Higher-order corrections for three-jet rate in electron-positron annihilation using the (anti-)$k_\perp$ algorithm

Z. Szőr

in collaboration with

A. Kardos, G.Somogyi and Z. Trócsányi

University of Debrecen

MTA-DE Particle Physics Research Group

QCD@LHC2017

29th August 2017

Why are jets important?

- In high-energy collider experiments detectors measure hadrons.
- In perturbation theory however we make calculations at the parton level.
- Assuming local hadron-parton duality, the degrees of freedom that can be used both in experiment and theory are *jets*.

Jets can be used to

- measure the strong coupling α_S.
- test the standard model.
- search for new physics.

Most popular process to extract α_S: three-jet production in e^+e^- annihilation.

$$O = \frac{\alpha_S}{2\pi} A + \mathcal{O}(\alpha_S^2)$$

The increasing precision of experiments require accurate theoretical predictions!

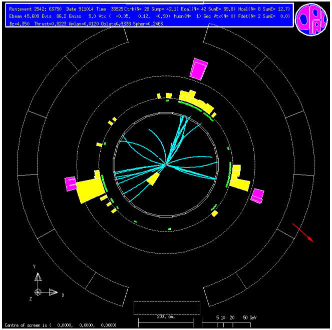


Figure: e^+e^- annihilation into three jets captured by the OPAL experiment

Jet rates

Relative production rate of n-jets

$$R_n(\vec{a}) = \frac{\sigma_{e^+e^- \to n\,\text{jets}}(\vec{a})}{\sigma_{e^+e^- \to \text{hadrons}}}$$

where $\vec{a}$ is some set of jet resolution parameters.

- Jets are defined according to jet clustering algorithms.
- Two type of jet algorithms: cone algorithms (usually infrared unsafe) and sequential algorithms.
- Widely used algorithm at LEP/LHC: $k_\perp$/anti-$k_\perp$

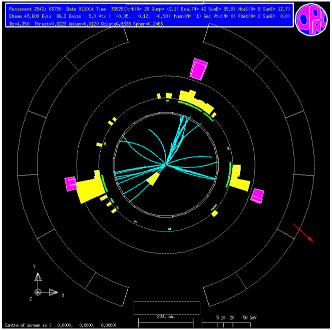


Figure: e^+e^- annihilation into three jets captured by the OPAL experiment

Fixed order calculations for $k_\perp$ and anti-$k_\perp$ are available at NNLO and next-to-double logarithmic resummation is known as well but no matched predictions for either at $\sqrt{Q^2} = 91.2$ GeV.

The general-$k_\perp$ algorithm for e^+e^- colliders[3]

Originally developed for hadron collisions, but can be adapted to e^+e^- colliders.
Two jet resolution parameters: $y_{\rm cut}$ and $E_{\rm cut}$

1. Calculate the following measures for every object i and j:

$$d_{ij} = \frac{\min(E_i^{2p}, E_j^{2p})(1 - \cos\theta_{ij})}{y_{\rm cut}}$$
$$d_{iB} = E_i^{2p}$$

   Repeat until there are no objects in the list, then go to Step 4.

2. If d_{ij} is the smaller one, combine object i and j and go to Step 1.
3. If d_{iB} is the smaller one, object i becomes a protojet, remove it from the clusterization list and go to Step 1.
4. Every protojet with $E_i > E_{\rm cut}$ is considered a resolved jet.

