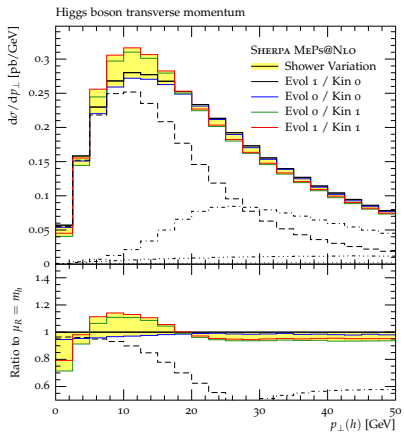


Analytical and Numerical Approaches to (N)LL resummation

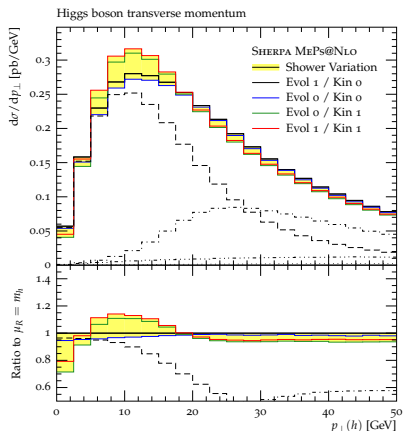
Stefan Hoeche, Daniel Reichelt, Frank Siegert



August 29, 2017
QCD@LHC 2017



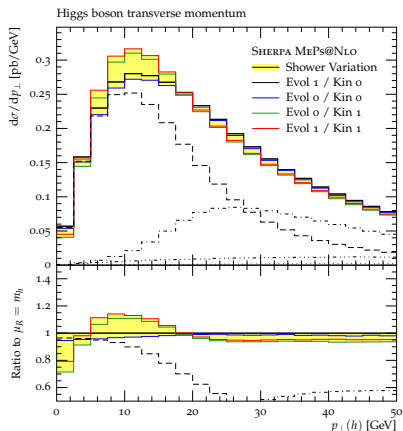
[Höhe, Krauss, Schönherr 2014]



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Large Resummation uncertainty:

- ▶ Motivates work on showers with better accuracy.

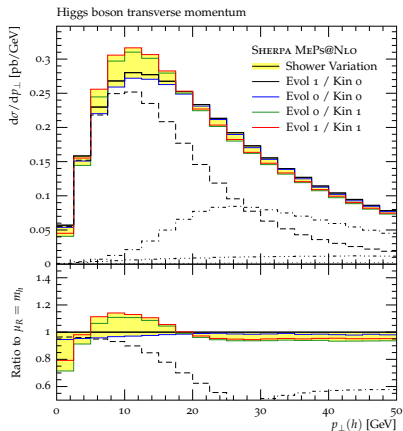


[Höche, Krauss, Schönherr 2014]

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- Analytical calculations:
 - ▶ Higher accuracy calculations exist.

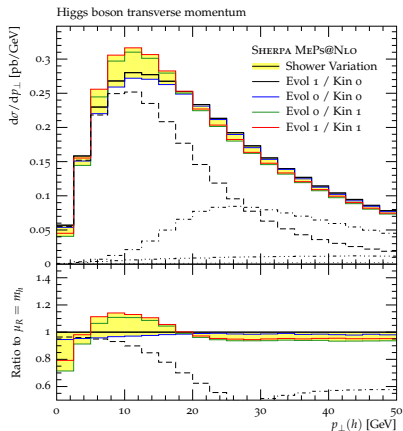


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- Numerical (parton shower) side:
 - ▶ Formally only lowest approximation.
 - ▶ Parton showers "are better than formally expected":
 - ★ Momentum conservation
 - ★ coherence, ...



[Höche, Krauss, Schönherr 2014]

Large Resummation uncertainty:

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- Analytical calculations:
 - ▶ Higher accuracy calculations exist.
- Numerical (parton shower) side:
 - ▶ Formally only lowest approximation.
 - ▶ Parton showers "are better than formally expected":
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 - ★ coherence, ...
- No straightforward comparison possible.

