Initial conditions for heavy ion collisions

Aleksas Mazeliauskas

Institut für Theoretische Physik
Universität Heidelberg

October 02, 2017

Indico programme

Initial Stages 2017

Polish Academy of Arts and Sciences
September 18-22 2017, Cracow, Poland

Initial conditions with QCD kinetic theory

Aleksas Mazeliauskas

Institut für Theoretische Physik
Universität Heidelberg

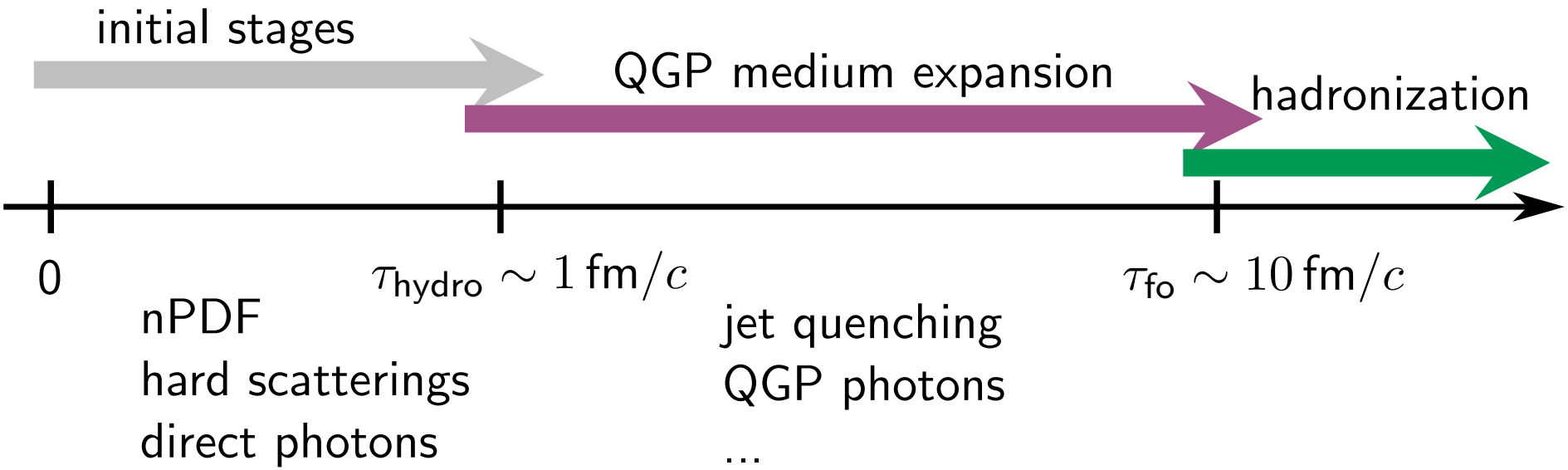
October 02, 2017

L. Keegan, A. Kurkela, A.M., and D. Teaney, JHEP 08, 171 (2016)[1].

A. Kurkela, A.M., J.-F. Paquet, S. Schlichting, and D. Teaney, *almost done*.

see also Teaney 18/09 IS'17

Ultra-relativistic heavy ion collisions in a nutshell



Boundary between initial stages and medium physics τ_{hydro} is artificial.

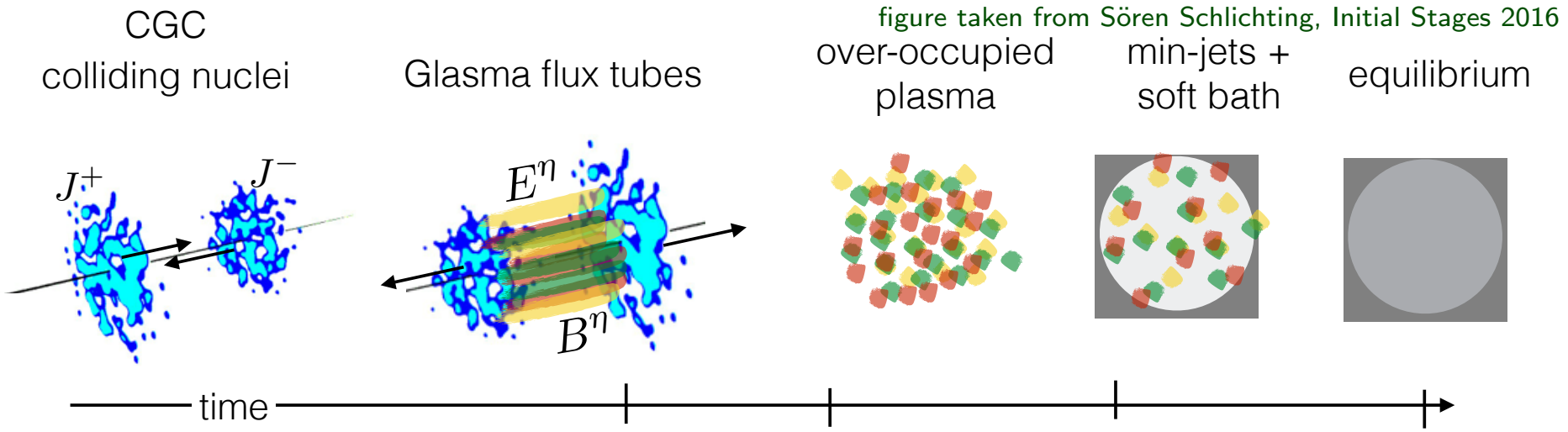
- when does jet quenching begin?
- when does QGP start emitting photons?

Our goal: consistent theoretical overlap of initial stages and the onset of hydrodynamics.

Initial stages in weak coupling picture

At high collision energies and densities $\alpha_s \ll 1$

figure taken from Sören Schlichting, Initial Stages 2016



strong fields

quasi particles

18/09 IS'17
Gelis
Boguslavski
Wu 20/19

classical-statistical
lattice gauge theory
$f \sim \frac{1}{\alpha_s} \gg 1$

eff. kinetic theory
$1 \ll f \ll \frac{1}{\alpha_s}$

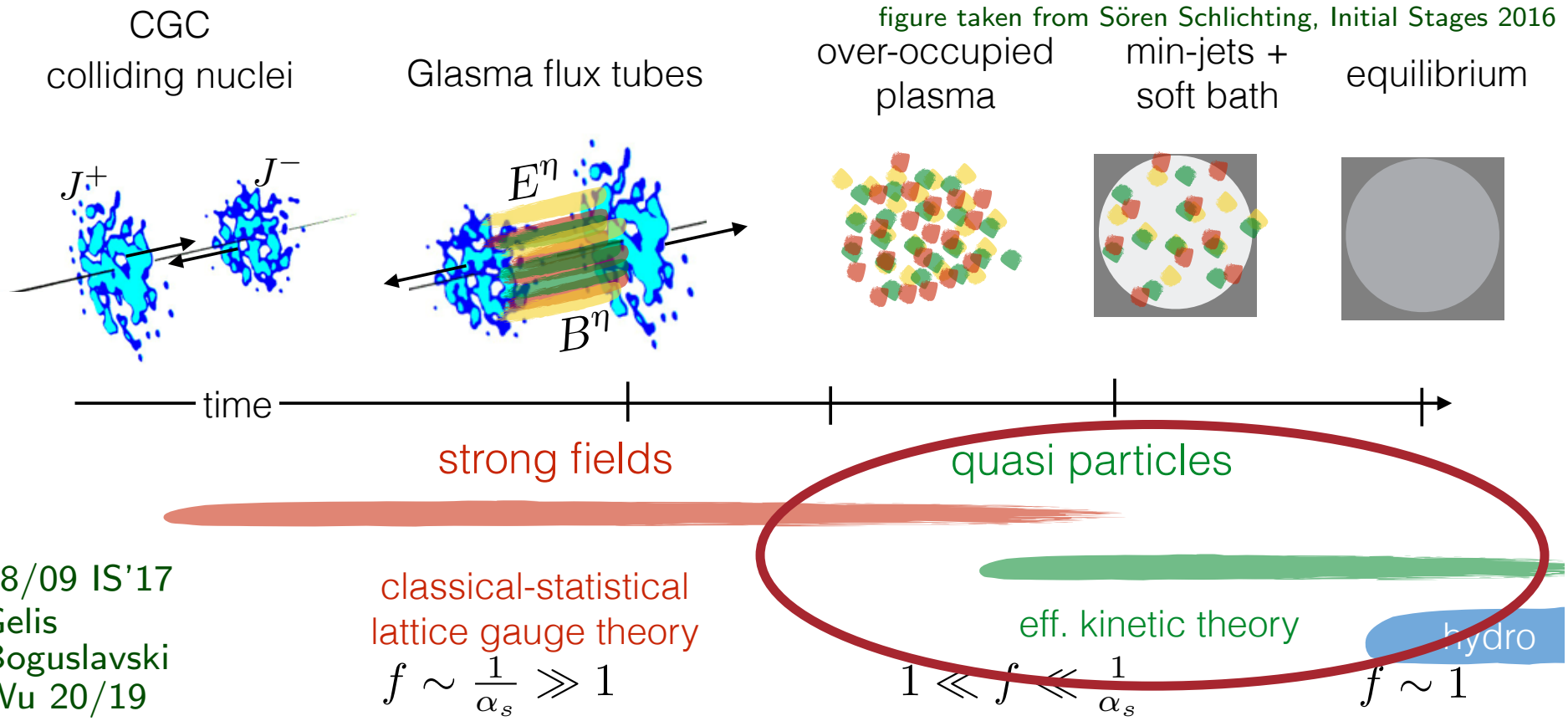
hydro
$f \sim 1$

QCD kinetic theory interpolates between strong fields and hydrodynamics

Initial stages in weak coupling picture

At high collision energies and densities $\alpha_s \ll 1$

figure taken from Sören Schlichting, Initial Stages 2016



18/09 IS'17
Gelis
Boguslavski
Wu 20/19

QCD kinetic theory interpolates between strong fields and hydrodynamics

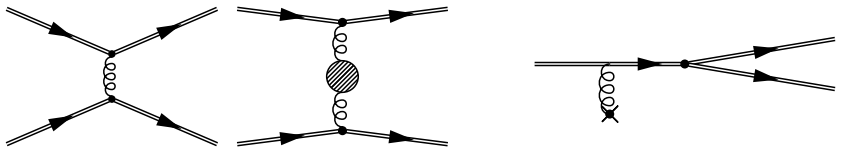
Thermalization in QCD effective kinetic theory

QCD effective kinetic theory — Arnold, Moore, Yaffe (2003)[2]

"Bottom-up" thermalization scenario — Baier, Mueller, Schiff, and Son (2001)[5]

Prescription: solve Boltzmann equation for distribution function $f(\mathbf{p})$

$$\partial_\tau f + \frac{\mathbf{p}}{|p|}\cdot\nabla f - \frac{p_z}{\tau}\partial_{p_z} f = -\underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]}_{} - \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]}_{}$$



- elastic $2 \leftrightarrow 2$ and *inelastic* $1 \leftrightarrow 2$ scatterings (with LPM suppression)

  the same QCD processes as in jet quenching

- at late times equilibrates to conformal hydrodynamics
- single parameter — the coupling constant $\lambda = 4\pi\alpha_s N_c$.

In practise need to extrapolate λ to realistic values.

High-pT QCD kinetic theory $\Rightarrow$ parton energy loss

Low-pT QCD kinetic theory $\Rightarrow$ thermalization

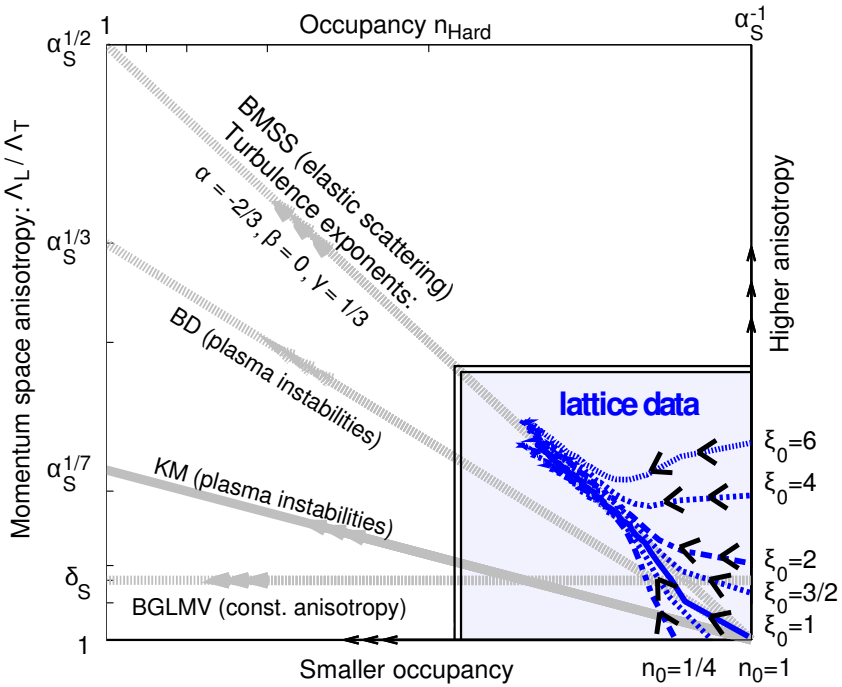
"Jet quenching and fluid dynamics = two manifestations of the same physics" – Wiedemann

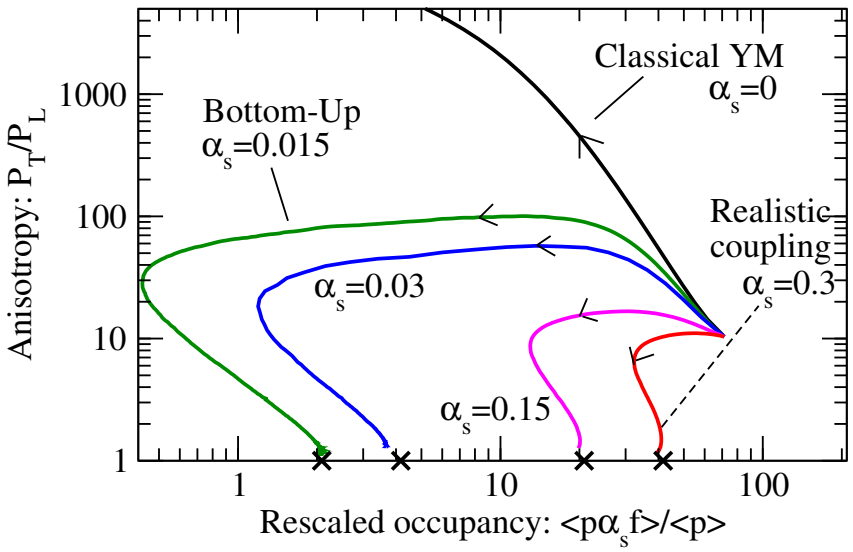
From classical simulations to kinetic theory

but anisotropies should create plasma instabilities.

classical-statistical

effective kinetic theory





Berges, Boguslavski, Schlichting, Venugopalan (2014)[3]

Kurkela and Zhu (2015)[4]

Anisotropy

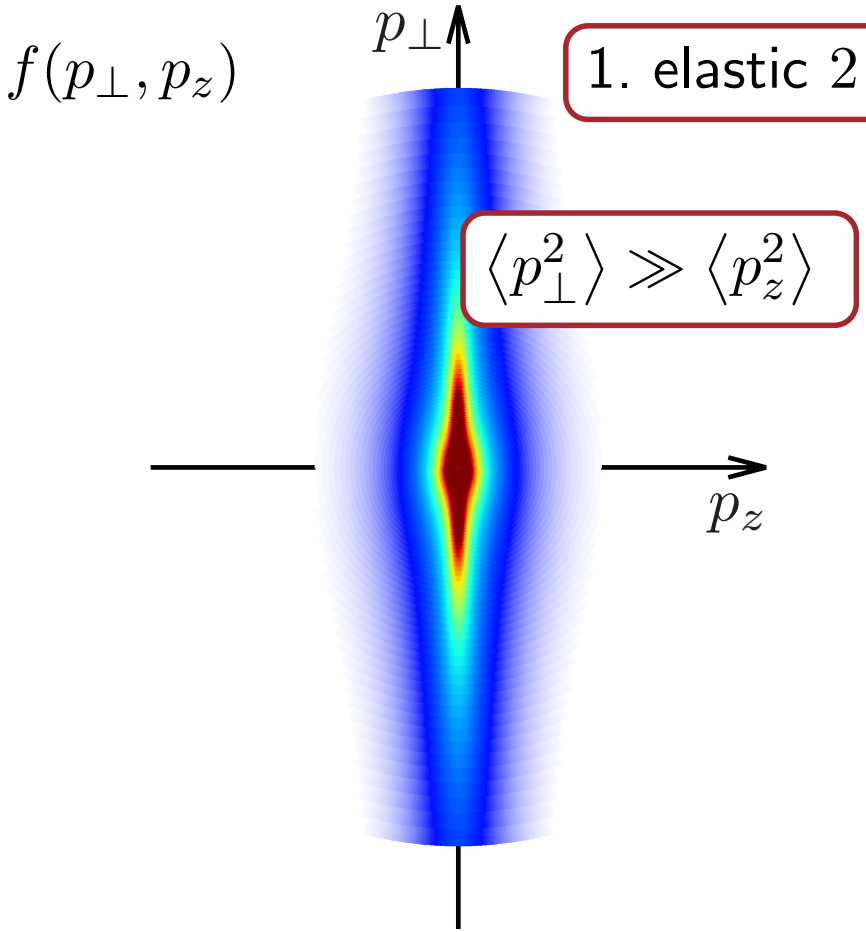
Occupancy

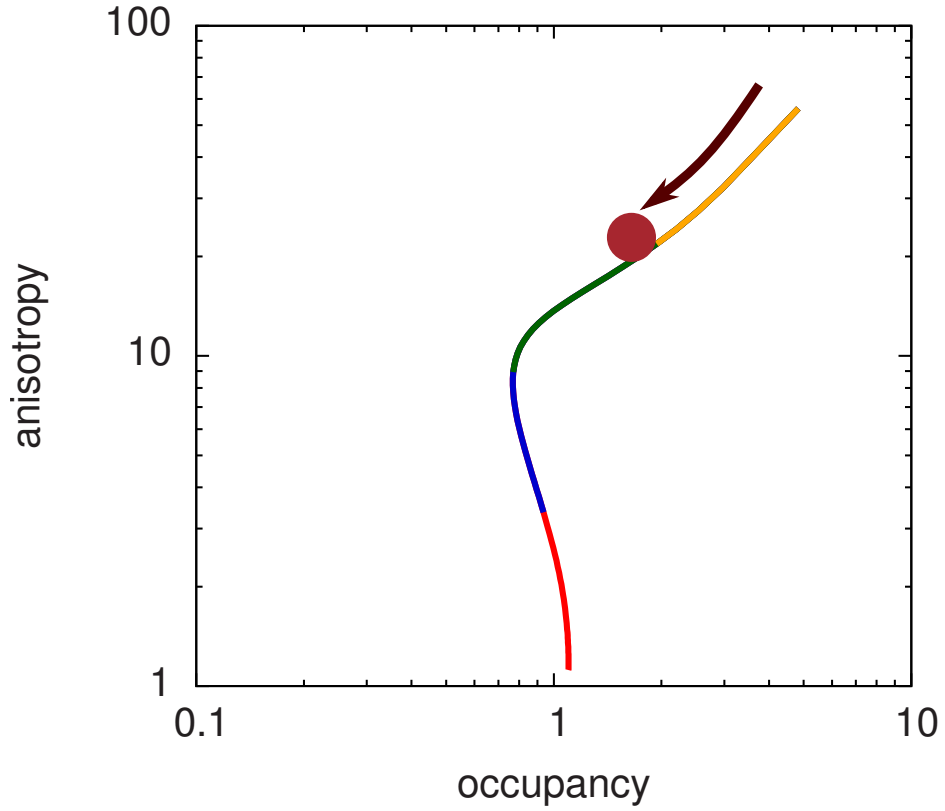
Classical-statistical lattice simulations find the "bottom-up" attractor.

