Improvements of the sector-improved residue subtraction scheme

Rene Poncelet
in collaboration with Michal Czakon and Arnd Behring

Institute for Theoretical Particle Physics and Cosmology
RWTH Aachen University

2017-08-29

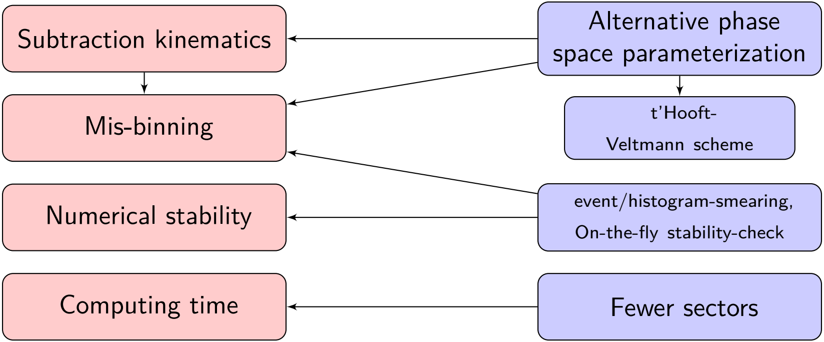
NNLO subtraction schemes

Handling real radiation contribution in NNLO calculations
cancellation of infrared divergences

increasing number of available NNLO calculations with a variety of schemes

- qT-slicing [Catani,Grazzini, '07] ,[Ferrera,Grazzini,Tramontano, '11], [Catani,Cieri,DeFlorian,Ferrera,Grazzini,'12], [Gehrmann,Grazzini,Kallweit,Maierhofer,Manteuffel,Rathlev,Torre,'14-15'], [Bonciani,Catani,Grazzini,Sargsyan,Torre,'14-'15]
- N-jettiness slicing [Gaunt,Stahlhofen,Tackmann,Walsh, '15], [Boughezal,Focke,Giele,Liu,Petriello,'15-'16] , [Bougezal,Campell,Ellis,Focke,Giele,Liu,Petriello,'15], [Campell,Ellis,Williams,'16]
- Antenna subtraction [Gehrmann, GehrmannDeRidder,Glover,Heinrich,'05-'08] , [Weinzierl,'08,'09], [Currie,Gehrmann,GehrmannDeRidder,Glover,Pires,'13-'17], [Bernreuther,Bogner,Dekkers,'11,'14], [Abelof,(Dekkers),GehrmannDeRidder,'11-'15], [Abelof,GehrmannDeRidder,Maierhofer,Pozzorini,'14], [Chen,Gehrmann,Glover,Jaquier,'15]
- Colorful subtraction [DelDuca,Somogyi,Troscanyi,'05-'13], [DelDuca,Duhr,Somogyi,Tramontano,Troscanyi,'15]
- Sector-improved residue subtraction (STRIPPER) [Czakon,'10,'11] , [Czakon,Fiedler,Mitov,'13,'15], [Czakon,Heymes,'14] [Czakon,Fiedler,Heymes,Mitov,'16,'17], [Bughezal,Caola,Melnikov,Petriello,Schulze,'13,'14], [Bughezal,Melnikov,Petriello,'11], [Caola,Czernecki,Liang,Melnikov,Szafron,'14], [Bruchseifer,Caola,Melnikov,'13-'14], [Caola, Melnikov, Röntsch,'17]

How to improve the STRIPPER subtraction scheme?



Motto

Optimizing by minimizing

Sector-decomposition

Formulation

Hadronic cross section:

$$\sigma_{h_1 h_2}(P_1, P_2) = \sum \iint_0^1 \mathrm{d}x_1 \, \mathrm{d}x_2 \, f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

partonic cross section:

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \hat{\sigma}_{ab}^{(1)} + \hat{\sigma}_{ab}^{(2)} + \mathcal{O}\left(\alpha_S^3\right)$$

Contributions with different final state multiplicities and convolutions:

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\mathrm{RR}} + \hat{\sigma}_{ab}^{\mathrm{RV}} + \hat{\sigma}_{ab}^{\mathrm{VV}} + \hat{\sigma}_{ab}^{\mathrm{C2}} + \hat{\sigma}_{ab}^{\mathrm{C1}}$$

