12th International Workshop
on High Physics in RHIC/LHC era

Effects of hadronic rescattering in mini-jet production and energy loss at the LHC

Sangwook Ryu

In collaboration with
Scott McDonald, Chun Shen, Sangyong Jeon and Charles Gale

FIAS Frankfurt Institute for Advanced Studies

McGill

Introduction

- By colliding heavy nuclei (at RHIC and the LHC), we can
  ➤ create quark-gluon plasma (QGP) and
  ➤ study its properties (e.g. transport coefficients).

- This can be done by creating a realistic dynamical model of heavy ion collisions

- Different aspects of QGP and hadronic matter influence each other. e.g.,
  ➤ non-zero ζ/s alters the estimate of η/s
  ➤ jet quenching in hadronic phase changes determination of the jet-medium interaction in QGP

- Goal : hybrid model covering all these different aspects.

PART 1

Hybrid approach and
description of soft (low-p_T) physics

PART 2

Jet production and energy loss
for hard (high-p_T) physics

PART 1

Hybrid approach and
description of soft (low-p_T) physics

PART 2

Jet production and energy loss
for hard (high-p_T) physics

Hybrid approach

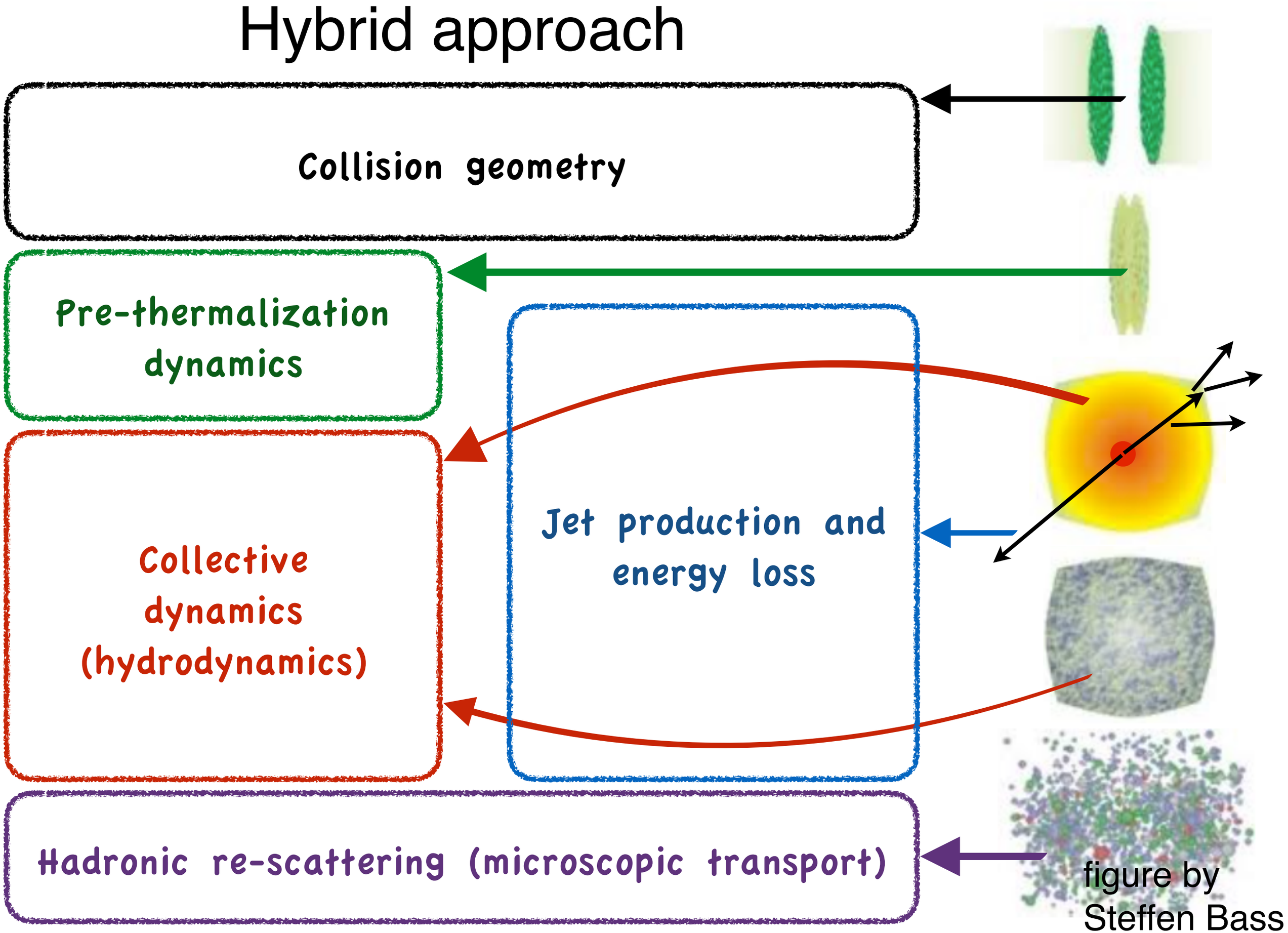
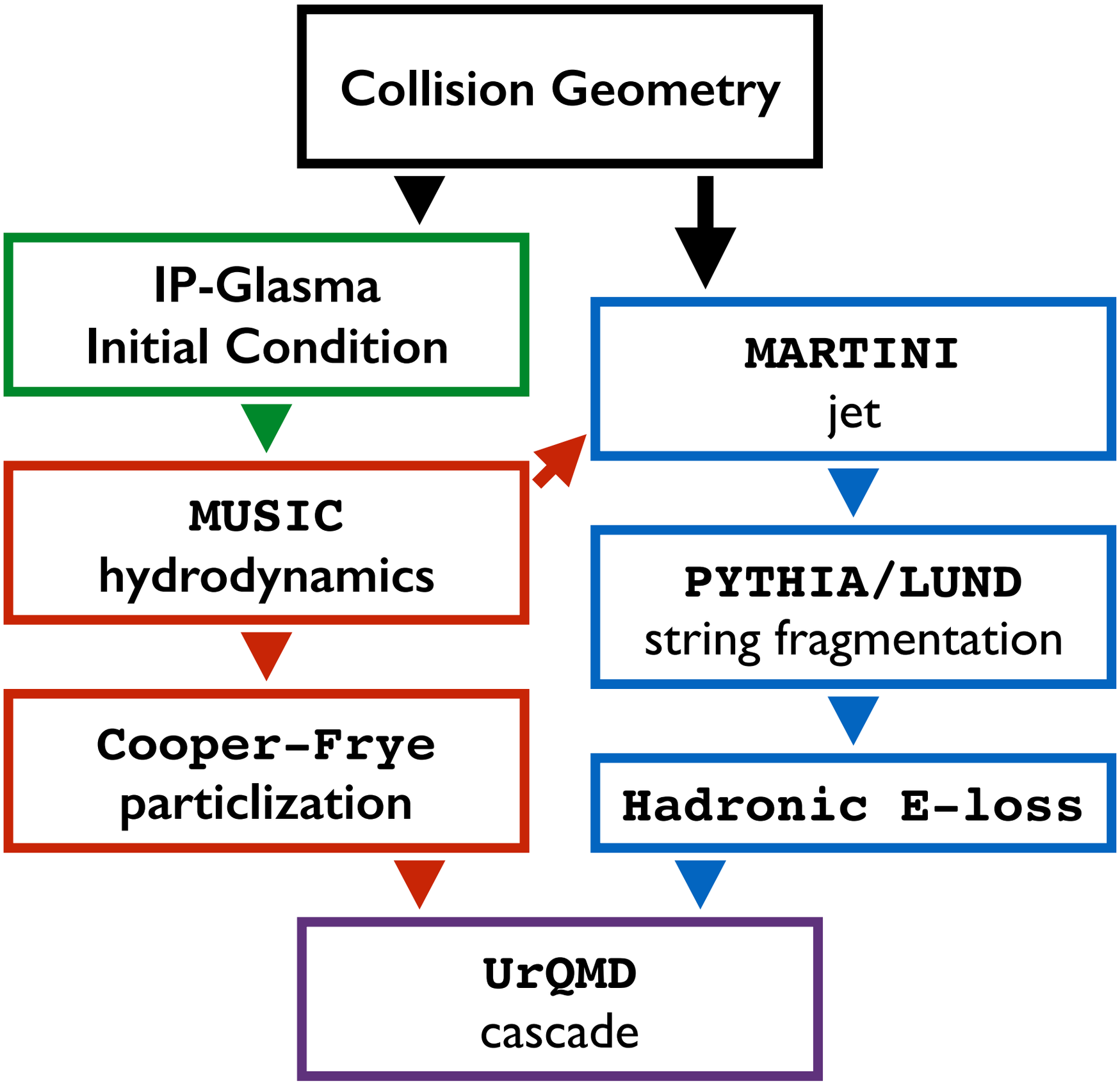
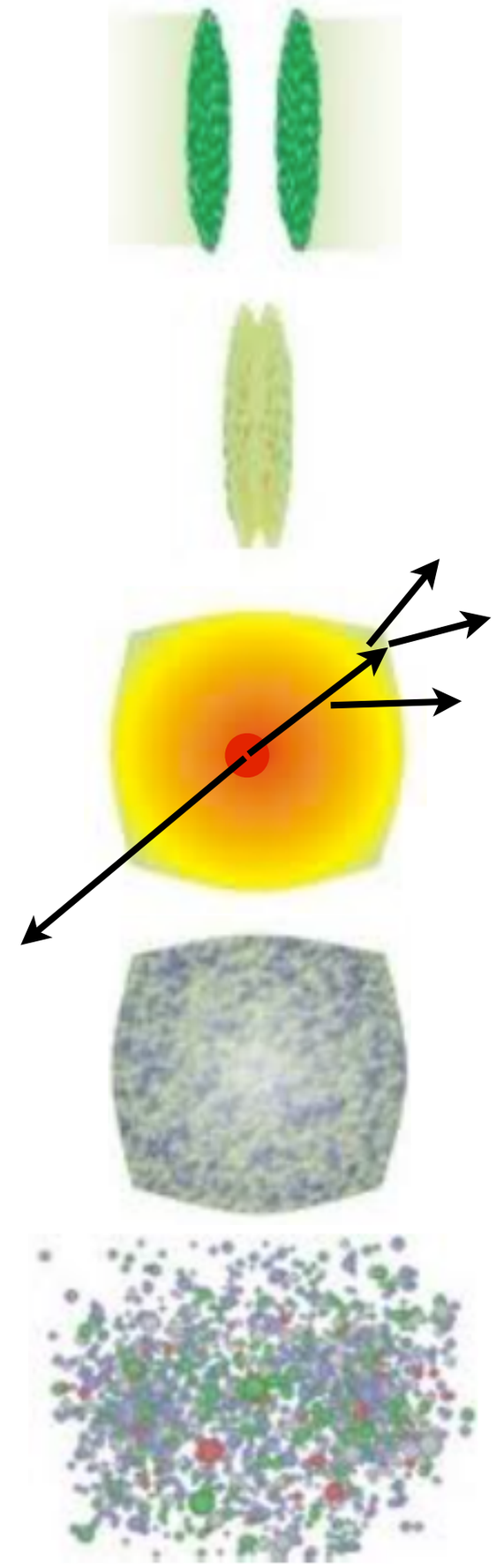


figure by Steffen Bass

Model Structure





Model Structure

Collision Geometry

IP-Glasma
Initial Condition

MARTINI
jet

MUSIC
hydrodynamics

PYTHIA/LUND
string fragmentation

Cooper-Frye
particlization

Hadronic E-loss

UrQMD
cascade

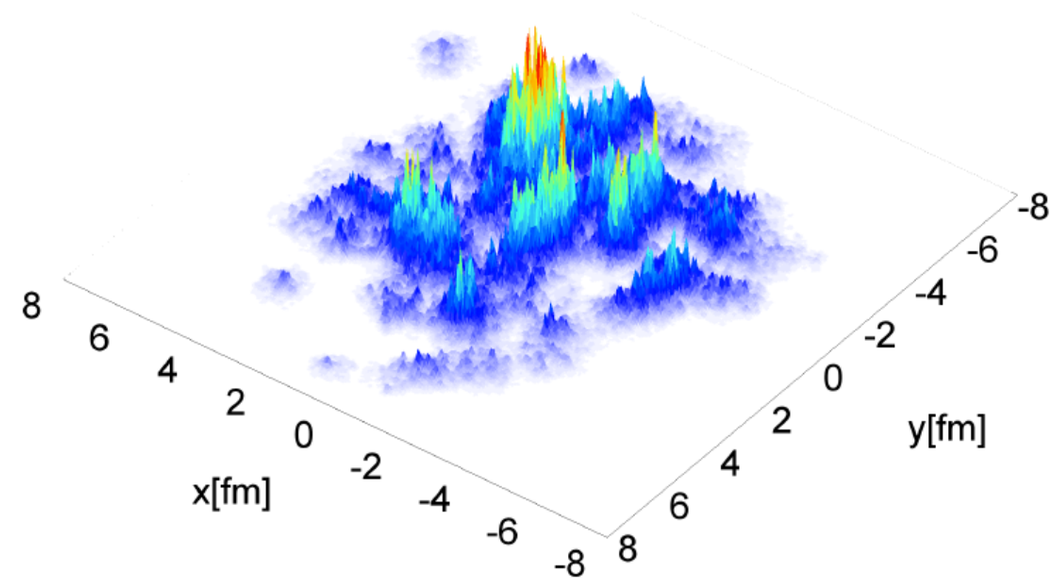
Model : IP-Glasma I.C.