- In this talk:

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 - ▶ Build a toy shower that emulates NLL resummation exactly.

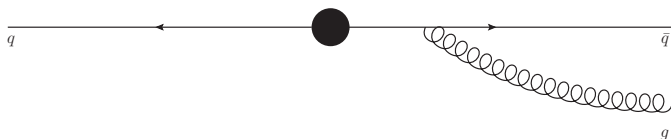
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- In this talk:
 - ▶ Build a toy shower that emulates NLL resummation exactly.
 - ▶ Turn on different contributions step by step
⇒ finally recover full parton shower.
 - ▶ Determine sizes of individual contributions.

Outline

- 1 General Setup
- 2 PS vs. Resummation
- 3 Conclusion

- Start from $q\bar{q}$ pair.
- Add soft and/or collinear gluons.
- Observable is zero in $q\bar{q}$ configuration.



- Consider additive observable, i.e. in presence of several soft gluons (In this talk \Rightarrow Thrust $1 - T$):

$$V(k_1, \dots, k_n) = \sum_{i=1}^n V(k_i)$$

- Why do we need Resummation?
- Matrix element with collinear gluon:
 - ▶ Notation:

$$\underbrace{d\Phi_3}_{\propto \text{Probability for } n+1 \text{ partons.}} \approx d\Phi_2 \underbrace{\left[\text{Diagram} \right]}_{\propto \text{Probability for } n \text{ partons.}} \times \underbrace{\frac{dk_T^2}{k_T^2} dz d\Phi \frac{\alpha_s}{2\pi} P_{qg}(z)}_{\text{Probability for emission of additional parton.}}$$

The diagram in the middle shows a wavy line (gluon) splitting into two straight lines (quarks) and another straight line (gluon). The first two straight lines are enclosed in a vertical bar with a superscript 2, representing the n-parton state. The gluon line is the additional parton.

- Why do we need Resummation?
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 - ▶ Notation:

$$d\Phi_3 \left| \begin{array}{c} \text{wavy line} \rightarrow \text{gluon} \rightarrow \text{two lines} \end{array} \right|^2 \approx d\Phi_2 \left| \begin{array}{c} \text{wavy line} \rightarrow \text{two lines} \end{array} \right|^2 \times \underbrace{\frac{dk_T^2}{k_T^2} dz d\Phi}_{:=dk} \frac{\alpha_s}{2\pi} P_{\text{qg}}(z)$$

$$R(t, t') := \int_{t'}^t dk \frac{\alpha_s}{2\pi} P_{\text{qg}}(z)$$

$-\partial_{t'} R(t, t') =$ Probability for Splitting at scale t' .

Total no-splitting probability:

- Probability for no splitting between two scales:

$$\begin{aligned}
 \Pi(t, t') &= 1 - \underbrace{\int_{t'}^t dk \frac{\alpha_s}{2\pi} P_{\text{qg}}(z)}_{\text{Probability of one emission.}} \\
 &\quad - \underbrace{\frac{1}{2} \int_{t'}^t dk_1 \frac{\alpha_s}{2\pi} P_{\text{qg}}(z_1) \int_{t'}^t dk_2 \frac{\alpha_s}{2\pi} P_{\text{qg}}(z_2)}_{\text{Probability of two emissions.}} - \dots \\
 &= \exp\left(-\int_{t'}^t dk \frac{\alpha_s}{2\pi} P_{\text{qg}}(z)\right) \\
 &= \exp(-R(t, t'))
 \end{aligned}$$

\Rightarrow Sudakov factor.

- Central to the Parton Shower:
 - ▶ Probability for splitting at scale t' starting from scale t with no intermediate splittings:

$$P(t, t') = \underbrace{-\partial_{t'} R(t, t')}_{\text{Probability for splitting at } t'} \underbrace{\exp(-R(t, t'))}_{\text{Probability for no splitting between } t \text{ and } t'}$$

- Iterate with this probability over all (hard) final state particles.

