ANALYSIS OF QED EFFECTS IN HADRONIC PROCESSES BEYOND LO

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Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10 (2016) 056; and work in progress

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Outline

- Motivation and introduction
- QED corrections to splitting kernels
  - Extending DGLAP equations
  - QCD-QED corrections to AP kernels
  - *Abelianization technique*
- NLO QED effects to $\gamma\gamma$ production
- Conclusions and perspectives

Specific references:

1)- H.O. QED splittings: de Florian, Rodrigo and GS, JHEP 01(2014)018, JHEP 10(2014)161; JHEP 03(2015)021
2)- QCD-QED AP kernels: de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056
3)- Diphoton production: Cieri, Ferrera and GS, **work in progress**

Motivation and introduction

Why we need QED corrections?

- More precise experimental data is available!! We need to include (previously neglected) small theoretical effects!!
- NLO QCD is the standard; **NNLO QCD** calculations starting to appear!
- **QED effects might compete with NNLO QCD (since $\alpha_S^2 \sim \alpha$)**
- Inclusion of QED beyond LO could lead to novel effects:
  - Quark-gluon interacting with leptons and photons
  - Charge separation
  - **Dependence on the photon content of the proton!**

    Manohar, Nason, Salam, Zanderighi, '16
  - Enhanced contributions at high-energies (due to the running EM coupling)
- **Thus, QED corrections MUST be taken into account!**

Extending DGLAP equations

Introducing QED corrections

- *DGLAP equations dictate the evolution of PDFs*
- **QED** interactions connects **QCD** partons with **photons and leptons.**

New PDFs (leptons & photon) + *New splitting functions* → ***Modified DGLAP equations***

- **Extend original DGLAP equations to deal with new *objects*:**

Kernels with fermions

$$\frac{dg}{dt} = \sum_f P_{gf} \otimes f + \sum_f P_{g\bar{f}} \otimes \bar{f} + P_{gg} \otimes g + P_{g\gamma} \otimes \gamma$$

Photon distributions

$$\frac{d\gamma}{dt} = \sum_f P_{\gamma f} \otimes f + \sum_f P_{\gamma\bar{f}} \otimes \bar{f} + P_{\gamma g} \otimes g + P_{\gamma\gamma} \otimes \gamma$$

Kernels with photons

$$\frac{dq_i}{dt} = \sum_f P_{q_i f} \otimes f + \sum_f P_{q_i\bar{f}} \otimes \bar{f} + P_{q_i g} \otimes g + P_{q_i\gamma} \otimes \gamma$$

Lepton distributions

$$\frac{dl_i}{dt} = \sum_f P_{l_i f} \otimes f + \sum_f P_{l_i\bar{f}} \otimes \bar{f} + P_{l_i g} \otimes g + P_{l_i\gamma} \otimes \gamma$$

Kernels with leptons

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Extending DGLAP equations

Introducing QED corrections

- Change **PDFs basis** to simplify the system of coupled integro-differential equations *Roth, Weinzierl '04*

New lepton distributions!

$$\mathcal{B}_c = \{u, \bar{u}, \ldots, t, \bar{t}, e, \bar{e}, \ldots, \tau, \bar{\tau}, g, \gamma\}$$

Photon distribution!

$$\Delta_{uc} = u + \bar{u} - c - \bar{c}\,,$$
$$\Delta_{ds} = d + \bar{d} - s - \bar{s}\,,$$
$$\Delta_{sb} = s + \bar{s} - b - \bar{b}\,,$$
$$\Delta_2^l = e + \bar{e} - \mu - \bar{\mu}\,,$$
$$\Delta_{UD} = u + \bar{u} + c + \bar{c} - d - \bar{d} - s - \bar{s} - b - \bar{b}$$
$$\Delta_3^l = e + \bar{e} + \mu + \bar{\mu} - 2(\tau + \bar{\tau})\,,$$

$$\Sigma = \sum_{i=1}^{n_F} (q_i + \bar{q}_i)\,,$$
$$\Sigma^l = \sum_{i=1}^{n_L} (l_i + \bar{l}_i)\,,$$

CANONICAL BASIS

$$q_v = q_i - \bar{q}_i\,,\; l_v = l_i - \bar{l}_i\,,$$

Valence distributions

- Photon and gluon distributions are not altered
- Straightforward extension to deal with n_F=6

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Extending DGLAP equations

- New optimized DGLAP equations (I)

$$\frac{dq_{v_i}}{dt} = P^-_{q_i} \otimes q_{v_i} + \sum_{j=1}^{n_F} \Delta P^S_{q_i q_j} \otimes q_{v_j} + \Delta P^S_{q_i l} \otimes \left(\sum_{j=1}^{n_L} l_{v_j} \right) ,$$

$$\frac{dl_{v_i}}{dt} = P^-_{l} \otimes l_{v_i} + \sum_{j=1}^{n_F} \Delta P^S_{l q_j} \otimes q_{v_j} + \Delta P^S_{ll} \otimes \left(\sum_{j=1}^{n_L} l_{v_j} \right) ,$$

Valence PDFs

$$\frac{d\{\Delta_{uc}, \Delta_{ct}\}}{dt} = P^+_u \otimes \{\Delta_{uc}, \Delta_{ct}\} ,$$

$$\frac{d\{\Delta_{ds}, \Delta_{sb}\}}{dt} = P^+_d \otimes \{\Delta_{ds}, \Delta_{sb}\} ,$$

$$\frac{d\Delta^l_{\{2,3\}}}{dt} = P^+_l \otimes \Delta^l_{\{2,3\}} ,$$

Diagonal equations

$$\frac{d\Sigma}{dt} = \frac{P^+_u + P^+_d}{2} \otimes \Sigma + \frac{P^+_u - P^+_d}{2} \otimes \Delta_{UD} + \frac{n_u \bar{P}^S_{uu} + n_d \bar{P}^S_{dd} + (n_u + n_d)\bar{P}^S_{ud}}{2} \otimes \Sigma + 2(n_u P_{ug} + n_d P_{dg}) \otimes g$$
$$+ \frac{n_u \bar{P}^S_{uu} - n_d \bar{P}^S_{dd} - (n_u - n_d)\bar{P}^S_{ud}}{2} \otimes \Delta_{UD} + \left(n_u \bar{P}^S_{ul} + n_d \bar{P}^S_{dl}\right) \otimes \Sigma^l + 2(n_u P_{u\gamma} + n_d P_{d\gamma}) \otimes \gamma ,$$

$$\frac{d\Sigma^l}{dt} = n_L \frac{\bar{P}^S_{lu} + \bar{P}^S_{ld}}{2} \otimes \Sigma + n_L \frac{\bar{P}^S_{lu} - \bar{P}^S_{ld}}{2} \otimes \Delta_{UD} + \left(P^+_l + n_L \bar{P}^S_{ll}\right) \otimes \Sigma^l + 2n_L (P_{lg} \otimes g + P_{l\gamma} \otimes \gamma) ,$$

