Initial conditions for heavy ion collisions

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Initial conditions with QCD kinetic theory

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see also Teaney 18/09 IS’17
Ultra-relativistic heavy ion collisions in a nutshell

Boundary between initial stages and medium physics $\tau_{\text{hydro}}$ is artificial.

- when does jet quenching begin?
- when does QGP start emitting photons?

Our goal: consistent theoretical overlap of initial stages and the onset of hydrodynamics.
Initial stages in weak coupling picture

At high collision energies and densities $\alpha_s \ll 1$

QCD kinetic theory interpolates between strong fields and hydrodynamics

18/09 IS’17
Gelis
Boguslavski
Wu 20/19

hydro

$\alpha_s \ll 1$

$1 \ll f \ll \frac{1}{\alpha_s}$

$f \sim 1$

$\sim \frac{1}{\alpha_s} \gg 1$
Initial stages in weak coupling picture

At high collision energies and densities $\alpha_s \ll 1$

QCD kinetic theory interpolates between strong fields and hydrodynamics
Thermalization in QCD effective kinetic theory

QCD effective kinetic theory  
Arnold, Moore, Yaffe (2003)[2]  

“Bottom-up” thermalization scenario  
Baier, Mueller, Schiff, and Son (2001)[5]  

Prescription: solve Boltzmann equation for distribution function $f(p)$

$$\partial_\tau f + \frac{p}{|p|} \cdot \nabla f - \frac{p_z}{\tau} \partial_{p_z} f = - \frac{C_2}{f} f - \frac{C_1}{f} f$$

- elastic $2 \leftrightarrow 2$ and inelastic $1 \leftrightarrow 2$ scatterings (with LPM suppression)

  *the same QCD processes as in jet quenching*

- at late times equilibrates to conformal hydrodynamics

- single parameter — the coupling constant $\lambda = 4\pi \alpha_s N_c$.

In practise need to extrapolate $\lambda$ to realistic values.

*High-$pT$ QCD kinetic theory $\Rightarrow$ parton energy loss*

*Low-$pT$ QCD kinetic theory $\Rightarrow$ thermalization*

“Jet quenching and fluid dynamics = two manifestations of the same physics” – Wiedemann

21/09 IS’17
From classical simulations to kinetic theory but anisotropies should create plasma instabilities.

Classical-statistical lattice simulations find the “bottom-up” attractor.

Numerical results for thermalization in kinetic theory

“Bottom-up” thermalization scenario

Numerical realization

\[ f(p_\perp, p_z) \]

\[ \langle p_{\perp}^2 \rangle \gg \langle p_z^2 \rangle \]

1. elastic 2 $\leftrightarrow$ 2 scatterings

Baier, Mueller, Schiff, and Son (2001)[5]
Kurkela and Zhu (2015)[4]

see also Teaney 18/09 IS’17
Numerical results for thermalization in kinetic theory

“Bottom-up” thermalization scenario

Numerical realization

\[ f(p_\perp, p_z) \]

2. soft $1 \leftrightarrow 2$ splitting

see also Teaney 18/09 IS’17
Numerical results for thermalization in kinetic theory

“Bottom-up” thermalization scenario
Numerical realization

$f(p_\perp, p_z)$

3. mini-jet quenching

see also Teaney 18/09 IS’17
Numerical results for thermalization in kinetic theory

“Bottom-up” thermalization scenario
Numerical realization

\[ f(p_{\perp}, p_z) \]

\[ \langle p_{\perp}^2 \rangle \approx \langle p_z^2 \rangle \]

4. isotropization

see also Teaney 18/09 IS’17
Scaling of pressure evolution for boost invariant system

Pressure isotropization in kinetic theory

$$\frac{\tau}{T_{Id.}} / (4\pi \eta / s)$$

$\lambda = 10$ ($\eta / s \approx 0.62$)  
$\lambda = 15$ ($\eta / s \approx 0.34$)  
$\lambda = 20$ ($\eta / s \approx 0.22$)  
$\lambda = 25$ ($\eta / s \approx 0.16$)

