Multidimensional observables and Mustraal frame

Separating electroweak and strong interaction effect: angular coefficients, effective Born, genuine weak QED and QCD effects

version with improvements introduced thanks to discussions after the talk.

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My goal is to look at some old, even very old results whether they can be useful to organize predictions of Standard Model measurements in general. And in particular for weighted events especially in perpective of Mashine Learning techniques.

Heritage: effective Born $\times$ genuine weak effects $\times$ ISR/FSR QED. Anything of use for LHC?

- References:
  New: Eur.Phys.J. C77(2017)111, Eur.Phys.J. C76 (2016)473,
  Old: Comput.Phys.Commun. 29 (1983) 185, E. Mirkes, Nucl.Phys. B387 (1992) 3, Phys.Rev. D50 (1994) 5692, D51 (1995) 4891, R. Kleiss Nucl.Phys. B347 (1990) 67

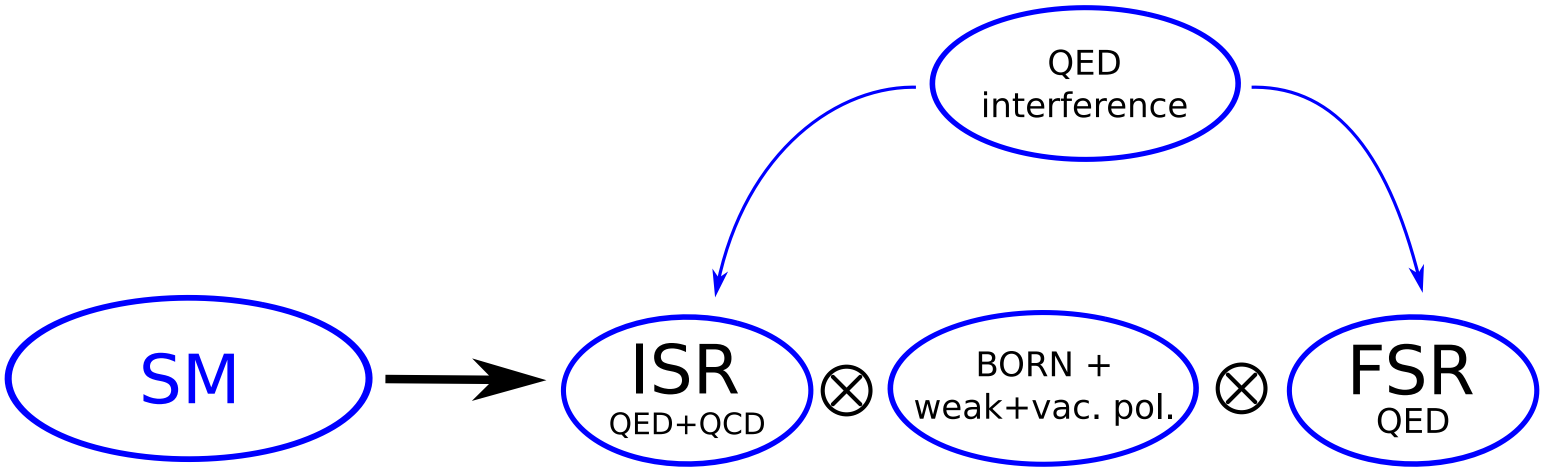
What Z, W, H signatures may mean?

- Even if physical gauge is chosen and bosons acquire masses, at Born level of SM, W, H and Z propagators are singular: $\frac{1}{s-M^2}$.
  This seems trivial:
  Replace propagator with the effective one $\frac{1}{s-M^2+i\Gamma M}$.
  Partial resummation of loop corrections to all orders must be performed to get $i\Gamma M$!

- Resulting approach, make bosons into physics states of definite properties, including width. It required massive effort at LEP. Results are used by CDF D0 as state-of-art today also. See Arie Bodek talk, CERN Jan 31, 2017, https://indico.cern.ch/event/571075/

- **I will not go into all details necessary for fundaments. I will concentrate on practical aspects/results.**

Production and decay for Bosons



- That is the picture we inherit.
- Let me present some details and later how it works in case when $2 \rightarrow 4$ matrix elements are used.
- For the precison to be controlled one must be able to define for each program:
  - phase space parametrization (which is best to be precise and explicit)
  - matrix elements (there approximations can be then numerically evaluated).

Topics:

A. Effective Born

B. Effective Born and jets ..

C. Electroweak form-factors

D. Projection operators, and other applications.

part A: Effective Born

Let us start with the lowest order coupling constants (without EW corrections) of the Z boson to fermions, where $s_W^2 = 1 - m_W^2/m_Z^2$ denotes $\sin\theta_W^2$ in the on-line scheme and T_3^f denotes third component of the isospin.

The vector v_e, v_f and axial a_e, a_f couplings for leptons and quarks respectively are defined with formulas below.

$$\begin{aligned} v_e &= (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2)/\Delta \\ v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2)/\Delta \\ a_e &= (2 \cdot T_3^e)/\Delta \\ a_f &= (2 \cdot T_3^f)/\Delta \end{aligned} \tag{1}$$

where

$$\Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)} \tag{2}$$

part A: Effective Born

With this notation, matrix element for the $q\bar{q} \to Z/\gamma^* \to l^+l^-$, denoted as ME_{Born}, can be written as:

$$
\begin{aligned}
ME_{Born} &= [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu u] \cdot (q_e \cdot q_f) \cdot \frac{\chi_\gamma(s)}{s} \\
&+ [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{\nu}\gamma^\nu u \cdot (v_e \cdot v_f) + \bar{u}\gamma^\mu v g_{\mu\nu} \bar{\nu}\gamma^\nu\gamma^5 u \cdot (v_e \cdot a_f) \\
&+ \bar{u}\gamma^\mu\gamma^5 v g_{\mu\nu} \bar{\nu}\gamma^\nu u \cdot (a_e \cdot v_f) + \bar{u}\gamma^\mu\gamma^5 v g_{\mu\nu} \bar{\nu}\gamma^\nu\gamma^5 u \cdot (a_e \cdot a_f)] \cdot \frac{\chi_Z(s)}{s}
\end{aligned} \tag{3}
$$

and Z-boson and photon propagators defined respectively as

$$\chi_\gamma(s) = 1 \tag{4}$$

$$\chi_Z(s) = \frac{G_\mu \dot{M}_z^2}{\sqrt{2} \cdot 8\pi \cdot \alpha_{QED}(0)} \cdot \Delta^2 \cdot \frac{s}{s - M_Z^2 + i \cdot \Gamma_Z \cdot M_Z} \tag{5}$$

At the peak of resonance $|\chi_Z(s)|(v_e \cdot v_f) > (q_e \cdot q_f)$ and as a consequence angular distribution asymmetries of leptons are proportional to $v_e = (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2)$. This gives good sensitivity for s_W^2 measurement.

Above and below resonance we are sensitive to lepton charge instead ...

Born cross-section, for $q\bar{q} \to Z/\gamma^* \to \ell^+\ell^-$ can be expressed as:

$$\frac{d\sigma^{q\bar{q}}_{Born}}{d\cos\theta}(s,\cos\theta,p) = (1+\cos^2\theta)\, F_0(s) + 2\cos\theta\, F_1(s) - p[(1+\cos^2\theta)\, F_2(s) + 2\cos\theta\, F_3(s)] \quad (6)$$

p denotes polarization of the outgoing leptons, and form-factors read:

$$\begin{aligned}
F_0(s) &= \frac{\pi\alpha^2}{2s}[q_f^2 q_\ell^2 \cdot \chi_\gamma^2(s) + 2\cdot\chi_\gamma(s) Re\chi_Z(s)\, q_f q_\ell v_f v_\ell + |\chi_Z^2(s)|^2 (v_f^2 + a_f^2)(v_\ell^2 + a_\ell^2)],\\
F_1(s) &= \frac{\pi\alpha^2}{2s}[2\chi_\gamma(s) Re\chi(s)\, q_f q_\ell v_f v_\ell + |\chi^2(s)|^2\, 2 v_f a_f 2 v_\ell a_\ell], \quad (7)\\
F_2(s) &= \frac{\pi\alpha^2}{2s}[2\chi_\gamma(s) Re\chi(s)\, q_f q_\ell v_f v_\ell + |\chi^2(s)|^2\, (v_f^2 + a_f^2) 2 v_\ell a_\ell],\\
F_3(s) &= \frac{\pi\alpha^2}{2s}[2\chi_\gamma(s) Re\chi(s)\, q_f q_\ell v_f v_\ell + |\chi^2(s)|^2\, (v_f^2 + a_f^2) 2 v_\ell a_\ell],
\end{aligned}$$

$\cos\theta$ denotes angle between incoming quark and outgoing lepton in the rest frame of outgoing leptons. That is rather simple spherical harmonics of the second order.

