Effects of hadronic rescattering in mini-jet production and energy loss at the LHC

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In collaboration with
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By colliding heavy nuclei (at RHIC and the LHC), we can
create quark-gluon plasma (QGP) and
study its properties (e.g. transport coefficients).

This can be done by creating a realistic dynamical model of heavy ion collisions.

Different aspects of QGP and hadronic matter influence each other. e.g.,
non-zero $\zeta/s$ alters the estimate of $\eta/s$
jet quenching in hadronic phase changes determination of the jet-medium interaction in QGP.

Goal: hybrid model covering all these different aspects.
PART 1
Hybrid approach and description of soft (low-$p_T$) physics

PART 2
Jet production and energy loss for hard (high-$p_T$) physics
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PART 2
Jet production and energy loss for hard (high-$p_T$) physics
Hybrid approach

Collision geometry

Pre-thermalization dynamics

Collective dynamics (hydrodynamics)

Jet production and energy loss

Hadronic re-scattering (microscopic transport)

figure by Steffen Bass
Model Structure

Collision Geometry

IP-Glasma
Initial Condition

MUSIC
hydrodynamics

Cooper-Frye
particlization

MARTINI
jet

PYTHIA/LUND
string fragmentation

Hadronic E-loss

UrQMD
cascade
Model Structure

- **Collision Geometry**

  - **IP-Glasma**
    - **Initial Condition**
      - **MUSIC**
        - hydrodynamics
      - Cooper-Frye particlization
      - **PYTHIA/LUND**
        - string fragmentation
      - **Hadronic E-loss**
        - **UrQMD**
          - cascade
      - **MARTINI**
        - jet
Model : IP-Glasma I.C.
B. Schenke, P. Tribedy and R. Venugopalan (2012)
Classical YM dynamics with color sources in nuclei

color charge distribution

\[ \langle \rho^a(x'_T) \rho^a(x''_T) \rangle = g^2 \mu_A^2 \delta^{ab} \delta^2(x'_T - x''_T) \]

\[ A^i_{(1,2)}(x_T) = \frac{i}{g} U_{(1,2)}(x_T) \partial_i U^\dagger_{(1,2)}(x_T) \]

\[ U_{(1,2)}(x_T) = \mathcal{P} \exp \left[ -ig \int dx^\pm \frac{\rho^{(1,2)}(x_T, x^\pm)}{\nabla^2_T - m^2} \right] \]

\[ A^i(\tau = +0) = A^i_{(1)} + A^i_{(2)} \]

\[ A^n(\tau = +0) = \frac{ig}{2} [A^i_{(1)}, A^i_{(2)}] \]

\[ \partial_\mu F^{\mu\nu} - ig [A_\mu, F^{\mu\nu}] = 0 \]

\[ T^{\mu \nu} (\tau = \tau_0) u^\nu = \epsilon u^\mu \]
Model: IP-Glasma I.C.

B. Schenke, P. Tribedy and R. Venugopalan (2012)

Classical YM dynamics with color sources in nuclei

well describes $v_n$ distribution

Model Structure

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Model: MUSIC hydro


Hydrodynamic equations of motion

Conservation equation \( \partial_\mu T^{\mu\nu} = 0 \)

Decomposition \( T^{\mu\nu} = \epsilon_0 u^\mu u^\nu - (P_0(\epsilon_0) + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu} \)

Local 3-metric \( \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \)

Local 3-gradient \( \nabla^\mu = \Delta^{\mu\nu} \partial_\nu \)
Model: MUSIC hydro


Equation of motion for viscous corrections

Shear viscosity relaxation equation

\[ \dot{\pi}^{\mu\nu} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + \frac{1}{\tau_\pi} \left( 2\eta \sigma^{\mu\nu} - \delta^{\mu\nu} \pi^{\rho\sigma} \theta + \varphi_7 \pi^{\langle \rho \sigma \rangle \alpha} + \lambda_\pi (\Pi \Pi \sigma^{\mu\nu}) \right) \]

Expansion rate

\[ \theta = \nabla_\mu u^\mu \]