Singlet distributions

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Extending DGLAP equations

- New optimized DGLAP equations (II)

$$\frac{dg}{dt} = \sum_f P_{gf} \otimes f + \sum_f P_{g\bar{f}} \otimes \bar{f} + P_{gg} \otimes g + P_{g\gamma} \otimes \gamma \,,$$

Gluon PDF evolution

$$\frac{d\gamma}{dt} = \sum_f P_{\gamma f} \otimes f + \sum_f P_{\gamma\bar{f}} \otimes \bar{f} + P_{\gamma g} \otimes g + P_{\gamma\gamma} \otimes \gamma \,,$$

Photon PDF evolution

$$\frac{d\Delta_{UD}}{dt} = \frac{P_u^+ + P_d^+}{2} \otimes \Delta_{UD} + \frac{P_u^+ - P_d^+}{2} \otimes \Sigma + \frac{n_u \bar{P}^S_{uu} - n_d \bar{P}^S_{dd} + (n_u - n_d)\bar{P}^S_{ud}}{2} \otimes \Sigma + 2(n_u P_{ug} - n_d P_{dg}) \otimes g$$
$$+ \frac{n_u \bar{P}^S_{uu} + n_d \bar{P}^S_{dd} - (n_u + n_d)\bar{P}^S_{ud}}{2} \otimes \Delta_{UD} + \left(n_u \bar{P}^S_{ul} - n_d \bar{P}^S_{dl}\right) \otimes \Sigma^l + 2(n_u P_{u\gamma} - n_d P_{d\gamma}) \otimes \gamma \,,$$

- Splitting kernels definitions (*introduced to simplify the notation*)

$$P_{q_i q_k} = \delta_{ik}\, P^V_{qq} + P^S_{qq} \,, \qquad P_{l_i l_k} = \delta_{ik}\, P^V_{ll} + P^S_{ll} \,,$$

$$P_{q_i \bar{q}_k} = \delta_{ik}\, P^V_{q\bar{q}} + P^S_{q\bar{q}} \,, \qquad P_{l_i \bar{l}_k} = \delta_{ik}\, P^V_{l\bar{l}} + P^S_{l\bar{l}} \,,$$

$$P^\pm_q = P^V_{qq} \pm P^V_{q\bar{q}} \,, \qquad P^\pm_l = P^V_{ll} \pm P^V_{l\bar{l}} \,,$$

$$\Delta P^S_{fF} \equiv P^S_{fF} - P^S_{f\bar{F}} \,,$$

$$\bar{P}^S_{fF} \equiv P^S_{fF} + P^S_{f\bar{F}} \,,$$

Vanishes at $\mathcal{O}(\alpha\, \alpha_S)$ and $\mathcal{O}(\alpha^2)$

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Extending DGLAP equations

Introducing QED corrections

- **Extended sum rules** *(impose physical constraints in AP kernels)*
  - *Fermion number conservation* ⇒ $\int_0^1 dx\, P_f^- = 0$
  - *Momentum conservation* ⇒ $0 = \frac{dP}{dt} = \int_0^1 dx\, x \left(\frac{dg}{dt} + \frac{d\gamma}{dt} + \sum_f \left(\frac{df}{dt} + \frac{d\bar{f}}{dt} \right) \right)$

- Some general remarks:
  - **Charge separation** effects introduced by QED
  - Non-trivial **quark-lepton mixing** (although simplified in the optimized basis)
  - **Explicit formulae** involving AP kernels can be obtained by replacing the **evolution equations**
  - Sum rules allow to **fix the behaviour** of AP kernels in the end-point (**x=1**)
  - *Also, they are useful for checking the consistency of the results.*

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

QCD-QED corrections to AP kernels

Definitions and previous results

- Perturbative expansion in QCD and QED couplings (non-trivial counting…)

$$P_{ij} = a_S\, P_{ij}^{(1,0)} + a\, P_{ij}^{(0,1)} + a_S^2\, P_{ij}^{(2,0)} + a_S\, a\, P_{ij}^{(1,1)} + a^2\, P_{ij}^{(0,2)} + \ldots$$

- Well-known LO results:

$$P_{qq}^{(1,0)}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\,\delta(1-x)\right] = C_F\left[p_{qq}(x) + \frac{3}{2}\,\delta(1-x)\right],$$
$$P_{qg}^{(1,0)}(x) = T_R\left[x^2 + (1-x)^2\right] = T_R\, p_{qg}(x)\,,$$
$$P_{gq}^{(1,0)}(x) = C_F\left[\frac{1+(1-x)^2}{x}\right] = C_F\, p_{gq}(x)\,,$$
$$P_{gg}^{(1,0)}(x) = 2\,C_A\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \frac{\beta_0}{2}\,\delta(1-x)\,,$$

Standard QCD AP-kernels at LO

$$P_{ff}^{(0,1)}(x) = e_f^2\left[p_{qq}(x) + \frac{3}{2}\delta(1-x)\right],$$
$$P_{f\gamma}^{(0,1)}(x) = e_f^2\, p_{qg}(x)\,,$$
$$P_{\gamma f}^{(0,1)}(x) = e_f^2\, p_{gq}(x)\,,$$
$$P_{\gamma\gamma}^{(0,1)}(x) = -\frac{2}{3}\sum_f e_f^2\,\delta(1-x)\,,$$

LO QED splitting functions

- Terms proportional to Dirac's **deltas** are originated by virtual (**loop**) corrections
- **Color** factors in **QCD** replaced with **EM charges** in **QED** $\Rightarrow$ Motivates **Abelianization algorithm**!

