WE-Heraeus Physics School

QCD – Old Challenges and New Opportunities

Bad Honnef, Sept 24–30, 2017

BFKL dynamics in QCD and the Soft Pomeron

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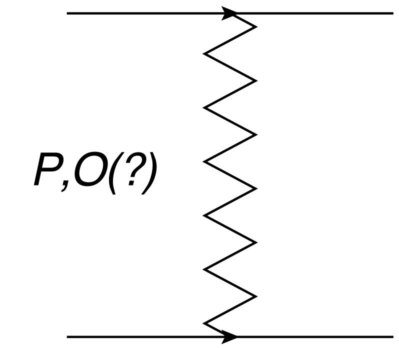
1. Introduction
2. The BFKL Pomeron
   i) Basics
   ii) Experimental tests
   iii) The discrete Pomeron
3. An attempt to interpolate between hard and soft Pomeron

Introduction

'Old' problem (ISR, SPS, Tevatron, HERA, RHIC, LHC):
pp-scattering, energy dependence of scattering amplitude, total cross section

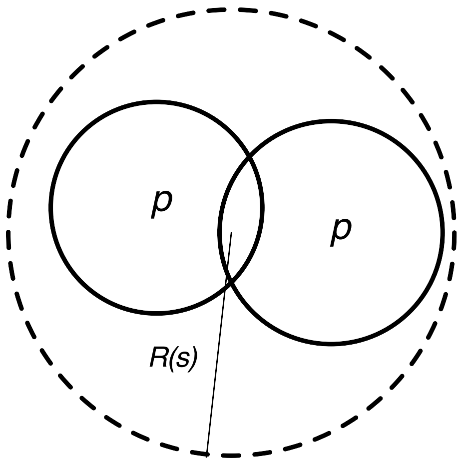


- Total cross section grows with energy (Pomeron exchange)

- t-dependence of elastic cross section shows difference between pp and ppbar (evidence for existence of Odderon)

- transverse extension of hadronic size -> nonperturbative physics
  in impact parameter exponential fall-off
  Pomeron slope = nonperturbative length scale
  (recent LHC data: evidence for slightly stronger growth)

Geometry at high energies:
separation between longitudinal and transverse degrees of freedom



$$R^2(s) = 2(B_0 + 2\alpha' \ln s) \qquad \alpha'_P \approx 0.25\ GeV^{-2}$$

Note: we are not yet in the asymptotic region!

How to find a solution in QCD to this nonperturbative problem?

Successful models:

-> talk of L.Jenkovsky

->talk of O.Nachtmann

geometric models: $\sigma_{tot} \sim (\ln s)^2,\ R(s) \sim\ lns$

Regge pole + cuts (DL): $\sigma_{tot} \sim s^{\alpha(0)-1},\ \alpha(0) - 1 \approx 0.1$ "Soft Pomeron"

Kaidalov: bare Pomeron with intercept above one + Reggeon field theory

Durham, Tel Aviv models: BFKL + Reggeon field theory

Models based upon AdS/CFT

-> talk of Chung-I Tan

Perturbative region (small projectiles): BFKL, not applicable to pp-scattering

On the other hand: perturbative QCD - the BFKL

Applicability:
small (transverse direction) projectiles, since strong coupling depends upon transverse momenta $\alpha_s(k_T^2)$:

- virtual photon
- heavy onium states
- jets with large transverse momenta

Gross features are different from soft Pomeron:

- high intercept 0.25...0.3
- small t-slope
- infinite radius

In the following:

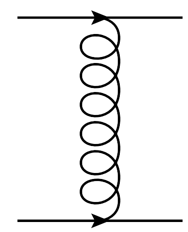
- explain BFKL,
- mention where it can be tested
- most recent development: discrete BFKL
- Novel attempt to connect with soft Pomeron

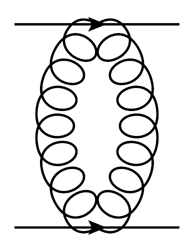
The BFKL Pomeron
I. Basics

Where does the BFKL come from: Balitsky, Fadin, Kuraev, Lipatov,
perturbation theory (in momentum space) , leading log approximation

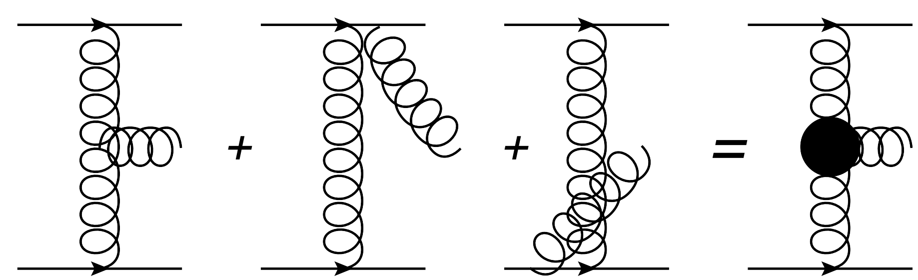
(Alternative: perturbation theory in configuration space, color dipole picture)

Mueller,Balitsky





reggeization



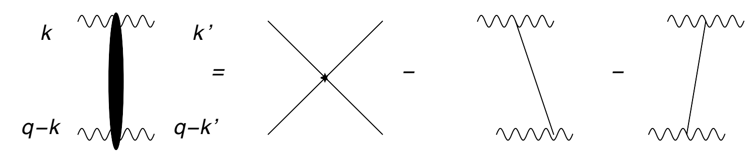
Lipatov production vertex

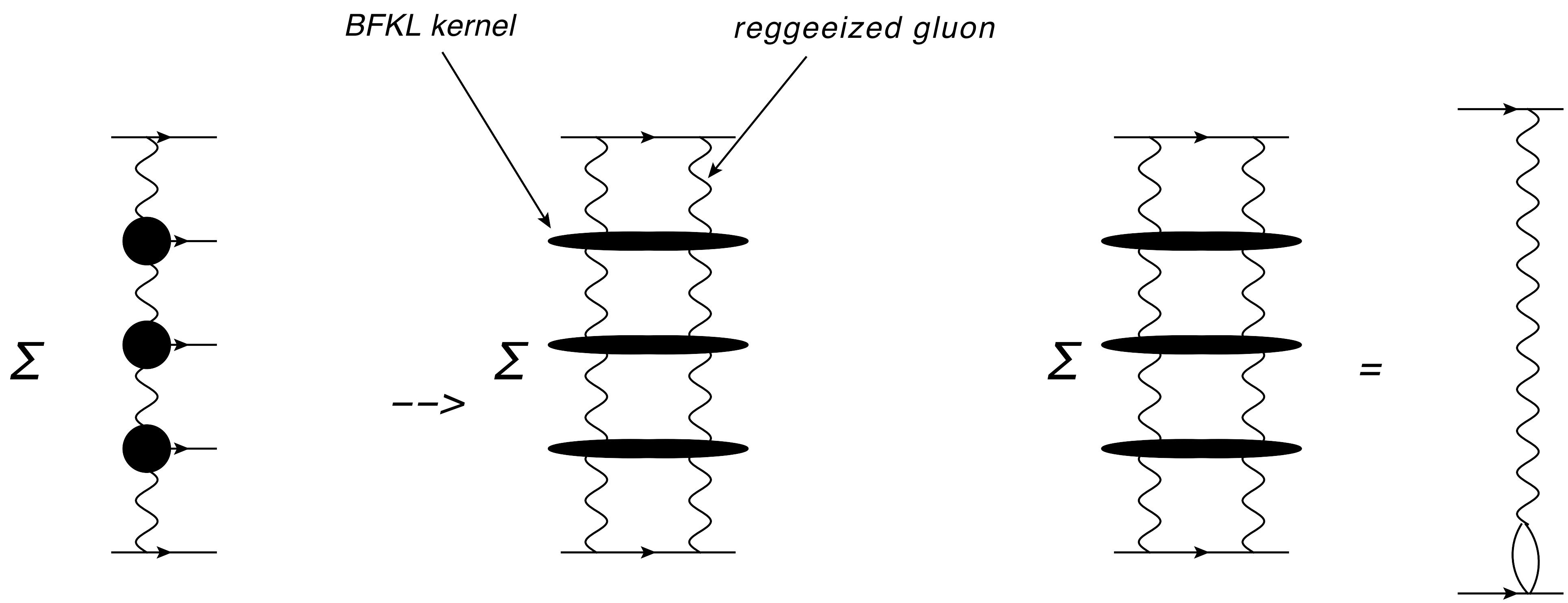
$$s\frac{1}{k^2} \to s\frac{s^{\omega(k^2)}}{k^2}, \ \omega(k^2) = -\frac{g^2}{(2\pi)^3}q^2 \int d^2k \frac{1}{k^2(q-k)^2}$$

$$\Gamma(k_1, k_2)$$

Use unitarity, take square of production amplitudes:

$$K(k,k',q) = g^2\left(q^2 - \frac{k^2(q-k'^2)+k'^2(q-k)^2}{(k-k')^2}\right)$$





color singlet: BFKL

color octet: bootstrap

$$A(s,t) = is\int \frac{d\omega}{2\pi i}(-s)^\omega \Phi_\omega(q^2)$$

important self consistency condition

$$\Phi_\omega(q^2) = \int \frac{d^2k d^2k'}{(2\pi)^6}\Phi_1(k,q)\Phi_\omega(k,k',q)\Phi_2(k',q)$$

$$\omega\Phi_\omega(k,k',q) = \frac{\delta^{(2)}(k-k')}{k^2{k'}^2} + K_{BFKL}\otimes\Phi_\omega(k,k') - (\beta(k^2)+\beta((q-k)^2)\Phi_\omega(k,k',q)$$

Some important features of BFKL ladders:

1) Infrared finiteness: cancellation between real and virtual contributions
   In NLO: running coupling

2) Scale (Moebius) invariance:

- in QCD only leading order, in NLO broken (running coupling)
- in N=4 SYM also beyond

3) Spectrum of eigenvalues :

$$e^{\nu,n}(k) = 2\pi\sqrt{2}(k^2)^{-i\nu-\frac{3}{2}} e^{-in\varphi}$$ (forward direction)

$$\chi(\nu,n) = \alpha_s \chi_1(\nu,n) + \alpha_s^2 \chi_2(\nu,n) + ...$$

$$\chi_1(\nu,n) = \frac{N_c}{\pi}\left(2\psi(1) - \psi(\frac{1+|n|}{2} + i\nu) - \psi(\frac{1+|n|}{2} - i\nu)\right)$$

Continuous spectrum, cut in the angular momentum plane with tip at

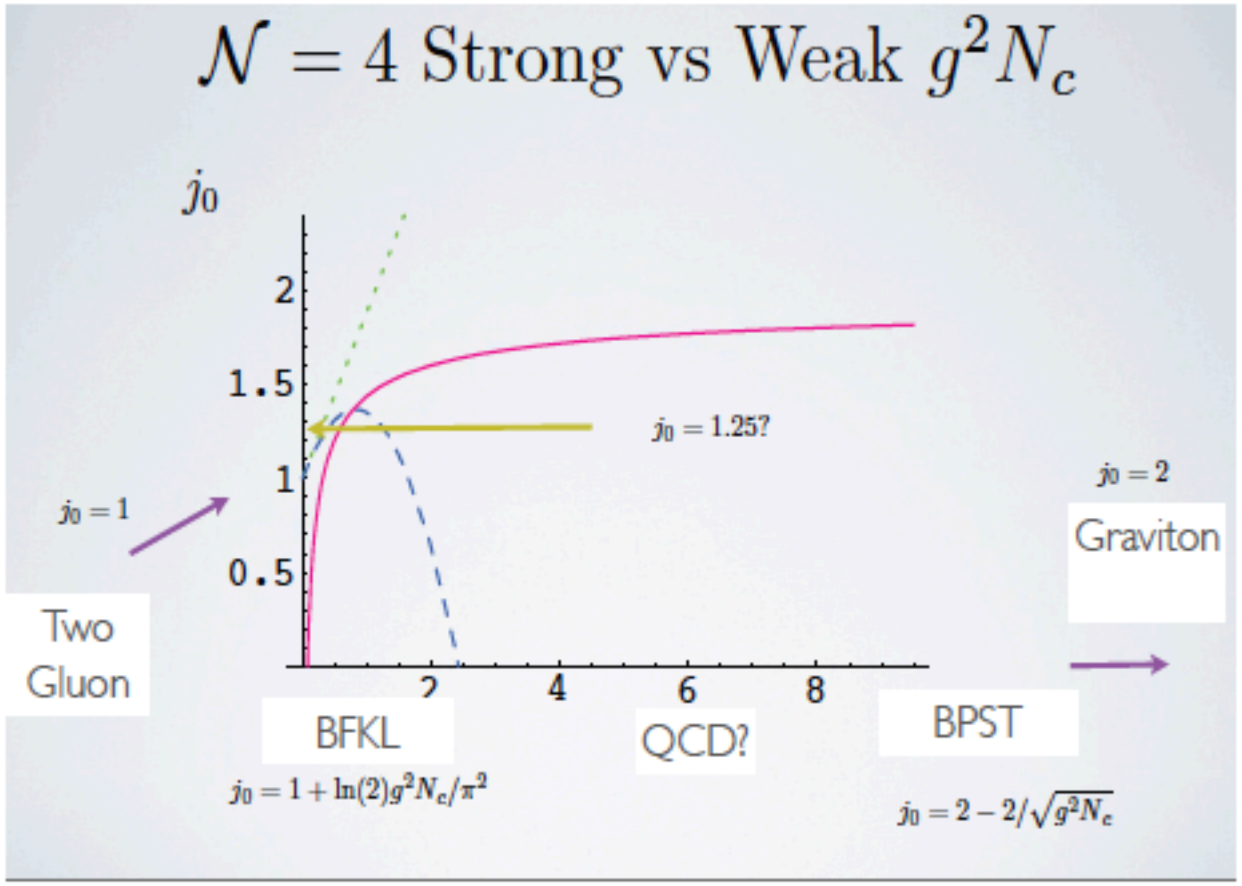
$$\chi_1(0,0) = \frac{N_c \alpha_s}{\pi} 4 \ln 2 \approx 0.25...0.3$$