B. Schenke, P. Tribedy and R. Venugopalan (2012)

Classical YM dynamics with color sources in nuclei

color charge distribution

$$\langle \rho^a(\mathbf{x}'_T)\,\rho^a(\mathbf{x}''_T)\rangle$$
$$= g^2\mu_A^2\delta^{ab}\delta^2(\mathbf{x}'_T - \mathbf{x}''_T)$$



▼ gluon field from each nucleus

$$A^i_{(1,2)}(\mathbf{x}_T)$$
$$= \frac{i}{g}U_{(1,2)}(\mathbf{x}_T)\,\partial_i U^\dagger_{(1,2)}(\mathbf{x}_T)$$
$$U_{(1,2)}(\mathbf{x}_T) = \mathcal{P}\exp\left[-ig\int dx^\pm \frac{\rho_{(1,2)}(\mathbf{x}_T, x^\pm)}{\nabla_T^2 - m^2}\right]$$

▼ initial gluon field after collision

$$A^i(\tau = +0) = A^i_{(1)} + A^i_{(2)}$$
$$A^\eta(\tau = +0) = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

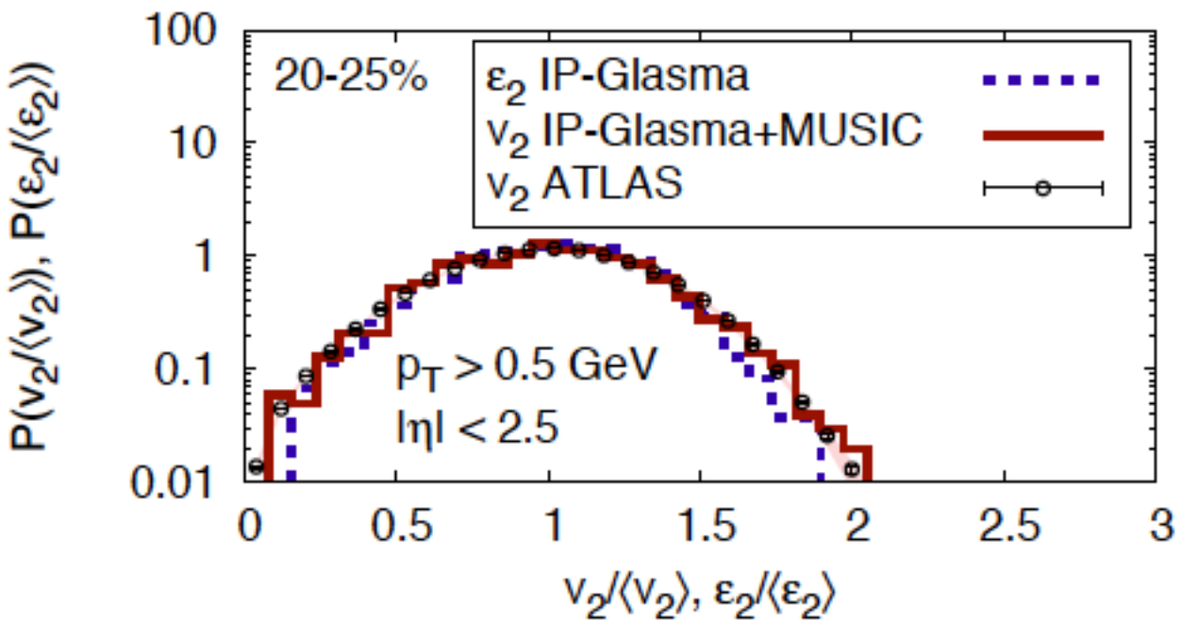
▶ energy density profile at $\tau = \tau_0$

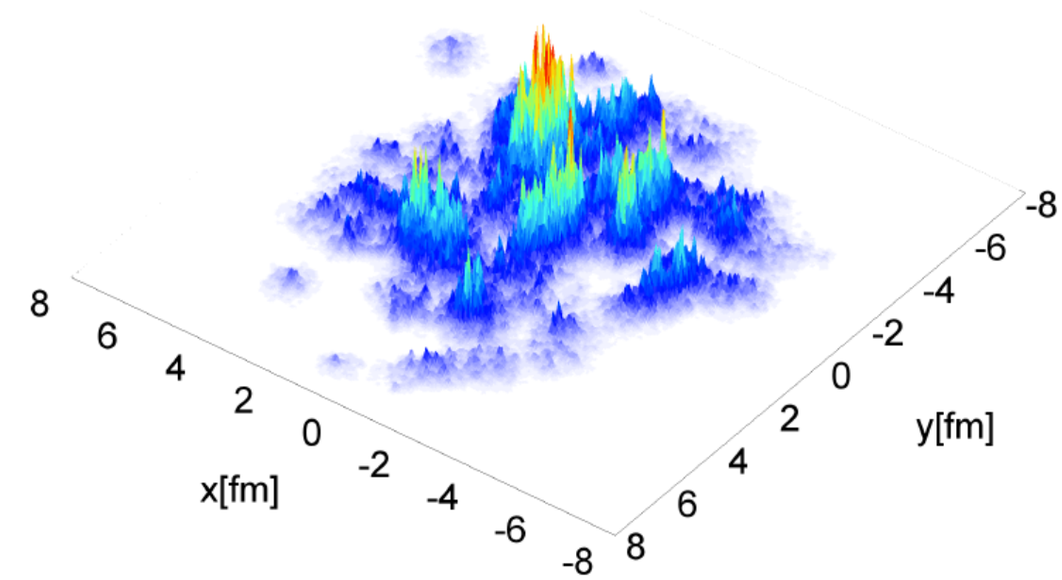
$$\partial_\mu F^{\mu\nu} - ig[A_\mu, F^{\mu\nu}] = 0$$
$$T^\mu{}_\nu(\tau = \tau_0)u^\nu = \epsilon\, u^\mu$$

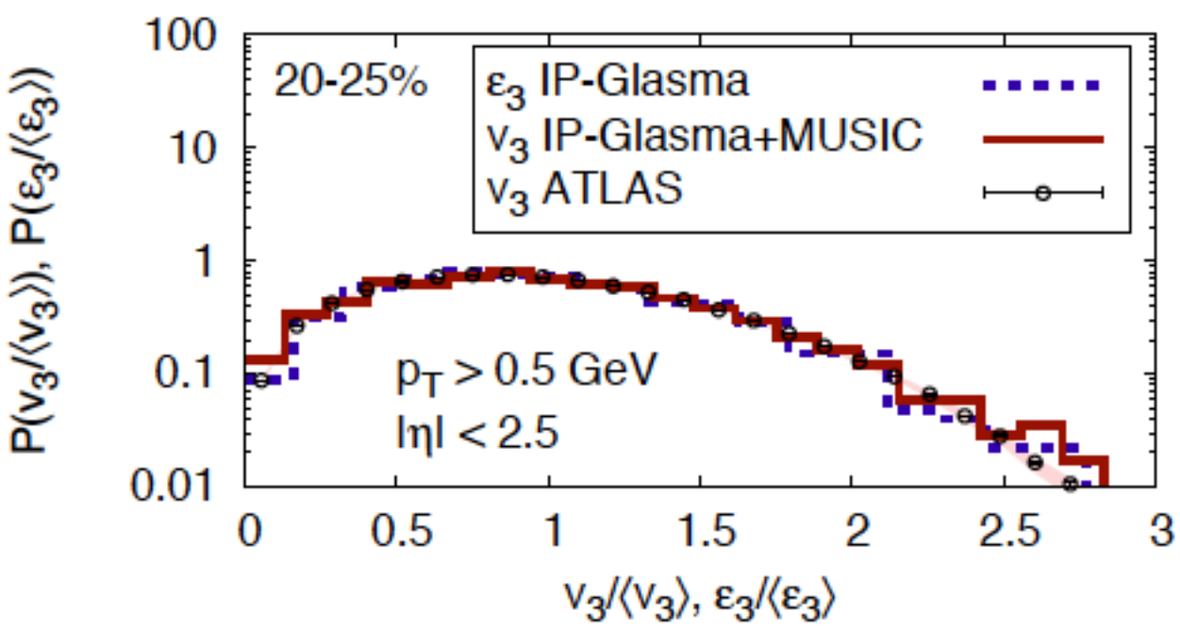
Model : IP-Glasma I.C.

B. Schenke, P. Tribedy and R. Venugopalan (2012)

Classical YM dynamics with color sources in nuclei







well describes v_n distribution

C. Gale, S. Jeon, B. Schenke,
P. Tribedy and R. Venugopalan (2012)

Model Structure

Collision Geometry

IP-Glasma
Initial Condition

MUSIC
hydrodynamics

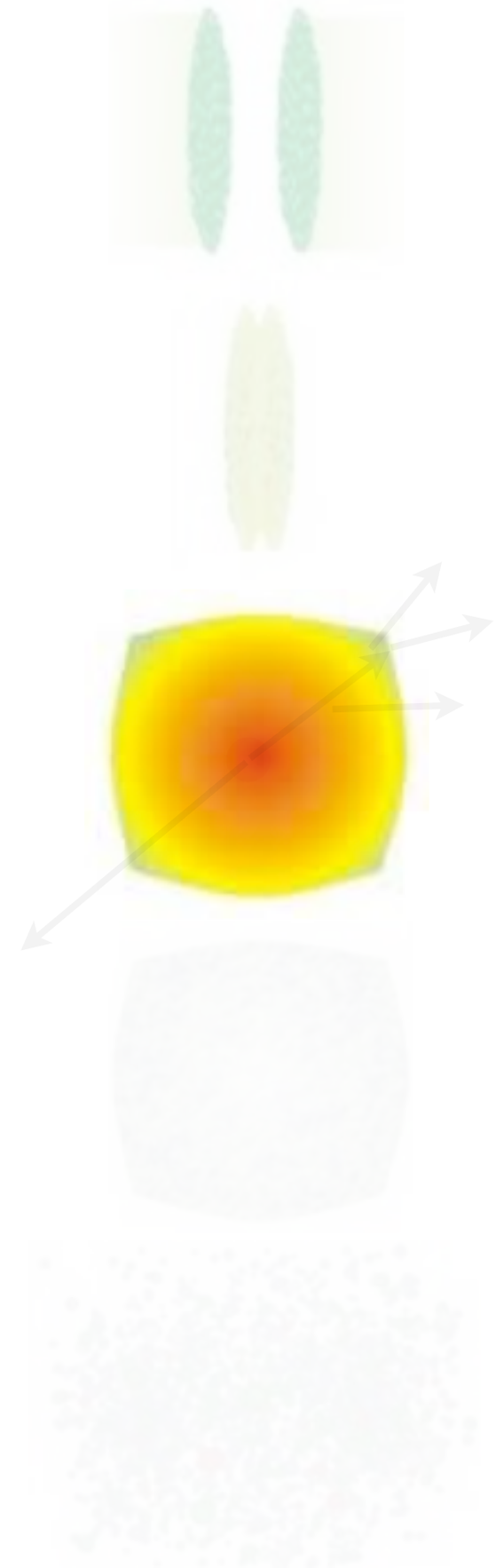
Cooper-Frye
particlization

UrQMD
cascade

MARTINI
jet

PYTHIA/LUND
string fragmentation

Hadronic E-loss



Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

hydrodynamic equations of motion

Conservation equation $\partial_\mu T^{\mu\nu} = 0$

Decomposition $T^{\mu\nu} = \epsilon_0 u^\mu u^\nu - (P_0(\epsilon_0) + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu}$

EoS bulk shear

Local 3-metric $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

Local 3-gradient $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

shear viscosity relaxation equation

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + \frac{1}{\tau_\pi}\Big(2\eta\,\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\,\sigma^{\mu\nu}\Big)$$

expansion rate

$$\theta = \nabla_\mu u^\mu$$

shear tensor

$$\sigma^{\mu\nu} = \frac{1}{2}\left[\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{3}{2}\Delta^{\mu\nu}\left(\nabla_\alpha u^\alpha\right)\right] \equiv \nabla^{\langle\mu}u^{\nu\rangle}$$

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

shear viscosity relaxation equation

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + \frac{1}{\tau_\pi}\Big(2\eta\,\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\,\sigma^{\mu\nu}\Big)$$

shear

$$\frac{\eta}{s} = \text{const}$$

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

shear viscosity relaxation equation

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + \frac{1}{\tau_\pi}\Big(2\eta\,\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\,\sigma^{\mu\nu}\Big)$$