Why is it of interest?

1. Condition: $s_W^2 = 1 - m_W^2/m_Z^2$ is important for some gauge cancellations, in case of multileg processes, but at the same time bring inconsistencies with measurements:
2. either m_W must be off by many experimental errors
3. or electroweak observables such as A_{FB} or P_τ by 50 % of their measurable values.
4. Nonetheless such on mass shell scheme is used by many programs of importance for QCD phenomenology.
5. If possible, technical solutions using calculation of weights are of interest...
6. ... at least for some time. Is it possible?

Mustraal frame

[18] F. A. Berends, R. Kleiss, and S. Jadach, *Comput. Phys. Commun.* **29** (1983) 185–200.

Mustraal: Monte Carlo for $e^+ e^- \to \mu^+ \mu^- \ (\gamma)$

$$s = 2p_+ \cdot p_-, \quad t = 2p_+ \cdot q_+, \quad u = 2p_+ \cdot q_-$$

$$s' = 2q_+ \cdot q_-, \quad t' = 2p_- \cdot q_-, \quad u' = 2p_- \cdot q_+$$

$$\sigma_{\mathrm{hard}} = \int \mathrm{d}\tau \left(X_{\mathrm{i}} + X_{\mathrm{f}} + X_{\mathrm{int}} \right),$$

The explicit forms of the three terms in σ_{hard} read:

$$X_{\mathrm{i}} = \frac{Q^2\alpha}{4\pi^2 s} \frac{1-\Delta}{k_+ k_-} s'^2 \left[\frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s', t, u) + \frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s', t', u') \right], \tag{3.4}$$

$$X_{\mathrm{f}} = \frac{Q'^2\alpha}{4\pi^2 s} \frac{1-\Delta'}{k'_+ k'_-} s^2 \left[\frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s, t, u') + \frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s, t', u) \right], \tag{3.5}$$

$$X_{\mathrm{int}} = \frac{QQ'\alpha}{4\pi^2 s} W \frac{\alpha^2}{2ss'} \left[(u^2 + u'^2 + t^2 + t'^2) \tilde{f}(s, s') + \tfrac{1}{2}(u^2 + u'^2 - t^2 - t'^2) \tilde{g}(s, s') \right]$$
$$+ \frac{QQ'\alpha^3}{4\pi^2 s} \frac{(s-s')M\Gamma}{k_+ k_- k'_+ k'_-} \epsilon_{\mu\nu\rho\sigma} p_+^\mu p_-^\nu q_+^\rho q_-^\sigma \left[\tilde{E}(s, s')(t^2 - t'^2) + \tilde{F}(s, s')(u^2 - u'^2) \right], \tag{3.6}$$

Resulting optimal frame used to minimise higher order corrections from initial state radiation in $e^+e^- \to Z/\gamma^* \to \mu\ \mu$ for algorithms of genuine EW corrections implementation in LEP time Monte Carlo's like Koral Z.

2

part B: Effective Born and jets ..

- We can see that distribution is a stochastic sum of Born-like distributions with coefficients which are *positive*. But it is only QED!

What are the Limitations and Perspectives for case of QCD jets:

- E. Mirkes and J. Ohnemus, “Angular distributions of Drell-Yan lepton pairs at the Tevatron: Order $\alpha - s^2$ corrections and Monte Carlo studies,” PRD **51** (1995) 4891
- R. Kleiss, “Inherent Limitations in the Effective Beam Technique for Algorithmic Solutions to Radiative Corrections,” Nucl. Phys. B **347**, 67 (1990).

If jets are present definition of angles θ, ϕ **(defined later)**, for effective Born becomes an **issue**. But we have $\alpha_s^2 \sim 0.01$ corrections only, to spherical harmonics independently of the choice (p_T transverse momentum of $\tau\tau$-pair, Y rapidity):

$$\begin{aligned}\frac{d\sigma}{dp_T^2 dY\, d\cos\theta d\phi} &= \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^2 dY}[(1+\cos^2\theta)+ \\ 1/2\, A_0(1-3\cos^2\theta) &+ A_1\sin(2\theta)\cos\phi + 1/2\, A_2\sin^2\theta\cos(2\phi) + A_3\sin\theta\cos\phi \\ +A_4\cos\theta &+ A_5\sin^2\theta\sin(2\phi) + A_6\sin(2\theta)\sin\phi + A_7\sin\theta\ \sin\phi] \quad (8)\end{aligned}$$

Extending definition of Mustraal frame

- We extended this frame to pp -> $l^+ l^-$ j (j) case
  - reconstruct x_1, x_2 of incoming partons from final state kinematics (information on jets used)
  - assume the quark is following x_1 direction (equivalent to what done in CS frame)
  - calculate (θ_1, ϕ_1), (θ_2, ϕ_2) of two Born's, weight with probability calculated not using couplings

$$wt_1 = \frac{E_{p1}^2(1+\cos\,\theta_1^2)}{E_{p1}^2(1+\cos\,\theta_1^2)+E_{p2}^2(1+\cos\,\theta_2^2)}, \quad wt_2 = \frac{E_{p2}^2(1+\cos\,\theta_2^2)}{E_{p1}^2(1+\cos\,\theta_1^2)+E_{p2}^2(1+\cos\,\theta_2^2)}$$

3

Collins-Soper and Mustraal frames for θ, ϕ definition.

Collins-Soper: the polar θ and azimuthal ϕ angles are constructed in lepton pair rest-frame. Since the Z-boson has usually a transverse momentum, the directions of initial protons are not collinear. The polar axis (z-axis) is bisecting the angle between the momentum of one of the proton and inverse of the momentum of the other one. The sign of the z-axis is defined by the sign of the lepton-pair momentum with respect to z-axis in the laboratory frame. The y-axis is defined as the normal vector to the plane spanned by the two incoming proton momenta.

Mustraal:

- **Definition below is for reference. It is important that every event may contribute with one of two configurations, defined either with the help of first or second beam (reconstructed parton) as seen in the rest frame of lepton pair. The final choice is made with probability independent of any couplings or PDFs.**
- We start from the following information, which turns out to be sufficient: (i) The 4-momenta and charges of outgoing leptons τ_1, τ_2. (ii) The sum of 4-momenta of all outgoing partons.
- The orientation of incoming beams b_1, b_2 is fixed as follows: b_1 is chosen to be always along positive z-axis of the laboratory frame and b_2 is anti-parallel to z axis. The information on incoming partons of p_1, p_2 is not taken from the event record. It is recalculated from kinematics of outgoing particles and knowledge of the center of mass energy of colliding protons. In this convention the energy fractions x_1 and x_2 of p_1, p_2 carried by colliding partons, define also the 3-momenta which are along b_1, b_2 respectively.
- The flavour of incoming partons (quark or antiquark) is attributed as follows: incoming parton of larger x_1 (x_2) is assumed to be the quark. This is equivalent to choice that the quark follow direction of the outgoing $\ell\ell$ system, similarly as it is defined for the Collins-Soper frame. This choice is necessary to fix sign of $\cos\theta_{1.2}$ defined later.
- The 4-vectors of incoming partons and outgoing leptons are boosted into lepton-pair rest frame.
- To fix orientation of the event we use versor $\hat{x}_{lab}$ of the laboratory reference frame. It is boosted into lepton-pair rest frame as well. It will be used in definition of azimuthal angle ϕ, which has to extend over the range $(0, 2\pi)$.

Collins-Soper and Mustraal frames for θ, ϕ definition.