- Central to the analytic Resummation:
 - ▶ Probability for all splittings leading to a value of the observable considered less than v .

$$\Sigma(v) = \int_{t_0} dt_1 \dots$$

- Central to the analytic Resummation:

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 & \times \theta \left(v - \underbrace{V(t_1)}_{\text{Value with splitting at scale } t_1} \right) + \dots
 \end{aligned}$$

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 & \times \theta \left(v - \underbrace{V(t_1)}_{\text{Value with splitting at scale } t_1} \right) \\
 & + \int_{t_0} dt_1 dt_2 P(Q, t_1) P(t_1, t_2) \exp(-R(t_2, t_0)) \\
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 &\quad \times \theta \left(v - \underbrace{V(t_1)}_{\text{Value with splitting at scale } t_1} \right) \\
 &+ \int_{t_0} dt_1 dt_2 P(Q, t_1) P(t_1, t_2) \exp(-R(t_2, t_0)) \\
 &\quad \times \theta(v - V(t_1, t_2)) + \dots \\
 &= \exp(-R(Q, t_0)) \underbrace{\int_{t_0} -\partial_{t_1} R(Q, t_1) \theta(v - V(t_1)) + \dots}_{\text{Integral over all splittings which lead to observable value less than } v.}
 \end{aligned}$$

- The Parton Shower result:

$$\Sigma(v) = \exp[-R(Q, t_0)] \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=1}^m \int \frac{dt_i}{t_i} R'(t_i) \right) \Theta \left(v - \sum_{j=1}^m V(t_j) \right)$$

- NLL calculation generic in CAESAR formalism [Banfi, Salam, Zanderighi 2005]:

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- Note: ϵ is a technical cutoff, see later.

- Write Shower and Analytic prediction in generic form (omitting the sum over hard legs):

$$\Sigma(v) = \exp \left[- \int_v \frac{d\xi}{\xi} \partial R_{>v}(\xi) - \int_{v_{\min}}^v \frac{d\xi}{\xi} \partial R_{<v}(\xi) \right]$$

$$\times \sum_{m=0}^{\infty} \frac{1}{m!} \prod_{i=1}^m \int_{v_{\min}} \frac{d\xi_i}{\xi_i} \partial R_{<v}(\xi_i) \Theta \left(v - \sum_{j=1}^m V(\xi_j) \right)$$

$$\partial R_{\leq v}(\xi) = \frac{\alpha_s^{\leq v, \text{SE}}(\mu_{\leq v}^2(\xi))}{\pi} \int_0^{z_{\leq v, \text{SE}}^{\max}(\xi)} dz \frac{C_F}{1-z}$$

$$- \frac{\alpha_s^{\leq v, \text{SF}}(\mu_{\leq v}^2(\xi))}{\pi} \int_0^{z_{\leq v, \text{SF}}^{\max}} dz C_F \frac{1+z}{2}$$

	Resummation	Shower
$\mu_{>v}^2$	$\xi(1-z)^{\frac{2b}{a+b}}$	$\xi(1-z)^{\frac{2b}{a+b}}$
$z_{>v,SE}^{\max}$	$1 - \xi^{\frac{a+b}{2a}}$	$1 - \xi^{\frac{a+b}{2a}}$
$\alpha_s^{\leq v,SE}$	2 Loop - CMW	2 Loop - CMW
$\mu_{<v}^2$	$v^{\frac{2}{a+b}}(1-z)^{\frac{2b}{a+b}}$	$\xi(1-z)^{\frac{2b}{a+b}}$
$z_{<v,SE}^{\max}$	$1 - v^{\frac{1}{a}}$	$1 - \xi^{\frac{a+b}{2a}}$
$z_{>v,SF}^{\max}$	1	$1 - \xi^{\frac{a+b}{2a}}$
$z_{<v,SF}^{\max}$	0	$1 - \xi^{\frac{a+b}{2a}}$
$\alpha_s^{\leq v,SF}, \alpha_s^{<v,SE}$	1 Loop	2 Loop - CMW

