Introduction to lattice QCD - Challenges and New Opportunities

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QCD - old challenges and new opportunities

HPC-LEAP
EUROPEAN JOINT DOCTORATES

Outline

Standard model

The Standard Model (SM) is a synthesis of three of the four forces of nature described by gauge theories with coupling constants:

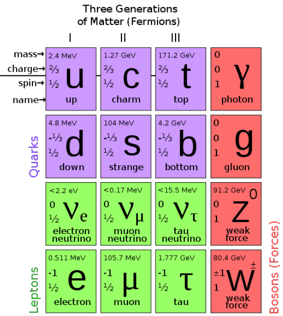
- Strong Interactions: $\alpha_s \sim 1$
- Electromagnetic interactions: $\alpha_{em} \approx 1/137$
- Weak interactions: $G_F \approx 10^{-5}\ GeV^{-2}$.

Basic constituents of matter:

- Six quarks, $u,\ d,\ s,\ c,\ b,\ t$, each in 3 colors, and six leptons $e,\ \nu_e,\ \mu,\ \nu_\mu,\ \tau,\ \nu_\tau$
- The quarks and leptons are classified into 3 generations of families.
- The interactions between the particles are mediated by vector bosons: the 8 gluons mediate strong interactions, the $W^\pm$ and Z mediate weak interactions, and the electromagnetic interactions are carried by the photon γ.
- The weak bosons acquire a mass through the Higgs mechanism.
- The SM is a local gauge field theory with the gauge group $SU(3) \times SU(2) \times U(1)$ specifying the interactions among these constituents.

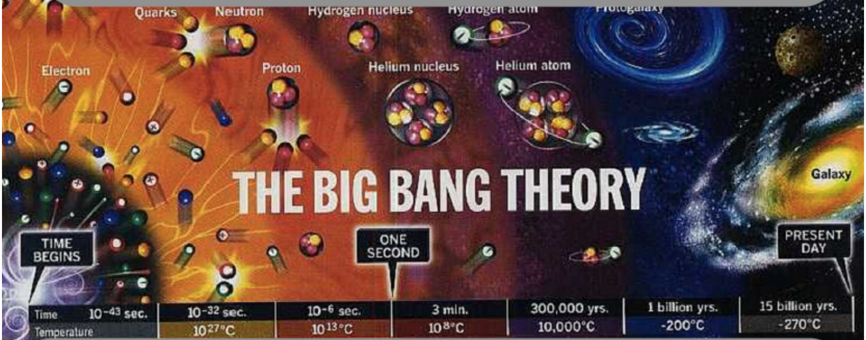
Masses in the Standard Model

Parameters	Number	Comments
Masses of quarks	6	$u,\ d,\ s$ light $c,\ b$ heavy $t = 175 \pm 6\ GeV$
Masses of leptons	6	$e,\ \mu,\ \tau$ $M_{\nu_e,\ \nu_\mu,\ \nu_\tau}$ non-zero
Mass of $W^\pm$	1	80.3 GeV
Mass of Z	1	91.2 GeV
Mass of gluons, γ		0 (Gauge symmetry)
Mass of Higgs	1	$125.03^{+0.26}_{-0.27}\rangle(\mathrm{stat})^{+0.13}_{-0.15}\rangle(\mathrm{sys})$ GeV discovered at LHC, 2012



Strong Interactions:

Evolution from Big-Bang to present Universe and beyond



★ **Quark-Gluon plasma at $t \sim 10^{-32} s$**
studied in heavy ion collisions at RHIC and LHC

★ **Hadron formation at $t \sim 10^{-6} s$**
studied in many experimental facilities word-wide e.g. CERN, JLab, Mainz

Quantum Chromodynamics
QCD

★ **Theoretical description of the strong interactions**

★ **Fundamental constituents:**
6 quarks, 8 gluons (force mediators)
bound states

★ **Quarks & gluons carry a color quantum number**
(Quarks: 3 colors)

★ **Few parameters to explain the spectrum of strong interactions:**
- **quark masses**
- **coupling constant**

Quantum ChromoDynamics (QCD)

QCD-Gauge theory of the strong interaction

Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f\right) \psi_f$$

$$D_\mu = \partial_\mu - ig\frac{\lambda^a}{2} A^a_\mu$$

Harald Fritzsch

Murray Gell-Mann

Heinrich Leutwyler

Phys.Lett. 47B (1973) 365-368

This “simple” Lagrangian produces the amazingly rich structure of strongly interacting matter in the universe.

Numerical simulation of QCD provides essential input for a wide class of complex strong interaction phenomena

Properties of QCD

Confinement

low energies

distances $>$ 1 fm

non-perturbative
e.g. soft hadronic processes
Lattice QCD

hadrons and glue balls

Asymptotic freedom

high energies

distances $<<$ 1 fm

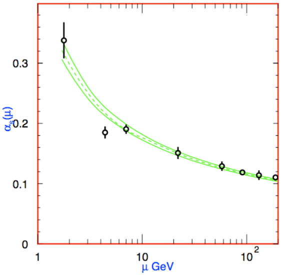
perturbative QCD

quarks and gluons

D. Gross H.D. Politzer F. Wilczek

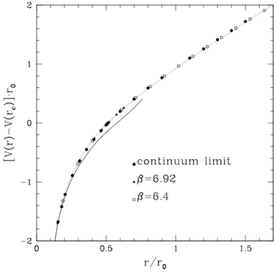
Properties of QCD

Asymptotic freedom: $g(\mu)$



[Yao et al., PDG 2006]

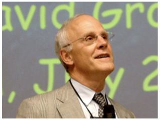
Confinement



[Necco & Sommer, Nucl Phys B622 (2002) 328]

Nobel Prize in Physics 2004

"...for the discovery of asymptotic freedom in the theory of the strong interaction"



David Gross

Frank Wilczek

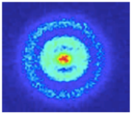
David Politzer

QCD vs QED

QED

Quantum theory of electromagnetic force mediated by exchange of photons

Photon couples to electric charge



A. Stodolna et al., PRL(13)213001

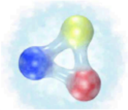
Hydrogen atom

$$m_{\rm Hydrogen} = \underbrace{0.51{\rm MeV}}_{m_{e^-}} + \underbrace{938.29{\rm MeV}}_{m_{p^+}} - \underbrace{13.6{\rm eV}}_{E_{\rm binding}}$$

QCD

Quantum theory of strong force mediated by exchange of gluons

Gluon couples to color charge of quark



artist's impression

Proton

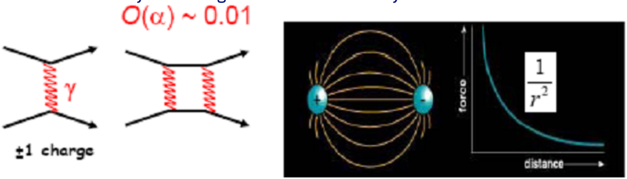
$$m_p = \underbrace{4.4{\rm MeV}}_{2\times m_u} + \underbrace{4.7{\rm MeV}}_{m_d} + \underbrace{929.2{\rm MeV}}_{\rm interaction} = 938{\rm MeV}$$

99% of mass comes from interaction!

QCD versus QED

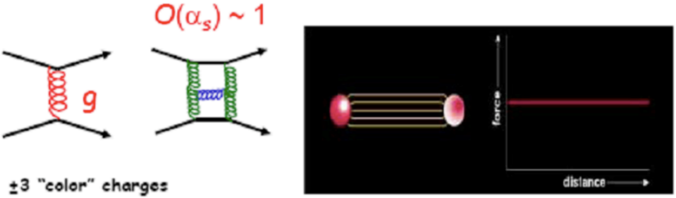
Quantum Electrodynamics (QED): The interaction is due to the exchange of photons. Every time there is an exchange of a photon there is a correction in the interaction of the order of 0.01.

→ we can apply perturbation theory reaching whatever accuracy we like



QCD: Interaction due to exchange of gluons. In the energy range of ~ 1GeV the coupling constant is ~1

→ We can no longer use perturbation theory



QCD on the lattice

- Discrete space-time lattice acts as a non-perturbative regularization scheme with the lattice spacing a providing an ultraviolet cutoff at $\pi/a \to$ no infinities. Furthermore, renormalized physical quantities have a finite well behaved limit as $a \to 0$.
- Can be simulated on the computer using methods analogous to those used for Statistical Mechanics systems. These simulations allow us to calculate correlation functions of hadronic operators and matrix elements of any operator between hadronic states in terms of the fundamental quark and gluon degrees of freedom.

Like continuum QCD lattice QCD has as unknown input parameters the coupling constant α_s and the masses of the up, down, strange, charm and bottom quarks (the top quark is too short lived).
$\Longrightarrow$Lattice QCD provides a well-defined approach to calculate observables non-perturbative starting directly from the QCD Langragian.

Lattice Quantum ChromoDynamics (QCD)

QCD-Gauge theory of the strong interaction

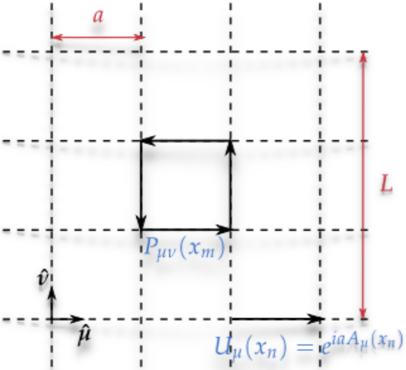
Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t}\bar{\psi}_f\left(i\gamma^\mu D_\mu - m_f\right)\psi_f$$

$$D_\mu = \partial_\mu - ig\frac{\lambda^a}{2}A^a_\mu$$

Choice of fermion discretisation scheme e.g. Clover, Twisted Mass, Staggered, Domain Wall, Overlap

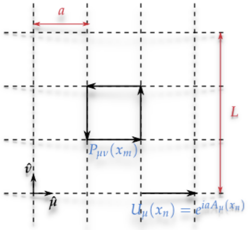
Each has its advantages and disadvantages



Eventually,

- all discretization schemes must agree in the continuum limit $a \to 0$
- observables extrapolated to the infinite volume limit $L \to \infty$

QCD on the lattice



Lattice QCD: K. Wilson, 1974 provided the formulation; M. Creutz, 1980 performed the first numerical simulation

- Discretization of space-time with lattice spacing a and implement gauge invariance
  - quark fields $\psi(x)$ and $\bar{\psi}(x)$ on lattice sites
  - Introduce parallel transporter connecting point x and $x + a\hat{\mu}$: $U_\mu(x) = e^{iaA_\mu(x)}$ i.e. gauge field $U_\mu(x)$ is defined on links $\longrightarrow$ lattice derivative $\mathcal{D}_\mu \psi(x) \to \frac{1}{a}\left[U_\mu(x)\psi(x + a\hat{\mu}) - \psi(x)\right]$
- Finite a provides an ultraviolet cutoff at $\pi/a \to$ non-perturbative regularization; Finite $L \to$ discrete momenta in units of $2\pi/L$ if periodic b.c.
- Construct an appropriate action S and rotate into imaginary time: $S = S_G + S_F$ where $S_F = \sum_x \bar{\psi}(x) D \psi(x)$ i.e. quadratic in the fermions
  $\longrightarrow$ can be integrated out
- Path integral over gauge fields:
  $Z \sim \int \mathcal{D}U_\mu(x) \prod_f det(D_f[U])\, e^{-S_G[U]}$
  $\to$ Monte Carlo simulation to produce a representative ensemble of $\{U_\mu(x)\}$ using the largest supercomputers
  $\to$ Observables: $\langle \mathcal{O} \rangle = \sum_{\{U_\mu\}} O(D^{-1}, U_\mu)$

Computational resources

Juelich SuperComputing Centre, Germany

Peak performance: 5.9 Petaflop/s

458 752 cores

Our time allocation: 65 Million core-h

Swiss National Supercomputing Centre, Switzerland

Peak performance: 7.8 PFlops/s

42 176 cores

Tesla Graphic cards

Our time allocation: 2 Million node-h

(equiv. to 200 Million core-h)

Europe's Fastest GPU SuperComputer

Gauss Centre, Stuttgart, Germany

Peak performance: 7.42 Petaflop/s

185 088 cores

Our time allocation: 48 Million core-h

With simulations at the physical point lattice QCD can provide essential input for the experimental programs.