$p = 1$: $k_\perp$ algorithm[1], $p = -1$: anti-$k_\perp$ algorithm[2]

[1] [Catani, Dokshitzer, Olsson, Turnock, Webber '91]
[2] [Cacciari,Salam,Soyez '08]
[3] [Cacciari,Salam,Soyez '11]

Fixed-order predictions for the three-jet rate

The three-jet rate is defined as

$$R_3(\vec{a},\mu) = \frac{\sigma_{3-\mathrm{jet}}(\vec{a},\mu)}{\sigma_{\mathrm{tot}}}$$

In perturbation theory up to NNLO accuracy

$$R_3^{\mathrm{FO}}(\vec{a},\mu) = \frac{\alpha_{\mathrm{S}}(\mu)}{2\pi} A_3(\vec{a},\mu) + \left(\frac{\alpha_{\mathrm{S}}(\mu)}{2\pi}\right)^2 B_3(\vec{a},\mu) + \left(\frac{\alpha_{\mathrm{S}}(\mu)}{2\pi}\right)^3 C_3(\vec{a},\mu)$$

- $\vec{a} = y_{\mathrm{cut}}$ for $k_\perp$ and $\vec{a} = (y_{\mathrm{cut}}, E_{\mathrm{cut}})$ for anti-$k_\perp$
- $\sqrt{Q^2} = 91.2$ GeV and $E_{\mathrm{cut}} = [0.077, 0.0385]\sqrt{Q^2}$
- Previous NNLO results:
  [Gehrmann-De Ridder, Gehrmann, Glover Heinrich '08], [Weinzierl '11]
- Results presented in this talk were obtained with the `MCCSM` [A. Kardos] partonic Monte Carlo program based on the CoLoRFulNNLO method.

The CoLoRFulNNLO method

CoLoRFulNNLO : Completely Local subtRactions for Fully differential predictions at NNLO accuracy

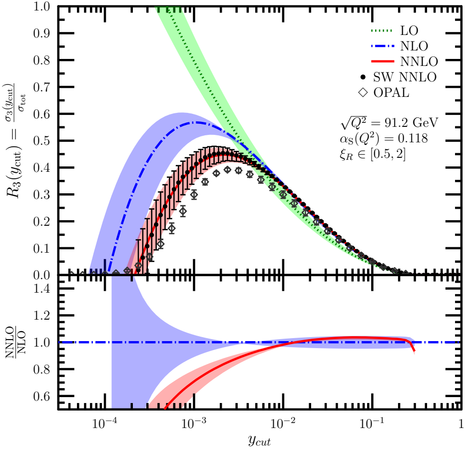
- Local subtraction terms, based on the universal infrared factorization properties of QCD squared matrix elements extended over the whole phase space via phase space mappings.
- Integrated counterterms: partially analytic and numeric.
- Poles cancel analytically.
- General for any number of colored partons in the final state.
- Implemented in the `MCCSM` partonic Monte Carlo code [A.Kardos].

$$\mathrm{d}\sigma_{m+2}^{\mathrm{NNLO}} = \left\{ \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2}^{(m)} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} J_{m}^{(m)} - \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} J_{m+1}^{(m)} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} J_{m}^{(m)} \right] \right\}_{d=4}$$

$$\mathrm{d}\sigma_{m+1}^{\mathrm{NNLO}} = \left\{ \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right] J_{m+1}^{(m)} - \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{A_1} \right] J_{m}^{(m)} \right\}_{d=4}$$

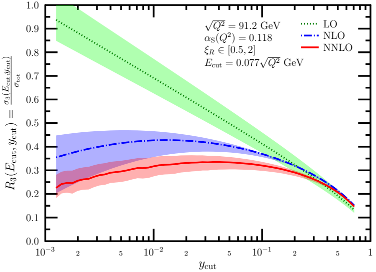
$$\mathrm{d}\sigma_{m}^{\mathrm{NNLO}} = \left\{ \mathrm{d}\sigma_{m}^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{A_1} \right] \right\}_{d=4} J_{m}^{(m}$$

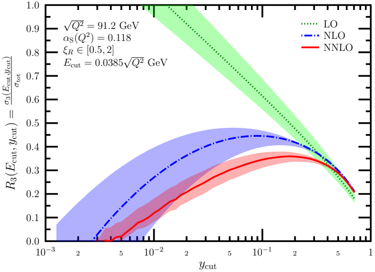
Fixed-order predictions for $k_\perp$ three-jet rate



Fixed order calculations using the CoLoRFulNNLO method compared with previous prediction [Weinzierl '11] and OPAL data.