Numerical results for thermalization in kinetic theory

"Bottom-up" thermalization scenario — Baier, Mueller, Schiff, and Son (2001)[5]

Numerical realization — Kurkela and Zhu (2015)[4]



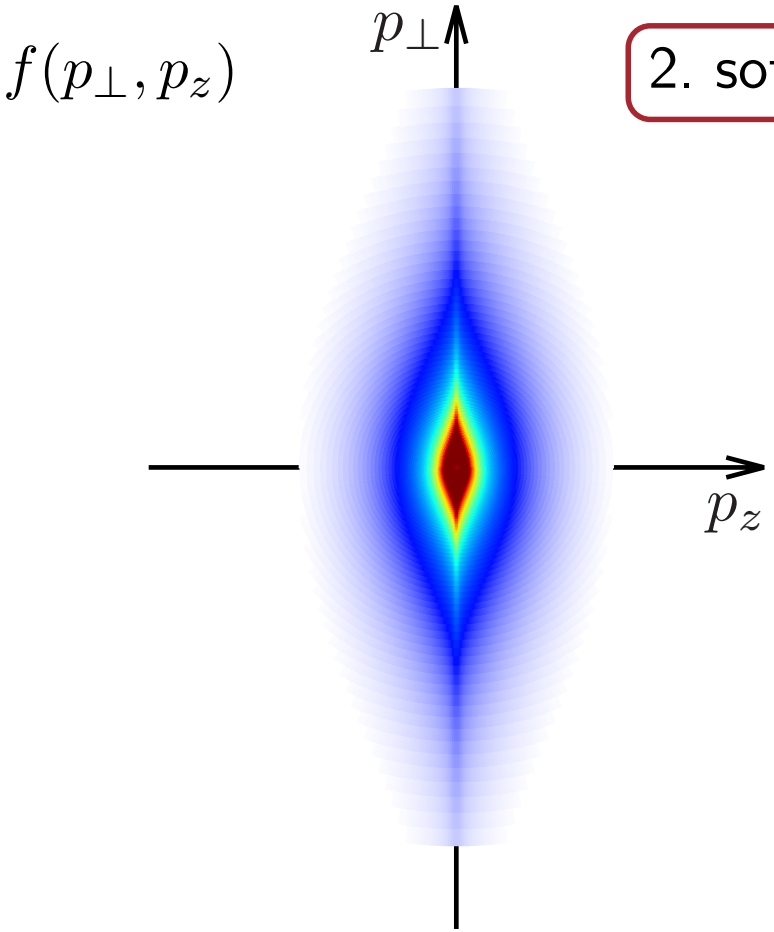


see also Teaney 18/09 IS'17

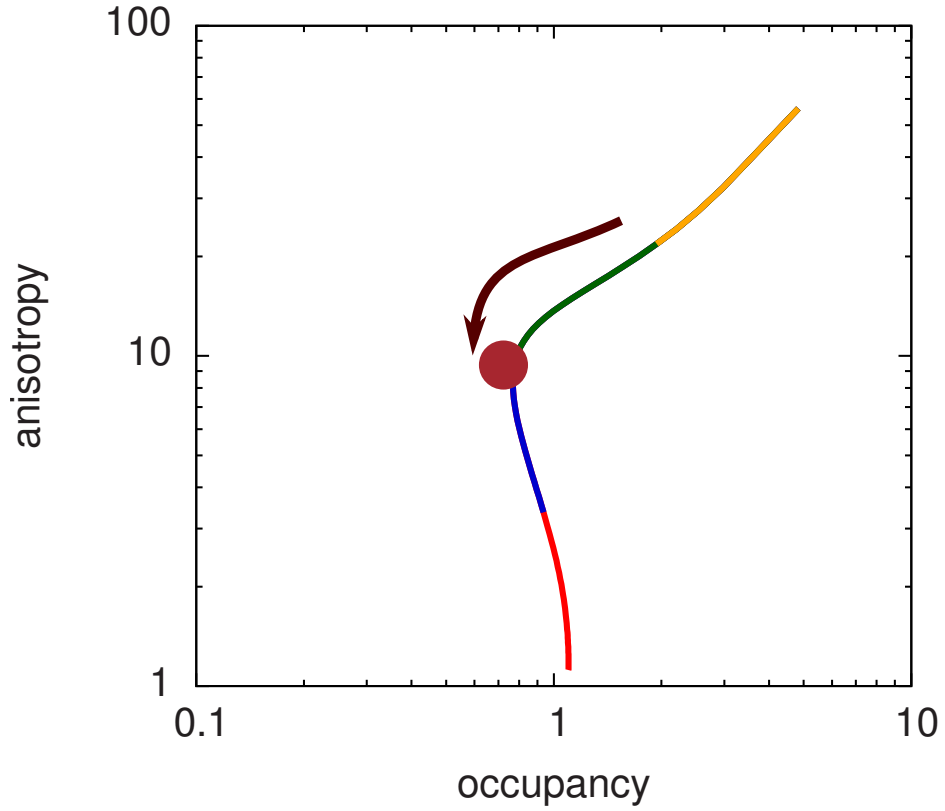
Numerical results for thermalization in kinetic theory

"Bottom-up" thermalization scenario — Baier, Mueller, Schiff, and Son (2001)[5]

Numerical realization — Kurkela and Zhu (2015)[4]



2. soft $1 \leftrightarrow 2$ splitting

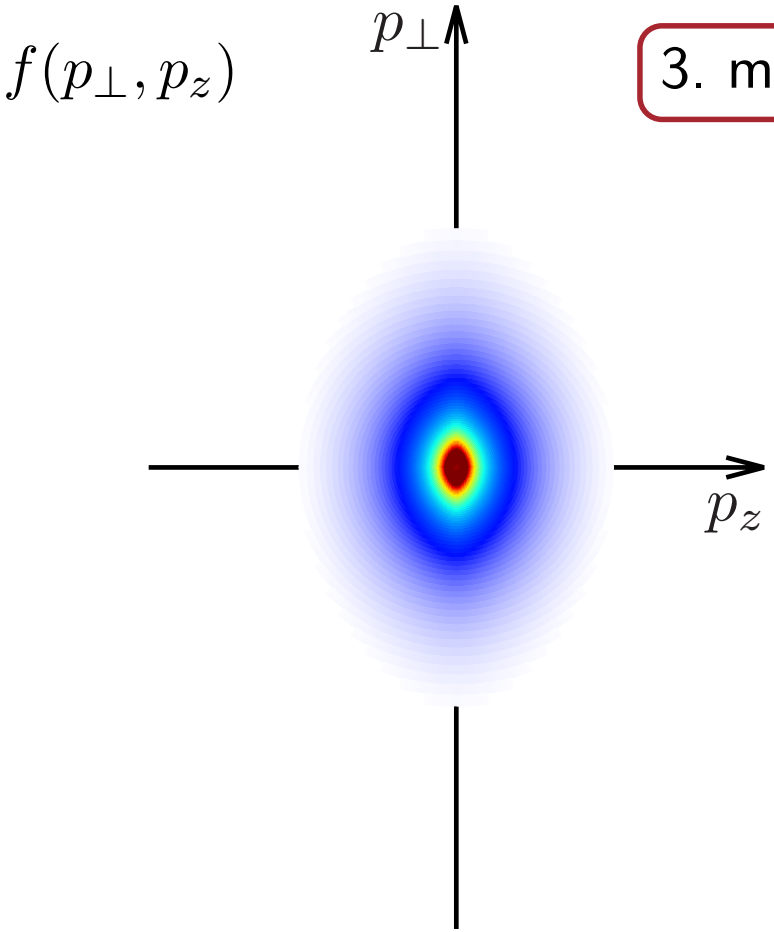


see also Teaney 18/09 IS'17

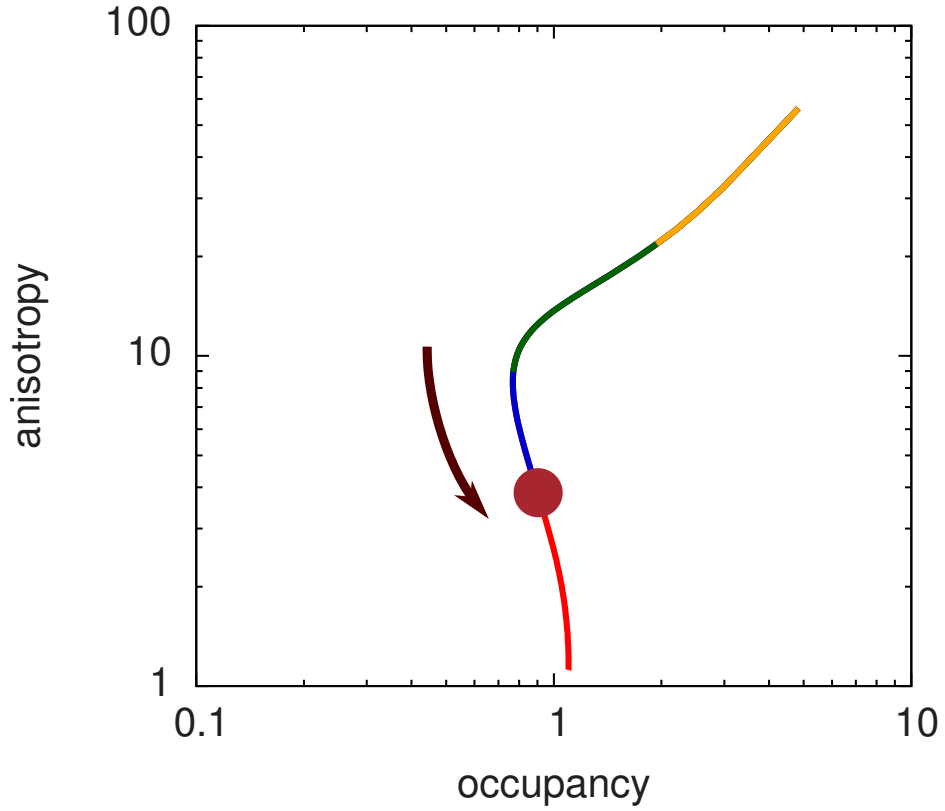
Numerical results for thermalization in kinetic theory

"Bottom-up" thermalization scenario — Baier, Mueller, Schiff, and Son (2001)[5]

Numerical realization — Kurkela and Zhu (2015)[4]



3. mini-jet quenching

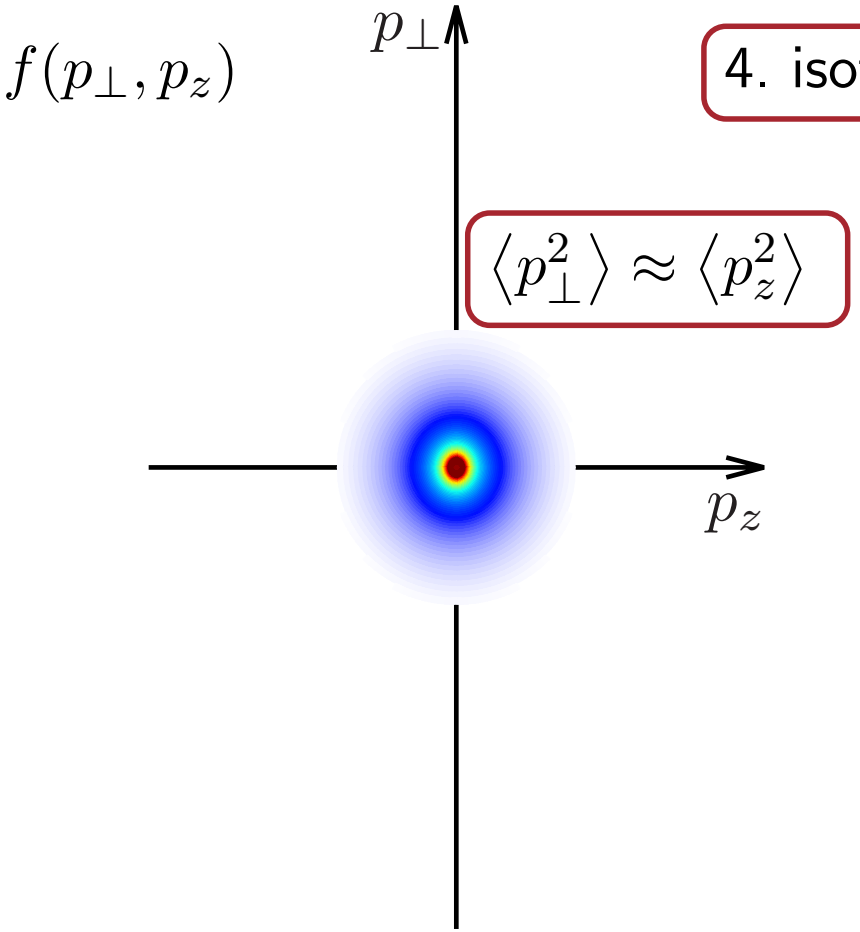


see also Teaney 18/09 IS'17

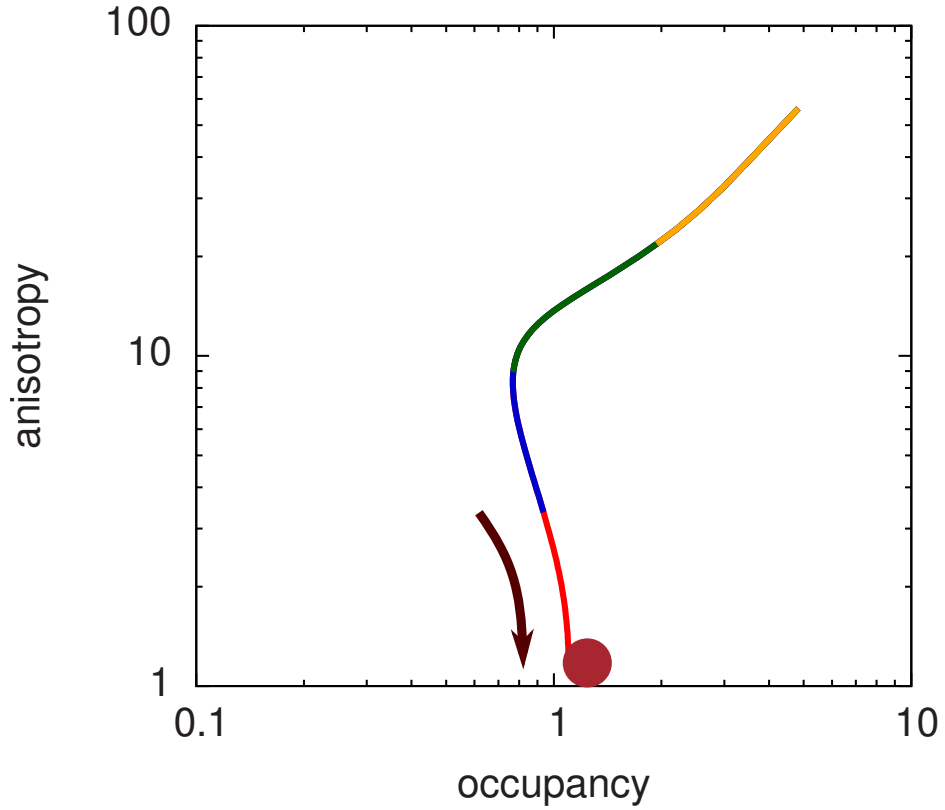
Numerical results for thermalization in kinetic theory

"Bottom-up" thermalization scenario — Baier, Mueller, Schiff, and Son (2001)[5]