Questions we would like to address

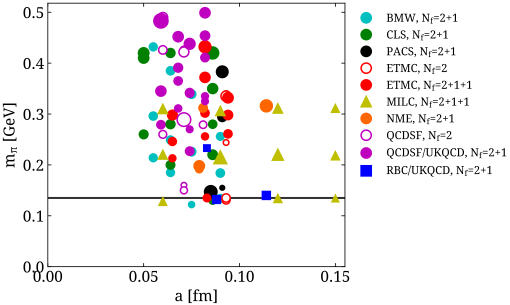
With simulations at the physical value of the pion mass there is a number of interesting questions we want to address:

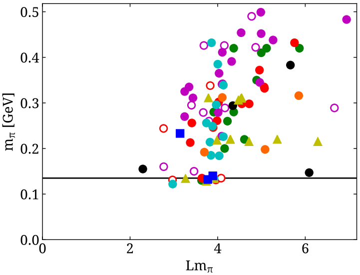
- Can we reproduce known quantities including the excited spectrum of the nucleon and its associated resonances?
- Can we resolve the long-standing issue of the spin content of the nucleon?
- Can we determine accurately enough the charge radius of the proton?
- Can we provide input for experimental searches for new physics?

In this talk I will address two topics:

- The nucleon scalar content or σ-terms as a probe of new physics
- The nucleon spin decomposition of the nucleon
- Nucleon form factors

Status of simulations



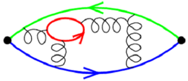


Size of the symbols according to the value of $m_\pi L$: smallest value $m_\pi L \sim 3$ and largest $m_\pi L \sim 6.7$.

Hadron mass

First goal: reproduce the low-lying masses

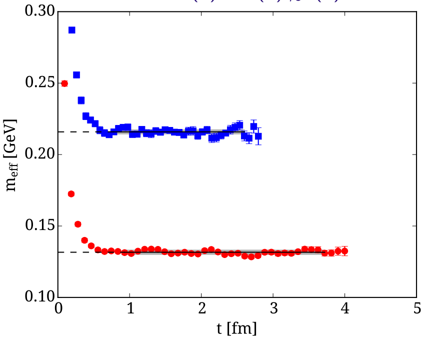
- Use Euclidean correlation functions:



$$G(\vec{q}, t_s) = \sum_{\vec{x}_s} e^{-i\vec{x}_s\cdot\vec{q}} \langle J(\vec{x}_s, t_s) J^\dagger(0)\rangle$$
$$= \sum_{n=0,\cdots,\infty} A_n e^{-E_n(\vec{q})t_s} \stackrel{t_s\to\infty}{\longrightarrow} A_0 e^{-E_0(\vec{q})t_s}$$

Interpolating field with the quantum numbers of π^+: $J(x) = \bar{d}(x)\gamma_5 u(x)$

- Large Euclidean time evolution gives ground state for given quantum numbers $\Longrightarrow$ enables determination of low-lying hadron properties
- $aE_{\rm eff}(\vec{q}, t_s) = \ln\left[G(\vec{q}, t_s)/G(\vec{q}, t_s + a)\right]$
  $= aE_0(\vec{q}) + \text{excited}\rangle\text{states}$
  $\to aE_0(\vec{q}) \stackrel{\vec{q}=0}{\to} am$

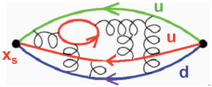


$N_f = 2+1+1$ TM fermions at $m_\pi = 210$ MeV
$N_f = 2$ TM plus clover fermions at physical pion mass

Hadron mass

First goal: reproduce the low-lying masses

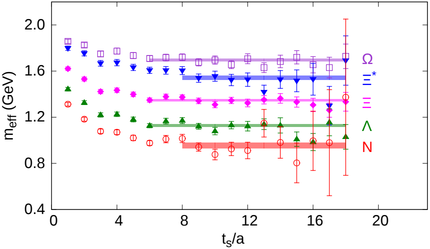
- Use Euclidean correlation functions:



$$G(\vec{q}, t_s) = \sum_{\vec{x}_s} e^{-i\vec{x}_s\cdot\vec{q}} \langle J(\vec{x}_s, t_s) J^\dagger(0)\rangle$$
$$= \sum_{n=0,\cdots,\infty} A_n e^{-E_n(\vec{q})t_s} \xrightarrow{t_s\to\infty} A_0 e^{-E_0(\vec{q})t_s}$$

Interpolating field with the quantum numbers of p: $J(x) = \epsilon^{abc}\left(u^{a\top}(x) C\gamma_5 d^b(x)\right) u^c(x)$

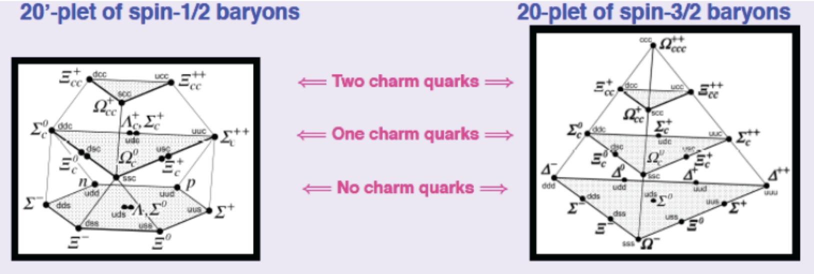
- Large Euclidean time evolution gives ground state for given quantum numbers $\Longrightarrow$ enables determination of low-lying hadron properties
- $aE_{\rm eff}(\vec{q}, t_s) = \ln\left[G(\vec{q}, t_s)/G(\vec{q}, t_s + a)\right]$
  $= aE_0(\vec{q}) + \text{excited}\rangle\text{states}$
  $\to aE_0(\vec{q}) \overset{\vec{q}=0}{\to} am$



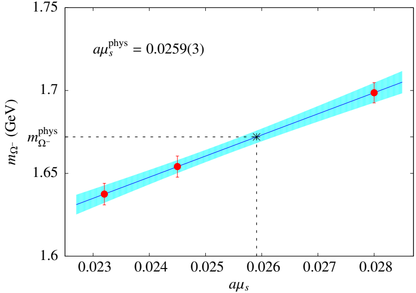
$N_f = 2$ TM plus clover fermions at physical pion mass

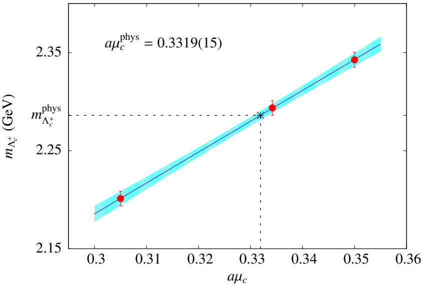
Noise to signal increases with t_s:$\sim e^{(m_h - \frac{3}{2}m_\pi)t_s}$

Hadron masses



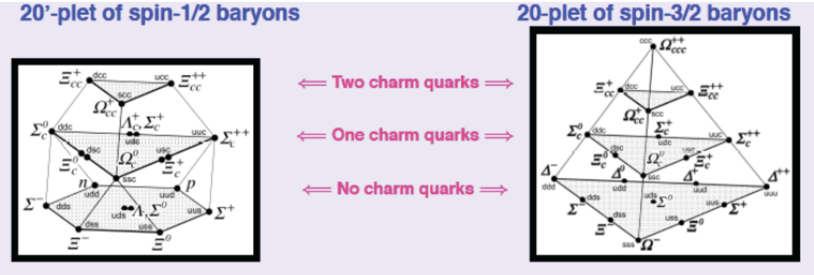
For our computation, the masses of the strange and charm quarks are fixed using the Ω^- and Λ_c^+.

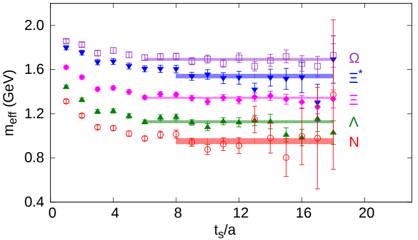


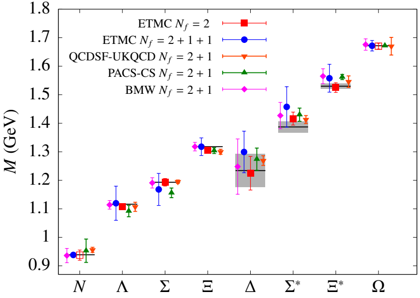


$m_s^R = 108.6(2.2)$ MeV and $m_c^R = 1392.6(23.5)$ MeV, in the $\overline{\rm MS}$-scheme at 2 GeV.