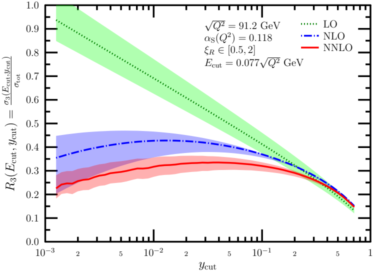
Fixed-order predictions for anti-$k_\perp$ three-jet rate

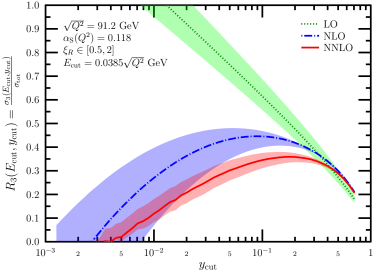




Fixed order predictions using the anti-$k_\perp$ algorithm with $E_{\rm cut} = 0.077\sqrt{Q^2}$ and $E_{\rm cut} = 0.0385\sqrt{Q^2}$.

Fixed-order predictions for anti-$k_\perp$ three-jet rate





Fixed order predictions using the anti-$k_\perp$ algorithm with $E_{\rm cut} = 0.077\sqrt{Q^2}$ and $E_{\rm cut} = 0.0385\sqrt{Q^2}$.

In the rest of the talk we will focus on the $k_\perp$-algorithm only, however the same steps can be repeated for anti-$k_\perp$.

Resummation of logarithmic terms

In small $y_{\rm cut}$ regions logarithmic terms become dominant and spoil the perturbative convergence, hence all order resummation of leading and next-to-leading logarithms is needed!

$$\alpha_{\rm S}(M_Z)\log^2(1/y_{\rm cut}) = 2.5 \text{ for } \alpha_{\rm S}(M_Z) = 0.118 \text{ and } y_{\rm cut} = 0.01$$

$$\alpha_{\rm S}(M_Z)\log(1/y_{\rm cut}) = 1.09 \text{ for } \alpha_{\rm S}(M_Z) = 0.118 \text{ and } y_{\rm cut} = 0.0001$$

Next-to-leading logarithmic accuracy (NLL)

$$R^{\rm NLL}(L) = \exp(Lg_1(\alpha_{\rm S}L) + g_2(\alpha_{\rm S}L) + \ldots)\mathcal{F}_{NLL}(L)\,,\ L = \log(1/y_{\rm cut})$$

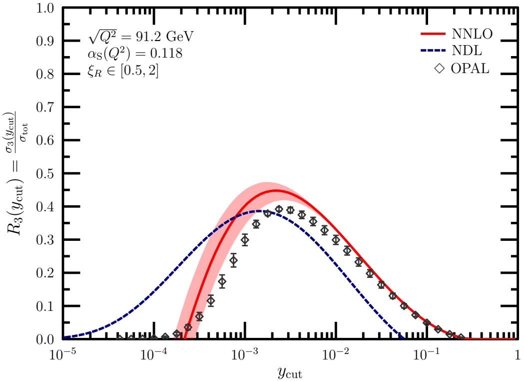
Next-to-double logarithmic accuracy (NDL)

$$R^{\rm NDL}(L) = \sum_{n=1}^{\infty} \alpha_{\rm S}^n \Big(G_{n,2n}L^{2n} + G_{n,2n-1}L^{2n-1} + \mathcal{O}(L^{2n-2})\Big)$$

For jet rates $\mathcal{F}_{NLL}$ is not known, only NDL resummation is available both for $k_\perp$[4] and anti-$k_\perp$[5].

[4] [Catani, Dokshitzer, Olsson, Turnock, Webber '91]
[5] [Gerwick, Gripaios, Schumann, Webber '13]

Resummation of logarithmic terms



Resummation provides physical behavior in the small y_{cut} limit, but does not describe the data. We need to combine the two kind of predictions!

