

FORWARD DI-JET PRODUCTION AS A PROBE OF SATURATION EFFECTS

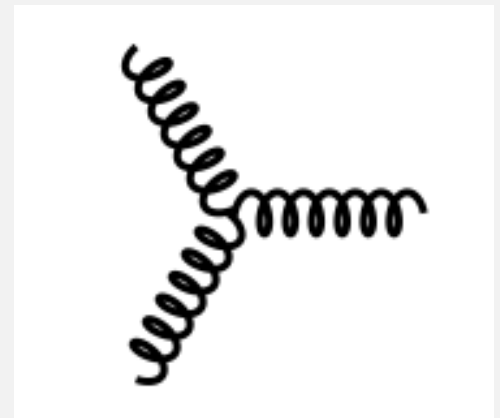
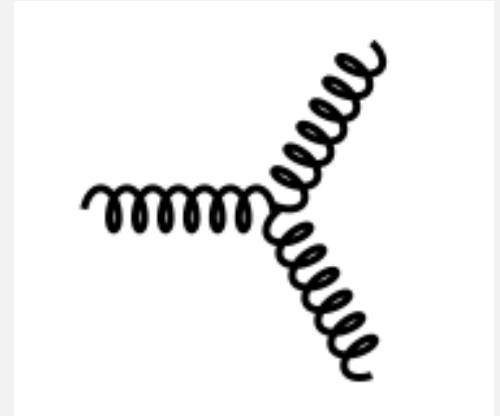
WHAT IS SATURATION?

GLUON BRANCHING

Gluon branching – The number of gluons grows towards lower Bjorken- x values.

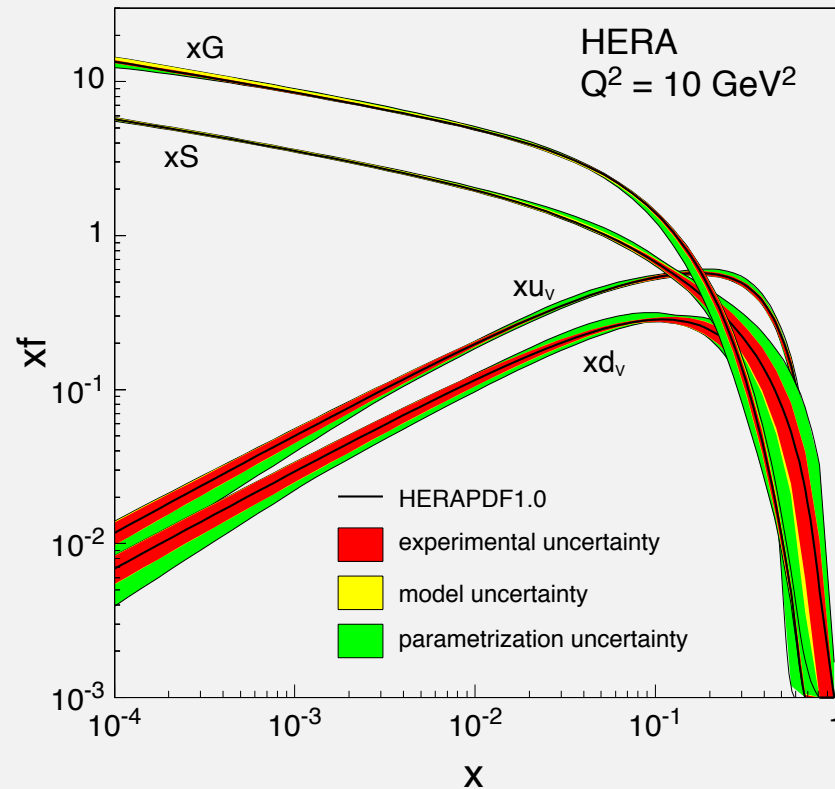
- Phase space inside hadron can be filled up.
- The unitarity of cross section would be violated.

Gluon recombination – High density of gluons can lead to overlapping of their wave functions and two gluons can merge into one.



PARTON DISTRIBUTION FUNCTIONS

- xS – Sea quark distribution
- xG – Gluon distribution
- xu_v – Valence u-quark distribution
- xd_v – Valence d-quark distribution



We can see the contribution of gluon branching effects in parton distribution functions.