Shear tensor

\[ \sigma^{\mu\nu} = \frac{1}{2} \left[ \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{3}{2} \Delta^{\mu\nu} (\nabla_\alpha u^\alpha) \right] \equiv \nabla^{\langle \mu \nu \rangle} \]
Model: MUSIC hydro


equation of motion for viscous corrections

Shear viscosity relaxation equation

\[
\frac{\dot{\pi}}{\pi} = -\frac{\pi^{\mu\nu}}{\tau_{\pi}} + \frac{1}{\tau_{\pi}} \left( 2\eta \sigma^{\mu\nu} - \delta_{\pi\pi} \pi^{\mu\nu} \theta + \varphi_{7} \pi^{\langle \mu \pi \rangle \alpha} \right) \\
- \tau_{\pi\pi} \pi^{\langle \mu \sigma^{\nu}\rangle \alpha} + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu}
\]

Shear viscosity

\[
\frac{\eta}{s} = \text{const}
\]
shear viscosity relaxation equation

\[ \dot{\pi} \langle \mu\nu \rangle = -\frac{\pi^{\mu\nu}}{\tau_{\pi}} + \frac{1}{\tau_{\pi}} \left( 2\eta \sigma^{\mu\nu} - \delta_{\pi\pi} \pi^{\mu\nu} \theta + \varphi \pi^{\mu\nu}\alpha \right. \\
\left. - \tau_{\pi\pi} \pi^{\mu\nu}\alpha + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} \right) \]

Model: MUSIC hydro


14-moment approximation in the small mass limit

G. Denicol, S. Jeon, and C. Gale (2014)

\[ \frac{\eta}{\tau_{\pi}} = \frac{1}{5} (\epsilon_0 + P_0) \quad \frac{\delta_{\pi\pi}}{\tau_{\pi}} = \frac{4}{3} \quad \frac{\lambda_{\pi\Pi}}{\tau_{\pi}} = \frac{6}{5} \quad \frac{\tau_{\pi\pi}}{\tau_{\pi}} = \frac{10}{7} \]

second-order transport coefficients
Model: MUSIC hydro


equation of motion for viscous corrections

bulk viscosity relaxation equation

\[
\dot{\Pi} = -\frac{\Pi}{\tau_{\Pi}} + \frac{1}{\tau_{\Pi}} \left( -\zeta \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} \right)
\]

Bulk viscosity relaxation equation

\[
\dot{\Pi} = -\frac{\Pi}{\tau_\Pi} + \frac{1}{\tau_\Pi} \left( -\zeta \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} \right)
\]

14-moment approximation in the small mass limit

G. Denicol, S. Jeon, and C. Gale (2014)

\[
\frac{\zeta}{\tau_\Pi} = 15 \left( \frac{1}{3} - c_s^2 \right)^2 (\epsilon_0 + P_0)
\]

\[
\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = \frac{2}{3} \quad \frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5} \left( \frac{1}{3} - c_s^2 \right)
\]

Model: MUSIC hydro


Equation of motion for viscous corrections

Second-order transport coefficients
Model : Equation of state

P. Huovinen, and P. Petreczky (2010)

Equation of state : **hadron gas** + **lattice data**

Only those included in UrQMD

Cross over phase transition around $T = 180$ MeV
Collision Geometry

IP-Glasma Initial Condition

MUSIC hydrodynamics

Cooper-Frye particlization

MARTINI jet

PYTHIA/LUND string fragmentation

Hadronic E-loss

UrQMD cascade
Model: Cooper-Frye sampling

F. Cooper and G. Frye (1974)

t 

\[ \sum (T = T_{sw}) \]

\[ \frac{dN}{d^3 p} = \int \sum \frac{p^\mu d^3 \Sigma_\mu}{E_p} \]

\( f(x, p) \)

sampling particles according to the Cooper-Frye formula
(transform hydrodynamic information into particles)

isothermal hypersurface

hydrodynamic information (temperature, flow velocity, ...)