The R-matching scheme

Combine predictions in a simple way:

$$R^{\mathrm{FO+NDL}} = R^{\mathrm{NDL}} - R^{\mathrm{NDL}}_{exp} + R^{\mathrm{FO}} \,,$$

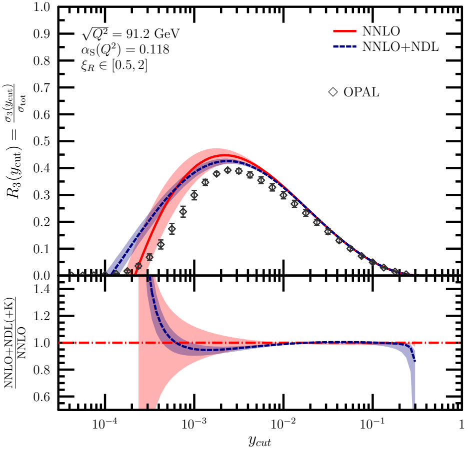
where

$$R^{\mathrm{NDL}}_{exp} = \frac{\alpha_{\mathrm{S}}}{2\pi} A_{exp} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^2 B_{exp} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^3 C_{exp}$$

Unknown subleading logs can spoil the expected physical behavior due to the remaining logaritmic singularities.

One can try to fit the coefficients of these subleading logs from fixed order predictions, however too many coefficients for jet rates.

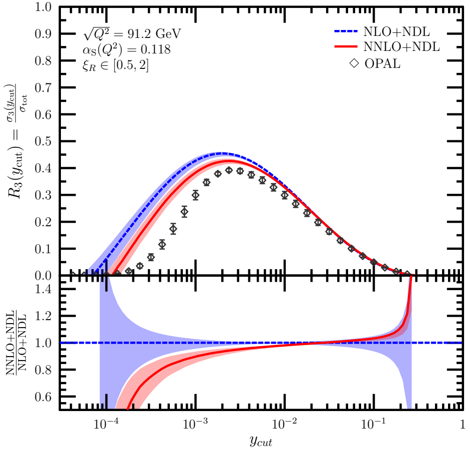
R-matching: NNLO vs. NNLO+NDL accuracy



NNLO vs. NNLO+NDL

Unphysical for $y_{\rm cut} < 10^{-4}$ due to sensitivity of the R-scheme to uncontrolled subleading logarithms.

R-matching: (N)NLO+NDL accuracy



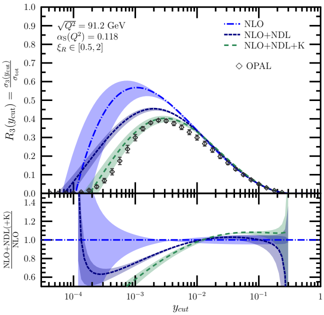
NLO+NDL vs. NNLO+NDL

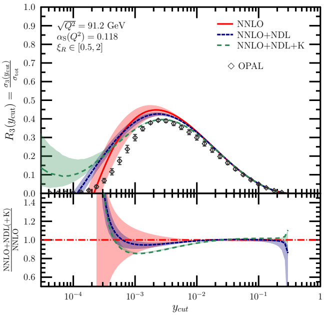
Unphysical for $y_{\rm cut} < 10^{-4}$ due to sensitivity of the R-scheme to uncontrolled subleading logarithms.

R-matching: NNLO+NDL+K accuracy

Include the cusp anomalous dimension to the resummation
[Nagy, Trócsányi '98]

$$R_3^{\mathrm{NDL+K}}(y_{\mathrm{cut}}) = \sum_{n=1}^{\infty} \alpha_{\mathrm{S}}^n \Big(G_{n,2n} \log^{2n} y_{\mathrm{cut}} + G_{n,2n-1} \log^{n-1} y_{\mathrm{cut}} + \mathcal{O}(\log^{2n-2} y_{\mathrm{cut}}) \Big)$$





Note that the improvement of the NLO+NDL+K prediction is only accidental below $y_{\mathrm{cut}} < 10^{-3}$.
NNLO+NDL+K is still unphysical for $y_{\mathrm{cut}} < 10^{-4}$.