4) BFKL contains DGLAP (in double log)

4) N=4 SYM: connection with graviton

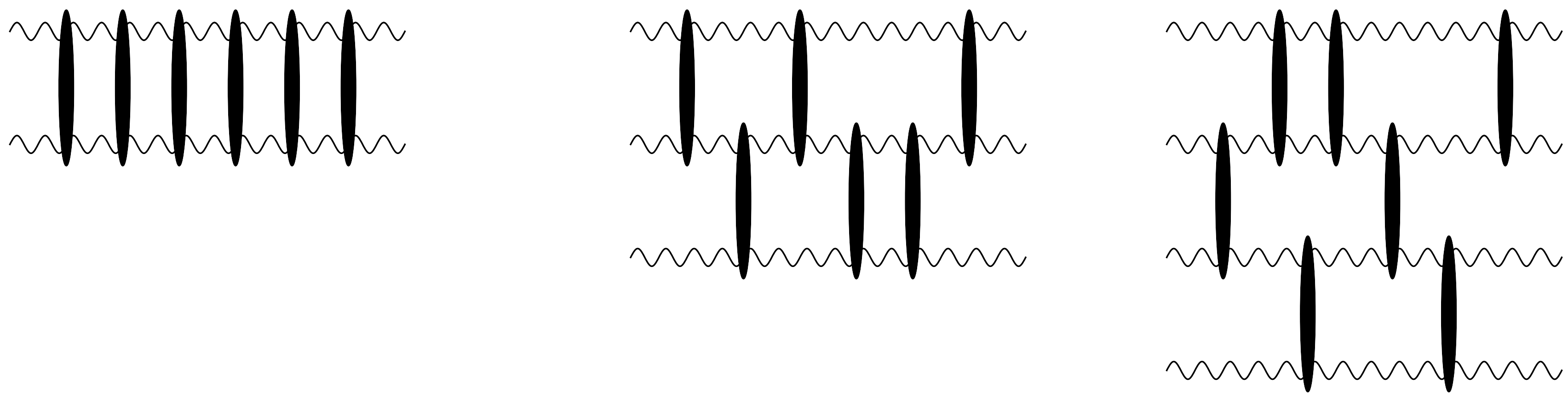
Conjecture of AdS/CFT correspondence:
N=4 SYM is dual to string theory in Anti-de Sitter space; contains graviton
BFKL appears in remainder function (BDS conjecture)

Weak coupling: BFKL ⟷ Strong coupling graviton



Brower et al

5) Integrability: BFKL (and its generalization, BKP) is first example of integrable system in quantum field theory



Odderon

BFKL

BKP

B,Kwiecinski, Praszalowicz

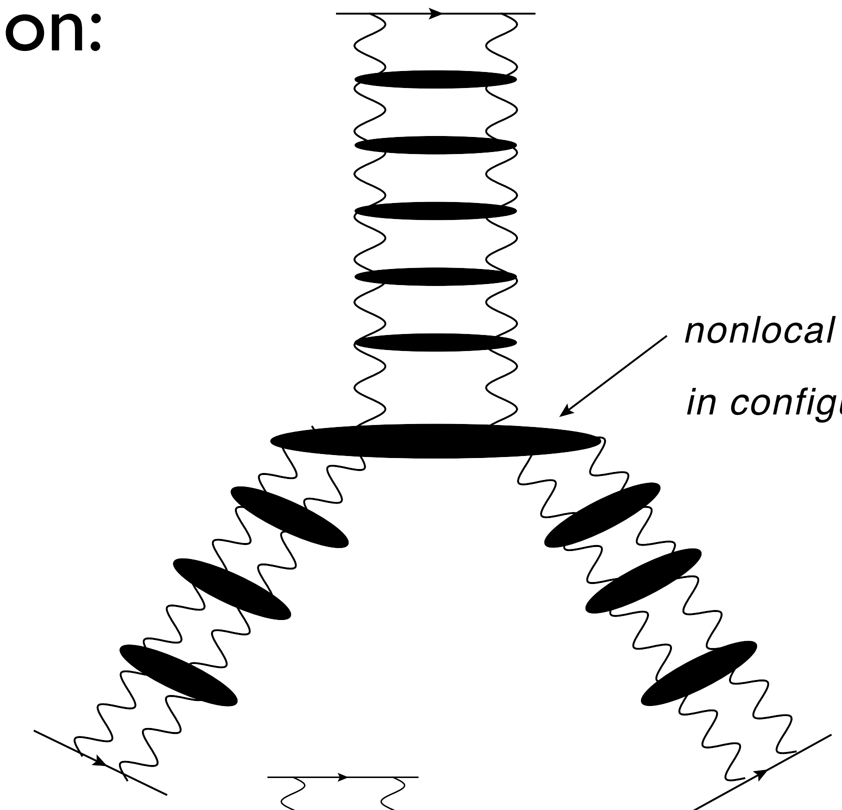
$$H_{BFKL} = H_{12} + \bar{H}_{12}$$

$$H_N = \sum_{k=1}^{N} (H_{kk+1}(z_k, z_{k+1}) + \bar{H}_{kk+1}(\bar{z}_k, \bar{z}_{k+1}))$$

Integrable spin chain

6) Back to QCD: BFKL is the beginning of a QCD reggeon field theory

Triple Regge region:
pp -> X+p



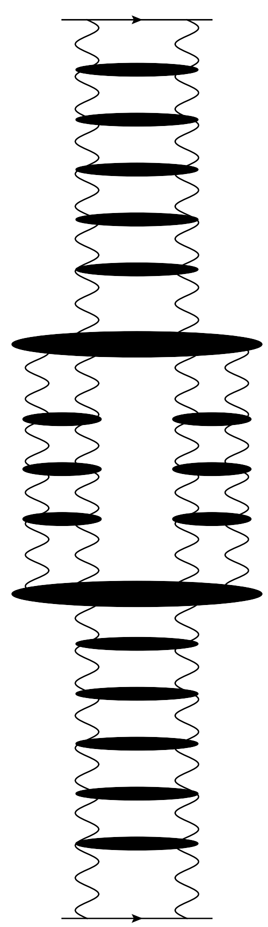
nonlocal triple Pomeron vertex

in configuration space: BK kernel

JB, Wüsthoff, Ewerz,

Balitsky, Kovchegov

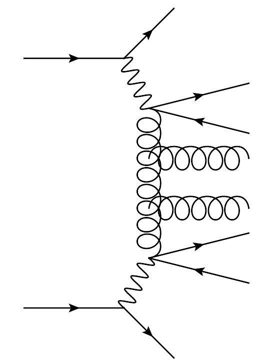
Field theory, e.g.
corrections to BFKL

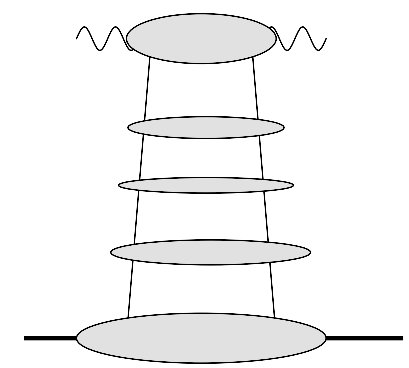


2. BFKL tests

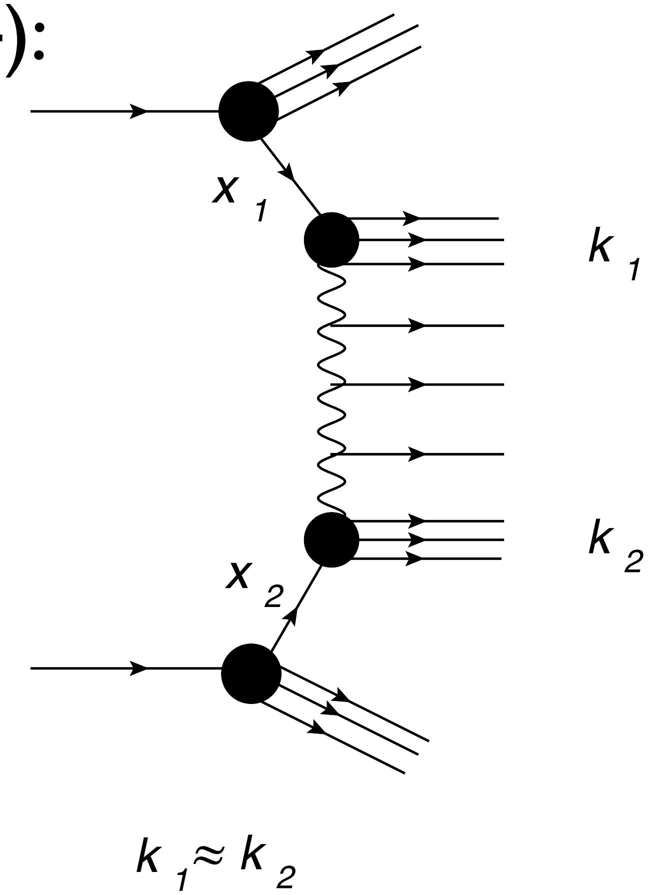
1) LEP: $\gamma^*\gamma^*$ scattering
2) HERA: structure function at small x
3) HERA: forward jets
4) LHC: Mueller-Navelet jets
5) BFKL Monte Carlo (Sabio-Vera, Chachamis)
6) HERA fit: the discrete Pomeron

...

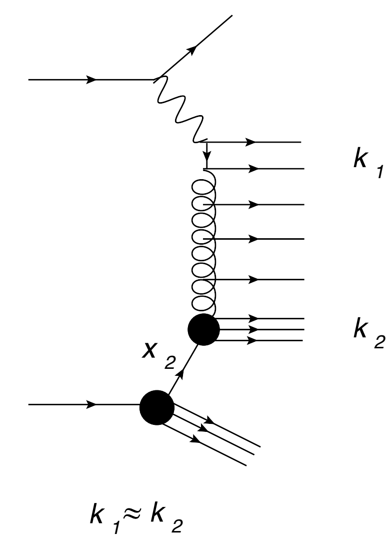




ad 4):



see Celiberto's talk



ad 5): BFKL at large but finite energies!

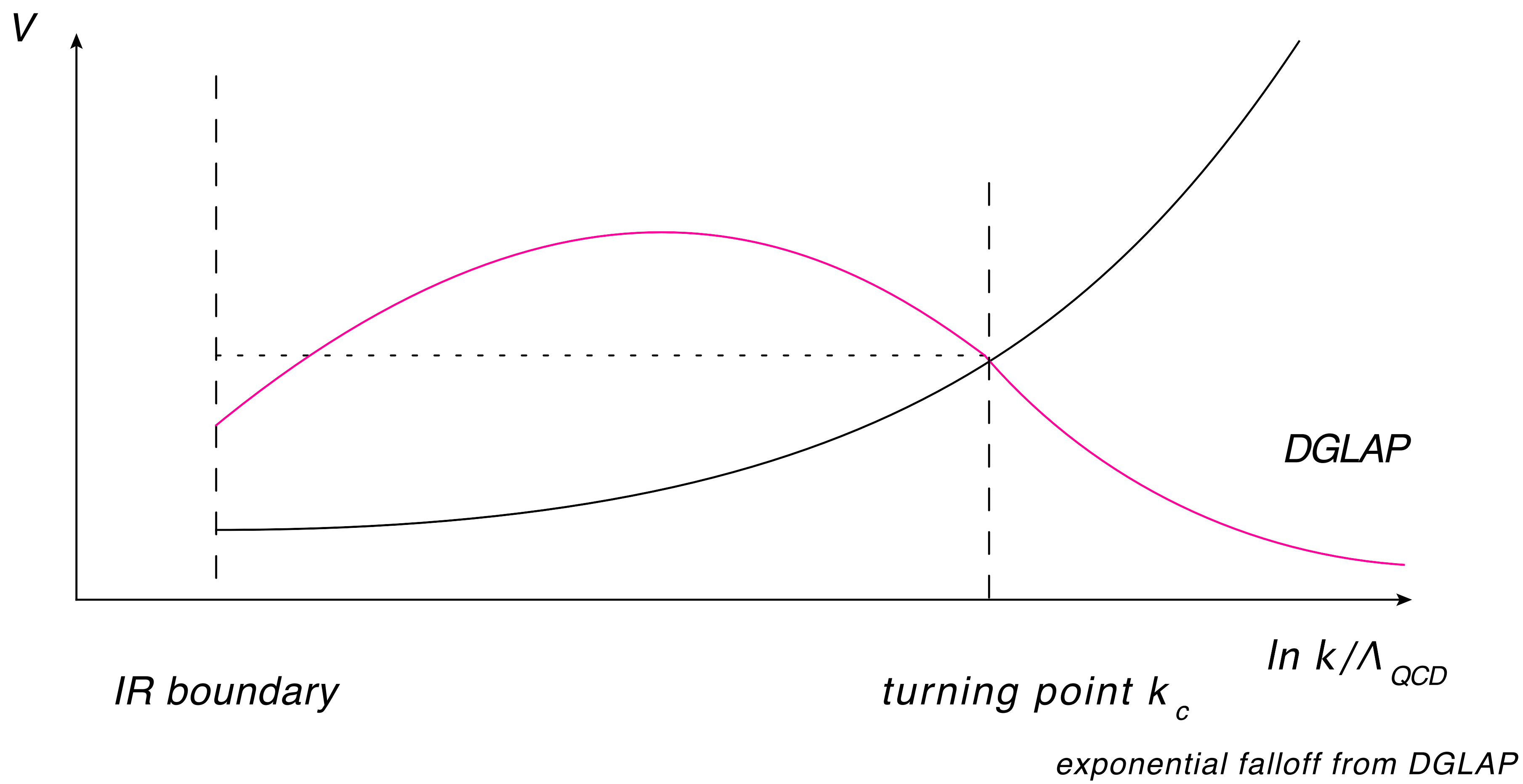
3.The discrete Pomeron

Lipatov 1986

In NLO: QCD coupling becomes running

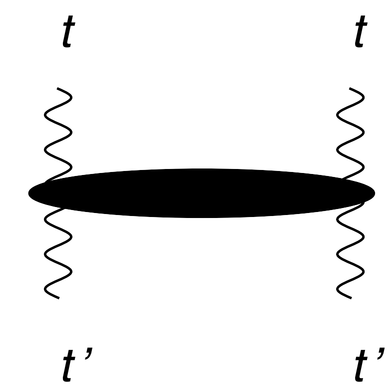
- scale invariance is lost Λ_{QCD}
- BFKL kernel is modified