2nd hydro

onset of hydrodynamics

c.f. attractor solutions:
Strickland 18/09 IS'17
Romatschke 20/09 IS'17
Spalinski 21/09 IS'17

kinetic theory

Collapse to scaling solution for time

$$\frac{\tau}{T_{Id.}} / \eta / s \propto \frac{\tau}{\tau_R}$$ in units of relaxation time.
Out-of-equilibrium energy evolution for boost invariant system

scaling in hydro:

\[ e(\tau) = \nu g \frac{\pi^2}{30} T_{\text{Id.}}^4 \left( 1 - \frac{8}{3} \frac{\eta/s}{\tau T_{\text{Id.}}} \right) + C_2 \left( \frac{\eta/s}{\tau T_{\text{Id.}}} \right)^2 \]


\[(\nu g \frac{\pi^2}{30} T_{\text{Id.}}^4) \text{ “ideal” temp.} \]

\[(\frac{8}{3} \frac{\eta/s}{\tau T_{\text{Id.}}}) \text{ ideal} \]

\[(\frac{\eta/s}{\tau T_{\text{Id.}}}) \text{ viscous} \]

\[C_2 \left( \frac{\eta/s}{\tau T_{\text{Id.}}} \right)^2 \text{ 2nd order hydro} \]

\[e(\tau) \big/ \left( \nu g \frac{\pi^2}{30} T_{\text{Id.}}^4 \right) \]

\[\tau T_{\text{Id.}} \big/ (4\pi \eta/s) \]
Out-of-equilibrium energy evolution for boost invariant system

**Generalized scaling:**

\[ e(\tau) = \nu g \frac{\pi^2}{30} T_{Id}^4(\tau) \times \mathcal{E} \left[ x = \frac{\tau T_{Id}(\tau)}{\eta/s} \right] \]

- **Kinetic theory**
- **2nd order hydro**
- **Free streaming**
- **Ideal hydro**
- **Pre-equilibrium**
- **Onset of hydrodynamics**
Event by event pre-equilibrium dynamics
Realistic initial conditions for hydrodynamics

\[ \tau_{\text{hydro}} \sim 1 \text{ fm}/c \]

\[ \tau_{\text{fo}} \sim 10 \text{ fm}/c \]

Examples of initial stages:

A  MC-Glauber:
   ✓ e-by-e fluctuations, ✗ dynamics, ✗ flow, ✗ hydrodynamization

B  IP-Glasma: Schenke, Tribedy, Venugopalan (2013)[6]
   ✓ e-by-e fluct., ✓ 2+1D Yang-Mills, ✓ flow, ✗ hydrodynamization

   See also EKRT model (includes nPDFs), Niemi 19/09 IS’17

Use the out-of-equilibrium kinetic theory to map IP-Glasma initial conditions to hydrodynamics.
Evolution of perturbations with kinetic theory

IP-Glasma initial conditions at $\tau_{EKT} = 0.2$ fm

$\tau_{EKT} = 0.2$ fm

$\tau_{EKT} = 0.2$ fm

$\tau_{hydro} = 1.2$ fm

$\tau_{hydro} = 1.2$ fm

$e(x, \tau_{EKT}), \vec{v}(x, \tau_{EKT}) \implies f(\tau, p, x) \implies e(x, \tau_{hydro}), \vec{v}(x, \tau_{hydro})$
Plane wave perturbations in transverse plane

Simplify 5+1D problem for \( f(\tau, p, x_\perp) \) by linearization

\[
f(\tau, p, x_\perp) = \underbrace{\bar{f}_p}_{\text{uniform background}} + \underbrace{\delta f_{k_\perp, p} e^{i k_\perp \cdot x_\perp}}_{\text{transverse perturbations}}.
\]

Linearized Boltzmann equation for perturbations

\[
\left( \partial_\tau - \frac{p_z}{\tau} \partial_{p_z} \right) \bar{f}_p = -C[\bar{f}]
\]

background

\[
\left( \partial_\tau - \frac{p_z}{\tau} \partial_{p_z} + \frac{i p_\perp \cdot k_\perp}{p} \right) \delta f_{k_\perp, p} = -\delta C[\bar{f}, \delta f]
\]