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1} \, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution}) \, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution}) \, \mathrm{F}_{n}$$

Sector decomposition

Several layers of decomposition

Selector functions:

$$1 = \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right]$$

Factorization of double soft limits:

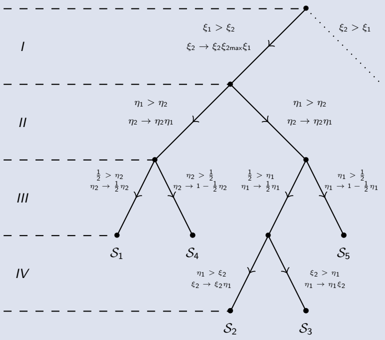
$$\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$$

Sector parameterization

Parameterization with respect to the reference parton r:
angles: $\hat{\eta}_i = \frac{1}{2}(1 - \cos\theta_{ir}) \in [0,1]$
energies: $\hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0,1]$

originally: 5 sub-sectors

Triple collinear factorization



Sector decomposition

Several layers of decomposition

Selector functions:

$$1 = \sum_{i,j} \left[\sum_{k} \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right]$$

Factorization of double soft limits:

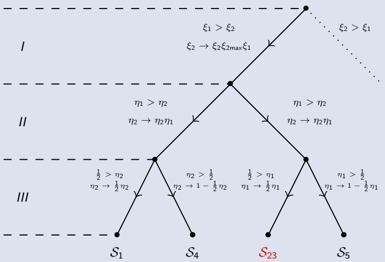
$$\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$$

Sector parameterization

Parameterization with respect to the reference parton r:
angles: $\hat{\eta}_i = \frac{1}{2}(1 - \cos\theta_{ir}) \in [0, 1]$
energies: $\hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0, 1]$

now: 4 sub-sectors

Triple collinear factorization

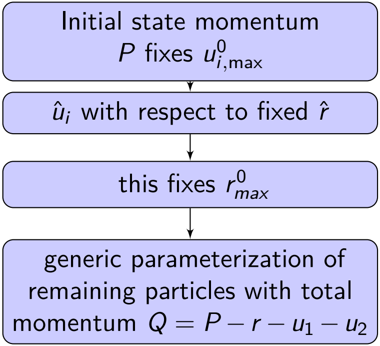


Caola, Melnikov, Röntsch [hep-ph:1702.01352v1]

Phase Space parameterization

Original phase space parameterization

triple coll. final state ref. case:



Other cases (single- and double-coll., initial state ref.) similar

type	unresolved config.	number
single	$\{r\}, \{r+u\}$	2
triple	double unres.	
	$\{r\}, \{r+u_1\}, \{r+u_1+u_2\}$	3
	single unres.	
1	$\{u_1, r\}, \{u_1, r+u_2\}$	2
2	$\{u_1, r\}$	1
3	$\{u_2, r+u_1\}$	1
4	$\{u_1, r\}, \{u_1+u_2, r\}$	2
5	$\{u_1, r\}, \{u_1+u_2, r\}$, $\{u_1 + \text{soft} u_2, r\}$	3
double	double unres.	
	$\{r_1, r_2\}, \{r_1+u_1, r_2\}$, $\{r_1+u_1, r_2+u_2\}$	3
	single unres.	
	$\{u_1, r_1, r_2\}, \{u_1, r_1, r_2+u_2\}$, $\{r_1+u_1, r_2, u_2\}$	3

Original phase space parameterization

Number of subtraction kinematics

- number is not minimal
- higher prob. of mis-binned subtraction events

⇓

- Better convergence of differential contributions
- Elegance

type	unresolved config.	number
single	$\{r\}, \{r+u\}$	2
triple	double unres.	
	$\{r\}, \{r+u_1\}, \{r+u_1+u_2\}$	3
	single unres.	
1	$\{u_1, r\}, \{u_1, r+u_2\}$	2
2	$\{u_1, r\}$	1
3	$\{u_2, r+u_1\}$	1
4	$\{u_1, r\}, \{u_1+u_2, r\}$	2
5	$\{u_1, r\}, \{u_1+u_2, r\},$	
	$\{u_1+\text{soft}u_2, r\}$	3
double	double unres.	
	$\{r_1, r_2\}, \{r_1+u_1, r_2\},$	
	$\{r_1+u_1, r_2+u_2\}$	3
	single unres.	
	$\{u_1, r_1, r_2\}, \{u_1, r_1, r_2+u_2\},$	
	$\{r_1+u_1, r_2, u_2\}$	3