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

QCD-QED corrections to AP kernels

Abelianization technique

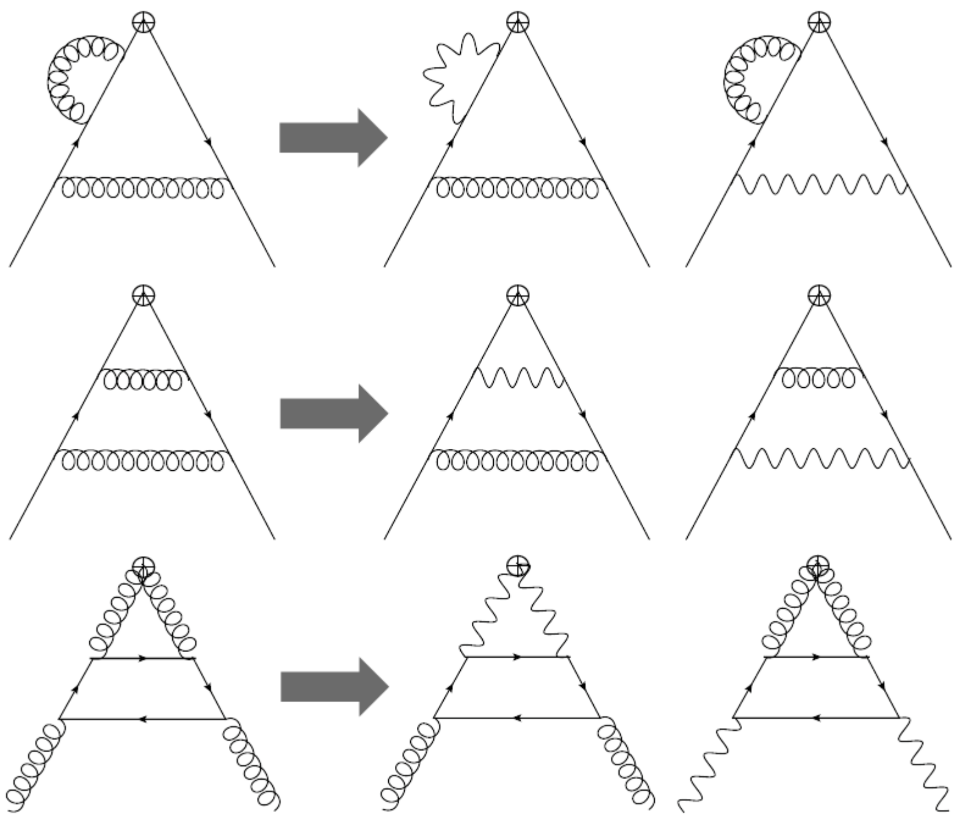
- The replacement has to be done comparing *carefully* each **topology**, **removing** the **QCD color factor** and multiplying by the **QCD-QED (or pure QED) factor**
- **Abelianization algorithm:** $\mathcal{O}(\alpha\,\alpha_S)$
  1. Remove the average over initial state colors
  2. Identify non-vanishing diagrams when replacing gluons by photons (both real and virtual terms). Replacement of non-observable gluons leads to non-equivalent topologies: multiply by 2!
  3. Express the color factors in terms of N_C, and keep only the leading terms when $N_C \to 0$ (complements step 2)
  4. For internal fermion loops, use $n_F \to \sum_{j=1}^{n_F} e_{q_j}^2$ (if external, multiply by its charge)
  5. Multiply the color stripped result by the proper color factor

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

QCD-QED corrections to AP kernels

Abelianization technique

- **Abelianization algorithm:** example at $\mathcal{O}(\alpha\,\alpha_S)$



$$P_{qq}^{(2,0)} \to P_{qq}^{(1,1)}$$

Non-observable gluon leads to non-equivalent diagrams contributing to the same kernel

$$P_{gg}^{(2,0)} \to P_{g\gamma}^{(1,1)} \oplus P_{\gamma g}^{(1,1)}$$

Replacement of external gluons leads to different kernels (no need of factor 2)

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

QCD-QED corrections to AP kernels

Abelianization technique

- **Important remarks:**
  - Use two-loop QCD results as starting point; keeping track of the different topologies contributing to the splittings is crucial to check the results

    Curci, Furmanski and Petronzio, Nucl. Phys. B 175 (1980) 27
    Furmanski and Petronzio, Phys. Lett. B 97 (1980) 437
    Ellis and Vogelsang, hep-ph/9602356

  - Mixed QCD-QED contributions (i.e. $\mathcal{O}(\alpha\,\alpha_S)$) obtained through the replacement of one gluon with one photon.
  - Two-loop QED contributions (i.e. $\mathcal{O}(\alpha^2)$) involve replacing two gluons; **internal fermion loops could contain leptons**

$$n_F \to \sum_f e_f^2 \qquad \text{with} \qquad \sum_f e_f^a = N_C \sum_{j=1}^{n_F} e_{q_j}^a + \sum_{j=1}^{n_L} e_{l_j}^a$$

  - Results have been cross-checked independently by another group!

    Manohar, Nason, Salam and Zanderighi, arXiv:1607.04266 [hep-ph]

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

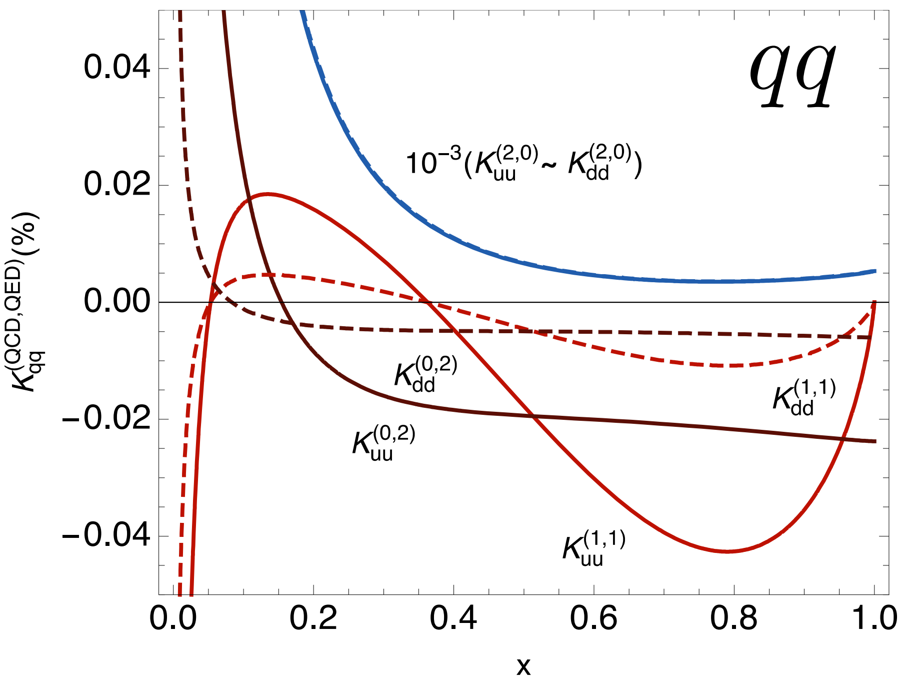
QCD-QED corrections to AP kernels

Phenomenological impact

- We define a ratio to quantify the effect of H.O. QED corrections

$$K_{ab}^{(i,j)} = a_S^i \, a^j \, \frac{P_{ab}^{(i,j)}(x)}{P_{ab}^{\mathrm{LO}}(x)} \qquad \text{with} \qquad P_{ab}^{\mathrm{LO}} = a_S \, P_{ab}^{(1,0)} + a \, P_{ab}^{(0,1)}$$