Numerical realization — Kurkela and Zhu (2015)[4]



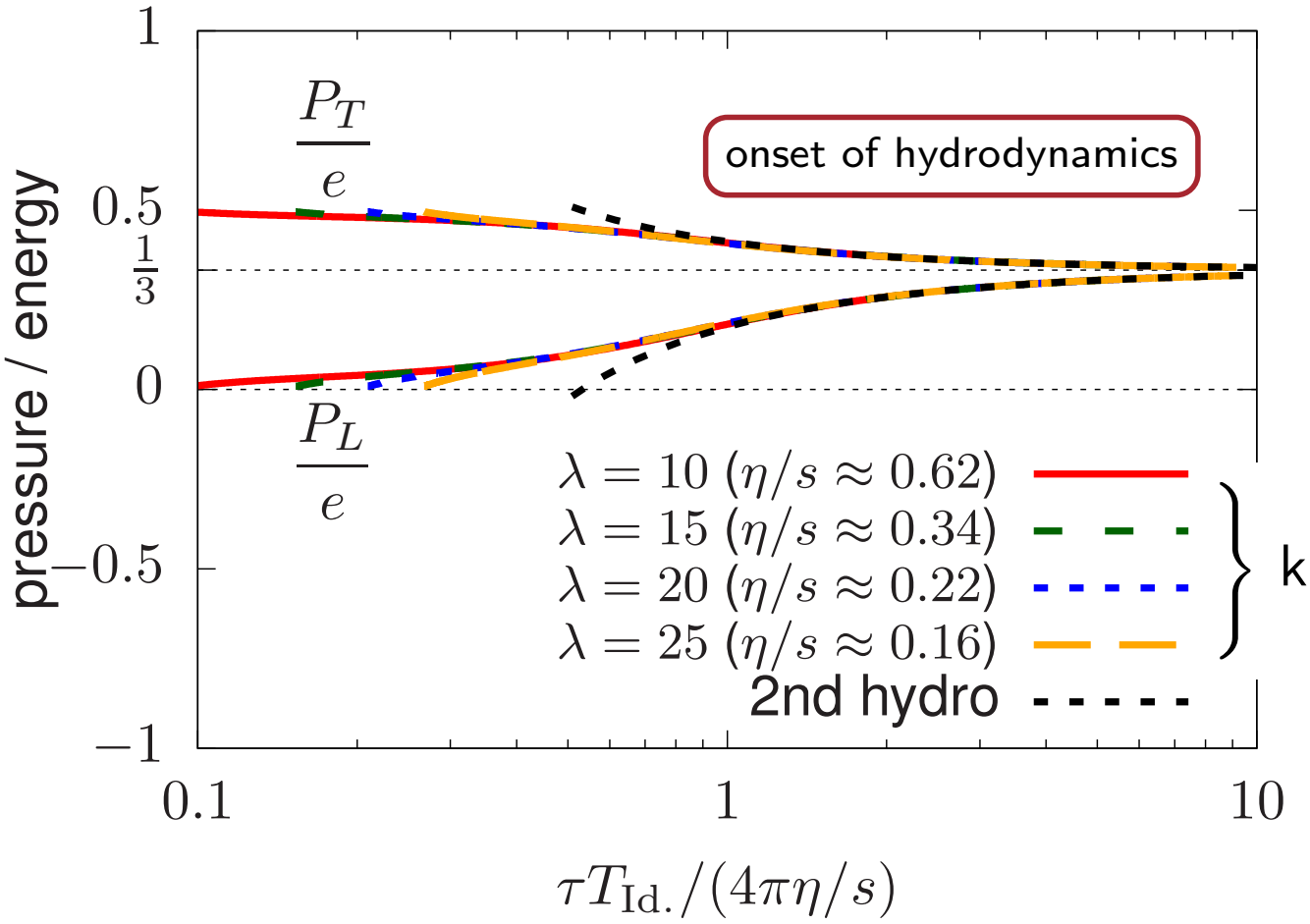
4. isotropization



see also Teaney 18/09 IS'17

Scaling of pressure evolution for boost invariant system

Pressure isotropization in kinetic theory



c.f. attractor solutions:

Strickland 18/09 IS'17

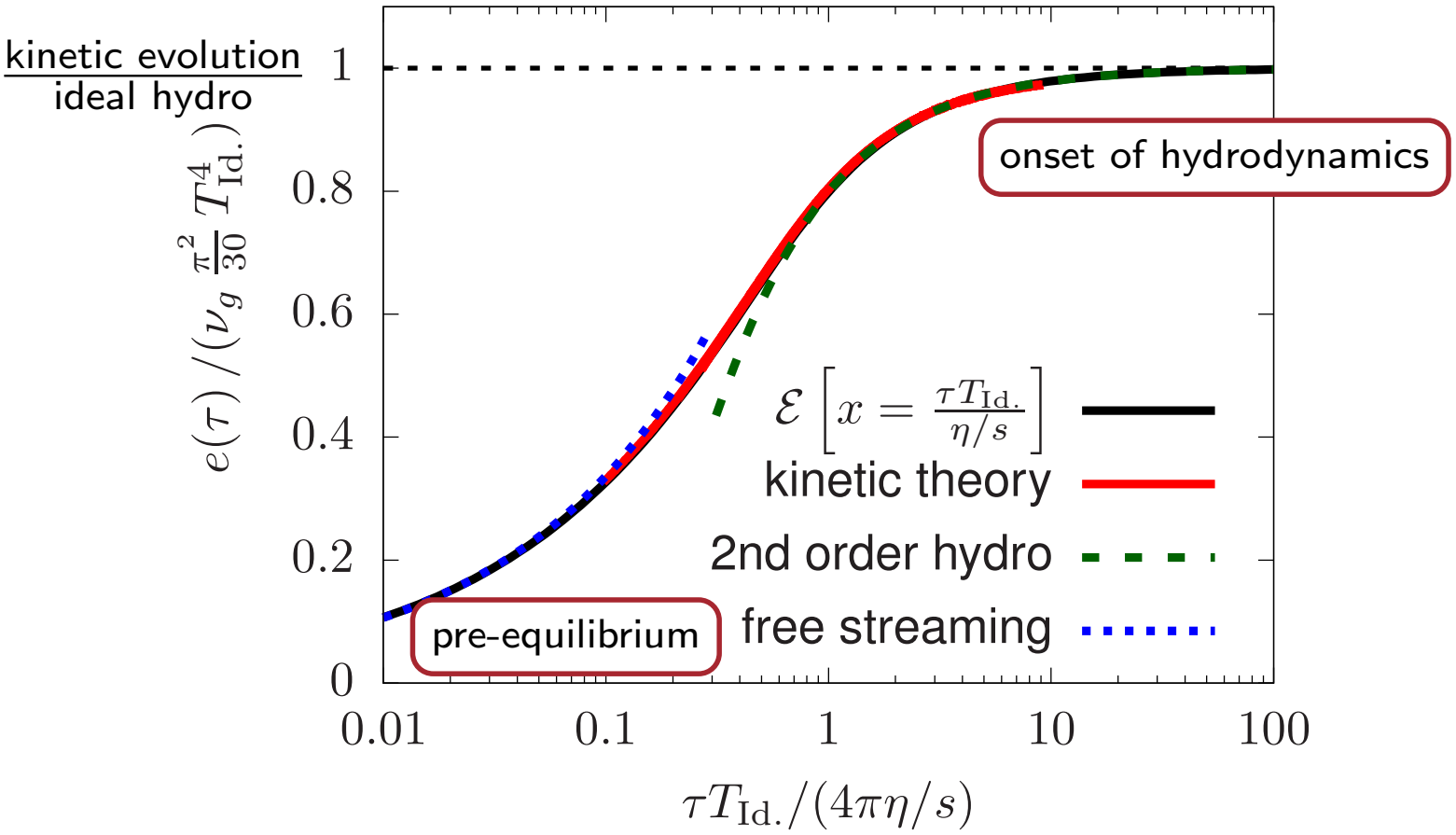
Romatschke 20/09 IS'17

Spalinski 21/09 IS'17

Collapse to scaling solution for time $\frac{\tau T_{\mathrm{Id.}}}{\eta/s} \propto \frac{\tau}{\tau_R}$ *in units of relaxation time.*

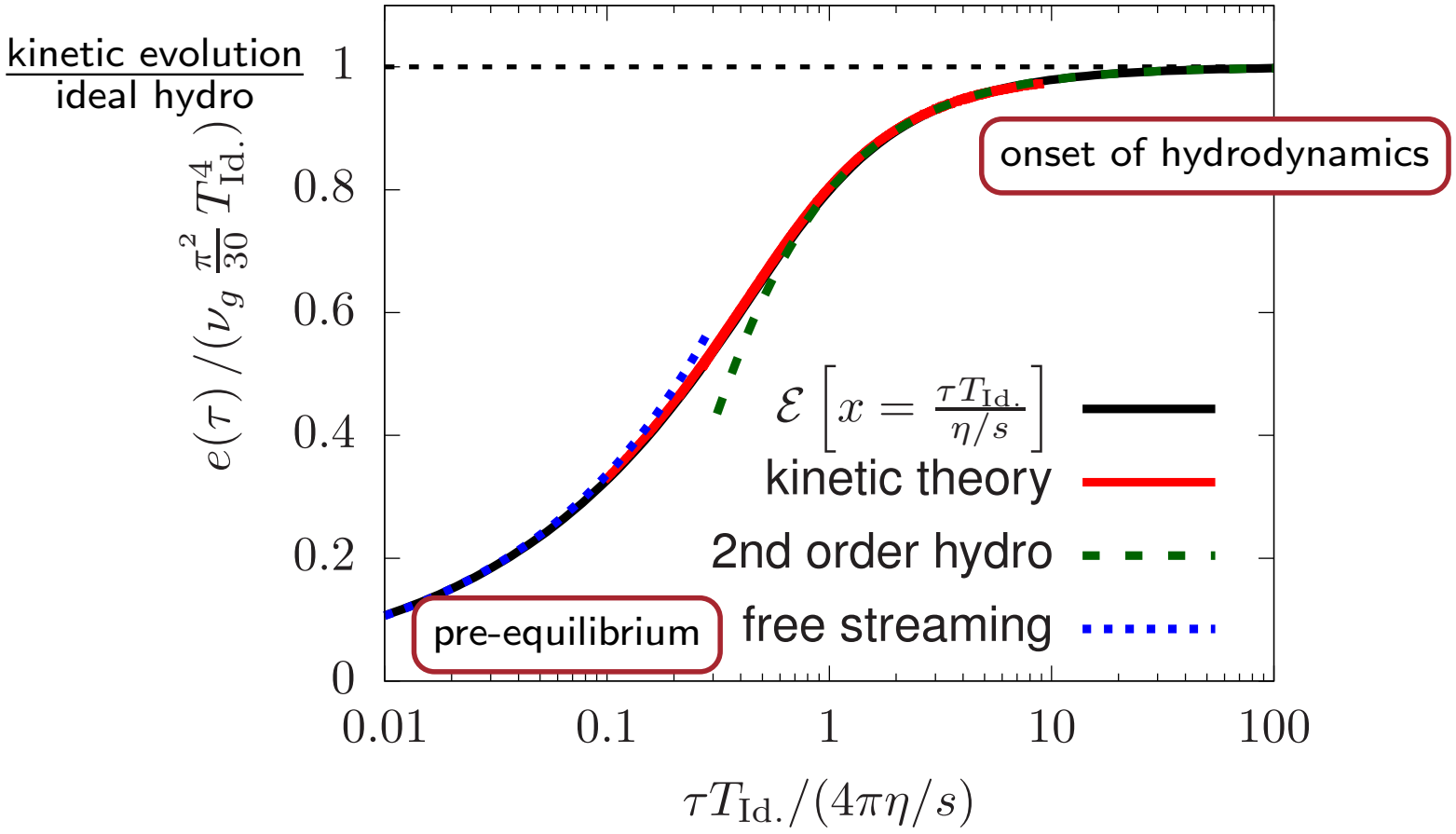
Out-of-equilibrium energy evolution for boost invariant system

scaling in hydro: $$e(\tau) = \underbrace{\nu_g \frac{\pi^2}{30} T_{\text{Id.}}^4}_{\text{“ideal” temp.}} \left(\underbrace{1}_{\text{ideal}} - \underbrace{\frac{8}{3}\frac{\eta/s}{\tau T_{\text{Id.}}}}_{\text{viscous}} + \underbrace{C_2 \left(\frac{\eta/s}{\tau T_{\text{Id.}}}\right)^2}_{\text{2nd order hydro}} \right)$$



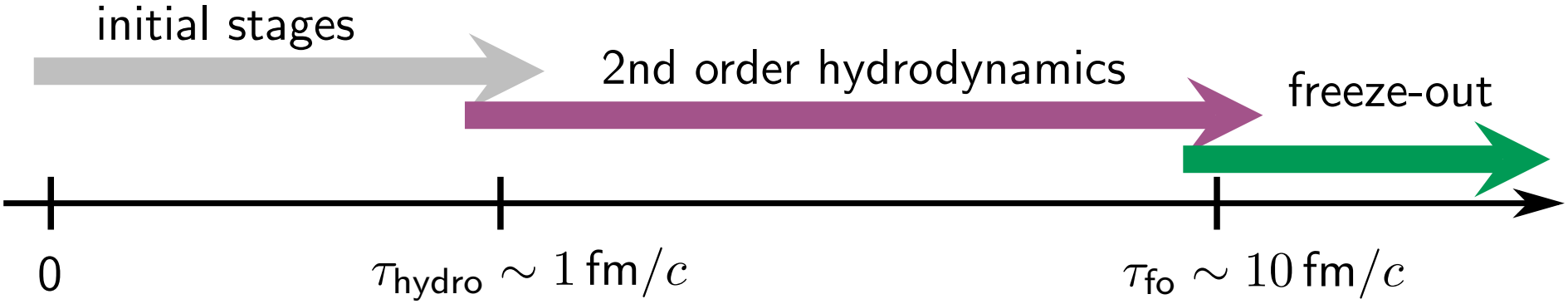
Out-of-equilibrium energy evolution for boost invariant system

generalized scaling: $$e(\tau) = \nu_g \frac{\pi^2}{30} T^4_{\text{Id.}}(\tau) \times \underbrace{\mathcal{E}\left[x = \frac{\tau T_{\text{Id.}}(\tau)}{\eta/s}\right]}_{\text{non-equilibrium evolution}}$$



Event by event pre-equilibrium dynamics

Realistic initial conditions for hydrodynamics



Examples of initial stages:

A MC-Glauber:
✓ e-by-e fluctuations, ✗ dynamics, ✗ flow, ✗ hydrodynamization

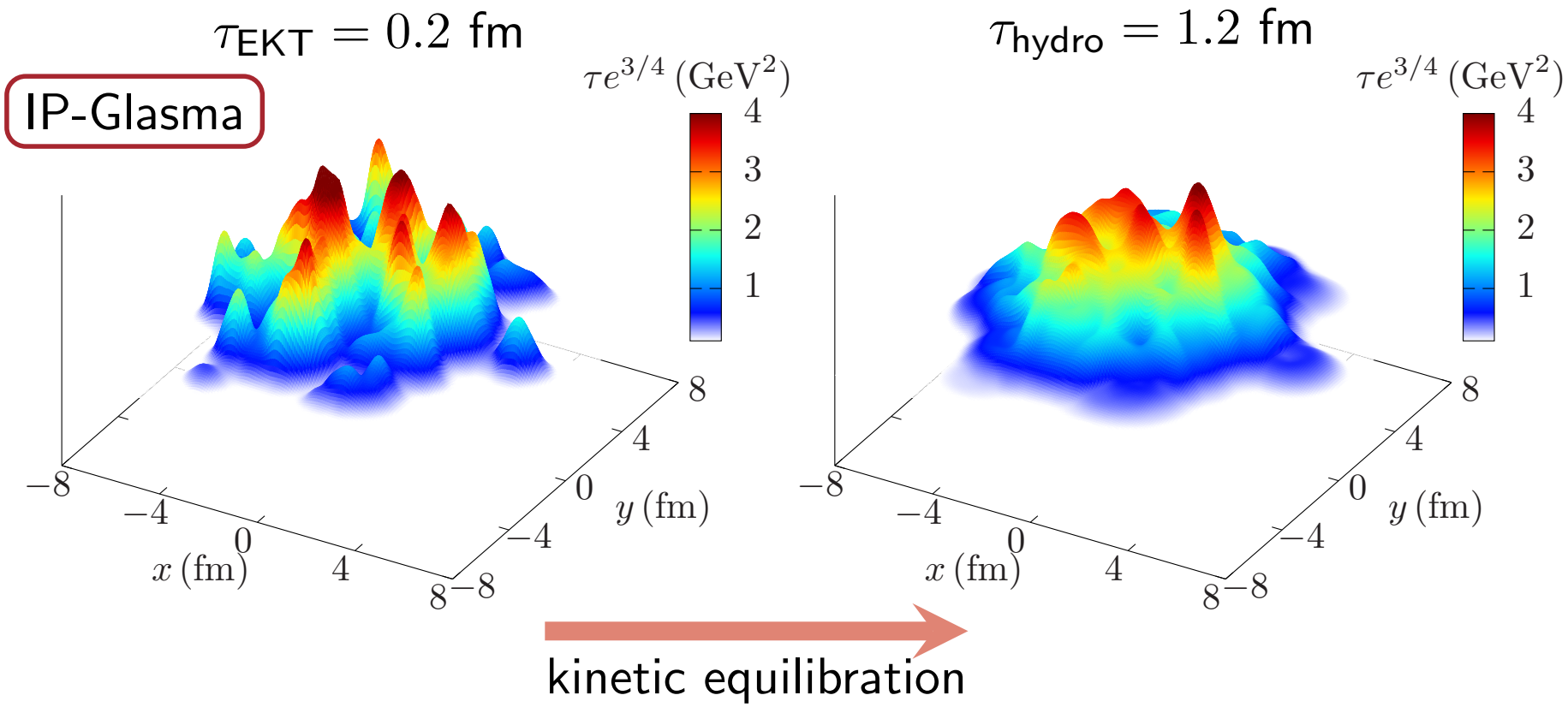
B IP-Glasma: Schenke, Tribedy, Venugopalan (2013)[6] Schenke 18/09 IS'17
✓ e-by-e fluct., ✓ 2+1D Yang-Mills, ✓ flow, ✗ hydrodynamization

see also EKRT model (includes nPDFs), Niemi 19/09 IS'17

Use the out-of-equilibrium kinetic theory to map IP-Glasma initial conditions to hydrodynamics.