Observables parametrised as (for $1 - T$: $a = b = 1$)

$$V(k) = \left(\frac{k_T}{Q}\right)^a e^{-b\eta}$$

Shower ordered in ξ

$$k_T = \xi(1-z)^{\frac{2b}{a+b}}$$

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 \Rightarrow need to deal with negative weights in shower,
methods e.g. in [Hoeche, Schumann, Siegert 2009]

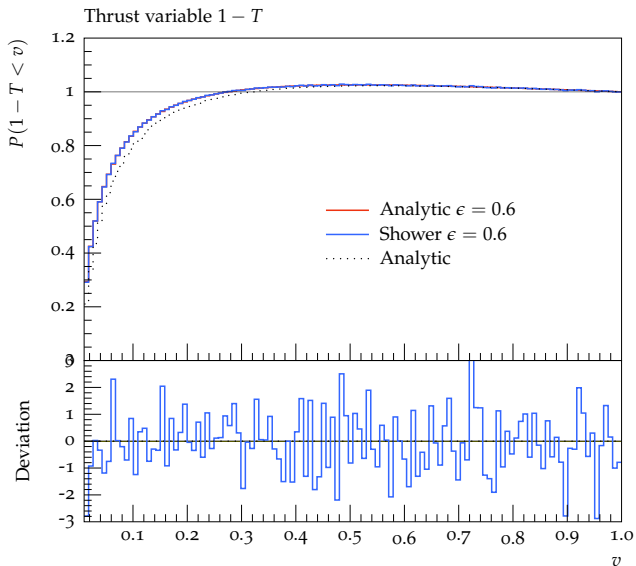
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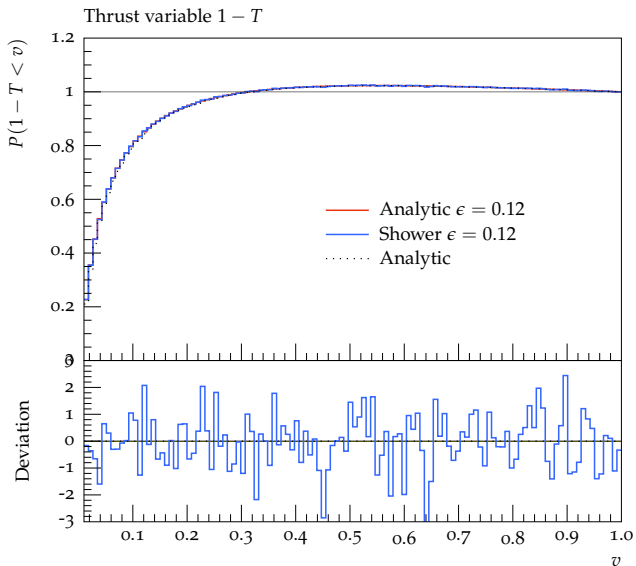
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- ▶ Weights will depend on v
⇒ compute cumulative distributions directly