The log R-matching scheme

Combine the logarithm of the predictions, such as

$$\log R^{\mathrm{FO+NDL}} = \log R^{\mathrm{NDL}} - (\log R^{\mathrm{NDL}})_{exp} + \tilde{R}^{\mathrm{FO}}$$

where

$$(\log R^{\mathrm{NDL}})_{exp} = \log\frac{\alpha_{\mathrm{S}}}{2\pi} + \log A_{exp} + \frac{\alpha_{\mathrm{S}}}{2\pi}\frac{B_{exp}}{A_{exp}} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^2 \frac{2A_{exp}C_{exp} - B_{exp}^2}{2A_{exp}^2} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^3 \frac{B_{exp}^3 - 3A_{exp}B_{exp}C_{exp} + 2A_{exp}^2 D_{exp}}{3A_{exp}^3}$$

and

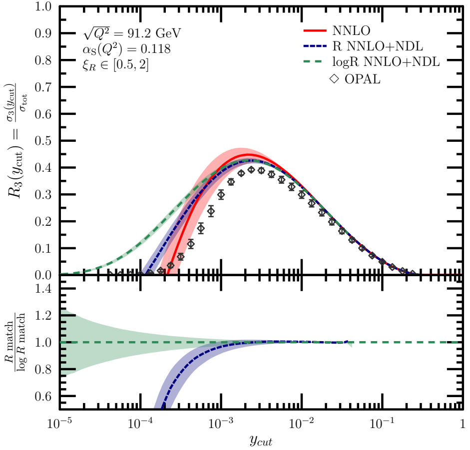
$$\tilde{R}^{\mathrm{FO}} = \log\frac{\alpha_{\mathrm{S}}}{2\pi} + \log A_{FO} + \frac{\alpha_{\mathrm{S}}}{2\pi}\frac{B_{FO}}{A_{FO}} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^2 \frac{2A_{FO}C_{FO} - B_{FO}^2}{2A_{FO}^2} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^3 \frac{B_{FO}^3 - 3A_{FO}B_{FO}C_{FO}}{3A_{FO}^3}$$

with

$$e^{\tilde{R}^{\mathrm{FO}}} \to \frac{\alpha_{\mathrm{S}}}{2\pi}A^{\mathrm{FO}} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^2 B^{\mathrm{FO}} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^3 C^{\mathrm{FO}}\,.$$

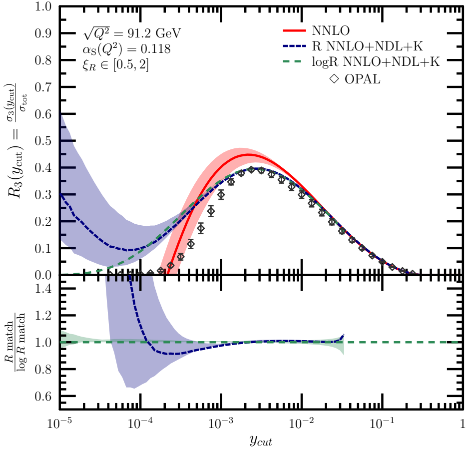
Multiplicative matching instead of additive, it is less sensitive to uncontrolled subleading logarithms.
Becomes unphysical in the LO kinematic limit.