- We first calculate $\cos\theta_1$ (and $\cos\theta_2$) of the angle between the outgoing lepton and incoming quark (outgoing anti-lepton and incoming anti-quark) directions.

$$\cos\ \theta_1 = \frac{\vec{\tau_1}\cdot\vec{p_1}}{|\vec{\tau_1}||\vec{p_1}|}, \qquad \cos\ \theta_2 = \frac{\vec{\tau_2}\cdot\vec{p_2}}{|\vec{\tau_1}||\vec{p_2}|} \tag{9}$$

- The azimuthal angles ϕ_1 and ϕ_2 corresponding to θ_1 and θ_2 are defined as follows. We first define $e_{\vec{y_{1,2}}}$ versors and with their help later $\phi_{1,2}$ as:

$$\vec{e_y} = \frac{\vec{x_{lab}}\times\vec{p_2}}{|\vec{e_y}|}, \quad \vec{e_x} = \frac{\vec{e_y}\times\vec{p_2}}{|\vec{e_x}|}$$

$$\begin{aligned}\cos\ \phi_1 &= \frac{\vec{e_x}\cdot\vec{\tau_1}}{\sqrt{(\vec{e_x}\cdot\vec{\tau_1})^2+(\vec{e_y}\cdot\vec{\tau_1})^2}}\\ \sin\ \phi_1 &= \frac{\vec{e_y}\cdot\vec{\tau_1}}{\sqrt{(\vec{e_x}\cdot\vec{\tau_1})^2+(\vec{e_y}\cdot\vec{\tau_1})^2}}\end{aligned} \tag{10}$$

and similarly for ϕ_2:

$$\vec{e_y} = \frac{\vec{x_{lab}}\times\vec{p_1}}{|\vec{e_y}|}, \quad \vec{e_x} = \frac{\vec{e_y}\times\vec{p_1}}{|\vec{e_x}|}$$

$$\begin{aligned}\cos\ \phi_2 &= \frac{\vec{e_x}\cdot\vec{\tau_2}}{\sqrt{(\vec{e_x}\cdot\vec{\tau_2})^2+(\vec{e_y}\cdot\vec{\tau_2})^2}}\\ \sin\ \phi_2 &= \frac{\vec{e_y}\cdot\vec{\tau_2}}{\sqrt{(\vec{e_x}\cdot\vec{\tau_2})^2+(\vec{e_y}\cdot\vec{\tau_2})^2}}.\end{aligned} \tag{11}$$

Collins-Soper and Mustraal frames for θ, ϕ definition.

- Each event contributes with two Born-like kinematics configurations $\theta_1 \phi_1$, $(\theta_2 \phi_2)$, respectively with wt_1 (and wt_2) weights; $wt_1 + wt_2 = 1$ where

$$
\begin{aligned}
wt_1 &= \frac{E_{p1}^2 (1+\cos^2\theta_1)}{E_{p1}^2 (1+\cos^2\theta_1) + E_{p2}^2 (1+\cos^2\theta_2)}, \\
wt_2 &= \frac{E_{p2}^2 (1+\cos^2\theta_2)}{E_{p1}^2 (1+\cos^2\theta_1) + E_{p2}^2 (1+\cos^2\theta_2)}.
\end{aligned} \qquad (12)
$$

In the calculation of the weight, incoming partons energies E_{p1}, E_{p2} in the rest frame of lepton pair are used, but not their couplings or flavours. That is also why, instead of $\sigma_B(s, \cos\theta)$ the simplification $(1+\cos^2\theta)$ is used in Eq. (12).

part B: Numerical results, is Born recovered?

We will use samples of events generated with the `MadGraph` matrix element Monte Carlo for Drell-Yan production of τ-lepton pairs, with $m_{\tau\tau} = 80 - 100$ GeV and 13 TeV pp collisions. Lowest order spin amplitudes are used in this program for the parton level process. For the EW scheme we have used default initialisation of the `MadGraph` with on-shell definition of $\sin^2\theta_W = 1 - m_W^2/m_Z^2 = 0.2222$, which determines value of the axial coupling for leptons and quarks to the Z-boson. The incoming partons are distributed accordingly to PDFs (using CTEQ6L1 PDFs).

Moments method

- We use the Monte Carlo sample of $Z \to \ell^\pm \ell^\mp$ ($W^\pm \to \ell^\pm \nu$) events and extract angular coefficients of Eq. (8) using moments methods [Mirkes:1994]. The first moment of a polynomial $P_i(\cos\theta, \phi)$, integrated over a specific range of p_T, Y is defined:

$$\langle P_i(\cos\theta,\phi)\rangle = \frac{\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi\, P_i(\cos\theta,\phi) d\sigma(\cos\theta,\phi)}{\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi\, d\sigma(\cos\theta,\phi)}. \qquad (13)$$

- Owing to the orthogonality of the spherical polynomials of Eq. (8), the weighted average of the angular distributions with respect to any specific polynomial, Eq. (20), isolates its corresponding coefficient, averaged over some phase-space region.

- As a consequence of Eq. (8) we obtain:

$$\begin{aligned}
\langle \frac{1}{2}(1-3\cos^2\theta)\rangle &= \tfrac{3}{20}(A_0 - \tfrac{2}{3}); & \langle \sin 2\theta \cos\phi\rangle &= \frac{1}{5}A_1; \\
\langle \sin^2\theta \cos 2\phi\rangle &= \tfrac{1}{10}A_2; & \langle \sin\theta \cos\phi\rangle &= \frac{1}{4}A_3; \\
\langle \cos\theta\rangle &= \tfrac{1}{4}A_4; & \langle \sin^2\theta \sin 2\phi\rangle &= \frac{1}{5}A_5; \\
\langle \sin 2\theta \sin\phi\rangle &= \tfrac{1}{5}A_6; & \langle \sin\theta \sin\phi\rangle &= \frac{1}{4}A_7.
\end{aligned} \qquad (14)$$

part B: Numerical results, is Born recovered?

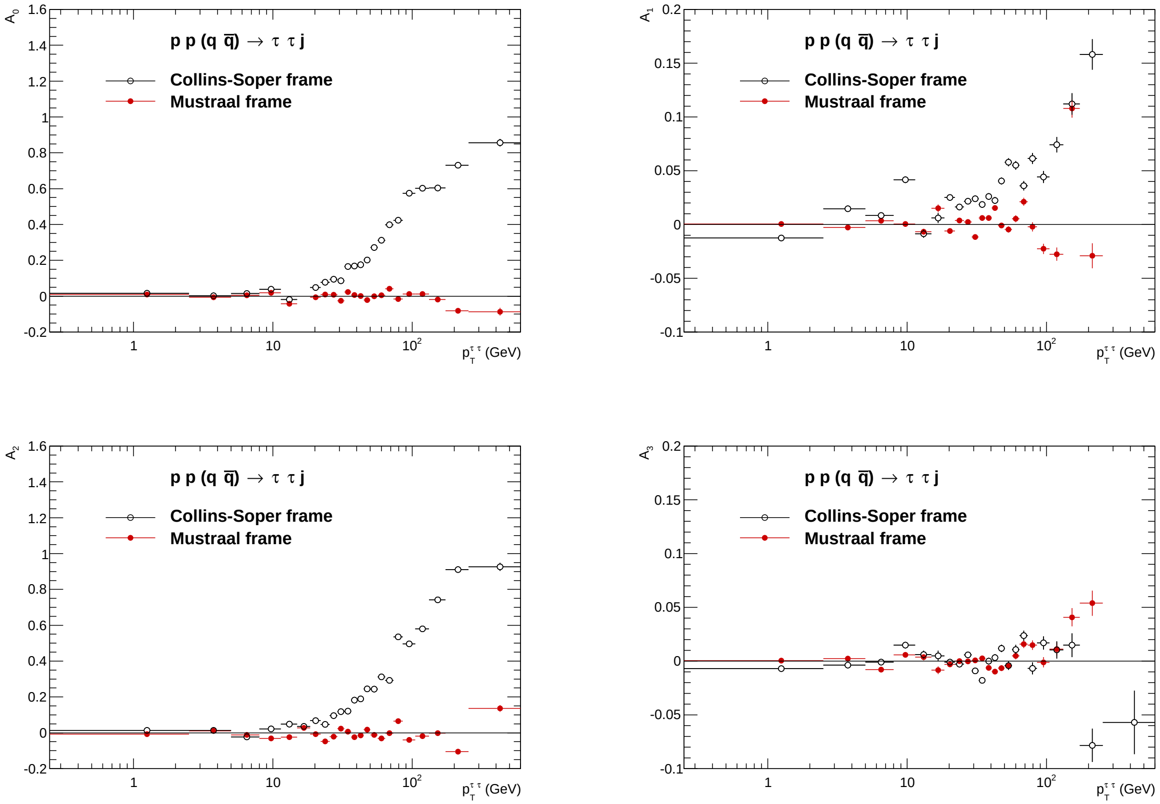


Figure 1: The A_i coefficients of Eq. (8)) calculated in Collins-Soper (black) and in `Mustraal` (red) frames for $pp(q\bar{q}) \to \tau\tau j$ process generated with `MadGraph`. From **Eur.Phys.J. C76 (2016) 473** . Tree level ME+ collinear pdf's used for analyzed sample.