Evolution of perturbations with kinetic theory

IP-Glasma initial conditions at $\tau_{\mathsf{EKT}} = 0.2\,\mathrm{fm}$



$$\underbrace{e(\boldsymbol{x}, \tau_{\mathsf{EKT}}), \vec{v}(\boldsymbol{x}, \tau_{\mathsf{EKT}})}_{\text{input to EKT}} \implies \underbrace{f(\tau, \mathbf{p}, \boldsymbol{x})}_{\text{kinetic evolution}} \implies \underbrace{e(\boldsymbol{x}, \tau_{\mathsf{hydro}}), \vec{v}(\boldsymbol{x}, \tau_{\mathsf{hydro}})}_{\text{input to hydro}}$$

Plane wave perturbations in transverse plane

Simplify 5+1D problem for $f(\tau, \mathbf{p}, \boldsymbol{x}_\perp)$ by linearization

$$f(\tau, \mathbf{p}, \boldsymbol{x}_\perp) = \underbrace{\bar{f}_{\mathbf{p}}}_{\text{uniform background}} + \underbrace{\delta f_{\boldsymbol{k}_\perp, \mathbf{p}} e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp}}_{\text{transverse perturbations}} .$$

Linearized Boltzmann equation for perturbations

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right)\bar{f}_{\mathbf{p}} = -\mathcal{C}[\bar{f}] \qquad \text{background}$$

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + \frac{i\mathbf{p}_\perp \cdot \boldsymbol{k}_\perp}{p}\right)\delta f_{\boldsymbol{k}_\perp, \mathbf{p}} = -\delta\mathcal{C}[\bar{f}, \delta f] \qquad \boldsymbol{k}_\perp \text{ perturbation}$$

Construct kinetic response functions to energy/momentum perturbations

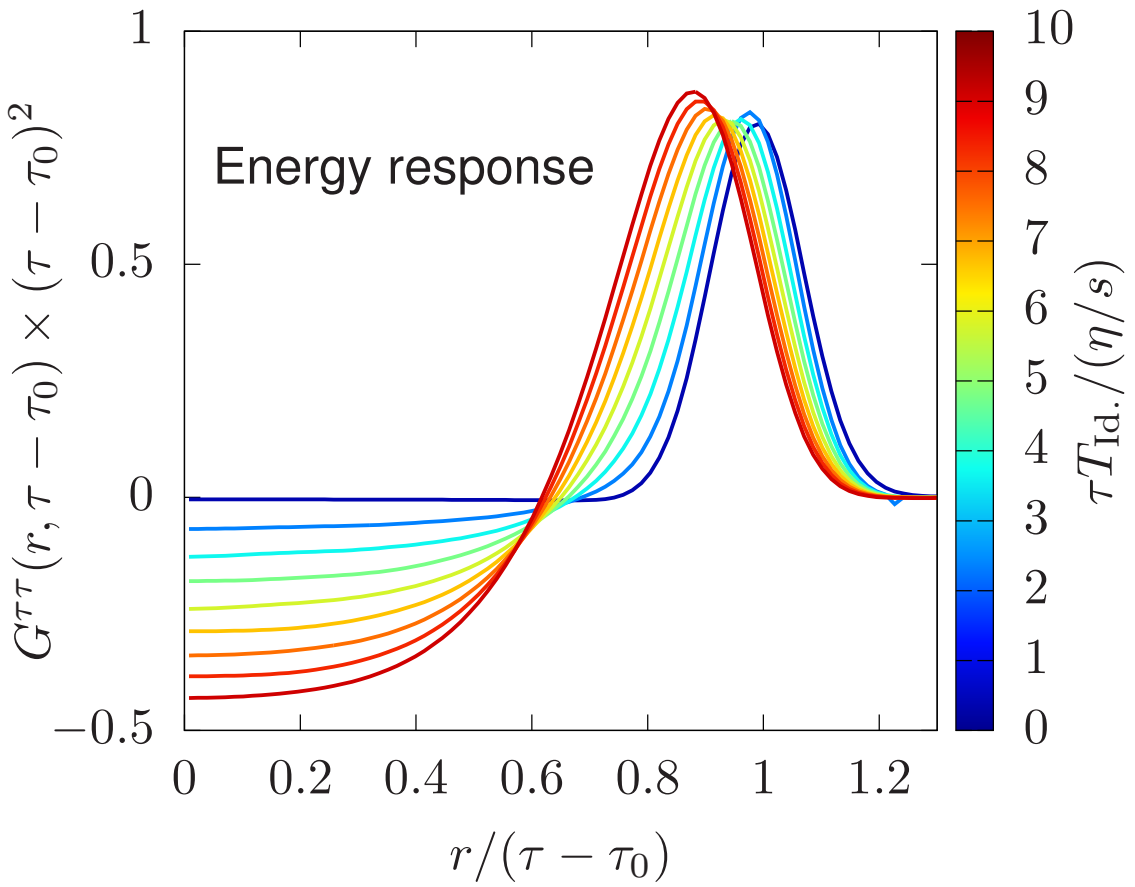
$$\underbrace{\frac{\delta T^{\mu\nu}(\tau, \boldsymbol{k})}{e(\tau)}}_{\text{output for hydro}} = \underbrace{\tilde{G}^{\mu\nu}_{(\tau,i)}\left(\boldsymbol{k}, \tau, \tau_0\right) \times}_{\text{non-equilibrium kinetic response}} \underbrace{\left[\frac{\delta T^{\tau\tau}(\tau_0, \boldsymbol{k})}{e(\tau_0)} \text{ or } \frac{\delta T^{\tau i}(\tau_0, \boldsymbol{k})}{e(\tau_0)}\right]}_{\text{initial perturbations}}$$

Scaling of kinetic theory response functions

- Response to perturbations also collapse to scaling solutions

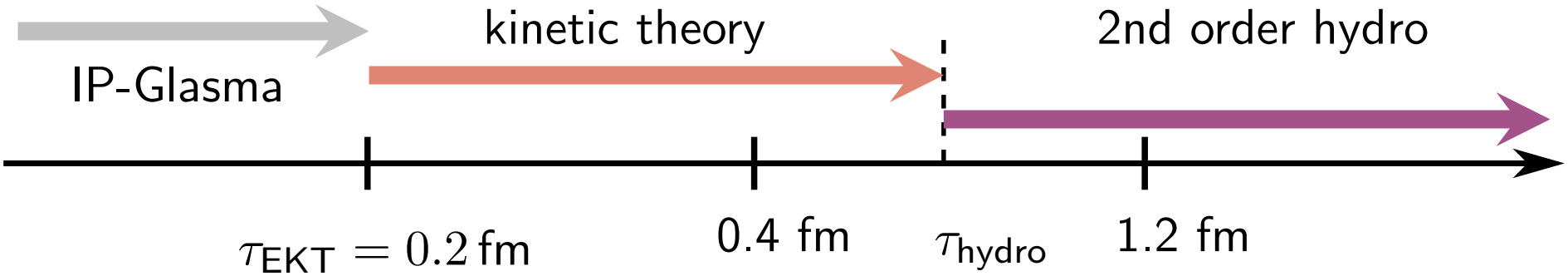
$$\tilde{G}^{\mu\nu}\Big(|\boldsymbol{k}|, \tau, \tau_0, e(\tau_0), \lambda\Big) = \tilde{G}^{\mu\nu,\mathrm{univ}}\left(\frac{\tau T_{\mathrm{ld.}}}{\eta/s}, |\boldsymbol{k}|(\tau - \tau_0)\right)$$

- Agrees with hydrodynamic scaling in $\tau \to \infty, |\boldsymbol{k}|\tau \to 0$ limit.
- Good scaling for a range of $\eta/s = 0.16{-}0.62$ values.

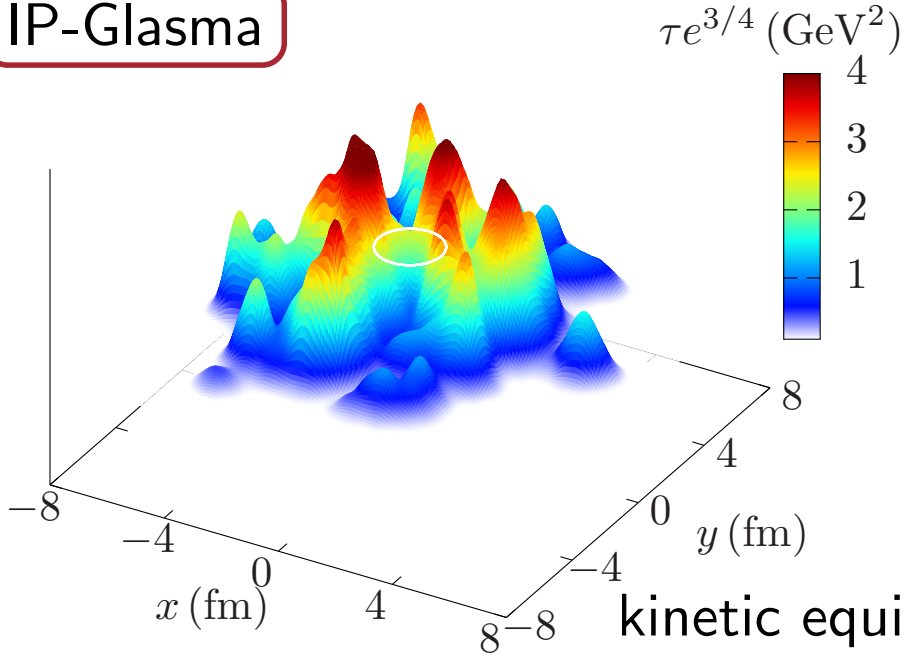


Kinetic evolution with IP-Glasma initial conditions

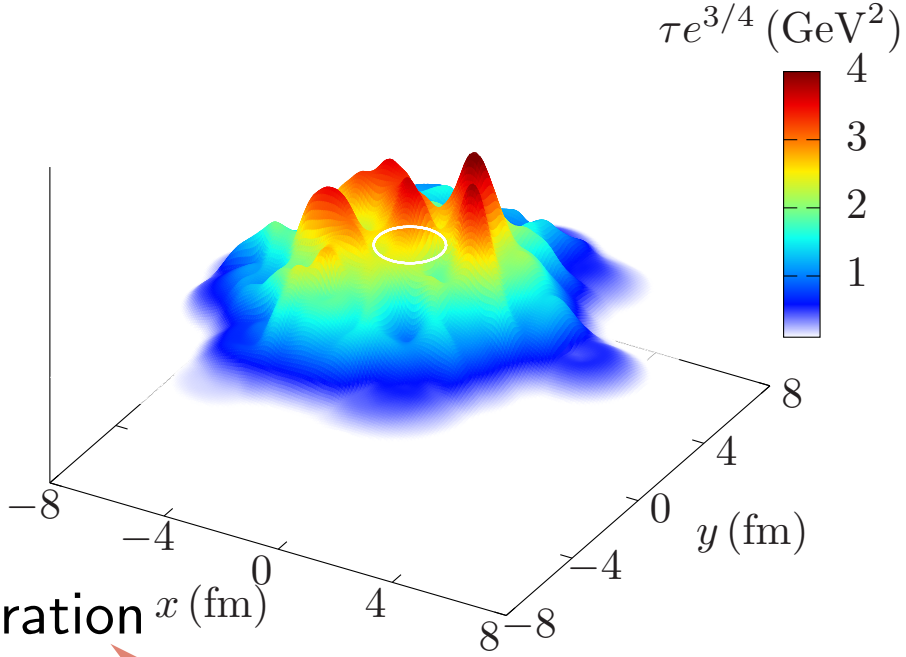
Equilibration with IP-Glasma initial conditions



IP-Glasma

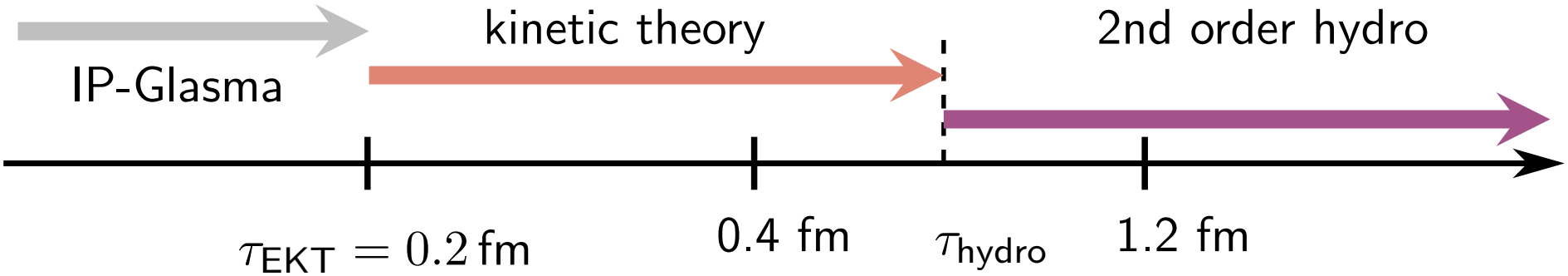


kinetic equilibration

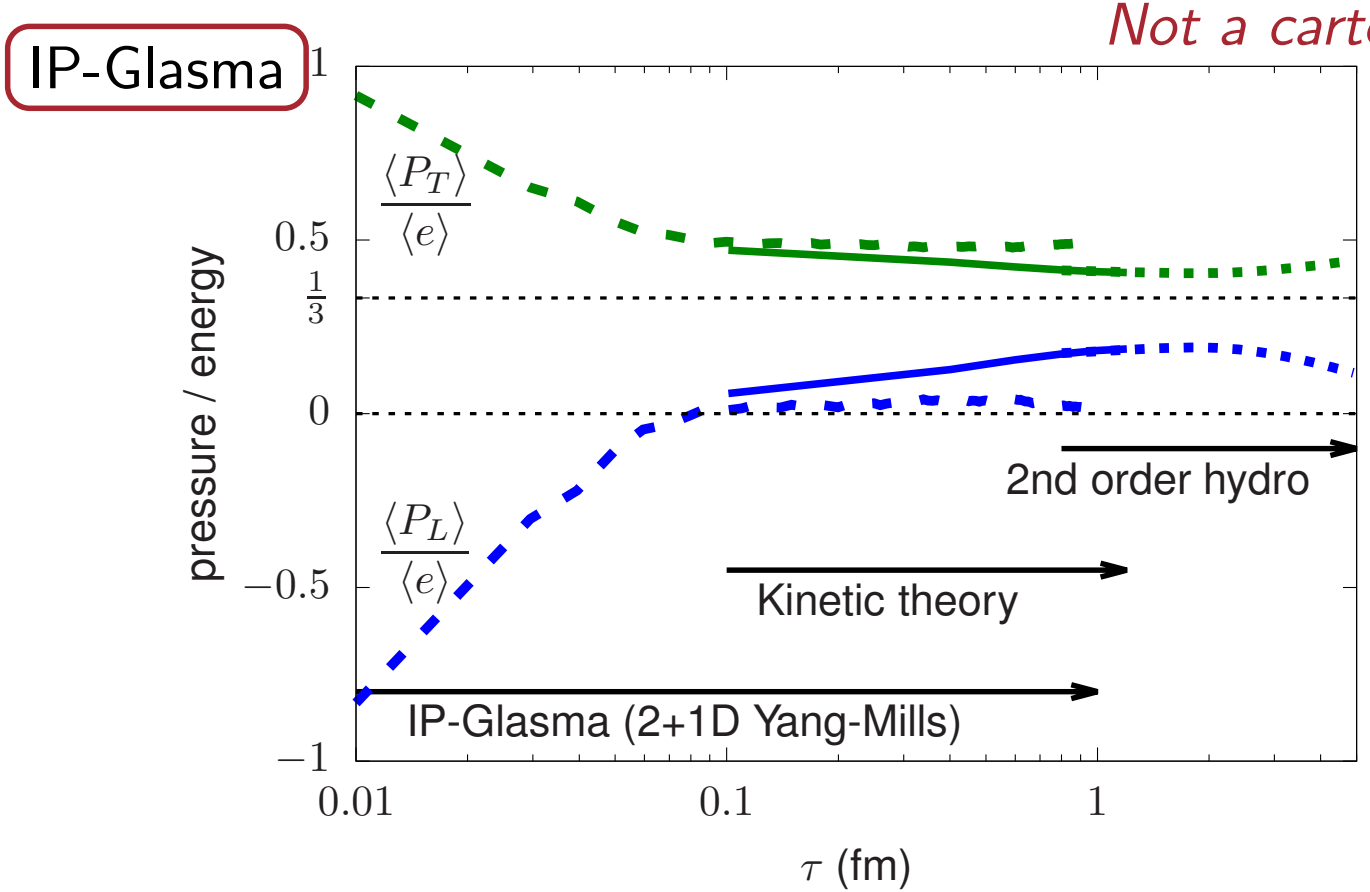


$$\underbrace{e(\boldsymbol{x}, \tau_{\mathrm{EKT}}),\ \vec{v}(\boldsymbol{x}, \tau_{\mathrm{EKT}})}_{\text{non-zero initial flow}} \Longrightarrow e(\tau) + \delta e(\tau, \boldsymbol{x}),\ \vec{v}(\tau, \boldsymbol{x}) \Longrightarrow \underbrace{T^{\mu\nu}(\boldsymbol{x}, \tau_{\mathrm{hydro}})}_{\text{full eng.-mom. tensor}}$$

Average pressure in the transverse plane

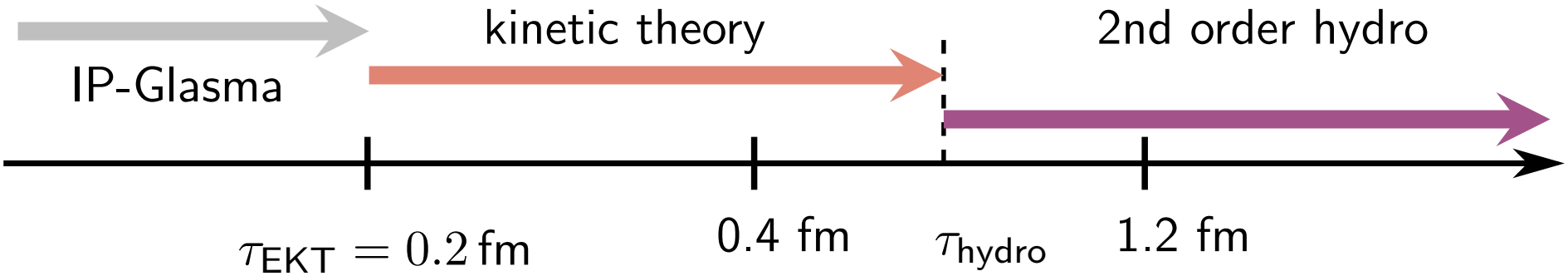


Not a cartoon!