- Quark-quark splittings



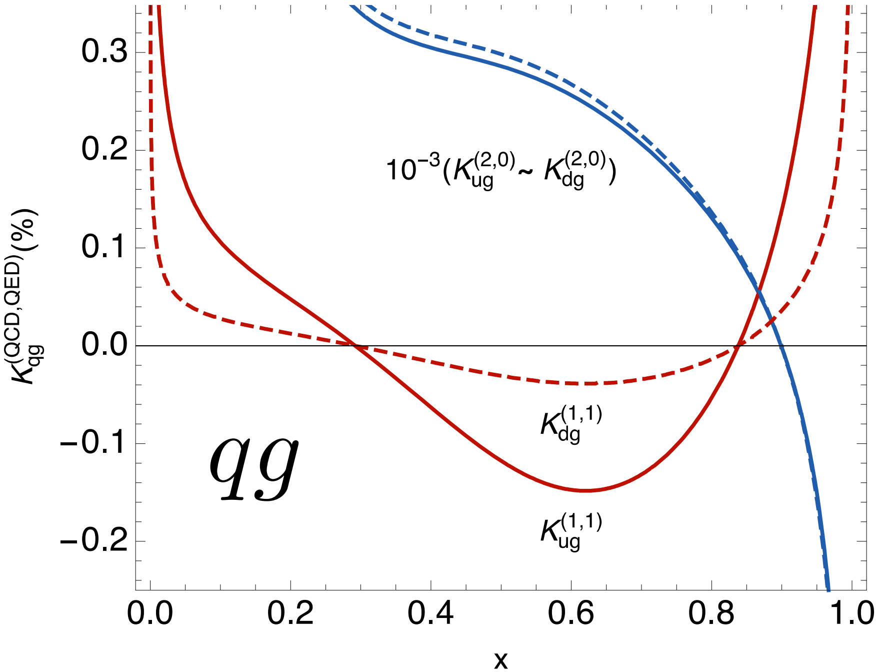
- Pure QCD contribution still dominant (x10^3)
- QED corrections introduce charge separation effects (specially at $\mathcal{O}(\alpha^2)$)
- Small corrections in intermediate x region

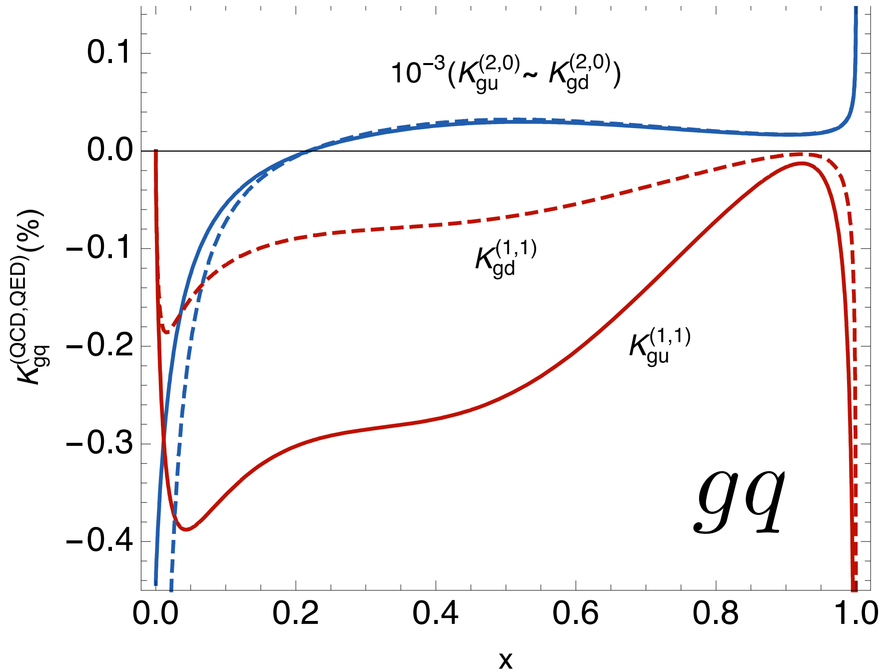
de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

QCD-QED corrections to AP kernels

Phenomenological impact

- Quark-gluon splittings





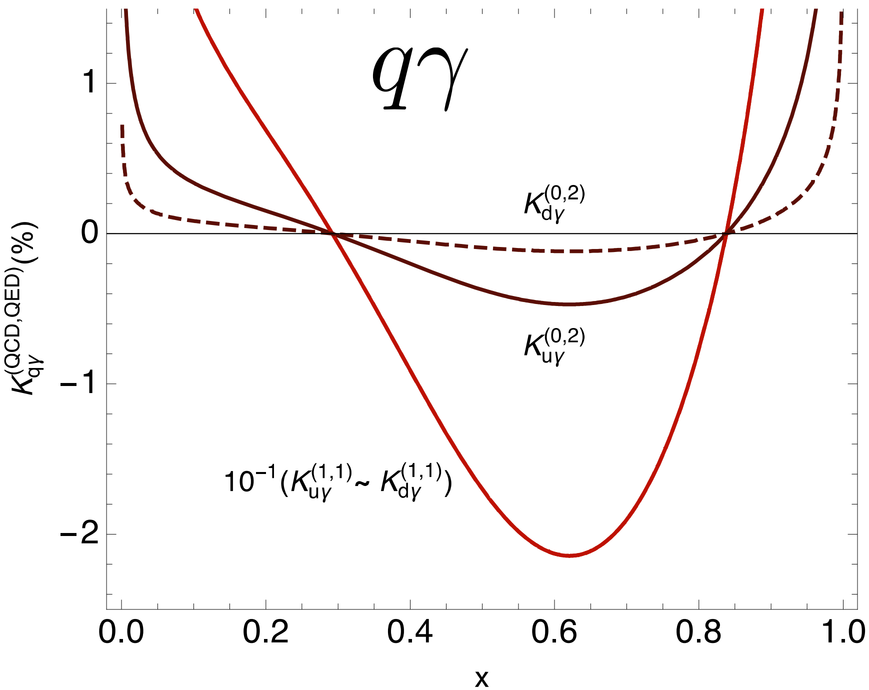
- Again, pure QCD contributions are dominant (x10^3)
- Small charge separation at $\mathcal{O}(\alpha\,\alpha_S)$ and no $\mathcal{O}(\alpha^2)$ corrections

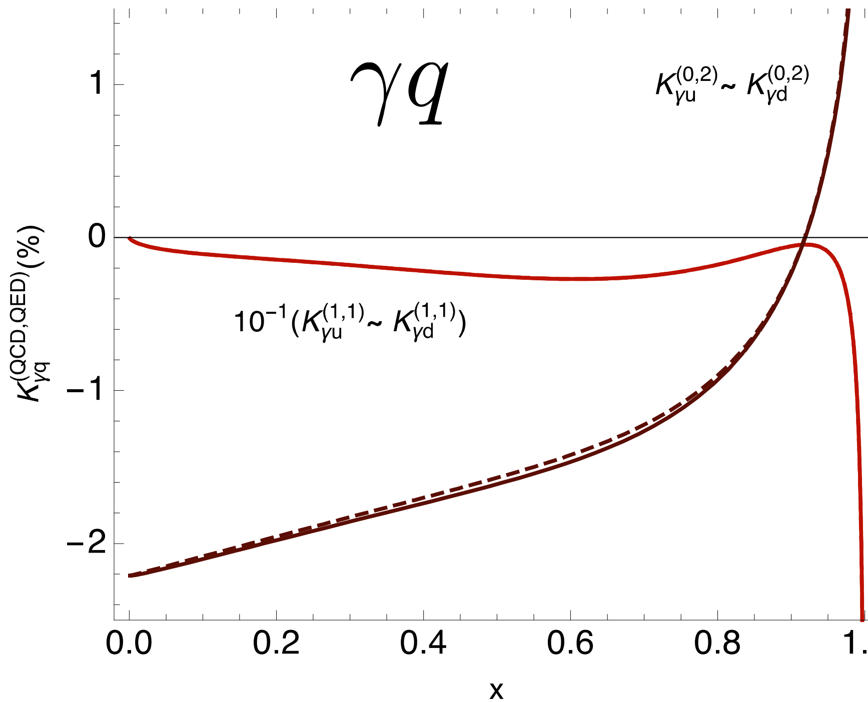
de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