14-moment approximation in the small mass limit

G. Denicol, S. Jeon, and C. Gale (2014)

$$\frac{\eta}{\tau_\pi} = \frac{1}{5}\left(\epsilon_0 + P_0\right) \qquad \frac{\delta_{\pi\pi}}{\tau_\pi} = \frac{4}{3} \qquad \frac{\lambda_{\pi\Pi}}{\tau_\pi} = \frac{6}{5} \qquad \frac{\tau_{\pi\pi}}{\tau_\pi} = \frac{10}{7}$$

second-order transport coefficients

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

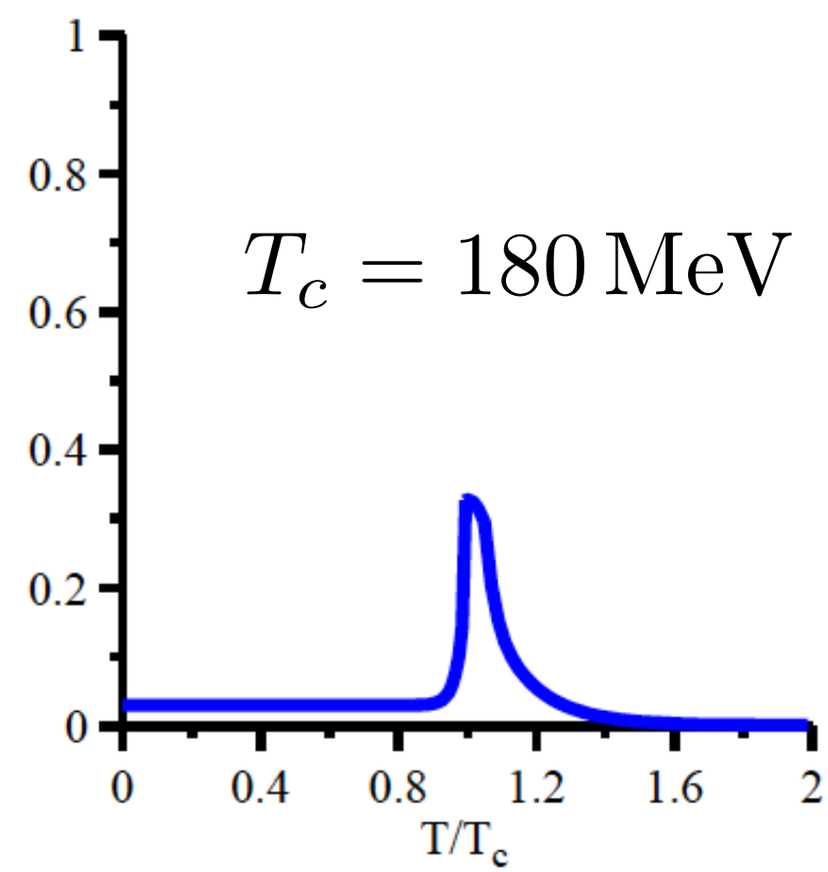
equation of motion for viscous corrections

bulk viscosity relaxation equation

$$\dot{\Pi} = -\frac{\Pi}{\tau_\Pi} + \frac{1}{\tau_\Pi}\left(-\zeta\,\theta - \delta_{\Pi\Pi}\Pi\,\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}\right)$$

bulk

ζ/s



F. Karsch,
D. Kharzeev and
K. Tuchin (2008)

J. Noronha-Hostler,
J. Noronha and
C. Greiner (2009)

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

bulk viscosity relaxation equation

$$\dot{\Pi} = -\frac{\Pi}{\tau_\Pi} + \frac{1}{\tau_\Pi}\left(-\zeta\,\theta - \delta_{\Pi\Pi}\Pi\,\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}\right)$$

14-moment approximation in the small mass limit

G. Denicol, S. Jeon, and C. Gale (2014)

$$\frac{\zeta}{\tau_\Pi} = 15\left(\frac{1}{3} - c_s^2\right)^2(\epsilon_0 + P_0)$$

$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = \frac{2}{3} \qquad \frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right)$$

second-order transport coefficients

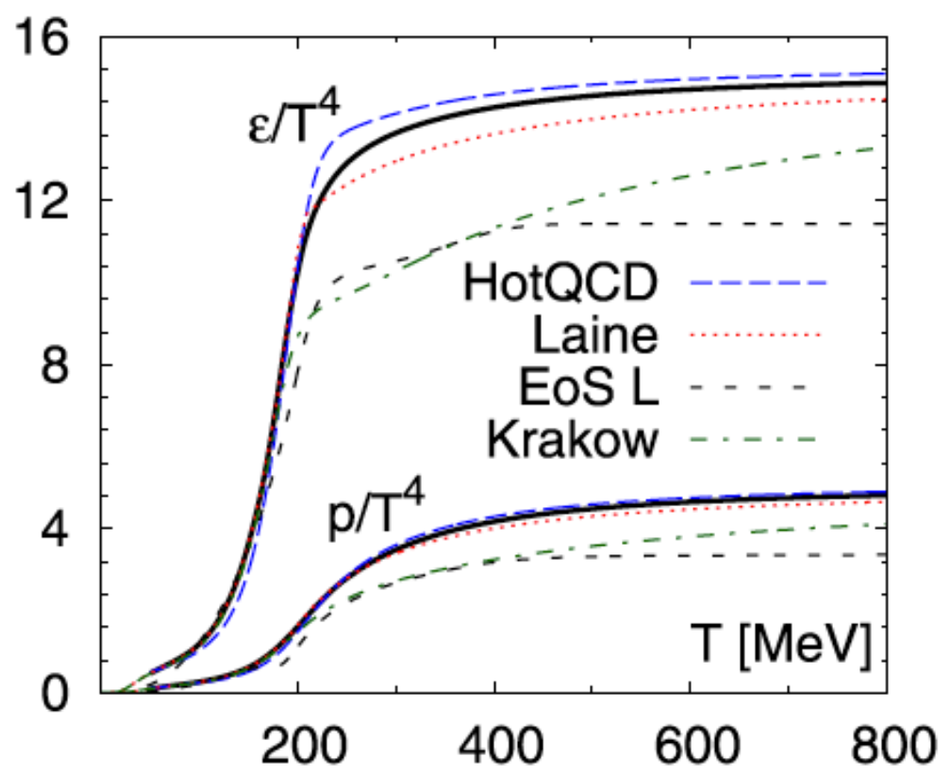
Model : Equation of state

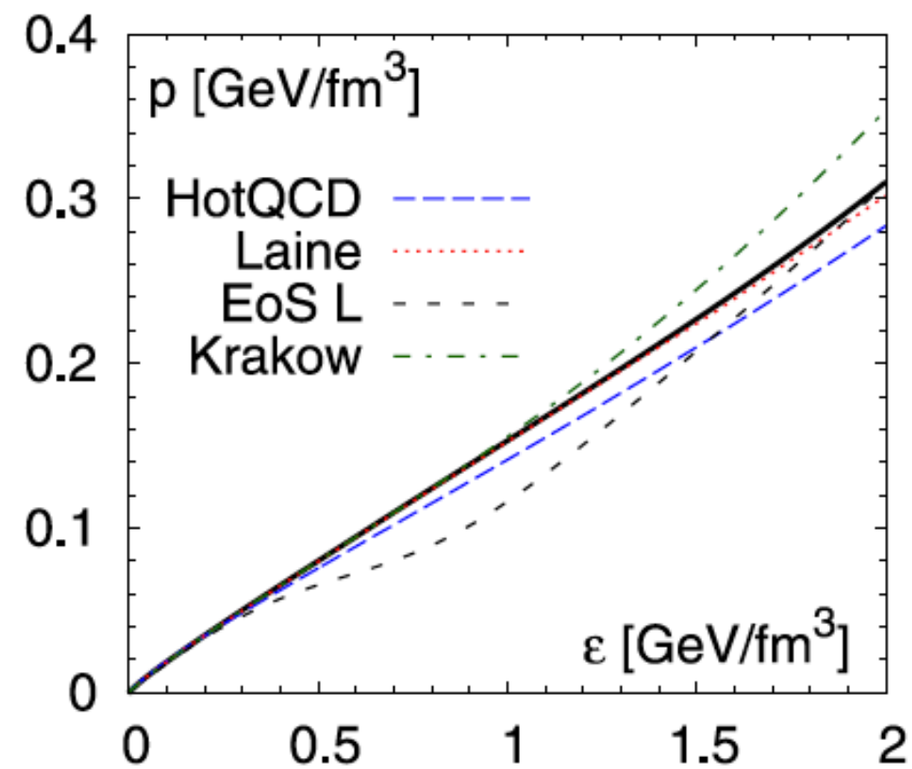
P. Huovinen, and P. Petreczky (2010)

Equation of state : hadron gas + lattice data

Only those included in UrQMD

Cross over phase transition around T = 180 MeV





Model Structure

Collision Geometry

IP-Glasma
Initial Condition

MUSIC
hydrodynamics

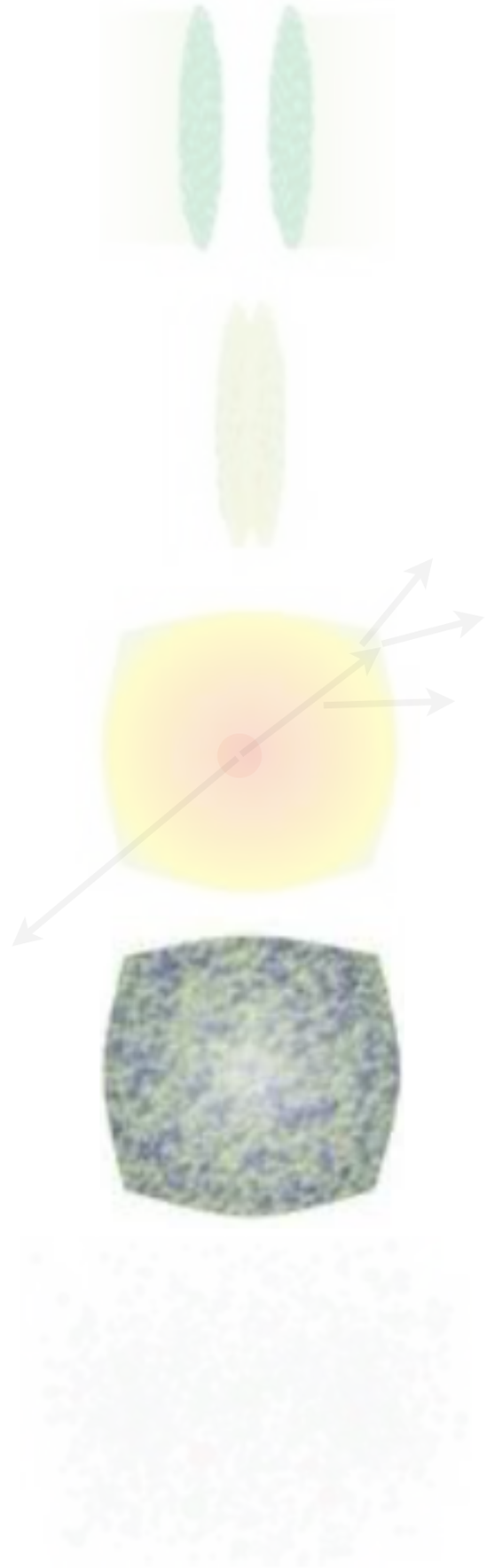
Cooper-Frye particlization

UrQMD
cascade

MARTINI
jet

PYTHIA/LUND
string fragmentation

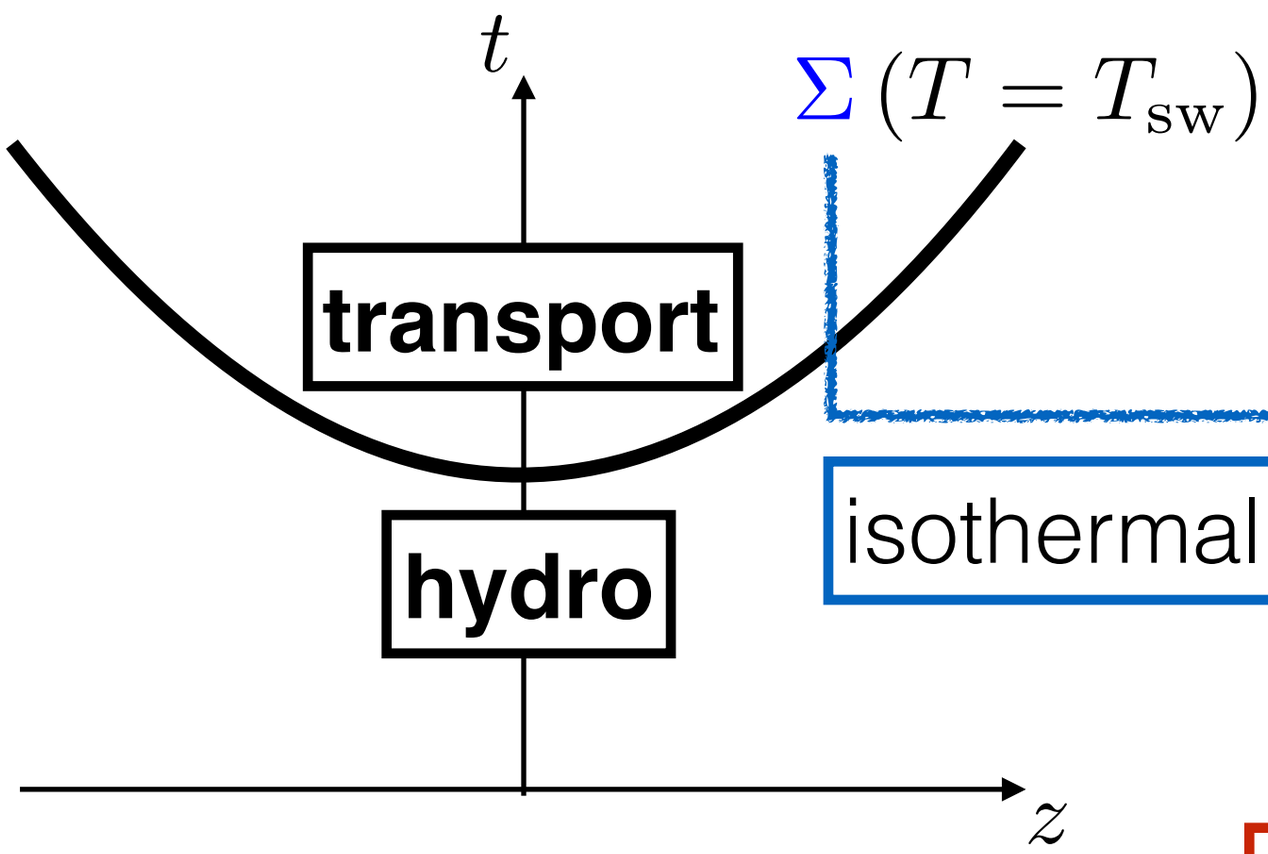
Hadronic E-loss



Model : Cooper-Frye sampling

F. Cooper and G. Frye (1974)

sampling particles according to the Cooper-Frye formula
(transform hydrodynamic information into particles)



$$\frac{dN}{d^3\mathbf{p}} = \int_{\Sigma} \frac{p^\mu d^3\Sigma_\mu}{E_\mathbf{p}} f(x, \mathbf{p})$$