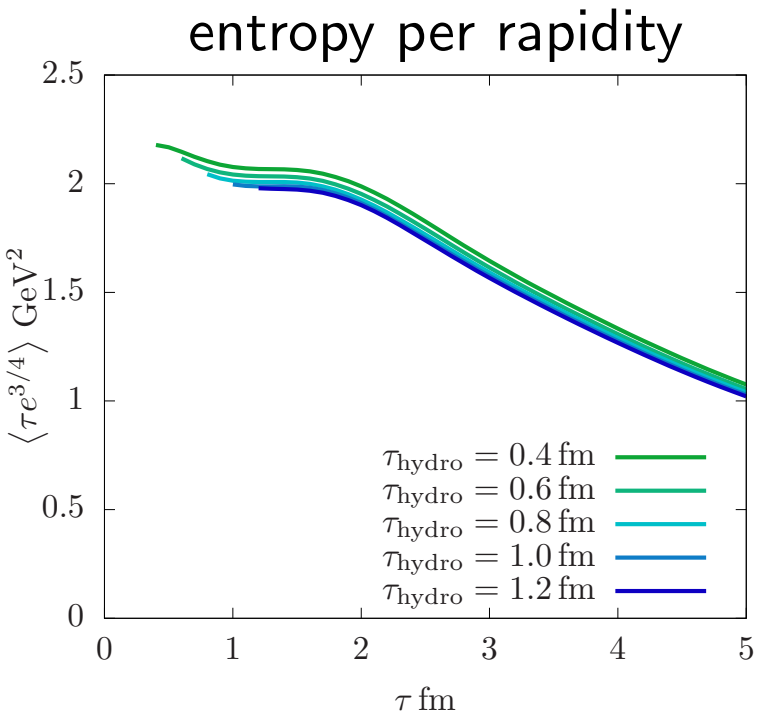


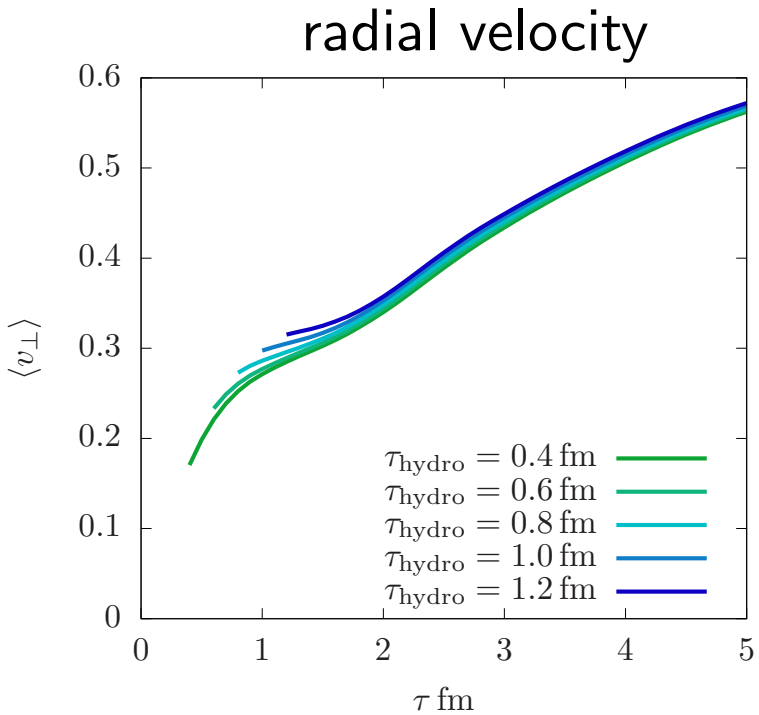
Overlapping and consistent pressure evolution between initial stages!

Transverse averaged density and flow

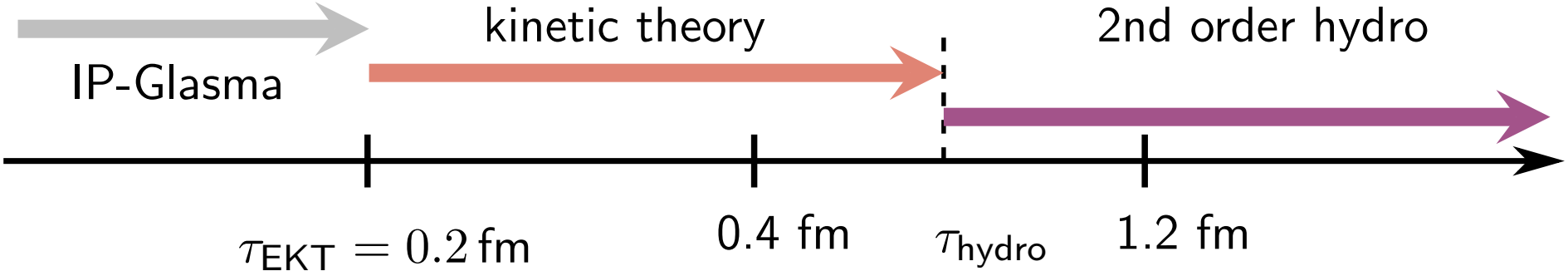


transverse averages $\quad \langle \ldots \rangle = \dfrac{\int d^2\boldsymbol{x}_\perp u^\tau e \ldots}{\int d^2\boldsymbol{x}_\perp u^\tau e}$

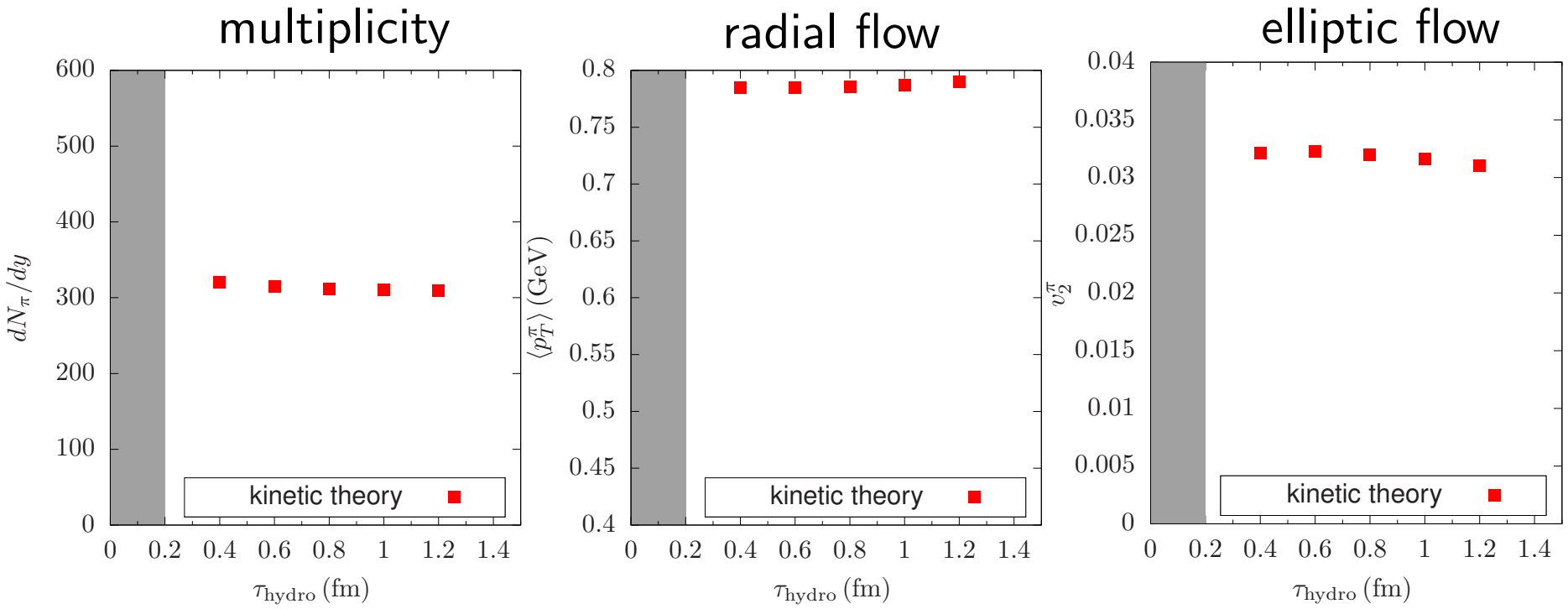




Hadronic observables: IP-Glasma



Thermal pions at freeze-out $T_{\mathrm{fo}} = 145\,\mathrm{MeV}$



Approximate independence of τ_{hydro} *for kinetic pre-equilibrium evolution!*

Summary

"Bottom-up" equilibration for realistic heavy ion initial conditions:

- Background evolution described by a scaling solution in $\tau T_{\text{ld.}}/(\eta/s)$
- Found linear kinetic response functions $G^{\mu\nu}(\tau T_{\text{ld.}}/(\eta/s), r/(\tau - \tau_0))$
- Demonstrated small dependence on crossover time τ_{hydro}

Smooth and consistent matching
between IP-Glasma initial conditions and hydrodynamics!

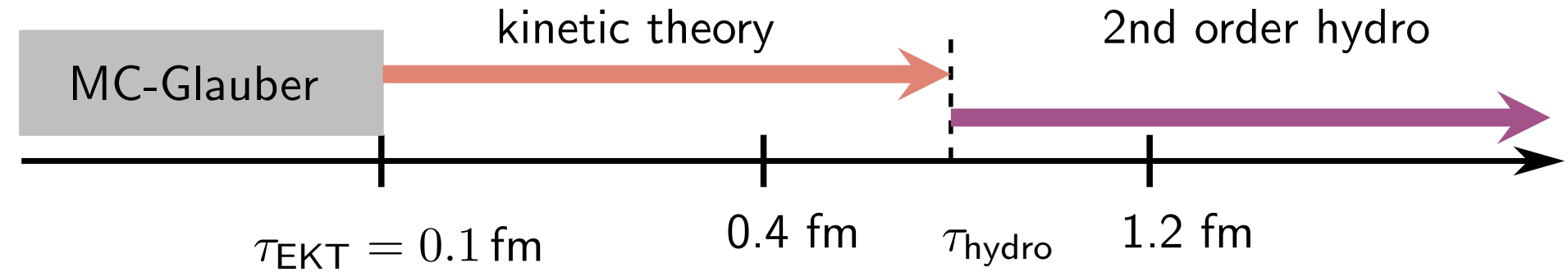
Outlook

- publish kinetic pre-equilibrium response code `KoMPoST`

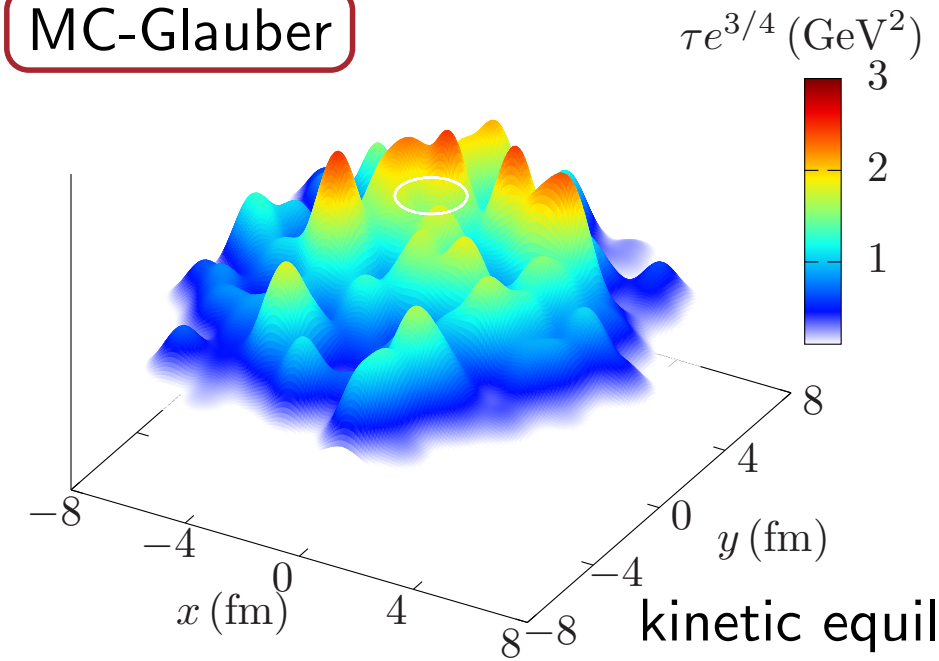
  Kurkela, Mazeliauskas, Schlichting and Teaney, to appear soon
- study chemical equilibration (i.e. add quarks)
- out-of-equilibrium photon production (c.f. Berges et al. (2017))
- study jet "thermalization", i.e. quenching, in the same setup

Backup

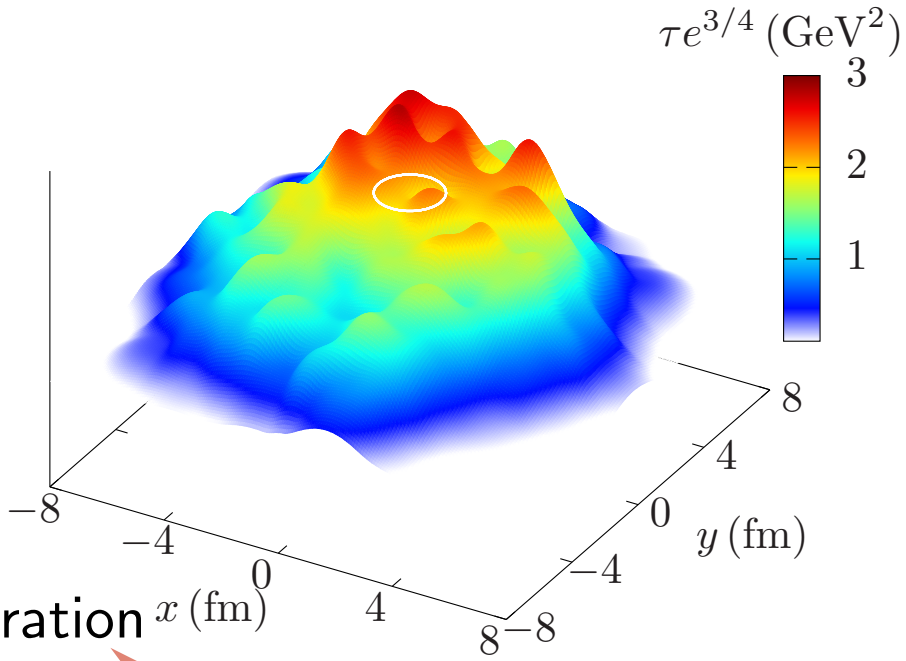
Evolution of transverse energy perturbations