Low-lying spectrum



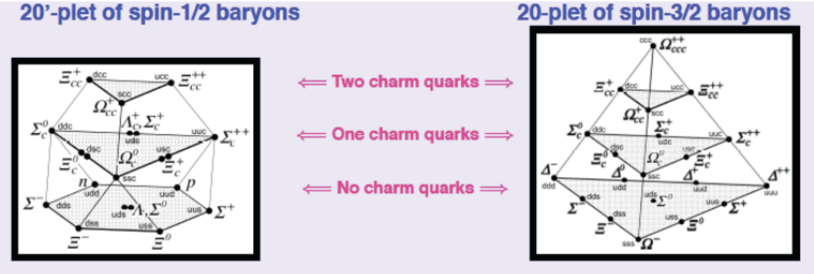


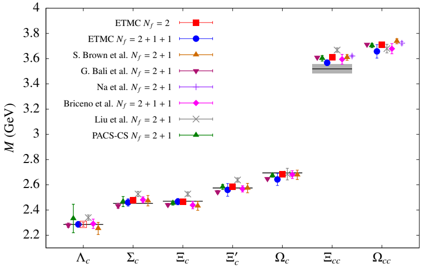


Using $N_f = 2$ simulations at a physical value of the pion mass

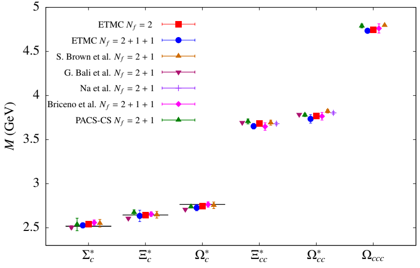
C. Alexandrou and C. Kallidonis, Phys. Rev. D96 (2017) 034511, arXiv:1704.02647

Low-lying spectrum





spin-1/2



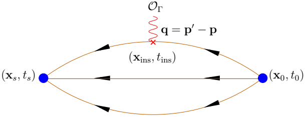
spin-3/2

C. Alexandrou and C. Kallidonis, Phys. Rev. D96 (2017) 034511, arXiv:1704.02647

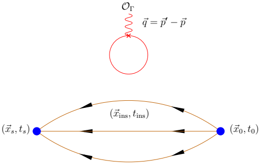
Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_s, t_s) \mathcal{O}_\Gamma^{\mu\nu}(\vec{x}_{\rm ins}, t_{\rm ins}) \overline{J}_\beta(\vec{x}_0, t_0) \rangle$$



connected contribution



disconnected contribution

Evaluation of matrix elements

Three-point functions:

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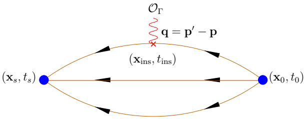

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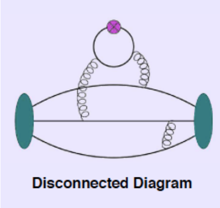

Evaluation of matrix elements

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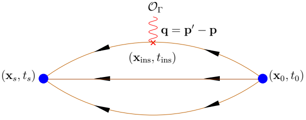
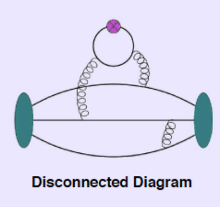


connected contribution



- Plateau method:

$$R(t_s,t_{\rm ins},t_0) \xrightarrow[(t_s-t_{\rm ins})\Delta\gg1]{(t_{\rm ins}-t_0)\Delta\gg1} \mathcal{M}[1+\ldots e^{-\Delta(\mathbf{p})(t_{\rm ins}-t_0)}+\ldots e^{-\Delta(\mathbf{p}')(t_s-t_{\rm ins})}]$$

- Summation method: Summing over $t_{\rm ins}$:

$$\sum_{t_{\rm ins}=t_0}^{t_s} R(t_s,t_{\rm ins},t_0) = \text{Const.} + \mathcal{M}[(t_s-t_0)+\mathcal{O}(e^{-\Delta(\mathbf{p})(t_s-t_0)})+\mathcal{O}(e^{-\Delta(\mathbf{p}')(t_s-t_0)})].$$

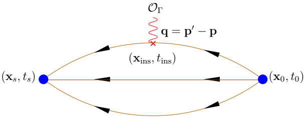
  Excited state contributions are suppressed by exponentials decaying with t_s-t_0, rather than $t_s-t_{\rm ins}$ and/or $t_{\rm ins}-t_0$
  However, one needs to fit the slope rather than to a constant or take differences and then fit to a constant
  L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys. B293, 420 (1987); S. Capitani *et al.*, arXiv:1205.0180
- Fit keeping the first excited state, T. Bhattacharya *et al.*, arXiv:1306.5435

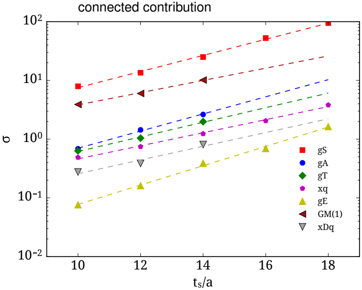
All should yield the same answer in the end of the day!

Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_s, t_s)\mathcal{O}^{\mu\nu}_\Gamma(\vec{x}_{\rm ins}, t_{\rm ins})\overline{J}_\beta(\vec{x}_0, t_0)\rangle$$





- $\mathcal{M}$ the desired matrix element
- t_s, $t_{\rm ins}$, t_0 the sink, insertion and source time-slices
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

To ensure ground state dominance need multiple sink-source time separations ranging from 0.9 fm to 1.5 fm

Nucleon isovector charges: g_A, g_S, g_T

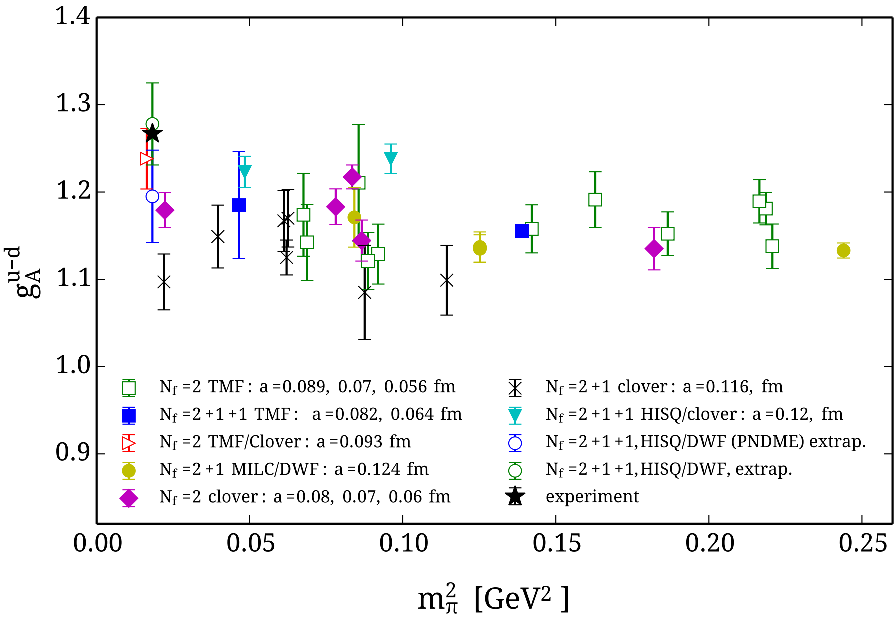
- axial-vector operator: $\mathcal{O}_A^a = \bar{\psi}(x)\gamma^\mu\gamma_5 \frac{\tau^a}{2}\psi(x)$
- scalar operator: $\mathcal{O}_S^a = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)$
- pseudoscalar: $\mathcal{O}_p^a = \bar{\psi}(x)\gamma_5\frac{\tau^a}{2}\psi(x)$
- tensor operator: $\mathcal{O}_T^a = \bar{\psi}(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$

$\Longrightarrow$ extract from matrix element: $\langle N(\vec{p}')\mathcal{O}_X N(\vec{p})\rangle|_{q^2=0}$

• Axial charge g_A • Scalar charge g_S • Pseudoscalar charge g_p, • Tensor charge g_T

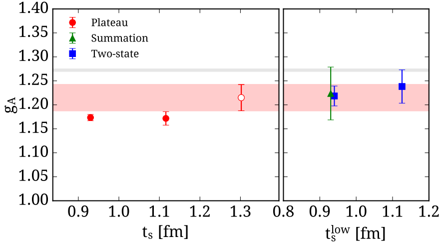
(i) isovector combination has no disconnect contributions; (ii) g_A well known experimentally, Goldberger-Treiman relation yields g_p, g_T to be measured at JLab, Predict g_S

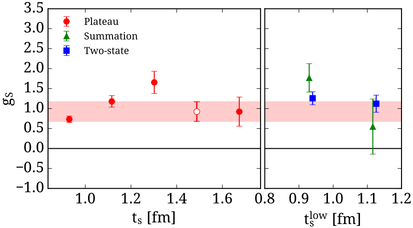
Nucleon axial charge g_A



Nucleon scalar charge

- $N_f = 2$ twisted mass plus clover, $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV
- ~ 9260 statistics for $t_s/a = 10, 12, 14$, ~ 48000 for $t_s/a = 16$ and ~ 70000 for $t_s/a = 18$
- 5 sink-source time separations ranging from 0.9 fm to 1.7 fm

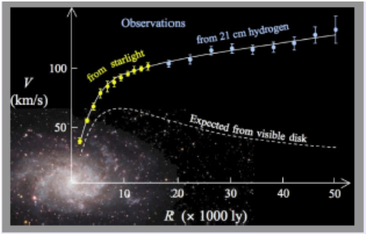




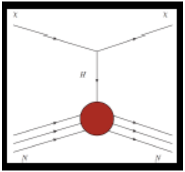
At the physical point we find from the plateau method:

- $g_A^{\rm isov} = 1.21(3)(3)$, $g_S^{\rm isov} = 0.93(25)(33)$, $g_T^{\rm isov} = 1.00(2)(1)$
- g_A further study for larger t_s. Important to keep constant error
  A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522
- New analysis of COMPASS and Belle data: $g_T^{u-d} = 0.81(44)$, M. R. A. Courtoy, A. Bacchettad, M. Guagnellia, arXiv: 1503.03495
- For g_S increasing the sink-source time separation to ~ 1.5 fm is crucial

The quark content of the nucleon



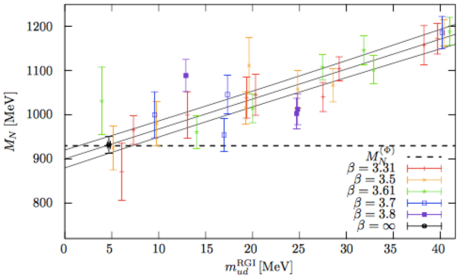
Rotational curve of M33 galaxy

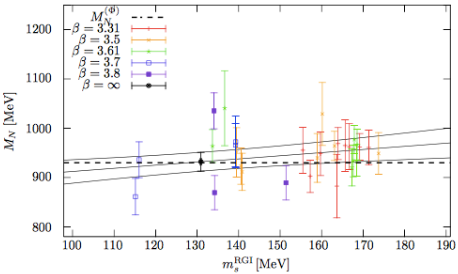


- $\sigma_f \equiv m_f \langle N|\bar{q}_f q_f|N\rangle$: measures the explicit breaking of chiral symmetry
  Largest uncertainty in interpreting experiments for direct dark matter searches - Higgs-nucleon coupling depends on σ,
  e.g. spin-independent cross-section can vary an order of magnitude if $\sigma_{\pi N}$ changes from 35 MeV to 60 MeV, J. Ellis, K. Olive, C. Savage, arXiv:0801.3656
- In lattice QCD:
  - Feynman-Hellmann theorem: $\sigma_l = m_l \frac{\partial m_N}{\partial m_l}$
    Similarly $\sigma_s = m_s \frac{\partial m_N}{\partial m_s}$, S. Dürr *et al.* (BMW$_c$) Phys.Rev.Lett. 116 (2016) 172001
  - Direct computation of the scalar matrix element
    G. Bali, *et al.* (RQCD) Phys.Rev. D93 (2016) 094504, arXiv:1603.00827; Yi-Bo Yang *et al.* (χQCD) Phys.Rev. D94 (2016) no.5, 054503; A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001;

The quark content of the nucleon via Feynman-Hellmann

BMW Collaboration: 47 lattice ensembles with $N_f = 2 + 1$ clover fermions, 5 lattice spacings down to 0.054 fm, lattice sizes up to 6 fm and pion masses down to 120 MeV.