QCD-QED corrections to AP kernels

Phenomenological impact

- Quark-photon splittings (***new!***)





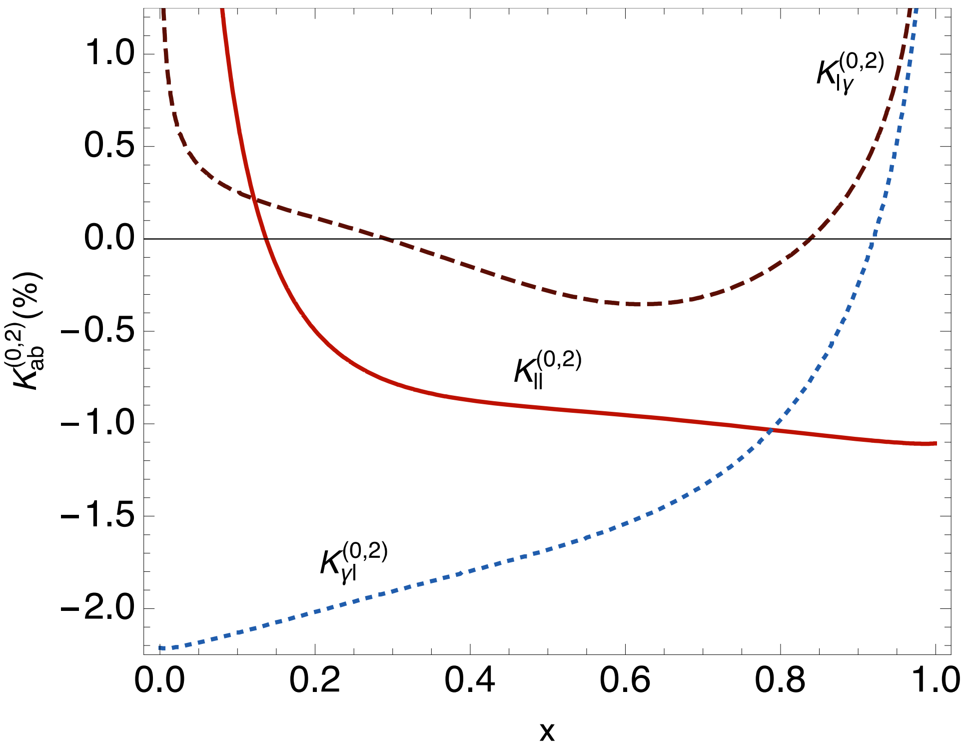
- No $\mathcal{O}(\alpha_S^2)$ contributions: QED corrections are crucial here!!!
- $\mathcal{O}(\alpha\,\alpha_S)$ is dominant but only $\mathcal{O}(\alpha^2)$ contributions are responsible of charge separation effects in $P_{q\gamma}$
- **Percent level corrections (could influence photon PDF)**

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

QCD-QED corrections to AP kernels

Phenomenological impact

- Splittings involving leptons (***new!***)



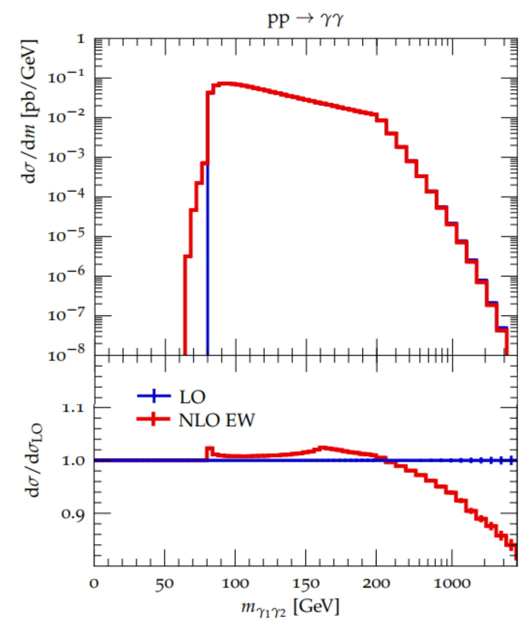
- Starting at $\mathcal{O}(\alpha)$
- Represent a few percent correction (no QCD contribution at LO)
- Lepton PDFs strongly suppressed (small phenomenological impact expected)

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

NLO QED corrections to $pp \to \gamma\gamma$

Applying QED Abelianization

- Application of Abelianization techniques to recover NLO QED.
- Full NLO EW corrections recently computed → Non-negligible effects found in the high-invariant mass region!!!
- Some subtleties to take into account:
  - *QED running:* **only** on-shell final state photons are present at LO; no need to include full QED running.
  - *Photon-ordering:* presence of photon radiation, cuts imposed on the two hardest photons → Dynamical constraint, minimum angular separation bigger than 120°!!!!)
  - *Photon-clustering:* collinear photons are merged; small phenomenological effects due to absence of collinear singularities (not the case in QCD…)