MC-Glauber

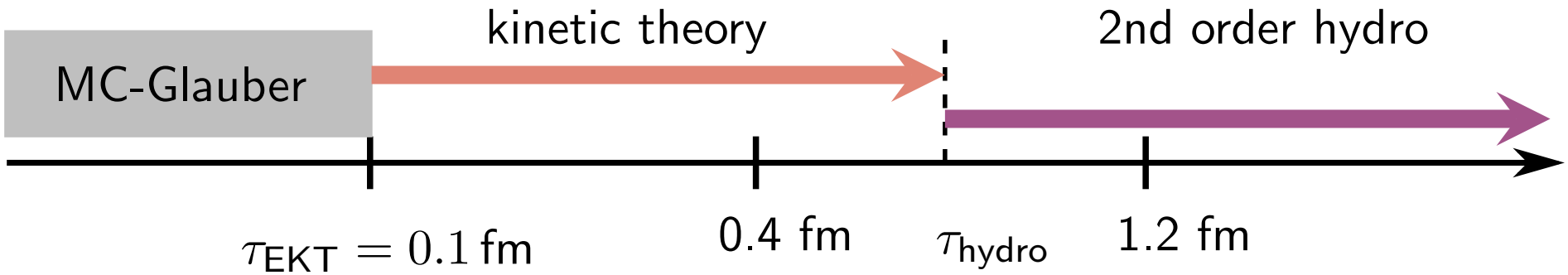


kinetic equilibration

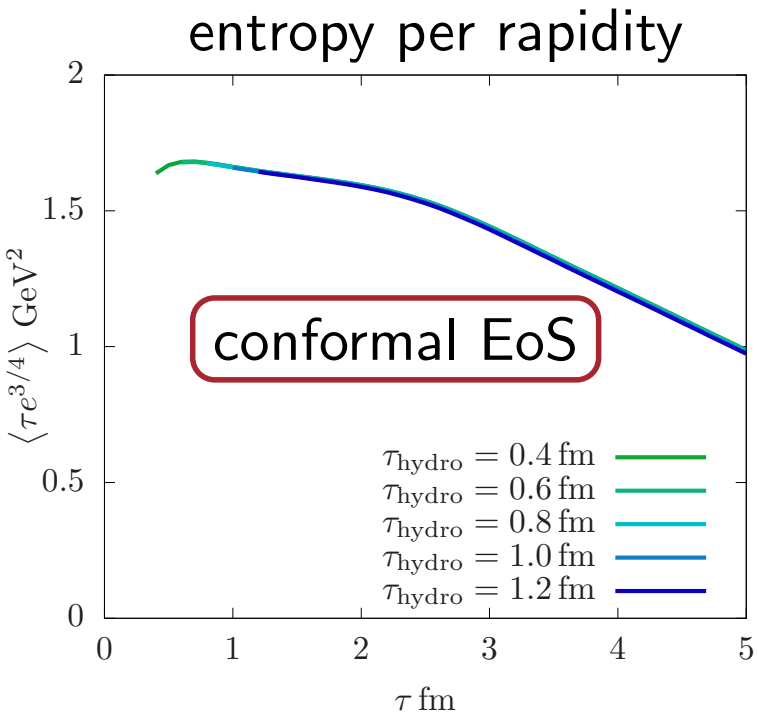


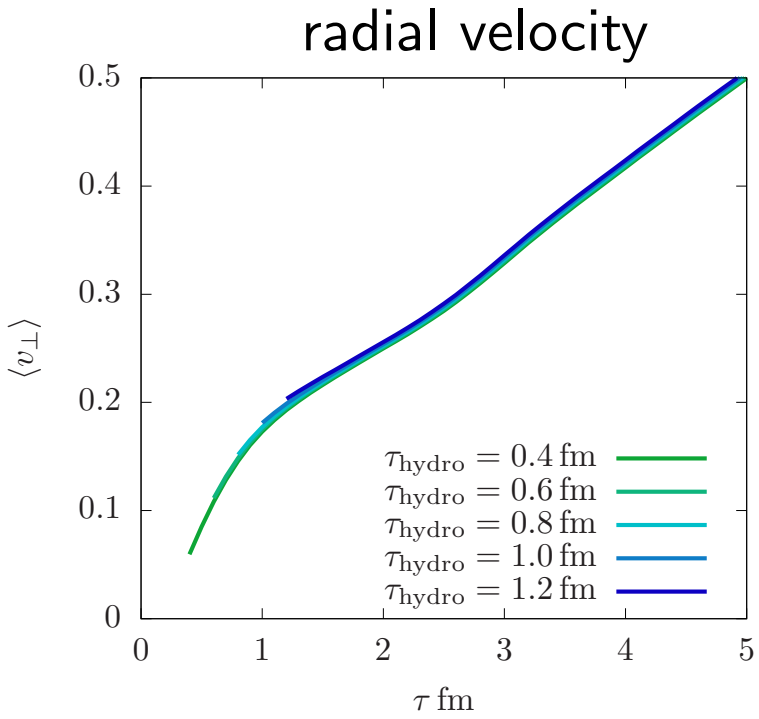
$$e(\boldsymbol{x}, \tau_{\mathsf{EKT}}) \implies \underbrace{e(\tau) + \delta e(\tau, \boldsymbol{x})}_{\text{linearization in causal circle}} \implies \underbrace{T^{\mu\nu}(\boldsymbol{x}, \tau_{\mathsf{hydro}})}_{\text{full energy momentum tensor}}$$

Transversly averaged density and flow

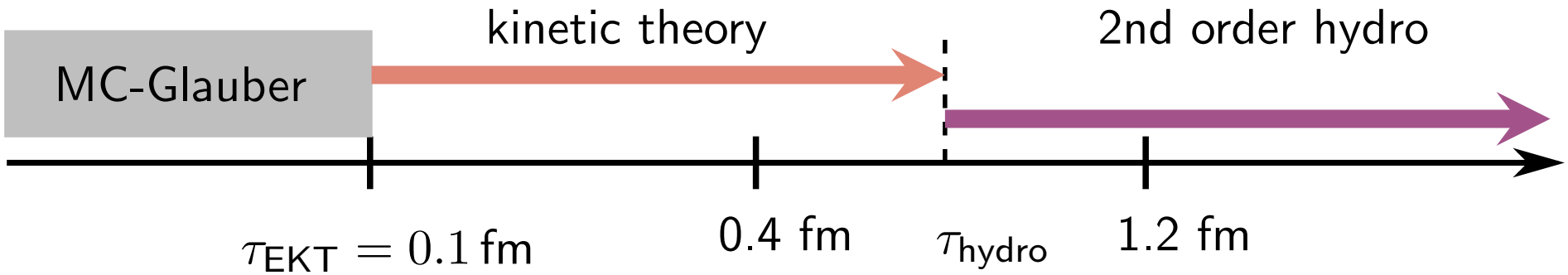


transverse averages $\langle \ldots \rangle = \dfrac{\int d^2\boldsymbol{x}_\perp u^\tau e \ldots}{\int d^2\boldsymbol{x}_\perp u^\tau e}$

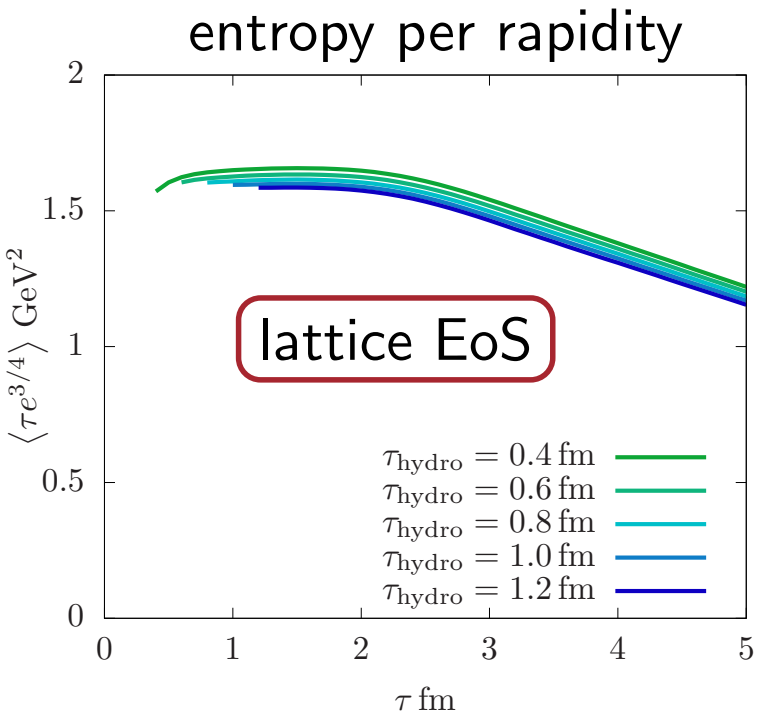


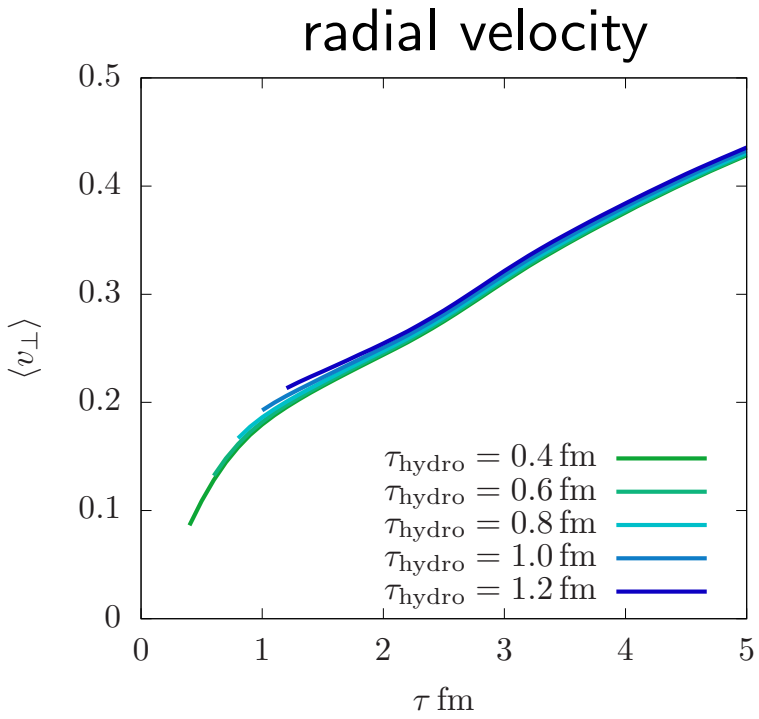


Transversly averaged density and flow

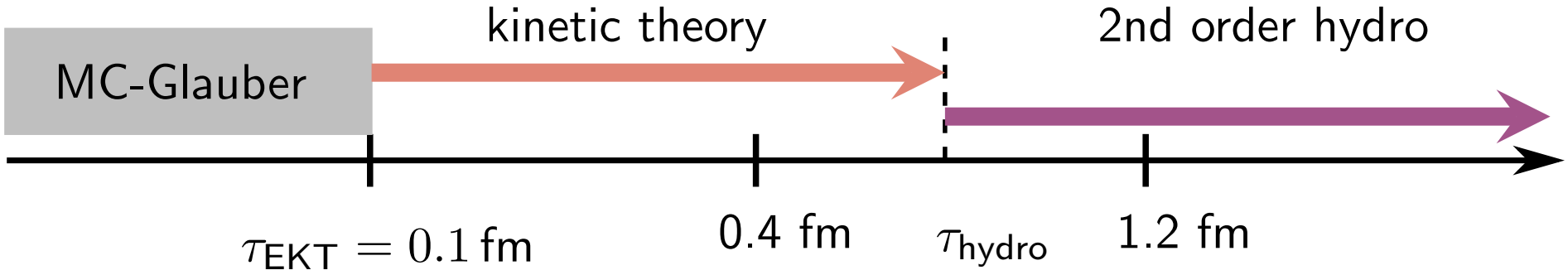


transverse averages $\langle \ldots \rangle = \dfrac{\int d^2\boldsymbol{x}_\perp u^\tau e \ldots}{\int d^2\boldsymbol{x}_\perp u^\tau e}$

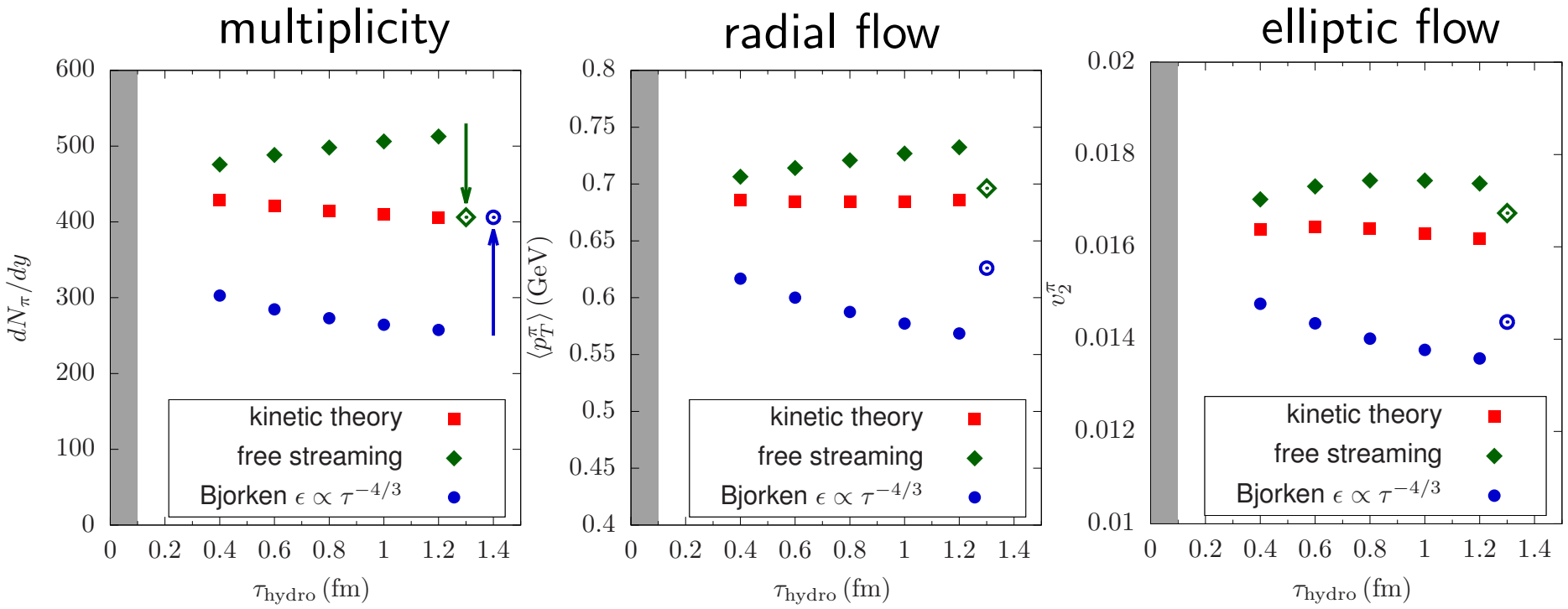




Hadronic observables: MC-Glauber

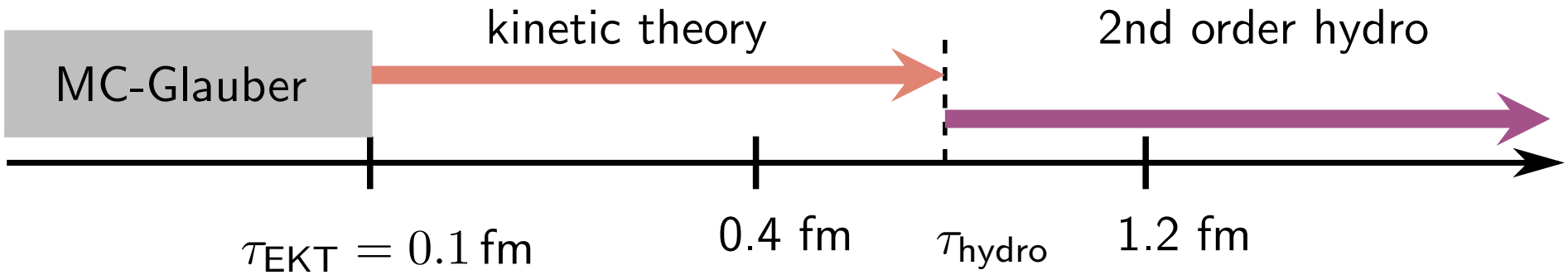


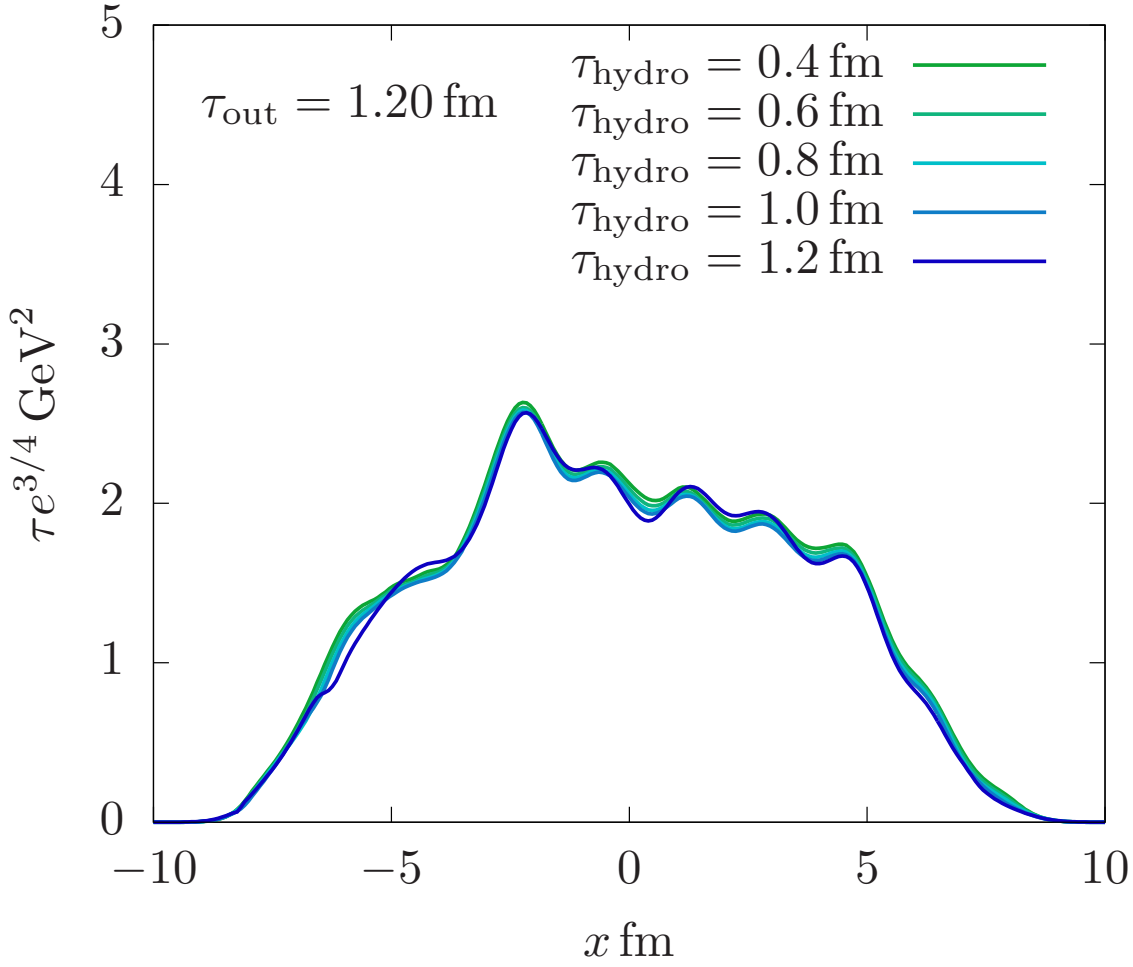
Thermal pions at freeze-out $T_{\text{fo}} = 145\,\text{MeV}$



