2) L'] \right. \\
& \quad \left. - \frac{1}{2} p^2 k^2 q^2 \left\{ [k^2 - k \cdot p] L - [p^2 - k \cdot p] L' \right\} - q^2 [p^4 (m^2 - k^2) L - k^4 (m^2 - p^2) L'] + 2m^2 k^2 p^2 (k^2 L - p^2 L') \right. \\
& \quad \left. + \frac{3m^2}{8\Delta^2} (p^4 - k^4) (p^2 - k^2) [p^2 (m^2 - k^2) L - k^2 (m^2 - p^2) L'] - \frac{3m^2}{4\Delta^2} p^2 k^2 q^2 (p^2 - k^2) [(m^2 + p^2) L + (m^2 + k^2) L'] \right. \\
& \quad \left. - \frac{3m^2}{4\Delta^2} p^2 k^2 (p^2 - k^2)^2 [(m^2 - p^2) L - (m^2 - k^2) L'] - \frac{3m^2}{8\Delta^2} p^2 q^2 (p^4 - k^4) [(m^2 + k^2) L - (m^2 + p^2) L'] \right. \\
& \quad \left. + \frac{3}{8\Delta^2} q^4 (p^2 - k^2) [(m^6 + p^4 k^2) L + (m^6 + p^2 k^4) L'] + \frac{3q^2}{8\Delta^2} (p^2 - k^2)^2 [(m^6 - p^4 k^2) L - (m^6 - p^2 k^4) L'] \right. \\
& \quad \left. + (p^2 - k^2) \left[-\frac{3m^4}{2\Delta^2} q^4 [m^2 + k \cdot p] + \frac{3}{2\Delta^2} k^2 p^2 q^4 [m^2 + k \cdot p] - 4m^4 q^2 + 2m^2 q^2 (p^2 + k^2) \right] S \right\}
\end{aligned}$$

Transverse components

QED



$$k^2, q^2 \gg p^2$$



$$k^2, p^2 \gg q^2$$

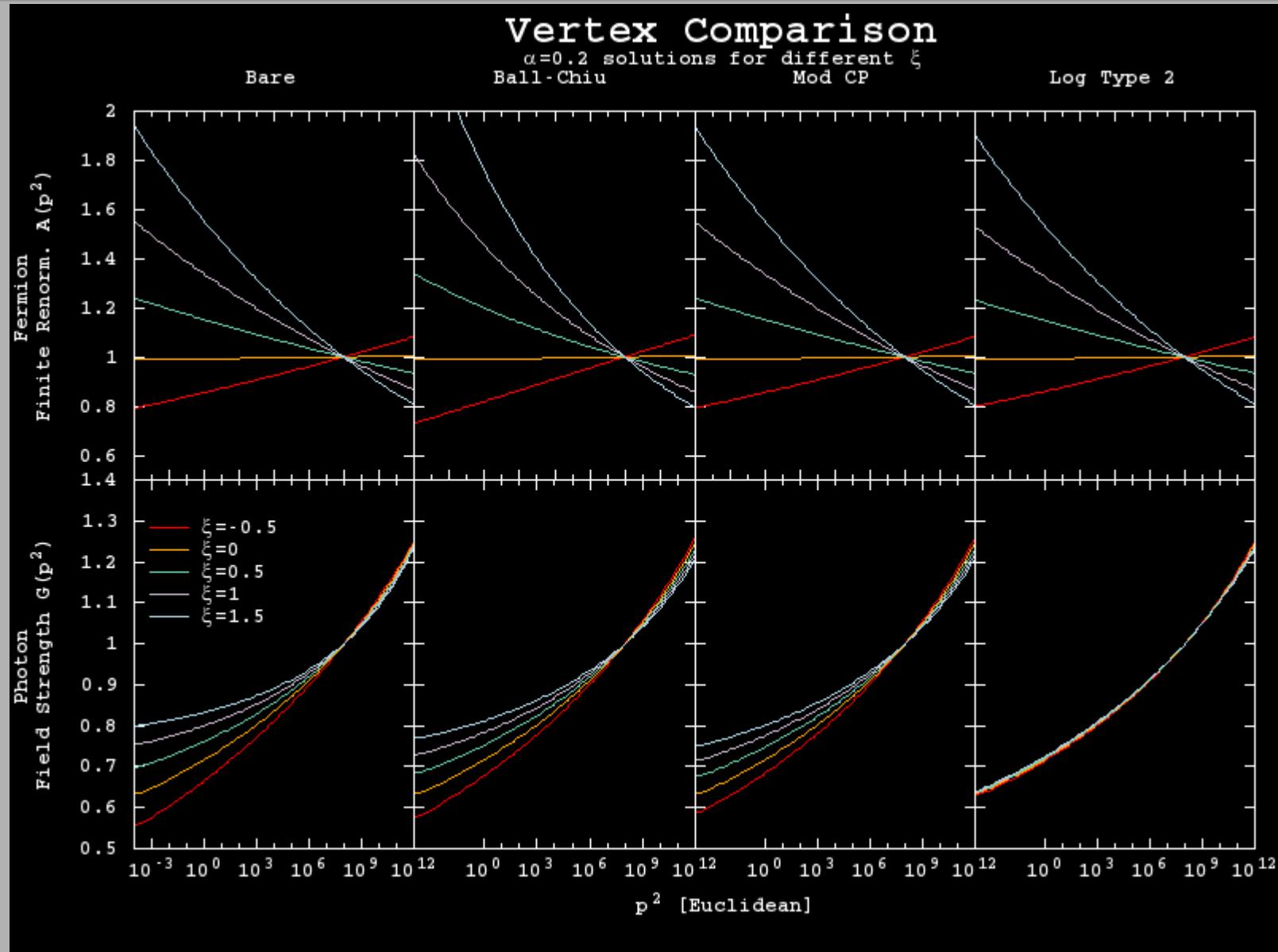
Gauge Invariance & Multiplicative Renormalizability

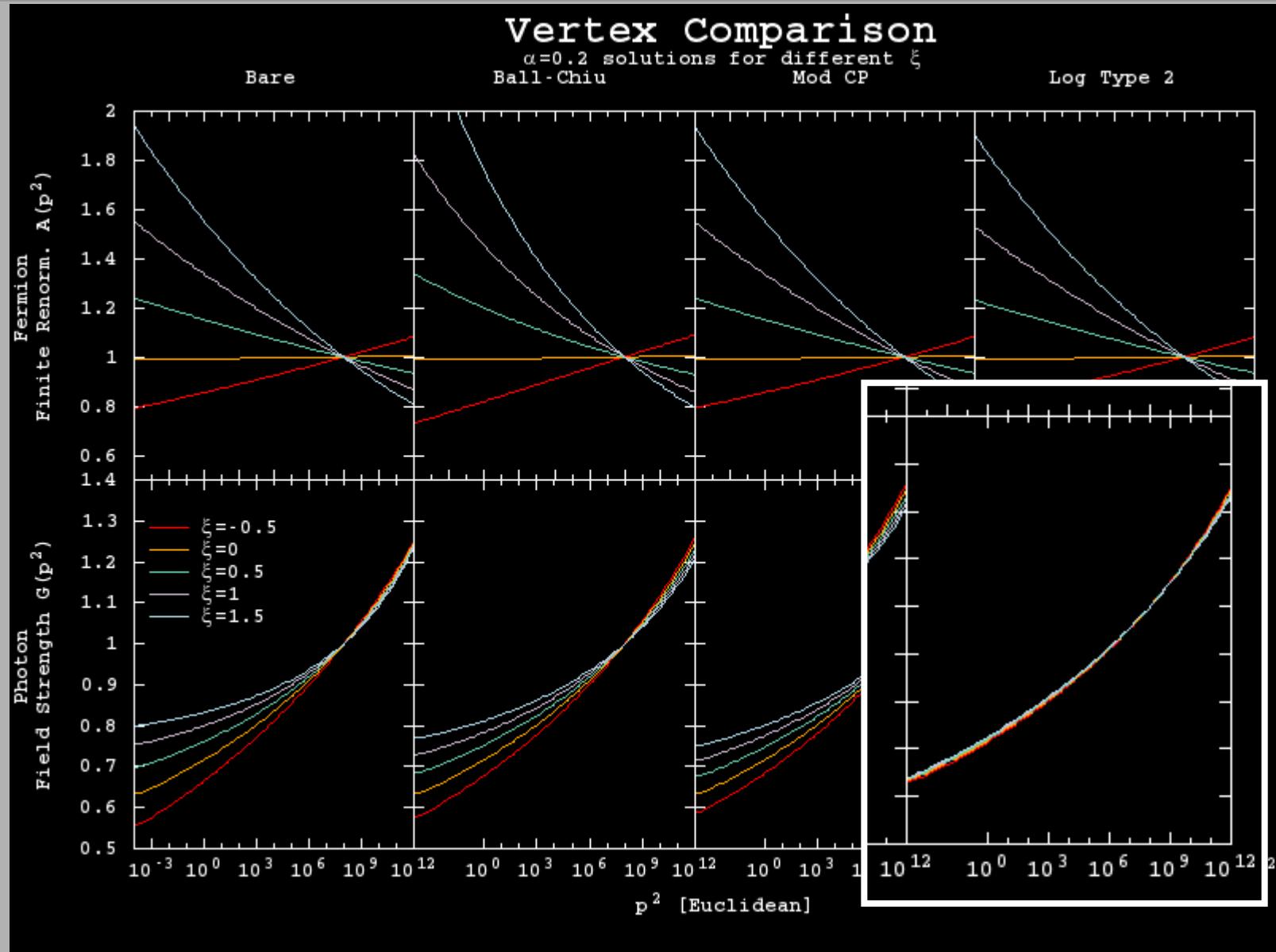
Transverse vertex

Kizilersu & P

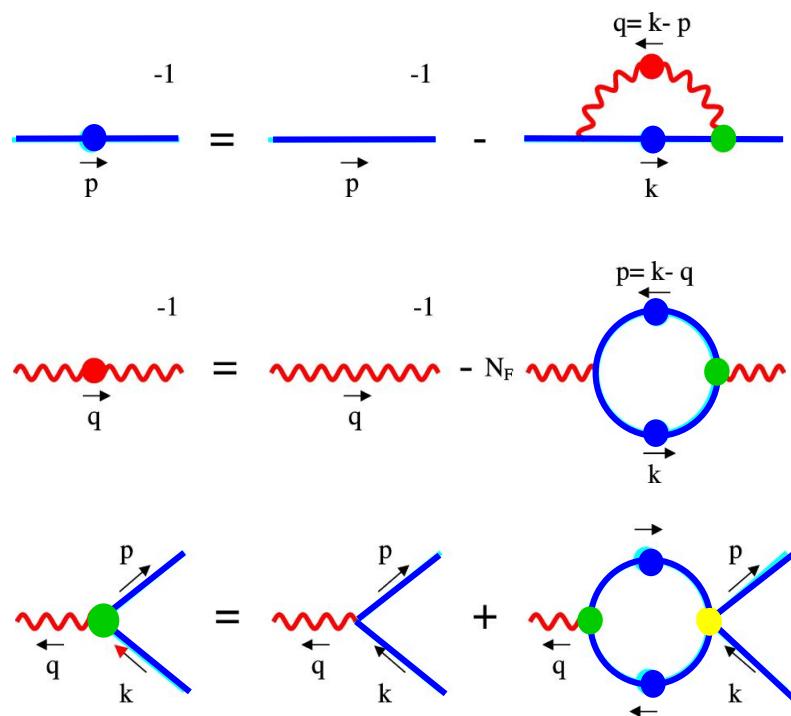
$$\Gamma_T^\mu(p, k; q) = \sum_{i=2,3,6,8} \tau_i(p^2, k^2, q^2) T_i^\mu(p, k; q)$$

$$\begin{aligned}\tau_2^E(p^2, k^2, q^2) &= \frac{1}{(k^4 - p^4)} \left(-\frac{4}{3} \right) \left(\frac{1}{F(k^2)} - \frac{1}{F(p^2)} \right) \\ &\quad + \frac{1}{(k^2 + p^2)^2} \left(-\frac{1}{3} \right) \left(\frac{1}{F(k^2)} + \frac{1}{F(p^2)} \right) \ln \left[\frac{1}{2} \left(\frac{F(q^2)}{F(k^2)} + \frac{F(q^2)}{F(p^2)} \right) \right] \\ \tau_3^E(p^2, k^2, q^2) &= -\frac{1}{(k^2 - p^2)} \left(\frac{5}{12} \right) \left(\frac{1}{F(k^2)} - \frac{1}{F(p^2)} \right) \\ &\quad - \frac{1}{(k^2 + p^2)} \left(\frac{1}{6} \right) \left(\frac{1}{F(k^2)} + \frac{1}{F(p^2)} \right) \ln \left[\frac{1}{2} \left(\frac{F(q^2)}{F(k^2)} + \frac{F(q^2)}{F(p^2)} \right) \right] \\ \tau_6^E(p^2, k^2, q^2) &= -\frac{1}{(k^2 + p^2)} \left(-\frac{1}{4} \right) \left(\frac{1}{F(k^2)} - \frac{1}{F(p^2)} \right), \quad \tau_8^E(p^2, k^2, q^2) = 0\end{aligned}$$





Schwinger-Dyson Equations



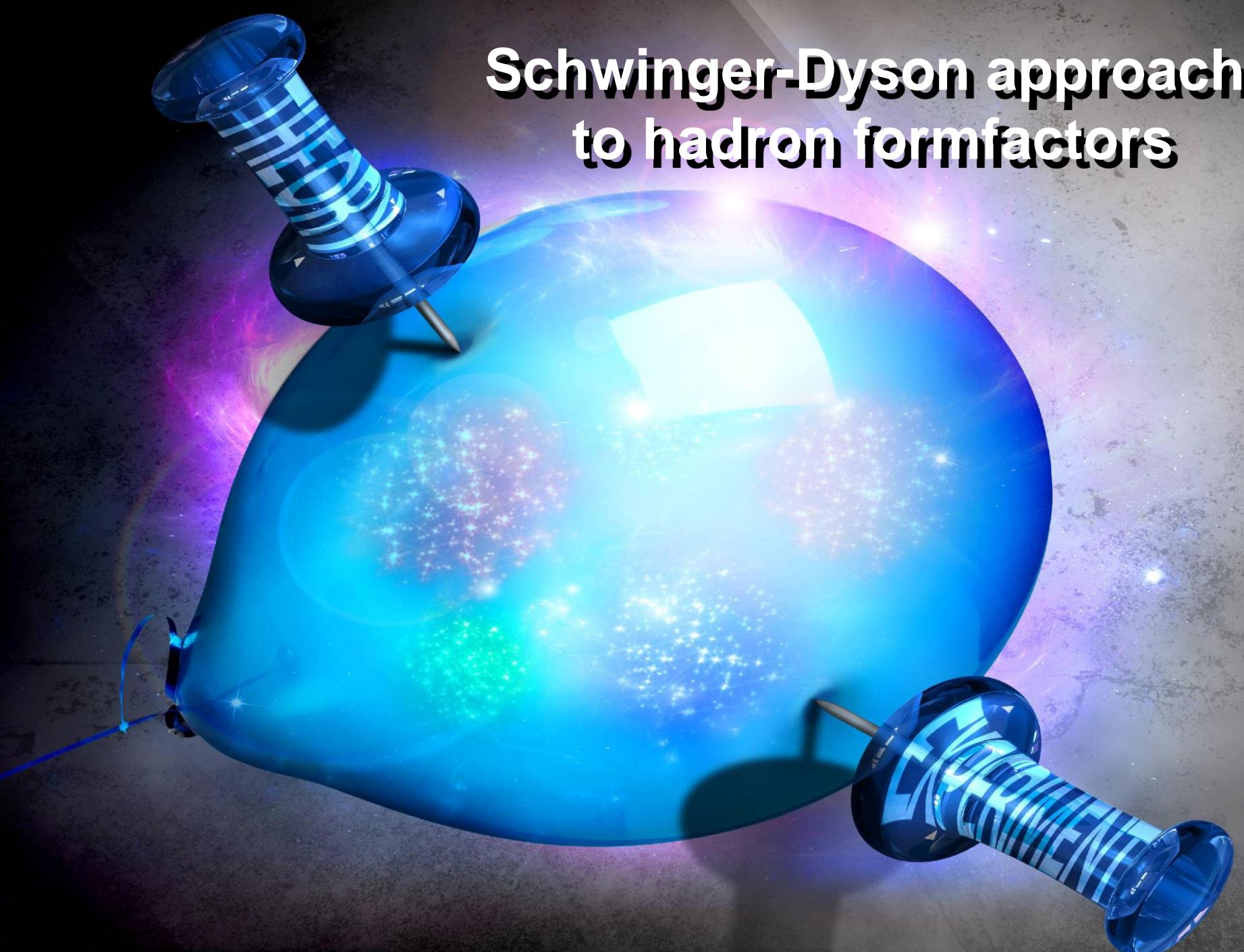
QED

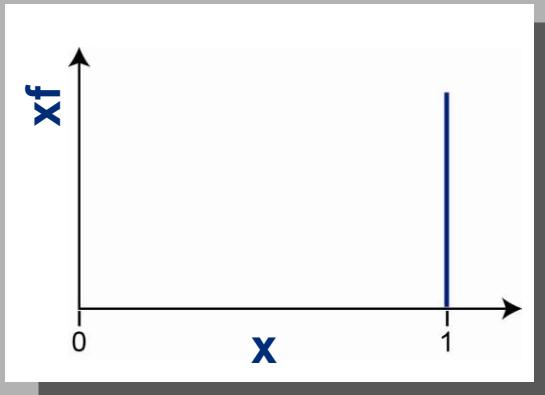
Consistent truncation

Gauge Invariance &
Multiplicative Renormalizability

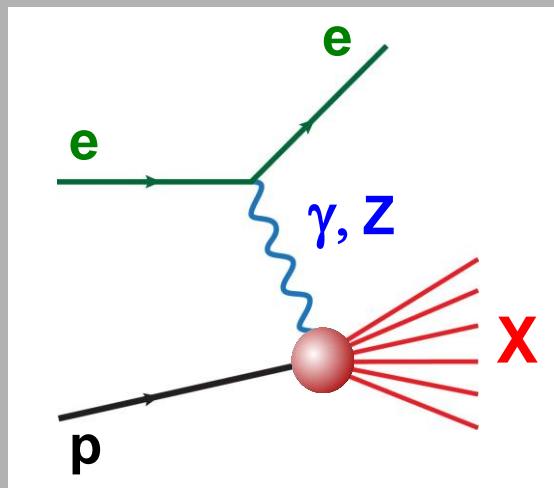
- (i) remove divergences (eg. quadratic div.)
- (ii) ensure correct gauge dependence (eg. transversality of boson)

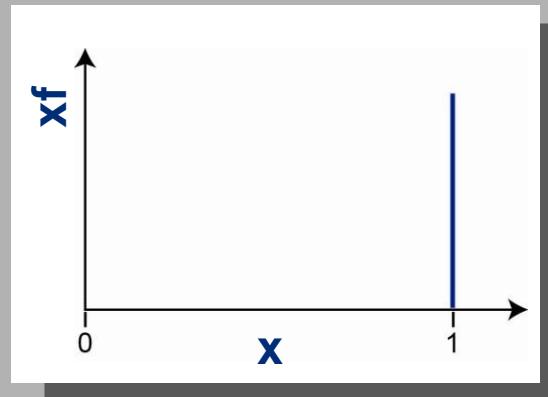
Schwinger-Dyson approach to hadron formfactors



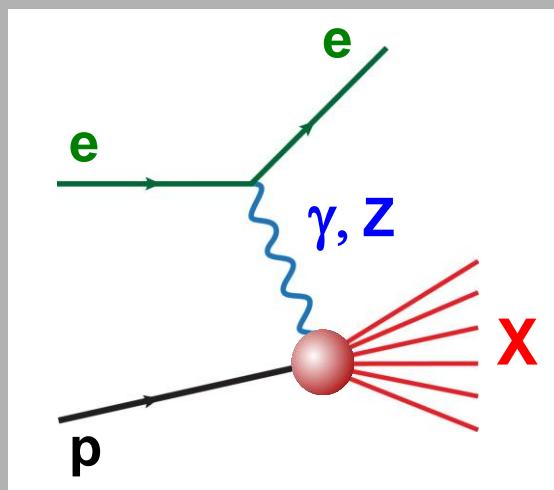
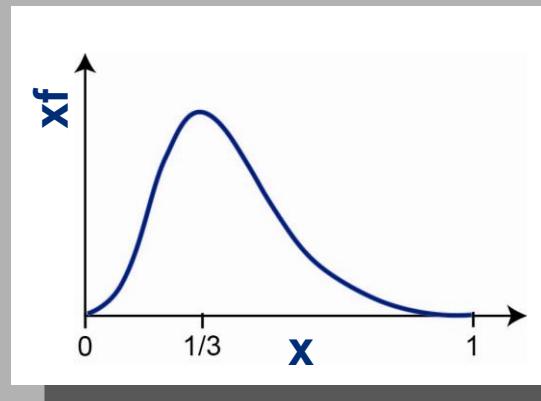
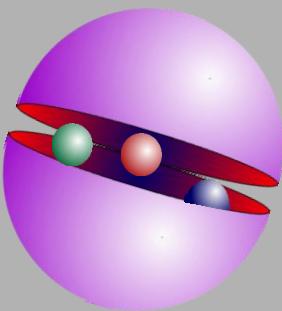


parton structure of the nucleon

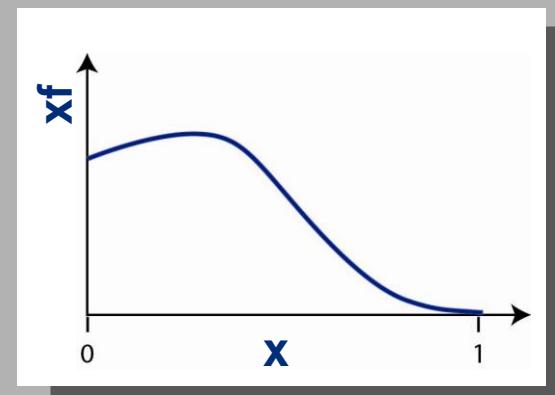
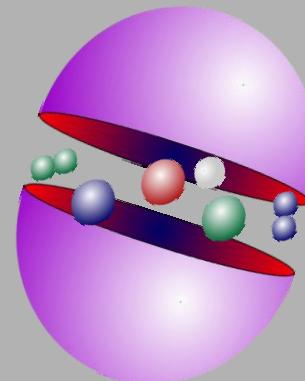
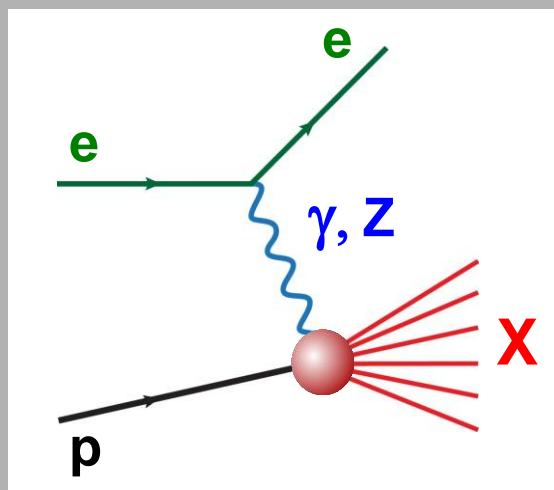
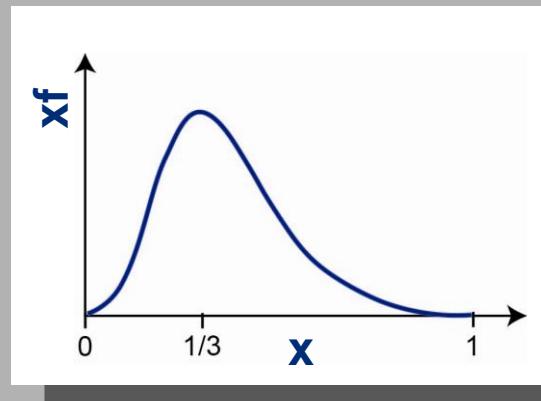
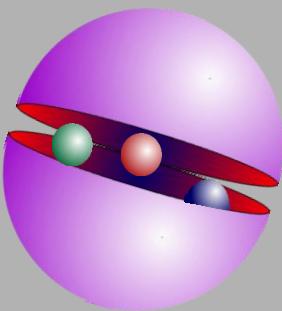
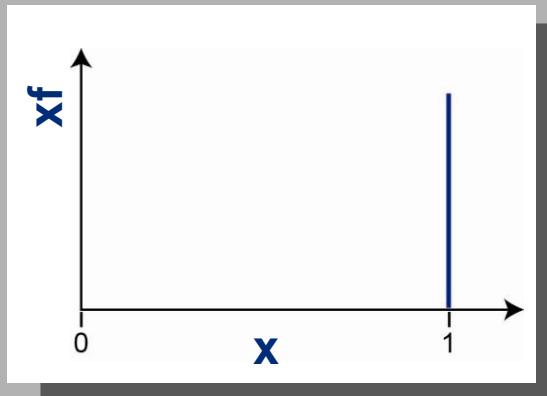




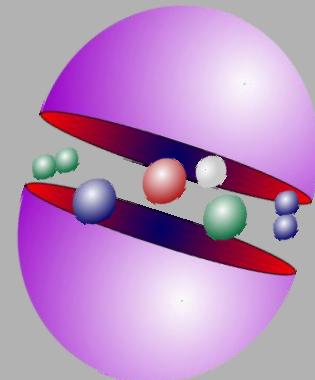
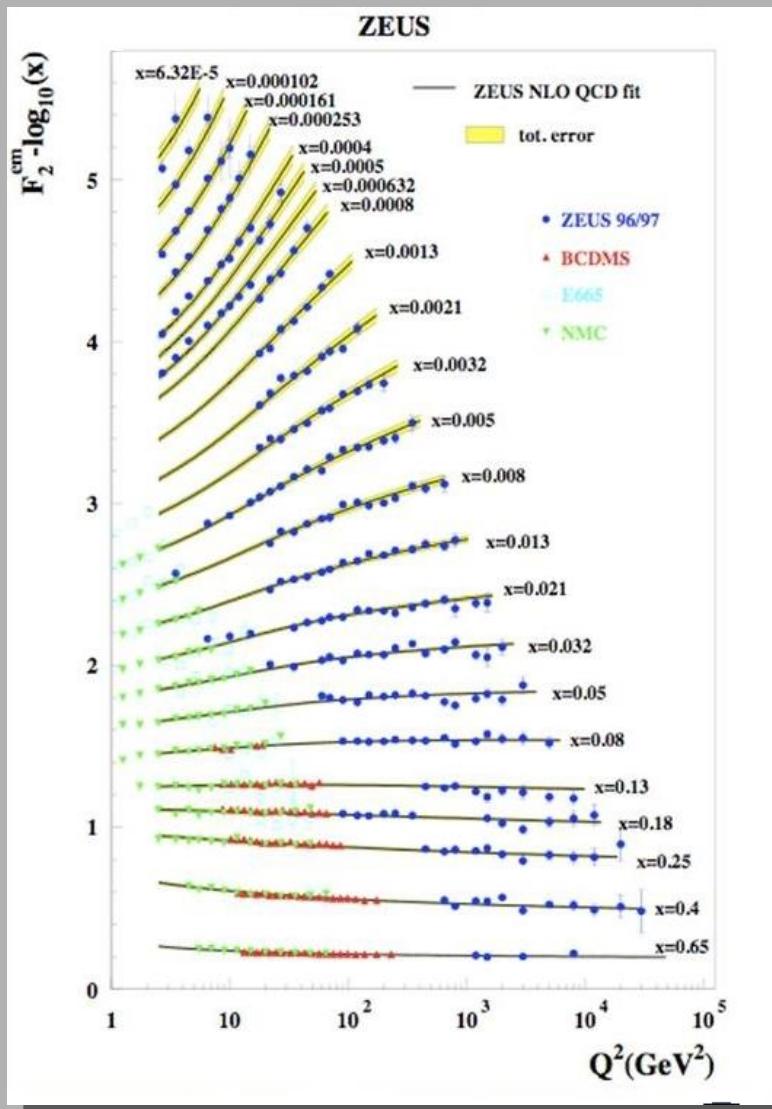
parton structure of the nucleon



parton structure of the nucleon



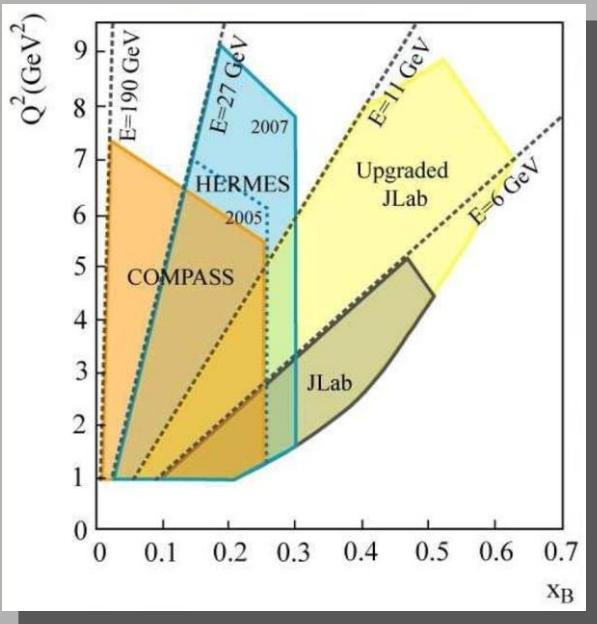
Parton Distribution Functions



pQCD

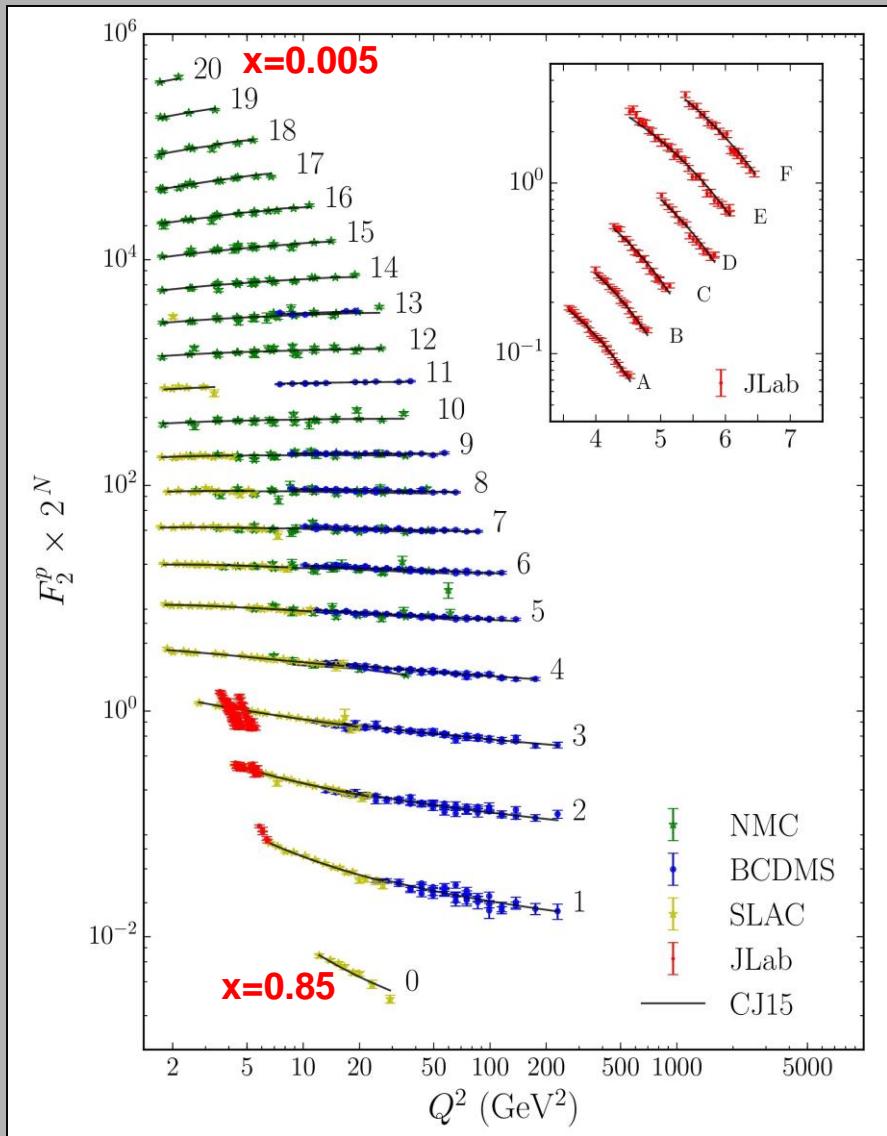
Q^2 evolution

Parton Distribution Functions



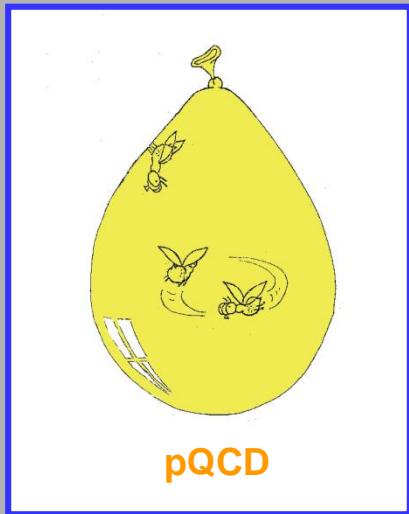
Accardi, Brady,
Melnitchouk, Owens, Sato