Do the gluon recombination effects also contribute?

WHERE TO LOOK FOR SATURATION EFFECTS

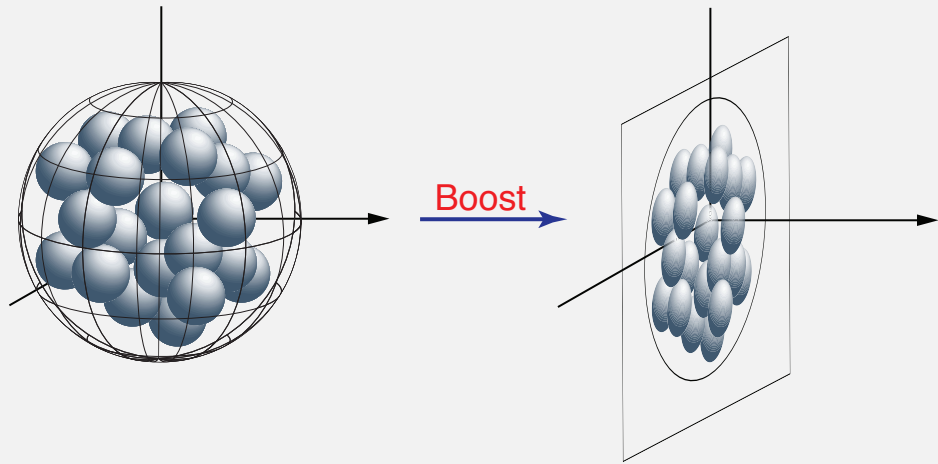
- For the observation of saturation effects, we need to reach low values of Bjorken- x .
- With a fixed collision energy, lower Bjorken- x corresponds to lower values of Q^2 .
- Low Q^2 means low k_t of the process and therefore difficult detection.

$$x = \frac{Q^2}{2P \cdot q}$$

SATURATION INSIDE NUCLEUS

Furthermore, if we look at the influence of nuclear effects on saturation , we find out that

$$Q_s^2 \sim A^{1/3}$$



Saturation scale Q_s^2 determines the momentum region where saturation effects start to dominate and suppress the distribution function.

Compared to proton, saturation scale inside a nucleus is higher due to higher overlapping because of Lorentz contraction.

These effects can be used to determine whether saturation is present in hadrons.

WHAT PROCESSES TO FOCUS ON?

STUDIED PROCESSES

For our computation, we focused on back-to-back jets in the forward region of rapidity.

Why?

STUDIED PROCESSES

Although $p_{1t}, p_{2t} \gg Q_s$

$$p(p_p) + A(p_A) \rightarrow j_1(p_1) + j_2(p_2) + X$$

If we focus on back to back jets in the transverse momentum plane, we can get

$$Q_s \sim k_t \quad k_t = |\vec{p}_{1t} + \vec{p}_{2t}|$$

Furthermore, it is necessary to reach the region where x_1 is large and $x_2 \ll 1$.

$x_2 \ll 1$ is necessary to detect saturation effects, large x_1 is required, because in this region of Bjorken- x , we can use parton distribution functions that are known with great precision from previous experiments. This can be achieved by looking into the forward region in rapidity.

HOW DO WE DETERMINE WHETHER
SATURATION EFFECTS ARE PRESENT?

USE OF NUCLEAR EFFECTS

We use the fact that in nuclei, saturation scale reaches higher values than in protons.

For the detection of saturation effects we use:

$$Q_s^2 \sim A^{1/3}$$

$$R_{pPb} = \frac{\frac{d\sigma^{p+Pb}}{d\mathcal{O}}}{A \frac{d\sigma^{p+p}}{d\mathcal{O}}}$$

When we take a look at the nuclear modification factor with respect to the angle between the two jets, we can see the non-linear effects as a decrease at about $\sim 180^\circ$ when $k_{1t} \sim k_{2t}$.

HOW DO WE PREDICT THESE
CROSS SECTIONS?