\( k_\perp \) perturbation

Construct kinetic response functions to energy/momentum perturbations

\[
\frac{\delta T^{\mu\nu}(\tau, k)}{e(\tau)} = \tilde{G}^{\mu\nu}_{(\tau, i)}(k, \tau, \tau_0) \times \begin{bmatrix} \frac{\delta T^{\tau\tau}(\tau_0, k)}{e(\tau_0)} \text{ or } \frac{\delta T^{\tau i}(\tau_0, k)}{e(\tau_0)} \end{bmatrix}
\]

output for hydro non-equilibrium kinetic response initial perturbations
Scaling of kinetic theory response functions

- Response to perturbations also collapse to scaling solutions
  \[ \tilde{G}^{\mu\nu}(|k|, \tau, \tau_0, e(\tau_0), \lambda) = \tilde{G}^{\mu\nu,\text{univ}}\left(\frac{\tau T_{\text{Id.}}}{\eta/s}, |k|(\tau - \tau_0)\right) \]

- Agrees with hydrodynamic scaling in \( \tau \to \infty, |k|\tau \to 0 \) limit.
- Good scaling for a range of \( \eta/s = 0.16-0.62 \) values.
Kinetic evolution with IP-Glasma initial conditions
Equilibration with IP-Glasma initial conditions

$\tau_{\text{EKT}} = 0.2 \, \text{fm}$

$0.4 \, \text{fm}$

$\tau_{\text{hydro}} = 1.2 \, \text{fm}$

**IP-Glasma**

**kinetic theory**

**2nd order hydro**

$\tau e^{3/4} (\text{GeV}^2)$

$k e^{3/4} (\text{GeV}^2)$

$x (\text{fm})$

$y (\text{fm})$

$\tau e^{3/4} (\text{GeV}^2)$

$z (\text{fm})$

**kinetic equilibration**

$e(x, \tau_{\text{EKT}}), \vec{v}(x, \tau_{\text{EKT}}) \rightarrow e(\tau) + \delta e(\tau, x), \vec{v}(\tau, x) \rightarrow T^{\mu\nu}(x, \tau_{\text{hydro}})$

**non-zero initial flow**

**full eng.-mom. tensor**
Average pressure in the transverse plane

$\tau_{\text{EKT}} = 0.2 \text{ fm}$

$0.4 \text{ fm}$

$\tau_{\text{hydro}} = 1.2 \text{ fm}$

Not a cartoon!

Overlapping and consistent pressure evolution between initial stages!
Transverse averaged density and flow

\( \tau_{\text{EKT}} = 0.2 \text{ fm} \quad 0.4 \text{ fm} \quad \tau_{\text{hydro}} = 1.2 \text{ fm} \)

kinetic theory

2nd order hydro

transverse averages

\[ \langle \ldots \rangle = \frac{\int d^2 x_\perp u^\tau e \ldots}{\int d^2 x_\perp u^\tau e} \]

entropy per rapidity

radial velocity
Hadronic observables: IP-Glasma

\[ \tau_{\text{EKT}} = 0.2 \text{ fm} \quad 0.4 \text{ fm} \quad \tau_{\text{hydro}} = 1.2 \text{ fm} \]

Thermal pions at freeze-out \( T_{\text{fo}} = 145 \text{ MeV} \)

**Multiplicity**

**Radial flow**

**Elliptic flow**

*Approximate independence of \( \tau_{\text{hydro}} \) for kinetic pre-equilibrium evolution!*
Summary

“Bottom-up” equilibration for realistic heavy ion initial conditions:
- Background evolution described by a scaling solution in $\tau T_{\text{Id.}}/(\eta/s)$
- Found linear kinetic response functions $G^{\mu\nu}(\tau T_{\text{Id.}}/(\eta/s), r/(\tau - \tau_0))$
- Demonstrated small dependence on crossover time $\tau_{\text{hydro}}$

*Smooth and consistent matching between IP-Glasma initial conditions and hydrodynamics!*