Mustraal frame works PERFECT. Note that our probablities/weights were stripped from dependence on EW parameters. It could be not so, but IS SO

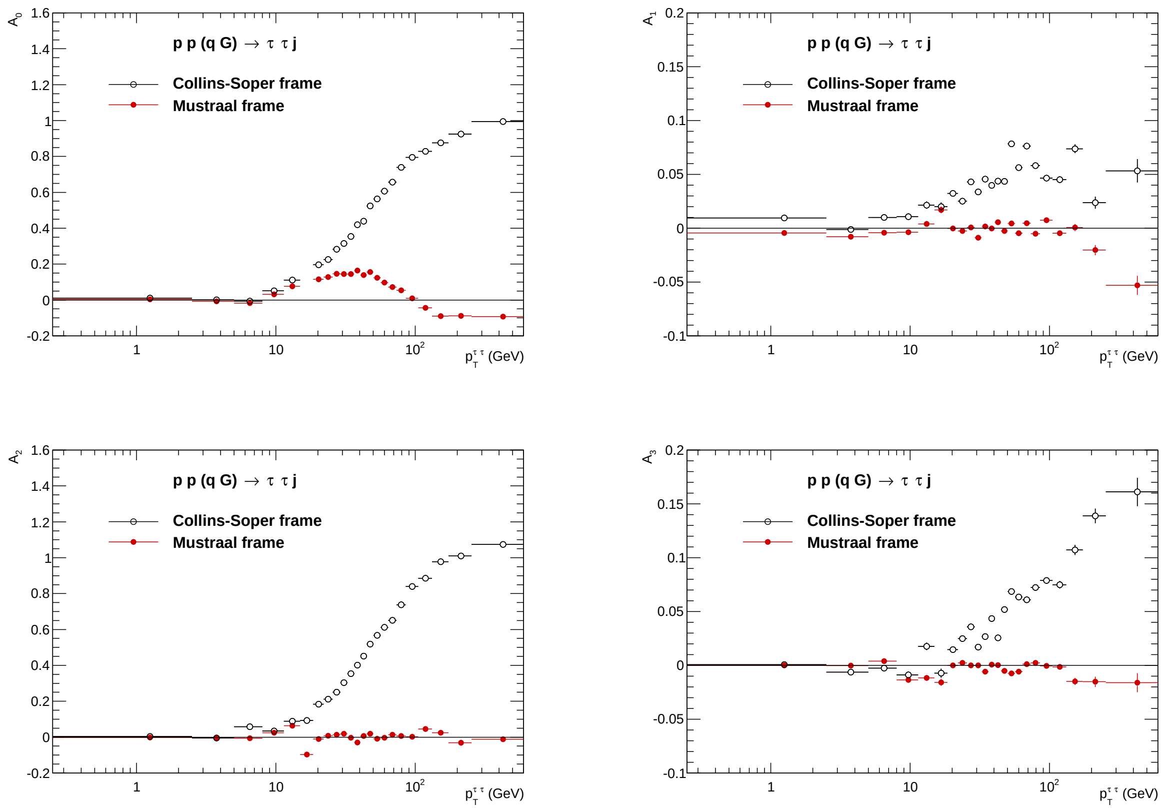


Figure 2: The A_i coefficients of Eq. (8)) calculated in Collins-Soper (black) and in `Mustraal` (red) frames for $pp(qG) \to \tau\tau j$ process generated with `MadGraph`. From **Eur.Phys.J. C76 (2016) 473** . Tree level ME+ collinear pdf's used for analyzed sample.

Mustraal frame works much better than Collins-Soper

part B: Numerical results, is Born recovered?

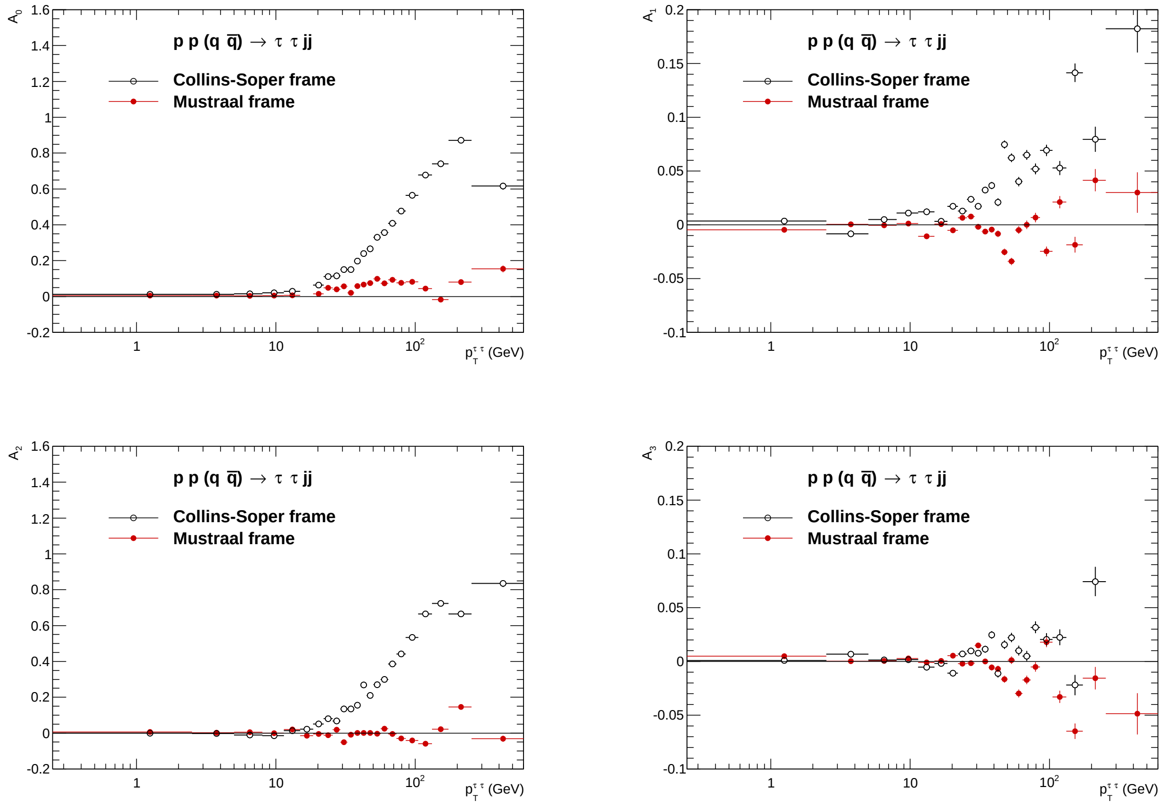


Figure 3: The A_i coefficients of Eq. (8)) calculated in Collins-Soper (black) and in `Mustraal` (red) frames for $pp(q\bar{q}) \to \tau\tau jj$ process generated with `MadGraph`. From **Eur.Phys.J. C76 (2016) 473** . Tree level ME+ collinear pdf's used for analyzed sample.

part B: Numerical results, is Born recovered?