Concept of new parameterization

Perform mapping $\mathrm{d}\Phi_{n+2}$ to Born configuration: $\{P, r_j, u_k\} \to \{\tilde{P}, \tilde{r}_j\}$

modification of [Frixone,Webber'02] or [Frixione,Nason,Oleari'07]

- keeping the direction of:
  sum of reference plus unresolved momenta $\leftrightarrow$ only reference momentum
- invertible: $\{\tilde{P}, \tilde{r}_j, u_k\} \to \{P, r_j, u_k\}$
- preserves $q^2 = \tilde{q}^2\,,\ \ \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Only rescaling of the reference(s) and Rotation+Boost of q necessary

New phase space parameterization

Create born phase space $\{\tilde{r}\}_i, \{\tilde{q}\}_i$

↓

create u_i with respect to fixed $\tilde{r}_i$'s within constrains

↓

determine rescaling parameter (and Jacobian) as well as boost

↓

boost the remaining system to obtain $q_i = \Lambda \tilde{q}_i$ (restores momentum conservation)

Additionally: $\mathcal{S}_4$,$\mathcal{S}_5$ energy parameterization by sum and portion of available energy

$\mathcal{S}$	unresolved config.	number
single	$\{r + u\}$	1
triple	double unres.	
	$\{r + u_1 + u_2\}$	1
	single unres.	
1	$\{u_1, r + u_2\}$	1
2,3	$\{u_1, r\}/\{u_2, r + u_1\}$	2
4	$\{u_1 + u_2, r\}$	1
5	$\{u_1, r\}, \{u_1 + u_2, r\}$	2
double	double unres.	
	$\{r_1 + u_1, r_2 + u_2\}$	1
	single unres.	
	$\{u_1, r_1, r_2 + u_2\}$,	
	$\{u_2, r_1 + u_1, r_2\}$,	2

Consequences

Features

- minimal number of subtraction kinematics
- only one DU configuration $\rightarrow$ pole cancellation for each Born phase space point
- expected improved convergence of invariant mass distributions, since $\tilde{q}^2 = q^2$

unintentional features

- construction in lab frame
- original construction of t'Hooft Veltmann corrections [Czakon,Heymes '14] is spoiled

t'Hooft-Veltmann scheme

Separately finite contributions

Finite parts:

σ_F^{RR}

σ_F^{RV}

σ_F^{VV}

Finite remainder parts:

$\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2}$

Single (SU) and double (DU) unresolved parts:

$\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}$

$\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{C1}, \sigma_{DU}^{VV}, \sigma_{DU}^{C2}$

$\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}$

$\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{C1}, \sigma_{DU}^{VV}, \sigma_{DU}^{C2}$

The measurement function

Observables: Implemented by infrared safe measurement function (MF) F_m

Infrared property in STRIPPER context:

- $\{x_i\} \to 0 \leftrightarrow$ single unresolved limit
  $\Rightarrow \mathrm{F}_{n+2} \to \mathrm{F}_{n+1}$
- $\{x_i\} \to 0 \leftrightarrow$ double unresolved limit
  $\Rightarrow \mathrm{F}_{n+2} \to \mathrm{F}_n$
  $\Rightarrow \mathrm{F}_{n+1} \to \mathrm{F}_n$

Tool for new t'Hooft Veltmann scheme formulation:

Parameterized MF $\mathrm{F}^{\alpha}_{n+1}$

- $\mathrm{F}^{\alpha}_{n} \equiv 0$ for $\alpha \neq 0$
  (NLO MF)
- 'arbitrary' F^{0}_{n}
  (NNLO MF)
- $\alpha \neq 0 \Rightarrow$ DU $= 0$ and SU separately finite