Connecting JLab to LHC

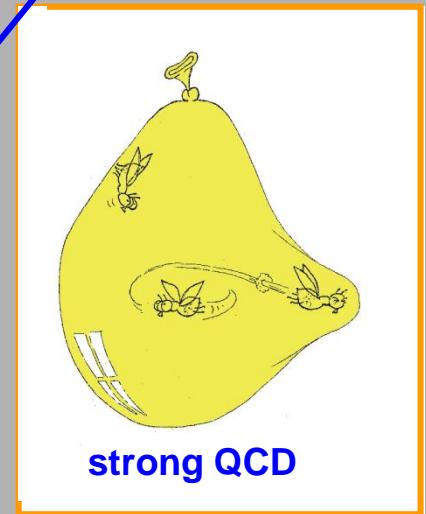


QCD

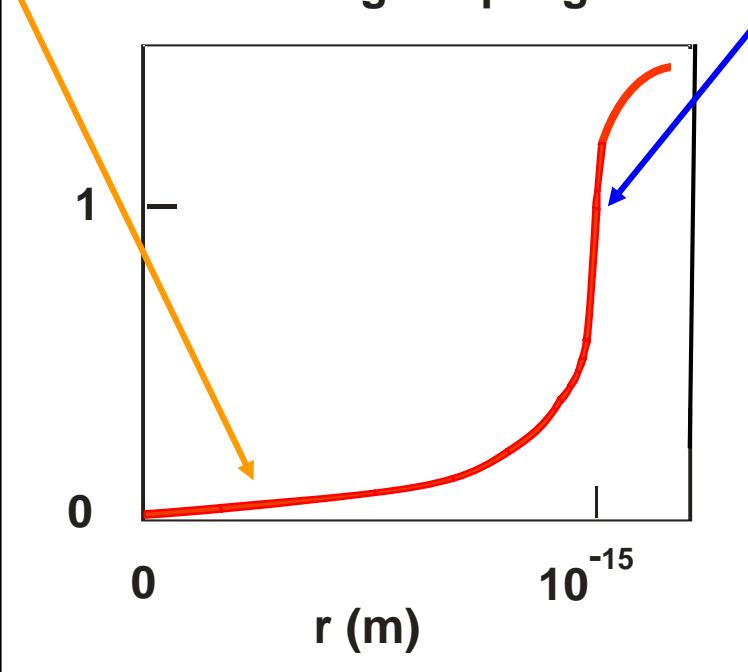
asymptotic freedom



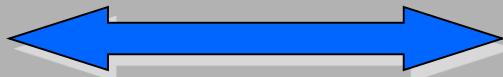
confinement



strong coupling



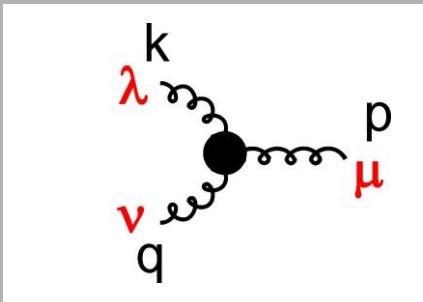
Route to Freedom



Route to Confinement

Schwinger-Dyson Equations

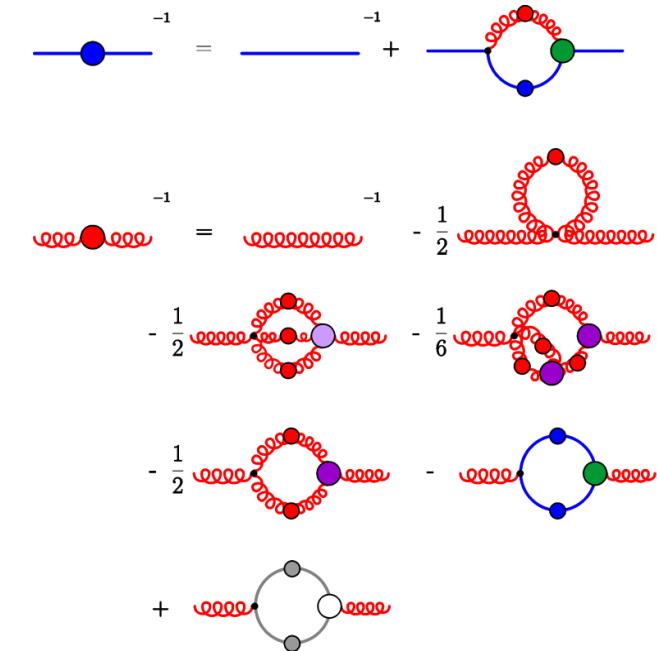
Slavnov-Taylor Identity



axial gauges

BBZ

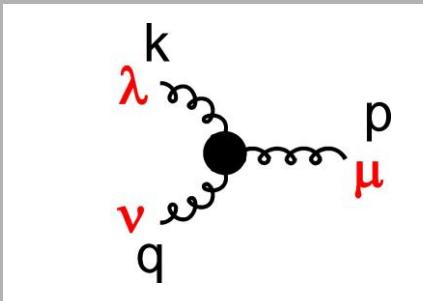
$$k_\lambda \Gamma^{\lambda\mu\nu}(k, p, q) = \Pi^{\mu\nu}(p) - \Pi^{\mu\nu}(q)$$



QCD

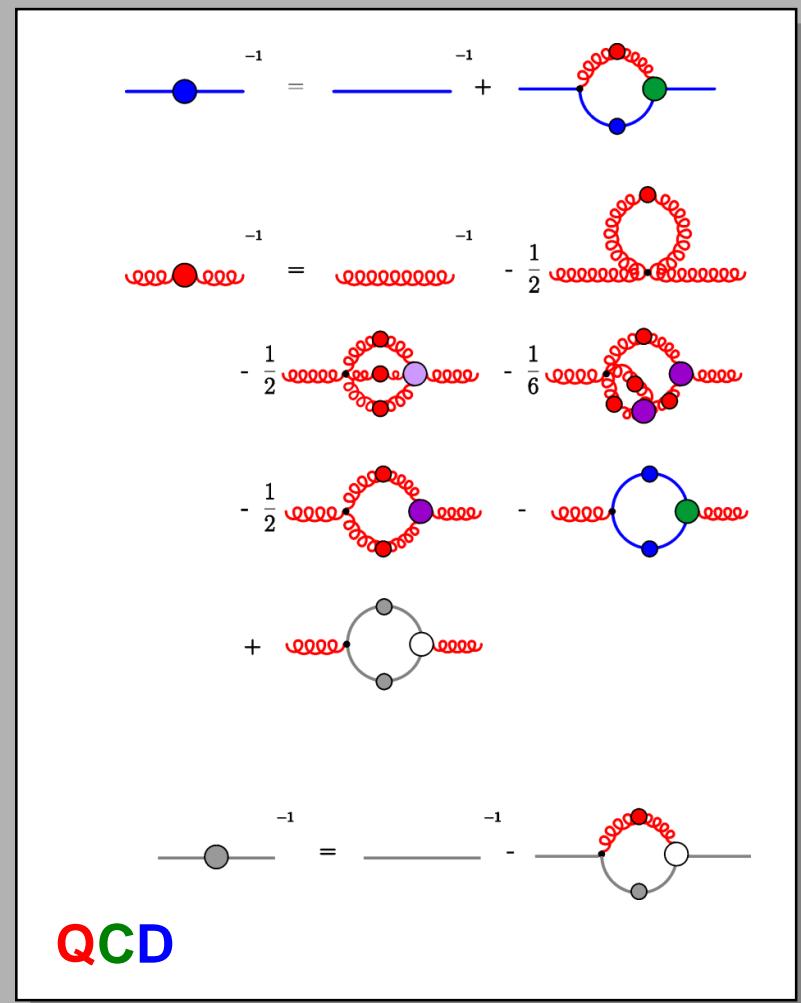
Schwinger-Dyson Equations

Slavnov-Taylor Identity

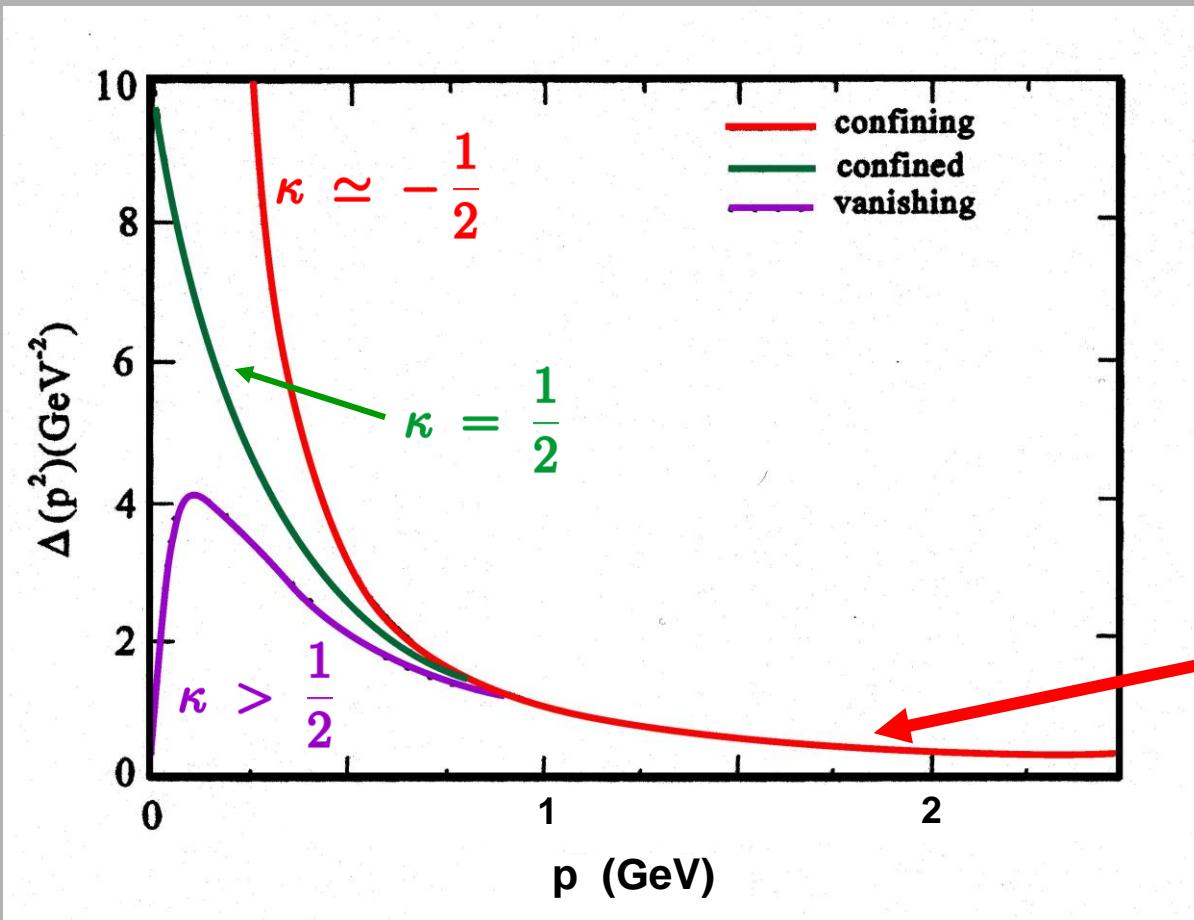


covariant gauges

$$k_\lambda \Gamma^{\lambda\mu\nu}(k, p, q) = H(k^2) [G_{\mu,\sigma}(q, -k) \Pi_{\sigma,\nu}^T(p) - G_{\nu\sigma}(p, -k) \Pi_{\sigma\mu}^T(q)]$$



gluon propagator



$$\Delta(p^2) \sim \frac{(p^2)^{2\kappa}}{p^2}$$

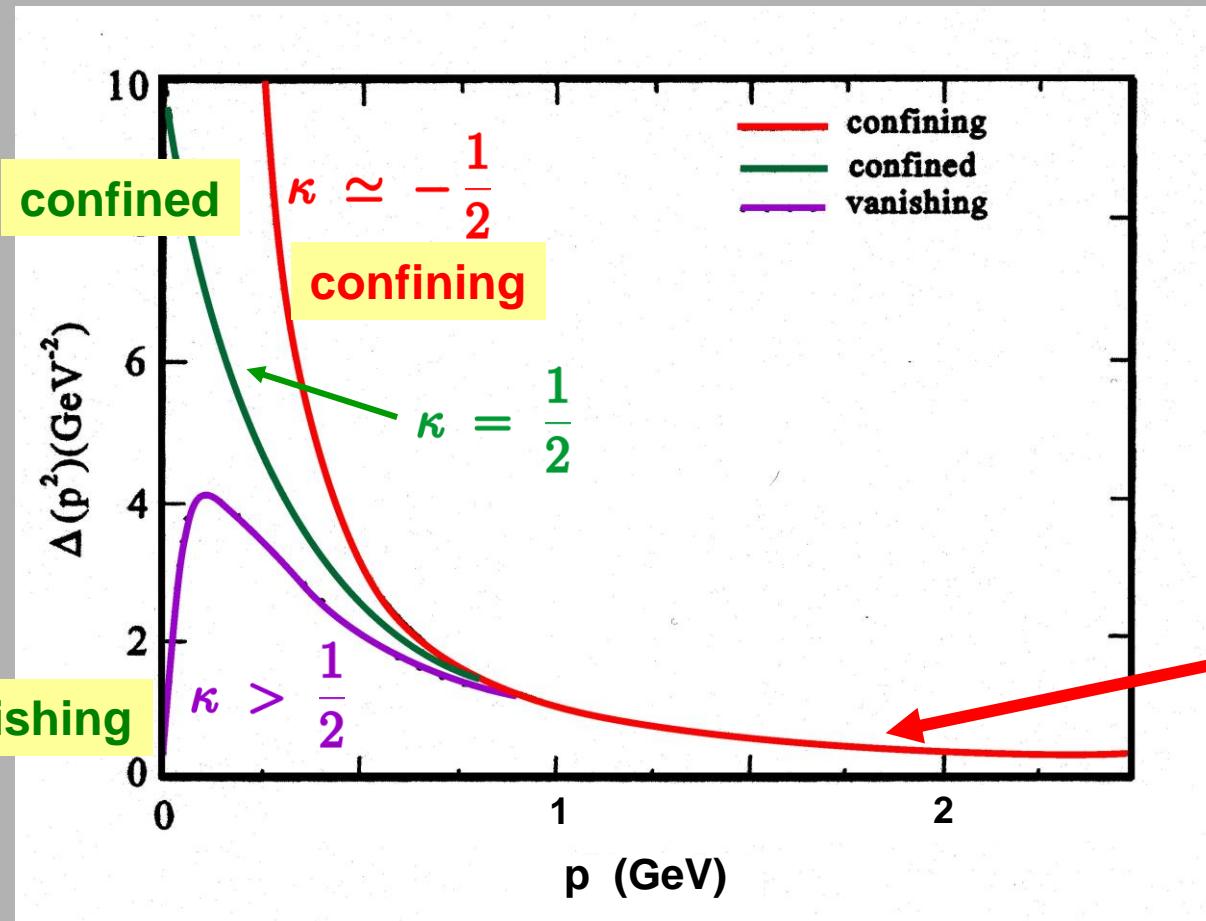
as $p^2 \rightarrow 0$

$\kappa = 0$ bare

perturbative

$$\Delta^{\mu\nu}(p) = \Delta(p^2) \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) + \xi \frac{p^\mu p^\nu}{p^4}$$

gluon propagator



$$\Delta(p^2) \sim \frac{(p^2)^{2\kappa}}{p^2}$$

as $p^2 \rightarrow 0$

$$\Delta^{\mu\nu}(p) = \Delta(p^2) \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) + \xi \frac{p^\mu p^\nu}{p^4}$$

Schwinger-Dyson Equations

gluons & ghosts

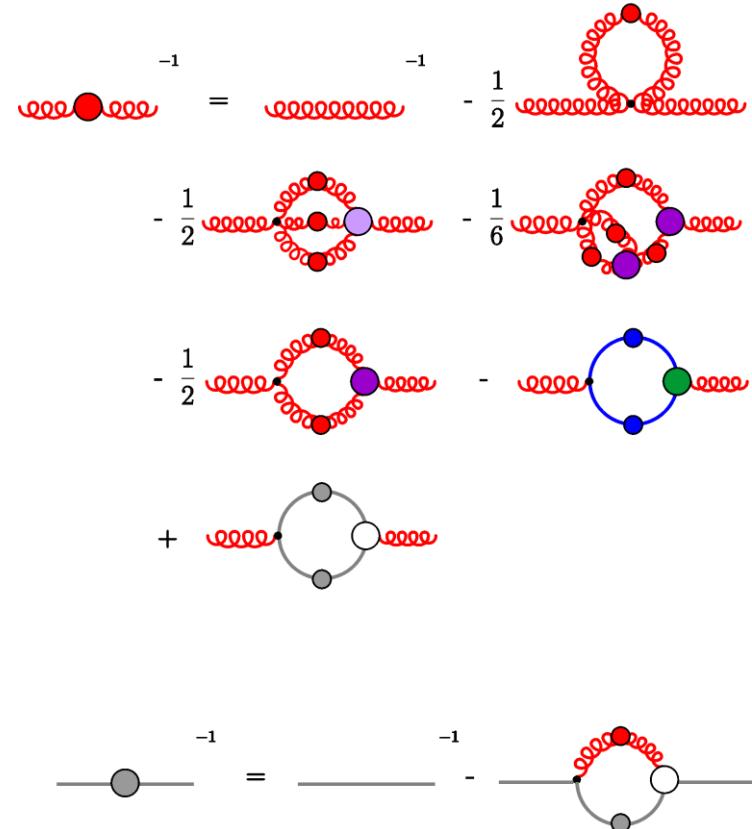
QCD

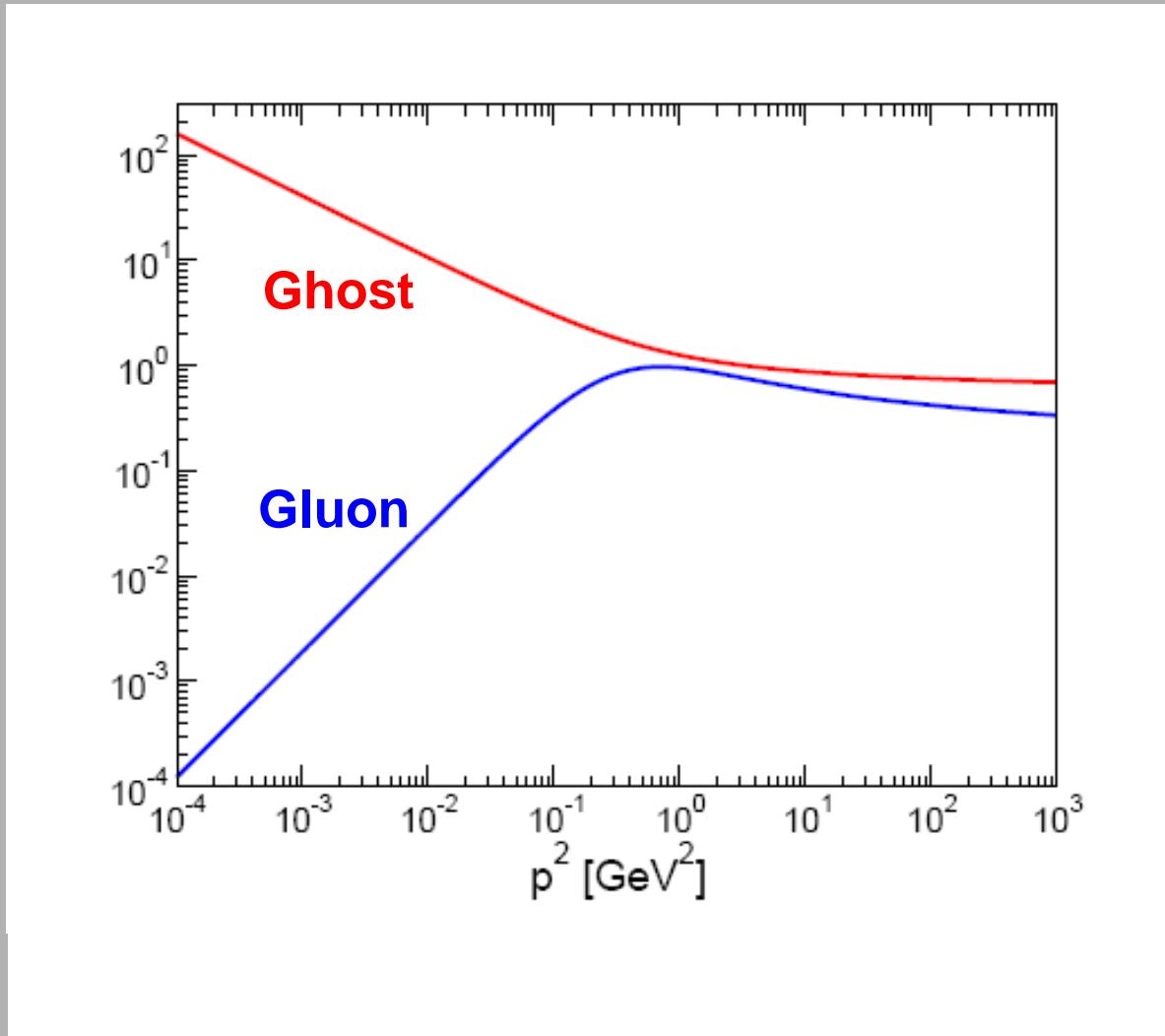
$$\Delta^{\mu\nu}(p) = \frac{g\ell(p^2)}{p^2} T^{\mu\nu}(p)$$