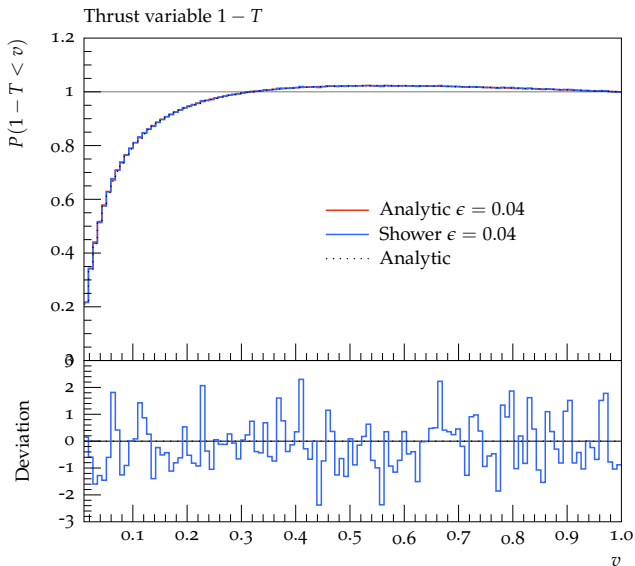
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methods e.g. in [Hoeche, Schumann, Siegert 2009]
- ▶ Weights will depend on ν
 \Rightarrow compute cumulative distributions directly
- ▶ Cutoff in resummation ν dependant $t_0 = \epsilon \nu$
 \Rightarrow Confirm $\epsilon \rightarrow 0$ limit in shower.

	Resummation	Shower
$\mu_{>v}^2$ $z_{>v,SE}^{\max}$ $\alpha_s^{>v,SE}$	$\xi(1-z)^{\frac{2b}{a+b}}$ $1 - \xi^{\frac{a+b}{2a}}$ 2 Loop - CMW	$\xi(1-z)^{\frac{2b}{a+b}}$ $1 - \xi^{\frac{a+b}{2a}}$ 2 Loop - CMW
$\mu_{<v}^2$ $z_{<v,SE}^{\max}$ $z_{>v,SF}^{\max}$	$v^{\frac{2}{a+b}}(1-z)^{\frac{2b}{a+b}}$ $1 - v^{\frac{1}{a}}$ 1	$\xi(1-z)^{\frac{2b}{a+b}}$ $1 - \xi^{\frac{a+b}{2a}}$ $1 - \xi^{\frac{a+b}{2a}}$
$z_{<v,SF}^{\max}$ $\alpha_s^{\leq v,SF}, \alpha_s^{<v,SE}$	0 1 Loop	$1 - \xi^{\frac{a+b}{2a}}$ 2 Loop - CMW