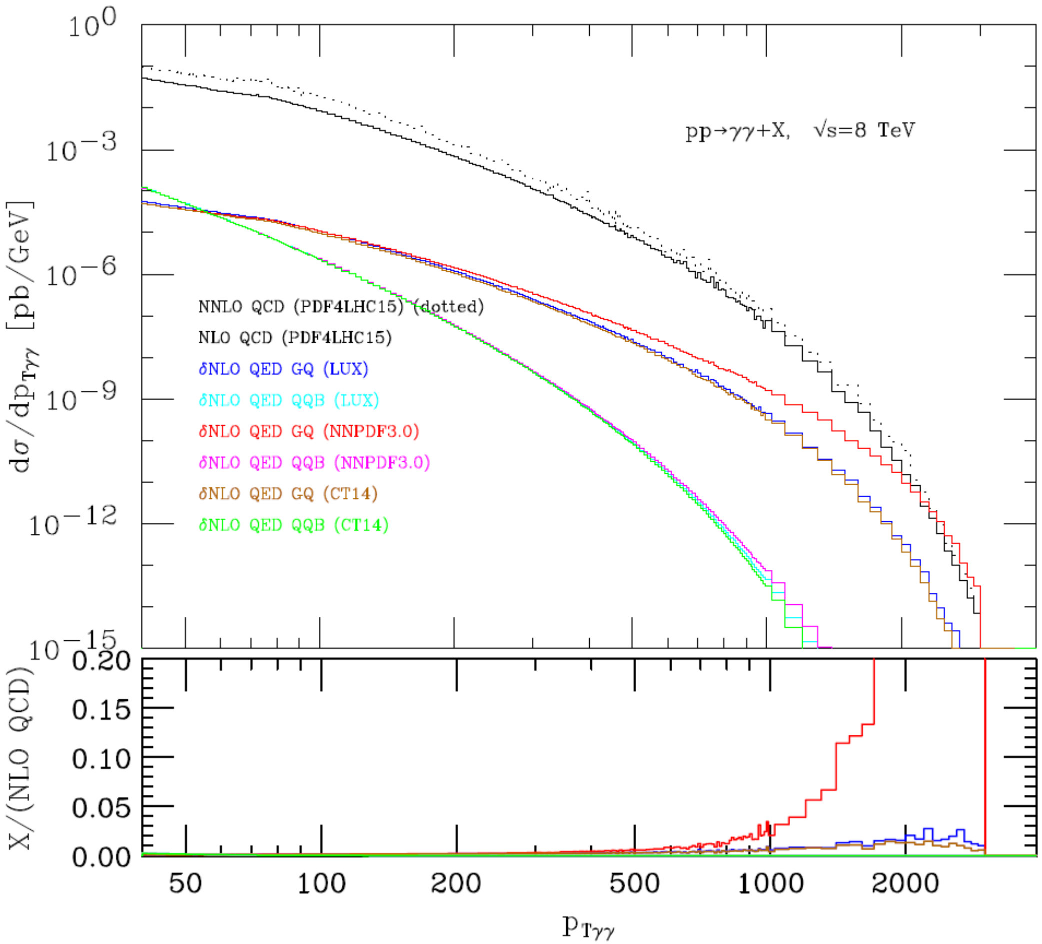
Chiesa et al, arXiv:1706.09022

Cieri, Ferrera and GS, work in progress

NLO QED corrections to $pp \rightarrow \gamma\gamma$

Applying QED Abelianization

- Some plots (comparison with NNLO and NLO QCD corrections)



Transverse-momentum distribution

- Quark channel remains (almost) unchanged by changing PDF (agreement in the quark sector)
- **Photon channel sensitive to PDFs (NNPDF3 vs LUX-CT14)**
- Differences among different sets enhanced in the high-pT region
- NLO QED effects small compared with NLO QCD, but the difference decreases as pT increases

QED effects are more important for high energies (even with static coupling…)

Cieri, Ferrera and GS, work in progress

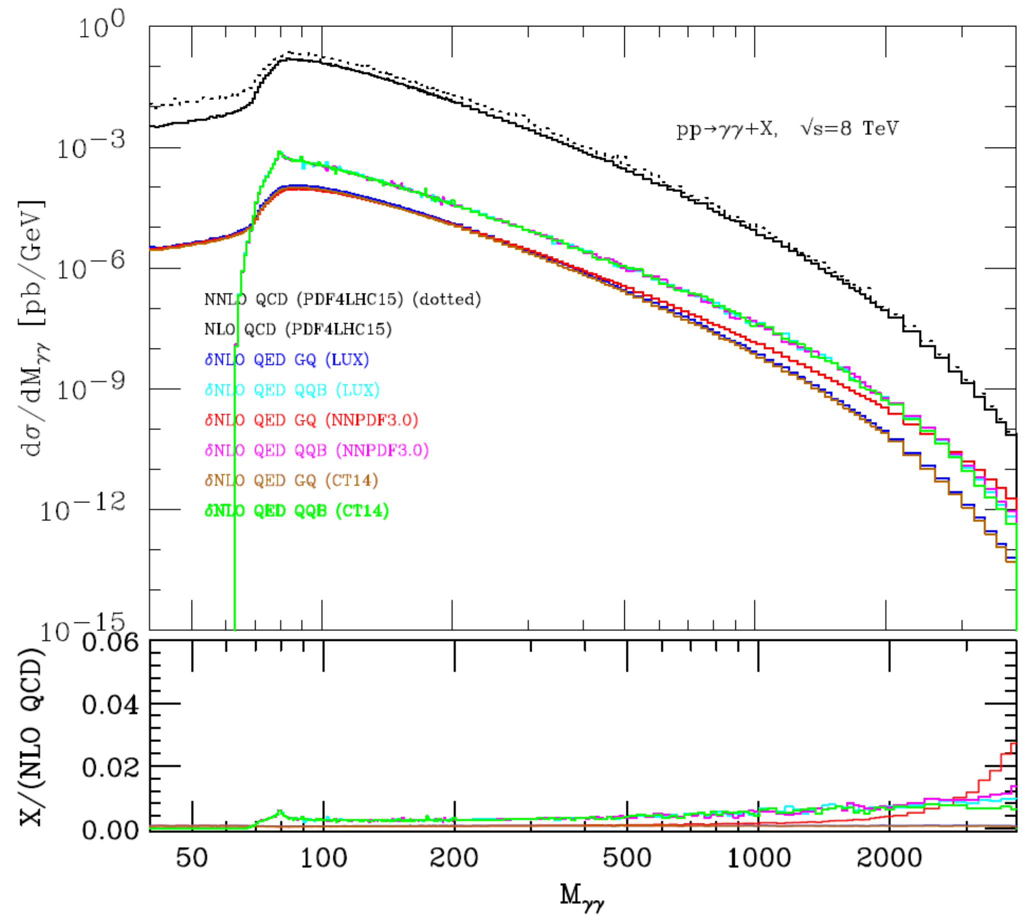
NLO QED corrections to $pp \to \gamma\gamma$

Applying QED Abelianization

- Some plots (comparison with NNLO and NLO QCD corrections)

- Quark channel remains (almost) unchanged by changing PDF (agreement in the quark sector)
- **Dynamical cuts generated by the tri-photon event selection**
- Differences among different sets noticeable in the photon channel (smaller than in the pT distribution)
- NLO QED effects small compared with NLO QCD

Again, QED effects are more important for high energies (even with static coupling…)



Invariant mass distribution (hardest photons)

Cieri, Ferrera and GS, work in progress

Conclusions and perspectives

- ✓ Splitting kernels are crucial to describe collinear limits (IR subtraction) and control PDF/FF evolution
- ✓ Mixed QCD-QED corrections computed!
- ✓ Fully consistent treatment of IR factorization
- ✓ **Percent level contributions to photon PDF evolution**

- ✓ Physical example: NLO QED corrections to diphoton production!
- ✓ Additional subtleties due to photon radiation (ordering, merging, identification)
- ✓ **QED corrections for the high-invariant mass region (a few percent level) ⇒ Full EW is crucial!**

Thanks !!!