R-matching vs. $\log R$-matching



Both the R-matching and the $\log R$-matching provides consitent prediction for $y_{\rm cut} \in (10^{-3}, 3 \times 10^{-2})$

R-matching vs. $\log R$-matching



Still both the R-matching and the $\log R$-matching provides consitent prediction for $y_{\text{cut}} \in (10^{-3}, 3 \times 10^{-2})$

Summary

- We calculated the three-jet rate at $\sqrt{Q^2} = 91.2$ GeV using the $k_\perp$ algorithm and the anti-$k_\perp$ clustering algorithm.
- We matched our fixed-order calculations with resummation at NDL accuracy in the R-scheme.
- We found that including the cusp anomalous dimension into the splitting kernels improves the prediction up to $y_{\mathrm{cut}} = 10^{-3}$ with NNLO predictions as well.
- We presented a way to match fixed-order predictions with resummation for jet rates in the $\log R$-matching scheme, which provides good description for the data over a wide range.

To improve further resummation for jet rates at higher accuracy would be necessary!

Thank you for your attention!

Backup slides

Resummation of next-to-double logarithms: $k_\perp$ algorithm

The resummed three-jet rate is

$$R_3^{\mathrm{NDL}} = 2[\Delta_q(Q)]^2 \int_{Q_0}^{Q} \mathrm{d}q \Gamma_q(Q,q)\Delta_g(q)\,, \quad Q_0 = \sqrt{Q^2}$$

[Catani, Dokshitzer, Olsson, Turnock, Webber '91]

Decay rates up to NDL:

$$\begin{aligned}
\Gamma_q(Q'',Q') &= \frac{2C_\mathrm{F}}{\pi}\frac{\alpha_\mathrm{S}(Q')}{Q'}\left[\log\frac{Q''}{Q'} - \frac{3}{4}\right] \\
\Gamma_g(Q'',Q') &= \frac{2C_\mathrm{A}}{\pi}\frac{\alpha_\mathrm{S}(Q')}{Q'}\left[\log\frac{Q''}{Q'} - \frac{11}{12}\right] \\
\Gamma_f(Q') &= \frac{n_\mathrm{f}}{3\pi}\frac{\alpha_\mathrm{S}(Q')}{Q'}\,.
\end{aligned}$$

Sudakov-factors:

$$\begin{aligned}
\Delta_q(Q'') &= \exp\left(-\int_{Q_0}^{Q''} \mathrm{d}Q' \Gamma_q(Q'',Q')\right) \\
\Delta_g(Q'') &= \exp\left(-\int_{Q_0}^{Q''} \mathrm{d}Q' \left[\Gamma_g(Q'',Q') + \Gamma_f(Q')\right]\right).
\end{aligned}$$

Resummation of next-to-double logarithms: $k_\perp$ algorithm

The resummed three-jet rate is

$$R_3^{\mathrm{NDL}} = 2[\Delta_q(Q)]^2 \int_{Q_0}^{Q} \mathrm{d}q \Gamma_q(Q,q)\Delta_g(q)\,, \quad Q_0 = \sqrt{Q^2}$$

[Catani, Dokshitzer, Olsson, Turnock, Webber '91]

At NDL accuracy it is sufficent to use the one-loop running formula for α_S

$$\alpha_\mathrm{S}(Q') = \frac{\alpha_\mathrm{S}(Q)}{1 - b_0 \alpha_\mathrm{S}(Q) \log \frac{Q}{Q'}}$$

where $b_0 = \beta_0/(2\pi)$. To include renormalization scale variation.

$$\alpha_\mathrm{S}(Q') = \frac{\alpha_\mathrm{S}(\mu)}{1 - b_0 \alpha_\mathrm{S}(\mu) \log \frac{\mu}{Q'}} = \frac{\alpha_\mathrm{S}(\mu)}{1 - b_0 \alpha_\mathrm{S}(\mu)(\log \frac{Q}{Q'} + \log \xi_R)}\,,$$

with $\xi_R = \mu/Q$.