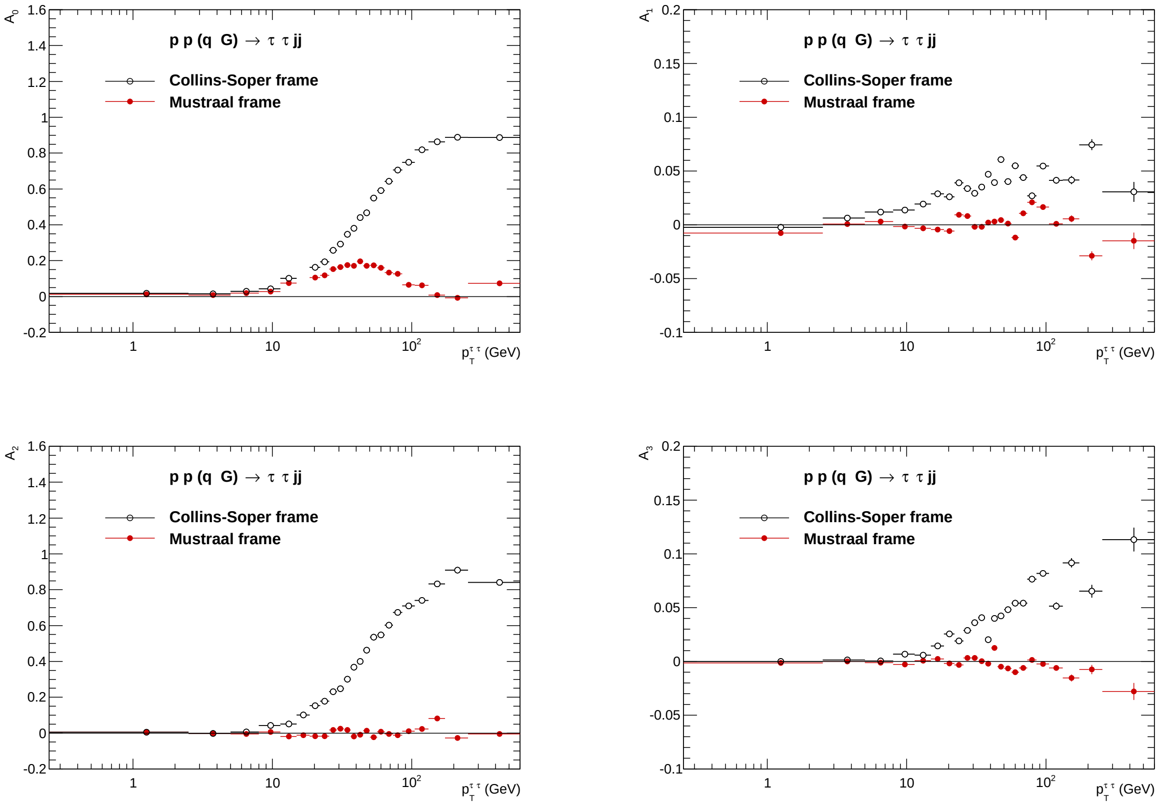


Figure 4: The A_i coefficients of Eq. (8)) calculated in Collins-Soper (black) and in `Mustraal` (red) frames for $pp(qG) \to \tau\tau jj$ process generated with `MadGraph`. From **Eur.Phys.J. C76 (2016) 473** . Tree level ME+ collinear pdf's used for analyzed sample.

part B: Numerical results, is Born recovered?

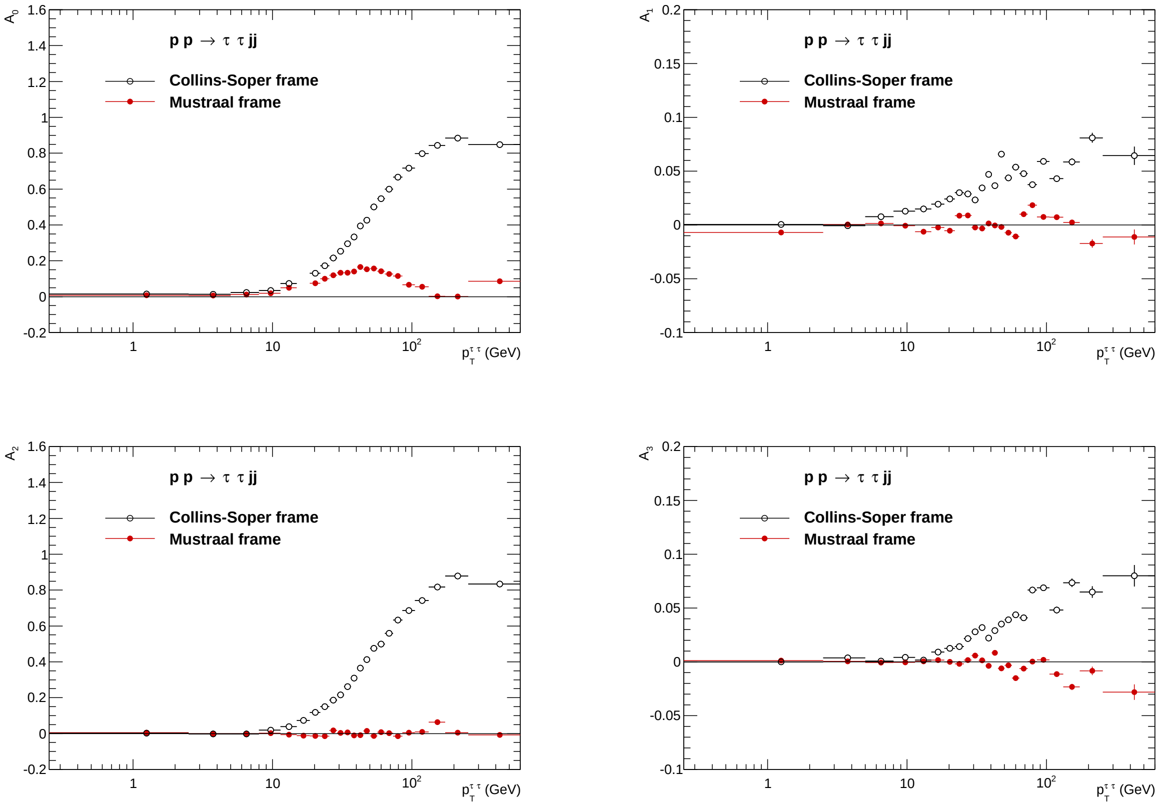


Figure 5: The A_i coefficients of Eq. (8)) calculated in Collins-Soper (black) and in `Mustraal` (red) frames for $pp \to \tau\tau jj$ process generated with `MadGraph`. From **Eur.Phys.J. C76 (2016) 473** . Tree level ME+ collinear pdf's used for analyzed sample.

part B: Numerical results, is Born recovered?

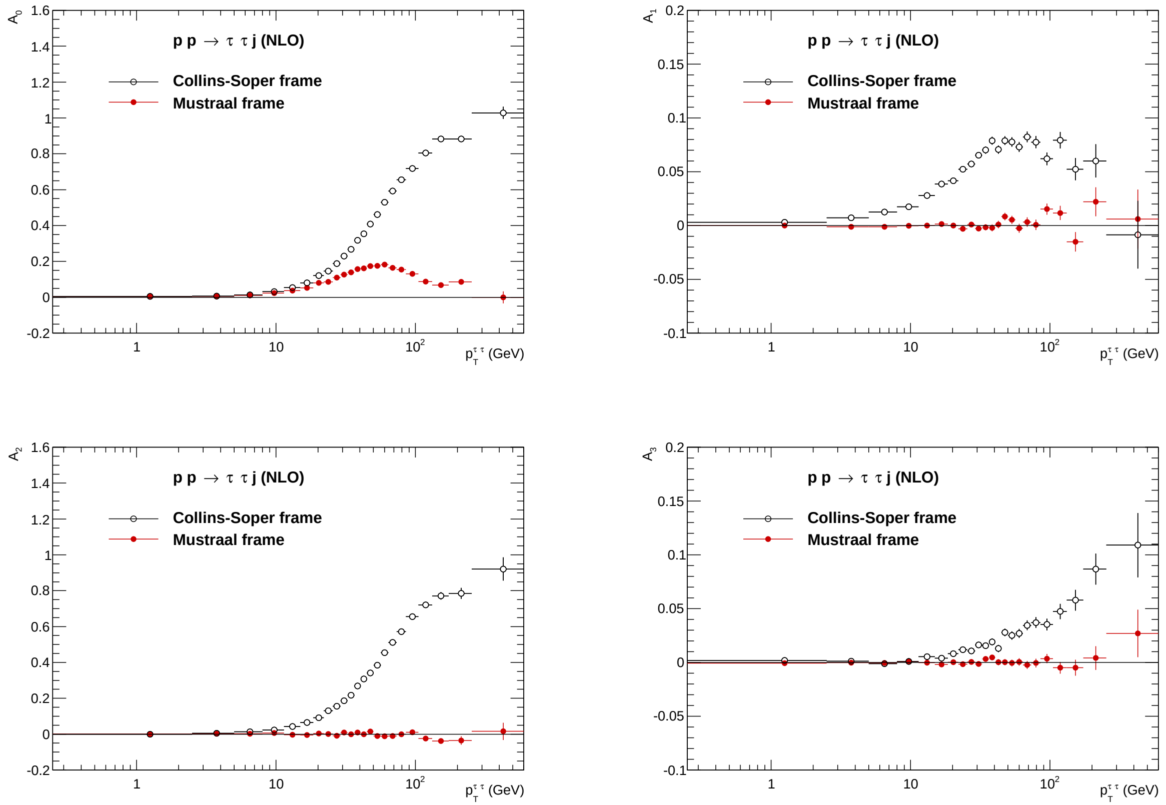


Figure 6: The A_i coefficients of Eq. (8)) calculated in Collins-Soper (black) and in `Mustraal` (red) frames for $pp \to \tau\tau j$ (NLO) process generated with `Powheg+MiNLO`. From **Eur.Phys.J. C76 (2016) 473**.