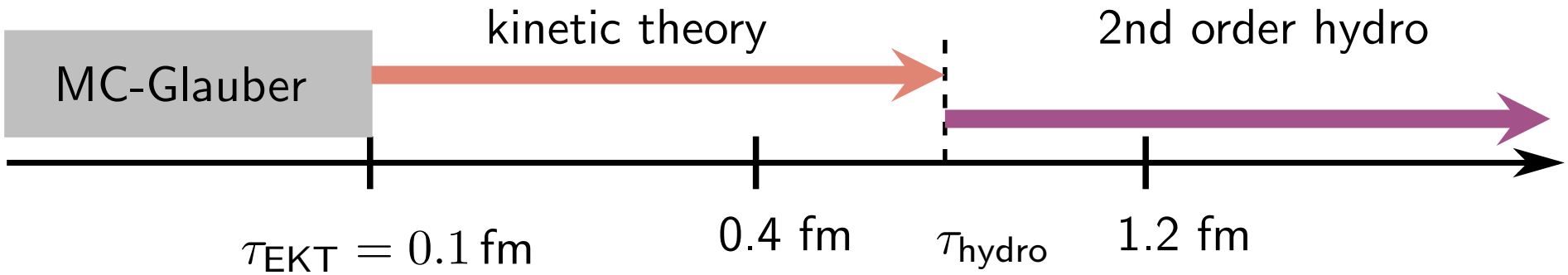
Approximate independence of τ_{hydro} *for kinetic pre-equilibrium evolution!*

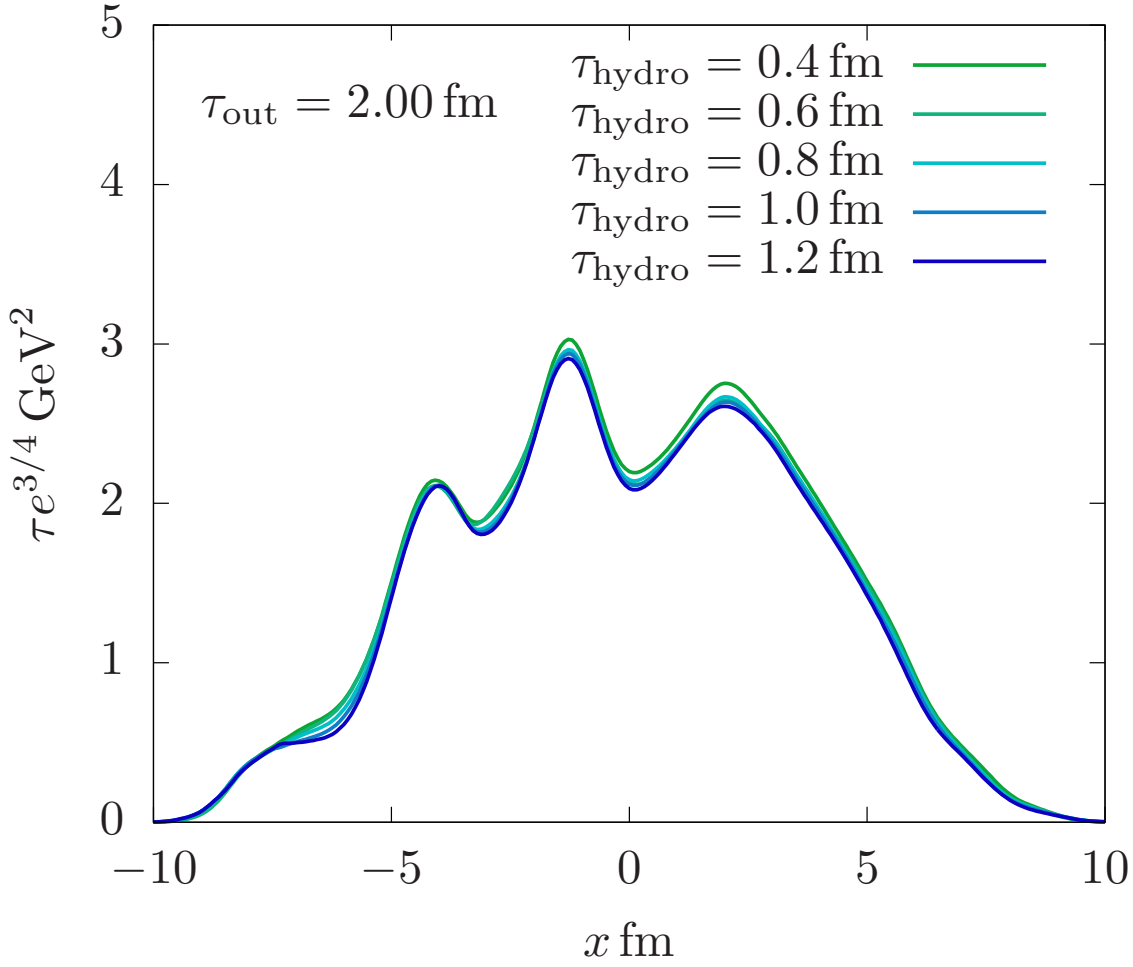
Entropy profile evolution for MC-Glauber



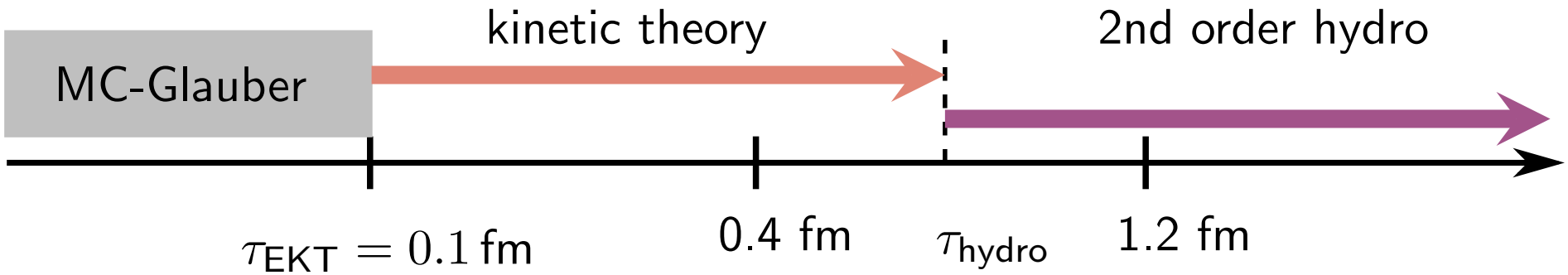


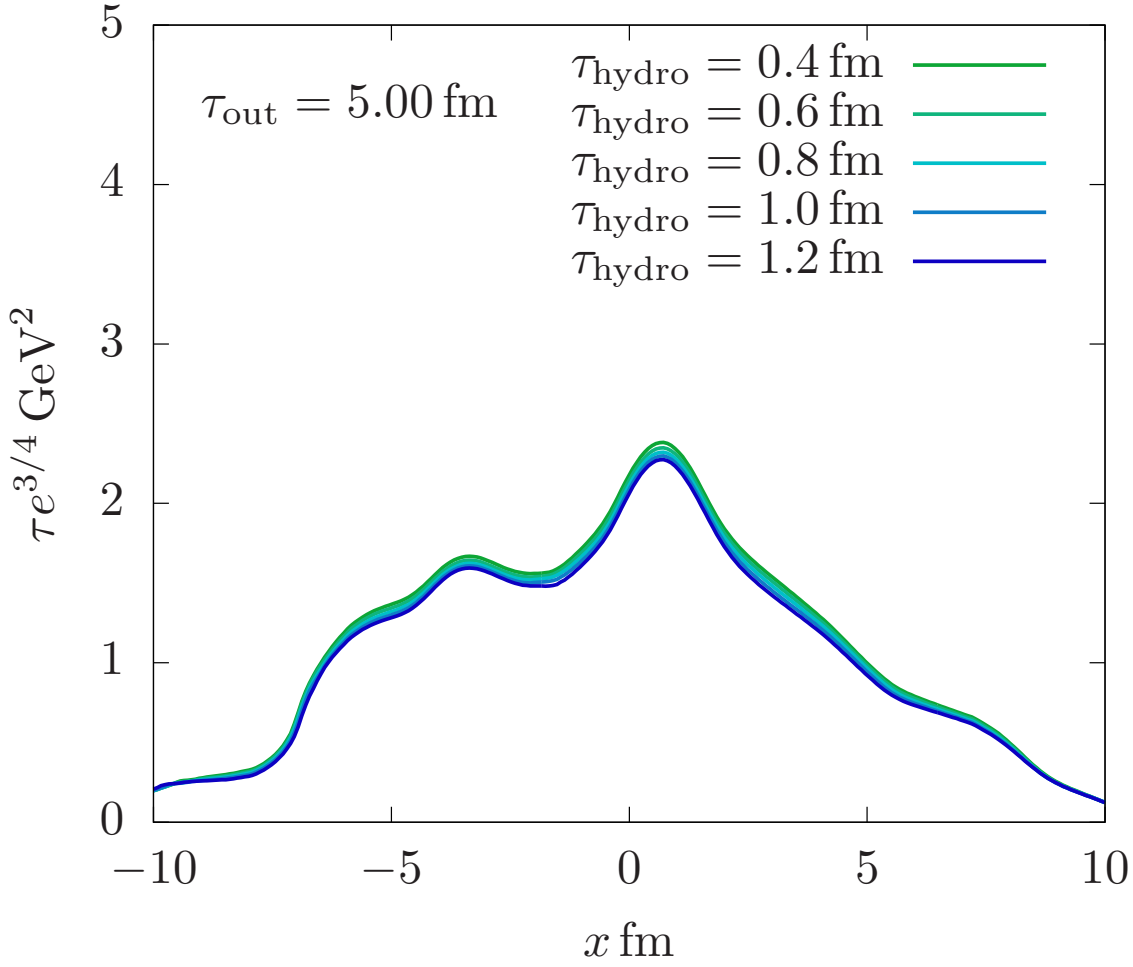
Entropy profile evolution for MC-Glauber



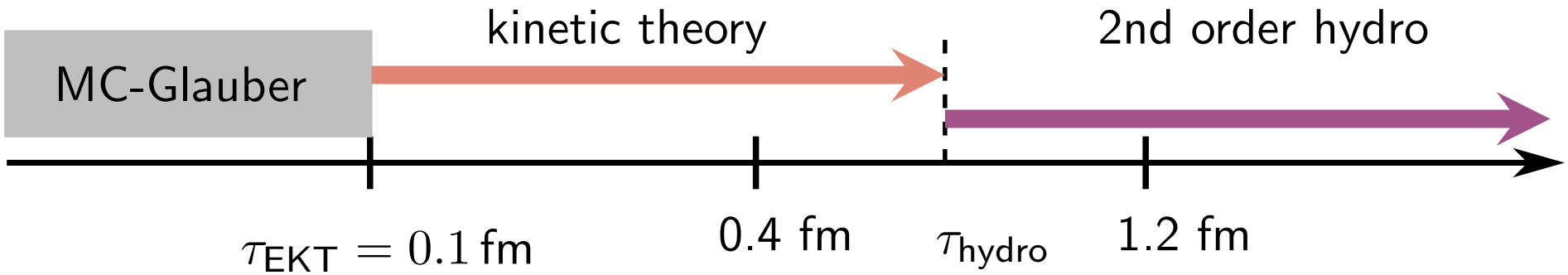


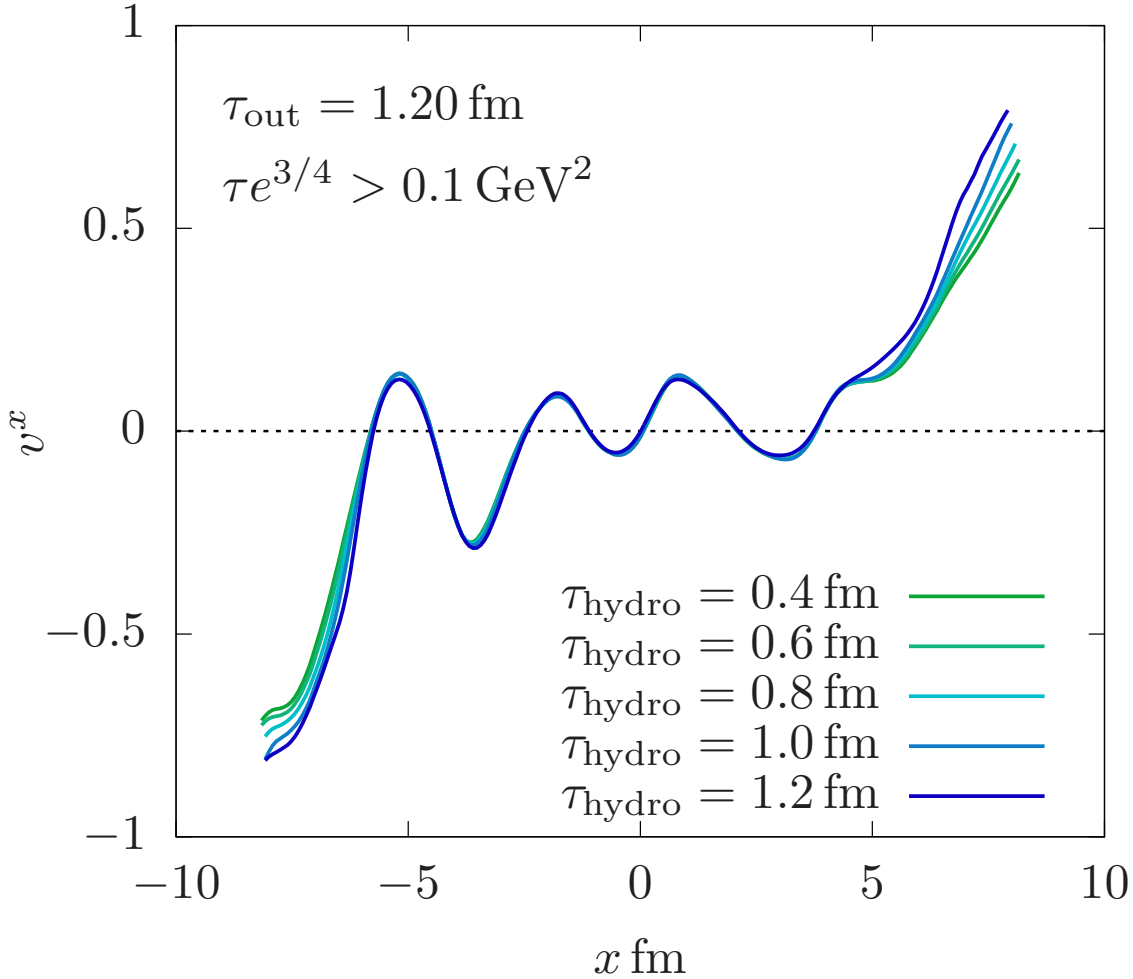
Entropy profile evolution for MC-Glauber