$$D(p) = \frac{gh(p^2)}{p^2}$$

$$T^{\mu\nu}(p) = g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$$

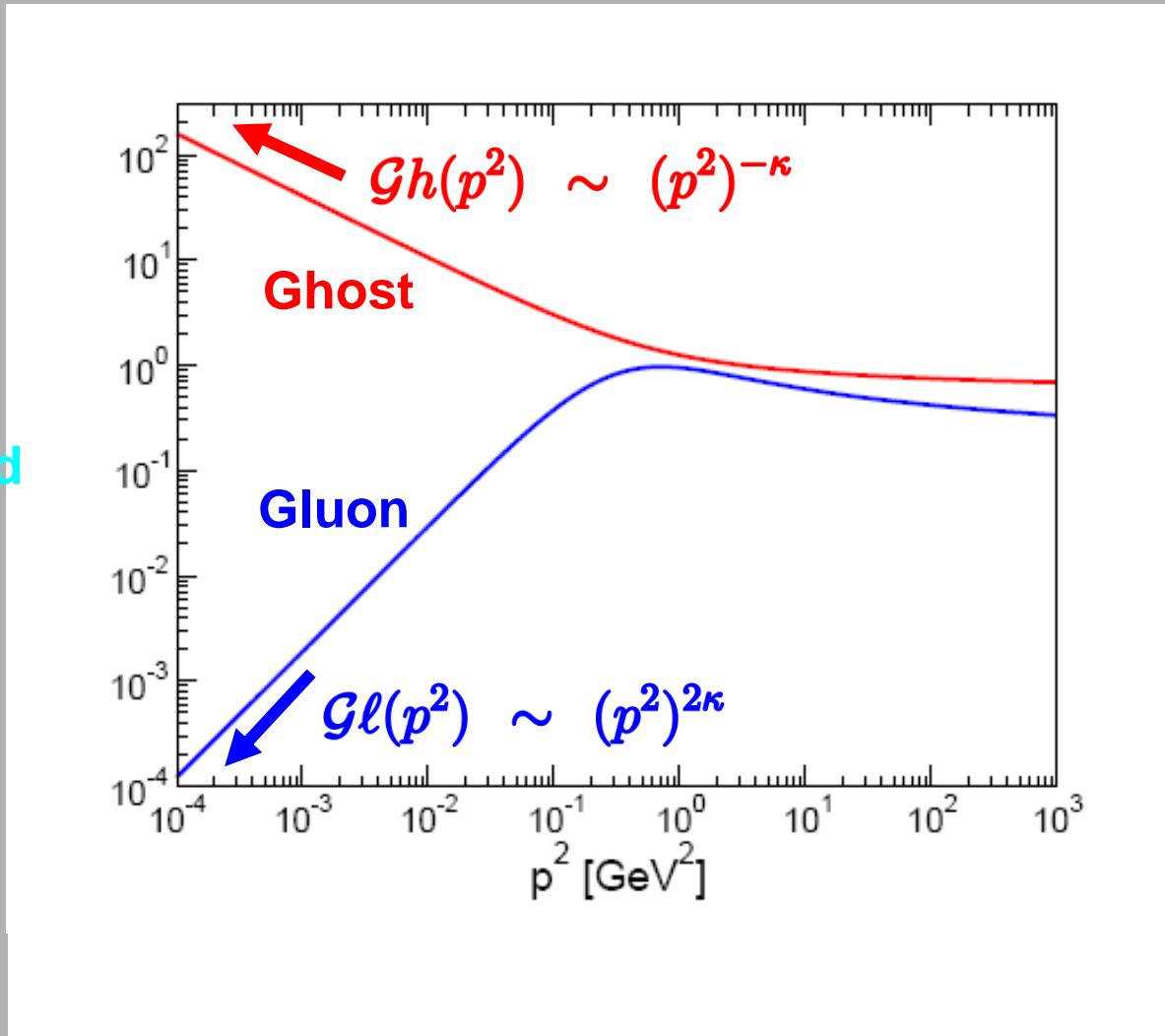
Landau gauge





von Smekal
Fischer

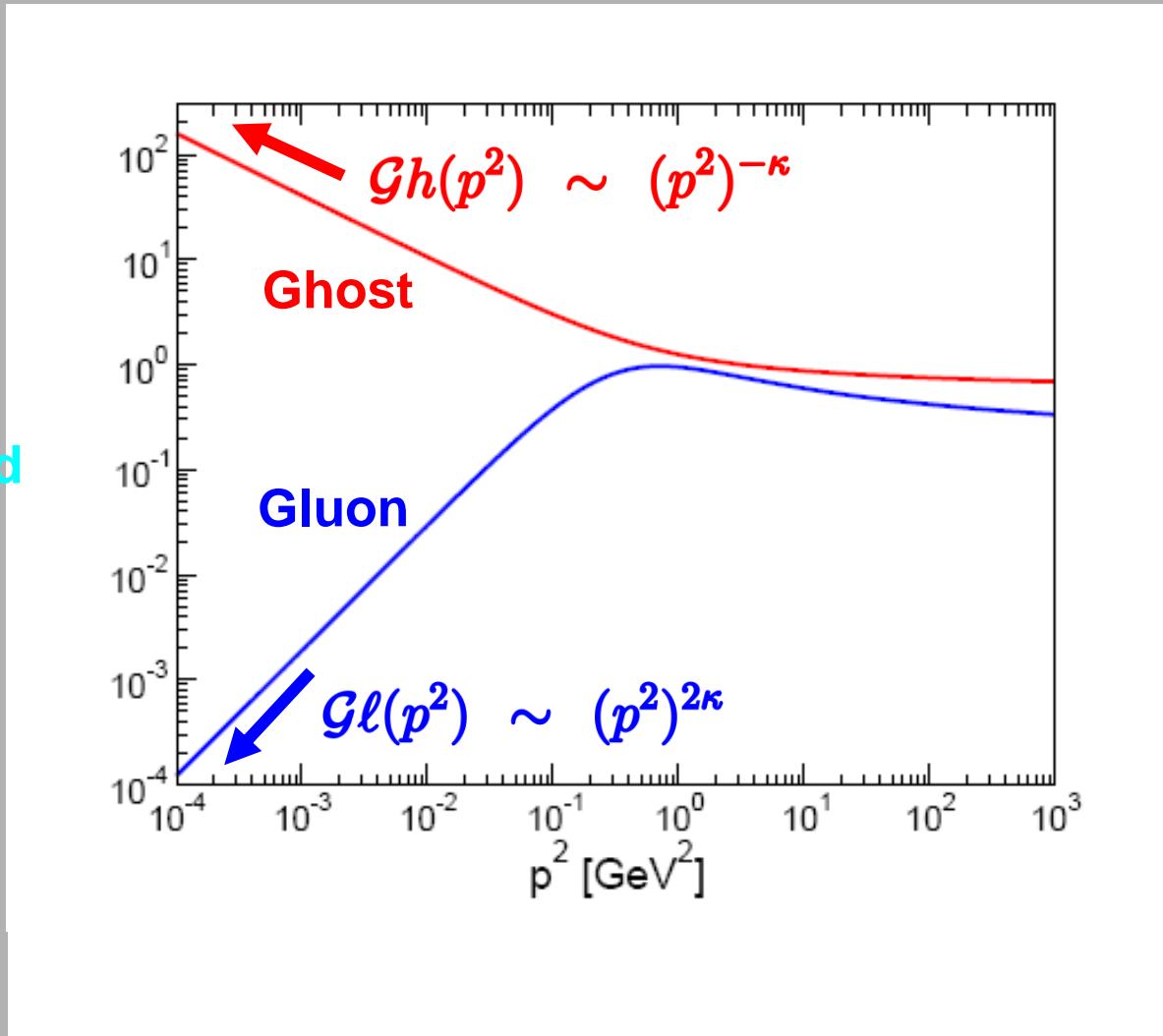
Deep Infrared

von Smekal
Fischer

Landau gauge

Deep Infrared

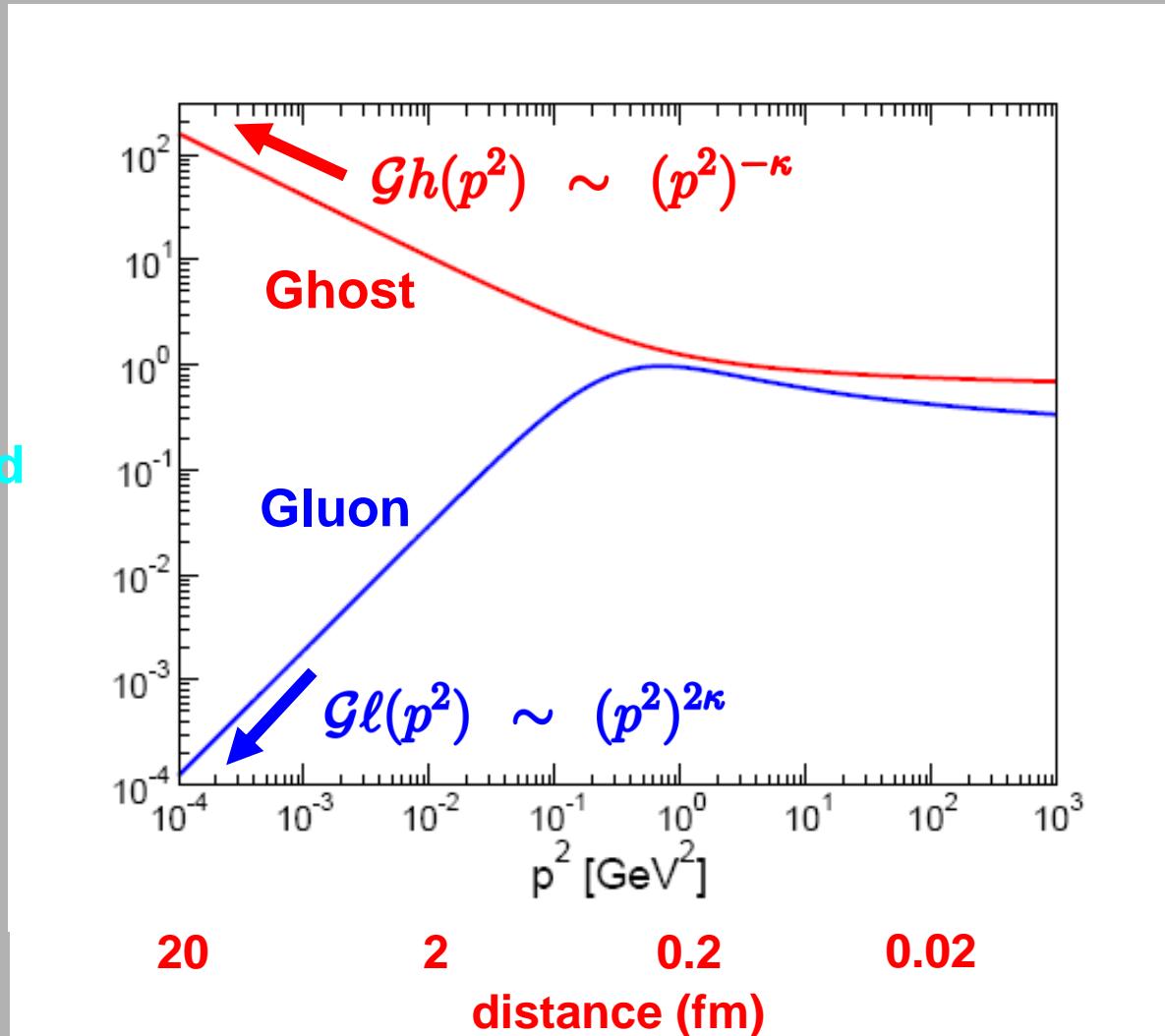
$$\kappa \simeq 0.6$$

von Smekal
Fischer

Landau gauge

Deep Infrared

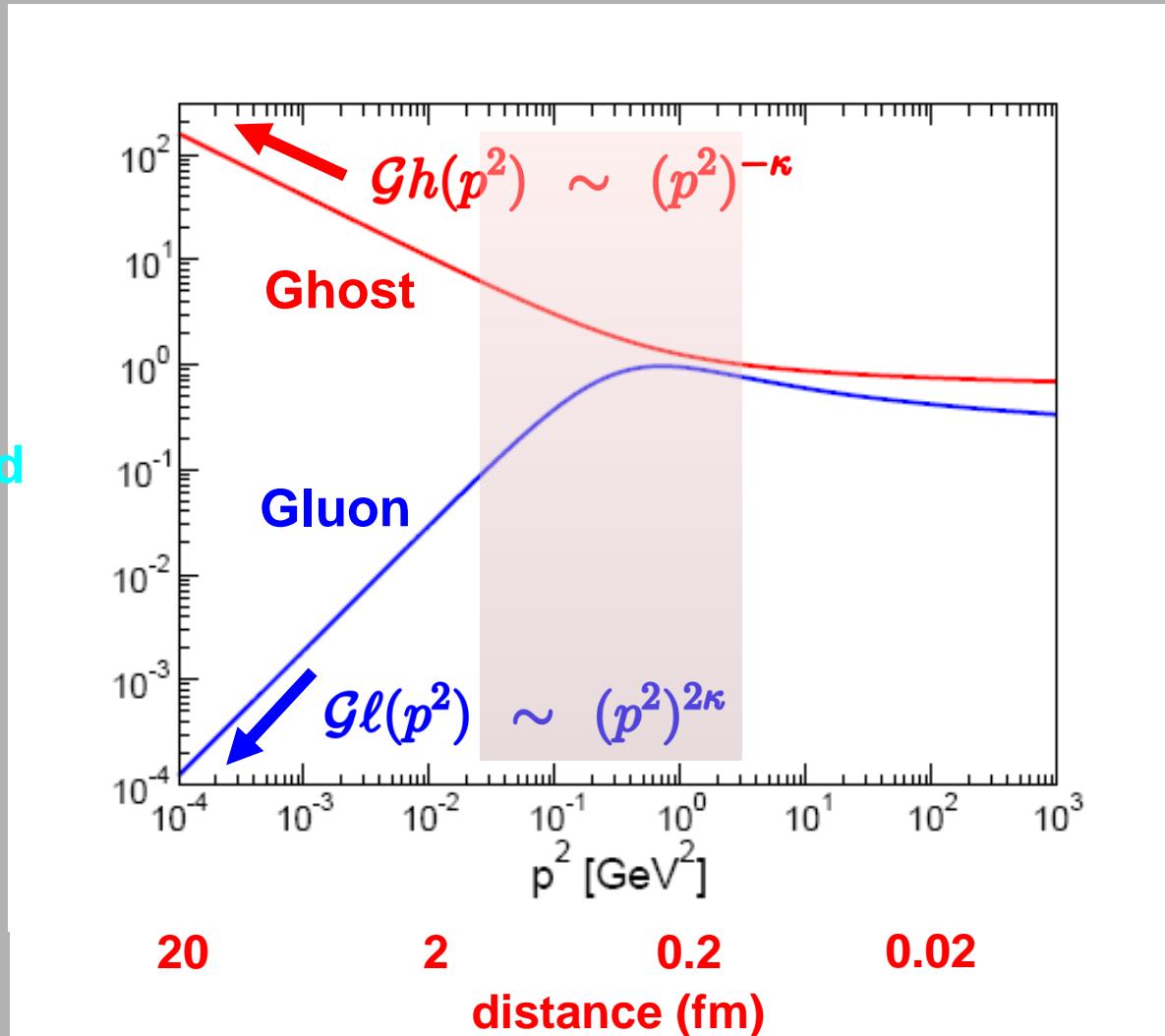
$$\kappa \simeq 0.6$$

von Smekal
Fischer

Landau gauge

Deep Infrared

$$\kappa \simeq 0.6$$

von Smekal
Fischer

Landau gauge

Schwinger-Dyson Equations

QCD

Quark propagator:

$$\text{---} \circ \text{---}^{-1} = \text{---} \circ \text{---}^{-1} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

Ghost propagator:

$$\text{---} \circ \text{---}^{-1} = \text{---} \circ \text{---}^{-1} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

Ghost-gluon vertex:

$$\text{---} \circ \text{---} \text{---} \circ \text{---} = \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---} \text{---} \circ \text{---}$$

Quark-gluon vertex:

$$\text{---} \circ \text{---} \text{---} \circ \text{---} = \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

$$+ \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

Gluon propagator:

$$\text{---} \circ \text{---}^{-1} = \text{---} \circ \text{---}^{-1} +$$

$$+ \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

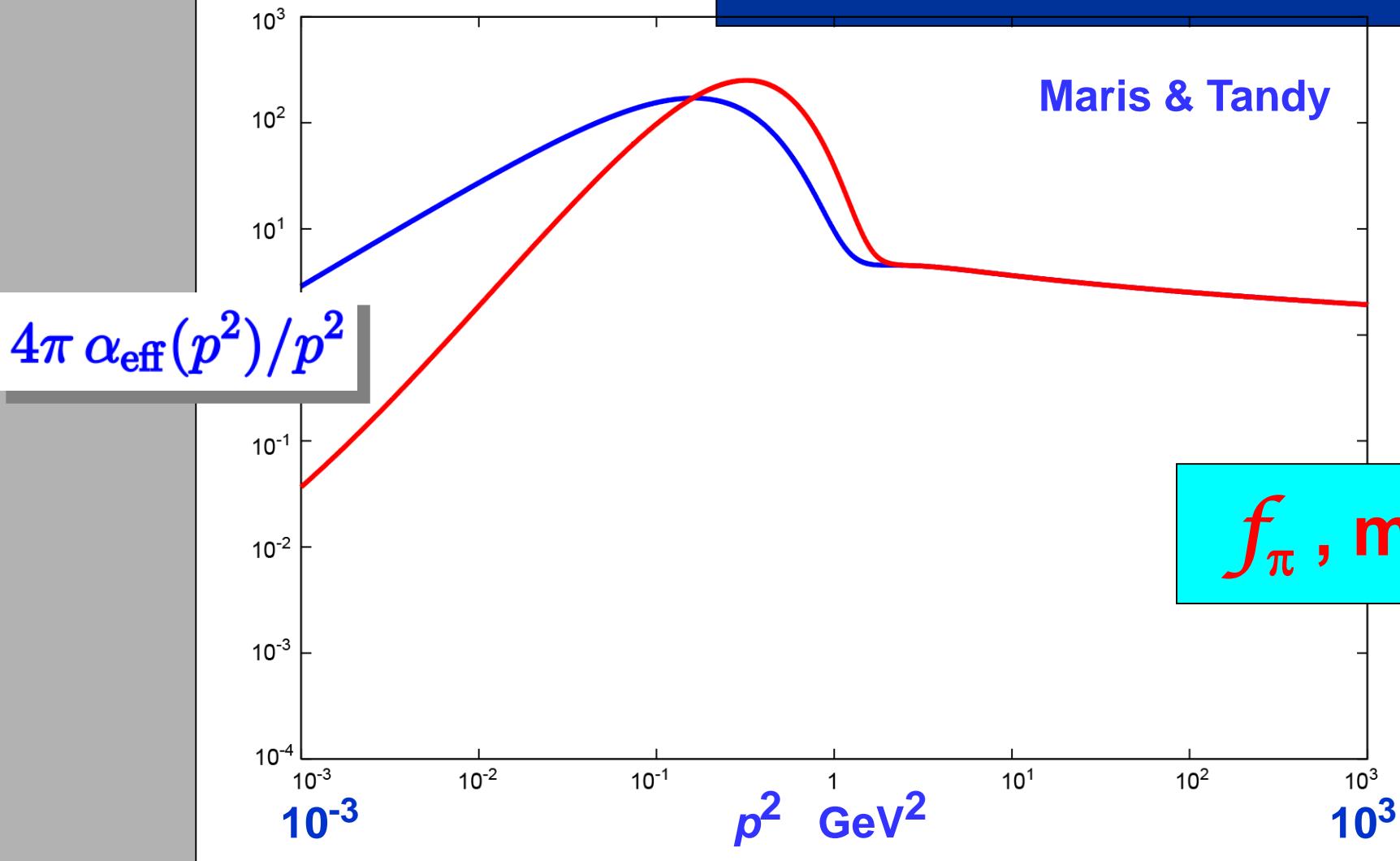
$$+ \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

$$+ \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

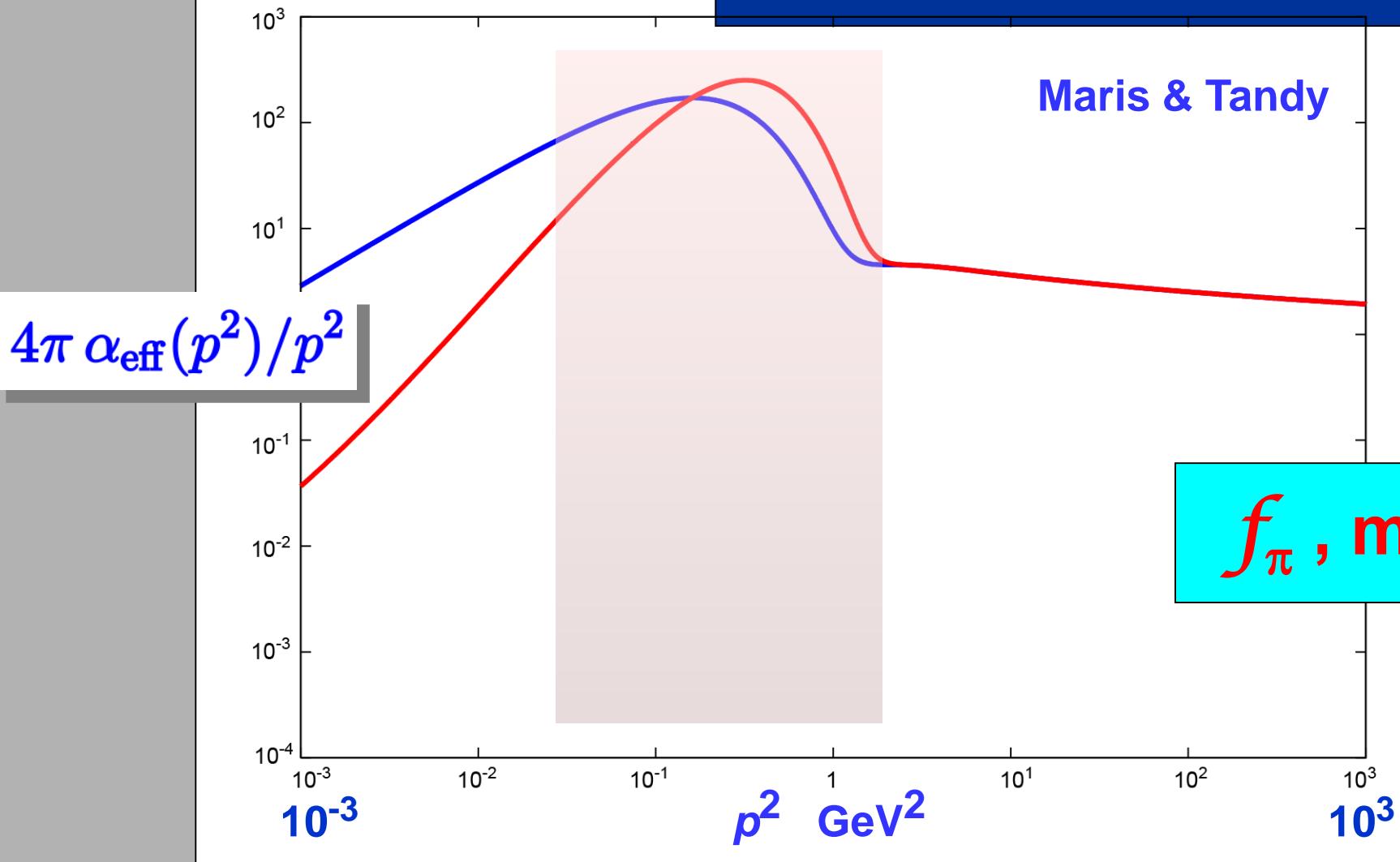
Consistent truncation essential

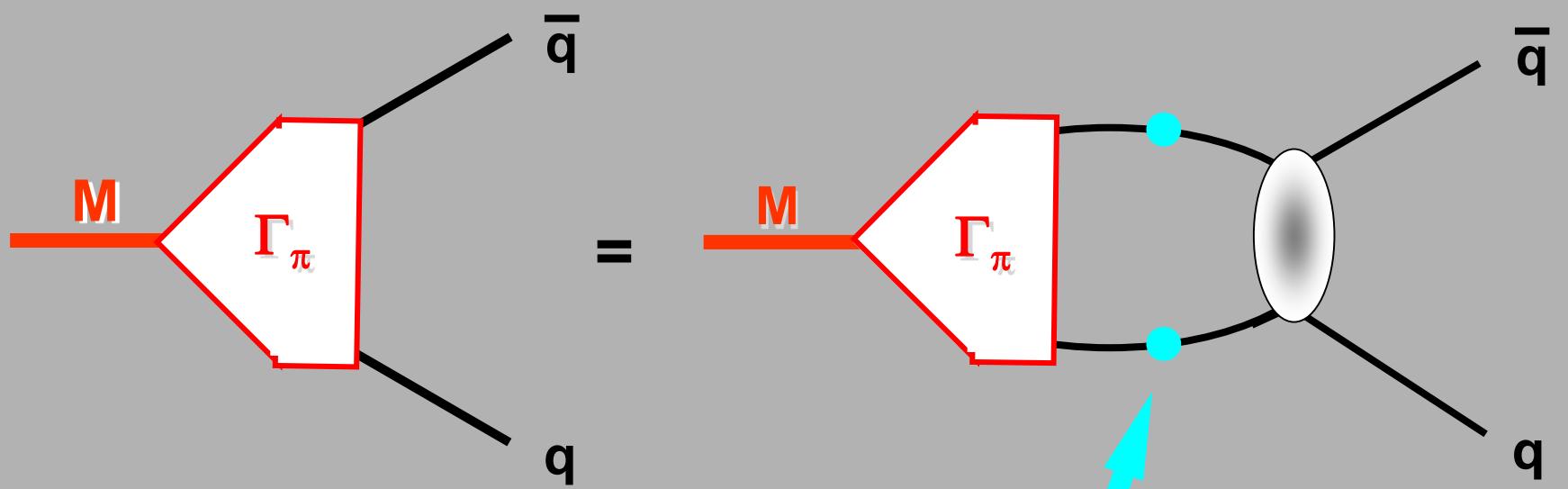
Gauge Invariance, Gauge Covariance
& Multiplicative Renormalizability

effective interaction strength

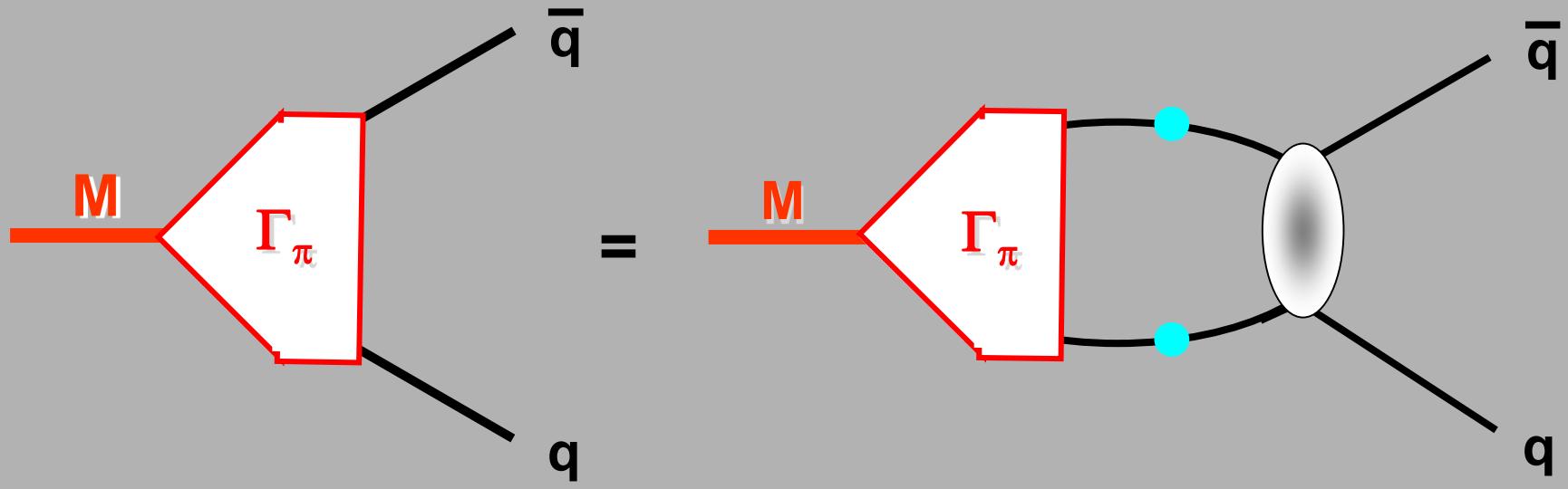


effective interaction strength

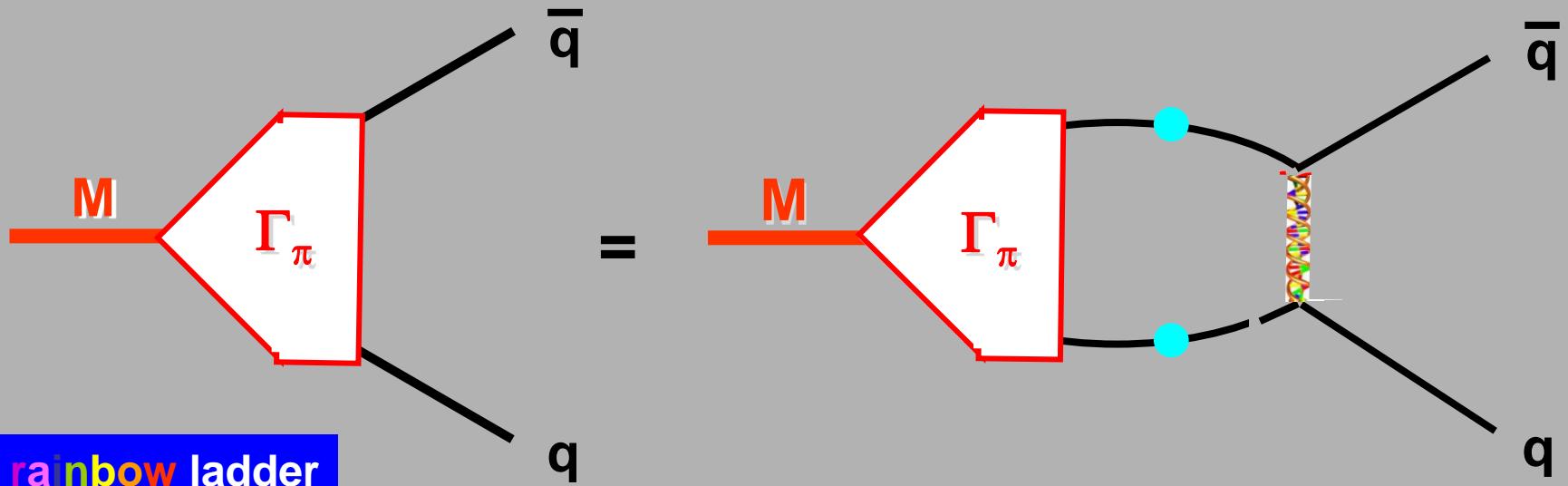




dressed quark



Maris & Tandy



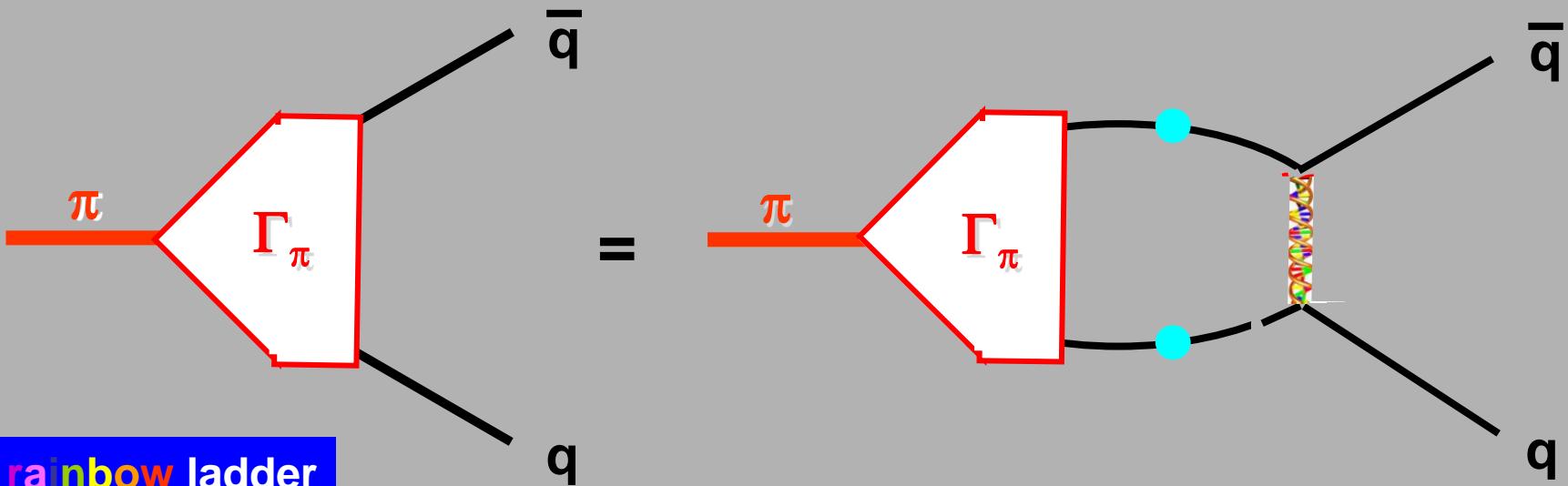
rainbow ladder

Axial Ward Identity

$$q_\mu = \Gamma_5 \begin{matrix} \nearrow q \\ k \longrightarrow p \end{matrix} = \begin{matrix} k \longrightarrow -1 \\ \gamma_5 \end{matrix} + \begin{matrix} \gamma_5 \\ k \longrightarrow -1 \\ p \end{matrix}$$

