Parton densities from a parton branching solution of QCD evolution equations

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QCD@LHC

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- solve DGLAP evolution equation to obtain collinear Parton Distribution Functions (PDFs)

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But not only! → further advantages:

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But not only! $\rightarrow$ further advantages:

- obtain Transverse Momentum Dependent (TMD) PDFs
  Moreover:
  study the role of the soft-gluon resolution scale choice in the TMDs

More details in new papers:

- Phys.Lett. B772 (2017) 446-451
- arXiv:1708.03279

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The project is not just a standalone evolution package!

- already available in xFitter for determination of PDFs
  https://www.xfitter.org/xFitter/
- TMDlib and TMDplotter web page for easy acces to TMDs and collinear PDFs and plotting tool
  https://tmdlib.hepforge.org/doxy/html/index.html
  http://tmdplotter.desy.de/
- comparison with measurements through interface with CASCADE and for example Rivet
  https://cascade.hepforge.org/
  https://rivet.hepforge.org/

Why Transverse Momentum Dependent PDFs?

Motivation:

Parton Shower crucial for obtaining predictions for processes at high-energy hadron colliders open problem: shower's transverse momentum kinematics.

One of the approaches to deal with the problem: Transverse Momentum Dependent (TMD) formalism based on TMD form of factorization .

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One of the approaches to deal with the problem: Transverse Momentum Dependent (TMD) formalism based on TMD form of factorization .

What is TMD PDF?

- TMD PDF is a generalization of a concept of the PDF.
- TMD: depends not only on x and μ^2 but also on k_T: $TMD(x, \mu^2, k_T)$

Goal: **TMD PDFs for all flavours, all x, μ^2 and k_T**

There are plenty of areas, where TMDs play an important role
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Introduction to the method

DGLAP evolution equation

DGLAP evolution equation for momentum weighted parton density $xf(x,\mu^2) = \widetilde{f}(x,\mu^2)$

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz P_{ab}\left(\mu^2,z\right) \widetilde{f}_b\left(\frac{x}{z},\mu^2\right) \quad (1)$$

a, b- quark (N_f flavours) or gluon, x- longitudinal momentum fraction of the proton carried by a parton a,

$z-$ splitting variable, μ- evolution mass scale

splitting function:

$$P_{ab}\left(\mu^2,z\right) = D_{ab}\left(\mu^2\right)\delta(1-z) + K_{ab}\left(\mu^2\right)\frac{1}{(1-z)_+} + R_{ab}\left(\mu^2,z\right), \quad (2)$$

$\int_0^1 f(x)g(x)_+ dx = \int_0^1 (f(x) - f(1))g(x)dx$

$R_{ab}\left(\mu^2,z\right)$ has no power divergences $(1-z)^{-n}$ for $z \to 1$.

As long as $P_{ab}\left(\mu^2,z\right)$ has this structure, the formalism presented today can be applied (LO, NLO, NNLO).

DGLAP evolution equation

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As long as $P_{ab}\left(\mu^2, z\right)$ has this structure, the formalism presented today can be applied (LO, NLO, NNLO).

Two potential problems for numerical solution: (details in Backup!)

- piece with $\delta(1-z)$ → treated with *momentum sum rule* $\sum_c \int_0^1 \mathrm{d}z z P_{ca}\left(\mu^2, z\right) = 0$,
- integrals separately divergent for $(z \to 1)$ → solved by a parameter z_M: $\int_x^1 \to \int_x^{z_M}$

Sudakov form factor

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^{z_M} \mathrm{d}z \; P_{ab}^R\left(\mu^2, z\right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) - \widetilde{f}_a\left(x, \mu^2\right) \sum_c \int_0^{z_M} \mathrm{d}z \; z P_{ca}^R\left(\mu^2, z\right)$$

where $P_{ab}^R\left(\mu^2, z\right) = R_{ab}\left(\mu^2, z\right) + K_{ab}\left(\mu^2\right)\frac{1}{1-z}$ - *real* part of the splitting function.