Note that for complete QCD Z+1jet NLO plus MiNLO pattern remained!

part B: Numerical results, is Born recovered?

- The choice of Mustraal frame is result of careful study of single photon (gluon) emission)
- In Ref of 1982 it was shown, that differential distribution is a sum of two born-like distributions convoluted with emission factors.
- This is a consequence of Lorentz group representation and that is why it generalizes to the case of double gluon or even double parton emissions.
- Presence of jets is like change of orientation of frames.
- That is why discussion if electroweak effects preserve, Born distributions second order spherical harmonics, is justified.

1. **Tools for discussions of the size of EW effects, $(\alpha\alpha_S)$ and observables for New Physics, also of relevance for W mass or s^2_W measurements are needed**
2. Complications due to weak Sudakovs etc. may favour other calculations, but it is of importance for the very high virtualities of $\tau\tau j$ system and may be dealt separately, J. H. Kuhn Nucl.Phys. B797 (2008) 27.
3. But one can not drop out effects which are known to be substantial. Also relation to phenomenology solutions of LEP and TEVATRON are of importance.
4. Form-factors for **"double deconvoluted gauge invariant set"** of contributions:
   - DIZET 6.21 as encapsulated in KKMC, LEP time Monte Carlo used e.g. in interpretatiuons of Z mass measurements. Comput.Phys.Commun. 59 (1990) 303
   - SANC as encapsulated in Tauola Universal Interface (no double loop QCD effects) Comput.Phys.Commun. 183 (2012) 821
   - Up to date SANC *_v_1.30* (which is to be available soon with the help of `TauSpinner`) A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, U. Klein, V. Kolesnikov, L. Rumyantsev, R. Sadykov, A. Sapronov.
     "Update of the MCSANC Monte Carlo integrator, v. 1.20".
     JETP Lett. 103 (2016) no.2, 131-136.

We can write amplitude for Born with EW loop corrections, $ME_{Born+EW}$, as:

$$\begin{aligned} ME_{Born+EW} &= [\bar{u}\gamma^{\mu}v g_{\mu\nu}\bar{v}\gamma^{\nu}u]\cdot(q_e\cdot q_f)\cdot\Gamma_{V_{\Pi}}\cdot\frac{\chi_{\gamma}(s)}{s} \\ &+ [\bar{u}\gamma^{\mu}v g_{\mu\nu}\bar{\nu}\gamma^{\nu}u\cdot(v_e\cdot v_f\cdot vv_{ef}) + \bar{u}\gamma^{\mu}v g_{\mu\nu}\bar{\nu}\gamma^{\nu}\gamma^{5}u\cdot(v_e\cdot a_f) \\ &+ \bar{u}\gamma^{\mu}\gamma^{5}v g_{\mu\nu}\bar{\nu}\gamma^{\nu}u\cdot(a_e\cdot v_f) + \bar{u}\gamma^{\mu}\gamma^{5}v g_{\mu\nu}\bar{\nu}\gamma^{\nu}\gamma^{5}u\cdot(a_e\cdot a_f)]\frac{\chi_Z(s)Z_{V_{\Pi}}}{s} \end{aligned} \tag{15}$$

One has to take into account, the angle dependent double-vector coupling extra correction, which breaks structure of the couplings into ones associated with Z boson production and decay:

$$\begin{aligned} vv_{ef} = \; & \frac{1}{v_e\cdot v_f}[(2\cdot T_3^e)(2\cdot T_3^f) - 4\cdot q_e\cdot s_W^2\cdot K_f(s,t) - 4\cdot q_f\cdot s_W^2\cdot K_e(s,t) \\ & +(4\cdot q_e\cdot s_W^2)(4\cdot q_f\cdot s_W^2)K_{ef}(s,t)]\frac{1}{\Delta^2} \end{aligned} \tag{16}$$

further terms are straightforward:

part C: Electroweak form-factors

$$
\begin{aligned}
v_e &= (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2 \cdot K_e(s,t))/\Delta \\
v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2 \cdot K_f(s,t))/\Delta \\
a_e &= (2 \cdot T_3^e)/\Delta \\
a_f &= (2 \cdot T_3^f)/\Delta
\end{aligned} \quad (17)
$$

The form-factors $K_e(s,t)$, $K_f(s,t)$ are functions of two Mandelstam invariants (s,t) due to the WW and ZZ box contributions.

Vacuum polarisation corrections Γ_{V_Π} to γ propagator are expressed as:

$$
\Gamma_{V_\Pi} = \frac{1}{2 - (1 + \Pi_{\gamma\gamma})} \quad (18)
$$

Normalisation correction Z_{V_Π} to Z-boson propagator is expressed as

$$
Z_{V_\Pi} = \rho_{e,f}(s,t) \quad (19)
$$

Form-factors vary with $\cos\theta$ by 0.1% at Z peak. Below 120 GeV by 1%

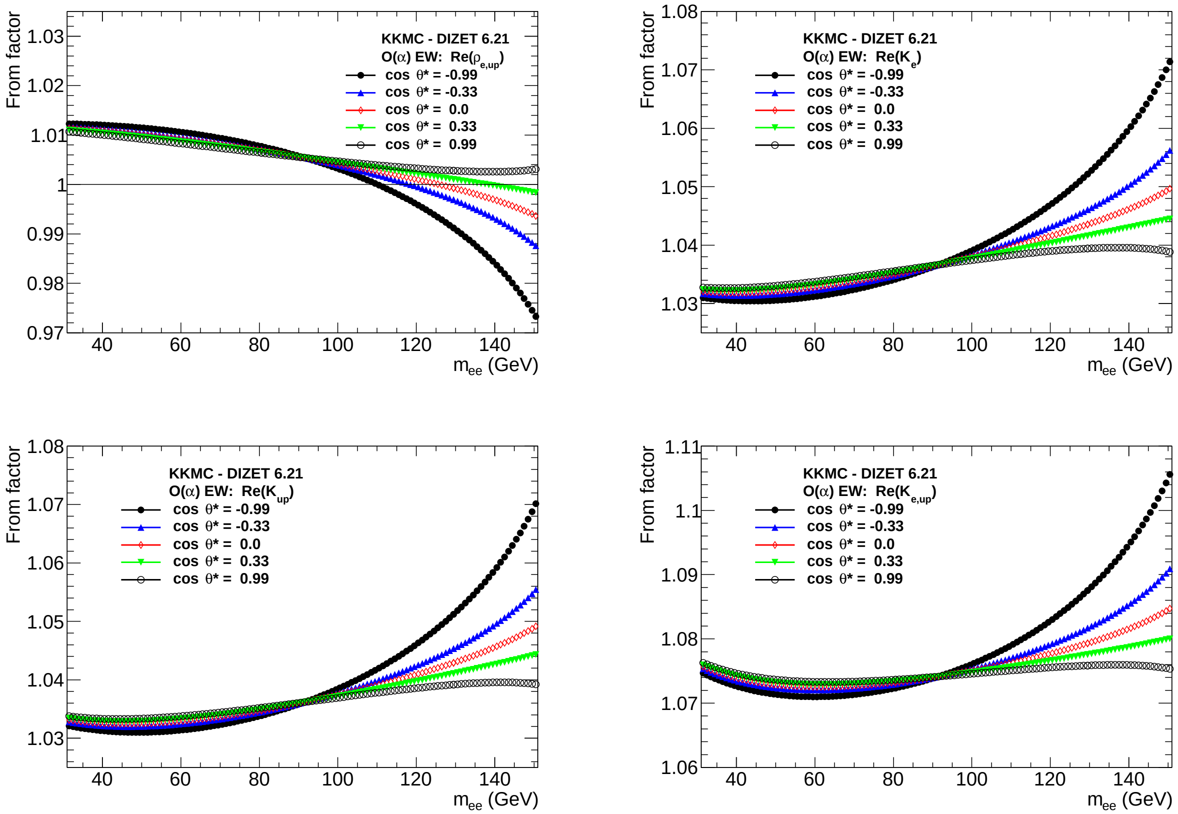


Figure 7: Real part of $\rho_{e,up}$, K_e, K_{up} and $K_{e,up}$ EW form-factors as a function of m_{ee} for few values of $\cos\theta$ and u-type quark flavour. Note that close to the Z peak angular dependence is minimal. For lower virtualities photon exchange dominates. Electroweak effects do not damage picture of spherical harmonics.