Resummation of next-to-double logarithms: anti-$k_\perp$ algorithm

The resummed three-jet rate is given by

$$R_3^{\mathrm{NDL}} = 2[\Delta_q(\kappa,\lambda)]^2 \int_0^\kappa \mathrm{d}\kappa' \int_0^\lambda \mathrm{d}\lambda' \Gamma_q(\kappa',\lambda',\kappa)\Delta_g(\kappa',\lambda')\,,$$

[Gerwick, Gripaios, Schumann, Webber '13]

with

$$\kappa = \log\frac{Q}{E_{\mathrm{cut}}}\,,\qquad \lambda = \log\frac{1}{y_{\mathrm{cut}}}\,.$$

One-loop running similarly to the $k_\perp$ case, but expressed with κ and λ

$$\alpha_{\mathrm{S}}(\kappa',\lambda') = \frac{\alpha_{\mathrm{S}}(\mu)}{1-\frac{b_0}{2}\alpha_{\mathrm{S}}(\mu)[2(\kappa-\kappa'+\log\xi_R)+\lambda-\lambda']}\,.$$

The decay rates up to NDL accuracy:

$$\Gamma_q(\kappa',\lambda',\kappa'') = \frac{C_{\mathrm{F}}}{\pi}\alpha_{\mathrm{S}}(\kappa',\lambda')\left(1-\frac{3}{4}e^{\kappa'-\kappa''}\right)$$

$$\Gamma_g(\kappa',\lambda',\kappa'') = \frac{C_{\mathrm{A}}}{\pi}\alpha_{\mathrm{S}}(\kappa',\lambda')\left(1-\frac{11}{12}e^{\kappa'-\kappa''}\right)$$

$$\Gamma_f(\kappa',\kappa'') = \frac{n_{\mathrm{f}}}{6\pi}\alpha_{\mathrm{S}}(\kappa',\lambda')e^{\kappa'-\kappa''}$$

The Sudakov-factors defined in the same manner as in the case of $k_\perp$ resummation.

R-matching - anti-$k_\perp$ algorithm

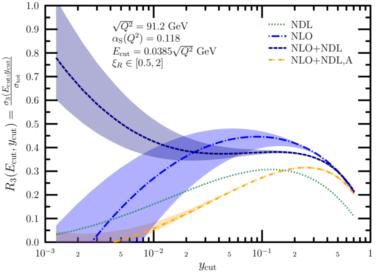


Figure: Matched predictions with NLO fixed order calculations using the anti-$k_\perp$ algorithm with $E_{\rm cut} = 0.0385\sqrt{Q^2}$.

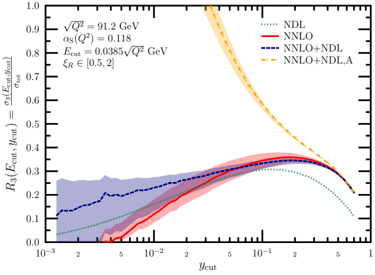


Figure: Matched predictions with NNLO fixed order calculations using the anti-$k_\perp$ algorithm with $E_{\rm cut} = 0.0385\sqrt{Q^2}$.

log R-matching: anti-$k_\perp$ algorithm

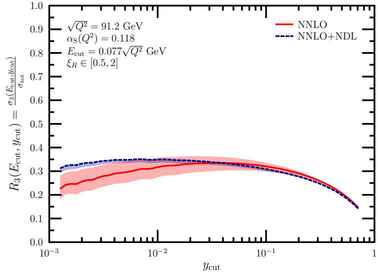


Figure: Matched predictions with NNLO fixed order calculations using the anti-$k_\perp$ algorithm with $E_{\rm cut} = 0.077\sqrt{Q^2}$.

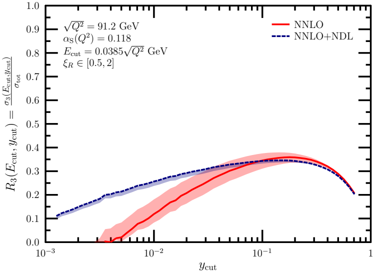


Figure: Matched predictions with NNLO fixed order calculations using the anti-$k_\perp$ algorithm with $E_{\rm cut} = 0.0385\sqrt{Q^2}$.