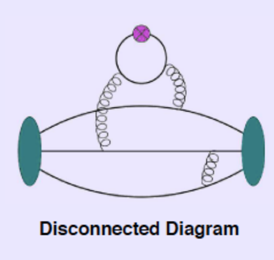




$\sigma_{\pi N} = 38(3)(3)$ MeV $\sigma_s = 105(41)(37)$ MeV

The quark content of the nucleon via direct determination

Need disconnected contributions



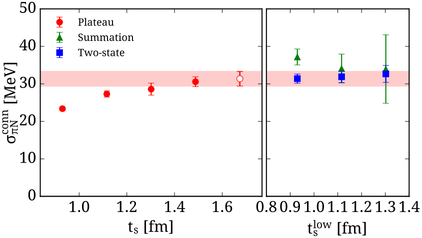
- RQCD: $N_f = 2$ clover fermions with a range of pion masses down to $m_\pi = 150$ MeV and $a = 0.06 - 0.08$ fm G. Bali, *et al.*, Phys.Rev. D93 (2016) 094504, arXiv:1603.00827
- χQCD: Valence overlap fermions on $N_f = 2 + 1$ flavor domain-wall fermion (DWF) configurations, 3 ensembles of $m_\pi = 330$ MeV, $m_\pi = 300$ MeV and $m_\pi = 139$ MeV Yi-Bo Yang *et al.*, Phys.Rev. D94 (2016) no.5, 054503; M/ Gong *et al.*, Phys. Rev. D 88 (2013) 014503 arXiv:1304.1194
- ETM Collaboration: $N_f = 2$ twisted mass plus clover, $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV, A. Abdel-Rehim *et al.*, arXiv:1601.3656, PRL116 (2016) 252001

The quark content of the nucleon from ETMC

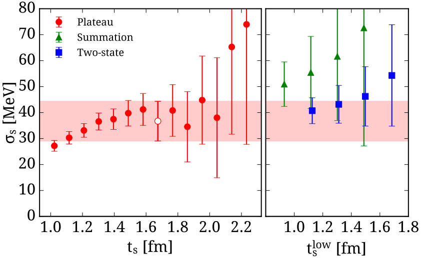
$N_f = 2$ twisted mass plus clover, $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV

- Connected: $t/a = 10, 12, 14$ 9264 statistics, $t/a = 16$ ∼47,600 statistics and $t/a = 18$ ∼70,000 statistics
- Disconnected: ∼213,700 statistics

A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001



Connected



Disconnected strange

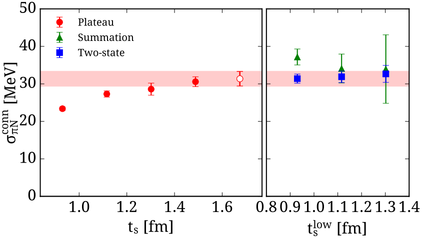
Our results are: $\sigma_{\pi N} = 36(2)$ MeV

The quark content of the nucleon from ETMC

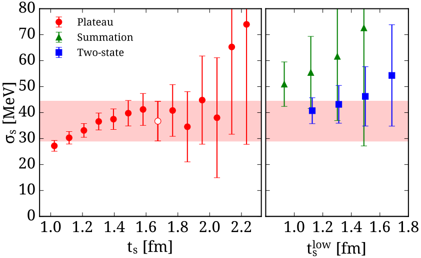
$N_f = 2$ twisted mass plus clover, $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV

- Connected: $t/a = 10, 12, 14$ 9264 statistics, $t/a = 16$ ~47,600 statistics and $t/a = 18$ ~70,000 statistics
- Disconnected: ~213,700 statistics

A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001



Connected

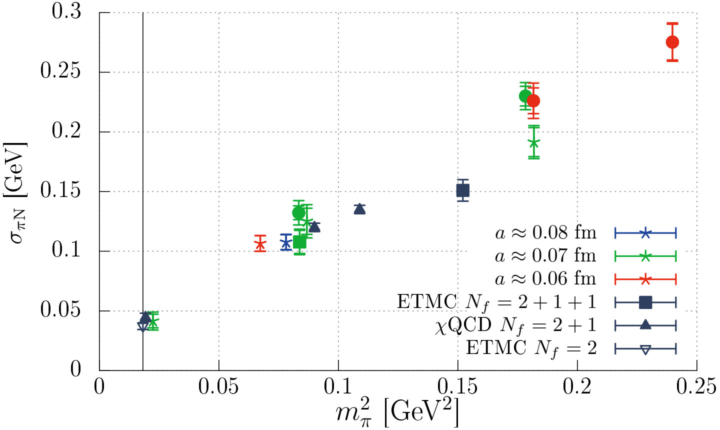


Disconnected strange

Our results are: $\sigma_{\pi N} = 36(2)$ MeV $\sigma_s = 37(8)$ MeV $\sigma_c = 83(17)$ MeV

The quark content of the nucleon

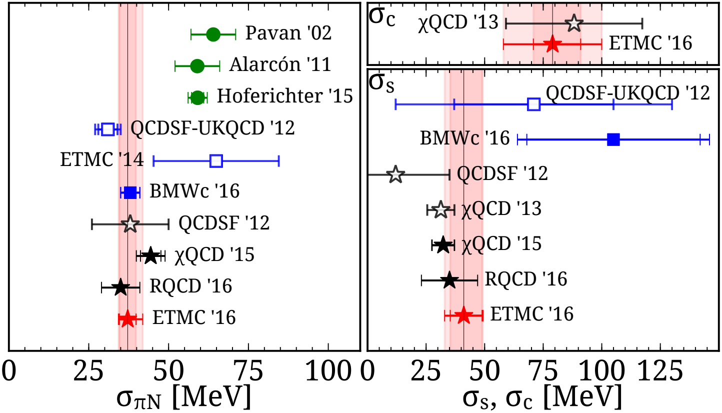
Comparison of results



G. Bali, *et al.*, Phys.Rev. D93 (2016) 094504, arXiv:1603.00827

The quark content of the nucleon

Comparison of results

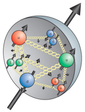


Recent results from lattice QCD at the physical point and from phenomenology. Filled symbols for lattice QCD results include simulations with pion mass close to its physical value, A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001

Nucleon spin

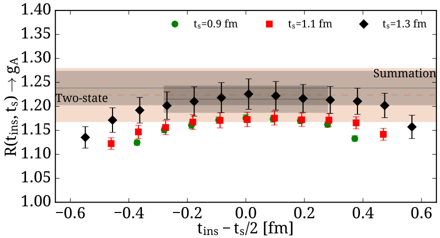
Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^g$

$J^q = \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0)\right)$ and $\Delta\Sigma^q = g_A^q$

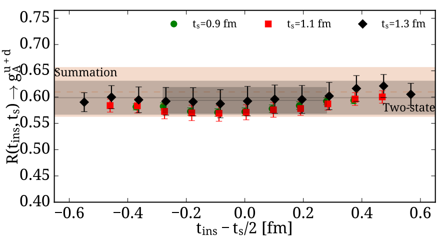


Need isoscalar g_A, which has disconnected contributions

- $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm
- Intrinsic quark spin: $\Delta\Sigma^q = g_A^q$



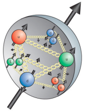
Isovector



Connected isoscalar

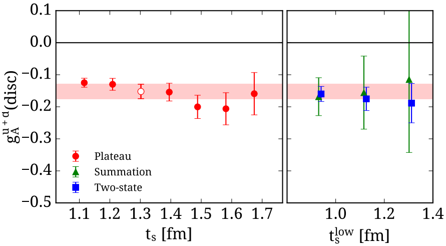
Nucleon spin

Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^g$

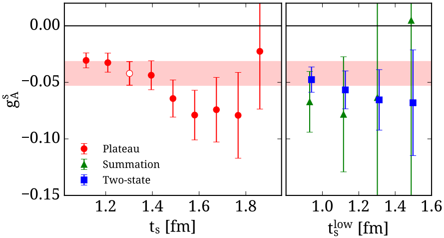


$J^q = \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0)\right)$ and $\Delta\Sigma^q = g_A^q$

Need isoscalar g_A, which has disconnected contributions



Isoscalar disconnected



Strange

We find from the plateau method:

- $g_A^{u+d} = -0.15(2)$ (disconnected only) with 854,400 statistics
- Combining with the isovector we find: $g_A^u = 0.828(21)$, $g_A^d = -0.387(21)$
- $g_A^s = -0.042(10)$ with 861,200 statistics

Quark total spin J^q

Generalized parton distributions functions (GPDs) are matrix elements of light cone operators that cannot be computed directly $\rightarrow$ Factorization leads to matrix elements of local operators:

- vector operator

$$\mathcal{O}_{V^a}^{\mu_1\cdots\mu_n} = \bar{\psi}(x)\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \ldots i \overleftrightarrow{D}^{\mu_n\}} \frac{\tau^a}{2}\psi(x)$$

- axial-vector operator

$$\mathcal{O}_{A^a}^{\mu_1\cdots\mu_n} = \bar{\psi}(x)\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \ldots i \overleftrightarrow{D}^{\mu_n\}} \gamma_5 \frac{\tau^a}{2}\psi(x)$$

- tensor operator

$$\mathcal{O}_{T^a}^{\mu_1\cdots\mu_n} = \bar{\psi}(x)\sigma^{\{\mu_1,\mu_2} i \overleftrightarrow{D}^{\mu_3} \ldots i \overleftrightarrow{D}^{\mu_n\}} \frac{\tau^a}{2}\psi(x)$$