First a simplified qualitative argument - one-dimensional Schrödinger equation:



In more detail:

BFKL equation (for n=0 only, forward direction):



$$\omega f_\omega(t) = \int dt' \sqrt{\bar{\alpha}(t)} K(t,t') \sqrt{\bar{\alpha}(t')} f_\omega(t')$$

$$t = \frac{k^2}{\Lambda^2_{QCD}}, \ \bar{\alpha}(t) = \frac{1}{\beta_0 t}$$

Ansatz:

$$f_\omega(t) = \sqrt{\frac{t}{2\pi\omega}} \int d\nu g_\omega(\nu) e^{it\nu}$$

Obtain:

$$i\omega\beta_0 \frac{\partial g_\omega(\nu)}{\partial \nu} = \chi_1(\nu) g_\omega(\nu), \ g_\omega(\nu) = \exp\left[\frac{1}{i\omega\beta_0}\int_0^\nu d\nu' \chi_1(\nu')\right]$$

For small t oscillatory behavior, unknown phase

Eigenvalue condition, turning point:

$$\omega = \bar{\alpha}(t)\chi_1(\nu(t)) \quad \omega = \bar{\alpha}(t_c)\chi_1(0):$$

$t < t_c : \nu(t) = real$, oscillatory behaviour

$t > t_c : \nu(t) = imaginary$, exponential falloff

Near turning point: Airy functions (similar to 1-dim S-equation)

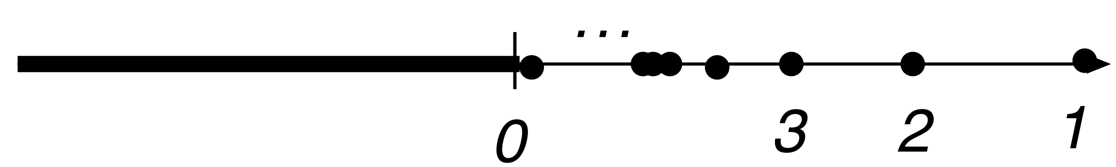
Numerical evaluation: fit to HERA data

Kowalski, Lipatov, Ross

$x < 0.001, Q^2 > 6\ GeV^2$

ω–plane

accumulation at zero



$$\omega_n = \frac{A}{n+B}, \text{with } A = 0.52,\ \ B = 1.62$$

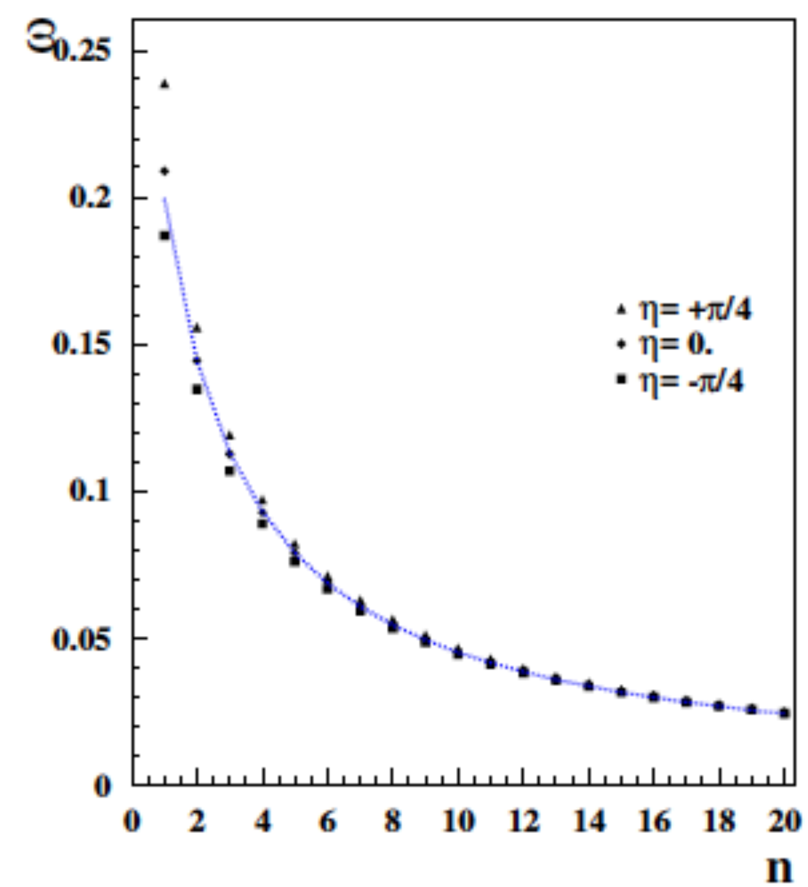
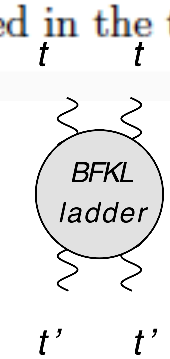


Figure 2: Eigenvalues ω_n determined in NLO for three fixed non-perturbative phases The dotted line shows a simple parametrisation described in the text.

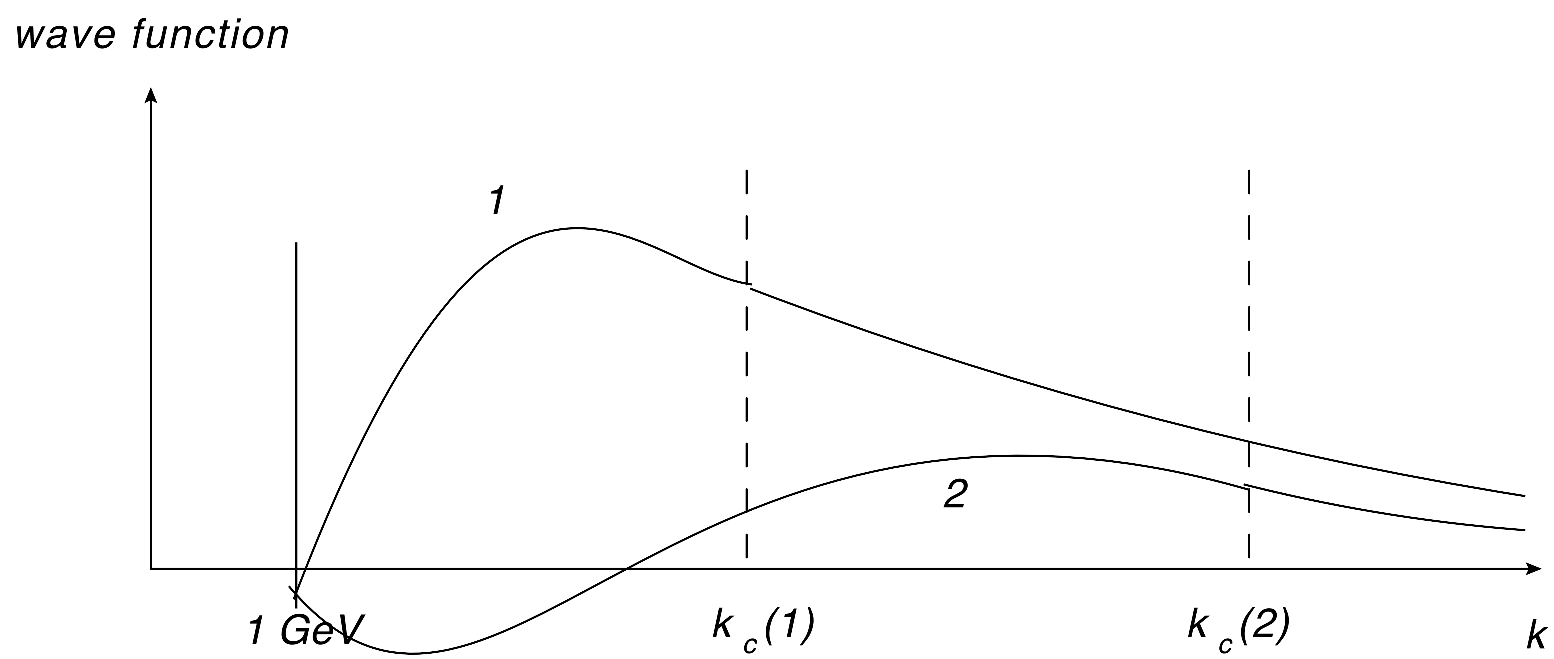
Fit needs 10 poles

$$G(t,t';\omega) = \sum_{n=1} \frac{f_{\omega_n}(t) f^*_{\omega_n}(t')}{\omega - \omega_n} + \int_{-\infty}^{0} d\omega' \frac{f_{-|\omega'|}(t) f^*_{-|\omega'|}(t')}{\omega - \omega'}$$



Eigenfunctions:

qualitatively:



$$k_c(4) = 2100\text{TeV}$$
$$k_c(3) = 260\text{TeV}$$
$$k_c(2) = 3.3\text{TeV}$$
$$k_c(1) = 50\text{GeV}$$

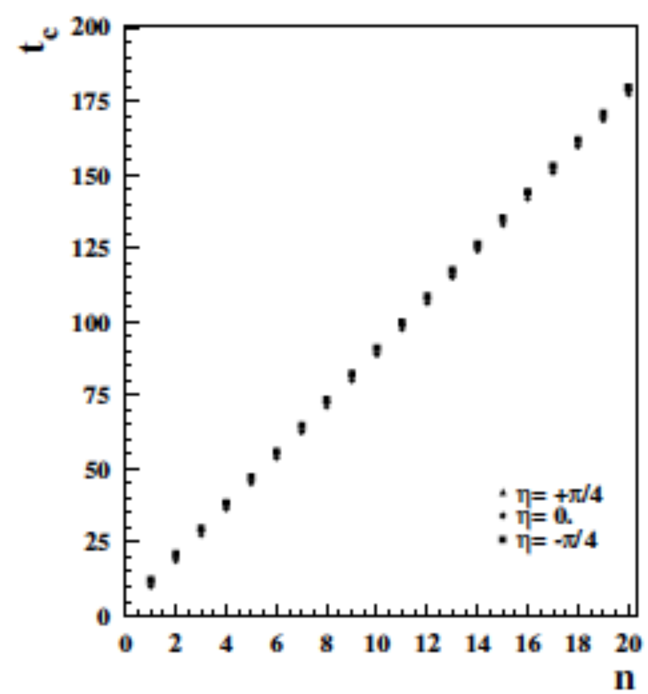


Figure 3: The critical momenta t_c determined in NLO for three fixed non-perturbative phases, η_n. $t_c = \ln k_c^2/\Lambda_{QCD}^2$ with $\Lambda_{QCD} = 275$ MeV.

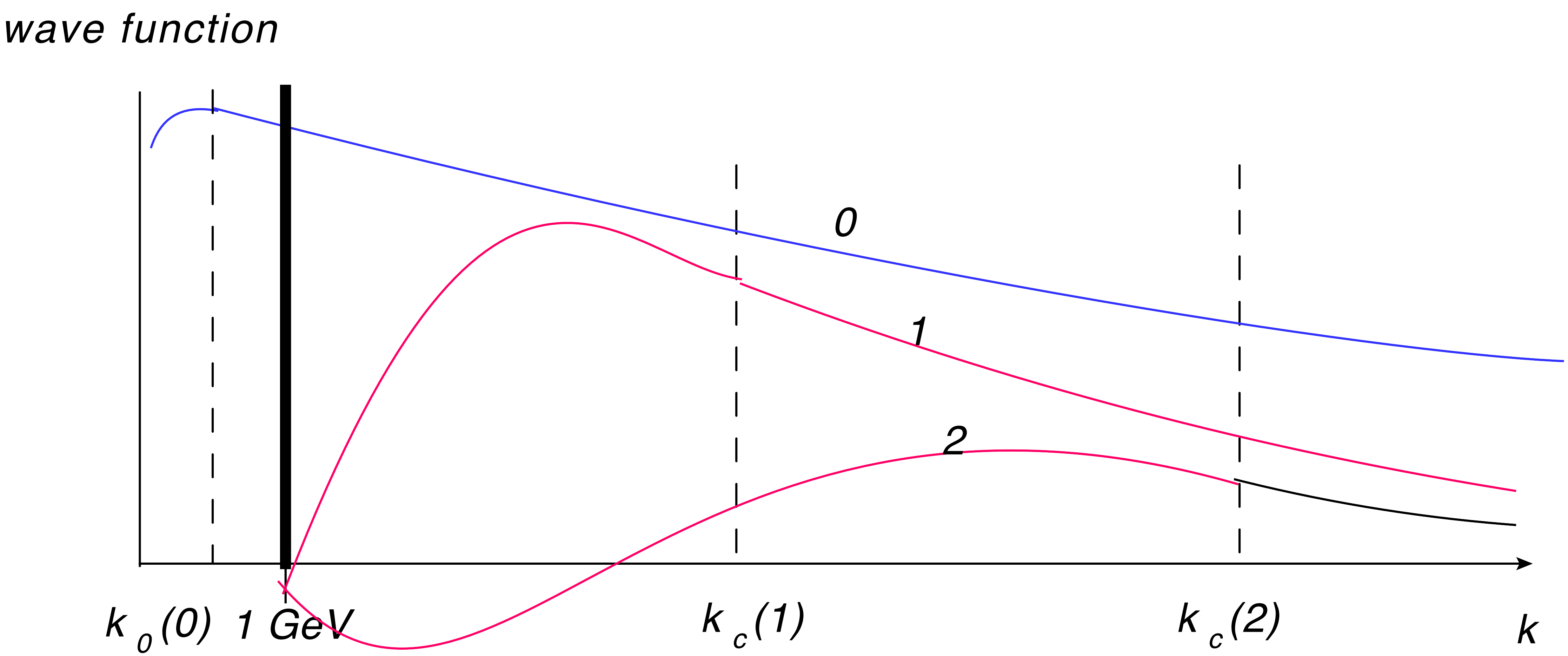
rapid growth of k_c

Quantum mechanics: connection between small and large momenta

Pecularities:

- leading pole decouples
- there should be a ground state, not seen in the fit

Possible picture:



What do we learn from this first numerical application of the discrete Pomeron:

- BFKL needs IR cutoff : spectrum becomes quite different: infinite sum of discrete poles
- details of the discrete spectrum are sensitive to large momentum region
- leading eigenvalue close to nonperturbative region (still needs ‘unitarization’)
- open questions: nonforward direction, dependence on IR cutoff,...