Sudakov form factor

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^{z_M} \mathrm{d}z\; P^R_{ab}\left(\mu^2, z\right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) - \widetilde{f}_a\left(x, \mu^2\right) \sum_c \int_0^{z_M} \mathrm{d}z\; z P^R_{ca}\left(\mu^2, z\right)$$

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Define the *Sudakov form factor*:

$$\Delta_a(\mu^2) = \exp\left(-\int_{\ln\mu_0^2}^{\ln\mu^2} d\left(\ln\mu'^2\right) \sum_b \int_0^{z_M} dz z P^R_{ba}\left(\mu'^2, z\right)\right)$$

Probability of evolving between μ_0^2 and μ^2 without any resolvable branching

Sudakov form factor

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^{z_M} \mathrm{d}z\; P^R_{ab}\left(\mu^2,z\right)\widetilde{f}_b\left(\frac{x}{z},\mu^2\right) - \widetilde{f}_a\left(x,\mu^2\right)\sum_c \int_0^{z_M} \mathrm{d}z\; zP^R_{ca}\left(\mu^2,z\right)$$

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$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^{z_M} dzP^R_{ab}\left(\mu^2,z\right)\widetilde{f}_b\left(\frac{x}{z},\mu^2\right) + \widetilde{f}_a\left(x,\mu^2\right)\frac{1}{\Delta_a(\mu^2)}\frac{d\Delta_a(\mu^2)}{d\ln\mu^2}$$

Sudakov form factor and Parton Branching

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^{z_M} dz P_{ab}^R\left(\mu^2, z\right)\widetilde{f}_b\left(\frac{x}{z},\mu^2\right) + \widetilde{f}_a\left(x,\mu^2\right)\frac{1}{\Delta_a(\mu^2)}\frac{d\Delta_a(\mu^2)}{d\ln\mu^2}$$

After integration:

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{\ln\mu_0^2}^{\ln\mu^2} d\ln\mu'^2 \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)}\sum_b \int_x^{z_M} dz P_{ab}^R\left(\mu'^2, z\right)\widetilde{f}_b\left(\frac{x}{z},\mu'^2\right)$$

Sudakov form factor and Parton Branching

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^{z_M} dz P^R_{ab}\left(\mu^2, z\right)\widetilde{f}_b\left(\frac{x}{z},\mu^2\right) + \widetilde{f}_a\left(x,\mu^2\right)\frac{1}{\Delta_a(\mu^2)}\frac{d\Delta_a(\mu^2)}{d\ln\mu^2}$$

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$\widetilde{f}_b\left(\frac{x}{z},\mu^{2\prime}\right)$ has it's own evolution history!

$$\widetilde{f}_b\left(\frac{x}{z},\mu'^2\right) = \widetilde{f}_b\left(\frac{x}{z},\mu_0^2\right)\Delta_b(\mu'^2) + \int_{\ln\mu_0^2}^{\ln\mu'^2} d\ln\mu''^2 \frac{\Delta_a(\mu'^2)}{\Delta_a(\mu''^2)} \sum_c \int_x^{z_M} dz' P^R_{bc}\left(\mu^{2\prime\prime}, z'\right)\widetilde{f}_c\left(\frac{x}{zz'},\mu''^2\right)$$

Sudakov form factor and Parton Branching

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^{z_M} dz P_{ab}^R\left(\mu^2, z\right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) + \widetilde{f}_a\left(x,\mu^2\right) \frac{1}{\Delta_a(\mu^2)} \frac{d\Delta_a(\mu^2)}{d\ln\mu^2}$$

After integration:

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{\ln\mu_0^2}^{\ln\mu^2} d\ln\mu'^2 \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \sum_b \int_x^{z_M} dz P_{ab}^R\left(\mu'^2, z\right) \widetilde{f}_b\left(\frac{x}{z}, \mu'^2\right)$$

