STRING/GAUGE DUALITY (ADS/CFT) AND HIGH ENERGY SCATTERING IN QCD

QCD - Old Challenges
and New Opportunities

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WE-Heraeus Physcics School
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BROWN

Outline

- Holographics Duality:
  - Duality in Physics
  - Gauge Theories and Scale Invaraince
  - ADS/CFT (String-Gauge Duality)
- QCD, High Energy Scattering:
  - unification of hard and soft physics - “Pomeron/Graviton”
  - Glueballs under AdS/CFT — masses and decays
  - DIS - BFKL vs DGLAPP
  - Inclusive and Exclusive Central Production
- QCD and Modern CFT/String Studies:

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- Holographics Duality - Historical Perspective:
  - Duality in Physics, Holographic Duality, etc.
  - Gauge Theories, Massless Limit and Scale Invaraince
  - Holographic Duality, ADS/CFT (String-Gauge Duality)
- Size and Shape of hadrons: QCD, High Energy Scattering
  - unification of hard and soft physics - "Pomeron/Graviton"
  - Glueballs under AdS/CFT — masses and decays
  - DIS — BPST Pomeron vs BFKL and DGLAPP
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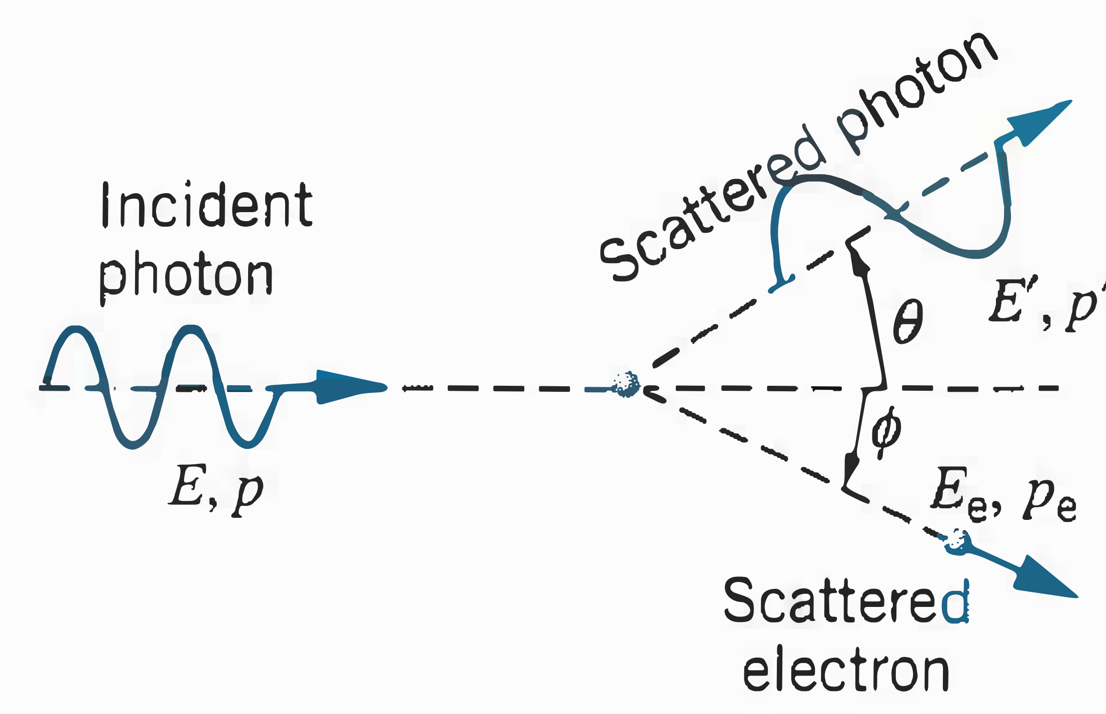
Duality:

“Particle-Wave Duality:

Illustration: Compton Scattering

$$\gamma + e^- \quad \rightarrow \quad \gamma + e^-$$



Particle: $E^2 = (pc)^2 + (m_0 c^2)^2$ photon: massless $\Leftrightarrow m_0 = 0, \quad E = p\,c$

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1 - \cos\theta)$$

Wave Length: λ

$$\frac{1}{\lambda'} - \frac{1}{\lambda} = \frac{h}{m_e c}(1 - \cos\theta)$$

Wave-Particle Duality: $E = \dfrac{hc}{\lambda}, \quad p = \dfrac{h}{\lambda}$

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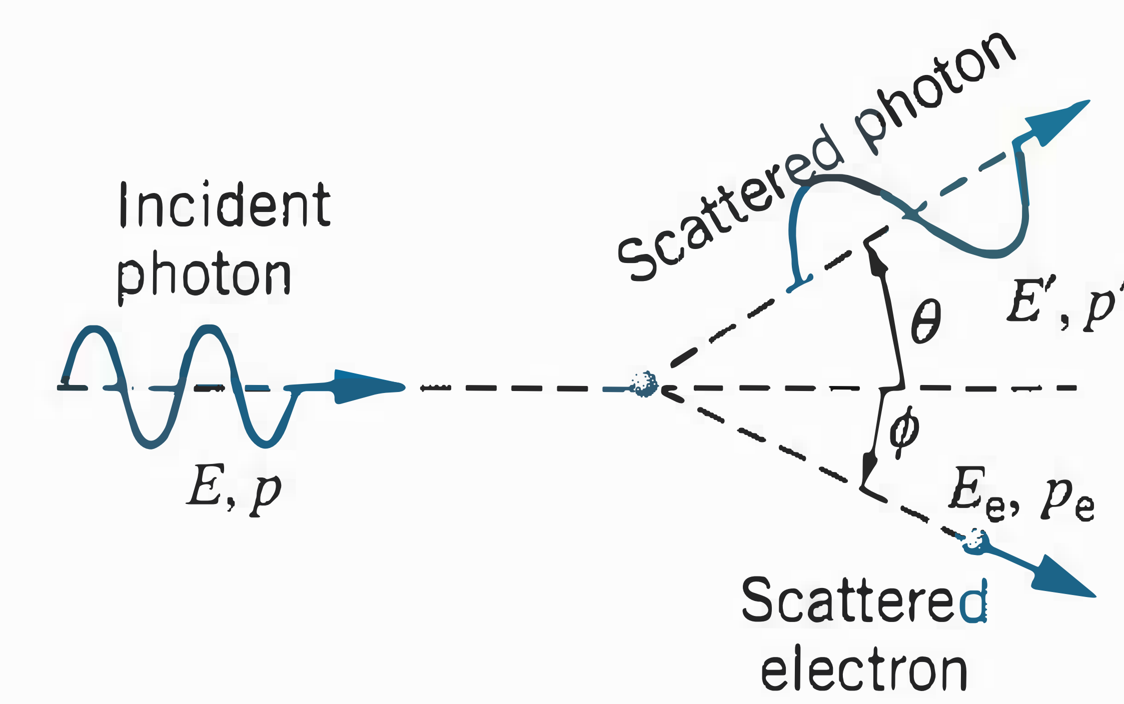

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Kinematical

Duality:

"High-Low Temperature Duality:"

"Ising Model:"

$$Z(\beta) = \sum_{\sigma=\pm 1} e^{-\beta\sigma_i\sigma_{i+1}}$$

$$Z(\beta) \quad \Leftrightarrow \quad Z(1/\beta)$$

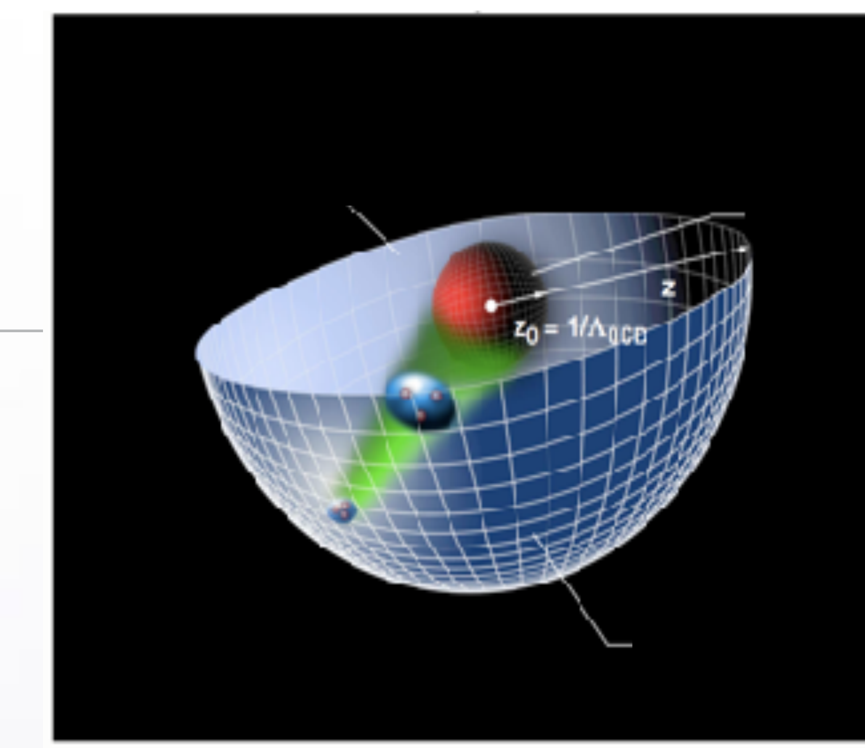
Understanding of symmetry, etc.,
leading to changed description of ground state,
new effective degrees of freedom.

Dynamical

Duality:

“Holographic Duality: Illustration: Potential Scattering:

$$\left\{-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})\right\}\Psi = E\Psi \qquad \Leftrightarrow \qquad \frac{d\sigma}{d\Omega}$$



Given $V(\vec{r}) \rightarrow$ Scattering S-Matrix: $S = I + iT$

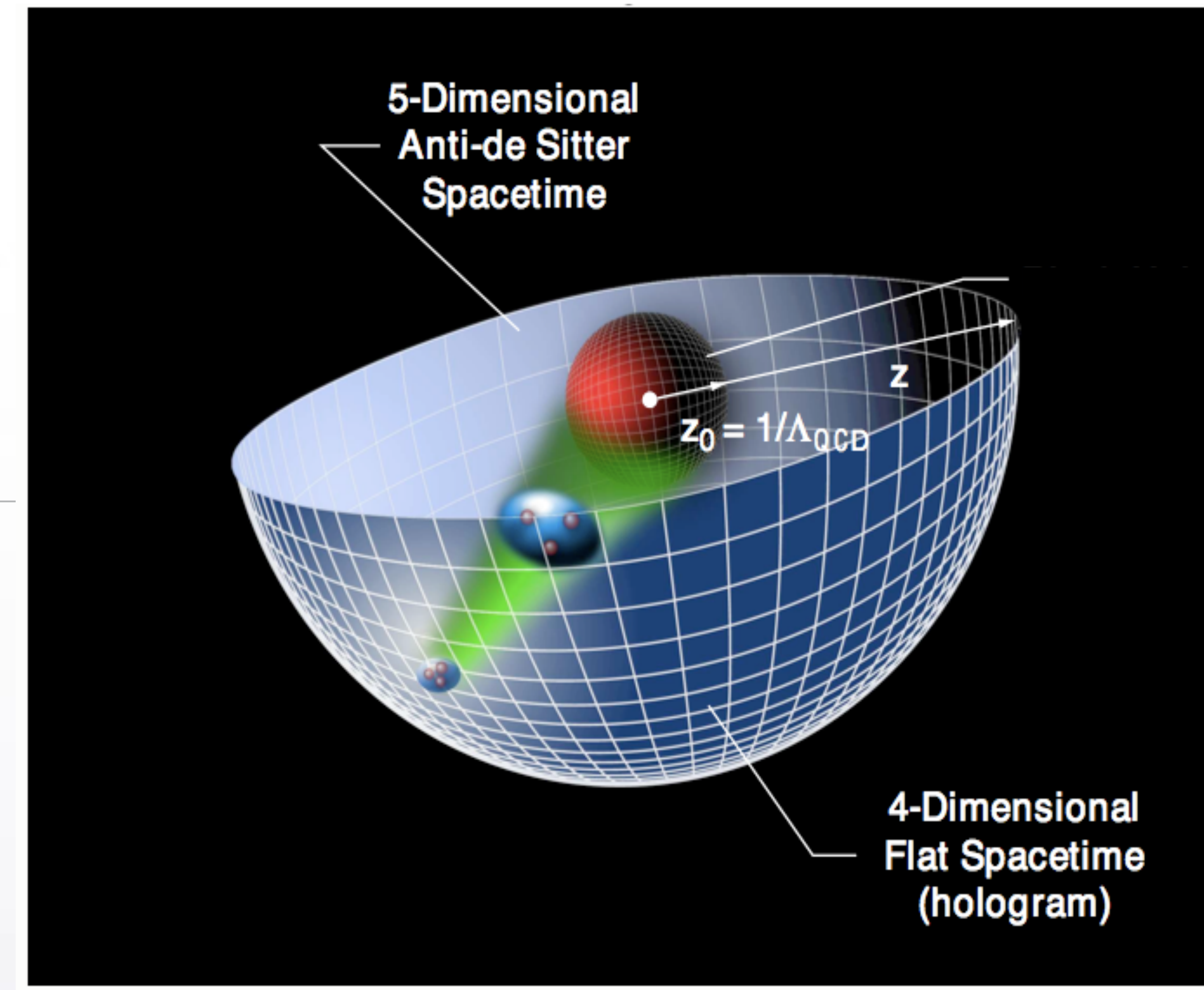
Given Scattering Matrix: $T \rightarrow$ Reconstruct $V(\vec{r})$ – Inverse Scattering problem

Physics at D dimension $\Leftrightarrow$ Equivalent Physics at $(D-1)$ dimension

Duality:

"Holographic Duality:

AdS/CFT Correspondence for Gauge Theories



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Physics at D dimension $\Leftrightarrow$ Equivalent Physics at (D+1) dimension

Symmetry, Unification and Universality:

- Symmetry: Lorentz Invariance.
- Unification: Quantum Physic.
- Unified Maxwell, Weak Interaction, and, QCD — Gauge Theories.
- Quantum Gravity ?? (Geometrical)

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Principle of equivalence $\Rightarrow$ General coordinate invariance.

Local isometry of metric: $dx^2 = -dt^2 + dx^2$

Gravity $\Rightarrow$ geometrical.

Symmetry, Unification and Universality:

- Symmetry: Conformal Invariance:

$$\boxed{O(1,1) \times O(1,3) \quad \Rightarrow \quad O(2,4)}$$

- Unification: Geometrization

symmetry as isometry of geometry of extended space-time.

$$\boxed{(t, \vec{x}) \oplus r \quad \Rightarrow \quad (t, \vec{x}, r)}$$

HIGH ENERGY SCATTERING AND SCALE INVARIANCE

Lagrangian for QED and QCD is scale invariant:

α_{qed}, α_{qcd}, etc., are dimensionless.

exceptions: mass for fermions.

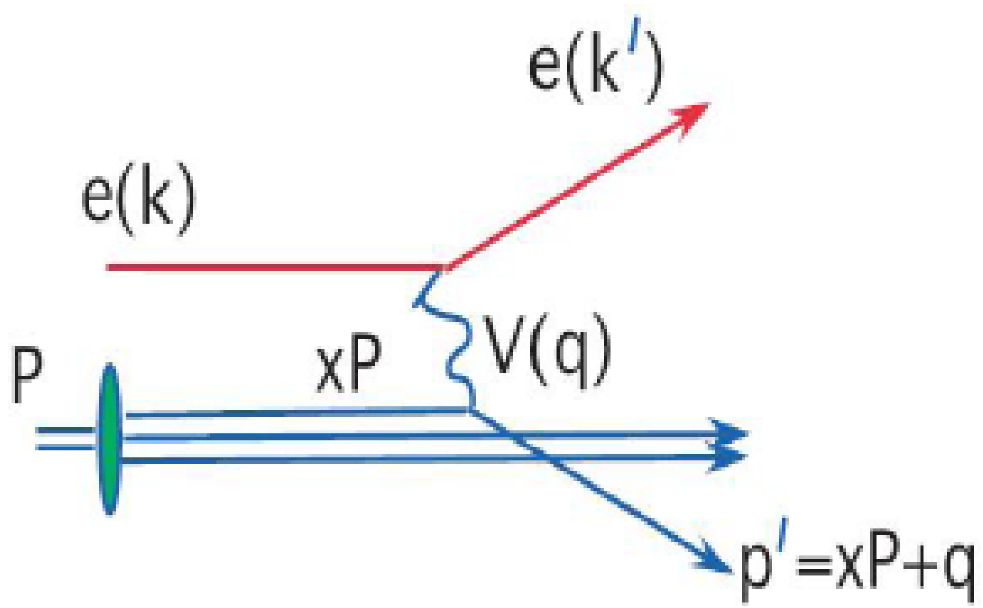
$$\frac{E}{pc} = \frac{\sqrt{(pc)^2 + m_0^2 c^4}}{pc} \simeq 1, \qquad p \to \infty$$

Modern approaches to fundamental physics begins with massless fermions, and masses are generated dynamically.

Lorentz + Scale invariance lead to large symmetry: Conformal Symmetry.

CFT: Conformal Invariant Field Theory

$Deep\ Inelastic\ Scattering\ (DIS)$



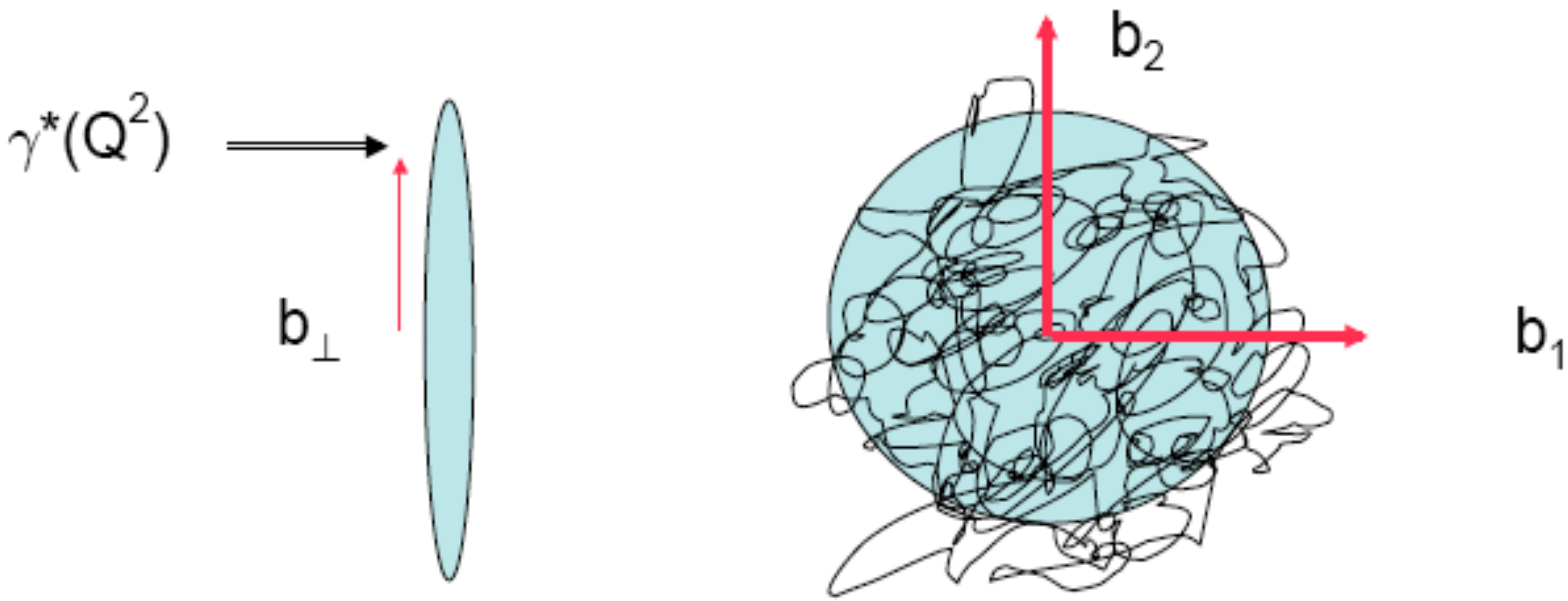
$$F_2(x, Q2) = \frac{Q^2}{4\pi^2\alpha_{em}}\left[\sigma_T(\gamma^* p) +_L (\gamma^* p)\right]$$

$$x \equiv \frac{Q^2}{s}$$

Scaling: $F(x, Q^2) \to F(x)$

$$Small\ x:\ \frac{Q^2}{s} \to 0$$

Geometry of High Energy Scattering and Scale Invariance

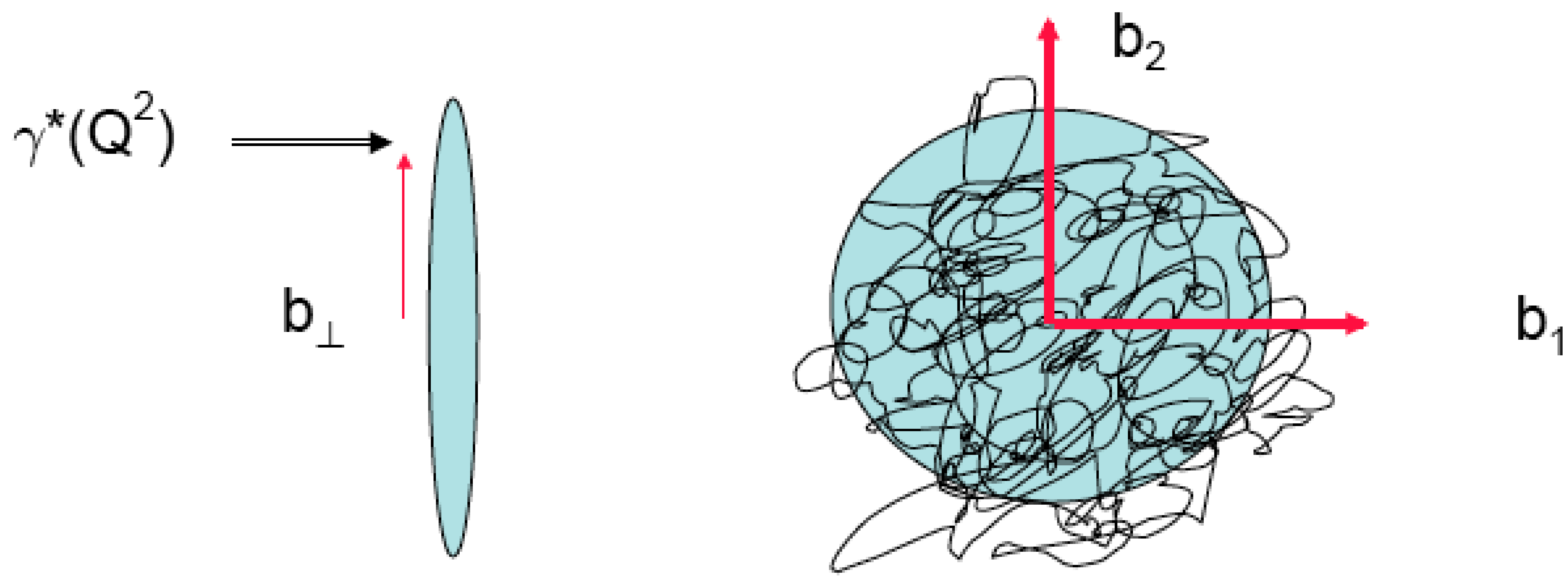


5 kinematical Parameters:

2-d Longitudinal	$p^\pm = p^0 \pm p^3 \simeq \exp[\pm \log(s/\Lambda_{qcd})]$
2-d Transverse space:	$x'_\perp - x_\perp = b_\perp$
1-d Resolution:	$z = 1/Q$ (or $z' = 1/Q'$)

QCD EMERGENCE OF 5-DIM

"Fifth" co-ordinate is size z / z' of proj/target



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Scale Invariance and AdS

What is the curved space?

Maldacena: UV (large r) is (almost) an $AdS_5 \times X$ space

$$ds^2 = r^2 dx_\mu dx^\mu + \frac{dr^2}{r^2}$$

Captures QCD's approximate UV conformal invariance

$$x \longrightarrow \zeta x \ , \ r \longrightarrow \frac{r}{\zeta}$$

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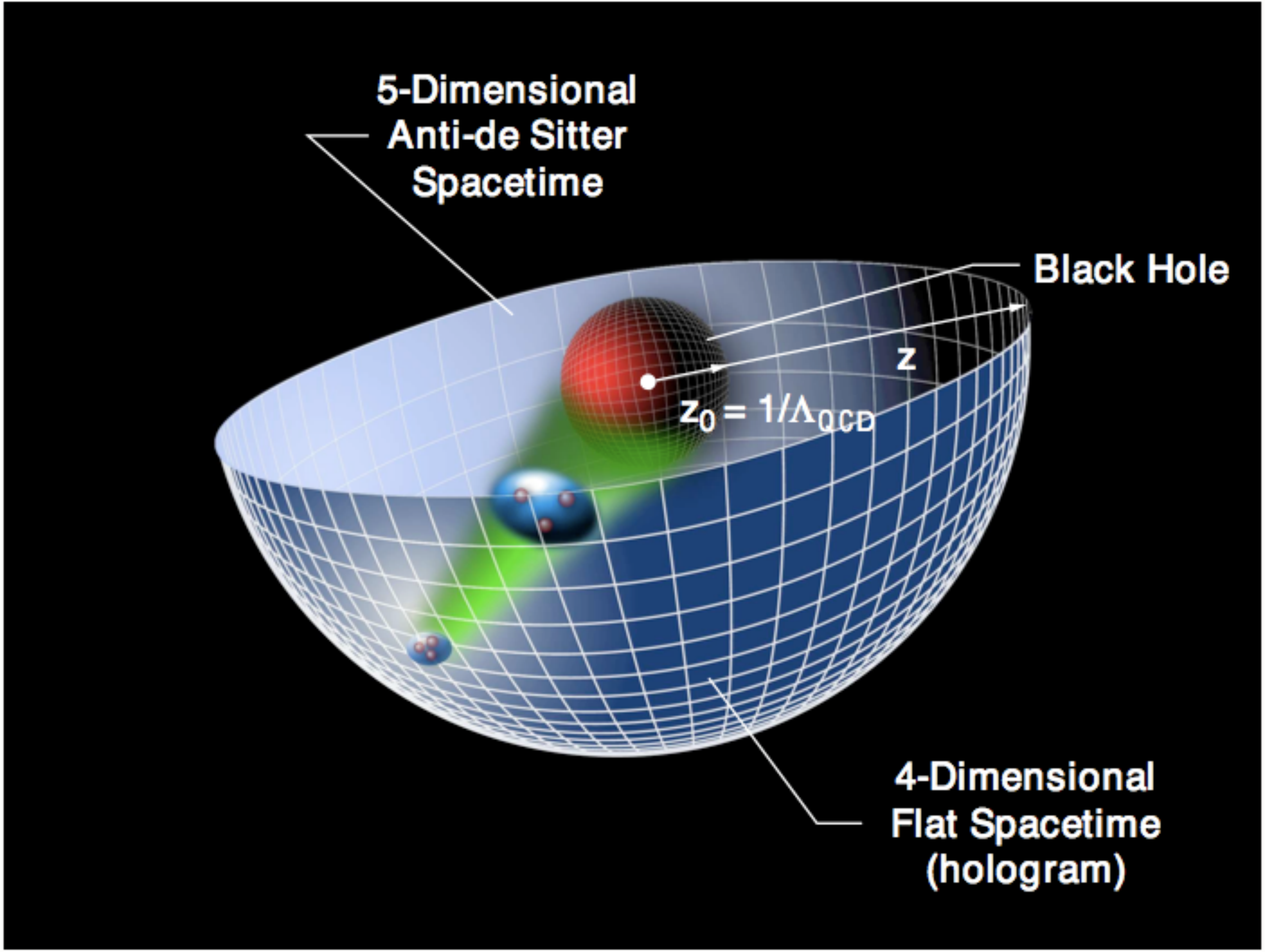
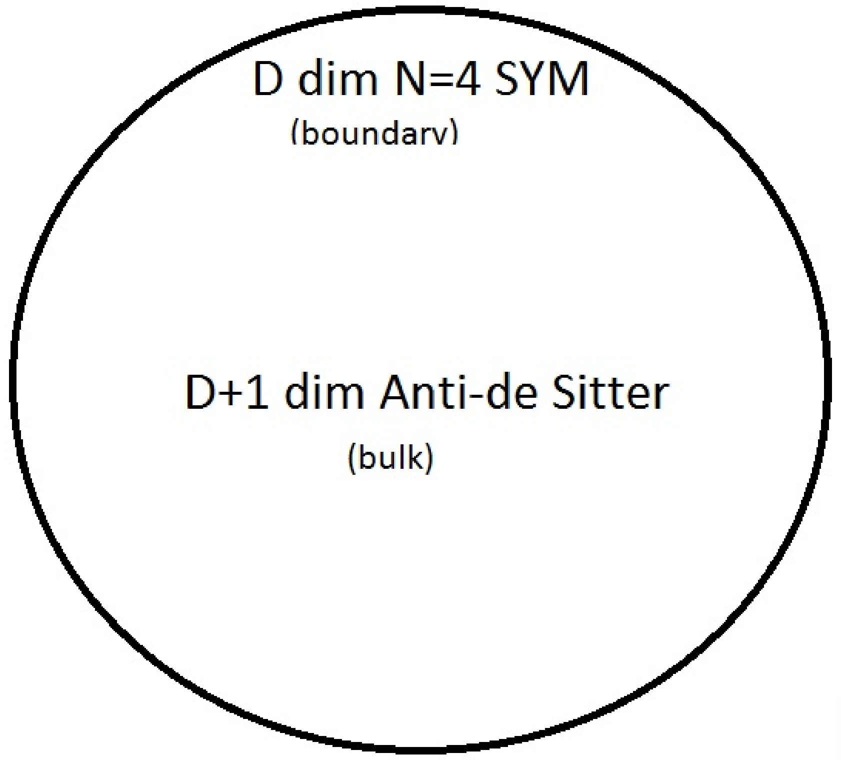
$$x \to \zeta x \ , \ \ r \to \frac{r}{\zeta} \qquad \text{(recall } r \sim \mu\text{)}$$

Confinement: IR (small r) is cut off in some way

$$r \sim \mu > r_{min} \sim \Lambda_{QCD}$$

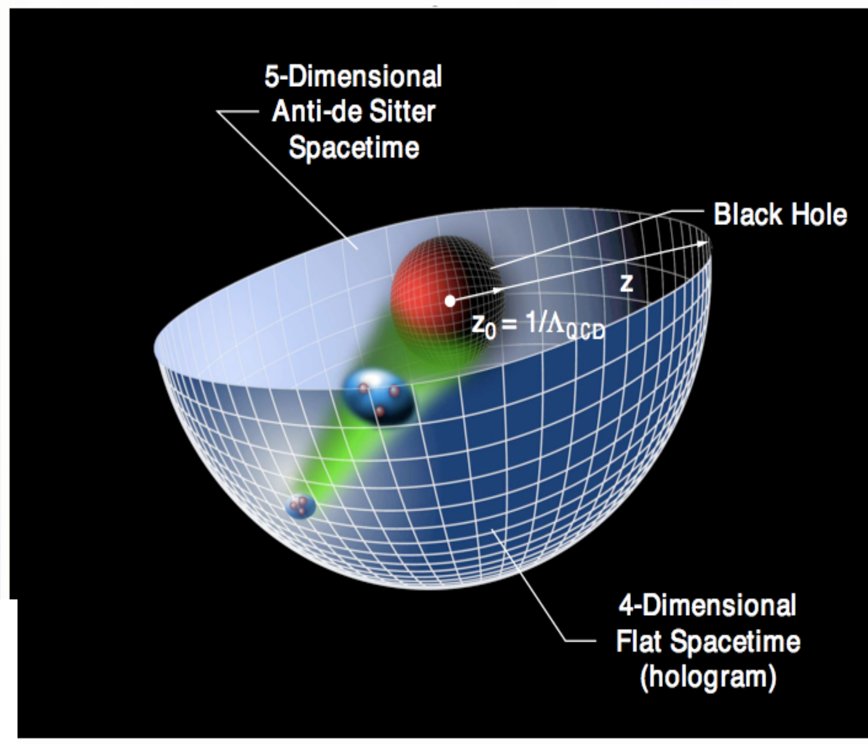

5-Dimensional
Anti-de Sitter
Spacetime
Black Hole
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

AdS Geometrization



The AdS/CFT is a holographic duality that equates a string theory (gravity) in high dimension with a conformal field theory (gauge) in 4 dimensions. Specifically, compactified 10 dimensional super string theory is conjectured to correspond to $\mathcal{N} = 4$ Super Yang Mills theory in 4 dimensions in the limit of large 't Hooft coupling:

$$\lambda = g_s N = g_{ym}^2 N_c = R^4/\alpha'^2 >> 1.$$



$$\boxed{ds^2 = \frac{R^2}{z^2}\left[dz^2 + dx \cdot dx\right] + R^2 d\Omega_5 \rightarrow e^{2A(z)}\left[dz^2 + dx \cdot dx\right] + R^2 d\Omega_5}$$

principle of equivalence: allows changing gravity into study of geometry!

Gauge-String Duality: AdS/CFT

Weak Coupling:

Gluons and Quarks: $A_\mu^{ab}(x), \psi_f^a(x)$

Gauge Invariant Operators: $\bar\psi(x)\psi(x), \;\; \bar\psi(x) D_\mu \psi(x)$

$S(x) = TrF_{\mu\nu}^2(x), \;\; O(x) = TrF^3(x)$

$T_{\mu\nu}(x) = TrF_{\mu\lambda}(x)F_{\lambda\nu}(x), \;\; etc.$

$$\mathcal{L}(x) = -TrF^2 + \bar\psi \not{D} \psi + \cdots$$

Strong Coupling:

Metric tensor: $G_{mn}(x) = g_{mn}^{(0)}(x) + h_{mn}(x)$

Anti-symmetric tensor (Kalb-Ramond fields): $b_{mn}(x)$

Dilaton, Axion, etc. $\phi(x), \; a(x), \; etc.$

Other differential forms: $C_{mn\cdots}(x)$

$$\mathcal{L}(x) = \mathcal{L}(G(x), b(x), C(x), \cdots)$$

$\mathcal{N} = 4$ SYM Scattering at High Energy

$$\langle e^{\int d^4x \phi_i(x) \mathcal{O}_i(x)} \rangle_{CFT} = \mathcal{Z}_{string} \left[\phi_i(x,z)|_{z \sim 0} \to \phi_i(x) \right]$$

Bulk Degrees of Freedom from type-IIB Supergravity on AdS_5:

- metric tensor: G_{MN}
- Kalb-Ramond 2 Forms: B_{MN}, C_{MN}
- Dilaton and zero form: ϕ and C_0

$$\lambda = g^2 N_c \to \infty$$

Supergravity limit

- Strong coupling
- Conformal
- Pomeron as Graviton in AdS

Partial list of successes of AdS/CFT:

Address quark-gluon plasma non-perturbatively.

New perspectives for String theories.

Unified treatment for High Energy Scattering.

Strong coupling for condensed matter physics.