Maybe: “BFKL is not so far from nonperturbative Pomeron”

3. An attempt to interpolate between BFKL and soft Pomeron

How to connect between BFKL and soft Pomeron?

On the BFKL side:

- without running coupling:
  scale invariance, far from nonperturbative QCD
- with a cutoff:
  a sequence of poles with high intercepts, need unitarization
  start from BFKL+screening = QCD reggeon field theory

On the soft pomeron side:

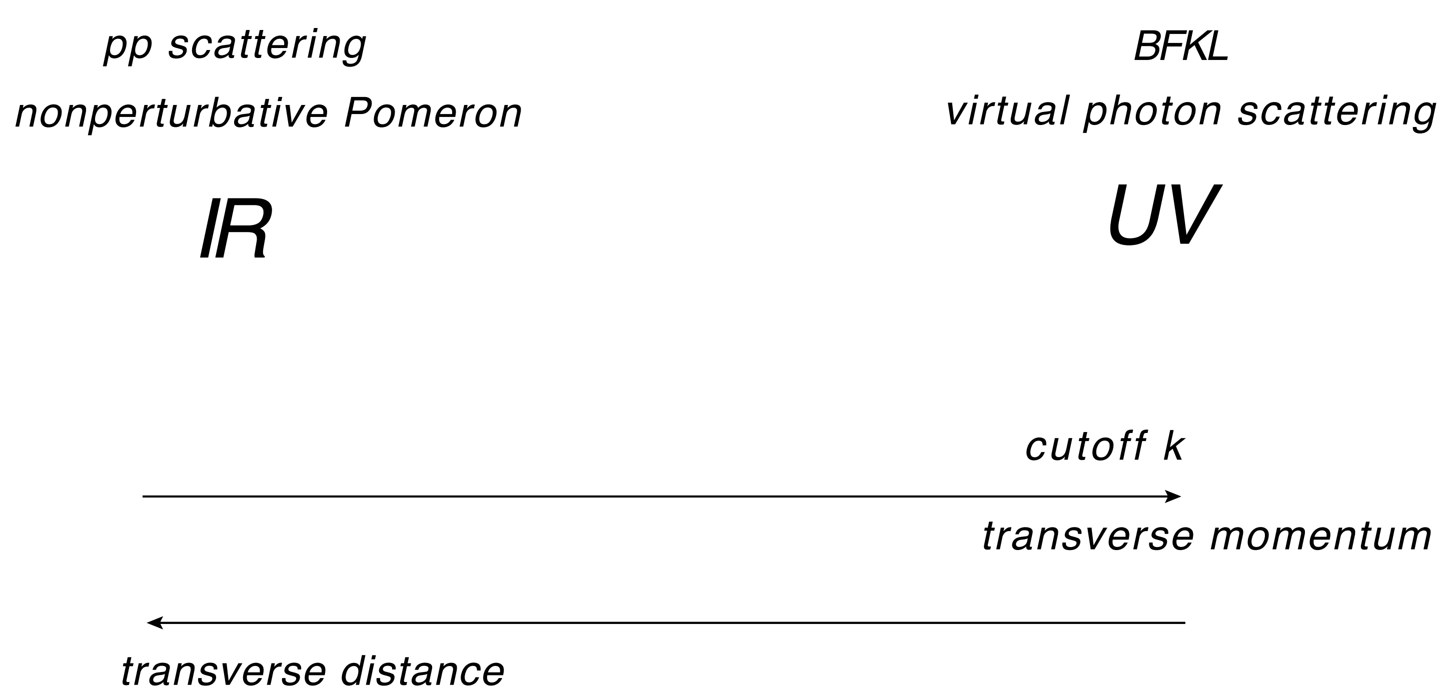
- DL Pomeron with smaller intercept, but still above one
- need Pomeron cuts, beginning of reggeon field theory

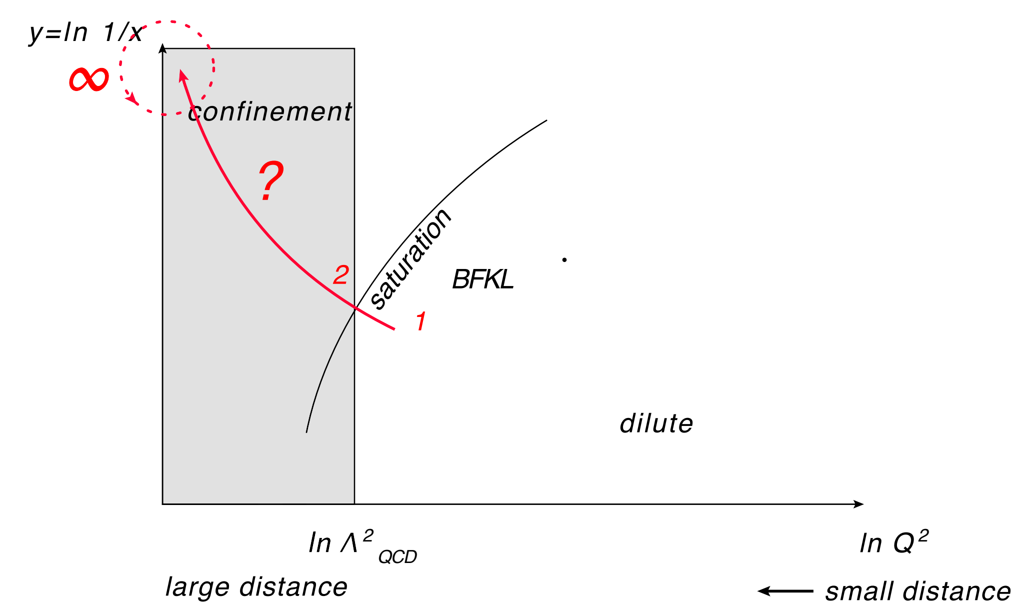
JB, Contreras, Vacca,
JHEP 1603 (2016) 201
PRD 95 (2017) 014013
work in progress

What this suggests:

1) find common framework
which fits for BFKL and for soft Pomeron: 2+1 dim: field theory = RFT

2) interpolate between short distance QCD (BFKL)
and long distances (soft Pomeron): RG equation (flow equations)





Steps:

1) Formulate BFKL (and QCD RFT) in terms of flow equations,
compute energy behavior as function of cutoff k

2) Fixpoint analysis in IR reggeon field theory

3) match steps 1) and 2)

On functional renormalization, flow equations

Reminder: Wilson approach

The standard Wilsonian action is defined by an iterative change in the UV-cutoff induced by a partial integration of quantum fluctuations:

$$\Lambda \to \Lambda' < \Lambda$$

$$\int [\mathrm{d}\varphi]^{\Lambda} e^{-S^{\Lambda}[\varphi]} = \int [\mathrm{d}\varphi]^{\Lambda'} e^{-S^{\Lambda'}[\varphi]}$$

--> flow of couplings constants etc

Alternatively: ERG-approach (Wetterich), sequence of theories, IR cutoff

(successful use in statistical mechanics, low energy QCD, and in gravity)

Wetterich;Gies: Berges;Pawlowski,...

define a bare theory at scale Λ, introduce IR cutoff k.
The integration of the modes in the interval $[k, \Lambda]$ defines a k-dependent average functional.
Letting k flowing down to 0 defines a flow for the functional which leads to full theory. k-dependent effective action:

regulator

$$e^{-\Gamma_k[\phi]} = \int [d\varphi]\mu_k e^{-S[\varphi]+\int_x (\varphi-\phi)_x \frac{\delta\Gamma_k[\phi]}{\delta\phi_x} - \Delta S_k[\varphi-\phi]}$$

$$e^{-\Gamma_k[\phi]} = \int [d\varphi]\mu_k e^{-S[\varphi]+\int_x (\varphi-\phi)_x \frac{\delta\Gamma_k[\phi]}{\delta\phi_x} - \Delta S_k[\varphi-\phi]}$$

Taking a derivative with respect the RG time $t = \ln k/k_0$ one obtains

$$\partial_t \Gamma_k = \frac{1}{2} Tr\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k\right] - \frac{\dot{\mu}_k}{\mu_k}$$

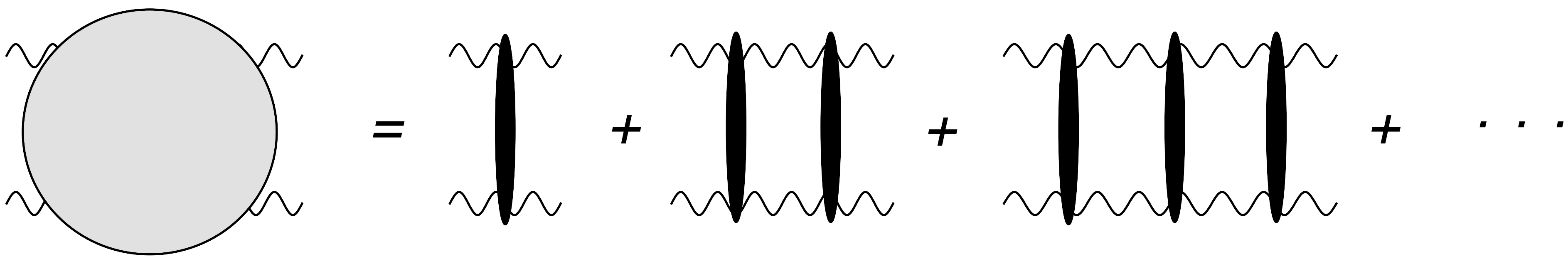
$\mathcal{R} = \text{regulator operator}$

UV and IR finite. All parameters (couplings etc) are k-dependent.
Quantum fluctuations $\longrightarrow$ coupled differential equations

$$\begin{aligned}
\partial_t \Gamma_k &= \frac{1}{2} G_{k;AB}\, \partial_t \mathcal{R}_{k;BA} \\
\partial_t \Gamma^{(1)}_{k;A_1} &= -\frac{1}{2} G_{k;AB} \Gamma^{(3)}_{k;A_1 BC} G_{k;CD}\, \partial_t \mathcal{R}_{k;DA} \\
\partial_t \Gamma^{(2)}_{k;A_1 A_2} &= \frac{1}{2} G_{k;AB} \Gamma^{(3)}_{k;A_1 BC} G_{k;CD} \Gamma^{(3)}_{k;A_2 DE} G_{k;EF}\, \partial_t \mathcal{R}_{k;FA} \\
&\quad + \frac{1}{2} G_{k;AB} \Gamma^{(3)}_{k;A_2 BC} G_{k;AB} \Gamma^{(3)}_{k;A_1 BC} G_{k;CD}\, \partial_t \mathcal{R}_{k;DA} \\
&\quad - \frac{1}{2} G_{k;AB} \Gamma^{(4)}_{k;A_1 A_2 BC} G_{k;CD}\, \partial_t \mathcal{R}_{k;DA}
\end{aligned}$$

A. The UV limit - setup

In the UV region (large momenta, short distances): need to begin with the (LO) BFKL Pomeron = bound state of two reggeized gluons:



non-local kernel, problem with LPA

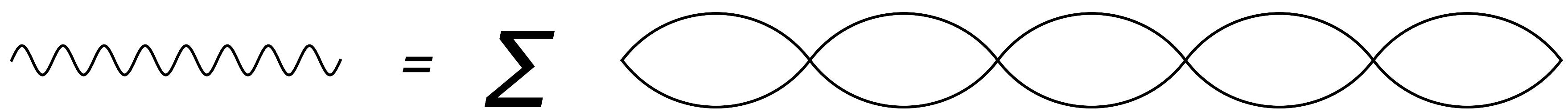
composite state: sum of ladder diagrams

 *= reggeized gluon*

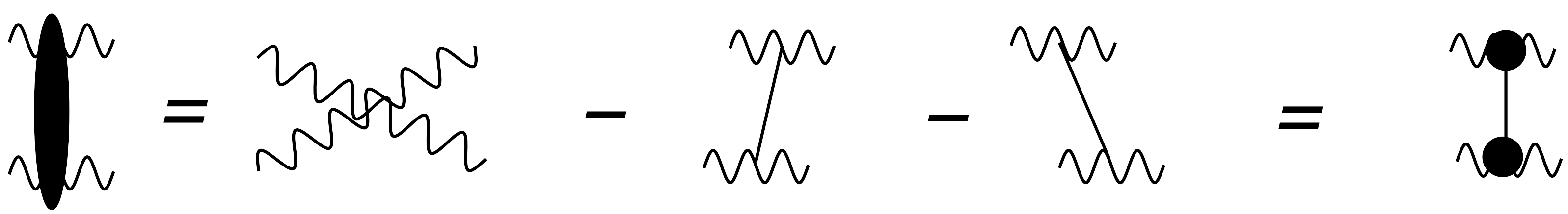
This equation contains:

- nontrival high energy behavior $T(s,t) \sim s^{1+\omega_{BFKL}}, \;\; \omega_{BFKL} = 0.2 \ldots 0.3$
- DGLAP evolution equation
- saturation

reggeized gluon = composite field of two elementary gluons



nonlocal kernel can be built by introduction of a complex scalar field



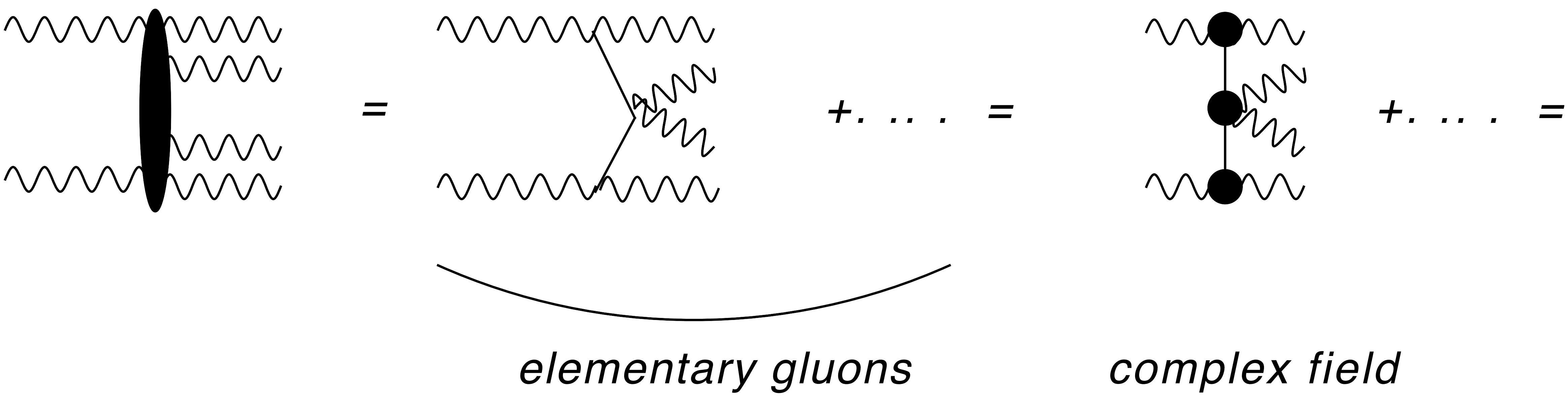
elementary gluon *complex scalar field*

Consistency beteen BFKL kernel and trajectory (bootstrap)

Field content:

- elementary gluon (for gluon) complex field for kernels (does not propagate in rapidity)
- reggeized gluon as composite state of elementary gluons
- Pomeron as bound state of reggeized gluons

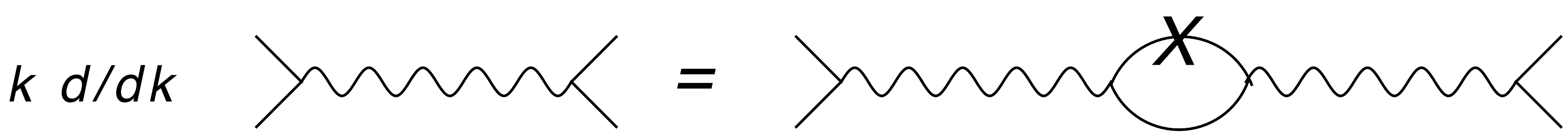
Triple vertex: highly nonlocal



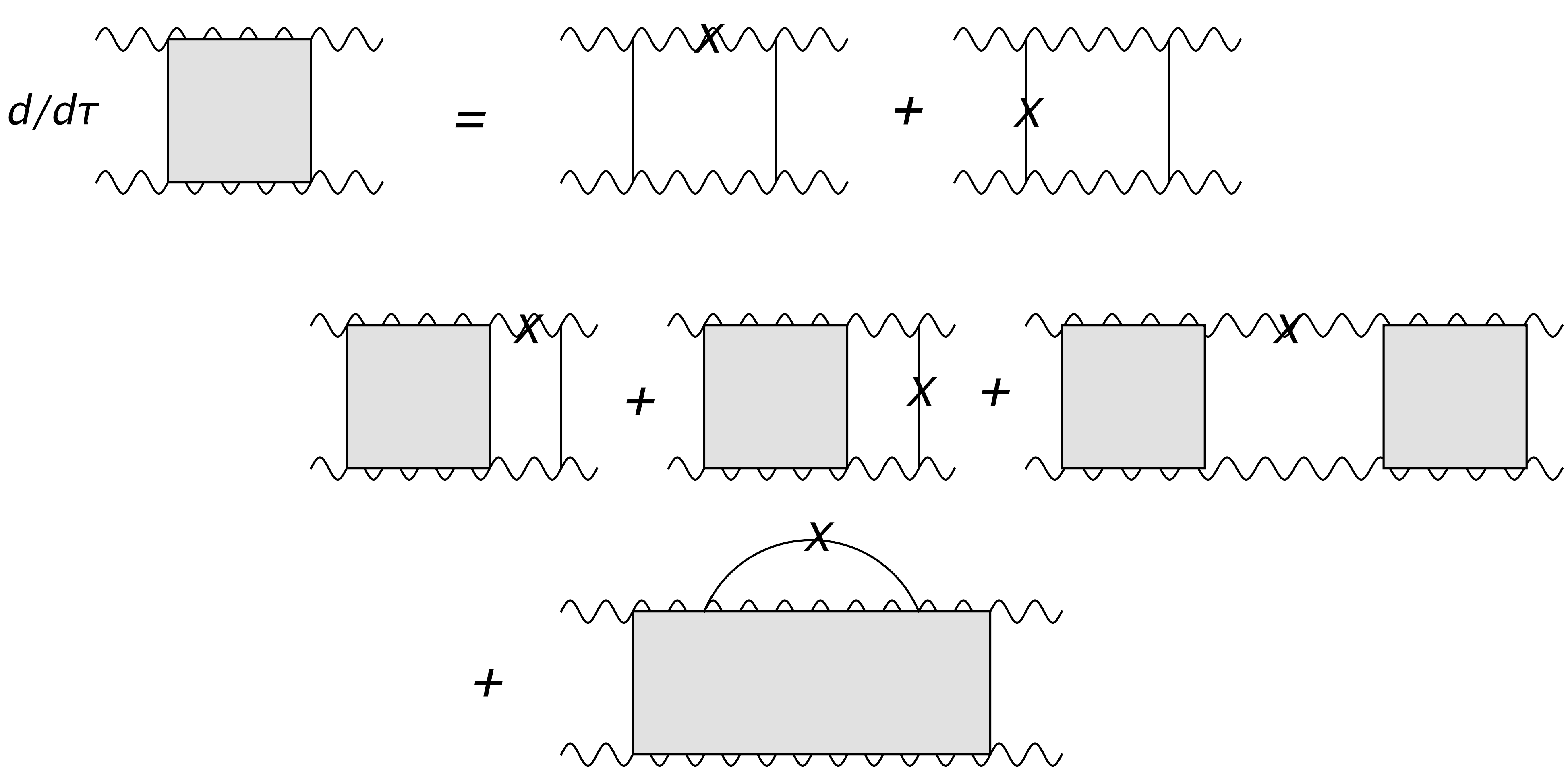
- complex field which does not propagate in rapidity
- elementary gluon for trajectory

RG-Equations:

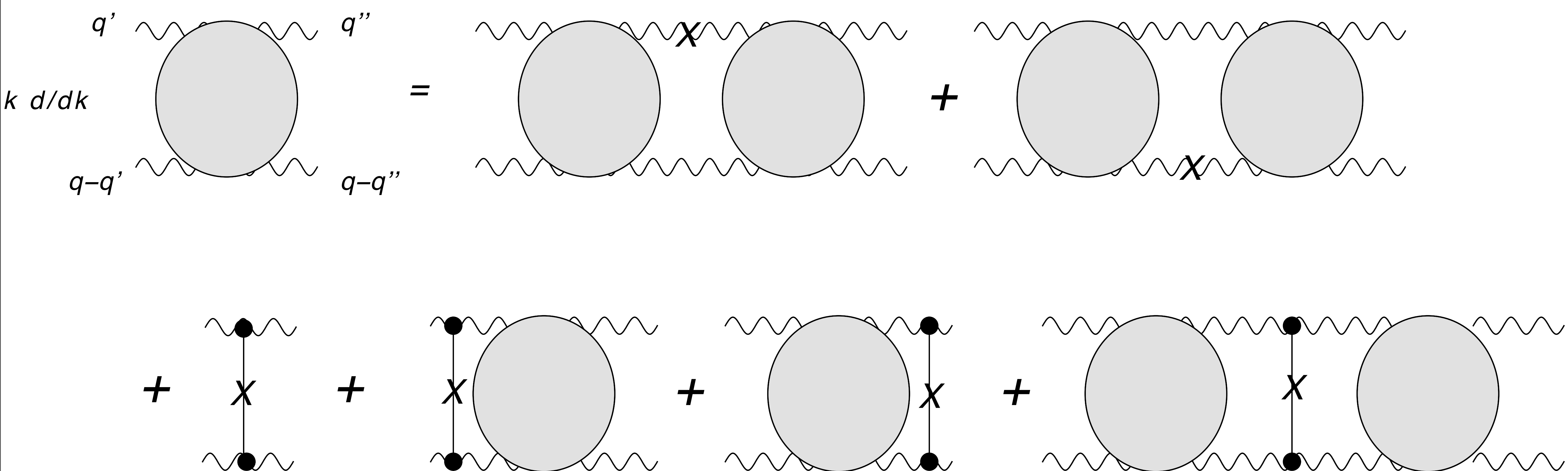
1) start from elementary gluons
2) introduce reggeized gluon as a composite field



3) introduce nonpropagating gluon field for BFKL kernel



4) For the BFKL 4-point function:
use feature of fields and derive a closed nonlinear equation, valid for for LO BFKL Greens function (with running coupling):



(Same structure as IR evolution equations)

Expect discrete poles, work on the solution is in progress

The IR region: search for fixed points

IR limit: region of small transverse momenta (large transverse distances)

Use reggeon field theory (2+1-dim field theory) and renormalization group, construct a flow from UV scale to IR scale

$$S = \int dy d^2x \mathcal{L}(\psi, \psi^\dagger) \qquad \psi = \text{Pomeron field}$$

Local approximation (LPA, strong assumption):

$$\mathcal{L} = (\tfrac{1}{2}\psi^\dagger \overset{\leftrightarrow}{\partial}_y \psi - \alpha' \psi^\dagger \nabla^2 \psi) + V(\psi, \psi^\dagger)$$

$$V(\psi, \psi^\dagger) = -\mu \psi^\dagger \psi + i\lambda \psi^\dagger (\psi^\dagger + \psi)\psi$$
$$+ g(\psi^\dagger \psi)^2 + g' \psi^\dagger ({\psi^\dagger}^2 + \psi^2)\psi + \cdots$$

Study the flow as function of IR cutoff k in transverse momentum, all fields and parameters become k-dependent.

Same universality class as Directed Percolation (Cardy, Sugar)

In a second step: include as second field for the Odderon (important restrictions on the couplings)

Derivative with respect the RG time t=log (k/k_0) leads to the Wetterich flow equation:

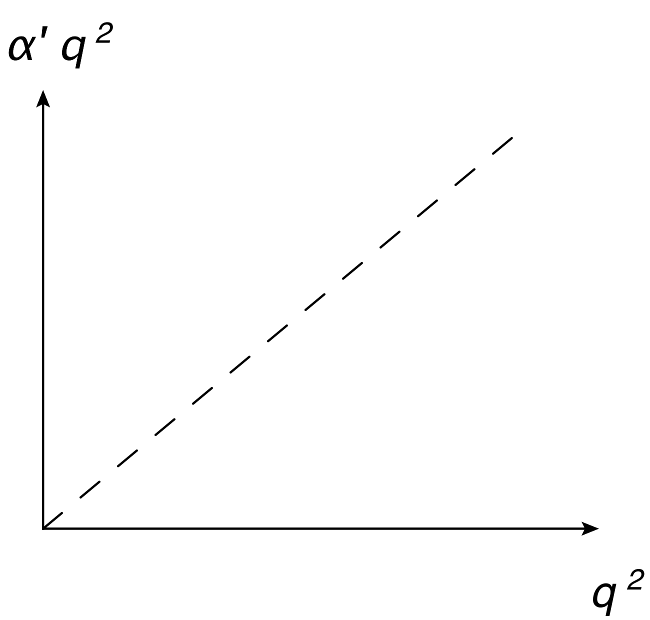
$$\partial_t \Gamma_k = \frac{1}{2} STr\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k\right]$$