$\widetilde{f}_b\left(\frac{x}{z}, \mu^{2\prime}\right)$ has it's own evolution history!

$$\widetilde{f}_b\left(\frac{x}{z}, \mu'^2\right) = \widetilde{f}_b\left(\frac{x}{z}, \mu_0^2\right)\Delta_b(\mu'^2) + \int_{\ln\mu_0^2}^{\ln\mu'^2} d\ln\mu''^2 \frac{\Delta_a(\mu'^2)}{\Delta_a(\mu''^2)} \sum_c \int_x^{z_M} dz' P_{bc}^R\left(\mu^{2\prime\prime}, z'\right) \widetilde{f}_c\left(\frac{x}{zz'}, \mu''^2\right)$$

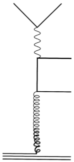
This problem has an iterative solution and an interpretation in terms of Parton Branching process!

Iterative solution

Example for a= gluon:

$$\widetilde{f}_a(x, \mu^2) = \widetilde{f}_a(x, \mu_0^2)\Delta_a(\mu^2)$$

probability of evolving from μ_0^2 to μ^2 without any resolvable branching.



Iterative solution

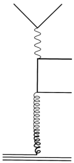
Example for a= gluon:

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{\ln\mu_0^2}^{\ln\mu^2} d\ln\mu'^2 \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \sum_b \int_x^{z_M} dz P_{ab}^R\left(\mu'^2, z\right) \widetilde{f}_b\left(\frac{x}{z}, \mu_0^2\right) \Delta_b(\mu'^2)$$

probability of evolving from μ_0^2 to μ^2 without any resolvable branching.

probability of a evolving from μ'^2 to μ^2 without any resolvable branching.

probability of b evolving from μ_0^2 to μ'^2 without any resolvable branching.



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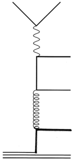

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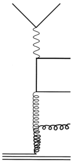

Iterative solution

Example for a= gluon:

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{\ln\mu_0^2}^{\ln\mu^2} d\ln\mu'^2 \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \sum_b \int_x^{z_M} dz P_{ab}^R\left(\mu'^2, z\right) \widetilde{f}_b\left(\frac{x}{z},\mu_0^2\right)\Delta_b(\mu'^2) + \ldots$$

probability of evolving from μ_0^2 to μ^2 without any resolvable branching.

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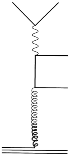

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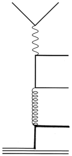

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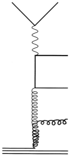


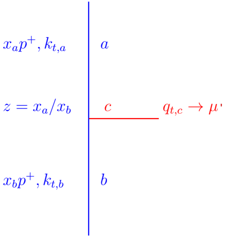
OR

...

This problem has an iterative solution which can be obtained from parton branching method.

k_t dependence

Parton branching method: for every branching μ'^2 is generated and available.



How to connect μ with $q_{t,c}$ of the emitted and $k_{t,a}$ of the propagating parton?

- "q_t- ordering": $\vec{q}_{t,c}^{\,2} = \mu'^2$.
- "angular ordering": $\vec{q}_{t,c}^{\,2} = (1-z)^2\mu'^2$
- "virtuality ordering": $\vec{q}_{t,c}^{\,2} = (1-z)\mu'^2$

$$\vec{k}_{t,a} = \vec{k}_{t,b} - \vec{q}_{t,c}$$

$k_{t,a}$ contains the whole history of the evolution.
In this method kinematics is treated properly at every branching.

z_M choice

Partons emitted with a transverse distance smaller than a certain value given by a resolution scale can not be resolved $\rightarrow$ branchings with $z > z_M$ are non- resolvable

Normally treated with the plus prescription but
integrals in evolution equation separately divergent $(z \rightarrow 1)$:
$\rightarrow$ solved by a parameter z_M: $\int_x^1 \rightarrow \int_x^{z_M}$

z_M- defines our resolution scale

Different choices of z_M:

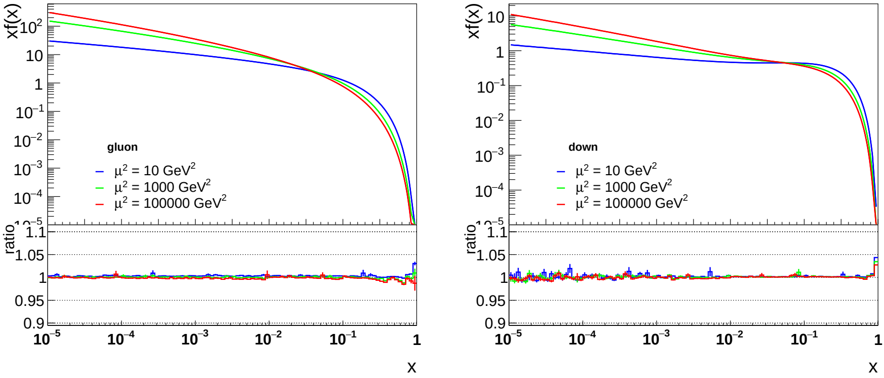
- z_M - fixed
- z_M - can change dynamically with the scale (resolution scale different for different scales) Replace $q_{t,c}$ with some minimum q_0:
  - "angular ordering": $\vec{q}_0^{\,2} = (1-z)^2 \mu'^2 \rightarrow z_M = 1 - \left(\frac{q_0}{\mu'}\right)$
  - "virtuality ordering": $\vec{q}_0^{\,2} = (1-z) \mu'^2 \rightarrow z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2$

Collinear PDFs from parton branching method

NLO comparison with semi analytical methods More details in: arXiv:1708.03279

Initial distribution: $\widetilde{f}_{b_0}(x_0, \mu_0^2)$ - from QCDnum

The evolution performed with parton branching method up to a given scale μ^2.
Obtained distribution compared with a pdf calculated at the same scale by semi analytical method (QCDnum)
Results for fixed $1 - z_M = 10^{-5}$.

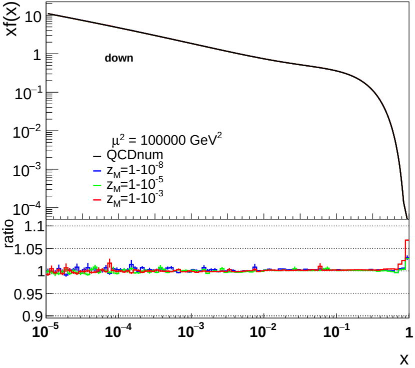


Upper plots: collinear pdfs from the parton branching method
Lower plots: ratios of the pdfs from a parton branching method and pdfs from QCDnum.

Very good agreement with the results coming from semi analytical methods (QCDnum).

Cross check for different fixed z_M More details in: arXiv:1708.03279

Comparison of the results for different fixed z_M values (all independent of branching scale).

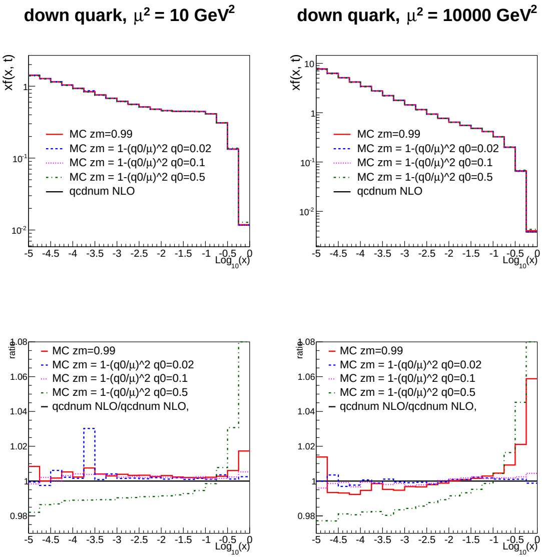


Upper plot: collinear pdfs from the parton branching method
Lower plot: ratios of the pdfs from the parton branching method and pdfs from QCDnum.