... above 160 GeV $\cos\theta$ dependence diverge. Reach 10 % at 240 GeV

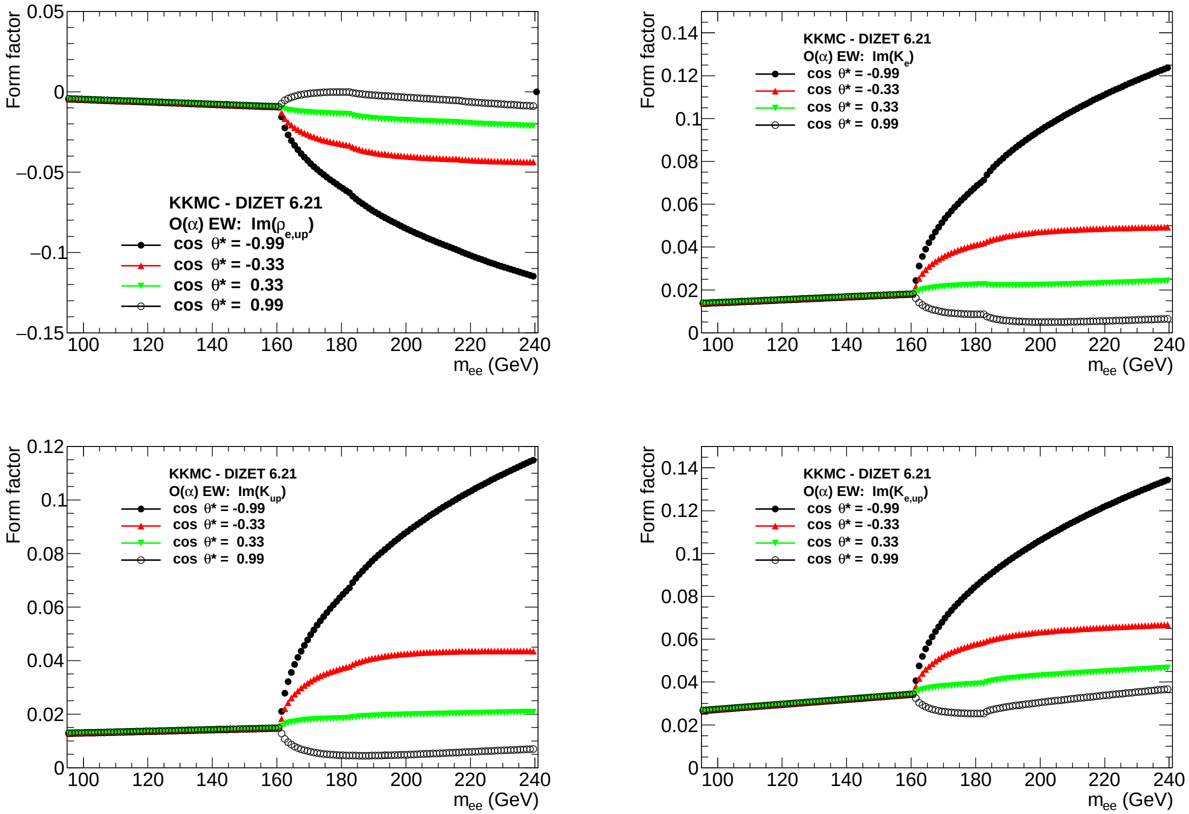


Figure 8: Imaginary part of $\rho_{e,u}$, K_e, K_{up} and $K_{e,up}$ a function of m_{ee} for few values of $\cos\theta$ and u-type quark flavour (left). Same for the down-type on the right. Note the WW and ZZ threshold effects which exhibits as discontinuity. Electroweak effects may complicate picture of spherical harmonics at virtualities above WW threshold.

part D: Applications.

Observations

- Form-factors break, but in numerically *non-significant manner*, the lepton angular distributions expansion into spherical harmonics of second order.
- In case of Mustraal frame only A_4 coefficent depend on electroweak couplings, other coefficents are close to zero.
- Independently of the choice for lepton pair definition directions in the rest frame of lepton pair remain to be distributed as polynomial of second order.
- This is also true for the case of W production and leptonic decays. Of course non-obsevability of neutrinos bring technical difficulties.
- **This open the way to last part of my talk. Applications:** reweighting methods and projection operators
- limits of *non-significant manner*.

Limitations

- Once invariant mass of the lepton pair is above the Z mass electroweak effects gradually degrade assumption that angular distributions of leptons in the rest-freame of lepton pair remain spherical harmoinics of the second order.
- Also effects due to electroweak Sudakov form-factors require attention.
- Keep in mind: real pair emission cancel dominant part of those effects.
- This can be manipulated with options for electroweak effects. Depending what is done with extra electroweak emitted pairs.
  - If in generated sample electroweak form-factors do not feature boxes we can deconvolute spherical harmonics all over the spectra
  - Otherwise only in region close or below Z peak.
  - Reweighting may use Mustraal frame without such a constraint.
  - In every case one need to take care of cancellations between Electroweak Sudakovs and extra electroweak-pair-emission

part D_1: Reweighting algorithms.

- Choice of Mustraal frame may be a good option to install electroweak loop corrections into generated Monte Carlo samples:
  - Electroweak scheme choices
  - Lineshape corrections
  - Electroweak boxes $\rightarrow$ Sudakov form-factors.
  - Note that sign of the $\cos\theta_{1,2}$ has to be attributed independently; on the basis of PDF's and Born level amplitudes.
- The last point need attention as depending of experimental cuts they may (or may not) cancel with real pair contributions.
- Similar mechanism as of photonic bremsstrahlung (infinite) contribution cancelling out with virtual corrections.
- Of course W and Z are not massless; effects/cancellations (between final states of distinct multiplicity) are far more subtle, less obvious and less universal too.

Next topic bring new contexts

- Eur.Phys.J. C761 (2016)473 for Z mediated processes
- Eur.Phys.J. C77(2017)111 for W mediated processes
- Formula of the next slide require a lot of refinements due to partial phase space coverage only.

part D_2: Moments methods and templates.

- We use the Monte Carlo sample of $W^\pm \to \ell^\pm \nu$ events and extract angular coefficients of Eq. (8) using moments methods [Mirkes:1994]. The first moment of a polynomial $P_i(\cos\theta, \phi)$, integrated over a specific range of p_T, Y is defined as follows:

$$\langle P_i(\cos\theta,\phi)\rangle = \frac{\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi\, P_i(\cos\theta,\phi) d\sigma(\cos\theta,\phi)}{\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi\, d\sigma(\cos\theta,\phi)}. \qquad (20)$$

- Owing to the orthogonality of the spherical polynomials of Eq. (8), the weighted average of the angular distributions with respect to any specific polynomial, Eq. (20), isolates its corresponding coefficient, averaged over some phase-space region.

- As a consequence of Eq. (8) we obtain:

$$\begin{aligned}
\langle \tfrac{1}{2}(1-3\cos^2\theta)\rangle &= \tfrac{3}{20}(A_0 - \tfrac{2}{3}); & \langle \sin 2\theta \cos\phi\rangle &= \frac{1}{5}A_1;\\
\langle \sin^2\theta \cos 2\phi\rangle &= \tfrac{1}{10}A_2; & \langle \sin\theta \cos\phi\rangle &= \frac{1}{4}A_3;\\
\langle \cos\theta\rangle &= \tfrac{1}{4}A_4; & \langle \sin^2\theta \sin 2\phi\rangle &= \frac{1}{5}A_5;\\
\langle \sin 2\theta \sin\phi\rangle &= \tfrac{1}{5}A_6; & \langle \sin\theta \sin\phi\rangle &= \frac{1}{4}A_7.
\end{aligned} \qquad (21)$$

part D_2: Moments methods and templates.