Example: $F^{\alpha}_{n+1} = F_{n+1}\Theta_{\alpha}(\{\alpha_i\})$
with $\Theta_{\alpha} = 0$ if some $\alpha_i < \alpha$

The single unresolved (SU) contribution

$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1}$ where $\sigma_{SU}^{c} = \int \mathrm{d}\Phi_{n+1} \left(I_{n+1}^{\mathrm{c}} F_{n+1} + I_{n}^{\mathrm{c}} F_{n} \right)$

NLO measurement function ($\alpha \neq 0$):

$$\int \mathrm{d}\Phi_{n+1} \left(I_{n+1}^{\mathrm{RR}} + I_{n+1}^{\mathrm{RV}} + I_{n+1}^{\mathrm{C1}} \right) F_{n+1}^{\alpha} = \text{finite in 4 dim.}$$

All divergences cancel in d-dimensions:

$$\sum_{c} \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1}^{\alpha} \equiv \sum_{c} \mathcal{I}^{c} = 0$$

SU finiteness for $\alpha = 0$

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \underbrace{- \mathcal{I}^{RR} - \mathcal{I}^{RV} - \mathcal{I}^{C1}}_{=0}$$

$$\sigma_{SU}^{c} - \mathcal{I}^{c} = \int \mathrm{d}\Phi_{n+1} \left\{ \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} + I_{n+1}^{c,(0)} \right] F_{n+1} + \left[\frac{I_{n}^{c,(-2)}}{\epsilon^2} + \frac{I_{n}^{c,(-1)}}{\epsilon} + I_{n}^{c,(0)} \right] F_{n} \right\}$$
$$- \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1} \Theta_{\alpha}(\{\alpha_i\})$$

SU finiteness for $\alpha = 0$

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \underbrace{-\mathcal{I}^{RR} - \mathcal{I}^{RV} - \mathcal{I}^{C1}}_{=0}$$

$$\begin{aligned}
\sigma_{SU}^{c} - \mathcal{I}^{c} &= \int \mathrm{d}\Phi_{n+1} \left\{ \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} + I_{n+1}^{c,(0)} \right] F_{n+1} + \left[\frac{I_{n}^{c,(-2)}}{\epsilon^2} + \frac{I_{n}^{c,(-1)}}{\epsilon} + I_{n}^{c,(0)} \right] F_{n} \right\} \\
&\quad - \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1} \Theta_\alpha(\{\alpha_i\}) \\
&= \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)} F_{n+1} + I_{n}^{c,(-2)} F_{n}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)} F_{n+1} + I_{n}^{c,(-1)} F_{n}}{\epsilon} \right] (1 - \Theta_\alpha(\{\alpha_i\})) \\
&\quad + \int \mathrm{d}\Phi_{n+1} \left[I_{n+1}^{c,(0)} F_{n+1} + I_{n}^{c,(0)} F_{n} \right] + \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n}^{c,(-2)}}{\epsilon^2} + \frac{I_{n}^{c,(-1)}}{\epsilon} \right] F_{n} \Theta_\alpha(\{\alpha_i\})
\end{aligned}$$

$$=: \underbrace{Z^c(\alpha)}_{\text{integrable, zero volume for } \alpha \to 0} + \underbrace{C^c}_{\text{no divergencies}} + \underbrace{N^c(\alpha)}_{\text{only } F_n \to \text{DU}}$$

The function $N^c(\alpha)$

Looks like slicing, but it is slicing *only* for divergences
$\rightarrow$ no actual slicing parameter in result

Powerlog-expansion:

$$N^c(\alpha) = \sum_{k=0}^{\ell_{\max}} \ln^k(\alpha) N_k^c(\alpha)$$

- all $N_k^c(\alpha)$ regular in α
- start expression independent of $\alpha \Rightarrow$ all logs cancel
- only $N_0^c(0)$ relevant

Putting parts together:

$$\sigma_{SU} - \sum_c N_0^c(0) \text{ and } \sigma_{DU} + \sum_c N_0^c(0)$$

are finite in 4 dimension

$\Downarrow$

SU contribution: $\sigma_{SU} - \sum_c N_0^c = \sum_c C^c$
original expression σ_{SU} in 4-dim without poles, no further ϵ pole cancellation