There is no dependence on z_M as long as z_M large enough.

Here results at NLO, at LO the same conclusion.

Dynamical z_M



Results for $z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2$

μ'- scale at which the branching happens
q_0 - a free parameter describing the resolution scale

On the plots: Comparison of collinear PDFs for different q_0 values

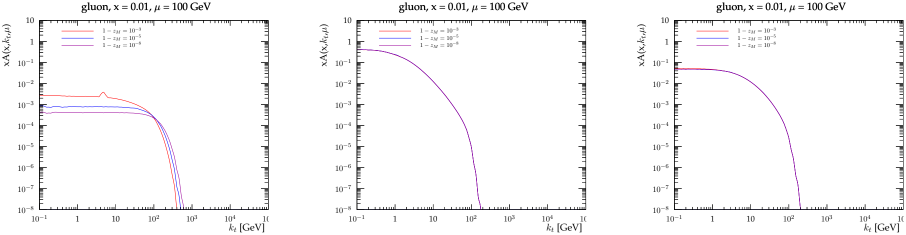
$q0 = 0.02$ GeV - the same behaviour as for fixed z_M being close to 1 (consistent with QCDNUM)

$q0 = 0.5$ GeV - similar behaviour as for fixed z_M being too far away from 1 (not consistent with QCDNUM)
Might be important for parton showers!

Results for TMDs

TMD PDFs from different k_t definition at NLO

Let's come back to fixed z_M
Reminder: for collinear PDFs there was no z_M dependence.
What about z_M dependence for TMDs?
Here the results for fixed z_M



q_t- ordering | angular ordering | virtuality ordering

large z_M - a lot of soft gluons!
q_t- ordering: for every z_M value we obtain different TMD
→ not physical behaviour, q_t- ordering shouldn't be used
For virtuality and angular ordering no z_M dependence (suppression of soft gluons because of the $(1 - z)$ term)

Fit of integrated TMDs for all flavours to HERA DIS data with xFitter

Procedure of the fit to the HERA 1+2 F_2 data More details in: arXiv:1708.03279

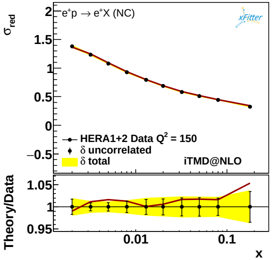
Goal: TMD PDF sets for all flavours, all x, Q^2 and k_T

- A kernel A_a^b is determined from the parton branching method from a toy starting distribution: $f_{0,b} = \delta(1-x)$.
- xFitter chooses a starting distribution $A_{0,b}$ and performs a convolution of the kernel A_a^b with the starting distribution $A_{0,b}$ to obtain a parton density

$$\widetilde{f}_a(x,\mu^2) = \int_0^\infty \frac{\mathrm{d}k_T^2}{k_T^2} \underbrace{\int \mathrm{d}x' A_{0,b}(x') \frac{x'}{x} A_a^b\left(\frac{x'}{x}, k_T^2, \mu^2\right)}_{\widetilde{f}_a(x,\mu^2,k_T^2)} \tag{3}$$

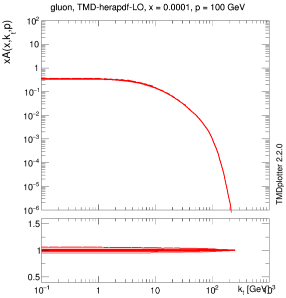
- Obtained parton density $\widetilde{f}_a(x,\mu^2)$ is fitted to the F_2 data and χ^2 is calculated. Data: arXiv:1506.06042v3, Abramowicz, H. and others.
- The procedure is repeated with the new starting distributions $A_{0,b}$ many times to minimize χ^2.

A very good $\chi^2/ndf \sim 1.2$ is obtained for $3.5 < Q^2 < 30000$ GeV^2.