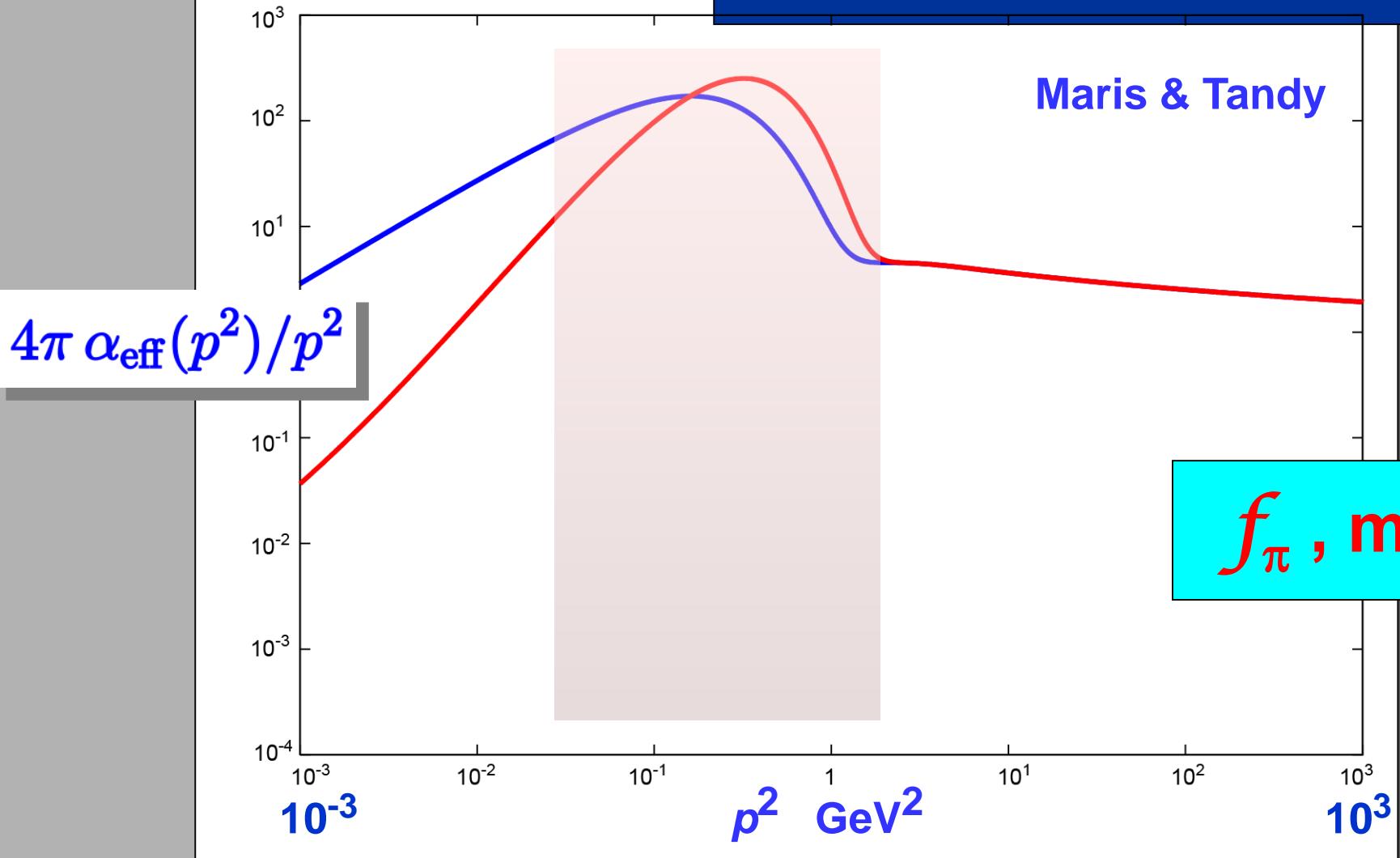
f_π, m_π

Maris, Roberts & Tandy



rainbow ladder

effective interaction strength

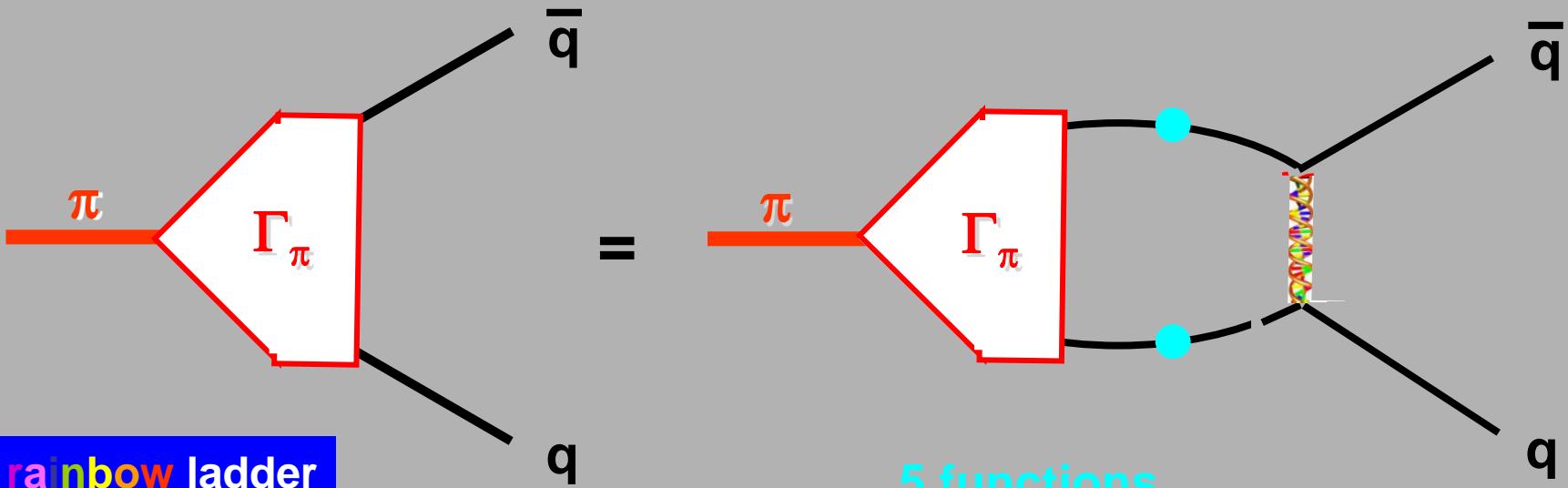


Axial Ward Identity

$$q_\mu = \Gamma_5(k, p) = \gamma_5(k, -1) + \gamma_5(p, -1)$$

π is massless as $m_q \rightarrow 0$

Maris, Roberts & Tandy



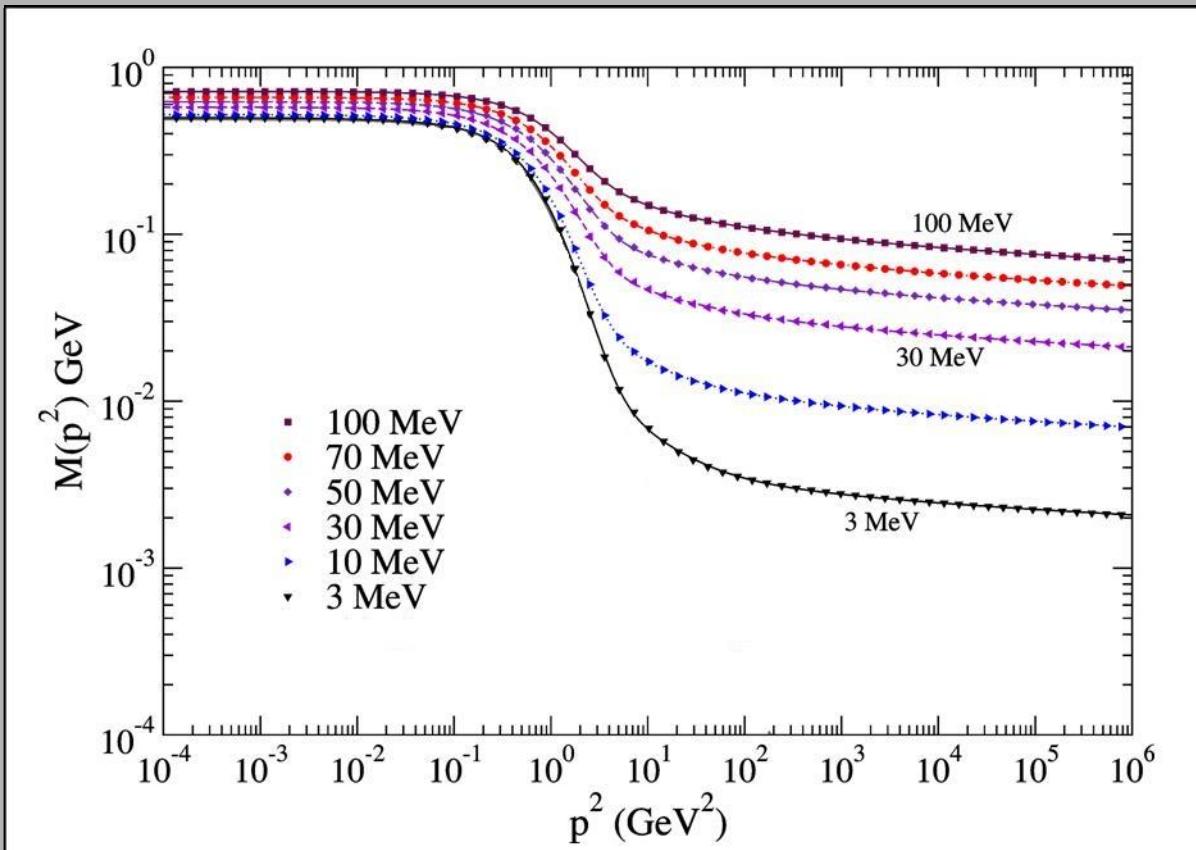
Quark mass function

$$\alpha_s > 1$$



χ SB

Williams,
Fischer,
P

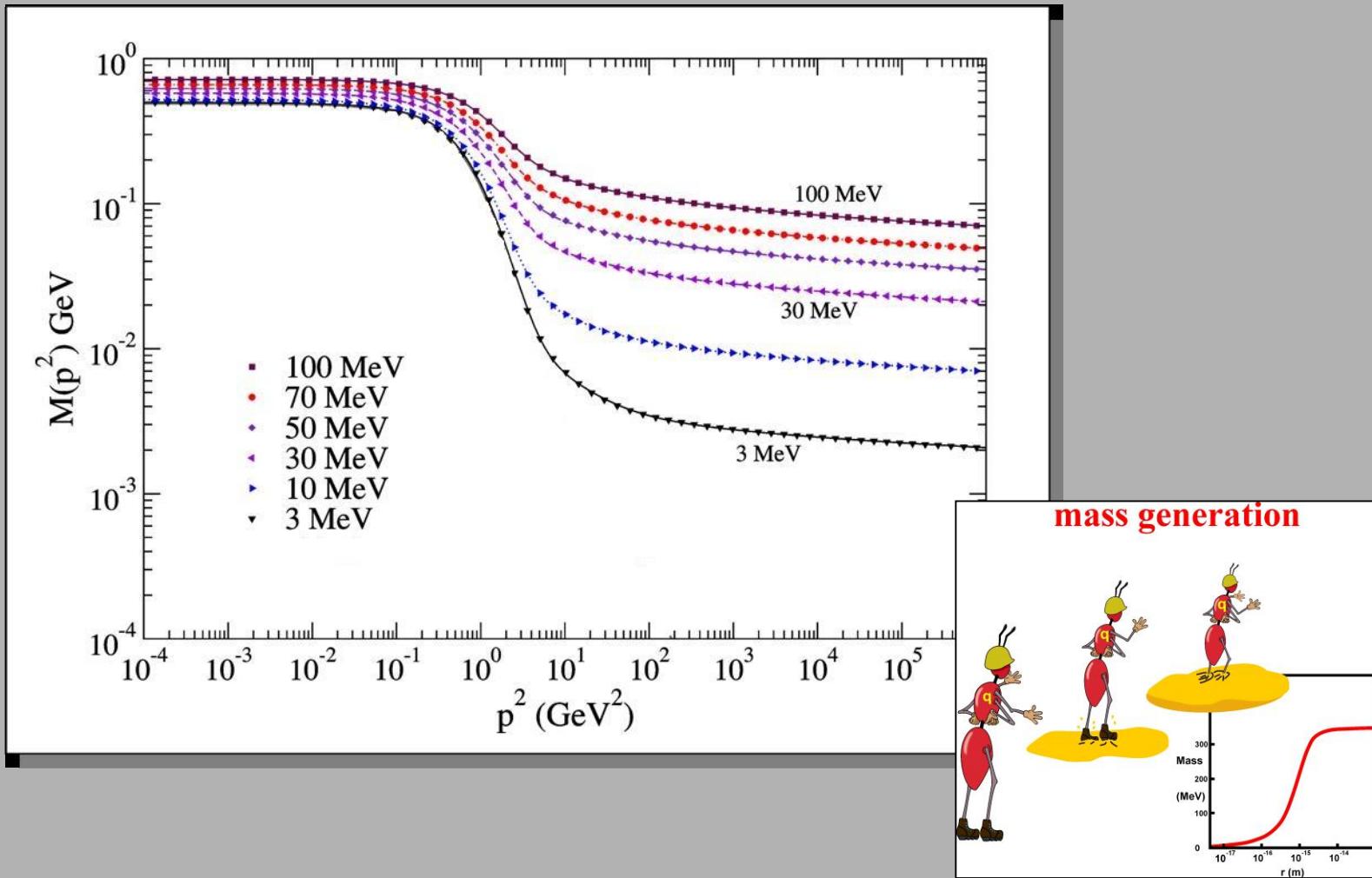


Quark mass function

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Quark mass function

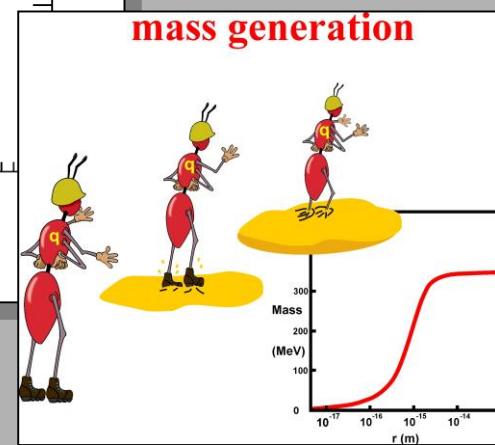
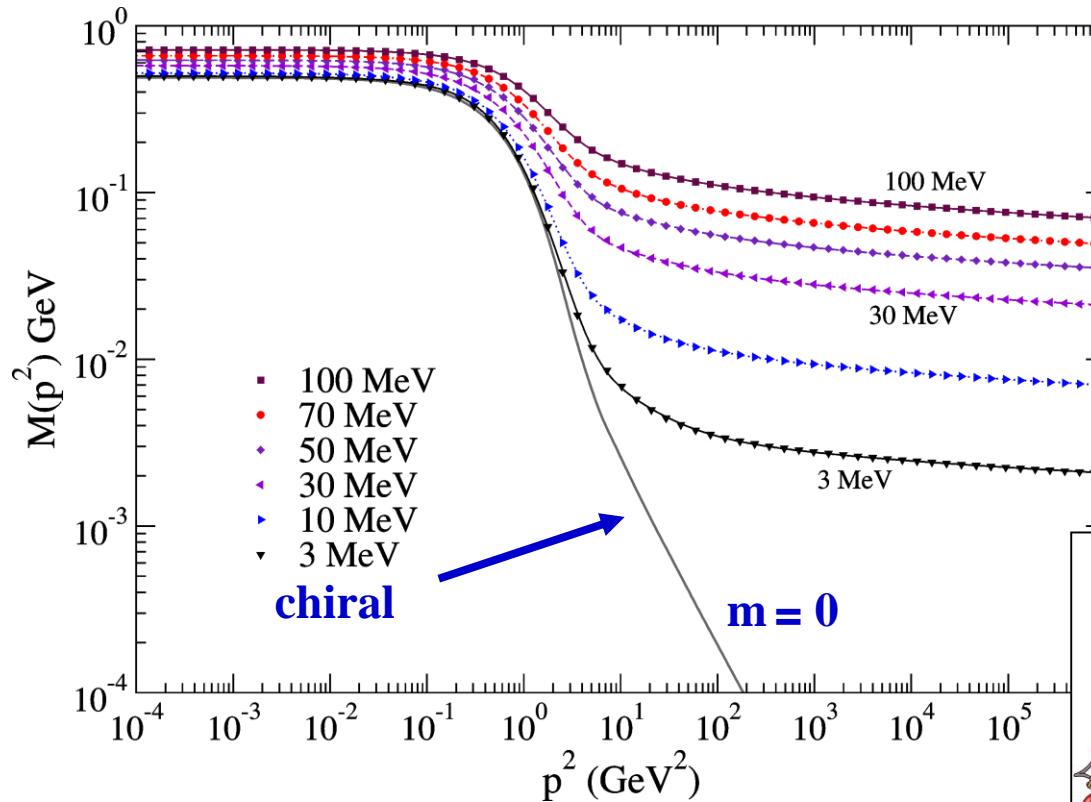
$$\alpha_s > 1$$

χ SB

Williams,
Fischer,
P

Maris &
Roberts

$$\langle \bar{q}q \rangle_0 \sim - (240 \text{ MeV})^3$$



constituent quarks

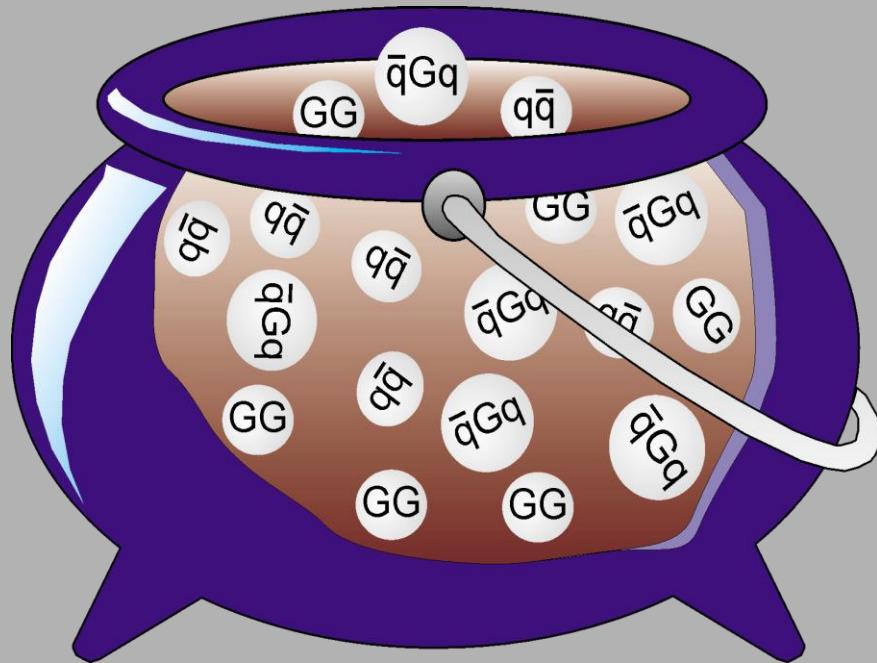
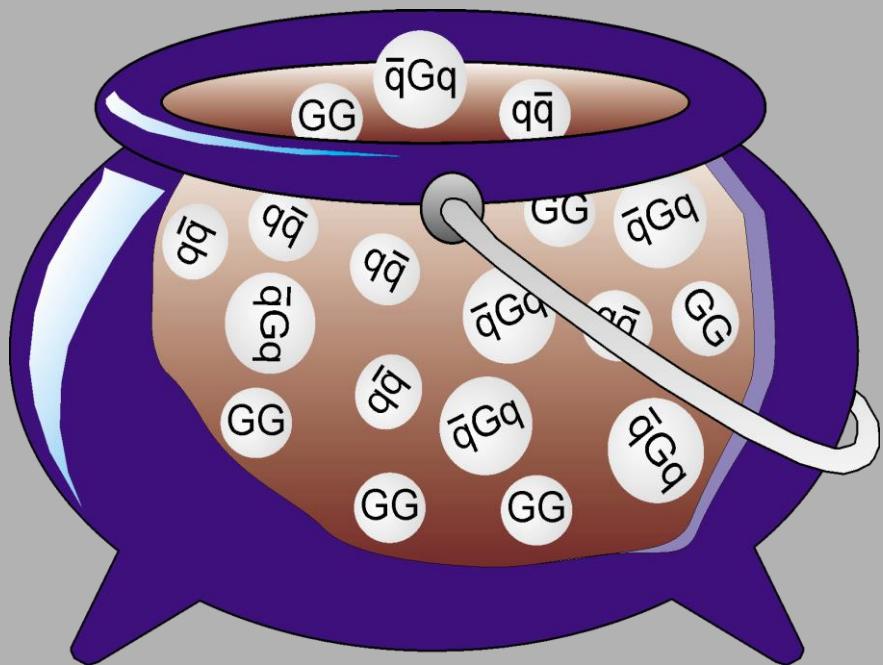


current quark

the glue that binds us

effective degrees of freedom

QCD vacuum



Schwinger-Dyson Equations

Bound State Equations

$$\begin{aligned} \text{---} \bullet \text{---}^{-1} &= \text{---} \text{---}^{-1} + \text{---} \text{---} \text{---} \text{---}^{-1} \\ \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} &= \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} - \frac{1}{2} \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} \\ -\frac{1}{2} \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} &- \frac{1}{6} \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} \\ -\frac{1}{2} \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} &- \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} \\ + \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} & \end{aligned}$$

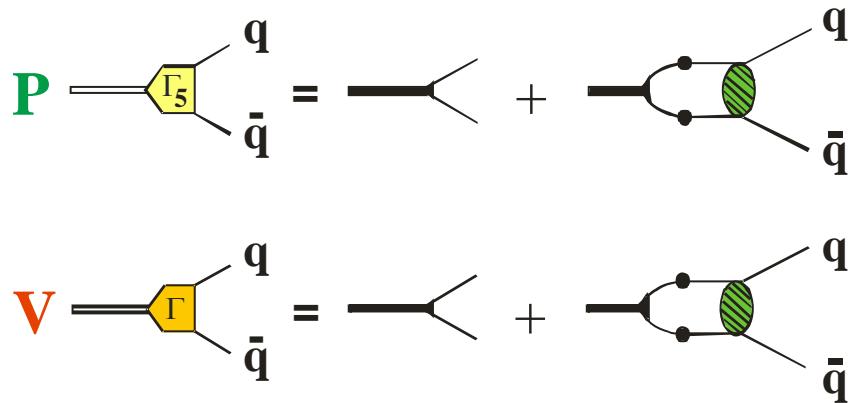
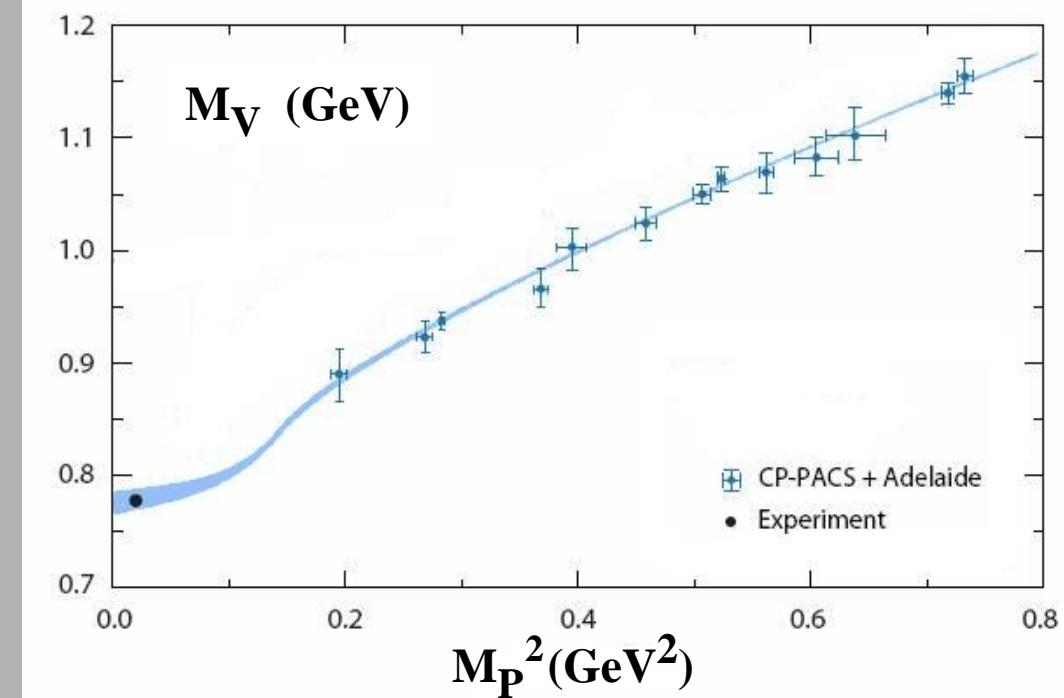
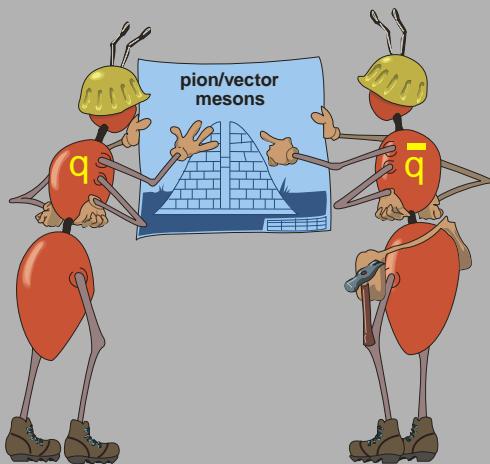
QCD

$$\begin{aligned} \mathbf{P} \quad \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} &= \text{---} \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} \\ \mathbf{V} \quad \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} &= \text{---} \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} \end{aligned}$$

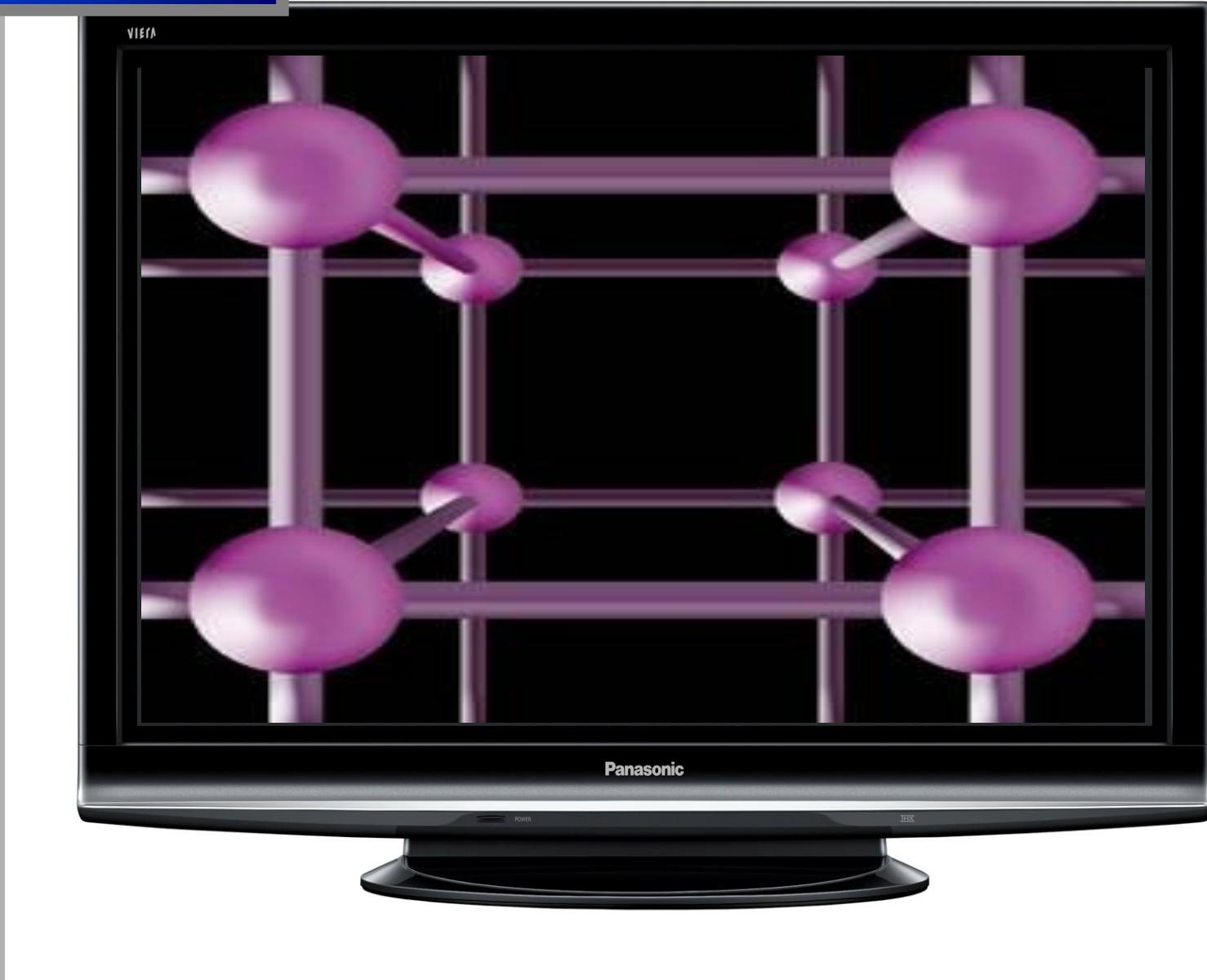
f_π, m_π

SDE/BSE – ANL/KSU

ρ v π

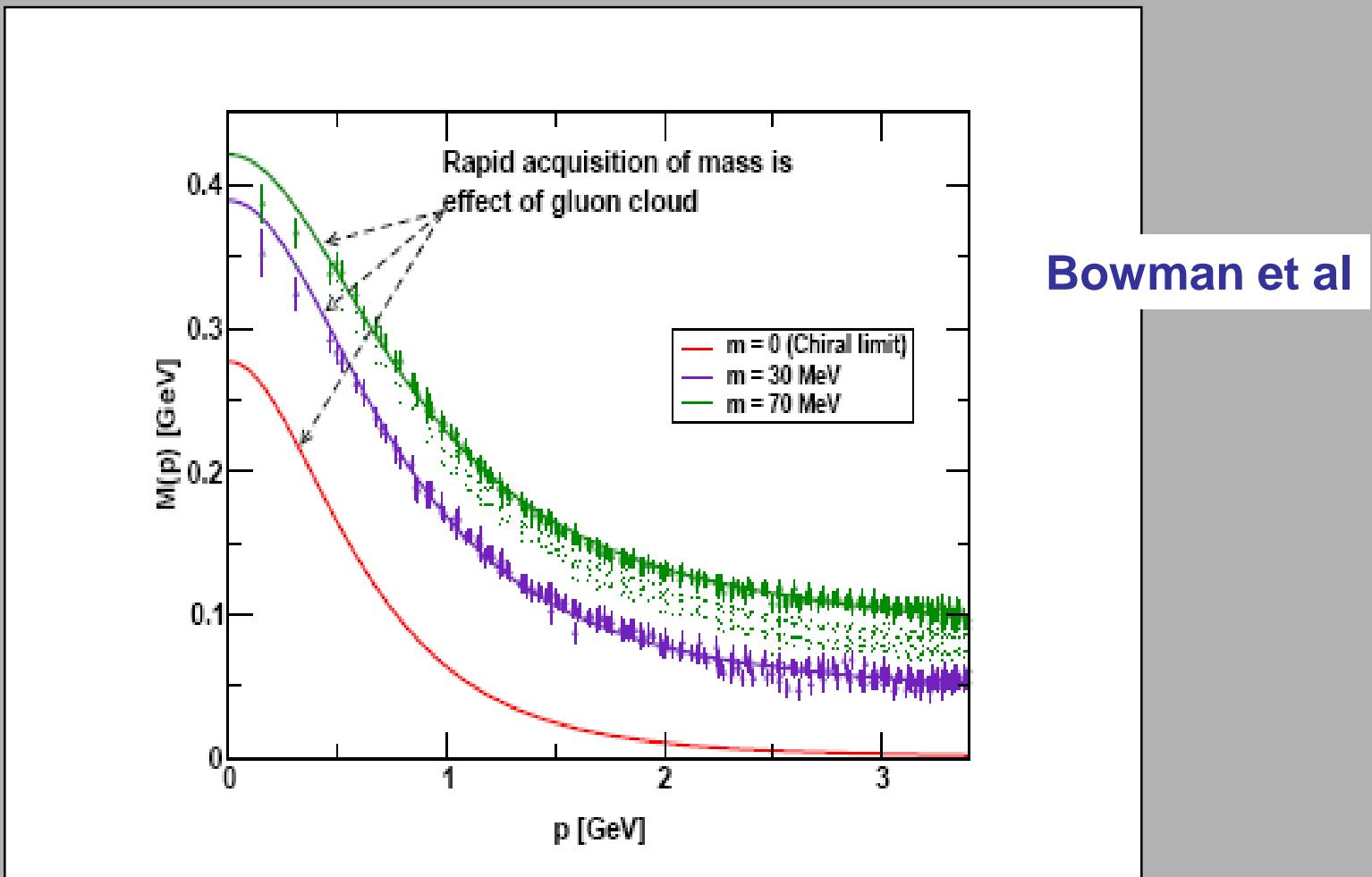


Lattice QCD





Lattice and SDE results



Bhagwat & Tandy / Roberts et al

Schwinger-Dyson Equations

gluons & ghosts

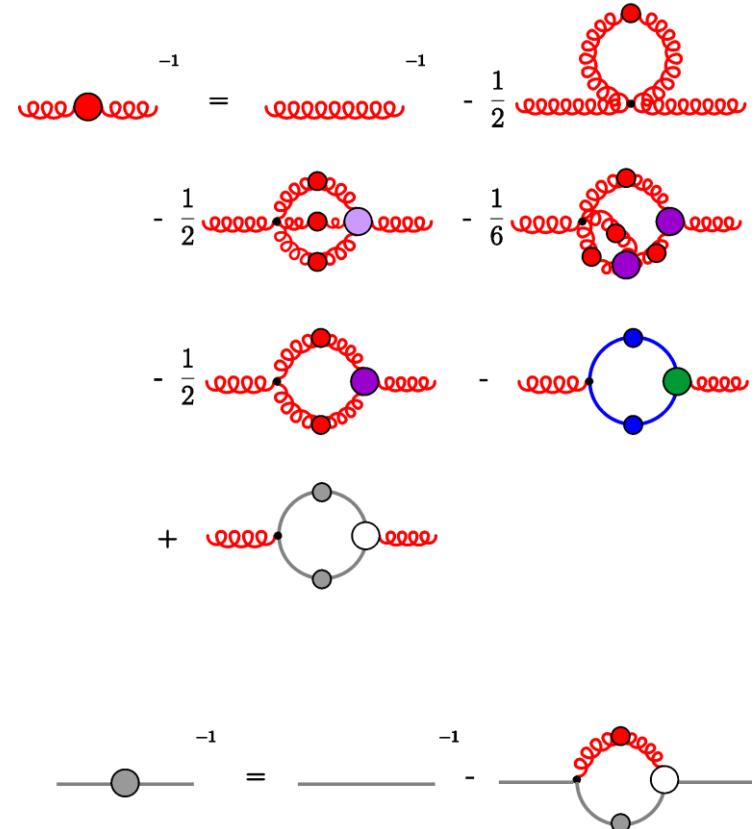
QCD

$$\Delta^{\mu\nu}(p) = \frac{g\ell(p^2)}{p^2} T^{\mu\nu}(p)$$

$$D(p) = \frac{gh(p^2)}{p^2}$$

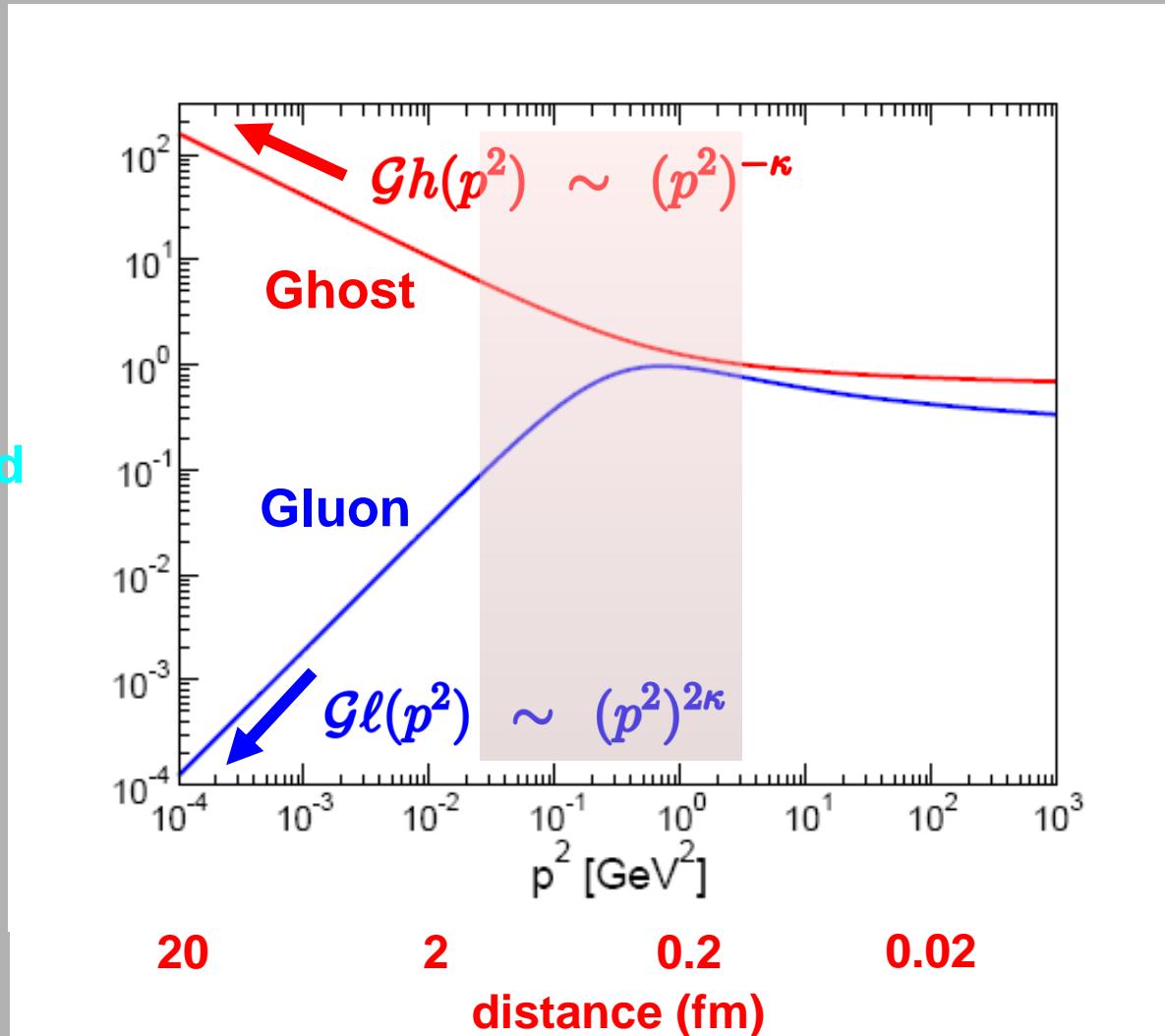
$$T^{\mu\nu}(p) = g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$$