Topological insulators, fractional quantum Hall effect, etc.

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Size and Shape of Hadrons

Rising of total cross sections with total energy

Shape of differential cross section

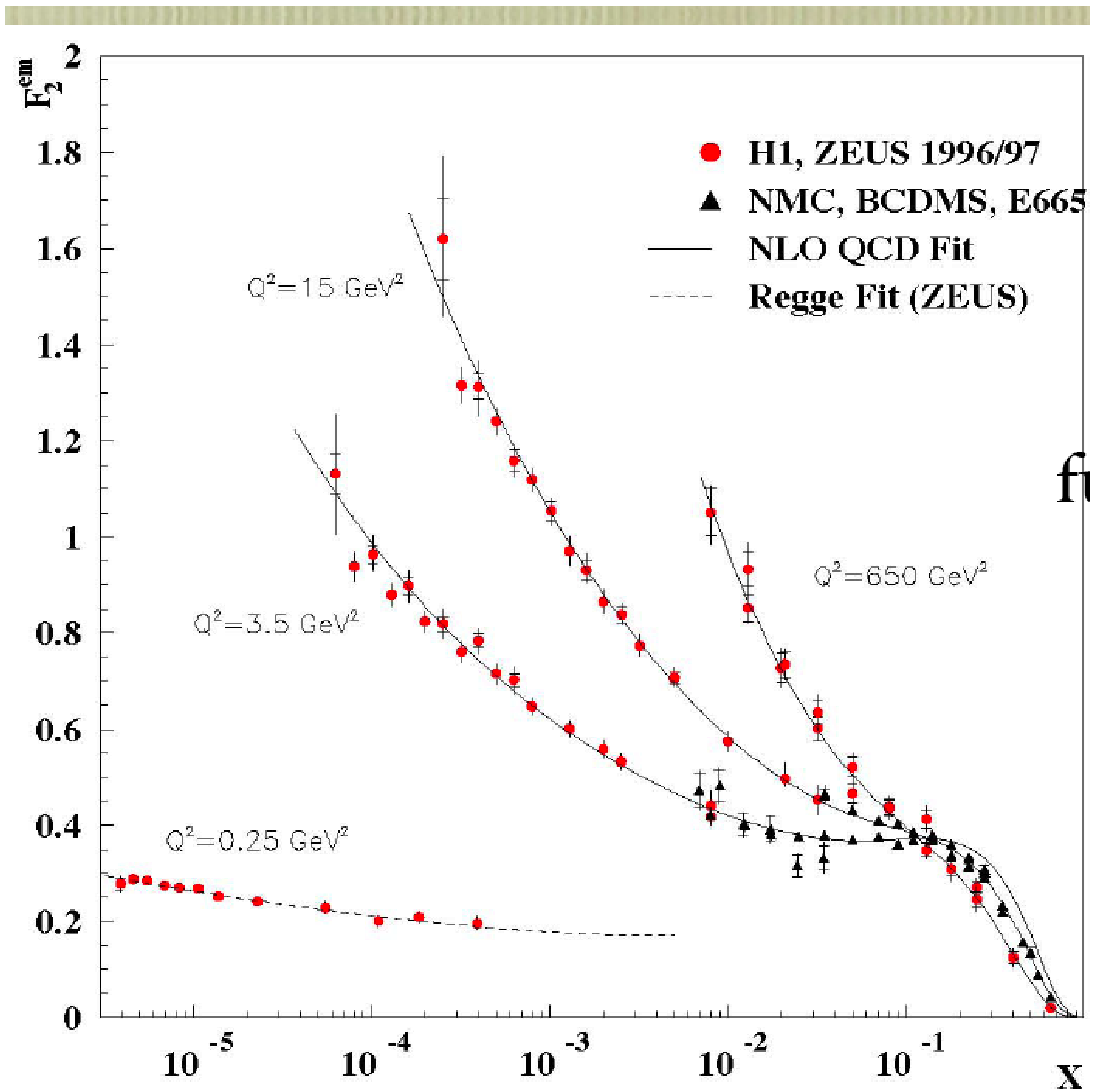
Scaling for DIS

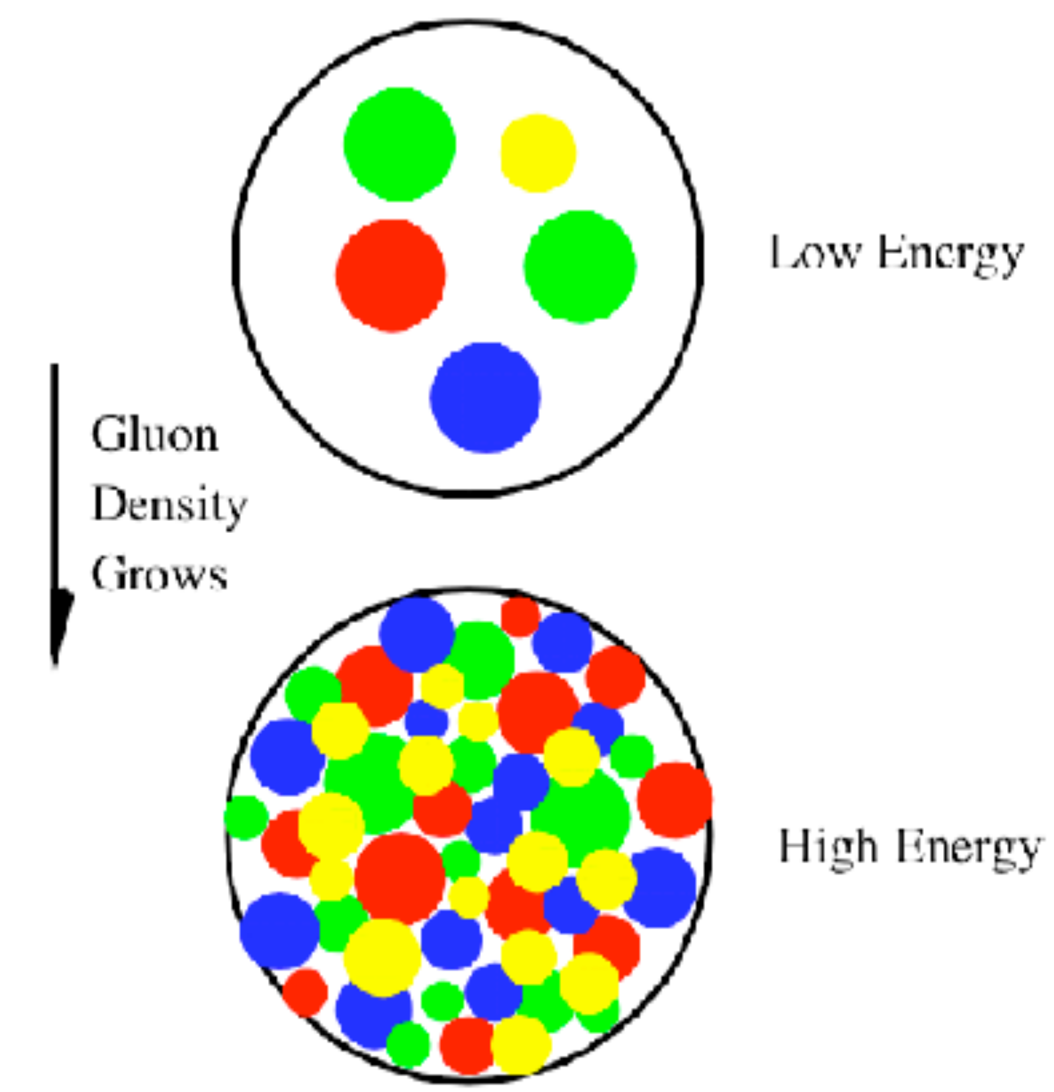
Correlations in particle production

Dimensional scaling

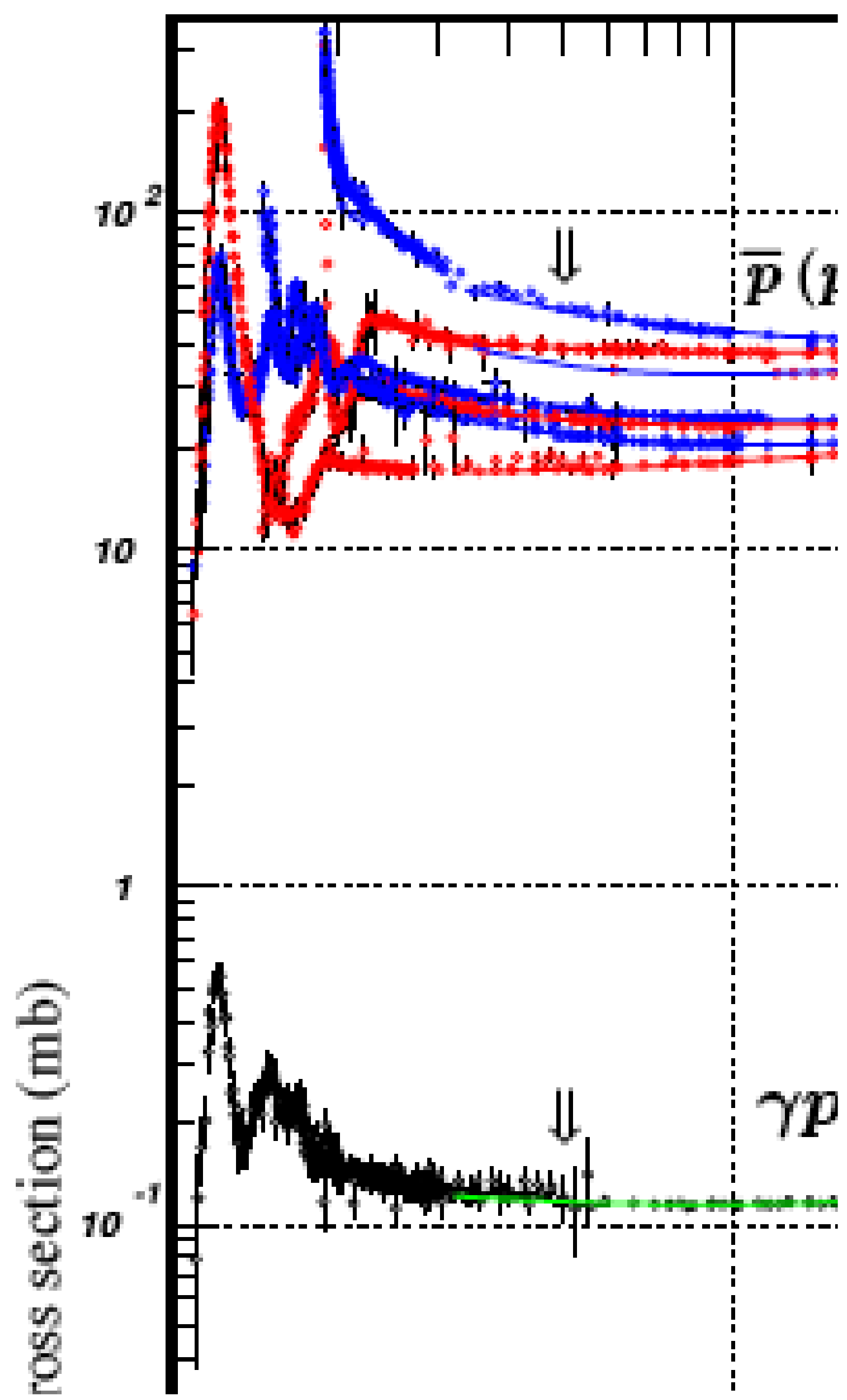
Diffractive production at LHC

Deep Inelastic Scattering (DIS)

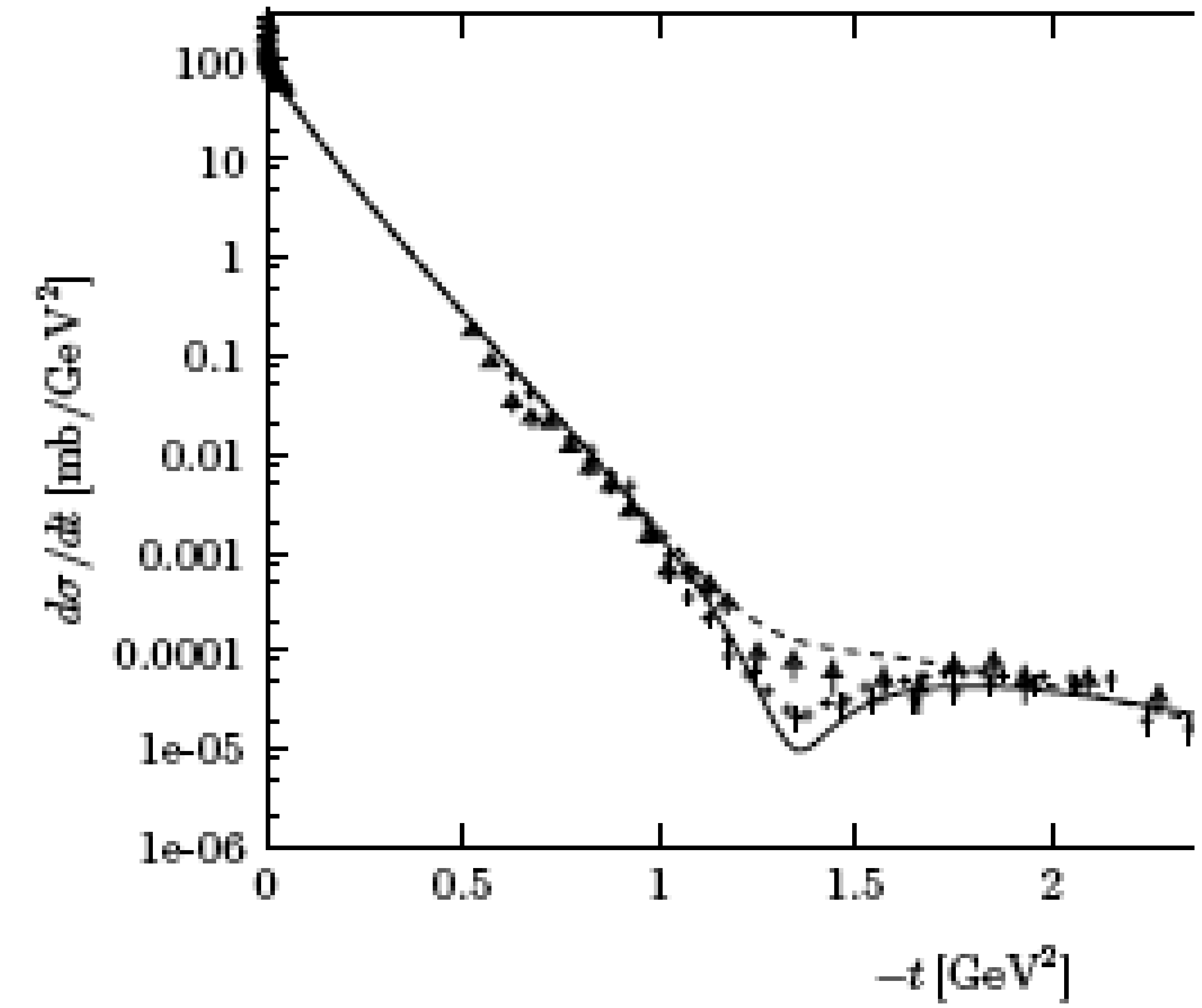




Total Cross Sections



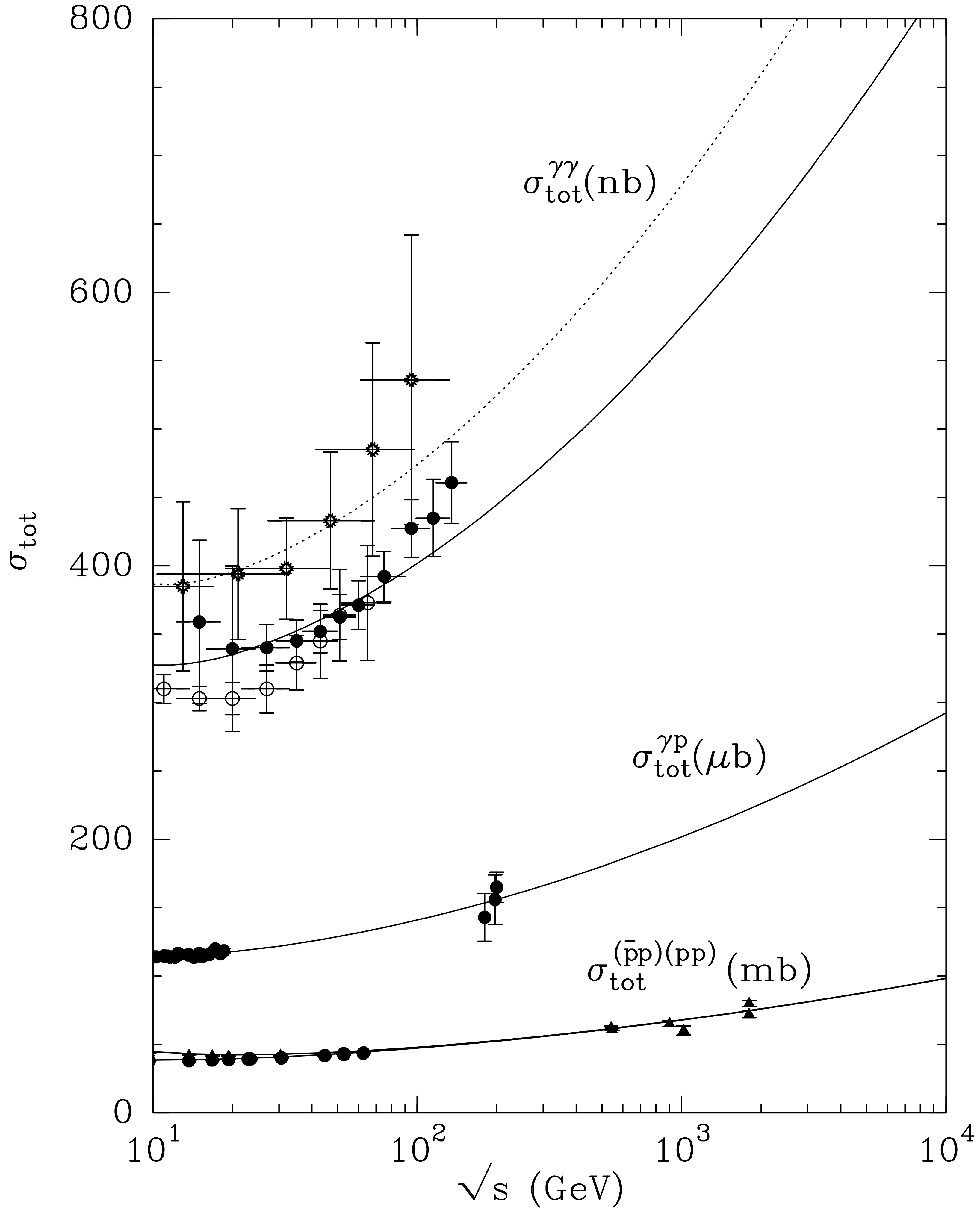
Differential Cross Sections



Near constant Size:

Diffraction Peak:

Total Cross Sections and Elastic Peaks



$$\sigma_{total}(s) \sim \mathrm{Im}\mathcal{A}(s, t=0)/s \sim s^{\epsilon}$$

$$\mathcal{A}(s, t=0) \sim s^{1+\epsilon}$$

$$\frac{d\sigma}{dt} \sim C(s)\; e^{B(s)t}$$

$$\epsilon \simeq 0.1 \sim 0.3$$

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Why does Total Cross Section increase with Energy?

Brief Review of Yukawa Picture:

$$V(r) = g^2 \frac{e^{-\mu r}}{r} \rightarrow \frac{g^2}{\mu^2 - t}$$

$$\mu \neq 0 \leftrightarrow \text{“}short - range\text{”}$$

$$\mu = 0 \leftrightarrow \text{“}long - range\text{”}$$

$$A = V + V * V + V * V * V + \cdots$$

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“Relativistic kinematics”

scalar exchange : $\hat{V}(s,t) \sim \dfrac{1}{\mu^2 - t}$

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vector exchange : $J_\mu J^\mu \rightarrow \hat{V}(s,t) \sim \frac{s}{\mu^2 - t}$ $\quad \sigma_{total} \sim$ constant

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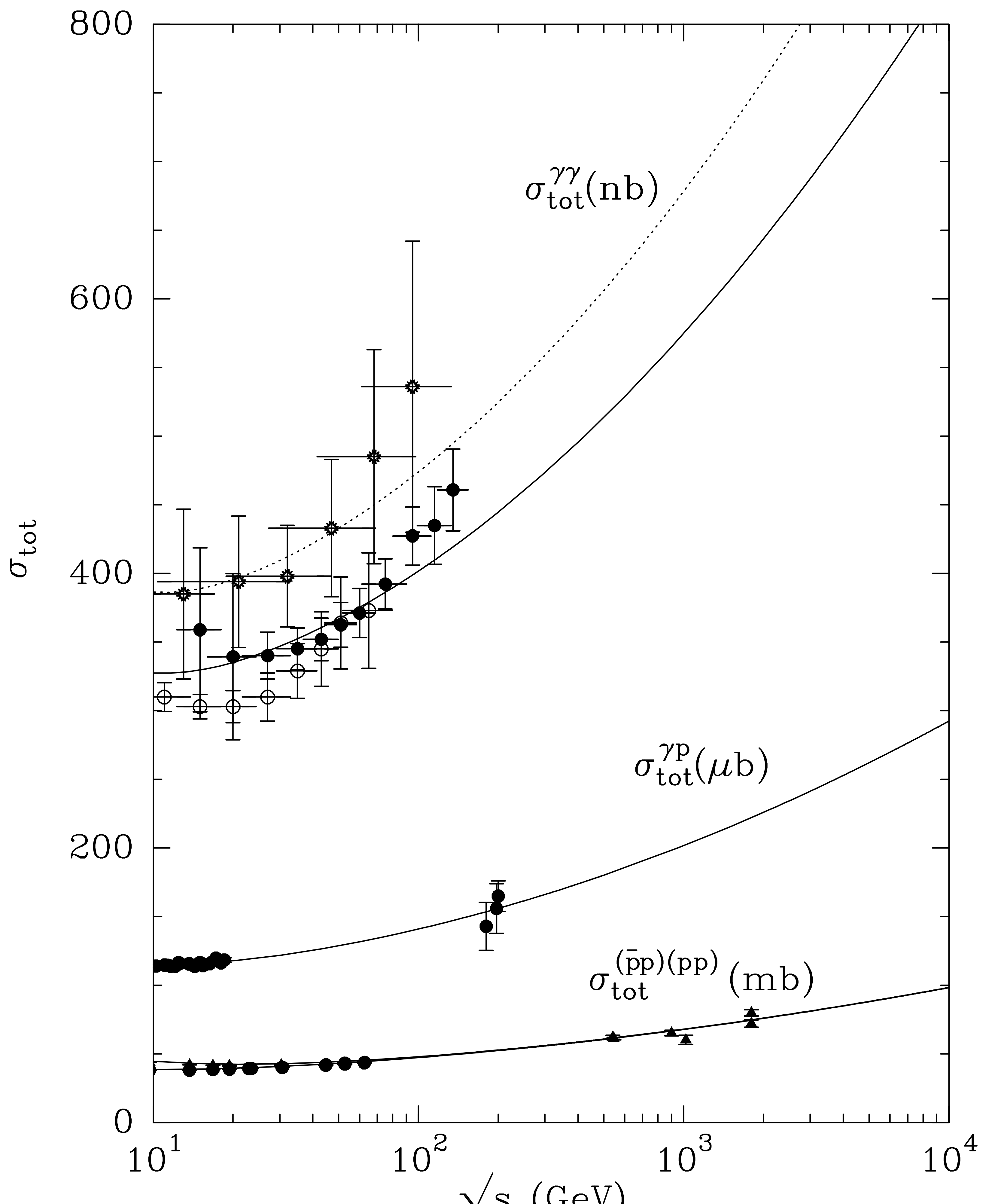
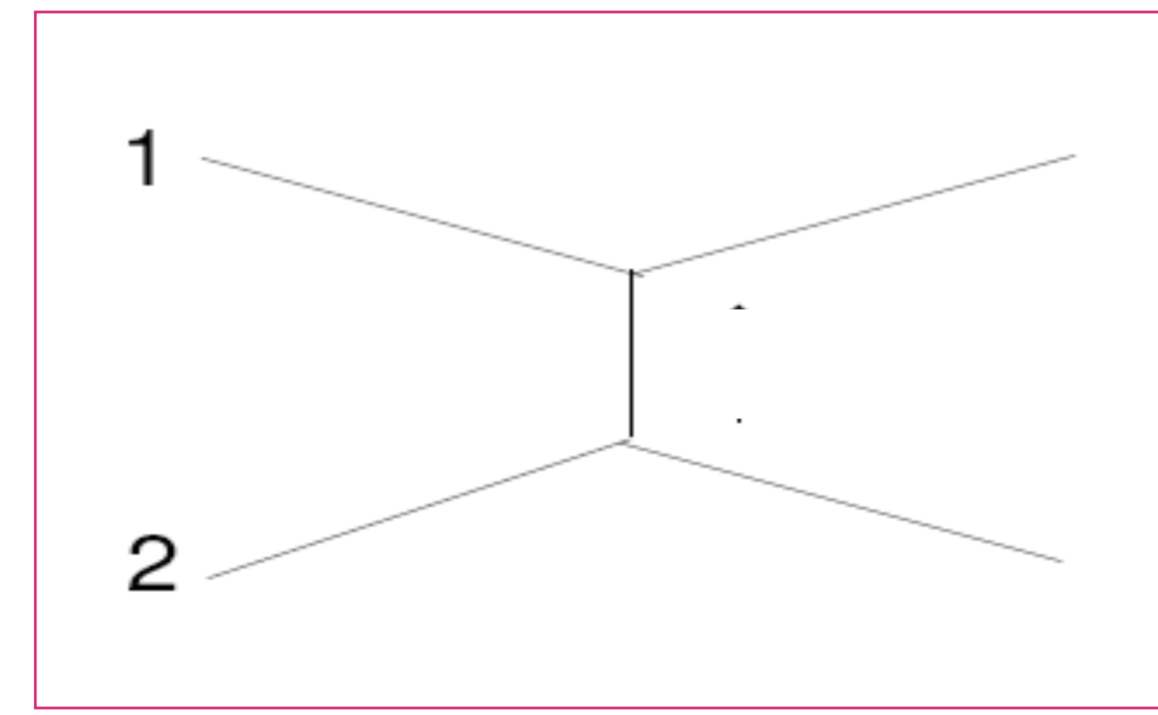
Total Cross Sections





$$\mathcal{A} \sim s^{J(t)} = s^{\alpha(0)+\alpha' t}$$

$$\sigma_{total} \sim \mathcal{A}(s,0)/s \sim S^{J(0)-1} \sim s^{\alpha(0)-1}$$

$$\alpha(0) > 1$$

effective spin exchange:
vector $\sim$ tensor

Need "Vector $\sim$ Tensor" exchange:

Need "none-zero Mass":

quarks and gluons ~ 2005

Need “Vector $\sim$ Tensor” exchange:

Need “none-zero Mass”:

Size and Shape: Dynamics of QCD

Parton Interpretation

quarks and gluons ~ 2005

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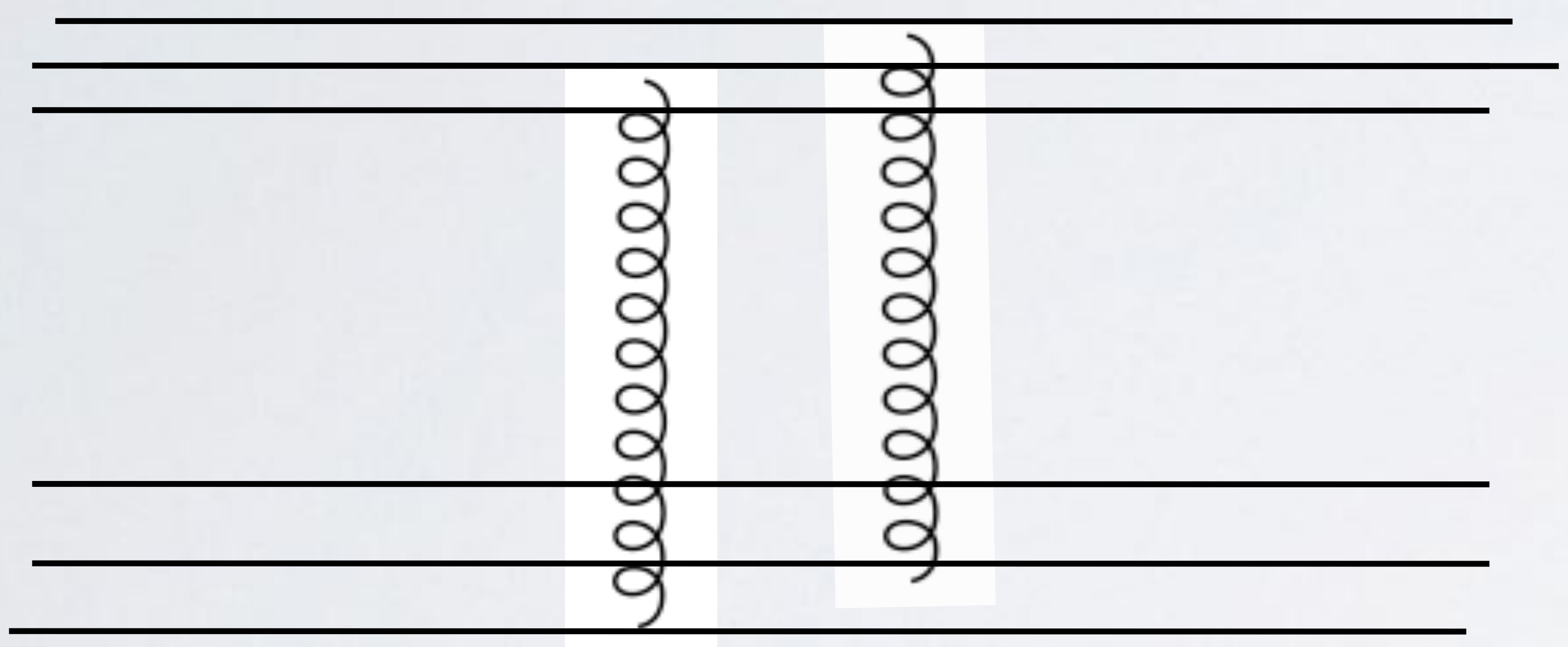
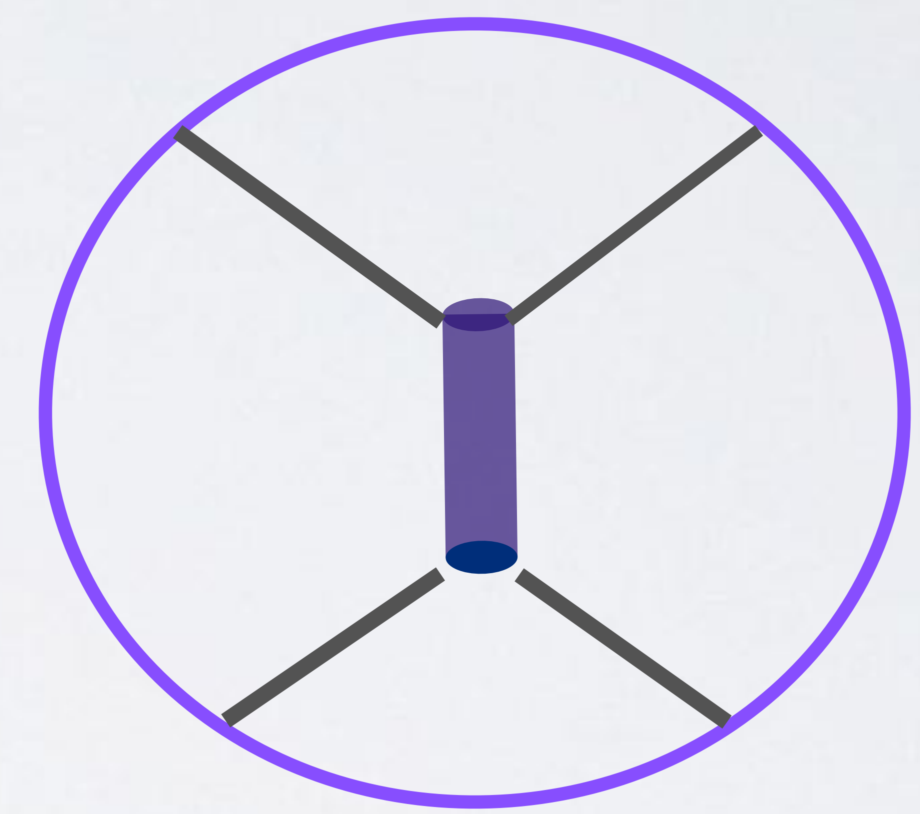
HIGH ENERGY SCATTERING <=> POMERON

WHAT IS THE POMERON ?

WEAK: TWO-GLUON <=> STRONG: ADS GRAVITON





$$J = 2$$

$$J_{cut} = 1 + 1 - 1 = 1$$

$$S = \frac{1}{2\kappa^2} \int d^4x dz \sqrt{-g(z)} \Big(-\mathcal{R} + \frac{12}{R^2} + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi \Big)$$

F.E. Low. Phys. Rev. D 12 (1975), p. 163.
S. Nussinov. Phys. Rev. Lett. 34 (1975), p. 1286.

AdS Witten Diagram: Adv. Theor. Math. Physics 2 (1998)253

Challenge for AdS/CFT for QCD

◆ Spin-2 leads to too rapid an increase for cross sections

Need to consider $\lambda = g^2 N$ finite. (stringy corrections)

◆ Confinement:

Conformal, therefore no scale and no particles, etc.