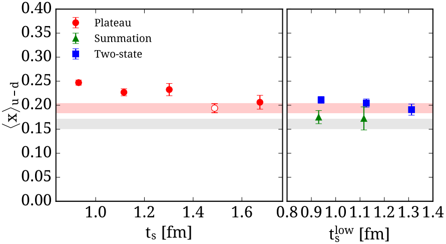
Special cases:

- no-derivative $\rightarrow$ nucleon form factors
- For $Q^2 = 0 \rightarrow$ parton distribution functions
  one-derivative $\rightarrow$ first moments e.g. average momentum fraction $\langle x \rangle$
  Generalized form factor decomposition:

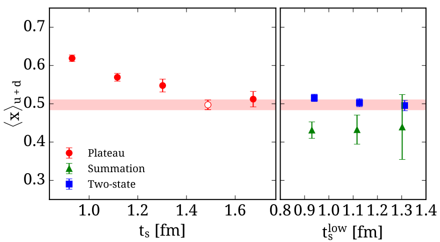
$$\langle N(p',s')|\mathcal{O}_{V3}^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]\frac{1}{2}u_N(p,s)$$

Total quark spin $J^q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$ and $\langle x \rangle_q = A_{20}^q(0)$

Momentum fraction $\langle x \rangle_{u-d}$

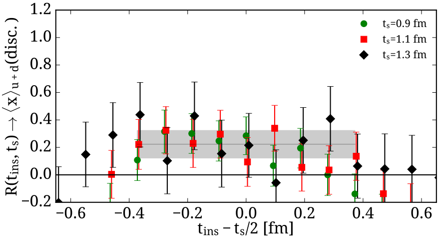


Results for the isovector in the $\overline{\mathrm{MS}}$ at 2 GeV

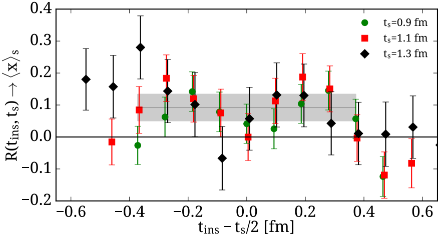


Results for the connected isoscalar in the $\overline{\mathrm{MS}}$ at 2 GeV

Momentum fraction $\langle \mathbf{x} \rangle_{u-d}$



Results for the disconnected isoscalar



Results for the strange

At the physical point we find in the $\overline{\text{MS}}$ at 2 GeV from the plateau method ($\mathcal{O}(860, 000)$ statistics):

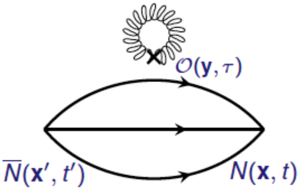
- $\langle x \rangle_{u-d} = 0.194(9)(10)$
- $\langle x \rangle_{u+d+s} = 0.80(12)_{\text{stat}}(10)_{\text{syst}}$

  $\langle x \rangle_{u+d+s}$ is perturbatively renormalized to one-loop due to its mixing with the gluon operator.

A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

Gluon content of the nucleon

- Gluons carry a significant amount of momentum and spin in the nucleon
  - Compute gluon momentum fraction : $\langle x \rangle_g = A_{20}^g$
  - Compute gluon spin: $J^g = \frac{1}{2}(A_{20}^g + B_{20}^g)$
- Nucleon matrix of the gluon operator: $O_{\mu\nu} = -G_{\mu\rho}G_{\nu\rho}$
  $\rightarrow$ gluon momentum fraction extracted from
  $\langle N(0)|O_{44} - \frac{1}{3}O_{jj}|N(0)\rangle = m_N < x >_g$



- Disconnected correlation function, known to be very noisy

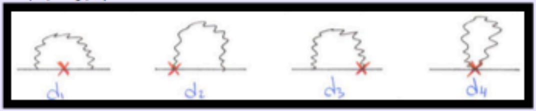
$\Rightarrow$ we employ several steps of stout smearing in order to remove fluctuations in the gauge field

- Results are computed on the $N_f = 2$ ensemble at the physical point, $m_\pi = 131$ MeV, $a = 0.093$ fm, $V = 48^3 \times 96$, A. Abdel-Rehim *et al.* (ETMC):1507.04936
- The methodology was tested for $N_f = 2+1+1$ twisted mass at $m_\pi = 373$ MeV, C. Alexandrou, V. Drach, K. Hadjiyiannakou, K. Jansen, B. Kostrzewa, C. Wiese, PoS LATTICE2013 (2014) 289

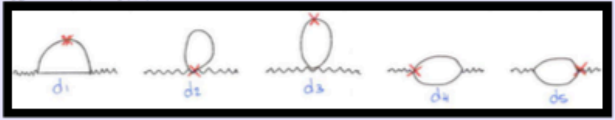
Nucleon gluon moment-Renormalization

Mixing with $\langle x \rangle_{u+d+s} \Longrightarrow$ Perturbation theory - M. Constantinou and H. Panagopoulos

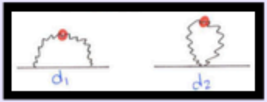
$\times Z_{qq}$: $\Lambda_{qq} = \langle q|\mathcal{O}_q|q\rangle$



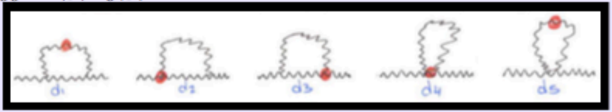
$\times Z_{qg}$: $\Lambda_{qg} = \langle g|\mathcal{O}_q|g\rangle$



$\bullet Z_{gq}$: $\Lambda_{gq} = \langle q|\mathcal{O}_g|q\rangle$



$\bullet Z_{gg}$: $\Lambda_{gg} = \langle g|\mathcal{O}_g|g\rangle$



Nucleon gluon moment-Renormalization

Mixing with $\langle x \rangle_{u+d+s} \Longrightarrow$ Perturbation theory - M. Constantinou and H. Panagopoulos

$\times Z_{qq}$: $\Lambda_{qq} = \langle q|\mathcal{O}_q|q\rangle$

$$\boxed{Z_{gg}=1+\frac{g^2}{16\pi^2}\left(1.0574\,N_f+\frac{-13.5627}{N_c}-\frac{2\,N_f}{3}\log(a^2\bar{\mu}^2)\right)}$$

$\times Z_{qg}$: $\Lambda_{qg} = \langle g|\mathcal{O}_q|g\rangle$

$$\boxed{Z_{gq}=0+\frac{g^2\,C_f}{16\pi^2}\left(0.8114+0.4434\,c_{SW}-0.2074\,c_{SW}^2+\frac{4}{3}\log(a^2\bar{\mu}^2)\right)}$$

$\bullet Z_{gq}$: $\Lambda_{gq} = \langle q|\mathcal{O}_g|q\rangle$

$$\boxed{Z_{qq}=1+\frac{g^2}{16\pi^2}\left(-1.8557+2.9582\,c_{SW}+0.3984\,c_{SW}^2-\frac{8}{3}\log(a^2\bar{\mu}^2)\right)}$$

$\bullet Z_{gg}$: $\Lambda_{gg} = \langle g|\mathcal{O}_g|g\rangle$

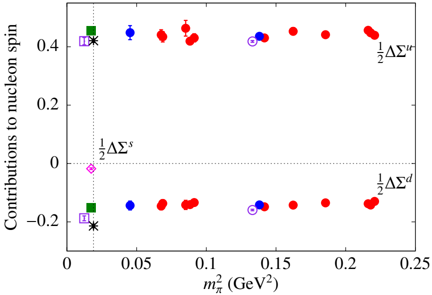
$$\boxed{Z_{qg}=0+\frac{g^2\,N_f}{16\pi^2}\left(0.2164+0.4511\,c_{SW}+1.4917\,c_{SW}^2-\frac{4}{3}\log(a^2\bar{\mu}^2)\right)}$$

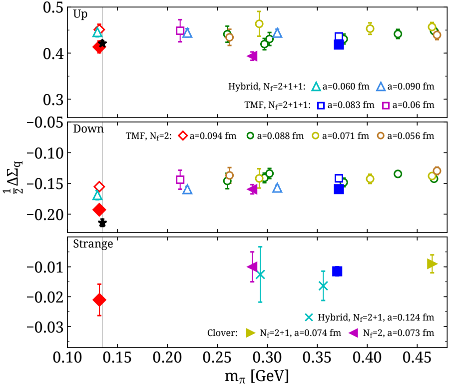
Results for the gluon content

- 2094 gauge configurations with 100 different source positions each $\rightarrow$ more than 200 000 measurements
- Due to mixing with the quark singlet operator, the renormalization and mixing coefficients had to be extracted from a one-loop perturbative lattice calculation, M. Constantinou and H. Panagopoulos
- $\langle x \rangle_{g,\text{bare}} = 0.318(24) \xrightarrow{\text{Renormalization}}$ $< x >_g^R = Z_{gg} < x >_g + Z_{gq} < x >_{u+d+s} = 0.267(12)_{\text{stat}}(10)_{\text{syst}}$. The renormalization is perturbatively done using two-levels of stout smearing. The systematic error is the difference between using one- and two-levels of stout smearing.
- Momentum sum is satisfied: $\sum_q \langle x \rangle_q + \langle x \rangle_g = \langle x \rangle_{u+d}^{CI} + \langle x \rangle_{u+d+s}^{DI} + \langle x \rangle_g = 1.07(12)_{\text{stat}}(10)_{\text{syst}}$

Nucleon spin

Disconnected contribution using $\mathcal{O}(860000)$ statistics





Nucleon spin

Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^g$

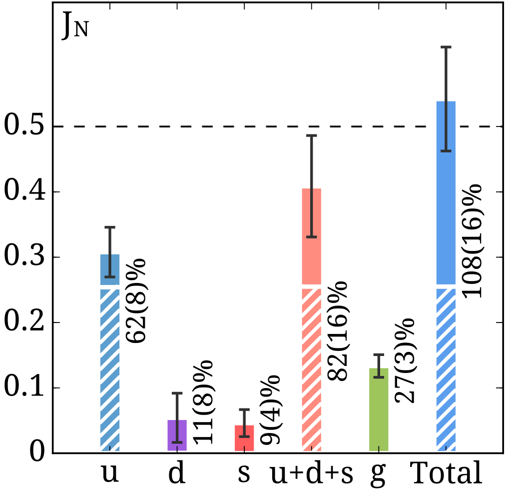
$$
\begin{aligned}
\frac{1}{2}\Delta\Sigma^u &= 0.415(13)(2), & \tfrac{1}{2}\Delta\Sigma^d &= -0.193(8)(3), & \frac{1}{2}\Delta\Sigma^s &= -0.021(5)(1) \\
J^u &= 0.308(30)(24), & J^d &= 0.054(29)(24), & J^s &= 0.046(21) \\
L^u &= -0.107(32)(24), & L^d &= 0.247(30)(24), & L^s &= 0.067(21)(1)
\end{aligned} \tag{1}
$$