- We extract coefficients A_i using generated neutrino momenta to calculate $\cos\theta$ and ϕ. As a technical test, 2-dimensional histogram of $(\cos\theta,\phi)$ distribution obtained from our events weighted with

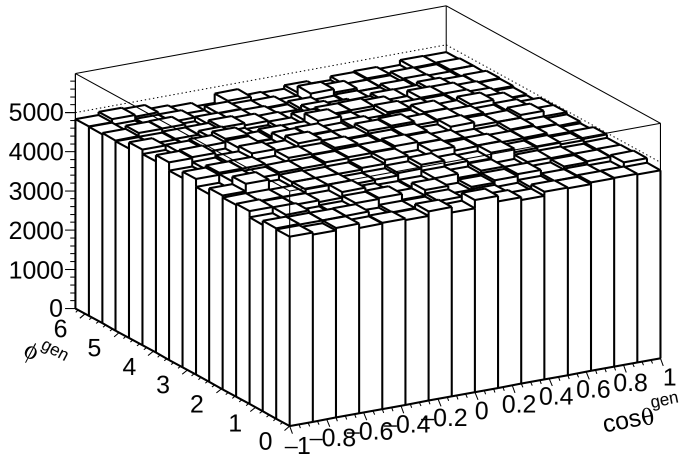
$$wt_{\Sigma A i P i} = \frac{1}{\Sigma_{i=0}^{i=8} A_i P_i(\cos\theta,\phi)} \tag{22}$$

where $A_8 = 1.0$ and $P_8 = 1+\cos^2\theta$ is used.

- By construction, thanks to Eqs. (21) and (20), weighted with (22) sample, feature unchanged Y, p_T distribution, but matrix element dependence of angular distribution of leptons in lepton pair rest-frame is completely removed.

- If averages for (20) are taken for sub-samples in appropriately narrow bins of Y and p_T this feature holds precisely for configurations of up single high p_T, thus degrading predictions of the Monte Carlo simulation results, to at worst NLO (NLL) level. We have found that for numerical results binning in p_T alone is sufficent.

- We fold events weighted with $wt_{\Sigma A i P i}$ into fiducial phase-space of the measurement: for the neutrino momentum reconstruction we use $m_W = m_W^{PDG}$ and take one of the two solutions for p_z^ν (neutrino momenum) at random, we recalculate θ, ϕ angles and we apply the kinematical selection of the fiducial phase-space. Right plot of Figure 9 shows how the initially flat distribution is distorted by this folding procedure.



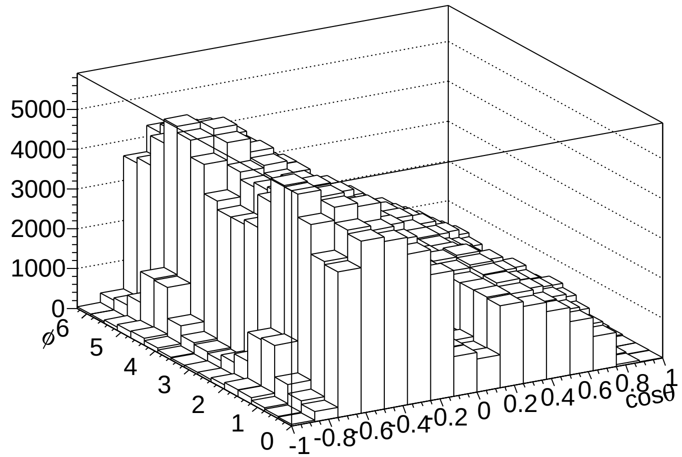


Figure 9: The 2D distribution of $\cos\theta$ and ϕ of charged lepton from $W^- \to \tau^-\nu$. Left, distribution of the full phase-space, with generated neutrino momentum used, and events weighted $wt_{\Sigma AiPi}$. Right, the same distribution is shown, but: m_W^{PDG} is used for solving Equations of neutrino momentum reconstruction. Randomly one of the two solutions for p_z^ν is taken and fiducial selection is applied. The weight $wt_{\Sigma AiPi}$ is calculated with generated neutrino momenta, as it should be.

part D_2: Moments methods and templates.

- We can now model any desired analytical polynomial shape of the generated full phase-space folded into fiducial phase-space of experimental measurement. It is enough to apply $wt_i = P_i \cdot wt_{\Sigma A i P i}$ to our events, to model the shape of the $P_i(\cos\theta, \phi)$ polynomial in the measurement fiducial phase-space.

- Linear combination of distributions obtained for all i, can be fitted to the data and in this way all A_i coefficents can be measured, also in W case.

- The templates obtained for P_i, deformed by experimental selections, reconstruction and theoretical effects of production of lepton pairs are obtained.

- Such deformations can be large, see Fig. 10, nonetheless template distributions are obtained and fits of A_i coefficients can be performed.

- Note, that thanks to the applied method, function to be fitted is linear in A_i.

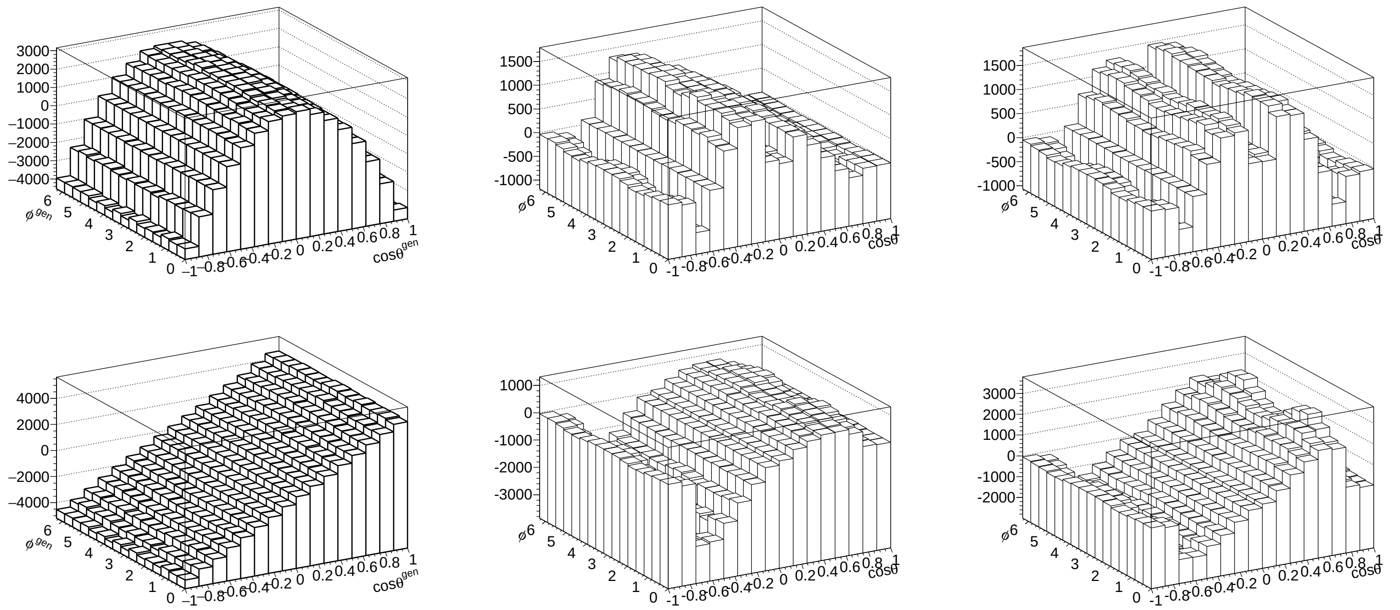


Figure 10: Analytical shape of the polynomial P_0 (top) and P_4 (bottom) in the full phase-space (left) and templates for polynomials after reconstructing p_Z^ν and fiducial selection for: W^- (middle) and W^+ (right).

Summary.

- We have demonstrated that Born-like distribution of lepton directions in lepton pair rest frame survive higher order QCD and electroweak corrections and remain a simple second order spherical harmonics.
- With special choice of coordinate frame only A_4 coefficent is non-trivial.
- We can use that for construction of templates.
  - For each Monte Carlo generated event we could use generated kinematic configuration and the reconstructed one.
  - This was used to construct template distributions to fit data.
  - Templates include not only detection effects, but all QCD/EW corrections embedded in Monte Carlo generated sample.

Summary.

- Alternatively (possible because of factorized Born kinematic configurations), one can attribute a weight for each event which would be a ratio of improved Born cross section to the one used in eg. QCD Monte Carlo.
- Such a weight can be a function of two variables only $s, \cos\theta_{Mustraal}$ reconstructed from momenta of outgoing leptons.
- Attributed to configurations with high p_T jets, of complex kinematic configurations.
- Sign of $\cos\theta_{Mustraal}$ has to be fixed independently.
- **With proper choice of reference freame one can to alarge degree separate electroweak and strong interaction effects. Alaso in case of high p_T configurations.**