◆ Short-distance: Running Coupling

Brower, Polchinski, Strassler, and Tan: "The Pomeron and Gauge/String Duality," hep-th/063115

“String Theory for QCD”

Need “Vector $\sim$ Tensor” exchange:

Need “none-zero Mass”:

quarks and gluons ~ 2005

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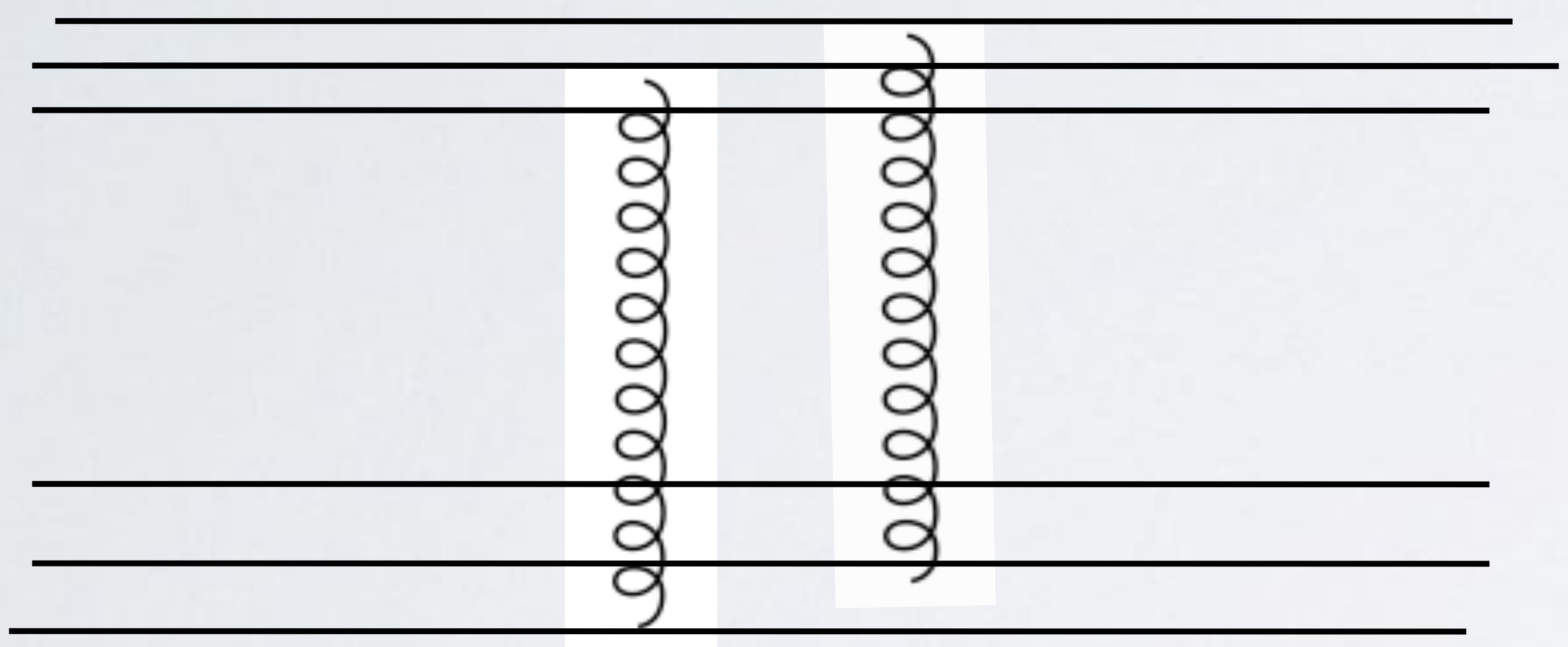

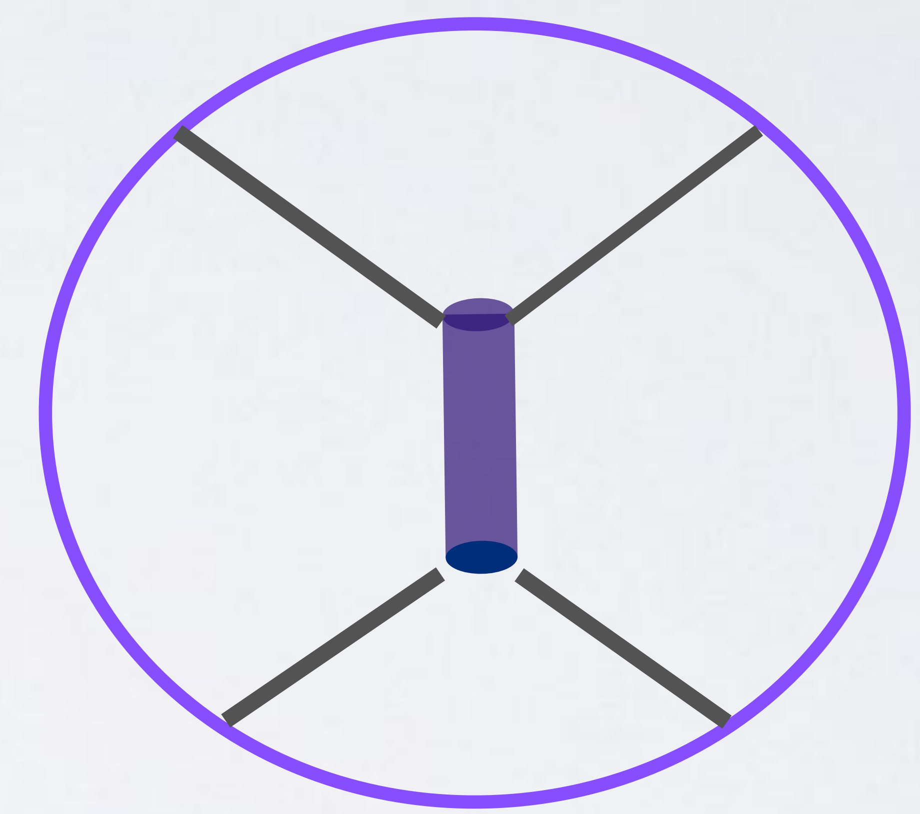

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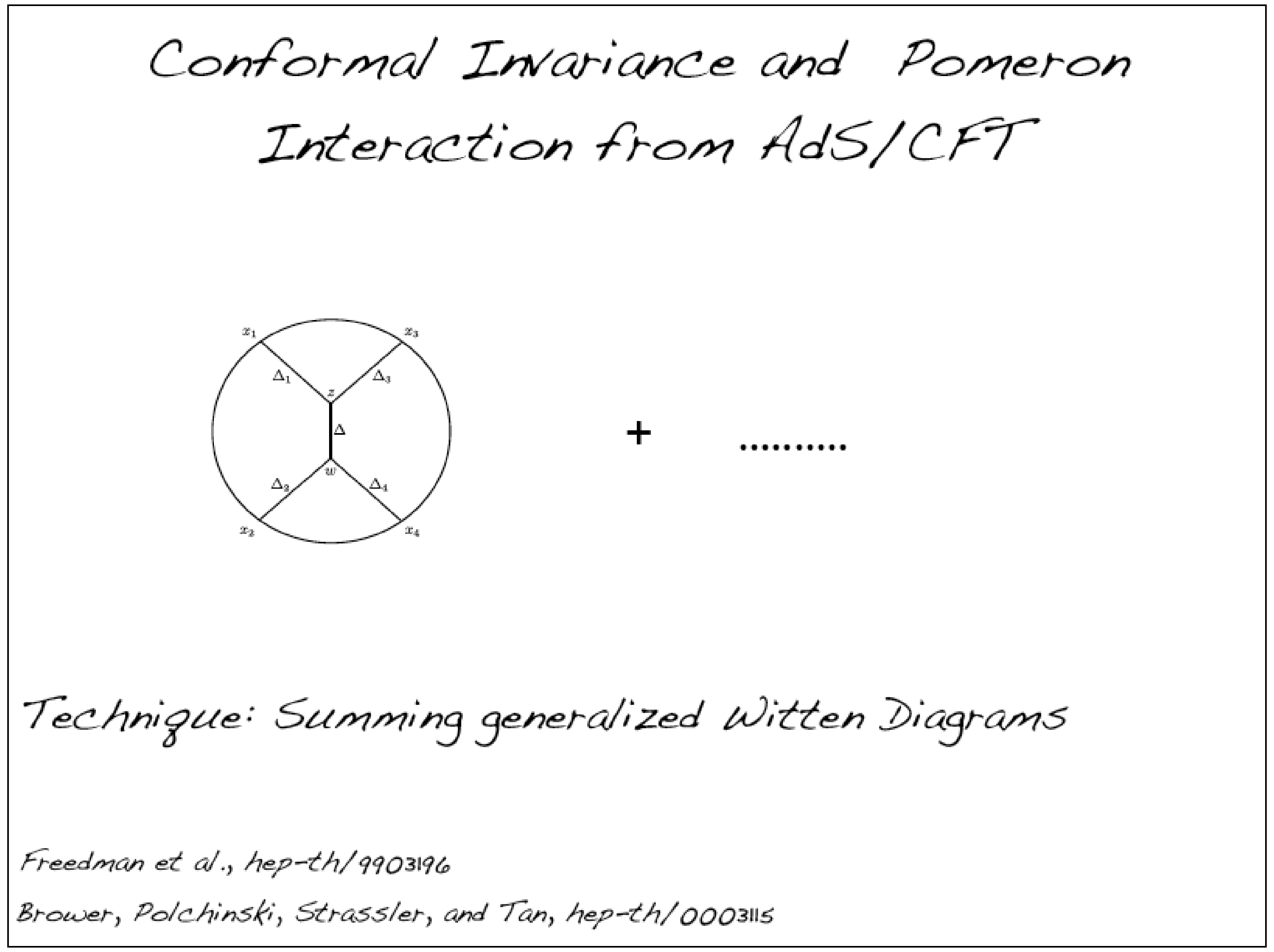
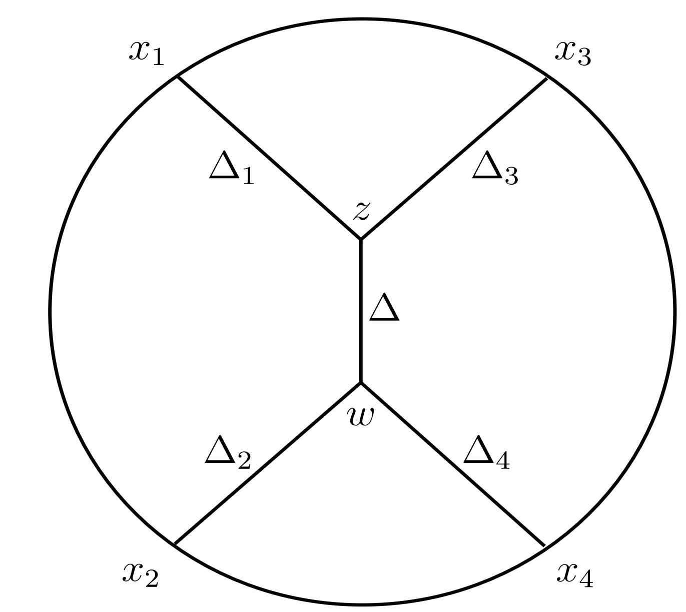


- Draw all "Witten-Feynman" Diagrams in AdS$_5$,
- High Energy Dominated by Spin-2 Exchanges:

$$p_1 + p_2 \to p_3 + p_4$$

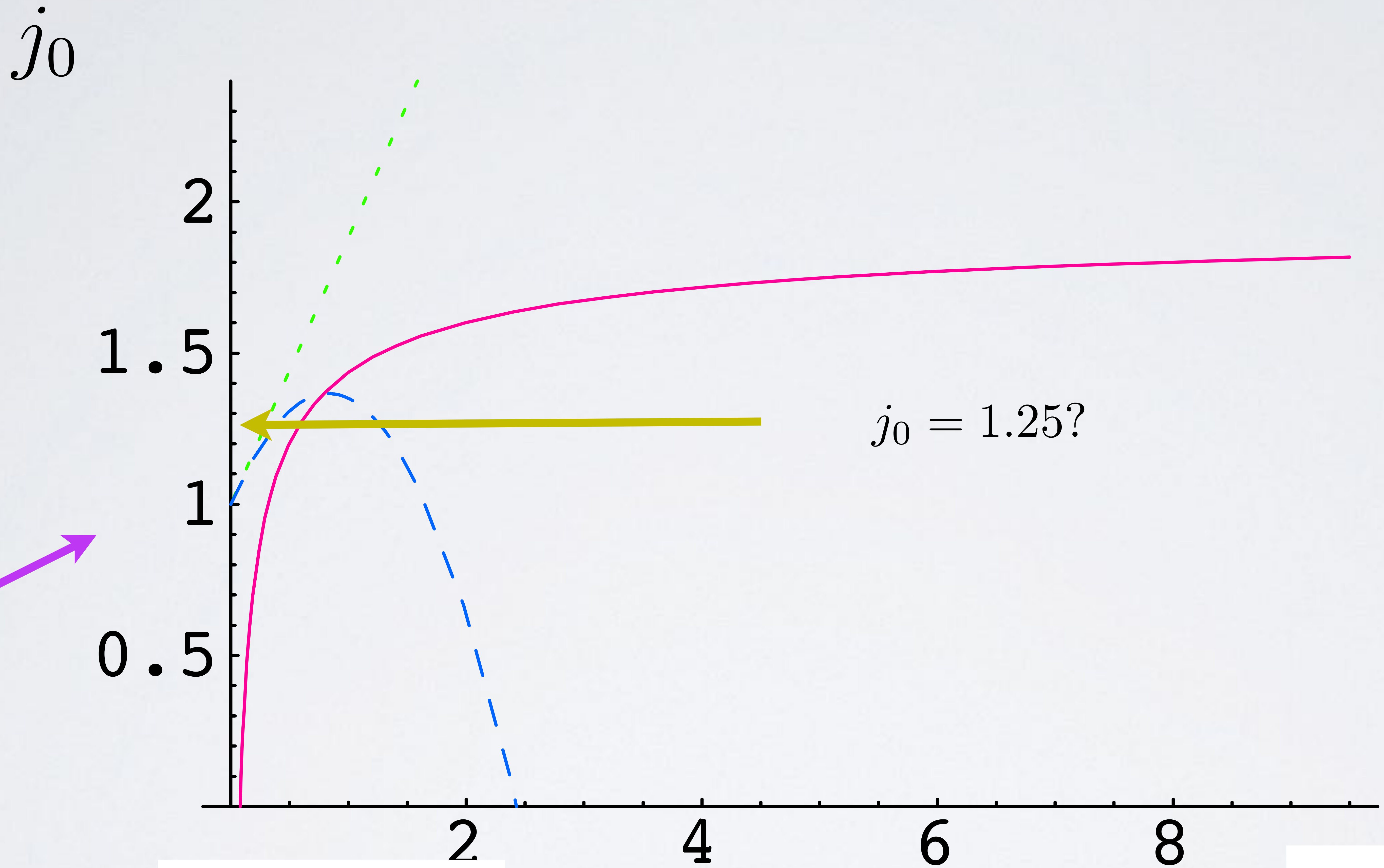


One Graviton Exchange at High Energy

$$T^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz}{z^5} \int \frac{dz'}{z'^5}\, \tilde\Phi_\Delta(p_1^2,z)\tilde\Phi_\Delta(p_3^2,z)\mathcal{T}^{(1)}(p_i,z,z')\tilde\Phi_\Delta(p_2^2,z')\tilde\Phi_\Delta(p_4^2,z')$$

$$\boxed{\mathcal{T}^{(1)}(p_i,z,z') = (z^2 z'^2 s)^2 G_{++,--}(q,z,z') = (zz's)^2 G^{(5)}_{\Delta=4}(q,z,z')}$$

$\mathcal{N} = 4$ Strong vs Weak $g^2 N_c$



$j_0 = 1.25?$

$j_0 = 2$

Graviton

$j_0 = 1$

Two Gluon

BFKL

QCD?

BPST

$j_0 = 1 + \ln(2) g^2 N_c / \pi^2$

$j_0 = 2 - 2/\sqrt{g^2 N_c}$

PHYSICS AT HIGH ENERGY

Confinement

Stringy Corrections

Pomeron as

Reggeized Graviton in AdS

Pomeron intercept due to diffusion

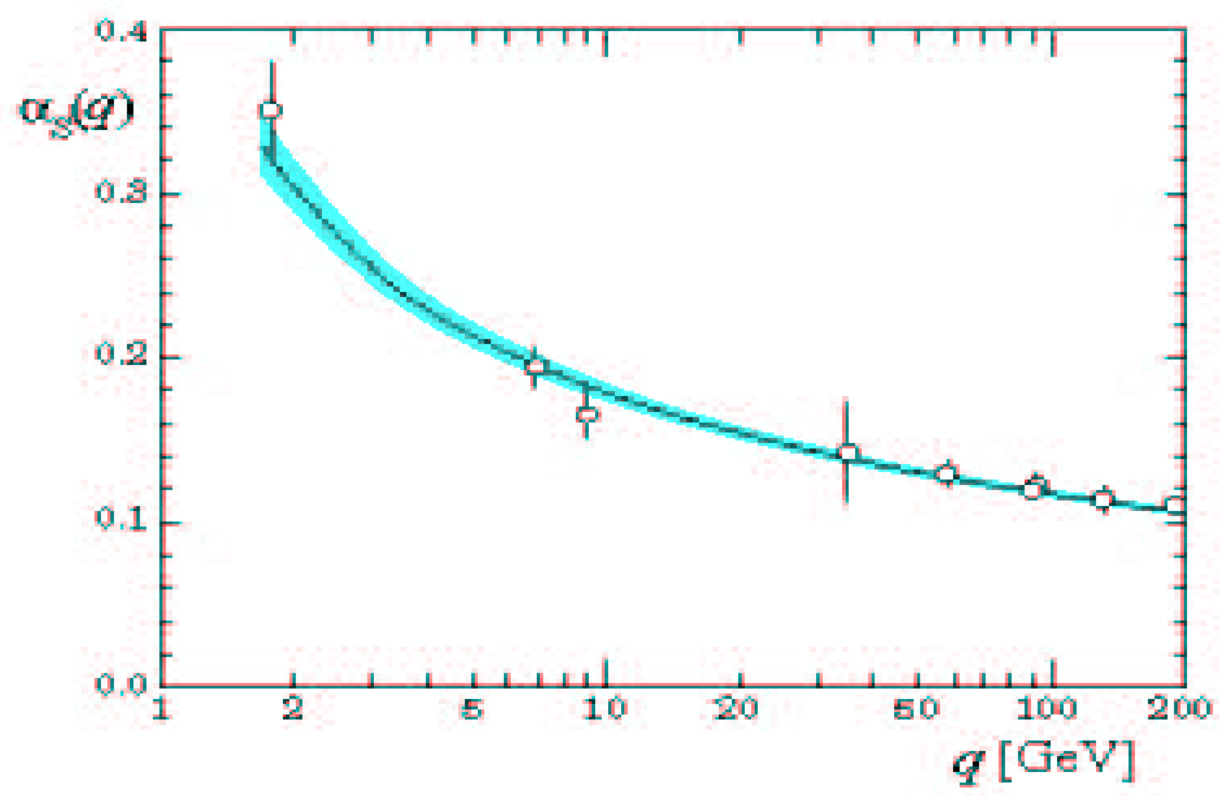
Diffusion takes place in both Impact space and in AdS

Diffusion in AdS relates anomalous dimension and to intercept

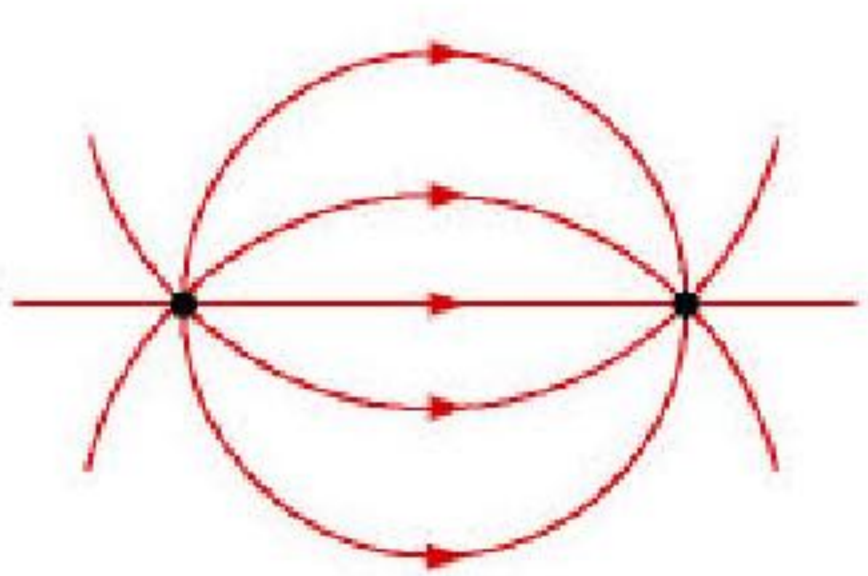
Diffusion in Impact space relates to expansion in transverse size

Asymptotic Freedom

perturbative



$$\alpha_s(q) \equiv \frac{\bar{g}(q)^2}{4\pi} = \frac{c}{\ln(q/\Lambda)} + \ldots$$



r < 0.1 fm

Confinement

non-perturbative

r >> 1 fm

Force at Long Distance--Constant Tension/Linear Potential, Coupling increasing, Quarks and Gluons strongly bound <==> "Stringy Behavior"

Outline

- Holographics Duality - Historical Perspective:
  - Duality in Physics, Holographic Duality, etc.
  - Gauge Theories, Massless Limit and Scale Invaraince
  - Holographic Duality, ADS/CFT (String-Gauge Duality)
- Size and Shape of hadrons: QCD, High Energy Scattering
  - "Pomeron/Graviton"
  - Glueballs masses and unification of hard and soft physics
  - DIS — BPST Pomeron vs BFKL and DGLAPP
  - Inclusive and Exclusive Central Production
- QCD and Modern CFT/String Studies:

Comparison of strong vs weak coupling kernel at t=0

Strong Coupling:

$$\mathcal{K}(r, r', s) = \frac{s^{j_0}}{\sqrt{4\pi \mathcal{D} \ln s}} e^{-(\ln r - \ln r')^2 / 4\mathcal{D} \ln s}$$

Diffusion in "warped co-ordinate"

$$j_0 = 2 - \frac{2}{\sqrt{g^2 N}} + O(1/g^2 N)$$

$$\mathcal{D} = \frac{1}{2\sqrt{g^2 N}} + O(1/g^2 N) \quad .$$

Weak Coupling:

$$K(s, k_\perp, k'_\perp) \approx \frac{s^{\alpha(0)-1}}{\sqrt{\pi \ln s}} e^{-[(\ln k'_\perp - \ln k_\perp)^2 / 4\mathcal{D} \ln s]}$$

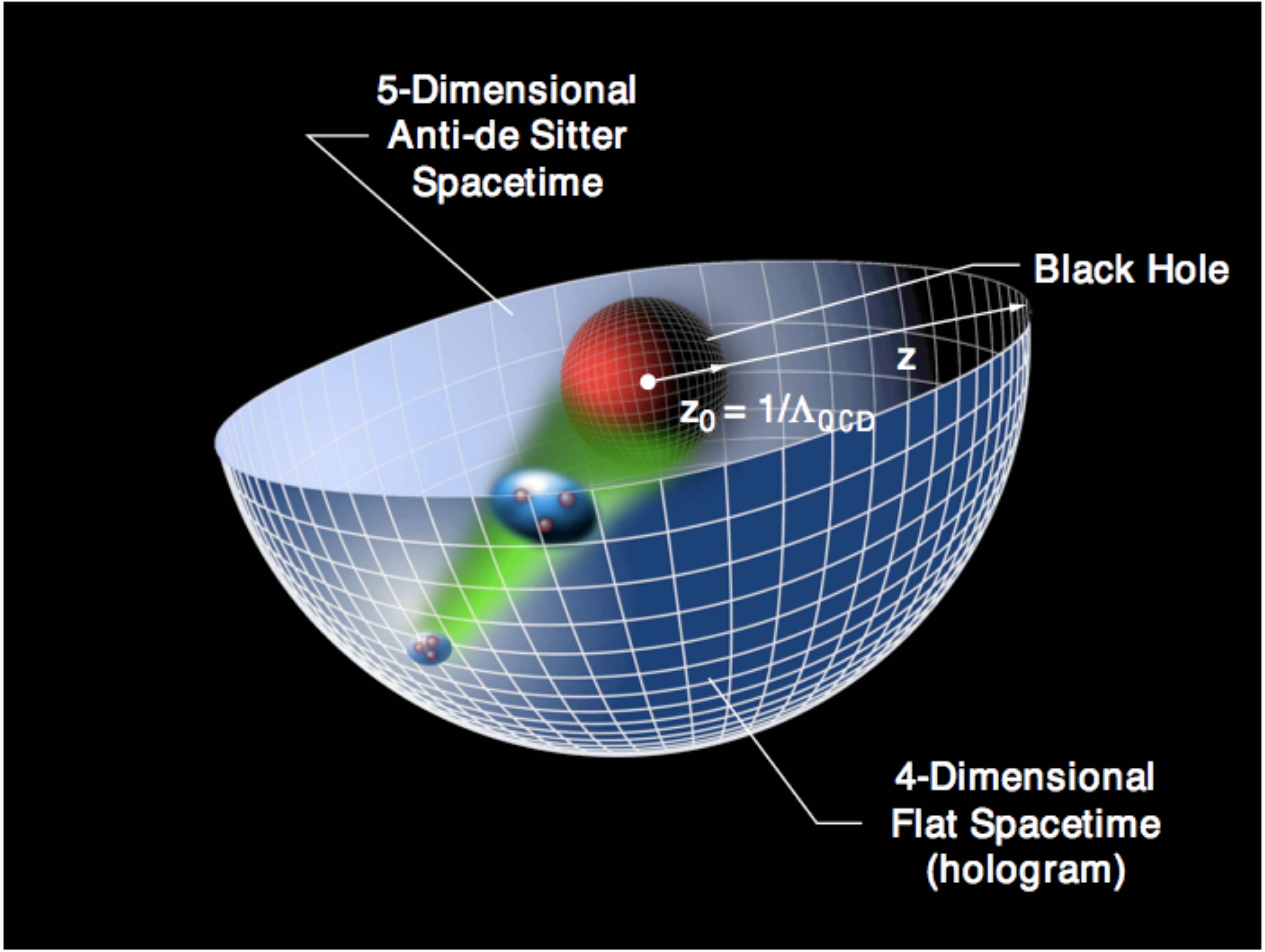
$$j_0 = 1 + \ln(2) g^2 N / \pi^2$$

$$\mathcal{D} = \frac{14\zeta(3)}{\pi} g^2 N / 4\pi^2 .$$

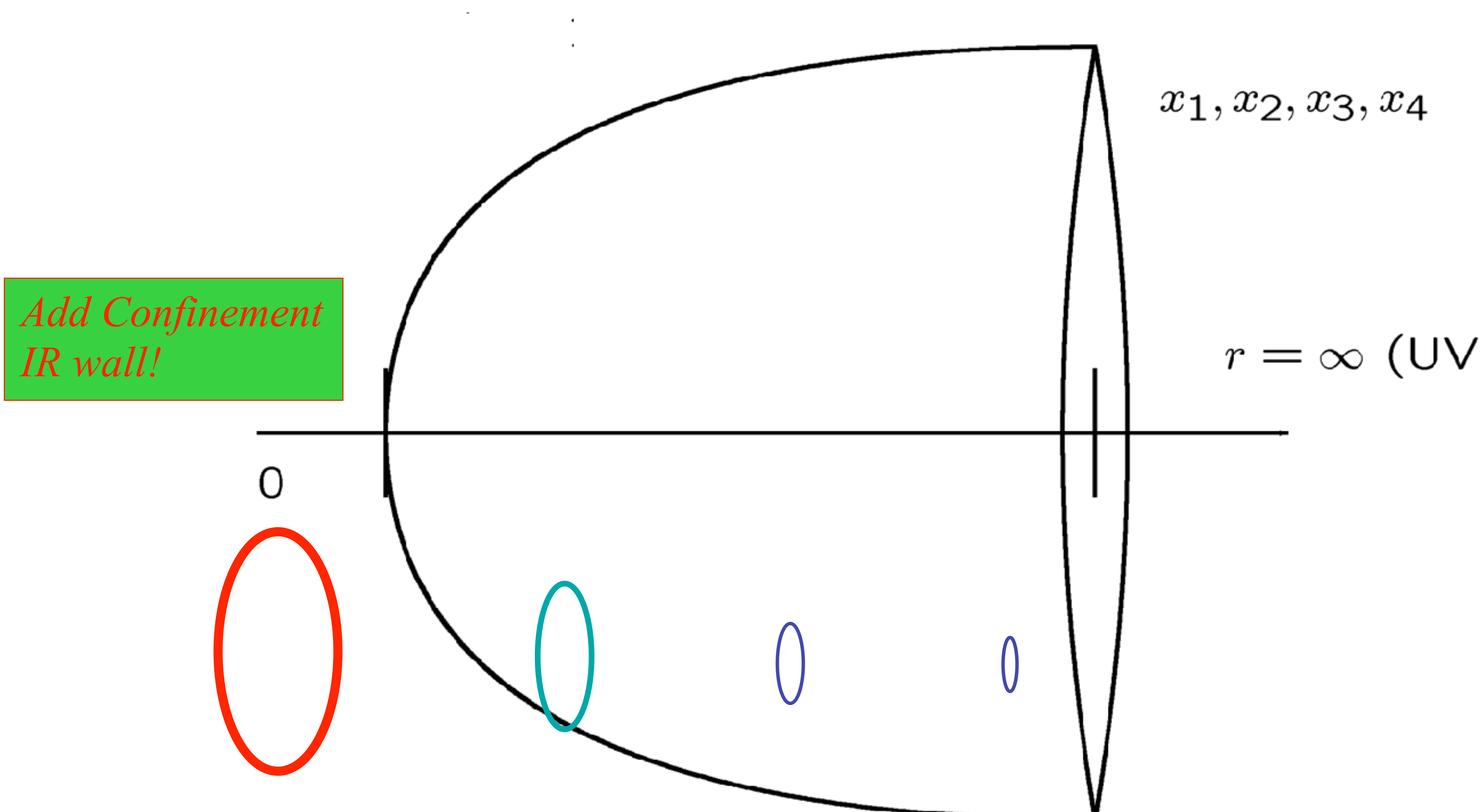
◆Confinement:

If strictly Conformal, therefore no scale and no particles,

Brower, Polchinski, Strassler, and Tan: "The Pomeron and Gauge/String Duality," hep-th/063115


5-Dimensional
Anti-de Sitter
Spacetime
Black Hole
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

Cutoff AdS$_5$



4-Dim Massive Graviton

5-Dim Massless Mode:

$$0 = E^2 - (p_1{}^2 + p_2{}^2 + p_3{}^2 + p_r{}^2)$$

If, due to Curvature in fifth-dim, $p_r{}^2 \neq 0$,

Four-Dimensional Mass:

$$E^2 = (p_1{}^2 + p_2{}^2 + p_3{}^2) + M^2$$

IIA Glueball Wave Equations

$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}T_4(r) - (m^2r^3)T_4(r) = 0$$

$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}V_4(r) - (m^2r^3 - \frac{9}{r(r^6 - 1)})V_4(r) = 0$$

$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}S_4(r) - (m^2r^3 + \frac{432r^5}{(5r^6 - 2)^2})S_4(r) = 0$$

$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}N_4(r) - (m^2r^3 - 27r^5 + \frac{9}{r})N_4(r) = 0$$

$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}M_4(r) - (m^2r^3 - 27r^5 - \frac{9r^5}{r^6 - 1})M_4(r) = 0$$

$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}L_4(r) - (m^2r^3 - 72r^5)L_4(r) = 0$$

IIA Classification of QCD_4

States from 11-d G_{MN}				States from 11-d A_{MNL}		
$G_{\mu\nu}$	$G_{\mu,11}$	$G_{11,11}$	m_0 (Eq.)	$A_{\mu\nu,11}$	$A_{\mu\nu\rho}$	m_0(Eq.)
G_{ij} 2^{++}	C_i $1^{++}{}_{(-)}$	ϕ 0^{++}	4.7007 (T_4)	B_{ij} 1^{+-}	C_{123} $0^{+-}{}_{(-)}$	7.3059(N_4)
$G_{i\tau}$ $1^{-+}{}_{(-)}$	C_τ 0^{-+}		5.6555 (V_4)	$B_{i\tau}$ $1^{--}{}_{(-)}$	$C_{ij\tau}$ 1^{--}	9.1129(M_4)
$G_{\tau\tau}$ 0^{++}			2.7034(S_4)		$G^\alpha{}_\alpha$ 0^{++}	10.7239(L_4)

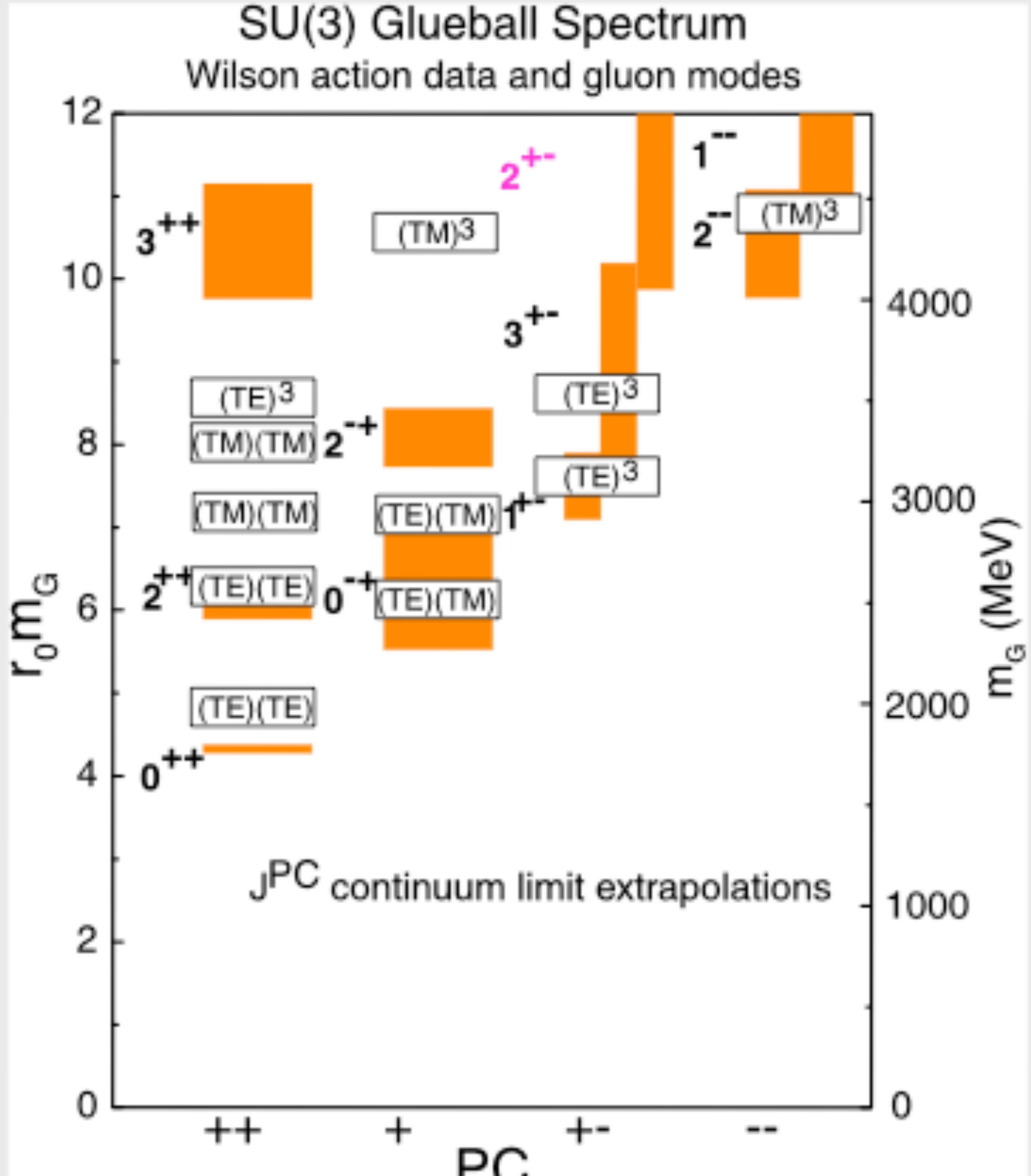
Subscripts to J^{PC} refer to $P_\tau = -1$ states