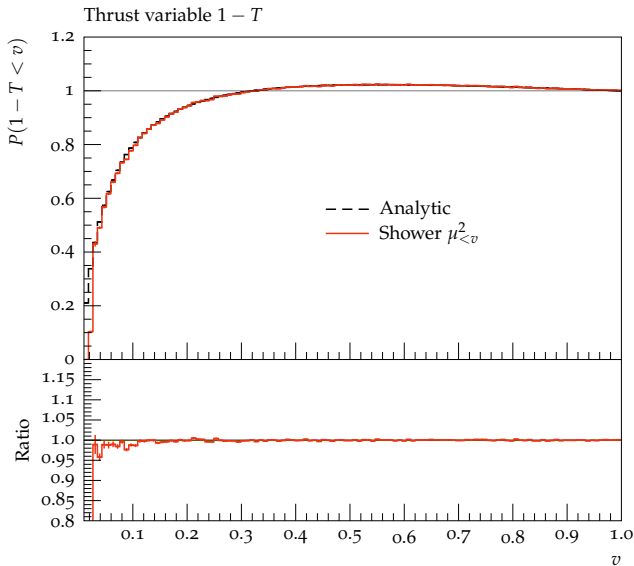




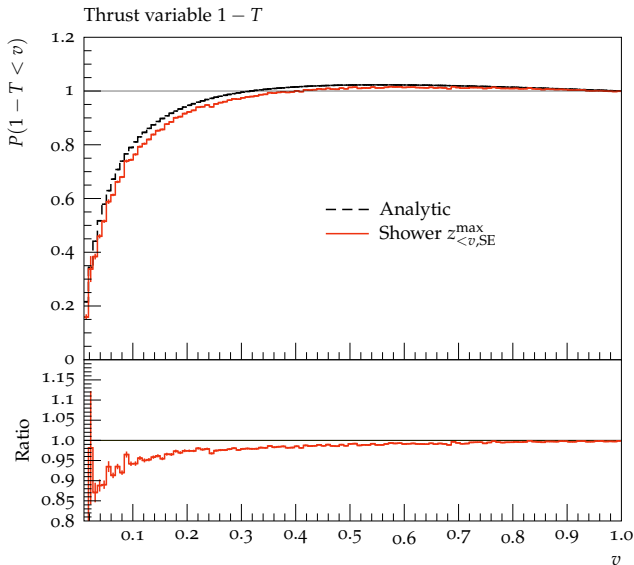


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$z_{<v,SF}^{\max}$ $\alpha_s^{\leq v,SF}, \alpha_s^{<v,SE}$	0 1 Loop	$1 - \xi^{\frac{a+b}{2a}}$ 2 Loop - CMW

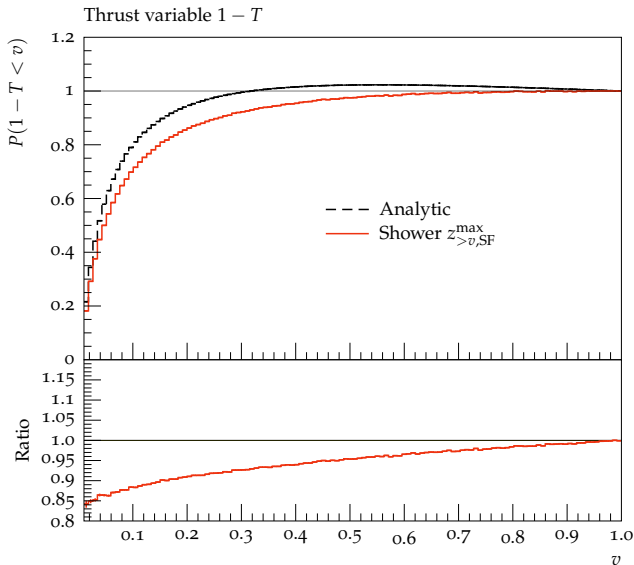
- $\mu_{<v}^2$: $v \frac{2}{a+b} (1-z)^{\frac{2b}{a+b}} \rightarrow \xi(1-z)^{\frac{2b}{a+b}}$