CROSS SECTION CALCULATION

Cross section is calculated as:

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

CROSS SECTION CALCULATION

$$d\sigma^{pA \rightarrow \text{dijets} + X} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

- P_t - Single jet transverse momentum, k_t - jet pair transverse momentum,

y_1, y_2 – jet rapidities

CROSS SECTION CALCULATION

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

- Running coupling – fixed at 1 for our computation

CROSS SECTION CALCULATION

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{x_1 x_2 s^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

- x_1 corresponds to the projectile particle, x_2 to the target particle and s is the energy of the collision

CROSS SECTION CALCULATION

I) Projectile gluon distribution

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

Is obtained from data from previous experiments.

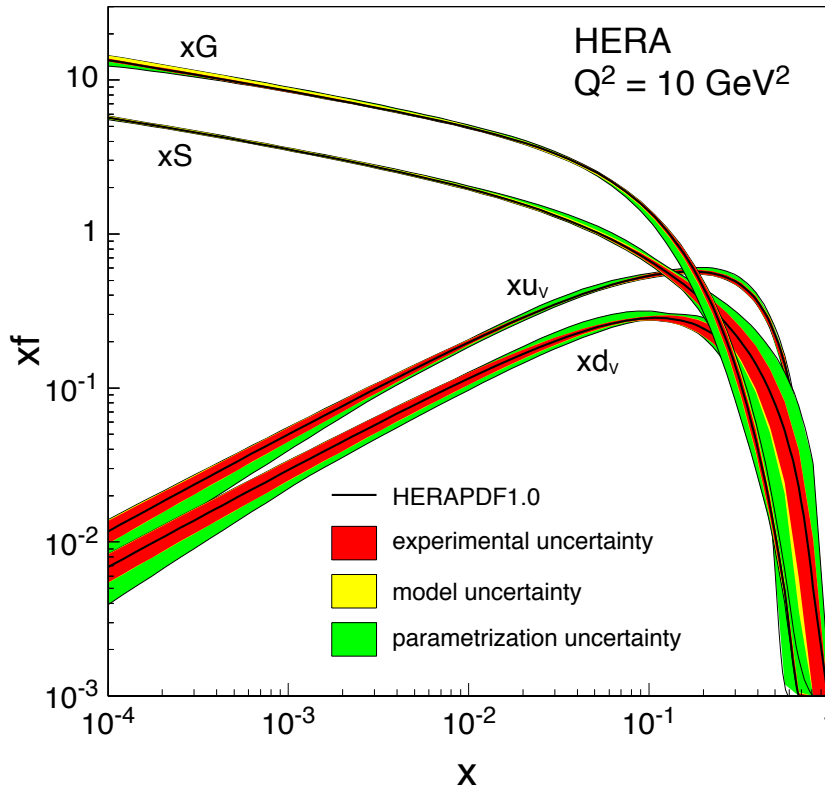
CROSS SECTION CALCULATION



Projectile gluon distrib

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha}{(x_1 x_2)}$$

Is obtained from data from



$k_t)$

CROSS SECTION CALCULATION

2) Matrix elements

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 \boxed{K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t)} \tilde{\Phi}_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

CROSS SECTION CALCULATION



Matrix elements

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 \boxed{K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t)} \hat{\mathcal{P}}_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

Take the following form for the
considered processes:

| i | 1 | 2 |
|---------------------------------------|---|---|
| $K_{gg^* \rightarrow gg}^{(i)}$ | $2 \frac{(\bar{s}^4 + \bar{t}^4 + \bar{u}^4)(\bar{u}\hat{u} + \bar{t}\hat{t})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$ | $-\frac{(\bar{s}^4 + \bar{t}^4 + \bar{u}^4)(\bar{u}\hat{u} + \bar{t}\hat{t} - \bar{s}\hat{s})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$ |
| $K_{gg^* \rightarrow q\bar{q}}^{(i)}$ | $\frac{1}{2N_c} \frac{(\bar{t}^2 + \bar{u}^2)(\bar{u}\hat{u} + \bar{t}\hat{t})}{\bar{s}\hat{s}\hat{t}\hat{u}}$ | $\frac{1}{2N_c^3} \frac{(\bar{t}^2 + \bar{u}^2)(\bar{u}\hat{u} + \bar{t}\hat{t} - \bar{s}\hat{s})}{\bar{s}\hat{s}\hat{t}\hat{u}}$ |
| $K_{qg^* \rightarrow qg}^{(i)}$ | $-\frac{\bar{u}(\bar{s}^2 + \bar{u}^2)}{2\bar{t}\hat{t}\hat{s}}$ | $-\frac{\bar{s}(\bar{s}^2 + \bar{u}^2)}{2\bar{t}\hat{t}\hat{u}}$ |

CROSS SECTION CALCULATION

3) Transverse momentum distributions

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

For these, we need to undergo several substeps.