$\Sigma\,(T = T_{\mathrm{sw}})$

isothermal hypersurface

hydrodynamic information
(temperature, flow velocity, …)

Model : Cooper-Frye sampling

F. Cooper and G. Frye (1974)

sampling particles according to the Cooper-Frye formula

$$\left.\frac{dN}{d^3\mathbf{p}}\right|_{\text{1-cell}} = \left[f_0(x,\mathbf{p}) + \delta f_{\text{shear}}(x,\mathbf{p}) + \delta f_{\text{bulk}}(x,\mathbf{p})\right] \frac{p^\mu \Delta^3 \Sigma_\mu}{E_{\mathbf{p}}}$$

$$f_0(x,\mathbf{p}) = \frac{1}{\exp\left[(p\cdot u)/T\right] \mp 1}$$

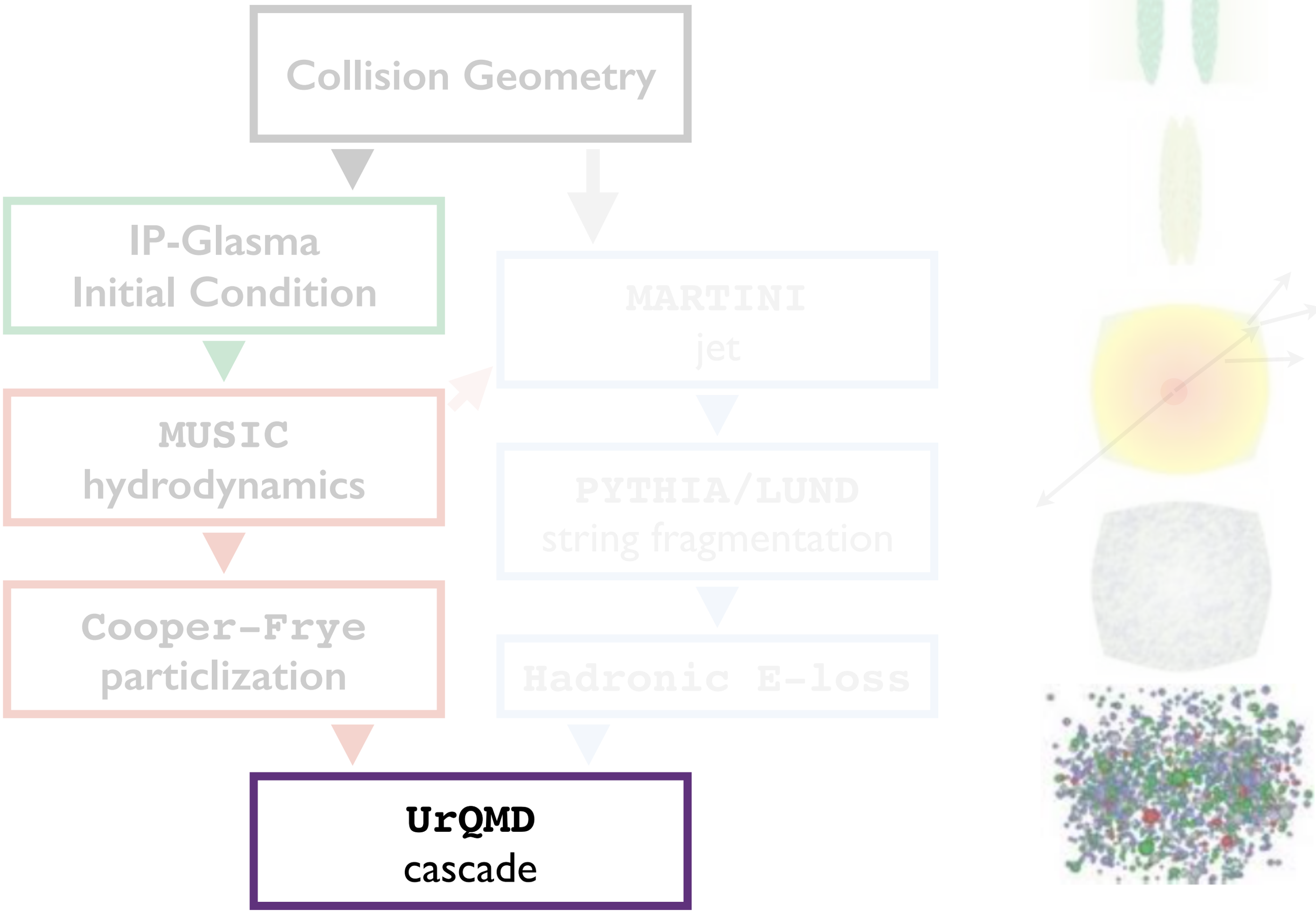
$$\delta f_{\text{shear}}(x,\mathbf{p}) = f_0(1 \pm f_0)\frac{p^\mu p^\nu \pi^{\mu\nu}}{2T^2(\epsilon_0 + P_0)}$$

P. Bozek (2010)

$$\delta f_{\text{bulk}}(x,\mathbf{p}) = -f_0(1 \pm f_0)\frac{C_{\text{bulk}}\Pi}{T}\left[c_s^2(p\cdot u) - \frac{(-p^\mu p^\nu \Delta_{\mu\nu})}{3(p\cdot u)}\right]$$

$$\frac{1}{C_{\text{bulk}}} = \frac{1}{3T}\sum_n m_n^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3 E_{\mathbf{k}}} f_{n,0}(1 \pm f_{n,0})\left(c_s^2 E_{\mathbf{k}} - \frac{|\mathbf{k}|^2}{3E_{\mathbf{k}}}\right)$$

Model Structure



Model : UrQMD cascade

Ultra-relativistic Quantum Molecular Dynamics

S. A. Bass *et al.* (1998)

Monte-Carlo implementation of transport theory

$$p^\mu \frac{\partial}{\partial x^\mu} f_i(x,p) = \mathcal{C}_i[f]$$

Which species? : 55 baryons + 32 mesons with masses up to 2.25 GeV

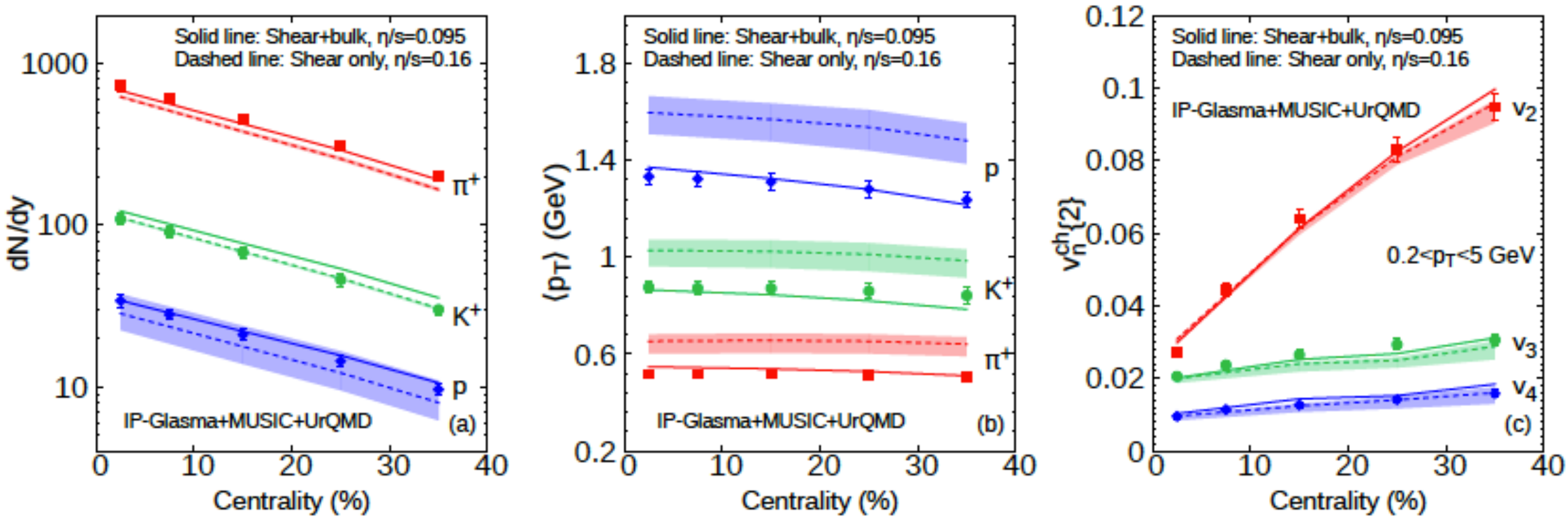
Cross sections : based on experimental data
Jet-hadron interaction by PYTHIA

Keeps track of particle trajectories

Description of soft physics

PRL 2015 (arXiv:1502.01675)

SR, J-F. Paquet, G. Denicol, C. Shen, B. Schenke, S. Jeon and C. Gale



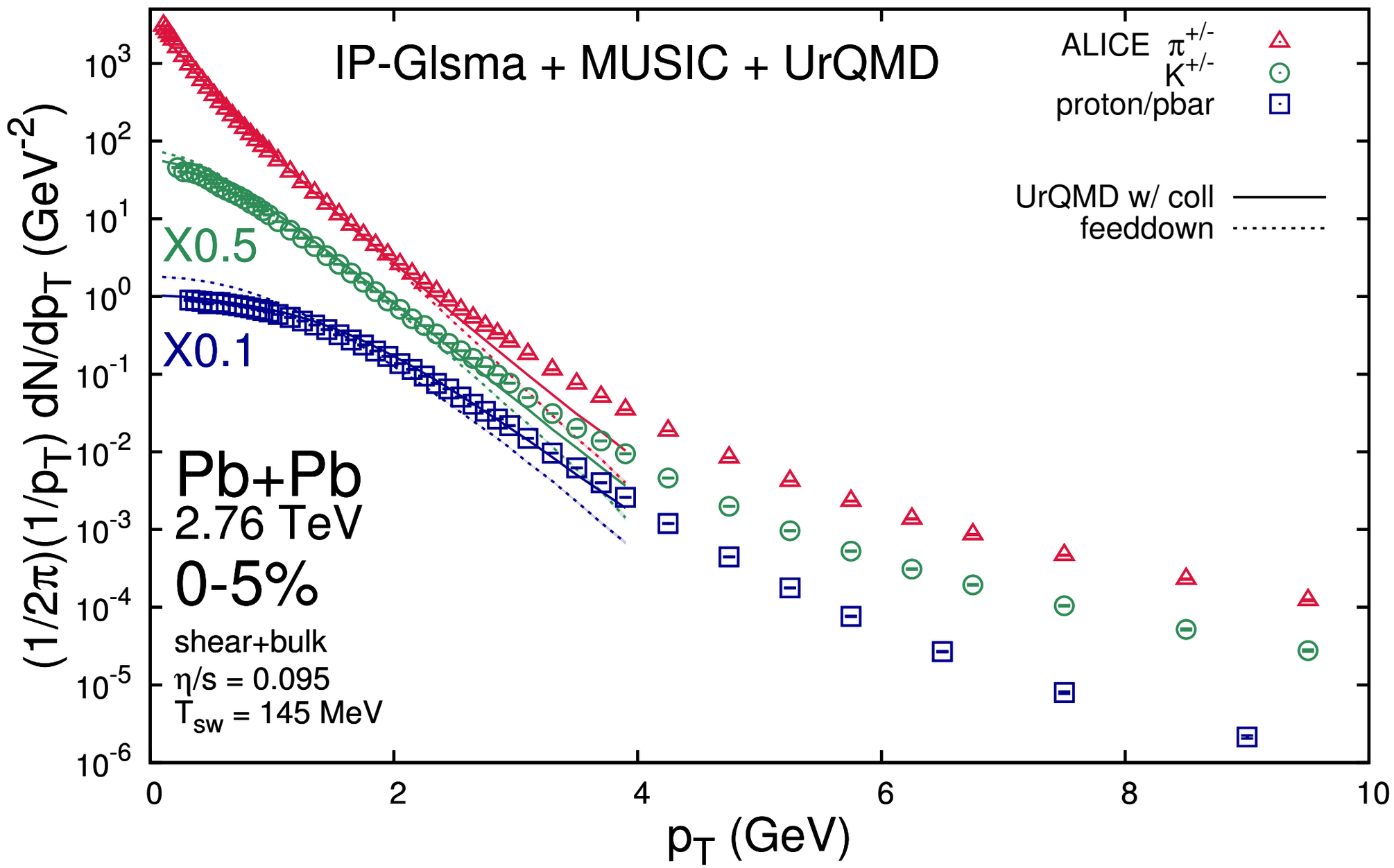
Parameters are tuned to fit **multiplicity, mean** p_T and **integrated flow coefficients** v_n.