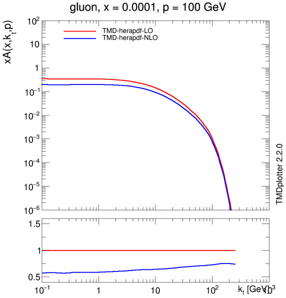


TMDs from fits

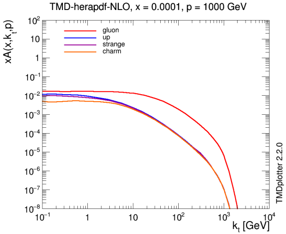
TMDs sets for all flavours with uncertainties were obtained from the fit



TMDs with experimental uncertainties from the fit.



Comparison of the LO and NLO TMDs from the fit.



At small k_T (no branching or just a few branchings), the difference in the quark TMDs comes from initial distributions.
At large k_T (many branchings) TMDs for quarks the same.

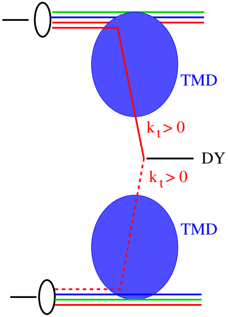
TMD sets released soon, working on model uncertainties

First applications of our TMDs

Current studies Work in progress

Use TMDs instead of PS for inclusive quantities:

LO Drell–Yan for $q\bar{q} \rightarrow Z_0$:
k_t according to TMD ($\hat{s}$ fixed, x_1, x_2 change)



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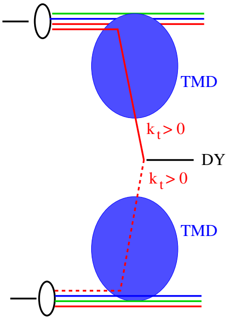
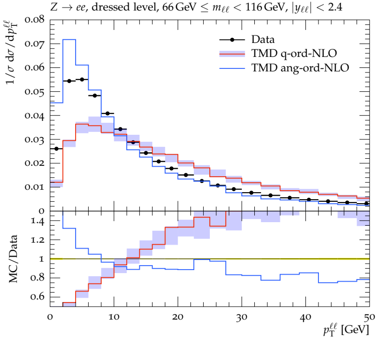
k_t according to TMD ($\hat{s}$ fixed, x_1, x_2 change)

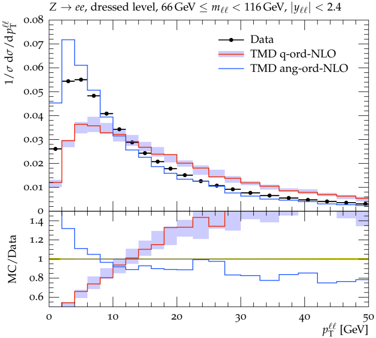




Current studies Work in progress

Use TMDs instead of PS for inclusive quantities:

- TMD fitted to HERA data reproduces correctly the shape of Z p_t spectrum
- NO tuning/adjustment of parameters is done
  all is coming from PDF fit, no free parameters after fit (in contrast to what is being done in MC tuning)
- transverse momentum originates directly from parton branching
- difference between angular ordering and virtuality ordering observed also in physical observable
- free parameters: intrinsic k_t (here gauss with width=0.7 GeV), scale in α_s (here it is μ but it could be k_t), fit to F2 (including k_t dependence of ME)



Summary

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New approach to solve coupled gluon and quark DGLAP evolution equation with a parton branching method at LO and NLO was shown.

Advantages:

- it reproduces exactly semi-analytical solution for collinear PDFs (results consistent with QCDNum),
  moreover:
- extraction of TMD PDFs
- options to study different orderings and different definitions of the resolution scale for collinear and TMD PDFs available within this framework
- TMDs are not defined consistently with pt ordering, but angular ordering and virtuality ordering give consistent definition
- fit to F2 Hera data at LO and NLO was performed within xFitter,
  TMDs sets for all flavours with uncertainties were obtained from the fit,
- application in measurements: use TMD instead of PS (first attempts look promising!)