CROSS SECTION CALCULATION

a) Scattering amplitude.

- Scattering amplitude corresponds to the cross section of the interaction of a color dipole with proton. It is a solution of the Balitsky-Kovchegov equation.

$$N_F(x, \mathbf{r}) = 1 - S_F(x, \mathbf{r})$$

b) Fourier transform

- We transform the scattering amplitude from the coordinate space to the momentum space with the Fourier transform.

$$F(x_2, k_t) = \int \frac{d^2\mathbf{r}}{(2\pi)^2} e^{-ik_t \cdot \mathbf{r}} S_F(x_2, \mathbf{r})$$

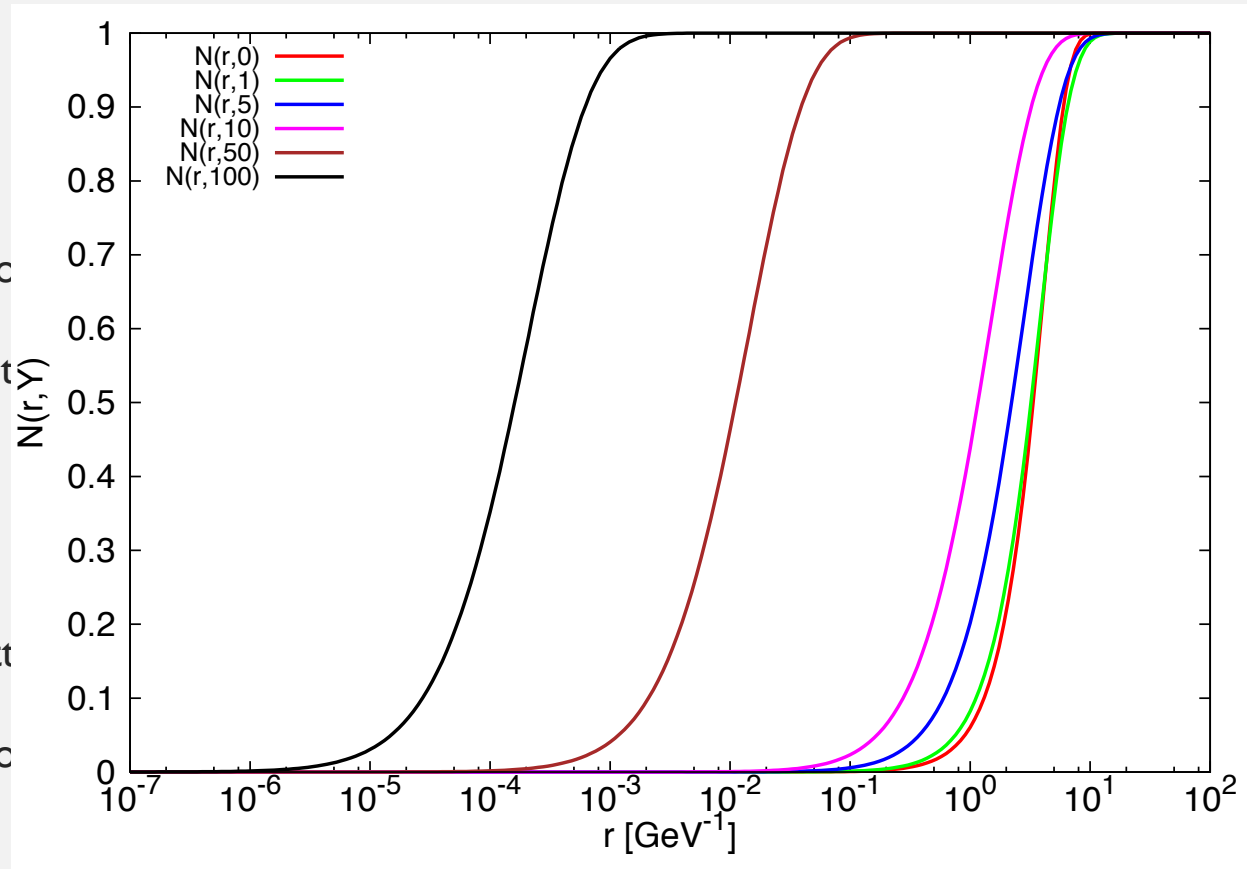
CROSS SECTION CALCULATION

✓ Scattering amplitude.