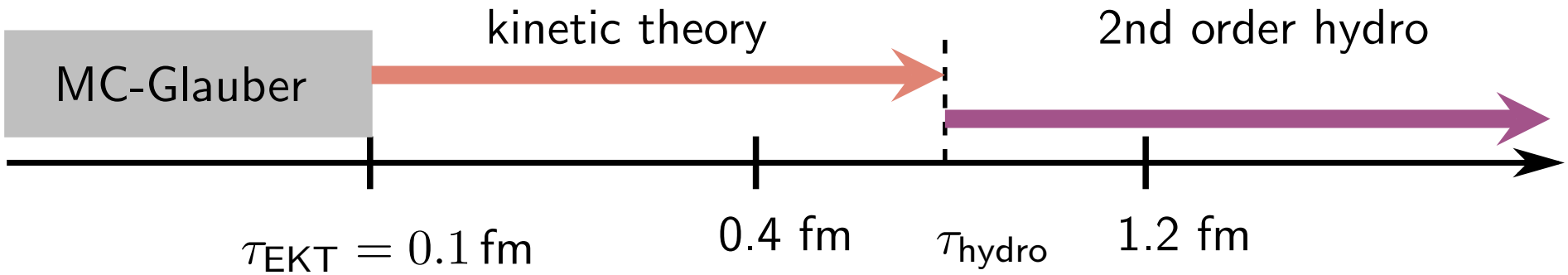


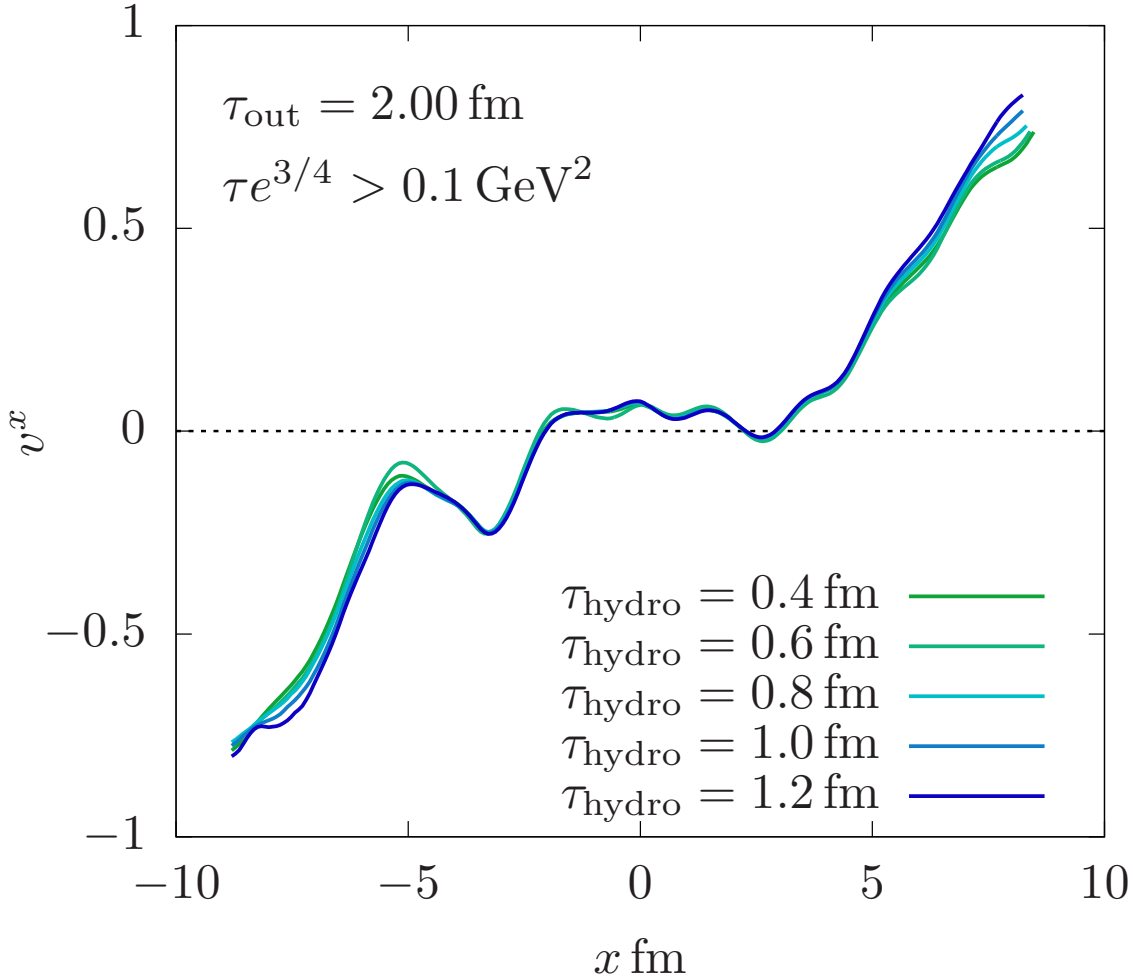
Transverse velocity evolution for MC-Glauber



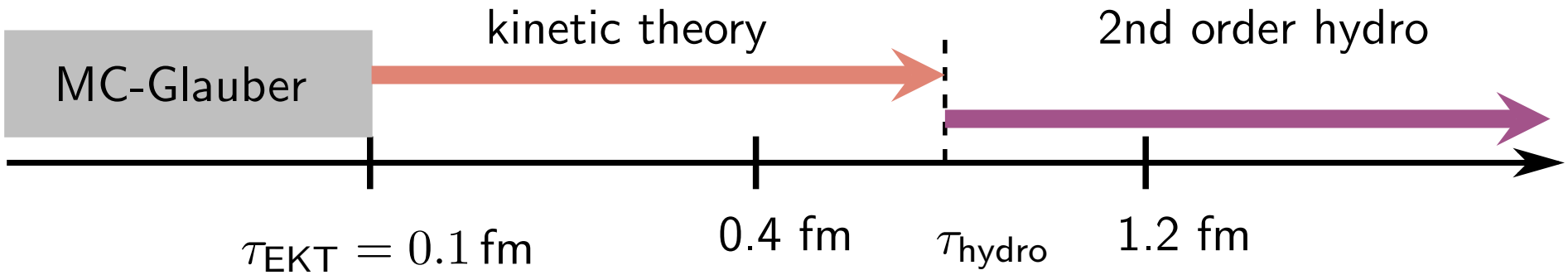


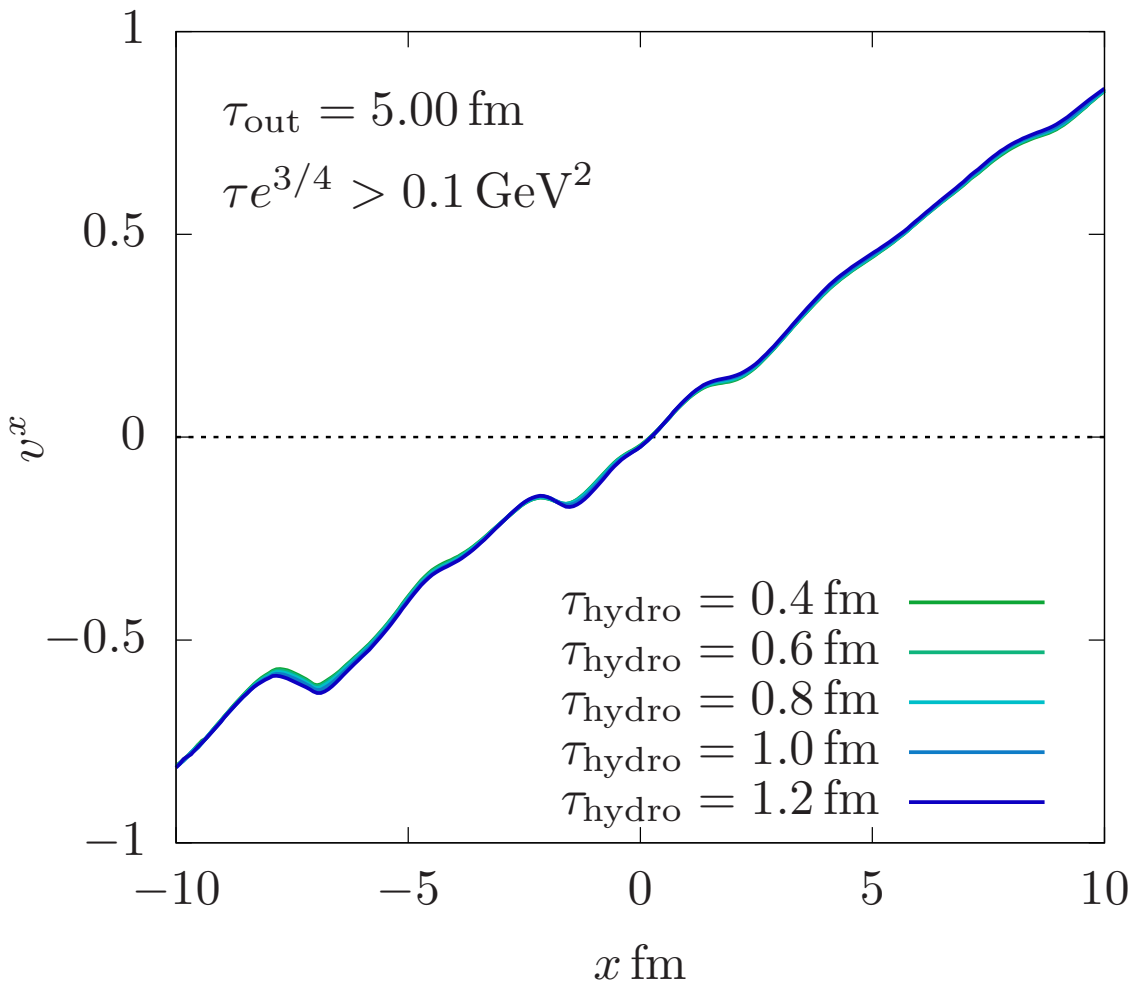
Transverse velocity evolution for MC-Glauber



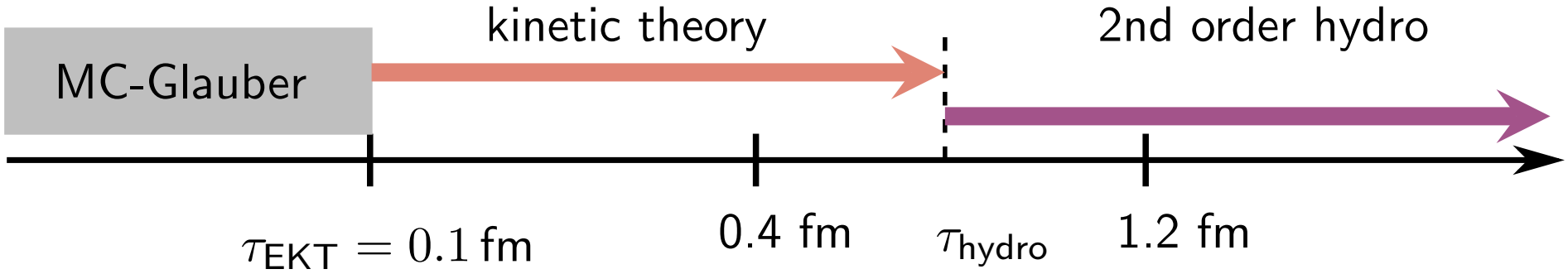


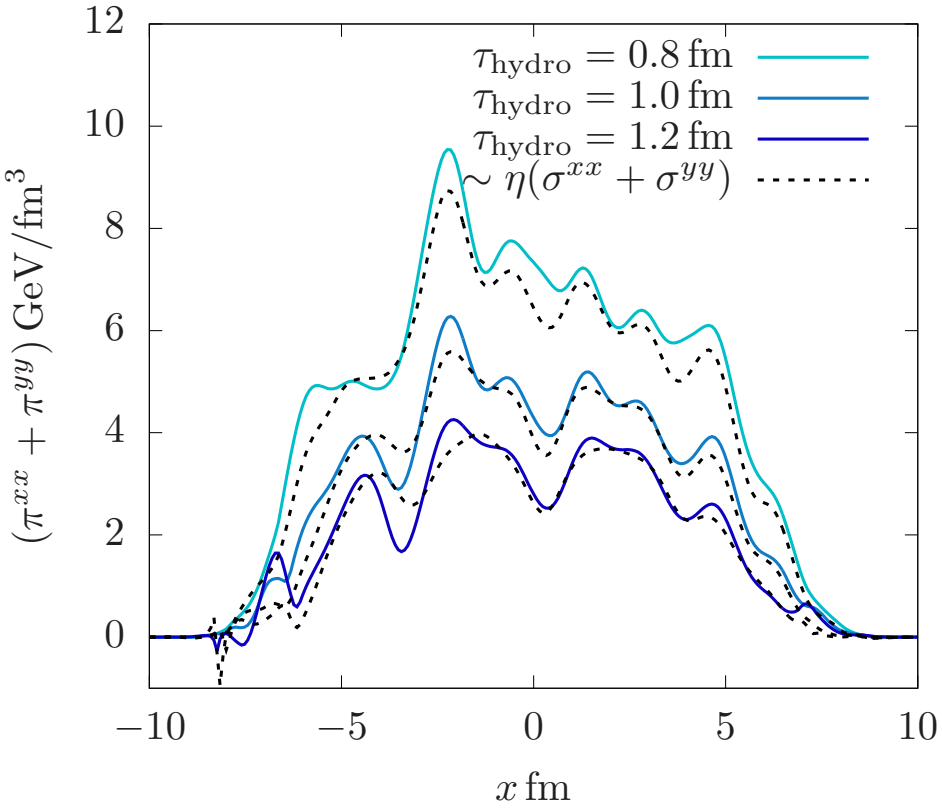
Transverse velocity evolution for MC-Glauber

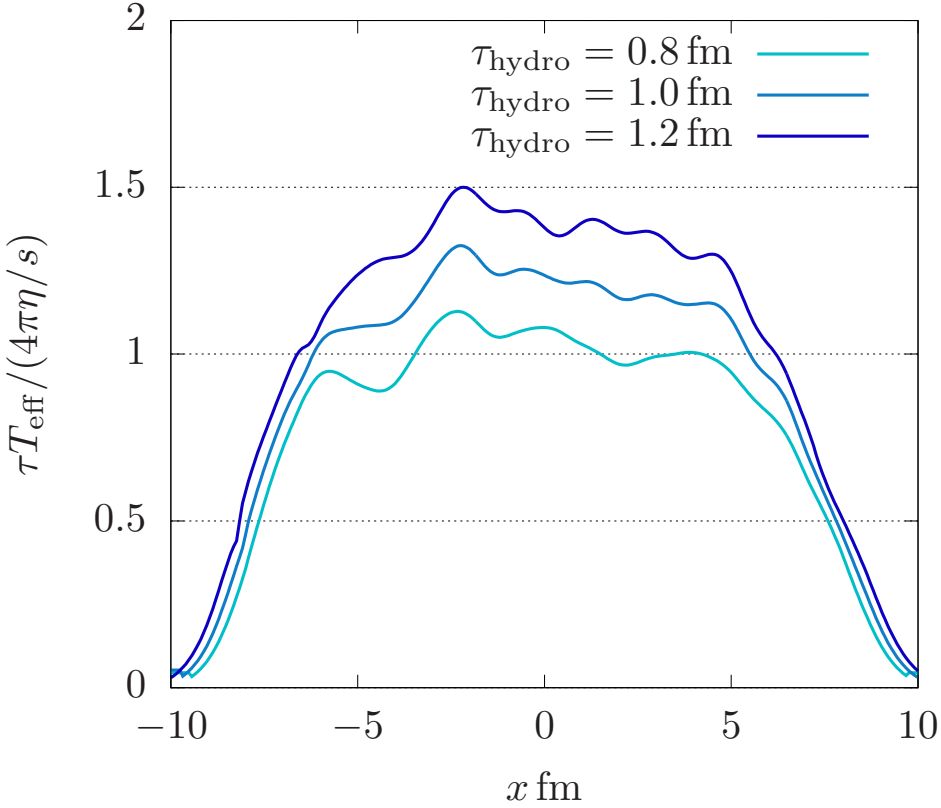




Shear-stress profile evolution for MC-Glauber







Parametrization of distribution function

"Bottom-up" thermalization scenario — Baier, Mueller, Schiff, and Son (2001)[5]

Universal-attractor — Berges, Boguslavski, Schlichting, Venugopalan (2014)[3]

Numerical realization — Kurkela and Zhu (2015)[4]

Initial anisotropic gluon distribution function

$$f_{\mathsf{BG}}(p_\perp, p_z) = \frac{A}{\lambda}\frac{p_0}{\tilde{p}} e^{-\frac{2}{3}\frac{\tilde{p}^2}{p_0^2}}, \quad \tilde{p} = \sqrt{p_\perp^2 + \xi^2 p_z^2}$$

with $\xi = 10$ and A and $p_0 = 1.8 Q_s$ adjusted to match lattice results [4, 7].

c.f. scaling solution for scalar theories [8]

$$f(p_\perp, p_z) = \frac{A}{\lambda}\frac{p}{p_T} e^{-\frac{p_z^2}{2\sigma^2}}$$

Scalar and vector perturbations for background

$f_{\mathsf{BG}}(p_\perp, p_z) \approx f_{\mathsf{BG}}(|\mathbf{p}_\perp|/Q_s)$

$$\delta f_{\boldsymbol{k}}^{(s)} = \frac{\delta Q_s(\boldsymbol{k})}{Q_s}\partial_{Q_s} f_{\mathsf{BG}}\left(\frac{|\mathbf{p}_\perp|}{Q_s}\right), \quad \delta f_{\boldsymbol{k}}^{(v),i} = \partial_{v_i} f_{\mathsf{BG}}\left(\frac{|\mathbf{p}_\perp - \mathbf{v}\mathbf{p}_\perp|}{Q_s}\right)\Bigg|_{\mathbf{v}=0}$$

Bibliography I

[1] Liam Keegan, Aleksi Kurkela, Aleksas Mazeliauskas, and Derek Teaney.
Initial conditions for hydrodynamics from weakly coupled pre-equilibrium evolution.
JHEP, 08:171, 2016, 1605.04287.

[2] Peter Brockway Arnold, Guy D. Moore, and Laurence G. Yaffe.
Effective kinetic theory for high temperature gauge theories.
JHEP, 01:030, 2003, hep-ph/0209353.

[3] Juergen Berges, Kirill Boguslavski, Soeren Schlichting, and Raju Venugopalan.
Universal attractor in a highly occupied non-Abelian plasma.
Phys. Rev., D89(11):114007, 2014, 1311.3005.

[4] Aleksi Kurkela and Yan Zhu.
Isotropization and hydrodynamization in weakly coupled heavy-ion collisions.
Phys. Rev. Lett., 115(18):182301, 2015, 1506.06647.

[5] R. Baier, Alfred H. Mueller, D. Schiff, and D. T. Son.
'Bottom up' thermalization in heavy ion collisions.
Phys. Lett., B502:51–58, 2001, hep-ph/0009237.

[6] Bjoern Schenke, Prithwish Tribedy, and Raju Venugopalan.
Fluctuating Glasma initial conditions and flow in heavy ion collisions.
Phys. Rev. Lett., 108:252301, 2012, 1202.6646.

Bibliography II

[7] T. Lappi.
Gluon spectrum in the glasma from JIMWLK evolution.
Phys. Lett., B703:325–330, 2011, 1105.5511.

[8] J. Berges, K. Boguslavski, S. Schlichting, and R. Venugopalan.
Nonequilibrium fixed points in longitudinally expanding scalar theories: infrared cascade, Bose condensation and a challenge for kinetic theory.
Phys. Rev., D92(9):096006, 2015, 1508.03073.