The **bulk viscosity is crucial** to describe those observables.

The **shear viscosity** $\frac{\eta}{s} = 0.095$ is favored.

Identified hadron p_T-spectra

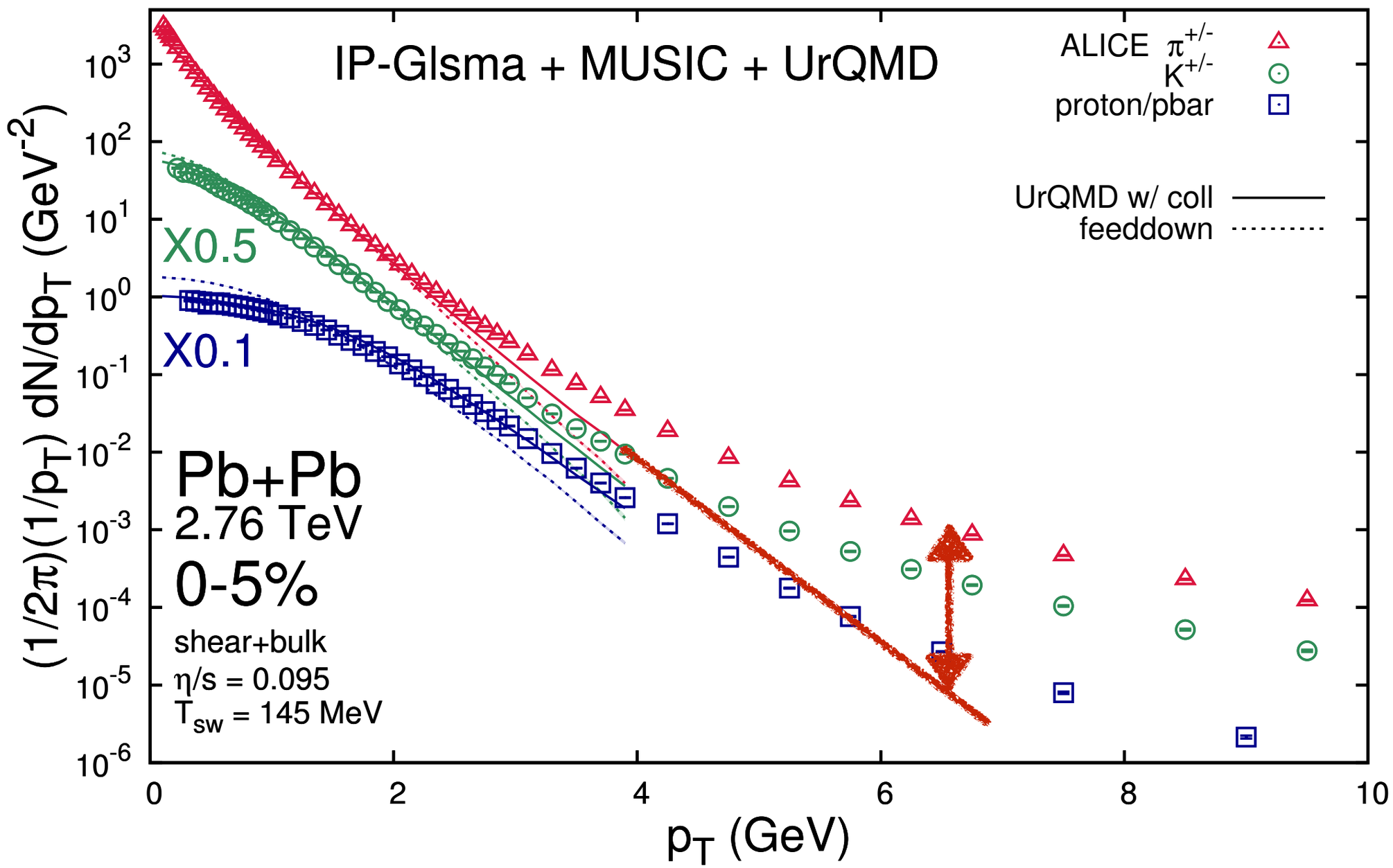
arXiv:1704.04216



The low-p_T spectra are well described.

Identified hadron p_T-spectra

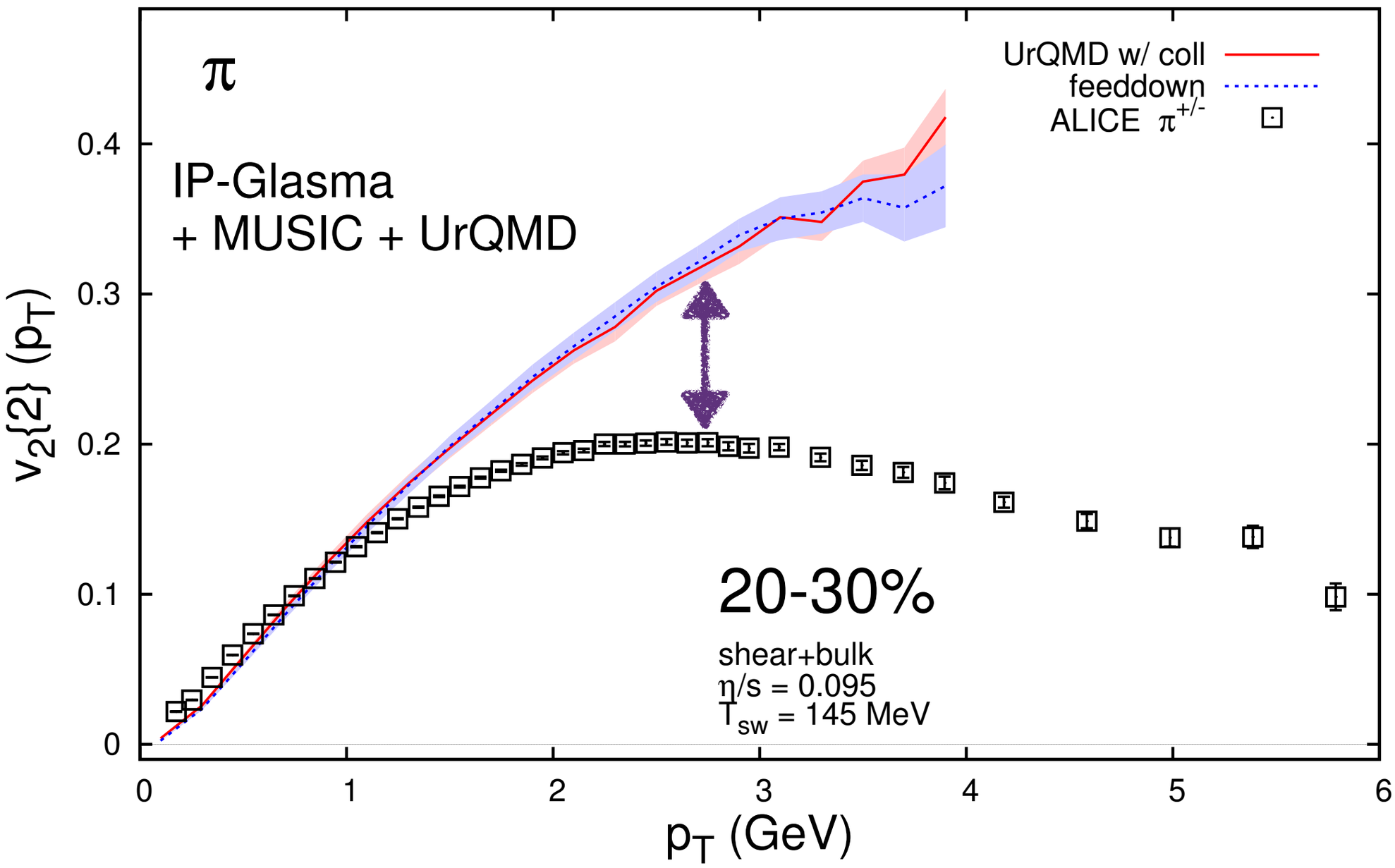
arXiv:1704.04216



Jet production and energy-loss are necessary.

Elliptic flow (identified hadrons)

arXiv:1704.04216



Jet production and energy-loss are necessary.

PART 1

Hybrid approach and
description of soft (low-p_T) physics

PART 2

Jet production and energy loss
for hard (high-p_T) physics

Model Structure

Collision Geometry

IP-Glasma
Initial Condition

MUSIC
hydrodynamics

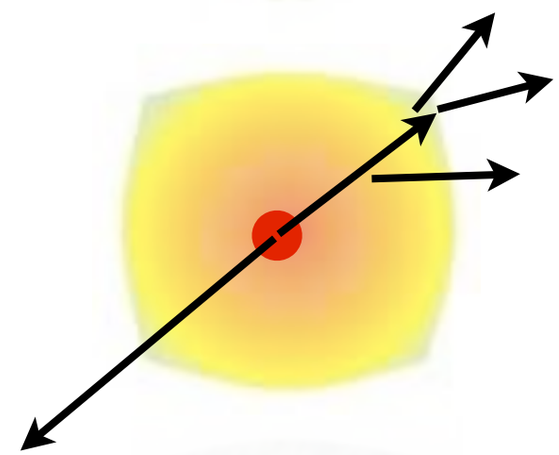
Cooper-Frye
particlization

UrQMD
cascade

MARTINI
jet

PYTHIA/LUND
string fragmentation

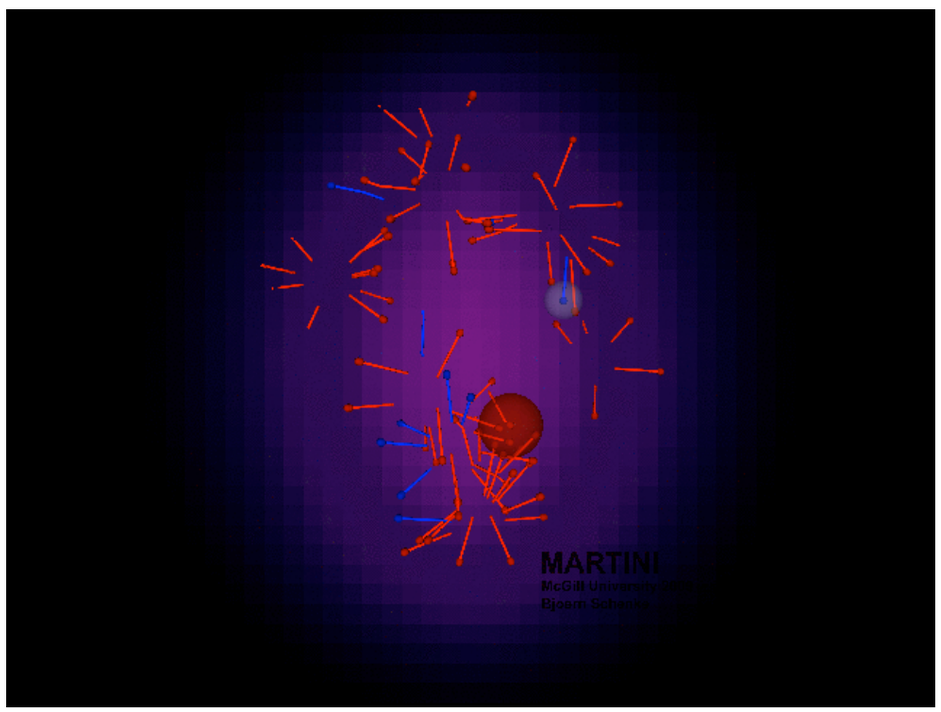
Hadronic E-loss



Model : MARTINI jets

Modular Algorithm for Relativistic Treatment of heavy IoN Interaction

B. Schenke, C. Gale and S. Jeon (2010)



Hard process at the position of binary collision (PYTHIA)

↓

Energy loss
- Radiation (AMY)
- Collision (with thermal partons)

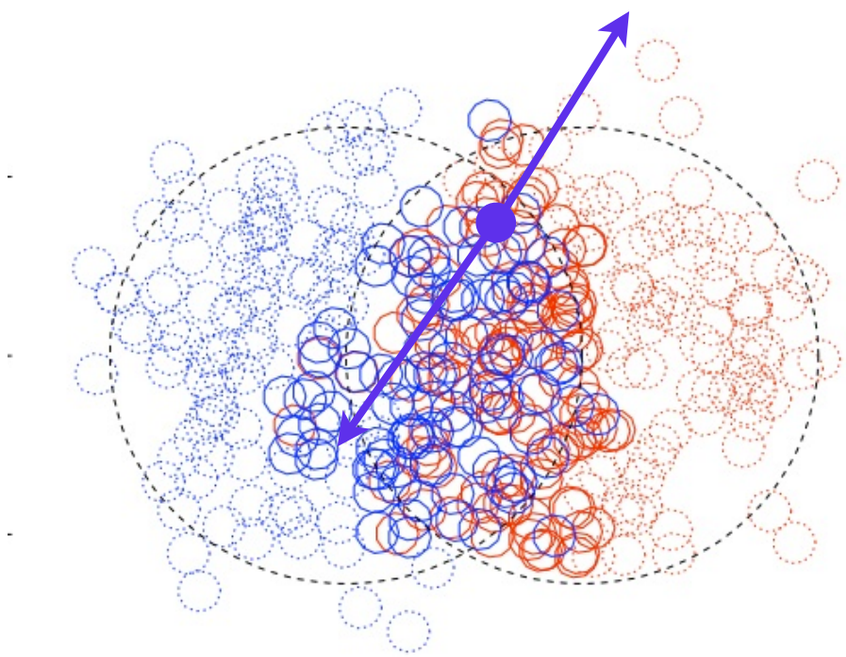
↓

Fragmentation into hadrons (PYTHIA / LUND string model)