- Scattering amplitude calculated with proton. It is a solution

✓ Fourier transform

- We transform the scattering amplitude with the Fourier transform



CROSS SECTION CALCULATION

- c) Compute the dipole gluon distribution and Weizacker Williams gluon distribution.

$$x_2 G^{(2)}(x_2, k_t) = \frac{N_c k_t^2 S_\perp}{2\pi^2 \alpha_s} F(x_2, k_t)$$

$$x_2 G^{(1)}(x_2, k_t) = \frac{C_F}{2\alpha_s \pi^4} \int d^2 b \int \frac{d^2 \mathbf{r}}{\mathbf{r}^2} e^{-i k_t \cdot \mathbf{r}} [1 - S_A(x_2, \mathbf{r})]$$

$$S_A(x, \mathbf{r}) = [S_F(x, \mathbf{r})]^2$$

CROSS SECTION CALCULATION

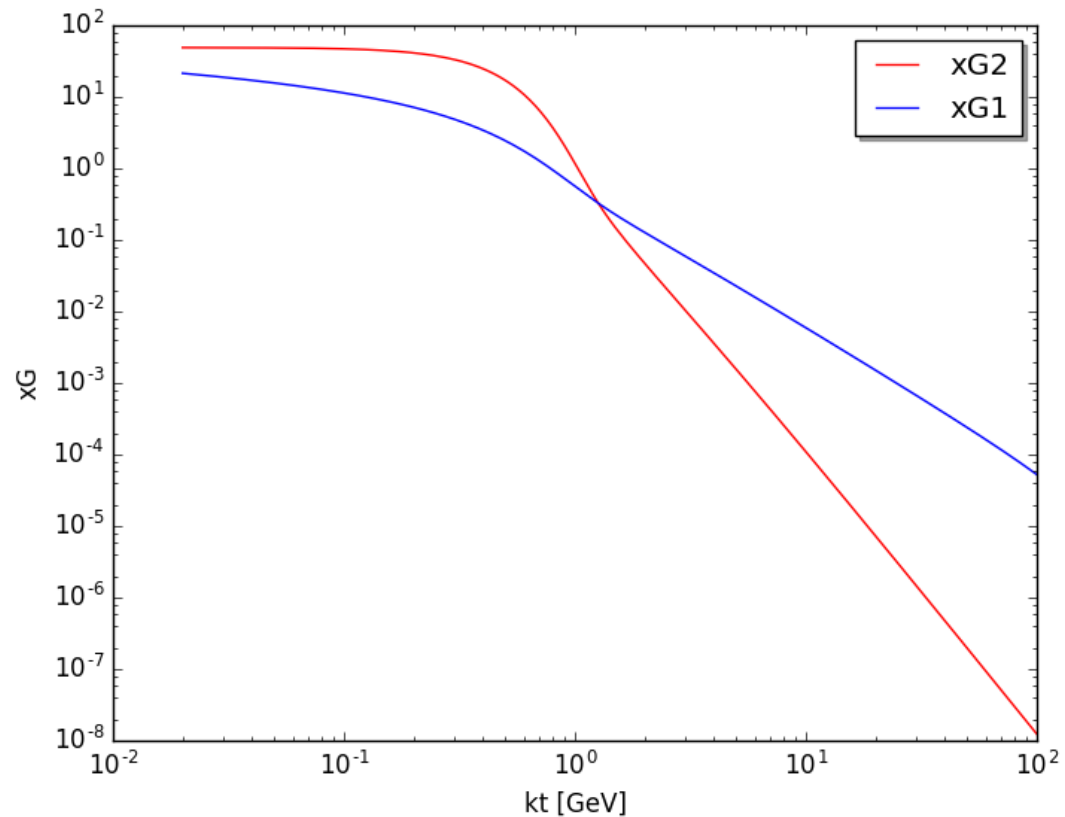


Compute the dipole

$$x_2 G^{(2)}(x_1, x_2, kt)$$

$$x_2 G^{(1)}(x_1, x_2, kt)$$

$Y = 0$, where $Y = \ln \frac{1}{x}$



on.