Landau gauge



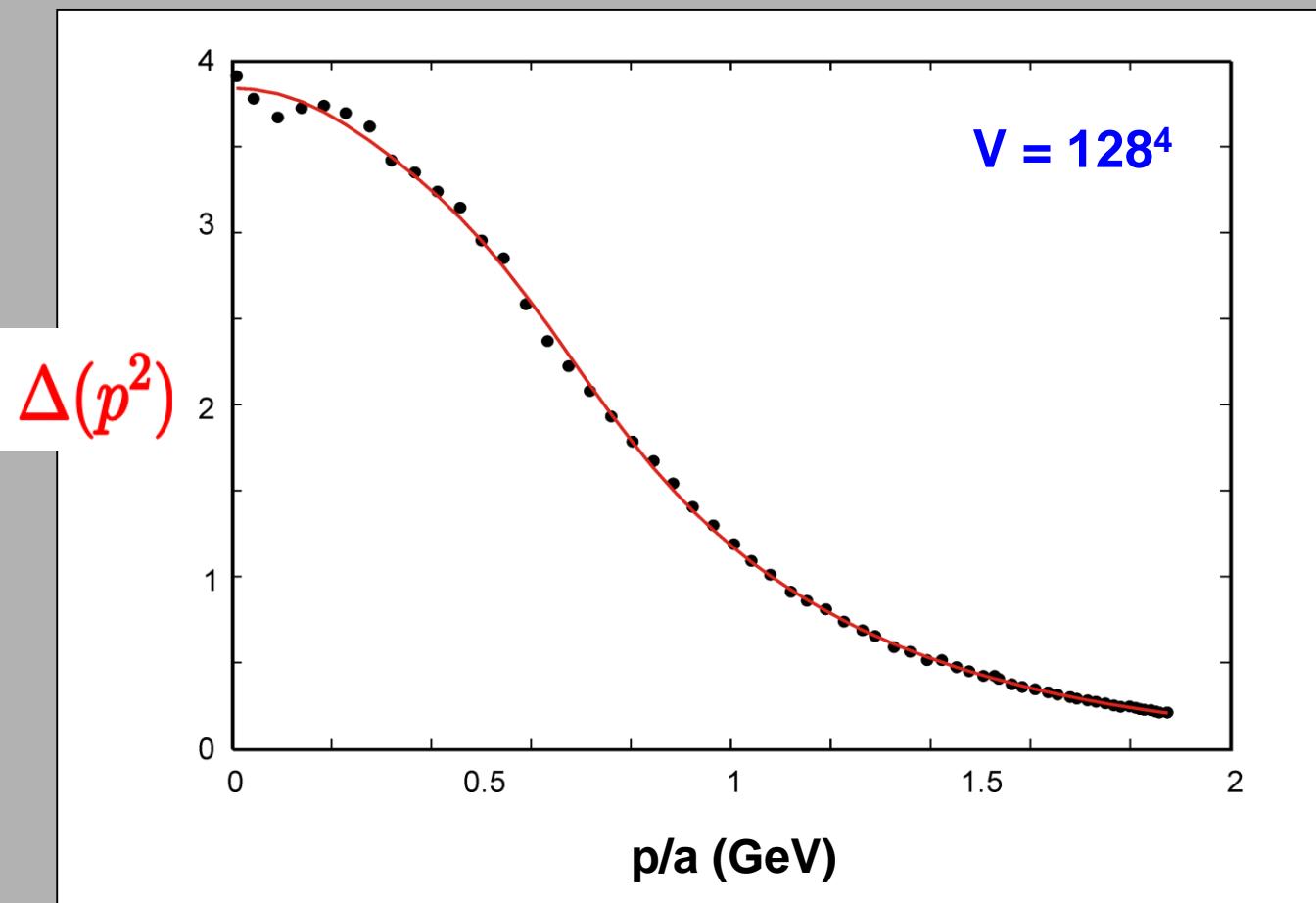
Deep Infrared

$$\kappa \simeq 0.6$$

von Smekal
Fischer

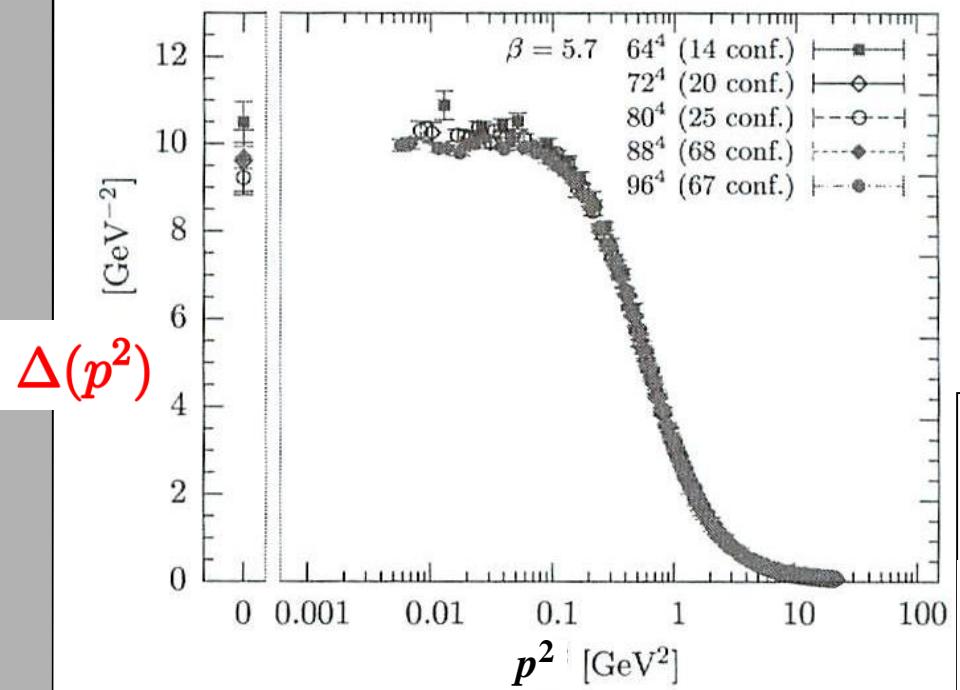
Landau gauge

Lattice Results: Cucchieri, Mendes



$$\Delta(p^2) = \frac{g\ell(p^2)}{p^2}$$

Bogolubsky et al. 2009

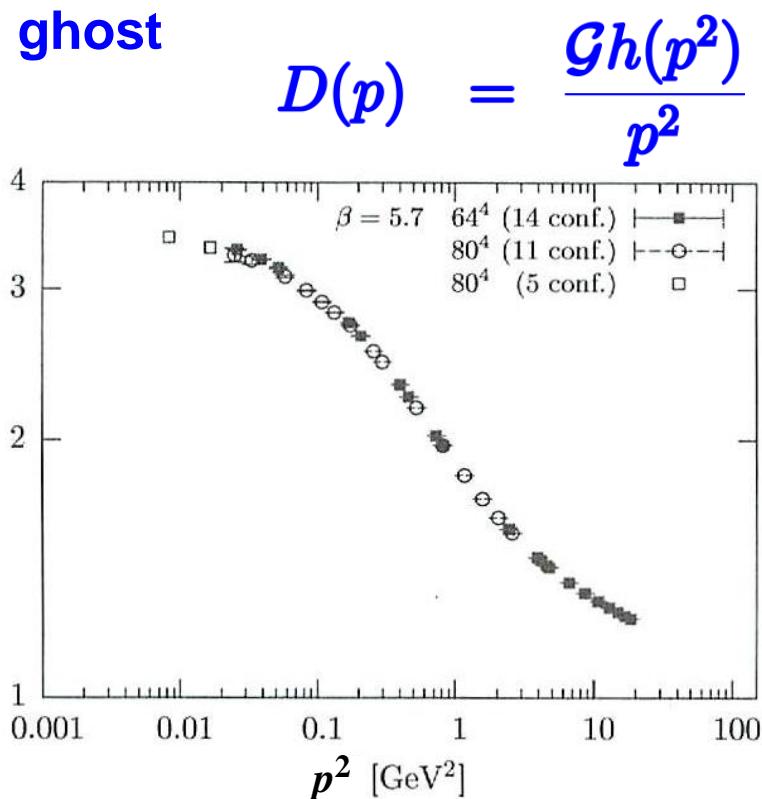


gluon

$$\Delta(p^2) = \frac{g\ell(p^2)}{p^2}$$

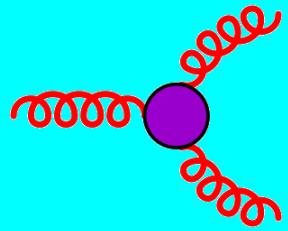
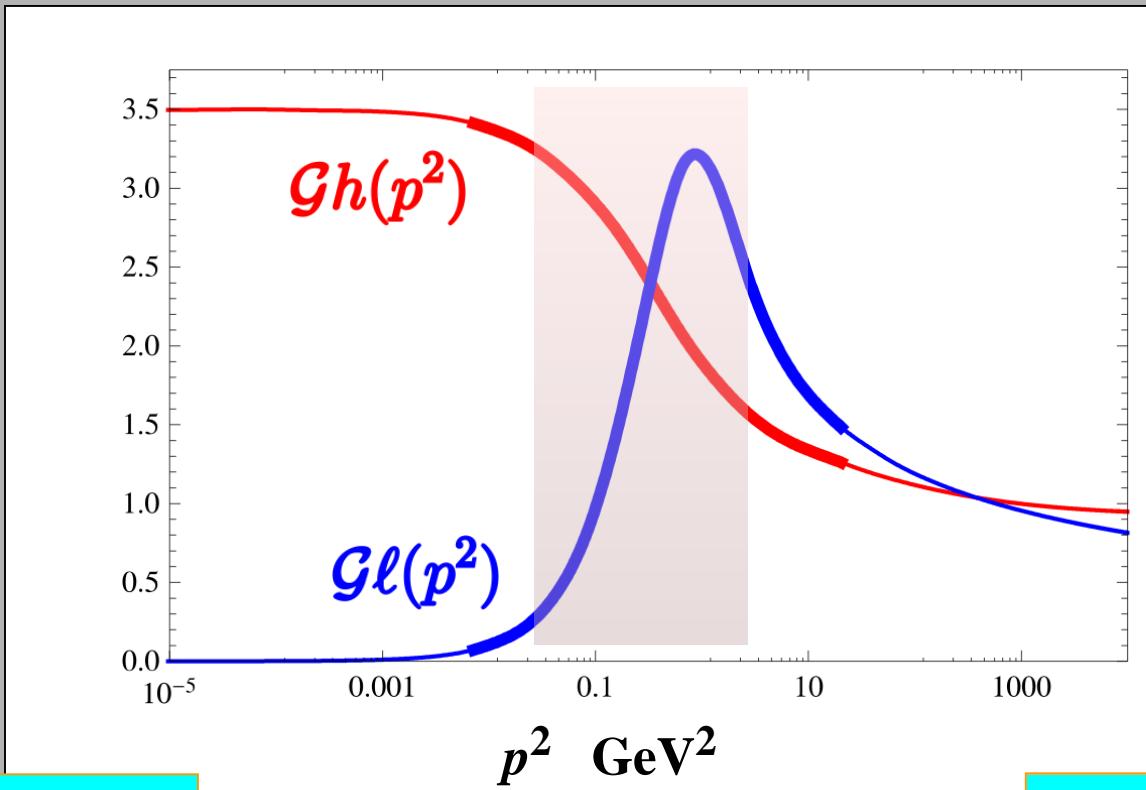
$gh(p^2)$

Oliveira & Silva



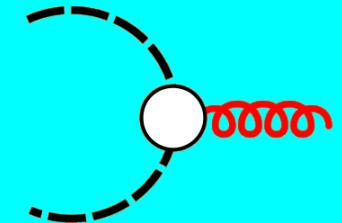
Solution of Gluon & Ghost SDEs

“massive”

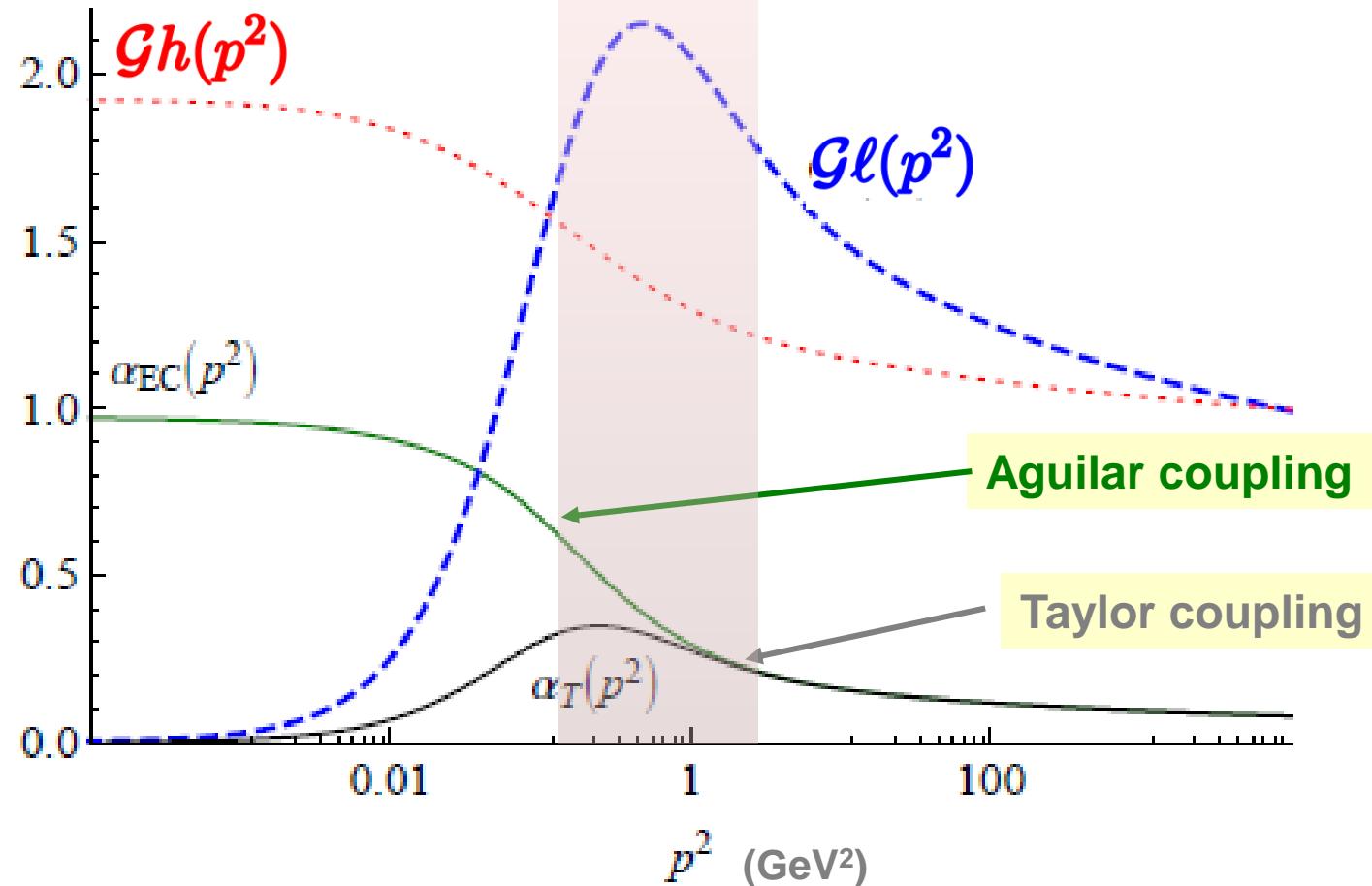


Aguilar, Papavassiliou, Binosi
Boucaud et al
Rodriguez Quintero

Wilson & P



Running coupling



Schwinger-Dyson Equations

QCD

Quark propagator:

$$\text{---} \circ \text{---}^{-1} = \text{---} \circ \text{---}^{-1} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

Ghost propagator:

$$\text{---} \circ \text{---}^{-1} = \text{---} \circ \text{---}^{-1} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

Ghost-gluon vertex:

$$\text{---} \circ \text{---} \text{---} \circ \text{---} = \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---} \text{---} \circ \text{---}$$

Quark-gluon vertex:

$$\text{---} \circ \text{---} \text{---} \circ \text{---} = \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

$$+ \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

Gluon propagator:

$$\text{---} \circ \text{---}^{-1} = \text{---} \circ \text{---}^{-1} +$$

$$+ \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

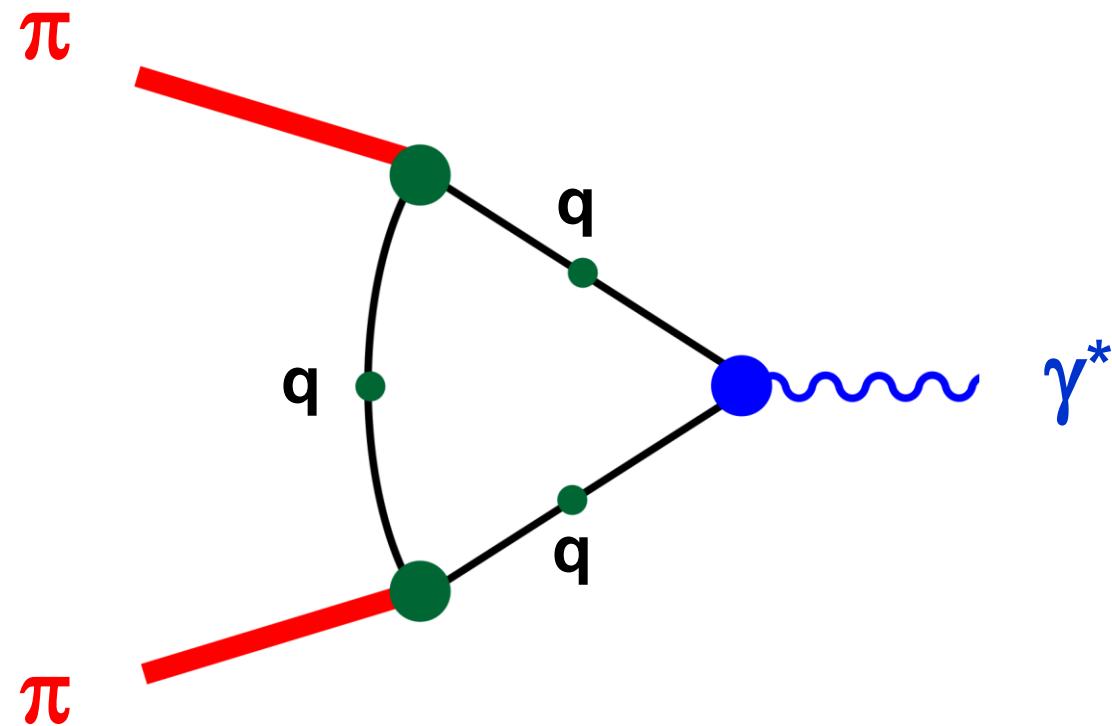
$$+ \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

$$+ \text{---} \circ \text{---} \text{---} \circ \text{---} + \text{---} \circ \text{---} \text{---} \circ \text{---}$$

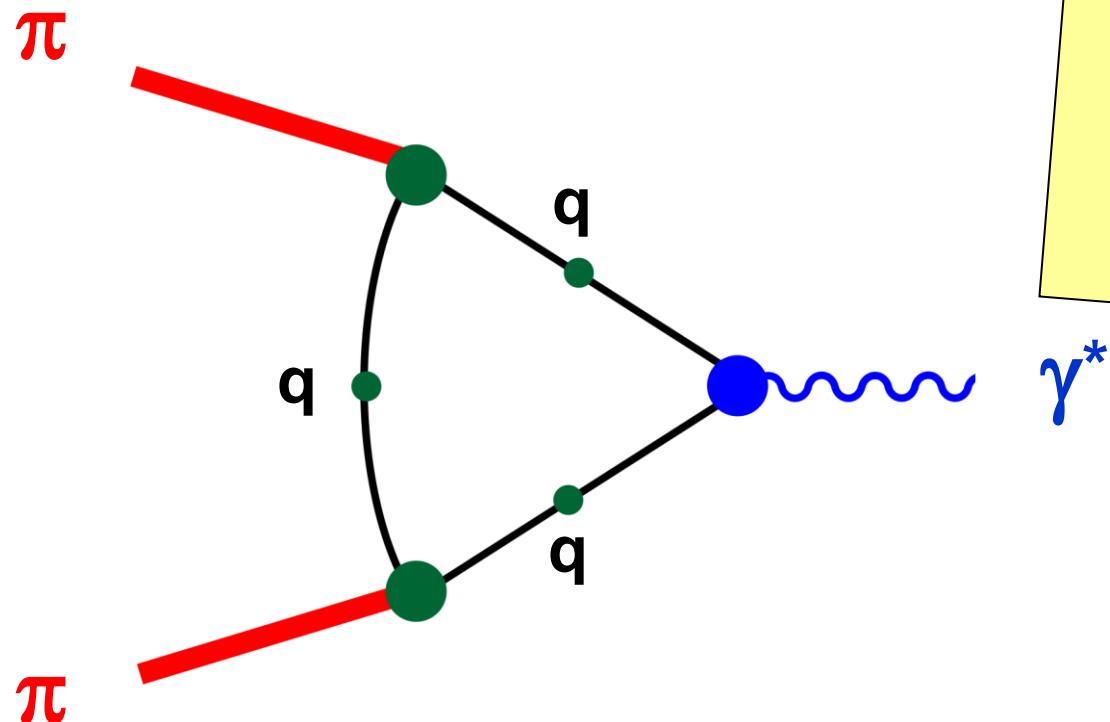
Consistent truncation essential

Gauge Invariance, Gauge Covariance
& Multiplicative Renormalizability

Pion electromagnetic formfactor

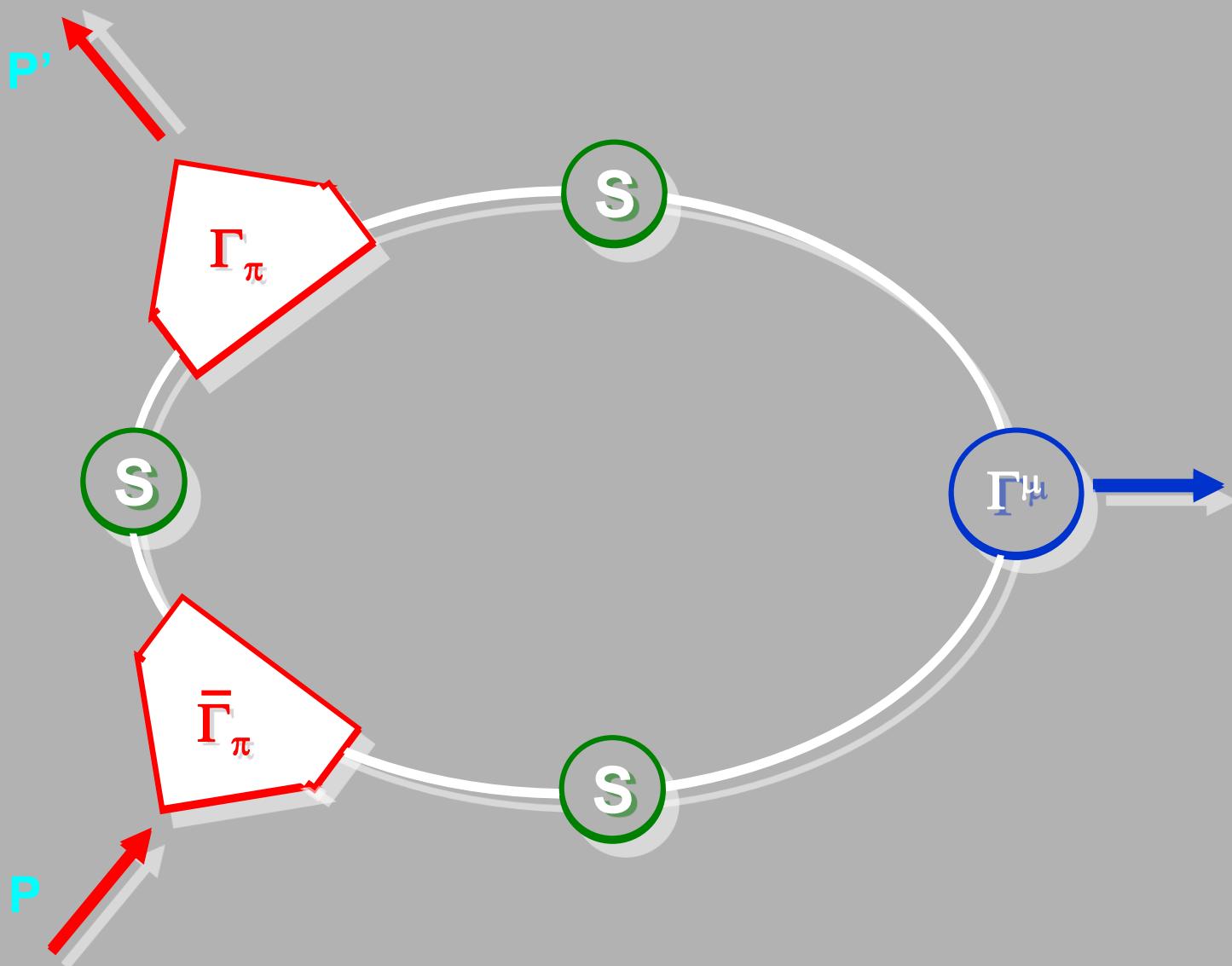


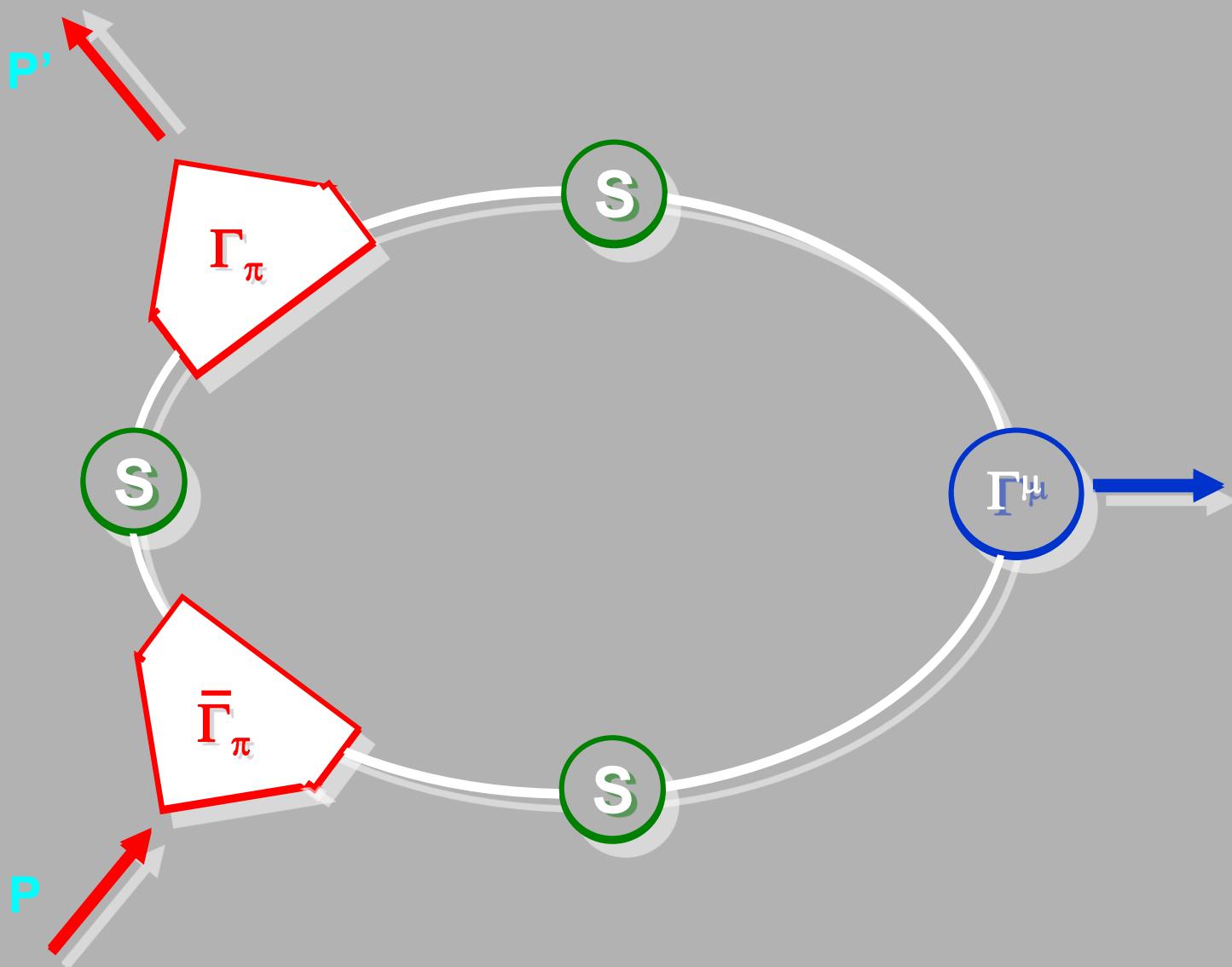
Pion electromagnetic formfactor



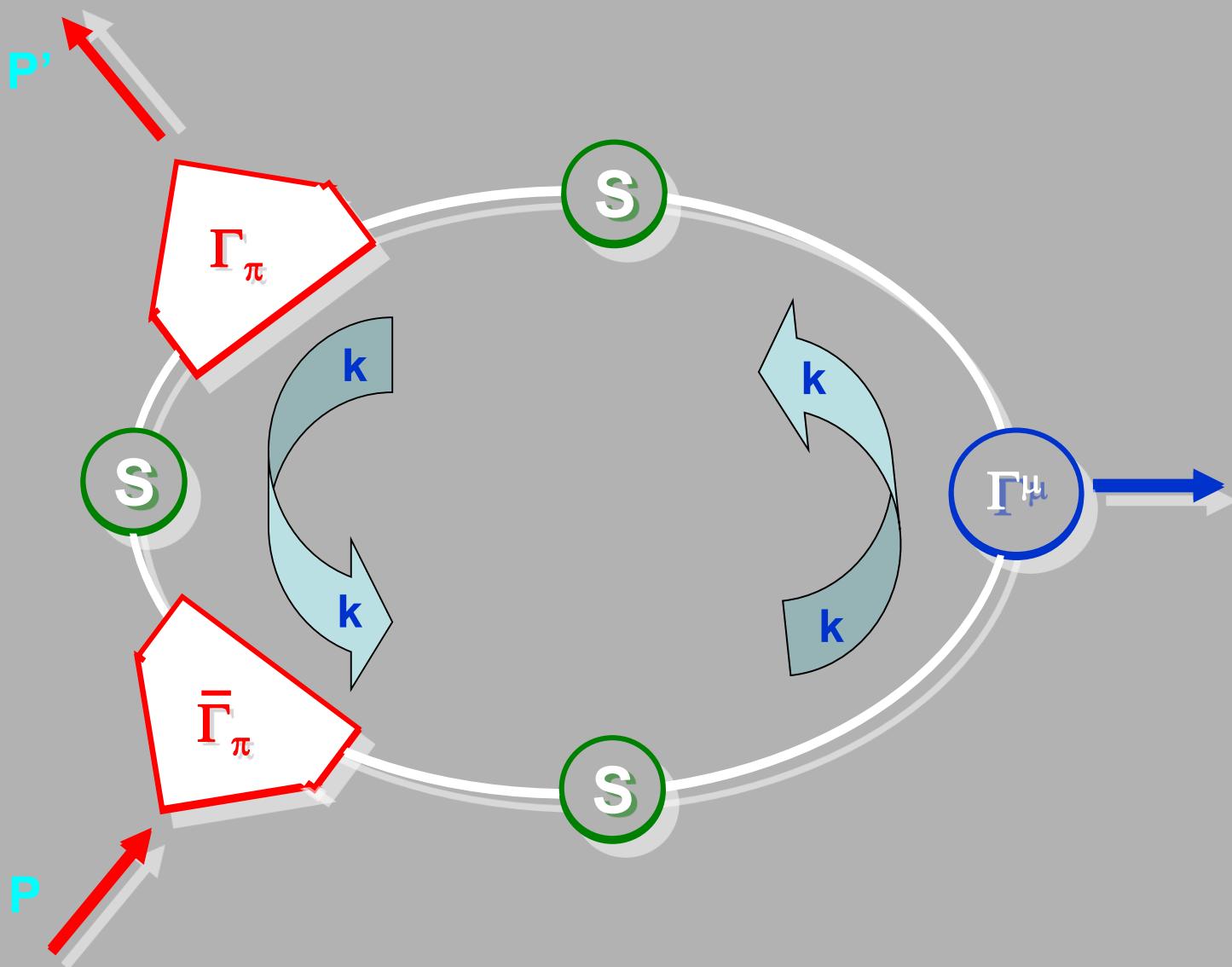
pQCD $Q^2 \gg \Lambda_{\text{QCD}}^2$

$$F_\pi(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2}$$

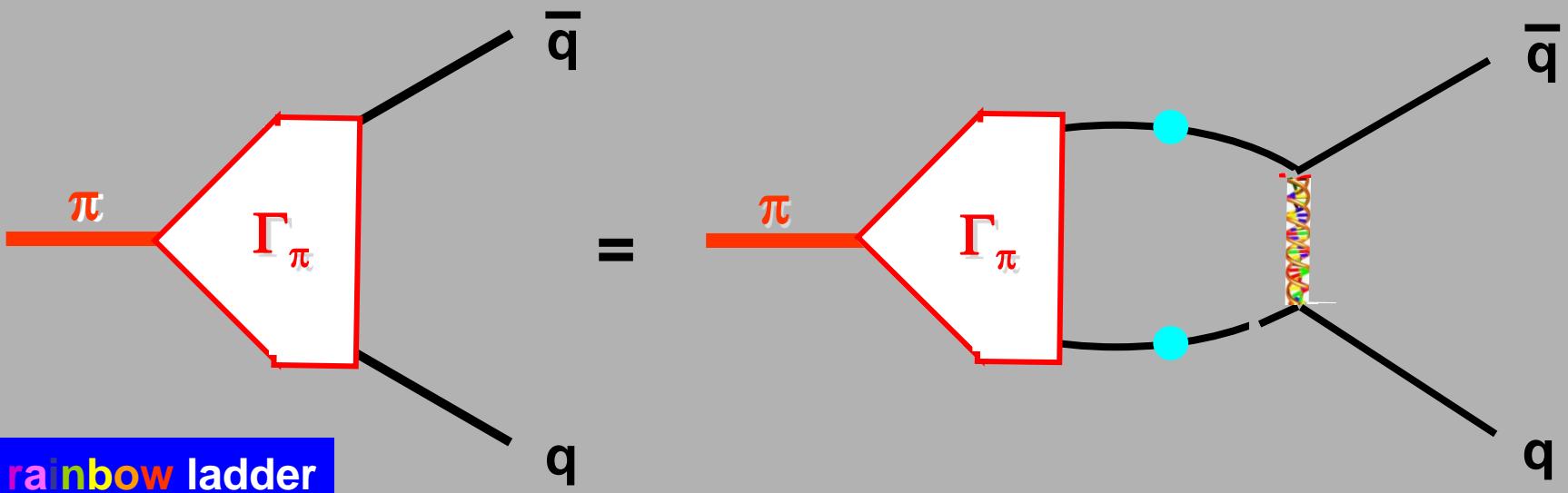




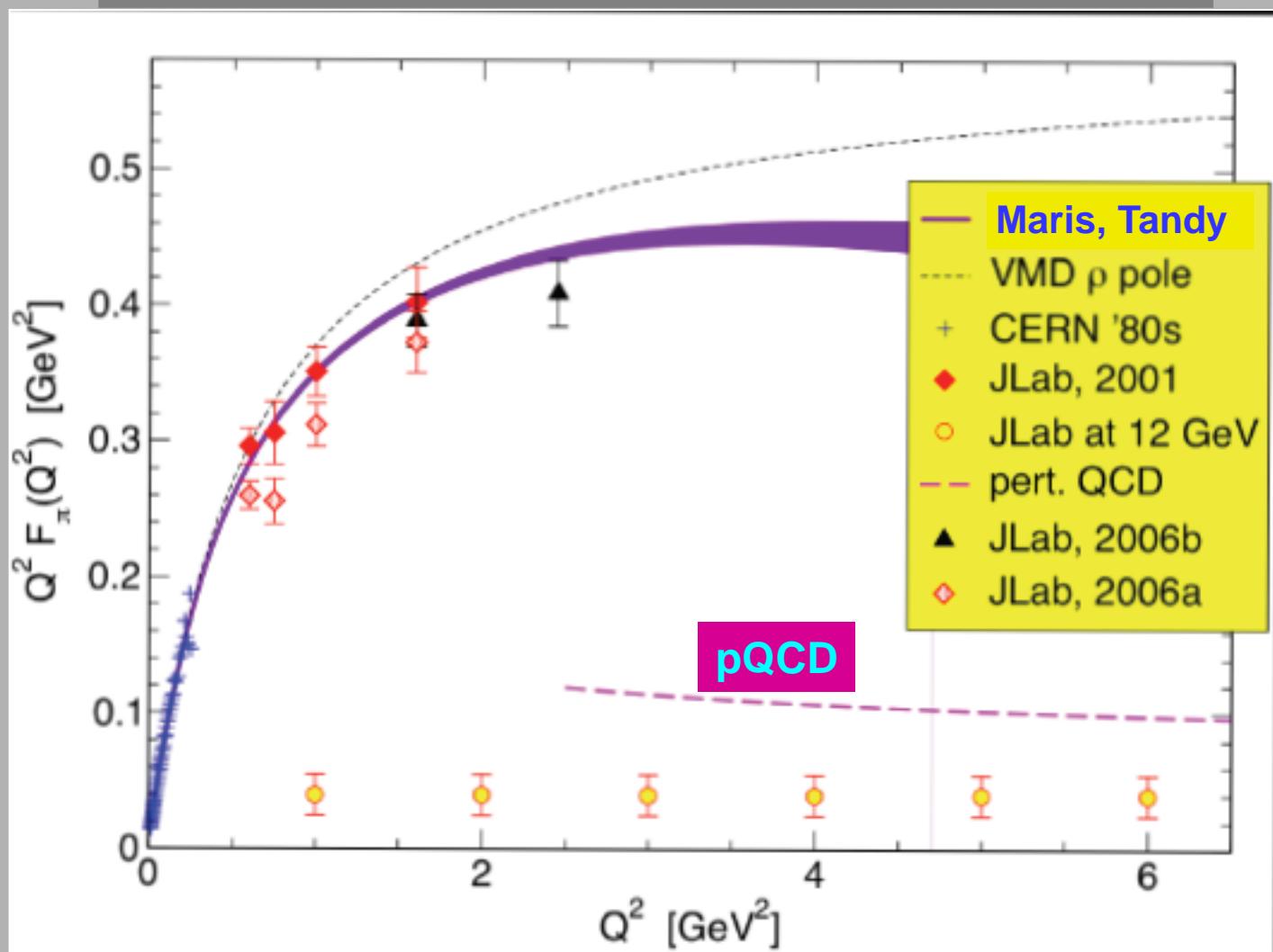
$$\Lambda_\mu = (P' + P)_\mu F_\pi(Q^2) = N_c \int \frac{d^4 k}{(2\pi)^4} \text{Tr} [\bar{\Gamma}^\pi S i\Gamma_\mu S \Gamma^\pi S]$$