Backup: Extended DGLAP equations

Introducing QED corrections: explicit expressions

- **Simplified formulae at** $\mathcal{O}(\alpha\,\alpha_S)$
  - Lepton distributions decouple
  - Charge separation still present
  - $\Delta P^S = 0$ after explicit computation

$$\int_0^1 dx \;\; \left(P_{qq}^{V\,(1,1)} - P_{q\bar{q}}^{V\,(1,1)}\right) = 0\,,$$

$$\int_0^1 dx\,x\left(2\,n_u P_{ug}^{(1,1)} + 2\,n_d P_{dg}^{(1,1)} + P_{\gamma g}^{(1,1)} + P_{gg}^{(1,1)}\right) = 0\,,$$

$$\int_0^1 dx\,x\left(2\,n_u P_{u\gamma}^{(1,1)} + 2\,n_d P_{d\gamma}^{(1,1)} + P_{g\gamma}^{(1,1)} + P_{\gamma\gamma}^{(1,1)}\right) = 0\,,$$

$$\int_0^1 dx\,x\left(P_{qq}^{V\,(1,1)} + P_{q\bar{q}}^{V\,(1,1)} + P_{gq}^{(1,1)} + P_{\gamma q}^{(1,1)}\right) = 0\,.$$

Sum rules

$$\frac{dq_{v_i}}{dt} = P_{q_i}^{-} \otimes q_{v_i} + \sum_{j=1}^{n_F} \Delta P^S \otimes q_{v_j}\,,$$

$$\frac{d\{\Delta_{uc},\Delta_{ct}\}}{dt} = P_u^{+} \otimes \{\Delta_{uc},\Delta_{ct}\}\,,$$

$$\frac{d\{\Delta_{ds},\Delta_{sb}\}}{dt} = P_d^{+} \otimes \{\Delta_{ds},\Delta_{sb}\}\,,$$

$$\frac{d\Delta_{UD}}{dt} = \frac{P_u^{+}+P_d^{+}}{2} \otimes \Delta_{UD} + \frac{P_u^{+}-P_d^{+}}{2} \otimes \Sigma + (n_u - n_d) P^S \otimes \Sigma$$
$$+\, 2(n_u P_{ug} - n_d P_{dg}) \otimes g + 2(n_u P_{u\gamma} - n_d P_{d\gamma}) \otimes \gamma\,,$$

$$\frac{d\Sigma}{dt} = \frac{P_u^{+}+P_d^{+}}{2} \otimes \Sigma + \frac{P_u^{+}-P_d^{+}}{2} \otimes \Delta_{UD} + n_F P^S \otimes \Sigma$$
$$+\, 2(n_u P_{ug} + n_d P_{dg}) \otimes g + 2(n_u P_{u\gamma} + n_d P_{d\gamma}) \otimes \gamma\,.$$

Evolution equations

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Backup: Extended DGLAP equations

Introducing QED corrections: complete sum rules

- **Explicit formulae at** $\mathcal{O}(\alpha^2)$

$$\int_0^1 dx\, x \left(\frac{P_u^+ - P_d^+}{2} + n_L \frac{\bar{P}_{lu}^S - \bar{P}_{ld}^S}{2} + \frac{n_u \bar{P}_{uu}^S - n_d \bar{P}_{dd}^S}{2} - \frac{(n_u - n_d)\bar{P}_{ud}^S}{2} + \frac{P_{gu} - P_{gd}}{2} + \frac{P_{\gamma u} - P_{\gamma d}}{2} \right) = 0\,,$$

From Δ_{UD} evolution

$$\int_0^1 dx\, x\, (2n_d P_{dg} + 2n_u P_{ug} + 2n_L P_{lg} + P_{\gamma g} + P_{gg}) = 0\,,$$

$$\int_0^1 dx\, x\, (2n_d P_{d\gamma} + 2n_u P_{u\gamma} + 2n_L P_{l\gamma} + P_{g\gamma} + P_{\gamma\gamma}) = 0\,;$$

From gluon and photon evolution

$$\int_0^1 dx\, x \left(\frac{P_u^+ + P_d^+}{2} + n_L \frac{\bar{P}_{lu}^S + \bar{P}_{ld}^S}{2} + \frac{n_u \bar{P}_{uu}^S + n_d \bar{P}_{dd}^S}{2} + \frac{n_F \bar{P}_{ud}^S}{2} + \frac{P_{gu} + P_{gd}}{2} + \frac{P_{\gamma u} + P_{\gamma d}}{2} \right) = 0\,,$$

$$\int_0^1 dx\, x\, (n_u \bar{P}_{ul}^S + n_d \bar{P}_{dl}^S + n_L \bar{P}_{ll}^S + P_l^+ + P_{gl} + P_{\gamma l}) = 0\,.$$

From singlet evolution

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Backup slides: $\mathcal{O}(\alpha\,\alpha_S)$ splittings

Explicit formulae (I)

$$P_{q\gamma}^{(1,1)} = \frac{C_F\, C_A\, e_q^2}{2} \left\{ 4 - 9x - (1-4x)\ln(x) - (1-2x)\ln^2(x) + 4\ln(1-x) \right.$$
$$\left. + \, p_{qg}(x) \left[2\ln^2\left(\frac{1-x}{x}\right) - 4\ln\left(\frac{1-x}{x}\right) - \frac{2\pi^2}{3} + 10 \right] \right\},$$

$$P_{g\gamma}^{(1,1)} = C_F\, C_A \left(\sum_{j=1}^{n_F} e_{q_j}^2 \right) \left\{ -16 + 8x + \frac{20}{3}x^2 + \frac{4}{3x} - (6+10x)\ln(x) - 2(1+x)\ln^2(x) \right\},$$

$$P_{\gamma\gamma}^{(1,1)} = -C_F\, C_A \left(\sum_{j=1}^{n_F} e_{q_j}^2 \right) \delta(1-x)\,,$$

$$P_{qg}^{(1,1)} = \frac{T_R\, e_q^2}{2} \left\{ 4 - 9x - (1-4x)\ln(x) - (1-2x)\ln^2(x) + 4\ln(1-x) \right.$$
$$\left. + \, p_{qg}(x) \left[2\ln^2\left(\frac{1-x}{x}\right) - 4\ln\left(\frac{1-x}{x}\right) - \frac{2\pi^2}{3} + 10 \right] \right\},$$

$$P_{\gamma g}^{(1,1)} = T_R \left(\sum_{j=1}^{n_F} e_{q_j}^2 \right) \left\{ -16 + 8x + \frac{20}{3}x^2 + \frac{4}{3x} - (6+10x)\ln(x) - 2(1+x)\ln^2(x) \right\},$$

$$P_{gg}^{(1,1)} = -T_R \left(\sum_{j=1}^{n_F} e_{q_j}^2 \right) \delta(1-x)\,,$$