CROSS SECTION CALCULATION

- d) Compute the transverse momentum distributions as a convolution of the Fourier transform and the gluon distributions.

$$\mathcal{F}_{qg}^{(1)}(x_2, k_t) = x_2 G^{(2)}(x_2, q_t),$$

$$\mathcal{F}_{qg}^{(2)}(x_2, k_t) = \int d^2 q_t x_2 G^{(1)}(x_2, q_t) F(x_2, k_t - q_t),$$

$$\mathcal{F}_{gg}^{(1)}(x_2, k_t) = \int d^2 q_t x_2 G^{(2)}(x_2, q_t) F(x_2, k_t - q_t),$$

$$\mathcal{F}_{gg}^{(2)}(x_2, k_t) = - \int d^2 q_t \frac{(k_t - q_t) \cdot q_t}{q_t^2} x_2 G^{(2)}(x_2, q_t) F(x_2, k_t - q_t),$$

$$\mathcal{F}_{gg}^{(6)}(x_2, k_t) = \int d^2 q_t d^2 q'_t x_2 G^{(1)}(x_2, q_t) F(x_2, q'_t) F(x_2, k_t - q_t - q'_t)$$

CROSS SECTION CALCULATION



Compute t

transform a

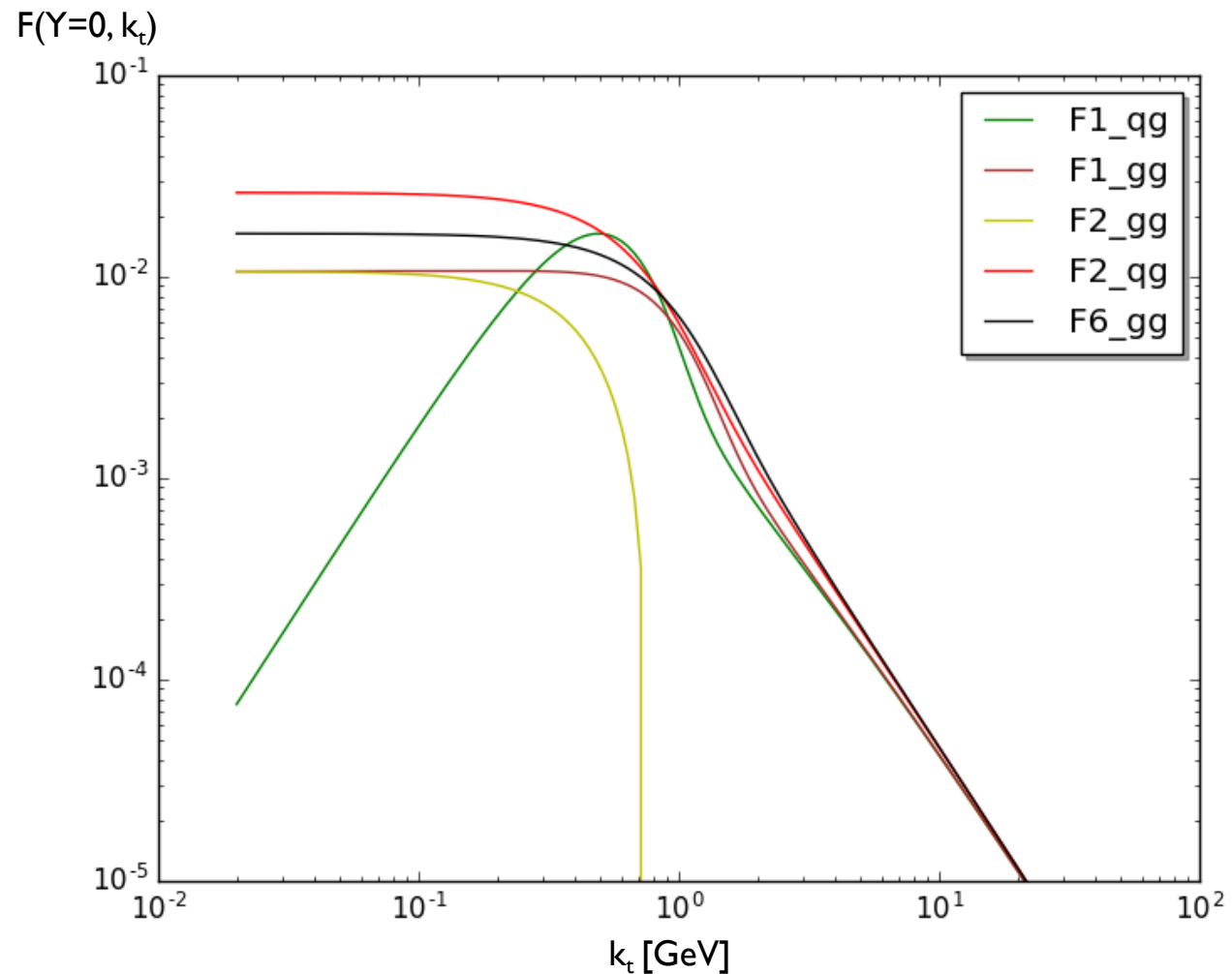
$$\mathcal{F}_{qg}^{(1)}(x_2, k_t)$$

$$\mathcal{F}_{qg}^{(2)}(x_2, k_t)$$

$$\mathcal{F}_{gg}^{(1)}(x_2, k_t)$$

$$\mathcal{F}_{gg}^{(2)}(x_2, k_t)$$

$$\mathcal{F}_{gg}^{(6)}(x_2, k_t)$$



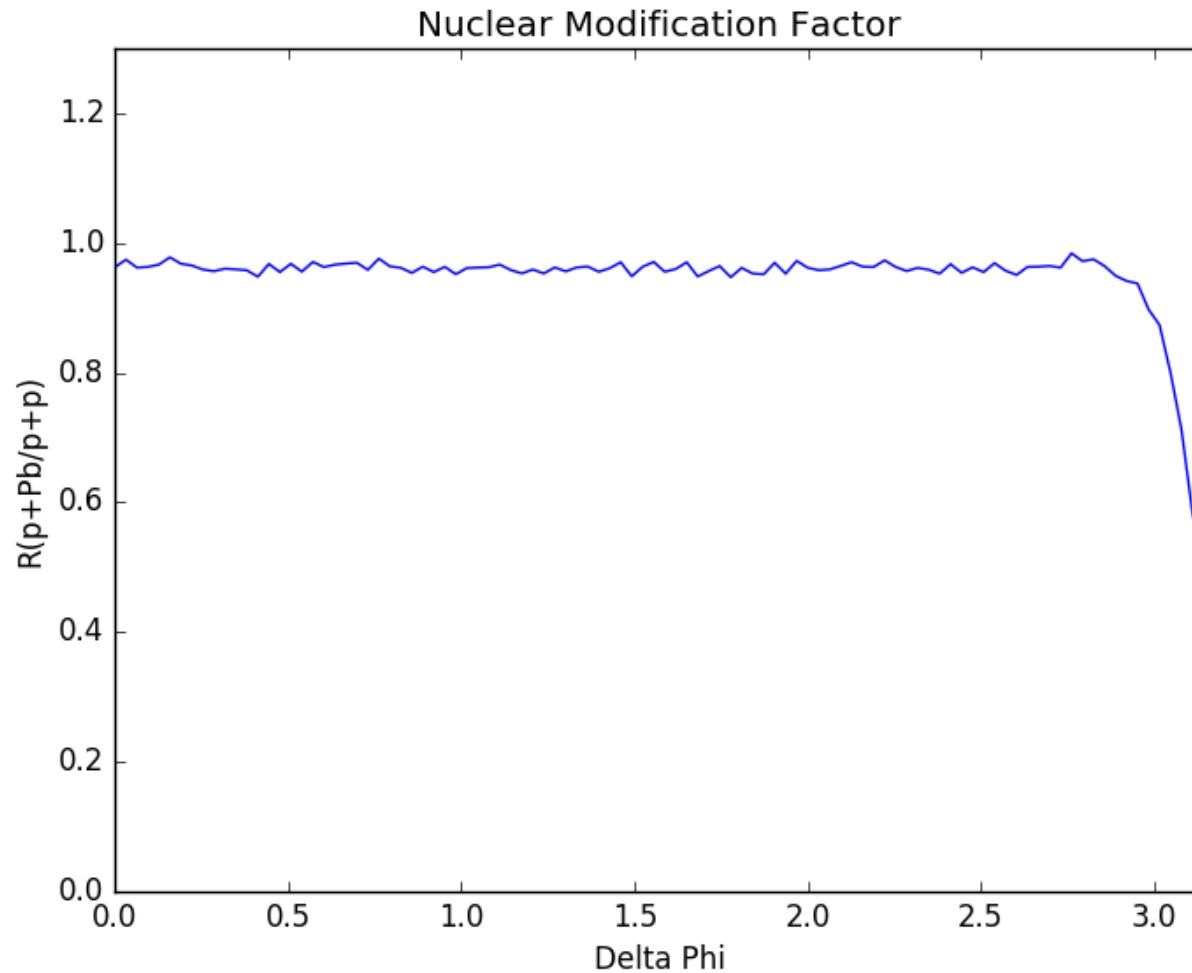
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CROSS SECTION CALCULATION