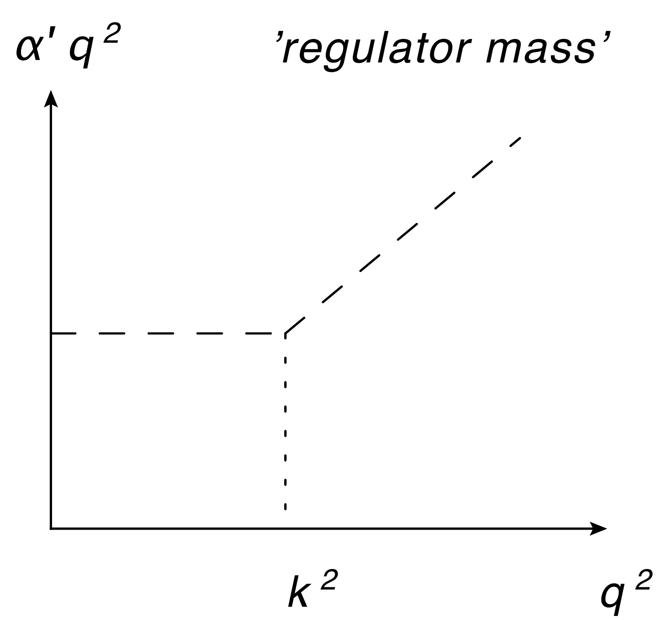
$$\mathcal{R}_k = \text{regulator operator, e.g.}$$

$$\mathcal{R}_k(q) = (k^2 - q^2)\Theta(k^2 - q^2)$$

which is UV and IR finite

From this derive coupled differential equations for Green's and vertex functions (see below)





Search for fixed points:

$$\Gamma[\psi^\dagger,\psi] = \int \mathrm{d}^2x\,\mathrm{d}\tau \left(Z(\frac{1}{2}\psi^\dagger \overset{\leftrightarrow}{\partial_\tau} \psi - \alpha' \psi^\dagger \nabla^2 \psi) + V[\psi^\dagger,\psi] \right),$$

$$V[\psi^\dagger,\psi] = -\mu\psi^\dagger\psi + i\lambda\psi^\dagger(\psi^\dagger+\psi)\psi + g(\psi^\dagger\psi)^2 + g'\psi^\dagger({\psi^\dagger}^2+\psi^2)\psi$$
$$+ i\lambda_5 {\psi^\dagger}^2 \left(\psi^\dagger + \psi\right)\psi^2 + i\lambda_5'\psi^\dagger \left({\psi^\dagger}^3 + \psi^3\right)\psi + \ldots$$

$$\dot{\tilde{V}}_k[\tilde{\psi}^\dagger,\tilde{\psi}] = (-(D+2)+\zeta_k)\tilde{V}_k[\tilde{\psi}^\dagger,\tilde{\psi}] + (D/2+\eta_k/2)(\tilde{\psi}\frac{\partial \tilde{V}_k}{\partial\tilde{\psi}}|_t + \tilde{\psi}^\dagger\frac{\partial\tilde{V}_k}{\partial\tilde{\psi}^\dagger}|_t) + \frac{\dot{V}_k}{\alpha' k^{D+2}}.$$

$$\dot{V}_k = N_D A_D(\eta_k,\zeta_k)\alpha_k' k^{2+D}\frac{1+\tilde{V}_{k\tilde{\psi}\tilde{\psi}^\dagger}}{\sqrt{1+2\tilde{V}_{k\tilde{\psi}\tilde{\psi}^\dagger}+\tilde{V}^2_{k\tilde{\psi}\tilde{\psi}^\dagger} - \tilde{V}_{k\tilde{\psi}\tilde{\psi}}\tilde{V}_{k\tilde{\psi}^\dagger\tilde{\psi}^\dagger}}}.$$

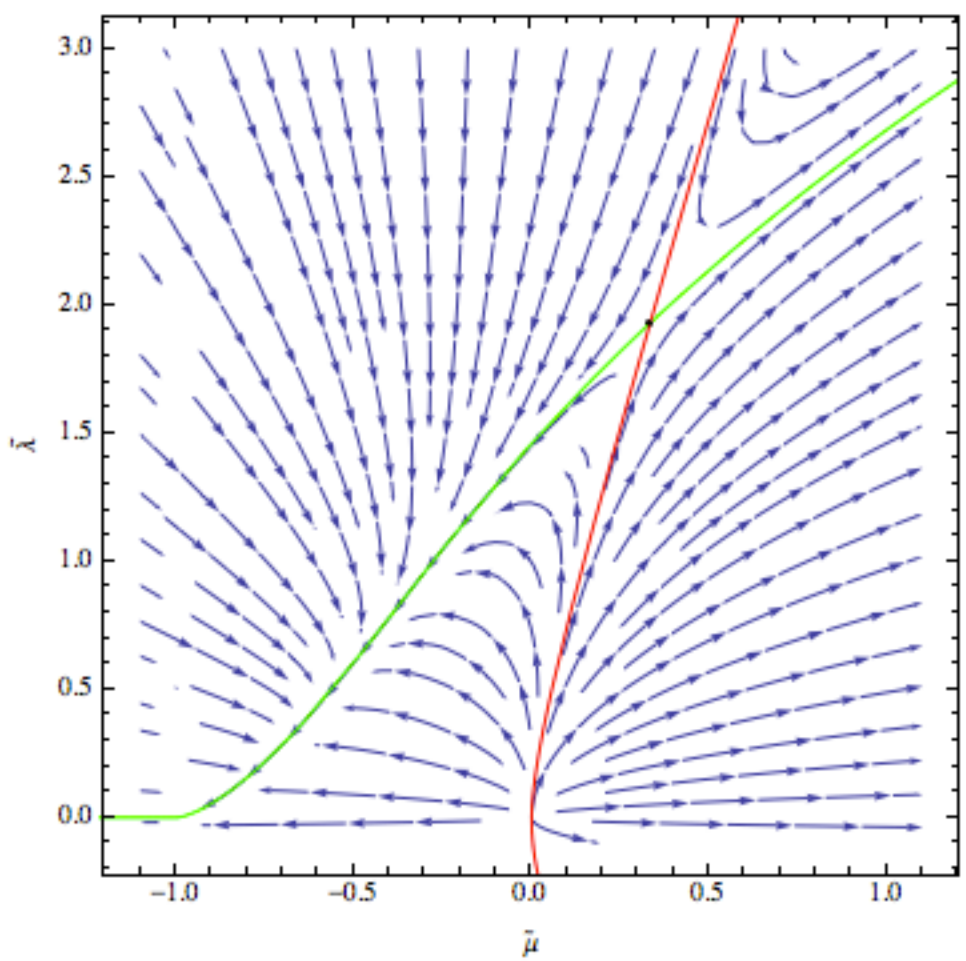
Possible approximations to solve for fixed points (for constant fields):

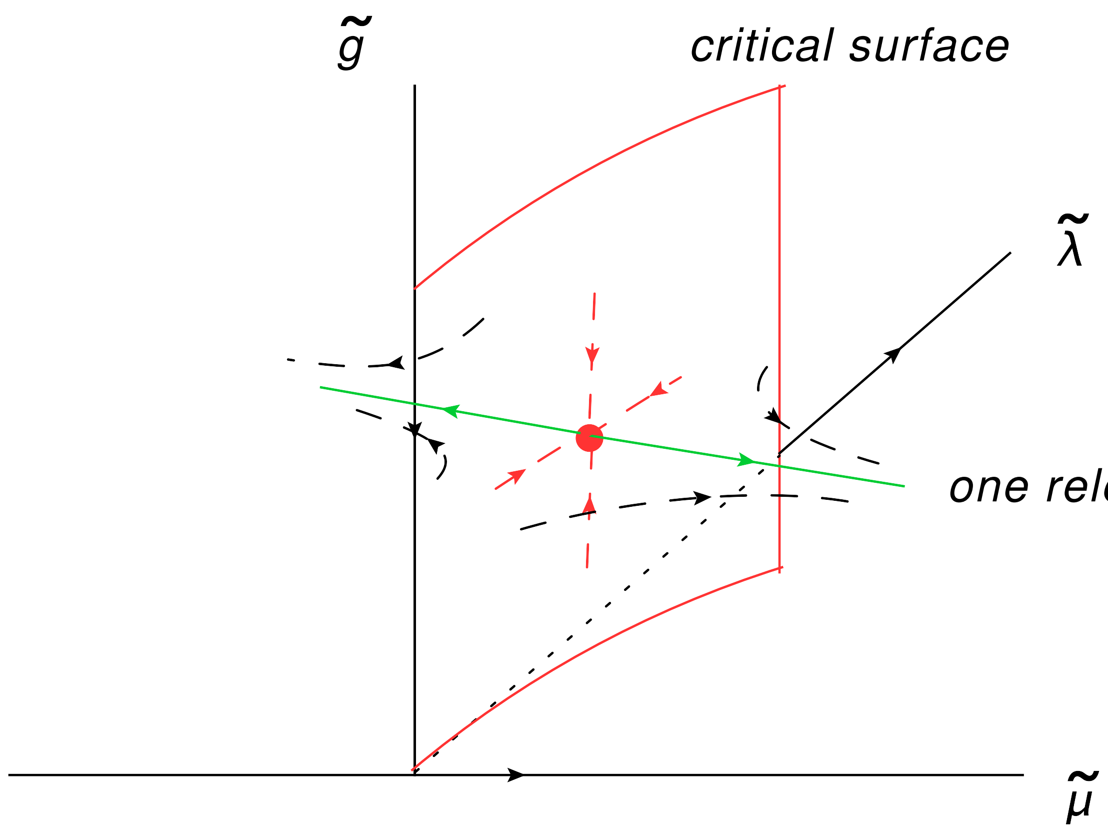
- polynomial expansion in fields around zero (beta-functions)
- polynomial expansion in fields around nonzero stationary point
- solve differential equations in the region of large fields

Different truncations (up to order16)

Results of the fixed point analysis

1) Existence of a fixed point with one relevant direction (independent of truncation)





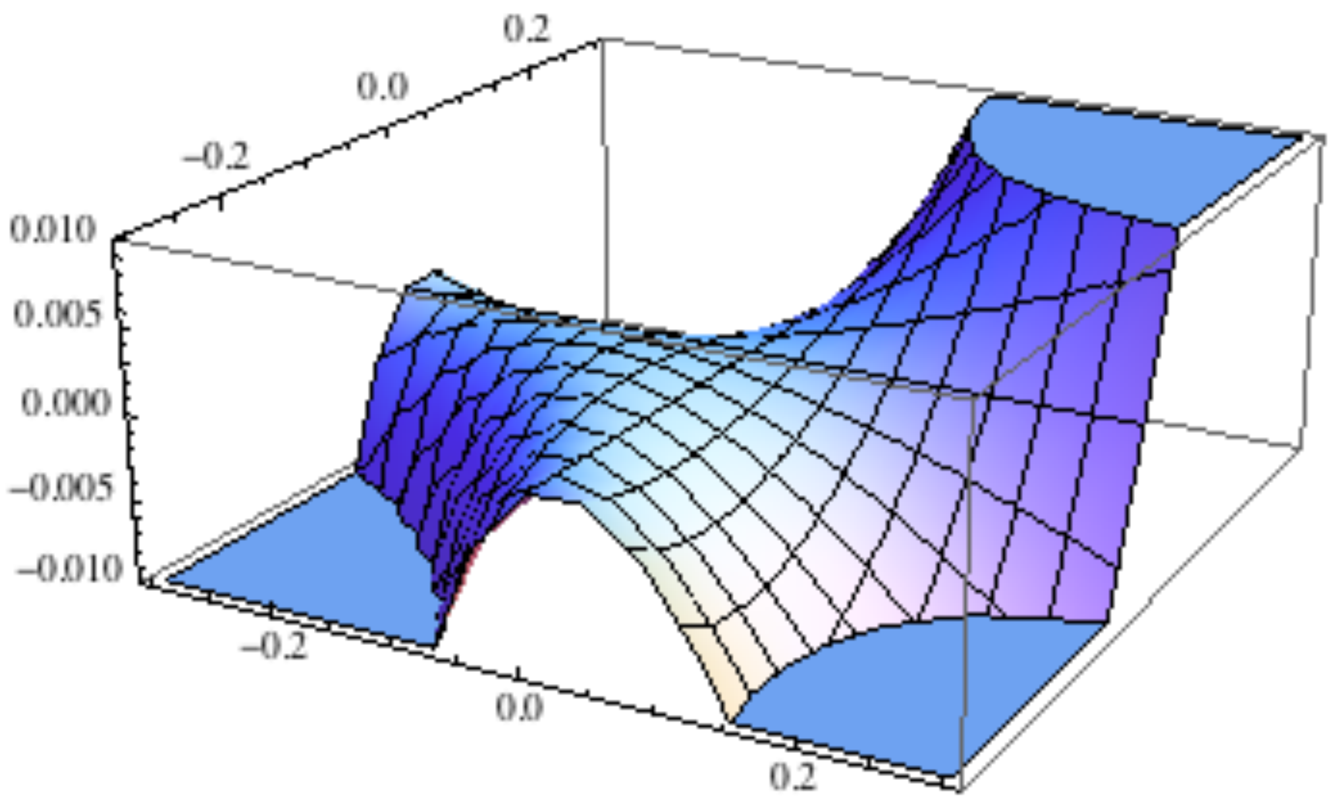
Flow in the space of parameters of the potential (couplings) :
reggeon mass (intercept) $\alpha(0) - 1 = \tilde{\mu}/Z$, triple coupling $\tilde{\lambda}$
fixed point IR attractive inside critical surface (red),
repulsive along one-dimensional relevant direction (green)

Convergence for higher truncations (expansion around nonzero stationary point) :

truncation	3	4	5	6	7	8
exponent ν	0.74	0.75	0.73	0.73	0.73	0.73
mass $\tilde{\mu}_{eff}$	0.33	0.362	0.384	0.383	0.397	0.397
$i\psi_{0,diag}$	0.058	0.072	0.074	0.074	0.0.074	0.074
iu_0	0.173	0.213	0.218	0.218	0.218	0.218

Compare with Monte Carlo result for Directed Percolation
(same universality class): $\nu = 0.73$

Shape of the effective potential (in the subspace of imaginary fields):



Extrema, location at lowest truncation:

$$(\tilde{\psi}_0, \tilde{\psi}_0^\dagger) = (0,0), \qquad (\frac{\tilde{\mu}}{i\tilde{\lambda}}, 0), \qquad (0, \frac{\tilde{\mu}}{i\tilde{\lambda}}), \qquad (\frac{\tilde{\mu}}{3i\tilde{\lambda}}, \frac{\tilde{\mu}}{3i\tilde{\lambda}}).$$

No further structure for larger fields

Main result of this part:

- found a candidate for fixed point (IR stable except for one relevant direction) which is robust when changing truncations

- know the effective potential

- Include Odderon: IR stable fixed point with two (three) relevant directions at the fixed point:
  new symmetry "Pomeron does not feel the Odderon, whereas Odderon has strong absorption".