Bag Classification of States

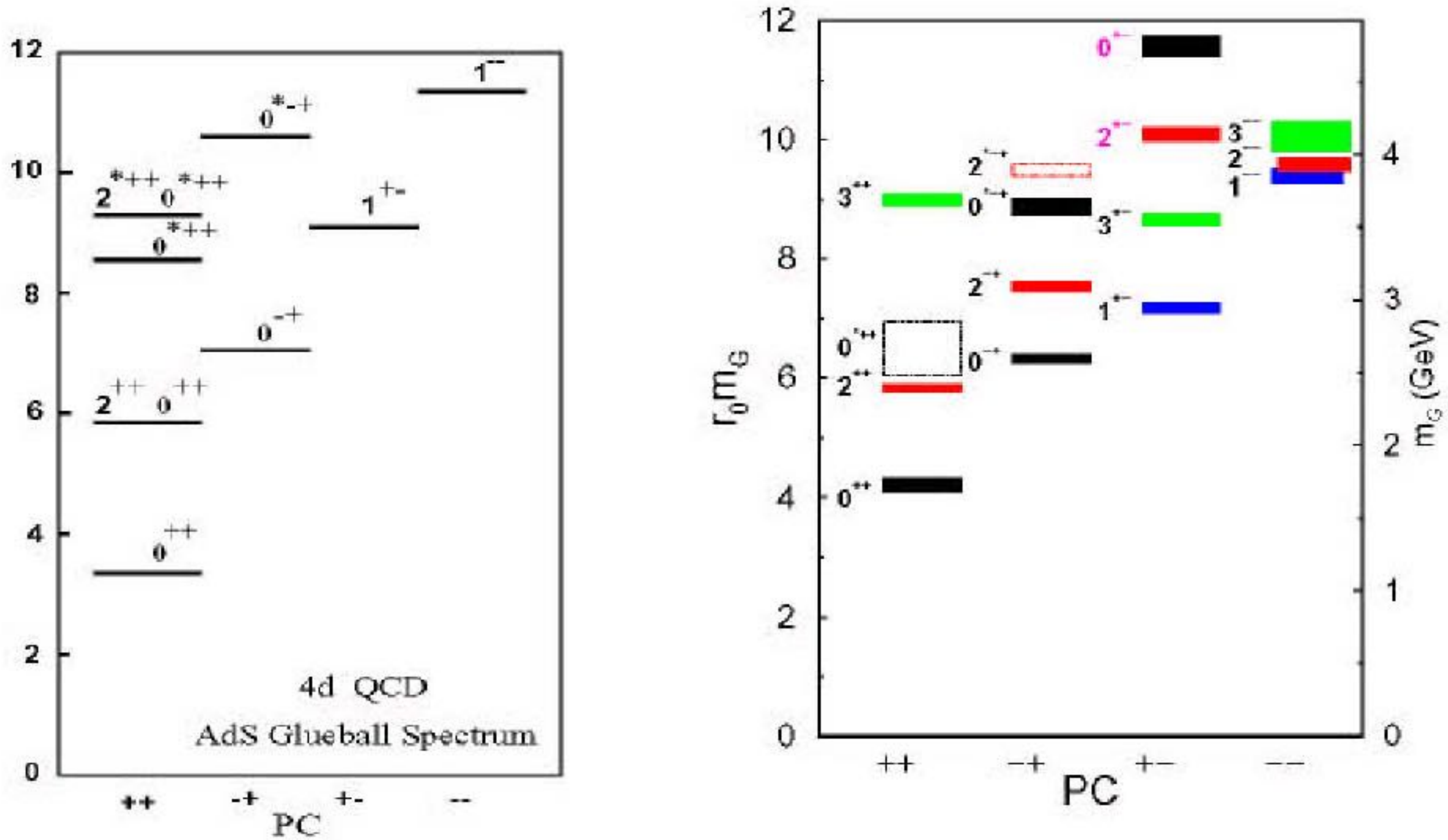
Dimension	State	Operator	Supergravity
d=4	0^{++}	$\mathrm{Tr}(FF) = E^a \cdot E^a - B^a \cdot B^a$	ϕ
d=4	2^{++}	$T_{ij} = E_i^a \cdot E_j^a + B_i^a \cdot B_j^a$ – trace	G_{ij}
d=4	0^{-+}	$\mathrm{Tr}(F^*F) = E^a \cdot B^a$	C_τ
d=4	0^{++}	$2T_{00} = E^a \cdot E^a + B^a \cdot B^a$	$G_{\tau\tau}$
d=4	2^{-+}	$E_i^a \cdot B_j^a + B_i^a \cdot E_j^a$ – trace	absent
d=4	2^{++}	$E_i^a \cdot E_j^a - B_i^a \cdot B_j^a$ – trace	absent
d=6	$(1,2,3)^{\pm +}$	$\mathrm{Tr}(F_{\mu\nu}\{F_{\rho\sigma},F_{\lambda\eta}\}] \sim d^{abc}F^a F^b F^c$	B_{ij} C_{ij}
d=6	$(1,2,3)^{\pm +}$	$\mathrm{Tr}(F_{\mu\nu}[F_{\rho\sigma},F_{\lambda\eta}]] \sim f^{abc}F^a F^b F^c$	absent

These are all the local d=4 and d=6 operators:See Jaffe, Johnson, Ryzak (JJR), "Qualitative Features of the Glueball Spectrum", Ann. Phys. 168 334 (1986)

Comparison with MIT Bag Calculation



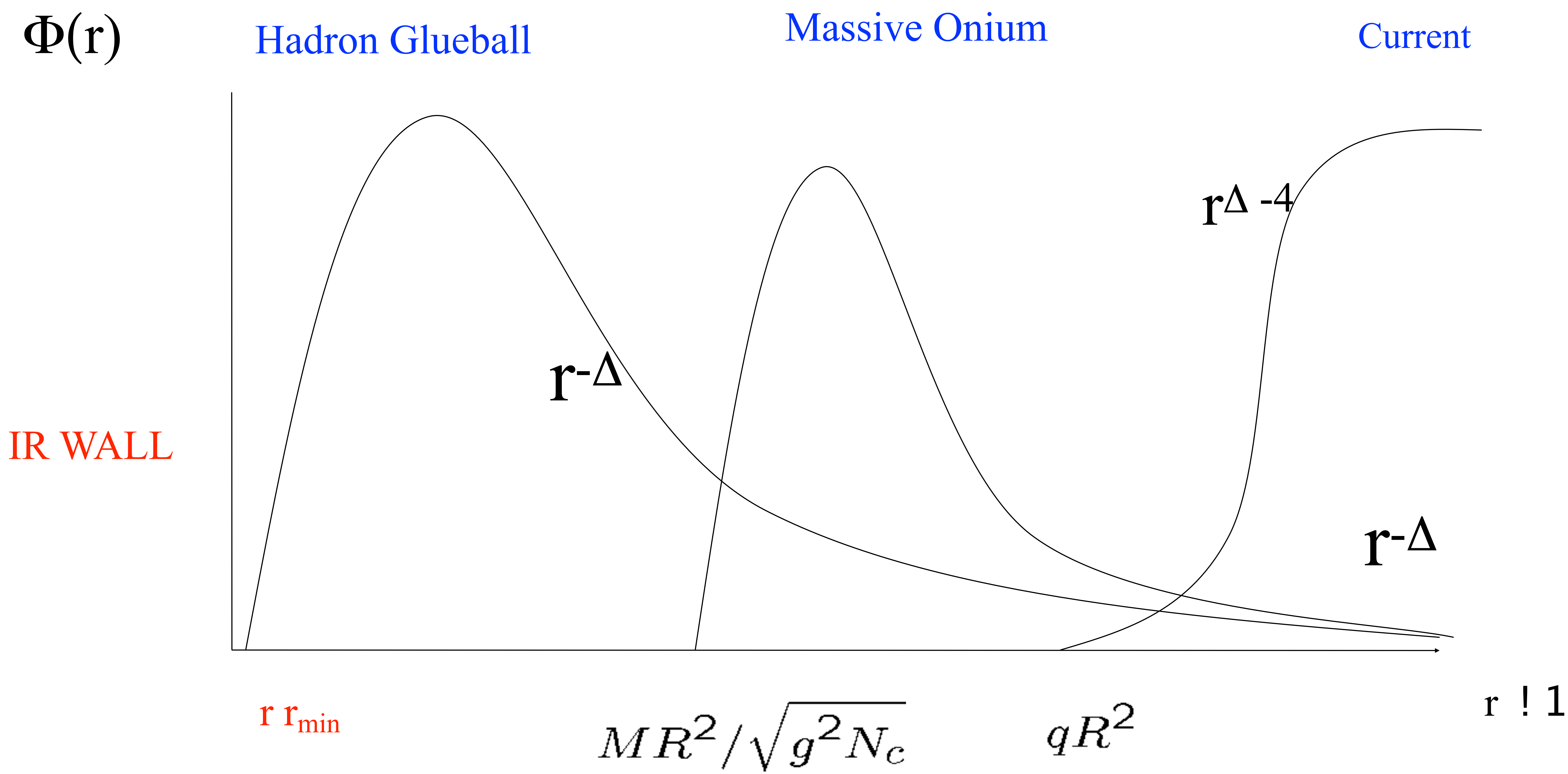
Glueball Spectrum



The AdS^7 glueball spectrum for QCD_4 in strong coupling (left) compared with the Morningstar/Peardon lattice spectrum for pure SU(3) QCD (right) with $1/r_0 = 410$ Mev.

R. Brower, S. Mathur, and C-I Tan, hep-th/0003115, "Glueball Spectrum of QCD from AdS Supergravity Duality".

Approx. Scale Invariance and the 5th dimension



Fixed-angle Scattering (Polchinski-Strassler) and also central inclusive production.

See also exhaustive work of Brodsky et al.

Quarks and Mesons

Holographic QCD

Glueballs to be identified through *decay patterns*, but not (yet) available from lattice $\rightarrow$ seeking help from holographic (large-N_c) QCD:

Witten-Sakai-Sugimoto model (Witten 1997, Sakai & Sugimoto 2004)

- almost parameter-free top-down construction from type-IIA superstrings:
  - N_c D4 branes provide nonconformal dual of 4+1-dim. super-Yang-Mills
  - KK compactification on circle in x_4 $\rightarrow$ nonsupersymmetric 3+1-dim. Yang-Mills theory at low energies
  - N_f chiral quarks from N_f D8-$\overline{\text{D8}}$ probe branes (Sakai & Sugimoto 2004ff)
- reproduces many aspects of low-energy QCD
  - chiral symmetry breaking $U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_V$ with Witten-Veneziano mass$^2 \propto 1/N_c$
  - effective Lagrangian: nonlinear σ-model with Skyrme term plus vector and axial-vector mesons similar to HLS models
  - correct WZW terms
- fitting $M_{\rm KK}$ and 't Hooft coupling $\lambda|_{M_{\rm KK}}$ yields quantitative predictions, often with only 10-30% errors:

Quantitative predictions

- Parameter-free: mass spectrum of vector and axial-vector mesons
  $m(\rho^*)/m(\rho)$, $m(a_1)/m(\rho)$, $m(a_1^*)/m(\rho)$ correct within $\lesssim 20\%$
- Other predictions, depending on value of 't Hooft coupling λ at scale $M_{\rm KK}$ with
  1. $m_\rho \approx 776$ MeV fixes $\boxed{M_{\rm KK} = 949 \text{ MeV}}$
  2. $f_\pi^2 = \frac{\lambda N_c}{54\pi^4} M_{\rm KK}^2$ gives $\boxed{\lambda = g_{\rm YM}^2 N_c \approx 16.63}$ [Sakai&Sugimoto 2005-7]
     (matching instead large-N_c lattice result [Bali et al. 2013] for $m_\rho/\sqrt{\sigma}$ gives $\lambda \approx 12.55$)

give (for $N_c = 3$ and $\lambda = 16.63 \ldots 12.55$):

- LO decay rate of ρ meson $\sim \lambda^{-1} N_c^{-1}$
  $\boxed{\Gamma_{\rho\rightarrow 2\pi}/m_\rho = 0.1535 \ldots 0.2034}$ (exp.: 0.191(1))
- decay rate for $\omega \rightarrow 3\pi$ (from Chern-Simons part of D8 action) $\sim \lambda^{-4} N_c^{-2}$
  $\boxed{\Gamma_{\omega\rightarrow 3\pi}/m_\omega = 0.0033 \ldots 0.0102}$ (exp.: 0.0097(1))
- Witten-Veneziano mass $m_{\eta_0} = \frac{\sqrt{N_f/N_c}}{3\sqrt{3}\pi} \lambda M_{\rm KK} \approx 967 \ldots 730$ MeV for $N_f = 3$;
  with explicit quark mass terms and fitted m_π, $m_K \rightarrow m_\eta$, $m_{\eta'}$ within 10%

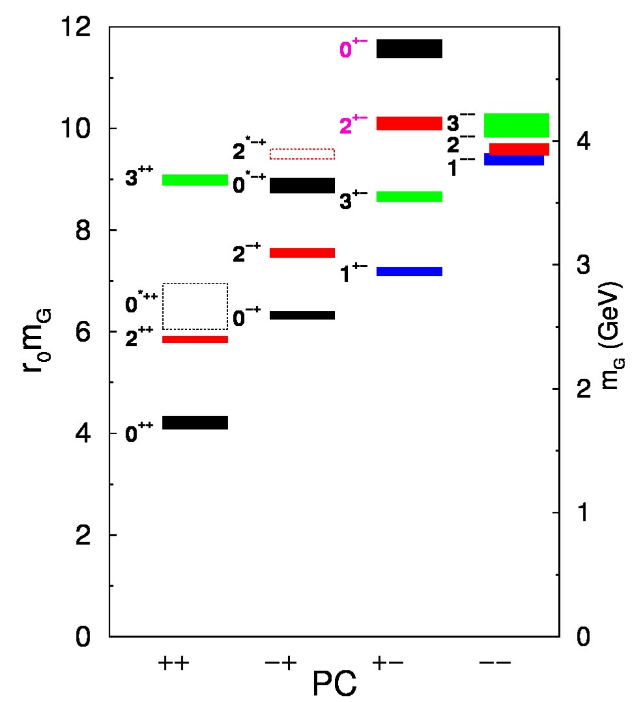
Glueballs from AdS/CFT

Longstanding, still elusive prediction of QCD: Glueballs

- Spectrum of *bare* glueballs (prior to mixing with $q\bar{q}$ states) more or less known from lattice:

$m_{0^{++}} \sim 1.7$ GeV
$m_{2^{++}} \sim 2.4$ GeV
$m_{0^{-+}} \sim 2.6$ GeV

Morningstar & Peardon hep-lat/9901004



- Interactions, decays of glueballs and mixing with $q\bar{q}$ however largely unknown:
$\rightarrow$ no conclusive identification of any glueball in meson spectrum

- most discussed scenario for lightest scalar glueball:

various phenomenological models describe $f_0(1500)$ or $f_0(1710)$ isoscalar 0^{++} mesons alternatingly as ∼50-70% or ∼75-90% G(lueball)

[G and two isoscalar $q\bar{q}$ states $u\bar{u} + d\bar{d}$ and $s\bar{s}$ can be shared by $f_0(1370)$, $f_0(1500)$, $f_0(1710)$]

Glueballs from supergravity background

$\exists$ scalar and tensor glueballs corresponding to 5D dilaton Φ and graviton G_{ij}
Csaki, Ooguri, Oz & Terning 1999

Type-IIA supergravity compactified on x_4-circle many more modes:
Constable & Myers 1999; Brower, Mathur & Tan 2000

Mode Sugra fields J^{PC}	S_4 G_{44} 0^{++}	T_4 Φ, G_{ij} $0^{++}/2^{++}$	V_4 C_1 0^{-+}	N_4 B_{ij} 1^{+-}	M_4 C_{ij4} 1^{--}	L_4 G^α_α 0^{++}
n=0	7.30835	22.0966	31.9853	53.3758	83.0449	115.002
n=1	46.9855	55.5833	72.4793	109.446	143.581	189.632
n=2	94.4816	102.452	126.144	177.231	217.397	277.283
n=3	154.963	162.699	193.133	257.959	304.531	378.099
n=4	228.709	236.328	273.482	351.895	405.011	492.171

Lowest mode not from dilaton, but from "exotic polarization" – in 11D notation:

$$\underline{\delta g_{44}} = -\frac{r^2}{L^2} f\, H(r) G(x), \quad \delta g_{\mu\nu} = \frac{r^2}{L^2}\left[\frac{1}{4}H(r)\eta_{\mu\nu} - \left(\frac{1}{4} + \frac{3R^6}{5r^6 - 2R^6}\right) H(r)\frac{\partial_\mu \partial_\nu}{M^2}\right] G(x)$$

$$\delta g_{11,11} = \frac{r^2}{L^2}\frac{1}{4}H(r)G(x), \quad \delta g_{rr} = -\frac{L^2}{r^2} f^{-1}\frac{3R^6 H(r) G(r)}{5r^6 - 2R^6}, \quad \delta g_{r\mu} = \frac{90 r^7 R^6 H(r) \partial_\mu G(x)}{M^2 L^2 (5r^6 - 2R^6)^2}$$

Quarks and Mesons and Glueball Production/ Decays in AdS/CFT

Glueball-$\bar{q}q$ couplings in Sakai-Sugimoto model

Gravitational modes stable in confined background, but can calculate *effective action for glueball-$\bar{q}q$ interactions*

done for lowest (exotic) mode by
Hashimoto, Tan & Terashima, Phys.Rev. D77 (2008) 086001 [arXiv:0709.2208]

revisited and extended to other modes by
Brünner, Parganlija & AR, Phys.Rev. D91 (2015) 106002 [arXiv:1501.07906]

For example: Vertices of one glueball and two (massless) pions for "exotic" mode:

$$S_{G_E\pi\pi} = \mathrm{Tr}\int d^4x \frac{1}{2}\partial_\mu\pi\,\partial_\nu\pi\left(\breve{c}_1\eta^{\mu\nu} - c_1\frac{\partial^\mu\partial^\nu}{M_E^2}\right)G_E$$

for "predominantly dilatonic" mode:

$$S_{G_D\pi\pi} = \mathrm{Tr}\int d^4x \frac{1}{2}\partial_\mu\pi\,\partial_\nu\pi\;\tilde{c}_1\left(\eta^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{M_D^2}\right)G_D$$

with $\{c_1, \breve{c}_1, \tilde{c}_1\} = \{62.66, 16.39, 17.23\} \times \lambda^{-1/2}N_c^{-1}M_{\mathrm{KK}}^{-1}$

and many more: $S_{G\rho\rho} \propto \lambda^{-1/2}N_c^{-1}$, $S_{G\rho\pi\pi} \propto \lambda^{-1}N_c^{-3/2}, \ldots$

Comparison with $f_0(1710)$

decay	Γ/M **(PDG)**	$\Gamma/M[G_D]$ (chiral)	$\Gamma/M[G_D]$ (massive)
$f_0(1710)$ (total)	**0.081(5)**	0.059...0.076	0.083...0.106
$f_0(1710) \to 2K$	(*) 0.029(10)	0.012...0.016	0.029...0.038
$f_0(1710) \to 2\eta$	0.014(6)	0.003...0.004	0.009...0.011
$f_0(1710) \to 2\pi$	$0.012(^{+5}_{-6})$	0.009...0.012	0.010...0.013
$f_0(1710) \to 2\rho, \rho\pi\pi \to 4\pi$	?	0.024...0.030	0.024...0.030
$f_0(1710) \to 2\omega$	$0.010(^{+6}_{-7})$	0.011...0.014	0.011...0.014
$f_0(1710) \to \eta\eta'$	?	0	if 0 : $\Updownarrow$
$\Gamma(\pi\pi)/\Gamma(K\bar{K})$	**0.41** $^{+0.11}_{-0.17}$	**3/4**	**0.35**
$\Gamma(\eta\eta)/\Gamma(K\bar{K})$	**0.48**± 0.15	1/4	0.28

* PDG ratios for decay rates + $\mathrm{Br}(f_0(1710) \to KK) = 0.36(12)$ [Albaladejo&Oller 2008]

- decays into 2 pseudoscalars: massive WSS perfectly compatible with PDG data!
- significant decay into 4 pions (after extrapolation to beyond 2ρ threshold): falsifiable prediction of this model!

  ($f_0(1710) \to 2\rho^0$ forthcoming from CMS-TOTEM!)

Summary – Glueballs in Witten-Sakai-Sugimoto model

After fitting just m_ρ to fix $M_{\mathrm{KK}} = 949$ MeV

- good prediction of higher vector and axial vector mesons masses,
- reasonable prediction of glueball masses if "exotic mode" discarded

after fitting f_π or $m_\rho/\sqrt{\sigma}$ to also fix 't Hooft coupling at $\lambda = 16.63 \ldots 12.55$

- good prediction of ρ and ω decay rates
- good prediction of anomalous $m'_\eta \propto N_c^{-\frac{1}{2}}\lambda M_{\mathrm{KK}}$

Holographic glueball decay rates:

- narrow partial width $G_D \to \pi\pi$,
  quite compatible with experimental data for $\boxed{f_0(1710) \text{ as nearly pure glueball}}$
- much stronger decay of $f_0(1710)$ into $K\bar{K}$ need not be indicative of $s\bar{s}$ nature – well reproduced if (so far unobserved) decay into $\eta\eta'$ small
- predictions for decay to 4π, $\eta\eta'$ falsifiable by CMS-TOTEM, BESIII, ...
- tensor glueball broad if at $\gtrsim 2$ GeV
- pseudoscalar glueball again narrow (in prep.)

Anton Rebhan

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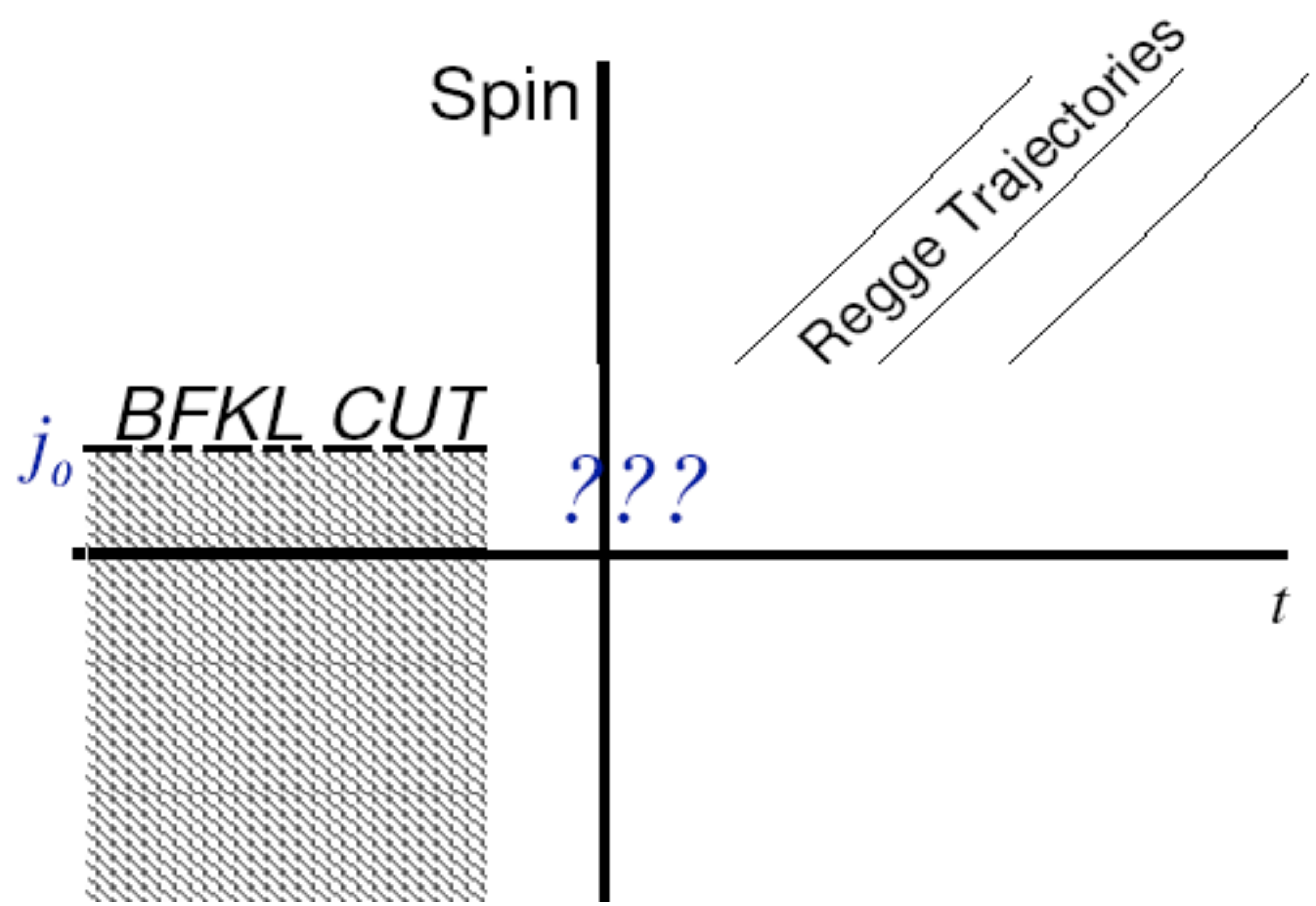
BPST PROGRAM
HE SCATTERING

VI. Synthesis of Hard (BFKL) & Soft (Regge) Pomeron

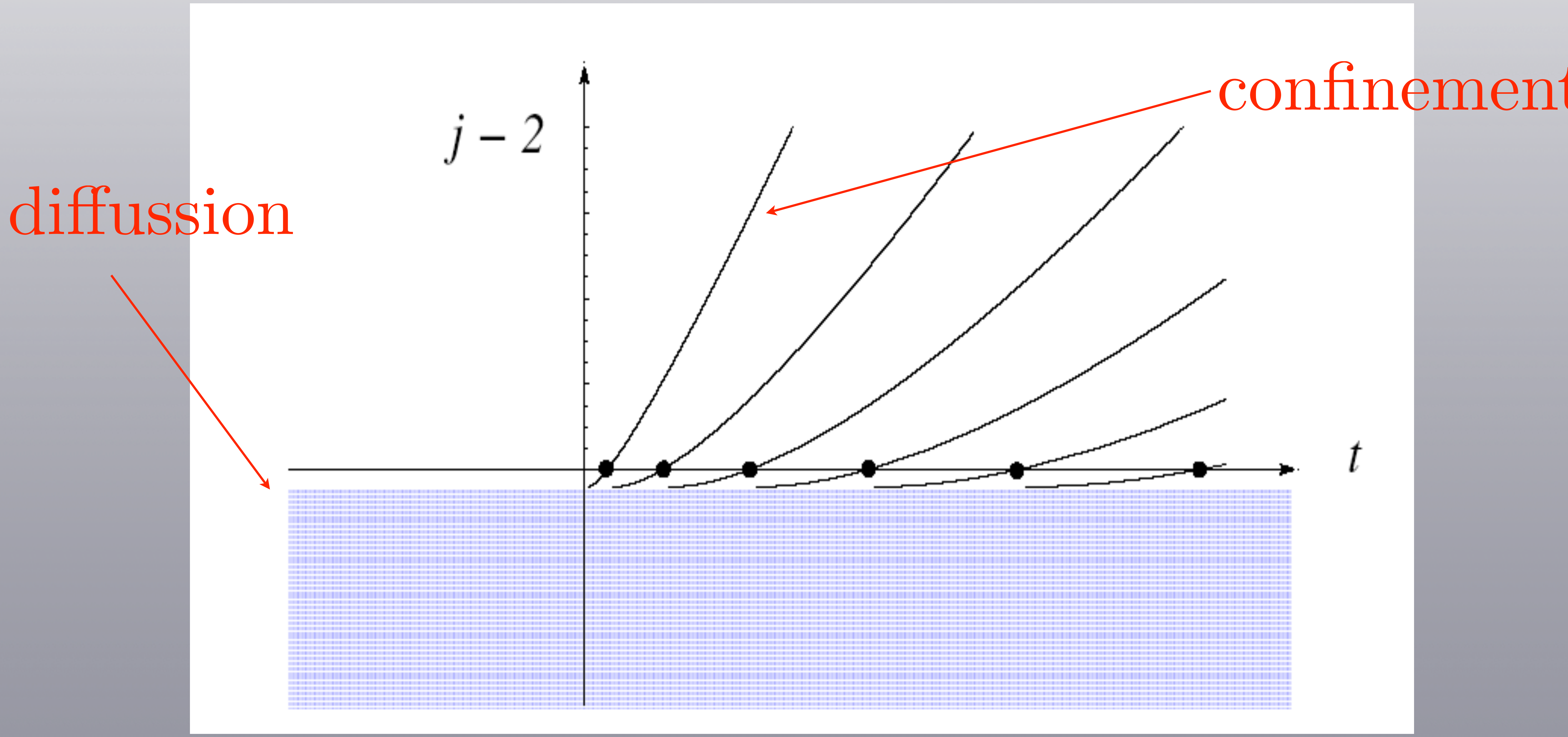
Analytic Structure of Pomeron Propagator

Conformal UV, Confining IR (large N)

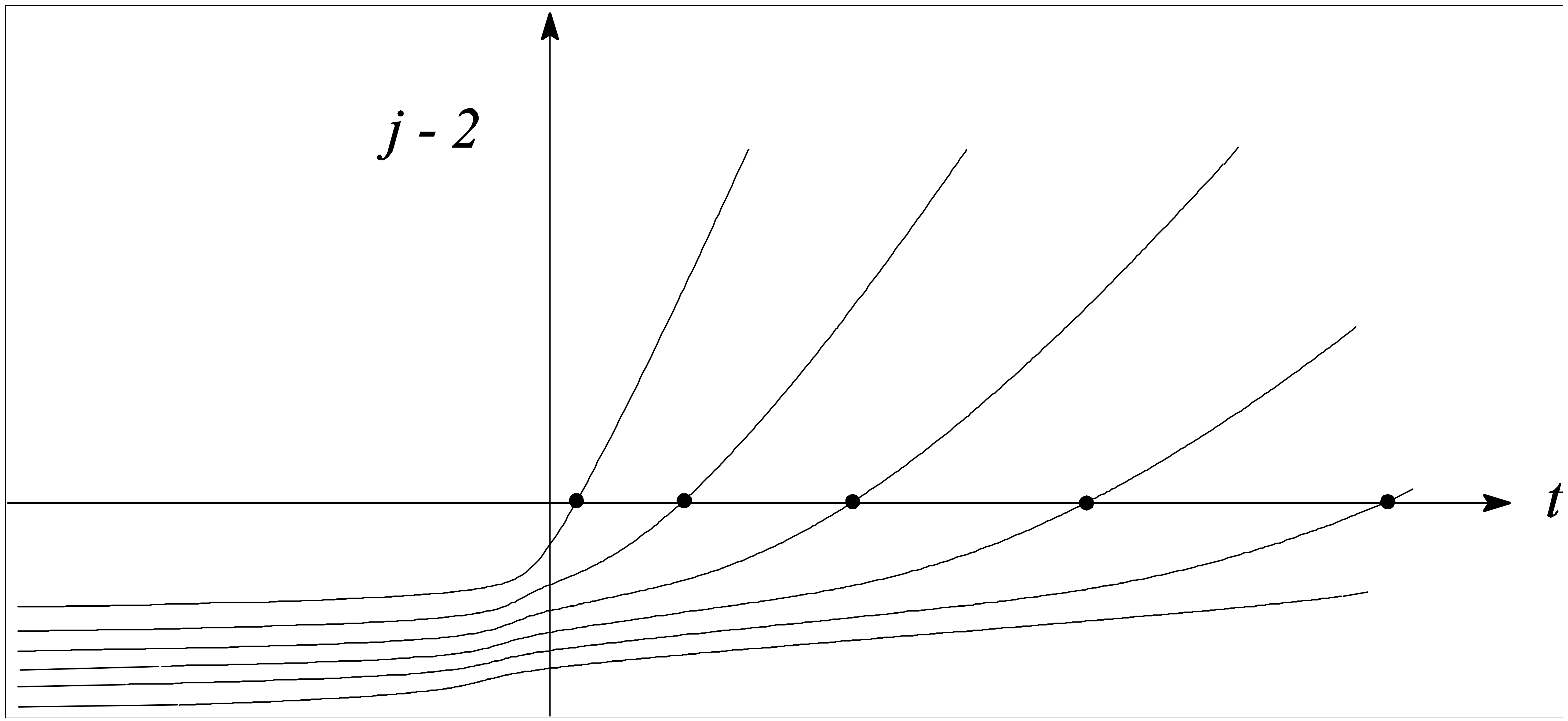


Unified Hard (conformal) and Soft (confining) Pomeron

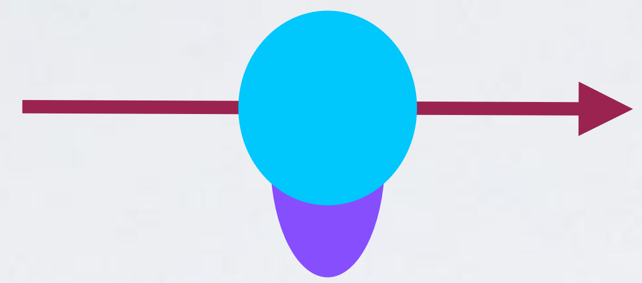
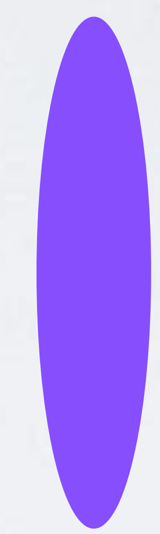
At finite λ, due to Confinement in AdS, *at* $t > 0$ aymptotical linear Regge trajectories



(Strong) Running Coupling



BASIC BUILDING BLOCK

- Elastic Vertex: 
- Pomeron/Graviton Propagator: 

$$\mathcal{K}(s,b,z,z') = -\left(\frac{(zz')^2}{R^4}\right)\int \frac{dj}{2\pi i}\left(\frac{1+e^{-i\pi j}}{\sin \pi j}\right)\widehat{s}^j\, G_j(z,x^\perp,z',x'^\perp;j)$$

conformal:
$$G_j(z,x^\perp,z',x'^\perp) = \frac{1}{4\pi z z'}\frac{e^{(2-\Delta(j))\xi}}{\sinh \xi}\ ,$$