transport

hydro
Model: Cooper-Frye sampling

F. Cooper and G. Frye (1974)

sampling particles according to the Cooper-Frye formula

\[
\begin{align*}
\frac{dN}{d^3\mathbf{p}}igg|_{\text{1-cell}} &= [f_0(x, p) + \delta f_{\text{shear}}(x, p) + \delta f_{\text{bulk}}(x, p)] \frac{p^\mu \Delta^3 \Sigma_\mu}{E_p} \\
f_0(x, p) &= \frac{1}{\exp \left(\frac{(p \cdot u)}{T}\right) + 1} \\
\delta f_{\text{shear}}(x, p) &= f_0(1 \pm f_0) \frac{p^\mu p^\nu \pi^{\mu\nu}}{2T^2(\epsilon_0 + P_0)} \\
\delta f_{\text{bulk}}(x, p) &= -f_0(1 \pm f_0) \frac{C_{\text{bulk}} \Pi}{T} \left[ c_s^2 (p \cdot u) - \frac{(-p^\mu p^\nu \Delta_{\mu\nu})}{3(p \cdot u)} \right] \\
\frac{1}{C_{\text{bulk}}} &= \frac{1}{3T} \sum_n m_n^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3 E_\mathbf{k}} \ f_{n,0}(1 \pm f_{n,0}) \left( c_s^2 E_\mathbf{k} - \frac{\left|\mathbf{k}\right|^2}{3E_\mathbf{k}} \right)
\end{align*}
\]

P. Bozek (2010)
Model: UrQMD cascade

Ultra-relativistic Quantum Molecular Dynamics

S. A. Bass et al. (1998)

Monte-Carlo implementation of transport theory

\[ p^\mu \frac{\partial}{\partial x^\mu} f_i(x, p) = C_i[f] \]

Which species? : 55 baryons + 32 mesons with masses up to 2.25 GeV

Cross sections : based on experimental data

Jet-hadron interaction by PYTHIA

Keeps track of particle trajectories
Parameters are tuned to fit multiplicity, mean $p_T$ and integrated flow coefficients $v_n$.

The bulk viscosity is crucial to describe those observables.

The shear viscosity $\frac{\eta}{s} = 0.095$ is favored.
The low- \( p_T \) spectra are well described.
Identified hadron $p_T$-spectra

arXiv:1704.04216

Jet production and energy-loss are necessary.
Jet production and energy-loss are necessary.

Elliptic flow (identified hadrons)

arXiv:1704.04216

Jet production and energy-loss are necessary.
PART 1
Hybrid approach and description of soft (low-$p_T$) physics

PART 2
Jet production and energy loss for hard (high-$p_T$) physics
Hard process at the position of binary collision (PYTHIA)

Energy loss
- Radiation (AMY)
- Collision (with thermal partons)

Fragmentation into hadrons (PYTHIA / LUND string model)

Model: MARTINI jets

Modular Algorithm for Relativistic Treatment of heavy IoN Interaction
B. Schenke, C. Gale and S. Jeon (2010)
Model: MARTINI jets

Modular Algorithm for Relativistic Treatment of heavy Ion Interaction
B. Schenke, C. Gale and S. Jeon (2010)

Hard process at the position of binary collision (PYTHIA)

\[ P_{\text{jet}}(\hat{p}_T, \text{min}) = \frac{\sigma_{\text{jet}}(\hat{p}_T, \text{min})}{\sigma_{\text{inel}}} \]

\[ \sigma_{\text{jet}}(\hat{p}_T, \text{min}) = \sum_{i,j} \int dx_1 f_i(x_1) \int dx_2 f_j(x_2) \hat{\sigma}_{(i,j)}(x_1, x_2; \hat{p}_T, \text{min}) \]
Model: MARTINI jets

Modular Algorithm for Relativistic Treatment of heavy Ion Interaction
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Hard process at the position of binary collision (PYTHIA)

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Hard process at the position of binary collision (PYTHIA)

Energy loss
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Fragmentation into hadrons (PYTHIA / LUND string model)
Model: jet energy loss

Radiative energy loss (AMY)


Collinear emission

Nearly on-shell intermediate propagator

Infinite number of diagrams ➤ Integral equations

figures by G-Y. Qin
Model: jet energy loss

Collisional energy loss (soft approximation)

B. Schenke, C. Gale and G-Y. Qin (2009)

G-Y. Qin et al. (2008)

\[
\begin{align*}
\frac{dE}{dt} \bigg|_{qq} &= \frac{2}{9} n_f \pi \alpha_s^2 T^2 \left[ \ln \frac{ET}{m_g^2} + c_f \frac{23}{12} + c_s \right] \\
\frac{dE}{dt} \bigg|_{gq} &= \frac{4}{3} \pi \alpha_s^2 T^2 \left[ \ln \frac{ET}{m_g^2} + c_b \frac{13}{6} + c_s \right] \\
\frac{dE}{dt} \bigg|_{gq} &= \frac{1}{2} n_f \pi \alpha_s^2 T^2 \left[ \ln \frac{ET}{m_g^2} + c_f \frac{13}{6} + c_s \right] \\
\frac{dE}{dt} \bigg|_{gg} &= 3\pi \alpha_s^2 T^2 \left[ \ln \frac{ET}{m_g^2} + c_b \frac{131}{48} + c_s \right]
\end{align*}
\]
Identified hadron $p_T$-spectra