Summary

Summary

- Minimizing the STRIPPER scheme
- alternative phase space parameterization
- new formulation of t'Hooft Veltmann scheme
- tests for a class of processes:
  $pp \to t\bar{t}$, $e^+e^- \to 2, 3j$, t decay, DIS, Drell-Yan, H decays, dijets

Summary

Summary

- Minimizing the STRIPPER scheme
- alternative phase space parameterization
- new formulation of t'Hooft Veltmann scheme
- tests for a class of processes:
  $pp \to t\bar{t}$, $e^+e^- \to 2, 3j$, t decay, DIS, Drell-Yan, H decays, dijets

Thank you for your attention

Supplements

Factorization and subtraction terms

Single unresolved phase space:

$$\iint_0^1 \mathrm{d}\eta \, \mathrm{d}\xi \; \eta^{a_1 - b_1 \epsilon} \xi^{a_2 - b_2 \epsilon}$$

Double unresolved phase space:

$$\iiiint_0^1 \mathrm{d}\eta_1 \, \mathrm{d}\xi_1 \, \mathrm{d}\eta_2 \, \mathrm{d}\xi_2 \; \eta_1^{a_1 - b_1 \epsilon} \xi_1^{a_2 - b_2 \epsilon} \eta_2^{a_3 - b_3 \epsilon} \xi_2^{a_4 - b_4 \epsilon}$$

Factorized singular limits:

$$\int \mathrm{d}\Phi_n \prod \mathrm{d}x_i \underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}} \tilde{\mu}(\{x_i\})$$

$$\underbrace{\prod x_i^{a_i+1} \langle \mathcal{M}_{n+2} | \mathcal{M}_{n+2} \rangle}_{\text{regular}} \mathrm{F}_{n+2}$$

Regularisation:

Master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\int_0^1 \mathrm{d}x \left[x^{-1-b\epsilon}\right]_+ f(x) = \int_0^1 \frac{f(x) - f(0)}{x^{1+b\epsilon}}$$

Calculation of $N_0^c(0)$

For each sector/contribution:

1. extraction of $\mathrm{d}\Phi_{n+1}$ from $\mathrm{d}\Phi_{n+2}\,|_{\text{SU pole}}$ (only for *RR* contribution)

$$\mathrm{d}\Phi_{n+2}\,|_{\text{SU pole}} = \Big(\underbrace{\mathrm{d}\Phi_n\,\mathrm{d}^d\mu(u_1)}_{\mathrm{d}\Phi_{n+1}}\mathrm{d}^d\mu(u_2)\,\Big)\Big|_{u_2\text{col/soft}}$$

2. expansion in ϵ up to ϵ^{-1} (except $\mathrm{d}\Phi_{n+1}$): $\mathrm{d}^d\Phi_{n+1}\left(\frac{\cdots}{\epsilon^2}+\frac{\cdots}{\epsilon}\right)$
3. Identifying $\ln^k(\alpha)$'s from x_i integrations over Θ function

$$\Theta_\alpha(\hat\eta, u^0) = \Theta(\hat\eta-\alpha)\Theta(\hat\xi u_{max}/E_{norm}-\alpha)$$

   $\rightarrow$ discard them

4. perform integration over Θ-functions of non-canceling and non-vanishing (in $\alpha \rightarrow 0$ limit) terms

PS

common starting point for all phase spaces :

$$\mathrm{d}\Phi_n = \mathrm{d}Q^2 \left[\prod_{j=1}^{n_{fr}} \mu_0(r_j) \prod_{k=1}^{n_u} \mu_0(u_k) \delta_+ \left(\left(P - \sum_{j=1}^{n_{fr}} r_j - \sum_{k=1}^{n_u} u_k \right)^2 - Q^2 \right) \right] \prod_{i=1}^{n_q} \mu_{m_i}(q_i)(2\pi)^d \delta^{(d)} \left(\sum_{i=1}^{n_q} q_i - q \right)$$

$$\text{with } \mu_m(k) \equiv \frac{\mathrm{d}^d k}{(2\pi)^d} 2\pi \delta\left(k^2 - m^2\right) \theta\left(k^0\right),$$

n : # final state particles, n_{fr} : # final state references, n_u : # additional partons