We find that $B_{20}^q(0) \sim 0 \longrightarrow$ taking $B_{20}(0)^g \sim 0$ we can directly check the nucleon spin sum:

$$\boxed{J_N = (0.308)_u + (0.054)_d + (0.046)_s + (0.133)_g = 0.54(6)(5)}$$

The proton spin puzzle

1987: the European Muon Collaboration showed that only a fraction of the proton spin is carried by the quarks $\Longrightarrow$ ETMC has now provided the solution



Recent results from lattice QCD at the physical point

C.A. *et al.*, Phys. Rev. Lett. (in press) arXiv:1706.02973





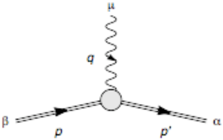


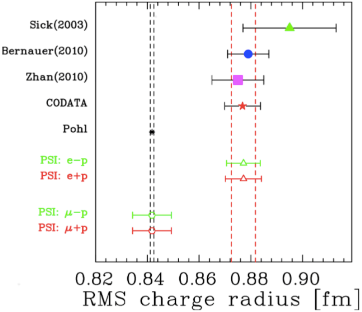
Electromagnetic form factors

$$\langle N(p',s')|j^\mu(0)|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u_N(p,s)$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2}F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$



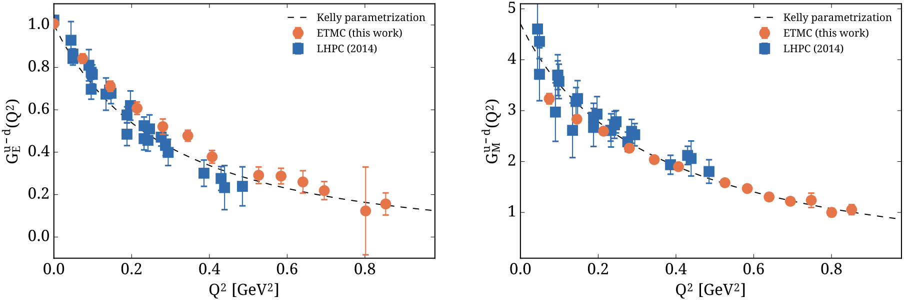


E. J. Downie, EPJ Conf. 113 (2016) 05021

- Proton radius extracted from muonic hydrogen is 7.9 σ different from the one extracted from electron scattering, R. Pohl *et al.*, Nature 466 (2010) 213
- Muonic measurement is ten times more accurate and a reanalysis of electron scattering data may give agreement with muonic measurement

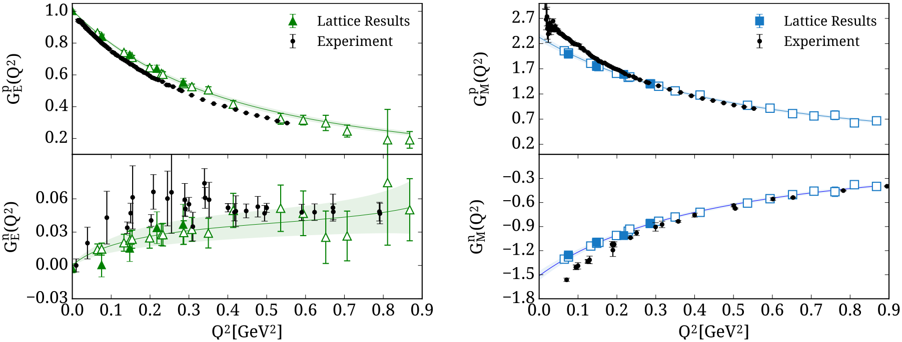
Recent results on the electric and magnetic form factors

Isovector form factors



- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$ G_E with $t_s = 1.7$ fm and 66,000 statistics, G_M with $t_s = 1.3$ fm and 9,300 statistics
- LHPC using $N_f = 2 + 1$ clover fermions, $a = 0.116$ fm, 48^4, summation method with 3 values of t_s from 0.9 fm to 1.4 fm and $\sim 7,800$ statistics, 1404.4029

Recent results on the electric and magnetic form factors



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only<1>C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, C. Kallidonis, G. Koutsou and A. Vaquero Aviles-Casco. Phys. Rev. D96 (2017) 034503, arXiv:1706.00469

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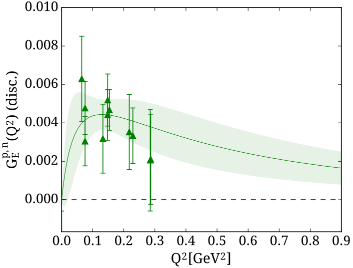

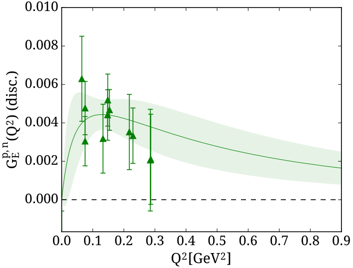

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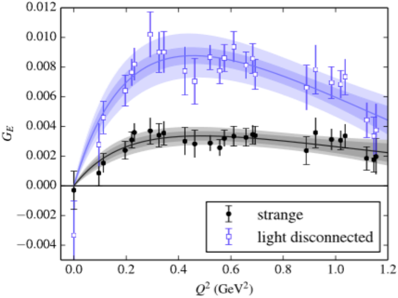
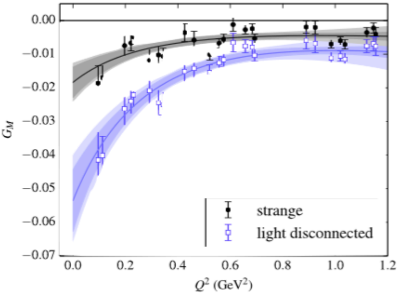
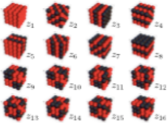
Strange Electromagnetic form factors

Experimental determination: Parity violating $e - N$ scattering
HAPPEX experiment finds $G_M^s(0.62) = -0.070(67)$
New methods for disconnected fermion loops: hierarchical probing, A. Stathopoulos, J. Laeuchli, K. Orginos, arXiv:1302.4018





$N_f = 2 + 1$ clover fermions, $m_\pi \sim 320$ MeV, J. Green et al., Phys.Rev. D92 (2015) 3, 031501, arXiv: 1505.01803



Sampling of the fermion propagator using site colouring schemes

Strange Electromagnetic form factors

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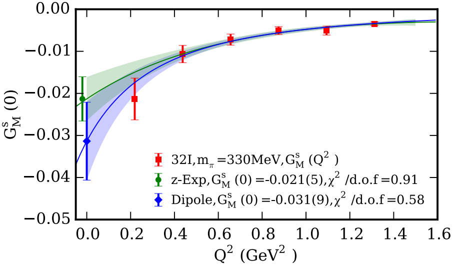
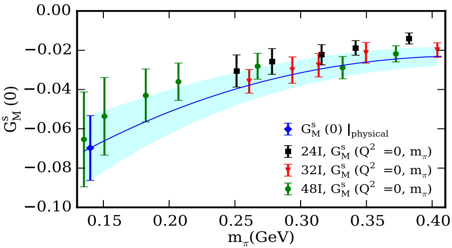
HAPPEX experiment finds $G_M^s(0.62) = -0.070(67)$

R. S. Sufian *et al.* (χQCD Collaboration) 1606.07075



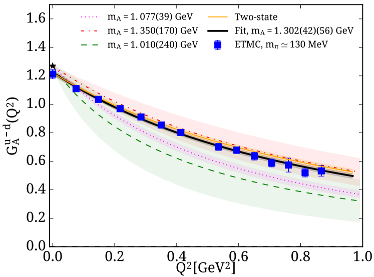


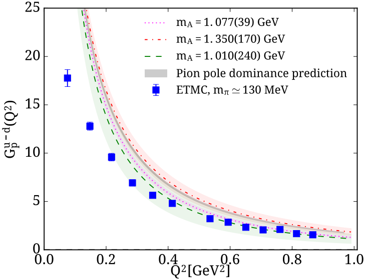
Overlap valence on $N_f = 2 + 1$ domain wall fermions, $24^3 \times 64$, $a = 0.11$ fm, $m_\pi = 330$ MeV; $32^3 \times 64$, $a = 0.083$ fm, $m_\pi = 300$ MeV and 48^3, a=0.11 fm, $m_\pi = 139$ MeV

Recent results on the axial form factors

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = i\sqrt{\frac{m_N^2}{E_N(\vec{p}\,')E_N(\vec{p})}}\,\bar{u}_N(p',s')\left(\gamma_\mu G_A(Q^2) - i\frac{Q_\mu}{2m_N}G_p(Q^2)\right)\gamma_5 u_N(p,s)$$

Isovector





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C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, C. Kallidonis, G. Koutsou and A. Vaquero Aviles-Casco. Phys. Rev. D, arXiv:1705.03399 [hep-lat]

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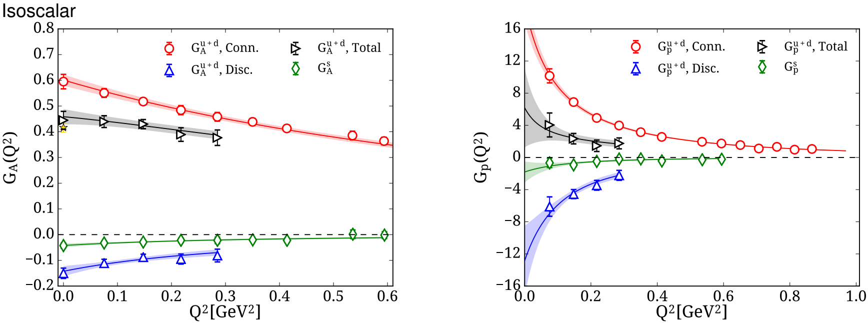

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C. A. *et al.* (ETMC), Phys. Rev. D, arXiv:1705.03399 [hep-lat]

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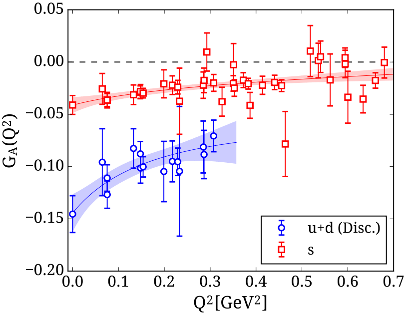
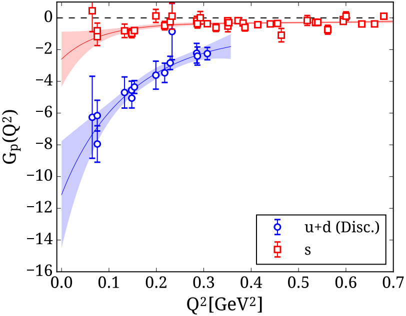
Isoscalar