Jet production and energy-loss are necessary.
Identified hadron $p_T$-spectra

Jet production and energy-loss are necessary.
Charged hadron $R_{AA}$ (inclusive)

Hadronic rescattering has significant effects on $p_T$ distribution.
Elliptic flow (identified hadrons)

Hadronic rescattering makes it more anisotropic.
Elliptic flow (identified hadrons)

Hadronic rescattering makes it more anisotropic.
Triangular flow (charged hadrons)

Hadronic rescattering makes it more anisotropic.

**Legend**
- UrQMD w/ coll feeddown
- CMS $v_3 \{EP\}$

**Graph Details**
- $h/-$
- Pb+Pb 2.76 TeV 20-30%
- $\alpha_S = 0.23$
- $\alpha_S = 0.2$
- $\eta/s = 0.095$
- $T_{sw} = 145$ MeV

**Equations**
- $v_3[2] (p_T)$
- $p_T$ (GeV)
Conclusion & Outlook

- A hybrid model, involving both the soft and hard physics of heavy ion collisions, is presented.

- The low-$p_T$ distribution is well reproduced, while we need jet production and energy-loss to extend toward the higher $p_T$ regime.

- Jet quenching in hadronic phase has significant effects in the intermediate-$p_T$ regime of mini-jets.

- Long-term plan: using SMASH as an afterburner
Backup Slides
Model: Cooper-Frye sampling

F. Cooper and G. Frye (1974)

sampling particles according to the Cooper-Frye formula

1. sample number of particles based on Poisson distribution

\[ \bar{N}|_{1\text{-cell}} = \begin{cases} 
[n_0(x) + \delta n_{\text{bulk}}(x)] u^\mu \Delta \Sigma_\mu & \text{if } u^\mu \Delta \Sigma_\mu \geq 0 \\
0 & \text{otherwise}
\end{cases} \]

\[ n_0(x) = d \int \frac{d^3k}{(2\pi)^3} f_0(k) \]

\[ \delta n_{\text{bulk}}(x) = d \int \frac{d^3k}{(2\pi)^3} \delta f_{\text{bulk}}(k) \]

2. sample momentum of each particles

according to the Cooper-Frye formula shown in the main slide
Model: jet energy loss

Radiative energy loss (AMY)


\[
\frac{d\Gamma}{dk}(p, k) = \frac{C_s g^2}{16\pi p^7} \frac{e^{k/T}}{e^{(p-k)/T} + 1} \left\{ \begin{array}{l}
\frac{1+(1-x)^2}{x^3(1-x)^2} \quad q \to qg \\
N_f \frac{x^2+(1-x)^2}{x^2(1-x)^2} \quad g \to q\bar{q} \\
\frac{1+x^4+(1-x)^4}{x^3(1-x)^3} \quad g \to gg
\end{array} \right. \\
\times \int \frac{d^2h}{(2\pi)^2} 2h \cdot \text{Re} \mathbf{F}(h, p, k)
\]

\[
2h = i \delta E(h, p, k) \mathbf{F}(h, p, k) + g_s^2 \int \frac{d^2q_\perp}{(2\pi)^2} \frac{m_D^2}{q_\perp^2(q_\perp^2 + m_D^2)} \times \left\{ (C_s - C_A/2)[\mathbf{F}(h) - \mathbf{F}(h - k \mathbf{q}_\perp)] + (C_A/2)[\mathbf{F}(h) - \mathbf{F}(h + p \mathbf{q}_\perp)] \right. \\
\left. + (C_A/2)[\mathbf{F}(h) - \mathbf{F}(h - (p - k) \mathbf{q}_\perp)] \right\}
\]

\[
\delta E(h, p, k) = \frac{h^2}{2pk(p-k)} + \frac{m_k^2}{2k} + \frac{m_{(p-k)}^2}{2(p-k)} - \frac{m_p^2}{2p} \\
h \equiv (k \times p) \times \mathbf{e}_\parallel
\]