Model : MARTINI jets

Modular Algorithm for Relativistic Treatment of heavy IoN Interaction

B. Schenke, C. Gale and S. Jeon (2010)



Hard process at the position of binary collision (PYTHIA)

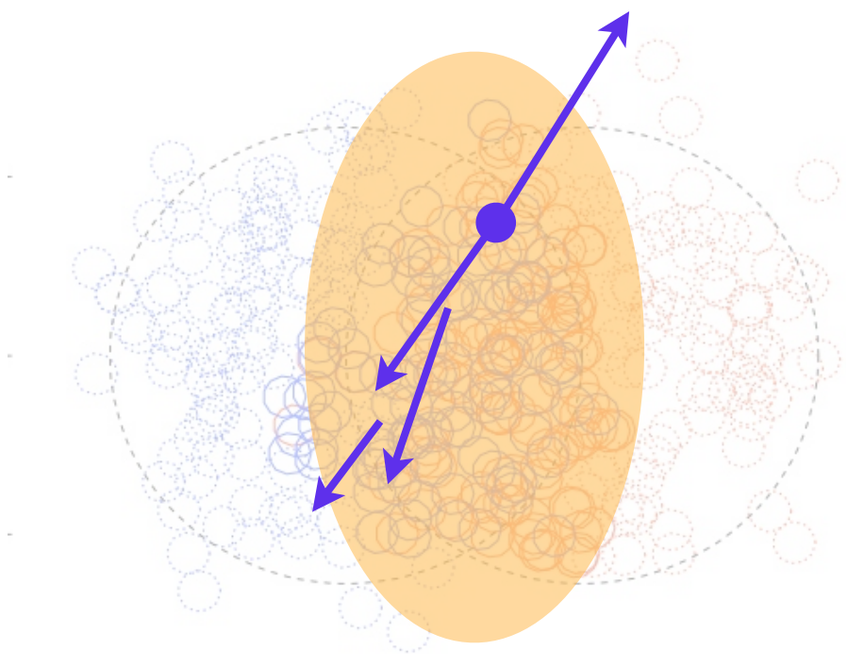
$$P_{\text{jet}}(\hat{p}_{T,\text{min}}) = \frac{\sigma_{\text{jet}}(\hat{p}_{T,\text{min}})}{\sigma_{\text{inel}}}$$

$$\sigma_{\text{jet}}(\hat{p}_{T,\text{min}}) = \sum_{i,j} \int dx_1 f_i(x_1) \int dx_2 f_j(x_2)\, \hat{\sigma}_{(i,j)}(x_1, x_2; \hat{p}_{T,\text{min}})$$

Model : MARTINI jets

Modular Algorithm for Relativistic Treatment of heavy IoN Interaction

B. Schenke, C. Gale and S. Jeon (2010)



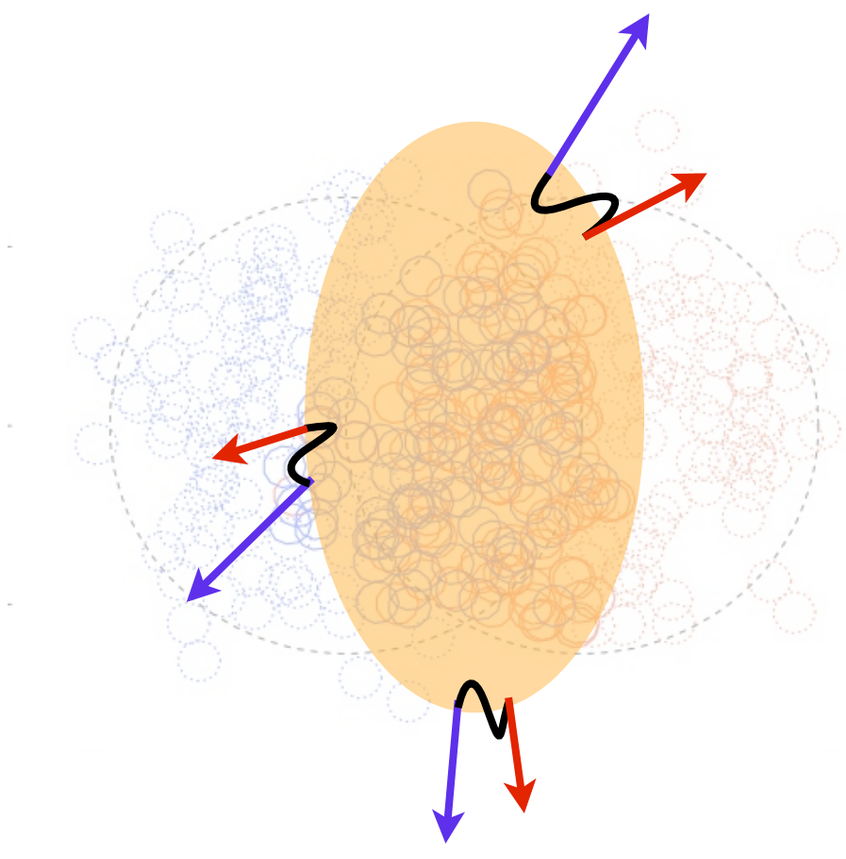
Hard process at the position of binary collision (PYTHIA)

Energy loss
- Radiation (AMY)
- Collision (with thermal partons)

Model : MARTINI jets

Modular Algorithm for Relativistic Treatment of heavy IoN Interaction

B. Schenke, C. Gale and S. Jeon (2010)



Hard process at the position of binary collision (PYTHIA)

↓

Energy loss
- Radiation (AMY)
- Collision (with thermal partons)

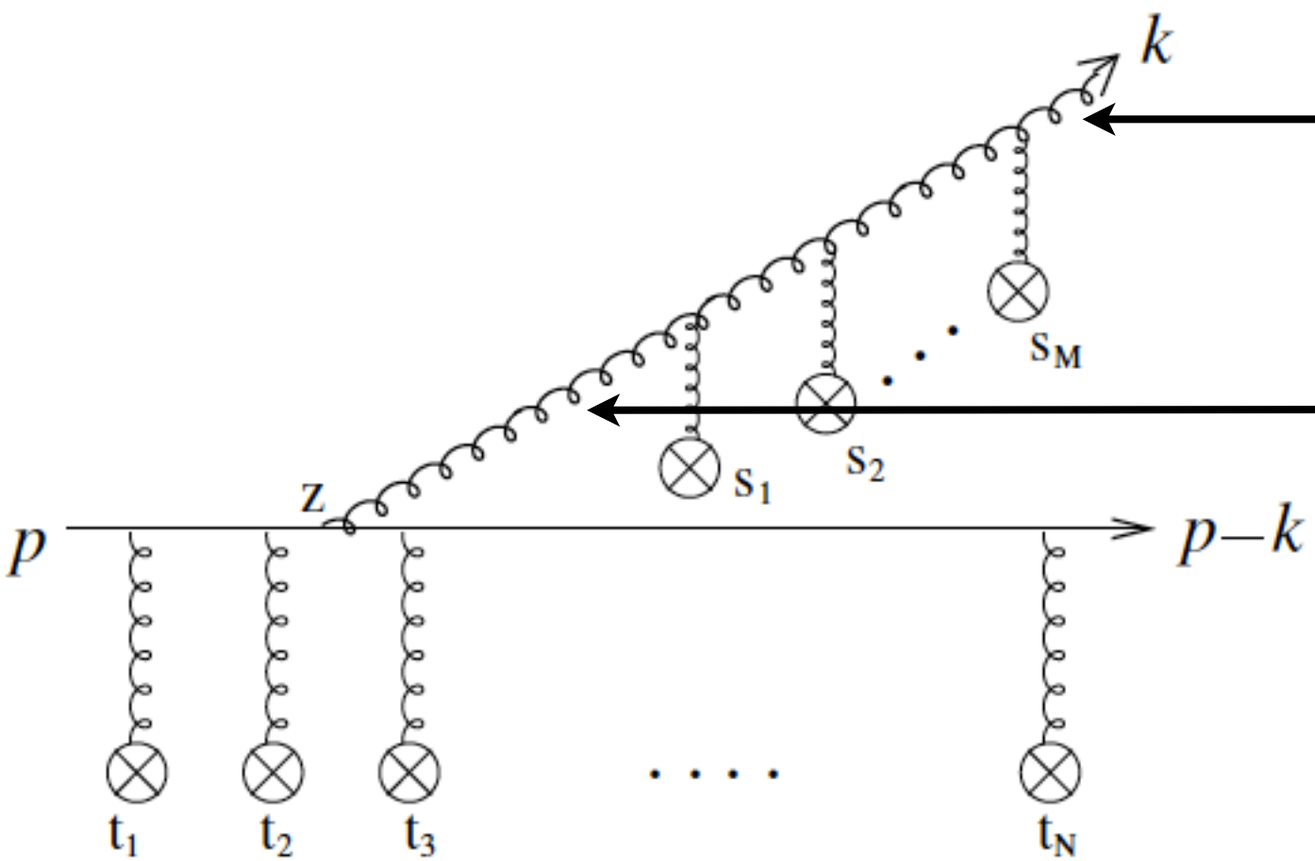
↓

Fragmentation into hadrons (PYTHIA / LUND string model)

Model : jet energy loss

Radiative energy loss (AMY)

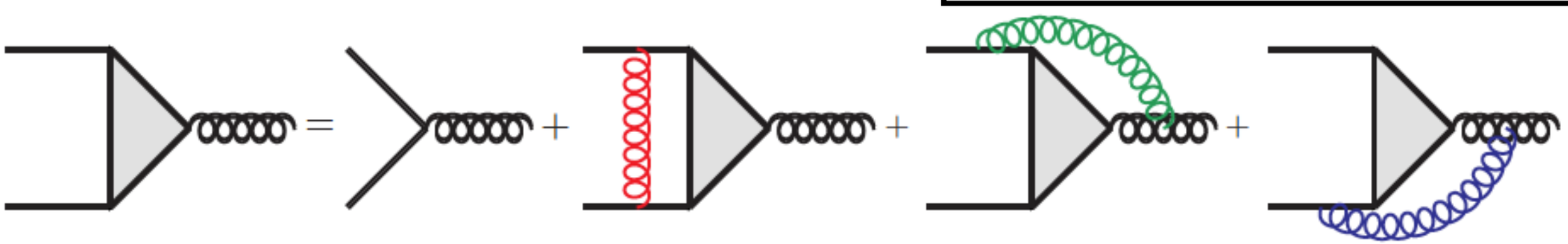
P. Arnold, G. Moore and L. Yaffe (2002)



Collinear emission

Nearly on-shell intermediate propagator

Infinite number of diagrams
➤ Integral equations



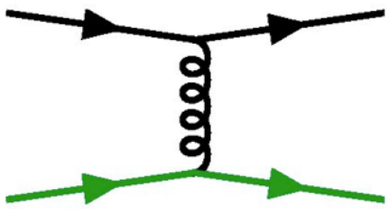
figures by G-Y. Qin

Model : jet energy loss

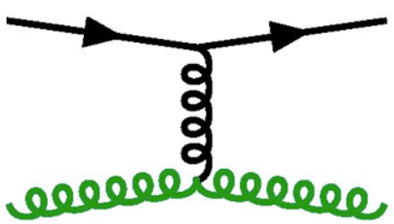
Collisional energy loss (soft approximation)

B. Schenke, C. Gale and G-Y. Qin (2009)

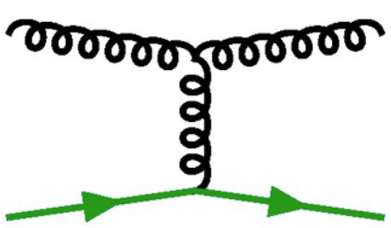
G-Y. Qin et al. (2008)



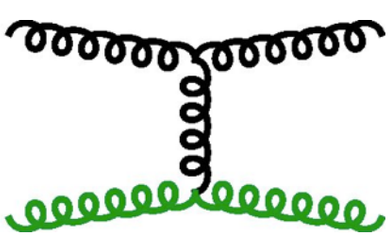
$$\left.\frac{dE}{dt}\right|_{qq} = \frac{2}{9} n_f \pi \alpha_s^2 T^2 \left[\ln \frac{ET}{m_g^2} + c_f \frac{23}{12} + c_s\right]$$



$$\left.\frac{dE}{dt}\right|_{qg} = \frac{4}{3} \pi \alpha_s^2 T^2 \left[\ln \frac{ET}{m_g^2} + c_b \frac{13}{6} + c_s\right]$$