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C. A.*et al.* (ETMC), Phys. Rev. D, arXiv:1705.03399 [hep-lat]

Direct evaluation of parton distribution functions - an exploratory study

$$\tilde{a}_n(x, \Lambda, P_3) = \int_{-\infty}^{+\infty} dx\, x^{n-1}\, \tilde{q}(x, \Lambda, P_3)\rangle,$$

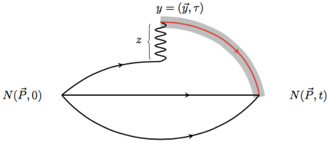
$$\tilde{q}(x, \Lambda, P_3) = \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{-izxP_3} \underbrace{\langle P|\bar{\psi}(z,0)\rangle\gamma_3\, W(z)\psi(0,0)|P\rangle}_{h(P_3,z)\rightarrow\ \text{can be computed in LQCD}}$$

is the quasi-distribution defined by X. Ji Phys.Rev.Lett. 110 (2013) 262002, arXiv:1305.1539
Exploratory calculations:

- Huey-Wen Lin *et al.* Phys. Rev. D91 (2015) 054510, Clover on $N_f = 2+1+1$ HISQ, $m_\pi = 310 MeV$ and Jiunn-Wei Chen *et al.*, arXiv:1603.06664
- C.A., K. Cichy, E. G. Ramos, V. Drach, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, Phys.Rev. D92 (2015) 014502
- $N_f = 2+1+1$, $V = 32^3 \times 64$, $m_\pi = 373$ MeV, $a \approx 0.082$ fm
- 1000 gauge configurations with 15 source positions each and 2 sets stochastic samples

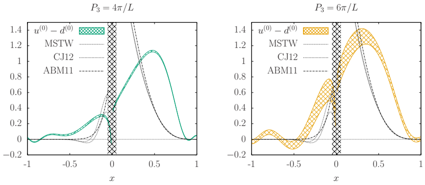
→ 30 000 measurements

- 5 steps of HYP smearing for the gauge links in the operator
- Stochastic method for the three-point functions
- Matching and mass corrections are included
- Currently under study for future applications:
  - Renormalization
  - A new smearing method indicates an improvement of errors that can enable us to reach larger momentum

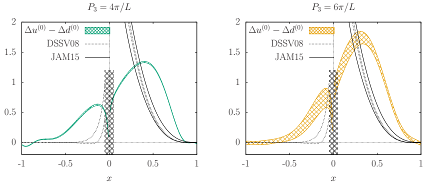


The momentum, helicity and transversity parton distributions

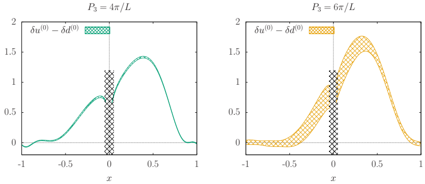
- $\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P|\bar{\psi}(z)\gamma_3 W_3(z,0)\psi(0)|P\rangle$
- crossing relation: $\bar{q}(x) = -q(-x)$
- negative $x \Rightarrow \bar{d} - \bar{u}$



- $\Delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$
- $\Delta\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P|\bar{\psi}(z)\gamma_5\gamma_3 W_3(z,0)\psi(0)|P\rangle$
- crossing relation: $\Delta\bar{q}(x) = \Delta q(-x)$
- negative x region $\Rightarrow \Delta\bar{u} - \Delta\bar{d}$



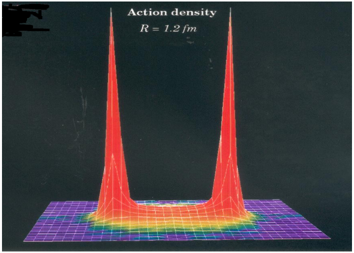
- $\delta q(x) = q^{\top}(x) - q^{\perp}(x)$
- $\delta\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P|\bar{\psi}(z)\gamma_j\gamma_3 W_3(z,0)\psi(0)|P\rangle$
- crossing relation: $\delta\bar{q}(x) = -\delta q(-x)$
- negative x region $\Rightarrow \delta\bar{d} - \delta\bar{u}$



C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Steffens and C. Wiese, arXiv:1610.03689 [hep-lat]

Challenges: *Ab Initio* Nuclear Physics?

From the $q\bar{q}$ potential to the determination of nuclear forces



K. Schilling, G. Bali and C. Schlichter, 1995

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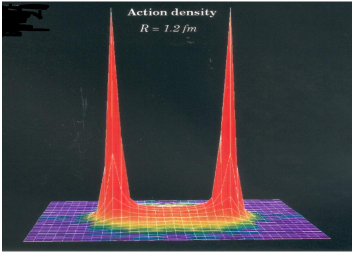

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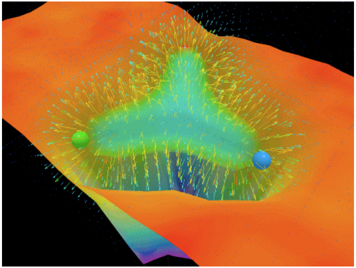

A.I. Signal, F.R.P. Bissey and D. Leinweber, arXiv:0806.0644

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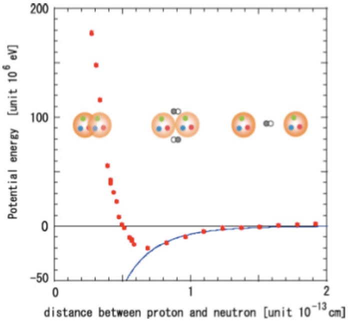
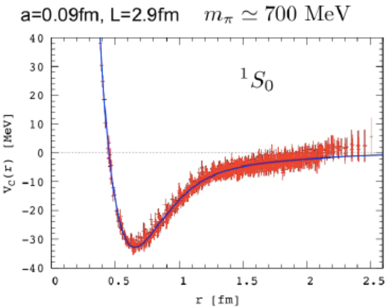




Two approaches:

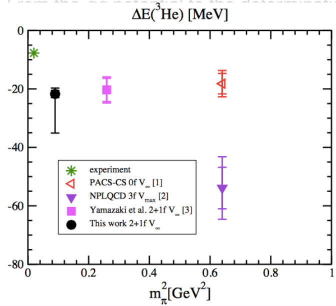
- Determine N-N energy as a function of $L \rightarrow$ extract phase shift - NPQCD
- Detemine BS wave function $\langle 0|N(\vec{r})N(\vec{0})|NN\rangle$ and exract asymptotically the phase shift - HALQCD

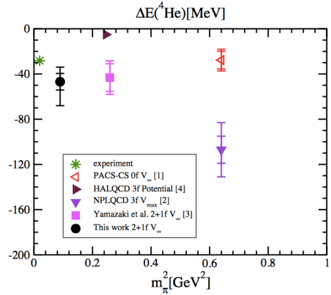
$\rightarrow$ study nuclear physics, neutron stars, ...

Only at the begining...

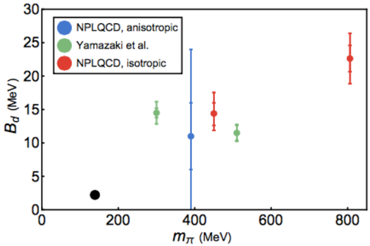
Challenges: *Ab Initio* Nuclear Physics?

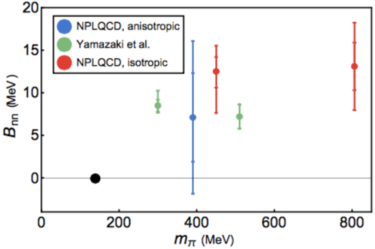
From the $q\bar{q}$ potential to the determination of nuclear for





T.Yamazaki, K. Ishikawa, Y. Kuramashi, A. Ukawa, 1502.04182





Deuteron and nn ($^1S_0$ channel) binding energy, K. Orginos *et al.* 1508.07583

Only at the begining...

Conclusions and Future Perspectives

- Computation of g_A, $\langle x \rangle_{u-d}$, etc, at the physical point allows direct comparison with experiment
- Provide predictions for g_s, g_T, tensor moment, σ-terms, etc.
- Resolution of the spin decomposition of the nucleon

A number of collaborations are now using simulations with close to physical values of the pion mass to:

- Compute gluonic observables
- Study excited states and resonances and scattering lengths
- Hadon-hadron interactions and multi-nucleon systems
- . . .

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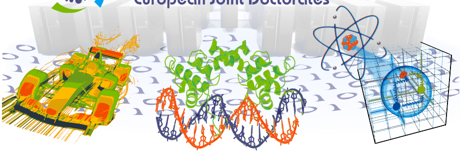

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European Twisted Mass Collaboration (ETMC)

Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool)

Collaborators:
A. Abdel-Rehim, S. Bacchio, K. Cichy, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K.Jansen, Ch. Kallidonis, G. Koutsou, K. Ottnad, M. Petschlies, F. Steffens, A. Vaquero, C. Wiese

Backup slides

Bibliography

1. J. W. Negele, *QCD and Hadrons on a lattice*, NATO ASI Series B: Physics vol. 228, 369, eds. D. Vautherin, F. Lenz and J.W. Negele.
2. M. Lüscher, *Advanced Lattice QCD*, Les Houches Summer School in Theoretical Physics, Session 68: Probing the Standard Model of Particle Interactions, Les Houches 1997, 229, hep-lat/9802029.
3. R. Gupta, *Introduction to Lattice QCD*, hep-lat/9807028.
4. P. Lepage, *Lattice for Novices*, hep-lat/0506036.
5. H. Wittig, Lecture week, SFB/TR16, 3-7 Aug. 2009, Bonn.
6. H. J. Rothe, *Quantum Gauge Theories: An introduction*, World Scientific, 1997.
7. I. Montvay and G. Münster, *Quantum Fields on a Lattice*, Cambridge University Press, 1994.
8. C.Gattringer and C. B. Lange, *Quantum Chromodynamics on the Lattice - An Introductory Presentation*, Springer, 2009.

Metropolis Algorithm

We need an algorithm to to create our set of random paths $x^{(\alpha)}$ with probability $\frac{e^{-S[x]}}{Z}$, where $Z = \int \mathcal{D}[x(t)]e^{-S[x]}$.

$\Longrightarrow$ a simple procedure, though not always the best, is the Metropolis Algorithm:

- Start with an arbitrary path $x^{(0)}$
- Modify by visiting each of the sites on the lattice, and randomizing the x_j's at those sites, one at a time, in a particular fashion as described below $\rightarrow$ generate a new random path from the old one: $x^{(0)} \rightarrow x^{(1)}$. This is called "updating" the path.
- Apply to $x^{(1)}$ to generate path $x^{(2)}$, and so on until we have N_{cf} random paths.