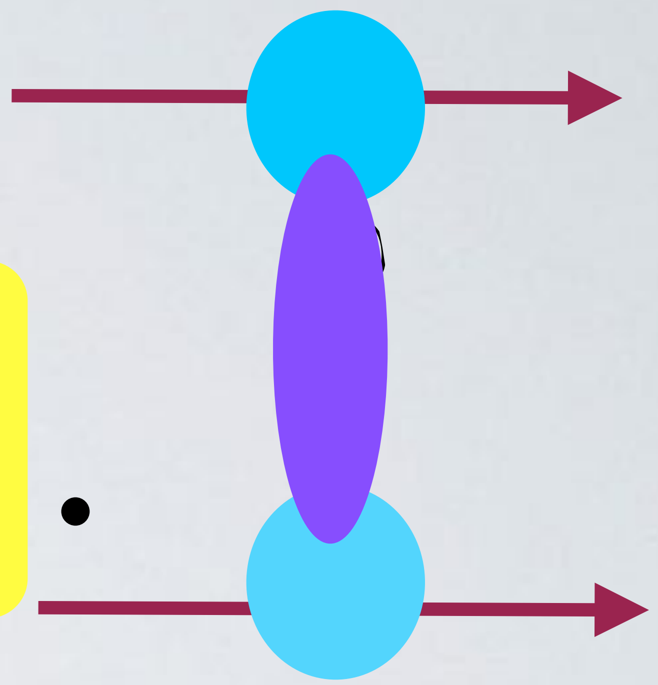
$$\Delta(j) = 2+\sqrt{2}\,\lambda^{1/4}\sqrt{(j-j_0)}$$

confinement: $G_j(z,x^\perp,z',x'^\perp;j) \longrightarrow$ discrete sum

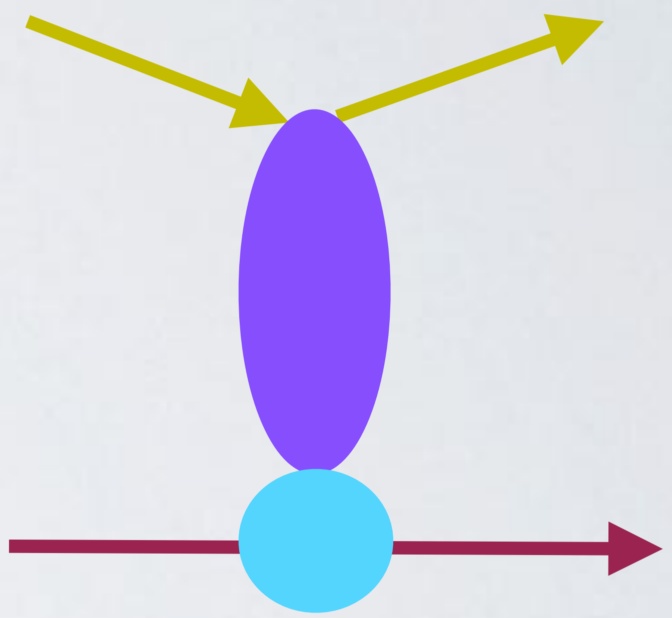
ADS BUILDING BLOCKS BLOCKS

For 2-to-2

$$A(s,t) = \Phi_{13} * \widetilde{\mathcal{K}}_P * \Phi_{24}\,.$$



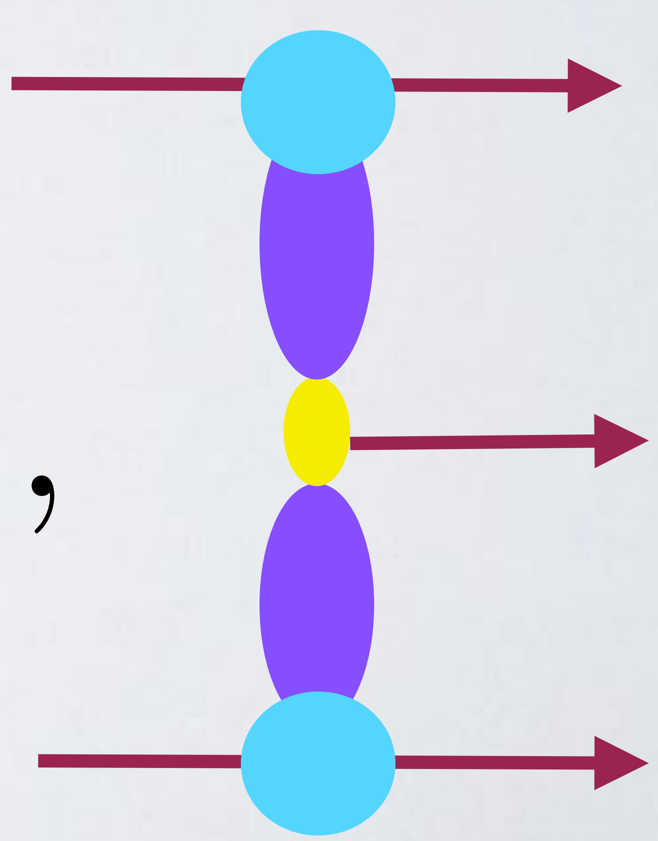
$$A(s,t) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}'\; e^{i\mathbf{q}_\perp \cdot (\mathbf{x}-\mathbf{x}')}\; \Phi_{13}(z)\; \mathcal{K}(s, \mathbf{x}-\mathbf{x}', z, z')\; \Phi_{24}(z')$$



$$d^3\mathbf{b} \equiv dz d^2x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$

For 2-to-3

$$A(s, s_1, s_2, t_1, t_2) = \Phi_{13} * \widetilde{\mathcal{K}}_P * V * \widetilde{\mathcal{K}}_P * \Phi_{24}\,,$$



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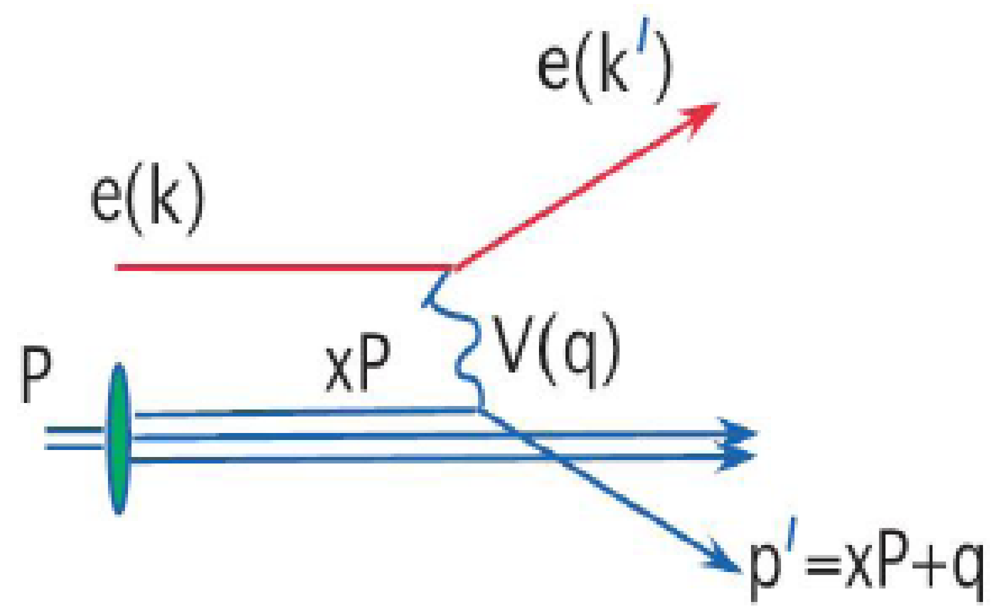


$$F_2(x, Q2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \left[\sigma_T(\gamma^* p) +_L (\gamma^* p)\right]$$

$$x \equiv \frac{Q^2}{s}$$

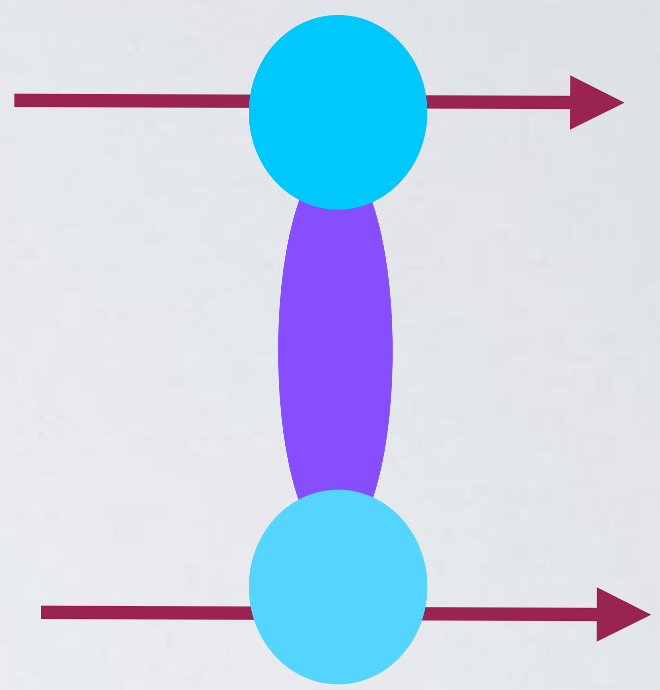
$$Small\ x : \ \frac{Q^2}{s} \to 0$$

$$Optical\ Theorem$$

$$\sigma_{total}(s, Q^2) = (1/s)\mathrm{Im}\ A(s, t = 0; Q^2)$$

ELASTIC VS DIS ADS BUILDING BLOCKS

$$A(s, x_\perp - x'_\perp) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}' \Phi_{12}(z) G(s, x_\perp - x'_\perp, z, z') \Phi_{34}(z')$$

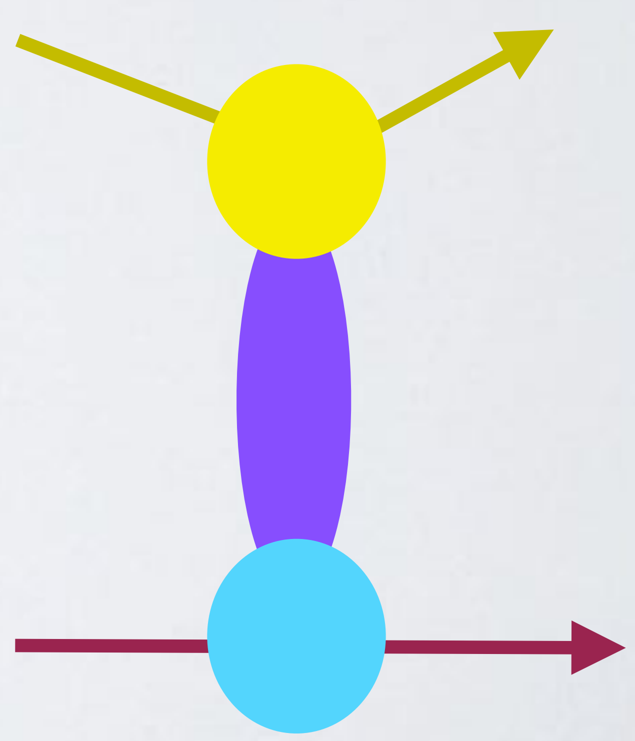


$$\sigma_T(s) = \frac{1}{s} Im A(s, 0)$$

---

for $\quad F_2(x, Q)$

$$\Phi_{13}(z) \to \Phi_{\gamma^*\gamma^*}(z, Q) = \frac{1}{z}[Qz)^4 (K_0^2(Qz) + K_1^2(Qz)]$$



$$d^3\mathbf{b} \equiv dz d^2 x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$

- We are interested in calculating the structure function $F_2(x, Q^2)$, which is simply the cross section for an off-shell photon. Using the optical theorem we obtain

$$\sigma_{tot} \simeq 2 \int d^2b \int dzdz' P_{13}(z) P_{24}(z')\ Im\ \chi(s, b, z, z')$$

- For DIS, P_{13} should present a photon on the boundary that couples to a spin 1 current in the bulk. This current then propagates through the bulk, and scatters off the target.
- The wave function, in the conformal limit, is

$$P_{13}(z) \to P_{13}(z, Q) = \frac{1}{z}(Qz)^4 (K_0^2(Qz) + K_1^2(Qz))$$

- For the proton, one for now treats it as a glueball of mass $\sim \Lambda = 1/Q'$.

Conformal Invariance as Isometry of AdS

Longitudinal Boost: $\tau = \log(\rho z z' s/2)$ $\quad \mathcal{K}(s, \vec{b}, z, z') = \int \frac{dj}{2\pi i} \left(\frac{e^{-i\pi j} + 1}{\sin \pi j} \right) e^{j\tau} \mathcal{K}(j, \vec{b}, z, z')$

Conformal Invariance in Transverse AdS_3: $\xi = \sinh^{-1} \left(\frac{b^2 + (z - z')^2}{2 z z'} \right)$

$$\mathcal{K}(j, \vec{b}, z, z') = \int \frac{d\nu}{2\pi} \left(\frac{e^{i\nu\xi}}{\sinh \xi} \right) G(j, \nu)$$

Pomeron as a pole in AdS: $\quad G(j, \nu) = \frac{1}{j - j_0 + \nu^2 / 2\sqrt{\lambda}}$

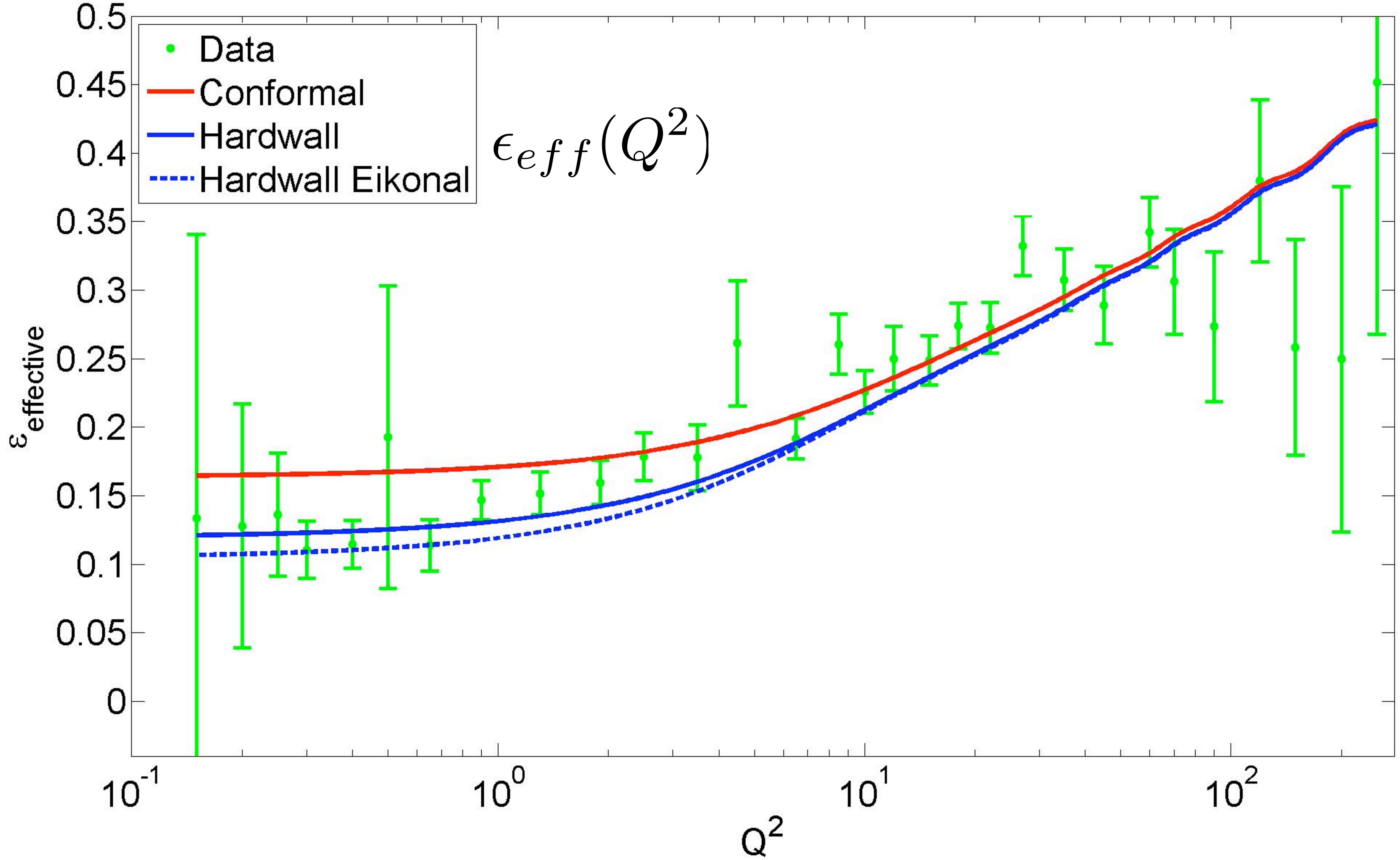
Full Conformal Invariance:

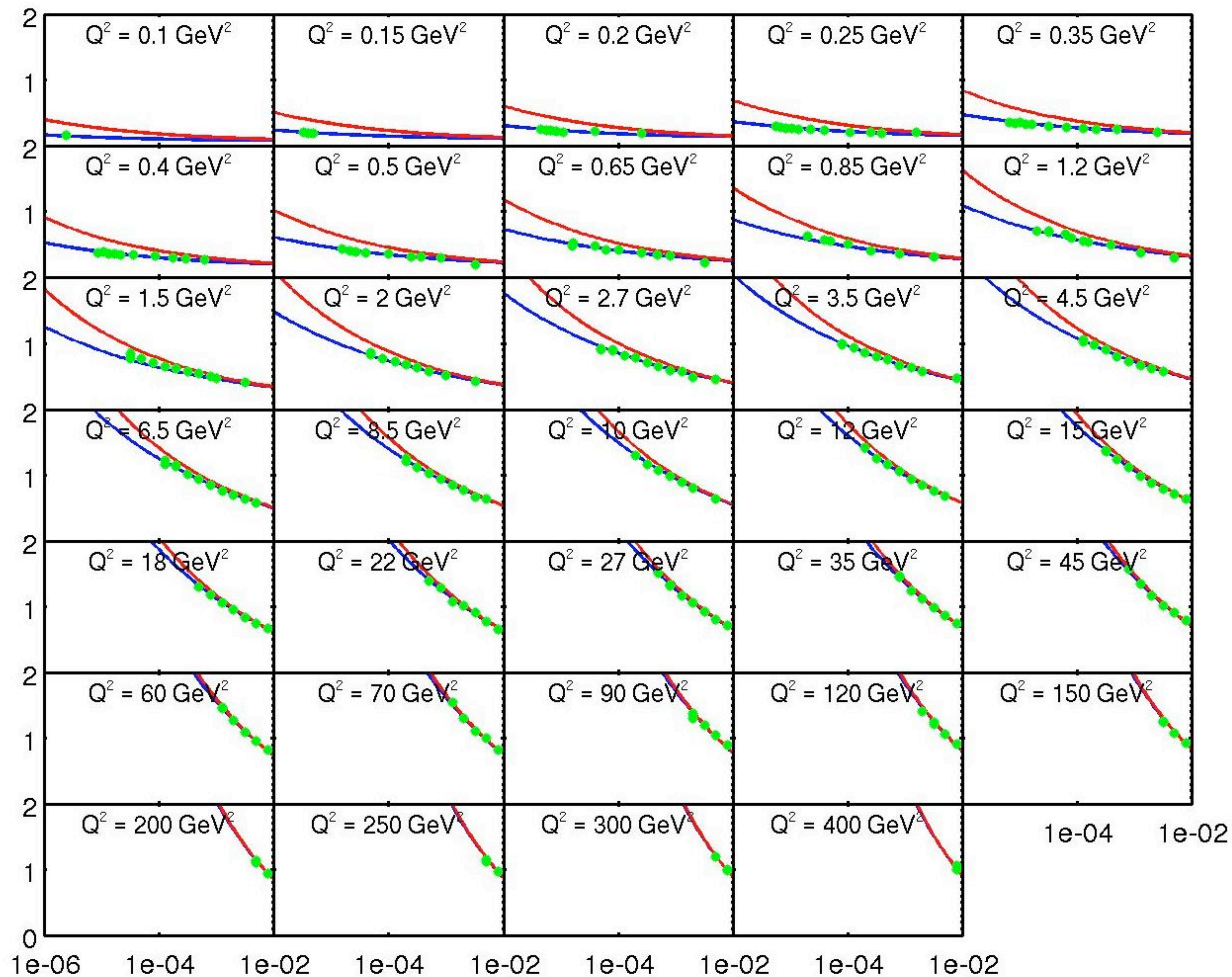
$$\mathrm{Im}\, \mathcal{K}(s, \vec{b}, z, z') = \int \frac{dj}{2\pi i} \int \frac{d\nu}{2\pi} \left(\frac{e^{j\tau} e^{i\nu\xi}}{\sinh \xi} \right) G(j, \nu)$$

$$\boxed{\Delta(j) = 2 + 2\sqrt{(j - j_0)/\rho}}$$

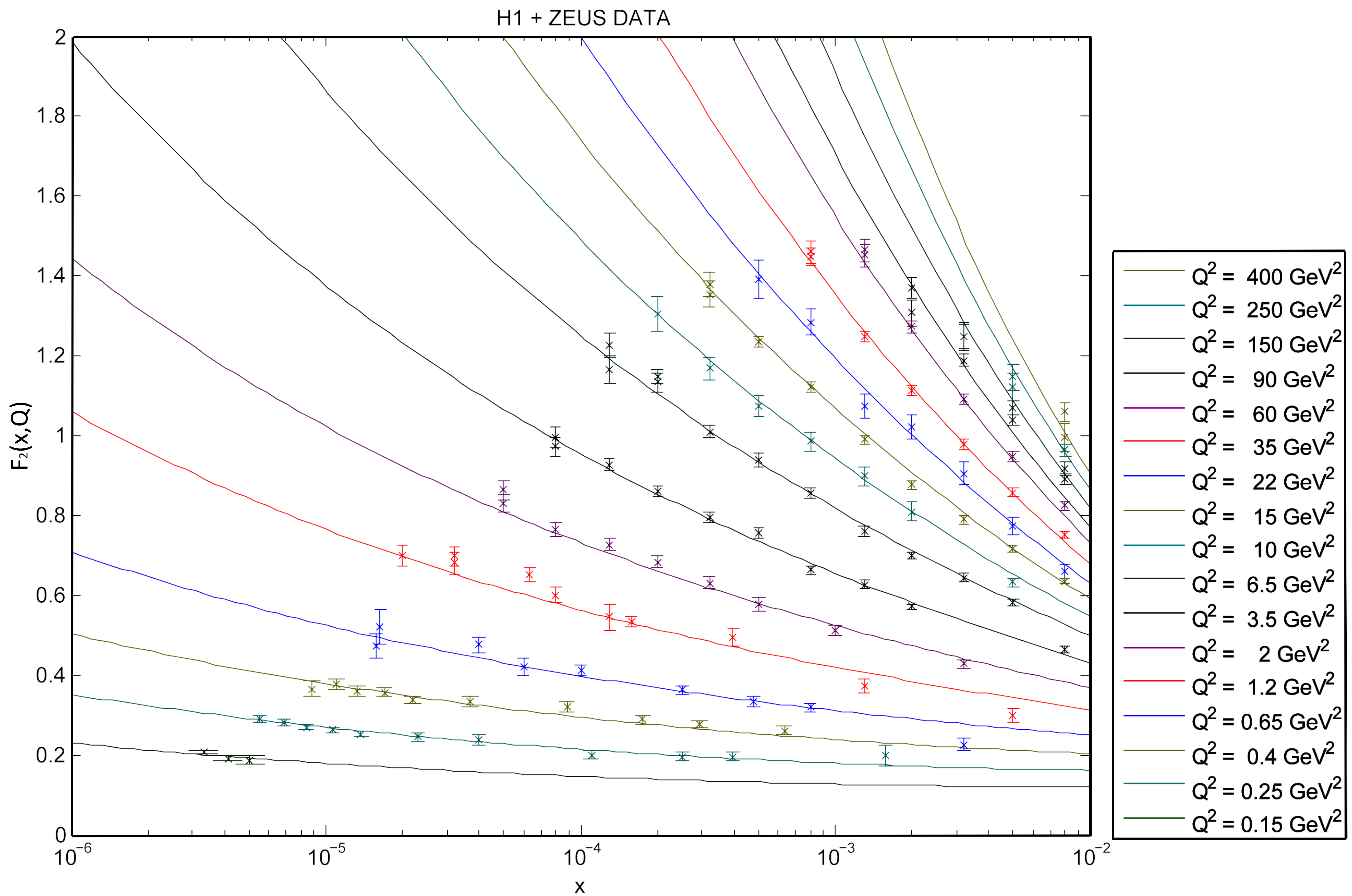
$$\mathcal{K}(j, \vec{b}, z, z') \sim \frac{e^{(2 - \Delta(j))\xi}}{\sinh \xi} \qquad \mathcal{K}(s, b, z, z') \sim e^{j_0} \left(\frac{\xi}{\sinh \xi} \frac{\exp(-\frac{\xi^2}{\rho\tau})}{\tau^{3/2}} \right)$$

$$F_2(x, Q^2) \sim (1/x)^{\epsilon_{effecti}}$$




Q² = 0.1 GeV²
Q² = 0.15 GeV²
Q² = 0.2 GeV²
Q² = 0.25 GeV²
Q² = 0.35 GeV²
Q² = 0.4 GeV²
Q² = 0.5 GeV²
Q² = 0.65 GeV²
Q² = 0.85 GeV²
Q² = 1.2 GeV²
Q² = 1.5 GeV²
Q² = 2 GeV²
Q² = 2.7 GeV²
Q² = 3.5 GeV²
Q² = 4.5 GeV²
Q² = 6.5 GeV²
Q² = 8.5 GeV²
Q² = 10 GeV²
Q² = 12 GeV²
Q² = 15 GeV²
Q² = 18 GeV²
Q² = 22 GeV²
Q² = 27 GeV²
Q² = 35 GeV²
Q² = 45 GeV²
Q² = 60 GeV²
Q² = 70 GeV²
Q² = 90 GeV²
Q² = 120 GeV²
Q² = 150 GeV²
Q² = 200 GeV²
Q² = 250 GeV²
Q² = 300 GeV²
Q² = 400 GeV²
0
1
2
1e-06
1e-04
1e-02

Plots



The structure function $F_2(x, Q^2)$ plotted for farious values of Q^2. The data points are from the H1-Zeus collaboration and the solid lines are the soft wall fit values.

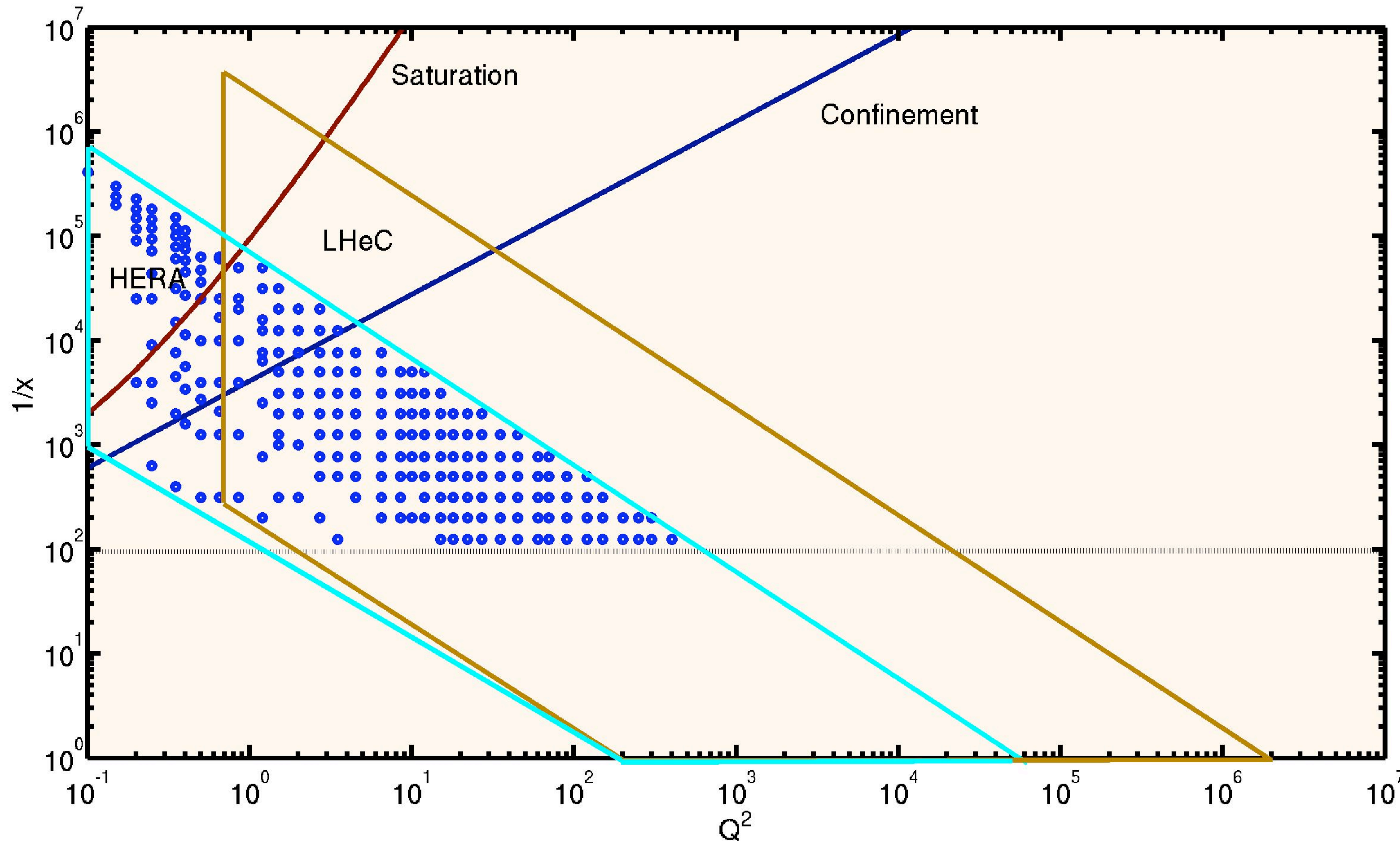
Questions on HERA DIS small-x data:

- Why $\alpha_{eff} = 1 + \epsilon_{eff}(Q^2)$?
- Confinement? (Perturbative vs. Non-perturbative?)
- Saturation? (evolution vs. non-linear evolution?)

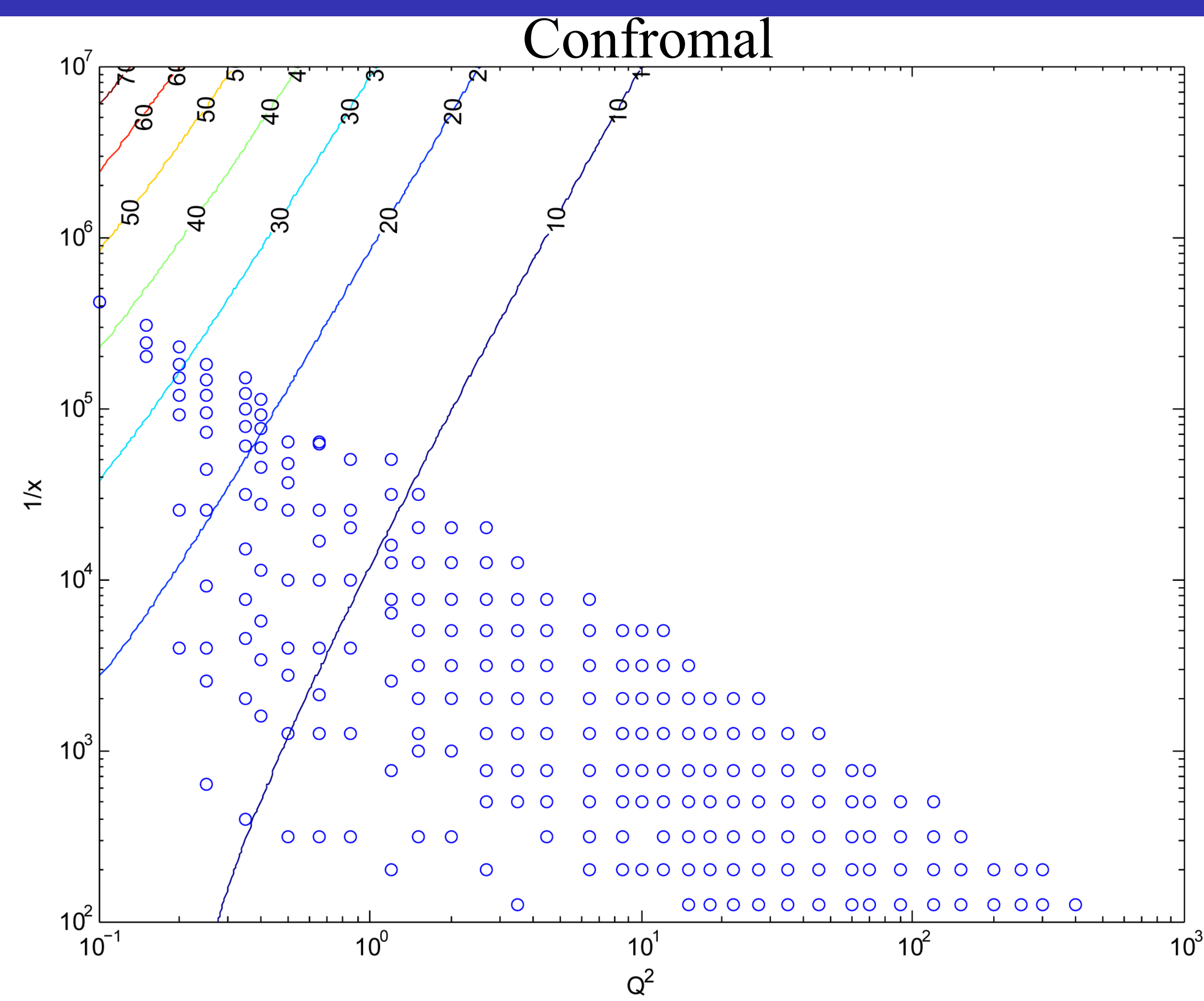
Pomeron as Reggeized Grviton in AdS vs BFKL and DGLAPP