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Pion electromagnetic formfactor



Parton Distribution Amplitude of the Pion : $\Phi_\pi(x)$

$\Phi_\pi(x)$ *is a probability amplitude describing the momentum distribution of a quark and an antiquark in the valence Fock state of the pion*

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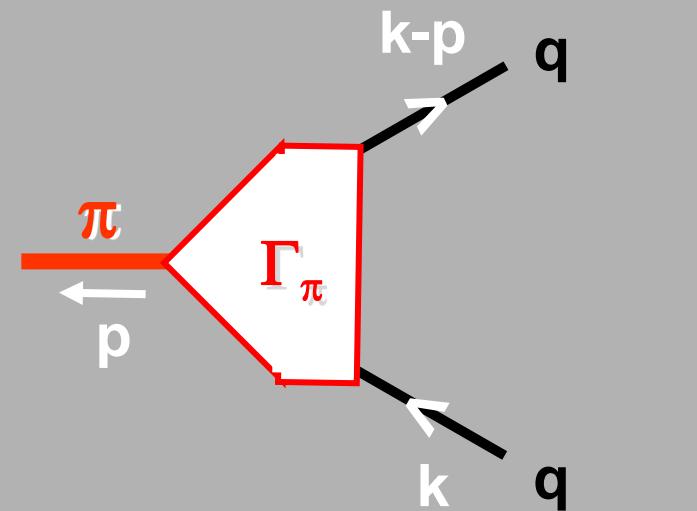
$$\Phi_\pi(x) = \Phi_\pi(1-x)$$

--- charge conjugation

Parton Distribution Amplitude of the Pion : $\Phi_\pi(x)$

$$f_\pi \Phi_\pi(x) = Z_2 \int \frac{d^4 k}{(2\pi)^2} \delta(k^+ - x p^+) \text{Tr} [\gamma^+ \gamma_5 S(k) \Gamma_\pi(k, p) S(k-p)]$$

↑
Bethe-Salpeter wave function

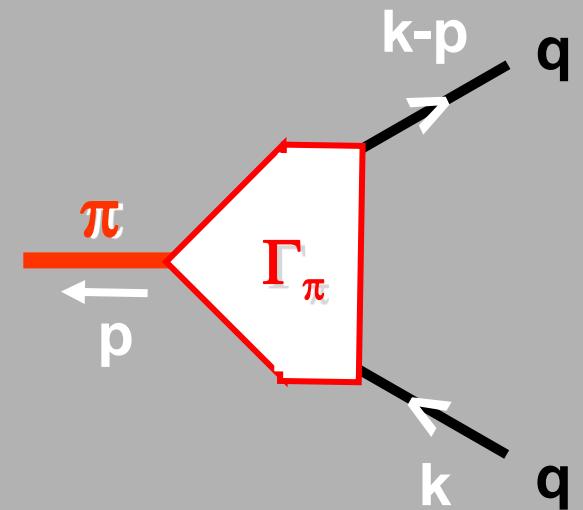


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axial vector projection of
 π Bethe-Salpeter wave function

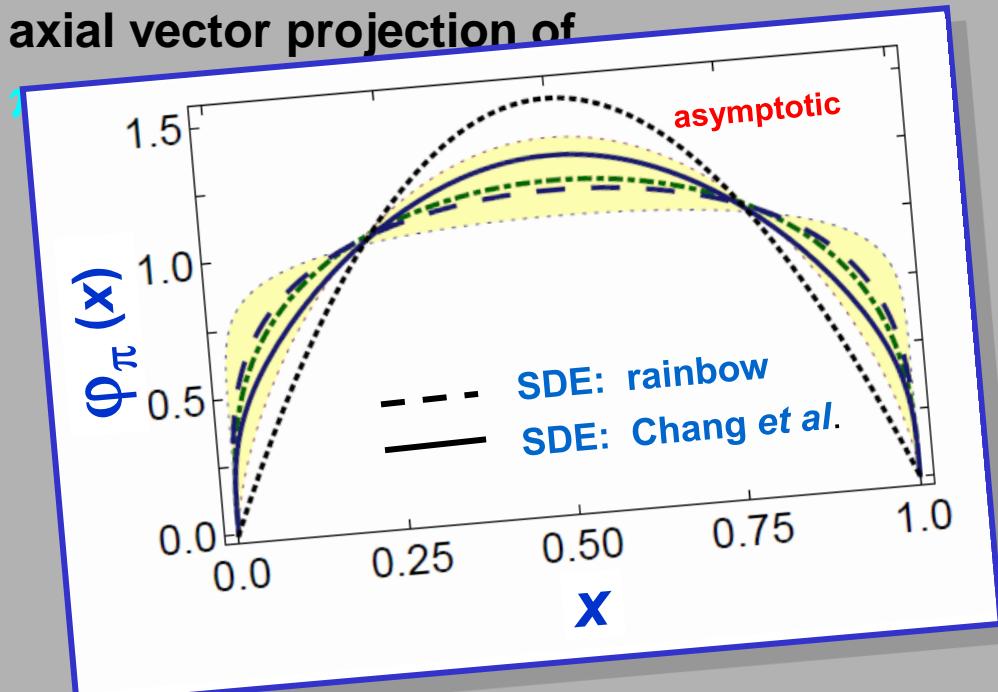
Bethe-Salpeter wave function



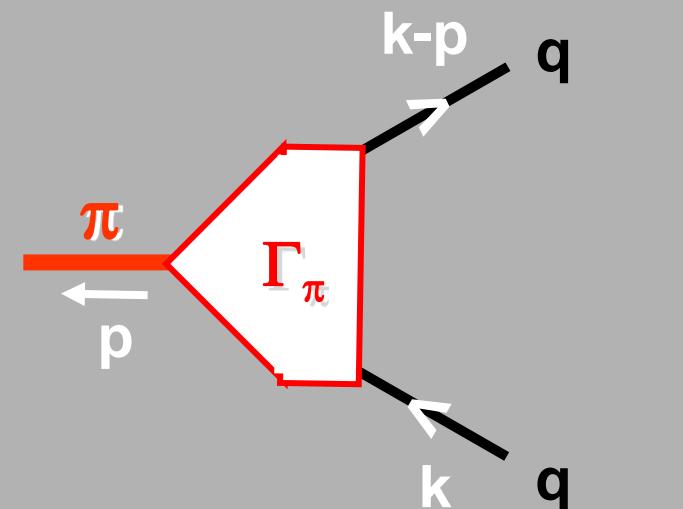
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axial vector projection of



Bethe-Salpeter wave function



Pion electromagnetic formfactor

pQCD

$$Q^2 \ F_\pi(Q^2) \rightsquigarrow 16\pi f_\pi^2 \alpha_s(Q^2) \ w_\pi^2$$

$$Q^2 \gg \Lambda_{\text{QCD}}^2$$

$$w_\pi = \frac{1}{3} \int_0^1 \frac{dx}{x} \ \Phi_\pi(x)$$

QCD evolution with momentum

$$\mu \frac{d}{d\mu} \Phi(x,\mu) = \int_0^1 dy V(x,y) \Phi(y,\mu)$$

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Solution: Efremov & Radyushkin (Brodsky & Lepage)

$$\Phi_\pi(x, Q^2) = 6x(1-x) \left[1 + \sum_{n=2,4,\dots} a_n^{3/2}(Q^2) C_n^{3/2}(2x-1) \right]$$

$a_n^{3/2}(Q^2) \rightarrow 0$ with powers of $\ln Q^2$

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$Q^2 \rightarrow \infty$

$$\Phi_\pi(x) \stackrel{\text{asym}}{=} 6x(1-x)$$

Pion electromagnetic formfactor

pQCD

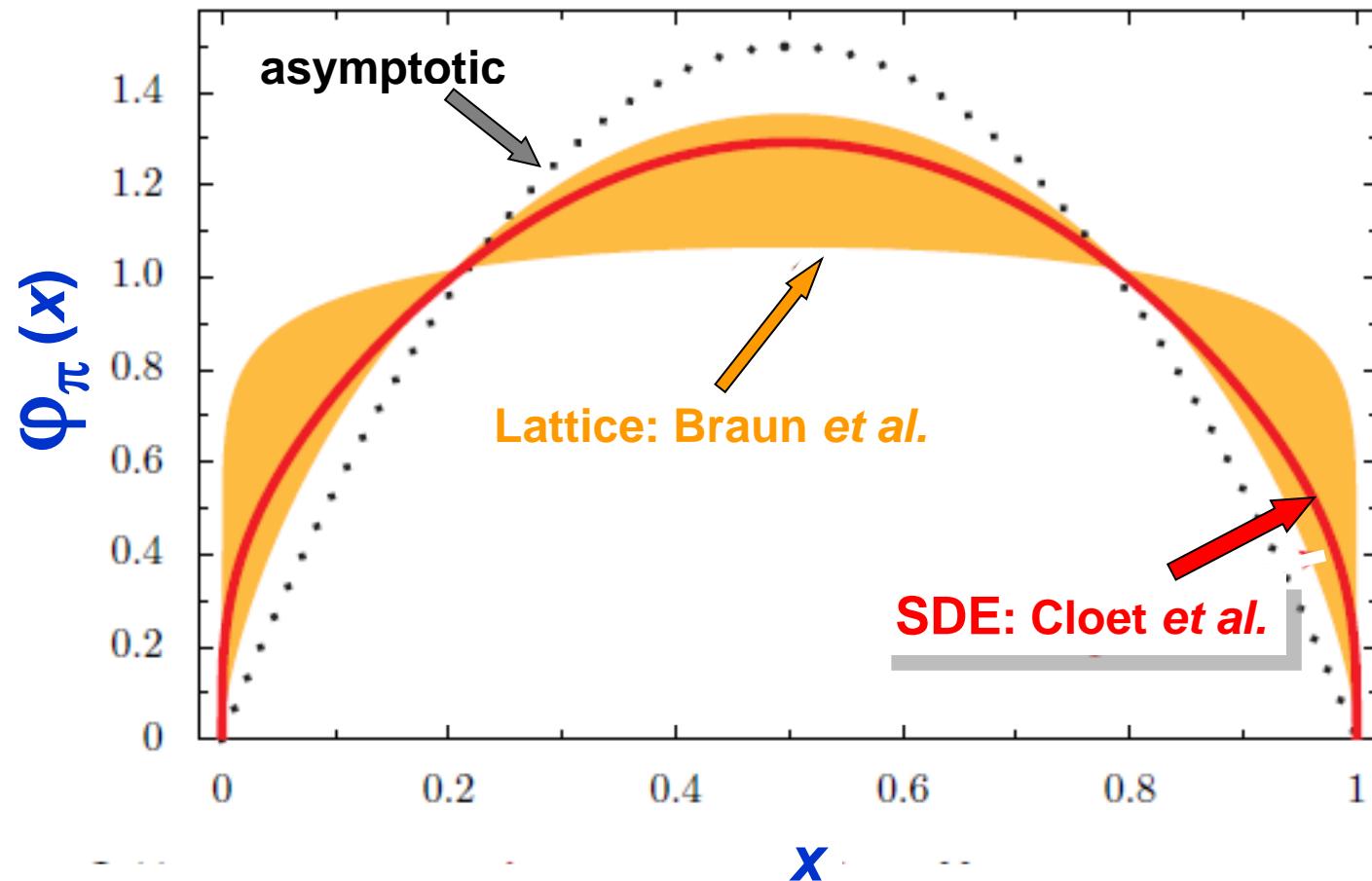
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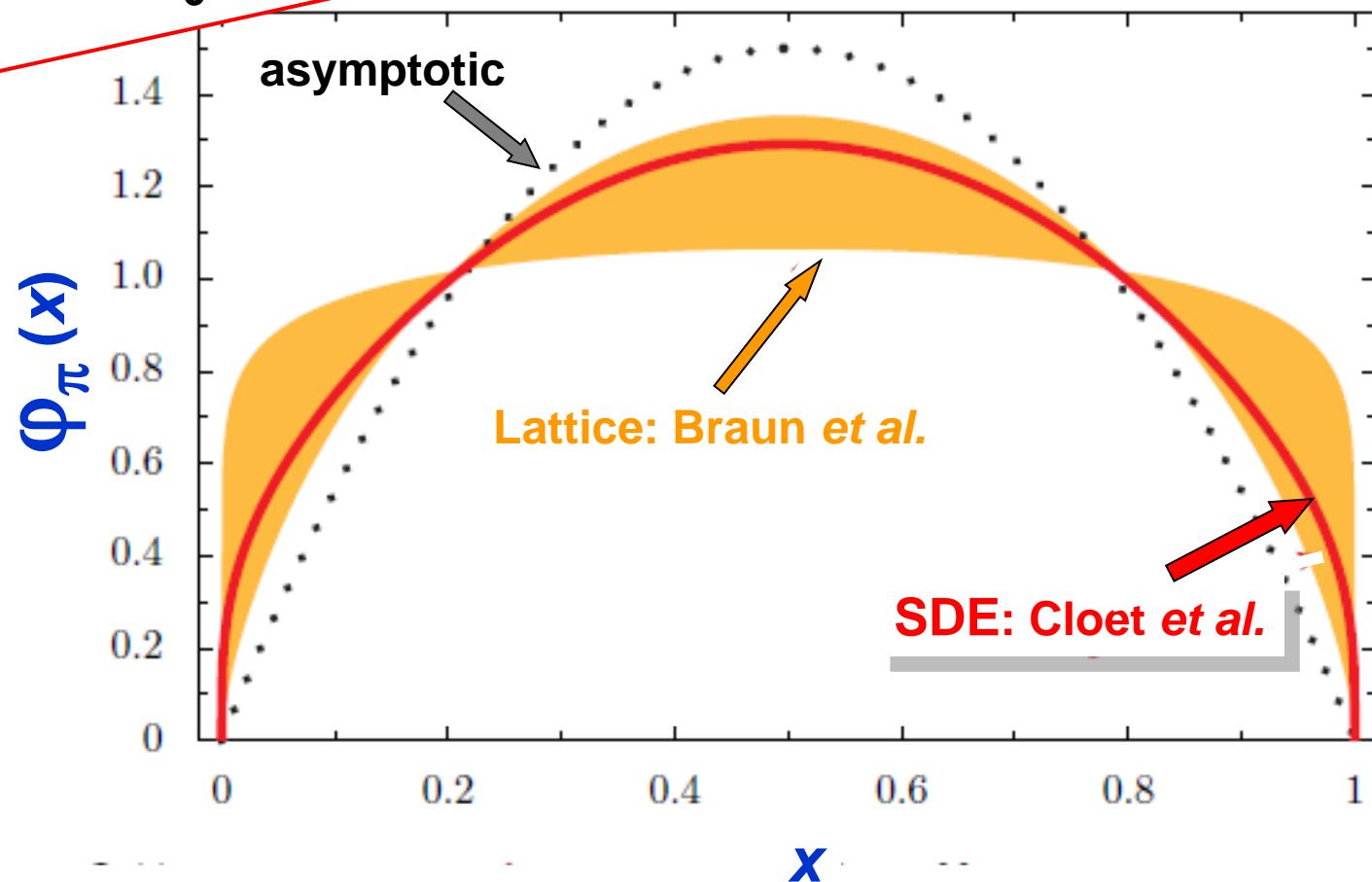
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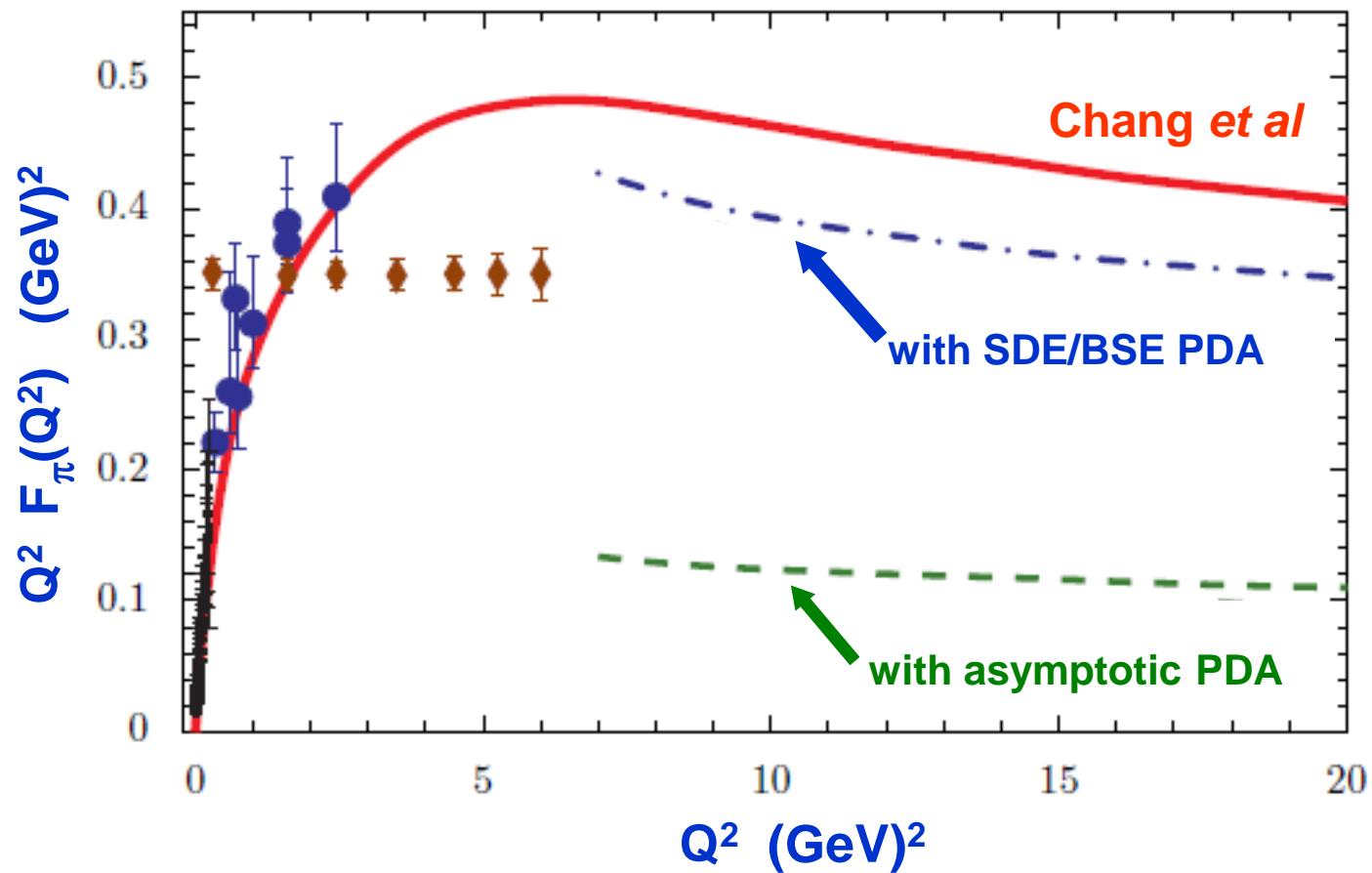


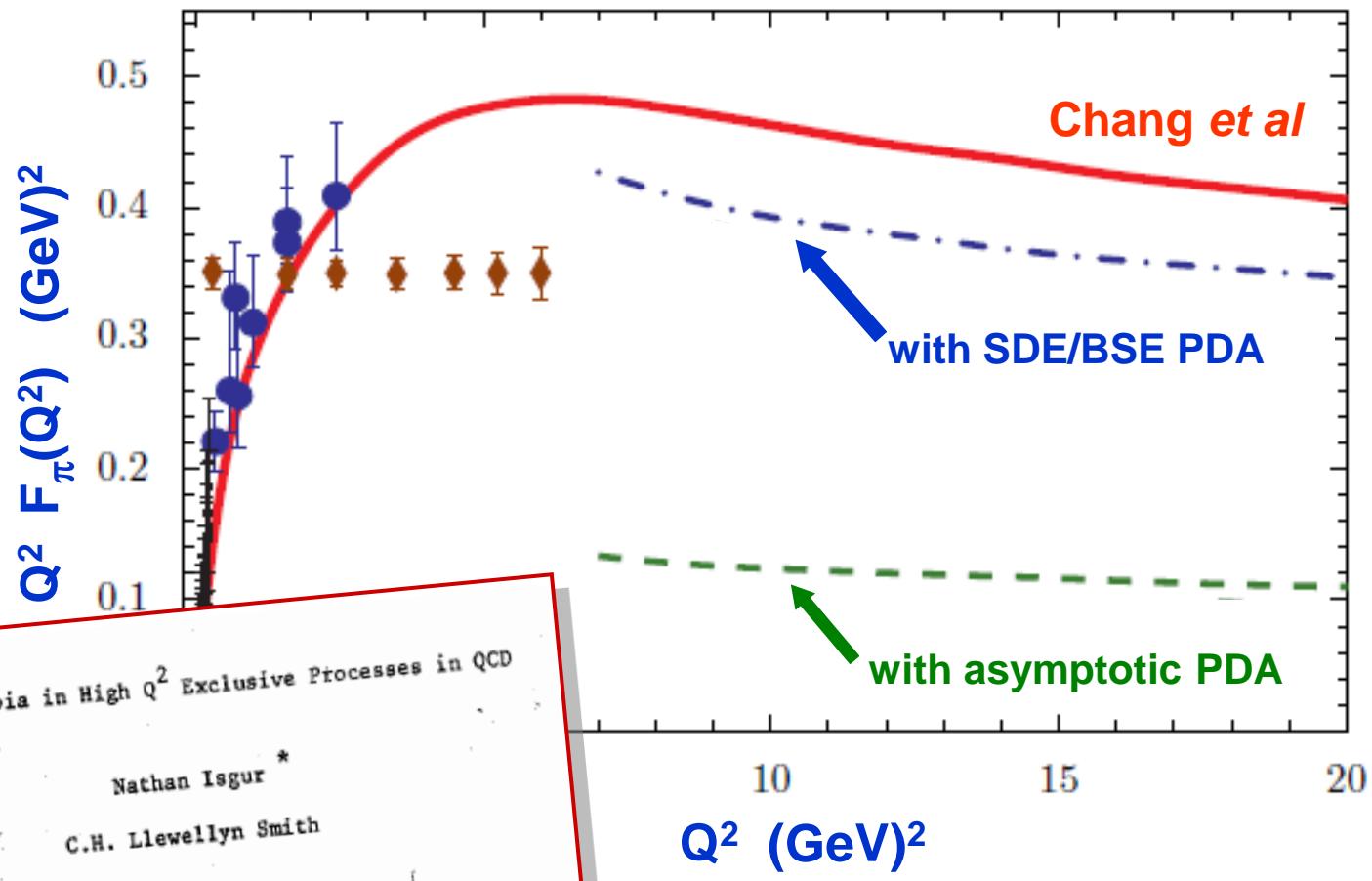
Parton Distribution

Amplitude of the Pion : $\Phi_\pi(x)$

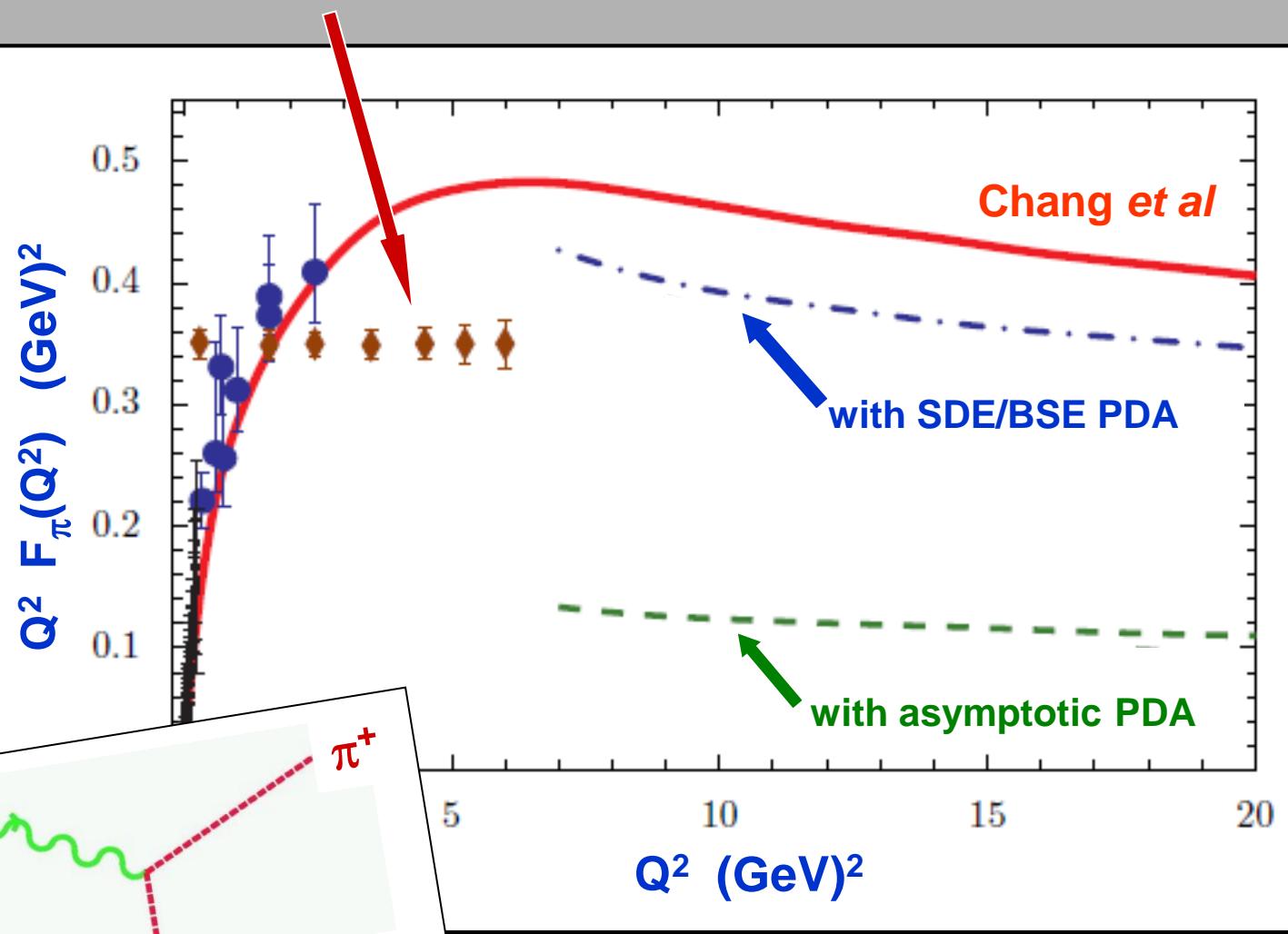
$$w_\pi = \frac{1}{3} \int_0^1 \frac{dx}{x} \Phi_\pi(x)$$