The algorithm for randomizing x_j at the j^{th} site is:

- Generate a random number $-\epsilon < \zeta \leq \epsilon$, with uniform probability;
- Let $x_j \rightarrow x_j + \zeta$ and compute the change ΔS in the action;
- If $\Delta S < 0$ retain the new value for x_j, and proceed to the next site;
- If $\Delta S > 0$ accept change with probability $\exp(-\Delta S)$ i.e. generate a random number η uniformly distributed between 0 and 1; retain the new value for x_j if $\exp(-\Delta S) > \eta$, otherwise restore the old value; proceed to the next site.

Comments:

- Choice of ϵ: should be tuned so that 40%–60% of the x_j's are changed on each pass (or "sweep") through the lattice. Then ϵ is of order the typical quantum fluctuations expected in the theory. Whatever the ϵ, successive paths are going to be quite similar and so contain rather similar information about the theory. Thus when we accumulate random paths $x^{(\alpha)}$ for our Monte Carlo estimates we should keep only every N_{cor}-th path; the intervening sweeps erase correlations, giving us configurations that are statistically independent. The optimal value for N_{cor} depends upon the theory, and can be found by experimentation. It also depends on the lattice spacing a.
- Initial configuration: Guess the first configuration $\rightarrow$ discard some number of configurations at the beginning, before starting to collect $x^{(\alpha)}$'s. This is called "thermalizing the lattice."

Metropolis Algorithm

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Comments:

- Choice of ϵ: should be tuned so that 40%–60% of the x_j's are changed on each pass (or "sweep") through the lattice. Then ϵ is of order the typical quantum fluctuations expected in the theory. Whatever the ϵ, successive paths are going to be quite similar and so contain rather similar information about the theory. Thus when we accumulate random paths $x^{(\alpha)}$ for our Monte Carlo estimates we should keep only every N_{cor}-th path; the intervening sweeps erase correlations, giving us configurations that are statistically independent. The optimal value for N_{cor} depends upon the theory, and can be found by experimentation. It also depends on the lattice spacing a.
- Initial configuration: Guess the first configuration $\rightarrow$ discard some number of configurations at the beginning, before starting to collect $x^{(\alpha)}$'s. This is called "thermalizing the lattice."

Gauge degrees of freedom

In the continuum a fermion moving from site x to y in the presence of a gauge field $A_\mu(x)$ picks up a phase factor given by the path ordered product

$$\psi(y) \;=\; \mathcal{P}\, e^{i\int_x^y gA_\mu(x)dx_\mu}\; \psi(x)\,.$$

$\Longrightarrow$ associate gauge fields with links that connect sites on the lattice. So, with each link associate a discrete version of the path ordered product:

$$U(x;x+\hat{\mu}) \;\equiv\; U_\mu(x) \;=\; e^{iagA_\mu(x)}\,,$$

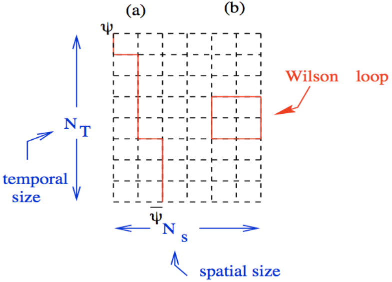
U is a 3×3 unitary matrix with unit determinant. It follows that

$$U(x;x-\hat{\mu}) \;\equiv\; U_{-\mu}(x) \;=\; e^{-iagA_\mu(x)} \;=\; U^\dagger(x-\hat{\mu};x)\;\;.$$

Local gauge symmetry

The effect of a local gauge transformation $V(x)$ on the variables $\psi(x)$ and U is defined as

$$\begin{aligned}\psi(x) &\rightarrow V(x)\psi(x)\\ \bar{\psi}(x) &\rightarrow \bar{\psi}(x)V^\dagger(x)\\ U_\mu(x) &\rightarrow V(x)U_\mu(x)V^\dagger(x+\hat{\mu})\end{aligned}$$



where $V(x)$ is in the same representation as the $U_\mu(x)$, *i.e.*, it is an $SU(3)$ matrix. With these definitions there are two types of gauge invariant objects that one can construct on the lattice.

- A string consisting of a path-ordered product of links capped by a fermion and an antifermion e.g.

$$\mathrm{Tr}\ \bar{\psi}(x)\ U_\mu(x)\ U_\nu(x+\hat{\mu})\ldots U_\rho(y-\hat{\rho})\ \psi(y)$$

  where the trace is over the color indices.
  If the string stretches across the lattice and is closed by the periodicity are called Polyakov lines.
- The simplest example of closed Wilson loops is the plaquette, a 1×1 loop,

$$W^{1\times 1}_{\mu\nu} = P_{\mu\nu}(x) = \mathrm{Re}\ \mathrm{Tr}\ \left(U_\mu(x)\ U_\nu(x+\hat{\mu})\ U^\dagger_\mu(x+\hat{\nu})\ U^\dagger_\nu(x)\right)\ .$$

Preserve gauge invariance at all $a \rightarrow$ protects from having many more parameters to tune (the zero gluon mass, and the equality of the quark-gluon, 3-gluon, and 4-gluon couplings) and there would arise many more operators at any given order in a.

U(1) gauge theory

Consider a Lagrangian of a complex field ϕ: $L = \partial_\mu \phi^* \partial^\mu \phi - V(\phi^*, \phi)$. If we require that the Lagrangian is invariant under a local gauge transformation $\phi'(x) = e^{-i\alpha(x)}\phi(x)$ then we need a field $A_\mu(x)$ to compensate the change in the derivative $\partial_\mu \phi$ that transforms as

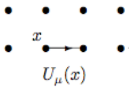
$$A'_\mu(x) = A_\mu(x) + \frac{1}{g}\partial_\mu \alpha(x)\ \partial_\mu \to D_\mu \equiv \partial_\mu + igA_\mu(x)$$

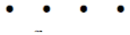
The gauge invariant Lagrangian is written as

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^* D_\mu \phi - V(\phi^*, \phi)$$

A scalar moving from site x to y in the presence of a gauge field $A_\mu(x)$ picks up a phase factor given by

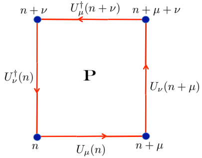
$$U(x;y) = e^{ig\int_x^y dx_\mu A^\mu(x)}$$





which removes the phase between the value of the field at the two points and yields a gauge invariant result.

The action is defined in terms of link variables assigned to links between sites of the space-time lattice.
The link variable from site n in the μ direction to site $n + a\hat{e}_\mu$ is defined as the discrete approximation to the integral $e^{ig\int_n^{n+\mu}}$: $U_\mu(n) = e^{i\theta_\mu(n)}$ with $\theta_\mu(n)$ the approximation of $g\int_n^{n+\mu} dx_\mu A^\mu(x)$.



The integral over the field variables is the invariant group measure for U(1): $\frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta$.
The action is the sum of all plaquettes $P_{\mu\nu} = U(n)_\mu U_\nu(n+\mu)U_\mu^\dagger(n+\nu)U_\nu^\dagger(n)$.
For U(1): $P_{\mu\nu}(n) = e^{i\theta_\mu(n)}e^{i\theta_\nu(n+\mu)}e^{-i\theta_\mu(n+\mu)}e^{-i\theta_\nu(n)} \equiv e^{iB_{\mu\nu}}$, $B_{\mu\nu} = \Delta_\mu\theta_\nu - \Delta_\nu\theta_\mu \overset{a\to 0}{\to} F_{\mu\nu}$.

Lattice action of U(1)

Since a plaquette produces $F_{\mu\nu}$ the action can be constructed by choosing a function of the plaquette such that it generates $F^2_{\mu\nu}$ in the continuum limit.

$$S = \beta \sum_n \sum_{\mu>\nu} (1 - Re\, P_{\mu\nu}(n)) = \beta \sum_n \sum_{\mu>\nu} (1 - \cos B_{\mu\nu})\,,$$

where $\beta = \frac{1}{g^2}$ and $B_{\mu\nu} = \Delta_\mu \theta_\nu - \Delta_\nu \theta_\mu \stackrel{a\to 0}{\to} F_{\mu\nu}$.

In the limit $a \to 0$ we recover continuum QED:
Taking $\theta_\mu(n) = agA_\mu(n)$ and expanding $\theta_\nu(n + \hat{e}_\mu a) = \theta_\nu(n) + a\partial_\mu\theta_\nu(n) + \mathcal{O}(a^2)$

$$\begin{aligned} S \quad &\sim \quad \frac{1}{g^2}\sum_P [1 - \cos(a\partial_\mu\theta_\nu - a\partial_\nu\theta_\mu)] = \frac{1}{g^2}\sum_P \left[1 - \cos(a^2 g F_{\mu\nu})\right] = \frac{1}{g^2}\sum_n \sum_{\mu>\nu}\left[\frac{a^4 g^2}{2} F^2_{\mu\nu} + \cdots\right] \\ &\to \quad \frac{1}{4}\int d^4x\, F^2_{\mu\nu}(x) \end{aligned}$$

Path integral for QED

The Hamiltonian does not constrain the charge state of the system $\rightarrow$ project the states appearing in the path integral onto the space satisfying $\vec{\nabla}.\vec{E} = \rho$ where ρ is the background charge. This is done by including in the path integral the δ-function:

$$\int \mathcal{D}\chi e^{i\int dxdt\,\chi(\vec{\nabla}.\vec{E}-\rho)}\,.$$

Consider $A_0 = 0$ and replacing $x \rightarrow A$ and the momentum $\rightarrow E$ in the coordinate path-integral we have

$$\begin{aligned} Z &= \int \mathcal{D}\chi\mathcal{D}\vec{A}\mathcal{D}\vec{E}\, e^{\int dxdt\left[i\vec{E}.\dot{\vec{A}}-\frac{1}{2}(E^2+B^2)+i\chi(\vec{\nabla}.\vec{E}-\rho)\right]} \\ &= \int \mathcal{D}\chi\mathcal{D}\vec{A}\, e^{-\int dxdt\left[\frac{1}{2}\left((\dot{\vec{A}}-\vec{\nabla}\chi)^2+B^2-i\chi\rho\right)\right]} \end{aligned}$$

Rename $\chi = A_0$ and take $\rho(x) = \sum_n q_n\delta(x - x_n)$

$$Z = \int \mathcal{D}A_\mu e^{-\int dxdt\,\frac{1}{4}F^2_{\mu\nu}} \prod_n e^{-iq_n\int dt\,A_0(x_n,t)}$$

$\Longrightarrow$ we obtain the Lagrangian path integral with a line of $\pm A_0$ fields at the positions of the fixed external $\pm$ charges.