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

Now we can compute the cross-section and from that the nuclear modification factor.

NUCLEAR MODIFICATION FACTOR



CONCLUSIONS

- The existence of saturation effects can be studied with the use of nuclear modification factor.
- That is caused by the fact that the transverse momentum of the outgoing back-to-back jet is similar to the nuclear saturation scale.
- These studies are all impact parameter independent. Future incorporation of non-trivial impact parameter dependence is highly desired, because it can have a major influence on the studied phenomena.

THANK YOU FOR YOUR ATTENTION

No matter what, don't lose hope. We are all bombastic.

- Dan Nekonečný

REFERENCES

- *Forward di-jet production in $p+Pb$ collisions in the small- x improved TMD factorization framework:* A. van Hameren, P. Kotko, K. Kutak, C. Marquet, E. Petreska and S. Sapeta, arXiv:1607.03121v1, 2016
- *Brief Review of Saturation Physics:* Yuri V. Kovchegov, arXiv:1410.7722v1 [hep-ph], 28 Oct 2014
- *Gluon saturation in dijet production in $p-Pb$ collisions at Large Hadron Collider:* Krzysztof Kutak, Sebastian Sapeta, arXiv:1205.5035v3, 2012

CROSS SECTION CALCULATION

e) Redefine the transverse momentum distributions in the limit of high number of

colors as:

$$\begin{aligned}
 \Phi_{qg \rightarrow qg}^{(1)} &= \mathcal{F}_{qg}^{(1)} & , & & \Phi_{qg \rightarrow qg}^{(2)} &\approx \mathcal{F}_{qg}^{(2)} \\
 \Phi_{gg \rightarrow q\bar{q}}^{(1)} &\approx \mathcal{F}_{gg}^{(1)} & , & & \Phi_{gg \rightarrow q\bar{q}}^{(2)} &\approx -N_c^2 \mathcal{F}_{gg}^{(2)} \\
 \Phi_{gg \rightarrow gg}^{(1)} &\approx \frac{1}{2} \left(\mathcal{F}_{gg}^{(1)} + \mathcal{F}_{gg}^{(6)} \right) & , & & \Phi_{gg \rightarrow gg}^{(2)} &\approx \mathcal{F}_{gg}^{(2)} + \mathcal{F}_{gg}^{(6)}
 \end{aligned}$$

STUDIED PROCESSES

For x_1 and x_2 holds:

$$x_1 = \frac{1}{\sqrt{s}} (|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}) , \quad x_2 = \frac{1}{\sqrt{s}} (|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2})$$

Therefore for $y_1, y_2 \gg 1$ we obtain $x_1 \sim 1$ and $x_2 \ll 1$.

That is why we shall focus on studying back-to-back jets in the forward region in rapidity to detect saturation effects.