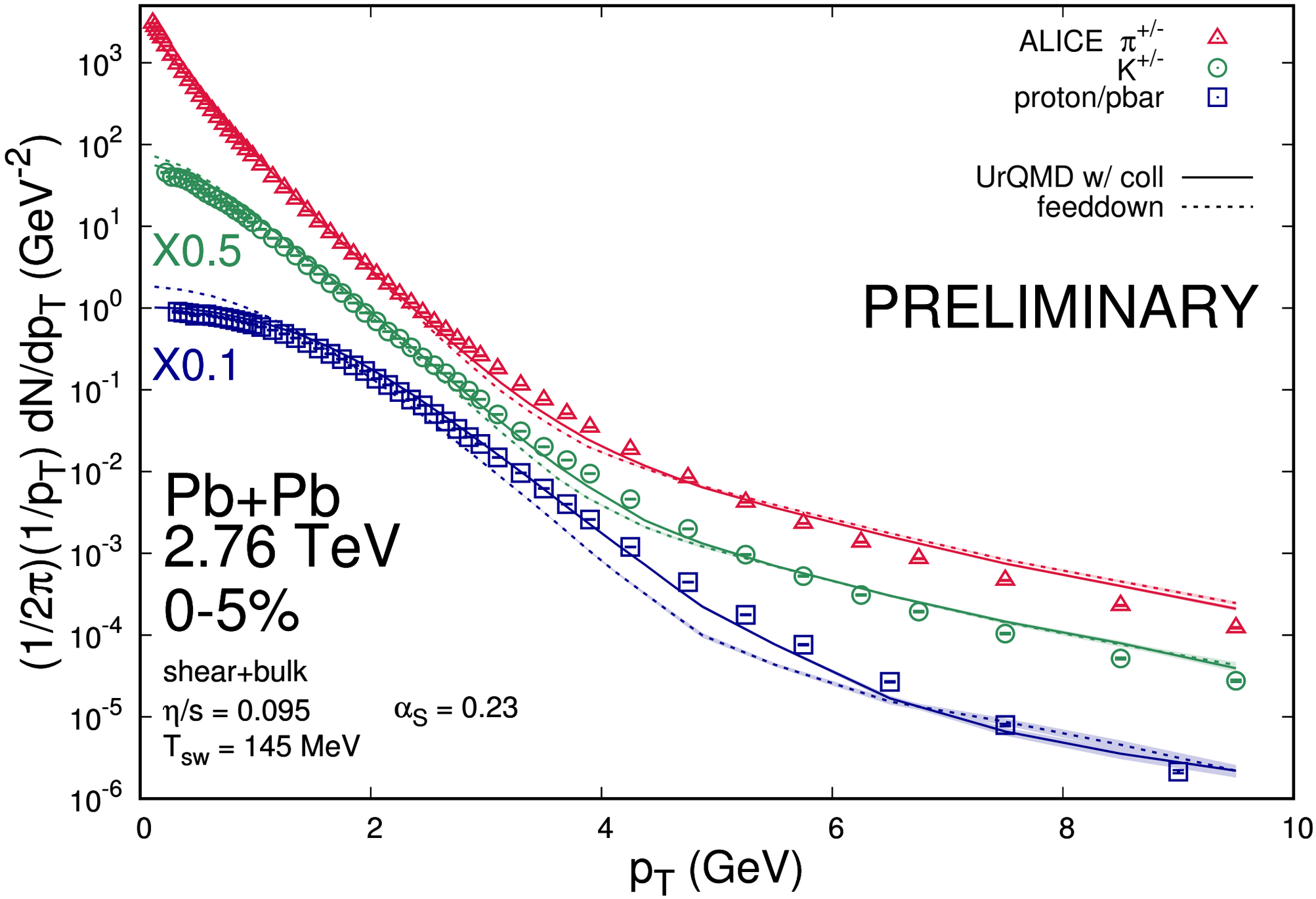


$$\left.\frac{dE}{dt}\right|_{gq} = \frac{1}{2} n_f \pi \alpha_s^2 T^2 \left[\ln \frac{ET}{m_g^2} + c_f \frac{13}{6} + c_s\right]$$



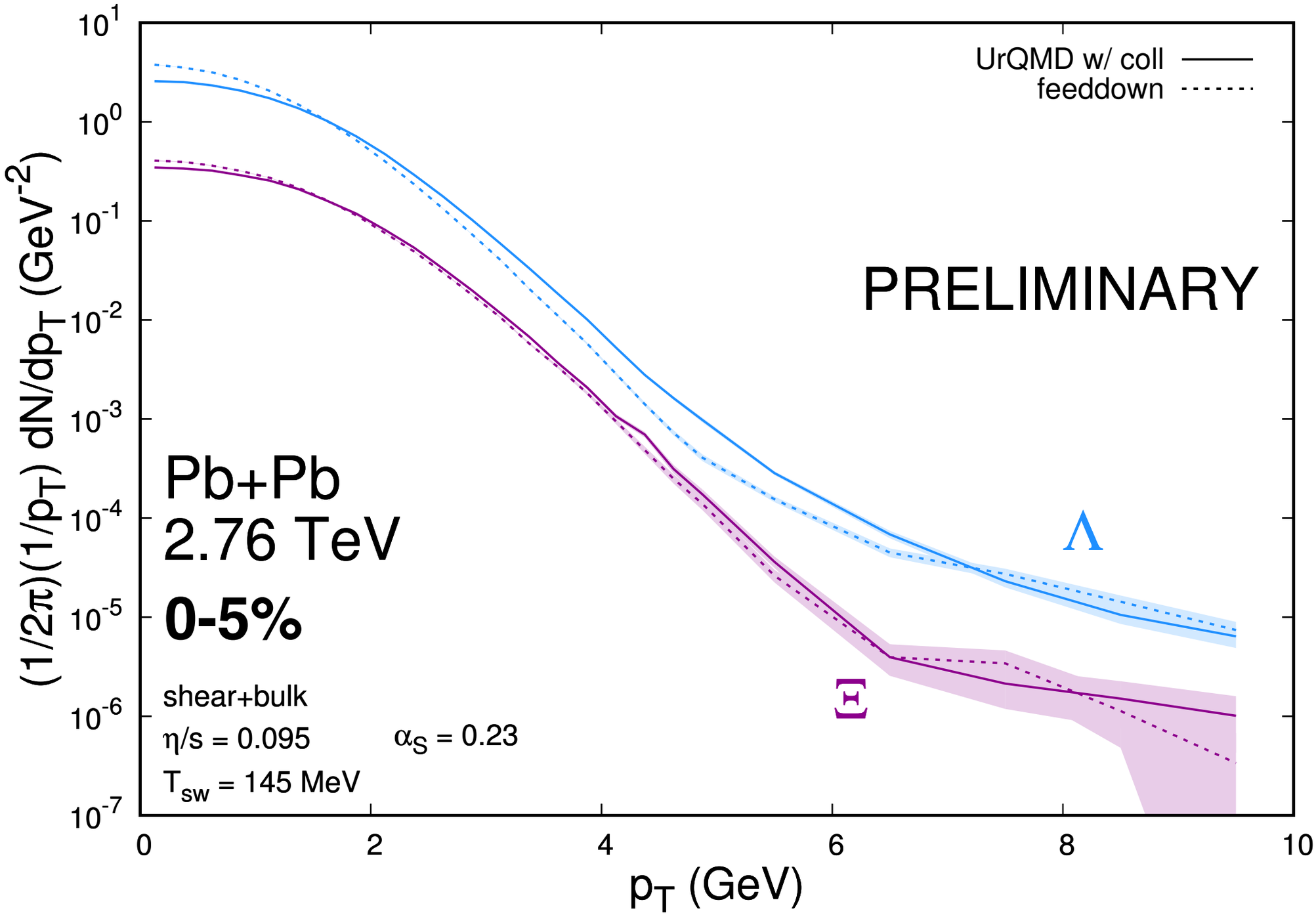
$$\left.\frac{dE}{dt}\right|_{gg} = 3 \pi \alpha_s^2 T^2 \left[\ln \frac{ET}{m_g^2} + c_b \frac{131}{48} + c_s\right]$$

Identified hadron p_T-spectra



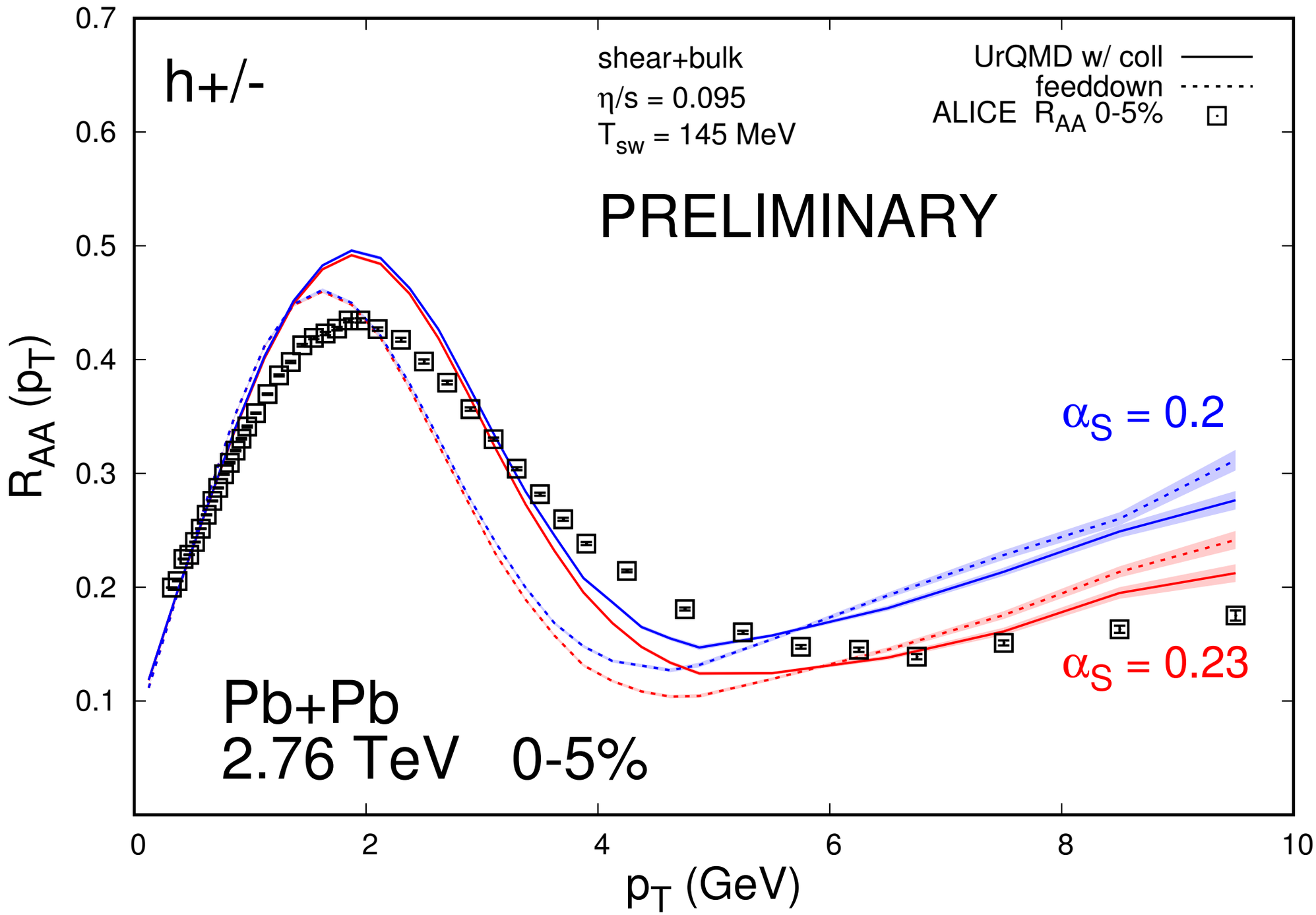
Jet production and energy-loss are necessary.