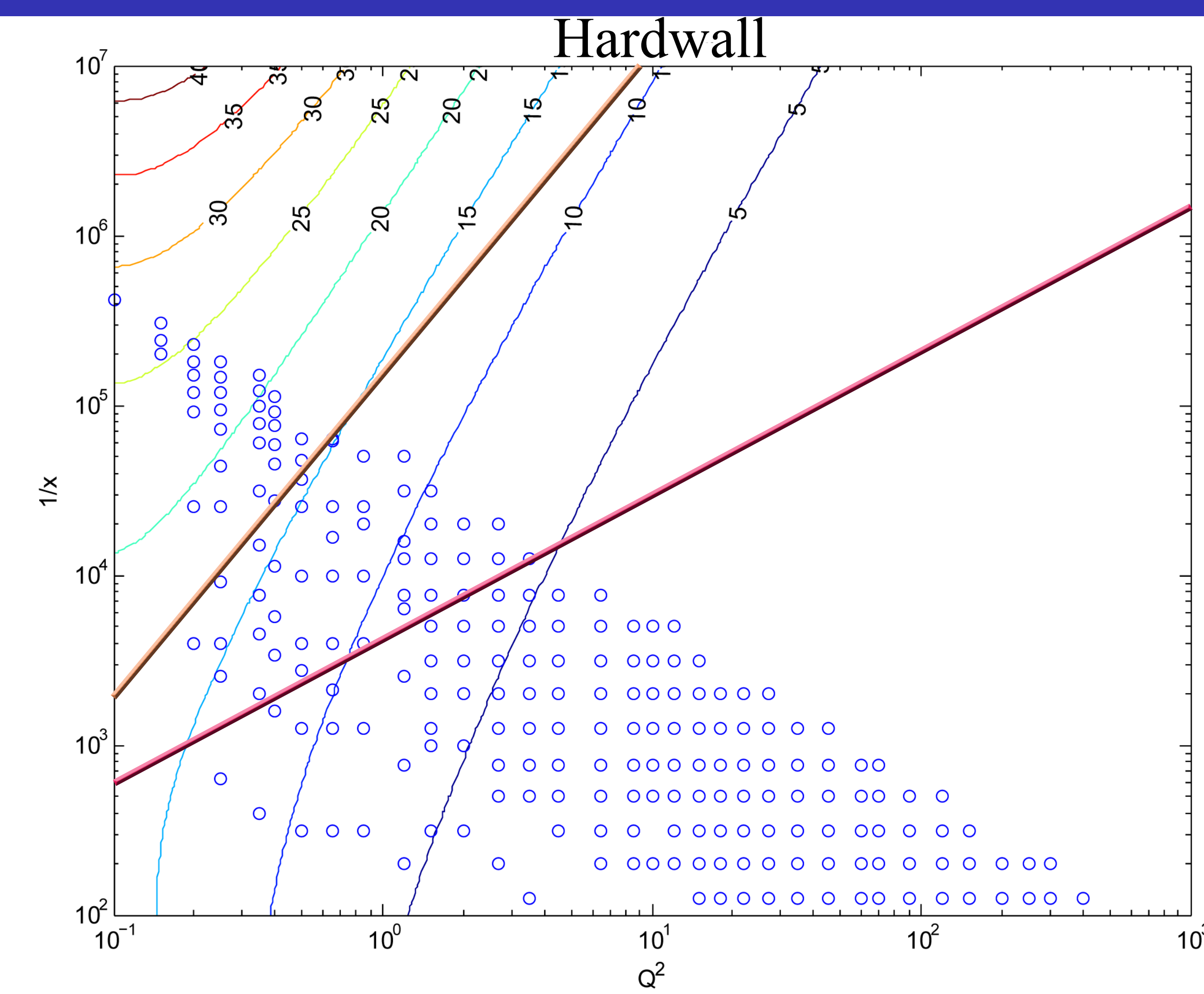
Nice Features of BPST Pomeron

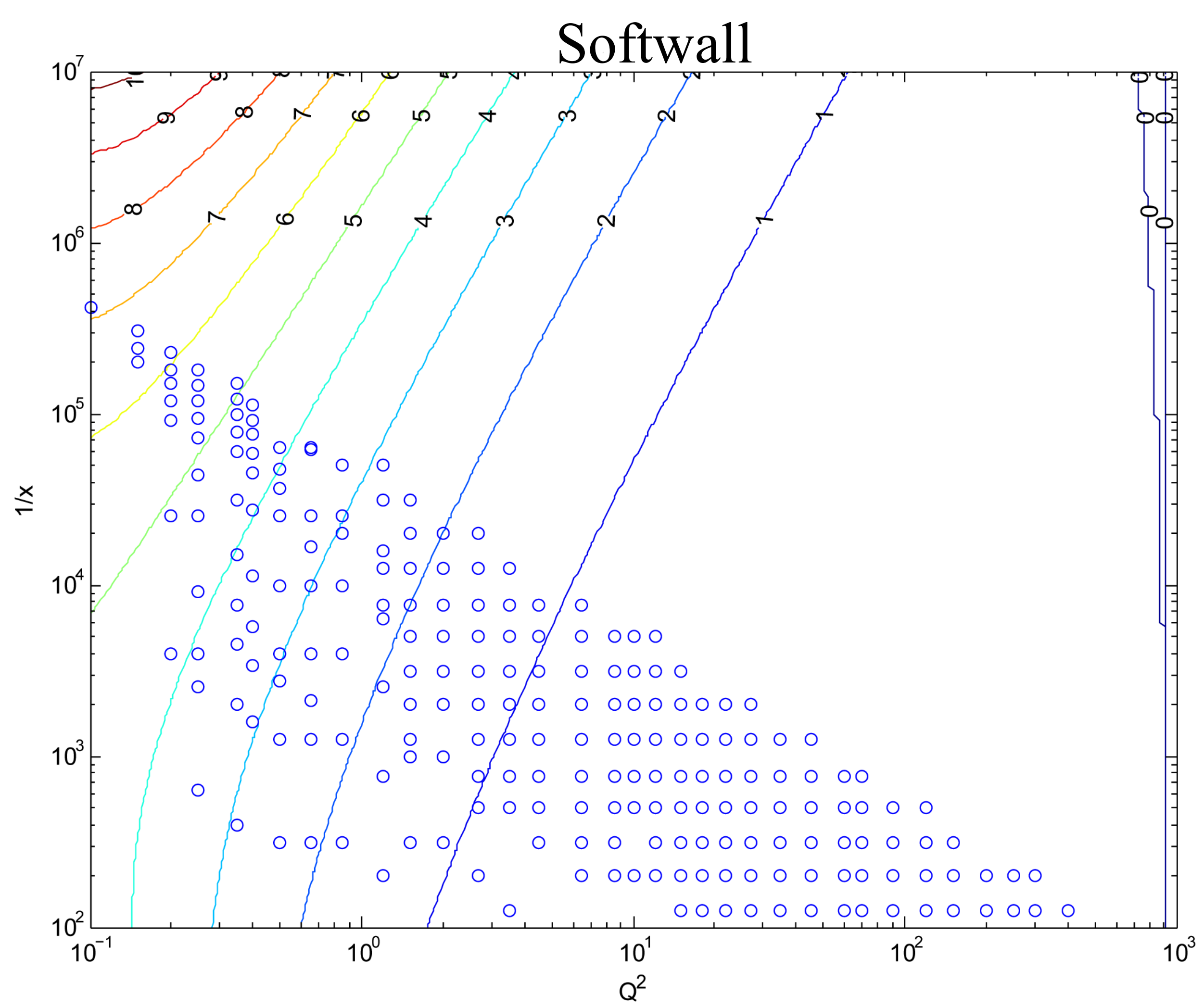
- Λ controls the strength of the soft wall and in the limit $\Lambda \to 0$ one recovers the conformal solution
$$\mathrm{Im}\chi_P^{conformal}(t=0) = \frac{g_0^2}{16}\sqrt{\frac{\rho^3}{\pi}}(zz')\frac{e^{(1-\rho)\tau}}{\tau^{1/2}} exp\left(\frac{-(\mathrm{Log}z - \mathrm{Log}z')^2}{\rho\tau}\right)$$
where $\tau = \mathrm{Log}(\rho zz's/2)$ and $\rho = 2 - j_0$. Note: this has a similar behavior to the weak coupling BFKL solution where
$$\mathrm{Im}\chi(p_\perp, p'_\perp, s) \sim \frac{s^{j_0}}{\sqrt{\pi\mathcal{D}\mathrm{Log}s}}\exp(-(\mathrm{Log}p'_\perp - \mathrm{Log}p_\perp)^2/\mathcal{D}\mathrm{Log}s)$$
- If we look at the energy dependence of the pomeron propagator, we can see a softened behavior in the forward regge limit.
$$\chi_{conformal} \sim -s^{\alpha_0}\mathrm{Log}^{-1/2}(s) \to \chi_{HW} \sim -s^{\alpha_0}/\mathrm{Log}^{-3/2}(s)$$
Analytically, this corresponded to the softening of a j-plane singularity from $1/\sqrt{j-j_0} \to \sqrt{j-j_0}$. Again, we see this same softened behavior in the soft wall model.
- (Possibly) interesting limit: conformal quantum mechanics. Here the EOM simplifies and takes the form of a model with 1+1 dimensional conformal symmetry[Fubini]


Saturation
Confinement
LHeC
HERA
1/x
Q^2
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7

More Plots: Saturation in DIS







Contour plots of Im$[\chi]$ as a function of $1/x$ vs Q^2 (Gev) for conformal, hard-wall, and softwall models. These plots are all in the forward limit, but the impact parameter representation can tell us about the onset of non-linear eikonal effects. The similar behavior for the softwall implies a similar conclusion about confinement vs saturation.

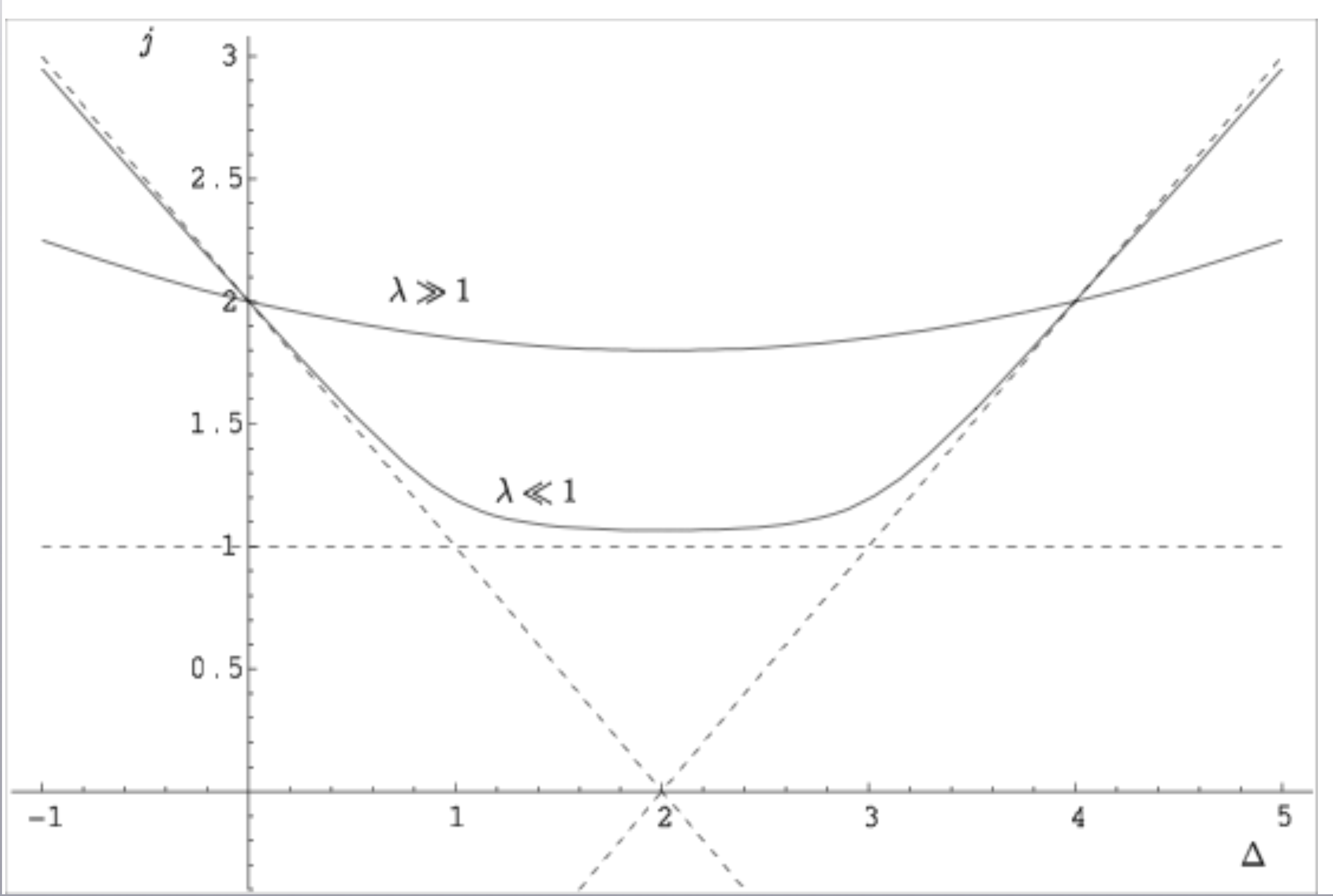
full conformal invariance

$$M_{2n} = \int_0^1 dx\, x^{2n-2}\, F_2(x, Q^2) \sim Q^{-\gamma(2n)}$$

$$\gamma(j) = \Delta(j) - j - \tau_{twist} \qquad\qquad \gamma(2) = \Delta(2) - 2 - 2 = 4 - 4 = 0$$

MOMENTS AND ANOMALOUS DIMENSION

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2} F_2(x, Q^2) \to Q^{-\gamma_n}$$



$$\gamma_2 = 0$$

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

Simultaneous compatible large Q^2 and small x evolutions!

Energy-Momentum Conservation built-in automatically.

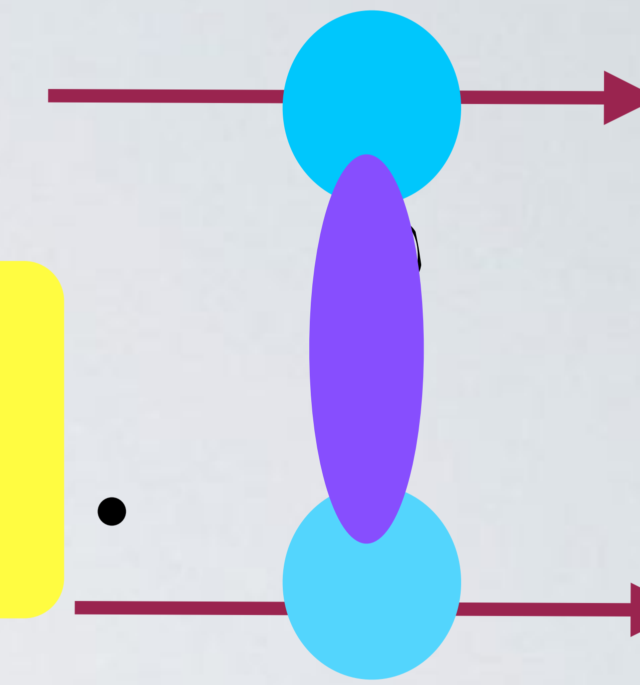
Outline

- Holographics Duality - Historical Perspective:
    - Duality in Physics, Holographic Duality, etc.
    - Gauge Theories, Massless Limit and Scale Invaraince
    - Holographic Duality, ADS/CFT (String-Gauge Duality)
- Size and Shape of hadrons: QCD, High Energy Scattering
    - “Pomeron/Graviton” — unification of hard and soft physics
    - Glueballs under AdS/CFT — masses and decays
    - DIS — BPST Pomeron vs BFKL and DGLAPP
    - Inclusive and Exclusive Central Production
- QCD and Modern CFT/String Studies:

ADS BUILDING BLOCKS BLOCKS

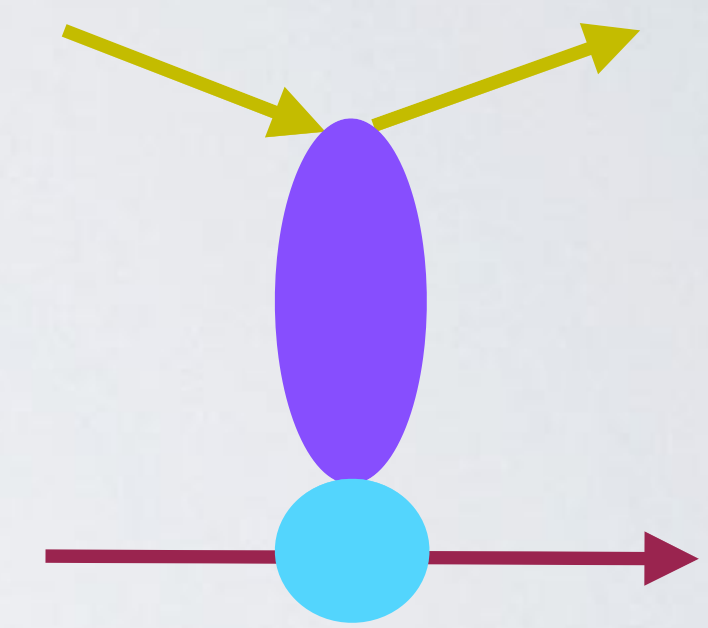
For 2-to-2

$$A(s,t) = \Phi_{13} * \widetilde{\mathcal{K}}_P * \Phi_{24} .$$



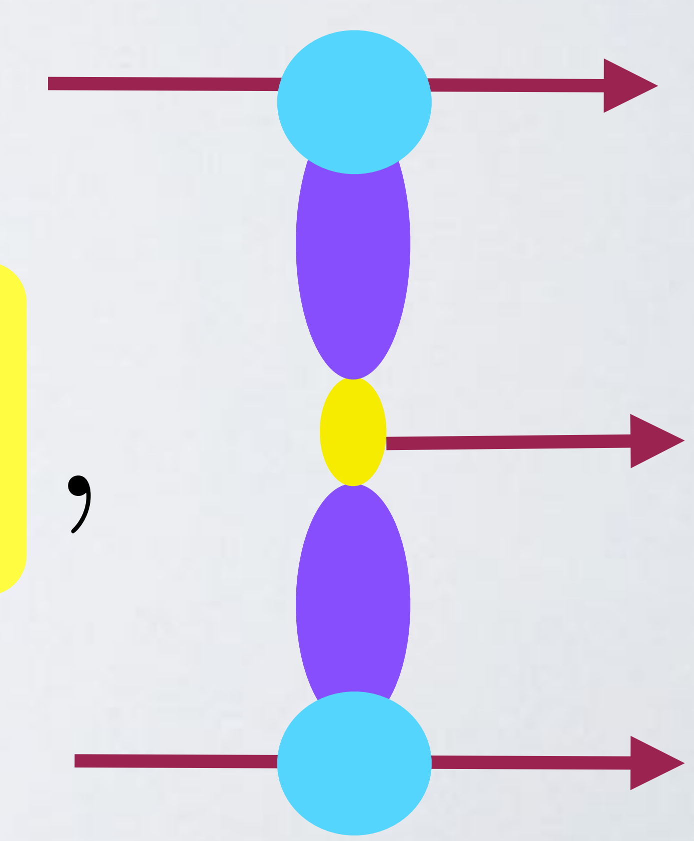
$$A(s,t) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}' \; e^{i\mathbf{q}_\perp \cdot (\mathbf{x}-\mathbf{x}')} \; \Phi_{13}(z) \; \mathcal{K}(s, \mathbf{x}-\mathbf{x}', z, z') \; \Phi_{24}(z')$$

$$d^3\mathbf{b} \equiv dz d^2 x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$



For 2-to-3

$$A(s, s_1, s_2, t_1, t_2) = \Phi_{13} * \widetilde{\mathcal{K}}_P * V * \widetilde{\mathcal{K}}_P * \Phi_{24} ,$$

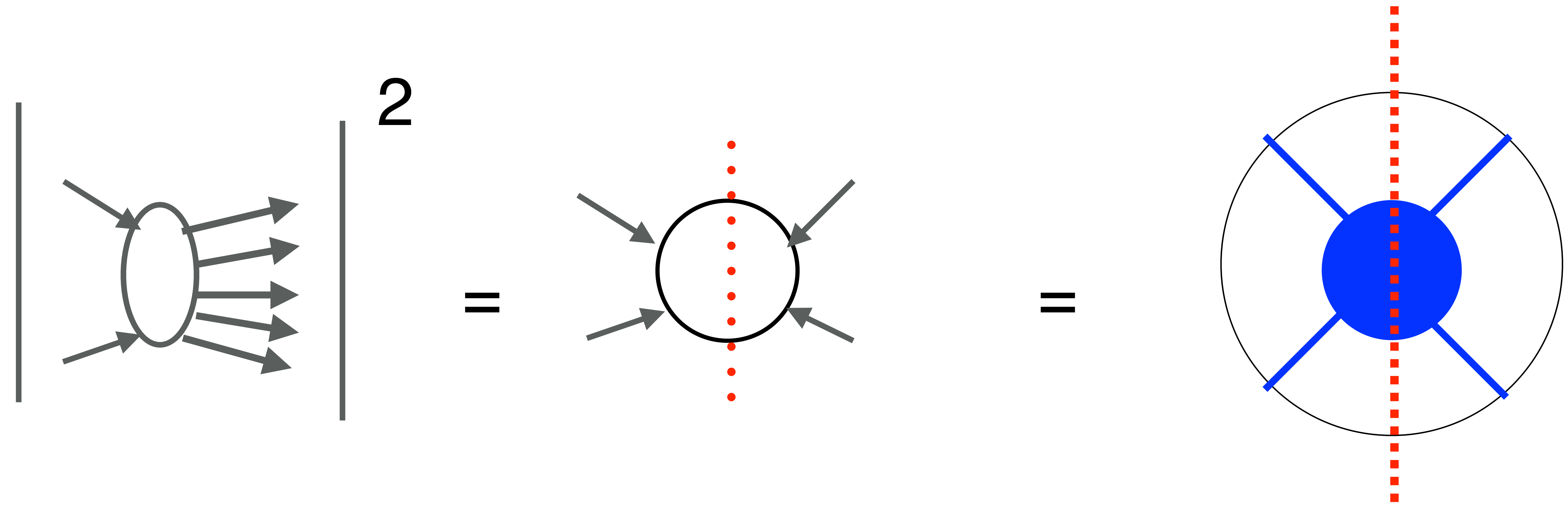


Central Inclusive and Exclusive Production:

Conformal Invariance?
Confinement?
Satuation?

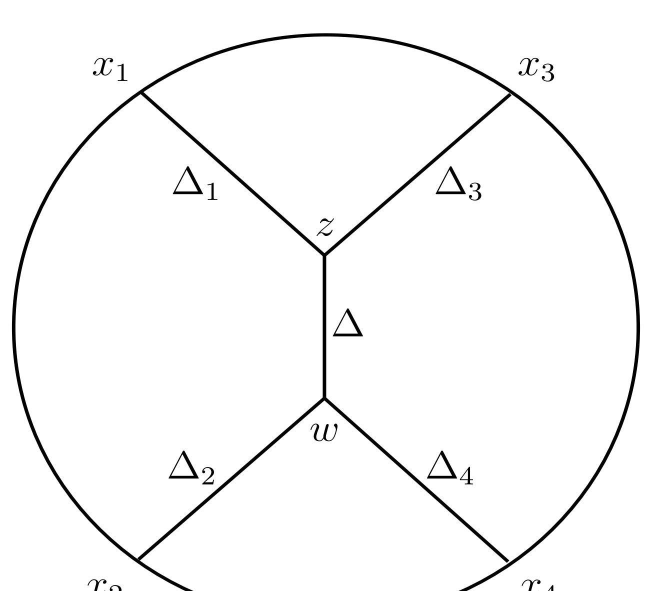
R. Nally, T. Raben, C-I Tan, (to appear).

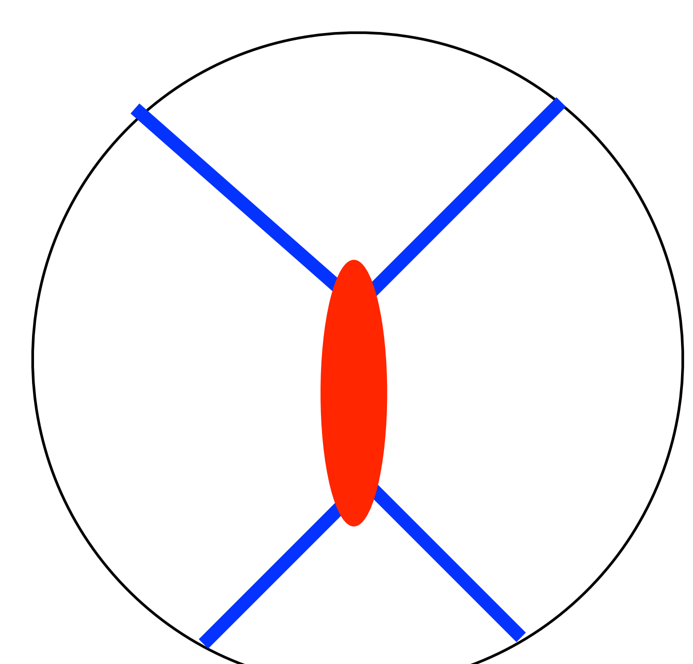
Total Cross Section and Optical Theorem in AdS/CFT

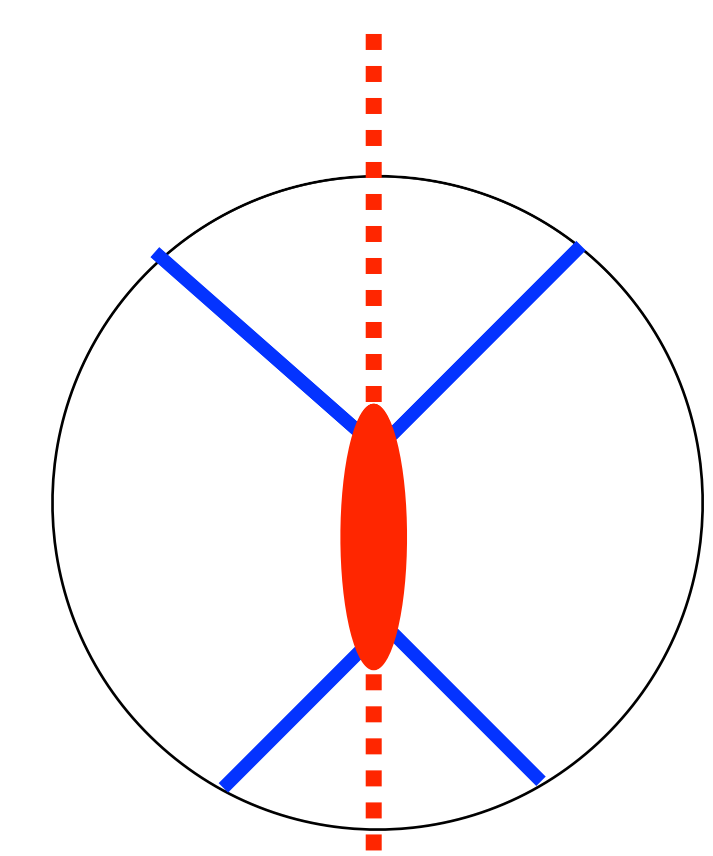


$$\sigma_{total}(s,Q^2) \sim (1/s)\sum_X |\langle X|p,q\rangle|^2 \sim \sum_X \int dx e^{-iq\cdot x}\langle p|J^\dagger(x)X^\dagger\rangle\langle XJ(0)|p\rangle \sim (1/s)Im\,T(s,t=0;Q^2)$$

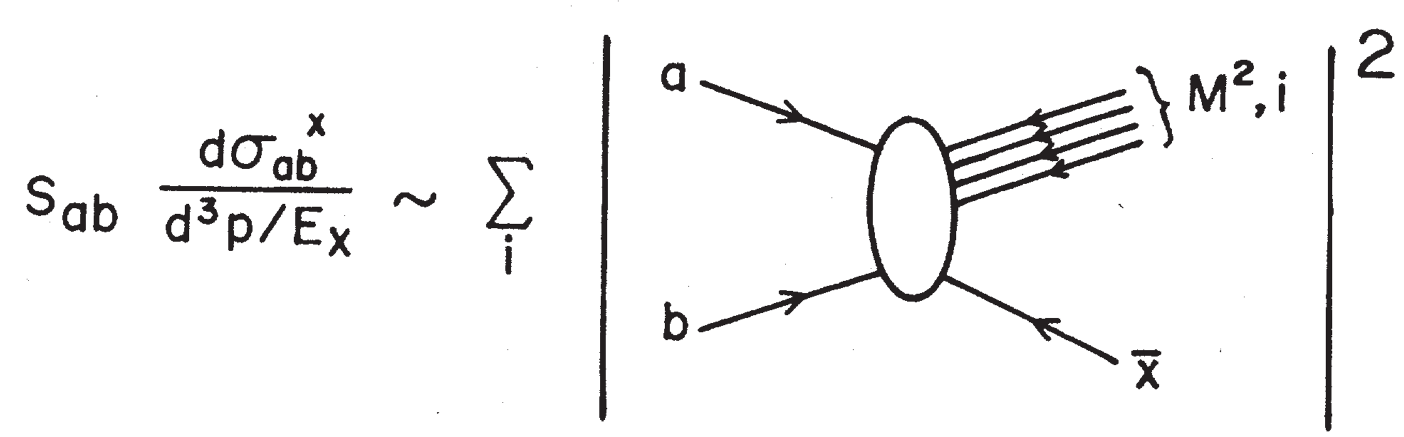
$p_1 + p_2 \to p_3 + p_4$

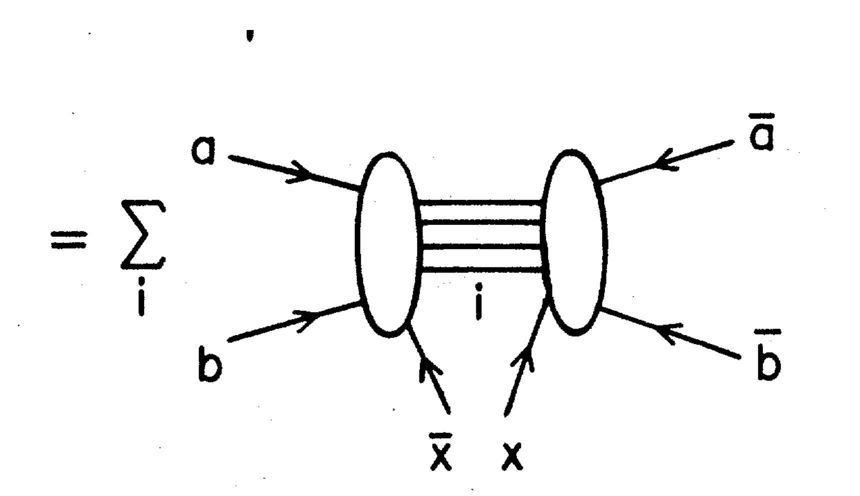


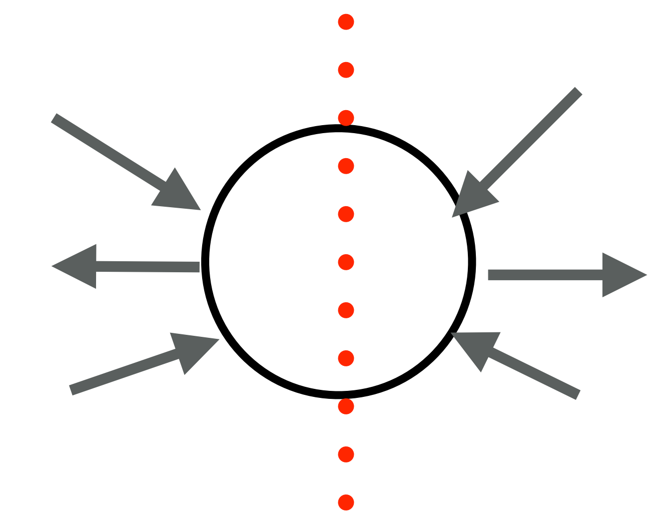




Single-Particle Inclusive Distribution and AdS/CFT

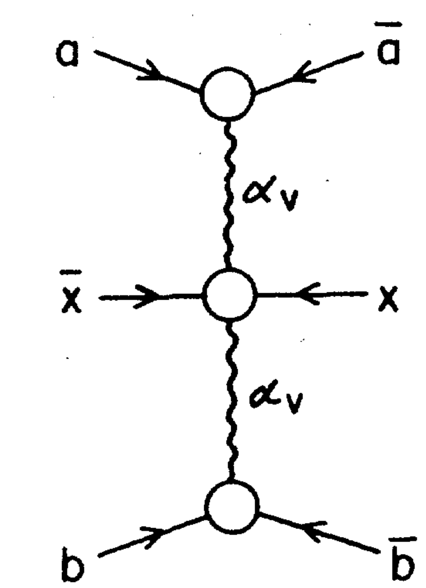


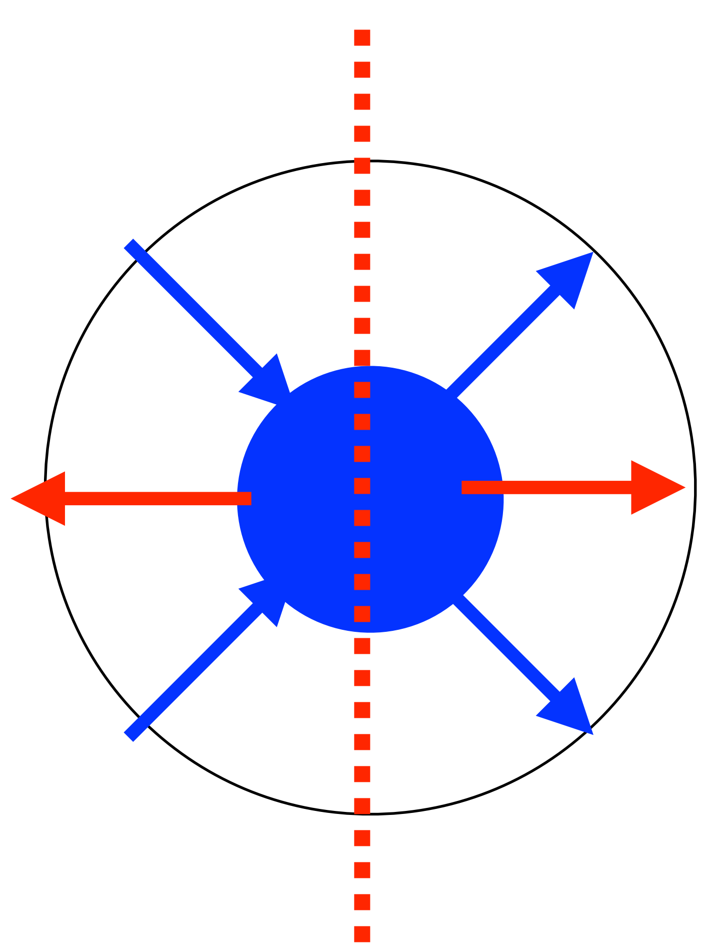


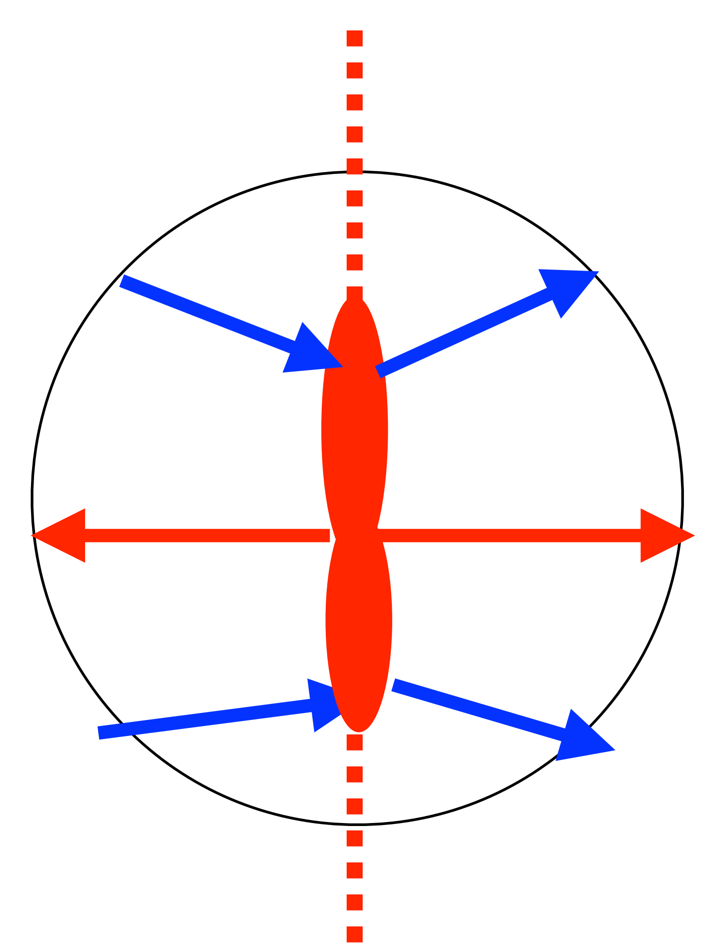


$$\frac{d\sigma}{dq} \sim (1/s) \sum_X |\langle q, X|p_1, p_2\rangle|^2 \sim \sum_X \int e^{iqx} \langle p_1, p_2|J^\dagger(x) X^\dagger\rangle X J(0)|p_1, p_2\rangle$$

$$\sim \int e^{iqx} \langle p_1'|J^\dagger(x)J(0)|p_1 p_2\rangle \sim Disc_{M^2} \int e^{iqx} \langle p_1'|T\{J^\dagger(x)J(0)\}|p_1 p_2\rangle$$