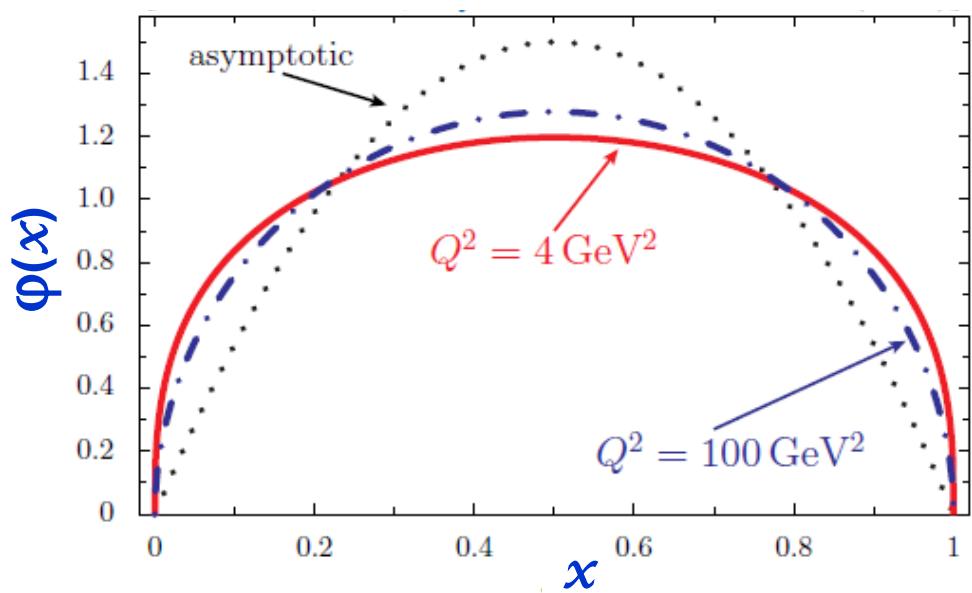






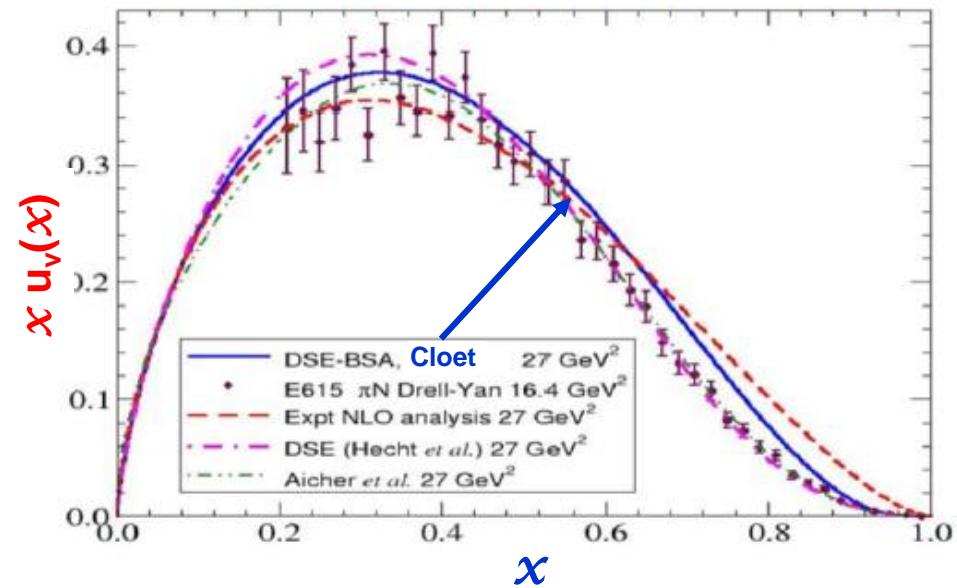
forthcoming JLab data





valence parton distribution

Cloet et al. 2013



lattice results: 2017

$$\langle \xi^n \rangle = \int_0^1 dx (2x-1)^n \varphi(x, \mu^2)$$

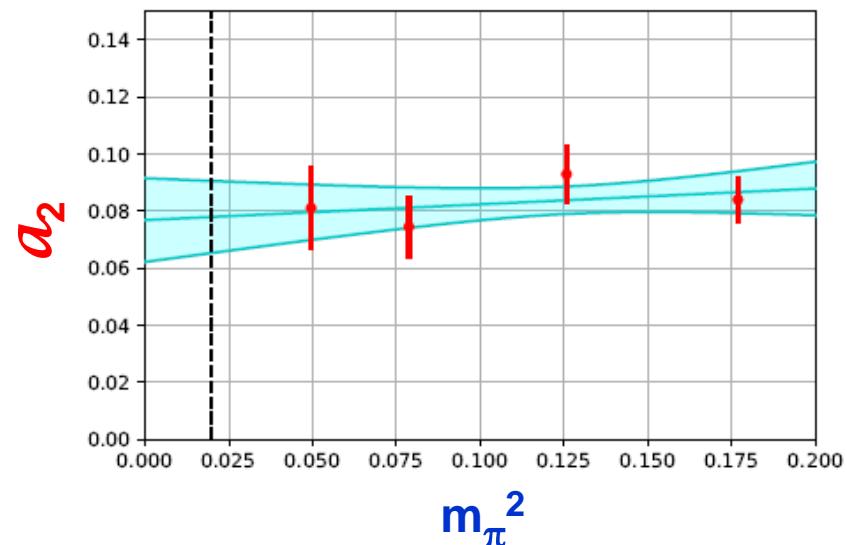
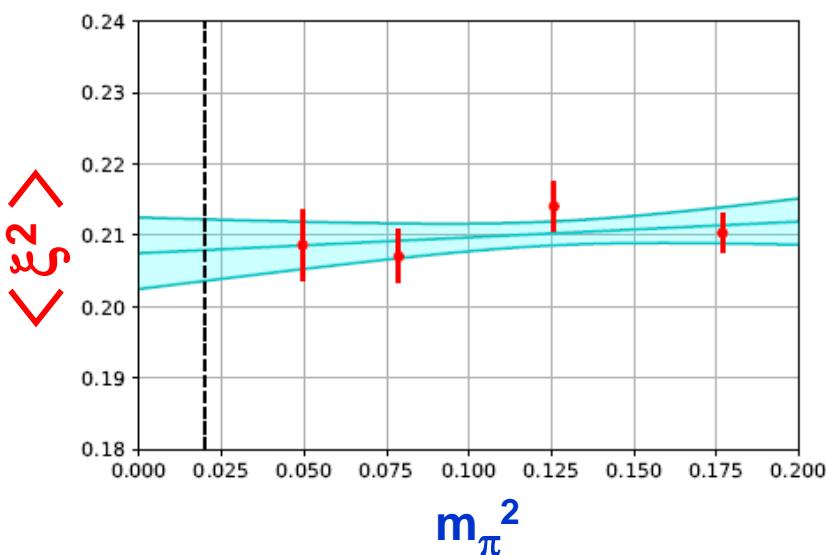
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Bali *et al.* 2017

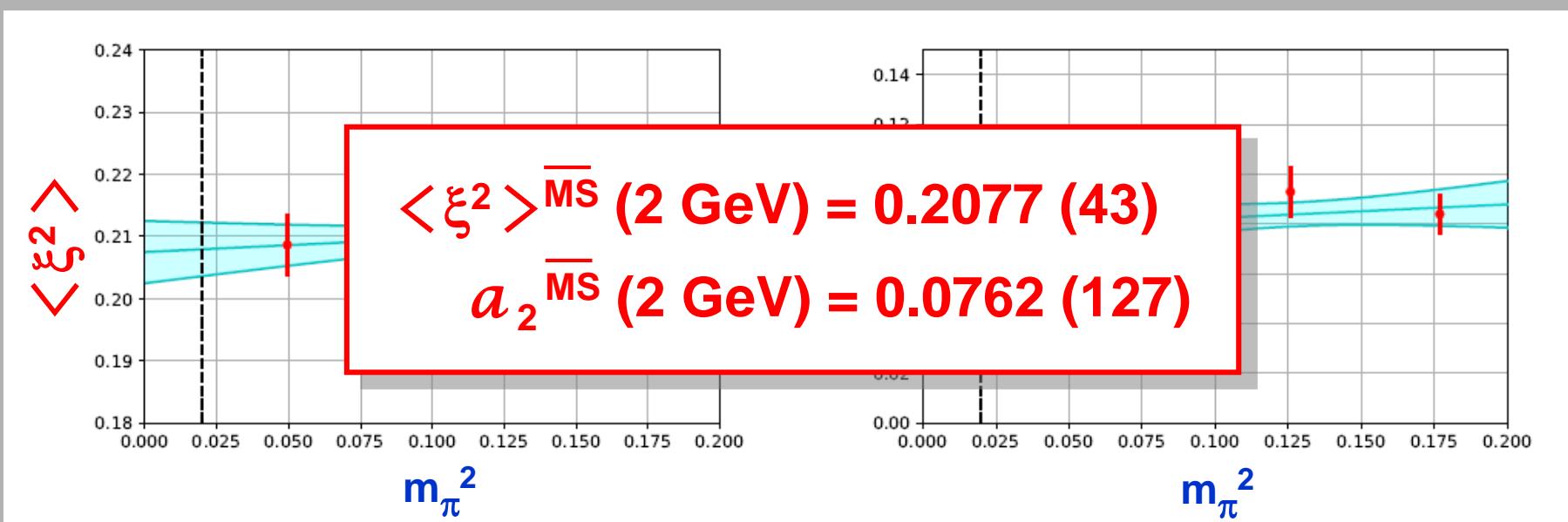


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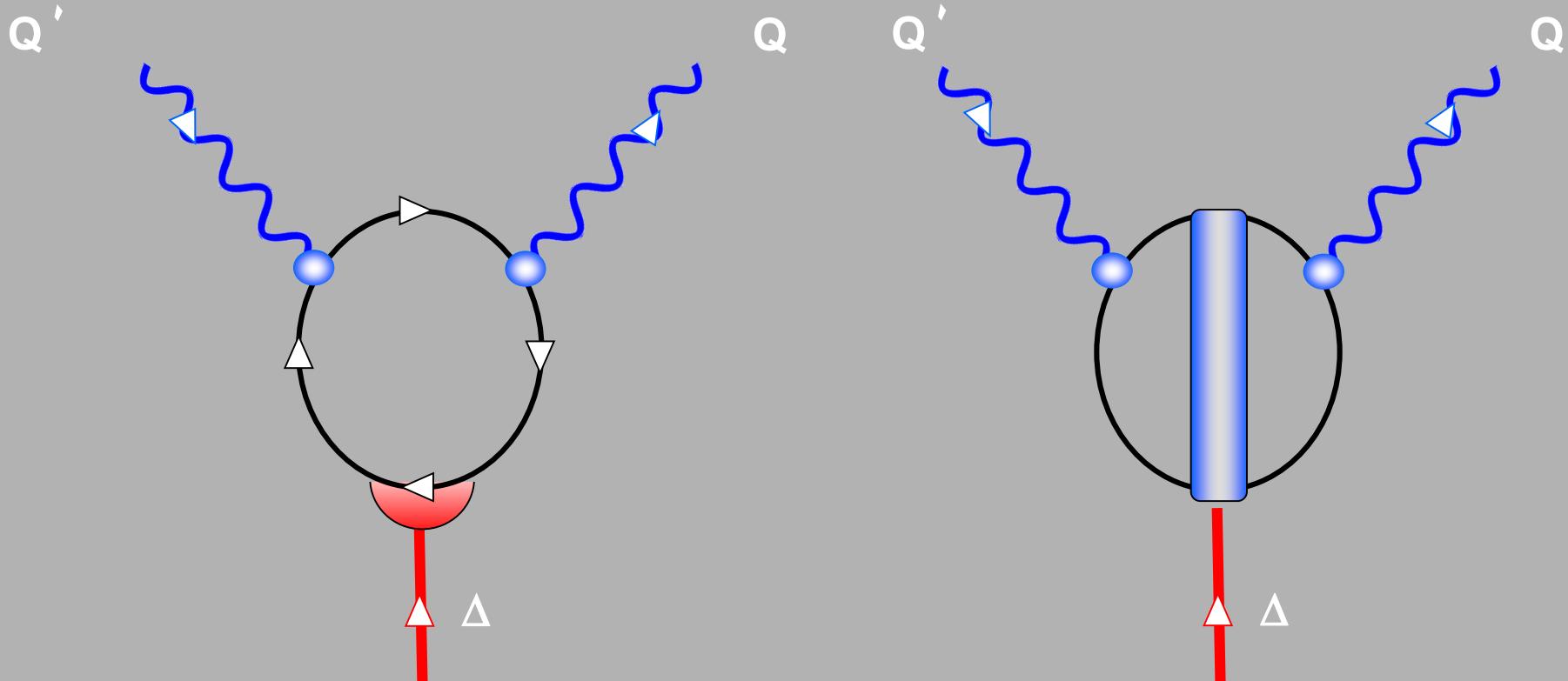
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Bali *et al.* 2017



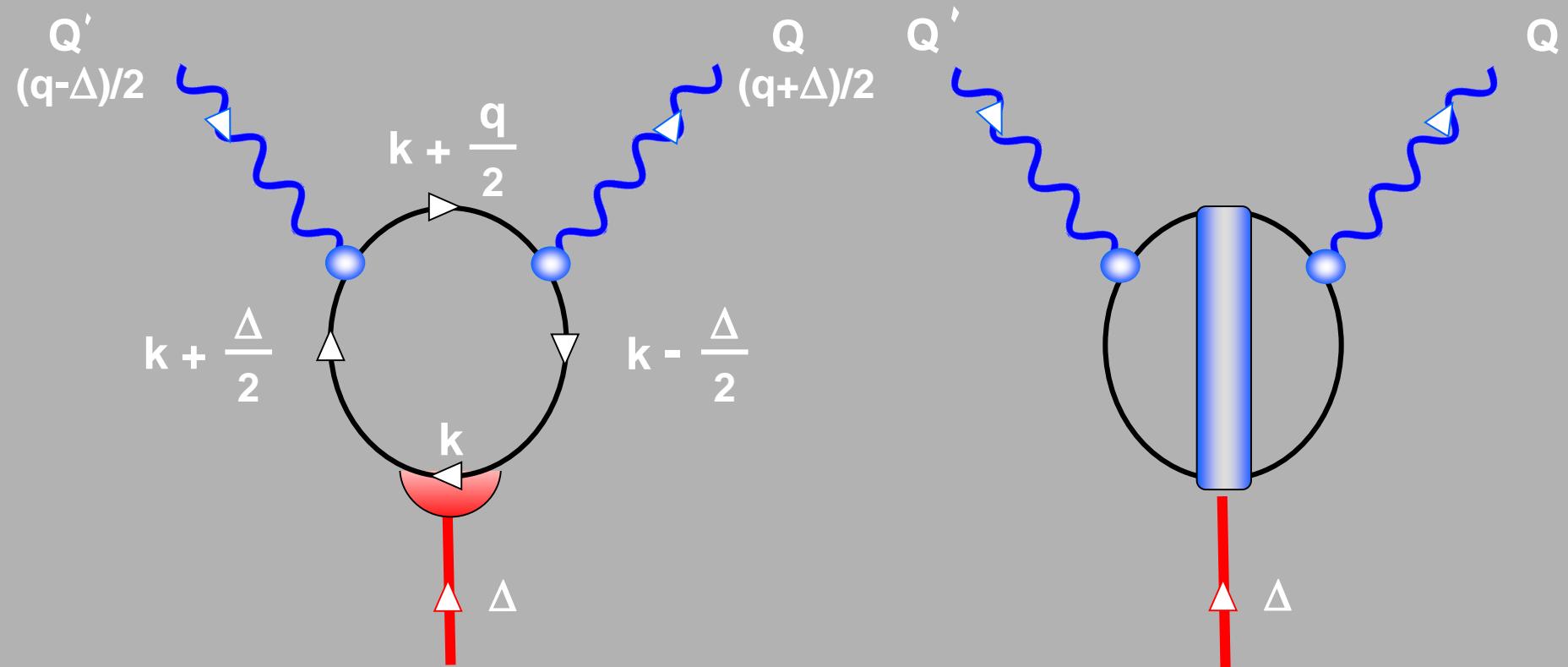


$\pi^0 \rightarrow \gamma\gamma$ transition formfactor





$\pi^0 \rightarrow \gamma\gamma$ transition formfactor



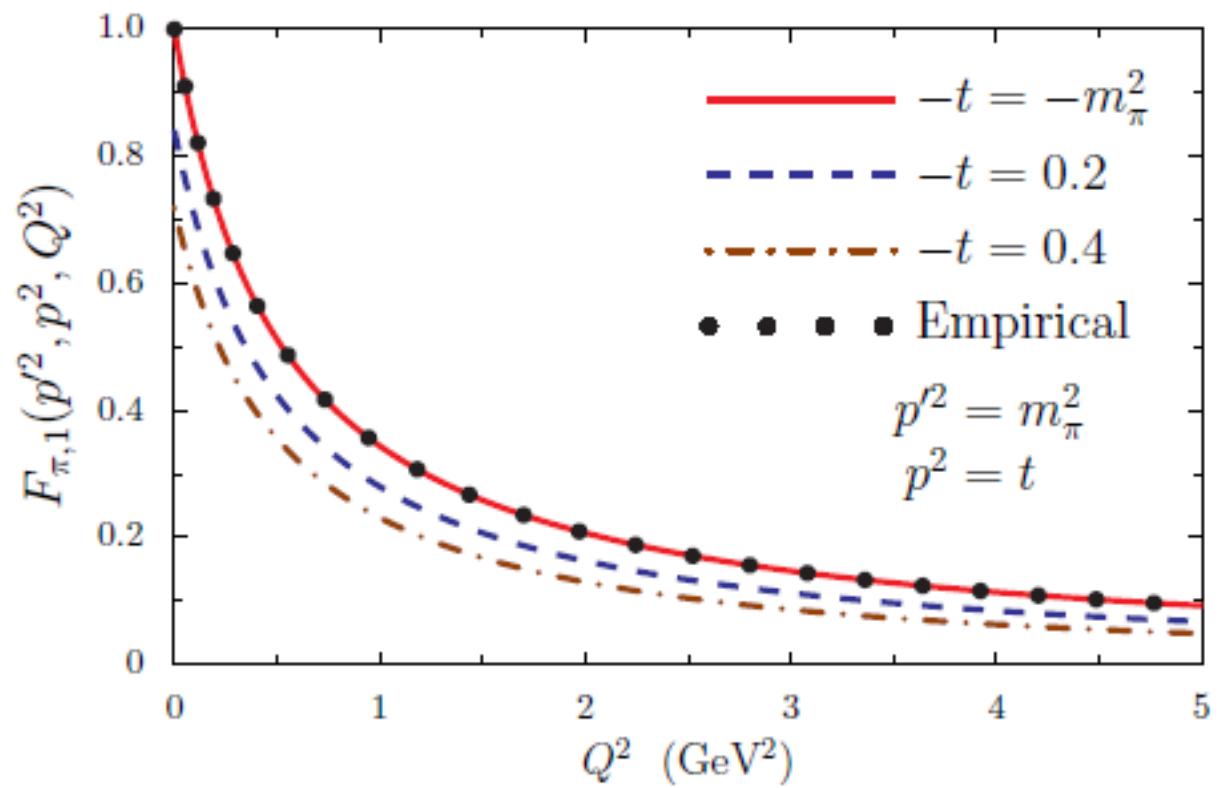
$$\Lambda^{\mu\nu} = \text{Tr} \int \underline{dk} \ S(k+\Delta/2) \Gamma_\pi(k, \Delta) \ S(k-\Delta/2) \Gamma^\mu(k-\Delta/2, k+q/2) \ S(k+q/2) \Gamma^\nu(k+q/2, k+\Delta/2)$$

$\pi^0 \rightarrow \gamma\gamma$ transition formfactor

$$\eta_+ = (Q'^2 + Q^2)/2 \quad \omega = (Q'^2 - Q^2)/2$$

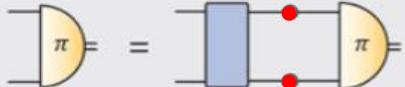
$$\tilde{F}(Q^2, Q'^2) = \frac{\eta_+ F(Q^2, Q'^2)}{4\pi^2 f_\pi^2} \xrightarrow{\eta_+ \gg 1} j(\omega)$$

$$j(\omega) = \frac{2}{3} \int_0^1 dx \frac{\eta_+^2}{\eta_+^2 - \omega^2 (2x-1)^2} \Phi_\pi(x)$$



Connecting to QCD

⇒ Bethe-Salpeter equation:



⇒ Faddeev equation:

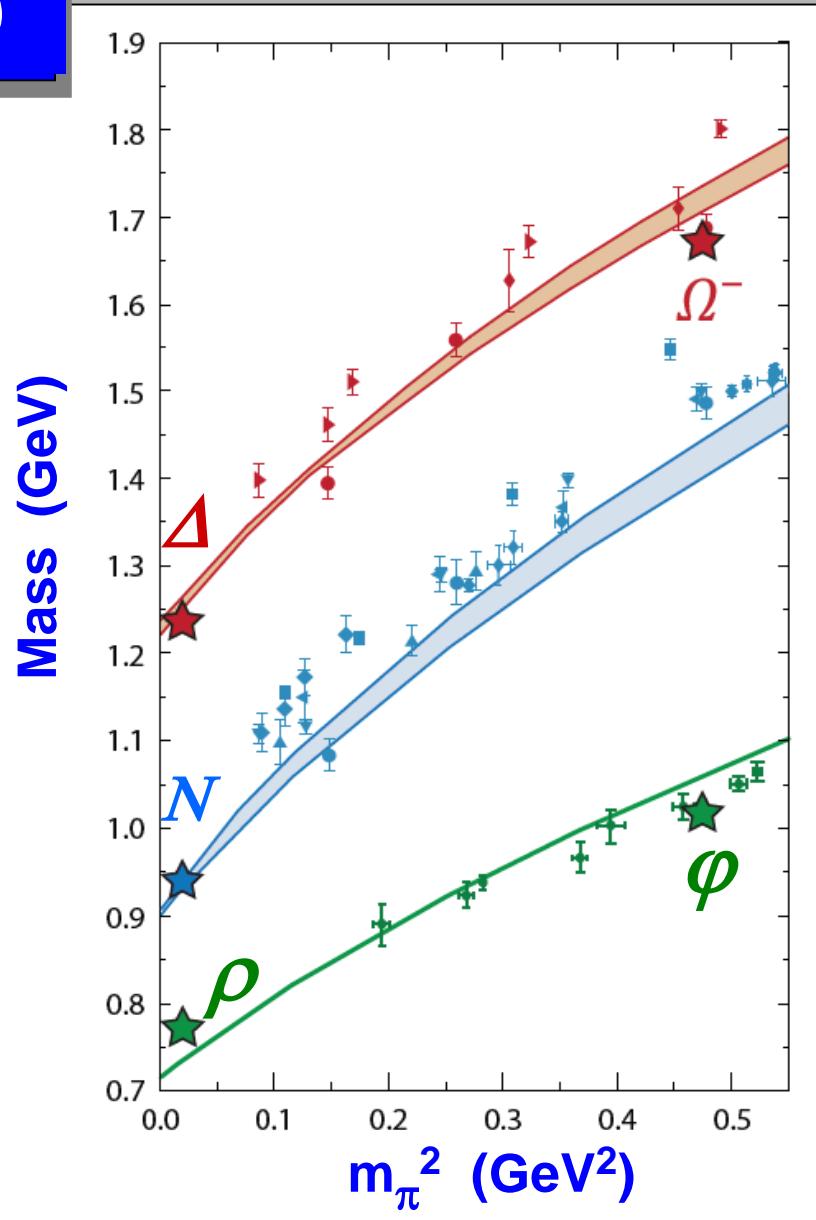


dressed quark propagator

Maris & Tandy

Eichmann et al

Sanchis-Alepuz et al



$\Delta-\Omega$

N

$\rho-\phi$

Connecting to QCD

Schwinger-Dyson for q,g

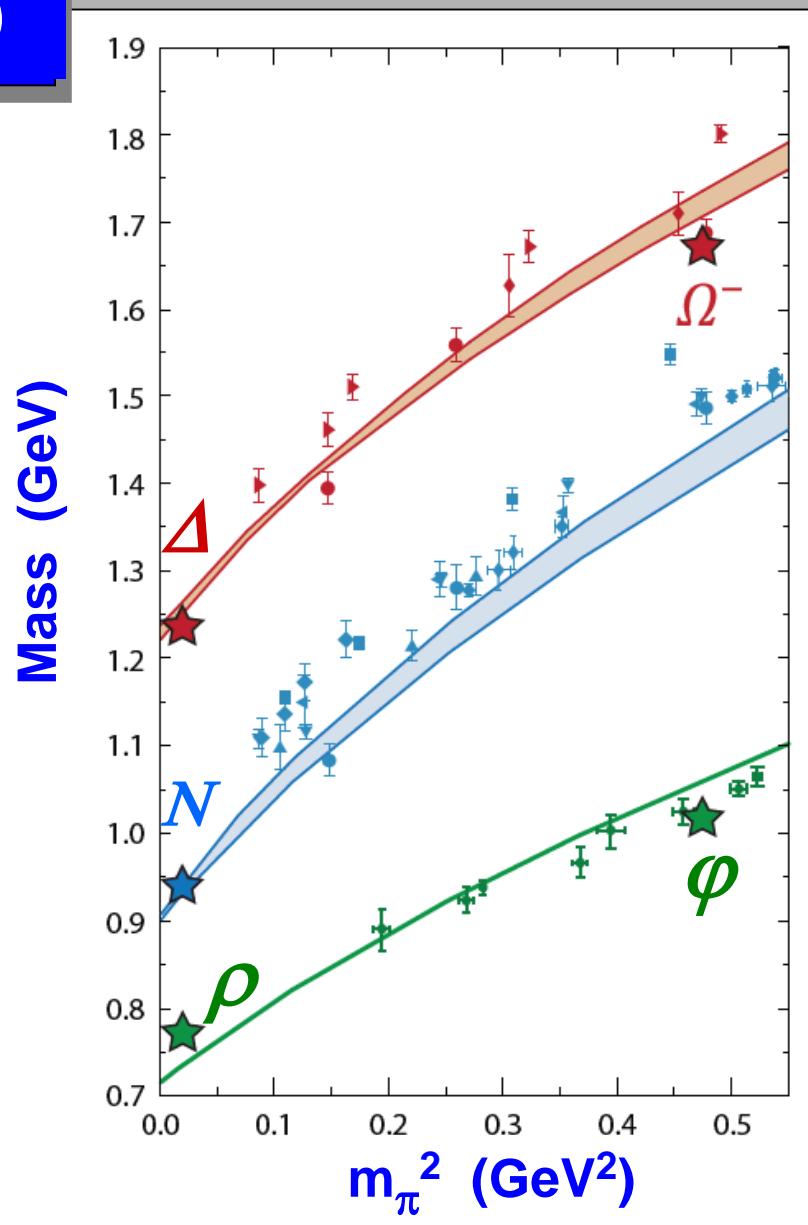
Bethe-Salpeter for mesons,

Fadeev for baryons

Maris & Tandy

Eichmann *et al*

Sanchis-Alepuz *et al*

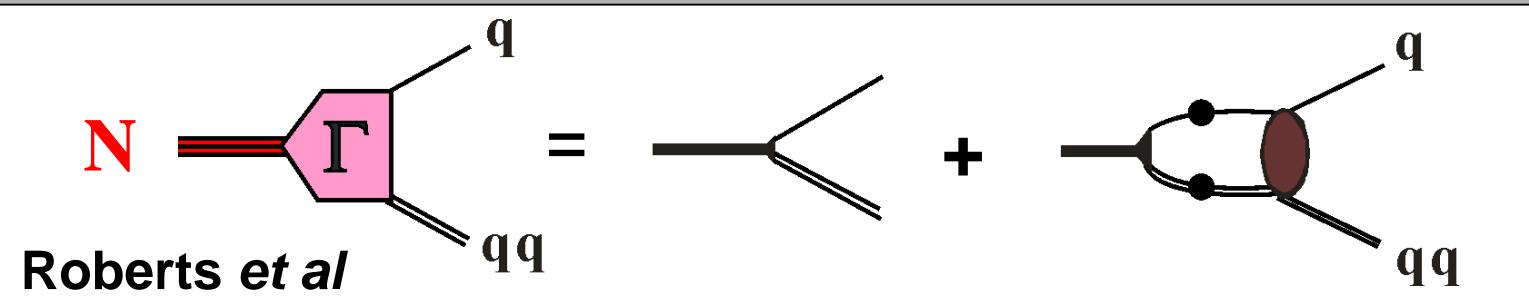
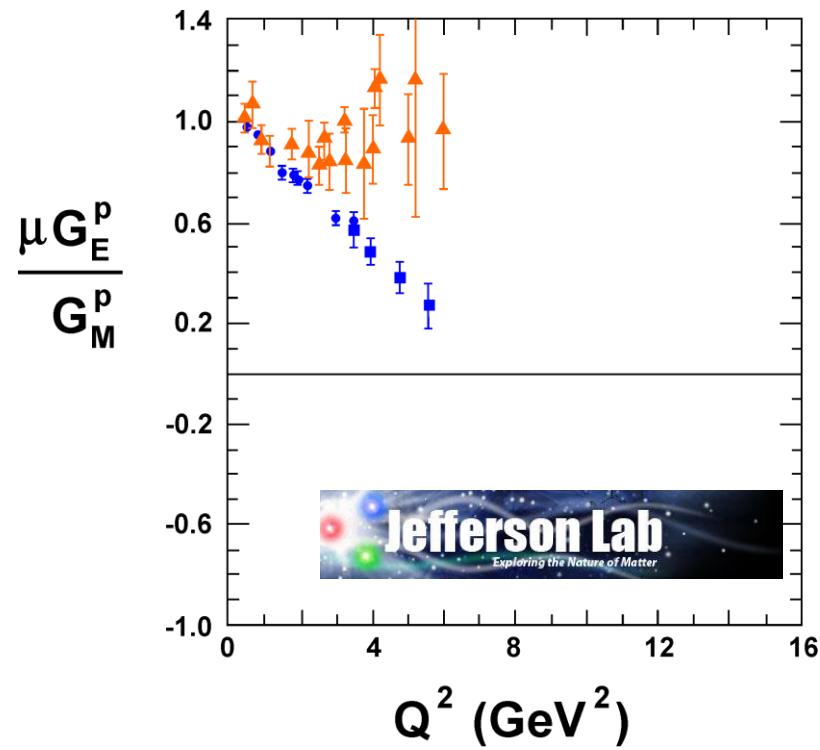
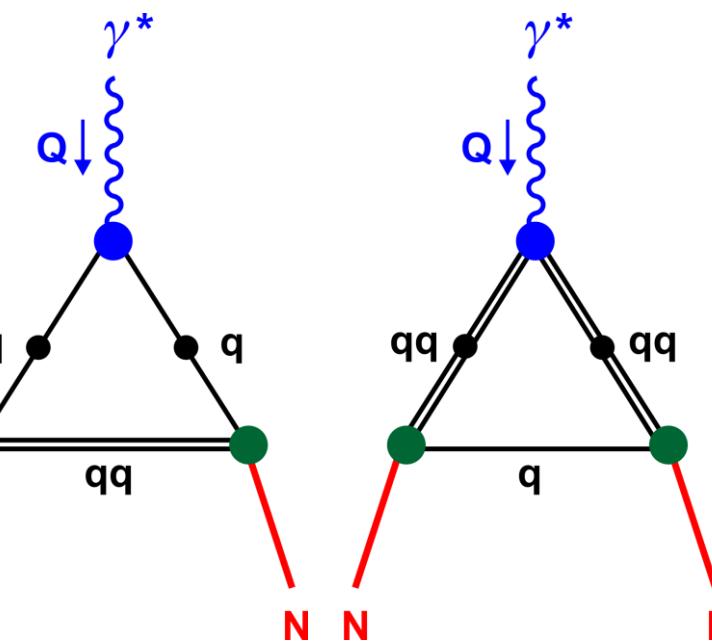


$\Delta-\Omega$

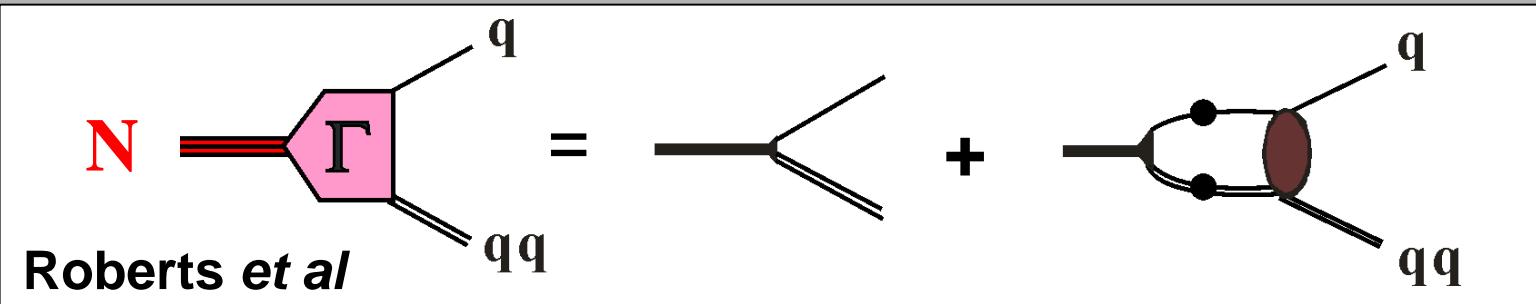
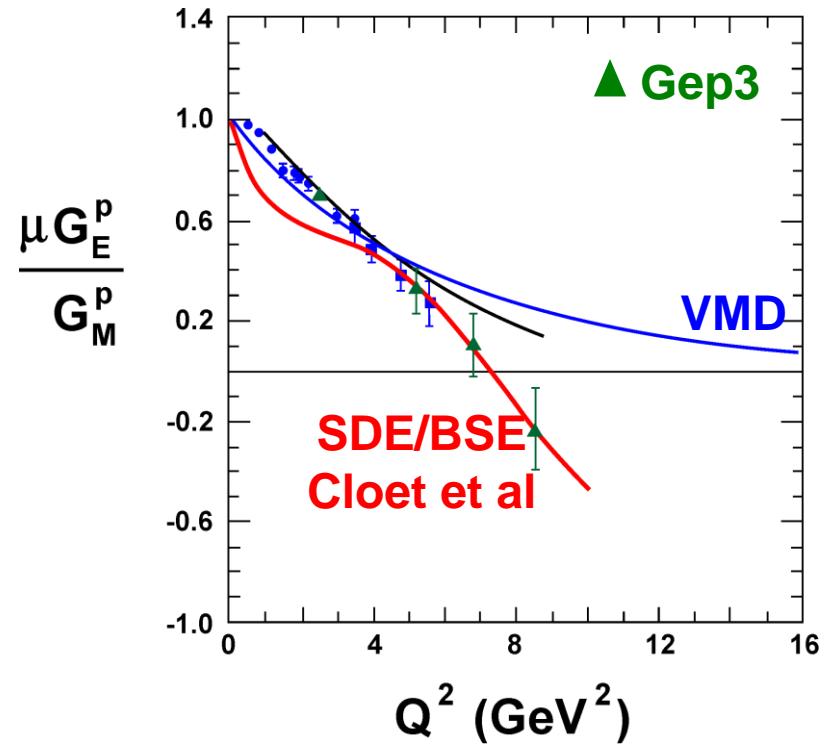
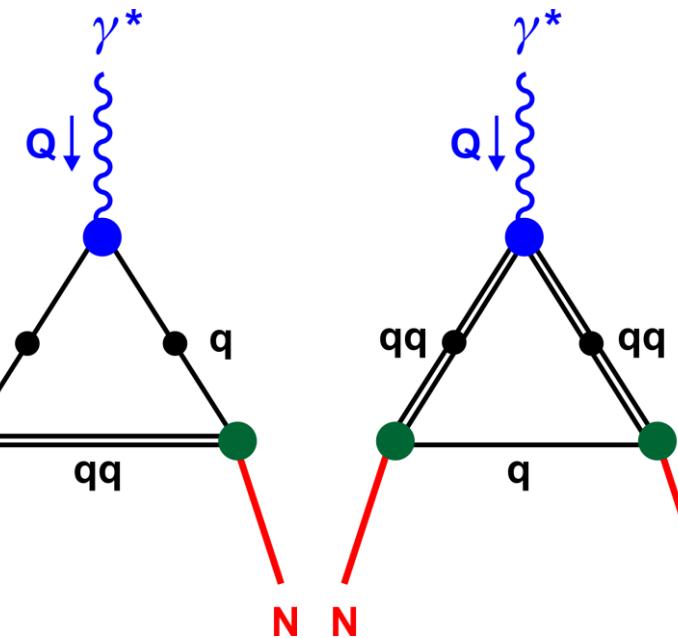
N