Prospects:

- TMD sets released soon,
- more applications in measurements and direct usage in PS matched calculation.

Thank you!

Back up

DGLAP evolution equation

DGLAP evolution equation for momentum weighted parton density $xf(x,\mu^2) = \widetilde{f}(x,\mu^2)$

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz P_{ab}\left(\mu^2, z\right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right)$$

a, b- quark (N_f flavours) or gluon, x- longitudinal momentum fraction of the proton carried by a parton a, z- splitting variable, μ- evolution variable

a structure of a splitting function:

$$P_{ab}\left(\mu^2, z\right) = D_{ab}\left(\mu^2\right)\delta(1-z) + K_{ab}\left(\mu^2\right)\frac{1}{(1-z)_+} + R_{ab}\left(\mu^2, z\right),$$

$\int_0^1 f(x)g(x)_+ dx = \int_0^1 (f(x) - f(1))g(x)dx$
$D_{ab}\left(\mu^2\right) = \delta_{ab} d_a\left(\mu^2\right)$, $K_{ab}\left(\mu^2\right) = \delta_{ab} k_a\left(\mu^2\right)$,
$R_{ab}\left(\mu^2, z\right)$ contains logarithmic terms in $\ln(1-z)$ and has no power divergences $(1-z)^{-n}$ for $z \to 1$.

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$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2}=\sum_b\int_x^1 dz\left(K_{ab}\left(\mu^2\right)\frac{1}{(1-z)}+R_{ab}\left(\mu^2,z\right)\right)\widetilde{f}_b\left(\frac{x}{z},\mu^2\right)+$$
$$-\sum_b\widetilde{f}_b\left(x,\mu^2\right)\int_0^1 dz\left(K_{ab}\left(\mu^2\right)\frac{1}{(1-z)}-D_{ab}\left(\mu^2\right)\delta(1-z)\right)$$

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$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz \left(K_{ab}\left(\mu^2\right)\frac{1}{(1-z)} + R_{ab}\left(\mu^2, z\right)\right)\widetilde{f}_b\left(\frac{x}{z},\mu^2\right) + \\ -\sum_b \widetilde{f}_b\left(x,\mu^2\right)\int_0^1 dz\left(K_{ab}\left(\mu^2\right)\frac{1}{(1-z)} - D_{ab}\left(\mu^2\right)\delta(1-z)\right)$$

Two potential problems for numerical solution:

- presence of the delta function,
- integrals separately divergent for $z \to 1$.

Momentum sum rule

To get rid of the delta function:

We use *momentum sum rule* $\sum_c \int_0^1 \mathrm{d}z z P_{ca}\left(\mu^2, z\right) = 0$:

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$$-\sum_b \widetilde{f}_b\left(x,\mu^2\right)\int_0^1 dz\left(K_{ab}\left(\mu^2\right)\frac{1}{(1-z)} - D_{ab}\left(\mu^2\right)\delta(1-z)\right) +$$

$$-\widetilde{f}_a(x,\mu^2)\sum_c \int_0^1 \mathrm{d}z z P_{ca}\left(\mu^2, z\right) =$$

$$= \sum_b \int_x^1 \mathrm{d}z \left(K_{ab}\left(\mu^2\right)\frac{1}{1-z} + R_{ab}\left(\mu^2, z\right)\right)\widetilde{f}_b\left(\frac{x}{z},\mu^2\right) +$$

$$-\widetilde{f}_a\left(x,\mu^2\right)\sum_c \int_0^1 \mathrm{d}z\, z\left(K_{ca}\left(\mu^2\right)\frac{1}{1-z} + R_{ca}\left(\mu^2, z\right)\right)$$