From this:
derive possible solutions for the behavior at very high energies

First glimpse at physics

Need to find out: on which trajectory is real physics?

Look at flow of physical physical observable: Pomeron intercept $\alpha(0) - 1 = \mu/Z$:

So far: fixed point analysis was done in terms of dimensionless variables: reggeon energy and momentum have different dimensions

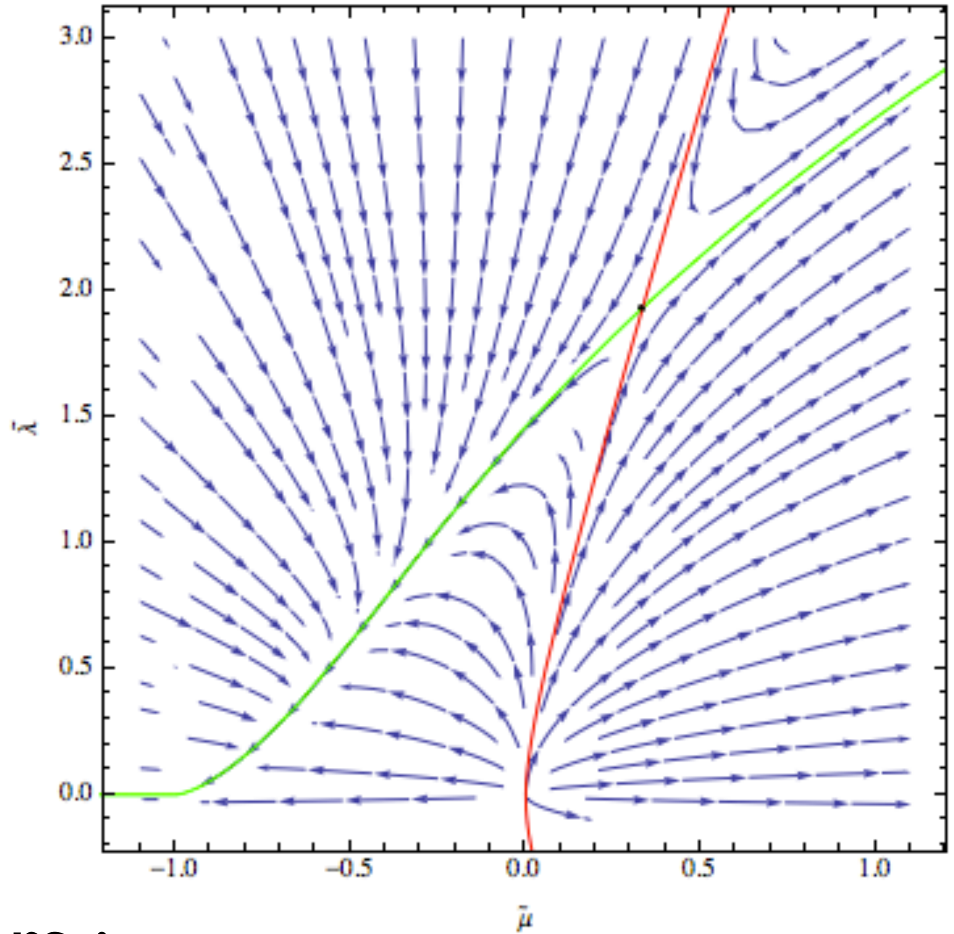
$$S = \int d^2x \, d\tau \left(Z(\frac{1}{2}\psi^\dagger \overleftrightarrow{\partial_\tau} \psi - \alpha' \psi^\dagger \nabla^2 \psi) + V[\psi^\dagger, \psi] \right), \qquad [\psi] = [\psi^\dagger] = k^{D/2}, \qquad [\alpha'] = E k^{-2}.$$

$$\tilde{\mu}_k = \frac{\mu_k}{Z_k \alpha'_k k^2}$$

$$\tilde{\lambda}_k = \frac{\lambda_k}{Z_k^{\frac{3}{2}} \alpha'_k k^2} k^{D/2}$$

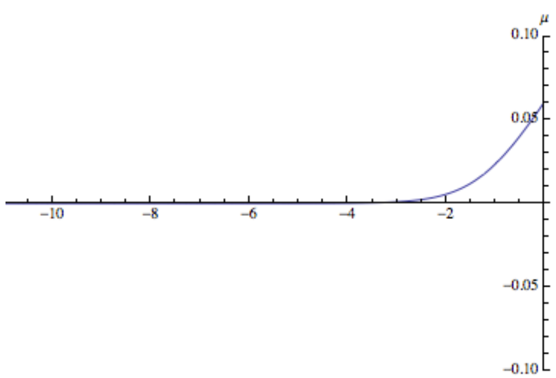
Evolution of physical (=dimensionful) parameters $\mu_k, \lambda_k, \ldots$ looks quite different from dimensionless ones $\tilde{\mu}_k, \tilde{\lambda}_k, \ldots$

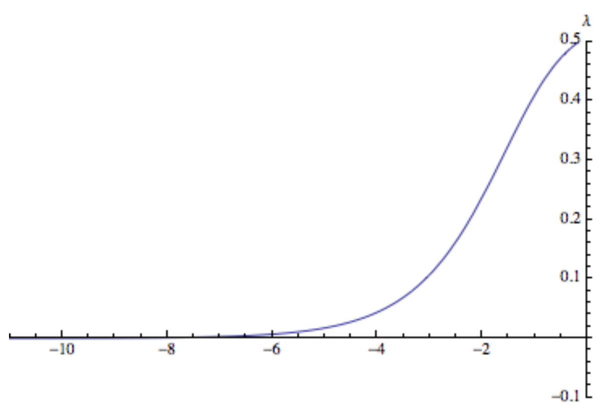
dimensionless parameters



physical parameters :

Critical subspace (red):

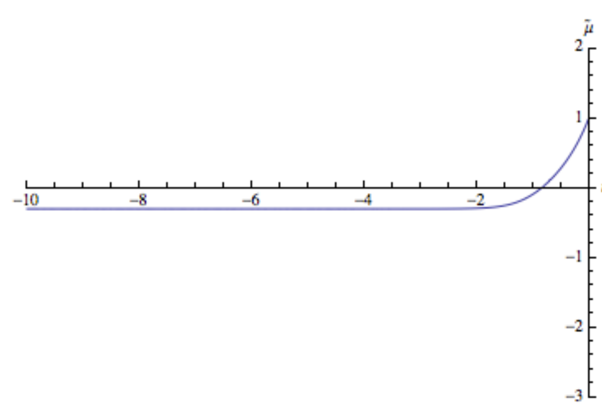




$\alpha(0) \to 1$ $\qquad\qquad$ $\lambda_{triple} \to 0$

But: theory not free!

Near critical subspace (blue): several possibilities, e.g.



$\alpha_k(0) \to \alpha_{k=0} < 1$

Main result: theory allows for different possibilities:

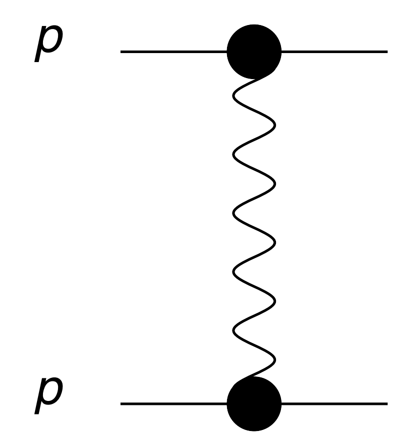
1) inside critical subspace: infrared stable fixed point with intercept one. But: need constraint at starting point in UV region
2) near critical surface: falling or rising total cross section. Need further study

In the following: consider a scenario inside the critical subspace

A simple model:
single Pomeron exchange - a scaling law

$$T_{el}(s,t) = is\int \frac{d\omega}{2\pi} s^{i\omega}\beta_p(t)\frac{1}{Z_k(i\omega+\alpha'_k q^2)-\mu_k}\beta_p(t)$$

$$= is\beta_p(t)Z_k^{-1}s^{\mu_k/Z-\alpha'_k q^2}\beta_p(t).$$



For small k:

$$T_{el}(s,t) \sim isk^{\eta}s^{k^{(2-\zeta)}\tilde{\mu_k}}f(\ln s\, q^2 k^{-\zeta})$$

$$\eta \approx -0.331\ (-0.6),\ \ \zeta \approx 0.172\ (0.28).$$

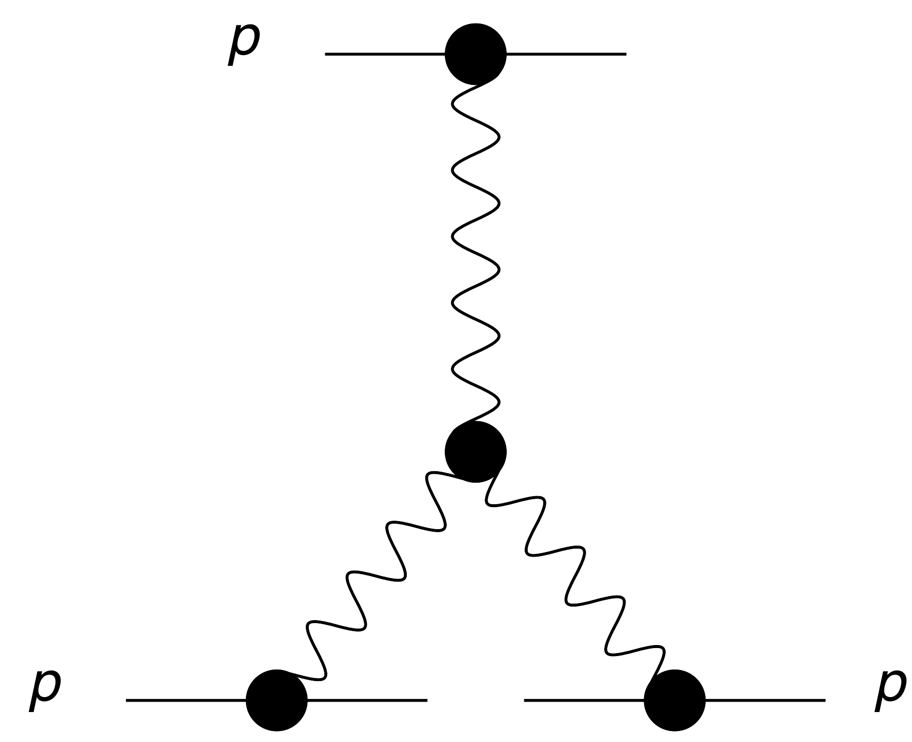
anomalous dimensions : directed percolation

Assume: for very large energies $\alpha'_k k^2 \sim \frac{1}{\ln s}$ $\qquad R^2(s) = 2(B_0 + 2\alpha'_k \ln s) \sim 1/k^2$

$$T_{el}(s,t) \sim is(\ln s)^{-\eta/(2-\zeta)}s^{(\ln s)^{-1}\tilde{\mu}_{fp}}f(t(\ln s)^{2/(2-\zeta)})$$

Triple Pomeron cross section:

$$\frac{d\sigma}{dtdM^2} = \frac{1}{16\pi M^2}\int\frac{d\omega}{2\pi i}\int\frac{d\omega_1}{2\pi i}\int\frac{d\omega_2}{2\pi i}\left(\frac{s}{M^2}\right)^{\omega_1+\omega_2}\left(\frac{M^2}{M_0^2}\right)^{\omega}$$
$$\beta(0)\frac{1}{Z_k i\omega-\mu_k}\lambda_k\frac{1}{Z_k(i\omega_1+\alpha'_k q^2)-\mu_k}\frac{1}{Z_k(i\omega_2+\alpha'_k q^2)-\mu_k}\beta(t)^2.$$



Additional energy dependence:

$$\lambda_k/Z_k^3 \sim (\ln s)^{-1+\frac{1-3/2\eta}{2-\zeta}}$$

Comparison with previous work:

$\rightarrow$ 2 x Gribov, Migdal
Abarbanel, Bronzan
Migdal, Polyakov, Ter-Martirosyan

RG analysis of RFT with triple coupling near D=4, impose the condition:

$$\alpha(0) = 1$$

$$T_{el}(s,t) \sim is(\ln s)^{\eta_O} F(t(\ln s)^{z_O}), \ \eta_0 \approx 0.35, \ z_0 \approx 1.165$$

$$= is(\ln s)^{-\eta/(2-\zeta)} F(t(\ln s)^{2/(2-\zeta)}) \qquad \eta_O = -\frac{\eta}{z}, \ z = 2-\zeta, \ z_O = \frac{2}{z}$$

Cannot apply to present data

For comparison: we did not impose conditions on intercept

$$T_{el}(s,t) \sim is(\ln s)^{-\eta/(2-\zeta)} s^{(\ln s)^{-1}\tilde{\mu}_{fp}} f(t(\ln s)^{2/(2-\zeta)})$$

Difference in intercept at finite enegies

$\uparrow$

Qualitative agreement with real physics!