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282

Backup slides: $\mathcal{O}(\alpha\,\alpha_S)$ splittings

Explicit formulae (II)

$$
\begin{aligned}
P_{qq}^{S(1,1)} &= P_{q\bar{q}}^{S(1,1)} = 0\,, \\
P_{qq}^{V(1,1)} &= -2\,C_F\,e_q^2 \left[\left(2\ln(1-x) + \frac{3}{2} \right) \ln(x)\, p_{qq}(x) + \frac{3+7x}{2} \ln(x) + \frac{1+x}{2} \ln^2(x) \right. \\
&\quad \left. + 5(1-x) + \left(\frac{\pi^2}{2} - \frac{3}{8} - 6\zeta_3 \right) \delta(1-x) \right]\,, \\
P_{q\bar{q}}^{V(1,1)} &= 2\,C_F\,e_q^2 \left[4(1-x) + 2(1+x)\ln(x) + 2p_{qq}(-x)S_2(x) \right]\,, \\
P_{gq}^{(1,1)} &= C_F\,e_q^2 \left[-(3\ln(1-x) + \ln^2(1-x))p_{gq}(x) + \left(2 + \frac{7}{2}x \right) \ln(x) \right. \\
&\quad \left. - \left(1 - \frac{x}{2} \right) \ln^2(x) - 2x\ln(1-x) - \frac{7}{2}x - \frac{5}{2} \right]\,, \\
P_{\gamma q}^{(1,1)} &= P_{gq}^{(1,1)}\,,
\end{aligned}
$$

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282

Backup slides: $\mathcal{O}(\alpha^2)$ splittings

Explicit formulae (I)

$$P_{q\gamma}^{(0,2)} = \frac{C_A\, e_q^4}{2} \left\{4 - 9x - (1-4x)\ln(x) - (1-2x)\ln^2(x) + 4\ln(1-x) \right.$$
$$\left. + \; p_{qg}(x)\left[2\ln^2\left(\frac{1-x}{x}\right) - 4\ln\left(\frac{1-x}{x}\right) - \frac{2\pi^2}{3} + 10\right]\right\} ,$$

$$P_{\gamma q}^{(0,2)} = e_q^4 \left[-\left(3\ln(1-x) + \ln^2(1-x)\right) p_{gq}(x) + \left(2 + \frac{7}{2}x\right)\ln(x) - \left(1 - \frac{x}{2}\right)\ln^2(x) \right.$$
$$\left. - \; 2x\ln(1-x) - \frac{7}{2}x - \frac{5}{2}\right] - e_q^2\left(\sum_f e_f^2\right)\left[\frac{4}{3}x + p_{gq}(x)\left(\frac{20}{9} + \frac{4}{3}\ln(1-x)\right)\right] ,$$

$$P_{qq}^{V(0,2)} = -e_q^4 \left[\left(2\ln(x)\ln(1-x) + \frac{3}{2}\ln(x)\right) p_{qq}(x) + \frac{3+7x}{2}\ln(x) \right.$$
$$\left. + \; \frac{1+x}{2}\ln^2(x) + 5(1-x) + \left(\frac{\pi^2}{2} - \frac{3}{8} - 6\zeta_3\right)\delta(1-x)\right]$$
$$- \; e_q^2\left(\sum_f e_f^2\right)\left[\frac{4}{3}(1-x) + p_{qq}(x)\left(\frac{2}{3}\ln(x) + \frac{10}{9}\right) + \left(\frac{2\pi^2}{9} + \frac{1}{6}\right)\delta(1-x)\right] ,$$

$$P_{q\bar{q}}^{V(0,2)} = e_q^4\left[4(1-x) + 2(1+x)\ln(x) + 2p_{qq}(-x)S_2(x)\right] ,$$

$$P_{qQ}^{S(0,2)} = P_{q\bar{Q}}^{S(0,2)} = C_A\, e_q^2\, e_Q^2\, p_s(x) ,$$

de Florian, Rodrigo and GS, JHEP 10(2016)056

Backup slides: $\mathcal{O}(\alpha^2)$ splittings

Explicit formulae (II)

$$
\begin{aligned}
P_{l\gamma}^{(0,2)} &= \frac{e_l^4}{C_A\, e_q^4}\, P_{q\gamma}^{(0,2)}\,, \\
P_{\gamma l}^{(0,2)} &= e_l^4 \left[-(3\ln(1-x) + \ln^2(1-x))p_{gq}(x) + \left(2 + \frac{7}{2}x\right)\ln(x) - \left(1 - \frac{x}{2}\right)\ln^2(x) \right. \\
&\quad \left. - \, 2x\ln(1-x) - \frac{7}{2}x - \frac{5}{2} \right] - e_l^2 \left(\sum_f e_f^2\right)\left[\frac{4}{3}x + p_{gq}(x)\left(\frac{20}{9} + \frac{4}{3}\ln(1-x)\right)\right]\,, \\
P_{ll}^{V(0,2)} &= -e_l^4 \left[\left(2\ln(x)\ln(1-x) + \frac{3}{2}\ln(x)\right)p_{qq}(x) + \frac{3+7x}{2}\ln(x) \right. \\
&\quad \left. + \, \frac{1+x}{2}\ln^2(x) + 5(1-x) + \left(\frac{\pi^2}{2} - \frac{3}{8} - 6\zeta_3\right)\delta(1-x) \right] \\
&\quad - \, e_l^2 \left(\sum_f e_f^2\right)\left[\frac{4}{3}(1-x) + p_{qq}(x)\left(\frac{2}{3}\ln(x) + \frac{10}{9}\right) + \left(\frac{2\pi^2}{9} + \frac{1}{6}\right)\delta(1-x)\right]\,, \\
P_{l\bar{l}}^{V(0,2)} &= \frac{e_l^4}{e_q^4}\, P_{q\bar{q}}^{V(0,2)}\,, \\
P_{lL}^{S(0,2)} &= P_{l\bar{L}}^{S(0,2)} = e_l^2\, e_L^2\, p_s(x)\,.
\end{aligned}
$$

de Florian, Rodrigo and GS, JHEP 10(2016)056

Backup slides: $\mathcal{O}(\alpha^2)$ splittings

Explicit formulae (III)

$$P_{\gamma\gamma}^{(0,2)} = \left(\sum_f e_f^4\right)\left[-16+8x+\frac{20}{3}x^2+\frac{4}{3x}-(6+10x)\ln(x) - 2(1+x)\ln^2(x)-\delta(1-x)\right],$$

$$P_{fg}^{(0,2)}=0\,,\quad P_{gf}^{(0,2)}=0\,,\quad P_{\gamma g}^{(0,2)}=0\,,\quad P_{lq}^{S(0,2)}=P_{l\bar{q}}^{S(0,2)}=e_l^2\,e_q^2\,p_s(x)\,,$$

$$P_{g\gamma}^{(0,2)}=0\,,\quad P_{gg}^{(0,2)}=0\,.\quad P_{ql}^{S(0,2)}=P_{q\bar{l}}^{S(0,2)}=C_A\,e_l^2\,e_q^2\,p_s(x)\,,$$

$$p_s(x)=\frac{20}{9x}-2+6x-\frac{56}{9}x^2+\left(1+5x+\frac{8}{3}x^2\right)\ln(x)-(1+x)\ln^2(x)$$

de Florian, Rodrigo and GS, JHEP 10(2016)056