$\rho-\phi$

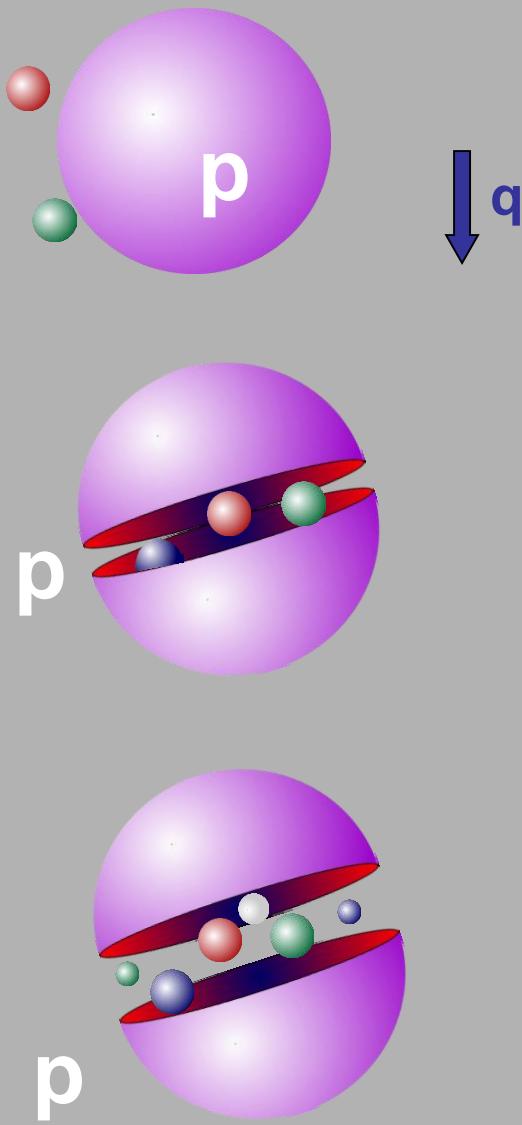
Nucleon electromagnetic formfactors



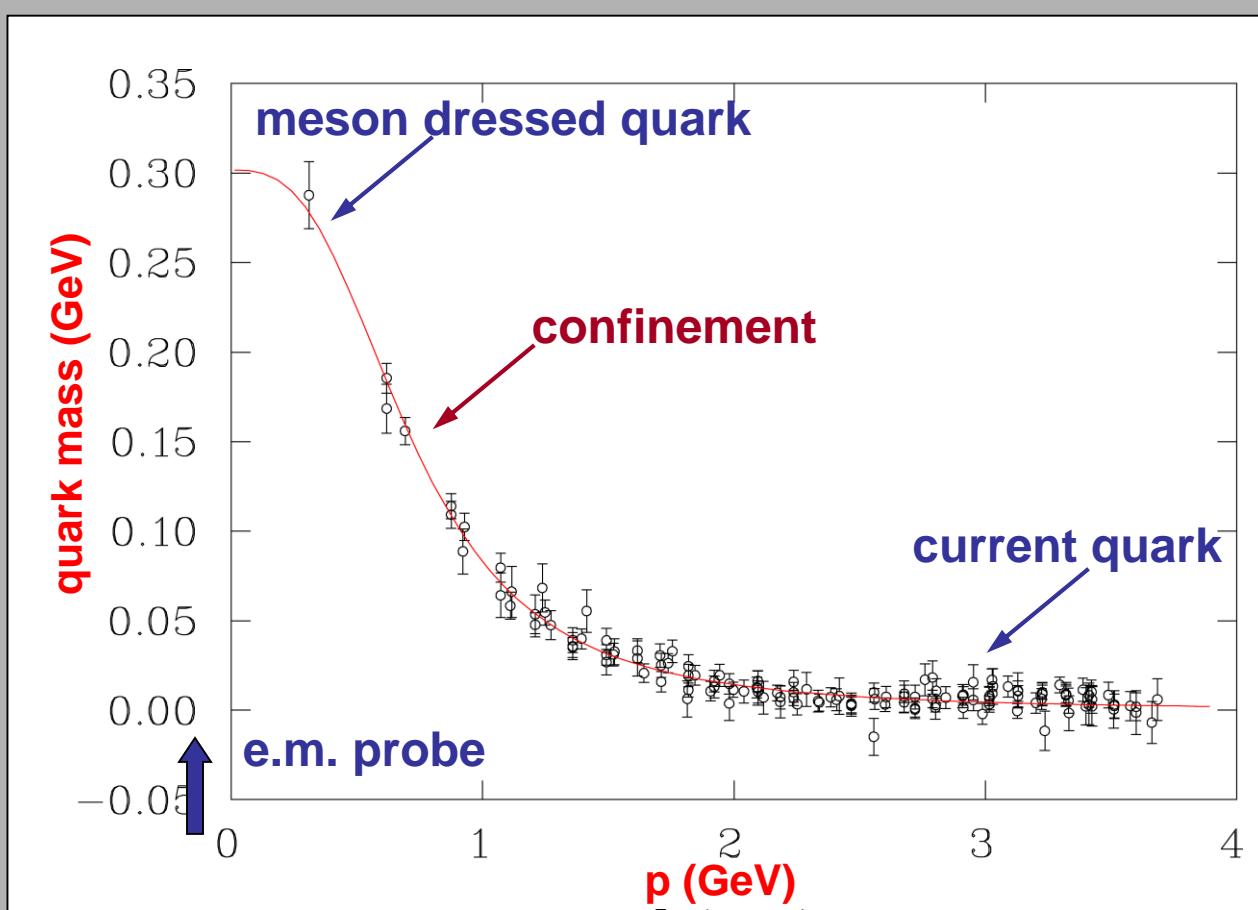
Nucleon electromagnetic formfactors



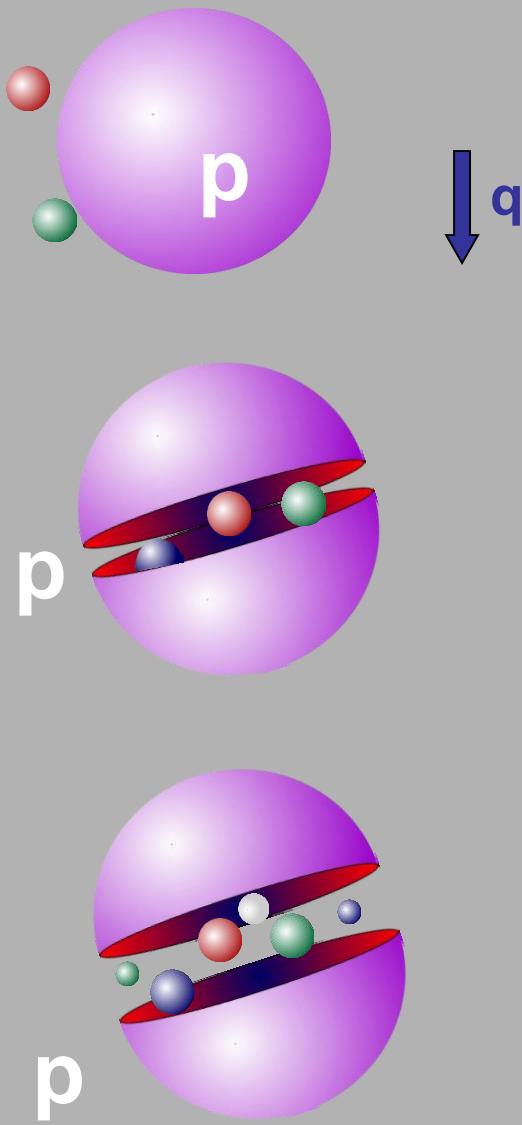
Electromagnetic probe of hadron structure



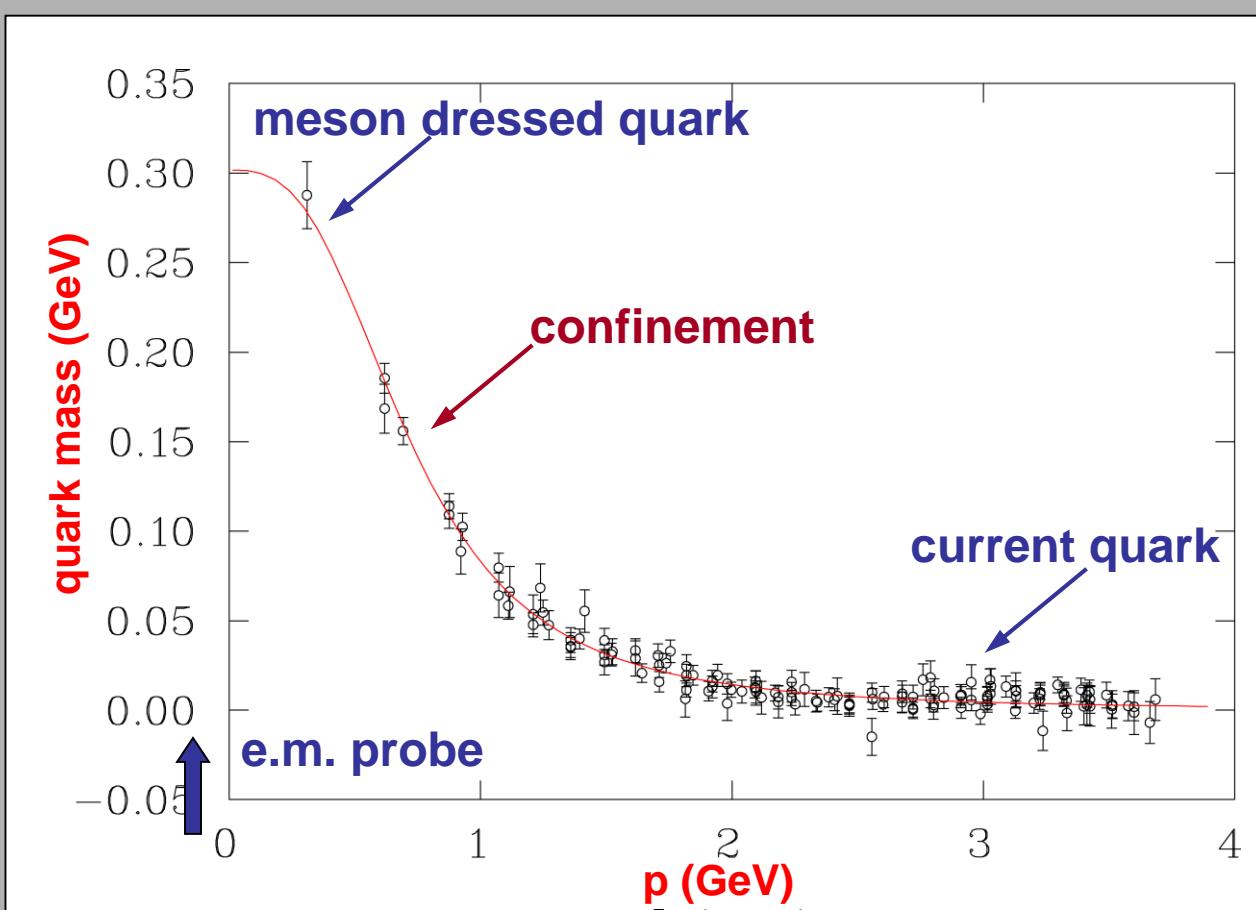
Quark mass function in the Landau gauge



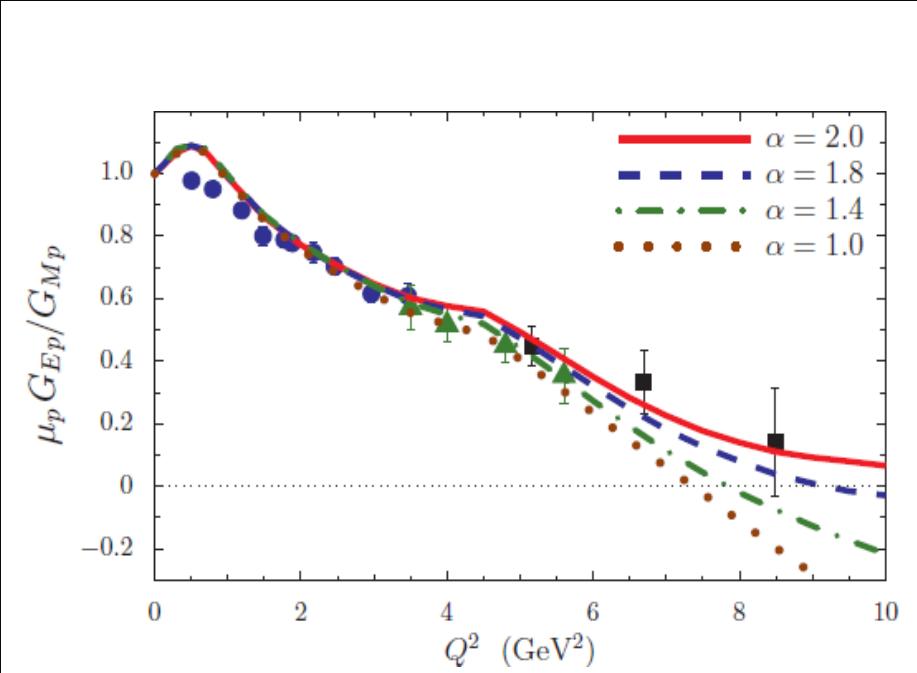
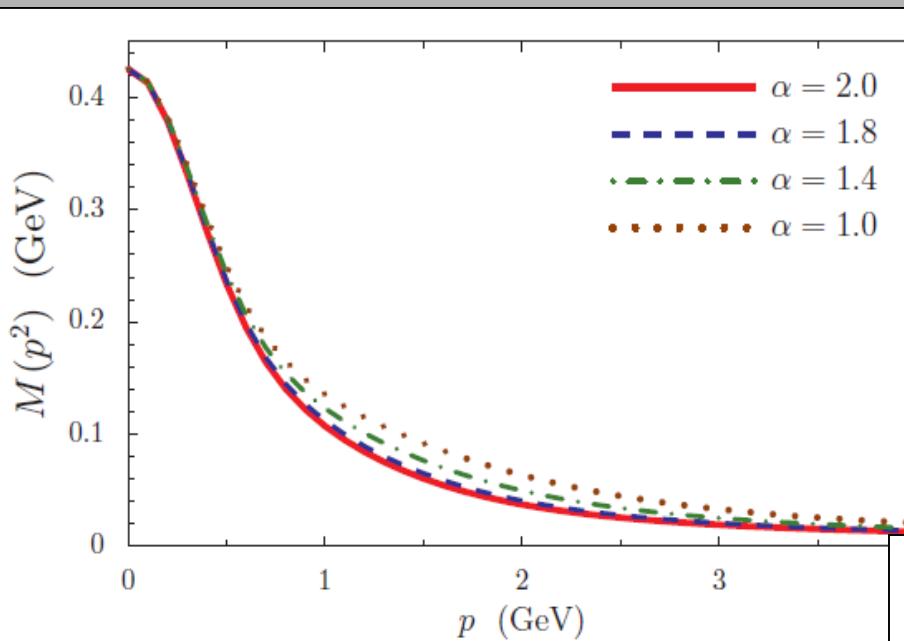
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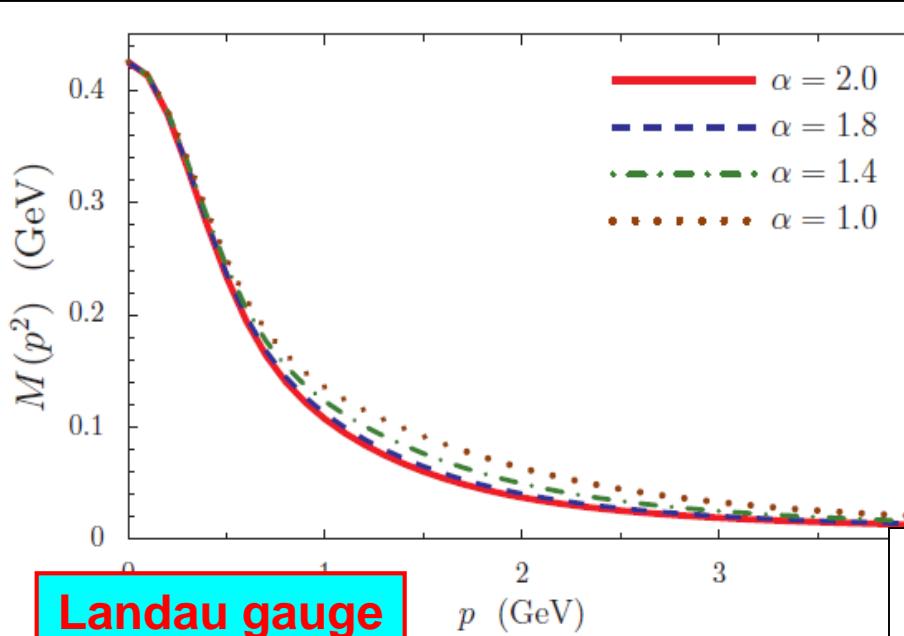
Electromagnetic probe of hadron structure



Cloet, Roberts & Thomas

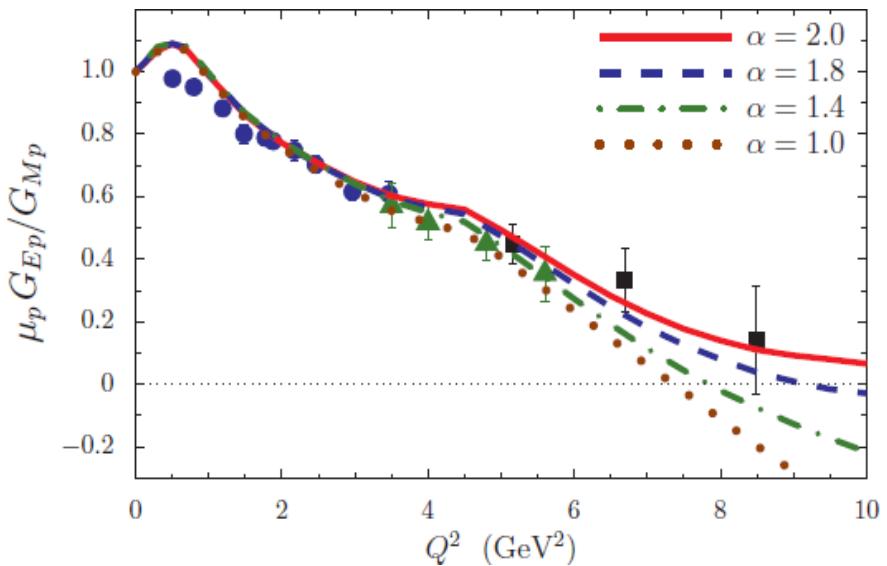
α = anomalous magnetic moment

Electromagnetic probe of hadron structure



Landau gauge

Physical, so gauge independent

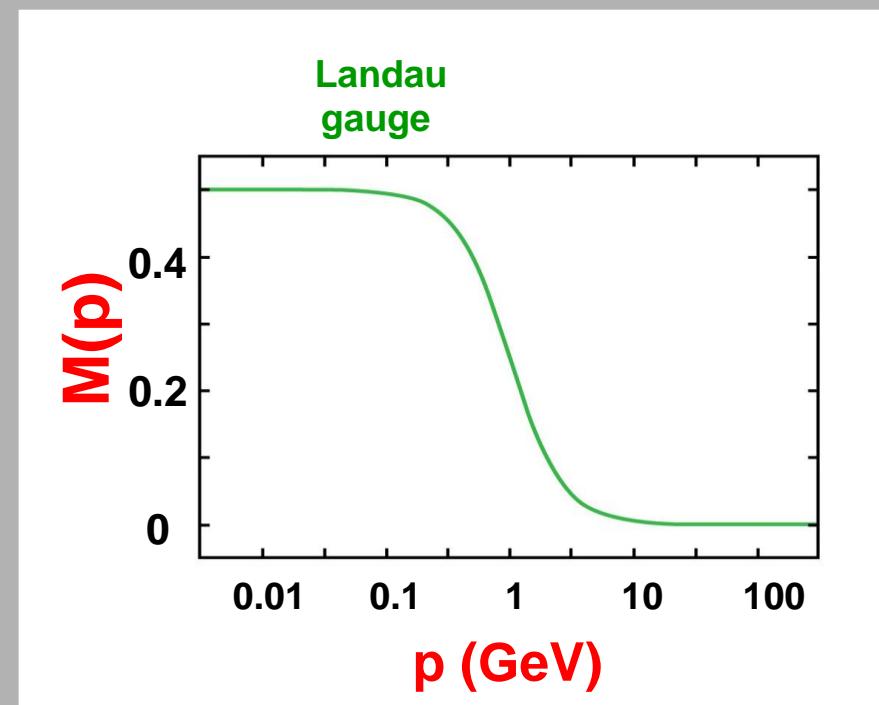
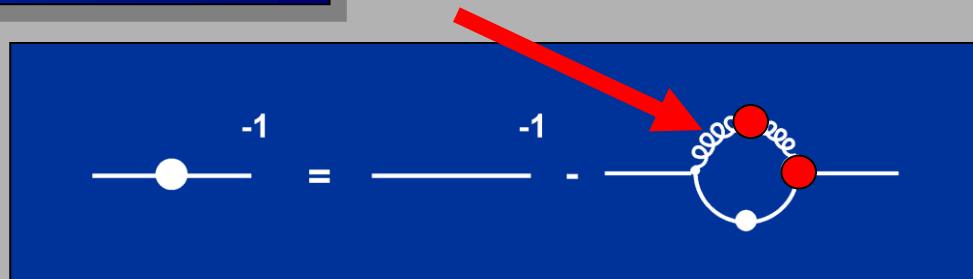


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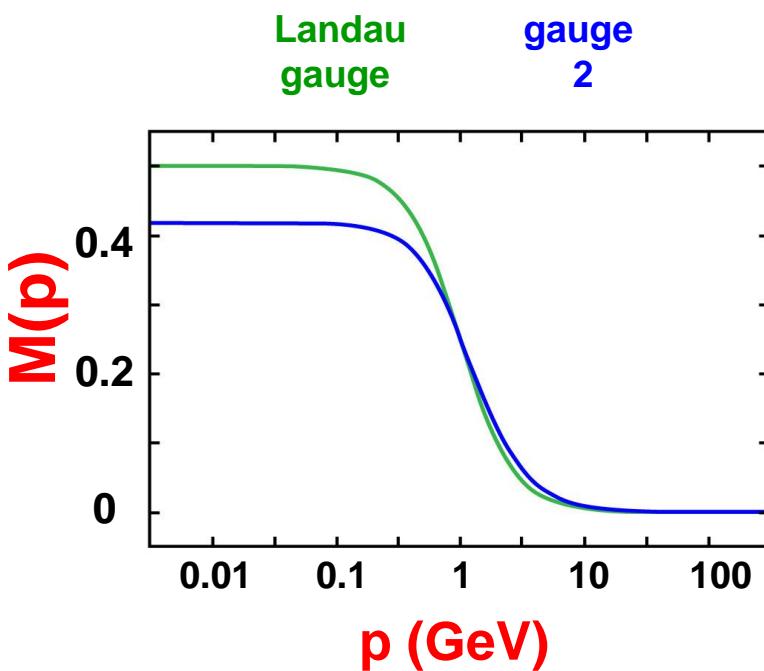
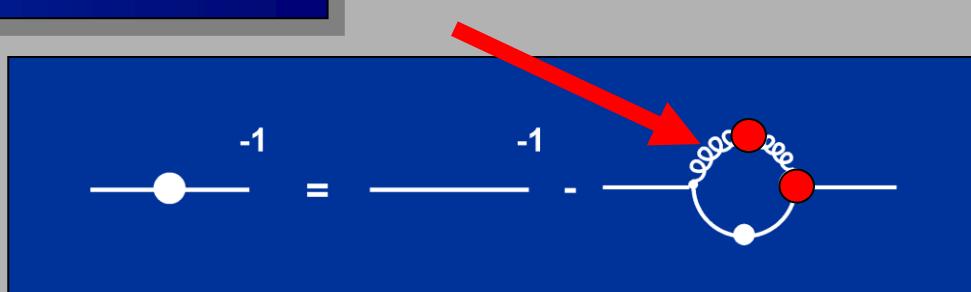
effective interaction strength

Maris & Tandy

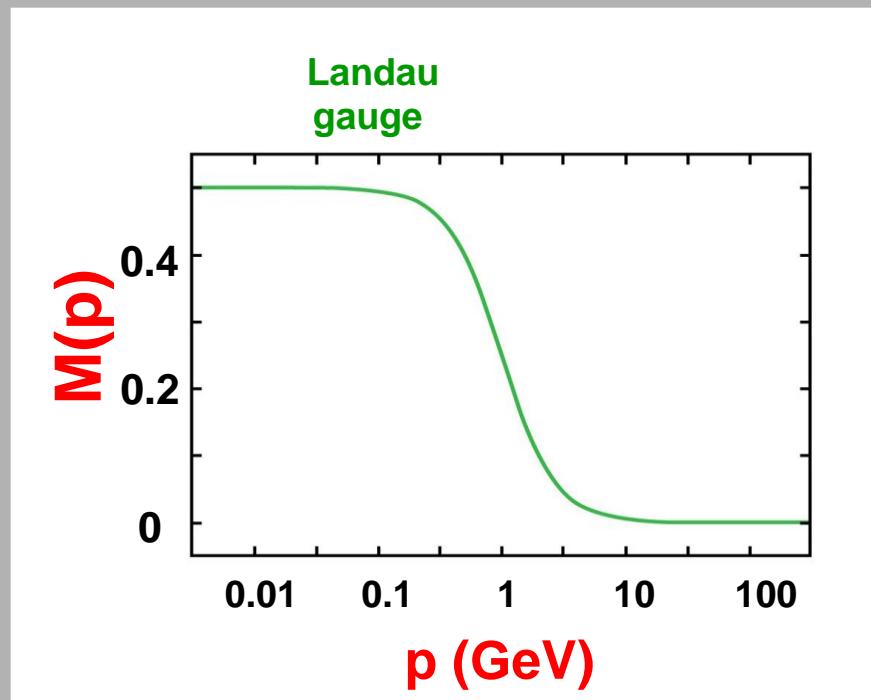


effective interaction strength

Maris & Tandy

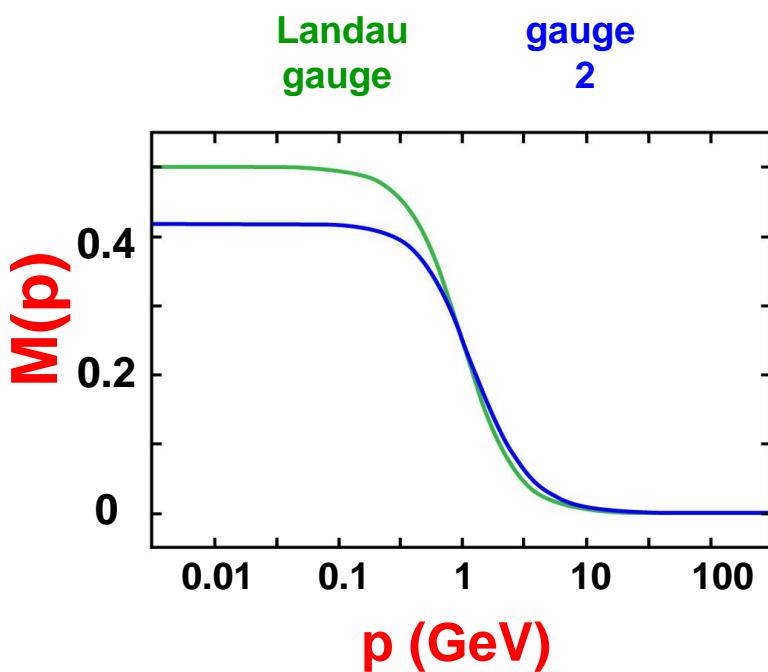
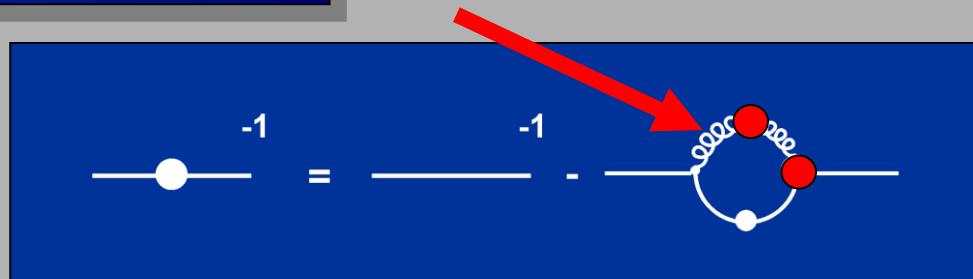


gauge covariance: Shaoyang Jia

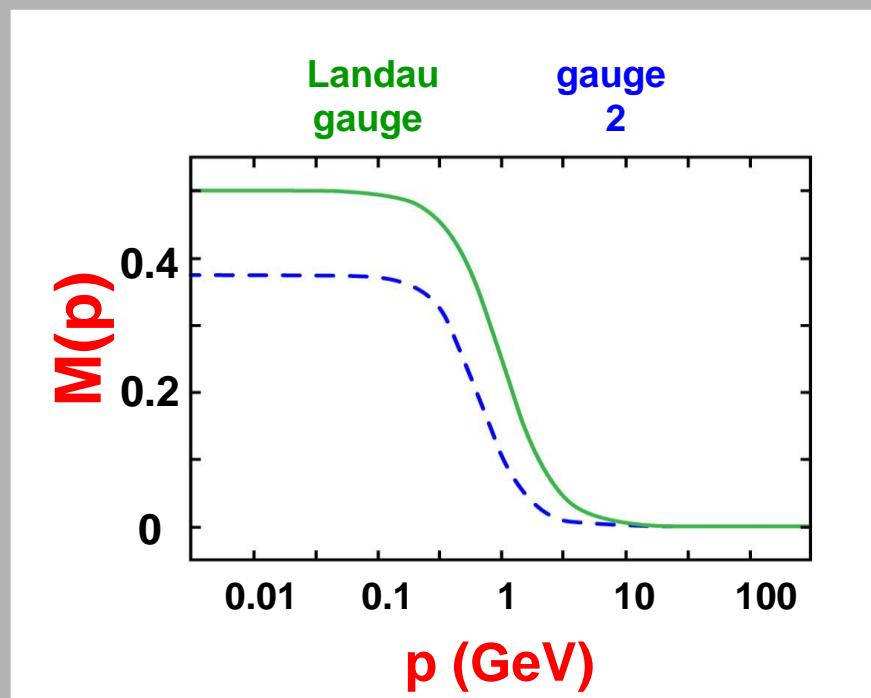


effective interaction strength

Maris & Tandy



gauge covariance: Shaoyang Jia



Building bridges



SDE/BSE in the continuum

Building bridges



SDE/BSE in the continuum

connects the lattice to the continuum

Building bridges



SDE/BSE in the continuum
connects the unphysical to physics

Building bridges



SDE/BSE in the continuum

connects theory to experiment

cool QCD