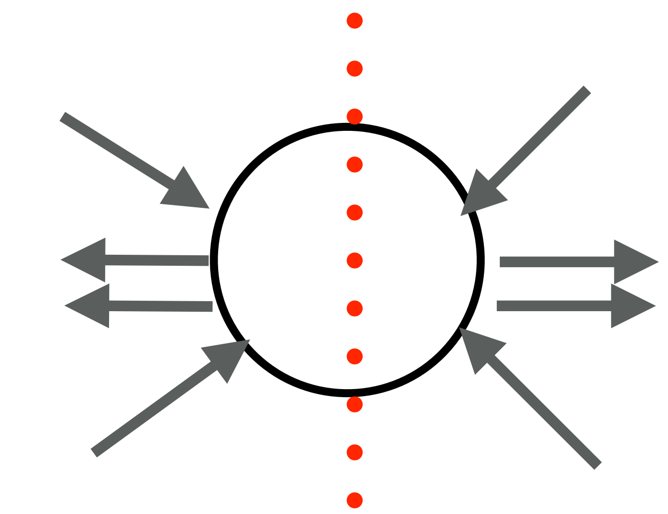


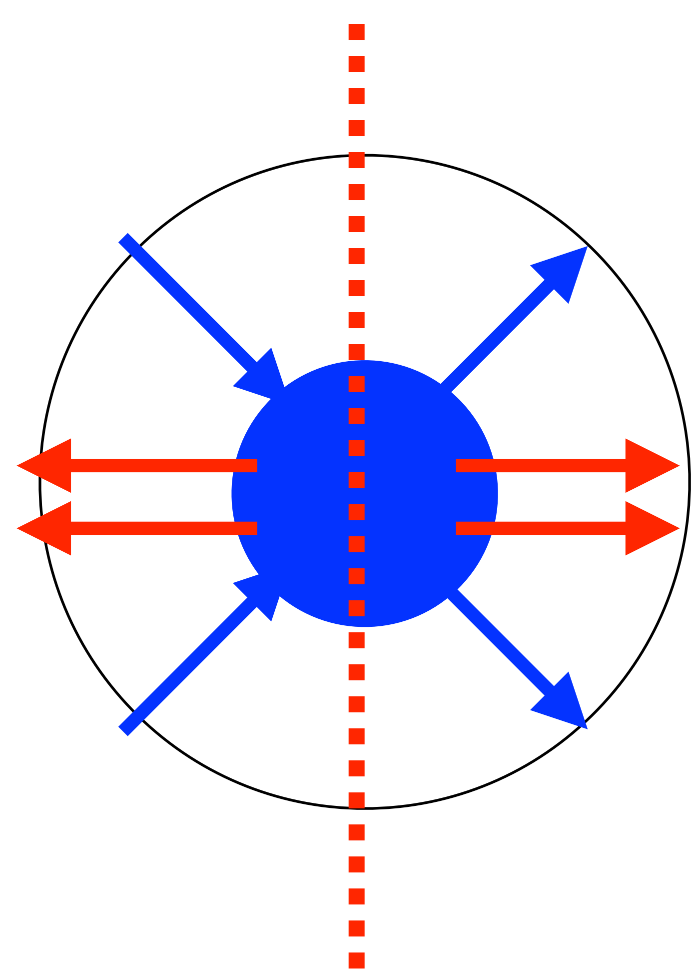


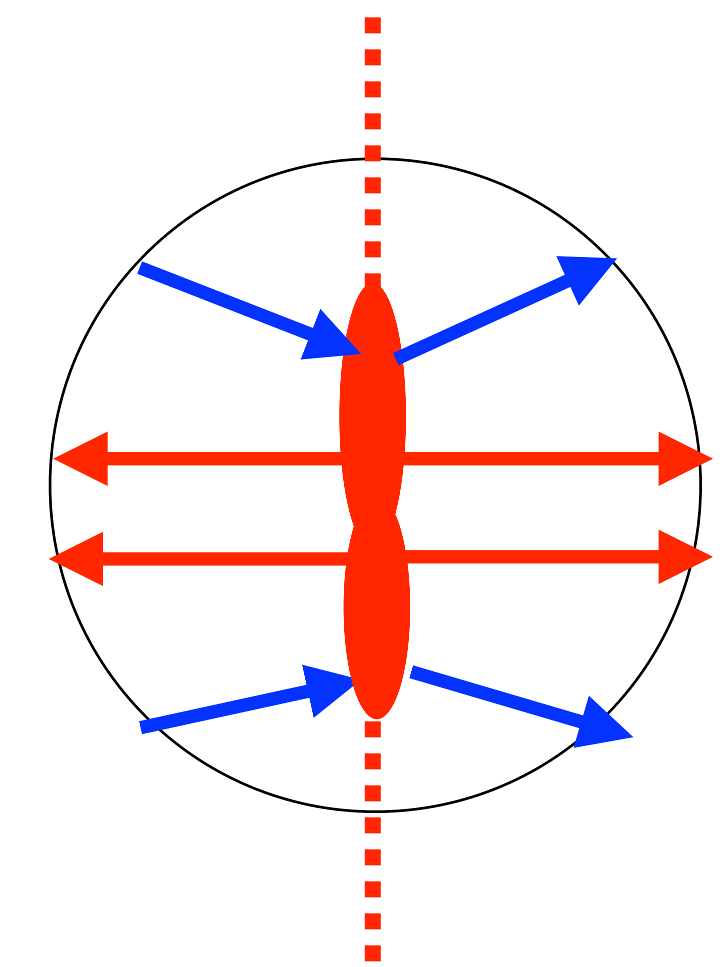
$$\mathcal{V}_{PP\phi}$$

Two-particle Correlation and AdS/CFT; Energy Correlations, etc.

$$\frac{d\sigma}{dq} \sim (1/s) \sum_X |\langle q_1, q_2, X | p_1, p_2 \rangle|^2 \sim Disc_{M^2} \langle p_1', p_2'; q_1, q_2 | p_1 p_2, q_1' q_2' \rangle \Big|_{p'=p;q'=q}$$







$$\mathcal{V}_{PP\phi_1\phi_2} = \mathcal{V}_{PP\phi_1}\mathcal{V}_{PP\phi_2} + \Delta\mathcal{V}_{PP\phi_1\phi_2}$$

CFT-based Prediction:

$$\rho(p_\perp, y, s) = \frac{1}{\sigma_{total}} \frac{d^3\sigma_{ab\to X}}{d\mathbf{p}_\perp^2\, dy} \sim p_\perp^{-8}.$$

arXiv:1702.05502v1

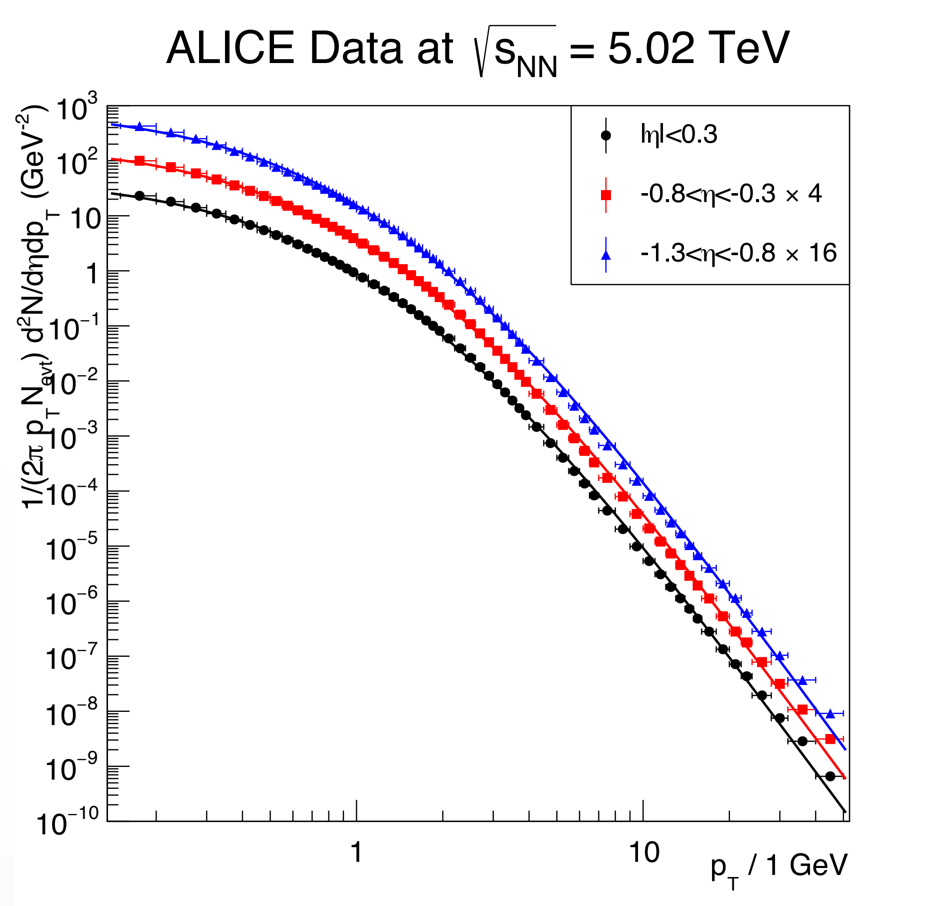


Figure 5.2: Fit of inclusive double-differential charged hadron production cross sections obtained in proton-lead collisions at center of mass energy $\sqrt{s} = 5.02$ by the ALICE Collaboration, presented in [93]. Two of the datasets are rescaled by factors of four and sixteen for visual clarity. The data are displayed alongside fits to the model in Eq. (5.2).

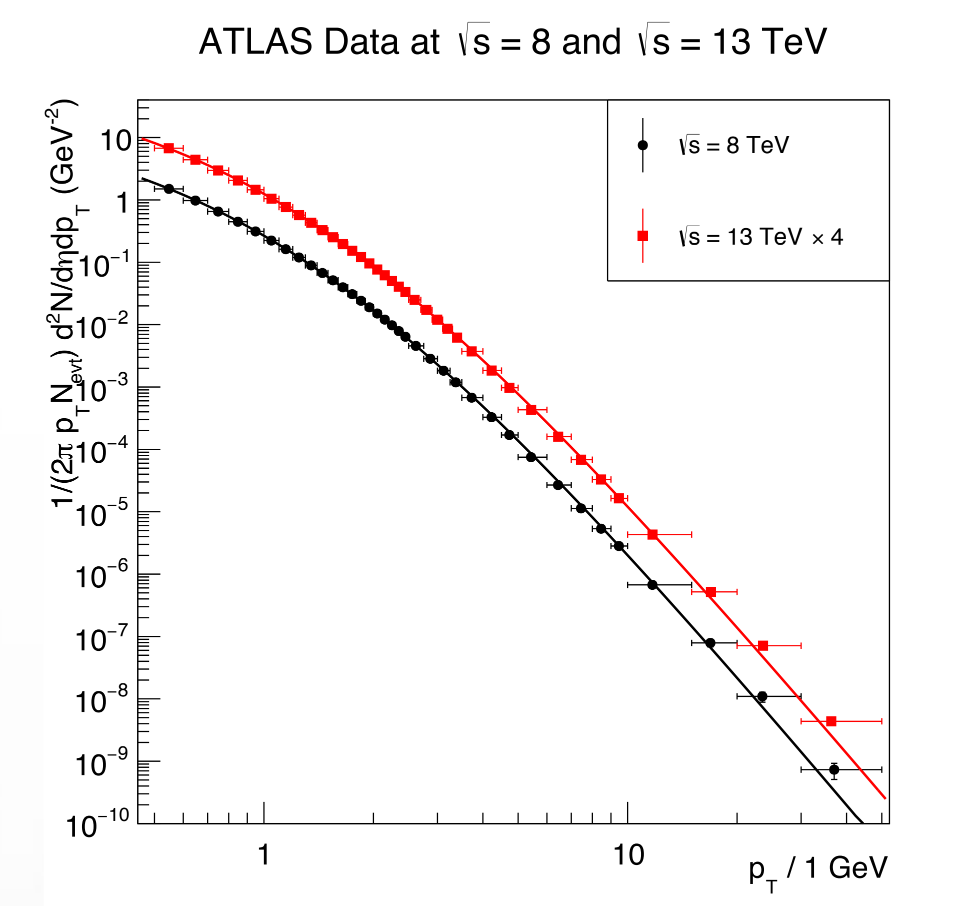


Figure 5.3: Fit of inclusive double-differential charged hadron production cross sections obtained in proton-proton collisions at center of mass energy $\sqrt{s} = 8$ and $\sqrt{s} = 13$ TeV by the ATLAS Collaboration, presented in [94] and [91], respectively. The 13 TeV dataset is rescaled by a factor of four for visual clarity. The data are displayed alongside fits to the model in Eq. (5.2)

$$\frac{1}{2\pi p_T} \frac{d^2\sigma}{dp_T d\eta} = \sum_i \frac{A_i}{(p_T + C)^{B_i}},$$

Dataset	A/10 (GeV^{-2})	B	C/(1 GeV)
ALICE 5.02 TeV, $\|\eta\| < 0.3$ [93]	38.48 ± 8.26	7.23 ± 0.09	1.32 ± 0.04
ALICE 5.02 TeV, $-0.8 < \eta < -0.3$ [93]	37.60 ± 7.97	7.22 ± 0.08	1.30 ± 0.04
ALICE 5.02 TeV, $-1.3 < \eta < -0.8$ [93]	43.00 ± 9.29	7.30 ± 0.09	1.31 ± 0.04
ATLAS 8 TeV [94]	4.46 ± 2.60	7.03 ± 0.264	1.07 ± 0.123
ATLAS 13 TeV [91]	5.77 ± 3.38	6.96 ± 0.265	1.12 ± 0.126

Table 1: Fitted values of parameters in Eq. (5.2) for three data sets. Both central values and statistical errors are quoted.

Summary and Outlook

- Provide meaning for Pomeron non-perturbatively from first principles.
- Realization of conformal invariance beyond perturbative QCD
- New starting point for unitarization, saturation, etc.
- First principle description of elastic/total cross sections, DIS at small-x, Central Diffractive Glueball production at LHC, etc.
- Inclusive Production and Dimensional Scalings.

Part-II

- **Holographics Duality - Historical Perspective:**
  - Duality in Physics, Holographic Duality, etc.
  - Gauge Theories, Massless Limit and Scale Invaraince
  - Holographic Duality, ADS/CFT (String-Gauge Duality)
- **Size and Shape of hadrons: QCD, High Energy Scattering**
  - "Pomeron/Graviton" — unification of hard and soft physics
  - Glueballs under AdS/CFT — masses and decays
  - DIS — BPST Pomeron vs BFKL and DGLAPP
  - Inclusive and Exclusive Central Production
- **QCD and Modern CFT/String Studies:**

Outline

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- QCD and Modern CFT/String Studies:

Pomeron, OPE and Anomalous Dimensions

$$G_{mn} = g^0_{mn} + h_{mn}$$

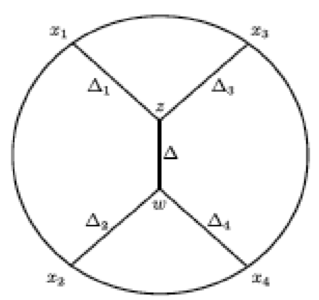
Massless modes of a closed string theory:

Need to keep higher string modes

As CFT, equivalence to OPE in strong coupling: using AdS

Anomalous Dimensions for leading twist operators

Conformal Invariance and Pomeron Interaction from AdS/CFT



+ ....

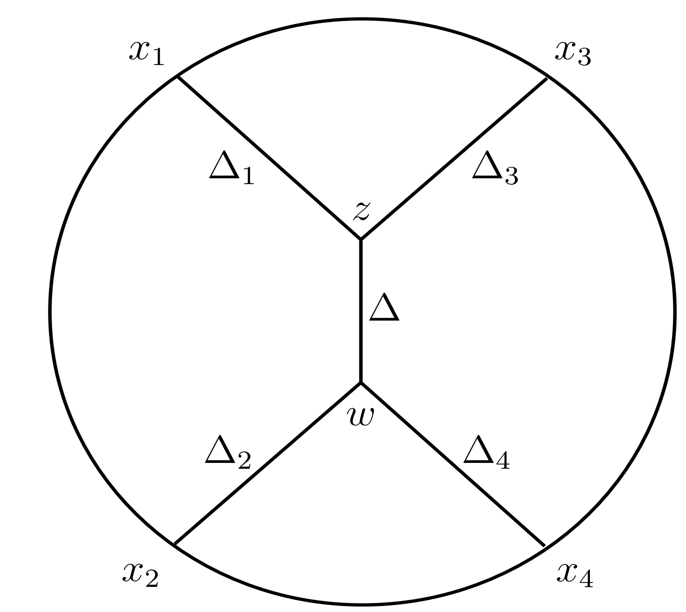
Technique: Summing generalized Witten Diagrams

Freedman et al., hep-th/9903196

Brower, Polchinski, Strassler, and Tan, hep-th/0003115

- Draw all "Witten-Feynman" Diagrams in AdS$_5$,
- High Energy Dominated by Spin-2 Exchanges:

$$p_1 + p_2 \to p_3 + p_4$$

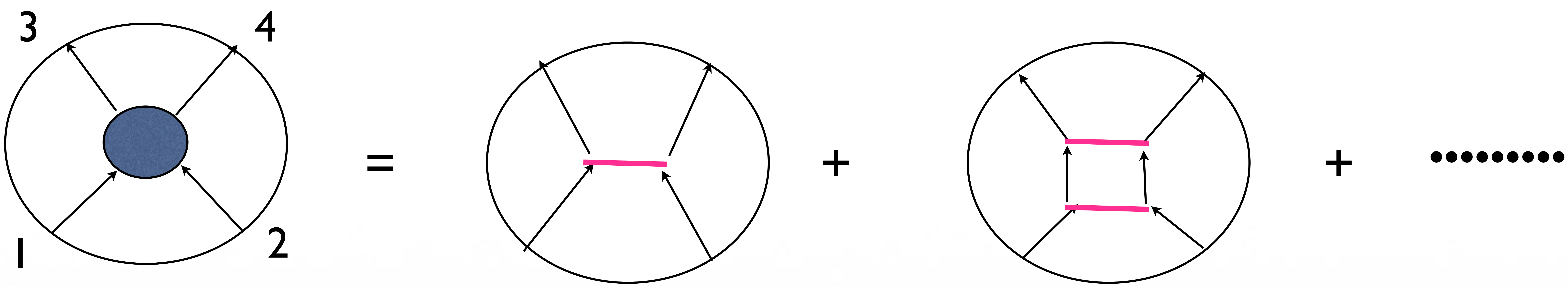


One Graviton Exchange at High Energy

$$T^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz}{z^5} \int \frac{dz'}{z'^5}\, \tilde\Phi_\Delta(p_1^2,z)\tilde\Phi_\Delta(p_3^2,z)\mathcal{T}^{(1)}(p_i,z,z')\tilde\Phi_\Delta(p_2^2,z')\tilde\Phi_\Delta(p_4^2,z')$$

$$\boxed{\mathcal{T}^{(1)}(p_i,z,z') = (z^2 z'^2 s)^2 G_{++,--}(q,z,z') = (zz's)^2 G^{(5)}_{\Delta=4}(q,z,z')}$$

Higher Orders Witten Diagrams:



$$s \to \infty,\ t = -q_\perp^2 < 0$$

$$A_4(s,t) \simeq \int d^2b\, e^{-i\mathbf{b}\cdot\mathbf{q}_\perp} \int d\mu(z) \int d\mu(z')$$

$$\times \quad \phi_1(z,\mathbf{b})\phi_3(z,\mathbf{b})\mathcal{K}(s,\mathbf{b}-\mathbf{b}',z,z')\phi_2(z',\mathbf{b}')\phi_4(z',\mathbf{b}')$$

$$A(s,t) = g_0^2 \int dz \int dz' \left(\sqrt{-g(z)}e^{-2A(z)}\phi_1(z)\phi_3(z)\right) \times \left((\alpha' s)^2 G^{(0)}(z,z';t)\right) \left(\sqrt{-g(z')}e^{-2A(z')}\phi_2(z')\phi_4(z')\right)$$

Scalar Bulk-Bulk Propagator:

$$s \to \infty,\ t = -q_\perp^2 < 0$$

$$G^{(0)}(z,z';t) = \left(\frac{zz'}{R^2}\right)^2 \int_0^\infty k dk \frac{J_2(kz)J_2(kz')}{k^2-t}$$

Scalar Bulk-Boundary Propagator:

$$G^{(0)}(z,z';t)\text{: limit } z' \to 0$$

$$A(s,t) = g_0^2 \int dz \int dz' \left(\sqrt{-g(z)}e^{-2A(z)}\phi_1(z)\phi_3(z)\right) \times \left((\alpha' s)^2 G^{(0)}(z,z';t)\right) \left(\sqrt{-g(z')}e^{-2A(z')}\phi_2(z')\phi_4(z')\right)$$

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Scalar Bulk-Boundary Propagator:

$G^{(0)}(z,z';t)$: limit $z' \to 0$

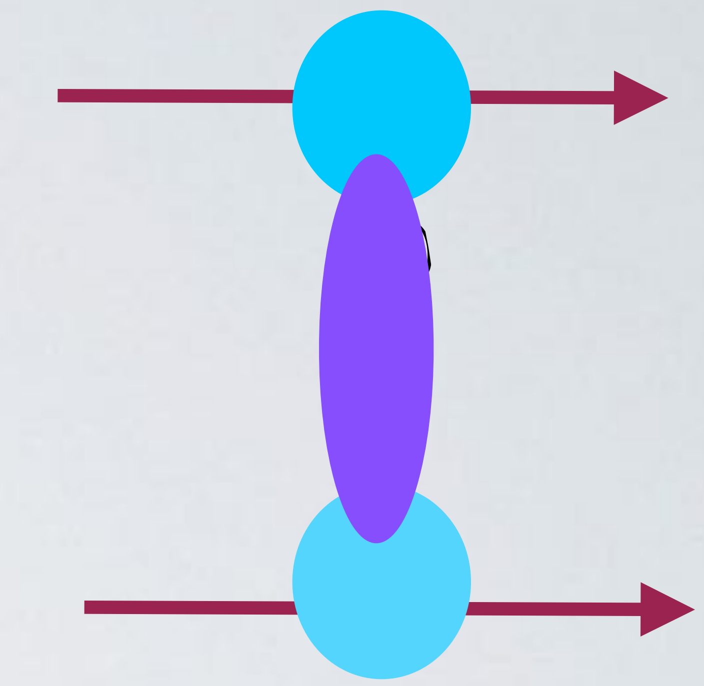
Discrete Spectrum

$$G^{(0)}(z,z';t) = R^{-1} \sum_n \frac{\phi_n(z,2)\phi_n(z',2)}{m_n^2 - t}$$

$$\sqrt{-g}\ (z/R)^2 \sum_n \phi_n(z,2)\phi_n(z',2) = \delta(z-z') \qquad \int_0^{z_{IR}} dz \sqrt{-g} e^{-2A} \phi_n(z)\phi_m(z) = \delta_{mn}$$

ELASTIC, DIS, 2-TO-3, ETC. ADS BUILDING BLOCKS

$$A(s,t) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}' \; e^{i\mathbf{q}_\perp\cdot(\mathbf{x}-\mathbf{x}')} \; \Phi_{13}(z) \; \mathcal{K}(s, \mathbf{x}-\mathbf{x}', z, z') \; \Phi_{24}(z')$$

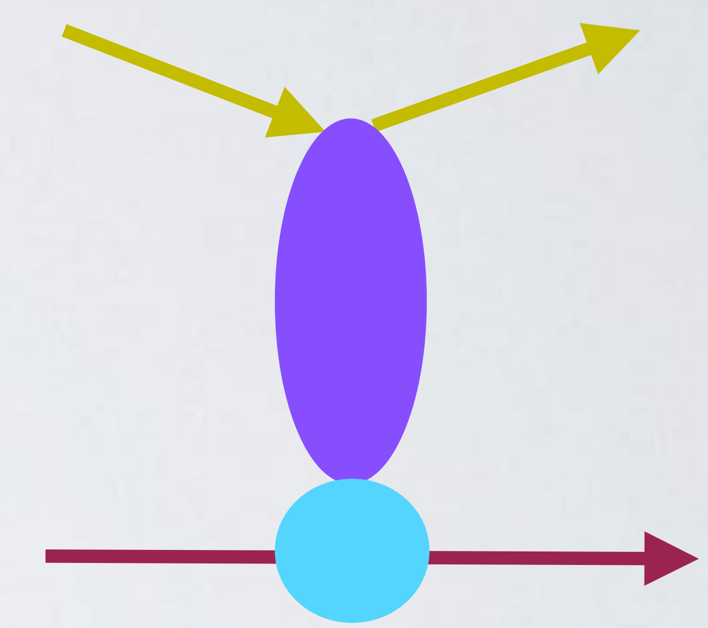


$$\sigma_T(s) = \frac{1}{s} Im A(s,0)$$

$$d^3\mathbf{b} \equiv dz d^2x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$

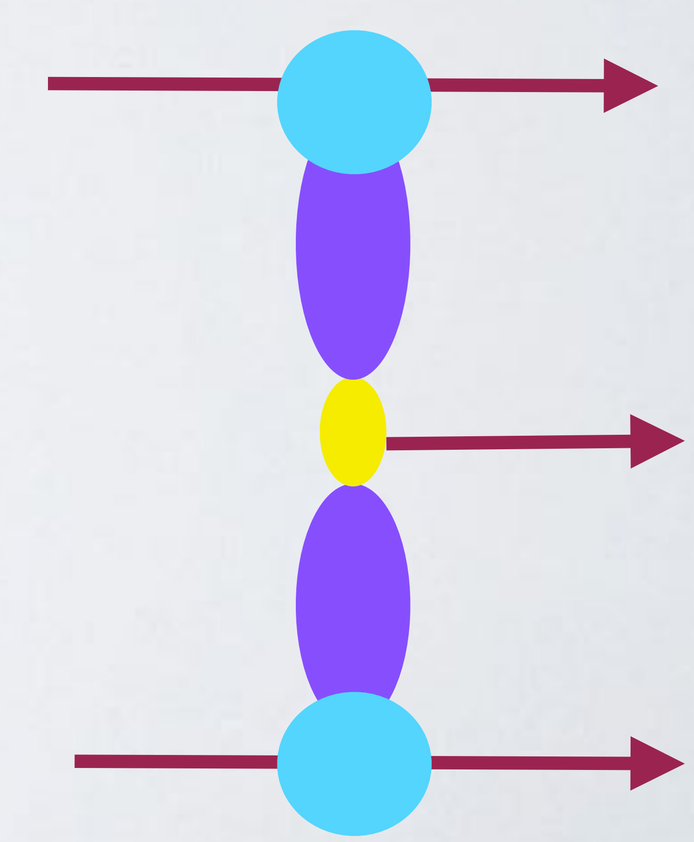
for $\quad F_2(x,Q)$

$$\Phi_{13}(z) \to \Phi_{\gamma^*\gamma^*}(z,Q) = \frac{1}{z}[Qz)^4(K_0^2(Qz) + K_1^2(Qz)]$$



For Double Diffractive Higgs

$$A(s_1, s_2, s, t_1, t_2) = \Phi_{13} * \mathcal{K}_1 * \mathcal{V}_H * \mathcal{K}_2 * \Phi_{24}$$



$$\mathcal{V}_H \to V_H \; \Phi_H = V_H \; (m_H z)^2 K_2(m_H z)$$

Comparison of strong vs weak coupling kernel at t=0

Strong Coupling:

$$\mathcal{K}(r,r',s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\ln s}}e^{-(\ln r-\ln r')^2/4\mathcal{D}\ln s}$$

Diffusion in "warped co-ordinate"

$$j_0 = 2 - \frac{2}{\sqrt{g^2N}} + O(1/g^2N)$$

$$\mathcal{D} = \frac{1}{2\sqrt{g^2N}} + O(1/g^2N) \quad .$$

Weak Coupling:

$$K(s,k_\perp,k'_\perp) \approx \frac{s^{\alpha(0)-1}}{\sqrt{\pi\ln s}}e^{-[(\ln k'_\perp-\ln k_\perp)^2/4\mathcal{D}\ln s]}$$

$$j_0 = 1 + \ln(2)g^2N/\pi^2$$

$$\mathcal{D} = \frac{14\zeta(3)}{\pi}g^2N/4\pi^2.$$

Impact Representation:

$$T^{(1)}(s; x_\perp - y_\perp) = (1/2\pi)^2 \int d^2 q_\perp e^{i(x_\perp - y_\perp)\cdot q_\perp} T^{(1)}(s, -q_\perp^2)$$

$$T^{(1)}(s; x_\perp - y_\perp) = g_s^2 \int \frac{dz dz'}{z^5 \; z'^5} \tilde{\Phi}_\Delta(p_1^2, z) \tilde{\Phi}_\Delta(p_3^2, z') \mathcal{K}(s, x_\perp - y_\perp, z, z') \tilde{\Phi}_\Delta(p_2^2, z') \tilde{\Phi}_\Delta(p_4^2, z')$$

j-plane Representation:

$$\mathcal{K}(s, x_\perp - y_\perp, z, z') = (zz') \int \frac{dj}{2\pi i} \frac{(1 + e^{-i\pi j})}{\sin \pi j} (\tilde{s})^j \; G^{(3)}_{\Delta_2}(j, x_\perp - y_\perp, z, z')$$

Reduction to AdS-3:

$$G^{(3)}_{\Delta_2}(j, x_\perp - y_\perp, z, z') = \frac{1}{(2\pi)^2} \int d^2 q_\perp e^{i(x_\perp - y_\perp)\cdot q_\perp} \tilde{G}^{(3)}_{\Delta_2}(j, -q_\perp^2, z, z')$$

D.E. for Propagator:

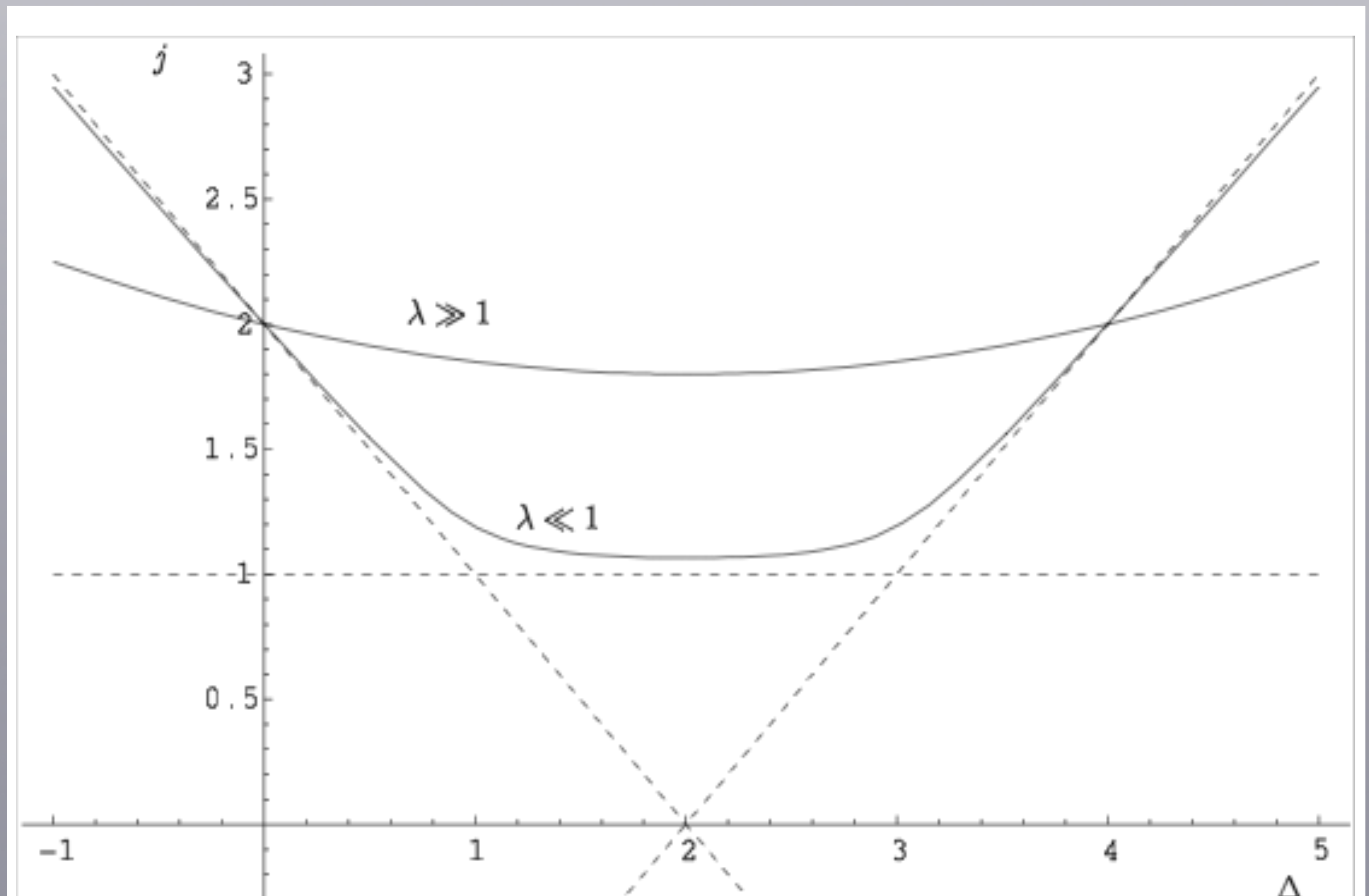
$$\boxed{\{2\sqrt{\lambda}(j-2) - z^3 \partial_z z^{-1} \partial_z - z^2 \partial^2_{x\perp} + 3\} G^{(3)}_{(\Delta(j)-1)}(x_\perp, x'_\perp, z, z') \quad = \quad z^3 \delta(z - z') \delta^{(2)}(x_\perp - x'_\perp)}$$

MOMENTS AND ANOMALOUS DIMENSION

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2} F_2(x, Q^2) \to Q^{-\gamma_n}$$

Simultaneous compatible large Q^2 and small x evolutions!