Outlook

- publish kinetic pre-equilibrium response code KoMPoST
  Kurkela, Mazeliauskas, Schlichting and Teaney, to appear soon
- study chemical equilibration (i.e. add quarks)
- out-of-equilibrium photon production (c.f. Berges et al. (2017))
- study jet “thermalization”, i.e. quenching, in the same setup
Backup
Evolution of transverse energy perturbations

\[ \tau_{EKT} = 0.1 \text{ fm} \quad 0.4 \text{ fm} \quad \tau_{\text{hydro}} = 1.2 \text{ fm} \]

**MC-Glauber**

\[ \tau e^{3/4} (\text{GeV}^2) \]

**kinetic theory**

**2nd order hydro**

**MC-Glauber kinetic equilibration**

\[ e(x, \tau_{EKT}) \quad \Rightarrow \quad e(\tau) + \delta e(\tau, x) \quad \Rightarrow \quad T^{\mu\nu}(x, \tau_{\text{hydro}}) \]

linearization in causal circle

full energy momentum tensor
Transversely averaged density and flow

\[ \tau_{\text{EKT}} = 0.1 \text{ fm} \quad 0.4 \text{ fm} \quad \tau_{\text{hydro}} = 1.2 \text{ fm} \]

transverse averages

\[ \langle ... \rangle = \frac{\int d^2 \mathbf{x}_\perp u^\tau e \ldots}{\int d^2 \mathbf{x}_\perp u^\tau e} \]

entropy per rapidity

radial velocity

conformal EoS
Transversely averaged density and flow

\[ \tau_{\text{EKT}} = 0.1 \text{ fm} \]

\[ \tau_{\text{hydro}} = 0.4 \text{ fm} \]

\[ \tau_{\text{hydro}} = 1.2 \text{ fm} \]