Identified hadron p_T-spectra



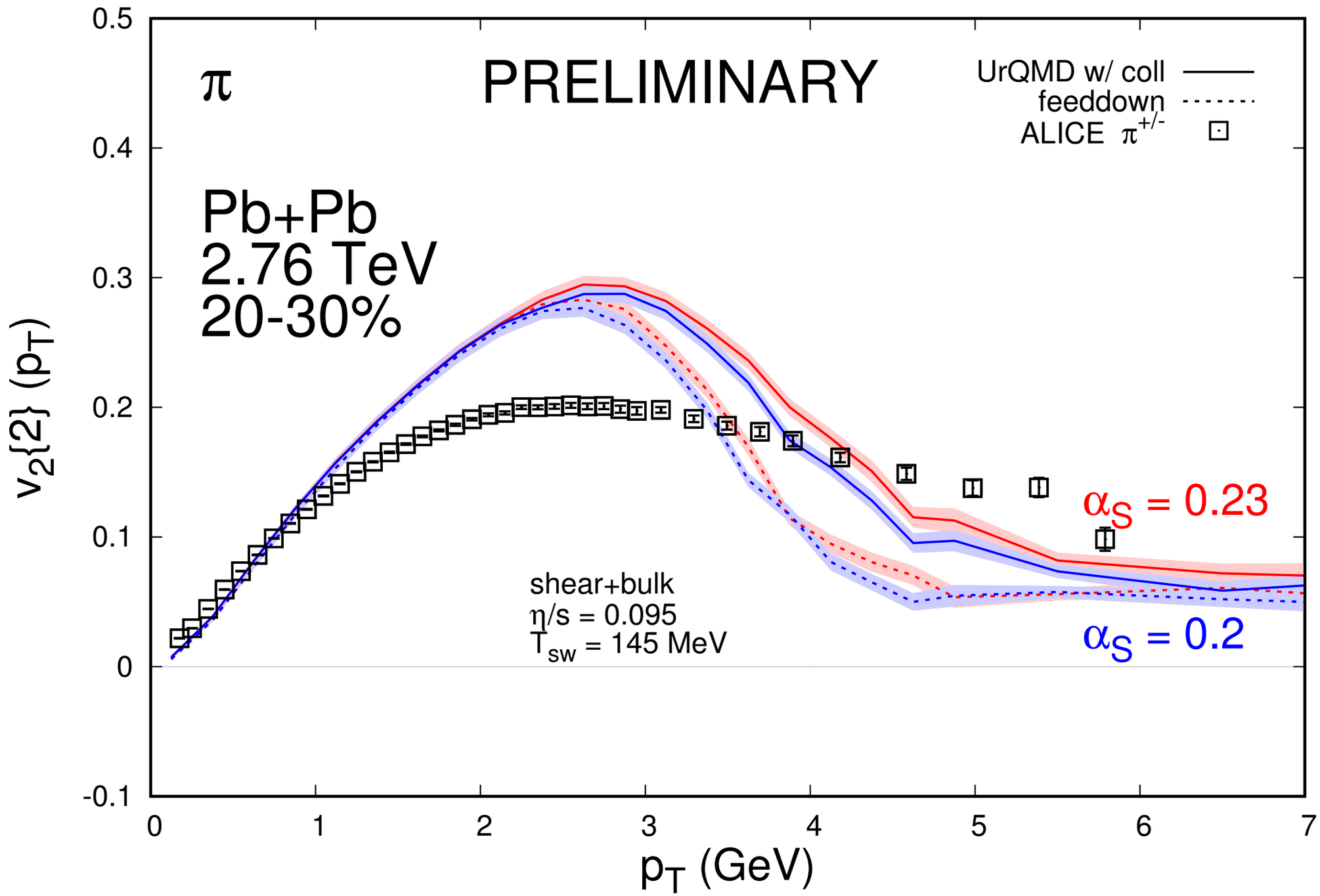
Jet production and energy-loss are necessary.

Charged hadron R_{AA} (inclusive)



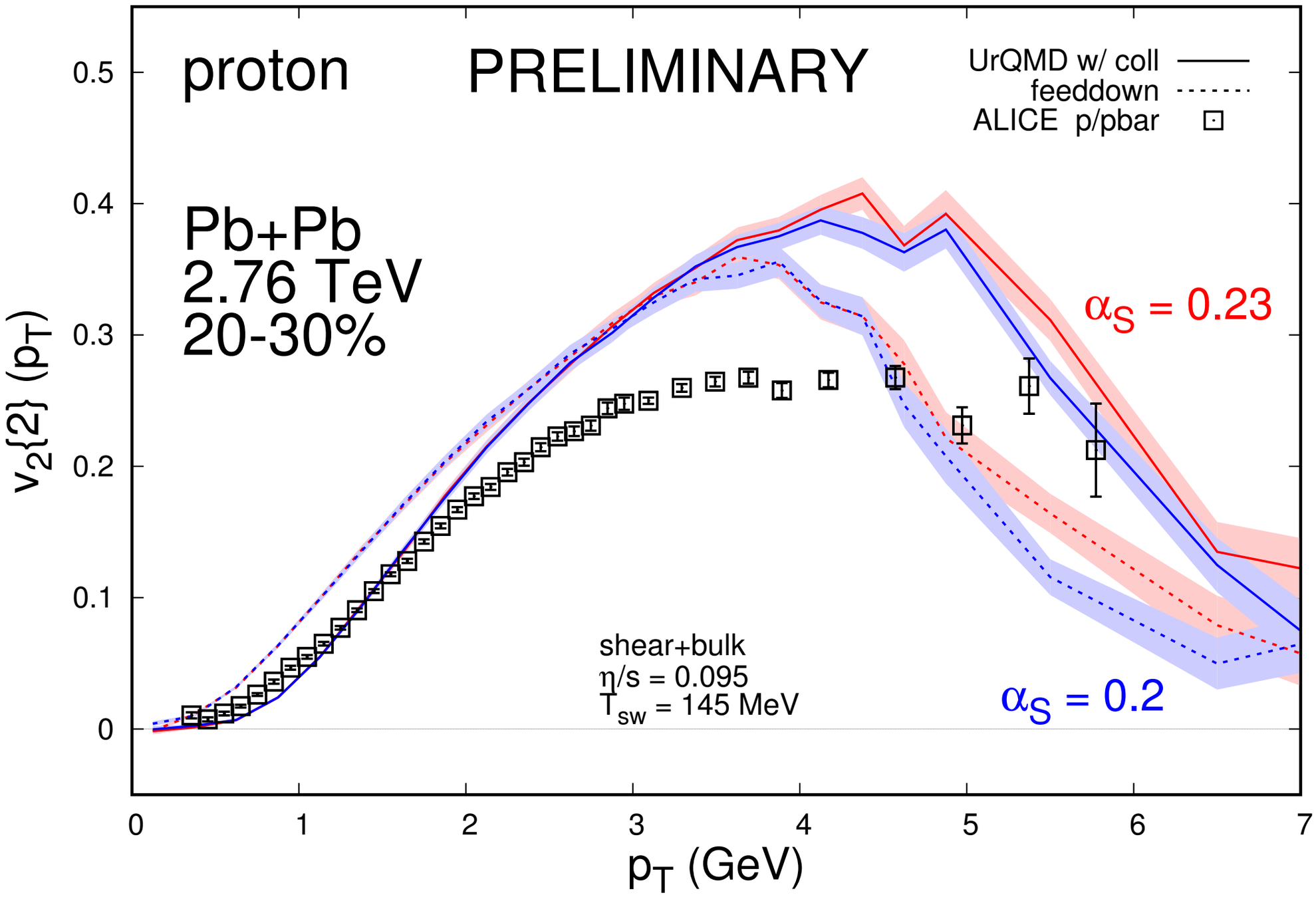
Hadronic rescattering has significant effects on p_T distribution.

Elliptic flow (identified hadrons)



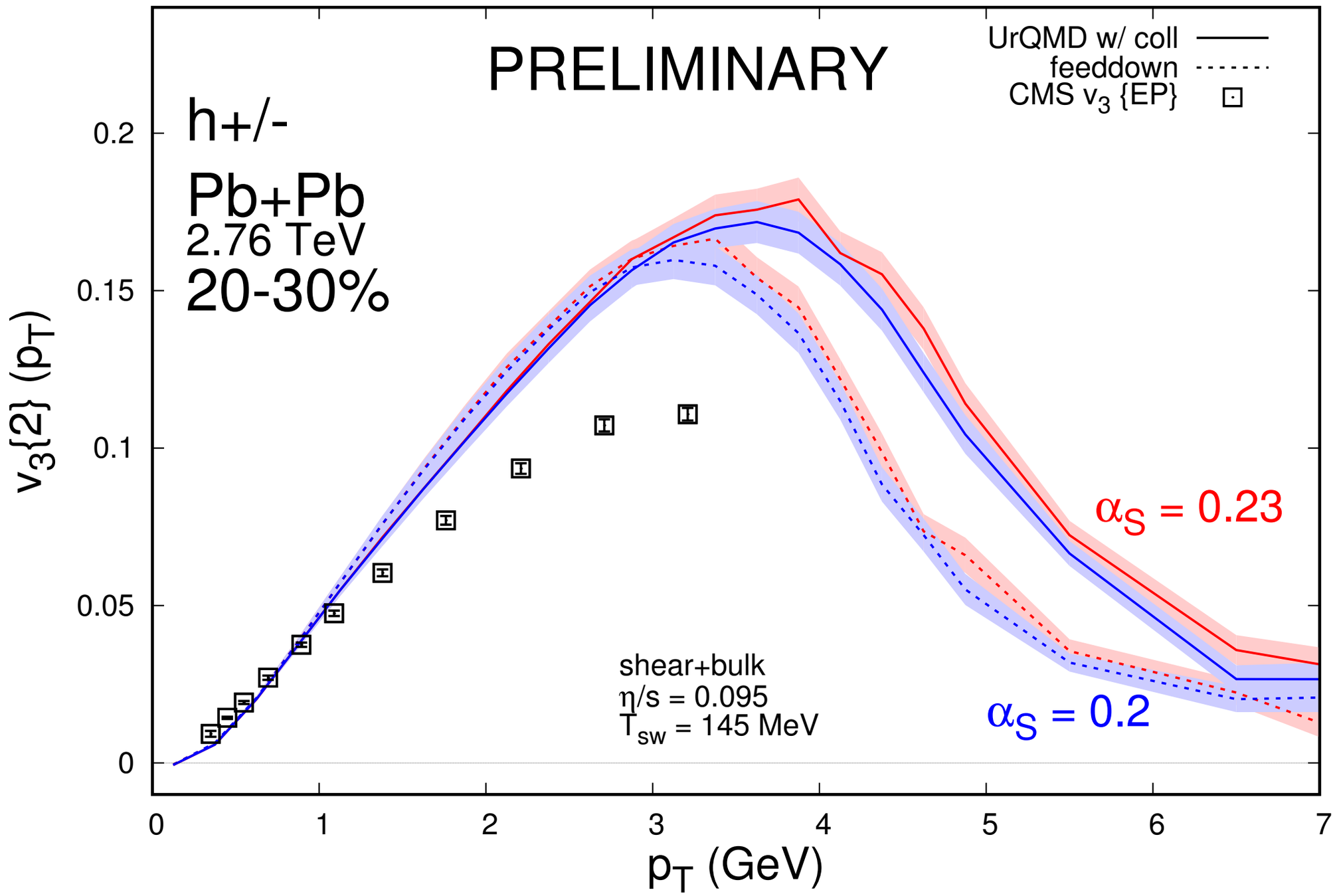
Hadronic rescattering makes it more anisotropic.

Elliptic flow (identified hadrons)



Hadronic rescattering makes it more anisotropic.

Triangular flow (charged hadrons)



Hadronic rescattering makes it more anisotropic.

Conclusion & Outlook

- A hybrid model, involving both the soft and hard physics of heavy ion collisions, is presented.
- The low-p_T distribution is well reproduced, while we need jet production and energy-loss to extend toward the higher p_T regime.
- Jet quenching in hadronic phase has significant effects in the intermediate-p_T regime of mini-jets.
- Long-term plan : using SMASH as an afterburner

Backup Slides

Model : Cooper-Frye sampling

F. Cooper and G. Frye (1974)

sampling particles according to the Cooper-Frye formula

1. sample number of particles based on Poisson distribution

$$\bar{N}|_{\text{1-cell}} = \begin{cases} [n_0(x) + \delta n_{\text{bulk}}(x)]\, u^\mu \Delta\Sigma_\mu & \text{if} \quad u^\mu \Delta\Sigma_\mu \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$n_0(x) = d \int \frac{d^3\mathbf{k}}{(2\pi)^3} f_0(\mathbf{k})$$

$$\delta n_{\text{bulk}}(x) = d \int \frac{d^3\mathbf{k}}{(2\pi)^3} \delta f_{\text{bulk}}(\mathbf{k})$$

2. sample momentum of each particles
according to the Cooper-Frye formula shown in the main slide

Model : jet energy loss

Radiative energy loss (AMY)

P. Arnold, G. Moore and L. Yaffe (2002)

$$\frac{d\Gamma}{dk}(p,k) = \frac{C_s g^2}{16\pi p^7}\frac{e^{k/T}}{e^{k/T}\mp 1}\frac{e^{(p-k)/T}}{e^{(p-k)/T}\mp 1}\left\{\begin{array}{cc} \frac{1+(1-x)^2}{x^3(1-x)^2} & q\to qg \\ N_f\frac{x^2+(1-x)^2}{x^2(1-x)^2} & g\to q\bar{q} \\ \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g\to gg \end{array}\right\}$$

$$\times\int\frac{d^2\mathbf{h}}{(2\pi)^2}2\mathbf{h}\cdot\mathrm{Re}\,\mathbf{F}(\mathbf{h},p,k)$$

$$2\mathbf{h} = i\,\delta E(\mathbf{h},p,k)\mathbf{F}(\mathbf{h},p,k) + g_s^2\int\frac{d^2\mathbf{q}_\perp}{(2\pi)^2}\frac{m_D^2}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2+m_D^2)}$$

$$\times\big\{(C_s-C_A/2)[\mathbf{F}(\mathbf{h})-\mathbf{F}(\mathbf{h}-k\,\mathbf{q}_\perp)]+(C_A/2)[\mathbf{F}(\mathbf{h})-\mathbf{F}(\mathbf{h}+p\,\mathbf{q}_\perp)]$$

$$+(C_A/2)[\mathbf{F}(\mathbf{h})-\mathbf{F}(\mathbf{h}-(p-k)\,\mathbf{q}_\perp)]\big\}$$

$$\delta E(\mathbf{h},p,k) = \frac{\mathbf{h}^2}{2pk(p-k)}+\frac{m_k^2}{2k}+\frac{m_{(p-k)}^2}{2(p-k)}-\frac{m_p^2}{2p}$$

$$\mathbf{h}\equiv(\mathbf{k}\times\mathbf{p})\times\mathbf{e}_{||}$$