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$$-\sum_b \widetilde{f}_b\left(x,\mu^2\right)\int_0^1 dz \left(K_{ab}\left(\mu^2\right)\frac{1}{(1-z)} - D_{ab}\left(\mu^2\right)\delta(1-z)\right) +$$

$$-\widetilde{f}_a(x,\mu^2)\sum_c \int_0^1 \mathrm{d}z z P_{ca}\left(\mu^2, z\right) =$$

$$= \sum_b \int_x^1 \mathrm{d}z \left(K_{ab}\left(\mu^2\right)\frac{1}{1-z} + R_{ab}\left(\mu^2, z\right)\right)\widetilde{f}_b\left(\frac{x}{z},\mu^2\right) +$$

$$-\widetilde{f}_a\left(x,\mu^2\right)\sum_c \int_0^1 \mathrm{d}z\, z\left(K_{ca}\left(\mu^2\right)\frac{1}{1-z} + R_{ca}\left(\mu^2, z\right)\right)$$

We got rid of the delta function,
both pieces of the equation written in the same way.
Virtual and non-resolvable pieces still included.

Divergence for $z \to 1$

To avoid divergence when $z \to 1$ a cut off must be introduced.
z_M - defines resolvable branching

$$\sum_b \int_x^1 \mathrm{dz} \left(K_{ab}\left(\mu^2\right) \frac{1}{1-z} + R_{ab}\left(\mu^2, z\right) \right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) +$$

$$- \widetilde{f}_a\left(x, \mu^2\right) \sum_c \int_0^1 \mathrm{dz}\, z \left(K_{ca}\left(\mu^2\right) \frac{1}{1-z} + R_{ca}\left(\mu^2, z\right) \right)$$

$$\to \sum_b \int_x^{z_M} \mathrm{dz} \left(K_{ab}\left(\mu^2\right) \frac{1}{1-z} + R_{ab}\left(\mu^2, z\right) \right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) +$$

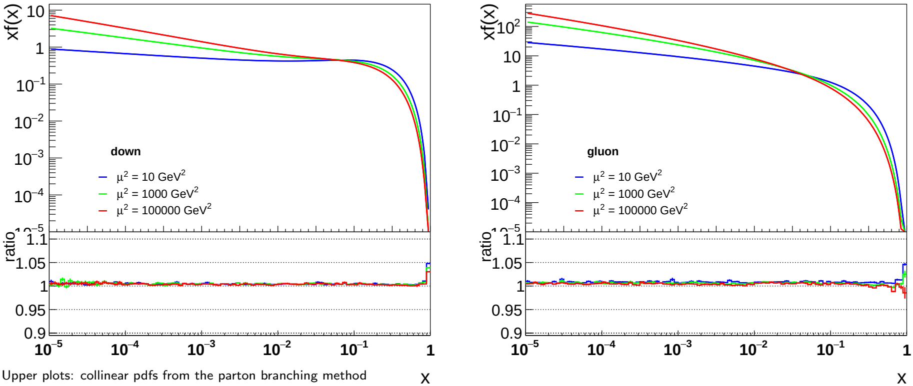
$$- \widetilde{f}_a\left(x, \mu^2\right) \sum_c \int_0^{z_M} \mathrm{dz}\, z \left(K_{ca}\left(\mu^2\right) \frac{1}{1-z} + R_{ca}\left(\mu^2, z\right) \right)$$

It can be shown that terms $\int_{z_M}^1$ skipped are of order $\mathcal{O}(1 - z_M)$.

LO comparison with semi analytical methods

Initial distribution: $\widetilde{f}_{b_0}(x_0, \mu_0^2)$ - from QCDnum

The evolution performed with parton branching method up to a given scale μ^2.
Obtained distribution compared with a pdf calculated at the same scale by semi analytical method (QCDnum)



Upper plots: collinear pdfs from the parton branching method
Lower plots: ratios of the pdfs from a parton branching method and pdfs from QCDnum.

Very good agreement with the results coming from semi analytical methods (QCDnum).