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transverse averages
\langle \ldots \rangle = \frac{\int d^2 \mathbf{x}_\perp u^\tau e \ldots}{\int d^2 \mathbf{x}_\perp u^\tau e}
```

entropy per rapidity

radial velocity

**lattice EoS**
Hadronic observables: MC-Glauber

\( \tau_{\text{EKT}} = 0.1 \text{ fm} \)

\( 0.4 \text{ fm} \)

\( \tau_{\text{hydro}} = 1.2 \text{ fm} \)

Thermal pions at freeze-out \( T_{\text{fo}} = 145 \text{ MeV} \)

Approximate independence of \( \tau_{\text{hydro}} \) for kinetic pre-equilibrium evolution!
Entropy profile evolution for MC-Glauber

\[ \tau_{EKT} = 0.1 \text{ fm} \quad 0.4 \text{ fm} \quad \tau_{\text{hydro}} \quad 1.2 \text{ fm} \]

\[ \tau_{\text{out}} = 1.20 \text{ fm} \]

\[ \tau_{\text{hydro}} = 0.4 \text{ fm} \]
\[ \tau_{\text{hydro}} = 0.6 \text{ fm} \]
\[ \tau_{\text{hydro}} = 0.8 \text{ fm} \]
\[ \tau_{\text{hydro}} = 1.0 \text{ fm} \]
\[ \tau_{\text{hydro}} = 1.2 \text{ fm} \]
Entropy profile evolution for MC-Glauber

\[ \tau_{EKT} = 0.1 \text{ fm} \quad 0.4 \text{ fm} \quad \tau_{\text{hydro}} = 1.2 \text{ fm} \]

\[ \tau_{\text{hydro}} = 0.4 \text{ fm} \quad \tau_{\text{hydro}} = 0.6 \text{ fm} \quad \tau_{\text{hydro}} = 0.8 \text{ fm} \quad \tau_{\text{hydro}} = 1.0 \text{ fm} \quad \tau_{\text{hydro}} = 1.2 \text{ fm} \]

\[ \tau_{\text{out}} = 2.00 \text{ fm} \]
Entropy profile evolution for MC-Glauber

\[ \tau_{EKT} = 0.1 \text{ fm} \quad 0.4 \text{ fm} \quad \tau_{\text{hydro}} \quad 1.2 \text{ fm} \]

\[ \tau_{\text{out}} = 5.00 \text{ fm} \]

\[ \tau_{e^{3/4} \text{ GeV}^2} \]

\( x \text{ fm} \)
Transverse velocity evolution for MC-Glauber

\[ \tau_{\text{EKT}} = 0.1 \text{ fm} \]

[0.4 fm]  \[ \tau_{\text{hydro}} \]

[1.2 fm]

\[ \tau_{\text{out}} = 1.20 \text{ fm} \]

\[ \tau_{e^{3/4}} > 0.1 \text{ GeV}^2 \]

\[ \tau_{\text{hydro}} = 0.4 \text{ fm} \]

\[ \tau_{\text{hydro}} = 0.6 \text{ fm} \]

\[ \tau_{\text{hydro}} = 0.8 \text{ fm} \]

\[ \tau_{\text{hydro}} = 1.0 \text{ fm} \]

\[ \tau_{\text{hydro}} = 1.2 \text{ fm} \]
Transverse velocity evolution for MC-Glauber

MC-Glauber

\[ \tau_{\text{EKT}} = 0.1 \text{ fm} \]

kinetic theory

2nd order hydro

\[ 0.4 \text{ fm} \]

\[ \tau_{\text{hydro}} \]

\[ 1.2 \text{ fm} \]

\[ \tau_{\text{out}} = 2.00 \text{ fm} \]

\[ \tau e^{3/4} > 0.1 \text{ GeV}^2 \]
Transverse velocity evolution for MC-Glauber

\[ \tau_{\text{EKT}} = 0.1 \text{ fm} \quad 0.4 \text{ fm} \quad \tau_{\text{hydro}} \quad 1.2 \text{ fm} \]

\[ \tau_{\text{out}} = 5.00 \text{ fm} \]

\[ \tau e^{3/4} > 0.1 \text{ GeV}^2 \]
Shear-stress profile evolution for MC-Glauber

$\tau_{EKT} = 0.1 \text{ fm}$

$0.4 \text{ fm}$

$\tau_{\text{hydro}} = 1.2 \text{ fm}$

$\tau_{\text{hydro}} = 0.8 \text{ fm}$

$\tau_{\text{hydro}} = 1.0 \text{ fm}$

$\tau_{\text{hydro}} = 1.2 \text{ fm}$

$\sim \eta(\sigma^{xx} + \sigma^{yy})$

$(\pi^{xx} + \pi^{yy}) \text{ GeV/fm}^3$

$x \text{ fm}$

$\tau_{T\text{eff}}/(4\pi\eta/s)$

$x \text{ fm}$
Parametrization of distribution function

“Bottom-up” thermalization scenario  
Baier, Mueller, Schiff, and Son (2001)[5]

Universal-attractor  
Berges, Boguslavski, Schlichting, Venugopalan (2014)[3]

Numerical realization  
Kurkela and Zhu (2015)[4]

Initial anisotropic gluon distribution function

\[
\begin{align*}
\mathcal{f}_{BG}(p_\perp, p_z) &= \frac{A}{\lambda} \frac{p_0}{\tilde{p}} e^{-\frac{2}{3} \frac{\tilde{p}^2}{p_0}}, \\
\tilde{p} &= \sqrt{p_\perp^2 + \xi^2 p_z^2}
\end{align*}
\]

with \( \xi = 10 \) and \( A \) and \( p_0 = 1.8Q_s \) adjusted to match lattice results [4, 7].

c.f. scaling solution for scalar theories [8]

\[
\begin{align*}
\mathcal{f}(p_\perp, p_z) &= \frac{A}{\lambda} \frac{p}{p_T} e^{-\frac{p_z^2}{2\sigma^2}}
\end{align*}
\]

Scalar and vector perturbations for background

\[
\delta f^{(s)}_k = \frac{\delta Q_s(k)}{Q_s} \partial_{Q_s} \mathcal{f}_{BG} \left( \frac{|p_\perp|}{Q_s} \right), \quad \delta f^{(v),i}_k = \partial_{v_i} \mathcal{f}_{BG} \left( \frac{|p_\perp - v_p p_\perp|}{Q_s} \right) \bigg|_{v=0}
\]


Gluon spectrum in the glasma from JIMWLK evolution.

Nonequilibrium fixed points in longitudinally expanding scalar theories: infrared cascade, Bose condensation and a challenge for kinetic theory.