- $z_{<v,SE}^{\max}$: $1 - v^{\frac{1}{a}} \rightarrow 1 - \xi^{\frac{a+b}{2a}}$

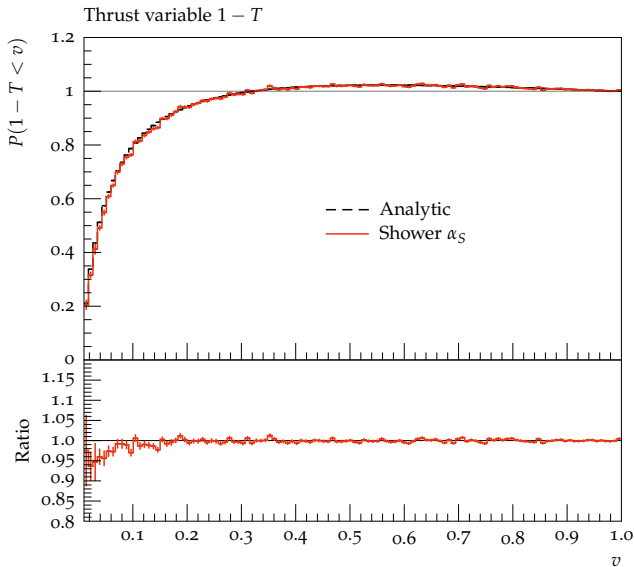


- $z_{>v,SF}^{\max}: 1 \rightarrow 1 - \xi^{\frac{a+b}{2a}}$

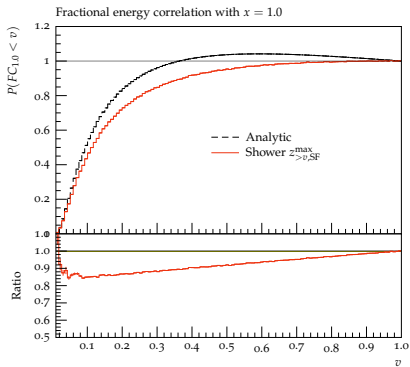
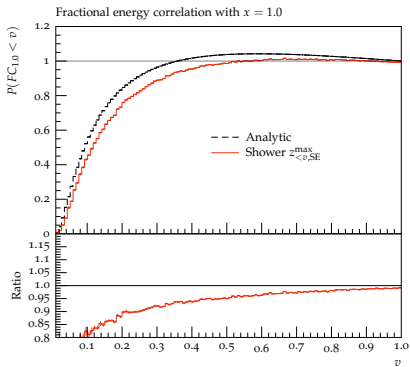


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$z_{<v,SF}^{\max}$ $\alpha_s^{\leq v,SF}, \alpha_s^{<v,SE}$	0 1 Loop	$1 - \xi^{\frac{a+b}{2a}}$ 2 Loop - CMW

- $\alpha_s^{\leq v, \text{SF}}$, $\alpha_s^{< v, \text{SE}}$: 1 Loop \rightarrow 2 Loop - CMW



- Most contributions relatively small
- except momentum conservation constraints
- results hold for other observables

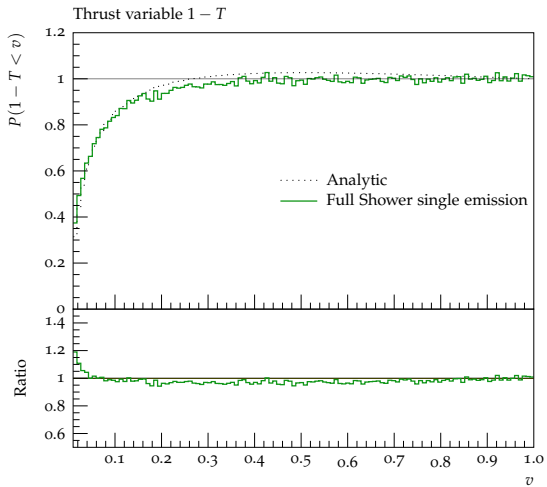


- So far: Momentum conserving effects for single emissions.

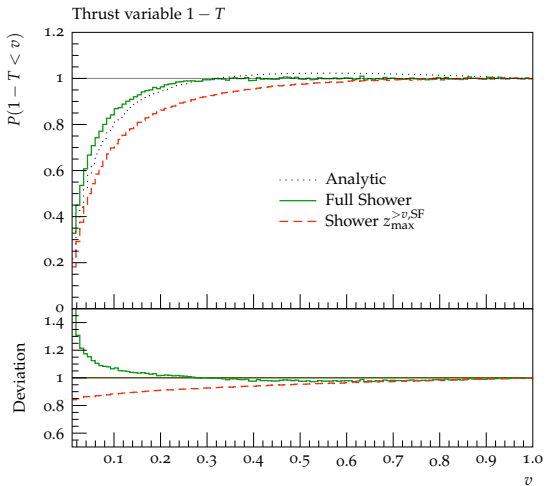
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⇒ keep all particles on-shell and preserve total momentum in every step.

- So far: Momentum conserving effects for single emissions.
- Full parton shower: local momentum conservation
⇒ keep all particles on-shell and preserve total momentum in every step.
- Still calculate Observable in Approximation corresponding to Resummation ⇒ fix thrust axis and normalization of $1 - T$ according to hard quark pair

- First emission result $\exp(-R)$:



- Full shower with local momentum conservation:



Outline

- 1 General Setup
- 2 PS vs. Resummation
- 3 Conclusion**

- Conclusion:

- ▶ Shower has been constructed to emulate NLL resummation.
- ▶ Provides framework to compare size of individual contributions of e.g. momentum conservation effects.

- Outlook:

- ▶ Generalise to more observables, i.e. in hadron collisions.

Backup