Energy-Momentum Conservation built-in automatically.



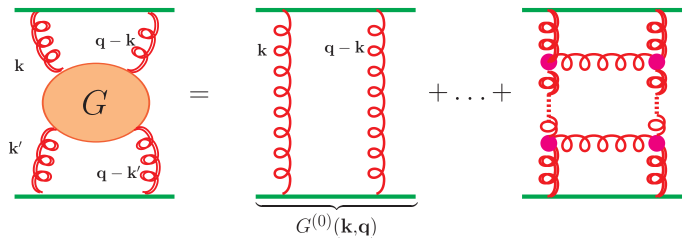
$$\gamma_2 = 0$$

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

BFKL

Balitsky, Fadin, Kuraev, Lipatov (BFKL): perturbative Pomeron. Large logs get in the way of usual perturbation theory: resum $\alpha_s\, log(s)$ to all orders. Bfkl equation – integral equation for Green's function in Mellin space



$$G(\mathbf{k},\mathbf{k}',\mathbf{q},Y) = \int_{-i\infty}^{+i\infty} \frac{d\omega}{2\pi i} e^{Y\omega} f_\omega(\mathbf{k},\mathbf{k}',\mathbf{q}) \to \int_{-i\infty}^{+i\infty} \frac{d\omega}{2\pi i} e^{Y\omega} \sum_{n\in\mathbb{Z}} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi\, i} \frac{E_{\gamma,n}(k)E^*_{\gamma,n}(k')}{\omega - \bar{\alpha}_s \chi(\gamma,n)}$$

where in Leading Log (LL)

$$\chi(\gamma,n) = 2\psi(1) - \psi(\gamma + \frac{|n|}{2}) - \psi(1-\gamma+\frac{|n|}{2}) \quad and \quad \omega_0 = \frac{4\alpha_s N_c}{\pi} ln(2)$$

Surprising conformal symmetry greatly simplifies things in coordinate space

Full $O(4,2)$ Conformal Group

$$SO(4,2) = SO(1,1) \times SO(3,1)$$

Anomalous dimensions

$$\gamma(j) = \Delta(j) - j - \tau_{twist}$$

DGLAPP

BFKL

BPST Pomeron

Graviton/Pomeron Regge trajectory [Brower, Polchinski, Strassler, Tan 06]

- Operators that contribute are the twist 2 operators

$$\mathcal{O}_J \sim F_{\alpha[\beta_1} D_{\beta_2} \ldots D_{\beta_{J-1}} F_{\beta_J]}{}^{\alpha}$$

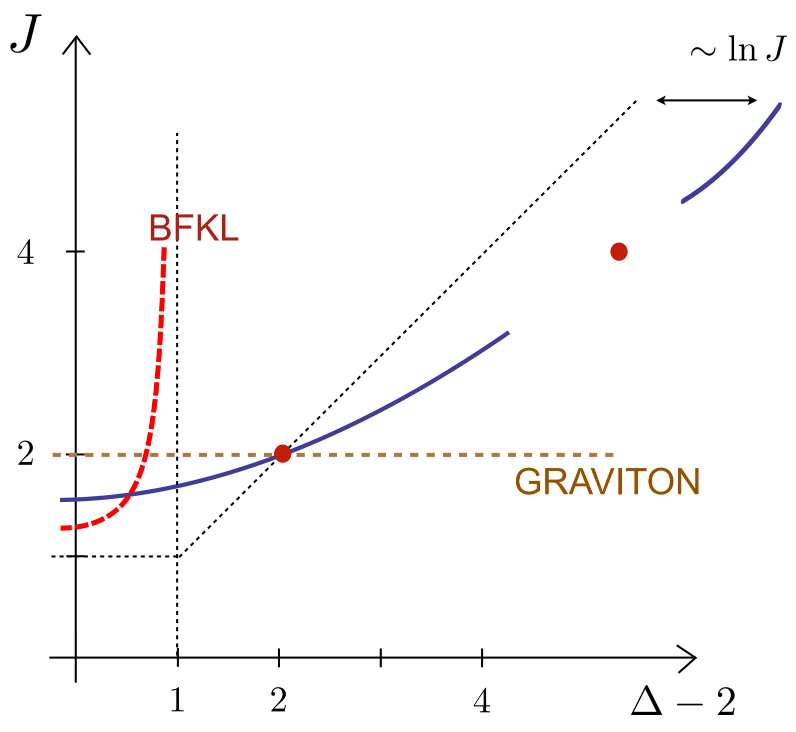
- Dual to string theory spin J field in leading Regge trajectory

$$(D^2 - m^2)\, h_{a_1 \ldots a_J} = 0$$

$$m^2 = \Delta(\Delta-4) - J\,, \qquad \Delta = \Delta(J)$$

- Diffusion limit

$$J(\Delta) = J_0 + \mathcal{D}\,(\Delta-2)^2 \implies m^2 = \frac{2}{\alpha'}(J-2) - \frac{J}{L^2}$$



Pomeron and Odderon Intercepts in the conformal Limit

Massless modes of a closed string theory:

metric tensor, $G_{mn} = g^0_{mn} + h_{mn}$

Kolb-Ramond anti-sym. tensor, $b_{mn} = -b_{nm}$

dilaton, etc. $\phi, \chi, \cdots$

Gauge-String Duality: AdS/CFT

Weak Coupling:

Gluons and Quarks: $A_\mu^{ab}(x), \psi_f^a(x)$

Gauge Invariant Operators: $\bar{\psi}(x)\psi(x), \;\; \bar{\psi}(x) D_\mu \psi(x)$

$S(x) = TrF_{\mu\nu}^2(x), \;\; O(x) = TrF^3(x)$

$T_{\mu\nu}(x) = TrF_{\mu\lambda}(x)F_{\lambda\nu}(x), \;\; etc.$

$$\mathcal{L}(x) = -TrF^2 + \bar{\psi}\not{D}\psi + \cdots$$

Strong Coupling:

Metric tensor: $G_{mn}(x) = g_{mn}^{(0)}(x) + h_{mn}(x)$

Anti-symmetric tensor (Kalb-Ramond fields): $b_{mn}(x)$

Dilaton, Axion, etc. $\phi(x), \; a(x), \; etc.$

Other differential forms: $C_{mn\cdots}(x)$

$$\mathcal{L}(x) = \mathcal{L}(G(x), b(x), C(x), \cdots)$$

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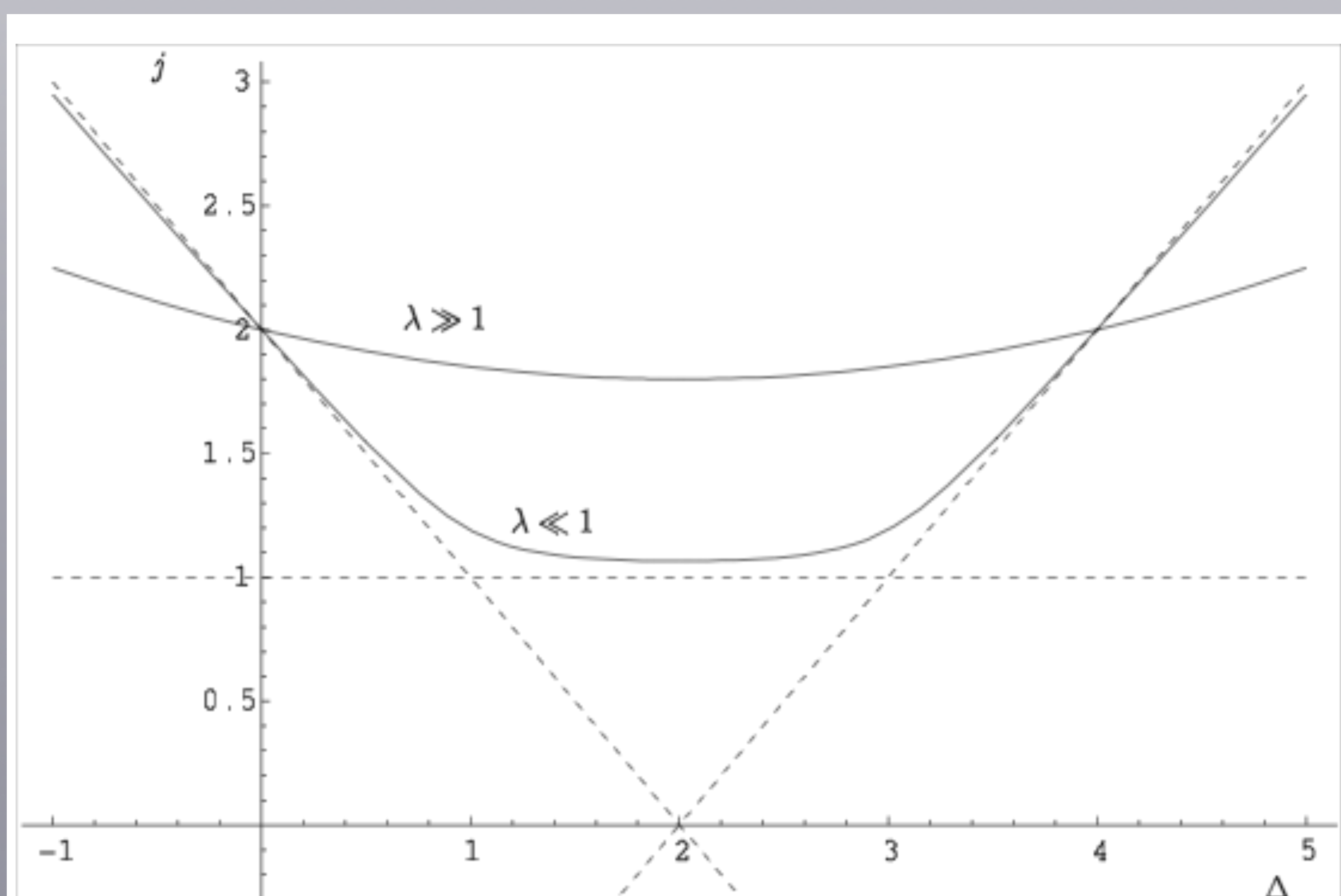

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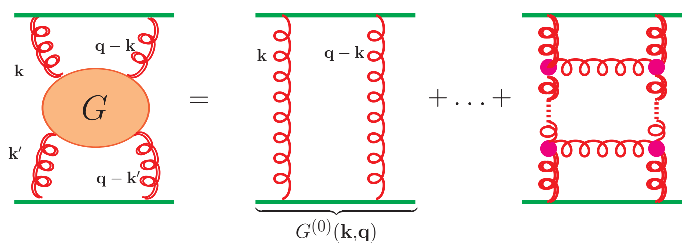

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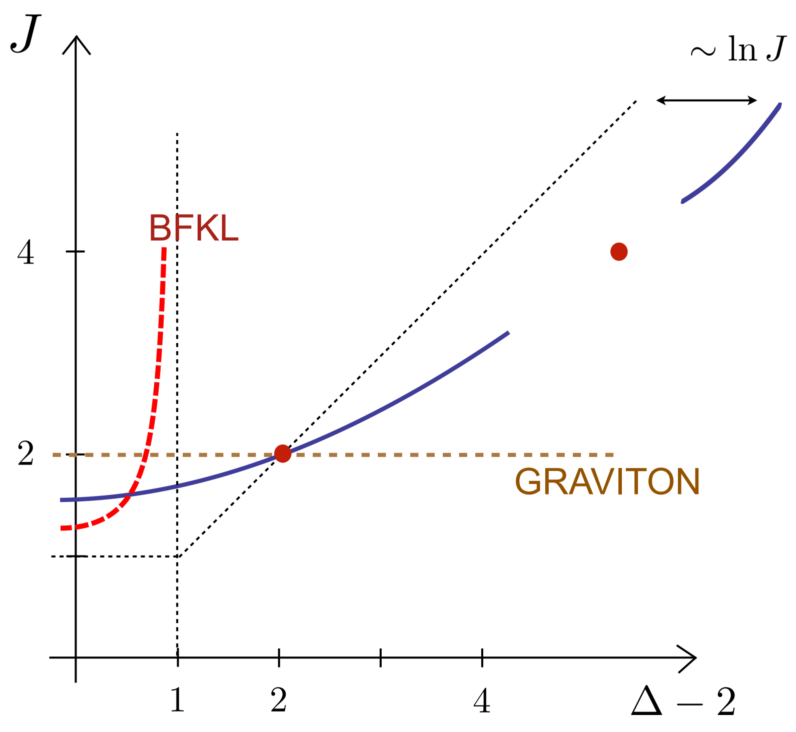

POMERON AND ODDERON IN STRONG COUPLING:

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

B.Basso, 1109.3154v2

POMERON

$$\alpha_p = 2 - \frac{2}{\lambda^{1/2}}$$

Brower, Polchinski, Strassler, Tan

Kotikov, Lipatov, et al.

ODDERON

Solution-a: $\alpha_O = 1 - \frac{8}{\lambda^{1/2}} -$

Solution-b: $\alpha_O = 1 - \frac{0}{\lambda^{1/2}} -$

Brower, Djuric, Tan

Avsar, Hatta, Matsuo

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$$\alpha_p = 2 - \frac{2}{\lambda^{1/2}} - \frac{1}{\lambda} + \frac{1}{4\lambda^{3/2}} + \frac{6\zeta(3)+2}{\lambda^2} + \frac{18\zeta(3)+\frac{361}{64}}{\lambda^{5/2}} + \frac{39\zeta(3)+\frac{447}{32}}{\lambda^3} + \cdots$$

Brower, Polchinski, Strassler, Tan

Gromov et al.

ODDERON

Kotikov, Lipatov, et al.

Costa, Goncalves, Penedones (1209.4355)
Kotikov, Lipatov (1301.0882)

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Solution-b: $$\alpha_O = 1 - \frac{0}{\lambda^{1/2}} -$$

Brower, Djuric, Tan

Avsar, Hatta, Matsuo

Brower, Costa, Djuric, Raben, Tan

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Solution-b:

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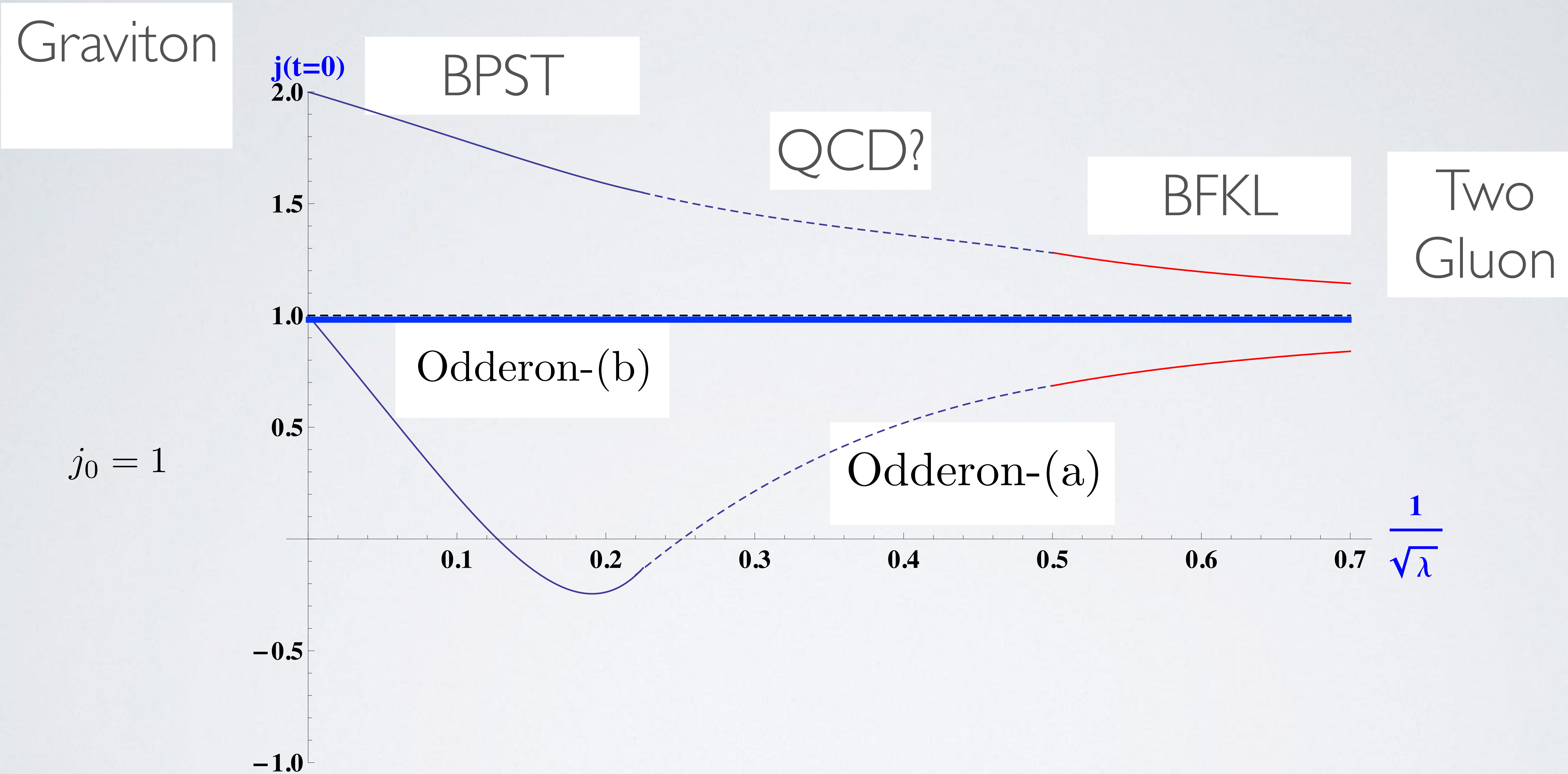
$\mathcal{N} = 4$ Strong vs Weak $g^2 N_c$



$j_0 = 1$

Formal Treatment via World-Sheet OPE

$$(L_0 - 1)V_P = (\bar{L}_0 - 1)V_P = 0$$

- Flat Space Pomeron Vertex Operator

$$\mathcal{V}_P^{\pm} = (2\partial X^{\pm}\bar{\partial}X^{\pm}/\alpha')^{1+\alpha' t/4} e^{\mp ik\cdot X} \ .$$

- Flat Space Odderon Vertex Operator

$$\mathcal{V}_O^{\pm} = (2\epsilon_{\pm,\perp}\partial X^{\pm}\bar{\partial}X^{\perp}/\alpha')(2\partial X^{\pm}\bar{\partial}X^{\pm}/\alpha')^{\alpha' t/4} e^{\mp ik\cdot X}$$

- Pomeron Vertex Operator in AdS

$$\mathcal{V}_P(j,\nu,k,\pm) \sim (\partial X^{\pm}\overline{\partial}X^{\pm})^{\frac{j}{2}} e^{\mp ik\cdot X} e^{(j-2)u} K_{\pm 2i\nu}(|t|^{1/2}e^{-u})$$

- Odderon Vertex Operator in AdS

$$\mathcal{V}_O(j,\nu,k,\pm) \sim (\partial X^{\pm}\overline{\partial}X^{\perp} - \partial X^{\perp}\overline{\partial}X^{\pm})(\partial X^{\pm}\overline{\partial}X^{\pm})^{\frac{j-1}{2}} e^{\mp ik\cdot X} e^{(j-1)u} K_{\pm 2i\nu}(|t|^{1/2}e^{-u})$$

CFT, OPE, and Regge Limit

$Symmetry \leftrightarrow Isometry$

full $O(4,2)$ conformal group as isometries of AdS_5

15 generators: $P_\mu, M_{\mu\nu}, D, K_\mu$

collinear group $SL_L(2,R) \times SL_R(2,R)$ used in DGLAP

generators: $D \pm M_{+-}$, $P_\pm$, $K_\mp$

$SL(2,C)$ Möbius invariance

generators: $iD \pm M_{12}$, $P_1 \pm iP_2$, $K_1 \mp iK_2$

isometries of the Euclidean (transverse) AdS_3 subspace of AdS_5

CFT correlate function – coordinate representation

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)\rangle$$

OPE: $\phi(x_1)\phi_2(x_2) \simeq \sum_k C_{1,2;k}(x_{12}, \partial_1)\mathcal{O}_k(x_1)$

Bootstrap: s-channel OPE = t-channel OPE

unitarity, positivity, locality, analyticity, etc.

Dynamics: $\boxed{\mathcal{O}_{(\Delta,j)_k}(x)}$ Conformal Dimension, Spin

Conformal Partial-Wave Expansion

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle_c = \frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_0}} \mathcal{A}(u,v)$$

Conformal inv. cross-ratios $\quad u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$

t-Channel partial-wave $\quad \mathcal{A}(u,v) = \sum_k \sum_{\Delta_k, j} c_{(12,(\Delta_k,j))} c_{(34,\Delta_(k,j))} \mathcal{G}_{(\Delta_k,j)}(u,v)$

Conformal Block $\quad \mathcal{G}_{(\Delta,j)}(u,v)$

Dynamics: $\{(\Delta_k(j), j)\}, k = 1, 2, \cdots, j = 0, 1, \cdots$ Conformal Data

Conformal Partial-Wave Expansion

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$\mathcal{N} = 4 \quad SYM$ *Integrability* AdS-Dual, Large-N, etc.

Conformal Partial-Wave Expansion and Regge Limit:

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$\mathcal{N} = 4 \quad SYM$ *Integrability* AdS-Dual, Large-N, etc.

Regge Limit: $u \to 0\,, \quad v \to 1\,,$ with $\sqrt{u}/(1-v)$ fixed

Conformal Partial-Wave Expansion and Regge Limit:

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Conformal Block $\quad \mathcal{G}_{(\Delta,j)}(u,v)$

Dynamics: $\quad \{(\Delta_k(j), j)\}\,, k = 1, 2, \cdots, \; j = 0, 1, \cdots$ $\quad$ Conformal Data

$\mathcal{N} = 4 \quad SYM$ $\quad$ *Integrability* $\quad$ AdS-Dual, Large-N, etc.

Regge Limit: $\quad u \to 0\,, \quad v \to 1\,, \quad$ with $\quad \sqrt{u}/(1-v) \quad$ fixed

Euclidean vs Minkowski?

Conformal Partial-Wave Expansion and Regge Limit:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle_c = \frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_0}} \mathcal{A}(u,v)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$\mathcal{A}(u,v) = \sum_k \sum_{\Delta_k, j} c_{(12,(\Delta_k,j))} c_{(34,\Delta_(k,j))} \mathcal{G}_{(\Delta_k,j)}(u,v)$$

Conformal Block $\mathcal{G}_{(\Delta,j)}(u,v)$

Regge Limit:

$u \to 0\,, \quad v \to 1\,, \quad$ with $\quad \sqrt{u}/(1-v) \quad$ fixed

---

Euclidean Regge limit:

$$\mathcal{G}_{(\Delta,j)}(u,v) \sim u^{\Delta/2}\, g(\tilde{b}^2)$$

$$\tilde{b}^2 \sim \frac{1-v}{\sqrt{u}} \sim \cos\theta$$

Minkowski Regge limit:

$$\mathcal{G}_{(\Delta,j)}(u,v) \sim u^{(1-j)/2}\, \mathcal{Y}(\tilde{b}^2)$$

$$\sqrt{u} \sim s^{-1}$$

$$\mathcal{Y}(\tilde{b}^2) \sim \tilde{b}^{-2(\Delta-1)}$$

$\tilde{b}^2 \sim \dfrac{1-v}{\sqrt{u}}$ large

Full $O(4,2)$ Conformal Group

$$\mathcal{A}(u,v) = \sum_k \sum_{\Delta_k, j} c_{(12,(\Delta_k,j))} c_{(34,\Delta_(k,j))} \mathcal{G}_{(\Delta_k,j)}(u,v)$$

$$\boxed{SO(4,2) = SO(1,1) \times SO(3,1)}$$

Full $O(4,2)$ Conformal Group

$$\boxed{SO(4,2) = SO(1,1) \times SO(3,1)}$$

Euclidean CFT

$$\mathcal{A}(u,v) = \sum_k \sum_{\Delta_k, j} c_{(12,(\Delta_k,j))} c_{(34,\Delta_(k,j))} \mathcal{G}_{(\Delta_k,j)}(u,v)$$

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Full $O(4,2)$ Conformal Group

$$\boxed{SO(4,2) = SO(1,1) \times SO(3,1)}$$

Euclidean CFT

$$\boxed{SO(4,2) = SO(1,1) \times SO(4)}$$

$$\mathcal{A}(u,v) \leftrightarrow \int_{d/2-i\infty}^{d/2+i\infty} \frac{d\Delta}{2\pi i} \sum_j \quad a_j(\Delta)\, G_{\Delta,j}(u,v)$$

Full $O(4,2)$ Conformal Group

$$\boxed{SO(4,2) = SO(1,1) \times SO(3,1)}$$

Euclidean CFT

$$\boxed{SO(5,1) = SO(1,1) \times SO(4)}$$

$$\mathcal{A}(u,v) \leftrightarrow \int_{d/2-i\infty}^{d/2+i\infty} \frac{d\Delta}{2\pi i} \sum_j \quad a_j(\Delta)\, G_{\Delta,j}(u,v)$$

Dynamics

$$a_j(\Delta) \sim \frac{1}{\Delta - \Delta_j}$$

Full $O(4,2)$ Conformal Group

$$\boxed{SO(4,2) = SO(1,1) \times SO(3,1)}$$

$$\mathcal{A}(u,v) \leftrightarrow \int_{d/2-i\infty}^{d/2+i\infty} \frac{d\Delta}{2\pi i} \int_{-1/2-i\infty}^{-1/2+i\infty} \frac{dj}{2\pi i} \quad a(\Delta,j)\, \mathcal{G}(u,v;\Delta,j)$$

Euclidean CFT

$$\boxed{SO(4,2) = SO(1,1) \times SO(4)}$$

$$\mathcal{A}(u,v) \leftrightarrow \int_{d/2-i\infty}^{d/2+i\infty} \frac{d\Delta}{2\pi i} \sum_j \quad a_j(\Delta)\, G_{\Delta,j}(u,v)$$

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$$a_j(\Delta) \sim \frac{1}{\Delta - \Delta_j} \quad \to \quad \frac{1}{\Delta - \Delta(j)}$$

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$$u \to 0\,, \quad v \to 1\,, \quad \text{with} \quad \sqrt{u}/(1-v) \quad \text{fixed}$$

Minkowski Regge limit:

"Sommerfeld-Watson resummation"

$$\mathcal{A}(u,v) = \sum_{\xi=\pm} \int \frac{d\Delta}{2\pi i} \int \frac{dj}{2\pi i} \frac{1+\xi e^{-i\pi j}}{\sin \pi j} a_\xi(\Delta, j) \mathcal{G}_{(\Delta,j)}(u,v)$$

Conformal Data:

$$a_\pm \sim \sum_k \frac{c_k(j)}{\Delta - \Delta_k^\pm(j)}$$

$$\mathcal{A}(u,v) = \sum_\xi \sum_k \int \frac{dj}{2\pi i} \frac{1+\xi e^{-i\pi j}}{\sin \pi j} c_k(j,\xi) \mathcal{G}_{(\Delta_k^\xi(j),j)}(u,v)$$

Dynamics

$$\boxed{a_j(\Delta) \sim \frac{1}{\Delta - \Delta_j} \quad \to \quad \frac{1}{\Delta - \Delta(j)}}$$

Single Trace Gauge Invariant Operators of $\mathcal{N} = 4$ SYM,

$$\boxed{Tr[F^2], \quad Tr[F_{\mu\rho}F_{\rho\nu}], \quad Tr[F_{\mu\rho}D_\pm^S F_{\rho\nu}], \quad Tr[Z^\tau], \quad Tr[D_\pm^S Z^\tau], \cdots}$$

Super-gravity in the $\lambda \to \infty$:

$$\boxed{Tr[F^2] \leftrightarrow \phi, \quad Tr[F_{\mu\rho}F_{\rho\nu}] \leftrightarrow G_{\mu\nu}, \quad \cdots}$$

Symmetry of Spectral Curve:

$$\boxed{\Delta(j) \leftrightarrow 4 - \Delta(j)}$$

Conformal Partial-Wave Expansion and Regge Limit:

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Conformal Data:

$$a_\pm \sim \sum_k \frac{c_k(j)}{\Delta - \Delta_k^\pm(j)}$$

$$\mathcal{A}(u,v) = \sum_\xi \sum_k \int \frac{dj}{2\pi i} \frac{1+\xi e^{-i\pi j}}{\sin \pi j} c_k(j,\xi) \mathcal{G}_{(\Delta_k^\xi(j),j)}(u,v)$$

Regge Limit:

$$\mathcal{A} \sim u^{(1-j_0)/2}$$

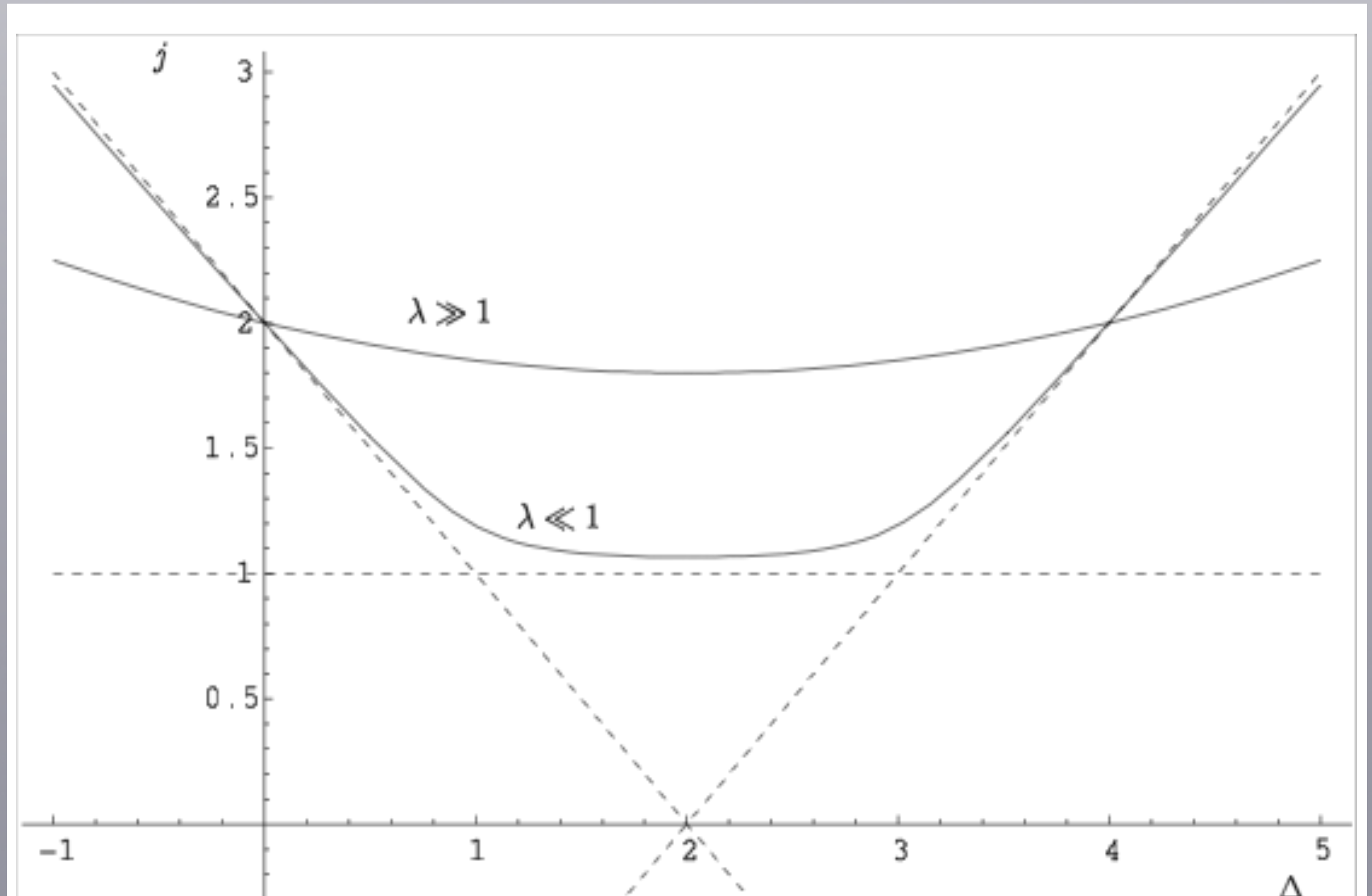
j_0 is the leading singularity of "anomalous dimensions", $\Delta(j) - j - \tau_0$.

MOMENTS AND ANOMALOUS DIMENSION

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2} F_2(x, Q^2) \to Q^{-\gamma_n}$$

Simultaneous compatible large Q^2 and small x evolutions!

Energy-Momentum Conservation built-in automatically.



$$\gamma_2 = 0$$

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

VII. Summary and Outlook

- Provide meaning for Pomeron non-perturbatively from first principles.
- Realization of conformal invariance beyond perturbative QCD
- New starting point for unitarization, saturation, etc.
- First principle description of elastic/total cross sections, DIS at small-x, Central Diffractive Glueball production at LHC, etc.
- Inclusive Production and Dimensional Scalings.
- "non-perturbative" (e.g., blackhole physics, locality in the bulk).

Partial list of successes of AdS/CFT:

Address quark-gluon plasma non-perturbatively.

New perspectives for String theories.

Unified treatment for High Energy Scattering.

Strong coupling for condensed matter physics.

Topological insulators, fractional quantum Hall effect, etc.

SYK Model for Chaos Bound and Falling into Black Holes

Stephen H. Shenker and Douglas Stanford. Stringy effects in scrambling. JHEP, 05:132, 532 2015.

Jeff Murugan, Douglas Stanford, and Edward Witten. More on Supersymmetric and 2d Analogs of the SYK Model. JHEP, 08:146, 2017.

Greatest Equations Ever:

according to J. Polchinski

Euler's Equation:

$$e^{i\pi} + 1 = 0$$

Greatest Equations Ever:

Maxwell:

$$d * F = *j \qquad dF = 0$$

Euler's Equation:

$$e^{i\pi} + 1 = 0$$

according to J. Polchinski

Greatest Equations Ever:

Maxwell:

$$d * F = *j \qquad dF = 0$$

Euler's Equation:

$$e^{i\pi} + 1 = 0$$

Gauge/String Duality:

$$AdS = CFT$$

according to J. Polchinski

For High Energy Collisions

What is AdS/CFT in the context of High Energy Scattering?

From perturbative QCD, dominance of two gluon exchange at HE.

$$\boxed{\text{CFT} = \text{AdS}}$$

For High Energy Collisions

$\boxed{\text{2-GLUONS in 4d = GRAVITON in 5d}}$

↕

$\boxed{\text{CFT = AdS}}$

For High Energy Collisions

2-GLUONS in 4d = GRAVITON in 5d

CFT = AdS

Dominant "Quasi-particle" exchange for High Energy is Graviton propagating in AdS