In the (mathematical) limit of infinite energies agrees with CFRT.
At present we are not at infinite energies:

$$R^2(s) = 2(B_0 + 2\alpha' \ln s), B_0 \sim 2\alpha' \ln s \approx 9\mathrm{GeV}^{-2}$$

A comment on other possible scenarios:
if the evolution misses the fixed point
(i.e. lies outside the critical surface, very sensitive to the starting point):

- subcritical solution $\alpha(0) < 1$, falling cross section, $\sigma_{tot} \to 0$

- supercritical Pomeron: $\alpha(0) > 1$
  eikonalization, most likely leads to Froissart type cross section $\sigma_{tot} \to (\ln s)^2$

A comment on the Odderon:
- experimentally the existence is still under discussion
- in pQCD there exist an Odderon (as bound state of three gluons)
- our fixed point analysis indicates: Odderon with intercept one should exist

Conclusions

BFKL is the most promising starting point for a theory of high energy scattering

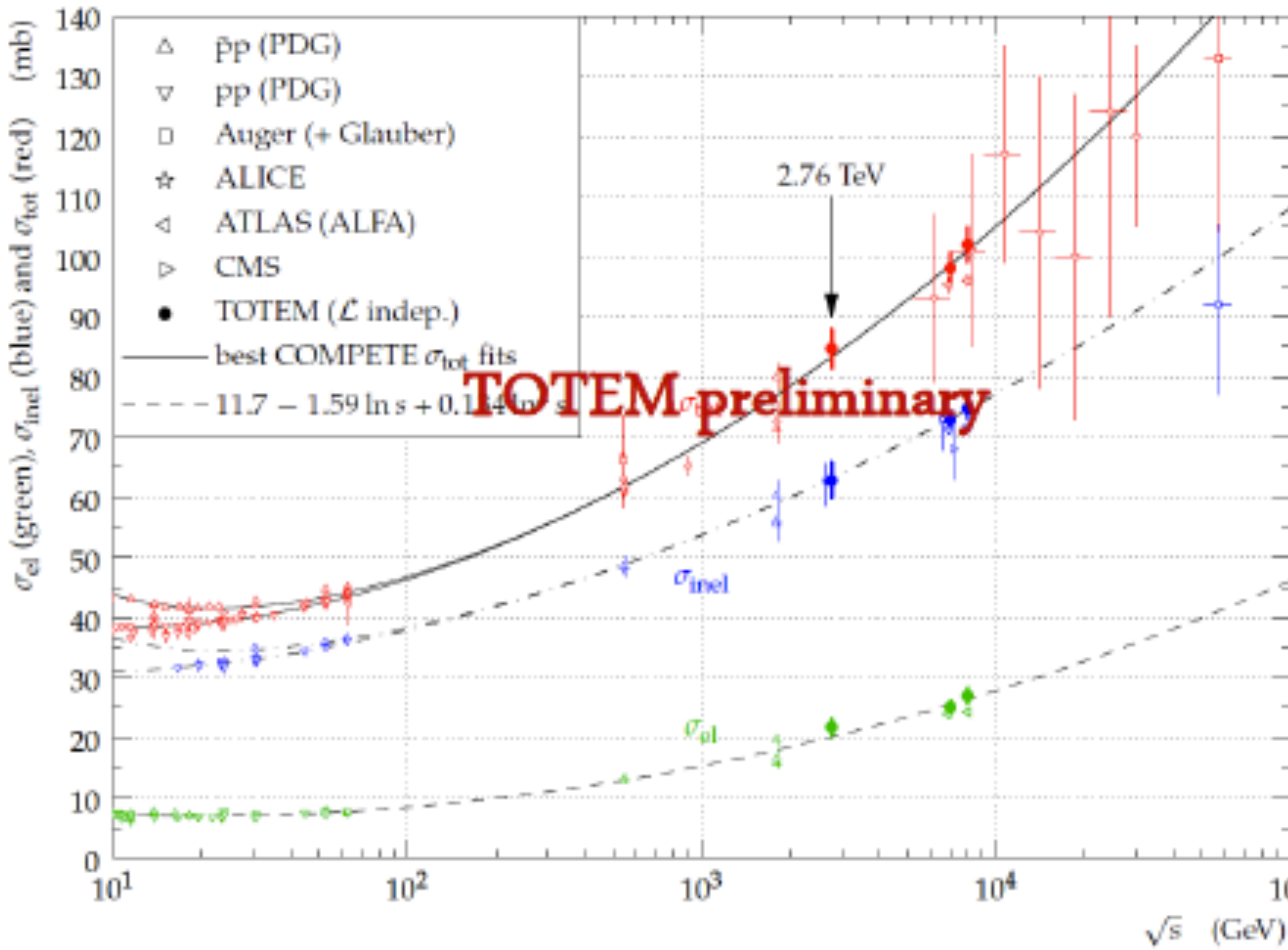
still some way to go, before we understand the transition to nonperturbative physics: ongoing attempt with first results

BFKL has broader relevance in the high energy limit of quantum field theory:
N=4 SYM, EW-theory, gravity

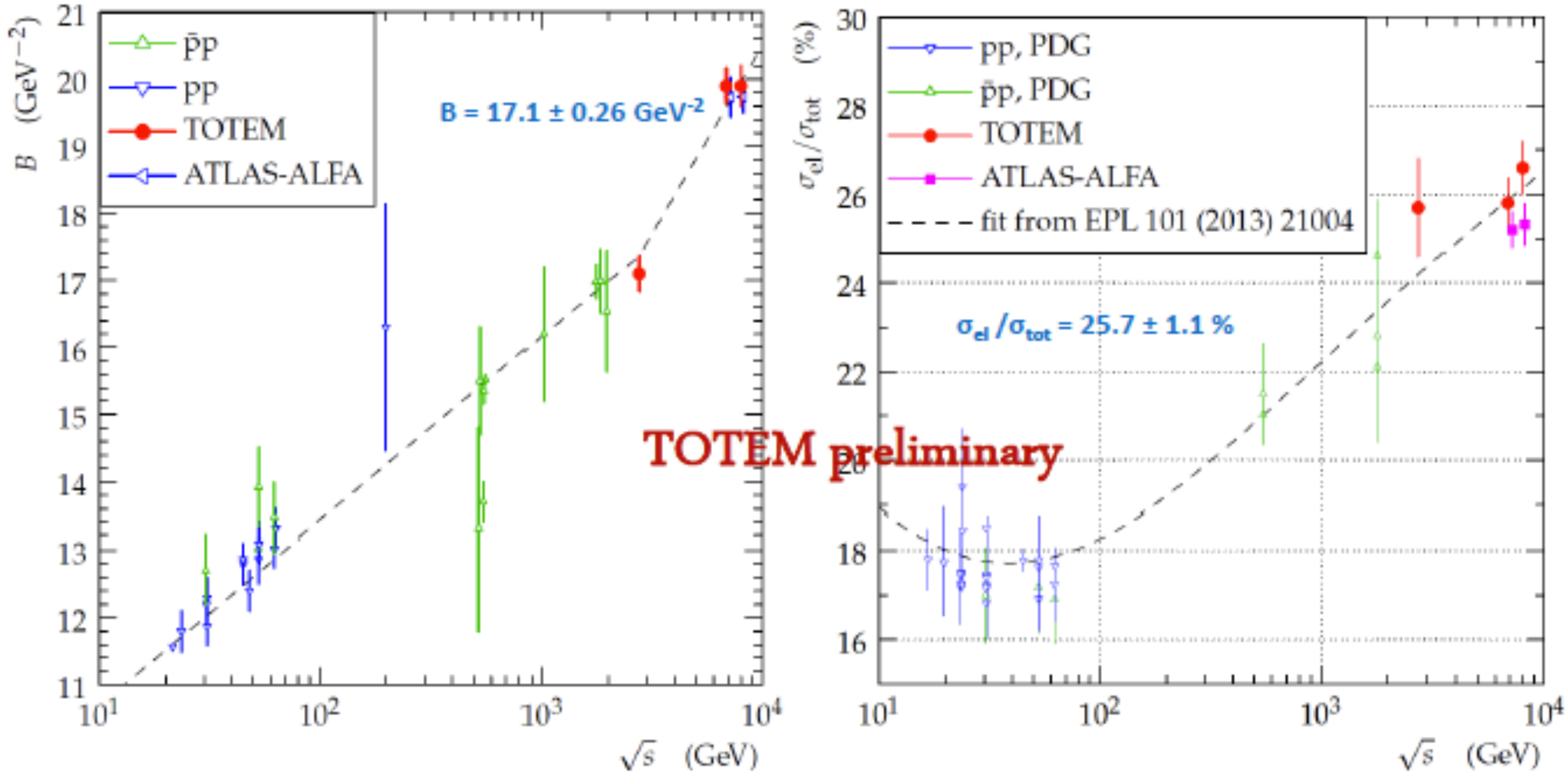
Backup slides

2.76 TeV luminosity independent cross-sections ($\beta^* = 11$ m optics)

σ_{tot} [mb]	σ_{el} [mb]	σ_{inel} [mb]
84.7 ± 3.3	21.8 ± 1.4	62.8 ± 2.9



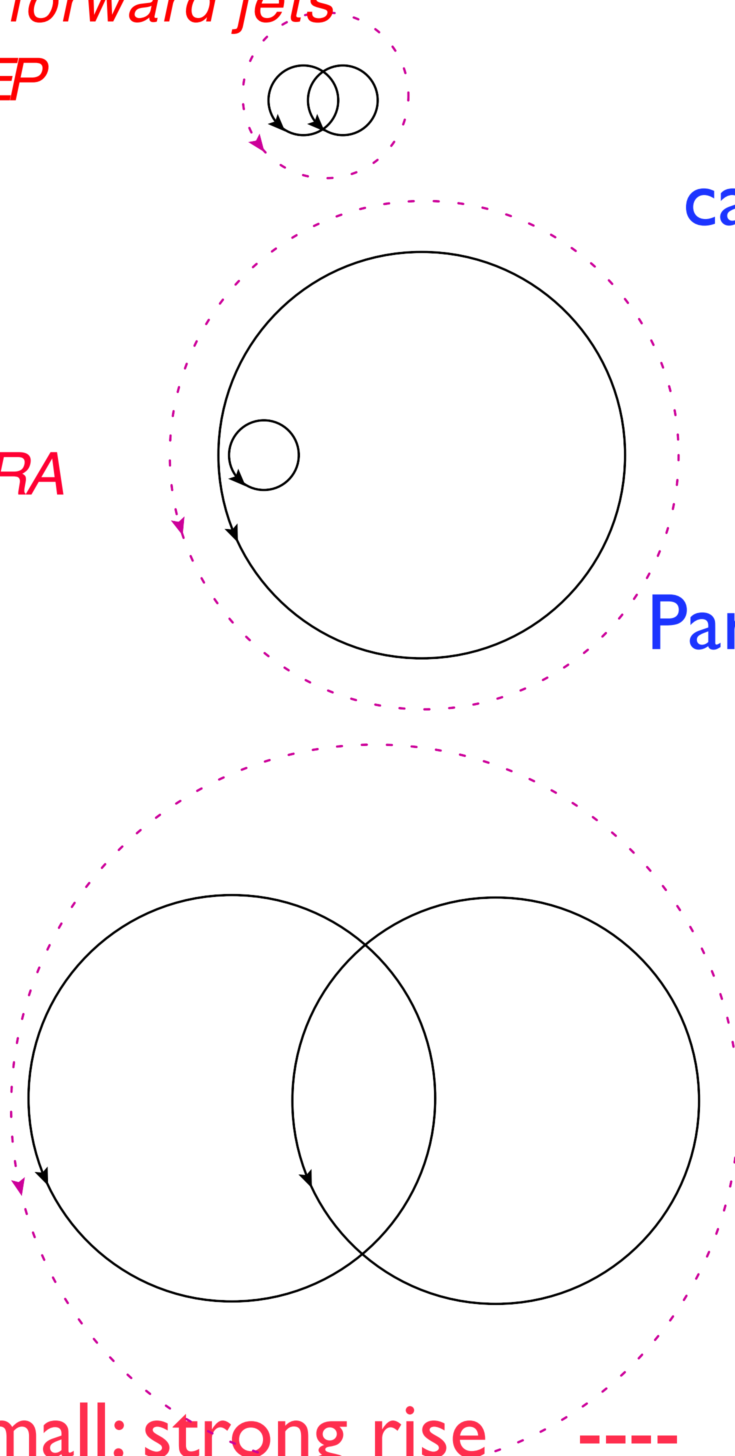
The nuclear slope B and the σ_{el}/σ_{tot} ratio at √s = 2.76 TeV



Energy dependence of total cross sections varies with transverse size:

HERA forward jets

LEP



$\gamma^*\gamma^* \quad \sigma_{tot} \approx s^{\omega_{BFKL}}$

calculable in pQCD

HERA

$\gamma^* p \quad \sigma_{tot} \approx (W^2)^{\lambda}$

Partly calculable in pQD

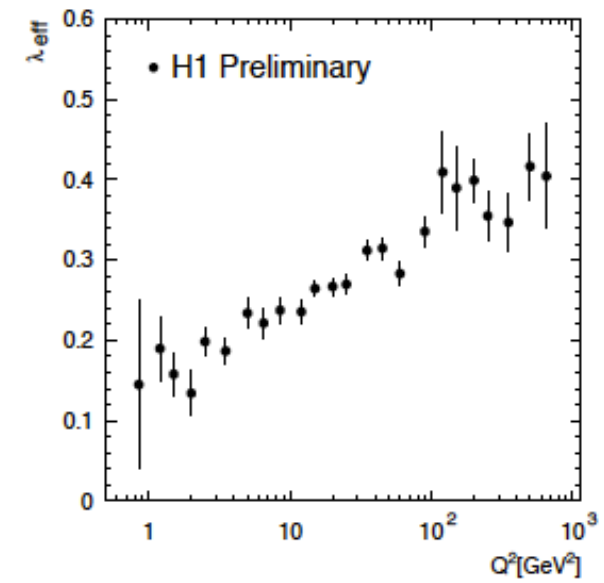


Figure 6: The slope $\lambda_{\rm eff}$ of F_2 as a function of Q^2 .

LHC

$p\,p \quad \sigma_{tot} \approx s^{0.08}$

nonperturbative

Small: strong rise ---- large: slow rise